Computer algebra independent integration tests
1_Algebraic_functions/1.1_Binomial_products/1.1.2Quadratic/1.1.2.4(ex)^m(a+bx^2)^p(c

Nasser M. Abbasi

December 3, 2018 Compiled on December 3, 2018 at 10:24am

Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

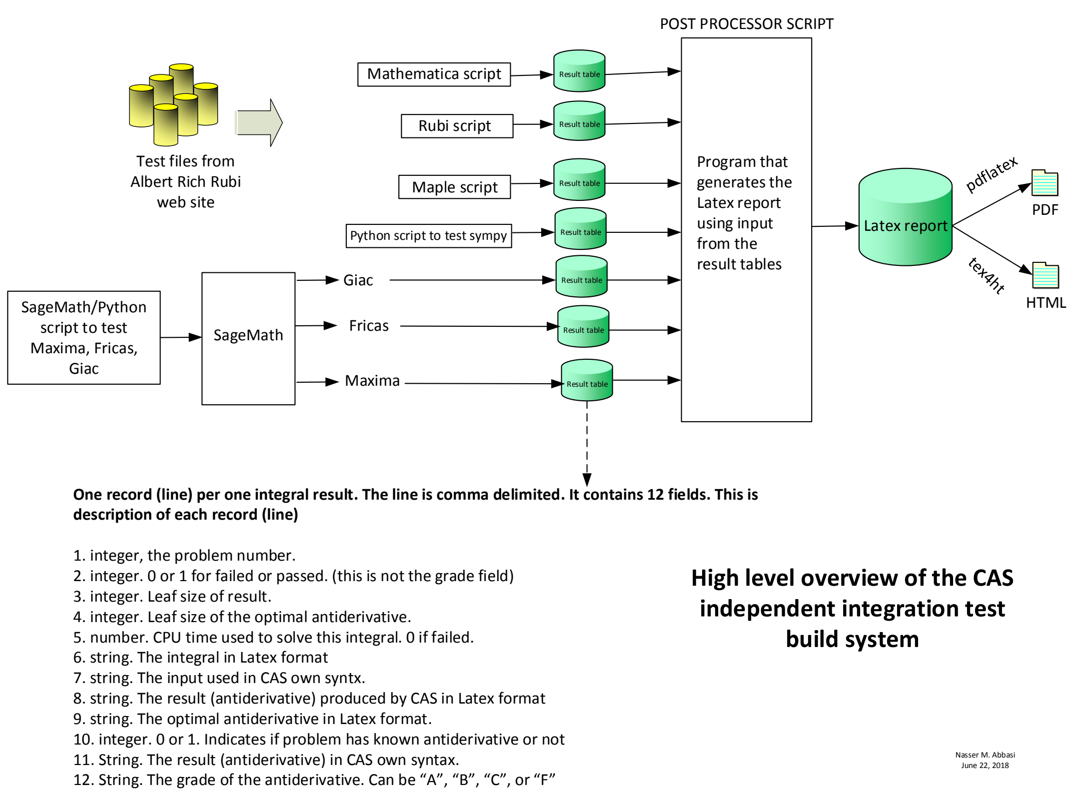
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expressi

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1156)	% 0. (0)
Rubi in Sympy	% 79.15 (915)	% 20.85 (241)
Mathematica	% 100. (1156)	% 0. (0)
Maple	% 86.68 (1002)	% 13.32 (154)
Maxima	% 24.74 (286)	% 75.26 (870)
Fricas	% 74.39 (860)	% 25.61 (296)
Sympy	% 44.9 (519)	% 55.1 (637)
Giac	% 69.03 (798)	% 30.97 (358)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

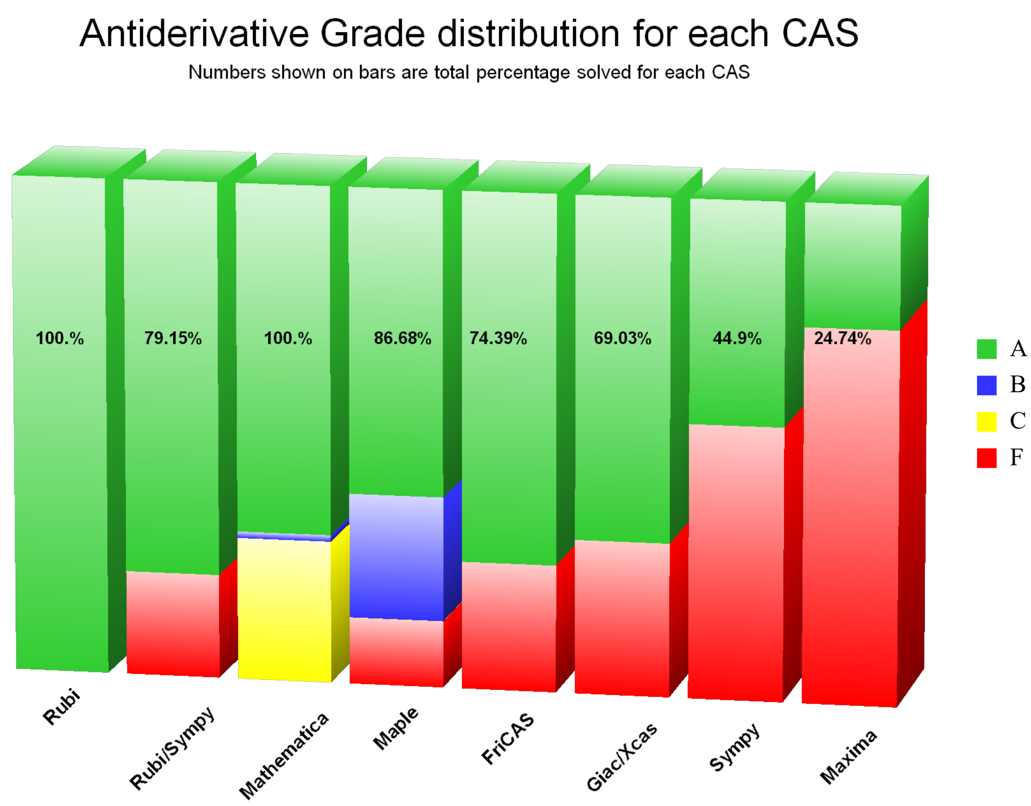
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

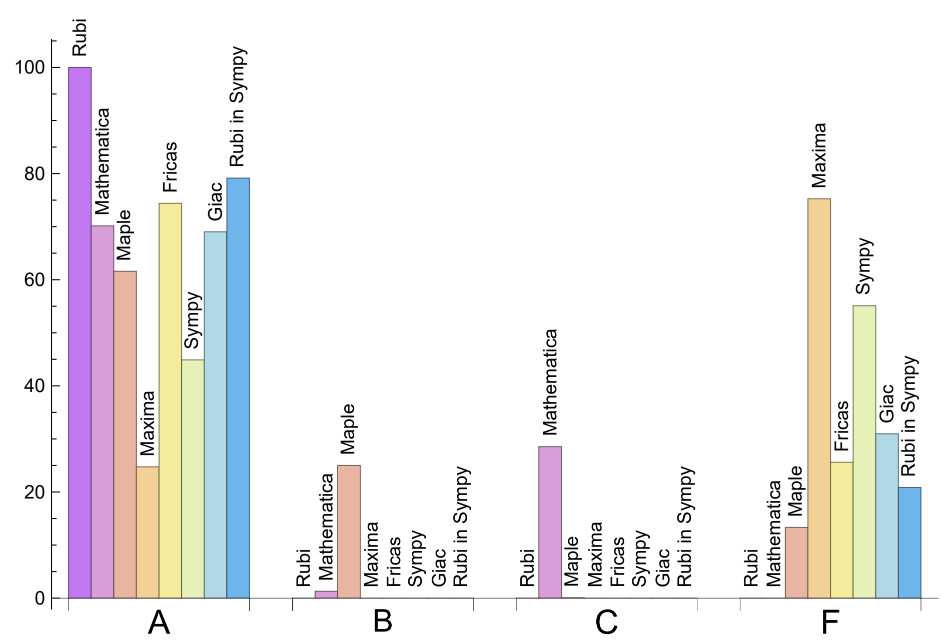
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	79.15	0.	0.	20.85
Mathematica	70.16	1.3	28.55	0.
Maple	61.59	25.	0.09	13.32
Maxima	24.74	0.	0.	75.26
Fricas	74.39	0.	0.	25.61
Sympy	44.9	0.	0.	55.1
Giac	69.03	0.	0.	30.97

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.49	179.5	1.	124.	1.
Rubi in Sympy	41.37	130.54	0.87	102.	0.9
Mathematica	0.38	163.5	0.98	120.	0.92
Maple	0.02	601.46	3.15	196.	1.31
Maxima	1.37	126.23	1.51	115.	1.38
Fricas	1.46	335.41	1.44	41.	1.1
Sympy	24.74	251.85	2.49	136.	1.49
Giac	0.36	218.64	1.72	167.	1.49

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 18, 19, 32, 33, 34, 35, 37, 39, 41, 42, 43, 44, 56, 57, 58, 59, 61, 71, 72, 73, 75, 77, 88, 89, 90, 91, 98, 106, 122, 127, 142, 145, 146, 147, 148, 149, 151, 154, 155, 157, 158, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 180, 181, 183, 184, 190, 198, 199, 200, 202, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 250, 258, 260, 263, 271, 272, 274, 275, 281, 283, 284, 285, 286, 297, 299, 309, 314, 316, 318, 336, 341, 342, 446, 448, 452, 456, 458, 461, 462, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 694, 775, 782, 783, 784, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 960, 1110, 1111, 1112, 1113, 1114, 1115, 1132, 1133, 1134, 1135, 1136, 1137, 1138}

Not solved by Mathematica {}

Not solved by Maple {322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156}

Not solved by Maxima {56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 98, 99, 100, 101, 102, 103, 104, 105, 106, 116, 128, 130, 132, 135, 137, 139, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 264, 266, 268, 270, 271, 273, 275, 277, 279, 280, 282, 284, 286, 288, 289, 291, 293, 295, 297, 299, 301, 303, 305, 307, 309, 310, 312, 314, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156}

Not solved by Fricas {322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 482, 484, 485, 486, 487, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1051, 1052, 1067, 1068, 1070, 1071, 1072, 1087, 1088, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156}

Not solved by Sympy {50, 51, 52, 53, 54, 55, 234, 236, 238, 240, 247, 248, 249, 250, 252, 256, 257, 258, 259, 260, 294, 295, 296, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 312, 314, 315, 316, 317, 318, 324, 329, 330, 335, 336, 337, 338, 340, 341, 342, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 407, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 520, 535, 537, 539, 552, 554, 596, 597, 613, 626, 648, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 790, 791, 792, 793, 794, 799, 800, 806, 807, 808, 809, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 827, 828, 829, 830, 831, 832, 833, 836, 837, 838, 839, 840, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1096, 1097, 1098, 1099, 1102, 1103, 1104, 1107, 1108, 1109, 1110, 1111, 1113, 1114, 1115, 1116, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156}

Not solved by Giac {228, 230, 232, 234, 236, 238, 240, 306, 308, 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 461, 462, 463, 464, 465, 466, 467, 468, 469, 488, 489, 490, 491, 492, 493, 494, 495, 719, 728, 730, 773, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 938, 939, 940, 941, 942, 943, 949, 950, 951, 952, 953, 960, 961, 962, 963, 964, 966, 968, 974, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {1014, 1015, 1017, 1018, 1026, 1027, 1028, 1029, 1030}

Mathematica {336, 341, 342, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 937, 938, 939, 947, 948, 949, 957, 958, 959, 965, 972, 973, 974, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	1	29	39	29
normalized size	1	1.	1.	0.85	1.09	0.03	0.88	1.18	0.88
time (sec)	N/A	0.06	0.01	0.001	1.372	0.21	0.083	0.216	10.085

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	1	29	39	0
normalized size	1	1.	1.	0.85	1.09	0.03	0.88	1.18	0.
time (sec)	N/A	0.078	0.012	0.001	1.336	0.197	0.088	0.213	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	1	26	35	0
normalized size	1	1.	1.	0.89	1.14	0.04	0.93	1.25	0.
time (sec)	N/A	0.036	0.008	0.002	1.334	0.21	0.083	0.211	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	38	34	27	41	0
normalized size	1	1.	1.	0.97	1.31	1.17	0.93	1.41	0.
time (sec)	N/A	0.059	0.015	0.005	1.333	0.229	1.088	0.217	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	32	38	20	31	0
normalized size	1	1.	1.	0.92	1.23	1.46	0.77	1.19	0.
time (sec)	N/A	0.048	0.014	0.006	1.345	0.233	1.077	0.241	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	38	41	26	57	0
normalized size	1	1.	1.	0.9	1.31	1.41	0.9	1.97	0.
time (sec)	N/A	0.069	0.019	0.008	1.373	0.241	1.35	0.263	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	35	39	26	38	0
normalized size	1	1.	1.04	0.96	1.35	1.5	1.	1.46	0.
time (sec)	N/A	0.05	0.02	0.007	1.345	0.225	1.447	0.254	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	28	41	42	27	53	31
normalized size	1	1.	1.07	0.97	1.41	1.45	0.93	1.83	1.07
time (sec)	N/A	0.063	0.031	0.008	1.346	0.244	2.063	0.246	9.459

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	39	39	32	42	27
normalized size	1	1.	1.06	0.9	1.26	1.26	1.03	1.35	0.87
time (sec)	N/A	0.05	0.021	0.007	1.384	0.228	2.092	0.269	7.621

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	28	39	39	32	42	31
normalized size	1	1.	1.06	0.85	1.18	1.18	0.97	1.27	0.94
time (sec)	N/A	0.07	0.016	0.007	1.331	0.228	2.618	0.282	9.802

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	1	56	72	49
normalized size	1	1.	1.	0.95	1.25	0.02	1.02	1.31	0.89
time (sec)	N/A	0.116	0.014	0.	1.346	0.212	0.111	0.25	16.858

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	69	1	53	72	34
normalized size	1	1.	1.21	1.24	1.64	0.02	1.26	1.71	0.81
time (sec)	N/A	0.159	0.022	0.001	1.35	0.22	0.121	0.25	15.725

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	53	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.06	1.36	0.
time (sec)	N/A	0.061	0.013	0.001	1.34	0.206	0.113	0.236	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	70	66	49	72	0
normalized size	1	1.	1.19	1.19	1.63	1.53	1.14	1.67	0.
time (sec)	N/A	0.081	0.026	0.003	1.352	0.245	1.19	0.227	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	65	72	48	65	0
normalized size	1	1.	1.	1.02	1.35	1.5	1.	1.35	0.
time (sec)	N/A	0.081	0.027	0.006	1.349	0.24	1.214	0.231	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	50	70	73	48	95	0
normalized size	1	1.	0.96	0.98	1.37	1.43	0.94	1.86	0.
time (sec)	N/A	0.134	0.04	0.01	1.352	0.234	1.543	0.232	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	68	70	49	68	0
normalized size	1	1.	1.04	0.96	1.42	1.46	1.02	1.42	0.
time (sec)	N/A	0.082	0.035	0.007	1.35	0.243	1.65	0.228	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	51	73	74	49	97	0
normalized size	1	1.	0.98	1.	1.43	1.45	0.96	1.9	0.
time (sec)	N/A	0.123	0.041	0.008	1.352	0.236	2.773	0.238	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	69	72	51	72	0
normalized size	1	1.	1.	0.94	1.44	1.5	1.06	1.5	0.
time (sec)	N/A	0.082	0.034	0.008	1.348	0.229	3.007	0.232	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	52	74	74	53	89	51
normalized size	1	1.	1.06	1.02	1.45	1.45	1.04	1.75	1.
time (sec)	N/A	0.099	0.048	0.009	1.346	0.239	4.576	0.229	15.633

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	48	72	72	56	74	48
normalized size	1	1.	1.06	0.91	1.36	1.36	1.06	1.4	0.91
time (sec)	N/A	0.082	0.031	0.007	1.345	0.221	4.822	0.227	12.902

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	48	72	72	56	74	51
normalized size	1	1.	1.15	1.	1.5	1.5	1.17	1.54	1.06
time (sec)	N/A	0.108	0.029	0.008	1.345	0.235	6.602	0.221	15.694

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	1	136	169	114
normalized size	1	1.	1.	1.06	1.38	0.01	1.16	1.44	0.97
time (sec)	N/A	0.411	0.029	0.003	1.337	0.218	0.174	0.22	38.824

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	1	138	169	114
normalized size	1	1.	1.	1.06	1.38	0.01	1.18	1.44	0.97
time (sec)	N/A	0.239	0.031	0.003	1.35	0.221	0.174	0.218	28.311

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	117	124	161	1	136	169	114
normalized size	1	1.	0.96	1.02	1.32	0.01	1.11	1.39	0.93
time (sec)	N/A	0.651	0.027	0.002	1.346	0.217	0.174	0.225	37.547

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	1	136	169	112
normalized size	1	1.	1.	1.06	1.38	0.01	1.16	1.44	0.96
time (sec)	N/A	0.237	0.028	0.001	1.352	0.212	0.178	0.223	27.327

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	161	1	133	167	85
normalized size	1	1.	1.13	1.31	1.69	0.01	1.4	1.76	0.89
time (sec)	N/A	0.53	0.043	0.002	1.347	0.204	0.178	0.224	33.685

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	1	136	169	114
normalized size	1	1.	1.	1.06	1.38	0.01	1.16	1.44	0.97
time (sec)	N/A	0.239	0.029	0.002	1.351	0.21	0.178	0.23	27.718

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	114	124	159	1	131	166	58
normalized size	1	1.	1.7	1.85	2.37	0.01	1.96	2.48	0.87
time (sec)	N/A	0.394	0.027	0.003	1.355	0.208	0.17	0.225	28.538

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	1	134	167	114
normalized size	1	1.	1.	1.06	1.38	0.01	1.15	1.43	0.97
time (sec)	N/A	0.237	0.029	0.003	1.349	0.206	0.171	0.228	29.405

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	107	124	161	1	133	167	34
normalized size	1	1.	2.55	2.95	3.83	0.02	3.17	3.98	0.81
time (sec)	N/A	0.182	0.042	0.002	1.357	0.207	0.169	0.227	20.497

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	121	155	1	129	163	0
normalized size	1	1.	1.	1.11	1.42	0.01	1.18	1.5	0.
time (sec)	N/A	0.17	0.028	0.003	1.348	0.218	0.171	0.222	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	113	124	162	158	134	170	0
normalized size	1	1.	1.28	1.41	1.84	1.8	1.52	1.93	0.
time (sec)	N/A	0.141	0.05	0.003	1.352	0.241	1.58	0.222	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	121	157	163	126	162	0
normalized size	1	1.	1.	1.12	1.45	1.51	1.17	1.5	0.
time (sec)	N/A	0.185	0.054	0.006	1.347	0.229	1.565	0.227	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	162	166	131	196	0
normalized size	1	1.	1.02	1.09	1.43	1.47	1.16	1.73	0.
time (sec)	N/A	0.295	0.094	0.008	1.346	0.219	1.961	0.221	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	110	118	159	163	126	165	104
normalized size	1	1.	1.02	1.09	1.47	1.51	1.17	1.53	0.96
time (sec)	N/A	0.191	0.062	0.008	1.348	0.226	2.027	0.244	27.094

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	124	165	166	126	201	0
normalized size	1	1.	1.	1.11	1.47	1.48	1.12	1.79	0.
time (sec)	N/A	0.278	0.067	0.01	1.366	0.225	3.271	0.235	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	113	162	163	126	166	107
normalized size	1	1.	1.	1.02	1.46	1.47	1.14	1.5	0.96
time (sec)	N/A	0.189	0.073	0.009	1.357	0.219	3.643	0.241	27.998

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	116	124	166	166	124	204	0
normalized size	1	1.	1.02	1.09	1.46	1.46	1.09	1.79	0.
time (sec)	N/A	0.273	0.068	0.01	1.339	0.224	6.229	0.24	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	108	162	163	126	167	107
normalized size	1	1.	1.	0.97	1.46	1.47	1.14	1.5	0.96
time (sec)	N/A	0.192	0.073	0.008	1.357	0.216	6.677	0.235	27.008

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	116	124	166	166	124	203	0
normalized size	1	1.	1.04	1.11	1.48	1.48	1.11	1.81	0.
time (sec)	N/A	0.269	0.103	0.013	1.351	0.223	11.366	0.226	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	115	102	161	163	122	166	0
normalized size	1	1.	1.06	0.94	1.49	1.51	1.13	1.54	0.
time (sec)	N/A	0.202	0.063	0.009	1.365	0.232	12.594	0.23	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	116	123	166	166	122	198	0
normalized size	1	1.	1.03	1.09	1.47	1.47	1.08	1.75	0.
time (sec)	N/A	0.246	0.104	0.012	1.334	0.238	21.584	0.228	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	122	101	161	163	122	169	0
normalized size	1	1.	1.13	0.94	1.49	1.51	1.13	1.56	0.
time (sec)	N/A	0.196	0.077	0.01	1.342	0.23	25.864	0.227	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	118	124	166	166	124	186	95
normalized size	1	1.	1.3	1.36	1.82	1.82	1.36	2.04	1.04
time (sec)	N/A	0.141	0.12	0.011	1.342	0.246	43.022	0.228	21.575

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	163	163	128	171	109
normalized size	1	1.	1.05	0.92	1.44	1.44	1.13	1.51	0.96
time (sec)	N/A	0.201	0.056	0.008	1.352	0.237	51.315	0.222	24.167

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	163	163	128	171	41
normalized size	1	1.	2.46	2.17	3.4	3.4	2.67	3.56	0.85
time (sec)	N/A	0.116	0.057	0.009	1.337	0.24	72.538	0.221	11.048

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	163	128	171	116
normalized size	1	1.	1.03	0.89	1.39	1.39	1.09	1.46	0.99
time (sec)	N/A	0.197	0.055	0.009	1.326	0.24	105.46	0.227	24.237

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	121	104	163	163	128	171	68
normalized size	1	1.	1.59	1.37	2.14	2.14	1.68	2.25	0.89
time (sec)	N/A	0.172	0.057	0.009	1.335	0.222	152.951	0.23	14.167

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	163	0	171	116
normalized size	1	1.	1.	0.89	1.39	1.39	0.	1.46	0.99
time (sec)	N/A	0.192	0.084	0.009	1.352	0.212	0.	0.219	24.429

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	163	0	171	114
normalized size	1	1.	1.03	0.89	1.39	1.39	0.	1.46	0.97
time (sec)	N/A	0.255	0.059	0.01	1.38	0.219	0.	0.225	29.764

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	163	0	171	114
normalized size	1	1.	1.	0.89	1.39	1.39	0.	1.46	0.97
time (sec)	N/A	0.197	0.095	0.008	1.377	0.204	0.	0.239	24.535

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	163	0	171	116
normalized size	1	1.	1.03	0.89	1.39	1.39	0.	1.46	0.99
time (sec)	N/A	0.25	0.057	0.009	1.339	0.22	0.	0.234	29.73

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	163	0	171	116
normalized size	1	1.	1.	0.89	1.39	1.39	0.	1.46	0.99
time (sec)	N/A	0.196	0.084	0.008	1.339	0.218	0.	0.234	25.733

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	163	0	171	116
normalized size	1	1.	1.03	0.89	1.39	1.39	0.	1.46	0.99
time (sec)	N/A	0.242	0.061	0.01	1.357	0.219	0.	0.224	30.111

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	116	0	1	173	146	0
normalized size	1	1.	1.	1.18	0.	0.01	1.77	1.49	0.
time (sec)	N/A	0.171	0.113	0.005	0.	0.239	2.024	0.23	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	86	100	101	65	104	0
normalized size	1	1.	0.95	1.15	1.33	1.35	0.87	1.39	0.
time (sec)	N/A	0.194	0.054	0.005	1.353	0.228	1.815	0.245	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	92	0	1	150	115	0
normalized size	1	1.	1.	1.19	0.	0.01	1.95	1.49	0.
time (sec)	N/A	0.137	0.087	0.004	0.	0.233	1.944	0.242	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	68	69	44	70	0
normalized size	1	1.	0.87	1.15	1.26	1.28	0.81	1.3	0.
time (sec)	N/A	0.134	0.031	0.003	1.35	0.229	1.691	0.23	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	68	0	1	90	77	49
normalized size	1	1.	0.98	1.17	0.	0.02	1.55	1.33	0.84
time (sec)	N/A	0.102	0.063	0.003	0.	0.25	1.798	0.231	15.717

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	42	41	27	43	0
normalized size	1	1.	0.89	1.14	1.2	1.17	0.77	1.23	0.
time (sec)	N/A	0.077	0.018	0.005	1.355	0.229	1.527	0.245	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	1	82	46	34
normalized size	1	1.	1.03	1.15	0.	0.03	2.1	1.18	0.87
time (sec)	N/A	0.049	0.047	0.003	0.	0.236	1.624	0.232	8.515

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	47	43	26	49	29
normalized size	1	1.	1.	1.09	1.38	1.26	0.76	1.44	0.85
time (sec)	N/A	0.09	0.021	0.007	1.35	0.215	2.255	0.231	13.51

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	48	0	1	82	49	36
normalized size	1	1.	0.98	1.12	0.	0.02	1.91	1.14	0.84
time (sec)	N/A	0.065	0.044	0.005	0.	0.236	1.854	0.226	9.863

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	56	65	63	41	96	44
normalized size	1	1.	0.98	1.12	1.3	1.26	0.82	1.92	0.88
time (sec)	N/A	0.121	0.034	0.01	1.347	0.227	3.041	0.229	16.032

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	72	0	1	129	77	49
normalized size	1	1.	1.02	1.22	0.	0.02	2.19	1.31	0.83
time (sec)	N/A	0.105	0.091	0.009	0.	0.238	2.261	0.225	14.691

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	81	95	99	61	135	63
normalized size	1	1.	1.01	1.17	1.38	1.43	0.88	1.96	0.91
time (sec)	N/A	0.152	0.048	0.01	1.356	0.237	3.805	0.229	20.01

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	96	0	1	163	109	66
normalized size	1	1.	0.98	1.2	0.	0.01	2.04	1.36	0.82
time (sec)	N/A	0.131	0.087	0.009	0.	0.24	2.748	0.227	18.568

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	96	107	130	132	88	170	83
normalized size	1	1.	1.03	1.15	1.4	1.42	0.95	1.83	0.89
time (sec)	N/A	0.192	0.065	0.011	1.341	0.237	4.395	0.231	24.23

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	120	0	1	187	143	85
normalized size	1	1.	1.02	1.21	0.	0.01	1.89	1.44	0.86
time (sec)	N/A	0.167	0.115	0.01	0.	0.247	3.668	0.222	23.966

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	113	146	177	232	126	215	0
normalized size	1	1.	0.9	1.16	1.4	1.84	1.	1.71	0.
time (sec)	N/A	0.373	0.136	0.016	1.352	0.233	3.821	0.24	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	134	155	0	1	233	188	0
normalized size	1	1.	1.02	1.18	0.	0.01	1.78	1.44	0.
time (sec)	N/A	0.287	0.179	0.013	0.	0.248	3.524	0.231	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	122	144	200	102	182	0
normalized size	1	1.	0.89	1.17	1.38	1.92	0.98	1.75	0.
time (sec)	N/A	0.281	0.122	0.017	1.348	0.232	3.404	0.225	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	132	0	1	206	155	102
normalized size	1	1.	1.01	1.2	0.	0.01	1.87	1.41	0.93
time (sec)	N/A	0.231	0.149	0.012	0.	0.237	3.325	0.227	59.428

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	98	111	163	78	143	0
normalized size	1	1.	0.88	1.2	1.35	1.99	0.95	1.74	0.
time (sec)	N/A	0.215	0.114	0.015	1.351	0.233	3.205	0.24	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	105	0	1	128	119	80
normalized size	1	1.	1.02	1.21	0.	0.01	1.47	1.37	0.92
time (sec)	N/A	0.174	0.119	0.012	0.	0.238	3.076	0.224	43.972

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	74	81	109	56	123	0
normalized size	1	1.	0.83	1.23	1.35	1.82	0.93	2.05	0.
time (sec)	N/A	0.151	0.057	0.015	1.352	0.227	2.778	0.224	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	82	0	1	114	80	60
normalized size	1	1.	1.01	1.22	0.	0.01	1.7	1.19	0.9
time (sec)	N/A	0.132	0.115	0.012	0.	0.232	2.633	0.222	23.375

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	54	59	36	88	32
normalized size	1	1.	1.	1.15	1.32	1.44	0.88	2.15	0.78
time (sec)	N/A	0.087	0.02	0.013	1.352	0.215	2.055	0.232	13.383

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	1	112	77	51
normalized size	1	1.	1.	1.08	0.	0.02	1.78	1.22	0.81
time (sec)	N/A	0.062	0.081	0.01	0.	0.238	2.119	0.226	9.446

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	69	95	46	85	44
normalized size	1	1.	0.9	1.04	1.35	1.86	0.9	1.67	0.86
time (sec)	N/A	0.114	0.05	0.017	1.348	0.233	2.198	0.227	16.503

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	85	0	1	114	84	61
normalized size	1	1.	0.99	1.2	0.	0.01	1.61	1.18	0.86
time (sec)	N/A	0.149	0.056	0.014	0.	0.243	2.505	0.223	19.361

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	86	103	158	70	111	68
normalized size	1	1.	0.84	1.13	1.36	2.08	0.92	1.46	0.89
time (sec)	N/A	0.177	0.077	0.021	1.336	0.233	3.868	0.234	21.227

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	110	0	1	184	115	80
normalized size	1	1.	1.	1.22	0.	0.01	2.04	1.28	0.89
time (sec)	N/A	0.262	0.133	0.017	0.	0.222	3.171	0.23	40.404

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	114	143	208	100	203	94
normalized size	1	1.	0.88	1.18	1.47	2.14	1.03	2.09	0.97
time (sec)	N/A	0.228	0.16	0.022	1.36	0.229	4.972	0.234	26.748

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	112	136	0	1	218	151	102
normalized size	1	1.	0.99	1.2	0.	0.01	1.93	1.34	0.9
time (sec)	N/A	0.342	0.134	0.017	0.	0.241	4.053	0.23	74.322

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	143	184	248	129	240	119
normalized size	1	1.	0.89	1.15	1.48	2.	1.04	1.94	0.96
time (sec)	N/A	0.299	0.186	0.023	1.359	0.233	6.22	0.233	34.936

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	136	182	223	312	163	247	0
normalized size	1	1.	0.91	1.21	1.49	2.08	1.09	1.65	0.
time (sec)	N/A	0.474	0.146	0.02	1.361	0.228	7.209	0.246	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	116	158	190	277	138	215	0
normalized size	1	1.	0.91	1.23	1.48	2.16	1.08	1.68	0.
time (sec)	N/A	0.371	0.137	0.018	1.345	0.22	6.7	0.233	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	134	157	242	116	178	0
normalized size	1	1.	0.86	1.23	1.44	2.22	1.06	1.63	0.
time (sec)	N/A	0.296	0.109	0.016	1.351	0.216	6.315	0.239	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	109	127	192	94	126	0
normalized size	1	1.	1.05	1.24	1.44	2.18	1.07	1.43	0.
time (sec)	N/A	0.224	0.062	0.017	1.338	0.223	5.536	0.227	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	80	97	120	70	82	56
normalized size	1	1.	0.97	1.21	1.47	1.82	1.06	1.24	0.85
time (sec)	N/A	0.168	0.041	0.012	1.344	0.219	4.217	0.233	20.825

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	57	57	42	38	26
normalized size	1	1.	0.94	1.22	1.78	1.78	1.31	1.19	0.81
time (sec)	N/A	0.062	0.022	0.01	1.342	0.219	2.705	0.223	8.502

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	68	104	161	75	103	60
normalized size	1	1.	0.87	1.	1.53	2.37	1.1	1.51	0.88
time (sec)	N/A	0.143	0.077	0.018	1.342	0.232	2.949	0.225	20.195

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	87	118	147	266	107	186	90
normalized size	1	1.	0.86	1.17	1.46	2.63	1.06	1.84	0.89
time (sec)	N/A	0.245	0.096	0.022	1.344	0.22	5.119	0.224	26.639

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	108	150	185	309	136	180	119
normalized size	1	1.	0.87	1.21	1.49	2.49	1.1	1.45	0.96
time (sec)	N/A	0.297	0.131	0.023	1.351	0.231	7.306	0.238	32.989

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	135	180	230	360	165	271	143
normalized size	1	1.	0.91	1.21	1.54	2.42	1.11	1.82	0.96
time (sec)	N/A	0.377	0.223	0.025	1.353	0.233	10.577	0.238	43.465

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	198	0	1	274	219	0
normalized size	1	1.	1.	1.25	0.	0.01	1.73	1.39	0.
time (sec)	N/A	0.519	0.15	0.016	0.	0.235	6.636	0.229	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	174	0	1	250	186	133
normalized size	1	1.	0.96	1.26	0.	0.01	1.81	1.35	0.96
time (sec)	N/A	0.403	0.199	0.015	0.	0.24	6.232	0.229	129.032

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	147	0	1	212	150	110
normalized size	1	1.	0.97	1.27	0.	0.01	1.83	1.29	0.95
time (sec)	N/A	0.313	0.15	0.016	0.	0.234	5.873	0.225	88.713

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	122	0	1	194	108	88
normalized size	1	1.	0.97	1.3	0.	0.01	2.06	1.15	0.94
time (sec)	N/A	0.209	0.121	0.014	0.	0.228	4.976	0.228	44.788

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	89	0	1	153	105	78
normalized size	1	1.	0.93	1.	0.	0.01	1.72	1.18	0.88
time (sec)	N/A	0.162	0.141	0.012	0.	0.247	3.543	0.225	23.909

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	90	0	1	150	105	78
normalized size	1	1.	0.91	0.98	0.	0.01	1.63	1.14	0.85
time (sec)	N/A	0.091	0.107	0.012	0.	0.243	2.829	0.223	12.774

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	125	0	1	194	111	88
normalized size	1	1.	0.99	1.29	0.	0.01	2.	1.14	0.91
time (sec)	N/A	0.274	0.098	0.017	0.	0.241	3.548	0.232	30.312

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	116	152	0	1	226	146	109
normalized size	1	1.	0.99	1.3	0.	0.01	1.93	1.25	0.93
time (sec)	N/A	0.432	0.148	0.02	0.	0.246	4.741	0.251	63.224

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	139	177	0	1	260	182	0
normalized size	1	1.	0.98	1.25	0.	0.01	1.83	1.28	0.
time (sec)	N/A	0.591	0.169	0.02	0.	0.251	7.12	0.264	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	16	16	26	16	8
normalized size	1	1.	1.	1.17	1.33	1.33	2.17	1.33	0.67
time (sec)	N/A	0.023	0.009	0.003	1.499	0.23	1.241	0.252	5.062

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	28	34	31	31	22	34	8
normalized size	1	1.	2.55	3.09	2.82	2.82	2.	3.09	0.73
time (sec)	N/A	0.023	0.015	0.005	1.327	0.234	1.282	0.232	5.533

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	14	14	7	15	5
normalized size	1	1.	0.91	1.45	1.27	1.27	0.64	1.36	0.45
time (sec)	N/A	0.007	0.006	0.008	1.339	0.23	0.166	0.219	3.673

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	12	5	9	5
normalized size	1	1.	1.	1.11	1.33	1.33	0.56	1.	0.56
time (sec)	N/A	0.007	0.006	0.009	1.338	0.218	0.175	0.24	4.16

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	27	14	20	14
normalized size	1	1.	1.	0.84	1.05	1.42	0.74	1.05	0.74
time (sec)	N/A	0.017	0.012	0.008	1.483	0.236	0.221	0.222	4.102

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	28	15	20	15
normalized size	1	1.	1.	0.84	1.05	1.47	0.79	1.05	0.79
time (sec)	N/A	0.015	0.012	0.007	1.497	0.224	0.219	0.223	3.879

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	26	10	19	10
normalized size	1	1.	1.	1.07	1.36	1.86	0.71	1.36	0.71
time (sec)	N/A	0.015	0.01	0.01	1.486	0.227	0.212	0.222	3.877

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	15	19	19	8	19	7
normalized size	1	1.	1.17	1.25	1.58	1.58	0.67	1.58	0.58
time (sec)	N/A	0.01	0.012	0.009	1.347	0.223	1.294	0.229	6.79

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	15	19	19	8	19	7
normalized size	1	1.	1.17	1.25	1.58	1.58	0.67	1.58	0.58
time (sec)	N/A	0.009	0.009	0.006	1.345	0.215	1.3	0.219	6.115

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	0	1	75	49	34
normalized size	1	1.	1.	0.95	0.	0.03	1.92	1.26	0.87
time (sec)	N/A	0.054	0.038	0.004	0.	0.236	1.639	0.228	9.53

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	25	41	53	27	34	29
normalized size	1	1.	1.	0.71	1.17	1.51	0.77	0.97	0.83
time (sec)	N/A	0.023	0.015	0.011	1.508	0.241	0.312	0.227	4.134

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	30	22	22	17	15	14
normalized size	1	1.	1.	2.14	1.57	1.57	1.21	1.07	1.
time (sec)	N/A	0.01	0.01	0.016	1.343	0.204	0.358	0.227	6.696

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	0
normalized size	1	1.	1.	2.	1.	1.	0.	1.	0.
time (sec)	N/A	0.003	0.	0.	1.341	0.202	0.072	0.218	0.042

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.62
time (sec)	N/A	0.007	0.001	0.	1.346	0.217	0.105	0.225	3.57

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.62
time (sec)	N/A	0.006	0.001	0.001	1.335	0.213	0.105	0.223	3.59

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.
time (sec)	N/A	0.006	0.	0.002	1.35	0.215	0.098	0.228	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	2	4	2
normalized size	1	1.	1.	1.33	1.33	1.33	0.67	1.33	0.67
time (sec)	N/A	0.005	0.	0.	1.332	0.22	0.087	0.222	3.149

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	9	5	3	7	3
normalized size	1	1.	1.	1.25	2.25	1.25	0.75	1.75	0.75
time (sec)	N/A	0.005	0.001	0.002	1.34	0.22	0.115	0.222	3.593

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	8	3	8	3
normalized size	1	1.	1.	1.17	1.33	1.33	0.5	1.33	0.5
time (sec)	N/A	0.006	0.001	0.001	1.343	0.216	0.123	0.223	3.595

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.006	0.001	0.	1.346	0.217	0.116	0.224	3.574

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	34	32	22	63	0
normalized size	1	1.	1.	0.9	1.17	1.1	0.76	2.17	0.
time (sec)	N/A	0.053	0.007	0.003	1.366	0.208	1.188	0.243	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	1	58	38	29
normalized size	1	1.	1.	0.88	0.	0.03	1.76	1.15	0.88
time (sec)	N/A	0.043	0.015	0.003	0.	0.238	1.224	0.223	10.064

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	19	12	85	12
normalized size	1	1.	1.	0.94	1.19	1.19	0.75	5.31	0.75
time (sec)	N/A	0.012	0.003	0.001	1.33	0.227	0.271	0.222	4.778

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	0	1	54	22	24
normalized size	1	1.	1.	0.68	0.	0.04	2.16	0.88	0.96
time (sec)	N/A	0.023	0.007	0.003	0.	0.229	0.327	0.222	5.43

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	34	28	17	35	22
normalized size	1	1.	1.	0.96	1.42	1.17	0.71	1.46	0.92
time (sec)	N/A	0.037	0.009	0.006	1.333	0.232	0.537	0.224	9.099

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	0	1	66	42	31
normalized size	1	1.	1.	0.89	0.	0.03	1.83	1.17	0.86
time (sec)	N/A	0.041	0.022	0.006	0.	0.248	1.329	0.221	9.881

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	49	49	32	63	37
normalized size	1	1.	0.97	0.92	1.29	1.29	0.84	1.66	0.97
time (sec)	N/A	0.063	0.011	0.007	1.322	0.231	1.635	0.225	12.403

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	32	46	54	31	43	29
normalized size	1	1.	0.8	0.91	1.31	1.54	0.89	1.23	0.83
time (sec)	N/A	0.062	0.016	0.007	1.337	0.227	1.396	0.223	11.479

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	0	1	80	50	39
normalized size	1	1.	1.	0.81	0.	0.02	1.7	1.06	0.83
time (sec)	N/A	0.045	0.035	0.009	0.	0.237	1.441	0.227	9.958

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	22	22	15	20	12
normalized size	1	1.	1.	0.94	1.29	1.29	0.88	1.18	0.71
time (sec)	N/A	0.013	0.004	0.002	1.343	0.211	1.297	0.23	4.768

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	0	1	80	50	39
normalized size	1	1.	1.	0.81	0.	0.02	1.7	1.06	0.83
time (sec)	N/A	0.037	0.039	0.005	0.	0.245	1.475	0.229	7.209

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	38	54	73	36	69	37
normalized size	1	1.	0.83	0.93	1.32	1.78	0.88	1.68	0.9
time (sec)	N/A	0.066	0.022	0.011	1.329	0.227	1.728	0.227	12.562

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	49	0	1	92	68	51
normalized size	1	1.	0.93	0.82	0.	0.02	1.53	1.13	0.85
time (sec)	N/A	0.06	0.062	0.012	0.	0.24	1.76	0.229	13.879

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	50	77	108	51	76	51
normalized size	1	1.	0.79	0.94	1.45	2.04	0.96	1.43	0.96
time (sec)	N/A	0.089	0.06	0.013	1.352	0.228	2.093	0.243	15.628

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	1	56	72	49
normalized size	1	1.	1.	0.95	1.25	0.02	1.02	1.31	0.89
time (sec)	N/A	0.113	0.015	0.001	1.341	0.197	0.115	0.221	13.974

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	1	53	72	0
normalized size	1	1.	1.	0.95	1.25	0.02	0.96	1.31	0.
time (sec)	N/A	0.182	0.013	0.001	1.324	0.199	0.116	0.222	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	1	56	72	49
normalized size	1	1.	1.	0.95	1.25	0.02	1.02	1.31	0.89
time (sec)	N/A	0.106	0.013	0.002	1.351	0.201	0.115	0.246	16.024

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	69	1	53	72	34
normalized size	1	1.	1.21	1.24	1.64	0.02	1.26	1.71	0.81
time (sec)	N/A	0.151	0.021	0.001	1.323	0.195	0.115	0.22	15.611

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	53	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.06	1.36	0.
time (sec)	N/A	0.065	0.012	0.	1.333	0.187	0.117	0.248	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	70	66	49	72	0
normalized size	1	1.	1.19	1.19	1.63	1.53	1.14	1.67	0.
time (sec)	N/A	0.08	0.024	0.003	1.349	0.218	1.225	0.231	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	65	72	48	65	0
normalized size	1	1.	1.	1.02	1.35	1.5	1.	1.35	0.
time (sec)	N/A	0.078	0.027	0.006	1.354	0.222	1.22	0.221	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	50	70	73	48	95	0
normalized size	1	1.	0.96	0.98	1.37	1.43	0.94	1.86	0.
time (sec)	N/A	0.125	0.042	0.009	1.353	0.221	1.618	0.223	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	68	70	49	68	0
normalized size	1	1.	1.04	0.96	1.42	1.46	1.02	1.42	0.
time (sec)	N/A	0.086	0.033	0.008	1.35	0.226	1.654	0.22	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	115	1	100	127	87
normalized size	1	1.	1.	1.03	1.32	0.01	1.15	1.46	1.
time (sec)	N/A	0.187	0.026	0.002	1.326	0.206	0.152	0.224	24.155

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	115	1	92	127	0
normalized size	1	1.	0.93	1.03	1.32	0.01	1.06	1.46	0.
time (sec)	N/A	0.318	0.045	0.001	1.35	0.192	0.152	0.223	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	115	1	100	127	87
normalized size	1	1.	1.	1.03	1.32	0.01	1.15	1.46	1.
time (sec)	N/A	0.174	0.029	0.	1.346	0.207	0.15	0.223	28.662

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	81	90	115	1	94	127	60
normalized size	1	1.	1.14	1.27	1.62	0.01	1.32	1.79	0.85
time (sec)	N/A	0.274	0.043	0.001	1.357	0.208	0.153	0.222	29.252

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	1	97	123	0
normalized size	1	1.	1.	1.06	1.35	0.01	1.18	1.5	0.
time (sec)	N/A	0.107	0.028	0.001	1.352	0.206	0.145	0.224	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	90	115	111	85	124	0
normalized size	1	1.	1.	1.12	1.44	1.39	1.06	1.55	0.
time (sec)	N/A	0.169	0.043	0.004	1.359	0.228	1.414	0.228	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	91	112	117	92	122	80
normalized size	1	1.	1.	1.12	1.38	1.44	1.14	1.51	0.99
time (sec)	N/A	0.127	0.064	0.006	1.353	0.216	1.383	0.228	25.328

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	93	115	119	87	154	0
normalized size	1	1.	0.99	1.11	1.37	1.42	1.04	1.83	0.
time (sec)	N/A	0.213	0.079	0.009	1.358	0.211	1.743	0.227	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	81	113	117	90	119	0
normalized size	1	1.	1.	1.01	1.41	1.46	1.12	1.49	0.
time (sec)	N/A	0.146	0.073	0.008	1.348	0.222	1.847	0.222	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	171	1	143	182	124
normalized size	1	1.	1.	1.01	1.35	0.01	1.13	1.43	0.98
time (sec)	N/A	0.251	0.045	0.001	1.355	0.206	0.179	0.223	34.223

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	119	128	171	1	138	182	94
normalized size	1	1.	1.12	1.21	1.61	0.01	1.3	1.72	0.89
time (sec)	N/A	0.539	0.059	0.002	1.349	0.209	0.177	0.229	41.076

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	171	1	143	182	124
normalized size	1	1.	1.	1.01	1.35	0.01	1.13	1.43	0.98
time (sec)	N/A	0.232	0.04	0.001	1.361	0.208	0.169	0.222	36.885

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	119	128	171	1	136	181	60
normalized size	1	1.	1.68	1.8	2.41	0.01	1.92	2.55	0.85
time (sec)	N/A	0.323	0.053	0.001	1.348	0.201	0.172	0.225	29.988

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	1	136	177	0
normalized size	1	1.	1.	1.02	1.37	0.01	1.11	1.45	0.
time (sec)	N/A	0.149	0.038	0.002	1.35	0.201	0.175	0.223	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	132	173	169	133	181	0
normalized size	1	1.	1.	1.07	1.41	1.37	1.08	1.47	0.
time (sec)	N/A	0.224	0.055	0.004	1.354	0.235	1.538	0.23	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	131	167	174	131	176	117
normalized size	1	1.	1.	1.09	1.39	1.45	1.09	1.47	0.98
time (sec)	N/A	0.156	0.072	0.007	1.353	0.226	1.522	0.22	34.878

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	120	134	173	177	133	216	0
normalized size	1	1.	0.98	1.09	1.41	1.44	1.08	1.76	0.
time (sec)	N/A	0.267	0.087	0.009	1.355	0.23	1.936	0.238	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	124	170	174	129	174	0
normalized size	1	1.	1.	1.03	1.42	1.45	1.08	1.45	0.
time (sec)	N/A	0.167	0.081	0.008	1.331	0.229	2.046	0.221	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	176	0	1	240	207	0
normalized size	1	1.	1.	1.69	0.	0.01	2.31	1.99	0.
time (sec)	N/A	0.177	0.158	0.004	0.	0.245	2.726	0.221	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	82	124	135	136	83	144	0
normalized size	1	1.	1.04	1.57	1.71	1.72	1.05	1.82	0.
time (sec)	N/A	0.203	0.069	0.006	1.348	0.226	2.199	0.222	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	135	0	1	192	153	0
normalized size	1	1.	1.	1.63	0.	0.01	2.31	1.84	0.
time (sec)	N/A	0.15	0.115	0.005	0.	0.243	2.44	0.231	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	85	88	89	51	90	0
normalized size	1	1.	0.8	1.39	1.44	1.46	0.84	1.48	0.
time (sec)	N/A	0.12	0.038	0.005	1.34	0.214	1.979	0.224	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	1	172	97	0
normalized size	1	1.	0.94	1.51	0.	0.02	2.73	1.54	0.
time (sec)	N/A	0.092	0.079	0.001	0.	0.244	2.128	0.227	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	69	82	80	41	84	0
normalized size	1	1.	0.98	1.35	1.61	1.57	0.8	1.65	0.
time (sec)	N/A	0.12	0.036	0.008	1.358	0.235	5.133	0.23	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	85	0	1	165	85	0
normalized size	1	1.	1.	1.55	0.	0.02	3.	1.55	0.
time (sec)	N/A	0.115	0.074	0.007	0.	0.232	2.885	0.234	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	81	95	100	49	123	53
normalized size	1	1.	1.03	1.4	1.64	1.72	0.84	2.12	0.91
time (sec)	N/A	0.149	0.047	0.01	1.361	0.239	5.636	0.228	22.997

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	98	0	1	172	96	56
normalized size	1	1.	0.97	1.48	0.	0.02	2.61	1.45	0.85
time (sec)	N/A	0.129	0.097	0.01	0.	0.236	3.139	0.224	21.394

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	116	130	132	66	188	68
normalized size	1	1.	0.96	1.55	1.73	1.76	0.88	2.51	0.91
time (sec)	N/A	0.166	0.075	0.011	1.327	0.229	5.31	0.226	26.776

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	143	0	1	207	151	73
normalized size	1	1.	0.99	1.64	0.	0.01	2.38	1.74	0.84
time (sec)	N/A	0.152	0.119	0.01	0.	0.242	3.912	0.222	27.011

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	108	160	181	184	105	248	88
normalized size	1	1.	1.1	1.63	1.85	1.88	1.07	2.53	0.9
time (sec)	N/A	0.197	0.112	0.012	1.364	0.238	6.429	0.232	32.117

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	138	196	0	1	280	211	0
normalized size	1	1.	0.95	1.35	0.	0.01	1.93	1.46	0.
time (sec)	N/A	0.335	0.153	0.014	0.	0.245	4.569	0.22	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	142	144	217	97	220	0
normalized size	1	1.	0.97	1.58	1.6	2.41	1.08	2.44	0.
time (sec)	N/A	0.258	0.105	0.016	1.348	0.23	4.363	0.224	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	156	0	1	245	154	100
normalized size	1	1.	0.89	1.32	0.	0.01	2.08	1.31	0.85
time (sec)	N/A	0.293	0.12	0.013	0.	0.248	3.945	0.228	44.085

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	97	100	136	68	149	0
normalized size	1	1.	0.9	1.56	1.61	2.19	1.1	2.4	0.
time (sec)	N/A	0.15	0.076	0.015	1.332	0.221	3.614	0.236	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	89	129	0	1	236	128	0
normalized size	1	1.	1.09	1.57	0.	0.01	2.88	1.56	0.
time (sec)	N/A	0.221	0.097	0.	0.	0.24	3.41	0.226	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	70	94	116	157	80	134	60
normalized size	1	1.	1.04	1.4	1.73	2.34	1.19	2.	0.9
time (sec)	N/A	0.154	0.072	0.021	1.344	0.236	5.232	0.227	29.737

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	131	0	1	238	138	90
normalized size	1	1.	0.86	1.24	0.	0.01	2.25	1.3	0.85
time (sec)	N/A	0.19	0.098	0.016	0.	0.24	3.97	0.224	24.311

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	114	135	215	92	147	70
normalized size	1	1.	0.89	1.41	1.67	2.65	1.14	1.81	0.86
time (sec)	N/A	0.2	0.163	0.02	1.326	0.235	5.864	0.235	27.846

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	161	0	1	248	150	110
normalized size	1	1.	0.85	1.28	0.	0.01	1.97	1.19	0.87
time (sec)	N/A	0.336	0.106	0.016	0.	0.228	4.632	0.226	42.332

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	223	0	1	238	208	150
normalized size	1	1.	0.91	1.37	0.	0.01	1.46	1.28	0.92
time (sec)	N/A	0.392	0.154	0.016	0.	0.242	8.474	0.228	106.856

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	114	155	162	240	122	144	0
normalized size	1	1.	1.15	1.57	1.64	2.42	1.23	1.45	0.
time (sec)	N/A	0.263	0.093	0.016	1.391	0.236	9.538	0.235	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	130	196	0	1	223	180	114
normalized size	1	1.	1.02	1.54	0.	0.01	1.76	1.42	0.9
time (sec)	N/A	0.323	0.179	0.015	0.	0.24	6.686	0.23	70.527

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	75	105	117	146	87	103	56
normalized size	1	1.	1.12	1.57	1.75	2.18	1.3	1.54	0.84
time (sec)	N/A	0.151	0.05	0.014	1.334	0.234	5.97	0.231	25.546

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	121	147	0	1	223	170	105
normalized size	1	1.	1.04	1.27	0.	0.01	1.92	1.47	0.91
time (sec)	N/A	0.183	0.175	0.001	0.	0.245	4.594	0.225	24.152

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	103	112	147	220	107	149	76
normalized size	1	1.	1.2	1.3	1.71	2.56	1.24	1.73	0.88
time (sec)	N/A	0.192	0.074	0.019	1.351	0.23	5.569	0.229	34.281

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	151	133	199	0	1	224	182	136
normalized size	1	0.99	0.88	1.31	0.	0.01	1.47	1.2	0.89
time (sec)	N/A	0.269	0.148	0.017	0.	0.253	5.61	0.223	29.95

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	99	149	192	346	139	239	100
normalized size	1	1.	0.93	1.41	1.81	3.26	1.31	2.25	0.94
time (sec)	N/A	0.277	0.16	0.023	1.362	0.228	8.219	0.227	36.583

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	148	227	0	1	240	204	150
normalized size	1	1.	0.92	1.41	0.	0.01	1.49	1.27	0.93
time (sec)	N/A	0.474	0.119	0.02	0.	0.248	6.925	0.227	68.002

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	86	100	101	65	104	0
normalized size	1	1.	0.95	1.15	1.33	1.35	0.87	1.39	0.
time (sec)	N/A	0.201	0.051	0.004	1.337	0.226	1.77	0.23	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	92	0	1	150	113	0
normalized size	1	1.	1.	1.19	0.	0.01	1.95	1.47	0.
time (sec)	N/A	0.138	0.087	0.005	0.	0.233	1.91	0.219	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	68	69	44	70	0
normalized size	1	1.	0.87	1.15	1.26	1.28	0.81	1.3	0.
time (sec)	N/A	0.131	0.032	0.004	1.346	0.232	1.654	0.233	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	68	0	1	90	78	49
normalized size	1	1.	0.98	1.17	0.	0.02	1.55	1.34	0.84
time (sec)	N/A	0.102	0.07	0.005	0.	0.246	1.781	0.227	16.123

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	42	39	27	43	0
normalized size	1	1.	0.89	1.14	1.2	1.11	0.77	1.23	0.
time (sec)	N/A	0.077	0.017	0.004	1.345	0.224	1.544	0.228	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	1	82	45	34
normalized size	1	1.	1.03	1.15	0.	0.03	2.1	1.15	0.87
time (sec)	N/A	0.047	0.042	0.	0.	0.243	1.609	0.224	8.634

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	47	45	26	49	29
normalized size	1	1.	1.	1.09	1.38	1.32	0.76	1.44	0.85
time (sec)	N/A	0.09	0.02	0.007	1.348	0.237	2.234	0.245	13.987

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	48	0	1	82	50	34
normalized size	1	1.	0.98	1.12	0.	0.02	1.91	1.16	0.79
time (sec)	N/A	0.064	0.042	0.005	0.	0.246	1.826	0.231	9.922

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	56	65	65	41	97	44
normalized size	1	1.	0.98	1.12	1.3	1.3	0.82	1.94	0.88
time (sec)	N/A	0.12	0.042	0.01	1.351	0.242	3.019	0.224	16.064

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	72	0	1	129	77	51
normalized size	1	1.	1.02	1.22	0.	0.02	2.19	1.31	0.86
time (sec)	N/A	0.101	0.095	0.008	0.	0.236	2.218	0.221	14.067

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	116	165	185	186	119	200	0
normalized size	1	1.	1.13	1.6	1.8	1.81	1.16	1.94	0.
time (sec)	N/A	0.274	0.087	0.006	1.347	0.235	2.282	0.226	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	176	0	1	240	207	0
normalized size	1	1.	1.	1.68	0.	0.01	2.29	1.97	0.
time (sec)	N/A	0.166	0.199	0.004	0.	0.248	2.505	0.235	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	124	136	138	83	144	0
normalized size	1	1.	1.02	1.55	1.7	1.72	1.04	1.8	0.
time (sec)	N/A	0.186	0.063	0.003	1.352	0.227	2.162	0.222	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	135	0	1	192	153	0
normalized size	1	1.	1.	1.61	0.	0.01	2.29	1.82	0.
time (sec)	N/A	0.145	0.125	0.005	0.	0.245	2.361	0.222	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	85	89	90	51	90	0
normalized size	1	1.	0.8	1.39	1.46	1.48	0.84	1.48	0.
time (sec)	N/A	0.114	0.039	0.004	1.34	0.234	1.976	0.225	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	1	172	97	0
normalized size	1	1.	0.94	1.51	0.	0.02	2.73	1.54	0.
time (sec)	N/A	0.092	0.083	0.	0.	0.233	2.19	0.221	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	69	82	80	41	84	0
normalized size	1	1.	0.98	1.35	1.61	1.57	0.8	1.65	0.
time (sec)	N/A	0.126	0.036	0.006	1.356	0.239	5.21	0.225	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	85	0	1	165	85	0
normalized size	1	1.	1.	1.55	0.	0.02	3.	1.55	0.
time (sec)	N/A	0.117	0.094	0.007	0.	0.242	2.765	0.223	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	81	93	99	49	122	53
normalized size	1	1.	1.03	1.4	1.6	1.71	0.84	2.1	0.91
time (sec)	N/A	0.149	0.046	0.01	1.355	0.238	5.811	0.226	22.918

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	98	0	1	172	97	56
normalized size	1	1.	1.03	1.53	0.	0.02	2.69	1.52	0.88
time (sec)	N/A	0.133	0.112	0.01	0.	0.241	3.109	0.225	21.021

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	128	263	296	297	187	321	0
normalized size	1	1.	0.93	1.91	2.14	2.15	1.36	2.33	0.
time (sec)	N/A	0.377	0.139	0.006	1.344	0.212	2.854	0.228	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	276	0	1	338	325	0
normalized size	1	1.	1.	1.97	0.	0.01	2.41	2.32	0.
time (sec)	N/A	0.218	0.079	0.006	0.	0.24	3.186	0.223	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	205	227	228	136	243	0
normalized size	1	1.	1.09	1.78	1.97	1.98	1.18	2.11	0.
time (sec)	N/A	0.276	0.094	0.005	1.349	0.221	2.656	0.226	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	218	0	1	275	248	0
normalized size	1	1.	0.99	1.83	0.	0.01	2.31	2.08	0.
time (sec)	N/A	0.191	0.067	0.005	0.	0.239	2.947	0.23	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	149	161	162	88	167	0
normalized size	1	1.	0.94	1.71	1.85	1.86	1.01	1.92	0.
time (sec)	N/A	0.168	0.054	0.005	1.33	0.227	2.444	0.222	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	92	161	0	1	240	174	0
normalized size	1	1.	0.94	1.64	0.	0.01	2.45	1.78	0.
time (sec)	N/A	0.13	0.114	0.	0.	0.233	2.784	0.226	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	116	132	136	63	134	0
normalized size	1	1.	0.89	1.59	1.81	1.86	0.86	1.84	0.
time (sec)	N/A	0.178	0.051	0.008	1.341	0.227	7.288	0.24	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	135	0	1	221	140	0
normalized size	1	1.	0.99	1.75	0.	0.01	2.87	1.82	0.
time (sec)	N/A	0.154	0.054	0.008	0.	0.237	3.651	0.234	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	114	131	142	63	162	0
normalized size	1	1.	1.03	1.56	1.79	1.95	0.86	2.22	0.
time (sec)	N/A	0.182	0.06	0.011	1.339	0.233	8.894	0.224	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	135	0	1	221	135	0
normalized size	1	1.	1.	1.82	0.	0.01	2.99	1.82	0.
time (sec)	N/A	0.158	0.063	0.01	0.	0.242	4.922	0.231	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	92	97	201	0	0
normalized size	1	1.	0.94	0.93	1.31	1.39	2.87	0.	0.
time (sec)	N/A	0.175	0.05	0.01	1.333	0.247	11.707	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	73	0	1	921	576	65
normalized size	1	1.	0.95	0.94	0.	0.01	11.81	7.38	0.83
time (sec)	N/A	0.223	0.149	0.012	0.	0.248	15.711	0.292	35.584

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	66	57	144	0	39
normalized size	1	1.	0.81	0.94	1.25	1.08	2.72	0.	0.74
time (sec)	N/A	0.13	0.031	0.01	1.347	0.232	6.394	0.	20.898

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	0	1	570	176	60
normalized size	1	1.	0.87	0.79	0.	0.01	8.14	2.51	0.86
time (sec)	N/A	0.098	0.069	0.008	0.	0.257	7.462	0.259	19.005

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	55	42	138	0	36
normalized size	1	1.	0.69	0.93	1.22	0.93	3.07	0.	0.8
time (sec)	N/A	0.073	0.027	0.009	1.338	0.232	3.242	0.	12.81

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	0	1	712	257	60
normalized size	1	1.	0.87	0.79	0.	0.01	10.17	3.67	0.86
time (sec)	N/A	0.069	0.072	0.	0.	0.252	8.13	0.253	15.62

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	82	73	0	0	49
normalized size	1	1.	0.87	0.95	1.32	1.18	0.	0.	0.79
time (sec)	N/A	0.155	0.044	0.013	1.345	0.283	0.	0.	25.999

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	76	0	1	1093	520	66
normalized size	1	1.	0.94	0.94	0.	0.01	13.49	6.42	0.81
time (sec)	N/A	0.214	0.143	0.012	0.	0.258	17.052	0.299	42.597

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	87	117	134	0	0	76
normalized size	1	1.	1.01	1.	1.34	1.54	0.	0.	0.87
time (sec)	N/A	0.226	0.066	0.016	1.385	0.439	0.	0.	33.736

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	98	0	1	1353	728	85
normalized size	1	1.	1.01	0.98	0.	0.01	13.53	7.28	0.85
time (sec)	N/A	0.455	0.252	0.014	0.	0.278	30.852	0.296	86.347

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	124	158	171	0	0	109
normalized size	1	1.	1.	1.04	1.33	1.44	0.	0.	0.92
time (sec)	N/A	0.293	0.096	0.02	1.35	1.252	0.	0.	43.685

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	135	141	0	1	1504	890	112
normalized size	1	1.	1.01	1.05	0.	0.01	11.22	6.64	0.84
time (sec)	N/A	0.66	0.242	0.016	0.	0.313	63.895	0.314	136.503

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	147	184	223	209	0	0	141
normalized size	1	1.	0.95	1.19	1.44	1.35	0.	0.	0.91
time (sec)	N/A	0.386	0.107	0.022	1.361	1.691	0.	0.	53.311

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	91	136	176	219	348	205	76
normalized size	1	1.	0.98	1.46	1.89	2.35	3.74	2.2	0.82
time (sec)	N/A	0.215	0.077	0.019	1.343	0.26	20.744	0.254	35.898

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	144	0	1	1850	163	94
normalized size	1	1.	1.	1.33	0.	0.01	17.13	1.51	0.87
time (sec)	N/A	0.244	0.219	0.014	0.	0.307	42.621	0.289	38.509

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	95	142	158	253	123	58
normalized size	1	1.	1.	1.28	1.92	2.14	3.42	1.66	0.78
time (sec)	N/A	0.16	0.056	0.017	1.358	0.224	8.448	0.296	25.56

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	90	134	0	1	1530	149	88
normalized size	1	1.	0.87	1.29	0.	0.01	14.71	1.43	0.85
time (sec)	N/A	0.182	0.214	0.014	0.	0.281	23.449	0.271	34.883

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	90	134	139	248	115	56
normalized size	1	1.	0.94	1.29	1.91	1.99	3.54	1.64	0.8
time (sec)	N/A	0.122	0.048	0.016	1.355	0.235	9.398	0.27	21.938

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	144	0	1	2033	165	94
normalized size	1	1.	0.87	1.32	0.	0.01	18.65	1.51	0.86
time (sec)	N/A	0.213	0.308	0.	0.	0.349	46.414	0.296	42.735

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	139	186	296	0	250	85
normalized size	1	1.	0.98	1.39	1.86	2.96	0.	2.5	0.85
time (sec)	N/A	0.23	0.163	0.023	1.354	1.01	0.	0.283	38.844

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	123	169	0	1	0	221	121
normalized size	1	1.	0.85	1.17	0.	0.01	0.	1.53	0.84
time (sec)	N/A	0.53	0.48	0.02	0.	0.615	0.	0.283	102.365

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	170	254	408	0	347	116
normalized size	1	1.	0.93	1.35	2.02	3.24	0.	2.75	0.92
time (sec)	N/A	0.329	0.55	0.026	1.364	2.369	0.	0.292	51.511

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	142	191	0	1	0	223	0
normalized size	1	1.	0.75	1.01	0.	0.01	0.	1.18	0.
time (sec)	N/A	0.754	0.905	0.022	0.	1.387	0.	0.274	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	218	319	392	418	313	97
normalized size	1	1.	0.85	1.88	2.75	3.38	3.6	2.7	0.84
time (sec)	N/A	0.265	0.197	0.019	1.363	0.243	17.376	0.29	43.688

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	154	299	0	1	0	275	139
normalized size	1	1.	0.98	1.9	0.	0.01	0.	1.75	0.89
time (sec)	N/A	0.435	0.424	0.017	0.	0.597	0.	0.259	73.485

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	77	177	293	346	410	235	82
normalized size	1	1.	0.77	1.77	2.93	3.46	4.1	2.35	0.82
time (sec)	N/A	0.218	0.218	0.019	1.353	0.245	16.318	0.276	35.077

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	151	298	0	1	3386	278	136
normalized size	1	1.	0.97	1.92	0.	0.01	21.85	1.79	0.88
time (sec)	N/A	0.361	0.398	0.017	0.	0.617	164.665	0.289	70.586

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	176	285	343	391	235	80
normalized size	1	1.	1.	1.8	2.91	3.5	3.99	2.4	0.82
time (sec)	N/A	0.167	0.079	0.018	1.367	0.241	15.712	0.259	31.772

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	158	310	0	1	0	293	146
normalized size	1	1.	0.99	1.94	0.	0.01	0.	1.83	0.91
time (sec)	N/A	0.459	0.508	0.002	0.	1.016	0.	0.262	99.184

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	141	286	375	702	0	425	133
normalized size	1	1.	0.95	1.92	2.52	4.71	0.	2.85	0.89
time (sec)	N/A	0.348	0.47	0.026	1.375	3.674	0.	0.258	64.122

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	172	335	0	1	0	319	0
normalized size	1	1.	0.82	1.59	0.	0.	0.	1.51	0.
time (sec)	N/A	0.827	0.677	0.025	0.	2.367	0.	0.254	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	171	322	491	864	0	482	167
normalized size	1	1.	0.96	1.81	2.76	4.85	0.	2.71	0.94
time (sec)	N/A	0.467	1.324	0.031	1.382	8.594	0.	0.254	75.206

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	196	362	0	1	0	346	0
normalized size	1	1.	0.73	1.34	0.	0.	0.	1.28	0.
time (sec)	N/A	1.202	0.816	0.028	0.	6.282	0.	0.24	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	23	15
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.1	0.71
time (sec)	N/A	0.038	0.007	0.009	1.333	0.223	0.205	0.234	6.707

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	105	0	1	128	119	80
normalized size	1	1.	1.02	1.21	0.	0.01	1.47	1.37	0.92
time (sec)	N/A	0.18	0.12	0.013	0.	0.234	3.077	0.235	42.241

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	74	80	105	56	122	0
normalized size	1	1.	0.83	1.23	1.33	1.75	0.93	2.03	0.
time (sec)	N/A	0.15	0.058	0.015	1.34	0.217	2.766	0.239	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	82	0	1	114	78	60
normalized size	1	1.	1.01	1.22	0.	0.01	1.7	1.16	0.9
time (sec)	N/A	0.131	0.113	0.012	0.	0.228	2.586	0.249	24.151

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	54	61	36	88	32
normalized size	1	1.	1.	1.15	1.32	1.49	0.88	2.15	0.78
time (sec)	N/A	0.088	0.02	0.013	1.346	0.22	2.039	0.233	14.052

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	1	112	77	51
normalized size	1	1.	1.	1.08	0.	0.02	1.78	1.22	0.81
time (sec)	N/A	0.063	0.073	0.001	0.	0.226	2.112	0.253	9.733

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	69	96	46	85	44
normalized size	1	1.	0.9	1.04	1.35	1.88	0.9	1.67	0.86
time (sec)	N/A	0.114	0.049	0.017	1.347	0.232	2.2	0.251	16.69

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	85	0	1	114	86	60
normalized size	1	1.	0.99	1.2	0.	0.01	1.61	1.21	0.85
time (sec)	N/A	0.145	0.055	0.014	0.	0.241	2.498	0.255	20.082

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	86	105	165	70	113	68
normalized size	1	1.	0.84	1.13	1.38	2.17	0.92	1.49	0.89
time (sec)	N/A	0.178	0.083	0.02	1.344	0.231	3.868	0.239	22.236

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	110	0	1	184	116	82
normalized size	1	1.	1.	1.22	0.	0.01	2.04	1.29	0.91
time (sec)	N/A	0.262	0.117	0.016	0.	0.246	3.16	0.229	40.582

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	138	196	0	1	280	211	0
normalized size	1	1.	0.95	1.35	0.	0.01	1.93	1.46	0.
time (sec)	N/A	0.342	0.145	0.014	0.	0.24	4.458	0.25	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	142	144	216	97	220	0
normalized size	1	1.	0.99	1.61	1.64	2.45	1.1	2.5	0.
time (sec)	N/A	0.261	0.106	0.015	1.337	0.225	4.256	0.238	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	105	156	0	1	245	154	100
normalized size	1	1.	0.91	1.34	0.	0.01	2.11	1.33	0.86
time (sec)	N/A	0.289	0.113	0.012	0.	0.248	4.021	0.237	45.041

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	97	99	136	68	150	0
normalized size	1	1.	0.92	1.59	1.62	2.23	1.11	2.46	0.
time (sec)	N/A	0.148	0.076	0.014	1.328	0.219	3.609	0.242	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	88	129	0	1	236	127	0
normalized size	1	1.	1.07	1.57	0.	0.01	2.88	1.55	0.
time (sec)	N/A	0.217	0.102	0.002	0.	0.234	3.47	0.242	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	70	94	116	158	80	134	60
normalized size	1	1.	1.04	1.4	1.73	2.36	1.19	2.	0.9
time (sec)	N/A	0.159	0.071	0.017	1.338	0.23	5.359	0.26	28.667

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	131	0	1	238	139	88
normalized size	1	1.	0.88	1.27	0.	0.01	2.31	1.35	0.85
time (sec)	N/A	0.184	0.112	0.015	0.	0.238	4.006	0.242	24.163

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	114	135	215	92	147	70
normalized size	1	1.	0.9	1.42	1.69	2.69	1.15	1.84	0.88
time (sec)	N/A	0.201	0.174	0.02	1.339	0.232	6.192	0.238	27.963

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	125	107	161	0	1	248	151	110
normalized size	1	0.98	0.84	1.27	0.	0.01	1.95	1.19	0.87
time (sec)	N/A	0.363	0.114	0.017	0.	0.245	4.748	0.249	42.118

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	151	302	0	1	382	325	163
normalized size	1	1.	0.89	1.79	0.	0.01	2.26	1.92	0.96
time (sec)	N/A	0.384	0.134	0.017	0.	0.242	6.082	0.254	64.709

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	106	229	235	343	158	336	0
normalized size	1	1.	0.91	1.96	2.01	2.93	1.35	2.87	0.
time (sec)	N/A	0.329	0.152	0.017	1.347	0.225	6.186	0.247	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	247	0	1	337	248	150
normalized size	1	1.	0.85	1.68	0.	0.01	2.29	1.69	1.02
time (sec)	N/A	0.448	0.115	0.013	0.	0.243	5.57	0.249	69.505

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	127	168	167	244	112	247	0
normalized size	1	1.	1.44	1.91	1.9	2.77	1.27	2.81	0.
time (sec)	N/A	0.218	0.074	0.014	1.356	0.216	5.273	0.237	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	205	0	1	313	205	0
normalized size	1	1.	1.	1.93	0.	0.01	2.95	1.93	0.
time (sec)	N/A	0.205	0.1	0.	0.	0.24	4.905	0.261	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	111	146	165	240	110	203	0
normalized size	1	1.	1.26	1.66	1.88	2.73	1.25	2.31	0.
time (sec)	N/A	0.204	0.176	0.023	1.354	0.239	10.253	0.232	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	94	189	0	1	309	193	0
normalized size	1	1.	0.72	1.44	0.	0.01	2.36	1.47	0.
time (sec)	N/A	0.335	0.097	0.017	0.	0.241	7.054	0.248	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	156	190	282	128	212	92
normalized size	1	1.	0.89	1.59	1.94	2.88	1.31	2.16	0.94
time (sec)	N/A	0.243	0.157	0.022	1.357	0.239	13.553	0.251	41.422

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	109	209	0	1	321	203	131
normalized size	1	1.	0.74	1.42	0.	0.01	2.18	1.38	0.89
time (sec)	N/A	0.351	0.102	0.019	0.	0.241	8.509	0.252	56.627

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	144	0	1	1850	165	94
normalized size	1	1.	0.87	1.32	0.	0.01	16.97	1.51	0.86
time (sec)	N/A	0.235	0.249	0.016	0.	0.311	42.89	0.242	36.81

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	95	142	158	253	124	58
normalized size	1	1.	1.	1.28	1.92	2.14	3.42	1.68	0.78
time (sec)	N/A	0.159	0.056	0.018	1.351	0.233	8.546	0.242	24.567

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	134	0	1	1530	149	88
normalized size	1	1.	1.	1.29	0.	0.01	14.71	1.43	0.85
time (sec)	N/A	0.175	0.245	0.014	0.	0.3	23.581	0.236	34.173

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	90	134	139	248	115	56
normalized size	1	1.	0.94	1.29	1.91	1.99	3.54	1.64	0.8
time (sec)	N/A	0.12	0.048	0.017	1.353	0.232	8.123	0.266	21.814

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	109	144	0	1	2033	163	94
normalized size	1	1.	1.01	1.33	0.	0.01	18.82	1.51	0.87
time (sec)	N/A	0.196	0.235	0.	0.	0.346	47.293	0.235	41.644

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	139	185	294	0	247	85
normalized size	1	1.	0.98	1.4	1.87	2.97	0.	2.49	0.86
time (sec)	N/A	0.249	0.191	0.022	1.359	1.027	0.	0.267	37.388

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	123	169	0	1	0	221	121
normalized size	1	1.	0.85	1.17	0.	0.01	0.	1.53	0.84
time (sec)	N/A	0.517	0.322	0.019	0.	0.62	0.	0.235	94.956

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	170	255	409	0	347	116
normalized size	1	1.	0.94	1.35	2.02	3.25	0.	2.75	0.92
time (sec)	N/A	0.325	0.272	0.027	1.368	2.467	0.	0.239	48.372

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	142	191	0	1	0	223	0
normalized size	1	1.	0.75	1.01	0.	0.01	0.	1.18	0.
time (sec)	N/A	0.762	0.609	0.023	0.	1.41	0.	0.284	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	155	209	348	481	0	379	151
normalized size	1	1.	0.97	1.31	2.17	3.01	0.	2.37	0.94
time (sec)	N/A	0.419	0.365	0.029	1.363	6.046	0.	0.227	58.454

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	179	234	0	1	0	279	0
normalized size	1	1.	0.72	0.94	0.	0.	0.	1.12	0.
time (sec)	N/A	1.103	0.568	0.024	0.	4.122	0.	0.239	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	202	268	458	554	0	478	202
normalized size	1	1.	0.96	1.28	2.18	2.64	0.	2.28	0.96
time (sec)	N/A	0.54	0.58	0.03	1.367	9.305	0.	0.24	83.461

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	133	222	0	1	2378	1	138
normalized size	1	1.	0.82	1.37	0.	0.01	14.68	0.01	0.85
time (sec)	N/A	0.411	0.324	0.02	0.	0.399	83.168	0.34	77.951

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	86	188	308	400	507	240	87
normalized size	1	1.	0.8	1.76	2.88	3.74	4.74	2.24	0.81
time (sec)	N/A	0.247	0.111	0.024	1.36	0.245	16.269	0.235	39.324

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	137	222	0	1	2399	1	126
normalized size	1	1.	0.93	1.51	0.	0.01	16.32	0.01	0.86
time (sec)	N/A	0.339	0.299	0.02	0.	0.523	97.893	0.308	68.392

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	77	143	290	342	408	220	76
normalized size	1	1.	0.84	1.55	3.15	3.72	4.43	2.39	0.83
time (sec)	N/A	0.177	0.119	0.024	1.365	0.255	14.46	0.251	33.125

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	136	238	0	1	0	1	141
normalized size	1	1.	0.81	1.43	0.	0.01	0.	0.01	0.84
time (sec)	N/A	0.457	0.556	0.002	0.	0.994	0.	0.394	99.673

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	133	225	398	729	0	0	124
normalized size	1	1.	0.94	1.6	2.82	5.17	0.	0.	0.88
time (sec)	N/A	0.358	0.41	0.033	1.368	4.122	0.	0.	62.593

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	158	261	0	1	0	1	189
normalized size	1	1.	0.72	1.2	0.	0.	0.	0.	0.87
time (sec)	N/A	0.813	0.587	0.026	0.	2.233	0.	0.461	164.269

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	157	254	514	900	0	0	143
normalized size	1	1.	1.01	1.63	3.29	5.77	0.	0.	0.92
time (sec)	N/A	0.449	0.368	0.037	1.371	9.231	0.	0.	71.608

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	178	285	0	1	0	1	0
normalized size	1	1.	0.66	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	1.17	0.696	0.03	0.	6.59	0.	0.458	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	166	388	0	1	0	406	187
normalized size	1	1.	0.8	1.87	0.	0.	0.	1.96	0.9
time (sec)	N/A	0.63	0.612	0.023	0.	1.107	0.	0.247	119.91

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	121	283	560	807	780	360	121
normalized size	1	1.	0.85	1.99	3.94	5.68	5.49	2.54	0.85
time (sec)	N/A	0.33	0.161	0.026	1.383	0.265	57.483	0.24	52.14

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	171	391	0	1	0	428	178
normalized size	1	1.	0.86	1.96	0.	0.	0.	2.14	0.89
time (sec)	N/A	0.603	0.707	0.022	0.	1.965	0.	0.257	117.255

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	234	532	684	643	309	109
normalized size	1	1.	0.85	1.86	4.22	5.43	5.1	2.45	0.87
time (sec)	N/A	0.248	0.226	0.024	1.367	0.236	56.269	0.233	47.84

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	197	403	0	1	0	448	0
normalized size	1	1.	0.86	1.75	0.	0.	0.	1.95	0.
time (sec)	N/A	0.728	0.971	0.002	0.	4.325	0.	0.239	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	187	374	711	1428	0	635	173
normalized size	1	1.	0.97	1.95	3.7	7.44	0.	3.31	0.9
time (sec)	N/A	0.49	0.545	0.037	1.389	14.561	0.	0.258	101.995

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	210	428	0	1	0	581	0
normalized size	1	1.	0.71	1.44	0.	0.	0.	1.96	0.
time (sec)	N/A	1.272	1.068	0.03	0.	9.148	0.	0.26	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	208	405	879	1656	0	861	202
normalized size	1	1.	0.97	1.88	4.09	7.7	0.	4.	0.94
time (sec)	N/A	0.613	0.624	0.04	1.405	29.675	0.	0.276	124.796

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	230	455	0	1	0	495	0
normalized size	1	1.	0.61	1.21	0.	0.	0.	1.31	0.
time (sec)	N/A	1.783	1.056	0.035	0.	20.727	0.	0.248	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	88	474	0	512	2069	909	87
normalized size	1	1.	0.92	4.94	0.	5.33	21.55	9.47	0.91
time (sec)	N/A	0.157	0.12	0.01	0.	0.237	9.576	0.271	20.949

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	262	0	290	1044	513	63
normalized size	1	1.	0.92	3.69	0.	4.08	14.7	7.23	0.89
time (sec)	N/A	0.116	0.076	0.009	0.	0.235	5.535	0.278	15.987

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	110	0	124	410	225	37
normalized size	1	1.	0.91	2.44	0.	2.76	9.11	5.	0.82
time (sec)	N/A	0.061	0.041	0.005	0.	0.235	2.866	0.308	9.423

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0	49
normalized size	1	1.	0.83	0.	0.	0.	2.88	0.	0.74
time (sec)	N/A	0.096	0.068	0.046	0.	0.	15.575	0.	12.68

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0	71
normalized size	1	1.	0.86	0.	0.	0.	9.74	0.	0.76
time (sec)	N/A	0.122	0.071	0.058	0.	0.	137.154	0.	14.871

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	73
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.12	0.08	0.079	0.	0.	0.	0.	14.74

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	141	976	0	1044	4345	1	144
normalized size	1	1.	0.93	6.46	0.	6.91	28.77	0.01	0.95
time (sec)	N/A	0.23	0.16	0.012	0.	0.238	17.108	0.285	38.005

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	101	569	0	597	2363	1071	100
normalized size	1	1.	0.93	5.22	0.	5.48	21.68	9.83	0.92
time (sec)	N/A	0.168	0.111	0.009	0.	0.238	9.856	0.228	28.235

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	262	0	290	1044	513	63
normalized size	1	1.	0.92	3.69	0.	4.08	14.7	7.23	0.89
time (sec)	N/A	0.111	0.082	0.008	0.	0.24	5.475	0.263	15.864

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	0	0	0	299	0	76
normalized size	1	1.	1.26	0.	0.	0.	3.18	0.	0.81
time (sec)	N/A	0.159	0.151	0.058	0.	0.	34.561	0.	25.942

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	118	0	0	0	0	0	104
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.279	0.162	0.066	0.	0.	0.	0.	41.955

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	166	118	0	0	0	0	0	148
normalized size	1	0.97	0.69	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.387	0.164	0.086	0.	0.	0.	0.	41.732

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	159	0	0	0	411	0	116
normalized size	1	1.	1.2	0.	0.	0.	3.09	0.	0.87
time (sec)	N/A	0.209	0.267	0.064	0.	0.	64.645	0.	37.481

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	0	0	0	299	0	76
normalized size	1	1.	1.26	0.	0.	0.	3.18	0.	0.81
time (sec)	N/A	0.152	0.141	0.058	0.	0.	34.314	0.	26.775

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0	49
normalized size	1	1.	0.83	0.	0.	0.	2.88	0.	0.74
time (sec)	N/A	0.107	0.066	0.046	0.	0.	15.456	0.	12.933

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	85	0	0	0	354	0	76
normalized size	1	1.	0.83	0.	0.	0.	3.47	0.	0.75
time (sec)	N/A	0.131	0.076	0.07	0.	0.	93.47	0.	21.275

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	127	0	0	0	0	0	128
normalized size	1	1.	0.81	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.5	0.128	0.095	0.	0.	0.	0.	96.117

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	196	0	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.003	0.415	0.078	0.	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	159	0	0	0	0	0	206
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	1.02
time (sec)	N/A	0.552	0.268	0.072	0.	0.	0.	0.	139.365

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	118	0	0	0	0	0	104
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.28	0.176	0.065	0.	0.	0.	0.	42.436

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0	71
normalized size	1	1.	0.86	0.	0.	0.	9.74	0.	0.76
time (sec)	N/A	0.122	0.072	0.056	0.	0.	137.2	0.	14.857

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	127	0	0	0	0	0	128
normalized size	1	1.	0.81	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.469	0.113	0.	0.	0.	0.	0.	95.473

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	195	0	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.964	0.481	0.072	0.	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	197	0	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.602	0.663	0.101	0.	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	43	46	39	41
normalized size	1	1.	0.85	0.82	0.92	1.1	1.18	1.	1.05
time (sec)	N/A	0.052	0.02	0.005	1.345	0.219	36.923	0.213	6.975

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	43	46	39	41
normalized size	1	1.	0.85	0.82	0.92	1.1	1.18	1.	1.05
time (sec)	N/A	0.05	0.018	0.005	1.348	0.225	20.012	0.217	7.109

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	43	46	39	41
normalized size	1	1.	0.85	0.82	0.92	1.1	1.18	1.	1.05
time (sec)	N/A	0.05	0.017	0.005	1.352	0.211	7.926	0.217	7.065

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	41	37	39	41
normalized size	1	1.	0.85	0.82	0.92	1.05	0.95	1.	1.05
time (sec)	N/A	0.047	0.017	0.005	1.351	0.22	2.646	0.251	7.192

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	36	39	44	39	39
normalized size	1	1.	0.89	0.86	0.97	1.05	1.19	1.05	1.05
time (sec)	N/A	0.048	0.017	0.006	1.345	0.222	2.224	0.214	6.973

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	36	39	44	39	39
normalized size	1	1.	0.89	0.86	0.97	1.05	1.19	1.05	1.05
time (sec)	N/A	0.05	0.018	0.005	1.347	0.223	3.483	0.21	6.895

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	36	39	42	39	37
normalized size	1	1.	0.89	0.86	0.97	1.05	1.14	1.05	1.
time (sec)	N/A	0.05	0.02	0.006	1.33	0.224	4.073	0.213	6.971

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	39	39	42	42	37
normalized size	1	1.	0.89	0.86	1.05	1.05	1.14	1.14	1.
time (sec)	N/A	0.049	0.024	0.004	1.339	0.221	6.172	0.209	6.964

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	76	80	72	63
normalized size	1	1.	0.84	0.89	1.1	1.21	1.27	1.14	1.
time (sec)	N/A	0.092	0.032	0.008	1.347	0.213	70.796	0.21	12.605

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	76	80	72	63
normalized size	1	1.	1.	0.89	1.1	1.21	1.27	1.14	1.
time (sec)	N/A	0.088	0.03	0.009	1.373	0.212	37.517	0.215	12.645

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	76	80	72	63
normalized size	1	1.	0.84	0.89	1.1	1.21	1.27	1.14	1.
time (sec)	N/A	0.09	0.033	0.008	1.381	0.216	20.496	0.251	12.928

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	73	66	72	63
normalized size	1	1.	0.84	0.89	1.1	1.16	1.05	1.14	1.
time (sec)	N/A	0.085	0.033	0.008	1.335	0.217	4.81	0.227	12.905

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	72	78	72	61
normalized size	1	1.	0.87	0.92	1.13	1.18	1.28	1.18	1.
time (sec)	N/A	0.088	0.033	0.009	1.337	0.217	6.717	0.21	12.649

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	72	78	72	61
normalized size	1	1.	0.87	0.92	1.13	1.18	1.28	1.18	1.
time (sec)	N/A	0.089	0.034	0.007	1.343	0.209	8.574	0.222	12.76

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	72	76	72	61
normalized size	1	1.	0.87	0.92	1.13	1.18	1.25	1.18	1.
time (sec)	N/A	0.09	0.034	0.008	1.323	0.224	10.212	0.212	12.692

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	72	72	76	74	61
normalized size	1	1.	0.93	0.92	1.18	1.18	1.25	1.21	1.
time (sec)	N/A	0.089	0.029	0.009	1.33	0.218	15.456	0.212	12.775

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	105	114	104	85
normalized size	1	1.	1.	0.94	1.16	1.24	1.34	1.22	1.
time (sec)	N/A	0.118	0.042	0.01	1.339	0.213	123.375	0.212	16.591

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	105	114	104	85
normalized size	1	1.	1.	0.94	1.16	1.24	1.34	1.22	1.
time (sec)	N/A	0.116	0.038	0.009	1.353	0.213	71.564	0.214	16.528

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	105	114	104	85
normalized size	1	1.	1.	0.94	1.16	1.24	1.34	1.22	1.
time (sec)	N/A	0.117	0.038	0.008	1.368	0.216	38.153	0.213	16.631

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	80	99	103	95	104	85
normalized size	1	1.	0.84	0.94	1.16	1.21	1.12	1.22	1.
time (sec)	N/A	0.112	0.041	0.009	1.33	0.212	10.218	0.213	16.941

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	80	99	101	112	104	83
normalized size	1	1.	0.86	0.96	1.19	1.22	1.35	1.25	1.
time (sec)	N/A	0.112	0.042	0.009	1.342	0.226	16.951	0.211	16.635

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	80	99	101	110	104	82
normalized size	1	1.	0.86	0.96	1.19	1.22	1.33	1.25	0.99
time (sec)	N/A	0.115	0.045	0.008	1.343	0.225	20.248	0.211	16.736

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	80	99	101	110	104	83
normalized size	1	1.	0.86	0.96	1.19	1.22	1.33	1.25	1.
time (sec)	N/A	0.112	0.044	0.008	1.336	0.22	24.298	0.218	16.691

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	80	101	101	107	107	82
normalized size	1	1.	0.96	0.99	1.25	1.25	1.32	1.32	1.01
time (sec)	N/A	0.113	0.037	0.008	1.338	0.224	33.238	0.223	16.632

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	264	330	0	805	0	402	258
normalized size	1	1.	0.96	1.2	0.	2.92	0.	1.46	0.93
time (sec)	N/A	0.567	0.307	0.017	0.	0.249	0.	0.23	85.802

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	243	308	0	1054	0	356	240
normalized size	1	1.	0.95	1.2	0.	4.1	0.	1.39	0.93
time (sec)	N/A	0.477	0.234	0.011	0.	0.255	0.	0.257	78.285

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	243	299	0	737	0	355	238
normalized size	1	1.	0.95	1.17	0.	2.89	0.	1.39	0.93
time (sec)	N/A	0.443	0.222	0.011	0.	0.251	0.	0.26	78.649

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	213	280	0	977	0	339	221
normalized size	1	1.	0.9	1.18	0.	4.12	0.	1.43	0.93
time (sec)	N/A	0.406	0.197	0.011	0.	0.252	0.	0.325	70.07

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	212	277	0	709	371	339	219
normalized size	1	1.	0.9	1.18	0.	3.02	1.58	1.44	0.93
time (sec)	N/A	0.375	0.203	0.011	0.	0.251	41.071	0.295	71.462

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	221	277	0	996	374	339	219
normalized size	1	1.	0.94	1.18	0.	4.24	1.59	1.44	0.93
time (sec)	N/A	0.399	0.302	0.014	0.	0.25	95.465	0.32	71.351

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	223	280	0	722	379	339	221
normalized size	1	1.	0.94	1.18	0.	3.05	1.6	1.43	0.93
time (sec)	N/A	0.388	0.322	0.014	0.	0.241	170.036	0.265	67.408

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	243	299	0	1031	0	362	240
normalized size	1	1.	0.95	1.17	0.	4.04	0.	1.42	0.94
time (sec)	N/A	0.444	0.392	0.017	0.	0.253	0.	0.252	74.84

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	277	339	0	851	0	402	289
normalized size	1	1.	0.89	1.09	0.	2.75	0.	1.3	0.93
time (sec)	N/A	0.519	0.394	0.021	0.	0.255	0.	0.249	87.267

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	256	317	0	1095	0	382	269
normalized size	1	1.	0.89	1.1	0.	3.79	0.	1.32	0.93
time (sec)	N/A	0.47	0.45	0.02	0.	0.255	0.	0.254	80.059

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	252	323	0	817	0	382	260
normalized size	1	1.	0.89	1.14	0.	2.88	0.	1.35	0.92
time (sec)	N/A	0.47	0.546	0.019	0.	0.255	0.	0.254	79.069

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	228	305	0	1072	0	369	240
normalized size	1	1.	0.87	1.17	0.	4.11	0.	1.41	0.92
time (sec)	N/A	0.399	0.308	0.019	0.	0.245	0.	0.244	71.625

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	228	305	0	801	0	369	240
normalized size	1	1.	0.87	1.17	0.	3.07	0.	1.41	0.92
time (sec)	N/A	0.397	0.299	0.017	0.	0.25	0.	0.244	69.822

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	253	323	0	1087	0	375	260
normalized size	1	1.	0.88	1.12	0.	3.76	0.	1.3	0.9
time (sec)	N/A	0.464	0.459	0.023	0.	0.259	0.	0.256	80.158

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	256	317	0	838	0	382	269
normalized size	1	1.	0.89	1.1	0.	2.9	0.	1.32	0.93
time (sec)	N/A	0.457	0.6	0.023	0.	0.259	0.	0.24	78.759

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	277	339	0	1164	0	409	289
normalized size	1	1.	0.89	1.09	0.	3.75	0.	1.32	0.93
time (sec)	N/A	0.522	0.463	0.026	0.	0.263	0.	0.27	88.718

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	285	363	0	909	0	410	298
normalized size	1	1.	0.9	1.15	0.	2.88	0.	1.3	0.94
time (sec)	N/A	0.526	0.504	0.024	0.	0.245	0.	0.261	88.581

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	272	325	0	1177	0	396	275
normalized size	1	1.	0.93	1.11	0.	4.02	0.	1.35	0.94
time (sec)	N/A	0.472	0.634	0.023	0.	0.259	0.	0.248	80.718

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	274	334	0	921	0	402	277
normalized size	1	1.	0.92	1.12	0.	3.09	0.	1.35	0.93
time (sec)	N/A	0.452	0.499	0.023	0.	0.258	0.	0.252	78.919

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	267	335	0	1189	0	402	277
normalized size	1	1.	0.9	1.12	0.	3.99	0.	1.35	0.93
time (sec)	N/A	0.464	0.37	0.023	0.	0.26	0.	0.246	81.306

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	263	325	0	903	0	396	275
normalized size	1	1.	0.9	1.11	0.	3.08	0.	1.35	0.94
time (sec)	N/A	0.454	0.368	0.022	0.	0.254	0.	0.245	78.289

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	285	363	0	1177	0	405	298
normalized size	1	1.	0.89	1.13	0.	3.66	0.	1.26	0.93
time (sec)	N/A	0.522	0.495	0.026	0.	0.258	0.	0.272	90.228

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	286	357	0	929	0	410	306
normalized size	1	1.	0.89	1.11	0.	2.89	0.	1.27	0.95
time (sec)	N/A	0.538	0.498	0.026	0.	0.257	0.	0.264	86.996

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	308	381	0	1257	0	440	326
normalized size	1	1.	0.9	1.11	0.	3.66	0.	1.28	0.95
time (sec)	N/A	0.579	0.54	0.033	0.	0.261	0.	0.262	95.638

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	76	80	72	63
normalized size	1	1.	0.84	0.89	1.1	1.21	1.27	1.14	1.
time (sec)	N/A	0.093	0.035	0.008	1.352	0.216	70.997	0.228	12.058

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	76	80	72	63
normalized size	1	1.	1.	0.89	1.1	1.21	1.27	1.14	1.
time (sec)	N/A	0.093	0.03	0.008	1.35	0.218	37.752	0.235	12.08

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	76	80	72	63
normalized size	1	1.	0.84	0.89	1.1	1.21	1.27	1.14	1.
time (sec)	N/A	0.088	0.034	0.009	1.354	0.208	20.441	0.228	12.084

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	73	66	72	63
normalized size	1	1.	0.84	0.89	1.1	1.16	1.05	1.14	1.
time (sec)	N/A	0.085	0.032	0.007	1.372	0.225	4.855	0.226	12.351

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	72	78	72	61
normalized size	1	1.	0.87	0.92	1.13	1.18	1.28	1.18	1.
time (sec)	N/A	0.084	0.031	0.008	1.358	0.22	6.718	0.241	12.075

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	72	78	72	61
normalized size	1	1.	0.87	0.92	1.13	1.18	1.28	1.18	1.
time (sec)	N/A	0.087	0.036	0.01	1.378	0.214	8.572	0.23	12.162

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	72	76	72	61
normalized size	1	1.	0.87	0.92	1.13	1.18	1.25	1.18	1.
time (sec)	N/A	0.087	0.033	0.008	1.341	0.227	10.295	0.233	12.125

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	72	72	76	74	61
normalized size	1	1.	0.93	0.92	1.18	1.18	1.25	1.21	1.
time (sec)	N/A	0.088	0.03	0.008	1.343	0.222	15.527	0.23	12.164

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	97	115	122	136	127	102
normalized size	1	1.	1.	1.	1.19	1.26	1.4	1.31	1.05
time (sec)	N/A	0.145	0.054	0.01	1.356	0.211	125.044	0.23	22.181

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	97	115	122	136	127	102
normalized size	1	1.	1.	1.	1.19	1.26	1.4	1.31	1.05
time (sec)	N/A	0.142	0.051	0.01	1.336	0.219	72.403	0.236	22.045

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	97	115	122	136	127	102
normalized size	1	1.	1.	1.	1.19	1.26	1.4	1.31	1.05
time (sec)	N/A	0.138	0.051	0.009	1.338	0.223	38.599	0.244	22.2

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	97	115	119	110	127	102
normalized size	1	1.	0.86	1.	1.19	1.23	1.13	1.31	1.05
time (sec)	N/A	0.133	0.062	0.009	1.344	0.222	10.214	0.227	22.715

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	97	115	117	134	127	100
normalized size	1	1.	1.	1.02	1.21	1.23	1.41	1.34	1.05
time (sec)	N/A	0.132	0.051	0.009	1.341	0.222	17.235	0.234	22.323

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	97	115	117	134	127	100
normalized size	1	1.	0.87	1.02	1.21	1.23	1.41	1.34	1.05
time (sec)	N/A	0.137	0.051	0.01	1.332	0.214	20.59	0.229	22.47

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	97	115	117	133	127	100
normalized size	1	1.	0.87	1.02	1.21	1.23	1.4	1.34	1.05
time (sec)	N/A	0.134	0.052	0.009	1.329	0.219	24.6	0.243	22.242

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	97	117	117	133	130	100
normalized size	1	1.	0.87	1.02	1.23	1.23	1.4	1.37	1.05
time (sec)	N/A	0.135	0.062	0.01	1.348	0.222	33.709	0.25	22.218

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	178	0	182	144
normalized size	1	1.	1.	0.99	1.23	1.28	0.	1.31	1.04
time (sec)	N/A	0.184	0.069	0.012	1.327	0.22	0.	0.237	29.748

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	178	192	182	144
normalized size	1	1.	1.	0.99	1.23	1.28	1.38	1.31	1.04
time (sec)	N/A	0.178	0.064	0.01	1.366	0.218	125.663	0.23	29.487

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	178	192	182	144
normalized size	1	1.	1.	0.99	1.23	1.28	1.38	1.31	1.04
time (sec)	N/A	0.176	0.061	0.011	1.329	0.221	73.279	0.23	29.827

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	138	171	176	155	182	144
normalized size	1	1.	1.	0.99	1.23	1.27	1.12	1.31	1.04
time (sec)	N/A	0.172	0.063	0.01	1.344	0.214	16.222	0.234	30.521

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	137	138	171	174	190	182	143
normalized size	1	1.	1.	1.01	1.25	1.27	1.39	1.33	1.04
time (sec)	N/A	0.171	0.064	0.01	1.349	0.219	36.206	0.251	29.906

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	137	138	171	174	189	182	143
normalized size	1	1.	1.	1.01	1.25	1.27	1.38	1.33	1.04
time (sec)	N/A	0.173	0.088	0.01	1.37	0.218	41.994	0.238	29.934

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	137	138	171	174	189	182	143
normalized size	1	1.	1.	1.01	1.25	1.27	1.38	1.33	1.04
time (sec)	N/A	0.17	0.094	0.01	1.329	0.218	44.208	0.23	30.208

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	138	174	174	185	185	143
normalized size	1	1.	0.88	1.01	1.27	1.27	1.35	1.35	1.04
time (sec)	N/A	0.173	0.09	0.012	1.34	0.22	58.573	0.232	29.833

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	299	545	0	1485	0	589	292
normalized size	1	1.	0.96	1.75	0.	4.77	0.	1.89	0.94
time (sec)	N/A	0.631	0.224	0.023	0.	0.247	0.	0.255	102.279

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	276	504	0	1980	0	520	272
normalized size	1	1.	0.95	1.74	0.	6.83	0.	1.79	0.94
time (sec)	N/A	0.533	0.173	0.017	0.	0.263	0.	0.249	96.331

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	276	495	0	1401	0	520	270
normalized size	1	1.	0.96	1.72	0.	4.86	0.	1.81	0.94
time (sec)	N/A	0.507	0.163	0.016	0.	0.26	0.	0.252	97.49

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	249	461	0	1894	0	487	252
normalized size	1	1.	0.93	1.72	0.	7.07	0.	1.82	0.94
time (sec)	N/A	0.459	0.176	0.014	0.	0.248	0.	0.265	88.84

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	452	0	1365	612	486	250
normalized size	1	1.	0.94	1.7	0.	5.13	2.3	1.83	0.94
time (sec)	N/A	0.448	0.181	0.015	0.	0.259	158.427	0.239	84.712

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	261	439	0	1902	597	464	241
normalized size	1	1.	1.	1.69	0.	7.32	2.3	1.78	0.93
time (sec)	N/A	0.559	0.199	0.017	0.	0.253	129.418	0.252	100.607

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	261	439	0	1368	597	464	241
normalized size	1	1.	1.	1.69	0.	5.26	2.3	1.78	0.93
time (sec)	N/A	0.555	0.202	0.018	0.	0.26	174.05	0.234	99.123

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	254	452	0	1910	0	477	250
normalized size	1	1.	0.95	1.69	0.	7.15	0.	1.79	0.94
time (sec)	N/A	0.597	0.196	0.02	0.	0.266	0.	0.26	100.367

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	254	461	0	1370	0	478	252
normalized size	1	1.	0.94	1.71	0.	5.09	0.	1.78	0.94
time (sec)	N/A	0.587	0.193	0.02	0.	0.257	0.	0.239	97.172

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	277	495	0	1958	0	527	270
normalized size	1	1.	0.96	1.72	0.	6.8	0.	1.83	0.94
time (sec)	N/A	0.699	0.23	0.021	0.	0.264	0.	0.259	110.706

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	372	563	0	1538	0	594	348
normalized size	1	1.	0.99	1.5	0.	4.1	0.	1.58	0.93
time (sec)	N/A	0.949	0.698	0.024	0.	0.264	0.	0.254	114.981

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	337	523	0	2029	0	558	321
normalized size	1	1.	0.97	1.51	0.	5.86	0.	1.61	0.93
time (sec)	N/A	0.697	0.312	0.025	0.	0.278	0.	0.247	111.288

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	333	523	0	1482	0	551	313
normalized size	1	1.	0.96	1.51	0.	4.28	0.	1.59	0.9
time (sec)	N/A	0.679	0.304	0.023	0.	0.27	0.	0.247	109.982

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	319	499	0	2021	0	524	286
normalized size	1	1.	1.03	1.61	0.	6.52	0.	1.69	0.92
time (sec)	N/A	0.615	0.301	0.026	0.	0.271	0.	0.255	103.585

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	318	496	0	1497	0	524	291
normalized size	1	1.	1.02	1.59	0.	4.8	0.	1.68	0.93
time (sec)	N/A	0.71	0.304	0.023	0.	0.27	0.	0.239	96.448

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	317	495	0	2039	0	525	308
normalized size	1	1.	0.95	1.49	0.	6.12	0.	1.58	0.92
time (sec)	N/A	0.714	0.304	0.026	0.	0.268	0.	0.26	96.639

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	315	498	0	1494	0	518	304
normalized size	1	1.	0.95	1.5	0.	4.5	0.	1.56	0.92
time (sec)	N/A	0.698	0.321	0.027	0.	0.27	0.	0.258	95.701

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	333	524	0	2043	0	541	332
normalized size	1	1.	0.92	1.44	0.	5.63	0.	1.49	0.91
time (sec)	N/A	0.806	0.325	0.028	0.	0.277	0.	0.259	108.071

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	383	590	0	1605	0	609	425
normalized size	1	1.	0.87	1.34	0.	3.65	0.	1.38	0.97
time (sec)	N/A	0.824	0.891	0.028	0.	0.28	0.	0.248	124.299

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	363	562	0	2140	0	576	388
normalized size	1	1.	0.91	1.4	0.	5.34	0.	1.44	0.97
time (sec)	N/A	0.758	0.402	0.029	0.	0.275	0.	0.262	115.602

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	361	568	0	1604	0	575	389
normalized size	1	1.	0.9	1.41	0.	3.99	0.	1.43	0.97
time (sec)	N/A	0.726	0.793	0.027	0.	0.27	0.	0.257	115.505

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	339	514	0	2126	0	562	350
normalized size	1	1.	0.93	1.41	0.	5.84	0.	1.54	0.96
time (sec)	N/A	0.644	0.335	0.026	0.	0.276	0.	0.283	107.693

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	339	514	0	1593	0	562	350
normalized size	1	1.	0.93	1.41	0.	4.38	0.	1.54	0.96
time (sec)	N/A	0.662	0.331	0.026	0.	0.275	0.	0.264	104.197

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	398	364	568	0	2147	0	576	381
normalized size	1	1.	0.91	1.42	0.	5.38	0.	1.44	0.95
time (sec)	N/A	0.83	0.744	0.031	0.	0.279	0.	0.336	110.094

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	401	365	562	0	1617	0	575	384
normalized size	1	1.	0.91	1.4	0.	4.02	0.	1.43	0.96
time (sec)	N/A	0.846	0.447	0.03	0.	0.273	0.	0.303	106.729

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	438	382	590	0	2168	0	599	420
normalized size	1	1.	0.87	1.34	0.	4.94	0.	1.36	0.96
time (sec)	N/A	0.979	0.768	0.034	0.	0.281	0.	0.317	117.404

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	314	721	0	2951	0	717	311
normalized size	1	1.	0.96	2.2	0.	9.	0.	2.19	0.95
time (sec)	N/A	0.638	0.26	0.015	0.	0.279	0.	0.293	106.645

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	314	712	0	2106	0	717	309
normalized size	1	1.	0.96	2.18	0.	6.46	0.	2.2	0.95
time (sec)	N/A	0.576	0.22	0.015	0.	0.264	0.	0.317	105.181

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	291	659	0	2844	0	662	291
normalized size	1	1.	0.95	2.15	0.	9.29	0.	2.16	0.95
time (sec)	N/A	0.524	0.248	0.015	0.	0.277	0.	0.303	98.209

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	291	650	0	2052	0	662	289
normalized size	1	1.	0.96	2.14	0.	6.75	0.	2.18	0.95
time (sec)	N/A	0.507	0.261	0.014	0.	0.255	0.	0.285	96.747

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	283	622	0	2843	0	624	265
normalized size	1	1.	1.	2.19	0.	10.01	0.	2.2	0.93
time (sec)	N/A	0.59	0.186	0.018	0.	0.28	0.	0.287	110.388

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	283	616	0	2052	0	622	0
normalized size	1	1.	1.	2.17	0.	7.23	0.	2.19	0.
time (sec)	N/A	0.541	0.165	0.018	0.	0.268	0.	0.275	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	287	616	0	2842	0	614	265
normalized size	1	1.	1.01	2.18	0.	10.04	0.	2.17	0.94
time (sec)	N/A	0.586	0.215	0.021	0.	0.283	0.	0.281	112.6

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	287	622	0	2043	0	614	0
normalized size	1	1.	1.01	2.2	0.	7.22	0.	2.17	0.
time (sec)	N/A	0.558	0.192	0.022	0.	0.271	0.	0.283	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	292	650	0	2858	0	652	289
normalized size	1	1.	0.96	2.15	0.	9.43	0.	2.15	0.95
time (sec)	N/A	0.623	0.254	0.022	0.	0.273	0.	0.281	119.585

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	292	659	0	2053	0	652	291
normalized size	1	1.	0.96	2.16	0.	6.73	0.	2.14	0.95
time (sec)	N/A	0.59	0.244	0.021	0.	0.273	0.	0.302	129.571

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	314	712	0	2927	0	724	309
normalized size	1	1.	0.97	2.19	0.	9.01	0.	2.23	0.95
time (sec)	N/A	0.656	0.285	0.024	0.	0.268	0.	0.322	170.11

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	378	804	0	2257	0	810	0
normalized size	1	1.	0.92	1.97	0.	5.52	0.	1.98	0.
time (sec)	N/A	0.986	0.395	0.027	0.	0.275	0.	0.279	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	345	748	0	2978	0	745	359
normalized size	1	1.	0.92	2.	0.	7.96	0.	1.99	0.96
time (sec)	N/A	0.908	0.409	0.026	0.	0.287	0.	0.318	153.945

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	345	748	0	2183	0	745	372
normalized size	1	1.	0.89	1.94	0.	5.66	0.	1.93	0.96
time (sec)	N/A	1.075	0.441	0.025	0.	0.272	0.	0.292	172.812

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	323	706	0	2955	0	697	347
normalized size	1	1.	0.86	1.88	0.	7.86	0.	1.85	0.92
time (sec)	N/A	0.911	0.411	0.024	0.	0.284	0.	0.308	152.817

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	323	697	0	2157	0	690	0
normalized size	1	1.	0.95	2.05	0.	6.34	0.	2.03	0.
time (sec)	N/A	0.768	0.389	0.023	0.	0.266	0.	0.293	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	327	682	0	2971	0	680	338
normalized size	1	1.	0.89	1.85	0.	8.07	0.	1.85	0.92
time (sec)	N/A	0.919	0.414	0.03	0.	0.286	0.	0.266	153.543

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	327	682	0	2183	0	676	0
normalized size	1	1.	0.89	1.86	0.	5.95	0.	1.84	0.
time (sec)	N/A	0.911	0.389	0.029	0.	0.265	0.	0.289	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	323	697	0	2982	0	682	354
normalized size	1	1.	0.86	1.85	0.	7.93	0.	1.81	0.94
time (sec)	N/A	0.94	0.417	0.03	0.	0.287	0.	0.306	157.362

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	323	706	0	2178	0	687	347
normalized size	1	1.	0.86	1.88	0.	5.79	0.	1.83	0.92
time (sec)	N/A	0.904	0.413	0.03	0.	0.279	0.	0.271	153.168

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	411	351	0	1777	0	0	0
normalized size	1	1.	0.86	0.73	0.	3.72	0.	0.	0.
time (sec)	N/A	1.191	0.503	0.019	0.	2.352	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	409	339	0	1499	0	0	0
normalized size	1	1.	0.86	0.71	0.	3.15	0.	0.	0.
time (sec)	N/A	1.03	0.403	0.019	0.	0.503	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	364	328	0	1737	0	0	420
normalized size	1	1.	0.79	0.71	0.	3.75	0.	0.	0.91
time (sec)	N/A	0.779	0.206	0.018	0.	0.343	0.	0.	149.778

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	364	304	0	1327	0	0	420
normalized size	1	1.	0.79	0.66	0.	2.87	0.	0.	0.91
time (sec)	N/A	0.767	0.196	0.017	0.	0.271	0.	0.	147.127

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	364	304	0	1624	0	0	420
normalized size	1	1.	0.79	0.66	0.	3.51	0.	0.	0.91
time (sec)	N/A	0.777	0.272	0.016	0.	0.278	0.	0.	148.044

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	364	328	0	1467	0	0	420
normalized size	1	1.	0.79	0.71	0.	3.17	0.	0.	0.91
time (sec)	N/A	0.717	0.255	0.016	0.	0.418	0.	0.	145.862

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	409	339	0	1797	0	0	0
normalized size	1	1.	0.86	0.71	0.	3.78	0.	0.	0.
time (sec)	N/A	1.157	0.497	0.02	0.	0.649	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	411	351	0	1557	0	0	0
normalized size	1	1.	0.86	0.73	0.	3.26	0.	0.	0.
time (sec)	N/A	1.023	0.518	0.02	0.	5.244	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	437	375	0	1852	0	0	0
normalized size	1	1.	0.88	0.75	0.	3.72	0.	0.	0.
time (sec)	N/A	1.485	0.639	0.023	0.	8.379	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	563	582	0	3723	0	969	0
normalized size	1	1.	0.99	1.02	0.	6.53	0.	1.7	0.
time (sec)	N/A	1.741	0.619	0.026	0.	50.736	0.	0.366	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	527	566	0	4362	0	919	0
normalized size	1	1.	0.98	1.06	0.	8.14	0.	1.71	0.
time (sec)	N/A	1.296	0.588	0.025	0.	12.283	0.	0.336	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	523	533	0	3505	0	903	0
normalized size	1	1.	0.98	1.	0.	6.59	0.	1.7	0.
time (sec)	N/A	1.131	0.516	0.024	0.	7.54	0.	0.346	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	522	528	0	4201	0	922	0
normalized size	1	1.	0.99	1.	0.	7.96	0.	1.75	0.
time (sec)	N/A	1.23	0.503	0.024	0.	3.352	0.	0.345	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	522	528	0	3430	0	884	0
normalized size	1	1.	0.99	1.	0.	6.5	0.	1.67	0.
time (sec)	N/A	0.987	0.495	0.023	0.	3.977	0.	0.335	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	523	533	0	4293	0	946	0
normalized size	1	1.	0.98	0.99	0.	8.01	0.	1.76	0.
time (sec)	N/A	1.264	0.535	0.023	0.	5.908	0.	0.362	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	526	566	0	3611	0	909	0
normalized size	1	1.	0.98	1.06	0.	6.74	0.	1.7	0.
time (sec)	N/A	1.118	0.55	0.023	0.	23.563	0.	0.335	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	540	582	0	4498	0	979	0
normalized size	1	1.	0.95	1.02	0.	7.89	0.	1.72	0.
time (sec)	N/A	1.697	1.69	0.029	0.	21.8	0.	0.358	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	542	588	0	3791	0	969	0
normalized size	1	1.	0.95	1.03	0.	6.65	0.	1.7	0.
time (sec)	N/A	1.724	1.639	0.028	0.	120.048	0.	0.341	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	563	612	0	4652	0	965	0
normalized size	1	1.	0.91	0.99	0.	7.53	0.	1.56	0.
time (sec)	N/A	2.146	1.51	0.033	0.	88.257	0.	0.368	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	640	839	0	6067	0	1	0
normalized size	1	1.	1.01	1.33	0.	9.61	0.	0.	0.
time (sec)	N/A	1.676	1.057	0.029	0.	101.111	0.	0.395	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	544	839	0	7329	0	1	0
normalized size	1	1.	0.87	1.34	0.	11.67	0.	0.	0.
time (sec)	N/A	1.658	1.662	0.028	0.	64.551	0.	0.444	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	543	848	0	0	0	1	0
normalized size	1	1.	0.87	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	1.494	1.66	0.027	0.	0.	0.	0.399	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	620	855	0	7324	0	1	0
normalized size	1	1.	0.98	1.35	0.	11.57	0.	0.	0.
time (sec)	N/A	1.773	2.206	0.027	0.	83.491	0.	0.426	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	620	882	0	0	0	1	0
normalized size	1	1.	0.98	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.738	1.687	0.027	0.	0.	0.	0.394	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	637	900	0	0	0	1	0
normalized size	1	1.	0.94	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	2.277	2.819	0.033	0.	0.	0.	0.42	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	639	906	0	0	0	1	0
normalized size	1	1.	0.94	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	2.019	2.617	0.035	0.	0.	0.	0.402	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	660	933	0	0	0	1	0
normalized size	1	1.	0.89	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	2.758	3.258	0.038	0.	0.	0.	0.457	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	585	740	0	5917	0	0	0
normalized size	1	1.	0.94	1.19	0.	9.48	0.	0.	0.
time (sec)	N/A	1.601	2.326	0.03	0.	27.94	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	583	740	0	7214	0	0	0
normalized size	1	1.	0.96	1.22	0.	11.85	0.	0.	0.
time (sec)	N/A	1.657	2.825	0.03	0.	24.385	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	575	770	0	6036	0	0	0
normalized size	1	1.	0.96	1.28	0.	10.04	0.	0.	0.
time (sec)	N/A	1.413	2.76	0.031	0.	75.942	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	589	778	0	7488	0	0	0
normalized size	1	1.	0.94	1.25	0.	12.	0.	0.	0.
time (sec)	N/A	1.838	3.308	0.03	0.	73.85	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	593	808	0	0	0	0	0
normalized size	1	1.	0.94	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	1.741	2.362	0.029	0.	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	606	825	0	0	0	0	0
normalized size	1	1.	0.9	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	2.31	3.805	0.036	0.	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	610	825	0	0	0	0	0
normalized size	1	1.	0.9	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	2.177	3.758	0.034	0.	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	630	849	0	0	0	0	0
normalized size	1	1.	0.86	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	2.842	2.743	0.041	0.	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	604	1066	0	0	0	1	0
normalized size	1	1.	0.84	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	2.107	2.502	0.033	0.	0.	0.	0.508	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	703	703	604	1067	0	0	0	1	0
normalized size	1	1.	0.86	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	2.199	3.227	0.035	0.	0.	0.	0.53	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	703	703	603	1094	0	0	0	1	0
normalized size	1	1.	0.86	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	2.081	4.447	0.034	0.	0.	0.	0.561	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	691	1100	0	0	0	1	0
normalized size	1	1.	0.94	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	2.46	5.428	0.034	0.	0.	0.	0.567	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	692	1124	0	0	0	1	0
normalized size	1	1.	0.94	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	2.053	5.708	0.033	0.	0.	0.	0.499	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	805	706	1143	0	0	0	1	0
normalized size	1	1.	0.88	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	3.056	6.145	0.043	0.	0.	0.	0.665	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	805	707	1143	0	0	0	1	0
normalized size	1	1.	0.88	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	2.929	4.372	0.049	0.	0.	0.	0.535	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	881	881	729	1170	0	0	0	1	0
normalized size	1	1.	0.83	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	3.691	4.515	0.046	0.	0.	0.	0.573	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	75	77	0	134	212	144	92
normalized size	1	1.	0.73	0.75	0.	1.3	2.06	1.4	0.89
time (sec)	N/A	0.232	0.083	0.009	0.	0.235	4.519	0.24	25.586

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	123	181	0	1	286	178	144
normalized size	1	1.	0.79	1.17	0.	0.01	1.85	1.15	0.93
time (sec)	N/A	0.224	0.132	0.011	0.	0.306	46.697	0.237	22.888

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	0	101	162	107	63
normalized size	1	1.	0.78	0.73	0.	1.38	2.22	1.47	0.86
time (sec)	N/A	0.166	0.057	0.008	0.	0.227	2.159	0.229	19.077

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	99	139	0	1	226	135	107
normalized size	1	1.	0.81	1.14	0.	0.01	1.85	1.11	0.88
time (sec)	N/A	0.182	0.097	0.01	0.	0.257	29.615	0.253	19.44

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	0	68	110	63	37
normalized size	1	1.	0.74	0.67	0.	1.48	2.39	1.37	0.8
time (sec)	N/A	0.1	0.039	0.006	0.	0.213	0.983	0.24	13.126

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	78	96	0	1	144	93	75
normalized size	1	1.	0.9	1.1	0.	0.01	1.66	1.07	0.86
time (sec)	N/A	0.082	0.07	0.008	0.	0.226	17.382	0.235	9.797

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	70	57	0	1	117	81	49
normalized size	1	1.	1.19	0.97	0.	0.02	1.98	1.37	0.83
time (sec)	N/A	0.134	0.102	0.01	0.	0.224	12.446	0.225	13.636

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	93	0	1	107	113	71
normalized size	1	1.	0.77	1.11	0.	0.01	1.27	1.35	0.85
time (sec)	N/A	0.102	0.064	0.013	0.	0.222	10.901	0.241	10.814

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	106	0	1	107	92	68
normalized size	1	1.	0.96	1.26	0.	0.01	1.27	1.1	0.81
time (sec)	N/A	0.18	0.116	0.012	0.	0.23	31.792	0.242	16.03

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	75	0	1	107	204	56
normalized size	1	1.	1.06	1.14	0.	0.02	1.62	3.09	0.85
time (sec)	N/A	0.085	0.07	0.013	0.	0.224	7.746	0.248	10.905

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	99	153	0	1	144	162	76
normalized size	1	1.	1.12	1.74	0.	0.01	1.64	1.84	0.86
time (sec)	N/A	0.196	0.111	0.013	0.	0.228	70.492	0.232	16.302

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	37	0	74	119	313	46
normalized size	1	1.	0.75	0.7	0.	1.4	2.25	5.91	0.87
time (sec)	N/A	0.082	0.05	0.008	0.	0.221	5.897	0.231	9.266

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	123	197	0	1	226	189	107
normalized size	1	1.	1.02	1.64	0.	0.01	1.88	1.58	0.89
time (sec)	N/A	0.253	0.165	0.013	0.	0.247	101.49	0.229	21.099

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	63	59	0	109	442	389	78
normalized size	1	1.	0.75	0.7	0.	1.3	5.26	4.63	0.93
time (sec)	N/A	0.118	0.069	0.008	0.	0.25	8.588	0.242	12.427

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	147	239	0	1	286	262	144
normalized size	1	1.	0.94	1.53	0.	0.01	1.83	1.68	0.92
time (sec)	N/A	0.315	0.193	0.024	0.	0.289	178.175	0.235	25.992

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	81	83	0	142	957	464	112
normalized size	1	1.	0.69	0.71	0.	1.21	8.18	3.97	0.96
time (sec)	N/A	0.167	0.086	0.01	0.	0.322	13.526	0.235	16.574

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	169	281	0	1	0	311	177
normalized size	1	1.	0.89	1.49	0.	0.01	0.	1.65	0.94
time (sec)	N/A	0.372	0.252	0.038	0.	0.411	0.	0.24	31.88

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	0	167	260	323	92
normalized size	1	1.	0.76	0.75	0.	1.62	2.52	3.14	0.89
time (sec)	N/A	0.228	0.096	0.008	0.	0.216	12.882	0.249	25.747

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	140	219	0	1	345	215	170
normalized size	1	1.	0.74	1.16	0.	0.01	1.84	1.14	0.9
time (sec)	N/A	0.286	0.158	0.012	0.	0.367	112.338	0.234	27.89

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	0	134	209	247	63
normalized size	1	1.	0.78	0.73	0.	1.84	2.86	3.38	0.86
time (sec)	N/A	0.167	0.065	0.007	0.	0.215	7.303	0.224	19.668

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	122	177	0	1	287	180	143
normalized size	1	1.	0.79	1.14	0.	0.01	1.85	1.16	0.92
time (sec)	N/A	0.211	0.126	0.01	0.	0.279	73.71	0.239	24.476

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	0	99	158	162	37
normalized size	1	1.	0.74	0.67	0.	2.15	3.43	3.52	0.8
time (sec)	N/A	0.102	0.05	0.006	0.	0.212	3.7	0.246	13.051

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	131	0	1	253	138	102
normalized size	1	1.	0.83	1.11	0.	0.01	2.14	1.17	0.86
time (sec)	N/A	0.11	0.124	0.008	0.	0.238	43.742	0.249	12.006

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	93	70	0	1	134	107	65
normalized size	1	1.	1.22	0.92	0.	0.01	1.76	1.41	0.86
time (sec)	N/A	0.165	0.151	0.01	0.	0.232	20.	0.247	16.013

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	84	125	0	1	216	154	100
normalized size	1	1.	0.77	1.15	0.	0.01	1.98	1.41	0.92
time (sec)	N/A	0.126	0.125	0.012	0.	0.229	26.962	0.247	12.629

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	100	132	0	1	184	139	94
normalized size	1	1.	0.91	1.2	0.	0.01	1.67	1.26	0.85
time (sec)	N/A	0.22	0.189	0.013	0.	0.23	42.762	0.234	18.938

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	168	0	1	202	279	105
normalized size	1	1.	0.72	1.41	0.	0.01	1.7	2.34	0.88
time (sec)	N/A	0.142	0.1	0.013	0.	0.229	17.009	0.242	13.912

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	184	0	1	216	177	104
normalized size	1	1.	0.87	1.6	0.	0.01	1.88	1.54	0.9
time (sec)	N/A	0.233	0.223	0.014	0.	0.229	89.04	0.23	19.276

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	88	115	0	1	184	319	75
normalized size	1	1.	1.02	1.34	0.	0.01	2.14	3.71	0.87
time (sec)	N/A	0.111	0.109	0.015	0.	0.231	12.714	0.251	13.867

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	233	0	1	253	215	105
normalized size	1	1.	1.	1.94	0.	0.01	2.11	1.79	0.88
time (sec)	N/A	0.25	0.157	0.013	0.	0.259	139.909	0.242	20.543

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	37	0	105	518	464	46
normalized size	1	1.	0.75	0.7	0.	1.98	9.77	8.75	0.87
time (sec)	N/A	0.084	0.071	0.009	0.	0.249	13.926	0.252	9.251

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	145	275	0	1	0	262	143
normalized size	1	1.	0.93	1.76	0.	0.01	0.	1.68	0.92
time (sec)	N/A	0.31	0.201	0.014	0.	0.296	0.	0.253	25.606

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	63	59	0	142	1408	540	78
normalized size	1	1.	0.75	0.7	0.	1.69	16.76	6.43	0.93
time (sec)	N/A	0.119	0.089	0.008	0.	0.333	21.206	0.257	12.454

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	164	317	0	1	0	286	170
normalized size	1	1.	0.89	1.72	0.	0.01	0.	1.55	0.92
time (sec)	N/A	0.368	0.291	0.046	0.	0.42	0.	0.252	31.943

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	0	198	313	545	92
normalized size	1	1.	0.76	0.75	0.	1.92	3.04	5.29	0.89
time (sec)	N/A	0.225	0.105	0.008	0.	0.222	30.849	0.234	26.04

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	166	257	0	1	0	263	206
normalized size	1	1.	0.75	1.16	0.	0.	0.	1.19	0.93
time (sec)	N/A	0.314	0.187	0.012	0.	0.553	0.	0.248	33.553

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	0	165	260	431	63
normalized size	1	1.	0.78	0.73	0.	2.26	3.56	5.9	0.86
time (sec)	N/A	0.166	0.078	0.009	0.	0.219	19.411	0.25	19.79

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	144	215	0	1	348	223	173
normalized size	1	1.	0.77	1.14	0.	0.01	1.85	1.19	0.92
time (sec)	N/A	0.252	0.167	0.01	0.	0.365	138.628	0.25	29.735

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	0	131	209	304	37
normalized size	1	1.	0.74	0.67	0.	2.85	4.54	6.61	0.8
time (sec)	N/A	0.101	0.058	0.006	0.	0.215	11.829	0.237	13.214

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	166	0	1	316	181	134
normalized size	1	1.	0.81	1.11	0.	0.01	2.12	1.21	0.9
time (sec)	N/A	0.138	0.121	0.008	0.	0.309	87.463	0.249	14.512

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	115	85	0	1	151	131	82
normalized size	1	1.	1.21	0.89	0.	0.01	1.59	1.38	0.86
time (sec)	N/A	0.19	0.232	0.011	0.	0.24	33.4	0.225	19.026

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	108	158	0	1	306	197	128
normalized size	1	1.	0.79	1.16	0.	0.01	2.25	1.45	0.94
time (sec)	N/A	0.155	0.178	0.012	0.	0.264	57.155	0.251	14.989

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	117	161	0	1	296	188	119
normalized size	1	1.	0.87	1.19	0.	0.01	2.19	1.39	0.88
time (sec)	N/A	0.267	0.273	0.012	0.	0.257	57.228	0.243	22.277

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	106	204	0	1	299	321	138
normalized size	1	1.	0.73	1.4	0.	0.01	2.05	2.2	0.95
time (sec)	N/A	0.169	0.177	0.013	0.	0.251	34.913	0.25	16.2

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	119	213	0	1	279	231	134
normalized size	1	1.	0.83	1.49	0.	0.01	1.95	1.62	0.94
time (sec)	N/A	0.277	0.326	0.012	0.	0.248	105.41	0.239	22.685

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	105	251	0	1	292	433	136
normalized size	1	1.	0.69	1.65	0.	0.01	1.92	2.85	0.89
time (sec)	N/A	0.181	0.169	0.013	0.	0.249	23.396	0.26	18.233

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	266	0	1	306	225	136
normalized size	1	1.	0.81	1.79	0.	0.01	2.05	1.51	0.91
time (sec)	N/A	0.284	0.297	0.013	0.	0.233	163.34	0.254	24.342

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	155	0	1	592	432	95
normalized size	1	1.	0.98	1.44	0.	0.01	5.48	4.	0.88
time (sec)	N/A	0.138	0.168	0.023	0.	0.274	22.199	0.245	17.563

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	143	311	0	1	0	263	139
normalized size	1	1.	0.94	2.05	0.	0.01	0.	1.73	0.91
time (sec)	N/A	0.302	0.273	0.015	0.	0.296	0.	0.235	25.256

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	37	0	138	1489	616	46
normalized size	1	1.	0.75	0.7	0.	2.6	28.09	11.62	0.87
time (sec)	N/A	0.084	0.089	0.007	0.	0.322	28.288	0.269	9.31

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	167	353	0	1	0	311	173
normalized size	1	1.	0.88	1.87	0.	0.01	0.	1.65	0.92
time (sec)	N/A	0.368	0.326	0.013	0.	0.42	0.	0.247	30.862

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	77	0	103	172	140	90
normalized size	1	1.	0.78	0.77	0.	1.03	1.72	1.4	0.9
time (sec)	N/A	0.228	0.072	0.009	0.	0.229	4.776	0.233	25.085

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	105	143	0	1	235	144	114
normalized size	1	1.	0.86	1.17	0.	0.01	1.93	1.18	0.93
time (sec)	N/A	0.171	0.115	0.011	0.	0.258	33.109	0.234	18.142

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	53	0	70	121	99	61
normalized size	1	1.	0.79	0.75	0.	0.99	1.7	1.39	0.86
time (sec)	N/A	0.168	0.057	0.007	0.	0.234	2.95	0.24	19.072

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	101	0	1	150	101	82
normalized size	1	1.	0.87	1.13	0.	0.01	1.69	1.13	0.92
time (sec)	N/A	0.127	0.073	0.01	0.	0.244	19.907	0.247	15.157

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	30	0	39	70	58	36
normalized size	1	1.	0.77	0.7	0.	0.91	1.63	1.35	0.84
time (sec)	N/A	0.101	0.025	0.005	0.	0.227	1.99	0.235	13.146

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	61	62	0	1	126	65	49
normalized size	1	1.	1.05	1.07	0.	0.02	2.17	1.12	0.84
time (sec)	N/A	0.055	0.04	0.007	0.	0.235	8.338	0.237	8.058

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	45	0	1	136	51	36
normalized size	1	1.	1.26	1.05	0.	0.02	3.16	1.19	0.84
time (sec)	N/A	0.112	0.064	0.01	0.	0.237	8.069	0.229	11.814

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	41	0	1	99	78	39
normalized size	1	1.	1.06	0.87	0.	0.02	2.11	1.66	0.83
time (sec)	N/A	0.064	0.042	0.012	0.	0.233	3.035	0.245	8.698

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	79	0	1	66	84	48
normalized size	1	1.	1.4	1.36	0.	0.02	1.14	1.45	0.83
time (sec)	N/A	0.142	0.072	0.013	0.	0.236	23.598	0.245	13.108

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	46	70	162	44
normalized size	1	1.	0.74	0.68	0.	0.87	1.32	3.06	0.83
time (sec)	N/A	0.085	0.043	0.006	0.	0.225	4.646	0.246	9.526

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	99	119	0	1	150	163	82
normalized size	1	1.	1.1	1.32	0.	0.01	1.67	1.81	0.91
time (sec)	N/A	0.195	0.123	0.013	0.	0.245	49.724	0.242	16.538

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	63	59	0	78	355	238	76
normalized size	1	1.	0.75	0.7	0.	0.93	4.23	2.83	0.9
time (sec)	N/A	0.117	0.056	0.008	0.	0.238	7.167	0.244	12.875

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	128	161	0	1	235	213	114
normalized size	1	1.	1.04	1.31	0.	0.01	1.91	1.73	0.93
time (sec)	N/A	0.251	0.166	0.016	0.	0.263	86.287	0.239	20.994

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	83	0	111	819	313	110
normalized size	1	1.	0.73	0.71	0.	0.95	7.	2.68	0.94
time (sec)	N/A	0.158	0.077	0.008	0.	0.268	10.668	0.248	16.929

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	185	0	1	233	184	148
normalized size	1	1.	0.81	1.22	0.	0.01	1.53	1.21	0.97
time (sec)	N/A	0.217	0.217	0.011	0.	0.277	61.554	0.239	23.289

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	77	0	119	172	131	88
normalized size	1	1.	0.78	0.78	0.	1.2	1.74	1.32	0.89
time (sec)	N/A	0.226	0.075	0.009	0.	0.216	5.346	0.24	25.002

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	141	0	1	177	140	114
normalized size	1	1.	0.83	1.18	0.	0.01	1.49	1.18	0.96
time (sec)	N/A	0.165	0.16	0.01	0.	0.233	36.947	0.231	18.028

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	52	0	85	117	88	60
normalized size	1	1.	0.82	0.78	0.	1.27	1.75	1.31	0.9
time (sec)	N/A	0.166	0.06	0.008	0.	0.212	3.43	0.228	18.99

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	97	0	1	114	95	75
normalized size	1	1.	0.9	1.17	0.	0.01	1.37	1.14	0.9
time (sec)	N/A	0.164	0.103	0.01	0.	0.238	20.568	0.26	22.544

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	30	0	54	66	46	34
normalized size	1	1.	0.73	0.73	0.	1.32	1.61	1.12	0.83
time (sec)	N/A	0.098	0.024	0.006	0.	0.23	2.251	0.228	12.957

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	58	54	0	1	60	69	46
normalized size	1	1.	1.07	1.	0.	0.02	1.11	1.28	0.85
time (sec)	N/A	0.052	0.062	0.007	0.	0.237	11.011	0.243	8.161

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	64	60	0	1	212	70	42
normalized size	1	1.	1.21	1.13	0.	0.02	4.	1.32	0.79
time (sec)	N/A	0.128	0.119	0.011	0.	0.24	15.015	0.226	13.183

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	58	68	77	39
normalized size	1	1.	0.77	0.77	0.	1.23	1.45	1.64	0.83
time (sec)	N/A	0.071	0.04	0.007	0.	0.224	14.145	0.235	8.236

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	109	0	1	262	134	73
normalized size	1	1.	1.06	1.27	0.	0.01	3.05	1.56	0.85
time (sec)	N/A	0.19	0.227	0.013	0.	0.246	29.431	0.228	16.877

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	58	0	92	284	244	75
normalized size	1	1.	0.74	0.71	0.	1.12	3.46	2.98	0.91
time (sec)	N/A	0.108	0.063	0.007	0.	0.231	25.683	0.241	11.403

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	115	153	0	1	180	185	114
normalized size	1	1.	0.96	1.27	0.	0.01	1.5	1.54	0.95
time (sec)	N/A	0.242	0.276	0.015	0.	0.245	56.035	0.234	20.476

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	84	83	0	127	593	397	109
normalized size	1	1.	0.73	0.72	0.	1.1	5.16	3.45	0.95
time (sec)	N/A	0.146	0.087	0.008	0.	0.234	39.92	0.241	15.447

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	143	197	0	1	236	243	148
normalized size	1	1.	0.93	1.29	0.	0.01	1.54	1.59	0.97
time (sec)	N/A	0.306	0.337	0.018	0.	0.234	91.675	0.231	25.674

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	105	107	0	158	1030	549	143
normalized size	1	1.	0.71	0.72	0.	1.07	6.96	3.71	0.97
time (sec)	N/A	0.185	0.108	0.01	0.	0.289	83.832	0.251	20.11

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	98	101	0	166	437	167	117
normalized size	1	1.	0.77	0.79	0.	1.3	3.41	1.3	0.91
time (sec)	N/A	0.276	0.1	0.009	0.	0.234	10.017	0.23	31.703

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	181	0	1	804	200	144
normalized size	1	1.	0.81	1.21	0.	0.01	5.4	1.34	0.97
time (sec)	N/A	0.205	0.176	0.011	0.	0.291	108.445	0.233	23.287

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	76	0	132	337	124	88
normalized size	1	1.	0.75	0.78	0.	1.36	3.47	1.28	0.91
time (sec)	N/A	0.224	0.081	0.008	0.	0.235	6.34	0.232	25.266

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	98	134	0	1	675	151	105
normalized size	1	1.	0.86	1.18	0.	0.01	5.92	1.32	0.92
time (sec)	N/A	0.232	0.13	0.012	0.	0.258	65.915	0.235	43.94

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	54	53	0	101	240	82	60
normalized size	1	1.	0.79	0.78	0.	1.49	3.53	1.21	0.88
time (sec)	N/A	0.171	0.064	0.008	0.	0.226	4.122	0.231	19.696

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	92	0	1	352	93	66
normalized size	1	1.	0.97	1.19	0.	0.01	4.57	1.21	0.86
time (sec)	N/A	0.107	0.147	0.009	0.	0.238	44.145	0.237	15.787

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	30	68	70	143	43	37
normalized size	1	1.	0.77	0.68	1.55	1.59	3.25	0.98	0.84
time (sec)	N/A	0.101	0.026	0.006	1.354	0.214	3.851	0.257	13.177

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	92	73	144	54	41
normalized size	1	1.	0.79	0.72	1.96	1.55	3.06	1.15	0.87
time (sec)	N/A	0.037	0.039	0.005	1.355	0.211	36.094	0.234	6.347

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	75	0	1	790	89	60
normalized size	1	1.	1.1	1.04	0.	0.01	10.97	1.24	0.83
time (sec)	N/A	0.155	0.26	0.012	0.	0.228	62.402	0.237	16.279

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	60	59	0	104	265	136	68
normalized size	1	1.	0.78	0.77	0.	1.35	3.44	1.77	0.88
time (sec)	N/A	0.093	0.063	0.007	0.	0.234	77.442	0.231	10.57

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	140	0	1	1608	136	99
normalized size	1	1.	1.01	1.24	0.	0.01	14.23	1.2	0.88
time (sec)	N/A	0.241	0.319	0.013	0.	0.251	119.557	0.236	21.394

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	79	82	0	136	524	302	99
normalized size	1	1.	0.73	0.76	0.	1.26	4.85	2.8	0.92
time (sec)	N/A	0.146	0.084	0.009	0.	0.255	141.831	0.245	13.961

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	136	187	0	1	0	223	143
normalized size	1	1.	0.91	1.25	0.	0.01	0.	1.49	0.95
time (sec)	N/A	0.295	0.346	0.018	0.	0.259	0.	0.253	25.375

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	105	107	0	174	0	454	141
normalized size	1	1.	0.72	0.73	0.	1.19	0.	3.11	0.97
time (sec)	N/A	0.176	0.114	0.01	0.	0.317	0.	0.247	18.927

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	132	149	0	242	389	248	144
normalized size	1	1.	0.84	0.95	0.	1.54	2.48	1.58	0.92
time (sec)	N/A	0.339	0.109	0.011	0.	0.221	8.713	0.229	44.665

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	108	0	189	308	192	100
normalized size	1	1.	0.87	0.95	0.	1.66	2.7	1.68	0.88
time (sec)	N/A	0.266	0.102	0.011	0.	0.216	4.912	0.237	31.965

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	69	0	139	226	130	66
normalized size	1	1.	0.87	0.9	0.	1.81	2.94	1.69	0.86
time (sec)	N/A	0.158	0.065	0.009	0.	0.215	2.353	0.227	23.066

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	105	100	0	1	153	136	82
normalized size	1	1.	1.14	1.09	0.	0.01	1.66	1.48	0.89
time (sec)	N/A	0.22	0.233	0.014	0.	0.239	15.198	0.236	24.599

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	103	132	0	1	148	120	94
normalized size	1	1.	0.94	1.21	0.	0.01	1.36	1.1	0.86
time (sec)	N/A	0.265	0.224	0.016	0.	0.24	31.666	0.23	24.731

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	140	130	207	0	1	219	207	128
normalized size	1	0.98	0.91	1.45	0.	0.01	1.53	1.45	0.9
time (sec)	N/A	0.407	0.298	0.016	0.	0.233	66.934	0.24	28.889

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	161	281	0	1	291	300	133
normalized size	1	1.	1.08	1.89	0.	0.01	1.95	2.01	0.89
time (sec)	N/A	0.383	0.206	0.018	0.	0.285	96.07	0.247	29.117

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	188	157	259	0	1	411	235	182
normalized size	1	0.98	0.82	1.36	0.	0.01	2.15	1.23	0.95
time (sec)	N/A	0.482	0.16	0.02	0.	0.325	57.138	0.247	40.269

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	122	190	0	1	291	173	143
normalized size	1	1.	0.82	1.28	0.	0.01	1.95	1.16	0.96
time (sec)	N/A	0.211	0.112	0.012	0.	0.258	34.623	0.246	23.32

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	130	99	163	0	1	219	170	114
normalized size	1	0.98	0.74	1.23	0.	0.01	1.65	1.28	0.86
time (sec)	N/A	0.235	0.184	0.016	0.	0.237	21.786	0.243	24.213

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	91	122	0	1	170	254	99
normalized size	1	1.	0.82	1.1	0.	0.01	1.53	2.29	0.89
time (sec)	N/A	0.198	0.143	0.016	0.	0.234	12.903	0.25	23.683

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	123	0	1	199	544	92
normalized size	1	1.	1.01	1.19	0.	0.01	1.93	5.28	0.89
time (sec)	N/A	0.188	0.131	0.017	0.	0.251	10.467	0.26	24.651

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	76	78	0	144	510	662	94
normalized size	1	1.01	0.77	0.79	0.	1.45	5.15	6.69	0.95
time (sec)	N/A	0.214	0.083	0.01	0.	0.308	10.697	0.253	22.091

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	108	117	0	198	1061	782	134
normalized size	1	1.01	0.76	0.82	0.	1.38	7.42	5.47	0.94
time (sec)	N/A	0.337	0.105	0.011	0.	0.477	15.848	0.269	26.973

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	190	141	158	0	250	1856	902	187
normalized size	1	1.01	0.75	0.84	0.	1.32	9.82	4.77	0.99
time (sec)	N/A	0.421	0.122	0.012	0.	0.8	24.067	0.266	32.614

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	278	225	389	0	1	0	355	270
normalized size	1	0.99	0.8	1.38	0.	0.	0.	1.26	0.96
time (sec)	N/A	0.669	0.244	0.03	0.	0.837	0.	0.247	49.216

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	100	108	0	242	384	437	100
normalized size	1	1.	0.88	0.95	0.	2.12	3.37	3.83	0.88
time (sec)	N/A	0.263	0.133	0.012	0.	0.222	13.352	0.239	32.159

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	232	193	321	0	1	505	296	224
normalized size	1	0.99	0.82	1.37	0.	0.	2.15	1.26	0.95
time (sec)	N/A	0.565	0.205	0.015	0.	0.497	138.387	0.23	47.126

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	69	0	190	303	315	66
normalized size	1	1.	0.87	0.9	0.	2.47	3.94	4.09	0.86
time (sec)	N/A	0.16	0.081	0.008	0.	0.217	7.627	0.232	23.178

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	159	249	0	1	440	236	192
normalized size	1	1.	0.81	1.27	0.	0.01	2.24	1.2	0.98
time (sec)	N/A	0.27	0.164	0.011	0.	0.327	88.159	0.236	27.378

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	116	115	0	1	172	163	99
normalized size	1	1.	1.05	1.04	0.	0.01	1.55	1.47	0.89
time (sec)	N/A	0.253	0.215	0.015	0.	0.241	26.904	0.24	27.487

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	172	135	221	0	1	367	234	155
normalized size	1	0.98	0.77	1.26	0.	0.01	2.1	1.34	0.89
time (sec)	N/A	0.297	0.238	0.016	0.	0.28	56.696	0.249	28.068

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	161	0	1	303	170	121
normalized size	1	1.	0.94	1.18	0.	0.01	2.23	1.25	0.89
time (sec)	N/A	0.315	0.271	0.016	0.	0.24	47.005	0.232	27.957

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	181	118	241	0	1	352	354	170
normalized size	1	0.98	0.64	1.31	0.	0.01	1.91	1.92	0.92
time (sec)	N/A	0.345	0.153	0.018	0.	0.259	33.911	0.249	28.38

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	178	150	256	0	1	332	246	167
normalized size	1	0.98	0.83	1.41	0.	0.01	1.83	1.36	0.92
time (sec)	N/A	0.497	0.284	0.018	0.	0.243	89.04	0.242	32.816

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	113	203	0	1	304	549	134
normalized size	1	1.	0.77	1.38	0.	0.01	2.07	3.73	0.91
time (sec)	N/A	0.252	0.187	0.018	0.	0.282	22.944	0.255	26.959

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	166	335	0	1	367	350	170
normalized size	1	0.98	0.89	1.79	0.	0.01	1.96	1.87	0.91
time (sec)	N/A	0.528	0.33	0.02	0.	0.239	134.915	0.242	34.2

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	108	0	292	468	1	100
normalized size	1	1.	0.87	0.95	0.	2.56	4.11	0.01	0.88
time (sec)	N/A	0.265	0.165	0.011	0.	0.233	31.548	0.239	32.608

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	278	226	383	0	1	0	358	270
normalized size	1	0.99	0.8	1.36	0.	0.	0.	1.27	0.96
time (sec)	N/A	0.653	0.249	0.023	0.	0.827	0.	0.242	53.602

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	69	0	240	384	564	66
normalized size	1	1.	0.87	0.9	0.	3.12	4.99	7.32	0.86
time (sec)	N/A	0.161	0.104	0.009	0.	0.233	19.903	0.237	23.527

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	192	308	0	1	537	298	238
normalized size	1	1.	0.8	1.28	0.	0.	2.24	1.24	0.99
time (sec)	N/A	0.329	0.205	0.012	0.	0.532	170.432	0.247	32.185

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	128	132	0	1	190	190	117
normalized size	1	1.	0.97	1.	0.	0.01	1.44	1.44	0.89
time (sec)	N/A	0.283	0.259	0.014	0.	0.25	48.622	0.248	31.318

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	174	278	0	1	496	296	199
normalized size	1	0.99	0.8	1.28	0.	0.	2.29	1.36	0.92
time (sec)	N/A	0.361	0.202	0.017	0.	0.363	117.643	0.248	32.438

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	156	193	0	1	518	223	146
normalized size	1	1.	0.96	1.19	0.	0.01	3.2	1.38	0.9
time (sec)	N/A	0.363	0.376	0.018	0.	0.259	71.787	0.246	31.966

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	219	155	298	0	1	490	414	218
normalized size	1	0.98	0.7	1.34	0.	0.	2.2	1.86	0.98
time (sec)	N/A	0.443	0.206	0.019	0.	0.354	74.201	0.246	32.012

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	219	186	305	0	1	473	327	207
normalized size	1	0.99	0.84	1.37	0.	0.	2.13	1.47	0.93
time (sec)	N/A	0.595	0.444	0.018	0.	0.251	116.474	0.246	38.043

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	225	158	369	0	1	474	689	216
normalized size	1	0.99	0.69	1.62	0.	0.	2.08	3.02	0.95
time (sec)	N/A	0.425	0.301	0.024	0.	0.344	48.125	0.251	33.58

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	219	186	387	0	1	468	386	211
normalized size	1	0.99	0.84	1.74	0.	0.	2.11	1.74	0.95
time (sec)	N/A	0.619	0.404	0.023	0.	0.279	166.573	0.25	38.297

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	159	265	0	1	422	240	187
normalized size	1	1.	0.82	1.37	0.	0.01	2.18	1.24	0.96
time (sec)	N/A	0.462	0.193	0.023	0.	0.375	63.904	0.242	36.456

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	108	0	139	240	203	99
normalized size	1	1.	0.88	0.96	0.	1.24	2.14	1.81	0.88
time (sec)	N/A	0.268	0.095	0.01	0.	0.249	5.108	0.229	31.988

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	125	197	0	1	301	182	138
normalized size	1	1.	0.86	1.35	0.	0.01	2.06	1.25	0.95
time (sec)	N/A	0.39	0.138	0.014	0.	0.294	39.84	0.236	34.084

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	69	0	92	158	132	65
normalized size	1	1.	0.89	0.93	0.	1.24	2.14	1.78	0.88
time (sec)	N/A	0.163	0.068	0.009	0.	0.244	3.108	0.232	23.112

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	91	131	0	1	238	123	102
normalized size	1	1.	0.85	1.22	0.	0.01	2.22	1.15	0.95
time (sec)	N/A	0.15	0.088	0.011	0.	0.264	20.389	0.242	20.384

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	76	87	0	1	167	111	66
normalized size	1	1.	1.01	1.16	0.	0.01	2.23	1.48	0.88
time (sec)	N/A	0.198	0.13	0.015	0.	0.259	21.71	0.236	22.457

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	88	0	1	155	126	70
normalized size	1	1.	0.93	1.07	0.	0.01	1.89	1.54	0.85
time (sec)	N/A	0.143	0.096	0.015	0.	0.258	10.679	0.249	21.122

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	92	100	0	1	99	109	68
normalized size	1	1.	1.15	1.25	0.	0.01	1.24	1.36	0.85
time (sec)	N/A	0.226	0.174	0.015	0.	0.26	42.817	0.232	22.14

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	85	0	1	158	211	75
normalized size	1	1.	0.86	1.01	0.	0.01	1.88	2.51	0.89
time (sec)	N/A	0.185	0.118	0.015	0.	0.313	5.539	0.253	22.214

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	126	157	0	1	178	189	95
normalized size	1	1.	1.19	1.48	0.	0.01	1.68	1.78	0.9
time (sec)	N/A	0.295	0.15	0.016	0.	0.261	74.437	0.239	24.515

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	74	78	0	99	391	421	92
normalized size	1	1.01	0.75	0.79	0.	1.	3.95	4.25	0.93
time (sec)	N/A	0.224	0.077	0.01	0.	0.257	9.157	0.242	22.924

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	168	224	0	1	301	325	138
normalized size	1	1.	1.11	1.48	0.	0.01	1.99	2.15	0.91
time (sec)	N/A	0.399	0.225	0.019	0.	0.294	130.835	0.234	29.733

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	158	263	0	1	0	236	189
normalized size	1	1.	0.8	1.34	0.	0.01	0.	1.2	0.96
time (sec)	N/A	0.417	0.257	0.025	0.	0.316	0.	0.241	45.97

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	108	0	155	236	180	97
normalized size	1	1.	0.9	1.	0.	1.44	2.19	1.67	0.9
time (sec)	N/A	0.262	0.103	0.01	0.	0.227	5.672	0.232	32.079

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	124	192	0	1	0	177	143
normalized size	1	1.	0.82	1.26	0.	0.01	0.	1.16	0.94
time (sec)	N/A	0.337	0.181	0.015	0.	0.251	0.	0.238	44.444

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	69	0	107	155	108	63
normalized size	1	1.	0.89	0.95	0.	1.47	2.12	1.48	0.86
time (sec)	N/A	0.158	0.075	0.008	0.	0.229	3.545	0.236	23.357

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	93	123	0	1	0	124	95
normalized size	1	1.	0.88	1.16	0.	0.01	0.	1.17	0.9
time (sec)	N/A	0.142	0.162	0.012	0.	0.234	0.	0.248	21.661

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	102	0	1	0	111	88
normalized size	1	1.	1.17	1.36	0.	0.01	0.	1.48	1.17
time (sec)	N/A	0.207	0.19	0.014	0.	0.238	0.	0.236	36.416

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	87	81	99	0	1	0	140	78
normalized size	1	0.96	0.89	1.09	0.	0.01	0.	1.54	0.86
time (sec)	N/A	0.185	0.138	0.014	0.	0.236	0.	0.244	21.818

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	135	0	1	0	189	92
normalized size	1	1.	1.02	1.31	0.	0.01	0.	1.83	0.89
time (sec)	N/A	0.284	0.25	0.016	0.	0.244	0.	0.259	24.658

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	66	77	0	115	0	269	90
normalized size	1	1.01	0.68	0.79	0.	1.19	0.	2.77	0.93
time (sec)	N/A	0.211	0.114	0.009	0.	0.236	0.	0.245	21.08

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	143	211	0	1	0	220	136
normalized size	1	1.	0.99	1.46	0.	0.01	0.	1.52	0.94
time (sec)	N/A	0.439	0.319	0.018	0.	0.243	0.	0.243	29.731

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	105	117	0	163	0	610	136
normalized size	1	1.	0.74	0.83	0.	1.16	0.	4.33	0.96
time (sec)	N/A	0.306	0.121	0.011	0.	0.287	0.	0.245	26.218

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	193	190	281	0	1	0	360	184
normalized size	1	1.01	0.99	1.46	0.	0.01	0.	1.88	0.96
time (sec)	N/A	0.526	0.559	0.02	0.	0.247	0.	0.241	35.483

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	156	255	0	1	0	257	190
normalized size	1	1.	0.77	1.26	0.	0.	0.	1.27	0.94
time (sec)	N/A	0.416	0.227	0.029	0.	0.309	0.	0.247	46.534

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	108	0	167	454	173	97
normalized size	1	1.	0.89	0.98	0.	1.52	4.13	1.57	0.88
time (sec)	N/A	0.267	0.108	0.01	0.	0.22	6.743	0.239	32.586

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	118	185	0	1	0	176	110
normalized size	1	1.	0.98	1.53	0.	0.01	0.	1.45	0.91
time (sec)	N/A	0.322	0.189	0.015	0.	0.256	0.	0.243	58.815

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	68	0	123	303	105	63
normalized size	1	1.	0.93	0.94	0.	1.71	4.21	1.46	0.88
time (sec)	N/A	0.159	0.076	0.008	0.	0.216	4.251	0.234	23.801

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	101	136	0	1	0	142	94
normalized size	1	1.	0.96	1.3	0.	0.01	0.	1.35	0.9
time (sec)	N/A	0.136	0.222	0.012	0.	0.236	0.	0.238	22.431

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	120	0	1	0	138	73
normalized size	1	1.	1.2	1.36	0.	0.01	0.	1.57	0.83
time (sec)	N/A	0.24	0.445	0.014	0.	0.231	0.	0.239	40.588

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	78	0	124	0	158	83
normalized size	1	1.	0.84	0.87	0.	1.38	0.	1.76	0.92
time (sec)	N/A	0.146	0.085	0.009	0.	0.23	0.	0.239	19.504

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	128	169	0	1	0	173	121
normalized size	1	1.	0.98	1.29	0.	0.01	0.	1.32	0.92
time (sec)	N/A	0.335	0.438	0.018	0.	0.239	0.	0.246	29.43

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	130	107	116	0	176	0	348	117
normalized size	1	0.99	0.82	0.89	0.	1.34	0.	2.66	0.89
time (sec)	N/A	0.324	0.114	0.01	0.	0.281	0.	0.237	24.849

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	179	265	0	1	0	284	175
normalized size	1	1.	0.97	1.43	0.	0.01	0.	1.54	0.95
time (sec)	N/A	0.533	0.479	0.02	0.	0.255	0.	0.252	35.673

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	142	158	0	231	0	687	180
normalized size	1	1.	0.78	0.86	0.	1.26	0.	3.75	0.98
time (sec)	N/A	0.418	0.16	0.011	0.	0.41	0.	0.253	30.005

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	0	1	0	95	70
normalized size	1	1.	0.78	0.74	0.	0.01	0.	1.32	0.97
time (sec)	N/A	0.066	0.04	0.01	0.	0.22	0.	0.236	22.348

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	38	0	1	0	62	51
normalized size	1	1.	0.85	0.73	0.	0.02	0.	1.19	0.98
time (sec)	N/A	0.045	0.026	0.008	0.	0.222	0.	0.237	17.419

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	24	0	1	0	31	39
normalized size	1	1.	1.	0.71	0.	0.03	0.	0.91	1.15
time (sec)	N/A	0.023	0.012	0.005	0.	0.225	0.	0.238	12.234

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	36	0	1	0	65	53
normalized size	1	1.	0.92	0.72	0.	0.02	0.	1.3	1.06
time (sec)	N/A	0.042	0.027	0.008	0.	0.244	0.	0.23	17.711

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	58	0	1	0	92	66
normalized size	1	1.	0.85	0.85	0.	0.01	0.	1.35	0.97
time (sec)	N/A	0.061	0.036	0.009	0.	0.245	0.	0.232	23.953

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	148	1088	0	1	0	254	143
normalized size	1	1.	0.94	6.93	0.	0.01	0.	1.62	0.91
time (sec)	N/A	0.608	0.359	0.022	0.	0.412	0.	0.242	66.781

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	963	0	1	0	130	73
normalized size	1	1.	0.97	10.94	0.	0.01	0.	1.48	0.83
time (sec)	N/A	0.235	0.189	0.02	0.	0.263	0.	0.227	26.702

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	1010	0	1	0	185	99
normalized size	1	1.	0.96	9.02	0.	0.01	0.	1.65	0.88
time (sec)	N/A	0.271	0.215	0.018	0.	0.332	0.	0.248	40.557

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	936	0	1	0	86	53
normalized size	1	1.	1.	14.4	0.	0.02	0.	1.32	0.82
time (sec)	N/A	0.148	0.052	0.015	0.	0.259	0.	0.239	20.018

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	948	0	1	0	151	70
normalized size	1	1.	1.04	11.7	0.	0.01	0.	1.86	0.86
time (sec)	N/A	0.12	0.054	0.014	0.	0.276	0.	0.244	21.162

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	229	984	0	1	0	117	66
normalized size	1	1.	2.86	12.3	0.	0.01	0.	1.46	0.82
time (sec)	N/A	0.207	0.602	0.017	0.	0.317	0.	0.233	26.017

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	1017	0	1	0	158	56
normalized size	1	1.	1.01	14.53	0.	0.01	0.	2.26	0.8
time (sec)	N/A	0.146	0.122	0.024	0.	0.26	0.	0.788	23.253

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	281	1054	0	1	0	163	97
normalized size	1	1.	2.49	9.33	0.	0.01	0.	1.44	0.86
time (sec)	N/A	0.362	1.646	0.018	0.	0.34	0.	0.253	46.908

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	1059	0	1	0	290	90
normalized size	1	1.	0.89	10.09	0.	0.01	0.	2.76	0.86
time (sec)	N/A	0.353	0.195	0.021	0.	0.311	0.	0.734	54.96

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	196	2081	0	1	0	358	197
normalized size	1	1.	0.93	9.91	0.	0.	0.	1.7	0.94
time (sec)	N/A	1.019	0.38	0.022	0.	2.467	0.	0.258	135.424

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	108	1897	0	1	0	204	97
normalized size	1	1.	0.94	16.5	0.	0.01	0.	1.77	0.84
time (sec)	N/A	0.296	0.206	0.019	0.	0.258	0.	0.239	34.361

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	1973	0	1	0	267	146
normalized size	1	1.	0.88	12.49	0.	0.01	0.	1.69	0.92
time (sec)	N/A	0.653	0.285	0.018	0.	0.786	0.	0.243	84.216

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	83	1856	0	1	0	151	75
normalized size	1	1.	0.91	20.4	0.	0.01	0.	1.66	0.82
time (sec)	N/A	0.195	0.151	0.017	0.	0.271	0.	0.235	26.227

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	110	1875	0	1	0	205	102
normalized size	1	1.	0.97	16.59	0.	0.01	0.	1.81	0.9
time (sec)	N/A	0.252	0.285	0.016	0.	0.425	0.	0.252	44.337

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	271	1919	0	1	0	158	80
normalized size	1	1.	2.82	19.99	0.	0.01	0.	1.65	0.83
time (sec)	N/A	0.344	0.799	0.018	0.	0.552	0.	0.24	41.831

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	105	1956	0	1	0	220	85
normalized size	1	1.	1.03	19.18	0.	0.01	0.	2.16	0.83
time (sec)	N/A	0.261	0.213	0.019	0.	0.38	0.	0.241	42.138

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	284	2003	0	1	0	182	100
normalized size	1	1.	2.49	17.57	0.	0.01	0.	1.6	0.88
time (sec)	N/A	0.412	1.339	0.02	0.	0.581	0.	0.239	46.414

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	90	2089	0	1	0	346	88
normalized size	1	1.	0.88	20.48	0.	0.01	0.	3.39	0.86
time (sec)	N/A	0.373	0.18	0.021	0.	0.287	0.	0.644	47.745

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	247	3373	0	1	0	483	0
normalized size	1	1.	0.85	11.59	0.	0.	0.	1.66	0.
time (sec)	N/A	1.436	0.411	0.032	0.	9.067	0.	0.26	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	298	3127	0	1	0	308	122
normalized size	1	1.	2.07	21.72	0.	0.01	0.	2.14	0.85
time (sec)	N/A	0.377	0.674	0.022	0.	0.294	0.	0.249	43.981

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	187	3235	0	1	0	373	207
normalized size	1	1.	0.86	14.91	0.	0.	0.	1.72	0.95
time (sec)	N/A	1.002	0.245	0.021	0.	3.013	0.	0.264	126.215

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	268	3078	0	1	0	248	99
normalized size	1	1.	2.25	25.87	0.	0.01	0.	2.08	0.83
time (sec)	N/A	0.257	0.57	0.019	0.	0.287	0.	0.236	34.998

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	140	3101	0	1	0	290	146
normalized size	1	1.	0.9	19.88	0.	0.01	0.	1.86	0.94
time (sec)	N/A	0.464	0.179	0.018	0.	1.342	0.	0.252	70.241

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	288	3148	0	1	0	227	105
normalized size	1	1.	2.32	25.39	0.	0.01	0.	1.83	0.85
time (sec)	N/A	0.551	0.555	0.021	0.	1.343	0.	0.236	63.028

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	132	3191	0	1	0	279	128
normalized size	1	1.	0.91	22.01	0.	0.01	0.	1.92	0.88
time (sec)	N/A	0.499	0.149	0.022	0.	0.911	0.	0.245	72.77

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	311	3247	0	1	0	231	126
normalized size	1	1.	2.16	22.55	0.	0.01	0.	1.6	0.88
time (sec)	N/A	0.639	0.654	0.022	0.	1.509	0.	0.241	69.741

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	125	3346	0	1	0	410	114
normalized size	1	1.	0.96	25.74	0.	0.01	0.	3.15	0.88
time (sec)	N/A	0.521	0.171	0.023	0.	0.7	0.	0.252	72.475

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	89	362	0	1	0	142	85
normalized size	1	1.	0.89	3.62	0.	0.01	0.	1.42	0.85
time (sec)	N/A	0.286	0.16	0.02	0.	0.257	0.	0.239	34.047

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	318	0	1	0	86	56
normalized size	1	1.	1.	4.68	0.	0.01	0.	1.26	0.82
time (sec)	N/A	0.193	0.071	0.017	0.	0.267	0.	0.234	21.686

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	300	0	1	0	53	41
normalized size	1	1.	1.	6.12	0.	0.02	0.	1.08	0.84
time (sec)	N/A	0.114	0.035	0.015	0.	0.245	0.	0.236	15.8

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	229	331	0	1	0	107	68
normalized size	1	1.	2.86	4.14	0.	0.01	0.	1.34	0.85
time (sec)	N/A	0.215	0.528	0.017	0.	0.317	0.	0.229	25.273

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	292	385	0	1	0	159	97
normalized size	1	1.	2.54	3.35	0.	0.01	0.	1.38	0.84
time (sec)	N/A	0.362	0.668	0.02	0.	0.394	0.	0.248	47.861

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	112	386	0	1	0	184	100
normalized size	1	1.	0.98	3.39	0.	0.01	0.	1.61	0.88
time (sec)	N/A	0.279	0.202	0.019	0.	0.384	0.	0.25	39.612

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	85	337	0	1	0	138	70
normalized size	1	1.	1.04	4.11	0.	0.01	0.	1.68	0.85
time (sec)	N/A	0.156	0.085	0.015	0.	0.355	0.	0.263	23.747

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	306	0	1	0	95	42
normalized size	1	1.	1.	6.24	0.	0.02	0.	1.94	0.86
time (sec)	N/A	0.058	0.038	0.006	0.	0.302	0.	0.247	11.152

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	334	0	1	0	150	61
normalized size	1	1.	1.	4.51	0.	0.01	0.	2.03	0.82
time (sec)	N/A	0.155	0.08	0.019	0.	0.324	0.	0.243	25.111

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	96	379	0	1	0	263	95
normalized size	1	1.	0.87	3.45	0.	0.01	0.	2.39	0.86
time (sec)	N/A	0.373	0.17	0.021	0.	0.324	0.	0.739	58.262

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	111	720	0	1	0	201	92
normalized size	1	1.	1.02	6.61	0.	0.01	0.	1.84	0.84
time (sec)	N/A	0.311	0.27	0.021	0.	0.466	0.	0.257	42.823

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	653	0	1	0	105	63
normalized size	1	1.	1.01	8.48	0.	0.01	0.	1.36	0.82
time (sec)	N/A	0.207	0.128	0.018	0.	0.254	0.	0.229	24.337

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	653	0	1	0	138	61
normalized size	1	1.	1.	8.82	0.	0.01	0.	1.86	0.82
time (sec)	N/A	0.148	0.133	0.017	0.	0.307	0.	0.235	24.082

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	618	0	1	0	96	61
normalized size	1	1.	1.	8.58	0.	0.01	0.	1.33	0.85
time (sec)	N/A	0.156	0.12	0.017	0.	0.262	0.	0.234	21.159

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	628	0	1	0	146	66
normalized size	1	1.	0.99	7.95	0.	0.01	0.	1.85	0.84
time (sec)	N/A	0.114	0.128	0.	0.	0.332	0.	0.239	20.066

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	316	681	0	1	0	158	87
normalized size	1	1.	2.95	6.36	0.	0.01	0.	1.48	0.81
time (sec)	N/A	0.347	2.327	0.018	0.	0.526	0.	0.232	48.03

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	102	695	0	1	0	0	100
normalized size	1	1.	0.82	5.6	0.	0.01	0.	0.	0.81
time (sec)	N/A	0.363	0.366	0.02	0.	0.359	0.	0.	58.233

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	355	763	0	1	0	248	136
normalized size	1	1.	2.28	4.89	0.	0.01	0.	1.59	0.87
time (sec)	N/A	0.647	4.387	0.021	0.	0.794	0.	0.249	77.492

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	124	762	0	1	0	373	151
normalized size	1	1.	0.7	4.33	0.	0.01	0.	2.12	0.86
time (sec)	N/A	0.685	0.364	0.025	0.	0.478	0.	1.896	96.118

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	94	1207	0	1	0	410	99
normalized size	1	1.	0.8	10.32	0.	0.01	0.	3.5	0.85
time (sec)	N/A	0.326	0.324	0.029	0.	0.571	0.	0.251	51.351

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	99	1123	0	1	0	171	85
normalized size	1	1.	0.96	10.9	0.	0.01	0.	1.66	0.83
time (sec)	N/A	0.264	0.317	0.02	0.	0.279	0.	0.244	31.666

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	103	1134	0	1	0	393	97
normalized size	1	1.	0.9	9.86	0.	0.01	0.	3.42	0.84
time (sec)	N/A	0.261	0.334	0.02	0.	0.502	0.	0.239	48.362

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	91	1086	0	1	0	159	82
normalized size	1	1.	0.93	11.08	0.	0.01	0.	1.62	0.84
time (sec)	N/A	0.191	0.243	0.018	0.	0.272	0.	0.232	28.724

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	111	1086	0	1	0	433	107
normalized size	1	1.	0.91	8.9	0.	0.01	0.	3.55	0.88
time (sec)	N/A	0.271	0.285	0.019	0.	0.565	0.	0.243	55.95

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	365	1186	0	1	0	242	121
normalized size	1	1.	2.52	8.18	0.	0.01	0.	1.67	0.83
time (sec)	N/A	0.588	1.811	0.019	0.	1.215	0.	0.236	76.255

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	143	1192	0	1	0	0	153
normalized size	1	1.	0.8	6.7	0.	0.01	0.	0.	0.86
time (sec)	N/A	0.686	0.435	0.025	0.	0.674	0.	0.	99.885

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	409	1289	0	1	0	294	189
normalized size	1	1.	1.94	6.11	0.	0.	0.	1.39	0.9
time (sec)	N/A	0.949	3.139	0.022	0.	2.319	0.	0.248	119.089

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	160	1285	0	1	0	0	224
normalized size	1	1.	0.65	5.24	0.	0.	0.	0.	0.91
time (sec)	N/A	1.088	0.707	0.027	0.	1.03	0.	0.	174.2

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	134	2615	0	1	0	4	134
normalized size	1	1.	0.89	17.43	0.	0.01	0.	0.03	0.89
time (sec)	N/A	0.49	0.294	0.024	0.	0.396	0.	0.609	65.897

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	91	2543	0	1	0	150	110
normalized size	1	1.	0.67	18.7	0.	0.01	0.	1.1	0.81
time (sec)	N/A	0.297	0.157	0.021	0.	0.274	0.	0.231	35.126

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	118	2547	0	1	0	4	105
normalized size	1	1.	0.98	21.22	0.	0.01	0.	0.03	0.88
time (sec)	N/A	0.256	0.134	0.02	0.	0.342	0.	0.569	39.684

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	1617	0	1	0	107	65
normalized size	1	1.	1.	20.21	0.	0.01	0.	1.34	0.81
time (sec)	N/A	0.162	0.085	0.017	0.	0.249	0.	0.238	20.629

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	2559	0	1	0	4	68
normalized size	1	1.	1.	31.21	0.	0.01	0.	0.05	0.83
time (sec)	N/A	0.098	0.149	0.02	0.	0.305	0.	2.17	18.539

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	313	2585	0	1	0	170	99
normalized size	1	1.	2.63	21.72	0.	0.01	0.	1.43	0.83
time (sec)	N/A	0.341	1.029	0.023	0.	0.361	0.	0.25	39.649

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	2618	0	1	0	4	95
normalized size	1	1.	0.89	23.17	0.	0.01	0.	0.04	0.84
time (sec)	N/A	0.335	0.127	0.025	0.	0.336	0.	2.938	42.718

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	343	2669	0	1	0	258	133
normalized size	1	1.	2.16	16.79	0.	0.01	0.	1.62	0.84
time (sec)	N/A	0.619	1.989	0.025	0.	0.442	0.	0.243	61.262

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	120	2667	0	1	0	487	129
normalized size	1	1.	0.82	18.14	0.	0.01	0.	3.31	0.88
time (sec)	N/A	0.597	0.248	0.026	0.	0.385	0.	7.39	77.589

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	192	4795	0	1	0	4	187
normalized size	1	1.	0.97	24.34	0.	0.01	0.	0.02	0.95
time (sec)	N/A	0.87	0.283	0.026	0.	0.554	0.	0.726	104.804

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	115	4673	0	1	0	234	136
normalized size	1	1.	0.71	28.67	0.	0.01	0.	1.44	0.83
time (sec)	N/A	0.366	0.287	0.024	0.	0.28	0.	0.227	42.809

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	155	4685	0	1	0	4	134
normalized size	1	1.	1.04	31.44	0.	0.01	0.	0.03	0.9
time (sec)	N/A	0.44	0.269	0.023	0.	0.402	0.	0.616	62.468

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	96	2821	0	1	0	161	85
normalized size	1	1.	0.97	28.49	0.	0.01	0.	1.63	0.86
time (sec)	N/A	0.203	0.211	0.018	0.	0.279	0.	0.274	25.619

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	141	4689	0	1	0	4	114
normalized size	1	1.	1.08	35.79	0.	0.01	0.	0.03	0.87
time (sec)	N/A	0.254	0.259	0.021	0.	0.374	0.	0.674	42.512

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	381	4718	0	1	0	223	109
normalized size	1	1.	2.95	36.57	0.	0.01	0.	1.73	0.84
time (sec)	N/A	0.411	0.635	0.024	0.	0.546	0.	0.221	45.201

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	100	4764	0	1	0	4	105
normalized size	1	1.	0.78	37.22	0.	0.01	0.	0.03	0.82
time (sec)	N/A	0.388	0.205	0.026	0.	0.281	0.	2.625	59.32

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	405	4820	0	1	0	300	153
normalized size	1	1.	2.38	28.35	0.	0.01	0.	1.76	0.9
time (sec)	N/A	0.695	0.839	0.026	0.	0.523	0.	0.225	83.375

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	131	4908	0	1	0	4	144
normalized size	1	1.	0.79	29.57	0.	0.01	0.	0.02	0.87
time (sec)	N/A	0.703	0.223	0.029	0.	0.36	0.	6.315	105.109

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	219	7611	0	1	0	4	246
normalized size	1	1.	0.85	29.5	0.	0.	0.	0.02	0.95
time (sec)	N/A	1.18	0.577	0.041	0.	3.033	0.	0.581	148.015

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	332	7443	0	1	0	356	170
normalized size	1	1.	1.68	37.59	0.	0.01	0.	1.8	0.86
time (sec)	N/A	0.464	0.929	0.027	0.	0.305	0.	0.229	53.152

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	173	7459	0	1	0	4	182
normalized size	1	1.	0.89	38.25	0.	0.01	0.	0.02	0.93
time (sec)	N/A	0.635	0.374	0.026	0.	1.429	0.	0.557	94.102

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	289	4363	0	1	0	257	110
normalized size	1	1.	2.29	34.63	0.	0.01	0.	2.04	0.87
time (sec)	N/A	0.267	0.674	0.023	0.	0.286	0.	0.228	33.167

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	143	7451	0	1	0	4	155
normalized size	1	1.	0.82	42.82	0.	0.01	0.	0.02	0.89
time (sec)	N/A	0.528	0.326	0.025	0.	0.986	0.	0.559	74.451

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	344	7477	0	1	0	290	138
normalized size	1	1.	2.15	46.73	0.	0.01	0.	1.81	0.86
time (sec)	N/A	0.627	0.804	0.03	0.	1.548	0.	0.234	68.921

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	150	7529	0	1	0	4	144
normalized size	1	1.	0.89	44.82	0.	0.01	0.	0.02	0.86
time (sec)	N/A	0.537	0.244	0.035	0.	0.712	0.	0.602	73.809

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	349	7590	0	1	0	390	155
normalized size	1	1.	1.94	42.17	0.	0.01	0.	2.17	0.86
time (sec)	N/A	0.72	1.185	0.031	0.	1.649	0.	0.238	72.755

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	121	7705	0	1	0	4	156
normalized size	1	1.	0.69	43.78	0.	0.01	0.	0.02	0.89
time (sec)	N/A	0.702	0.277	0.032	0.	0.356	0.	4.992	106.82

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	129	846	0	1	0	4	116
normalized size	1	1.	0.98	6.41	0.	0.01	0.	0.03	0.88
time (sec)	N/A	0.326	0.319	0.021	0.	0.615	0.	0.572	43.118

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	807	0	1	0	157	78
normalized size	1	1.	1.	8.15	0.	0.01	0.	1.59	0.79
time (sec)	N/A	0.245	0.127	0.019	0.	0.268	0.	0.236	26.565

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	817	0	1	0	4	73
normalized size	1	1.	0.97	9.18	0.	0.01	0.	0.04	0.82
time (sec)	N/A	0.159	0.152	0.019	0.	0.341	0.	2.009	24.484

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	513	0	1	0	124	70
normalized size	1	1.	0.97	5.9	0.	0.01	0.	1.43	0.8
time (sec)	N/A	0.172	0.148	0.017	0.	0.258	0.	0.239	21.73

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	823	0	1	0	304	83
normalized size	1	1.	1.	8.23	0.	0.01	0.	3.04	0.83
time (sec)	N/A	0.147	0.175	0.008	0.	0.366	0.	0.241	21.743

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	360	838	0	1	0	207	110
normalized size	1	1.	2.77	6.45	0.	0.01	0.	1.59	0.85
time (sec)	N/A	0.394	1.197	0.021	0.	0.728	0.	0.233	51.556

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	116	841	0	1	0	535	124
normalized size	1	1.	0.79	5.72	0.	0.01	0.	3.64	0.84
time (sec)	N/A	0.413	0.268	0.024	0.	0.404	0.	2.135	61.012

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	387	899	0	1	0	362	158
normalized size	1	1.	2.09	4.86	0.	0.01	0.	1.96	0.85
time (sec)	N/A	0.677	2.119	0.023	0.	1.23	0.	0.244	80.616

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	136	893	0	1	0	506	180
normalized size	1	1.	0.66	4.33	0.	0.	0.	2.46	0.87
time (sec)	N/A	0.742	0.41	0.026	0.	0.558	0.	4.631	128.861

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	106	1498	0	1	0	4	110
normalized size	1	1.	0.82	11.52	0.	0.01	0.	0.03	0.85
time (sec)	N/A	0.283	0.436	0.032	0.	0.504	0.	3.341	52.072

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	111	1456	0	1	0	244	107
normalized size	1	1.	0.83	10.87	0.	0.01	0.	1.82	0.8
time (sec)	N/A	0.308	0.285	0.02	0.	0.323	0.	0.236	36.702

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1453	0	1	0	4	104
normalized size	1	1.	0.89	11.81	0.	0.01	0.	0.03	0.85
time (sec)	N/A	0.271	0.246	0.022	0.	0.595	0.	3.354	45.001

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	989	0	1	0	203	97
normalized size	1	1.	0.9	8.75	0.	0.01	0.	1.8	0.86
time (sec)	N/A	0.223	0.32	0.019	0.	0.282	0.	0.222	29.21

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	120	1461	0	1	0	4	121
normalized size	1	1.	0.85	10.29	0.	0.01	0.	0.03	0.85
time (sec)	N/A	0.289	0.373	0.02	0.	0.668	0.	3.298	55.48

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	406	1672	0	1	0	319	144
normalized size	1	1.	2.39	9.84	0.	0.01	0.	1.88	0.85
time (sec)	N/A	0.654	1.841	0.023	0.	2.846	0.	0.237	82.179

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	145	1524	0	1	0	0	178
normalized size	1	1.	0.71	7.43	0.	0.	0.	0.	0.87
time (sec)	N/A	0.777	0.612	0.026	0.	0.772	0.	0.	135.396

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	451	1778	0	1	0	510	216
normalized size	1	1.	1.87	7.38	0.	0.	0.	2.12	0.9
time (sec)	N/A	1.024	3.814	0.025	0.	5.381	0.	0.239	123.639

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	167	1608	0	1	0	4	0
normalized size	1	1.	0.6	5.81	0.	0.	0.	0.01	0.
time (sec)	N/A	1.163	0.745	0.028	0.	1.258	0.	9.98	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	158	2463	0	1	0	4	151
normalized size	1	1.	0.91	14.16	0.	0.01	0.	0.02	0.87
time (sec)	N/A	0.578	0.451	0.036	0.	1.036	0.	5.018	88.841

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	137	2400	0	1	0	351	141
normalized size	1	1.	0.81	14.12	0.	0.01	0.	2.06	0.83
time (sec)	N/A	0.492	0.732	0.023	0.	0.406	0.	0.238	47.589

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	2369	0	1	0	803	143
normalized size	1	1.	1.	14.53	0.	0.01	0.	4.93	0.88
time (sec)	N/A	0.444	0.384	0.024	0.	1.186	0.	4.499	79.037

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	1639	0	1	0	301	122
normalized size	1	1.	0.98	11.71	0.	0.01	0.	2.15	0.87
time (sec)	N/A	0.274	0.353	0.02	0.	0.336	0.	0.228	38.696

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	170	2405	0	1	0	4	178
normalized size	1	1.	0.85	11.97	0.	0.	0.	0.02	0.89
time (sec)	N/A	0.543	0.737	0.024	0.	1.403	0.	4.868	117.102

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	461	2837	0	1	0	410	197
normalized size	1	1.	2.05	12.61	0.	0.	0.	1.82	0.88
time (sec)	N/A	1.028	2.412	0.026	0.	7.748	0.	0.24	124.364

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	188	2513	0	1	0	4	0
normalized size	1	1.	0.67	9.01	0.	0.	0.	0.01	0.
time (sec)	N/A	1.267	0.91	0.029	0.	1.764	0.	7.085	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	489	2980	0	1	0	684	0
normalized size	1	1.	1.61	9.8	0.	0.	0.	2.25	0.
time (sec)	N/A	1.47	6.33	0.028	0.	14.266	0.	0.259	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	210	2623	0	1	0	0	0
normalized size	1	1.	0.58	7.25	0.	0.	0.	0.	0.
time (sec)	N/A	1.664	1.134	0.032	0.	3.276	0.	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	159	276	0	0	97	0	199
normalized size	1	1.	0.75	1.3	0.	0.	0.46	0.	0.94
time (sec)	N/A	0.378	0.499	0.059	0.	0.	134.715	0.	34.736

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	234	414	0	0	95	0	308
normalized size	1	1.	0.69	1.23	0.	0.	0.28	0.	0.91
time (sec)	N/A	0.634	1.666	0.047	0.	0.	9.645	0.	62.874

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	132	246	0	0	97	0	158
normalized size	1	1.	0.75	1.4	0.	0.	0.55	0.	0.9
time (sec)	N/A	0.293	0.36	0.037	0.	0.	10.797	0.	27.97

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	186	391	0	0	100	0	309
normalized size	1	1.	0.56	1.17	0.	0.	0.3	0.	0.93
time (sec)	N/A	0.636	1.156	0.049	0.	0.	12.735	0.	63.985

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	120	234	0	0	100	0	156
normalized size	1	1.	0.7	1.36	0.	0.	0.58	0.	0.91
time (sec)	N/A	0.289	0.365	0.041	0.	0.	65.555	0.	28.832

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	217	417	0	0	0	0	313
normalized size	1	1.	0.64	1.23	0.	0.	0.	0.	0.93
time (sec)	N/A	0.646	0.895	0.049	0.	0.	0.	0.	65.625

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	139	242	0	0	0	0	139
normalized size	1	1.	0.91	1.59	0.	0.	0.	0.	0.91
time (sec)	N/A	0.23	0.248	0.058	0.	0.	0.	0.	20.348

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	237	439	0	0	0	0	309
normalized size	1	1.	0.72	1.33	0.	0.	0.	0.	0.93
time (sec)	N/A	0.533	0.442	0.064	0.	0.	0.	0.	50.707

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	163	270	0	0	0	0	177
normalized size	1	1.	0.87	1.44	0.	0.	0.	0.	0.95
time (sec)	N/A	0.278	0.32	0.046	0.	0.	0.	0.	25.431

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	178	300	0	0	0	0	230
normalized size	1	1.	0.71	1.19	0.	0.	0.	0.	0.91
time (sec)	N/A	0.431	0.585	0.037	0.	0.	0.	0.	41.808

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	214	438	0	0	197	0	352
normalized size	1	1.	0.57	1.16	0.	0.	0.52	0.	0.93
time (sec)	N/A	0.718	1.47	0.037	0.	0.	57.871	0.	72.712

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	155	272	0	0	199	0	192
normalized size	1	1.	0.72	1.27	0.	0.	0.93	0.	0.9
time (sec)	N/A	0.358	0.408	0.022	0.	0.	51.051	0.	34.239

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	206	421	0	0	202	0	343
normalized size	1	1.	0.56	1.15	0.	0.	0.55	0.	0.93
time (sec)	N/A	0.72	1.1	0.027	0.	0.	68.574	0.	73.966

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	140	255	0	0	202	0	197
normalized size	1	1.	0.67	1.21	0.	0.	0.96	0.	0.94
time (sec)	N/A	0.362	0.39	0.025	0.	0.	127.336	0.	35.271

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	232	422	0	0	0	0	337
normalized size	1	1.	0.64	1.16	0.	0.	0.	0.	0.92
time (sec)	N/A	0.702	1.028	0.027	0.	0.	0.	0.	75.06

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	237	417	0	0	0	0	318
normalized size	1	1.	0.7	1.23	0.	0.	0.	0.	0.94
time (sec)	N/A	0.65	1.517	0.041	0.	0.	0.	0.	64.976

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	134	250	0	0	94	0	162
normalized size	1	1.	0.77	1.44	0.	0.	0.54	0.	0.93
time (sec)	N/A	0.294	0.431	0.039	0.	0.	75.103	0.	28.614

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	181	379	0	0	92	0	277
normalized size	1	1.	0.61	1.27	0.	0.	0.31	0.	0.93
time (sec)	N/A	0.546	1.099	0.022	0.	0.	8.378	0.	53.926

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	116	214	0	0	94	0	124
normalized size	1	1.	0.83	1.54	0.	0.	0.68	0.	0.89
time (sec)	N/A	0.234	0.47	0.023	0.	0.	7.684	0.	22.135

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	193	378	0	0	97	0	265
normalized size	1	1.	0.67	1.3	0.	0.	0.33	0.	0.91
time (sec)	N/A	0.553	0.412	0.026	0.	0.	18.366	0.	55.463

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	118	223	0	0	97	0	126
normalized size	1	1.	0.86	1.62	0.	0.	0.7	0.	0.91
time (sec)	N/A	0.239	0.438	0.025	0.	0.	151.268	0.	23.175

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	221	417	0	0	0	0	321
normalized size	1	1.	0.65	1.22	0.	0.	0.	0.	0.94
time (sec)	N/A	0.638	0.486	0.027	0.	0.	0.	0.	68.68

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	168	252	0	0	0	0	199
normalized size	1	1.	0.8	1.19	0.	0.	0.	0.	0.94
time (sec)	N/A	0.364	0.29	0.053	0.	0.	0.	0.	36.77

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	229	391	0	0	0	0	316
normalized size	1	1.	0.68	1.16	0.	0.	0.	0.	0.94
time (sec)	N/A	0.645	1.445	0.05	0.	0.	0.	0.	65.604

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	143	225	0	0	0	0	160
normalized size	1	1.	0.82	1.29	0.	0.	0.	0.	0.92
time (sec)	N/A	0.298	0.23	0.027	0.	0.	0.	0.	29.389

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	216	382	0	0	94	0	270
normalized size	1	1.	0.72	1.27	0.	0.	0.31	0.	0.9
time (sec)	N/A	0.584	1.02	0.026	0.	0.	46.277	0.	56.752

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	133	213	0	0	94	0	126
normalized size	1	1.	0.92	1.48	0.	0.	0.65	0.	0.88
time (sec)	N/A	0.248	0.155	0.029	0.	0.	71.039	0.	23.753

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	202	386	0	0	0	0	303
normalized size	1	1.	0.61	1.16	0.	0.	0.	0.	0.91
time (sec)	N/A	0.642	0.401	0.031	0.	0.	0.	0.	66.839

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	146	232	0	0	0	0	163
normalized size	1	1.	0.83	1.32	0.	0.	0.	0.	0.93
time (sec)	N/A	0.303	0.224	0.032	0.	0.	0.	0.	30.44

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	233	417	0	0	0	0	359
normalized size	1	1.	0.61	1.1	0.	0.	0.	0.	0.95
time (sec)	N/A	0.729	0.626	0.033	0.	0.	0.	0.	79.654

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	163	439	0	0	0	0	194
normalized size	1	1.	0.78	2.11	0.	0.	0.	0.	0.93
time (sec)	N/A	0.359	0.393	0.056	0.	0.	0.	0.	38.181

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	249	767	0	0	0	0	313
normalized size	1	1.	0.71	2.2	0.	0.	0.	0.	0.9
time (sec)	N/A	0.664	1.026	0.064	0.	0.	0.	0.	69.787

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	429	0	0	0	0	163
normalized size	1	1.	0.88	2.32	0.	0.	0.	0.	0.88
time (sec)	N/A	0.307	0.32	0.028	0.	0.	0.	0.	32.089

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	247	764	0	0	0	0	304
normalized size	1	1.	0.72	2.22	0.	0.	0.	0.	0.88
time (sec)	N/A	0.642	0.941	0.027	0.	0.	0.	0.	67.182

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	164	425	0	0	0	0	163
normalized size	1	1.	0.88	2.27	0.	0.	0.	0.	0.87
time (sec)	N/A	0.313	0.255	0.032	0.	0.	0.	0.	31.292

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	182	771	0	0	0	0	342
normalized size	1	1.	0.48	2.05	0.	0.	0.	0.	0.91
time (sec)	N/A	0.719	0.539	0.034	0.	0.	0.	0.	78.19

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	166	446	0	0	0	0	197
normalized size	1	1.	0.78	2.09	0.	0.	0.	0.	0.92
time (sec)	N/A	0.369	0.446	0.035	0.	0.	0.	0.	37.832

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	225	448	0	0	0	0	277
normalized size	1	1.	0.78	1.56	0.	0.	0.	0.	0.96
time (sec)	N/A	0.688	0.446	0.095	0.	0.	0.	0.	57.615

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	282	658	0	0	148	0	405
normalized size	1	1.	0.66	1.55	0.	0.	0.35	0.	0.95
time (sec)	N/A	0.981	1.242	0.065	0.	0.	23.681	0.	89.41

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	189	401	0	0	150	0	228
normalized size	1	1.	0.77	1.64	0.	0.	0.61	0.	0.93
time (sec)	N/A	0.513	0.316	0.041	0.	0.	26.926	0.	48.939

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	260	624	0	0	153	0	396
normalized size	1	1.	0.62	1.48	0.	0.	0.36	0.	0.94
time (sec)	N/A	0.947	0.852	0.055	0.	0.	30.703	0.	94.065

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	171	383	0	0	153	0	216
normalized size	1	1.	0.73	1.64	0.	0.	0.65	0.	0.92
time (sec)	N/A	0.507	0.295	0.046	0.	0.	83.4	0.	51.04

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	226	648	0	0	0	0	394
normalized size	1	1.	0.54	1.54	0.	0.	0.	0.	0.94
time (sec)	N/A	0.964	1.4	0.074	0.	0.	0.	0.	94.137

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	210	160	385	0	0	0	0	199
normalized size	1	0.99	0.75	1.81	0.	0.	0.	0.	0.93
time (sec)	N/A	0.432	0.325	0.079	0.	0.	0.	0.	37.722

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	383	283	659	0	0	0	0	360
normalized size	1	0.99	0.73	1.71	0.	0.	0.	0.	0.93
time (sec)	N/A	0.768	0.846	0.084	0.	0.	0.	0.	65.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	213	187	403	0	0	0	0	204
normalized size	1	0.98	0.86	1.86	0.	0.	0.	0.	0.94
time (sec)	N/A	0.455	0.245	0.049	0.	0.	0.	0.	37.861

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	437	241	706	0	0	0	0	415
normalized size	1	0.99	0.55	1.6	0.	0.	0.	0.	0.94
time (sec)	N/A	0.887	0.702	0.054	0.	0.	0.	0.	74.456

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	304	743	0	0	0	1	510
normalized size	1	1.	0.57	1.4	0.	0.	0.	0.	0.96
time (sec)	N/A	1.289	2.044	0.054	0.	0.	0.	0.451	119.482

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	259	489	0	0	0	1	328
normalized size	1	1.	0.76	1.44	0.	0.	0.	0.	0.96
time (sec)	N/A	0.779	0.443	0.041	0.	0.	0.	0.339	67.083

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	316	699	0	0	304	1	464
normalized size	1	1.	0.66	1.45	0.	0.	0.63	0.	0.96
time (sec)	N/A	1.114	1.467	0.025	0.	0.	138.9	0.433	102.842

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	223	444	0	0	306	0	270
normalized size	1	1.	0.78	1.55	0.	0.	1.07	0.	0.94
time (sec)	N/A	0.659	0.378	0.023	0.	0.	150.981	0.	56.943

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	261	669	0	0	0	0	454
normalized size	1	1.	0.55	1.41	0.	0.	0.	0.	0.95
time (sec)	N/A	1.082	1.651	0.032	0.	0.	0.	0.	105.974

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	202	415	0	0	0	0	272
normalized size	1	1.	0.7	1.44	0.	0.	0.	0.	0.94
time (sec)	N/A	0.623	0.366	0.028	0.	0.	0.	0.	59.478

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	240	668	0	0	0	0	444
normalized size	1	1.	0.51	1.43	0.	0.	0.	0.	0.95
time (sec)	N/A	1.079	1.516	0.031	0.	0.	0.	0.	105.925

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	237	661	0	0	0	0	413
normalized size	1	1.	0.55	1.54	0.	0.	0.	0.	0.96
time (sec)	N/A	0.991	1.746	0.044	0.	0.	0.	0.	92.29

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	190	405	0	0	0	0	230
normalized size	1	1.	0.79	1.69	0.	0.	0.	0.	0.96
time (sec)	N/A	0.552	0.346	0.044	0.	0.	0.	0.	49.525

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	249	604	0	0	143	0	354
normalized size	1	1.	0.66	1.61	0.	0.	0.38	0.	0.94
time (sec)	N/A	0.864	1.075	0.027	0.	0.	16.971	0.	78.593

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	148	350	0	0	144	0	178
normalized size	1	1.	0.77	1.81	0.	0.	0.75	0.	0.92
time (sec)	N/A	0.414	0.374	0.028	0.	0.	18.392	0.	41.071

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	200	595	0	0	148	0	347
normalized size	1	1.	0.54	1.6	0.	0.	0.4	0.	0.93
time (sec)	N/A	0.826	1.704	0.032	0.	0.	25.124	0.	81.275

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	165	352	0	0	0	0	165
normalized size	1	1.	0.9	1.91	0.	0.	0.	0.	0.9
time (sec)	N/A	0.39	0.288	0.029	0.	0.	0.	0.	42.289

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	217	626	0	0	0	0	360
normalized size	1	1.	0.56	1.62	0.	0.	0.	0.	0.93
time (sec)	N/A	0.838	1.437	0.03	0.	0.	0.	0.	84.316

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	159	370	0	0	0	0	178
normalized size	1	1.	0.82	1.92	0.	0.	0.	0.	0.92
time (sec)	N/A	0.432	0.303	0.051	0.	0.	0.	0.	42.63

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	288	667	0	0	0	0	410
normalized size	1	1.	0.66	1.52	0.	0.	0.	0.	0.94
time (sec)	N/A	0.981	0.695	0.055	0.	0.	0.	0.	98.466

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	196	411	0	0	0	0	231
normalized size	1	1.	0.81	1.7	0.	0.	0.	0.	0.95
time (sec)	N/A	0.574	0.376	0.051	0.	0.	0.	0.	52.467

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	226	407	0	0	0	0	282
normalized size	1	1.	0.76	1.38	0.	0.	0.	0.	0.95
time (sec)	N/A	0.617	0.434	0.056	0.	0.	0.	0.	69.602

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	276	618	0	0	0	0	410
normalized size	1	1.	0.63	1.42	0.	0.	0.	0.	0.94
time (sec)	N/A	0.912	1.163	0.056	0.	0.	0.	0.	104.712

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	191	363	0	0	0	0	228
normalized size	1	1.	0.78	1.48	0.	0.	0.	0.	0.93
time (sec)	N/A	0.522	0.329	0.032	0.	0.	0.	0.	60.185

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	244	597	0	0	0	0	357
normalized size	1	1.	0.64	1.55	0.	0.	0.	0.	0.93
time (sec)	N/A	0.802	0.879	0.031	0.	0.	0.	0.	91.03

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	174	341	0	0	0	0	177
normalized size	1	1.	0.9	1.77	0.	0.	0.	0.	0.92
time (sec)	N/A	0.421	0.248	0.033	0.	0.	0.	0.	51.157

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	250	594	0	0	0	0	359
normalized size	1	1.	0.64	1.51	0.	0.	0.	0.	0.91
time (sec)	N/A	0.861	0.569	0.034	0.	0.	0.	0.	84.901

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	181	353	0	0	0	0	189
normalized size	1	1.	0.87	1.71	0.	0.	0.	0.	0.91
time (sec)	N/A	0.473	0.297	0.036	0.	0.	0.	0.	45.487

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	277	638	0	0	0	0	411
normalized size	1	1.	0.64	1.47	0.	0.	0.	0.	0.95
time (sec)	N/A	1.015	0.575	0.036	0.	0.	0.	0.	96.834

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	222	696	0	0	0	0	286
normalized size	1	1.	0.74	2.3	0.	0.	0.	0.	0.95
time (sec)	N/A	0.61	0.476	0.062	0.	0.	0.	0.	71.672

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	298	1191	0	0	0	0	413
normalized size	1	1.	0.67	2.69	0.	0.	0.	0.	0.93
time (sec)	N/A	0.923	1.37	0.059	0.	0.	0.	0.	106.014

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	204	674	0	0	0	0	226
normalized size	1	1.	0.82	2.72	0.	0.	0.	0.	0.91
time (sec)	N/A	0.506	0.424	0.032	0.	0.	0.	0.	61.548

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	281	1176	0	0	0	0	369
normalized size	1	1.	0.7	2.92	0.	0.	0.	0.	0.92
time (sec)	N/A	0.822	1.334	0.033	0.	0.	0.	0.	93.825

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	169	660	0	0	0	0	194
normalized size	1	1.	0.79	3.1	0.	0.	0.	0.	0.91
time (sec)	N/A	0.438	0.45	0.035	0.	0.	0.	0.	53.821

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	222	1187	0	0	0	0	406
normalized size	1	1.	0.5	2.69	0.	0.	0.	0.	0.92
time (sec)	N/A	1.012	0.799	0.039	0.	0.	0.	0.	100.651

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	211	686	0	0	0	0	236
normalized size	1	1.	0.82	2.66	0.	0.	0.	0.	0.91
time (sec)	N/A	0.582	0.448	0.037	0.	0.	0.	0.	56.411

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	246	1231	0	0	0	0	461
normalized size	1	1.	0.5	2.52	0.	0.	0.	0.	0.94
time (sec)	N/A	1.088	0.824	0.04	0.	0.	0.	0.	110.417

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	382	1479	0	0	0	0	0
normalized size	1	1.	1.03	3.98	0.	0.	0.	0.	0.
time (sec)	N/A	2.059	1.744	0.097	0.	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	418	1491	0	0	0	0	0
normalized size	1	1.	1.01	3.6	0.	0.	0.	0.	0.
time (sec)	N/A	2.195	1.201	0.065	0.	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	418	1286	0	0	0	0	0
normalized size	1	1.	1.33	4.08	0.	0.	0.	0.	0.
time (sec)	N/A	1.379	0.654	0.026	0.	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	164	701	0	0	0	0	0
normalized size	1	1.	0.45	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	1.522	0.345	0.024	0.	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	162	651	0	0	0	0	255
normalized size	1	1.	0.57	2.3	0.	0.	0.	0.	0.9
time (sec)	N/A	0.991	0.344	0.046	0.	0.	0.	0.	164.329

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	337	1274	0	0	0	0	0
normalized size	1	1.	0.86	3.25	0.	0.	0.	0.	0.
time (sec)	N/A	2.044	0.924	0.057	0.	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	338	1167	0	0	0	0	0
normalized size	1	1.	1.1	3.79	0.	0.	0.	0.	0.
time (sec)	N/A	1.36	1.037	0.05	0.	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	381	1553	0	0	0	0	0
normalized size	1	1.	0.83	3.4	0.	0.	0.	0.	0.
time (sec)	N/A	2.791	1.375	0.058	0.	0.	0.	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	378	2183	0	0	0	0	0
normalized size	1	1.	0.78	4.5	0.	0.	0.	0.	0.
time (sec)	N/A	2.942	1.177	0.05	0.	0.	0.	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	378	1920	0	0	0	0	0
normalized size	1	1.	1.02	5.16	0.	0.	0.	0.	0.
time (sec)	N/A	2.052	1.172	0.032	0.	0.	0.	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	427	1927	0	0	0	0	0
normalized size	1	1.	1.01	4.58	0.	0.	0.	0.	0.
time (sec)	N/A	2.253	0.811	0.03	0.	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	425	1721	0	0	0	0	0
normalized size	1	1.	1.3	5.25	0.	0.	0.	0.	0.
time (sec)	N/A	1.435	0.811	0.033	0.	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	436	1754	0	0	0	0	0
normalized size	1	1.	1.05	4.21	0.	0.	0.	0.	0.
time (sec)	N/A	2.189	0.792	0.036	0.	0.	0.	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	438	1740	0	0	0	0	0
normalized size	1	1.	1.33	5.27	0.	0.	0.	0.	0.
time (sec)	N/A	1.546	0.764	0.035	0.	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	380	2028	0	0	0	0	0
normalized size	1	1.	0.83	4.42	0.	0.	0.	0.	0.
time (sec)	N/A	2.921	1.329	0.038	0.	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	423	853	0	0	0	0	0
normalized size	1	1.	1.39	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.325	0.732	0.052	0.	0.	0.	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	165	472	0	0	0	0	0
normalized size	1	1.	0.47	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	1.471	0.224	0.049	0.	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	165	417	0	0	0	0	238
normalized size	1	1.	0.63	1.6	0.	0.	0.	0.	0.91
time (sec)	N/A	1.002	0.23	0.034	0.	0.	0.	0.	156.888

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	165	337	0	0	0	0	187
normalized size	1	1.	0.81	1.66	0.	0.	0.	0.	0.92
time (sec)	N/A	0.811	0.224	0.032	0.	0.	0.	0.	101.271

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	163	346	0	0	0	0	170
normalized size	1	1.	0.87	1.84	0.	0.	0.	0.	0.9
time (sec)	N/A	0.744	0.232	0.038	0.	0.	0.	0.	116.801

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	338	835	0	0	0	0	0
normalized size	1	1.	0.89	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	1.947	0.632	0.041	0.	0.	0.	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	338	740	0	0	0	0	0
normalized size	1	1.	1.14	2.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.306	0.562	0.04	0.	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	383	1109	0	0	0	0	0
normalized size	1	1.	0.86	2.5	0.	0.	0.	0.	0.
time (sec)	N/A	2.648	1.355	0.043	0.	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	424	1041	0	0	0	0	0
normalized size	1	1.	0.95	2.34	0.	0.	0.	0.	0.
time (sec)	N/A	2.208	0.722	0.066	0.	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	424	826	0	0	0	0	0
normalized size	1	1.	1.25	2.44	0.	0.	0.	0.	0.
time (sec)	N/A	1.433	0.767	0.061	0.	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	327	839	0	0	0	0	0
normalized size	1	1.	0.79	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	1.994	0.523	0.038	0.	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	328	704	0	0	0	0	0
normalized size	1	1.	1.04	2.24	0.	0.	0.	0.	0.
time (sec)	N/A	1.237	0.485	0.039	0.	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	356	830	0	0	0	0	0
normalized size	1	1.	0.85	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	2.069	0.798	0.038	0.	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	357	708	0	0	0	0	0
normalized size	1	1.	1.09	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	1.305	0.765	0.046	0.	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	401	1058	0	0	0	0	0
normalized size	1	1.	0.81	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	2.766	1.849	0.043	0.	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	413	896	0	0	0	0	0
normalized size	1	1.	1.04	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	2.111	1.605	0.046	0.	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	426	2561	0	0	0	0	0
normalized size	1	1.	1.18	7.07	0.	0.	0.	0.	0.
time (sec)	N/A	1.851	0.788	0.067	0.	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	318	2542	0	0	0	0	0
normalized size	1	1.	0.77	6.15	0.	0.	0.	0.	0.
time (sec)	N/A	2.043	0.363	0.064	0.	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	318	2255	0	0	0	0	0
normalized size	1	1.	0.97	6.88	0.	0.	0.	0.	0.
time (sec)	N/A	1.313	0.375	0.027	0.	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	317	2534	0	0	0	0	0
normalized size	1	1.	0.76	6.08	0.	0.	0.	0.	0.
time (sec)	N/A	2.027	0.305	0.028	0.	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	317	2251	0	0	0	0	0
normalized size	1	1.	0.95	6.72	0.	0.	0.	0.	0.
time (sec)	N/A	1.232	0.315	0.037	0.	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	340	2568	0	0	0	0	0
normalized size	1	1.	0.77	5.78	0.	0.	0.	0.	0.
time (sec)	N/A	2.529	1.007	0.041	0.	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	361	2316	0	0	0	0	0
normalized size	1	1.	1.02	6.52	0.	0.	0.	0.	0.
time (sec)	N/A	1.779	0.971	0.038	0.	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	392	3790	0	0	0	0	0
normalized size	1	1.	0.91	8.83	0.	0.	0.	0.	0.
time (sec)	N/A	2.543	1.062	0.042	0.	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	353	3886	0	0	0	0	0
normalized size	1	1.	0.73	8.01	0.	0.	0.	0.	0.
time (sec)	N/A	2.731	0.966	0.038	0.	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	353	3466	0	0	0	0	0
normalized size	1	1.	0.93	9.1	0.	0.	0.	0.	0.
time (sec)	N/A	1.837	0.996	0.038	0.	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	428	3858	0	0	0	0	0
normalized size	1	1.	0.9	8.14	0.	0.	0.	0.	0.
time (sec)	N/A	2.295	0.693	0.037	0.	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	428	2531	0	0	0	0	0
normalized size	1	1.	1.17	6.92	0.	0.	0.	0.	0.
time (sec)	N/A	1.49	0.648	0.04	0.	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	454	3879	0	0	0	0	0
normalized size	1	1.	0.87	7.47	0.	0.	0.	0.	0.
time (sec)	N/A	2.986	1.229	0.049	0.	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	453	3484	0	0	0	0	0
normalized size	1	1.	1.1	8.46	0.	0.	0.	0.	0.
time (sec)	N/A	2.167	1.201	0.043	0.	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	414	2956	0	0	0	0	0
normalized size	1	1.	0.86	6.11	0.	0.	0.	0.	0.
time (sec)	N/A	2.292	0.587	0.051	0.	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	414	2520	0	0	0	0	0
normalized size	1	1.	1.1	6.7	0.	0.	0.	0.	0.
time (sec)	N/A	1.466	0.598	0.044	0.	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	325	2548	0	0	0	0	0
normalized size	1	1.	0.71	5.54	0.	0.	0.	0.	0.
time (sec)	N/A	2.181	0.411	0.039	0.	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	325	2258	0	0	0	0	0
normalized size	1	1.	0.9	6.22	0.	0.	0.	0.	0.
time (sec)	N/A	1.397	0.386	0.038	0.	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	335	2545	0	0	0	0	0
normalized size	1	1.	0.72	5.48	0.	0.	0.	0.	0.
time (sec)	N/A	2.157	0.518	0.037	0.	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	336	2266	0	0	0	0	0
normalized size	1	1.	0.92	6.17	0.	0.	0.	0.	0.
time (sec)	N/A	1.365	0.521	0.043	0.	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	390	2982	0	0	0	0	0
normalized size	1	1.	0.73	5.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.912	1.591	0.051	0.	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	390	2622	0	0	0	0	0
normalized size	1	1.	0.91	6.11	0.	0.	0.	0.	0.
time (sec)	N/A	2.125	1.476	0.048	0.	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	432	2964	0	0	0	0	0
normalized size	1	1.	0.82	5.6	0.	0.	0.	0.	0.
time (sec)	N/A	2.95	0.985	0.046	0.	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	422	2530	0	0	0	0	0
normalized size	1	1.	1.	6.02	0.	0.	0.	0.	0.
time (sec)	N/A	1.829	0.93	0.046	0.	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	339	2561	0	0	0	0	0
normalized size	1	1.	0.7	5.28	0.	0.	0.	0.	0.
time (sec)	N/A	2.702	1.012	0.044	0.	0.	0.	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	2277	0	0	0	0	0
normalized size	1	1.	0.87	5.82	0.	0.	0.	0.	0.
time (sec)	N/A	1.712	0.816	0.043	0.	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	482	2950	0	0	0	0	0
normalized size	1	1.	0.91	5.56	0.	0.	0.	0.	0.
time (sec)	N/A	2.96	1.325	0.049	0.	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	472	2554	0	0	0	0	0
normalized size	1	1.	1.11	6.	0.	0.	0.	0.	0.
time (sec)	N/A	1.919	1.321	0.056	0.	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	476	3385	0	0	0	0	0
normalized size	1	1.	0.76	5.39	0.	0.	0.	0.	0.
time (sec)	N/A	3.765	2.557	0.053	0.	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	476	2871	0	0	0	0	0
normalized size	1	1.	0.93	5.61	0.	0.	0.	0.	0.
time (sec)	N/A	2.85	2.489	0.06	0.	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	522	5126	0	0	0	0	0
normalized size	1	1.	0.92	9.02	0.	0.	0.	0.	0.
time (sec)	N/A	3.71	1.486	0.09	0.	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	520	4403	0	0	0	0	0
normalized size	1	1.	1.15	9.7	0.	0.	0.	0.	0.
time (sec)	N/A	2.275	1.434	0.082	0.	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	568	5078	0	0	0	0	0
normalized size	1	1.	1.03	9.22	0.	0.	0.	0.	0.
time (sec)	N/A	3.565	1.88	0.063	0.	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	547	4403	0	0	0	0	0
normalized size	1	1.	1.22	9.85	0.	0.	0.	0.	0.
time (sec)	N/A	2.368	1.646	0.06	0.	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	626	5689	0	0	0	0	0
normalized size	1	1.	1.	9.1	0.	0.	0.	0.	0.
time (sec)	N/A	3.878	2.429	0.066	0.	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	629	4776	0	0	0	0	0
normalized size	1	1.	1.22	9.29	0.	0.	0.	0.	0.
time (sec)	N/A	2.482	2.795	0.067	0.	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	735	735	582	6334	0	0	0	1	0
normalized size	1	1.	0.79	8.62	0.	0.	0.	0.	0.
time (sec)	N/A	4.751	3.593	0.079	0.	0.	0.	1.243	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	606	606	582	5248	0	0	0	1	0
normalized size	1	1.	0.96	8.66	0.	0.	0.	0.	0.
time (sec)	N/A	3.825	3.756	0.079	0.	0.	0.	1.251	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	174	532	0	1	0	304	194
normalized size	1	1.	0.83	2.55	0.	0.	0.	1.45	0.93
time (sec)	N/A	0.598	0.183	0.072	0.	0.274	0.	0.253	42.99

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	339	0	1	0	207	119
normalized size	1	1.	0.92	2.47	0.	0.01	0.	1.51	0.87
time (sec)	N/A	0.322	0.126	0.02	0.	0.263	0.	0.248	27.86

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	198	0	1	0	143	73
normalized size	1	1.	1.17	2.3	0.	0.01	0.	1.66	0.85
time (sec)	N/A	0.177	0.087	0.014	0.	0.25	0.	0.25	18.16

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	238	177	0	1	0	208	83
normalized size	1	1.	2.59	1.92	0.	0.01	0.	2.26	0.9
time (sec)	N/A	0.274	0.555	0.036	0.	0.356	0.	0.25	26.649

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	188	207	0	1	0	0	76
normalized size	1	1.	2.11	2.33	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.251	0.42	0.043	0.	0.278	0.	0.	20.945

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	224	355	0	1	0	0	126
normalized size	1	1.	1.57	2.48	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.376	0.388	0.04	0.	0.335	0.	0.	29.428

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	246	526	0	0	0	0	308
normalized size	1	1.	0.72	1.53	0.	0.	0.	0.	0.9
time (sec)	N/A	0.856	0.76	0.029	0.	0.	0.	0.	99.332

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	199	335	0	0	0	0	219
normalized size	1	1.	0.77	1.29	0.	0.	0.	0.	0.85
time (sec)	N/A	0.495	0.44	0.025	0.	0.	0.	0.	63.018

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	111	168	0	0	0	0	197
normalized size	1	1.	0.48	0.72	0.	0.	0.	0.	0.85
time (sec)	N/A	0.433	0.478	0.038	0.	0.	0.	0.	57.479

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	228	418	0	0	0	0	269
normalized size	1	1.	0.74	1.36	0.	0.	0.	0.	0.88
time (sec)	N/A	0.772	1.89	0.038	0.	0.	0.	0.	98.688

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	224	770	0	1	0	410	260
normalized size	1	1.	0.81	2.79	0.	0.	0.	1.49	0.94
time (sec)	N/A	0.763	0.261	0.046	0.	0.3	0.	0.254	61.635

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	163	532	0	1	0	293	163
normalized size	1	1.	0.87	2.84	0.	0.01	0.	1.57	0.87
time (sec)	N/A	0.429	0.153	0.024	0.	0.273	0.	0.26	39.467

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	119	337	0	1	0	201	110
normalized size	1	1.	0.95	2.7	0.	0.01	0.	1.61	0.88
time (sec)	N/A	0.244	0.093	0.018	0.	0.283	0.	0.247	25.99

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	400	287	0	1	0	284	119
normalized size	1	1.	3.01	2.16	0.	0.01	0.	2.14	0.89
time (sec)	N/A	0.44	0.999	0.021	0.	0.888	0.	0.25	42.339

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	327	298	0	1	0	4	122
normalized size	1	1.	2.4	2.19	0.	0.01	0.	0.03	0.9
time (sec)	N/A	0.441	0.513	0.023	0.	0.745	0.	0.603	42.962

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	208	352	0	1	0	0	117
normalized size	1	1.	1.59	2.69	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.349	0.421	0.029	0.	0.339	0.	0.	29.051

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	305	782	0	0	0	0	398
normalized size	1	1.	0.71	1.82	0.	0.	0.	0.	0.93
time (sec)	N/A	1.401	1.099	0.029	0.	0.	0.	0.	162.498

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	245	544	0	0	0	0	309
normalized size	1	1.	0.73	1.62	0.	0.	0.	0.	0.92
time (sec)	N/A	0.929	0.764	0.027	0.	0.	0.	0.	106.902

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	206	352	0	0	0	0	211
normalized size	1	1.	0.84	1.44	0.	0.	0.	0.	0.86
time (sec)	N/A	0.485	0.452	0.025	0.	0.	0.	0.	63.941

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	227	433	0	0	0	0	279
normalized size	1	1.	0.73	1.39	0.	0.	0.	0.	0.9
time (sec)	N/A	0.825	0.605	0.027	0.	0.	0.	0.	106.261

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	274	1054	0	1	0	536	323
normalized size	1	1.	0.81	3.1	0.	0.	0.	1.58	0.95
time (sec)	N/A	0.931	0.306	0.05	0.	0.307	0.	0.265	81.789

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	204	770	0	1	0	397	212
normalized size	1	1.	0.86	3.25	0.	0.	0.	1.68	0.89
time (sec)	N/A	0.536	0.208	0.027	0.	0.296	0.	0.256	54.326

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	152	529	0	1	0	284	146
normalized size	1	1.	0.93	3.23	0.	0.01	0.	1.73	0.89
time (sec)	N/A	0.331	0.141	0.022	0.	0.271	0.	0.249	35.839

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	357	446	0	1	0	354	173
normalized size	1	1.	1.91	2.39	0.	0.01	0.	1.89	0.93
time (sec)	N/A	0.674	0.944	0.026	0.	2.732	0.	0.262	65.934

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	358	423	0	1	0	4	167
normalized size	1	1.	1.91	2.26	0.	0.01	0.	0.02	0.89
time (sec)	N/A	0.683	0.836	0.025	0.	2.128	0.	0.619	71.37

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	359	464	0	1	0	4	180
normalized size	1	1.	1.87	2.42	0.	0.01	0.	0.02	0.94
time (sec)	N/A	0.636	0.929	0.024	0.	1.782	0.	1.885	64.181

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	379	1047	0	0	0	0	0
normalized size	1	1.	0.69	1.89	0.	0.	0.	0.	0.
time (sec)	N/A	1.854	3.056	0.037	0.	0.	0.	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	306	782	0	0	0	0	410
normalized size	1	1.	0.7	1.79	0.	0.	0.	0.	0.94
time (sec)	N/A	1.277	1.101	0.03	0.	0.	0.	0.	149.728

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	254	568	0	0	0	0	308
normalized size	1	1.	0.77	1.72	0.	0.	0.	0.	0.93
time (sec)	N/A	0.781	0.789	0.027	0.	0.	0.	0.	99.076

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	261	583	0	0	0	0	304
normalized size	1	1.	0.78	1.74	0.	0.	0.	0.	0.9
time (sec)	N/A	0.826	0.815	0.026	0.	0.	0.	0.	99.716

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	135	0	0	0	0	92
normalized size	1	1.	0.93	1.36	0.	0.	0.	0.	0.93
time (sec)	N/A	0.298	0.144	0.031	0.	0.	0.	0.	34.841

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	81	62	78	0	54	75
normalized size	1	1.	0.92	1.25	0.95	1.2	0.	0.83	1.15
time (sec)	N/A	0.147	0.068	0.035	1.517	0.241	0.	0.229	14.324

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	86	129	0	0	0	0	61
normalized size	1	1.	1.23	1.84	0.	0.	0.	0.	0.87
time (sec)	N/A	0.162	0.145	0.017	0.	0.	0.	0.	25.616

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	60	36	69	66	45	49
normalized size	1	1.	0.95	1.54	0.92	1.77	1.69	1.15	1.26
time (sec)	N/A	0.089	0.03	0.013	1.497	0.233	10.969	0.233	11.111

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	127	306	0	0	0	0	218
normalized size	1	1.	0.53	1.27	0.	0.	0.	0.	0.9
time (sec)	N/A	0.445	0.19	0.025	0.	0.	0.	0.	51.665

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	340	0	1	0	212	128
normalized size	1	1.	0.96	2.41	0.	0.01	0.	1.5	0.91
time (sec)	N/A	0.42	0.183	0.046	0.	0.277	0.	0.245	33.477

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	103	200	0	1	0	140	75
normalized size	1	1.	1.17	2.27	0.	0.01	0.	1.59	0.85
time (sec)	N/A	0.237	0.088	0.024	0.	0.264	0.	0.242	21.986

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	63	103	0	1	0	73	41
normalized size	1	1.	1.4	2.29	0.	0.02	0.	1.62	0.91
time (sec)	N/A	0.121	0.036	0.02	0.	0.244	0.	0.241	14.841

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	153	103	0	1	0	120	42
normalized size	1	1.	3.33	2.24	0.	0.02	0.	2.61	0.91
time (sec)	N/A	0.173	0.083	0.026	0.	0.26	0.	0.229	16.622

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	192	209	0	1	0	558	78
normalized size	1	1.	2.11	2.3	0.	0.01	0.	6.13	0.86
time (sec)	N/A	0.269	0.322	0.03	0.	0.293	0.	0.261	22.445

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	224	355	0	1	0	0	131
normalized size	1	1.	1.5	2.38	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.431	0.379	0.034	0.	0.324	0.	0.	46.084

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	249	546	0	0	0	0	313
normalized size	1	1.	0.73	1.6	0.	0.	0.	0.	0.92
time (sec)	N/A	0.854	0.827	0.036	0.	0.	0.	0.	97.822

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	201	333	0	0	0	0	226
normalized size	1	1.	0.77	1.28	0.	0.	0.	0.	0.87
time (sec)	N/A	0.502	0.475	0.033	0.	0.	0.	0.	63.58

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	122	129	0	0	0	0	97
normalized size	1	1.	1.05	1.11	0.	0.	0.	0.	0.84
time (sec)	N/A	0.169	0.112	0.027	0.	0.	0.	0.	22.446

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	146	224	0	0	0	0	126
normalized size	1	1.	0.95	1.46	0.	0.	0.	0.	0.82
time (sec)	N/A	0.286	0.529	0.031	0.	0.	0.	0.	39.373

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	229	435	0	0	0	0	274
normalized size	1	1.	0.75	1.42	0.	0.	0.	0.	0.89
time (sec)	N/A	0.757	0.596	0.034	0.	0.	0.	0.	105.076

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	129	511	0	1	0	4	112
normalized size	1	1.	1.	3.96	0.	0.01	0.	0.03	0.87
time (sec)	N/A	0.42	0.25	0.054	0.	0.377	0.	0.56	35.851

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	101	292	0	1	0	4	71
normalized size	1	1.	1.22	3.52	0.	0.01	0.	0.05	0.86
time (sec)	N/A	0.232	0.121	0.031	0.	0.338	0.	0.593	21.141

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	30	0	65	0	95	26
normalized size	1	1.	0.97	0.88	0.	1.91	0.	2.79	0.76
time (sec)	N/A	0.098	0.042	0.007	0.	0.253	0.	0.247	10.162

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	133	609	0	1	0	4	124
normalized size	1	1.	0.97	4.45	0.	0.01	0.	0.03	0.91
time (sec)	N/A	0.374	0.329	0.054	0.	0.402	0.	0.599	38.348

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	63	0	173	0	289	71
normalized size	1	1.	0.61	0.71	0.	1.94	0.	3.25	0.8
time (sec)	N/A	0.218	0.088	0.009	0.	0.267	0.	0.257	21.166

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	60	0	170	0	174	61
normalized size	1	1.	0.7	0.81	0.	2.3	0.	2.35	0.82
time (sec)	N/A	0.148	0.063	0.007	0.	0.271	0.	0.24	15.309

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	91	119	0	354	0	806	141
normalized size	1	1.	0.59	0.77	0.	2.3	0.	5.23	0.92
time (sec)	N/A	0.472	0.153	0.012	0.	0.327	0.	0.281	43.939

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	91	125	0	363	0	637	116
normalized size	1	1.	0.66	0.91	0.	2.63	0.	4.62	0.84
time (sec)	N/A	0.295	0.12	0.013	0.	0.335	0.	0.285	31.027

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	83	113	0	350	0	328	97
normalized size	1	1.	0.73	1.	0.	3.1	0.	2.9	0.86
time (sec)	N/A	0.196	0.102	0.01	0.	0.335	0.	0.265	23.191

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	151	213	0	609	0	1	204
normalized size	1	1.	0.7	0.98	0.	2.81	0.	0.	0.94
time (sec)	N/A	0.648	0.216	0.016	0.	0.494	0.	0.316	62.371

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	72	108	0	1	0	77	41
normalized size	1	1.	1.53	2.3	0.	0.02	0.	1.64	0.87
time (sec)	N/A	0.126	0.126	0.057	0.	0.242	0.	0.24	14.808

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	67	111	0	1	0	77	42
normalized size	1	1.	1.4	2.31	0.	0.02	0.	1.6	0.88
time (sec)	N/A	0.134	0.065	0.053	0.	0.244	0.	0.241	19.146

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	72	70	0	0	0	0	99
normalized size	1	1.	0.65	0.64	0.	0.	0.	0.	0.9
time (sec)	N/A	0.157	0.072	0.028	0.	0.	0.	0.	19.501

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	59	59	0	0	0	0	71
normalized size	1	1.	0.68	0.68	0.	0.	0.	0.	0.82
time (sec)	N/A	0.234	0.083	0.024	0.	0.	0.	0.	39.336

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	70	76	0	0	0	0	75
normalized size	1	1.	0.8	0.86	0.	0.	0.	0.	0.85
time (sec)	N/A	0.126	0.072	0.023	0.	0.	0.	0.	17.839

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	31	0	0	0	0	29
normalized size	1	1.	0.77	1.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.107	0.044	0.022	0.	0.	0.	0.	16.401

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	37	29	0	0	0	0	26
normalized size	1	1.	1.19	0.94	0.	0.	0.	0.	0.84
time (sec)	N/A	0.11	0.051	0.018	0.	0.	0.	0.	16.798

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	35	0	0	0	0	29
normalized size	1	1.	0.8	1.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.106	0.043	0.031	0.	0.	0.	0.	17.268

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	45	0	0	0	0	26
normalized size	1	1.	1.09	1.29	0.	0.	0.	0.	0.74
time (sec)	N/A	0.11	0.047	0.031	0.	0.	0.	0.	16.559

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	35	0	0	0	0	32
normalized size	1	1.	0.8	1.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.109	0.046	0.029	0.	0.	0.	0.	18.214

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	33	0	0	0	0	29
normalized size	1	1.	0.8	0.94	0.	0.	0.	0.	0.83
time (sec)	N/A	0.112	0.045	0.033	0.	0.	0.	0.	16.583

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	47	0	0	0	0	42
normalized size	1	1.	0.88	1.12	0.	0.	0.	0.	1.
time (sec)	N/A	0.101	0.043	0.02	0.	0.	0.	0.	15.639

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	47	0	0	0	0	46
normalized size	1	1.	0.88	1.09	0.	0.	0.	0.	1.07
time (sec)	N/A	0.101	0.042	0.027	0.	0.	0.	0.	15.501

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	47	0	0	0	0	42
normalized size	1	1.	0.85	1.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.111	0.046	0.027	0.	0.	0.	0.	16.676

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	34	36	0	0	0	0	70
normalized size	1	1.	0.42	0.45	0.	0.	0.	0.	0.88
time (sec)	N/A	0.092	0.04	0.019	0.	0.	0.	0.	13.604

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	38	34	0	0	0	0	68
normalized size	1	1.	0.46	0.41	0.	0.	0.	0.	0.83
time (sec)	N/A	0.095	0.04	0.023	0.	0.	0.	0.	13.834

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	48	0	0	0	0	70
normalized size	1	1.	0.57	0.55	0.	0.	0.	0.	0.8
time (sec)	N/A	0.107	0.043	0.025	0.	0.	0.	0.	13.736

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	34	0	0	0	0	14
normalized size	1	1.	2.18	2.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.102	0.052	0.018	0.	0.	0.	0.	18.797

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	63	0	146	135	0	0	100
normalized size	1	1.	0.58	0.	1.34	1.24	0.	0.	0.92
time (sec)	N/A	0.221	0.06	0.063	1.493	0.235	0.	0.	13.586

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	58	0	131	165	0	0	88
normalized size	1	1.	0.62	0.	1.39	1.76	0.	0.	0.94
time (sec)	N/A	0.165	0.043	0.057	1.503	0.234	0.	0.	11.505

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	84	0	116	109	0	0	73
normalized size	1	1.	1.06	0.	1.47	1.38	0.	0.	0.92
time (sec)	N/A	0.12	0.067	0.039	1.508	0.236	0.	0.	8.781

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	111	0	0	257	0	0	121
normalized size	1	1.	0.82	0.	0.	1.89	0.	0.	0.89
time (sec)	N/A	0.232	0.213	0.057	0.	0.24	0.	0.	13.653

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	115	0	0	159	0	0	85
normalized size	1	1.	1.19	0.	0.	1.64	0.	0.	0.88
time (sec)	N/A	0.183	0.203	0.064	0.	0.233	0.	0.	13.01

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	215	0	0	319	0	0	148
normalized size	1	1.	1.25	0.	0.	1.85	0.	0.	0.86
time (sec)	N/A	0.353	0.293	0.081	0.	0.243	0.	0.	22.755

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	236	0	0	0	0	0	19
normalized size	1	1.	0.44	0.	0.	0.	0.	0.	0.04
time (sec)	N/A	0.621	0.335	0.069	0.	0.	0.	0.	7.599

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	120	0	0	0	0	0	19
normalized size	1	1.	0.23	0.	0.	0.	0.	0.	0.04
time (sec)	N/A	0.448	0.139	0.05	0.	0.	0.	0.	8.708

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	0	0	0	144
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	1.27
time (sec)	N/A	0.061	0.073	0.	0.	0.	0.	0.	12.832

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	243	0	0	0	0	0	20
normalized size	1	1.	0.45	0.	0.	0.	0.	0.	0.04
time (sec)	N/A	0.599	0.275	0.069	0.	0.	0.	0.	8.22

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	245	0	0	0	0	0	24
normalized size	1	1.	0.44	0.	0.	0.	0.	0.	0.04
time (sec)	N/A	0.811	0.268	0.075	0.	0.	0.	0.	8.06

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	82	0	170	174	0	0	121
normalized size	1	1.	0.62	0.	1.28	1.31	0.	0.	0.91
time (sec)	N/A	0.28	0.079	0.069	1.508	0.236	0.	0.	15.405

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	77	0	155	204	0	0	105
normalized size	1	1.	0.66	0.	1.34	1.76	0.	0.	0.91
time (sec)	N/A	0.229	0.061	0.064	1.522	0.238	0.	0.	13.917

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	0	140	158	0	0	94
normalized size	1	1.	0.69	0.	1.39	1.56	0.	0.	0.93
time (sec)	N/A	0.184	0.047	0.064	1.499	0.237	0.	0.	11.417

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	0	140	173	0	0	87
normalized size	1	1.	0.69	0.	1.39	1.71	0.	0.	0.86
time (sec)	N/A	0.159	0.043	0.054	1.52	0.236	0.	0.	10.122

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-2)	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	205	0	0	332	0	0	141
normalized size	1	1.	1.3	0.	0.	2.1	0.	0.	0.89
time (sec)	N/A	0.315	0.153	0.082	0.	0.238	0.	0.	18.459

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-2)	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	213	0	0	343	0	0	156
normalized size	1	1.	1.16	0.	0.	1.87	0.	0.	0.85
time (sec)	N/A	0.388	0.28	0.087	0.	0.242	0.	0.	22.732

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-2)	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	234	0	0	383	0	0	177
normalized size	1	1.	1.12	0.	0.	1.84	0.	0.	0.85
time (sec)	N/A	0.459	0.286	0.095	0.	0.241	0.	0.	26.949

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	231	0	0	0	0	0	19
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.03
time (sec)	N/A	0.62	0.224	0.093	0.	0.	0.	0.	7.281

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	231	0	0	0	0	0	19
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.03
time (sec)	N/A	0.63	0.203	0.084	0.	0.	0.	0.	8.431

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	231	0	0	0	0	0	456
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.595	0.205	0.063	0.	0.	0.	0.	39.682

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	241	0	0	0	0	0	20
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.04
time (sec)	N/A	0.804	0.28	0.079	0.	0.	0.	0.	7.828

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	246	0	0	0	0	0	24
normalized size	1	1.	0.42	0.	0.	0.	0.	0.	0.04
time (sec)	N/A	0.974	0.406	0.076	0.	0.	0.	0.	7.602

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	76	0	204	351	0	216	175
normalized size	1	1.	0.56	0.	1.5	2.58	0.	1.59	1.29
time (sec)	N/A	0.26	0.104	0.109	1.496	0.254	0.	0.248	35.457

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	71	0	189	342	0	189	162
normalized size	1	1.	0.59	0.	1.56	2.83	0.	1.56	1.34
time (sec)	N/A	0.194	0.076	0.09	1.507	0.258	0.	0.247	32.968

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	0	174	335	0	174	144
normalized size	1	1.	0.62	0.	1.64	3.16	0.	1.64	1.36
time (sec)	N/A	0.146	0.047	0.086	1.501	0.268	0.	0.241	30.931

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	100	0	159	250	0	159	128
normalized size	1	1.	1.1	0.	1.75	2.75	0.	1.75	1.41
time (sec)	N/A	0.059	0.072	0.052	1.51	0.271	0.	0.237	27.695

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	140	0	0	344	0	292	178
normalized size	1	1.	0.97	0.	0.	2.37	0.	2.01	1.23
time (sec)	N/A	0.233	0.239	0.077	0.	0.261	0.	0.258	35.692

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	263	0	0	400	0	259	204
normalized size	1	1.	1.61	0.	0.	2.45	0.	1.59	1.25
time (sec)	N/A	0.335	0.441	0.119	0.	0.272	0.	0.276	41.603

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	273	0	0	0	0	0	27
normalized size	1	1.	1.66	0.	0.	0.	0.	0.	0.16
time (sec)	N/A	0.205	0.352	0.098	0.	0.	0.	0.	8.296

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	140	0	0	0	0	0	27
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.18
time (sec)	N/A	0.164	0.056	0.066	0.	0.	0.	0.	9.36

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	135	0	0	0	0	0	88
normalized size	1	1.	1.12	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.061	0.06	0.	0.	0.	0.	0.	75.848

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	152	0	0	0	0	0	29
normalized size	1	1.	0.92	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.189	0.174	0.098	0.	0.	0.	0.	8.851

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	156	0	0	0	0	0	32
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.243	0.248	0.115	0.	0.	0.	0.	8.986

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	0	100	86	0	101	70
normalized size	1	1.	0.95	0.	1.28	1.1	0.	1.29	0.9
time (sec)	N/A	0.159	0.082	0.12	1.499	0.235	0.	0.242	17.313

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	69	0	85	77	0	86	56
normalized size	1	1.	1.1	0.	1.35	1.22	0.	1.37	0.89
time (sec)	N/A	0.147	0.065	0.098	1.507	0.235	0.	0.239	16.196

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	0	70	70	0	72	42
normalized size	1	1.	0.71	0.	1.46	1.46	0.	1.5	0.88
time (sec)	N/A	0.118	0.029	0.095	1.499	0.235	0.	0.234	13.965

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	0	55	55	48	57	26
normalized size	1	1.	1.67	0.	1.67	1.67	1.45	1.73	0.79
time (sec)	N/A	0.077	0.015	0.074	1.498	0.238	4.191	0.239	11.238

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	137	0	0	284	0	209	148
normalized size	1	1.	0.79	0.	0.	1.64	0.	1.21	0.86
time (sec)	N/A	0.335	0.217	0.095	0.	0.25	0.	0.24	31.731

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	252	0	0	335	0	228	173
normalized size	1	1.	1.32	0.	0.	1.75	0.	1.19	0.91
time (sec)	N/A	0.403	0.37	0.137	0.	0.252	0.	0.246	37.562

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	257	0	0	0	0	0	41
normalized size	1	1.	1.05	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.434	0.362	0.108	0.	0.	0.	0.	21.459

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	132	0	0	0	0	0	41
normalized size	1	1.	0.59	0.	0.	0.	0.	0.	0.18
time (sec)	N/A	0.277	0.054	0.086	0.	0.	0.	0.	24.745

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	127	0	0	139	0	0	168
normalized size	1	1.	2.08	0.	0.	2.28	0.	0.	2.75
time (sec)	N/A	0.041	0.048	0.	0.	2.667	0.	0.	51.547

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	144	0	0	0	0	0	42
normalized size	1	1.	0.59	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.32	0.179	0.104	0.	0.	0.	0.	23.788

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	148	0	0	0	0	0	46
normalized size	1	1.	0.56	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.523	0.229	0.131	0.	0.	0.	0.	23.875

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	142	0	0	351	0	0	31
normalized size	1	1.	1.1	0.	0.	2.72	0.	0.	0.24
time (sec)	N/A	0.097	0.234	0.043	0.	0.241	0.	0.	9.652

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	142	0	0	351	0	0	27
normalized size	1	1.	1.18	0.	0.	2.92	0.	0.	0.22
time (sec)	N/A	0.095	0.223	0.08	0.	0.242	0.	0.	10.446

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	150	0	0	532	0	0	31
normalized size	1	1.	1.21	0.	0.	4.29	0.	0.	0.25
time (sec)	N/A	0.119	0.23	0.07	0.	0.246	0.	0.	10.337

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	151	0	0	545	0	0	27
normalized size	1	1.	1.27	0.	0.	4.58	0.	0.	0.23
time (sec)	N/A	0.129	0.222	0.074	0.	0.247	0.	0.	11.632

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	162	0	0	212	0	0	53
normalized size	1	1.	1.35	0.	0.	1.77	0.	0.	0.44
time (sec)	N/A	0.109	0.273	0.077	0.	0.242	0.	0.	32.786

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	162	0	0	212	0	0	49
normalized size	1	1.	1.35	0.	0.	1.77	0.	0.	0.41
time (sec)	N/A	0.105	0.263	0.076	0.	0.241	0.	0.	33.652

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	171	0	0	251	0	0	53
normalized size	1	1.	1.49	0.	0.	2.18	0.	0.	0.46
time (sec)	N/A	0.133	0.288	0.071	0.	0.24	0.	0.	36.214

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	168	0	0	257	0	0	49
normalized size	1	1.	1.41	0.	0.	2.16	0.	0.	0.41
time (sec)	N/A	0.136	0.29	0.076	0.	0.239	0.	0.	40.372

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	76	0	204	277	0	216	175
normalized size	1	1.	0.4	0.	1.09	1.47	0.	1.15	0.93
time (sec)	N/A	0.496	0.096	0.082	1.499	0.243	0.	0.243	34.95

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	74	0	189	270	0	189	162
normalized size	1	1.	0.43	0.	1.09	1.56	0.	1.09	0.94
time (sec)	N/A	0.425	0.08	0.072	1.511	0.244	0.	0.249	33.057

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	66	0	174	261	0	174	141
normalized size	1	1.	0.42	0.	1.1	1.65	0.	1.1	0.89
time (sec)	N/A	0.373	0.053	0.07	1.537	0.241	0.	0.241	31.412

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	100	0	159	282	0	159	128
normalized size	1	1.	0.7	0.	1.11	1.97	0.	1.11	0.9
time (sec)	N/A	0.266	0.076	0.062	1.509	0.24	0.	0.237	27.322

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	139	0	0	381	0	284	178
normalized size	1	1.	0.71	0.	0.	1.93	0.	1.44	0.9
time (sec)	N/A	0.447	0.282	0.072	0.	0.248	0.	0.258	34.284

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	136	0	0	440	0	259	204
normalized size	1	1.	0.63	0.	0.	2.05	0.	1.2	0.95
time (sec)	N/A	0.559	0.311	0.091	0.	0.249	0.	0.263	40.118

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	282	0	0	0	0	0	27
normalized size	1	1.	1.55	0.	0.	0.	0.	0.	0.15
time (sec)	N/A	0.337	0.531	0.083	0.	0.	0.	0.	8.666

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	277	0	0	0	0	0	27
normalized size	1	1.	1.69	0.	0.	0.	0.	0.	0.16
time (sec)	N/A	0.282	0.195	0.081	0.	0.	0.	0.	8.608

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	142	0	0	351	0	0	27
normalized size	1	1.	1.18	0.	0.	2.92	0.	0.	0.22
time (sec)	N/A	0.091	0.072	0.	0.	0.238	0.	0.	9.763

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	0	0	0	0	0	87
normalized size	1	1.	0.93	0.	0.	0.	0.	0.	0.59
time (sec)	N/A	0.134	0.186	0.065	0.	0.	0.	0.	81.78

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	140	0	0	0	0	0	29
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.267	0.253	0.083	0.	0.	0.	0.	9.005

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	142	0	0	0	0	0	32
normalized size	1	1.	0.77	0.	0.	0.	0.	0.	0.17
time (sec)	N/A	0.326	0.273	0.087	0.	0.	0.	0.	9.199

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	134	0	0	115	0	0	41
normalized size	1	1.	2.2	0.	0.	1.89	0.	0.	0.67
time (sec)	N/A	0.065	0.244	0.099	0.	0.227	0.	0.	25.704

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	134	0	0	158	0	0	46
normalized size	1	1.	2.2	0.	0.	2.59	0.	0.	0.75
time (sec)	N/A	0.069	0.229	0.056	0.	0.224	0.	0.	24.98

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	138	0	0	1	0	0	41
normalized size	1	1.	1.92	0.	0.	0.01	0.	0.	0.57
time (sec)	N/A	0.091	0.297	0.07	0.	0.234	0.	0.	28.762

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	143	0	0	1	0	0	46
normalized size	1	1.	1.93	0.	0.	0.01	0.	0.	0.62
time (sec)	N/A	0.099	0.243	0.069	0.	0.234	0.	0.	29.579

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	164	0	0	185	0	0	49
normalized size	1	1.	1.93	0.	0.	2.18	0.	0.	0.58
time (sec)	N/A	0.102	0.307	0.102	0.	0.233	0.	0.	33.717

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	164	0	0	185	0	0	54
normalized size	1	1.	1.93	0.	0.	2.18	0.	0.	0.64
time (sec)	N/A	0.096	0.277	0.075	0.	0.236	0.	0.	34.36

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	169	0	0	252	0	0	49
normalized size	1	1.	1.76	0.	0.	2.62	0.	0.	0.51
time (sec)	N/A	0.132	0.347	0.084	0.	0.233	0.	0.	38.338

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	174	0	0	258	0	0	54
normalized size	1	1.	1.78	0.	0.	2.63	0.	0.	0.55
time (sec)	N/A	0.133	0.293	0.09	0.	0.232	0.	0.	39.11

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	0	100	86	0	101	70
normalized size	1	1.	0.95	0.	1.28	1.1	0.	1.29	0.9
time (sec)	N/A	0.16	0.088	0.106	1.506	0.226	0.	0.237	15.856

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	69	0	85	80	0	86	56
normalized size	1	1.	1.1	0.	1.35	1.27	0.	1.37	0.89
time (sec)	N/A	0.147	0.071	0.093	1.501	0.225	0.	0.249	14.536

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	0	70	70	0	72	42
normalized size	1	1.	0.71	0.	1.46	1.46	0.	1.5	0.88
time (sec)	N/A	0.115	0.028	0.095	1.508	0.227	0.	0.237	12.291

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	0	55	55	0	57	27
normalized size	1	1.	1.67	0.	1.67	1.67	0.	1.73	0.82
time (sec)	N/A	0.07	0.013	0.083	1.507	0.225	0.	0.234	9.606

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	139	0	0	284	0	209	148
normalized size	1	1.	0.8	0.	0.	1.64	0.	1.21	0.86
time (sec)	N/A	0.333	0.282	0.091	0.	0.239	0.	0.241	28.401

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	136	0	0	335	0	228	173
normalized size	1	1.	0.71	0.	0.	1.75	0.	1.19	0.91
time (sec)	N/A	0.4	0.303	0.14	0.	0.242	0.	0.241	35.03

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	266	0	0	0	0	0	41
normalized size	1	1.	1.61	0.	0.	0.	0.	0.	0.25
time (sec)	N/A	0.437	0.498	0.109	0.	0.	0.	0.	21.682

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	261	0	0	0	0	0	41
normalized size	1	1.	1.78	0.	0.	0.	0.	0.	0.28
time (sec)	N/A	0.348	0.217	0.105	0.	0.	0.	0.	21.495

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	134	0	0	115	0	0	41
normalized size	1	1.	2.2	0.	0.	1.89	0.	0.	0.67
time (sec)	N/A	0.07	0.073	0.	0.	0.22	0.	0.	24.929

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	129	0	0	0	0	0	170
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	1.34
time (sec)	N/A	0.143	0.16	0.098	0.	0.	0.	0.	49.153

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	0	0	0	0	0	42
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.28
time (sec)	N/A	0.307	0.279	0.107	0.	0.	0.	0.	22.158

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	134	0	0	0	0	0	46
normalized size	1	1.	0.81	0.	0.	0.	0.	0.	0.28
time (sec)	N/A	0.404	0.278	0.114	0.	0.	0.	0.	22.611

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	97	0	0	0	0	0	165
normalized size	1	1.	0.56	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.36	0.132	0.065	0.	0.	0.	0.	35.499

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	80	0	0	0	92	0	124
normalized size	1	1.	0.59	0.	0.	0.	0.68	0.	0.91
time (sec)	N/A	0.271	0.097	0.046	0.	0.	5.852	0.	29.174

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	77	0	0	0	85	0	104
normalized size	1	1.	0.68	0.	0.	0.	0.75	0.	0.92
time (sec)	N/A	0.237	0.071	0.05	0.	0.	40.781	0.	28.329

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	82	58	0	0	63
normalized size	1	1.	0.66	0.58	1.22	0.87	0.	0.	0.94
time (sec)	N/A	0.115	0.066	0.008	1.435	0.226	0.	0.	12.135

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	72	62	130	89	0	0	99
normalized size	1	1.	0.69	0.6	1.25	0.86	0.	0.	0.95
time (sec)	N/A	0.17	0.1	0.01	1.423	0.224	0.	0.	17.692

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	94	86	176	122	0	0	136
normalized size	1	1.	0.67	0.61	1.25	0.87	0.	0.	0.96
time (sec)	N/A	0.222	0.119	0.009	1.432	0.223	0.	0.	23.416

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	123	0	0	0	0	0	165
normalized size	1	1.	0.68	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.409	0.157	0.057	0.	0.	0.	0.	40.839

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	97	0	0	0	94	0	124
normalized size	1	1.	0.7	0.	0.	0.	0.68	0.	0.89
time (sec)	N/A	0.32	0.116	0.044	0.	0.	59.314	0.	33.194

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	77	0	0	0	78	0	90
normalized size	1	1.	0.75	0.	0.	0.	0.76	0.	0.88
time (sec)	N/A	0.264	0.094	0.05	0.	0.	10.564	0.	28.389

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	84	0	0	0	0	0	99
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.271	0.106	0.055	0.	0.	0.	0.	29.87

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	107	0	0	0	0	0	134
normalized size	1	1.	0.74	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.337	0.186	0.057	0.	0.	0.	0.	35.922

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	132	0	0	0	0	0	173
normalized size	1	1.	0.73	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.402	0.218	0.062	0.	0.	0.	0.	43.248

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	84	0	0	999	94	0	158
normalized size	1	1.	0.49	0.	0.	5.84	0.55	0.	0.92
time (sec)	N/A	0.305	0.122	0.111	0.	0.257	162.9	0.	33.332

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	71	0	0	482	83	0	110
normalized size	1	1.	0.58	0.	0.	3.95	0.68	0.	0.9
time (sec)	N/A	0.203	0.067	0.066	0.	0.25	60.757	0.	26.304

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	39	0	57	0	0	61
normalized size	1	1.	0.67	0.58	0.	0.85	0.	0.	0.91
time (sec)	N/A	0.114	0.067	0.008	0.	0.223	0.	0.	13.067

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	71	62	0	88	0	0	97
normalized size	1	1.	0.68	0.6	0.	0.85	0.	0.	0.93
time (sec)	N/A	0.165	0.105	0.009	0.	0.218	0.	0.	17.545

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	94	86	0	122	0	0	134
normalized size	1	1.	0.67	0.61	0.	0.87	0.	0.	0.95
time (sec)	N/A	0.219	0.16	0.009	0.	0.218	0.	0.	23.334

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	111	0	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	0.165	0.108	0.	0.	0.	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	84	0	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.125	0.088	0.	0.	0.	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	0	0	0	94	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.95	0.	0.
time (sec)	N/A	0.167	0.096	0.055	0.	0.	32.652	0.	0.

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.121	0.094	0.	0.	0.	0.	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	114	0	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.178	0.103	0.	0.	0.	0.	0.

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	143	0	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	0.236	0.109	0.	0.	0.	0.	0.

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	85	0	0	0	0	0	158
normalized size	1	1.	0.46	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.373	0.143	0.084	0.	0.	0.	0.	36.135

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	73	0	0	0	87	0	112
normalized size	1	1.	0.58	0.	0.	0.	0.7	0.	0.9
time (sec)	N/A	0.244	0.066	0.054	0.	0.	173.65	0.	29.349

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	40	0	76	0	0	58
normalized size	1	1.	0.68	0.62	0.	1.17	0.	0.	0.89
time (sec)	N/A	0.117	0.057	0.009	0.	0.242	0.	0.	11.879

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	66	62	0	109	0	0	97
normalized size	1	1.	0.63	0.6	0.	1.05	0.	0.	0.93
time (sec)	N/A	0.171	0.1	0.009	0.	0.238	0.	0.	17.105

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	89	86	0	142	0	0	134
normalized size	1	1.	0.63	0.61	0.	1.01	0.	0.	0.95
time (sec)	N/A	0.22	0.149	0.01	0.	0.252	0.	0.	22.723

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	110	0	0	0	0	0	167
normalized size	1	1.	0.57	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.419	0.145	0.087	0.	0.	0.	0.	40.781

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	85	0	0	0	0	0	124
normalized size	1	1.	0.56	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.335	0.113	0.077	0.	0.	0.	0.	35.031

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	0	0	0	126
normalized size	1	1.	0.68	0.	0.	0.	0.	0.	1.09
time (sec)	N/A	0.275	0.099	0.064	0.	0.	0.	0.	34.722

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	91	0	0	0	0	0	131
normalized size	1	1.	0.63	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.328	0.138	0.095	0.	0.	0.	0.	36.275

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	121	0	0	0	0	0	170
normalized size	1	1.	0.67	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.394	0.241	0.096	0.	0.	0.	0.	42.58

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	116	0	0	1108	0	0	194
normalized size	1	1.	0.52	0.	0.	5.01	0.	0.	0.88
time (sec)	N/A	0.392	0.187	0.103	0.	0.284	0.	0.	41.555

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	96	0	0	564	0	0	138
normalized size	1	1.	0.64	0.	0.	3.79	0.	0.	0.93
time (sec)	N/A	0.247	0.128	0.053	0.	0.271	0.	0.	33.322

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	84	0	0	90
normalized size	1	1.	0.56	0.49	0.	1.06	0.	0.	1.14
time (sec)	N/A	0.128	0.054	0.007	0.	0.239	0.	0.	16.339

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	65	62	0	107	0	0	99
normalized size	1	1.	0.62	0.6	0.	1.03	0.	0.	0.95
time (sec)	N/A	0.17	0.09	0.01	0.	0.246	0.	0.	17.435

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	94	86	0	142	0	0	133
normalized size	1	1.	0.67	0.61	0.	1.01	0.	0.	0.94
time (sec)	N/A	0.218	0.149	0.009	0.	0.246	0.	0.	22.813

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	115	110	0	174	0	0	170
normalized size	1	1.	0.65	0.62	0.	0.98	0.	0.	0.96
time (sec)	N/A	0.277	0.21	0.01	0.	0.245	0.	0.	29.058

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	139	0	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.404	0.233	0.104	0.	0.	0.	0.	0.

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	116	0	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.322	0.195	0.085	0.	0.	0.	0.	0.

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	107	0	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.179	0.053	0.	0.	0.	0.	0.

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	111	0	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.158	0.05	0.	0.	0.	0.	0.

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	120	0	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	0.184	0.088	0.	0.	0.	0.	0.

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	140	0	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	0.244	0.097	0.	0.	0.	0.	0.

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	171	0	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	0.462	0.107	0.	0.	0.	0.	0.

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	218	0	0	0	0	0	78
normalized size	1	1.	2.16	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.234	0.606	0.206	0.	0.	0.	0.	32.798

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	176	0	0	0	0	0	65
normalized size	1	1.	2.1	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.222	0.36	0.095	0.	0.	0.	0.	29.176

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	174	0	0	0	0	0	65
normalized size	1	1.	2.07	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.216	0.374	0.087	0.	0.	0.	0.	35.481

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	61
normalized size	1	1.	2.18	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.118	0.113	0.	0.	0.	0.	0.	29.119

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	171	0	0	0	0	0	65
normalized size	1	1.	2.09	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.203	0.355	0.086	0.	0.	0.	0.	29.365

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	173	0	0	0	0	0	70
normalized size	1	1.	2.06	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.208	0.457	0.089	0.	0.	0.	0.	28.93

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	160	0	0	0	0	0	206
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.718	0.442	0.097	0.	0.	0.	0.	101.978

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	159	0	0	0	0	0	117
normalized size	1	1.	1.09	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.318	0.41	0.088	0.	0.	0.	0.	46.587

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	65
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.16	0.116	0.08	0.	0.	0.	0.	27.031

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	225	0	0	0	0	0	73
normalized size	1	1.	2.32	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.246	0.427	0.081	0.	0.	0.	0.	29.526

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	225	0	0	0	0	0	75
normalized size	1	1.	2.3	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.26	0.47	0.086	0.	0.	0.	0.	29.064

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	228	0	0	0	0	0	78
normalized size	1	1.	2.28	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.261	0.527	0.091	0.	0.	0.	0.	30.611

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	179	0	0	0	0	0	71
normalized size	1	1.	1.97	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.22	0.407	0.059	0.	0.	0.	0.	32.693

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	181	0	0	0	0	0	71
normalized size	1	1.	1.99	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.217	0.366	0.058	0.	0.	0.	0.	33.013

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	181	0	0	0	0	0	71
normalized size	1	1.	1.99	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.216	0.366	0.057	0.	0.	0.	0.	32.411

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	179	0	0	0	0	0	70
normalized size	1	1.	2.01	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.216	0.374	0.059	0.	0.	0.	0.	32.157

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	179	0	0	0	0	0	73
normalized size	1	1.	2.01	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.217	0.387	0.06	0.	0.	0.	0.	32.863

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	180	0	0	0	0	0	75
normalized size	1	1.	1.98	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.221	0.383	0.059	0.	0.	0.	0.	33.052

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1066] had the largest ratio of [0.5833]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	18	0.056
2	A	3	2	1.	16	0.125
3	A	2	1	1.	15	0.067
4	A	3	2	1.	18	0.111
5	A	2	1	1.	18	0.056
6	A	3	2	1.	18	0.111
7	A	2	1	1.	18	0.056
8	A	3	2	1.	18	0.111
9	A	2	1	1.	18	0.056
10	A	3	2	1.	18	0.111
11	A	2	1	1.	20	0.05
12	A	3	2	1.	18	0.111
13	A	2	1	1.	17	0.059
14	A	4	3	1.	20	0.15
15	A	2	1	1.	20	0.05
16	A	3	2	1.	20	0.1
17	A	2	1	1.	20	0.05
18	A	3	2	1.	20	0.1
19	A	2	1	1.	20	0.05
20	A	3	2	1.	20	0.1
21	A	2	1	1.	20	0.05
22	A	3	3	1.	20	0.15
23	A	3	2	1.	20	0.1
24	A	2	1	1.	20	0.05
25	A	3	2	1.	20	0.1
26	A	2	1	1.	20	0.05

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	3	2	1.	20	0.1
28	A	2	1	1.	20	0.05
29	A	3	2	1.	20	0.1
30	A	2	1	1.	20	0.05
31	A	3	2	1.	18	0.111
32	A	2	1	1.	17	0.059
33	A	4	3	1.	20	0.15
34	A	2	1	1.	20	0.05
35	A	3	2	1.	20	0.1
36	A	2	1	1.	20	0.05
37	A	3	2	1.	20	0.1
38	A	2	1	1.	20	0.05
39	A	3	2	1.	20	0.1
40	A	2	1	1.	20	0.05
41	A	3	2	1.	20	0.1
42	A	2	1	1.	20	0.05
43	A	3	2	1.	20	0.1
44	A	2	1	1.	20	0.05
45	A	4	3	1.	20	0.15
46	A	2	1	1.	20	0.05
47	A	3	3	1.	20	0.15
48	A	2	1	1.	20	0.05
49	A	4	4	1.	20	0.2
50	A	2	1	1.	20	0.05
51	A	3	2	1.	20	0.1
52	A	2	1	1.	20	0.05
53	A	3	2	1.	20	0.1
54	A	2	1	1.	20	0.05
55	A	3	2	1.	20	0.1
56	A	4	3	1.	20	0.15
57	A	3	2	1.	20	0.1
58	A	4	3	1.	20	0.15
59	A	3	2	1.	20	0.1
60	A	3	3	1.	20	0.15
61	A	3	2	1.	18	0.111
62	A	2	2	1.	17	0.118
63	A	3	2	1.	20	0.1
64	A	2	2	1.	20	0.1
65	A	3	2	1.	20	0.1
66	A	3	3	1.	20	0.15
67	A	3	2	1.	20	0.1
68	A	4	3	1.	20	0.15
69	A	3	2	1.	20	0.1
70	A	5	3	1.	20	0.15
71	A	3	2	1.	20	0.1
72	A	4	3	1.	20	0.15
73	A	3	2	1.	20	0.1
74	A	4	3	1.	20	0.15
75	A	3	2	1.	20	0.1

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
76	A	4	3	1.	20	0.15
77	A	3	2	1.	20	0.1
78	A	3	3	1.	20	0.15
79	A	3	2	1.	18	0.111
80	A	2	2	1.	17	0.118
81	A	3	2	1.	20	0.1
82	A	3	3	1.	20	0.15
83	A	3	2	1.	20	0.1
84	A	4	3	1.	20	0.15
85	A	3	2	1.	20	0.1
86	A	4	3	1.	20	0.15
87	A	3	2	1.	20	0.1
88	A	3	2	1.	20	0.1
89	A	3	2	1.	20	0.1
90	A	3	2	1.	20	0.1
91	A	3	2	1.	20	0.1
92	A	3	2	1.	20	0.1
93	A	2	2	1.	18	0.111
94	A	3	2	1.	20	0.1
95	A	3	2	1.	20	0.1
96	A	3	2	1.	20	0.1
97	A	3	2	1.	20	0.1
98	A	5	4	1.	20	0.2
99	A	5	4	1.	20	0.2
100	A	5	4	1.	20	0.2
101	A	4	4	1.	20	0.2
102	A	3	3	1.	20	0.15
103	A	3	3	1.	17	0.176
104	A	4	3	1.	20	0.15
105	A	5	4	1.	20	0.2
106	A	5	4	1.	20	0.2
107	A	2	2	1.	15	0.133
108	A	2	2	1.	17	0.118
109	A	1	1	1.	13	0.077
110	A	1	1	1.	15	0.067
111	A	2	2	1.	15	0.133
112	A	2	2	1.	13	0.154
113	A	2	2	1.	13	0.154
114	A	1	1	1.	19	0.053
115	A	1	1	1.	18	0.056
116	A	2	2	1.	18	0.111
117	A	3	3	1.	13	0.231
118	A	1	1	1.	18	0.056
119	A	2	2	1.	17	0.118
120	A	2	2	1.	23	0.087
121	A	2	2	1.	23	0.087
122	A	2	2	1.	21	0.095
123	A	2	2	1.	20	0.1
124	A	2	2	1.	23	0.087

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.	23	0.087
126	A	2	2	1.	23	0.087
127	A	4	3	1.	23	0.13
128	A	3	3	1.	23	0.13
129	A	2	2	1.	21	0.095
130	A	2	2	1.	20	0.1
131	A	5	5	1.	23	0.217
132	A	3	3	1.	23	0.13
133	A	4	3	1.	23	0.13
134	A	4	3	1.	23	0.13
135	A	3	3	1.	23	0.13
136	A	2	2	1.	21	0.095
137	A	3	3	1.	20	0.15
138	A	4	3	1.	23	0.13
139	A	4	4	1.	23	0.174
140	A	4	3	1.	23	0.13
141	A	2	1	1.	20	0.05
142	A	3	2	1.	20	0.1
143	A	2	1	1.	20	0.05
144	A	3	2	1.	18	0.111
145	A	2	1	1.	17	0.059
146	A	4	3	1.	20	0.15
147	A	2	1	1.	20	0.05
148	A	3	2	1.	20	0.1
149	A	2	1	1.	20	0.05
150	A	2	1	1.	22	0.045
151	A	3	2	1.	22	0.091
152	A	2	1	1.	22	0.045
153	A	3	2	1.	20	0.1
154	A	2	1	1.	19	0.053
155	A	3	2	1.	22	0.091
156	A	2	1	1.	22	0.045
157	A	3	2	1.	22	0.091
158	A	2	1	1.	22	0.045
159	A	2	1	1.	22	0.045
160	A	3	2	1.	22	0.091
161	A	2	1	1.	22	0.045
162	A	3	2	1.	20	0.1
163	A	2	1	1.	19	0.053
164	A	3	2	1.	22	0.091
165	A	2	1	1.	22	0.045
166	A	3	2	1.	22	0.091
167	A	2	1	1.	22	0.045
168	A	3	2	1.	22	0.091
169	A	3	2	1.	22	0.091
170	A	3	2	1.	22	0.091
171	A	3	2	1.	20	0.1
172	A	3	2	1.	19	0.105
173	A	3	2	1.	22	0.091

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
174	A	3	2	1.	22	0.091
175	A	3	2	1.	22	0.091
176	A	3	2	1.	22	0.091
177	A	3	2	1.	22	0.091
178	A	3	2	1.	22	0.091
179	A	3	2	1.	22	0.091
180	A	5	4	1.	22	0.182
181	A	3	2	1.	22	0.091
182	A	4	4	1.	22	0.182
183	A	3	2	1.	20	0.1
184	A	4	3	1.	19	0.158
185	A	3	2	1.	22	0.091
186	A	3	3	1.	22	0.136
187	A	3	2	1.	22	0.091
188	A	4	4	1.	22	0.182
189	A	5	4	1.	22	0.182
190	A	3	2	1.	22	0.091
191	A	4	4	1.	22	0.182
192	A	3	2	1.	20	0.1
193	A	3	3	1.	19	0.158
194	A	3	2	1.	22	0.091
195	A	4	4	0.99	22	0.182
196	A	3	2	1.	22	0.091
197	A	5	4	1.	22	0.182
198	A	3	2	1.	20	0.1
199	A	4	3	1.	20	0.15
200	A	3	2	1.	20	0.1
201	A	3	3	1.	20	0.15
202	A	3	2	1.	18	0.111
203	A	2	2	1.	17	0.118
204	A	3	2	1.	20	0.1
205	A	2	2	1.	20	0.1
206	A	3	2	1.	20	0.1
207	A	3	3	1.	20	0.15
208	A	3	2	1.	22	0.091
209	A	3	2	1.	22	0.091
210	A	3	2	1.	22	0.091
211	A	3	2	1.	22	0.091
212	A	3	2	1.	20	0.1
213	A	3	2	1.	19	0.105
214	A	3	2	1.	22	0.091
215	A	3	2	1.	22	0.091
216	A	3	2	1.	22	0.091
217	A	3	2	1.	22	0.091
218	A	3	2	1.	22	0.091
219	A	3	2	1.	22	0.091
220	A	3	2	1.	22	0.091
221	A	3	2	1.	22	0.091
222	A	3	2	1.	20	0.1

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
223	A	3	2	1.	19	0.105
224	A	3	2	1.	22	0.091
225	A	3	2	1.	22	0.091
226	A	3	2	1.	22	0.091
227	A	3	2	1.	22	0.091
228	A	3	2	1.	22	0.091
229	A	4	3	1.	22	0.136
230	A	3	2	1.	22	0.091
231	A	3	2	1.	22	0.091
232	A	4	3	1.	20	0.15
233	A	3	2	1.	19	0.105
234	A	3	2	1.	22	0.091
235	A	4	3	1.	22	0.136
236	A	3	2	1.	22	0.091
237	A	5	4	1.	22	0.182
238	A	3	2	1.	22	0.091
239	A	6	4	1.	22	0.182
240	A	3	2	1.	22	0.091
241	A	3	2	1.	22	0.091
242	A	4	3	1.	22	0.136
243	A	3	2	1.	22	0.091
244	A	4	3	1.	22	0.136
245	A	3	2	1.	20	0.1
246	A	4	3	1.	19	0.158
247	A	3	2	1.	22	0.091
248	A	5	4	1.	22	0.182
249	A	3	2	1.	22	0.091
250	A	6	4	1.	22	0.182
251	A	3	2	1.	22	0.091
252	A	5	4	1.	22	0.182
253	A	3	2	1.	22	0.091
254	A	5	4	1.	22	0.182
255	A	3	2	1.	20	0.1
256	A	5	4	1.	19	0.21
257	A	3	2	1.	22	0.091
258	A	6	5	1.	22	0.227
259	A	3	2	1.	22	0.091
260	A	7	5	1.	22	0.227
261	A	4	3	1.	16	0.188
262	A	4	3	1.	20	0.15
263	A	3	2	1.	20	0.1
264	A	3	3	1.	20	0.15
265	A	3	2	1.	18	0.111
266	A	2	2	1.	17	0.118
267	A	3	2	1.	20	0.1
268	A	3	3	1.	20	0.15
269	A	3	2	1.	20	0.1
270	A	4	3	1.	20	0.15
271	A	5	4	1.	22	0.182

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	3	2	1.	22	0.091
273	A	4	4	1.	22	0.182
274	A	3	2	1.	20	0.1
275	A	4	3	1.	19	0.158
276	A	3	2	1.	22	0.091
277	A	3	3	1.	22	0.136
278	A	3	2	1.	22	0.091
279	A	4	4	0.98	22	0.182
280	A	4	3	1.	22	0.136
281	A	3	2	1.	22	0.091
282	A	5	4	1.	22	0.182
283	A	3	2	1.	20	0.1
284	A	4	3	1.	19	0.158
285	A	3	2	1.	22	0.091
286	A	4	3	1.	22	0.136
287	A	3	2	1.	22	0.091
288	A	4	3	1.	22	0.136
289	A	4	3	1.	22	0.136
290	A	3	2	1.	22	0.091
291	A	4	3	1.	22	0.136
292	A	3	2	1.	20	0.1
293	A	4	3	1.	19	0.158
294	A	3	2	1.	22	0.091
295	A	5	4	1.	22	0.182
296	A	3	2	1.	22	0.091
297	A	6	4	1.	22	0.182
298	A	3	2	1.	22	0.091
299	A	7	4	1.	22	0.182
300	A	3	2	1.	22	0.091
301	A	5	4	1.	22	0.182
302	A	3	2	1.	22	0.091
303	A	5	4	1.	22	0.182
304	A	3	2	1.	20	0.1
305	A	5	4	1.	19	0.21
306	A	3	2	1.	22	0.091
307	A	6	5	1.	22	0.227
308	A	3	2	1.	22	0.091
309	A	7	5	1.	22	0.227
310	A	6	4	1.	22	0.182
311	A	3	2	1.	22	0.091
312	A	6	4	1.	22	0.182
313	A	3	2	1.	20	0.1
314	A	6	4	1.	19	0.21
315	A	3	2	1.	22	0.091
316	A	7	5	1.	22	0.227
317	A	3	2	1.	22	0.091
318	A	8	5	1.	22	0.227
319	A	2	1	1.	20	0.05
320	A	2	1	1.	20	0.05

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
321	A	2	1	1.	18	0.056
322	A	2	2	1.	20	0.1
323	A	2	2	1.	20	0.1
324	A	2	2	1.	20	0.1
325	A	2	1	1.	22	0.045
326	A	2	1	1.	22	0.045
327	A	2	1	1.	20	0.05
328	A	3	2	1.	22	0.091
329	A	3	3	1.	22	0.136
330	A	3	3	0.97	22	0.136
331	A	3	2	1.	22	0.091
332	A	3	2	1.	22	0.091
333	A	2	2	1.	20	0.1
334	A	3	2	1.	22	0.091
335	A	5	3	1.	22	0.136
336	A	6	4	1.	22	0.182
337	A	4	3	1.	22	0.136
338	A	3	3	1.	22	0.136
339	A	2	2	1.	20	0.1
340	A	5	3	1.	22	0.136
341	A	6	4	1.	22	0.182
342	A	7	4	1.	22	0.182
343	A	2	1	1.	20	0.05
344	A	2	1	1.	20	0.05
345	A	2	1	1.	20	0.05
346	A	2	1	1.	20	0.05
347	A	2	1	1.	20	0.05
348	A	2	1	1.	20	0.05
349	A	2	1	1.	20	0.05
350	A	2	1	1.	20	0.05
351	A	2	1	1.	22	0.045
352	A	2	1	1.	22	0.045
353	A	2	1	1.	22	0.045
354	A	2	1	1.	22	0.045
355	A	2	1	1.	22	0.045
356	A	2	1	1.	22	0.045
357	A	2	1	1.	22	0.045
358	A	2	1	1.	22	0.045
359	A	2	1	1.	22	0.045
360	A	2	1	1.	22	0.045
361	A	2	1	1.	22	0.045
362	A	2	1	1.	22	0.045
363	A	2	1	1.	22	0.045
364	A	2	1	1.	22	0.045
365	A	2	1	1.	22	0.045
366	A	2	1	1.	22	0.045
367	A	13	9	1.	22	0.409
368	A	12	9	1.	22	0.409
369	A	12	9	1.	22	0.409

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
370	A	11	8	1.	22	0.364
371	A	11	8	1.	22	0.364
372	A	11	8	1.	22	0.364
373	A	11	8	1.	22	0.364
374	A	12	9	1.	22	0.409
375	A	13	9	1.	22	0.409
376	A	12	9	1.	22	0.409
377	A	12	9	1.	22	0.409
378	A	11	8	1.	22	0.364
379	A	11	8	1.	22	0.364
380	A	12	9	1.	22	0.409
381	A	12	9	1.	22	0.409
382	A	13	9	1.	22	0.409
383	A	13	10	1.	22	0.454
384	A	12	9	1.	22	0.409
385	A	12	9	1.	22	0.409
386	A	12	9	1.	22	0.409
387	A	12	9	1.	22	0.409
388	A	13	10	1.	22	0.454
389	A	13	10	1.	22	0.454
390	A	14	10	1.	22	0.454
391	A	2	1	1.	22	0.045
392	A	2	1	1.	22	0.045
393	A	2	1	1.	22	0.045
394	A	2	1	1.	22	0.045
395	A	2	1	1.	22	0.045
396	A	2	1	1.	22	0.045
397	A	2	1	1.	22	0.045
398	A	2	1	1.	22	0.045
399	A	2	1	1.	24	0.042
400	A	2	1	1.	24	0.042
401	A	2	1	1.	24	0.042
402	A	2	1	1.	24	0.042
403	A	2	1	1.	24	0.042
404	A	2	1	1.	24	0.042
405	A	2	1	1.	24	0.042
406	A	2	1	1.	24	0.042
407	A	2	1	1.	24	0.042
408	A	2	1	1.	24	0.042
409	A	2	1	1.	24	0.042
410	A	2	1	1.	24	0.042
411	A	2	1	1.	24	0.042
412	A	2	1	1.	24	0.042
413	A	2	1	1.	24	0.042
414	A	2	1	1.	24	0.042
415	A	14	9	1.	24	0.375
416	A	13	9	1.	24	0.375
417	A	13	9	1.	24	0.375
418	A	12	8	1.	24	0.333

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
419	A	12	8	1.	24	0.333
420	A	12	9	1.	24	0.375
421	A	12	9	1.	24	0.375
422	A	12	9	1.	24	0.375
423	A	12	9	1.	24	0.375
424	A	13	10	1.	24	0.417
425	A	14	10	1.	24	0.417
426	A	13	10	1.	24	0.417
427	A	13	10	1.	24	0.417
428	A	12	9	1.	24	0.375
429	A	12	9	1.	24	0.375
430	A	12	9	1.	24	0.375
431	A	12	9	1.	24	0.375
432	A	13	10	1.	24	0.417
433	A	14	10	1.	24	0.417
434	A	13	10	1.	24	0.417
435	A	13	10	1.	24	0.417
436	A	12	9	1.	24	0.375
437	A	12	9	1.	24	0.375
438	A	13	10	1.	24	0.417
439	A	13	10	1.	24	0.417
440	A	14	11	1.	24	0.458
441	A	13	9	1.	24	0.375
442	A	13	9	1.	24	0.375
443	A	12	8	1.	24	0.333
444	A	12	8	1.	24	0.333
445	A	12	8	1.	24	0.333
446	A	12	8	1.	24	0.333
447	A	12	8	1.	24	0.333
448	A	12	8	1.	24	0.333
449	A	12	8	1.	24	0.333
450	A	12	8	1.	24	0.333
451	A	12	8	1.	24	0.333
452	A	13	9	1.	24	0.375
453	A	13	9	1.	24	0.375
454	A	14	10	1.	24	0.417
455	A	13	9	1.	24	0.375
456	A	13	9	1.	24	0.375
457	A	13	9	1.	24	0.375
458	A	13	9	1.	24	0.375
459	A	13	9	1.	24	0.375
460	A	13	9	1.	24	0.375
461	A	22	9	1.	24	0.375
462	A	21	9	1.	24	0.375
463	A	20	8	1.	24	0.333
464	A	20	8	1.	24	0.333
465	A	20	8	1.	24	0.333
466	A	20	8	1.	24	0.333
467	A	22	9	1.	24	0.375

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	21	9	1.	24	0.375
469	A	23	10	1.	24	0.417
470	A	22	10	1.	24	0.417
471	A	22	9	1.	24	0.375
472	A	21	9	1.	24	0.375
473	A	22	9	1.	24	0.375
474	A	21	9	1.	24	0.375
475	A	22	9	1.	24	0.375
476	A	21	9	1.	24	0.375
477	A	23	10	1.	24	0.417
478	A	22	10	1.	24	0.417
479	A	24	10	1.	24	0.417
480	A	22	10	1.	24	0.417
481	A	23	10	1.	24	0.417
482	A	22	10	1.	24	0.417
483	A	23	10	1.	24	0.417
484	A	22	10	1.	24	0.417
485	A	24	11	1.	24	0.458
486	A	23	11	1.	24	0.458
487	A	25	11	1.	24	0.458
488	A	22	10	1.	24	0.417
489	A	23	10	1.	24	0.417
490	A	22	10	1.	24	0.417
491	A	23	10	1.	24	0.417
492	A	22	10	1.	24	0.417
493	A	24	11	1.	24	0.458
494	A	23	11	1.	24	0.458
495	A	25	11	1.	24	0.458
496	A	23	10	1.	24	0.417
497	A	24	10	1.	24	0.417
498	A	23	10	1.	24	0.417
499	A	24	10	1.	24	0.417
500	A	23	10	1.	24	0.417
501	A	25	11	1.	24	0.458
502	A	24	11	1.	24	0.458
503	A	26	11	1.	24	0.458
504	A	3	2	1.	22	0.091
505	A	6	5	1.	22	0.227
506	A	3	2	1.	22	0.091
507	A	5	5	1.	22	0.227
508	A	3	2	1.	20	0.1
509	A	4	4	1.	19	0.21
510	A	5	5	1.	22	0.227
511	A	4	4	1.	22	0.182
512	A	5	5	1.	22	0.227
513	A	4	4	1.	22	0.182
514	A	5	5	1.	22	0.227
515	A	2	2	1.	22	0.091
516	A	6	6	1.	22	0.273

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
517	A	3	3	1.	22	0.136
518	A	7	6	1.	22	0.273
519	A	4	3	1.	22	0.136
520	A	8	6	1.	22	0.273
521	A	3	2	1.	22	0.091
522	A	7	5	1.	22	0.227
523	A	3	2	1.	22	0.091
524	A	6	5	1.	22	0.227
525	A	3	2	1.	20	0.1
526	A	5	4	1.	19	0.21
527	A	6	5	1.	22	0.227
528	A	5	4	1.	22	0.182
529	A	6	5	1.	22	0.227
530	A	5	5	1.	22	0.227
531	A	6	6	1.	22	0.273
532	A	5	4	1.	22	0.182
533	A	6	5	1.	22	0.227
534	A	2	2	1.	22	0.091
535	A	7	6	1.	22	0.273
536	A	3	3	1.	22	0.136
537	A	8	6	1.	22	0.273
538	A	3	2	1.	22	0.091
539	A	8	5	1.	22	0.227
540	A	3	2	1.	22	0.091
541	A	7	5	1.	22	0.227
542	A	3	2	1.	20	0.1
543	A	6	4	1.	19	0.21
544	A	7	5	1.	22	0.227
545	A	6	4	1.	22	0.182
546	A	7	5	1.	22	0.227
547	A	6	5	1.	22	0.227
548	A	7	6	1.	22	0.273
549	A	6	5	1.	22	0.227
550	A	7	6	1.	22	0.273
551	A	6	4	1.	22	0.182
552	A	7	5	1.	22	0.227
553	A	2	2	1.	22	0.091
554	A	8	6	1.	22	0.273
555	A	3	2	1.	22	0.091
556	A	5	4	1.	22	0.182
557	A	3	2	1.	22	0.091
558	A	4	4	1.	22	0.182
559	A	3	2	1.	20	0.1
560	A	3	3	1.	19	0.158
561	A	4	4	1.	22	0.182
562	A	3	3	1.	22	0.136
563	A	4	4	1.	22	0.182
564	A	2	2	1.	22	0.091
565	A	5	5	1.	22	0.227

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	3	3	1.	22	0.136
567	A	6	5	1.	22	0.227
568	A	4	3	1.	22	0.136
569	A	6	5	1.	22	0.227
570	A	3	2	1.	22	0.091
571	A	5	5	1.	22	0.227
572	A	3	2	1.	22	0.091
573	A	4	4	1.	22	0.182
574	A	3	2	1.	20	0.1
575	A	3	3	1.	19	0.158
576	A	4	4	1.	22	0.182
577	A	2	2	1.	22	0.091
578	A	5	5	1.	22	0.227
579	A	3	3	1.	22	0.136
580	A	6	5	1.	22	0.227
581	A	4	3	1.	22	0.136
582	A	7	5	1.	22	0.227
583	A	5	3	1.	22	0.136
584	A	3	2	1.	22	0.091
585	A	6	5	1.	22	0.227
586	A	3	2	1.	22	0.091
587	A	5	5	1.	22	0.227
588	A	3	2	1.	22	0.091
589	A	4	4	1.	22	0.182
590	A	3	2	1.	20	0.1
591	A	2	2	1.	19	0.105
592	A	5	5	1.	22	0.227
593	A	3	3	1.	22	0.136
594	A	6	5	1.	22	0.227
595	A	4	4	1.	22	0.182
596	A	7	5	1.	22	0.227
597	A	5	4	1.	22	0.182
598	A	3	2	1.	24	0.083
599	A	3	2	1.	24	0.083
600	A	3	2	1.	22	0.091
601	A	6	5	1.	24	0.208
602	A	6	6	1.	24	0.25
603	A	6	6	0.98	24	0.25
604	A	6	6	1.	24	0.25
605	A	6	6	0.98	24	0.25
606	A	5	5	1.	21	0.238
607	A	5	5	0.98	24	0.208
608	A	5	5	1.	24	0.208
609	A	5	5	1.	24	0.208
610	A	3	3	1.01	24	0.125
611	A	4	4	1.01	24	0.167
612	A	5	4	1.01	24	0.167
613	A	8	6	0.99	24	0.25
614	A	3	2	1.	24	0.083

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
615	A	7	6	0.99	24	0.25
616	A	3	2	1.	22	0.091
617	A	6	5	1.	21	0.238
618	A	7	5	1.	24	0.208
619	A	6	5	0.98	24	0.208
620	A	7	6	1.	24	0.25
621	A	6	5	0.98	24	0.208
622	A	7	6	0.98	24	0.25
623	A	6	6	1.	24	0.25
624	A	7	7	0.98	24	0.292
625	A	3	2	1.	24	0.083
626	A	8	6	0.99	24	0.25
627	A	3	2	1.	22	0.091
628	A	7	5	1.	21	0.238
629	A	8	5	1.	24	0.208
630	A	7	5	0.99	24	0.208
631	A	8	6	1.	24	0.25
632	A	7	5	0.98	24	0.208
633	A	8	6	0.99	24	0.25
634	A	7	6	0.99	24	0.25
635	A	8	7	0.99	24	0.292
636	A	6	5	1.	24	0.208
637	A	3	2	1.	24	0.083
638	A	5	5	1.	24	0.208
639	A	3	2	1.	22	0.091
640	A	4	4	1.	21	0.19
641	A	5	4	1.	24	0.167
642	A	4	4	1.	24	0.167
643	A	5	5	1.	24	0.208
644	A	4	4	1.	24	0.167
645	A	5	5	1.	24	0.208
646	A	3	3	1.01	24	0.125
647	A	6	6	1.	24	0.25
648	A	6	5	1.	24	0.208
649	A	3	2	1.	24	0.083
650	A	5	5	1.	24	0.208
651	A	3	2	1.	22	0.091
652	A	4	4	1.	21	0.19
653	A	5	4	1.	24	0.167
654	A	4	4	0.96	24	0.167
655	A	5	5	1.	24	0.208
656	A	3	3	1.01	24	0.125
657	A	6	6	1.	24	0.25
658	A	4	4	1.	24	0.167
659	A	7	6	1.01	24	0.25
660	A	6	6	1.	24	0.25
661	A	3	2	1.	24	0.083
662	A	5	5	1.	24	0.208
663	A	3	2	1.	22	0.091

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
664	A	4	4	1.	21	0.19
665	A	5	4	1.	24	0.167
666	A	3	3	1.	24	0.125
667	A	6	6	1.	24	0.25
668	A	4	4	0.99	24	0.167
669	A	7	6	1.	24	0.25
670	A	5	5	1.	24	0.208
671	A	4	3	1.	22	0.136
672	A	3	3	1.	22	0.136
673	A	2	2	1.	20	0.1
674	A	3	3	1.	22	0.136
675	A	4	3	1.	22	0.136
676	A	7	7	1.	24	0.292
677	A	5	5	1.	24	0.208
678	A	6	6	1.	24	0.25
679	A	4	4	1.	22	0.182
680	A	5	5	1.	21	0.238
681	A	6	5	1.	24	0.208
682	A	4	4	1.	24	0.167
683	A	7	6	1.	24	0.25
684	A	5	5	1.	24	0.208
685	A	8	7	1.	24	0.292
686	A	6	5	1.	24	0.208
687	A	7	7	1.	24	0.292
688	A	5	4	1.	22	0.182
689	A	6	6	1.	21	0.286
690	A	7	6	1.	24	0.25
691	A	6	6	1.	24	0.25
692	A	7	6	1.	24	0.25
693	A	5	5	1.	24	0.208
694	A	9	8	1.	24	0.333
695	A	7	5	1.	24	0.208
696	A	8	8	1.	24	0.333
697	A	6	4	1.	22	0.182
698	A	7	7	1.	21	0.333
699	A	8	7	1.	24	0.292
700	A	7	7	1.	24	0.292
701	A	8	7	1.	24	0.292
702	A	7	7	1.	24	0.292
703	A	5	4	1.	24	0.167
704	A	4	4	1.	24	0.167
705	A	3	3	1.	22	0.136
706	A	6	5	1.	24	0.208
707	A	7	6	1.	24	0.25
708	A	6	6	1.	24	0.25
709	A	5	5	1.	24	0.208
710	A	2	2	1.	21	0.095
711	A	4	4	1.	24	0.167
712	A	5	5	1.	24	0.208

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
713	A	6	6	1.	24	0.25
714	A	4	4	1.	24	0.167
715	A	4	4	1.	24	0.167
716	A	4	4	1.	22	0.182
717	A	3	3	1.	21	0.143
718	A	7	6	1.	24	0.25
719	A	5	5	1.	24	0.208
720	A	8	7	1.	24	0.292
721	A	6	5	1.	24	0.208
722	A	5	5	1.	24	0.208
723	A	5	5	1.	24	0.208
724	A	5	5	1.	24	0.208
725	A	5	4	1.	22	0.182
726	A	5	5	1.	21	0.238
727	A	8	7	1.	24	0.292
728	A	6	6	1.	24	0.25
729	A	9	7	1.	24	0.292
730	A	7	6	1.	24	0.25
731	A	7	7	1.	24	0.292
732	A	5	5	1.	24	0.208
733	A	6	6	1.	24	0.25
734	A	4	4	1.	22	0.182
735	A	3	3	1.	21	0.143
736	A	7	6	1.	24	0.25
737	A	5	5	1.	24	0.208
738	A	8	7	1.	24	0.292
739	A	6	5	1.	24	0.208
740	A	8	8	1.	24	0.333
741	A	6	5	1.	24	0.208
742	A	7	7	1.	24	0.292
743	A	5	5	1.	22	0.227
744	A	6	6	1.	21	0.286
745	A	7	6	1.	24	0.25
746	A	5	5	1.	24	0.208
747	A	8	7	1.	24	0.292
748	A	6	5	1.	24	0.208
749	A	9	8	1.	24	0.333
750	A	7	5	1.	24	0.208
751	A	8	7	1.	24	0.292
752	A	6	5	1.	22	0.227
753	A	7	7	1.	21	0.333
754	A	8	7	1.	24	0.292
755	A	7	7	1.	24	0.292
756	A	8	7	1.	24	0.292
757	A	6	6	1.	24	0.25
758	A	6	6	1.	24	0.25
759	A	4	4	1.	24	0.167
760	A	4	4	1.	24	0.167
761	A	4	4	1.	22	0.182

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
762	A	3	3	1.	21	0.143
763	A	7	6	1.	24	0.25
764	A	5	5	1.	24	0.208
765	A	8	7	1.	24	0.292
766	A	6	5	1.	24	0.208
767	A	5	5	1.	24	0.208
768	A	5	5	1.	24	0.208
769	A	5	5	1.	24	0.208
770	A	5	5	1.	22	0.227
771	A	5	5	1.	21	0.238
772	A	8	7	1.	24	0.292
773	A	6	6	1.	24	0.25
774	A	9	8	1.	24	0.333
775	A	7	6	1.	24	0.25
776	A	6	5	1.	24	0.208
777	A	6	5	1.	24	0.208
778	A	6	5	1.	24	0.208
779	A	6	5	1.	22	0.227
780	A	6	5	1.	21	0.238
781	A	9	7	1.	24	0.292
782	A	7	6	1.	24	0.25
783	A	10	8	1.	24	0.333
784	A	8	6	1.	24	0.25
785	A	5	5	1.	26	0.192
786	A	6	6	1.	26	0.231
787	A	4	4	1.	26	0.154
788	A	6	6	1.	26	0.231
789	A	4	4	1.	26	0.154
790	A	6	6	1.	26	0.231
791	A	4	4	1.	24	0.167
792	A	7	7	1.	24	0.292
793	A	5	5	1.	24	0.208
794	A	6	5	1.	26	0.192
795	A	7	6	1.	26	0.231
796	A	5	4	1.	26	0.154
797	A	7	6	1.	26	0.231
798	A	5	4	1.	26	0.154
799	A	7	7	1.	26	0.269
800	A	6	6	1.	26	0.231
801	A	4	4	1.	26	0.154
802	A	5	5	1.	26	0.192
803	A	3	3	1.	26	0.115
804	A	5	5	1.	26	0.192
805	A	3	3	1.	26	0.115
806	A	6	6	1.	26	0.231
807	A	5	5	1.	26	0.192
808	A	6	6	1.	26	0.231
809	A	4	4	1.	26	0.154
810	A	5	5	1.	26	0.192

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
811	A	3	3	1.	26	0.115
812	A	6	6	1.	26	0.231
813	A	4	4	1.	26	0.154
814	A	7	7	1.	26	0.269
815	A	5	4	1.	26	0.154
816	A	6	6	1.	26	0.231
817	A	4	4	1.	26	0.154
818	A	6	6	1.	26	0.231
819	A	4	4	1.	26	0.154
820	A	7	6	1.	26	0.231
821	A	5	4	1.	26	0.154
822	A	6	6	1.	28	0.214
823	A	7	7	1.	28	0.25
824	A	5	5	1.	28	0.179
825	A	7	7	1.	28	0.25
826	A	5	5	1.	28	0.179
827	A	7	7	1.	28	0.25
828	A	5	5	0.99	26	0.192
829	A	7	7	0.99	26	0.269
830	A	5	5	0.98	26	0.192
831	A	8	8	0.99	26	0.308
832	A	9	8	1.	28	0.286
833	A	7	6	1.	28	0.214
834	A	8	7	1.	28	0.25
835	A	6	5	1.	28	0.179
836	A	8	7	1.	28	0.25
837	A	6	5	1.	28	0.179
838	A	8	7	1.	28	0.25
839	A	7	7	1.	28	0.25
840	A	5	5	1.	28	0.179
841	A	6	6	1.	28	0.214
842	A	4	4	1.	28	0.143
843	A	6	6	1.	28	0.214
844	A	4	4	1.	28	0.143
845	A	6	6	1.	28	0.214
846	A	4	4	1.	28	0.143
847	A	7	7	1.	28	0.25
848	A	5	5	1.	28	0.179
849	A	6	5	1.	28	0.179
850	A	7	7	1.	28	0.25
851	A	5	5	1.	28	0.179
852	A	6	6	1.	28	0.214
853	A	4	4	1.	28	0.143
854	A	6	6	1.	28	0.214
855	A	4	4	1.	28	0.143
856	A	7	7	1.	28	0.25
857	A	6	6	1.	28	0.214
858	A	7	7	1.	28	0.25
859	A	5	5	1.	28	0.179

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
860	A	6	6	1.	28	0.214
861	A	4	4	1.	28	0.143
862	A	7	7	1.	28	0.25
863	A	5	5	1.	28	0.179
864	A	8	7	1.	28	0.25
865	A	11	9	1.	30	0.3
866	A	15	12	1.	30	0.4
867	A	10	8	1.	30	0.267
868	A	13	11	1.	30	0.367
869	A	9	7	1.	30	0.233
870	A	15	12	1.	30	0.4
871	A	10	8	1.	30	0.267
872	A	16	13	1.	30	0.433
873	A	16	13	1.	30	0.433
874	A	11	9	1.	30	0.3
875	A	15	12	1.	30	0.4
876	A	10	8	1.	30	0.267
877	A	15	12	1.	30	0.4
878	A	10	8	1.	30	0.267
879	A	16	13	1.	30	0.433
880	A	10	8	1.	30	0.267
881	A	13	11	1.	30	0.367
882	A	9	7	1.	30	0.233
883	A	6	4	1.	30	0.133
884	A	6	4	1.	30	0.133
885	A	15	12	1.	30	0.4
886	A	10	8	1.	30	0.267
887	A	16	13	1.	30	0.433
888	A	15	12	1.	30	0.4
889	A	10	8	1.	30	0.267
890	A	15	12	1.	30	0.4
891	A	10	8	1.	30	0.267
892	A	15	12	1.	30	0.4
893	A	10	8	1.	30	0.267
894	A	16	13	1.	30	0.433
895	A	11	9	1.	30	0.3
896	A	11	9	1.	30	0.3
897	A	15	12	1.	30	0.4
898	A	10	8	1.	30	0.267
899	A	15	12	1.	30	0.4
900	A	10	8	1.	30	0.267
901	A	16	13	1.	30	0.433
902	A	11	9	1.	30	0.3
903	A	12	10	1.	30	0.333
904	A	16	13	1.	30	0.433
905	A	11	9	1.	30	0.3
906	A	15	12	1.	30	0.4
907	A	10	8	1.	30	0.267
908	A	16	13	1.	30	0.433

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
909	A	11	9	1.	30	0.3
910	A	15	12	1.	30	0.4
911	A	10	8	1.	30	0.267
912	A	15	12	1.	30	0.4
913	A	10	8	1.	30	0.267
914	A	15	12	1.	30	0.4
915	A	10	8	1.	30	0.267
916	A	16	13	1.	30	0.433
917	A	11	9	1.	30	0.3
918	A	16	13	1.	30	0.433
919	A	11	9	1.	30	0.3
920	A	16	13	1.	30	0.433
921	A	11	9	1.	30	0.3
922	A	16	13	1.	30	0.433
923	A	11	9	1.	30	0.3
924	A	17	14	1.	30	0.467
925	A	12	10	1.	30	0.333
926	A	17	13	1.	30	0.433
927	A	12	9	1.	30	0.3
928	A	17	13	1.	30	0.433
929	A	12	9	1.	30	0.3
930	A	17	13	1.	30	0.433
931	A	12	9	1.	30	0.3
932	A	18	14	1.	30	0.467
933	A	13	10	1.	30	0.333
934	A	6	6	1.	26	0.231
935	A	5	5	1.	26	0.192
936	A	4	4	1.	24	0.167
937	A	6	5	1.	26	0.192
938	A	4	4	1.	26	0.154
939	A	5	5	1.	26	0.192
940	A	6	6	1.	26	0.231
941	A	5	5	1.	26	0.192
942	A	6	6	1.	26	0.231
943	A	6	6	1.	26	0.231
944	A	7	6	1.	26	0.231
945	A	6	5	1.	26	0.192
946	A	5	4	1.	24	0.167
947	A	7	6	1.	26	0.231
948	A	7	6	1.	26	0.231
949	A	5	4	1.	26	0.154
950	A	7	6	1.	26	0.231
951	A	6	6	1.	26	0.231
952	A	5	5	1.	26	0.192
953	A	6	6	1.	26	0.231
954	A	8	6	1.	26	0.231
955	A	7	5	1.	26	0.192
956	A	6	4	1.	24	0.167
957	A	8	7	1.	26	0.269

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
958	A	8	7	1.	26	0.269
959	A	8	7	1.	26	0.269
960	A	8	7	1.	26	0.269
961	A	7	7	1.	26	0.269
962	A	6	6	1.	26	0.231
963	A	6	6	1.	26	0.231
964	A	5	5	1.	26	0.192
965	A	6	6	1.	26	0.231
966	A	4	4	1.	26	0.154
967	A	5	5	1.	24	0.208
968	A	5	5	1.	26	0.192
969	A	5	5	1.	26	0.192
970	A	4	4	1.	26	0.154
971	A	3	3	1.	24	0.125
972	A	3	3	1.	26	0.115
973	A	4	4	1.	26	0.154
974	A	6	6	1.	26	0.231
975	A	6	6	1.	26	0.231
976	A	5	5	1.	26	0.192
977	A	2	2	1.	26	0.077
978	A	4	4	1.	26	0.154
979	A	6	6	1.	26	0.231
980	A	5	5	1.	26	0.192
981	A	4	4	1.	26	0.154
982	A	2	2	1.	24	0.083
983	A	5	5	1.	26	0.192
984	A	3	3	1.	26	0.115
985	A	3	3	1.	24	0.125
986	A	4	4	1.	26	0.154
987	A	4	4	1.	26	0.154
988	A	4	3	1.	24	0.125
989	A	5	5	1.	26	0.192
990	A	3	3	1.	25	0.12
991	A	3	3	1.	26	0.115
992	A	2	2	1.	26	0.077
993	A	5	5	1.	26	0.192
994	A	2	2	1.	24	0.083
995	A	3	3	1.	26	0.115
996	A	3	3	1.	26	0.115
997	A	3	3	1.	26	0.115
998	A	3	3	1.	26	0.115
999	A	3	3	1.	26	0.115
1000	A	3	3	1.	26	0.115
1001	A	3	3	1.	24	0.125
1002	A	3	3	1.	24	0.125
1003	A	3	3	1.	26	0.115
1004	A	2	2	1.	24	0.083
1005	A	2	2	1.	24	0.083
1006	A	2	2	1.	26	0.077

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1007	A	3	3	1.	26	0.115
1008	A	7	6	1.	22	0.273
1009	A	6	6	1.	22	0.273
1010	A	5	5	1.	20	0.25
1011	A	10	7	1.	22	0.318
1012	A	7	7	1.	22	0.318
1013	A	12	9	1.	22	0.409
1014	A	7	7	1.	22	0.318
1015	A	6	6	1.	22	0.273
1016	A	1	1	1.	19	0.053
1017	A	7	7	1.	22	0.318
1018	A	8	8	1.	22	0.364
1019	A	7	7	1.	22	0.318
1020	A	7	7	1.	22	0.318
1021	A	6	6	1.	22	0.273
1022	A	6	6	1.	20	0.3
1023	A	11	8	1.	22	0.364
1024	A	12	9	1.	22	0.409
1025	A	13	9	1.	22	0.409
1026	A	7	7	1.	22	0.318
1027	A	7	7	1.	22	0.318
1028	A	7	7	1.	19	0.368
1029	A	8	8	1.	22	0.364
1030	A	9	8	1.	22	0.364
1031	A	10	5	1.	24	0.208
1032	A	7	5	1.	24	0.208
1033	A	4	3	1.	24	0.125
1034	A	1	1	1.	22	0.045
1035	A	8	7	1.	24	0.292
1036	A	14	8	1.	24	0.333
1037	A	6	4	1.	24	0.167
1038	A	4	3	1.	24	0.125
1039	A	1	1	1.	21	0.048
1040	A	5	4	1.	24	0.167
1041	A	8	4	1.	24	0.167
1042	A	7	6	1.	24	0.25
1043	A	7	6	1.	24	0.25
1044	A	6	6	1.	24	0.25
1045	A	5	5	1.	22	0.227
1046	A	16	12	1.	24	0.5
1047	A	17	13	1.	24	0.542
1048	A	12	7	1.	24	0.292
1049	A	7	6	1.	24	0.25
1050	A	1	1	1.	21	0.048
1051	A	8	7	1.	24	0.292
1052	A	14	7	1.	24	0.292
1053	A	1	1	1.	24	0.042
1054	A	1	1	1.	24	0.042
1055	A	1	1	1.	24	0.042

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1056	A	1	1	1.	26	0.038
1057	A	1	1	1.	26	0.038
1058	A	1	1	1.	26	0.038
1059	A	1	1	1.	26	0.038
1060	A	1	1	1.	28	0.036
1061	A	20	12	1.	24	0.5
1062	A	17	12	1.	24	0.5
1063	A	14	10	1.	24	0.417
1064	A	11	8	1.	22	0.364
1065	A	18	13	1.	24	0.542
1066	A	24	14	1.	24	0.583
1067	A	11	5	1.	24	0.208
1068	A	8	5	1.	24	0.208
1069	A	1	1	1.	24	0.042
1070	A	3	3	1.	21	0.143
1071	A	7	5	1.	24	0.208
1072	A	10	5	1.	24	0.208
1073	A	1	1	1.	24	0.042
1074	A	1	1	1.	24	0.042
1075	A	1	1	1.	24	0.042
1076	A	1	1	1.	26	0.038
1077	A	1	1	1.	28	0.036
1078	A	1	1	1.	28	0.036
1079	A	1	1	1.	28	0.036
1080	A	1	1	1.	30	0.033
1081	A	7	6	1.	24	0.25
1082	A	7	6	1.	24	0.25
1083	A	6	6	1.	24	0.25
1084	A	5	5	1.	22	0.227
1085	A	16	12	1.	24	0.5
1086	A	17	13	1.	24	0.542
1087	A	15	6	1.	24	0.25
1088	A	11	6	1.	24	0.25
1089	A	1	1	1.	24	0.042
1090	A	4	4	1.	21	0.19
1091	A	9	6	1.	24	0.25
1092	A	13	6	1.	24	0.25
1093	A	7	7	1.	26	0.269
1094	A	6	6	1.	26	0.231
1095	A	6	6	1.	26	0.231
1096	A	2	2	1.	26	0.077
1097	A	3	3	1.	26	0.115
1098	A	4	3	1.	26	0.115
1099	A	8	7	1.	26	0.269
1100	A	7	7	1.	26	0.269
1101	A	6	6	1.	26	0.231
1102	A	6	6	1.	26	0.231
1103	A	7	7	1.	26	0.269
1104	A	8	7	1.	26	0.269

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1105	A	7	7	1.	26	0.269
1106	A	6	6	1.	26	0.231
1107	A	2	2	1.	26	0.077
1108	A	3	3	1.	26	0.115
1109	A	4	3	1.	26	0.115
1110	A	6	5	1.	26	0.192
1111	A	5	5	1.	26	0.192
1112	A	4	4	1.	26	0.154
1113	A	4	4	1.	26	0.154
1114	A	5	5	1.	26	0.192
1115	A	6	5	1.	26	0.192
1116	A	7	7	1.	26	0.269
1117	A	6	6	1.	26	0.231
1118	A	2	2	1.	26	0.077
1119	A	3	3	1.	26	0.115
1120	A	4	3	1.	26	0.115
1121	A	8	7	1.	26	0.269
1122	A	7	7	1.	26	0.269
1123	A	6	6	1.	26	0.231
1124	A	7	7	1.	26	0.269
1125	A	8	8	1.	26	0.308
1126	A	8	8	1.	26	0.308
1127	A	7	7	1.	26	0.269
1128	A	2	2	1.	26	0.077
1129	A	3	3	1.	26	0.115
1130	A	4	3	1.	26	0.115
1131	A	5	3	1.	26	0.115
1132	A	7	5	1.	26	0.192
1133	A	6	5	1.	26	0.192
1134	A	5	5	1.	26	0.192
1135	A	4	4	1.	26	0.154
1136	A	5	5	1.	26	0.192
1137	A	6	6	1.	26	0.231
1138	A	7	6	1.	26	0.231
1139	A	3	2	1.	24	0.083
1140	A	3	2	1.	22	0.091
1141	A	3	2	1.	22	0.091
1142	A	3	2	1.	19	0.105
1143	A	3	2	1.	22	0.091
1144	A	3	2	1.	22	0.091
1145	A	5	5	1.	22	0.227
1146	A	4	4	1.	22	0.182
1147	A	3	3	1.	20	0.15
1148	A	3	3	1.	22	0.136
1149	A	3	3	1.	22	0.136
1150	A	3	3	1.	22	0.136
1151	A	3	2	1.	26	0.077
1152	A	3	2	1.	26	0.077
1153	A	3	2	1.	26	0.077

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1154	A	3	2	1.	26	0.077
1155	A	3	2	1.	26	0.077
1156	A	3	2	1.	26	0.077

3 Listing of integrals

3.1 $\int x^2 \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Rubi [A] time = 0.0602547, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)*(A + B*x^2),x]

[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Rubi in Sympy [A] time = 10.0854, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3}+\frac{Bbx^7}{7}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)*(B*x**2+A),x)

[Out] A*a*x**3/3 + B*b*x**7/7 + x**5*(A*b/5 + B*a/5)
```

Mathematica [A] time = 0.0100798, size = 33, normalized size = 1.

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)*(A + B*x^2),x]

[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Maple [A] time = 0.001, size = 28, normalized size = 0.9

$$\frac{aAx^3}{3}+\frac{(Ab+Ba)\,x^5}{5}+\frac{bBx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

Maxima [A] time = 1.37163, size = 36, normalized size = 1.09

$$\frac{1}{7}Bbx^7 + \frac{1}{5}(Ba + Ab)x^5 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^2,x, algorithm="maxima")
```

```
[Out] 1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A] time = 0.20973, size = 1, normalized size = 0.03

$$\frac{1}{7}x^7bB + \frac{1}{5}x^5aB + \frac{1}{5}x^5bA + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/3*x^3*a*A
```

Sympy [A] time = 0.082555, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3} + \frac{Bbx^7}{7} + x^5\left(\frac{Ab}{5} + \frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**3/3 + B*b*x**7/7 + x**5*(A*b/5 + B*a/5)
```

GIAC/XCAS [A] time = 0.215866, size = 39, normalized size = 1.18

$$\frac{1}{7}Bbx^7 + \frac{1}{5}Bax^5 + \frac{1}{5}Abx^5 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^2,x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/3*A*a*x^3
```

3.2 $\int x\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Rubi [A] time = 0.0777268, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^6}{6}+\frac{a\int^{x^2}A\,dx}{2}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)\int^{x^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] B*b*x**6/6 + a*Integral(A, (x, x**2))/2 + (A*b/2 + B*a/2)*Integra
l(x, (x, x**2))
```

Mathematica [A] time = 0.0115034, size = 33, normalized size = 1.

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Maple [A] time = 0.001, size = 28, normalized size = 0.9

$$\frac{aAx^2}{2}+\frac{(Ab+Ba)\,x^4}{4}+\frac{bBx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

Maxima [A] time = 1.33638, size = 36, normalized size = 1.09

$$\frac{1}{6}Bbx^6 + \frac{1}{4}(Ba + Ab)x^4 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A] time = 0.197312, size = 1, normalized size = 0.03

$$\frac{1}{6}x^6bB + \frac{1}{4}x^4aB + \frac{1}{4}x^4bA + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + 1/2*x^2*a*A
```

Sympy [A] time = 0.087729, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2} + \frac{Bbx^6}{6} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**6/6 + x**4*(A*b/4 + B*a/4)
```

GIAC/XCAS [A] time = 0.21267, size = 39, normalized size = 1.18

$$\frac{1}{6}Bbx^6 + \frac{1}{4}Bax^4 + \frac{1}{4}Abx^4 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x,x, algorithm="giac")
```

```
[Out] 1/6*B*b*x^6 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/2*A*a*x^2
```

3.3 $\int (a + bx^2) (A + Bx^2) dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(aB + Ab) + aAx + \frac{1}{5}bBx^5$$

```
[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Rubi [A] time = 0.0364509, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{1}{3}x^3(aB + Ab) + aAx + \frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(A + B*x^2),x]

[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^5}{5} + a\int A\,dx + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A),x)

[Out] B*b*x**5/5 + a*Integral(A, x) + x**3*(A*b/3 + B*a/3)
```

Mathematica [A] time = 0.00839187, size = 28, normalized size = 1.

$$\frac{1}{3}x^3(aB + Ab) + aAx + \frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(A + B*x^2),x]

[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Maple [A] time = 0.002, size = 25, normalized size = 0.9

$$aAx + \frac{(Ab + Ba)\,x^3}{3} + \frac{bBx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A),x)

[Out] a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

Maxima [A] time = 1.33378, size = 32, normalized size = 1.14

$$\frac{1}{5}Bbx^5 + \frac{1}{3}(Ba + Ab)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Fricas [A] time = 0.21031, size = 1, normalized size = 0.04

$$\frac{1}{5}x^5bB + \frac{1}{3}x^3aB + \frac{1}{3}x^3bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b*B + 1/3*x^3*a*B + 1/3*x^3*b*A + x*a*A
```

Sympy [A] time = 0.083425, size = 26, normalized size = 0.93

$$Aax + \frac{Bbx^5}{5} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x + B*b*x**5/5 + x**3*(A*b/3 + B*a/3)
```

GIAC/XCAS [A] time = 0.210535, size = 35, normalized size = 1.25

$$\frac{1}{5}Bbx^5 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

3.4 $\int \frac{(a+bx^2)(A+Bx^2)}{x} dx$

Optimal. Leaf size=29

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Rubi [A] time = 0.0588078, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Aa\log(x^2)}{2} + \frac{Bb\int^{x^2} x\,dx}{2} + \frac{a\int^{x^2} B\,dx}{2} + \frac{b\int^{x^2} A\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)/x,x)
```

```
[Out] A*a*log(x**2)/2 + B*b*Integral(x, (x, x**2))/2 + a*Integral(B, (x
, x**2))/2 + b*Integral(A, (x, x**2))/2
```

Mathematica [A] time = 0.0148098, size = 29, normalized size = 1.

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Maple [A] time = 0.005, size = 28, normalized size = 1.

$$\frac{bBx^4}{4} + \frac{Ax^2b}{2} + \frac{Bx^2a}{2} + aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)/x,x)
```

```
[Out] 1/4*b*B*x^4+1/2*A*x^2*b+1/2*B*x^2*a+a*A*ln(x)
```

Maxima [A] time = 1.33308, size = 38, normalized size = 1.31

$$\frac{1}{4} Bbx^4 + \frac{1}{2} (Ba + Ab)x^2 + \frac{1}{2} Aa \log \left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x,x, algorithm="maxima")

[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + 1/2*A*a*log(x^2)
```

Fricas [A] time = 0.229161, size = 34, normalized size = 1.17

$$\frac{1}{4} Bbx^4 + \frac{1}{2} (Ba + Ab)x^2 + Aa \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x,x, algorithm="fricas")

[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] time = 1.08803, size = 27, normalized size = 0.93

$$Aa \log (x) + \frac{Bbx^4}{4} + x^2 \left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x,x)

[Out] A*a*log(x) + B*b*x**4/4 + x**2*(A*b/2 + B*a/2)
```

GIAC/XCAS [A] time = 0.216739, size = 41, normalized size = 1.41

$$\frac{1}{4} Bbx^4 + \frac{1}{2} Bax^2 + \frac{1}{2} Abx^2 + \frac{1}{2} Aa\ln \left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x,x, algorithm="giac")

[Out] 1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + 1/2*A*a*ln(x^2)
```

3.5 $\int \frac{(a+bx^2)(A+Bx^2)}{x^2} dx$

Optimal. Leaf size=26

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Rubi [A] time = 0.0484547, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^2,x]

[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{x}+\frac{Bbx^3}{3}+\frac{(Ab+Ba)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(B*x**2+A)/x**2,x)

[Out] -A*a/x + B*b*x**3/3 + (A*b + B*a)*Integral(A, x)/A
```

Mathematica [A] time = 0.0138997, size = 26, normalized size = 1.

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^2,x]

[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Maple [A] time = 0.006, size = 24, normalized size = 0.9

$$\frac{bBx^3}{3}+Axb+Bxa-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^2,x)

[Out] 1/3*b*B*x^3+A*x*b+B*x*a-a*A/x
```

Maxima [A] time = 1.34501, size = 32, normalized size = 1.23

$$\frac{1}{3}\,Bbx^3 + (Ba + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*x^3 + (B*a + A*b)*x - A*a/x
```

Fricas [A] time = 0.232597, size = 38, normalized size = 1.46

$$\frac{Bbx^4 + 3\,(Ba + Ab)x^2 - 3\,Aa}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^4 + 3*(B*a + A*b)*x^2 - 3*A*a)/x
```

Sympy [A] time = 1.07721, size = 20, normalized size = 0.77

$$-\frac{Aa}{x} + \frac{Bbx^3}{3} + x\,(Ab + Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a/x + B*b*x**3/3 + x*(A*b + B*a)
```

GIAC/XCAS [A] time = 0.241363, size = 31, normalized size = 1.19

$$\frac{1}{3}\,Bbx^3 + Bax + Abx - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*B*b*x^3 + B*a*x + A*b*x - A*a/x
```

3.6 $\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=29

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

```
[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Rubi [A] time = 0.0685896, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x^2))/x^3,x]

[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{2x^2}+\frac{b\int^{x^2}B\,dx}{2}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)/x**3,x)

[Out] -A*a/(2*x**2) + b*Integral(B, (x, x**2))/2 + (A*b/2 + B*a/2)*log(
x**2)
```

Mathematica [A] time = 0.0185427, size = 29, normalized size = 1.

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x^2))/x^3,x]

[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Maple [A] time = 0.008, size = 26, normalized size = 0.9

$$\frac{bBx^2}{2}+A\ln(x)\,b+B\ln(x)\,a-\frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)/x^3,x)
```

```
[Out] 1/2*b*B*x^2+A*ln(x)*b+B*ln(x)*a-1/2*a*A/x^2
```

Maxima [A] time = 1.37265, size = 38, normalized size = 1.31

$$\frac{1}{2}Bbx^2 + \frac{1}{2}(Ba + Ab)\log(x^2) - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*A*a/x^2
```

Fricas [A] time = 0.24138, size = 41, normalized size = 1.41

$$\frac{Bbx^4 + 2(Ba + Ab)x^2\log(x) - Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b*x^4 + 2*(B*a + A*b)*x^2*log(x) - A*a)/x^2
```

Sympy [A] time = 1.3499, size = 26, normalized size = 0.9

$$-\frac{Aa}{2x^2} + \frac{Bbx^2}{2} + (Ab + Ba)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) + B*b*x**2/2 + (A*b + B*a)*log(x)
```

GIAC/XCAS [A] time = 0.262869, size = 57, normalized size = 1.97

$$\frac{1}{2}Bbx^2 + \frac{1}{2}(Ba + Ab)\ln(x^2) - \frac{Bax^2 + Abx^2 + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*ln(x^2) - 1/2*(B*a*x^2 + A*b*x^2 +
A*a)/x^2
```

3.7 $\int \frac{(a+bx^2)(A+Bx^2)}{x^4} dx$

Optimal. Leaf size=26

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

```
[Out] -(a*A)/(3*x^3) - (A*b + a*B)/x + b*B*x
```

Rubi [A] time = 0.0497919, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^4,x]

[Out] -(a*A)/(3*x^3) - (A*b + a*B)/x + b*B*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{3x^3}+b\int B\,dx-\frac{Ab+Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(B*x**2+A)/x**4,x)

[Out] -A*a/(3*x**3) + b*Integral(B, x) - (A*b + B*a)/x
```

Mathematica [A] time = 0.0204076, size = 27, normalized size = 1.04

$$\frac{-aB-Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^4,x]

[Out] -(a*A)/(3*x^3) + (-(A*b) - a*B)/x + b*B*x
```

Maple [A] time = 0.007, size = 25, normalized size = 1.

$$bBx-\frac{Aa}{3\,x^3}-\frac{Ab+Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^4,x)

[Out] b*B*x-1/3*a*A/x^3-(A*b+B*a)/x
```

Maxima [A] time = 1.34497, size = 35, normalized size = 1.35

$$Bbx - \frac{3\,(Ba + Ab)x^2 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^4,x, algorithm="maxima")
```

```
[Out] B*b*x - 1/3*(3*(B*a + A*b)*x^2 + A*a)/x^3
```

Fricas [A] time = 0.225187, size = 39, normalized size = 1.5

$$\frac{3\,Bbx^4 - 3\,(Ba + Ab)x^2 - Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b*x^4 - 3*(B*a + A*b)*x^2 - A*a)/x^3
```

Sympy [A] time = 1.44718, size = 26, normalized size = 1.

$$Bbx - \frac{Aa + x^2\,(3Ab + 3Ba)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**4,x)
```

```
[Out] B*b*x - (A*a + x**2*(3*A*b + 3*B*a))/(3*x**3)
```

GIAC/XCAS [A] time = 0.254484, size = 38, normalized size = 1.46

$$Bbx - \frac{3\,Bax^2 + 3\,Abx^2 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^4,x, algorithm="giac")
```

```
[Out] B*b*x - 1/3*(3*B*a*x^2 + 3*A*b*x^2 + A*a)/x^3
```

3.8 $\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=29

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/(2*x^2) + b*B*Log[x]
```

Rubi [A] time = 0.0634178, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x^2))/x^5,x]

[Out] -(a*A)/(4*x^4) - (A*b + a*B)/(2*x^2) + b*B*Log[x]
```

Rubi in Sympy [A] time = 9.45912, size = 31, normalized size = 1.07

$$-\frac{Aa}{4x^4}+\frac{Bb\log\left(x^2\right)}{2}-\frac{\frac{Ab}{2}+\frac{Ba}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)/x**5,x)

[Out] -A*a/(4*x**4) + B*b*log(x**2)/2 - (A*b/2 + B*a/2)/x**2
```

Mathematica [A] time = 0.0307526, size = 31, normalized size = 1.07

$$\frac{-aB-Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x^2))/x^5,x]

[Out] -(a*A)/(4*x^4) + (-(A*b) - a*B)/(2*x^2) + b*B*Log[x]
```

Maple [A] time = 0.008, size = 28, normalized size = 1.

$$bB\ln(x)-\frac{Aa}{4\,x^4}-\frac{Ab}{2\,x^2}-\frac{Ba}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)/x^5,x)

[Out] b*B*ln(x)-1/4*a*A/x^4-1/2/x^2*A*b-1/2/x^2*B*a
```

Maxima [A] time = 1.34606, size = 41, normalized size = 1.41

$$\frac{1}{2}\,Bb\log\left(x^2\right)-\frac{2\,(Ba+Ab)x^2+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*log(x^2) - 1/4*(2*(B*a + A*b)*x^2 + A*a)/x^4
```

Fricas [A] time = 0.244359, size = 42, normalized size = 1.45

$$\frac{4\,Bbx^4\log(x)-2\,(Ba+Ab)x^2-Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*B*b*x^4*log(x) - 2*(B*a + A*b)*x^2 - A*a)/x^4
```

Sympy [A] time = 2.06332, size = 27, normalized size = 0.93

$$Bb\log(x)-\frac{Aa+x^2\,(2Ab+2Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**5,x)
```

```
[Out] B*b*log(x) - (A*a + x**2*(2*A*b + 2*B*a))/(4*x**4)
```

GIAC/XCAS [A] time = 0.246021, size = 53, normalized size = 1.83

$$\frac{1}{2}\,Bb\ln\left(x^2\right)-\frac{3\,Bbx^4+2\,Bax^2+2\,Abx^2+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b*ln(x^2) - 1/4*(3*B*b*x^4 + 2*B*a*x^2 + 2*A*b*x^2 + A*a)/x
^4
```

3.9 $\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=31

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(3*x^3) - (b*B)/x
```

Rubi [A] time = 0.0504188, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(3*x^3) - (b*B)/x
```

Rubi in Sympy [A] time = 7.62054, size = 27, normalized size = 0.87

$$-\frac{Aa}{5x^5}-\frac{Bb}{x}-\frac{\frac{Ab}{3}+\frac{Ba}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a/(5*x**5) - B*b/x - (A*b/3 + B*a/3)/x**3
```

Mathematica [A] time = 0.0207695, size = 33, normalized size = 1.06

$$\frac{-aB-Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) + (-(A*b) - a*B)/(3*x^3) - (b*B)/x
```

Maple [A] time = 0.007, size = 28, normalized size = 0.9

$$-\frac{Ab+Ba}{3\,x^3}-\frac{Aa}{5\,x^5}-\frac{Bb}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/3*(A*b+B*a)/x^3-1/5*a*A/x^5-b*B/x
```

Maxima [A] time = 1.38374, size = 39, normalized size = 1.26

$$-\frac{15\,Bbx^4 + 5\,(Ba + Ab)x^2 + 3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

Fricas [A] time = 0.227657, size = 39, normalized size = 1.26

$$-\frac{15\,Bbx^4 + 5\,(Ba + Ab)x^2 + 3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

Sympy [A] time = 2.09165, size = 32, normalized size = 1.03

$$-\frac{3Aa + 15Bbx^4 + x^2\,(5Ab + 5Ba)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**6,x)
```

```
[Out] -(3*A*a + 15*B*b*x**4 + x**2*(5*A*b + 5*B*a))/(15*x**5)
```

GIAC/XCAS [A] time = 0.269024, size = 42, normalized size = 1.35

$$-\frac{15\,Bbx^4 + 5\,Bax^2 + 5\,Abx^2 + 3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*B*a*x^2 + 5*A*b*x^2 + 3*A*a)/x^5
```

3.10 $\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=33

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Rubi [A] time = 0.0698478, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^7,x]

[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Rubi in Sympy [A] time = 9.8017, size = 31, normalized size = 0.94

$$-\frac{Aa}{6x^6}-\frac{Bb}{2x^2}-\frac{\frac{Ab}{4}+\frac{Ba}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(B*x**2+A)/x**7,x)

[Out] -A*a/(6*x**6) - B*b/(2*x**2) - (A*b/4 + B*a/4)/x**4
```

Mathematica [A] time = 0.0155557, size = 35, normalized size = 1.06

$$\frac{-aB-Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^7,x]

[Out] -(a*A)/(6*x^6) + (-(A*b) - a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Maple [A] time = 0.007, size = 28, normalized size = 0.9

$$-\frac{Aa}{6\,x^6}-\frac{Ab+Ba}{4\,x^4}-\frac{Bb}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^7,x)

[Out] -1/6*a*A/x^6-1/4*(A*b+B*a)/x^4-1/2*b*B/x^2
```

Maxima [A] time = 1.33117, size = 39, normalized size = 1.18

$$-\frac{6 \, B b x^{4} + 3 \, (B a + A b) x^{2} + 2 \, A a}{12 \, x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^7,x, algorithm="maxima")

[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

Fricas [A] time = 0.2279, size = 39, normalized size = 1.18

$$-\frac{6 \, B b x^{4} + 3 \, (B a + A b) x^{2} + 2 \, A a}{12 \, x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^7,x, algorithm="fricas")

[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

Sympy [A] time = 2.61765, size = 32, normalized size = 0.97

$$-\frac{2Aa + 6Bbx^{4} + x^{2} (3Ab + 3Ba)}{12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**7,x)

[Out] -(2*A*a + 6*B*b*x**4 + x**2*(3*A*b + 3*B*a))/(12*x**6)
```

GIAC/XCAS [A] time = 0.282054, size = 42, normalized size = 1.27

$$-\frac{6 \, B b x^{4} + 3 \, B a x^{2} + 3 \, A b x^{2} + 2 \, A a}{12 \, x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^7,x, algorithm="giac")

[Out] -1/12*(6*B*b*x^4 + 3*B*a*x^2 + 3*A*b*x^2 + 2*A*a)/x^6
```

3.11 $\int x^2 \left(a+bx^2\right)^2 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7
 + (b^2*B*x^9)/9
```

Rubi [A] time = 0.115645, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7
 + (b^2*B*x^9)/9
```

Rubi in Sympy [A] time = 16.858, size = 49, normalized size = 0.89

$$\frac{Aa^2x^3}{3}+\frac{Bb^2x^9}{9}+\frac{ax^5\left(2Ab+Ba\right)}{5}+\frac{bx^7\left(Ab+2Ba\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**9/9 + a*x**5*(2*A*b + B*a)/5 + b*x**7*(
A*b + 2*B*a)/7
```

Mathematica [A] time = 0.0141036, size = 55, normalized size = 1.

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7
 + (b^2*B*x^9)/9
```

Maple [A] time = 0., size = 52, normalized size = 1.

$$\frac{b^2Bx^9}{9}+\frac{\left(b^2A+2\,abB\right)x^7}{7}+\frac{\left(2\,abA+a^2B\right)x^5}{5}+\frac{a^2Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/9*b^2*B*x^9+1/7*(A*b^2+2*B*a*b)*x^7+1/5*(2*A*a*b+B*a^2)*x^5+1/3
*a^2*A*x^3
```

Maxima [A] time = 1.34559, size = 69, normalized size = 1.25

$$\frac{1}{9}Bb^2x^9 + \frac{1}{7}\left(2Bab + Ab^2\right)x^7 + \frac{1}{3}Aa^2x^3 + \frac{1}{5}\left(Ba^2 + 2Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(
B*a^2 + 2*A*a*b)*x^5
```

Fricas [A] time = 0.211573, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9b^2B + \frac{2}{7}x^7baB + \frac{1}{7}x^7b^2A + \frac{1}{5}x^5a^2B + \frac{2}{5}x^5baA + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/5*x^5*a^2*B + 2
/5*x^5*b*a*A + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.111041, size = 56, normalized size = 1.02

$$\frac{Aa^2x^3}{3} + \frac{Bb^2x^9}{9} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**9/9 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**
5*(2*A*a*b/5 + B*a**2/5)
```

GIAC/XCAS [A] time = 0.250098, size = 72, normalized size = 1.31

$$\frac{1}{9}Bb^2x^9 + \frac{2}{7}Babx^7 + \frac{1}{7}Ab^2x^7 + \frac{1}{5}Ba^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/5*B*a^2*x^5 + 2
/5*A*a*b*x^5 + 1/3*A*a^2*x^3
```

3.12 $\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^3(Ab-aB)}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^3)/(6*b^2) + (B*(a + b*x^2)^4)/(8*b^2)
```

Rubi [A] time = 0.159326, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\left(a+bx^2\right)^3(Ab-aB)}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^3)/(6*b^2) + (B*(a + b*x^2)^4)/(8*b^2)
```

Rubi in Sympy [A] time = 15.7249, size = 34, normalized size = 0.81

$$\frac{B\left(a+bx^2\right)^4}{8b^2}+\frac{\left(a+bx^2\right)^3(Ab-Ba)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**4/(8*b**2) + (a + b*x**2)**3*(A*b - B*a)/(6*b**2)
```

Mathematica [A] time = 0.0216139, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2A+4bx^4(2aB+Ab)+6ax^2(aB+2Ab)+3b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (x^2*(12*a^2*A + 6*a*(2*A*b + a*B)*x^2 + 4*b*(A*b + 2*a*B)*x^4 +
3*b^2*B*x^6))/24
```

Maple [A] time = 0.001, size = 52, normalized size = 1.2

$$\frac{b^2Bx^8}{8}+\frac{\left(b^2A+2\,abB\right)x^6}{6}+\frac{\left(2\,abA+a^2B\right)x^4}{4}+\frac{a^2Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/8*b^2*B*x^8+1/6*(A*b^2+2*B*a*b)*x^6+1/4*(2*A*a*b+B*a^2)*x^4+1/2
*a^2*A*x^2
```

Maxima [A] time = 1.34988, size = 69, normalized size = 1.64

$$\frac{1}{8}\,Bb^2x^8 + \frac{1}{6}\,\left(2\,Bab + Ab^2\right)x^6 + \frac{1}{2}\,Aa^2x^2 + \frac{1}{4}\,\left(Ba^2 + 2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(
B*a^2 + 2*A*a*b)*x^4
```

Fricas [A] time = 0.220117, size = 1, normalized size = 0.02

$$\frac{1}{8}x^8b^2B + \frac{1}{3}x^6baB + \frac{1}{6}x^6b^2A + \frac{1}{4}x^4a^2B + \frac{1}{2}x^4baA + \frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2*B + 1/3*x^6*b*a*B + 1/6*x^6*b^2*A + 1/4*x^4*a^2*B + 1
/2*x^4*b*a*A + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.120903, size = 53, normalized size = 1.26

$$\frac{Aa^2x^2}{2} + \frac{Bb^2x^8}{8} + x^6\left(\frac{Ab^2}{6} + \frac{Bab}{3}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**8/8 + x**6*(A*b**2/6 + B*a*b/3) + x**4*
(A*a*b/2 + B*a**2/4)
```

GIAC/XCAS [A] time = 0.249948, size = 72, normalized size = 1.71

$$\frac{1}{8}\,Bb^2x^8 + \frac{1}{3}\,Babx^6 + \frac{1}{6}\,Ab^2x^6 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{2}\,Aabx^4 + \frac{1}{2}\,Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x,x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/4*B*a^2*x^4 + 1
/2*A*a*b*x^4 + 1/2*A*a^2*x^2
```

3.13 $\int \left(a+bx^2\right)^2 \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=50

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^
2*B*x^7)/7
```

Rubi [A] time = 0.0610966, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^
2*B*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bb^2x^7}{7}+a^2\int A\,dx+\frac{ax^3\,(2Ab+Ba)}{3}+\frac{bx^5\,(Ab+2Ba)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] B*b**2*x**7/7 + a**2*Integral(A, x) + a*x**3*(2*A*b + B*a)/3 + b*
x**5*(A*b + 2*B*a)/5
```

Mathematica [A] time = 0.0130988, size = 50, normalized size = 1.

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^
2*B*x^7)/7
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{b^2Bx^7}{7}+\frac{\left(b^2A+2\,abB\right)x^5}{5}+\frac{\left(2\,abA+a^2B\right)x^3}{3}+a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/7*b^2*B*x^7+1/5*(A*b^2+2*B*a*b)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2
*A*x
```

Maxima [A] time = 1.3395, size = 65, normalized size = 1.3

$$\frac{1}{7}Bb^2x^7 + \frac{1}{5}\left(2Bab + Ab^2\right)x^5 + Aa^2x + \frac{1}{3}\left(Ba^2 + 2Aab\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2
+ 2*A*a*b)*x^3
```

Fricas [A] time = 0.206069, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7b^2B + \frac{2}{5}x^5baB + \frac{1}{5}x^5b^2A + \frac{1}{3}x^3a^2B + \frac{2}{3}x^3baA + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2*B + 2/5*x^5*b*a*B + 1/5*x^5*b^2*A + 1/3*x^3*a^2*B + 2
/3*x^3*b*a*A + x*a^2*A
```

Sympy [A] time = 0.112546, size = 53, normalized size = 1.06

$$Aa^2x + \frac{Bb^2x^7}{7} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**7/7 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**3*(2*
A*a*b/3 + B*a**2/3)
```

GIAC/XCAS [A] time = 0.23633, size = 68, normalized size = 1.36

$$\frac{1}{7}Bb^2x^7 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2
/3*A*a*b*x^3 + A*a^2*x
```

3.14 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=43

$$a^2A\log(x)+aAbx^2+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4$$

```
[Out] a*A*b*x^2 + (A*b^2*x^4)/4 + (B*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x
]
```

Rubi [A] time = 0.0807077, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$a^2A\log(x)+aAbx^2+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] a*A*b*x^2 + (A*b^2*x^4)/4 + (B*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Aa^2\log\left(x^2\right)}{2}+Aabx^2+\frac{Ab^2\int^{x^2}x\,dx}{2}+\frac{B\left(a+bx^2\right)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*log(x**2)/2 + A*a*b*x**2 + A*b**2*Integral(x, (x, x**2))/2
 + B*(a + b*x**2)**3/(6*b)
```

Mathematica [A] time = 0.0258223, size = 51, normalized size = 1.19

$$a^2A\log(x)+\frac{1}{4}bx^4(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^6)/6
 + a^2*A*Log[x]
```

Maple [A] time = 0.003, size = 51, normalized size = 1.2

$$\frac{Bb^2x^6}{6}+\frac{Ab^2x^4}{4}+\frac{Bx^4ab}{2}+aAbx^2+\frac{Bx^2a^2}{2}+a^2A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x,x)
```

```
[Out] 1/6*B*b^2*x^6+1/4*A*b^2*x^4+1/2*B*x^4*a*b+a*A*b*x^2+1/2*B*x^2*a^2
+a^2*A*ln(x)
```

Maxima [A] time = 1.35194, size = 70, normalized size = 1.63

$$\frac{1}{6}Bb^2x^6 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + \frac{1}{2}Aa^2\log\left(x^2\right) + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/2*A*a^2*log(x^2) +
1/2*(B*a^2 + 2*A*a*b)*x^2
```

Fricas [A] time = 0.244783, size = 66, normalized size = 1.53

$$\frac{1}{6}Bb^2x^6 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + Aa^2\log\left(x\right) + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + A*a^2*log(x) + 1/2*(B
*a^2 + 2*A*a*b)*x^2
```

Sympy [A] time = 1.18992, size = 49, normalized size = 1.14

$$Aa^2\log\left(x\right) + \frac{Bb^2x^6}{6} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**6/6 + x**4*(A*b**2/4 + B*a*b/2) + x**2*
(A*a*b + B*a**2/2)
```

GIAC/XCAS [A] time = 0.227323, size = 72, normalized size = 1.67

$$\frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Ba^2x^2 + Aabx^2 + \frac{1}{2}Aa^2\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*B*a^2*x^2 + A
*a*b*x^2 + 1/2*A*a^2*ln(x^2)
```

3.15 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2
*B*x^5)/5
```

Rubi [A] time = 0.080669, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2
*B*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{x}+\frac{Bb^2x^5}{5}+\frac{bx^3\left(Ab+2Ba\right)}{3}+\frac{a\left(2Ab+Ba\right)\int B\,dx}{B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**5/5 + b*x**3*(A*b + 2*B*a)/3 + a*(2*A*b + B
*a)*Integral(B, x)/B
```

Mathematica [A] time = 0.0265244, size = 48, normalized size = 1.

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2
*B*x^5)/5
```

Maple [A] time = 0.006, size = 49, normalized size = 1.

$$\frac{b^2Bx^5}{5}+\frac{Ax^3b^2}{3}+\frac{2\,Bx^3ab}{3}+2\,Axab+Bxa^2-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^2,x)
```

```
[Out] 1/5*b^2*B*x^5+1/3*A*x^3*b^2+2/3*B*x^3*a*b+2*A*x*a*b+B*x*a^2-a^2*A
/x
```

Maxima [A] time = 1.34916, size = 65, normalized size = 1.35

$$\frac{1}{5}Bb^2x^5 + \frac{1}{3}\left(2Bab + Ab^2\right)x^3 - \frac{Aa^2}{x} + \left(Ba^2 + 2Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/3*(2*B*a*b + A*b^2)*x^3 - A*a^2/x + (B*a^2 + 2*
A*a*b)*x
```

Fricas [A] time = 0.240336, size = 72, normalized size = 1.5

$$\frac{3Bb^2x^6 + 5\left(2Bab + Ab^2\right)x^4 - 15Aa^2 + 15\left(Ba^2 + 2Aab\right)x^2}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^6 + 5*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 15*(B*a^
2 + 2*A*a*b)*x^2)/x
```

Sympy [A] time = 1.21373, size = 48, normalized size = 1.

$$-\frac{Aa^2}{x} + \frac{Bb^2x^5}{5} + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right) + x\left(2Aab + Ba^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**5/5 + x**3*(A*b**2/3 + 2*B*a*b/3) + x*(2*A*
a*b + B*a**2)
```

GIAC/XCAS [A] time = 0.230963, size = 65, normalized size = 1.35

$$\frac{1}{5}Bb^2x^5 + \frac{2}{3}Babx^3 + \frac{1}{3}Ab^2x^3 + Ba^2x + 2Aabx - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + B*a^2*x + 2*A*a*b
*x - A*a^2/x
```

3.16 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

```
[Out] -(a^2*A)/(2*x^2) + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^4)/4 + a*(2
*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.133702, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^4)/4 + a*(2
*A*b + a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{2x^2}+\frac{Bb^2\int^{x^2}x\,dx}{2}+\frac{a\left(2Ab+Ba\right)\log\left(x^2\right)}{2}+\frac{b\left(Ab+2Ba\right)\int^{x^2}A\,dx}{2A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*Integral(x, (x, x**2))/2 + a*(2*A*b + B
*a)*log(x**2)/2 + b*(A*b + 2*B*a)*Integral(A, (x, x**2))/(2*A)
```

Mathematica [A] time = 0.0398264, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2A}{x^2}+2bx^2(2aB+Ab)+4a\log(x)(aB+2Ab)+b^2Bx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*A)/x^2 + 2*b*(A*b + 2*a*B)*x^2 + b^2*B*x^4 + 4*a*(2*A*b
+ a*B)*Log[x])/4
```

Maple [A] time = 0.01, size = 50, normalized size = 1.

$$\frac{b^2Bx^4}{4}+\frac{Ax^2b^2}{2}+Bx^2ab+2\,A\ln(x)\,ab+B\ln(x)\,a^2-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(B*x^2+A)/x^3,x)
```

```
[Out] 1/4*b^2*B*x^4+1/2*A*x^2*b^2+B*x^2*a*b+2*A*ln(x)*a*b+B*ln(x)*a^2-1
/2*a^2*A/x^2
```

Maxima [A] time = 1.35219, size = 70, normalized size = 1.37

$$\frac{1}{4}\,Bb^2x^4 + \frac{1}{2}\left(2\,Bab + Ab^2\right)x^2 + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)\log\left(x^2\right) - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + 1/2*(2*B*a*b + A*b^2)*x^2 + 1/2*(B*a^2 + 2*A*a*b)
*log(x^2) - 1/2*A*a^2/x^2
```

Fricas [A] time = 0.233536, size = 73, normalized size = 1.43

$$\frac{Bb^2x^6 + 2\left(2\,Bab + Ab^2\right)x^4 + 4\left(Ba^2 + 2\,Aab\right)x^2\log\left(x\right) - 2\,Aa^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^6 + 2*(2*B*a*b + A*b^2)*x^4 + 4*(B*a^2 + 2*A*a*b)*x^
2*log(x) - 2*A*a^2)/x^2
```

Sympy [A] time = 1.54329, size = 48, normalized size = 0.94

$$-\frac{Aa^2}{2x^2} + \frac{Bb^2x^4}{4} + a\left(2Ab + Ba\right)\log\left(x\right) + x^2\left(\frac{Ab^2}{2} + Bab\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**2*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**4/4 + a*(2*A*b + B*a)*log(x) + x**2*
(A*b**2/2 + B*a*b)
```

GIAC/XCAS [A] time = 0.232184, size = 95, normalized size = 1.86

$$\frac{1}{4}\,Bb^2x^4 + Babx^2 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)\ln\left(x^2\right) - \frac{Ba^2x^2 + 2\,Aabx^2 + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + B*a*b*x^2 + 1/2*A*b^2*x^2 + 1/2*(B*a^2 + 2*A*a*b)
*ln(x^2) - 1/2*(B*a^2*x^2 + 2*A*a*b*x^2 + A*a^2)/x^2
```

3.17 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^4} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

```
[Out] -(a^2*A)/(3*x^3) - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2
*B*x^3)/3
```

Rubi [A] time = 0.0822763, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2
*B*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{3x^3}+\frac{Bb^2x^3}{3}-\frac{a\,(2Ab+Ba)}{x}+\frac{b\,(Ab+2Ba)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) + B*b**2*x**3/3 - a*(2*A*b + B*a)/x + b*(A*b + 2
*B*a)*Integral(A, x)/A
```

Mathematica [A] time = 0.0346548, size = 50, normalized size = 1.04

$$\frac{a^2(-B)-2aAb}{x}-\frac{a^2A}{3x^3}+bx(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) + (-2*a*A*b - a^2*B)/x + b*(A*b + 2*a*B)*x + (b^
2*B*x^3)/3
```

Maple [A] time = 0.007, size = 46, normalized size = 1.

$$\frac{b^2Bx^3}{3}+Axb^2+2\,Bxab-\frac{Aa^2}{3\,x^3}-\frac{a\,(2\,Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^4,x)
```

```
[Out] 1/3*b^2*B*x^3+A*x*b^2+2*B*x*a*b-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x
```

Maxima [A] time = 1.34977, size = 68, normalized size = 1.42

$$\frac{1}{3}Bb^2x^3 + \left(2\,Bab + Ab^2\right)x - \frac{Aa^2 + 3\left(Ba^2 + 2\,Aab\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*x - 1/3*(A*a^2 + 3*(B*a^2 + 2*A
*a*b)*x^2)/x^3
```

Fricas [A] time = 0.243247, size = 70, normalized size = 1.46

$$\frac{Bb^2x^6 + 3\left(2\,Bab + Ab^2\right)x^4 - Aa^2 - 3\left(Ba^2 + 2\,Aab\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + 3*(2*B*a*b + A*b^2)*x^4 - A*a^2 - 3*(B*a^2 + 2*A
*a*b)*x^2)/x^3
```

Sympy [A] time = 1.65045, size = 49, normalized size = 1.02

$$\frac{Bb^2x^3}{3} + x\left(Ab^2 + 2Bab\right) - \frac{Aa^2 + x^2\left(6Aab + 3Ba^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**2*x**3/3 + x*(A*b**2 + 2*B*a*b) - (A*a**2 + x**2*(6*A*a*b +
3*B*a**2))/(3*x**3)
```

GIAC/XCAS [A] time = 0.227837, size = 68, normalized size = 1.42

$$\frac{1}{3}Bb^2x^3 + 2\,Babx + Ab^2x - \frac{3\,Ba^2x^2 + 6\,Aabx^2 + Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + 2*B*a*b*x + A*b^2*x - 1/3*(3*B*a^2*x^2 + 6*A*a*b*
x^2 + A*a^2)/x^3
```

3.18 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/(2*x^2) + (b^2*B*x^2)/2 + b*
(A*b + 2*a*B)*Log[x]
```

Rubi [A] time = 0.122924, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/(2*x^2) + (b^2*B*x^2)/2 + b*
(A*b + 2*a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{4x^4}-\frac{a\left(2Ab+Ba\right)}{2x^2}+\frac{b^2\int^{x^2}B\,dx}{2}+\frac{b\left(Ab+2Ba\right)\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**5,x)
```

```
[Out] -A*a**2/(4*x**4) - a*(2*A*b + B*a)/(2*x**2) + b**2*Integral(B, (x
, x**2))/2 + b*(A*b + 2*B*a)*log(x**2)/2
```

Mathematica [A] time = 0.0411089, size = 50, normalized size = 0.98

$$b\log(x)(2aB+Ab)-\frac{a^2\left(A+2Bx^2\right)+4aAbx^2-2b^2Bx^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] -(4*a*A*b*x^2 - 2*b^2*B*x^6 + a^2*(A + 2*B*x^2))/(4*x^4) + b*(A*b
 + 2*a*B)*Log[x]
```

Maple [A] time = 0.008, size = 51, normalized size = 1.

$$\frac{b^2Bx^2}{2}+A\ln(x)\,b^2+2\,B\ln(x)\,ab-\frac{Aa^2}{4\,x^4}-\frac{abA}{x^2}-\frac{a^2B}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^5,x)
```

```
[Out] 1/2*b^2*B*x^2+A*ln(x)*b^2+2*B*ln(x)*a*b-1/4*a^2*A/x^4-a/x^2*A*b-1
/2*a^2/x^2*B
```

Maxima [A] time = 1.35162, size = 73, normalized size = 1.43

$$\frac{1}{2}\,Bb^2x^2 + \frac{1}{2}\left(2\,Bab + Ab^2\right)\log\left(x^2\right) - \frac{Aa^2 + 2\left(Ba^2 + 2\,Aab\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(A*a^2 + 2*(
B*a^2 + 2*A*a*b)*x^2)/x^4
```

Fricas [A] time = 0.235866, size = 74, normalized size = 1.45

$$\frac{2\,Bb^2x^6 + 4\left(2\,Bab + Ab^2\right)x^4\log\left(x\right) - Aa^2 - 2\left(Ba^2 + 2\,Aab\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^2*x^6 + 4*(2*B*a*b + A*b^2)*x^4*log(x) - A*a^2 - 2*(B*
a^2 + 2*A*a*b)*x^2)/x^4
```

Sympy [A] time = 2.77345, size = 49, normalized size = 0.96

$$\frac{Bb^2x^2}{2} + b\left(Ab + 2Ba\right)\log\left(x\right) - \frac{Aa^2 + x^2\left(4Aab + 2Ba^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**2*x**2/2 + b*(A*b + 2*B*a)*log(x) - (A*a**2 + x**2*(4*A*a*b
+ 2*B*a**2))/(4*x**4)
```

GIAC/XCAS [A] time = 0.237539, size = 97, normalized size = 1.9

$$\frac{1}{2}\,Bb^2x^2 + \frac{1}{2}\left(2\,Bab + Ab^2\right)\ln\left(x^2\right) - \frac{6\,Babx^4 + 3\,Ab^2x^4 + 2\,Ba^2x^2 + 4\,Aabx^2 + Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*ln(x^2) - 1/4*(6*B*a*b*x^4
+ 3*A*b^2*x^4 + 2*B*a^2*x^2 + 4*A*a*b*x^2 + A*a^2)/x^4
```

3.19 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/
x + b^2*B*x
```

Rubi [A] time = 0.0819563, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/
x + b^2*B*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{5x^5}-\frac{a\left(2Ab+Ba\right)}{3x^3}+b^2\int B\,dx-\frac{b\left(Ab+2Ba\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a**2/(5*x**5) - a*(2*A*b + B*a)/(3*x**3) + b**2*Integral(B, x)
 - b*(A*b + 2*B*a)/x
```

Mathematica [A] time = 0.0344791, size = 48, normalized size = 1.

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/
x + b^2*B*x
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$-\frac{Aa^2}{5\,x^5}-\frac{a\left(2\,Ab+Ba\right)}{3\,x^3}-\frac{b\left(Ab+2\,Ba\right)}{x}+b^2Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

Maxima [A] time = 1.34827, size = 69, normalized size = 1.44

$$Bb^2x - \frac{15\left(2\,Bab + Ab^2\right)x^4 + 3\,Aa^2 + 5\left(Ba^2 + 2\,Aab\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^6,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/15*(15*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 + 2
*A*a*b)*x^2)/x^5
```

Fricas [A] time = 0.22893, size = 72, normalized size = 1.5

$$\frac{15\,Bb^2x^6 - 15\left(2\,Bab + Ab^2\right)x^4 - 3\,Aa^2 - 5\left(Ba^2 + 2\,Aab\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*b^2*x^6 - 15*(2*B*a*b + A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^
2 + 2*A*a*b)*x^2)/x^5
```

Sympy [A] time = 3.00719, size = 51, normalized size = 1.06

$$Bb^2x - \frac{3Aa^2 + x^4\left(15Ab^2 + 30Bab\right) + x^2\left(10Aab + 5Ba^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**2*x - (3*A*a**2 + x**4*(15*A*b**2 + 30*B*a*b) + x**2*(10*A*a
*b + 5*B*a**2))/(15*x**5)
```

GIAC/XCAS [A] time = 0.232235, size = 72, normalized size = 1.5

$$Bb^2x - \frac{30\,Babx^4 + 15\,Ab^2x^4 + 5\,Ba^2x^2 + 10\,Aabx^2 + 3\,Aa^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^6,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/15*(30*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 + 10*A*
a*b*x^2 + 3*A*a^2)/x^5
```

3.20 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{2x^2}+b^2B\log(x)$$

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/
(2*x^2) + b^2*B*Log[x]
```

Rubi [A] time = 0.0988146, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{2x^2}+b^2B\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/
(2*x^2) + b^2*B*Log[x]
```

Rubi in Sympy [A] time = 15.6329, size = 51, normalized size = 1.

$$-\frac{Aa^2}{6x^6}+\frac{Bb^2\log\left(x^2\right)}{2}-\frac{a\left(2Ab+Ba\right)}{4x^4}-\frac{b\left(Ab+2Ba\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**2/(6*x**6) + B*b**2*log(x**2)/2 - a*(2*A*b + B*a)/(4*x**4)
- b*(A*b + 2*B*a)/(2*x**2)
```

Mathematica [A] time = 0.0482106, size = 54, normalized size = 1.06

$$b^2B\log(x)-\frac{a^2\left(2A+3Bx^2\right)+6abx^2\left(A+2Bx^2\right)+6Ab^2x^4}{12x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -(6*A*b^2*x^4 + 6*a*b*x^2*(A + 2*B*x^2) + a^2*(2*A + 3*B*x^2))/(1
2*x^6) + b^2*B*Log[x]
```

Maple [A] time = 0.009, size = 52, normalized size = 1.

$$b^2B\ln(x)-\frac{Aa^2}{6\,x^6}-\frac{abA}{2\,x^4}-\frac{a^2B}{4\,x^4}-\frac{b^2A}{2\,x^2}-\frac{abB}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^7,x)
```

```
[Out] b^2*B*ln(x)-1/6*a^2*A/x^6-1/2*a/x^4*A*b-1/4*a^2/x^4*B-1/2*b^2/x^2
*A-b/x^2*B*a
```

Maxima [A] time = 1.34602, size = 74, normalized size = 1.45

$$\frac{1}{2}\,Bb^2\log\left(x^2\right) - \frac{6\left(2\,Bab + Ab^2\right)x^4 + 2\,Aa^2 + 3\left(Ba^2 + 2\,Aab\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(6*(2*B*a*b + A*b^2)*x^4 + 2*A*a^2 + 3*
(B*a^2 + 2*A*a*b)*x^2)/x^6
```

Fricas [A] time = 0.238589, size = 74, normalized size = 1.45

$$\frac{12\,Bb^2x^6\log\left(x\right) - 6\left(2\,Bab + Ab^2\right)x^4 - 2\,Aa^2 - 3\left(Ba^2 + 2\,Aab\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*b^2*x^6*log(x) - 6*(2*B*a*b + A*b^2)*x^4 - 2*A*a^2 - 3
*(B*a^2 + 2*A*a*b)*x^2)/x^6
```

Sympy [A] time = 4.57595, size = 53, normalized size = 1.04

$$Bb^2\log\left(x\right) - \frac{2Aa^2 + x^4\left(6Ab^2 + 12Bab\right) + x^2\left(6Aab + 3Ba^2\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**2*log(x) - (2*A*a**2 + x**4*(6*A*b**2 + 12*B*a*b) + x**2*(6*
A*a*b + 3*B*a**2))/(12*x**6)
```

GIAC/XCAS [A] time = 0.229301, size = 89, normalized size = 1.75

$$\frac{1}{2}\,Bb^2\ln\left(x^2\right) - \frac{11\,Bb^2x^6 + 12\,Babx^4 + 6\,Ab^2x^4 + 3\,Ba^2x^2 + 6\,Aabx^2 + 2\,Aa^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*ln(x^2) - 1/12*(11*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x
^4 + 3*B*a^2*x^2 + 6*A*a*b*x^2 + 2*A*a^2)/x^6
```

3.21 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8} \, dx$

Optimal. Leaf size=53

$$-\frac{a^2 A}{7x^7} - \frac{a(aB + 2Ab)}{5x^5} - \frac{b(2aB + Ab)}{3x^3} - \frac{b^2 B}{x}$$

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/
(3*x^3) - (b^2*B)/x
```

Rubi [A] time = 0.0817739, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{a^2 A}{7x^7} - \frac{a(aB + 2Ab)}{5x^5} - \frac{b(2aB + Ab)}{3x^3} - \frac{b^2 B}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/
(3*x^3) - (b^2*B)/x
```

Rubi in Sympy [A] time = 12.9016, size = 48, normalized size = 0.91

$$-\frac{Aa^2}{7x^7} - \frac{Bb^2}{x} - \frac{a\,(2Ab + Ba)}{5x^5} - \frac{b\,(Ab + 2Ba)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**8,x)
```

```
[Out] -A*a**2/(7*x**7) - B*b**2/x - a*(2*A*b + B*a)/(5*x**5) - b*(A*b +
 2*B*a)/(3*x**3)
```

Mathematica [A] time = 0.0308534, size = 56, normalized size = 1.06

$$-\frac{3a^2\left(5A + 7Bx^2\right) + 14abx^2\left(3A + 5Bx^2\right) + 35b^2x^4\left(A + 3Bx^2\right)}{105x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -(35*b^2*x^4*(A + 3*B*x^2) + 14*a*b*x^2*(3*A + 5*B*x^2) + 3*a^2*(
5*A + 7*B*x^2))/(105*x^7)
```

Maple [A] time = 0.007, size = 48, normalized size = 0.9

$$-\frac{Aa^2}{7\,x^7} - \frac{a\,(2\,Ab + Ba)}{5\,x^5} - \frac{b\,(Ab + 2\,Ba)}{3\,x^3} - \frac{Bb^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^8,x)
```

```
[Out] -1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/
x
```

Maxima [A] time = 1.34511, size = 72, normalized size = 1.36

$$-\frac{105\,Bb^2x^6 + 35\,\left(2\,Bab + Ab^2\right)x^4 + 15\,Aa^2 + 21\,\left(Ba^2 + 2\,Aab\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*
(B*a^2 + 2*A*a*b)*x^2)/x^7
```

Fricas [A] time = 0.220799, size = 72, normalized size = 1.36

$$-\frac{105\,Bb^2x^6 + 35\,\left(2\,Bab + Ab^2\right)x^4 + 15\,Aa^2 + 21\,\left(Ba^2 + 2\,Aab\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*
(B*a^2 + 2*A*a*b)*x^2)/x^7
```

Sympy [A] time = 4.8222, size = 56, normalized size = 1.06

$$-\frac{15Aa^2 + 105Bb^2x^6 + x^4\left(35Ab^2 + 70Bab\right) + x^2\left(42Aab + 21Ba^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**8,x)
```

```
[Out] -(15*A*a**2 + 105*B*b**2*x**6 + x**4*(35*A*b**2 + 70*B*a*b) + x**
2*(42*A*a*b + 21*B*a**2))/(105*x**7)
```

GIAC/XCAS [A] time = 0.227301, size = 74, normalized size = 1.4

$$-\frac{105\,Bb^2x^6 + 70\,Babx^4 + 35\,Ab^2x^4 + 21\,Ba^2x^2 + 42\,Aabx^2 + 15\,Aa^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 70*B*a*b*x^4 + 35*A*b^2*x^4 + 21*B*a^2*x^
2 + 42*A*a*b*x^2 + 15*A*a^2)/x^7
```

3.22 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=48

$$\frac{(a+bx^2)^3(Ab-4aB)}{24a^2x^6}-\frac{A(a+bx^2)^3}{8ax^8}$$

```
[Out] -(A*(a + b*x^2)^3)/(8*a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(24*
a^2*x^6)
```

Rubi [A] time = 0.107831, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{(a+bx^2)^3(Ab-4aB)}{24a^2x^6}-\frac{A(a+bx^2)^3}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -(A*(a + b*x^2)^3)/(8*a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(24*
a^2*x^6)
```

Rubi in Sympy [A] time = 15.6937, size = 51, normalized size = 1.06

$$-\frac{Aa^2}{8x^8}-\frac{Bb^2}{2x^2}-\frac{a(2Ab+Ba)}{6x^6}-\frac{b(Ab+2Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**9,x)
```

```
[Out] -A*a**2/(8*x**8) - B*b**2/(2*x**2) - a*(2*A*b + B*a)/(6*x**6) - b
*(A*b + 2*B*a)/(4*x**4)
```

Mathematica [A] time = 0.0285649, size = 55, normalized size = 1.15

$$-\frac{a^2(3A+4Bx^2)+4abx^2(2A+3Bx^2)+6b^2x^4(A+2Bx^2)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -(6*b^2*x^4*(A + 2*B*x^2) + 4*a*b*x^2*(2*A + 3*B*x^2) + a^2*(3*A
+ 4*B*x^2))/(24*x^8)
```

Maple [A] time = 0.008, size = 48, normalized size = 1.

$$-\frac{a(2Ab+Ba)}{6x^6}-\frac{b(Ab+2Ba)}{4x^4}-\frac{Aa^2}{8x^8}-\frac{Bb^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^9,x)
```

```
[Out] -1/6*a*(2*A*b+B*a)/x^6-1/4*b*(A*b+2*B*a)/x^4-1/8*A*a^2/x^8-1/2*B*
b^2/x^2
```

Maxima [A] time = 1.34502, size = 72, normalized size = 1.5

$$-\frac{12\,Bb^2x^6+6\,\left(2\,Bab+Ab^2\right)x^4+3\,Aa^2+4\,\left(Ba^2+2\,Aab\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^
2 + 2*A*a*b)*x^2)/x^8
```

Fricas [A] time = 0.234853, size = 72, normalized size = 1.5

$$-\frac{12\,Bb^2x^6+6\,\left(2\,Bab+Ab^2\right)x^4+3\,Aa^2+4\,\left(Ba^2+2\,Aab\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^
2 + 2*A*a*b)*x^2)/x^8
```

Sympy [A] time = 6.60162, size = 56, normalized size = 1.17

$$-\frac{3Aa^2+12Bb^2x^6+x^4\left(6Ab^2+12Bab\right)+x^2\left(8Aab+4Ba^2\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**9,x)
```

```
[Out] -(3*A*a**2 + 12*B*b**2*x**6 + x**4*(6*A*b**2 + 12*B*a*b) + x**2*(
8*A*a*b + 4*B*a**2))/(24*x**8)
```

GIAC/XCAS [A] time = 0.221465, size = 74, normalized size = 1.54

$$-\frac{12\,Bb^2x^6+12\,Babx^4+6\,Ab^2x^4+4\,Ba^2x^2+8\,Aabx^2+3\,Aa^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 4*B*a^2*x^2 +
8*A*a*b*x^2 + 3*A*a^2)/x^8
```

3.23 $\int x^9 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b +
 a*B)*x^14)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b +
 2*a*B)*x^18)/18 + (b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Rubi [A] time = 0.41053, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b +
 a*B)*x^14)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b +
 2*a*B)*x^18)/18 + (b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Rubi in Sympy [A] time = 38.8242, size = 114, normalized size = 0.97

$$\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{22}}{22}+\frac{a^4x^{12}\left(5Ab+Ba\right)}{12}+\frac{5a^3bx^{14}\left(2Ab+Ba\right)}{14}+\frac{5a^2b^2x^{16}\left(Ab+Ba\right)}{8}+\frac{5ab^3x^{18}\left(Ab+2Ba\right)}{18}+\frac{b^4x^{20}\left(Ab+5Ba\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**22/22 + a**4*x**12*(5*A*b + B*a)/12 +
 5*a**3*b*x**14*(2*A*b + B*a)/14 + 5*a**2*b**2*x**16*(A*b + B*a)/
8 + 5*a*b**3*x**18*(A*b + 2*B*a)/18 + b**4*x**20*(A*b + 5*B*a)/20
```

Mathematica [A] time = 0.029266, size = 117, normalized size = 1.

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b +
 a*B)*x^14)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b +
 2*a*B)*x^18)/18 + (b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Maple [A] time = 0.003, size = 124, normalized size = 1.1

$$\frac{b^{5}Bx^{22}}{22}+\frac{\left(b^{5}A+5\,ab^{4}B\right)x^{20}}{20}+\frac{\left(5\,ab^{4}A+10\,a^{2}b^{3}B\right)x^{18}}{18}+\frac{\left(10\,a^{2}b^{3}A+10\,a^{3}b^{2}B\right)x^{16}}{16}+\frac{\left(10\,a^{3}b^{2}A+5\,a^{4}bB\right)x^{14}}{14}+\frac{\left(5\,a^{4}bA+a^{5}B\right)x^{12}}{12}+\frac{a^{5}Ax^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/22*b^5*B*x^22+1/20*(A*b^5+5*B*a*b^4)*x^20+1/18*(5*A*a*b^4+10*B*
a^2*b^3)*x^18+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/14*(10*A*a^
3*b^2+5*B*a^4*b)*x^14+1/12*(5*A*a^4*b+B*a^5)*x^12+1/10*a^5*A*x^10
```

Maxima [A] time = 1.33729, size = 161, normalized size = 1.38

$$\frac{1}{22}\,Bb^{5}x^{22}+\frac{1}{20}\left(5\,Bab^{4}+Ab^{5}\right)x^{20}+\frac{5}{18}\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{18}+\frac{5}{8}\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{16}+\frac{1}{10}\,Aa^{5}x^{10}+\frac{5}{14}\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{14}+\frac{1}{12}\left(Ba^{5}+5\,Aa^{4}b\right)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^9,x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b
^3 + A*a*b^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^
5*x^10 + 5/14*(B*a^4*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^
4*b)*x^12
```

Fricas [A] time = 0.218028, size = 1, normalized size = 0.01

$$\frac{1}{22}x^{22}b^{5}B+\frac{1}{4}x^{20}b^{4}aB+\frac{1}{20}x^{20}b^{5}A+\frac{5}{9}x^{18}b^{3}a^{2}B+\frac{5}{18}x^{18}b^{4}aA+\frac{5}{8}x^{16}b^{2}a^{3}B+\frac{5}{8}x^{16}b^{3}a^{2}A+\frac{5}{14}x^{14}ba^{4}B+\frac{5}{7}x^{14}b^{2}a^{3}A+\frac{1}{12}x^{12}a^{5}B+\frac{5}{12}x^{12}ba^{4}A+\frac{1}{10}x^{10}a^{5}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^9,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5*B + 1/4*x^20*b^4*a*B + 1/20*x^20*b^5*A + 5/9*x^18*b
^3*a^2*B + 5/18*x^18*b^4*a*A + 5/8*x^16*b^2*a^3*B + 5/8*x^16*b^3*
a^2*A + 5/14*x^14*b*a^4*B + 5/7*x^14*b^2*a^3*A + 1/12*x^12*a^5*B
+ 5/12*x^12*b*a^4*A + 1/10*x^10*a^5*A
```

Sympy [A] time = 0.173882, size = 136, normalized size = 1.16

$$\frac{Aa^{5}x^{10}}{10}+\frac{Bb^{5}x^{22}}{22}+x^{20}\left(\frac{Ab^{5}}{20}+\frac{Bab^{4}}{4}\right)+x^{18}\left(\frac{5Aab^{4}}{18}+\frac{5Ba^{2}b^{3}}{9}\right)+x^{16}\left(\frac{5Aa^{2}b^{3}}{8}+\frac{5Ba^{3}b^{2}}{8}\right)+x^{14}\left(\frac{5Aa^{3}b^{2}}{7}+\frac{5Ba^{4}b}{14}\right)+x^{12}\left(\frac{5Aa^{4}b}{12}+\frac{Ba^{5}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**22/22 + x**20*(A*b**5/20 + B*a*b**4/4
) + x**18*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**16*(5*A*a**2*b**
3/8 + 5*B*a**3*b**2/8) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14)
+ x**12*(5*A*a**4*b/12 + B*a**5/12)
```

GIAC/XCAS [A] time = 0.220382, size = 169, normalized size = 1.44

$$\frac{1}{22}\,Bb^5x^{22} + \frac{1}{4}\,Bab^4x^{20} + \frac{1}{20}\,Ab^5x^{20} + \frac{5}{9}\,Ba^2b^3x^{18} + \frac{5}{18}\,Aab^4x^{18} + \frac{5}{8}\,Ba^3b^2x^{16} + \frac{5}{8}\,Aa^2b^3x^{16} + \frac{5}{14}\,Ba^4bx^{14} + \frac{5}{7}\,Aa^3b^2x^{14} + \frac{1}{12}\,Ba^5x^{12} + \frac{5}{12}\,Aa^4bx^{12} + \frac{1}{10}\,Aa^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^9,x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 5/9*B*a^2*
b^3*x^18 + 5/18*A*a*b^4*x^18 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3
*x^16 + 5/14*B*a^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/12*B*a^5*x^12
+ 5/12*A*a^4*b*x^12 + 1/10*A*a^5*x^10
```

3.24 $\int x^8 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=117

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a
*B)*x^13)/13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2
*a*B)*x^17)/17 + (b^4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Rubi [A] time = 0.238852, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a
*B)*x^13)/13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2
*a*B)*x^17)/17 + (b^4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Rubi in Sympy [A] time = 28.3106, size = 114, normalized size = 0.97

$$\frac{Aa^5x^9}{9}+\frac{Bb^5x^{21}}{21}+\frac{a^4x^{11}\left(5Ab+Ba\right)}{11}+\frac{5a^3bx^{13}\left(2Ab+Ba\right)}{13}+\frac{2a^2b^2x^{15}\left(Ab+Ba\right)}{3}+\frac{5ab^3x^{17}\left(Ab+2Ba\right)}{17}+\frac{b^4x^{19}\left(Ab+5Ba\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**21/21 + a**4*x**11*(5*A*b + B*a)/11 + 5
*a**3*b*x**13*(2*A*b + B*a)/13 + 2*a**2*b**2*x**15*(A*b + B*a)/3
+ 5*a*b**3*x**17*(A*b + 2*B*a)/17 + b**4*x**19*(A*b + 5*B*a)/19
```

Mathematica [A] time = 0.0309702, size = 117, normalized size = 1.

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a
*B)*x^13)/13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2
*a*B)*x^17)/17 + (b^4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Maple [A] time = 0.003, size = 124, normalized size = 1.1

$$\frac{b^5 B x^{21}}{21}+\frac{\left(b^5 A+5\, a b^4 B\right) x^{19}}{19}+\frac{\left(5\, a b^4 A+10\, a^2 b^3 B\right) x^{17}}{17}+\frac{\left(10\, a^2 b^3 A+10\, a^3 b^2 B\right) x^{15}}{15}+\frac{\left(10\, a^3 b^2 A+5\, a^4 b B\right) x^{13}}{13}+\frac{\left(5\, a^4 b A+a^5 B\right) x^{11}}{11}+\frac{a^5 A x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^8*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/21*b^5*B*x^21+1/19*(A*b^5+5*B*a*b^4)*x^19+1/17*(5*A*a*b^4+10*B*
a^2*b^3)*x^17+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/13*(10*A*a^
3*b^2+5*B*a^4*b)*x^13+1/11*(5*A*a^4*b+B*a^5)*x^11+1/9*a^5*A*x^9
```

Maxima [A] time = 1.34997, size = 161, normalized size = 1.38

$$\frac{1}{21}\, B b^5 x^{21}+\frac{1}{19}\left(5\, B a b^4+A b^5\right) x^{19}+\frac{5}{17}\left(2\, B a^2 b^3+A a b^4\right) x^{17}+\frac{2}{3}\left(B a^3 b^2+A a^2 b^3\right) x^{15}+\frac{1}{9}\, A a^5 x^9+\frac{5}{13}\left(B a^4 b+2\, A a^3 b^2\right) x^{13}+\frac{1}{11}\left(B a^5+5\, A a^4 b\right) x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^5*x^8,x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b
^3 + A*a*b^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5
*x^9 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*
b)*x^11
```

Fricas [A] time = 0.220797, size = 1, normalized size = 0.01

$$\frac{1}{21} x^{21} b^5 B+\frac{5}{19} x^{19} b^4 a B+\frac{1}{19} x^{19} b^5 A+\frac{10}{17} x^{17} b^3 a^2 B+\frac{5}{17} x^{17} b^4 a A+\frac{2}{3} x^{15} b^2 a^3 B+\frac{2}{3} x^{15} b^3 a^2 A+\frac{5}{13} x^{13} b a^4 B+\frac{10}{13} x^{13} b^2 a^3 A+\frac{1}{11} x^{11} a^5 B+\frac{5}{11} x^{11} b a^4 A+\frac{1}{9} x^9 a^5 A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^5*x^8,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5*B + 5/19*x^19*b^4*a*B + 1/19*x^19*b^5*A + 10/17*x^1
7*b^3*a^2*B + 5/17*x^17*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b
^3*a^2*A + 5/13*x^13*b*a^4*B + 10/13*x^13*b^2*a^3*A + 1/11*x^11*a
^5*B + 5/11*x^11*b*a^4*A + 1/9*x^9*a^5*A
```

Sympy [A] time = 0.173894, size = 138, normalized size = 1.18

$$\frac{A a^5 x^9}{9}+\frac{B b^5 x^{21}}{21}+x^{19}\left(\frac{A b^5}{19}+\frac{5 B a b^4}{19}\right)+x^{17}\left(\frac{5 A a b^4}{17}+\frac{10 B a^2 b^3}{17}\right)+x^{15}\left(\frac{2 A a^2 b^3}{3}+\frac{2 B a^3 b^2}{3}\right)+x^{13}\left(\frac{10 A a^3 b^2}{13}+\frac{5 B a^4 b}{13}\right)+x^{11}\left(\frac{5 A a^4 b}{11}+\frac{B a^5}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**21/21 + x**19*(A*b**5/19 + 5*B*a*b**4/1
9) + x**17*(5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**15*(2*A*a**2*
b**3/3 + 2*B*a**3*b**2/3) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b
/13) + x**11*(5*A*a**4*b/11 + B*a**5/11)
```

GIAC/XCAS [A] time = 0.218383, size = 169, normalized size = 1.44

$$\frac{1}{21}\,Bb^{5}x^{21}+\frac{5}{19}\,Bab^{4}x^{19}+\frac{1}{19}\,Ab^{5}x^{19}+\frac{10}{17}\,Ba^{2}b^{3}x^{17}+\frac{5}{17}\,Aab^{4}x^{17}+\frac{2}{3}\,Ba^{3}b^{2}x^{15}+\frac{2}{3}\,Aa^{2}b^{3}x^{15}+\frac{5}{13}\,Ba^{4}bx^{13}+\frac{10}{13}\,Aa^{3}b^{2}x^{13}+\frac{1}{11}\,Ba^{5}x^{11}+\frac{5}{11}\,Aa^{4}bx^{11}+\frac{1}{9}\,Aa^{5}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^8,x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 10/17*B*a
^2*b^3*x^17 + 5/17*A*a*b^4*x^17 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*
b^3*x^15 + 5/13*B*a^4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/11*B*a^5*
x^11 + 5/11*A*a^4*b*x^11 + 1/9*A*a^5*x^9
```

3.25 $\int x^7 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=122

$$-\frac{a^3\left(a+bx^2\right)^6(Ab-aB)}{12b^5}+\frac{a^2\left(a+bx^2\right)^7(3Ab-4aB)}{14b^5}+\frac{\left(a+bx^2\right)^9(Ab-4aB)}{18b^5}-\frac{3a\left(a+bx^2\right)^8(Ab-2aB)}{16b^5}+\frac{B\left(a+bx^2\right)^{10}}{20b^5}$$

```
[Out] -(a^3*(A*b - a*B)*(a + b*x^2)^6)/(12*b^5) + (a^2*(3*A*b - 4*a*B)*
(a + b*x^2)^7)/(14*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(16*b
^5) + ((A*b - 4*a*B)*(a + b*x^2)^9)/(18*b^5) + (B*(a + b*x^2)^10)
/(20*b^5)
```

Rubi [A] time = 0.650576, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a^3\left(a+bx^2\right)^6(Ab-aB)}{12b^5}+\frac{a^2\left(a+bx^2\right)^7(3Ab-4aB)}{14b^5}+\frac{\left(a+bx^2\right)^9(Ab-4aB)}{18b^5}-\frac{3a\left(a+bx^2\right)^8(Ab-2aB)}{16b^5}+\frac{B\left(a+bx^2\right)^{10}}{20b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -(a^3*(A*b - a*B)*(a + b*x^2)^6)/(12*b^5) + (a^2*(3*A*b - 4*a*B)*
(a + b*x^2)^7)/(14*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(16*b
^5) + ((A*b - 4*a*B)*(a + b*x^2)^9)/(18*b^5) + (B*(a + b*x^2)^10)
/(20*b^5)
```

Rubi in Sympy [A] time = 37.547, size = 114, normalized size = 0.93

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{20}}{20}+\frac{a^4x^{10}(5Ab+Ba)}{10}+\frac{5a^3bx^{12}(2Ab+Ba)}{12}+\frac{5a^2b^2x^{14}(Ab+Ba)}{7}+\frac{5ab^3x^{16}(Ab+2Ba)}{16}+\frac{b^4x^{18}(Ab+5Ba)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**20/20 + a**4*x**10*(5*A*b + B*a)/10 + 5
*a**3*b*x**12*(2*A*b + B*a)/12 + 5*a**2*b**2*x**14*(A*b + B*a)/7
+ 5*a*b**3*x**16*(A*b + 2*B*a)/16 + b**4*x**18*(A*b + 5*B*a)/18
```

Mathematica [A] time = 0.0269963, size = 117, normalized size = 0.96

$$\frac{1}{8}a^5Ax^8+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{12}a^3bx^{12}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{18}b^4x^{18}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a
*B)*x^12)/12 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2
*a*B)*x^16)/16 + (b^4*(A*b + 5*a*B)*x^18)/18 + (b^5*B*x^20)/20
```

Maple [A] time = 0.002, size = 124, normalized size = 1.

$$\frac{b^5 B x^{20}}{20}+\frac{\left(b^5 A+5\, a b^4 B\right) x^{18}}{18}+\frac{\left(5\, a b^4 A+10\, a^2 b^3 B\right) x^{16}}{16}+\frac{\left(10\, a^2 b^3 A+10\, a^3 b^2 B\right) x^{14}}{14}+\frac{\left(10\, a^3 b^2 A+5\, a^4 b B\right) x^{12}}{12}+\frac{\left(5\, a^4 b A+a^5 B\right) x^{10}}{10}+\frac{a^5 A x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/20*b^5*B*x^20+1/18*(A*b^5+5*B*a*b^4)*x^18+1/16*(5*A*a*b^4+10*B*
a^2*b^3)*x^16+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/12*(10*A*a^
3*b^2+5*B*a^4*b)*x^12+1/10*(5*A*a^4*b+B*a^5)*x^10+1/8*a^5*A*x^8
```

Maxima [A] time = 1.34573, size = 161, normalized size = 1.32

$$\frac{1}{20}\, B b^5 x^{20}+\frac{1}{18}\left(5\, B a b^4+A b^5\right) x^{18}+\frac{5}{16}\left(2\, B a^2 b^3+A a b^4\right) x^{16}+\frac{5}{7}\left(B a^3 b^2+A a^2 b^3\right) x^{14}+\frac{1}{8}\, A a^5 x^8+\frac{5}{12}\left(B a^4 b+2\, A a^3 b^2\right) x^{12}+\frac{1}{10}\left(B a^5+5\, A a^4 b\right) x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^7,x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b
^3 + A*a*b^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5
*x^8 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*
b)*x^10
```

Fricas [A] time = 0.216696, size = 1, normalized size = 0.01

$$\frac{1}{20}x^{20}b^5B+\frac{5}{18}x^{18}b^4aB+\frac{1}{18}x^{18}b^5A+\frac{5}{8}x^{16}b^3a^2B+\frac{5}{16}x^{16}b^4aA+\frac{5}{7}x^{14}b^2a^3B+\frac{5}{7}x^{14}b^3a^2A+\frac{5}{12}x^{12}ba^4B+\frac{5}{6}x^{12}b^2a^3A+\frac{1}{10}x^{10}a^5B+\frac{1}{2}x^{10}ba^4A+\frac{1}{8}x^8a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^7,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 5/8*x^16*
b^3*a^2*B + 5/16*x^16*b^4*a*A + 5/7*x^14*b^2*a^3*B + 5/7*x^14*b^3
*a^2*A + 5/12*x^12*b*a^4*B + 5/6*x^12*b^2*a^3*A + 1/10*x^10*a^5*B
 + 1/2*x^10*b*a^4*A + 1/8*x^8*a^5*A
```

Sympy [A] time = 0.174133, size = 136, normalized size = 1.11

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{20}}{20}+x^{18}\left(\frac{Ab^5}{18}+\frac{5Bab^4}{18}\right)+x^{16}\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{14}\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{12}\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)+x^{10}\left(\frac{Aa^4b}{2}+\frac{Ba^5}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**20/20 + x**18*(A*b**5/18 + 5*B*a*b**4/1
8) + x**16*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**14*(5*A*a**2*b*
*3/7 + 5*B*a**3*b**2/7) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12)
 + x**10*(A*a**4*b/2 + B*a**5/10)
```

GIAC/XCAS [A] time = 0.225156, size = 169, normalized size = 1.39

$$\frac{1}{20}\,Bb^{5}x^{20}+\frac{5}{18}\,Bab^{4}x^{18}+\frac{1}{18}\,Ab^{5}x^{18}+\frac{5}{8}\,Ba^{2}b^{3}x^{16}+\frac{5}{16}\,Aab^{4}x^{16}+\frac{5}{7}\,Ba^{3}b^{2}x^{14}+\frac{5}{7}\,Aa^{2}b^{3}x^{14}+\frac{5}{12}\,Ba^{4}bx^{12}+\frac{5}{6}\,Aa^{3}b^{2}x^{12}+\frac{1}{10}\,Ba^{5}x^{10}+\frac{1}{2}\,Aa^{4}bx^{10}+\frac{1}{8}\,Aa^{5}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^7,x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 5/8*B*a^2
*b^3*x^16 + 5/16*A*a*b^4*x^16 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^
3*x^14 + 5/12*B*a^4*b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/10*B*a^5*x^10
 + 1/2*A*a^4*b*x^10 + 1/8*A*a^5*x^8
```

3.26 $\int x^6 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=117

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B
)*x^11)/11 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a
*B)*x^15)/3 + (b^4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Rubi [A] time = 0.236656, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B
)*x^11)/11 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a
*B)*x^15)/3 + (b^4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Rubi in Sympy [A] time = 27.3275, size = 112, normalized size = 0.96

$$\frac{Aa^5x^7}{7}+\frac{Bb^5x^{19}}{19}+\frac{a^4x^9\left(5Ab+Ba\right)}{9}+\frac{5a^3bx^{11}\left(2Ab+Ba\right)}{11}+\frac{10a^2b^2x^{13}\left(Ab+Ba\right)}{13}+\frac{ab^3x^{15}\left(Ab+2Ba\right)}{3}+\frac{b^4x^{17}\left(Ab+5Ba\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**19/19 + a**4*x**9*(5*A*b + B*a)/9 + 5*a
**3*b*x**11*(2*A*b + B*a)/11 + 10*a**2*b**2*x**13*(A*b + B*a)/13
+ a*b**3*x**15*(A*b + 2*B*a)/3 + b**4*x**17*(A*b + 5*B*a)/17
```

Mathematica [A] time = 0.0276702, size = 117, normalized size = 1.

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B
)*x^11)/11 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a
*B)*x^15)/3 + (b^4*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{19}}{19}+\frac{(b^5A+5ab^4B)x^{17}}{17}+\frac{(5ab^4A+10a^2b^3B)x^{15}}{15}+\frac{(10a^2b^3A+10a^3b^2B)x^{13}}{13}+\frac{(10a^3b^2A+5a^4bB)x^{11}}{11}+\frac{(5a^4bA+a^5B)x^9}{9}+\frac{a^5Ax^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/19*b^5*B*x^19+1/17*(A*b^5+5*B*a*b^4)*x^17+1/15*(5*A*a*b^4+10*B*
a^2*b^3)*x^15+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/11*(10*A*a^
3*b^2+5*B*a^4*b)*x^11+1/9*(5*A*a^4*b+B*a^5)*x^9+1/7*a^5*A*x^7
```

Maxima [A] time = 1.35183, size = 161, normalized size = 1.38

$$\frac{1}{19}Bb^5x^{19}+\frac{1}{17}(5Bab^4+Ab^5)x^{17}+\frac{1}{3}(2Ba^2b^3+Aab^4)x^{15}+\frac{10}{13}(Ba^3b^2+Aa^2b^3)x^{13}+\frac{1}{7}Aa^5x^7+\frac{5}{11}(Ba^4b+2Aa^3b^2)x^{11}+\frac{1}{9}(Ba^5+5Aa^4b)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^6,x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^
3 + A*a*b^4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^
5*x^7 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*
b)*x^9
```

Fricas [A] time = 0.212281, size = 1, normalized size = 0.01

$$\frac{1}{19}x^{19}b^5B+\frac{5}{17}x^{17}b^4aB+\frac{1}{17}x^{17}b^5A+\frac{2}{3}x^{15}b^3a^2B+\frac{1}{3}x^{15}b^4aA+\frac{10}{13}x^{13}b^2a^3B+\frac{10}{13}x^{13}b^3a^2A+\frac{5}{11}x^{11}ba^4B+\frac{10}{11}x^{11}b^2a^3A+\frac{1}{9}x^9a^5B+\frac{5}{9}x^9ba^4A+\frac{1}{7}x^7a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^6,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5*B + 5/17*x^17*b^4*a*B + 1/17*x^17*b^5*A + 2/3*x^15*
b^3*a^2*B + 1/3*x^15*b^4*a*A + 10/13*x^13*b^2*a^3*B + 10/13*x^13*
b^3*a^2*A + 5/11*x^11*b*a^4*B + 10/11*x^11*b^2*a^3*A + 1/9*x^9*a^
5*B + 5/9*x^9*b*a^4*A + 1/7*x^7*a^5*A
```

Sympy [A] time = 0.178191, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7}+\frac{Bb^5x^{19}}{19}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{15}\left(\frac{Aab^4}{3}+\frac{2Ba^2b^3}{3}\right)+x^{13}\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{11}\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right)+x^9\left(\frac{5Aa^4b}{9}+\frac{Ba^5}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**19/19 + x**17*(A*b**5/17 + 5*B*a*b**4/1
7) + x**15*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**13*(10*A*a**2*b**3
/13 + 10*B*a**3*b**2/13) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/
11) + x**9*(5*A*a**4*b/9 + B*a**5/9)
```

GIAC/XCAS [A] time = 0.223134, size = 169, normalized size = 1.44

$$\frac{1}{19}Bb^5x^{19} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{10}{13}Ba^3b^2x^{13} + \frac{10}{13}Aa^2b^3x^{13} + \frac{5}{11}Ba^4bx^{11} + \frac{10}{11}Aa^3b^2x^{11} + \frac{1}{9}Ba^5x^9 + \frac{5}{9}Aa^4bx^9 + \frac{1}{7}Aa^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^6,x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 2/3*B*a^2
*b^3*x^15 + 1/3*A*a*b^4*x^15 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2
*b^3*x^13 + 5/11*B*a^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/9*B*a^5*
x^9 + 5/9*A*a^4*b*x^9 + 1/7*A*a^5*x^7
```

3.27 $\int x^5 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=95

$$\frac{a^2\left(a+bx^2\right)^6(Ab-aB)}{12b^4}+\frac{\left(a+bx^2\right)^8(Ab-3aB)}{16b^4}-\frac{a\left(a+bx^2\right)^7(2Ab-3aB)}{14b^4}+\frac{B\left(a+bx^2\right)^9}{18b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^6)/(12*b^4) - (a*(2*A*b - 3*a*B)*(a
+ b*x^2)^7)/(14*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(16*b^4) + (
B*(a + b*x^2)^9)/(18*b^4)
```

Rubi [A] time = 0.529588, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a^2\left(a+bx^2\right)^6(Ab-aB)}{12b^4}+\frac{\left(a+bx^2\right)^8(Ab-3aB)}{16b^4}-\frac{a\left(a+bx^2\right)^7(2Ab-3aB)}{14b^4}+\frac{B\left(a+bx^2\right)^9}{18b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^6)/(12*b^4) - (a*(2*A*b - 3*a*B)*(a
+ b*x^2)^7)/(14*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(16*b^4) + (
B*(a + b*x^2)^9)/(18*b^4)
```

Rubi in Sympy [A] time = 33.6855, size = 85, normalized size = 0.89

$$\frac{B\left(a+bx^2\right)^9}{18b^4}+\frac{a^2\left(a+bx^2\right)^6(Ab-Ba)}{12b^4}-\frac{a\left(a+bx^2\right)^7(2Ab-3Ba)}{14b^4}+\frac{\left(a+bx^2\right)^8(Ab-3Ba)}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**9/(18*b**4) + a**2*(a + b*x**2)**6*(A*b - B*a)/(1
2*b**4) - a*(a + b*x**2)**7*(2*A*b - 3*B*a)/(14*b**4) + (a + b*x*
*2)**8*(A*b - 3*B*a)/(16*b**4)
```

Mathematica [A] time = 0.0430928, size = 107, normalized size = 1.13

$$\frac{x^6\left(168a^5A+126a^4x^2(aB+5Ab)+504a^3bx^4(aB+2Ab)+840a^2b^2x^6(aB+Ab)+63b^4x^{10}(5aB+Ab)+360ab^3x^8(2aB+Ab)\right.}{1008}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^6*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^2 + 504*a^3*b*(2*A*b +
a*B)*x^4 + 840*a^2*b^2*(A*b + a*B)*x^6 + 360*a*b^3*(A*b + 2*a*B)*
x^8 + 63*b^4*(A*b + 5*a*B)*x^10 + 56*b^5*B*x^12))/1008
```

Maple [A] time = 0.002, size = 124, normalized size = 1.3

$$\frac{b^5Bx^{18}}{18}+\frac{\left(b^5A+5\,ab^4B\right)x^{16}}{16}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{14}}{14}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{12}}{12}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{10}}{10}+\frac{\left(5\,a^4bA+a^5B\right)x^8}{8}+\frac{a^5Ax^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/18*b^5*B*x^18+1/16*(A*b^5+5*B*a*b^4)*x^16+1/14*(5*A*a*b^4+10*B*
a^2*b^3)*x^14+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/10*(10*A*a^
3*b^2+5*B*a^4*b)*x^10+1/8*(5*A*a^4*b+B*a^5)*x^8+1/6*a^5*A*x^6
```

Maxima [A] time = 1.34733, size = 161, normalized size = 1.69

$$\frac{1}{18}\,Bb^5x^{18} + \frac{1}{16}\left(5\,Bab^4 + Ab^5\right)x^{16} + \frac{5}{14}\left(2\,Ba^2b^3 + Aab^4\right)x^{14} + \frac{5}{6}\left(Ba^3b^2 + Aa^2b^3\right)x^{12} + \frac{1}{6}\,Aa^5x^6 + \frac{1}{2}\left(Ba^4b + 2\,Aa^3b^2\right)x^{10} + \frac{1}{8}\left(Ba^5 + 5\,Aa^4b\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^5,x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b
^3 + A*a*b^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5
*x^6 + 1/2*(B*a^4*b + 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)
*x^8
```

Fricas [A] time = 0.204274, size = 1, normalized size = 0.01

$$\frac{1}{18}x^{18}b^5B + \frac{5}{16}x^{16}b^4aB + \frac{1}{16}x^{16}b^5A + \frac{5}{7}x^{14}b^3a^2B + \frac{5}{14}x^{14}b^4aA + \frac{5}{6}x^{12}b^2a^3B + \frac{5}{6}x^{12}b^3a^2A + \frac{1}{2}x^{10}ba^4B + x^{10}b^2a^3A + \frac{1}{8}x^8a^5B + \frac{5}{8}x^8ba^4A + \frac{1}{6}x^6a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^5,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5*B + 5/16*x^16*b^4*a*B + 1/16*x^16*b^5*A + 5/7*x^14*
b^3*a^2*B + 5/14*x^14*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3
*a^2*A + 1/2*x^10*b*a^4*B + x^10*b^2*a^3*A + 1/8*x^8*a^5*B + 5/8*
x^8*b*a^4*A + 1/6*x^6*a^5*A
```

Sympy [A] time = 0.177622, size = 133, normalized size = 1.4

$$\frac{Aa^5x^6}{6} + \frac{Bb^5x^{18}}{18} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{14}\left(\frac{5Aab^4}{14} + \frac{5Ba^2b^3}{7}\right) + x^{12}\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^{10}\left(Aa^3b^2 + \frac{Ba^4b}{2}\right) + x^8\left(\frac{5Aa^4b}{8} + \frac{Ba^5}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**18/18 + x**16*(A*b**5/16 + 5*B*a*b**4/1
6) + x**14*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**12*(5*A*a**2*b*
*3/6 + 5*B*a**3*b**2/6) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**8
*(5*A*a**4*b/8 + B*a**5/8)
```

GIAC/XCAS [A] time = 0.224315, size = 167, normalized size = 1.76

$$\frac{1}{18}Bb^5x^{18} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{1}{2}Ba^4bx^{10} + Aa^3b^2x^{10} + \frac{1}{8}Ba^5x^8 + \frac{5}{8}Aa^4bx^8 + \frac{1}{6}Aa^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^5,x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 5/7*B*a^2
*b^3*x^14 + 5/14*A*a*b^4*x^14 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^
3*x^12 + 1/2*B*a^4*b*x^10 + A*a^3*b^2*x^10 + 1/8*B*a^5*x^8 + 5/8*
A*a^4*b*x^8 + 1/6*A*a^5*x^6
```

3.28 $\int x^4 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=117

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B
)*x^9)/9 + (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a
*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Rubi [A] time = 0.238899, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B
)*x^9)/9 + (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a
*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Rubi in Sympy [A] time = 27.718, size = 114, normalized size = 0.97

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{17}}{17}+\frac{a^4x^7\left(5Ab+Ba\right)}{7}+\frac{5a^3bx^9\left(2Ab+Ba\right)}{9}+\frac{10a^2b^2x^{11}\left(Ab+Ba\right)}{11}+\frac{5ab^3x^{13}\left(Ab+2Ba\right)}{13}+\frac{b^4x^{15}\left(Ab+5Ba\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**17/17 + a**4*x**7*(5*A*b + B*a)/7 + 5*a
**3*b*x**9*(2*A*b + B*a)/9 + 10*a**2*b**2*x**11*(A*b + B*a)/11 +
5*a*b**3*x**13*(A*b + 2*B*a)/13 + b**4*x**15*(A*b + 5*B*a)/15
```

Mathematica [A] time = 0.0287476, size = 117, normalized size = 1.

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B
)*x^9)/9 + (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a
*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Maple [A] time = 0.002, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{17}}{17}+\frac{\left(b^5A+5\,ab^4B\right)x^{15}}{15}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{13}}{13}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{11}}{11}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^9}{9}+\frac{\left(5\,a^4bA+a^5B\right)x^7}{7}+\frac{a^5Ax^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/17*b^5*B*x^17+1/15*(A*b^5+5*B*a*b^4)*x^15+1/13*(5*A*a*b^4+10*B*
a^2*b^3)*x^13+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/9*(10*A*a^3
*b^2+5*B*a^4*b)*x^9+1/7*(5*A*a^4*b+B*a^5)*x^7+1/5*a^5*A*x^5
```

Maxima [A] time = 1.35053, size = 161, normalized size = 1.38

$$\frac{1}{17}\,Bb^5x^{17}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{1}{5}\,Aa^5x^5+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{7}\left(Ba^5+5\,Aa^4b\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^4,x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b
^3 + A*a*b^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a
^5*x^5 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b
)*x^7
```

Fricas [A] time = 0.209979, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}b^5B+\frac{1}{3}x^{15}b^4aB+\frac{1}{15}x^{15}b^5A+\frac{10}{13}x^{13}b^3a^2B+\frac{5}{13}x^{13}b^4aA+\frac{10}{11}x^{11}b^2a^3B+\frac{10}{11}x^{11}b^3a^2A+\frac{5}{9}x^9ba^4B+\frac{10}{9}x^9b^2a^3A+\frac{1}{7}x^7a^5B+\frac{5}{7}x^7ba^4A+\frac{1}{5}x^5a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5*B + 1/3*x^15*b^4*a*B + 1/15*x^15*b^5*A + 10/13*x^13
*b^3*a^2*B + 5/13*x^13*b^4*a*A + 10/11*x^11*b^2*a^3*B + 10/11*x^1
1*b^3*a^2*A + 5/9*x^9*b*a^4*B + 10/9*x^9*b^2*a^3*A + 1/7*x^7*a^5*
B + 5/7*x^7*b*a^4*A + 1/5*x^5*a^5*A
```

Sympy [A] time = 0.177728, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{17}}{17}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{13}\left(\frac{5Aab^4}{13}+\frac{10Ba^2b^3}{13}\right)+x^{11}\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^9\left(\frac{10Aa^3b^2}{9}+\frac{5Ba^4b}{9}\right)+x^7\left(\frac{5Aa^4b}{7}+\frac{Ba^5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**17/17 + x**15*(A*b**5/15 + B*a*b**4/3)
+ x**13*(5*A*a*b**4/13 + 10*B*a**2*b**3/13) + x**11*(10*A*a**2*b*
*3/11 + 10*B*a**3*b**2/11) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/
9) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

GIAC/XCAS [A] time = 0.230039, size = 169, normalized size = 1.44

$$\frac{1}{17}Bb^5x^{17} + \frac{1}{3}Bab^4x^{15} + \frac{1}{15}Ab^5x^{15} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{7}Ba^5x^7 + \frac{5}{7}Aa^4bx^7 + \frac{1}{5}Aa^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^4,x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 10/13*B*a^
2*b^3*x^13 + 5/13*A*a*b^4*x^13 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a
^2*b^3*x^11 + 5/9*B*a^4*b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/7*B*a^5*x^
7 + 5/7*A*a^4*b*x^7 + 1/5*A*a^5*x^5
```

3.29 $\int x^3 (a+bx^2)^5 (A+Bx^2)\, dx$

Optimal. Leaf size=67

$$\frac{(a+bx^2)^7 (Ab-2aB)}{14b^3}-\frac{a(a+bx^2)^6 (Ab-aB)}{12b^3}+\frac{B(a+bx^2)^8}{16b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^6)/(12*b^3) + ((A*b - 2*a*B)*(a + b*x
^2)^7)/(14*b^3) + (B*(a + b*x^2)^8)/(16*b^3)
```

Rubi [A] time = 0.393699, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{(a+bx^2)^7 (Ab-2aB)}{14b^3}-\frac{a(a+bx^2)^6 (Ab-aB)}{12b^3}+\frac{B(a+bx^2)^8}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^6)/(12*b^3) + ((A*b - 2*a*B)*(a + b*x
^2)^7)/(14*b^3) + (B*(a + b*x^2)^8)/(16*b^3)
```

Rubi in Sympy [A] time = 28.5377, size = 58, normalized size = 0.87

$$\frac{B(a+bx^2)^8}{16b^3}-\frac{a(a+bx^2)^6 (Ab-Ba)}{12b^3}+\frac{(a+bx^2)^7 (Ab-2Ba)}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**8/(16*b**3) - a*(a + b*x**2)**6*(A*b - B*a)/(12*b
**3) + (a + b*x**2)**7*(A*b - 2*B*a)/(14*b**3)
```

Mathematica [A] time = 0.0270142, size = 114, normalized size = 1.7

$$\frac{1}{4}a^5Ax^4+\frac{1}{6}a^4x^6(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{12}ab^3x^{12}(2aB+Ab)+\frac{1}{16}b^5Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B
)*x^8)/8 + a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^12
)/12 + (b^4*(A*b + 5*a*B)*x^14)/14 + (b^5*B*x^16)/16
```

Maple [B] time = 0.003, size = 124, normalized size = 1.9

$$\frac{b^5Bx^{16}}{16}+\frac{(b^5A+5\,ab^4B)\,x^{14}}{14}+\frac{(5\,ab^4A+10\,a^2b^3B)\,x^{12}}{12}+\frac{(10\,a^2b^3A+10\,a^3b^2B)\,x^{10}}{10}+\frac{(10\,a^3b^2A+5\,a^4bB)\,x^8}{8}+\frac{(5\,a^4bA+a^5B)\,x^6}{6}+\frac{a^5Ax^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/16*b^5*B*x^16+1/14*(A*b^5+5*B*a*b^4)*x^14+1/12*(5*A*a*b^4+10*B*
a^2*b^3)*x^12+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/8*(10*A*a^3
*b^2+5*B*a^4*b)*x^8+1/6*(5*A*a^4*b+B*a^5)*x^6+1/4*a^5*A*x^4
```

Maxima [A] time = 1.35494, size = 159, normalized size = 2.37

$$\frac{1}{16}Bb^5x^{16}+\frac{1}{14}\left(5Bab^4+Ab^5\right)x^{14}+\frac{5}{12}\left(2Ba^2b^3+Aab^4\right)x^{12}+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{1}{4}Aa^5x^4+\frac{5}{8}\left(Ba^4b+2Aa^3b^2\right)x^8+\frac{1}{6}\left(Ba^5+5Aa^4b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b
^3 + A*a*b^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4
 + 5/8*(B*a^4*b + 2*A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Fricas [A] time = 0.208351, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}b^5B+\frac{5}{14}x^{14}b^4aB+\frac{1}{14}x^{14}b^5A+\frac{5}{6}x^{12}b^3a^2B+\frac{5}{12}x^{12}b^4aA+x^{10}b^2a^3B+x^{10}b^3a^2A+\frac{5}{8}x^8ba^4B+\frac{5}{4}x^8b^2a^3A+\frac{1}{6}x^6a^5B+\frac{5}{6}x^6ba^4A+\frac{1}{4}x^4a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5*B + 5/14*x^14*b^4*a*B + 1/14*x^14*b^5*A + 5/6*x^12*
b^3*a^2*B + 5/12*x^12*b^4*a*A + x^10*b^2*a^3*B + x^10*b^3*a^2*A +
 5/8*x^8*b*a^4*B + 5/4*x^8*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*
a^4*A + 1/4*x^4*a^5*A
```

Sympy [A] time = 0.170284, size = 131, normalized size = 1.96

$$\frac{Aa^5x^4}{4}+\frac{Bb^5x^{16}}{16}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{12}\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^{10}\left(Aa^2b^3+Ba^3b^2\right)+x^8\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^6\left(\frac{5Aa^4b}{6}+\frac{Ba^5}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**16/16 + x**14*(A*b**5/14 + 5*B*a*b**4/1
4) + x**12*(5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**10*(A*a**2*b**3
 + B*a**3*b**2) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**6*(5
*A*a**4*b/6 + B*a**5/6)
```

GIAC/XCAS [A] time = 0.224687, size = 166, normalized size = 2.48

$$\frac{1}{16}Bb^5x^{16} + \frac{5}{14}Bab^4x^{14} + \frac{1}{14}Ab^5x^{14} + \frac{5}{6}Ba^2b^3x^{12} + \frac{5}{12}Aab^4x^{12} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 + \frac{1}{6}Ba^5x^6 + \frac{5}{6}Aa^4bx^6 + \frac{1}{4}Aa^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^3,x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 5/6*B*a^2
*b^3*x^12 + 5/12*A*a*b^4*x^12 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 +
 5/8*B*a^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/6*B*a^5*x^6 + 5/6*A*a^4*
b*x^6 + 1/4*A*a^5*x^4
```

3.30 $\int x^2 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=117

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B
)*x^7)/7 + (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B
)*x^11)/11 + (b^4*(A*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Rubi [A] time = 0.236516, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B
)*x^7)/7 + (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B
)*x^11)/11 + (b^4*(A*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Rubi in Sympy [A] time = 29.4049, size = 114, normalized size = 0.97

$$\frac{Aa^5x^3}{3}+\frac{Bb^5x^{15}}{15}+\frac{a^4x^5\left(5Ab+Ba\right)}{5}+\frac{5a^3bx^7\left(2Ab+Ba\right)}{7}+\frac{10a^2b^2x^9\left(Ab+Ba\right)}{9}+\frac{5ab^3x^{11}\left(Ab+2Ba\right)}{11}+\frac{b^4x^{13}\left(Ab+5Ba\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**15/15 + a**4*x**5*(5*A*b + B*a)/5 + 5*a
**3*b*x**7*(2*A*b + B*a)/7 + 10*a**2*b**2*x**9*(A*b + B*a)/9 + 5*
a*b**3*x**11*(A*b + 2*B*a)/11 + b**4*x**13*(A*b + 5*B*a)/13
```

Mathematica [A] time = 0.0290692, size = 117, normalized size = 1.

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B
)*x^7)/7 + (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B
)*x^11)/11 + (b^4*(A*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Maple [A] time = 0.003, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{15}}{15}+\frac{(b^5A+5\,ab^4B)\,x^{13}}{13}+\frac{(5\,ab^4A+10\,a^2b^3B)\,x^{11}}{11}+\frac{(10\,a^2b^3A+10\,a^3b^2B)\,x^9}{9}+\frac{(10\,a^3b^2A+5\,a^4bB)\,x^7}{7}+\frac{(5\,a^4bA+a^5B)\,x^5}{5}+\frac{a^5Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/15*b^5*B*x^15+1/13*(A*b^5+5*B*a*b^4)*x^13+1/11*(5*A*a*b^4+10*B*
a^2*b^3)*x^11+1/9*(10*A*a^2*b^3+10*B*a^3*b^2)*x^9+1/7*(10*A*a^3*b
^2+5*B*a^4*b)*x^7+1/5*(5*A*a^4*b+B*a^5)*x^5+1/3*a^5*A*x^3
```

Maxima [A] time = 1.34936, size = 161, normalized size = 1.38

$$\frac{1}{15}\,Bb^5x^{15}+\frac{1}{13}\,(5\,Bab^4+Ab^5)\,x^{13}+\frac{5}{11}\,(2\,Ba^2b^3+Aab^4)\,x^{11}+\frac{10}{9}\,(Ba^3b^2+Aa^2b^3)\,x^9+\frac{1}{3}\,Aa^5x^3+\frac{5}{7}\,(Ba^4b+2\,Aa^3b^2)\,x^7+\frac{1}{5}\,(Ba^5+5\,Aa^4b)\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^2,x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b
^3 + A*a*b^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5
*x^3 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*
x^5
```

Fricas [A] time = 0.206406, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}b^5B+\frac{5}{13}x^{13}b^4aB+\frac{1}{13}x^{13}b^5A+\frac{10}{11}x^{11}b^3a^2B+\frac{5}{11}x^{11}b^4aA+\frac{10}{9}x^9b^2a^3B+\frac{10}{9}x^9b^3a^2A+\frac{5}{7}x^7ba^4B+\frac{10}{7}x^7b^2a^3A+\frac{1}{5}x^5a^5B+x^5ba^4A+\frac{1}{3}x^3a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^5*B + 5/13*x^13*b^4*a*B + 1/13*x^13*b^5*A + 10/11*x^1
1*b^3*a^2*B + 5/11*x^11*b^4*a*A + 10/9*x^9*b^2*a^3*B + 10/9*x^9*b
^3*a^2*A + 5/7*x^7*b*a^4*B + 10/7*x^7*b^2*a^3*A + 1/5*x^5*a^5*B +
 x^5*b*a^4*A + 1/3*x^3*a^5*A
```

Sympy [A] time = 0.171118, size = 134, normalized size = 1.15

$$\frac{Aa^5x^3}{3}+\frac{Bb^5x^{15}}{15}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)+x^{11}\left(\frac{5Aab^4}{11}+\frac{10Ba^2b^3}{11}\right)+x^9\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^7\left(\frac{10Aa^3b^2}{7}+\frac{5Ba^4b}{7}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**15/15 + x**13*(A*b**5/13 + 5*B*a*b**4/1
3) + x**11*(5*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**9*(10*A*a**2*
b**3/9 + 10*B*a**3*b**2/9) + x**7*(10*A*a**3*b**2/7 + 5*B*a**4*b/
7) + x**5*(A*a**4*b + B*a**5/5)
```

GIAC/XCAS [A] time = 0.228103, size = 167, normalized size = 1.43

$$\frac{1}{15}Bb^5x^{15} + \frac{5}{13}Bab^4x^{13} + \frac{1}{13}Ab^5x^{13} + \frac{10}{11}Ba^2b^3x^{11} + \frac{5}{11}Aab^4x^{11} + \frac{10}{9}Ba^3b^2x^9 + \frac{10}{9}Aa^2b^3x^9 + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x^7 + \frac{1}{5}Ba^5x^5 + Aa^4bx^5 + \frac{1}{3}Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x^2,x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + 10/11*B*a
^2*b^3*x^11 + 5/11*A*a*b^4*x^11 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2
*b^3*x^9 + 5/7*B*a^4*b*x^7 + 10/7*A*a^3*b^2*x^7 + 1/5*B*a^5*x^5 +
 A*a^4*b*x^5 + 1/3*A*a^5*x^3
```

3.31 $\int x\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^6(Ab-aB)}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^6)/(12*b^2) + (B*(a + b*x^2)^7)/(14*b^2)
```

Rubi [A] time = 0.18201, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\left(a+bx^2\right)^6(Ab-aB)}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5*(A + B*x^2),x]

[Out] ((A*b - a*B)*(a + b*x^2)^6)/(12*b^2) + (B*(a + b*x^2)^7)/(14*b^2)
```

Rubi in Sympy [A] time = 20.4966, size = 34, normalized size = 0.81

$$\frac{B\left(a+bx^2\right)^7}{14b^2}+\frac{\left(a+bx^2\right)^6(Ab-Ba)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(b*x**2+a)**5*(B*x**2+A),x)

[Out] B*(a + b*x**2)**7/(14*b**2) + (a + b*x**2)**6*(A*b - B*a)/(12*b**
2)
```

Mathematica [B] time = 0.0415072, size = 107, normalized size = 2.55

$$\frac{1}{84}x^2\left(42a^5A+21a^4x^2(aB+5Ab)+70a^3bx^4(aB+2Ab)+105a^2b^2x^6(aB+Ab)\right.$$
$$\left.+7b^4x^{10}(5aB+Ab)+42ab^3x^8(2aB+Ab)+6b^5Bx^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5*(A + B*x^2),x]

[Out] (x^2*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^2 + 70*a^3*b*(2*A*b + a*B
)*x^4 + 105*a^2*b^2*(A*b + a*B)*x^6 + 42*a*b^3*(A*b + 2*a*B)*x^8
+ 7*b^4*(A*b + 5*a*B)*x^10 + 6*b^5*B*x^12))/84
```

Maple [B] time = 0.002, size = 124, normalized size = 3.

$$\frac{b^5Bx^{14}}{14}+\frac{\left(b^5A+5\,ab^4B\right)x^{12}}{12}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{10}}{10}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^8}{8}$$
$$+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^6}{6}+\frac{\left(5\,a^4bA+a^5B\right)x^4}{4}+\frac{a^5Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/14*b^5*B*x^14+1/12*(A*b^5+5*B*a*b^4)*x^12+1/10*(5*A*a*b^4+10*B*
a^2*b^3)*x^10+1/8*(10*A*a^2*b^3+10*B*a^3*b^2)*x^8+1/6*(10*A*a^3*b
^2+5*B*a^4*b)*x^6+1/4*(5*A*a^4*b+B*a^5)*x^4+1/2*a^5*A*x^2
```

Maxima [A] time = 1.3572, size = 161, normalized size = 3.83

$$\frac{1}{14}Bb^5x^{14} + \frac{1}{12}\left(5\,Bab^4 + Ab^5\right)x^{12} + \frac{1}{2}\left(2\,Ba^2b^3 + Aab^4\right)x^{10} + \frac{5}{4}\left(Ba^3b^2 + Aa^2b^3\right)x^8 + \frac{1}{2}Aa^5x^2 + \frac{5}{6}\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + \frac{1}{4}\left(Ba^5 + 5\,Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^5*x,x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^
3 + A*a*b^4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x
^2 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^
4
```

Fricas [A] time = 0.206943, size = 1, normalized size = 0.02

$$\frac{1}{14}x^{14}b^5B + \frac{5}{12}x^{12}b^4aB + \frac{1}{12}x^{12}b^5A + x^{10}b^3a^2B + \frac{1}{2}x^{10}b^4aA + \frac{5}{4}x^8b^2a^3B + \frac{5}{4}x^8b^3a^2A + \frac{5}{6}x^6ba^4B + \frac{5}{3}x^6b^2a^3A + \frac{1}{4}x^4a^5B + \frac{5}{4}x^4ba^4A + \frac{1}{2}x^2a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^5*x,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^5*B + 5/12*x^12*b^4*a*B + 1/12*x^12*b^5*A + x^10*b^3*
a^2*B + 1/2*x^10*b^4*a*A + 5/4*x^8*b^2*a^3*B + 5/4*x^8*b^3*a^2*A
+ 5/6*x^6*b*a^4*B + 5/3*x^6*b^2*a^3*A + 1/4*x^4*a^5*B + 5/4*x^4*b
*a^4*A + 1/2*x^2*a^5*A
```

Sympy [A] time = 0.168778, size = 133, normalized size = 3.17

$$\frac{Aa^5x^2}{2} + \frac{Bb^5x^{14}}{14} + x^{12}\left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^8\left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^6\left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6}\right) + x^4\left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**14/14 + x**12*(A*b**5/12 + 5*B*a*b**4/1
2) + x**10*(A*a*b**4/2 + B*a**2*b**3) + x**8*(5*A*a**2*b**3/4 + 5
*B*a**3*b**2/4) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**4*(5
*A*a**4*b/4 + B*a**5/4)
```

GIAC/XCAS [A] time = 0.226793, size = 167, normalized size = 3.98

$$\frac{1}{14}Bb^5x^{14} + \frac{5}{12}Bab^4x^{12} + \frac{1}{12}Ab^5x^{12} + Ba^2b^3x^{10} + \frac{1}{2}Aab^4x^{10} + \frac{5}{4}Ba^3b^2x^8 + \frac{5}{4}Aa^2b^3x^8 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + \frac{1}{2}Aa^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5*x,x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + B*a^2*b^3
*x^10 + 1/2*A*a*b^4*x^10 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8
+ 5/6*B*a^4*b*x^6 + 5/3*A*a^3*b^2*x^6 + 1/4*B*a^5*x^4 + 5/4*A*a^4
*b*x^4 + 1/2*A*a^5*x^2
```

3.32 $\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=109

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (
b^4*(A*b + 5*a*B)*x^11)/11 + (b^5*B*x^13)/13
```

Rubi [A] time = 0.169955, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (
b^4*(A*b + 5*a*B)*x^11)/11 + (b^5*B*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bb^5x^{13}}{13}+a^5\int A\,dx+\frac{a^4x^3\,(5Ab+Ba)}{3}+a^3bx^5\,(2Ab+Ba)+\frac{10a^2b^2x^7\,(Ab+Ba)}{7}+\frac{5ab^3x^9\,(Ab+2Ba)}{9}+\frac{b^4x^{11}\,(Ab+5Ba)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] B*b**5*x**13/13 + a**5*Integral(A, x) + a**4*x**3*(5*A*b + B*a)/3
 + a**3*b*x**5*(2*A*b + B*a) + 10*a**2*b**2*x**7*(A*b + B*a)/7 +
5*a*b**3*x**9*(A*b + 2*B*a)/9 + b**4*x**11*(A*b + 5*B*a)/11
```

Mathematica [A] time = 0.0280715, size = 109, normalized size = 1.

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (
10*a^2*b^2*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (
```

```
b^4*(A*b + 5*a*B)*x^11)/11 + (b^5*B*x^13)/13
```

Maple [A] time = 0.003, size = 121, normalized size = 1.1

$$\frac{b^5Bx^{13}}{13}+\frac{\left(b^5A+5\,ab^4B\right)x^{11}}{11}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^9}{9}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^7}{7}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^5}{5}+\frac{\left(5\,a^4bA+a^5B\right)x^3}{3}+a^5Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/13*b^5*B*x^13+1/11*(A*b^5+5*B*a*b^4)*x^11+1/9*(5*A*a*b^4+10*B*a
^2*b^3)*x^9+1/7*(10*A*a^2*b^3+10*B*a^3*b^2)*x^7+1/5*(10*A*a^3*b^2
+5*B*a^4*b)*x^5+1/3*(5*A*a^4*b+B*a^5)*x^3+a^5*A*x
```

Maxima [A] time = 1.34763, size = 155, normalized size = 1.42

$$\frac{1}{13}\,Bb^5x^{13}+\frac{1}{11}\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+Aa^5x+\left(Ba^4b+2\,Aa^3b^2\right)x^5+\frac{1}{3}\left(Ba^5+5\,Aa^4b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5,x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^
3 + A*a*b^4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (
B*a^4*b + 2*A*a^3*b^2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Fricas [A] time = 0.218125, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}b^5B+\frac{5}{11}x^{11}b^4aB+\frac{1}{11}x^{11}b^5A+\frac{10}{9}x^9b^3a^2B+\frac{5}{9}x^9b^4aA+\frac{10}{7}x^7b^2a^3B+\frac{10}{7}x^7b^3a^2A+x^5ba^4B+2x^5b^2a^3A+\frac{1}{3}x^3a^5B+\frac{5}{3}x^3ba^4A+xa^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^5*B + 5/11*x^11*b^4*a*B + 1/11*x^11*b^5*A + 10/9*x^9*
b^3*a^2*B + 5/9*x^9*b^4*a*A + 10/7*x^7*b^2*a^3*B + 10/7*x^7*b^3*a
^2*A + x^5*b*a^4*B + 2*x^5*b^2*a^3*A + 1/3*x^3*a^5*B + 5/3*x^3*b*
a^4*A + x*a^5*A
```

Sympy [A] time = 0.170535, size = 129, normalized size = 1.18

$$Aa^5x+\frac{Bb^5x^{13}}{13}+x^{11}\left(\frac{Ab^5}{11}+\frac{5Bab^4}{11}\right)+x^9\left(\frac{5Aab^4}{9}+\frac{10Ba^2b^3}{9}\right)+x^7\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right)+x^5\left(2Aa^3b^2+Ba^4b\right)+x^3\left(\frac{5Aa^4b}{3}+\frac{Ba^5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**13/13 + x**11*(A*b**5/11 + 5*B*a*b**4/11) +
x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**7*(10*A*a**2*b**3/7 +
 10*B*a**3*b**2/7) + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**3*(5*A*
a**4*b/3 + B*a**5/3)
```

GIAC/XCAS [A] time = 0.22216, size = 163, normalized size = 1.5

$$\frac{1}{13}\,Bb^5x^{13} + \frac{5}{11}\,Bab^4x^{11} + \frac{1}{11}\,Ab^5x^{11} + \frac{10}{9}\,Ba^2b^3x^9 + \frac{5}{9}\,Aab^4x^9 + \frac{10}{7}\,Ba^3b^2x^7 + \frac{10}{7}\,Aa^2b^3x^7 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{3}\,Ba^5x^3 + \frac{5}{3}\,Aa^4bx^3 + Aa^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5,x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 10/9*B*a^
2*b^3*x^9 + 5/9*A*a*b^4*x^9 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3
*x^7 + B*a^4*b*x^5 + 2*A*a^3*b^2*x^5 + 1/3*B*a^5*x^3 + 5/3*A*a^4*
b*x^3 + A*a^5*x
```

3.33 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=88

$$a^5A\log(x)+\frac{5}{2}a^4Abx^2+\frac{5}{2}a^3Ab^2x^4+\frac{5}{3}a^2Ab^3x^6+\frac{5}{8}aAb^4x^8+\frac{B(a+bx^2)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

```
[Out] (5*a^4*A*b*x^2)/2 + (5*a^3*A*b^2*x^4)/2 + (5*a^2*A*b^3*x^6)/3 + (
5*a*A*b^4*x^8)/8 + (A*b^5*x^10)/10 + (B*(a + b*x^2)^6)/(12*b) + a
^5*A*Log[x]
```

Rubi [A] time = 0.141172, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$a^5A\log(x)+\frac{5}{2}a^4Abx^2+\frac{5}{2}a^3Ab^2x^4+\frac{5}{3}a^2Ab^3x^6+\frac{5}{8}aAb^4x^8+\frac{B(a+bx^2)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (5*a^4*A*b*x^2)/2 + (5*a^3*A*b^2*x^4)/2 + (5*a^2*A*b^3*x^6)/3 + (
5*a*A*b^4*x^8)/8 + (A*b^5*x^10)/10 + (B*(a + b*x^2)^6)/(12*b) + a
^5*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Aa^5\log(x^2)}{2}+\frac{5Aa^4bx^2}{2}+5Aa^3b^2\int^{x^2}x\,dx+\frac{5Aa^2b^3x^6}{3}+\frac{5Aab^4x^8}{8}+\frac{Ab^5x^{10}}{10}+\frac{B(a+bx^2)^6}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x,x)
```

```
[Out] A*a**5*log(x**2)/2 + 5*A*a**4*b*x**2/2 + 5*A*a**3*b**2*Integral(x
, (x, x**2)) + 5*A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**8/8 + A*b**5*
x**10/10 + B*(a + b*x**2)**6/(12*b)
```

Mathematica [A] time = 0.0497772, size = 113, normalized size = 1.28

$$a^5A\log(x)+\frac{1}{2}a^4x^2(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)$$
$$+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{12}b^5Bx^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^2)/2 + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (5*a^
2*b^2*(A*b + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(
A*b + 5*a*B)*x^10)/10 + (b^5*B*x^12)/12 + a^5*A*Log[x]
```

Maple [A] time = 0.003, size = 124, normalized size = 1.4

$$\frac{Bb^5x^{12}}{12}+\frac{Ab^5x^{10}}{10}+\frac{Bx^{10}ab^4}{2}+\frac{5\,aAb^4x^8}{8}+\frac{5\,Bx^8a^2b^3}{4}+\frac{5\,a^2Ab^3x^6}{3}+\frac{5\,Bx^6a^3b^2}{3}+\frac{5\,a^3Ab^2x^4}{2}+\frac{5\,Bx^4a^4b}{4}+\frac{5\,a^4Abx^2}{2}+\frac{Bx^2a^5}{2}+a^5A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x,x)
```

```
[Out] 1/12*B*b^5*x^12+1/10*A*b^5*x^10+1/2*B*x^10*a*b^4+5/8*a*A*b^4*x^8+
5/4*B*x^8*a^2*b^3+5/3*a^2*A*b^3*x^6+5/3*B*x^6*a^3*b^2+5/2*a^3*A*b
^2*x^4+5/4*B*x^4*a^4*b+5/2*a^4*A*b*x^2+1/2*B*x^2*a^5+a^5*A*ln(x)
```

Maxima [A] time = 1.35166, size = 162, normalized size = 1.84

$$\frac{1}{12}\,Bb^5x^{12}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+\frac{1}{2}\,Aa^5\log\left(x^2\right)+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1/2*A*a^5*lo
g(x^2) + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b
)*x^2
```

Fricas [A] time = 0.240674, size = 158, normalized size = 1.8

$$\frac{1}{12}\,Bb^5x^{12}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+Aa^5\log(x)+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + A*a^5*log(x)
 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Sympy [A] time = 1.57969, size = 134, normalized size = 1.52

$$Aa^5\log(x)+\frac{Bb^5x^{12}}{12}+x^{10}\left(\frac{Ab^5}{10}+\frac{Bab^4}{2}\right)+x^8\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^6\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right)+x^4\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x^2\left(\frac{5Aa^4b}{2}+\frac{Ba^5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**12/12 + x**10*(A*b**5/10 + B*a*b**4/2)
+ x**8*(5*A*a*b**4/8 + 5*B*a**2*b**3/4) + x**6*(5*A*a**2*b**3/3 +
 5*B*a**3*b**2/3) + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x**2*
(5*A*a**4*b/2 + B*a**5/2)
```

GIAC/XCAS [A] time = 0.221658, size = 170, normalized size = 1.93

$$\frac{1}{12}Bb^5x^{12} + \frac{1}{2}Bab^4x^{10} + \frac{1}{10}Ab^5x^{10} + \frac{5}{4}Ba^2b^3x^8 + \frac{5}{8}Aab^4x^8 + \frac{5}{3}Ba^3b^2x^6 + \frac{5}{3}Aa^2b^3x^6 + \frac{5}{4}Ba^4bx^4 + \frac{5}{2}Aa^3b^2x^4 + \frac{1}{2}Ba^5x^2 + \frac{5}{2}Aa^4bx^2 + \frac{1}{2}Aa^5\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 5/4*B*a^2*
b^3*x^8 + 5/8*A*a*b^4*x^8 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6
 + 5/4*B*a^4*b*x^4 + 5/2*A*a^3*b^2*x^4 + 1/2*B*a^5*x^2 + 5/2*A*a^
4*b*x^2 + 1/2*A*a^5*ln(x^2)
```

3.34 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2} \, dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/
3 + 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (
b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x^11)/11
```

Rubi [A] time = 0.185105, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/
3 + 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (
b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{x}+\frac{Bb^5x^{11}}{11}+\frac{5a^3bx^3\,(2Ab+Ba)}{3}+2a^2b^2x^5\,(Ab+Ba)+\frac{5ab^3x^7\,(Ab+2Ba)}{7}+\frac{b^4x^9\,(Ab+5Ba)}{9}+\frac{a^4\,(5Ab+Ba)\int B\,dx}{B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**11/11 + 5*a**3*b*x**3*(2*A*b + B*a)/3 + 2*a
**2*b**2*x**5*(A*b + B*a) + 5*a*b**3*x**7*(A*b + 2*B*a)/7 + b**4*
x**9*(A*b + 5*B*a)/9 + a**4*(5*A*b + B*a)*Integral(B, x)/B
```

Mathematica [A] time = 0.0541114, size = 108, normalized size = 1.

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/
3 + 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (
```

```
b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x^11)/11
```

Maple [A] time = 0.006, size = 121, normalized size = 1.1

$$\frac{b^5Bx^{11}}{11}+\frac{Ax^9b^5}{9}+\frac{5\,Bx^9ab^4}{9}+\frac{5\,Ax^7ab^4}{7}+\frac{10\,Bx^7a^2b^3}{7}+2\,Ax^5a^2b^3+2\,Bx^5a^3b^2+\frac{10\,Ax^3a^3b^2}{3}+\frac{5\,Bx^3a^4b}{3}+5\,Axa^4b+Bxa^5-\frac{Aa^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^2,x)
```

```
[Out] 1/11*b^5*B*x^11+1/9*A*x^9*b^5+5/9*B*x^9*a*b^4+5/7*A*x^7*a*b^4+10/
7*B*x^7*a^2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+10/3*A*x^3*a^3*b^
2+5/3*B*x^3*a^4*b+5*A*x*a^4*b+B*x*a^5-a^5*A/x
```

Maxima [A] time = 1.34748, size = 157, normalized size = 1.45

$$\frac{1}{11}\,Bb^5x^{11}+\frac{1}{9}\left(5\,Bab^4+Ab^5\right)x^9+\frac{5}{7}\left(2\,Ba^2b^3+Aab^4\right)x^7+2\left(Ba^3b^2+Aa^2b^3\right)x^5-\frac{Aa^5}{x}+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3+\left(Ba^5+5\,Aa^4b\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/7*(2*B*a^2*b^3
+ A*a*b^4)*x^7 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 - A*a^5/x + 5/3*(B
*a^4*b + 2*A*a^3*b^2)*x^3 + (B*a^5 + 5*A*a^4*b)*x
```

Fricas [A] time = 0.22893, size = 163, normalized size = 1.51

$$\frac{63\,Bb^5x^{12}+77\left(5\,Bab^4+Ab^5\right)x^{10}+495\left(2\,Ba^2b^3+Aab^4\right)x^8+1386\left(Ba^3b^2+Aa^2b^3\right)x^6-693\,Aa^5+1155\left(Ba^4b+2\,Aa^3b^2\right)}{693\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^12 + 77*(5*B*a*b^4 + A*b^5)*x^10 + 495*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 693*A*a^
5 + 1155*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 693*(B*a^5 + 5*A*a^4*b)*x^
2)/x
```

Sympy [A] time = 1.56495, size = 126, normalized size = 1.17

$$-\frac{Aa^5}{x}+\frac{Bb^5x^{11}}{11}+x^9\left(\frac{Ab^5}{9}+\frac{5Bab^4}{9}\right)+x^7\left(\frac{5Aab^4}{7}+\frac{10Ba^2b^3}{7}\right)+x^5\left(2Aa^2b^3+2Ba^3b^2\right)+x^3\left(\frac{10Aa^3b^2}{3}+\frac{5Ba^4b}{3}\right)+x\left(5Aa^4b+Ba^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**11/11 + x**9*(A*b**5/9 + 5*B*a*b**4/9) + x*
*7*(5*A*a*b**4/7 + 10*B*a**2*b**3/7) + x**5*(2*A*a**2*b**3 + 2*B*
a**3*b**2) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3) + x*(5*A*a**4
*b + B*a**5)
```

GIAC/XCAS [A] time = 0.226972, size = 162, normalized size = 1.5

$$\frac{1}{11}\,Bb^{5}x^{11} + \frac{5}{9}\,Bab^{4}x^{9} + \frac{1}{9}\,Ab^{5}x^{9} + \frac{10}{7}\,Ba^{2}b^{3}x^{7} + \frac{5}{7}\,Aab^{4}x^{7} + 2\,Ba^{3}b^{2}x^{5} + 2\,Aa^{2}b^{3}x^{5} + \frac{5}{3}\,Ba^{4}bx^{3} + \frac{10}{3}\,Aa^{3}b^{2}x^{3} + Ba^{5}x + 5\,Aa^{4}bx - \frac{Aa^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 10/7*B*a^2*b^
3*x^7 + 5/7*A*a*b^4*x^7 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/3
*B*a^4*b*x^3 + 10/3*A*a^3*b^2*x^3 + B*a^5*x + 5*A*a^4*b*x - A*a^5
/x
```

3.35 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3} \, dx$

Optimal. Leaf size=113

$$-\frac{a^5 A}{2x^2} + a^4 \log(x)(aB + 5Ab) + \frac{5}{2}a^3 bx^2(aB + 2Ab) + \frac{5}{2}a^2 b^2 x^4(aB + Ab) + \frac{1}{8}b^4 x^8(5aB + Ab) + \frac{5}{6}ab^3 x^6(2aB + Ab) + \frac{1}{10}b^5 Bx^{10}$$

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*
b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a
*B)*x^8)/8 + (b^5*B*x^10)/10 + a^4*(5*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.294954, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^5 A}{2x^2} + a^4 \log(x)(aB + 5Ab) + \frac{5}{2}a^3 bx^2(aB + 2Ab) + \frac{5}{2}a^2 b^2 x^4(aB + Ab) + \frac{1}{8}b^4 x^8(5aB + Ab) + \frac{5}{6}ab^3 x^6(2aB + Ab) + \frac{1}{10}b^5 Bx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*
b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a
*B)*x^8)/8 + (b^5*B*x^10)/10 + a^4*(5*A*b + a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{2x^2} + \frac{Bb^5 x^{10}}{10} + \frac{a^4 (5Ab + Ba) \log(x^2)}{2} + \frac{5a^3 bx^2 (2Ab + Ba)}{2} + 5a^2 b^2 (Ab + Ba) \int^{x^2} x \, dx + \frac{5ab^3 x^6 (Ab + 2Ba)}{6} + \frac{b^4 x^8 (Ab + 5Ba)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**10/10 + a**4*(5*A*b + B*a)*log(x**2)
/2 + 5*a**3*b*x**2*(2*A*b + B*a)/2 + 5*a**2*b**2*(A*b + B*a)*Inte
gral(x, (x, x**2)) + 5*a*b**3*x**6*(A*b + 2*B*a)/6 + b**4*x**8*(A
*b + 5*B*a)/8
```

Mathematica [A] time = 0.0940193, size = 115, normalized size = 1.02

$$-\frac{a^5 A}{2x^2} + \frac{5}{2}a^3 bx^2(aB + 2Ab) + \frac{5}{2}a^2 b^2 x^4(aB + Ab) + \log(x) \left(a^5 B + 5a^4 Ab\right) + \frac{1}{8}b^4 x^8(5aB + Ab) + \frac{5}{6}ab^3 x^6(2aB + Ab) + \frac{1}{10}b^5 Bx^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*
b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a
```

```
*B)*x^8)/8 + (b^5*B*x^10)/10 + (5*a^4*A*b + a^5*B)*Log[x]
```

Maple [A] time = 0.008, size = 123, normalized size = 1.1

$$\frac{b^{5}Bx^{10}}{10}+\frac{Ax^{8}b^{5}}{8}+\frac{5\,Bx^{8}ab^{4}}{8}+\frac{5\,Ax^{6}ab^{4}}{6}+\frac{5\,Bx^{6}a^{2}b^{3}}{3}+\frac{5\,Ax^{4}a^{2}b^{3}}{2}+\frac{5\,Bx^{4}a^{3}b^{2}}{2}+5\,Ax^{2}a^{3}b^{2}+\frac{5\,Bx^{2}a^{4}b}{2}+5\,A\ln\left(x\right)a^{4}b+B\ln\left(x\right)a^{5}-\frac{Aa^{5}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^3,x)
```

```
[Out] 1/10*b^5*B*x^10+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+5/6*A*x^6*a*b^4+5/3
*B*x^6*a^2*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+5*A*x^2*a^3*b^
2+5/2*B*x^2*a^4*b+5*A*ln(x)*a^4*b+B*ln(x)*a^5-1/2*a^5*A/x^2
```

Maxima [A] time = 1.34558, size = 162, normalized size = 1.43

$$\frac{1}{10}\,Bb^{5}x^{10}+\frac{1}{8}\,\left(5\,Bab^{4}+Ab^{5}\right)x^{8}+\frac{5}{6}\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{6}+\frac{5}{2}\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{4}-\frac{Aa^{5}}{2\,x^{2}}+\frac{5}{2}\,\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{2}+\frac{1}{2}\,\left(Ba^{5}+5\,Aa^{4}b\right)\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/6*(2*B*a^2*b^3
+ A*a*b^4)*x^6 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 - 1/2*A*a^5/x^2
+ 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x
^2)
```

Fricas [A] time = 0.219188, size = 166, normalized size = 1.47

$$\frac{12\,Bb^{5}x^{12}+15\,\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+100\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+300\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}-60\,Aa^{5}+300\,\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+}{120\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(12*B*b^5*x^12 + 15*(5*B*a*b^4 + A*b^5)*x^10 + 100*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 60*A*a^5
+ 300*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 120*(B*a^5 + 5*A*a^4*b)*x^2*l
og(x))/x^2
```

Sympy [A] time = 1.96108, size = 131, normalized size = 1.16

$$-\frac{Aa^{5}}{2x^{2}}+\frac{Bb^{5}x^{10}}{10}+a^{4}\left(5Ab+Ba\right)\log\left(x\right)+x^{8}\left(\frac{Ab^{5}}{8}+\frac{5Bab^{4}}{8}\right)+x^{6}\left(\frac{5Aab^{4}}{6}+\frac{5Ba^{2}b^{3}}{3}\right)+x^{4}\left(\frac{5Aa^{2}b^{3}}{2}+\frac{5Ba^{3}b^{2}}{2}\right)+x^{2}\left(5Aa^{3}b^{2}+\frac{5Ba^{4}b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**10/10 + a**4*(5*A*b + B*a)*log(x) +
x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b*
*3/3) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x**2*(5*A*a**3
*b**2 + 5*B*a**4*b/2)
```

GIAC/XCAS [A] time = 0.220962, size = 196, normalized size = 1.73

$$\frac{1}{10}\,Bb^{5}x^{10} + \frac{5}{8}\,Bab^{4}x^{8} + \frac{1}{8}\,Ab^{5}x^{8} + \frac{5}{3}\,Ba^{2}b^{3}x^{6} + \frac{5}{6}\,Aab^{4}x^{6} + \frac{5}{2}\,Ba^{3}b^{2}x^{4} + \frac{5}{2}\,Aa^{2}b^{3}x^{4} + \frac{5}{2}\,Ba^{4}bx^{2} + 5\,Aa^{3}b^{2}x^{2} + \frac{1}{2}\,\left(Ba^{5} + 5\,Aa^{4}b\right)\ln\left(x^{2}\right) - \frac{Ba^{5}x^{2} + 5\,Aa^{4}bx^{2} + Aa^{5}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 5/3*B*a^2*b^3
*x^6 + 5/6*A*a*b^4*x^6 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 +
5/2*B*a^4*b*x^2 + 5*A*a^3*b^2*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*ln(x^
2) - 1/2*(B*a^5*x^2 + 5*A*a^4*b*x^2 + A*a^5)/x^2
```

3.36 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{3x^3}-\frac{a^4(aB+5Ab)}{x}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

```
[Out] -(a^5*A)/(3*x^3) - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*
x + (10*a^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b
^4*(A*b + 5*a*B)*x^7)/7 + (b^5*B*x^9)/9
```

Rubi [A] time = 0.191129, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5A}{3x^3}-\frac{a^4(aB+5Ab)}{x}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*
x + (10*a^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b
^4*(A*b + 5*a*B)*x^7)/7 + (b^5*B*x^9)/9
```

Rubi in Sympy [A] time = 27.0943, size = 104, normalized size = 0.96

$$-\frac{Aa^5}{3x^3}+\frac{Bb^5x^9}{9}-\frac{a^4\left(5Ab+Ba\right)}{x}+5a^3bx\left(2Ab+Ba\right)+\frac{10a^2b^2x^3\left(Ab+Ba\right)}{3}+ab^3x^5\left(Ab+2Ba\right)+\frac{b^4x^7\left(Ab+5Ba\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**4,x)
```

```
[Out] -A*a**5/(3*x**3) + B*b**5*x**9/9 - a**4*(5*A*b + B*a)/x + 5*a**3*
b*x*(2*A*b + B*a) + 10*a**2*b**2*x**3*(A*b + B*a)/3 + a*b**3*x**5
*(A*b + 2*B*a) + b**4*x**7*(A*b + 5*B*a)/7
```

Mathematica [A] time = 0.062461, size = 110, normalized size = 1.02

$$-\frac{a^5A}{3x^3}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{a^5(-B)-5a^4Ab}{x}+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) + (-5*a^4*A*b - a^5*B)/x + 5*a^3*b*(2*A*b + a*B)
*x + (10*a^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (
```

```
b^4*(A*b + 5*a*B)*x^7)/7 + (b^5*B*x^9)/9
```

Maple [A] time = 0.008, size = 118, normalized size = 1.1

$$\frac{b^5Bx^9}{9}+\frac{Ax^7b^5}{7}+\frac{5\,Bx^7ab^4}{7}+Ax^5ab^4+2\,Bx^5a^2b^3+\frac{10\,Ax^3a^2b^3}{3}+\frac{10\,Bx^3a^3b^2}{3}+10\,Axa^3b^2+5\,Bxa^4b-\frac{Aa^5}{3\,x^3}-\frac{a^4\,(5\,Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^4,x)
```

```
[Out] 1/9*b^5*B*x^9+1/7*A*x^7*b^5+5/7*B*x^7*a*b^4+A*x^5*a*b^4+2*B*x^5*a
^2*b^3+10/3*A*x^3*a^2*b^3+10/3*B*x^3*a^3*b^2+10*A*x*a^3*b^2+5*B*x
*a^4*b-1/3*a^5*A/x^3-a^4*(5*A*b+B*a)/x
```

Maxima [A] time = 1.34786, size = 159, normalized size = 1.47

$$\frac{1}{9}\,Bb^5x^9+\frac{1}{7}\left(5\,Bab^4+Ab^5\right)x^7+\left(2\,Ba^2b^3+Aab^4\right)x^5+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+5\left(Ba^4b+2\,Aa^3b^2\right)x-\frac{Aa^5+3\left(Ba^5+5\,Aa^4b\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + (2*B*a^2*b^3 + A*a*
b^4)*x^5 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5*(B*a^4*b + 2*A*a^
3*b^2)*x - 1/3*(A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

Fricas [A] time = 0.226258, size = 163, normalized size = 1.51

$$\frac{7\,Bb^5x^{12}+9\left(5\,Bab^4+Ab^5\right)x^{10}+63\left(2\,Ba^2b^3+Aab^4\right)x^8+210\left(Ba^3b^2+Aa^2b^3\right)x^6-21\,Aa^5+315\left(Ba^4b+2\,Aa^3b^2\right)x^4-63}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^5*x^12 + 9*(5*B*a*b^4 + A*b^5)*x^10 + 63*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 + 31
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

Sympy [A] time = 2.02657, size = 126, normalized size = 1.17

$$\frac{Bb^5x^9}{9}+x^7\left(\frac{Ab^5}{7}+\frac{5Bab^4}{7}\right)+x^5\left(Aab^4+2Ba^2b^3\right)+x^3\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right)+x\left(10Aa^3b^2+5Ba^4b\right)-\frac{Aa^5+x^2\left(15Aa^4b+3Ba^5\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**5*x**9/9 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**5*(A*a*b**4 +
 2*B*a**2*b**3) + x**3*(10*A*a**2*b**3/3 + 10*B*a**3*b**2/3) + x*
(10*A*a**3*b**2 + 5*B*a**4*b) - (A*a**5 + x**2*(15*A*a**4*b + 3*B
*a**5))/(3*x**3)
```

GIAC/XCAS [A] time = 0.244244, size = 165, normalized size = 1.53

$$\frac{1}{9}\,Bb^5x^9 + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + 2\,Ba^2b^3x^5 + Aab^4x^5 + \frac{10}{3}\,Ba^3b^2x^3 + \frac{10}{3}\,Aa^2b^3x^3 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{3\,Ba^5x^2 + 15\,Aa^4bx^2 + Aa^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 2*B*a^2*b^3*x^5
 + A*a*b^4*x^5 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*B*a^
4*b*x + 10*A*a^3*b^2*x - 1/3*(3*B*a^5*x^2 + 15*A*a^4*b*x^2 + A*a^
5)/x^3
```

3.37 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b +
 a*B)*x^2 + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^
6)/6 + (b^5*B*x^8)/8 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.277694, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b +
 a*B)*x^2 + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^
6)/6 + (b^5*B*x^8)/8 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{4x^4}+\frac{Bb^5x^8}{8}-\frac{a^4\,(5Ab+Ba)}{2x^2}+\frac{5a^3b\,(2Ab+Ba)\log\left(x^2\right)}{2}+5a^2b^2x^2\,(Ab+Ba)+\frac{5ab^3\,(Ab+2Ba)\int^{x^2}x\,dx}{2}+\frac{b^4x^6\,(Ab+5Ba)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**5,x)
```

```
[Out] -A*a**5/(4*x**4) + B*b**5*x**8/8 - a**4*(5*A*b + B*a)/(2*x**2) +
5*a**3*b*(2*A*b + B*a)*log(x**2)/2 + 5*a**2*b**2*x**2*(A*b + B*a)
 + 5*a*b**3*(A*b + 2*B*a)*Integral(x, (x, x**2))/2 + b**4*x**6*(A
*b + 5*B*a)/6
```

Mathematica [A] time = 0.0665043, size = 112, normalized size = 1.

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b +
 a*B)*x^2 + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^
6)/6 + (b^5*B*x^8)/8 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Maple [A] time = 0.01, size = 124, normalized size = 1.1

$$\frac{b^5Bx^8}{8}+\frac{Ax^6b^5}{6}+\frac{5\,Bx^6ab^4}{6}+\frac{5\,Ax^4ab^4}{4}+\frac{5\,Bx^4a^2b^3}{2}+5\,Ax^2a^2b^3 +5\,Bx^2a^3b^2+10\,A\ln(x)\,a^3b^2+5\,B\ln(x)\,a^4b-\frac{Aa^5}{4\,x^4}-\frac{5\,a^4bA}{2\,x^2}-\frac{a^5B}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^5,x)
```

```
[Out] 1/8*b^5*B*x^8+1/6*A*x^6*b^5+5/6*B*x^6*a*b^4+5/4*A*x^4*a*b^4+5/2*B
*x^4*a^2*b^3+5*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2+10*A*ln(x)*a^3*b^2+5
*B*ln(x)*a^4*b-1/4*a^5*A/x^4-5/2*a^4/x^2*A*b-1/2*a^5/x^2*B
```

Maxima [A] time = 1.36597, size = 165, normalized size = 1.47

$$\frac{1}{8}\,Bb^5x^8+\frac{1}{6}\left(5\,Bab^4+Ab^5\right)x^6+\frac{5}{4}\left(2\,Ba^2b^3+Aab^4\right)x^4+5\left(Ba^3b^2+Aa^2b^3\right)x^2 +\frac{5}{2}\left(Ba^4b+2\,Aa^3b^2\right)\log\left(x^2\right)-\frac{Aa^5+2\left(Ba^5+5\,Aa^4b\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/4*(2*B*a^2*b^3 +
A*a*b^4)*x^4 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*log(x^2) - 1/4*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

Fricas [A] time = 0.22511, size = 166, normalized size = 1.48

$$\frac{3\,Bb^5x^{12}+4\left(5\,Bab^4+Ab^5\right)x^{10}+30\left(2\,Ba^2b^3+Aab^4\right)x^8+120\left(Ba^3b^2+Aa^2b^3\right)x^6-6\,Aa^5+120\left(Ba^4b+2\,Aa^3b^2\right)x^4\log(x}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^12 + 4*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 6*A*a^5 + 120
*(B*a^4*b + 2*A*a^3*b^2)*x^4*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x^2)
/x^4
```

Sympy [A] time = 3.27077, size = 126, normalized size = 1.12

$$\frac{Bb^5x^8}{8}+5a^3b\left(2Ab+Ba\right)\log\left(x\right)+x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^4\left(\frac{5Aab^4}{4}+\frac{5Ba^2b^3}{2}\right) +x^2\left(5Aa^2b^3+5Ba^3b^2\right)-\frac{Aa^5+x^2\left(10Aa^4b+2Ba^5\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**5*x**8/8 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**6*(A*b**5/6 +
5*B*a*b**4/6) + x**4*(5*A*a*b**4/4 + 5*B*a**2*b**3/2) + x**2*(5*A
*a**2*b**3 + 5*B*a**3*b**2) - (A*a**5 + x**2*(10*A*a**4*b + 2*B*a
**5))/(4*x**4)
```

GIAC/XCAS [A] time = 0.234827, size = 201, normalized size = 1.79

$$\frac{1}{8}Bb^5x^8 + \frac{5}{6}Bab^4x^6 + \frac{1}{6}Ab^5x^6 + \frac{5}{2}Ba^2b^3x^4 + \frac{5}{4}Aab^4x^4 + 5Ba^3b^2x^2 + 5Aa^2b^3x^2 + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)\ln\left(x^2\right) - \frac{15Ba^4bx^4 + 30Aa^3b^2x^4 + 2Ba^5x^2 + 10Aa^4bx^2 + Aa^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^5,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 5/2*B*a^2*b^3*x
^4 + 5/4*A*a*b^4*x^4 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 + 5/2*(B
*a^4*b + 2*A*a^3*b^2)*ln(x^2) - 1/4*(15*B*a^4*b*x^4 + 30*A*a^3*b^
2*x^4 + 2*B*a^5*x^2 + 10*A*a^4*b*x^2 + A*a^5)/x^4
```

3.38 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^6} \, dx$

Optimal. Leaf size=111

$$-\frac{a^5 A}{5x^5} - \frac{a^4(aB+5Ab)}{3x^3} - \frac{5a^3b(aB+2Ab)}{x} + 10a^2b^2x(aB+Ab) + \frac{1}{5}b^4x^5(5aB+Ab) + \frac{5}{3}ab^3x^3(2aB+Ab) + \frac{1}{7}b^5Bx^7$$

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b
+ a*B))/x + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3
)/3 + (b^4*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Rubi [A] time = 0.18945, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{a^5 A}{5x^5} - \frac{a^4(aB+5Ab)}{3x^3} - \frac{5a^3b(aB+2Ab)}{x} + 10a^2b^2x(aB+Ab) + \frac{1}{5}b^4x^5(5aB+Ab) + \frac{5}{3}ab^3x^3(2aB+Ab) + \frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b
+ a*B))/x + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3
)/3 + (b^4*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Rubi in Sympy [A] time = 27.9983, size = 107, normalized size = 0.96

$$-\frac{Aa^5}{5x^5} + \frac{Bb^5x^7}{7} - \frac{a^4\left(5Ab+Ba\right)}{3x^3} - \frac{5a^3b\left(2Ab+Ba\right)}{x} + 10a^2b^2x\left(Ab+Ba\right) + \frac{5ab^3x^3\left(Ab+2Ba\right)}{3} + \frac{b^4x^5\left(Ab+5Ba\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a**5/(5*x**5) + B*b**5*x**7/7 - a**4*(5*A*b + B*a)/(3*x**3) -
5*a**3*b*(2*A*b + B*a)/x + 10*a**2*b**2*x*(A*b + B*a) + 5*a*b**3*
x**3*(A*b + 2*B*a)/3 + b**4*x**5*(A*b + 5*B*a)/5
```

Mathematica [A] time = 0.0730572, size = 111, normalized size = 1.

$$-\frac{a^5 A}{5x^5} - \frac{a^4(aB+5Ab)}{3x^3} - \frac{5a^3b(aB+2Ab)}{x} + 10a^2b^2x(aB+Ab) + \frac{1}{5}b^4x^5(5aB+Ab) + \frac{5}{3}ab^3x^3(2aB+Ab) + \frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b
+ a*B))/x + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3
```

```
)/3 + (b^4*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Maple [A] time = 0.009, size = 113, normalized size = 1.

$$\frac{b^5Bx^7}{7}+\frac{Ax^5b^5}{5}+Bx^5ab^4+\frac{5\,Ax^3ab^4}{3}+\frac{10\,Bx^3a^2b^3}{3}+10\,Axa^2b^3+10\,Bxa^3b^2-\frac{a^4\,(5\,Ab+Ba)}{3\,x^3}-\frac{Aa^5}{5\,x^5}-5\,\frac{a^3b\,(2\,Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^6,x)
```

```
[Out] 1/7*b^5*B*x^7+1/5*A*x^5*b^5+B*x^5*a*b^4+5/3*A*x^3*a*b^4+10/3*B*x^
3*a^2*b^3+10*A*x*a^2*b^3+10*B*x*a^3*b^2-1/3*a^4*(5*A*b+B*a)/x^3-1
/5*a^5*A/x^5-5*a^3*b*(2*A*b+B*a)/x
```

Maxima [A] time = 1.35738, size = 162, normalized size = 1.46

$$\frac{1}{7}\,Bb^5x^7+\frac{1}{5}\left(5\,Bab^4+Ab^5\right)x^5+\frac{5}{3}\left(2\,Ba^2b^3+Aab^4\right)x^3+10\left(Ba^3b^2+Aa^2b^3\right)x-\frac{3\,Aa^5+75\left(Ba^4b+2\,Aa^3b^2\right)x^4+5\left(Ba^5+5\,Aa^4b\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/3*(2*B*a^2*b^3 +
A*a*b^4)*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/15*(3*A*a^5 + 75*
(B*a^4*b + 2*A*a^3*b^2)*x^4 + 5*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

Fricas [A] time = 0.219484, size = 163, normalized size = 1.47

$$\frac{15\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+175\left(2\,Ba^2b^3+Aab^4\right)x^8+1050\left(Ba^3b^2+Aa^2b^3\right)x^6-21\,Aa^5-525\left(Ba^4b+2\,Aa^3b^2\right)x^4}{105\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5
 - 525*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 35*(B*a^5 + 5*A*a^4*b)*x^2)/
x^5
```

Sympy [A] time = 3.64273, size = 126, normalized size = 1.14

$$\frac{Bb^5x^7}{7}+x^5\left(\frac{Ab^5}{5}+Bab^4\right)+x^3\left(\frac{5Aab^4}{3}+\frac{10Ba^2b^3}{3}\right)+x\left(10Aa^2b^3+10Ba^3b^2\right)-\frac{3Aa^5+x^4\left(150Aa^3b^2+75Ba^4b\right)+x^2\left(25Aa^4b+5Ba^5\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**5*x**7/7 + x**5*(A*b**5/5 + B*a*b**4) + x**3*(5*A*a*b**4/3 +
 10*B*a**2*b**3/3) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) - (3*A*a
**5 + x**4*(150*A*a**3*b**2 + 75*B*a**4*b) + x**2*(25*A*a**4*b +
5*B*a**5))/(15*x**5)
```

GIAC/XCAS [A] time = 0.240905, size = 166, normalized size = 1.5

$$\frac{1}{7}Bb^5x^7 + Bab^4x^5 + \frac{1}{5}Ab^5x^5 + \frac{10}{3}Ba^2b^3x^3 + \frac{5}{3}Aab^4x^3 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{75\,Ba^4bx^4 + 150\,Aa^3b^2x^4 + 5\,Ba^5x^2 + 25\,Aa^4bx^2 + 3\,Aa^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^6,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 10/3*B*a^2*b^3*x^3
+ 5/3*A*a*b^4*x^3 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/15*(75*B*
a^4*b*x^4 + 150*A*a^3*b^2*x^4 + 5*B*a^5*x^2 + 25*A*a^4*b*x^2 + 3*
A*a^5)/x^5
```

3.39 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b
+ a*B))/(2*x^2) + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^6)/6 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rubi [A] time = 0.27327, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b
+ a*B))/(2*x^2) + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^6)/6 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{6x^6}+\frac{Bb^5x^6}{6}-\frac{a^4\left(5Ab+Ba\right)}{4x^4}-\frac{5a^3b\left(2Ab+Ba\right)}{2x^2}+5a^2b^2\left(Ab+Ba\right)\log\left(x^2\right)+\frac{5ab^3x^2\left(Ab+2Ba\right)}{2}+\frac{b^4\left(Ab+5Ba\right)\int^{x^2}x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**5/(6*x**6) + B*b**5*x**6/6 - a**4*(5*A*b + B*a)/(4*x**4) -
5*a**3*b*(2*A*b + B*a)/(2*x**2) + 5*a**2*b**2*(A*b + B*a)*log(x**
2) + 5*a*b**3*x**2*(A*b + 2*B*a)/2 + b**4*(A*b + 5*B*a)*Integral(
x, (x, x**2))/2
```

Mathematica [A] time = 0.0683826, size = 116, normalized size = 1.02

$$\frac{1}{12}\left(-\frac{a^5\left(2A+3Bx^2\right)}{x^6}-\frac{15a^4b\left(A+2Bx^2\right)}{x^4}-\frac{60a^3Ab^2}{x^2}+120a^2b^2\log(x)(aB+Ab)+60a^2b^3Bx^2+15ab^4x^2\left(2A+Bx^2\right)+b^5x^4\left(3A+2Bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] ((-60*a^3*A*b^2)/x^2 + 60*a^2*b^3*B*x^2 + 15*a*b^4*x^2*(2*A + B*x
^2) - (15*a^4*b*(A + 2*B*x^2))/x^4 + b^5*x^4*(3*A + 2*B*x^2) - (a
^5*(2*A + 3*B*x^2))/x^6 + 120*a^2*b^2*(A*b + a*B)*Log[x])/12
```

Maple [A] time = 0.01, size = 124, normalized size = 1.1

$$\frac{b^5Bx^6}{6}+\frac{Ax^4b^5}{4}+\frac{5\,Bx^4ab^4}{4}+\frac{5\,Ax^2ab^4}{2}+5\,Bx^2a^2b^3+10\,A\ln(x)\,a^2b^3+10\,B\ln(x)\,a^3b^2-\frac{Aa^5}{6\,x^6}-\frac{5\,a^4bA}{4\,x^4}-\frac{a^5B}{4\,x^4}-5\,\frac{a^3b^2A}{x^2}-\frac{5\,a^4bB}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^7,x)
```

```
[Out] 1/6*b^5*B*x^6+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5/2*A*x^2*a*b^4+5*B*x
^2*a^2*b^3+10*A*ln(x)*a^2*b^3+10*B*ln(x)*a^3*b^2-1/6*a^5*A/x^6-5/
4*a^4/x^4*A*b-1/4*a^5/x^4*B-5*a^3*b^2/x^2*A-5/2*a^4*b/x^2*B
```

Maxima [A] time = 1.33862, size = 166, normalized size = 1.46

$$\frac{1}{6}\,Bb^5x^6+\frac{1}{4}\left(5\,Bab^4+Ab^5\right)x^4+\frac{5}{2}\left(2\,Ba^2b^3+Aab^4\right)x^2+5\left(Ba^3b^2+Aa^2b^3\right)\log\left(x^2\right)-\frac{2\,Aa^5+30\left(Ba^4b+2\,Aa^3b^2\right)x^4+3\left(Ba^5+5\,Aa^4b\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^5/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5/2*(2*B*a^2*b^3 +
A*a*b^4)*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(2*A*a^5
 + 30*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^
6
```

Fricas [A] time = 0.223637, size = 166, normalized size = 1.46

$$\frac{2\,Bb^5x^{12}+3\left(5\,Bab^4+Ab^5\right)x^{10}+30\left(2\,Ba^2b^3+Aab^4\right)x^8+120\left(Ba^3b^2+Aa^2b^3\right)x^6\log(x)-2\,Aa^5-30\left(Ba^4b+2\,Aa^3b^2\right)x^4}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^5/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^12 + 3*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6*log(x) - 2*A*a^
5 - 30*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 3*(B*a^5 + 5*A*a^4*b)*x^2)/x
^6
```

Sympy [A] time = 6.2285, size = 124, normalized size = 1.09

$$\frac{Bb^5x^6}{6}+10a^2b^2\left(Ab+Ba\right)\log\left(x\right)+x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right)+x^2\left(\frac{5Aab^4}{2}+5Ba^2b^3\right)-\frac{2Aa^5+x^4\left(60Aa^3b^2+30Ba^4b\right)+x^2\left(15Aa^4b+3Ba^5\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**5*x**6/6 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**4*(A*b**5/4
+ 5*B*a*b**4/4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*b**3) - (2*A*a**5
 + x**4*(60*A*a**3*b**2 + 30*B*a**4*b) + x**2*(15*A*a**4*b + 3*B*
a**5))/(12*x**6)
```

GIAC/XCAS [A] time = 0.240123, size = 204, normalized size = 1.79

$$\frac{1}{6}\,Bb^5x^6 + \frac{5}{4}\,Bab^4x^4 + \frac{1}{4}\,Ab^5x^4 + 5\,Ba^2b^3x^2 + \frac{5}{2}\,Aab^4x^2 + 5\,\left(Ba^3b^2 + Aa^2b^3\right)\ln\left(x^2\right) - \frac{110\,Ba^3b^2x^6 + 110\,Aa^2b^3x^6 + 30\,Ba^4bx^4 + 60\,Aa^3b^2x^4 + 3\,Ba^5x^2 + 15\,Aa^4bx^2 + 2\,Aa^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^7,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 5*B*a^2*b^3*x^2
 + 5/2*A*a*b^4*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*ln(x^2) - 1/12*(11
0*B*a^3*b^2*x^6 + 110*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b
^2*x^4 + 3*B*a^5*x^2 + 15*A*a^4*b*x^2 + 2*A*a^5)/x^6
```

3.40 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8} dx$

Optimal. Leaf size=111

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b
+ a*B))/(3*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a
*B)*x + (b^4*(A*b + 5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Rubi [A] time = 0.192304, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b
+ a*B))/(3*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a
*B)*x + (b^4*(A*b + 5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Rubi in Sympy [A] time = 27.008, size = 107, normalized size = 0.96

$$-\frac{Aa^5}{7x^7}+\frac{Bb^5x^5}{5}-\frac{a^4\left(5Ab+Ba\right)}{5x^5}-\frac{5a^3b\left(2Ab+Ba\right)}{3x^3}-\frac{10a^2b^2\left(Ab+Ba\right)}{x}+5ab^3x\left(Ab+2Ba\right)+\frac{b^4x^3\left(Ab+5Ba\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**8,x)
```

```
[Out] -A*a**5/(7*x**7) + B*b**5*x**5/5 - a**4*(5*A*b + B*a)/(5*x**5) -
5*a**3*b*(2*A*b + B*a)/(3*x**3) - 10*a**2*b**2*(A*b + B*a)/x + 5*
a*b**3*x*(A*b + 2*B*a) + b**4*x**3*(A*b + 5*B*a)/3
```

Mathematica [A] time = 0.0728265, size = 111, normalized size = 1.

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b
+ a*B))/(3*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a
*B)*x + (b^4*(A*b + 5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Maple [A] time = 0.008, size = 108, normalized size = 1.

$$\frac{b^5Bx^5}{5}+\frac{Ax^3b^5}{3}+\frac{5\,Bx^3ab^4}{3}+5\,Axab^4+10\,Bxa^2b^3 \\ -\frac{5\,a^3b\left(2\,Ab+Ba\right)}{3\,x^3}-\frac{a^4\left(5\,Ab+Ba\right)}{5\,x^5}-10\,\frac{a^2b^2\left(Ab+Ba\right)}{x}-\frac{Aa^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^8,x)
```

```
[Out] 1/5*b^5*B*x^5+1/3*A*x^3*b^5+5/3*B*x^3*a*b^4+5*A*x*a*b^4+10*B*x*a^
2*b^3-5/3*a^3*b*(2*A*b+B*a)/x^3-1/5*a^4*(5*A*b+B*a)/x^5-10*a^2*b^
2*(A*b+B*a)/x-1/7*a^5*A/x^7
```

Maxima [A] time = 1.35688, size = 162, normalized size = 1.46

$$\frac{1}{5}Bb^{5}x^{5}+\frac{1}{3}\left(5\,Bab^{4}+Ab^{5}\right)x^{3}+5\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x-\frac{1050\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+15\,Aa^{5}+175\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+21\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5*(2*B*a^2*b^3 + A*
a*b^4)*x - 1/105*(1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 15*A*a^5 + 1
75*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

Fricas [A] time = 0.215868, size = 163, normalized size = 1.47

$$\frac{21\,Bb^{5}x^{12}+35\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+525\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}-1050\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}-15\,Aa^{5}-175\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*B*b^5*x^12 + 35*(5*B*a*b^4 + A*b^5)*x^10 + 525*(2*B*a^2
*b^3 + A*a*b^4)*x^8 - 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 15*A*a^5
 - 175*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 21*(B*a^5 + 5*A*a^4*b)*x^2)/
x^7
```

Sympy [A] time = 6.67653, size = 126, normalized size = 1.14

$$\frac{Bb^{5}x^{5}}{5}+x^{3}\left(\frac{Ab^{5}}{3}+\frac{5Bab^{4}}{3}\right)+x\left(5Aab^{4}+10Ba^{2}b^{3}\right)-\frac{15Aa^{5}+x^{6}\left(1050Aa^{2}b^{3}+1050Ba^{3}b^{2}\right)+x^{4}\left(350Aa^{3}b^{2}+175Ba^{4}b\right)+x^{2}\left(105Aa^{4}b+21Ba^{5}\right)}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**8,x)
```

```
[Out] B*b**5*x**5/5 + x**3*(A*b**5/3 + 5*B*a*b**4/3) + x*(5*A*a*b**4 +
10*B*a**2*b**3) - (15*A*a**5 + x**6*(1050*A*a**2*b**3 + 1050*B*a
*3*b**2) + x**4*(350*A*a**3*b**2 + 175*B*a**4*b) + x**2*(105*A*a
*4*b + 21*B*a**5))/(105*x**7)
```

GIAC/XCAS [A] time = 0.235117, size = 167, normalized size = 1.5

$$\frac{1}{5}\,Bb^{5}x^{5}+\frac{5}{3}\,Bab^{4}x^{3}+\frac{1}{3}\,Ab^{5}x^{3}+10\,Ba^{2}b^{3}x+5\,Aab^{4}x-\frac{1050\,Ba^{3}b^{2}x^{6}+1050\,Aa^{2}b^{3}x^{6}+175\,Ba^{4}bx^{4}+350\,Aa^{3}b^{2}x^{4}+21\,Ba^{5}x^{2}+105\,Aa^{4}bx^{2}+15\,Aa^{5}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^8,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 10*B*a^2*b^3*x
+ 5*A*a*b^4*x - 1/105*(1050*B*a^3*b^2*x^6 + 1050*A*a^2*b^3*x^6 +
175*B*a^4*b*x^4 + 350*A*a^3*b^2*x^4 + 21*B*a^5*x^2 + 105*A*a^4*b*
x^2 + 15*A*a^5)/x^7
```

3.41 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=112

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b
+ a*B))/(4*x^4) - (5*a^2*b^2*(A*b + a*B))/x^2 + (b^4*(A*b + 5*a*B
)*x^2)/2 + (b^5*B*x^4)/4 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rubi [A] time = 0.268635, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b
+ a*B))/(4*x^4) - (5*a^2*b^2*(A*b + a*B))/x^2 + (b^4*(A*b + 5*a*B
)*x^2)/2 + (b^5*B*x^4)/4 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{8x^8}+\frac{Bb^5\int^{x^2}x\,dx}{2}-\frac{a^4(5Ab+Ba)}{6x^6}-\frac{5a^3b(2Ab+Ba)}{4x^4}-\frac{5a^2b^2(Ab+Ba)}{x^2}+\frac{5ab^3(Ab+2Ba)\log(x^2)}{2}+\frac{b^4(Ab+5Ba)\int^{x^2}A\,dx}{2A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**9,x)
```

```
[Out] -A*a**5/(8*x**8) + B*b**5*Integral(x, (x, x**2))/2 - a**4*(5*A*b
+ B*a)/(6*x**6) - 5*a**3*b*(2*A*b + B*a)/(4*x**4) - 5*a**2*b**2*(
A*b + B*a)/x**2 + 5*a*b**3*(A*b + 2*B*a)*log(x**2)/2 + b**4*(A*b
+ 5*B*a)*Integral(A, (x, x**2))/(2*A)
```

Mathematica [A] time = 0.103304, size = 116, normalized size = 1.04

$$5ab^3\log(x)(2aB+Ab)-\frac{a^5\left(3A+4Bx^2\right)+10a^4bx^2\left(2A+3Bx^2\right)+60a^3b^2x^4\left(A+2Bx^2\right)+120a^2Ab^3x^6-60ab^4Bx^{10}-6b^5x^{10}\left(2A+Bx^2\right)}{24x^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -(120*a^2*A*b^3*x^6 - 60*a*b^4*B*x^10 - 6*b^5*x^10*(2*A + B*x^2)
+ 60*a^3*b^2*x^4*(A + 2*B*x^2) + 10*a^4*b*x^2*(2*A + 3*B*x^2) + a
```

```
^5*(3*A + 4*B*x^2))/(24*x^8) + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Maple [A] time = 0.013, size = 124, normalized size = 1.1

$$\frac{b^5Bx^4}{4}+\frac{Ax^2b^5}{2}+\frac{5\,Bx^2ab^4}{2}+5\,A\ln(x)\,ab^4+10\,B\ln(x)\,a^2b^3-\frac{5\,a^4bA}{6\,x^6}-\frac{a^5B}{6\,x^6}-\frac{5\,a^3b^2A}{2\,x^4}-\frac{5\,a^4bB}{4\,x^4}-\frac{Aa^5}{8\,x^8}-5\,\frac{a^2b^3A}{x^2}-5\,\frac{a^3b^2B}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^9,x)
```

```
[Out] 1/4*b^5*B*x^4+1/2*A*x^2*b^5+5/2*B*x^2*a*b^4+5*A*ln(x)*a*b^4+10*B*
ln(x)*a^2*b^3-5/6*a^4/x^6*A*b-1/6*a^5/x^6*B-5/2*a^3*b^2/x^4*A-5/4
*a^4*b/x^4*B-1/8*a^5*A/x^8-5*a^2*b^3/x^2*A-5*a^3*b^2/x^2*B
```

Maxima [A] time = 1.35121, size = 166, normalized size = 1.48

$$\frac{1}{4}\,Bb^5x^4+\frac{1}{2}\left(5\,Bab^4+Ab^5\right)x^2+\frac{5}{2}\left(2\,Ba^2b^3+Aab^4\right)\log\left(x^2\right)-\frac{120\left(Ba^3b^2+Aa^2b^3\right)x^6+3\,Aa^5+30\left(Ba^4b+2\,Aa^3b^2\right)x^4+4\left(Ba^5+5\,Aa^4b\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 + 5/2*(2*B*a^2*b^3 +
A*a*b^4)*log(x^2) - 1/24*(120*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3*A*a
^5 + 30*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 4*(B*a^5 + 5*A*a^4*b)*x^2)/
x^8
```

Fricas [A] time = 0.223127, size = 166, normalized size = 1.48

$$\frac{6\,Bb^5x^{12}+12\left(5\,Bab^4+Ab^5\right)x^{10}+120\left(2\,Ba^2b^3+Aab^4\right)x^8\log(x)-120\left(Ba^3b^2+Aa^2b^3\right)x^6-3\,Aa^5-30\left(Ba^4b+2\,Aa^3b^2\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(6*B*b^5*x^12 + 12*(5*B*a*b^4 + A*b^5)*x^10 + 120*(2*B*a^2*b
^3 + A*a*b^4)*x^8*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 3*A*
a^5 - 30*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 4*(B*a^5 + 5*A*a^4*b)*x^2)
/x^8
```

Sympy [A] time = 11.3661, size = 124, normalized size = 1.11

$$\frac{Bb^5x^4}{4}+5ab^3\left(Ab+2Ba\right)\log(x)+x^2\left(\frac{Ab^5}{2}+\frac{5Bab^4}{2}\right)-\frac{3Aa^5+x^6\left(120Aa^2b^3+120Ba^3b^2\right)+x^4\left(60Aa^3b^2+30Ba^4b\right)+x^2\left(20Aa^4b+4Ba^5\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**9,x)
```

```
[Out] B*b**5*x**4/4 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**2*(A*b**5/2 +
5*B*a*b**4/2) - (3*A*a**5 + x**6*(120*A*a**2*b**3 + 120*B*a**3*b*
*2) + x**4*(60*A*a**3*b**2 + 30*B*a**4*b) + x**2*(20*A*a**4*b + 4
*B*a**5))/(24*x**8)
```

GIAC/XCAS [A] time = 0.226235, size = 203, normalized size = 1.81

$$\frac{1}{4}\,Bb^5x^4 + \frac{5}{2}\,Bab^4x^2 + \frac{1}{2}\,Ab^5x^2 + \frac{5}{2}\left(2\,Ba^2b^3 + Aab^4\right)\ln\left(x^2\right) - \frac{250\,Ba^2b^3x^8 + 125\,Aab^4x^8 + 120\,Ba^3b^2x^6 + 120\,Aa^2b^3x^6 + 30\,Ba^4bx^4 + 60\,Aa^3b^2x^4 + 4\,Ba^5x^2 + 20\,Aa^4bx^2 + 3\,Aa^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^9,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 + 5/2*(2*B*a^2*b^
3 + A*a*b^4)*ln(x^2) - 1/24*(250*B*a^2*b^3*x^8 + 125*A*a*b^4*x^8
+ 120*B*a^3*b^2*x^6 + 120*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a
^3*b^2*x^4 + 4*B*a^5*x^2 + 20*A*a^4*b*x^2 + 3*A*a^5)/x^8
```

3.42 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}} \, dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{10a^2b^2(aB+Ab)}{3x^3}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{3}b^5Bx^3$$

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b +
a*B))/x^5 - (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*
a*B))/x + b^4*(A*b + 5*a*B)*x + (b^5*B*x^3)/3
```

Rubi [A] time = 0.201864, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{10a^2b^2(aB+Ab)}{3x^3}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b +
a*B))/x^5 - (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*
a*B))/x + b^4*(A*b + 5*a*B)*x + (b^5*B*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{9x^9}+\frac{Bb^5x^3}{3}-\frac{a^4\left(5Ab+Ba\right)}{7x^7}-\frac{a^3b\left(2Ab+Ba\right)}{x^5}-\frac{10a^2b^2\left(Ab+Ba\right)}{3x^3}-\frac{5ab^3\left(Ab+2Ba\right)}{x}+\frac{b^4\left(Ab+5Ba\right)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**10,x)
```

```
[Out] -A*a**5/(9*x**9) + B*b**5*x**3/3 - a**4*(5*A*b + B*a)/(7*x**7) -
a**3*b*(2*A*b + B*a)/x**5 - 10*a**2*b**2*(A*b + B*a)/(3*x**3) - 5
*a*b**3*(A*b + 2*B*a)/x + b**4*(A*b + 5*B*a)*Integral(A, x)/A
```

Mathematica [A] time = 0.0634757, size = 115, normalized size = 1.06

$$-\frac{a^5\left(7A+9Bx^2\right)+9a^4bx^2\left(5A+7Bx^2\right)+42a^3b^2x^4\left(3A+5Bx^2\right)+210a^2b^3x^6\left(A+3Bx^2\right)+315ab^4x^8\left(A-Bx^2\right)-21b^5x^{10}}{63x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -(315*a*b^4*x^8*(A - B*x^2) - 21*b^5*x^10*(3*A + B*x^2) + 210*a^2
*b^3*x^6*(A + 3*B*x^2) + 42*a^3*b^2*x^4*(3*A + 5*B*x^2) + 9*a^4*b
*x^2*(5*A + 7*B*x^2) + a^5*(7*A + 9*B*x^2))/(63*x^9)
```

Maple [A] time = 0.009, size = 102, normalized size = 0.9

$$\frac{b^5Bx^3}{3}+Axb^5+5\,Bxab^4-\frac{Aa^5}{9\,x^9}-\frac{10\,a^2b^2\left(Ab+Ba\right)}{3\,x^3}-\frac{a^3b\left(2\,Ab+Ba\right)}{x^5}-5\,\frac{ab^3\left(Ab+2\,Ba\right)}{x}-\frac{a^4\left(5\,Ab+Ba\right)}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^10,x)
```

```
[Out] 1/3*b^5*B*x^3+A*x*b^5+5*B*x*a*b^4-1/9*a^5*A/x^9-10/3*a^2*b^2*(A*b
+B*a)/x^3-a^3*b*(2*A*b+B*a)/x^5-5*a*b^3*(A*b+2*B*a)/x-1/7*a^4*(5*
A*b+B*a)/x^7
```

Maxima [A] time = 1.36525, size = 161, normalized size = 1.49

$$\frac{1}{3}\,Bb^5x^3 + \left(5\,Bab^4 + Ab^5\right)x - \frac{315\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 210\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 7\,Aa^5 + 63\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 9\left(Ba^5 + 5\,Aa^4b\right)x^2}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*x - 1/63*(315*(2*B*a^2*b^3 +
A*a*b^4)*x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 7*A*a^5 + 63*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

Fricas [A] time = 0.231796, size = 163, normalized size = 1.51

$$\frac{21\,Bb^5x^{12} + 63\left(5\,Bab^4 + Ab^5\right)x^{10} - 315\left(2\,Ba^2b^3 + Aab^4\right)x^8 - 210\left(Ba^3b^2 + Aa^2b^3\right)x^6 - 7\,Aa^5 - 63\left(Ba^4b + 2\,Aa^3b^2\right)x^4 - 9}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*B*b^5*x^12 + 63*(5*B*a*b^4 + A*b^5)*x^10 - 315*(2*B*a^2*
b^3 + A*a*b^4)*x^8 - 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 7*A*a^5 -
63*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

Sympy [A] time = 12.5944, size = 122, normalized size = 1.13

$$\frac{Bb^5x^3}{3} + x\left(Ab^5 + 5Bab^4\right) - \frac{7Aa^5 + x^8\left(315Aab^4 + 630Ba^2b^3\right) + x^6\left(210Aa^2b^3 + 210Ba^3b^2\right) + x^4\left(126Aa^3b^2 + 63Ba^4b\right) + x^2\left(45Aa^4b + 9Ba^5\right)}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**10,x)
```

```
[Out] B*b**5*x**3/3 + x*(A*b**5 + 5*B*a*b**4) - (7*A*a**5 + x**8*(315*A
*a*b**4 + 630*B*a**2*b**3) + x**6*(210*A*a**2*b**3 + 210*B*a**3*b
**2) + x**4*(126*A*a**3*b**2 + 63*B*a**4*b) + x**2*(45*A*a**4*b +
 9*B*a**5))/(63*x**9)
```

GIAC/XCAS [A] time = 0.230042, size = 166, normalized size = 1.54

$$\frac{1}{3}\,Bb^5x^3 + 5\,Bab^4x + Ab^5x - \frac{630\,Ba^2b^3x^8 + 315\,Aab^4x^8 + 210\,Ba^3b^2x^6 + 210\,Aa^2b^3x^6 + 63\,Ba^4bx^4 + 126\,Aa^3b^2x^4 + 9\,Ba^5x^2 + 45\,Aa^4bx^2 + 7\,Aa^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^10,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + 5*B*a*b^4*x + A*b^5*x - 1/63*(630*B*a^2*b^3*x^8 +
 315*A*a*b^4*x^8 + 210*B*a^3*b^2*x^6 + 210*A*a^2*b^3*x^6 + 63*B*a
^4*b*x^4 + 126*A*a^3*b^2*x^4 + 9*B*a^5*x^2 + 45*A*a^4*b*x^2 + 7*A
*a^5)/x^9
```

3.43 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}} dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{5a^2b^2(aB+Ab)}{2x^4}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{2}b^5Bx^2$$

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*
b + a*B))/(6*x^6) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A
*b + 2*a*B))/(2*x^2) + (b^5*B*x^2)/2 + b^4*(A*b + 5*a*B)*Log[x]
```

Rubi [A] time = 0.245703, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{5a^2b^2(aB+Ab)}{2x^4}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*
b + a*B))/(6*x^6) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A
*b + 2*a*B))/(2*x^2) + (b^5*B*x^2)/2 + b^4*(A*b + 5*a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{10x^{10}}-\frac{a^4\left(5Ab+Ba\right)}{8x^8}-\frac{5a^3b\left(2Ab+Ba\right)}{6x^6}-\frac{5a^2b^2\left(Ab+Ba\right)}{2x^4}-\frac{5ab^3\left(Ab+2Ba\right)}{2x^2}+\frac{b^5\int^{x^2}B\,dx}{2}+\frac{b^4\left(Ab+5Ba\right)\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**11,x)
```

```
[Out] -A*a**5/(10*x**10) - a**4*(5*A*b + B*a)/(8*x**8) - 5*a**3*b*(2*A*
b + B*a)/(6*x**6) - 5*a**2*b**2*(A*b + B*a)/(2*x**4) - 5*a*b**3*(
A*b + 2*B*a)/(2*x**2) + b**5*Integral(B, (x, x**2))/2 + b**4*(A*b
 + 5*B*a)*log(x**2)/2
```

Mathematica [A] time = 0.103834, size = 116, normalized size = 1.03

$$b^4\log(x)(5aB+Ab)-\frac{3a^5\left(4A+5Bx^2\right)+25a^4bx^2\left(3A+4Bx^2\right)+100a^3b^2x^4\left(2A+3Bx^2\right)+300a^2b^3x^6\left(A+2Bx^2\right)+300aAb^4x^8-60b^5Bx^{12}}{120x^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -(300*a*A*b^4*x^8 - 60*b^5*B*x^12 + 300*a^2*b^3*x^6*(A + 2*B*x^2)
 + 100*a^3*b^2*x^4*(2*A + 3*B*x^2) + 25*a^4*b*x^2*(3*A + 4*B*x^2)
```

```
+ 3*a^5*(4*A + 5*B*x^2))/(120*x^10) + b^4*(A*b + 5*a*B)*Log[x]
```

Maple [A] time = 0.012, size = 123, normalized size = 1.1

$$\frac{b^5Bx^2}{2}+A\ln(x)\,b^5+5\,B\ln(x)\,ab^4-\frac{5\,a^3b^2A}{3\,x^6}-\frac{5\,a^4bB}{6\,x^6}-\frac{5\,a^2b^3A}{2\,x^4}-\frac{5\,a^3b^2B}{2\,x^4}-\frac{Aa^5}{10\,x^{10}}-\frac{5\,a^4bA}{8\,x^8}-\frac{a^5B}{8\,x^8}-\frac{5\,ab^4A}{2\,x^2}-5\,\frac{a^2b^3B}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^11,x)
```

```
[Out] 1/2*b^5*B*x^2+A*ln(x)*b^5+5*B*ln(x)*a*b^4-5/3*a^3*b^2/x^6*A-5/6*a
^4*b/x^6*B-5/2*a^2*b^3/x^4*A-5/2*a^3*b^2/x^4*B-1/10*a^5*A/x^10-5/
8*a^4/x^8*A*b-1/8*a^5/x^8*B-5/2*a*b^4/x^2*A-5*a^2*b^3/x^2*B
```

Maxima [A] time = 1.33405, size = 166, normalized size = 1.47

$$\frac{1}{2}\,Bb^5x^2+\frac{1}{2}\left(5\,Bab^4+Ab^5\right)\log\left(x^2\right)-\frac{300\left(2\,Ba^2b^3+Aab^4\right)x^8+300\left(Ba^3b^2+Aa^2b^3\right)x^6+12\,Aa^5+100\left(Ba^4b+2\,Aa^3b^2\right)x^4+15\left(Ba^5+5\,Aa^4b\right)x^2}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(300*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 12*A
*a^5 + 100*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 15*(B*a^5 + 5*A*a^4*b)*x
^2)/x^10
```

Fricas [A] time = 0.237817, size = 166, normalized size = 1.47

$$\frac{60\,Bb^5x^{12}+120\left(5\,Bab^4+Ab^5\right)x^{10}\log(x)-300\left(2\,Ba^2b^3+Aab^4\right)x^8-300\left(Ba^3b^2+Aa^2b^3\right)x^6-12\,Aa^5-100\left(Ba^4b+2\,Aa\right.}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(60*B*b^5*x^12 + 120*(5*B*a*b^4 + A*b^5)*x^10*log(x) - 300*
(2*B*a^2*b^3 + A*a*b^4)*x^8 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 1
2*A*a^5 - 100*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 15*(B*a^5 + 5*A*a^4*b
)*x^2)/x^10
```

Sympy [A] time = 21.584, size = 122, normalized size = 1.08

$$\frac{Bb^5x^2}{2}+b^4\left(Ab+5Ba\right)\log(x)-\frac{12Aa^5+x^8\left(300Aab^4+600Ba^2b^3\right)+x^6\left(300Aa^2b^3+300Ba^3b^2\right)+x^4\left(200Aa^3b^2+100Ba^4b\right)+x^2\left(75Aa^4b+15Ba^5\right)}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**11,x)
```

```
[Out] B*b**5*x**2/2 + b**4*(A*b + 5*B*a)*log(x) - (12*A*a**5 + x**8*(30
0*A*a*b**4 + 600*B*a**2*b**3) + x**6*(300*A*a**2*b**3 + 300*B*a**
3*b**2) + x**4*(200*A*a**3*b**2 + 100*B*a**4*b) + x**2*(75*A*a**4
*b + 15*B*a**5))/(120*x**10)
```

GIAC/XCAS [A] time = 0.227729, size = 198, normalized size = 1.75

$$\frac{1}{2}\,Bb^5x^2 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)\ln\left(x^2\right) - \frac{685\,Bab^4x^{10} + 137\,Ab^5x^{10} + 600\,Ba^2b^3x^8 + 300\,Aab^4x^8 + 300\,Ba^3b^2x^6 + 300\,Aa^2b^3x^6 + 100\,Ba^4bx^4 + 200\,Aa^3b^2x^4 + 15\,Ba}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*ln(x^2) - 1/120*(685*B*a*
b^4*x^10 + 137*A*b^5*x^10 + 600*B*a^2*b^3*x^8 + 300*A*a*b^4*x^8 +
 300*B*a^3*b^2*x^6 + 300*A*a^2*b^3*x^6 + 100*B*a^4*b*x^4 + 200*A*
a^3*b^2*x^4 + 15*B*a^5*x^2 + 75*A*a^4*b*x^2 + 12*A*a^5)/x^10
```

3.44 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{12}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^5Bx$$

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*
b + a*B))/(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b +
 2*a*B))/(3*x^3) - (b^4*(A*b + 5*a*B))/x + b^5*B*x
```

Rubi [A] time = 0.19647, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^5Bx$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*
b + a*B))/(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b +
 2*a*B))/(3*x^3) - (b^4*(A*b + 5*a*B))/x + b^5*B*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^5}{11x^{11}}-\frac{a^4\left(5Ab+Ba\right)}{9x^9}-\frac{5a^3b\left(2Ab+Ba\right)}{7x^7}-\frac{2a^2b^2\left(Ab+Ba\right)}{x^5}-\frac{5ab^3\left(Ab+2Ba\right)}{3x^3}+b^5\int B\,dx-\frac{b^4\left(Ab+5Ba\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**12,x)
```

```
[Out] -A*a**5/(11*x**11) - a**4*(5*A*b + B*a)/(9*x**9) - 5*a**3*b*(2*A*
b + B*a)/(7*x**7) - 2*a**2*b**2*(A*b + B*a)/x**5 - 5*a*b**3*(A*b
+ 2*B*a)/(3*x**3) + b**5*Integral(B, x) - b**4*(A*b + 5*B*a)/x
```

Mathematica [A] time = 0.0774845, size = 122, normalized size = 1.13

$$-\frac{a^5\left(9A+11Bx^2\right)}{99x^{11}}-\frac{5a^4b\left(7A+9Bx^2\right)}{63x^9}-\frac{2a^3b^2\left(5A+7Bx^2\right)}{7x^7}-\frac{2a^2b^3\left(3A+5Bx^2\right)}{3x^5}-\frac{5ab^4\left(A+3Bx^2\right)}{3x^3}-\frac{Ab^5}{x}+b^5Bx$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -((A*b^5)/x) + b^5*B*x - (5*a*b^4*(A + 3*B*x^2))/(3*x^3) - (2*a^2
*b^3*(3*A + 5*B*x^2))/(3*x^5) - (2*a^3*b^2*(5*A + 7*B*x^2))/(7*x^
7) - (5*a^4*b*(7*A + 9*B*x^2))/(63*x^9) - (a^5*(9*A + 11*B*x^2))/
(99*x^11)
```

Maple [A] time = 0.01, size = 101, normalized size = 0.9

$$-\frac{Aa^5}{11\,x^{11}}-\frac{a^4\,(5\,Ab+Ba)}{9\,x^9}-\frac{5\,a^3b\,(2\,Ab+Ba)}{7\,x^7}-2\,\frac{a^2b^2\,(Ab+Ba)}{x^5}-\frac{5\,ab^3\,(Ab+2\,Ba)}{3\,x^3}-\frac{b^4\,(Ab+5\,Ba)}{x}+b^5Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^12,x)
```

```
[Out] -1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^
7-2*a^2*b^2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*
a)/x+b^5*B*x
```

Maxima [A] time = 1.34185, size = 161, normalized size = 1.49

$$Bb^5x-\frac{693\left(5\,Bab^4+Ab^5\right)x^{10}+1155\left(2\,Ba^2b^3+Aab^4\right)x^8+1386\left(Ba^3b^2+Aa^2b^3\right)x^6+63\,Aa^5+495\left(Ba^4b+2\,Aa^3b^2\right)x^4+77\,(Ba}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^12,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/693*(693*(5*B*a*b^4 + A*b^5)*x^10 + 1155*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 63*A*a^5 + 4
95*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

Fricas [A] time = 0.230145, size = 163, normalized size = 1.51

$$\frac{693\,Bb^5x^{12}-693\left(5\,Bab^4+Ab^5\right)x^{10}-1155\left(2\,Ba^2b^3+Aab^4\right)x^8-1386\left(Ba^3b^2+Aa^2b^3\right)x^6-63\,Aa^5-495\left(Ba^4b+2\,Aa^3b^2\right.}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^12,x, algorithm="fricas")
```

```
[Out] 1/693*(693*B*b^5*x^12 - 693*(5*B*a*b^4 + A*b^5)*x^10 - 1155*(2*B*
a^2*b^3 + A*a*b^4)*x^8 - 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 63*A*
a^5 - 495*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 77*(B*a^5 + 5*A*a^4*b)*x^
2)/x^11
```

Sympy [A] time = 25.8644, size = 122, normalized size = 1.13

$$Bb^5x-\frac{63Aa^5+x^{10}\left(693Ab^5+3465Bab^4\right)+x^8\left(1155Aab^4+2310Ba^2b^3\right)+x^6\left(1386Aa^2b^3+1386Ba^3b^2\right)+x^4\left(990Aa^3b^2+495Ba\right.}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**12,x)
```

```
[Out] B*b**5*x - (63*A*a**5 + x**10*(693*A*b**5 + 3465*B*a*b**4) + x**8
*(1155*A*a*b**4 + 2310*B*a**2*b**3) + x**6*(1386*A*a**2*b**3 + 13
86*B*a**3*b**2) + x**4*(990*A*a**3*b**2 + 495*B*a**4*b) + x**2*(3
85*A*a**4*b + 77*B*a**5))/(693*x**11)
```

GIAC/XCAS [A] time = 0.226987, size = 169, normalized size = 1.56

$$Bb^5x - \frac{3465\,Bab^4x^{10} + 693\,Ab^5x^{10} + 2310\,Ba^2b^3x^8 + 1155\,Aab^4x^8 + 1386\,Ba^3b^2x^6 + 1386\,Aa^2b^3x^6 + 495\,Ba^4bx^4 + 990\,Aa^3b^2x^4 +}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^12,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/693*(3465*B*a*b^4*x^10 + 693*A*b^5*x^10 + 2310*B*a^2*
b^3*x^8 + 1155*A*a*b^4*x^8 + 1386*B*a^3*b^2*x^6 + 1386*A*a^2*b^3*
x^6 + 495*B*a^4*b*x^4 + 990*A*a^3*b^2*x^4 + 77*B*a^5*x^2 + 385*A*
a^4*b*x^2 + 63*A*a^5)/x^11
```

3.45 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{A\left(a+bx^2\right)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

```
[Out] -(a^5*B)/(10*x^10) - (5*a^4*b*B)/(8*x^8) - (5*a^3*b^2*B)/(3*x^6)
- (5*a^2*b^3*B)/(2*x^4) - (5*a*b^4*B)/(2*x^2) - (A*(a + b*x^2)^6)
/(12*a*x^12) + b^5*B*Log[x]
```

Rubi [A] time = 0.141153, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{A\left(a+bx^2\right)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -(a^5*B)/(10*x^10) - (5*a^4*b*B)/(8*x^8) - (5*a^3*b^2*B)/(3*x^6)
- (5*a^2*b^3*B)/(2*x^4) - (5*a*b^4*B)/(2*x^2) - (A*(a + b*x^2)^6)
/(12*a*x^12) + b^5*B*Log[x]
```

Rubi in Sympy [A] time = 21.5749, size = 95, normalized size = 1.04

$$-\frac{A\left(a+bx^2\right)^6}{12ax^{12}}-\frac{Ba^5}{10x^{10}}-\frac{5Ba^4b}{8x^8}-\frac{5Ba^3b^2}{3x^6}-\frac{5Ba^2b^3}{2x^4}-\frac{5Bab^4}{2x^2}+\frac{Bb^5\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**13,x)
```

```
[Out] -A*(a + b*x**2)**6/(12*a*x**12) - B*a**5/(10*x**10) - 5*B*a**4*b/
(8*x**8) - 5*B*a**3*b**2/(3*x**6) - 5*B*a**2*b**3/(2*x**4) - 5*B*
a*b**4/(2*x**2) + B*b**5*log(x**2)/2
```

Mathematica [A] time = 0.120187, size = 118, normalized size = 1.3

$$b^5B\log(x)-\frac{2a^5\left(5A+6Bx^2\right)+15a^4bx^2\left(4A+5Bx^2\right)+50a^3b^2x^4\left(3A+4Bx^2\right)+100a^2b^3x^6\left(2A+3Bx^2\right)+150ab^4x^8\left(A+2Bx^2\right)+60A}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -(60*A*b^5*x^10 + 150*a*b^4*x^8*(A + 2*B*x^2) + 100*a^2*b^3*x^6*(
2*A + 3*B*x^2) + 50*a^3*b^2*x^4*(3*A + 4*B*x^2) + 15*a^4*b*x^2*(4
*A + 5*B*x^2) + 2*a^5*(5*A + 6*B*x^2))/(120*x^12) + b^5*B*Log[x]
```

Maple [A] time = 0.011, size = 124, normalized size = 1.4

$$-\frac{Aa^{5}}{12\,x^{12}}+b^{5}B\ln\left(x\right)-\frac{5\,a^{2}b^{3}A}{3\,x^{6}}-\frac{5\,a^{3}b^{2}B}{3\,x^{6}}-\frac{5\,ab^{4}A}{4\,x^{4}}-\frac{5\,a^{2}b^{3}B}{2\,x^{4}}-\frac{a^{4}bA}{2\,x^{10}}-\frac{a^{5}B}{10\,x^{10}}-\frac{5\,a^{3}b^{2}A}{4\,x^{8}}-\frac{5\,a^{4}bB}{8\,x^{8}}-\frac{b^{5}A}{2\,x^{2}}-\frac{5\,ab^{4}B}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^13,x)
```

```
[Out] -1/12*A*a^5/x^12+b^5*B*ln(x)-5/3*a^2*b^3/x^6*A-5/3*a^3*b^2*B/x^6-
5/4*a*b^4/x^4*A-5/2*a^2*b^3*B/x^4-1/2*a^4/x^10*A*b-1/10*a^5*B/x^1
0-5/4*a^3*b^2/x^8*A-5/8*a^4*b*B/x^8-1/2*b^5/x^2*A-5/2*a*b^4*B/x^2
```

Maxima [A] time = 1.3423, size = 166, normalized size = 1.82

$$\frac{1}{2}\,Bb^{5}\log\left(x^{2}\right)-\frac{60\,\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+150\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+200\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+10\,Aa^{5}+75\,\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+12\,\left(Ba^{5}+5\right.}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(60*(5*B*a*b^4 + A*b^5)*x^10 + 150*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 10*A
*a^5 + 75*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 12*(B*a^5 + 5*A*a^4*b)*x^
2)/x^12
```

Fricas [A] time = 0.246158, size = 166, normalized size = 1.82

$$\frac{120\,Bb^{5}x^{12}\log\left(x\right)-60\,\left(5\,Bab^{4}+Ab^{5}\right)x^{10}-150\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}-200\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}-10\,Aa^{5}-75\,\left(Ba^{4}b+2\,Aa\right.}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*B*b^5*x^12*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^10 - 150*
(2*B*a^2*b^3 + A*a*b^4)*x^8 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 1
0*A*a^5 - 75*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 12*(B*a^5 + 5*A*a^4*b)
*x^2)/x^12
```

Sympy [A] time = 43.0222, size = 124, normalized size = 1.36

$$Bb^{5}\log\left(x\right)-\frac{10Aa^{5}+x^{10}\left(60Ab^{5}+300Bab^{4}\right)+x^{8}\left(150Aab^{4}+300Ba^{2}b^{3}\right)+x^{6}\left(200Aa^{2}b^{3}+200Ba^{3}b^{2}\right)+x^{4}\left(150Aa^{3}b^{2}+75Ba^{4}b\right)+x^{2}}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**13,x)
```

```
[Out] B*b**5*log(x) - (10*A*a**5 + x**10*(60*A*b**5 + 300*B*a*b**4) + x
**8*(150*A*a*b**4 + 300*B*a**2*b**3) + x**6*(200*A*a**2*b**3 + 20
```

```
0*B*a**3*b**2) + x**4*(150*A*a**3*b**2 + 75*B*a**4*b) + x**2*(60*
A*a**4*b + 12*B*a**5))/(120*x**12)
```

GIAC/XCAS [A] time = 0.228281, size = 186, normalized size = 2.04

$$\frac{1}{2}\,Bb^5\ln\left(x^2\right) - \frac{147\,Bb^5x^{12} + 300\,Bab^4x^{10} + 60\,Ab^5x^{10} + 300\,Ba^2b^3x^8 + 150\,Aab^4x^8 + 200\,Ba^3b^2x^6 + 200\,Aa^2b^3x^6 + 75\,Ba^4bx^4 + 150\,Aa^3b}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*ln(x^2) - 1/120*(147*B*b^5*x^12 + 300*B*a*b^4*x^10 + 60
*A*b^5*x^10 + 300*B*a^2*b^3*x^8 + 150*A*a*b^4*x^8 + 200*B*a^3*b^2
*x^6 + 200*A*a^2*b^3*x^6 + 75*B*a^4*b*x^4 + 150*A*a^3*b^2*x^4 + 1
2*B*a^5*x^2 + 60*A*a^4*b*x^2 + 10*A*a^5)/x^12
```

3.46 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{14}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{x}$$

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*
A*b + a*B))/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(
A*b + 2*a*B))/x^5 - (b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

Rubi [A] time = 0.201403, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*
A*b + a*B))/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(
A*b + 2*a*B))/x^5 - (b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

Rubi in Sympy [A] time = 24.1668, size = 109, normalized size = 0.96

$$-\frac{Aa^5}{13x^{13}}-\frac{Bb^5}{x}-\frac{a^4(5Ab+Ba)}{11x^{11}}-\frac{5a^3b(2Ab+Ba)}{9x^9}-\frac{10a^2b^2(Ab+Ba)}{7x^7}-\frac{ab^3(Ab+2Ba)}{x^5}-\frac{b^4(Ab+5Ba)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

```
[Out] -A*a**5/(13*x**13) - B*b**5/x - a**4*(5*A*b + B*a)/(11*x**11) - 5
*a**3*b*(2*A*b + B*a)/(9*x**9) - 10*a**2*b**2*(A*b + B*a)/(7*x**7
) - a*b**3*(A*b + 2*B*a)/x**5 - b**4*(A*b + 5*B*a)/(3*x**3)
```

Mathematica [A] time = 0.0556409, size = 119, normalized size = 1.05

$$-\frac{63a^5(11A+13Bx^2)+455a^4bx^2(9A+11Bx^2)+1430a^3b^2x^4(7A+9Bx^2)+2574a^2b^3x^6(5A+7Bx^2)+3003ab^4x^8(3A+5}{9009x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -(3003*b^5*x^10*(A + 3*B*x^2) + 3003*a*b^4*x^8*(3*A + 5*B*x^2) +
2574*a^2*b^3*x^6*(5*A + 7*B*x^2) + 1430*a^3*b^2*x^4*(7*A + 9*B*x^
2) + 455*a^4*b*x^2*(9*A + 11*B*x^2) + 63*a^5*(11*A + 13*B*x^2))/(
9009*x^13)
```

Maple [A] time = 0.008, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{13\,x^{13}}-\frac{a^4(5\,Ab+Ba)}{11\,x^{11}}-\frac{5\,a^3b(2\,Ab+Ba)}{9\,x^9}-\frac{10\,a^2b^2(Ab+Ba)}{7\,x^7}-\frac{ab^3(Ab+2\,Ba)}{x^5}-\frac{b^4(Ab+5\,Ba)}{3\,x^3}-\frac{Bb^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^14,x)
```

```
[Out] -1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/
x^9-10/7*a^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b
+5*B*a)/x^3-b^5*B/x
```

Maxima [A] time = 1.35236, size = 163, normalized size = 1.44

$$-\frac{9009\,Bb^{5}x^{12} + 3003\,\left(5\,Bab^{4} + Ab^{5}\right)x^{10} + 9009\,\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{8} + 12870\,\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{6} + 693\,Aa^{5} + 5005\,\left(Ba^{4}b + 2\right.}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 +
693*A*a^5 + 5005*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a
^4*b)*x^2)/x^13
```

Fricas [A] time = 0.237297, size = 163, normalized size = 1.44

$$-\frac{9009\,Bb^{5}x^{12} + 3003\,\left(5\,Bab^{4} + Ab^{5}\right)x^{10} + 9009\,\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{8} + 12870\,\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{6} + 693\,Aa^{5} + 5005\,\left(Ba^{4}b + 2\right.}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 +
693*A*a^5 + 5005*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a
^4*b)*x^2)/x^13
```

Sympy [A] time = 51.3153, size = 128, normalized size = 1.13

$$-\frac{693Aa^{5} + 9009Bb^{5}x^{12} + x^{10}\left(3003Ab^{5} + 15015Bab^{4}\right) + x^{8}\left(9009Aab^{4} + 18018Ba^{2}b^{3}\right) + x^{6}\left(12870Aa^{2}b^{3} + 12870Ba^{3}b^{2}\right) + x}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

```
[Out] -(693*A*a**5 + 9009*B*b**5*x**12 + x**10*(3003*A*b**5 + 15015*B*a
*b**4) + x**8*(9009*A*a*b**4 + 18018*B*a**2*b**3) + x**6*(12870*A
*a**2*b**3 + 12870*B*a**3*b**2) + x**4*(10010*A*a**3*b**2 + 5005*
B*a**4*b) + x**2*(4095*A*a**4*b + 819*B*a**5))/(9009*x**13)
```

GIAC/XCAS [A] time = 0.222474, size = 171, normalized size = 1.51

$$-\frac{9009\,Bb^{5}x^{12} + 15015\,Bab^{4}x^{10} + 3003\,Ab^{5}x^{10} + 18018\,Ba^{2}b^{3}x^{8} + 9009\,Aab^{4}x^{8} + 12870\,Ba^{3}b^{2}x^{6} + 12870\,Aa^{2}b^{3}x^{6} + 5005\,Ba}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 15015*B*a*b^4*x^10 + 3003*A*b^5*x^10 +
 18018*B*a^2*b^3*x^8 + 9009*A*a*b^4*x^8 + 12870*B*a^3*b^2*x^6 + 1
2870*A*a^2*b^3*x^6 + 5005*B*a^4*b*x^4 + 10010*A*a^3*b^2*x^4 + 819
*B*a^5*x^2 + 4095*A*a^4*b*x^2 + 693*A*a^5)/x^13
```

3.47 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{15}}\,dx$

Optimal. Leaf size=48

$$\frac{(a+bx^2)^6(Ab-7aB)}{84a^2x^{12}}-\frac{A(a+bx^2)^6}{14ax^{14}}$$

```
[Out] -(A*(a + b*x^2)^6)/(14*a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(8
4*a^2*x^12)
```

Rubi [A] time = 0.116166, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{(a+bx^2)^6(Ab-7aB)}{84a^2x^{12}}-\frac{A(a+bx^2)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -(A*(a + b*x^2)^6)/(14*a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(8
4*a^2*x^12)
```

Rubi in Sympy [A] time = 11.0481, size = 41, normalized size = 0.85

$$-\frac{A(a+bx^2)^6}{14ax^{14}}+\frac{(a+bx^2)^6(Ab-7Ba)}{84a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**15,x)
```

```
[Out] -A*(a + b*x**2)**6/(14*a*x**14) + (a + b*x**2)**6*(A*b - 7*B*a)/(
84*a**2*x**12)
```

Mathematica [B] time = 0.057132, size = 118, normalized size = 2.46

$$-\frac{a^5(6A+7Bx^2)+7a^4bx^2(5A+6Bx^2)+21a^3b^2x^4(4A+5Bx^2)+35a^2b^3x^6(3A+4Bx^2)+35ab^4x^8(2A+3Bx^2)+21b^5x^{10}}{84x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -(21*b^5*x^10*(A + 2*B*x^2) + 35*a*b^4*x^8*(2*A + 3*B*x^2) + 35*a
^2*b^3*x^6*(3*A + 4*B*x^2) + 21*a^3*b^2*x^4*(4*A + 5*B*x^2) + 7*a
^4*b*x^2*(5*A + 6*B*x^2) + a^5*(6*A + 7*B*x^2))/(84*x^14)
```

Maple [B] time = 0.009, size = 104, normalized size = 2.2

$$-\frac{a^4(5Ab+Ba)}{12x^{12}}-\frac{5ab^3(Ab+2Ba)}{6x^6}-\frac{b^4(Ab+5Ba)}{4x^4}-\frac{a^3b(2Ab+Ba)}{2x^{10}}-\frac{5a^2b^2(Ab+Ba)}{4x^8}-\frac{Aa^5}{14x^{14}}-\frac{Bb^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^15,x)
```

```
[Out] -1/12*a^4*(5*A*b+B*a)/x^12-5/6*a*b^3*(A*b+2*B*a)/x^6-1/4*b^4*(A*b
+5*B*a)/x^4-1/2*a^3*b*(2*A*b+B*a)/x^10-5/4*a^2*b^2*(A*b+B*a)/x^8-
1/14*A*a^5/x^14-1/2*B*b^5/x^2
```

Maxima [A] time = 1.33698, size = 163, normalized size = 3.4

$$-\frac{42\,Bb^5x^{12} + 21\,\left(5\,Bab^4 + Ab^5\right)x^{10} + 70\,\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 105\,\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 6\,Aa^5 + 42\,\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 7}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*
b^3 + A*a*b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 +
42*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

Fricas [A] time = 0.240491, size = 163, normalized size = 3.4

$$-\frac{42\,Bb^5x^{12} + 21\,\left(5\,Bab^4 + Ab^5\right)x^{10} + 70\,\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 105\,\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 6\,Aa^5 + 42\,\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 7}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*
b^3 + A*a*b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 +
42*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

Sympy [A] time = 72.5382, size = 128, normalized size = 2.67

$$-\frac{6Aa^5 + 42Bb^5x^{12} + x^{10}\left(21Ab^5 + 105Bab^4\right) + x^8\left(70Aab^4 + 140Ba^2b^3\right) + x^6\left(105Aa^2b^3 + 105Ba^3b^2\right) + x^4\left(84Aa^3b^2 + 42Ba\right.}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**15,x)
```

```
[Out] -(6*A*a**5 + 42*B*b**5*x**12 + x**10*(21*A*b**5 + 105*B*a*b**4) +
 x**8*(70*A*a*b**4 + 140*B*a**2*b**3) + x**6*(105*A*a**2*b**3 + 1
05*B*a**3*b**2) + x**4*(84*A*a**3*b**2 + 42*B*a**4*b) + x**2*(35*
A*a**4*b + 7*B*a**5))/(84*x**14)
```

GIAC/XCAS [A] time = 0.221296, size = 171, normalized size = 3.56

$$-\frac{42\,Bb^5x^{12} + 105\,Bab^4x^{10} + 21\,Ab^5x^{10} + 140\,Ba^2b^3x^8 + 70\,Aab^4x^8 + 105\,Ba^3b^2x^6 + 105\,Aa^2b^3x^6 + 42\,Ba^4bx^4 + 84\,Aa^3b^2x^4}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 105*B*a*b^4*x^10 + 21*A*b^5*x^10 + 140*B*a
^2*b^3*x^8 + 70*A*a*b^4*x^8 + 105*B*a^3*b^2*x^6 + 105*A*a^2*b^3*x
^6 + 42*B*a^4*b*x^4 + 84*A*a^3*b^2*x^4 + 7*B*a^5*x^2 + 35*A*a^4*b
*x^2 + 6*A*a^5)/x^14
```

3.48 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{3x^3}$$

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*
A*b + a*B))/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b
^3*(A*b + 2*a*B))/(7*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)
/(3*x^3)
```

Rubi [A] time = 0.196982, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*
A*b + a*B))/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b
^3*(A*b + 2*a*B))/(7*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)
/(3*x^3)
```

Rubi in Sympy [A] time = 24.2368, size = 116, normalized size = 0.99

$$-\frac{Aa^5}{15x^{15}}-\frac{Bb^5}{3x^3}-\frac{a^4(5Ab+Ba)}{13x^{13}}-\frac{5a^3b(2Ab+Ba)}{11x^{11}}-\frac{10a^2b^2(Ab+Ba)}{9x^9}-\frac{5ab^3(Ab+2Ba)}{7x^7}-\frac{b^4(Ab+5Ba)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

```
[Out] -A*a**5/(15*x**15) - B*b**5/(3*x**3) - a**4*(5*A*b + B*a)/(13*x**
13) - 5*a**3*b*(2*A*b + B*a)/(11*x**11) - 10*a**2*b**2*(A*b + B*a
)/(9*x**9) - 5*a*b**3*(A*b + 2*B*a)/(7*x**7) - b**4*(A*b + 5*B*a)
/(5*x**5)
```

Mathematica [A] time = 0.0545849, size = 121, normalized size = 1.03

$$-\frac{231a^5(13A+15Bx^2)+1575a^4bx^2(11A+13Bx^2)+4550a^3b^2x^4(9A+11Bx^2)+7150a^2b^3x^6(7A+9Bx^2)+6435ab^4x^8(5}{45045x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -(3003*b^5*x^10*(3*A + 5*B*x^2) + 6435*a*b^4*x^8*(5*A + 7*B*x^2)
+ 7150*a^2*b^3*x^6*(7*A + 9*B*x^2) + 4550*a^3*b^2*x^4*(9*A + 11*B
*x^2) + 1575*a^4*b*x^2*(11*A + 13*B*x^2) + 231*a^5*(13*A + 15*B*x
^2))/(45045*x^15)
```

Maple [A] time = 0.009, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{15\,x^{15}}-\frac{a^4\,(5\,Ab+Ba)}{13\,x^{13}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{11\,x^{11}}-\frac{10\,a^2b^2\,(Ab+Ba)}{9\,x^9}-\frac{5\,ab^3\,(Ab+2\,Ba)}{7\,x^7}-\frac{b^4\,(Ab+5\,Ba)}{5\,x^5}-\frac{Bb^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^16,x)
```

```
[Out] -1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)
/x^11-10/9*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^
4*(A*b+5*B*a)/x^5-1/3*b^5*B/x^3
```

Maxima [A] time = 1.32595, size = 163, normalized size = 1.39

$$-\frac{15015\,Bb^5x^{12}+9009\,\left(5\,Bab^4+Ab^5\right)x^{10}+32175\,\left(2\,Ba^2b^3+Aab^4\right)x^8+50050\,\left(Ba^3b^2+Aa^2b^3\right)x^6+3003\,Aa^5+20475\,\left(Ba^4\right.}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 3217
5*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6
 + 3003*A*a^5 + 20475*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 +
 5*A*a^4*b)*x^2)/x^15
```

Fricas [A] time = 0.240123, size = 163, normalized size = 1.39

$$-\frac{15015\,Bb^5x^{12}+9009\,\left(5\,Bab^4+Ab^5\right)x^{10}+32175\,\left(2\,Ba^2b^3+Aab^4\right)x^8+50050\,\left(Ba^3b^2+Aa^2b^3\right)x^6+3003\,Aa^5+20475\,\left(Ba^4\right.}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 3217
5*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6
 + 3003*A*a^5 + 20475*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 +
 5*A*a^4*b)*x^2)/x^15
```

Sympy [A] time = 105.46, size = 128, normalized size = 1.09

$$-\frac{3003Aa^5+15015Bb^5x^{12}+x^{10}\left(9009Ab^5+45045Bab^4\right)+x^8\left(32175Aab^4+64350Ba^2b^3\right)+x^6\left(50050Aa^2b^3+50050Ba^3b^2\right)}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

```
[Out] -(3003*A*a**5 + 15015*B*b**5*x**12 + x**10*(9009*A*b**5 + 45045*B
*a*b**4) + x**8*(32175*A*a*b**4 + 64350*B*a**2*b**3) + x**6*(5005
0*A*a**2*b**3 + 50050*B*a**3*b**2) + x**4*(40950*A*a**3*b**2 + 20
475*B*a**4*b) + x**2*(17325*A*a**4*b + 3465*B*a**5))/(45045*x**15
)
```

GIAC/XCAS [A] time = 0.227017, size = 171, normalized size = 1.46

$$-\frac{15015\,Bb^5x^{12} + 45045\,Bab^4x^{10} + 9009\,Ab^5x^{10} + 64350\,Ba^2b^3x^8 + 32175\,Aab^4x^8 + 50050\,Ba^3b^2x^6 + 50050\,Aa^2b^3x^6 + 20475}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 45045*B*a*b^4*x^10 + 9009*A*b^5*x^10
 + 64350*B*a^2*b^3*x^8 + 32175*A*a*b^4*x^8 + 50050*B*a^3*b^2*x^6
+ 50050*A*a^2*b^3*x^6 + 20475*B*a^4*b*x^4 + 40950*A*a^3*b^2*x^4 +
 3465*B*a^5*x^2 + 17325*A*a^4*b*x^2 + 3003*A*a^5)/x^15
```

3.49 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx$

Optimal. Leaf size=76

$$-\frac{b\left(a+bx^2\right)^6(Ab-4aB)}{336a^3x^{12}}+\frac{\left(a+bx^2\right)^6(Ab-4aB)}{56a^2x^{14}}-\frac{A\left(a+bx^2\right)^6}{16ax^{16}}$$

```
[Out] -(A*(a + b*x^2)^6)/(16*a*x^16) + ((A*b - 4*a*B)*(a + b*x^2)^6)/(5
6*a^2*x^14) - (b*(A*b - 4*a*B)*(a + b*x^2)^6)/(336*a^3*x^12)
```

Rubi [A] time = 0.171961, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{b\left(a+bx^2\right)^6(Ab-4aB)}{336a^3x^{12}}+\frac{\left(a+bx^2\right)^6(Ab-4aB)}{56a^2x^{14}}-\frac{A\left(a+bx^2\right)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -(A*(a + b*x^2)^6)/(16*a*x^16) + ((A*b - 4*a*B)*(a + b*x^2)^6)/(5
6*a^2*x^14) - (b*(A*b - 4*a*B)*(a + b*x^2)^6)/(336*a^3*x^12)
```

Rubi in Sympy [A] time = 14.1668, size = 68, normalized size = 0.89

$$-\frac{A\left(a+bx^2\right)^6}{16ax^{16}}+\frac{\left(a+bx^2\right)^6(Ab-4Ba)}{56a^2x^{14}}-\frac{b\left(a+bx^2\right)^6(Ab-4Ba)}{336a^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**17,x)
```

```
[Out] -A*(a + b*x**2)**6/(16*a*x**16) + (a + b*x**2)**6*(A*b - 4*B*a)/(
56*a**2*x**14) - b*(a + b*x**2)**6*(A*b - 4*B*a)/(336*a**3*x**12)
```

Mathematica [A] time = 0.0567621, size = 121, normalized size = 1.59

$$-\frac{3a^5\left(7A+8Bx^2\right)+20a^4bx^2\left(6A+7Bx^2\right)+56a^3b^2x^4\left(5A+6Bx^2\right)+84a^2b^3x^6\left(4A+5Bx^2\right)+70ab^4x^8\left(3A+4Bx^2\right)+28b^5}{336x^{16}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -(28*b^5*x^10*(2*A + 3*B*x^2) + 70*a*b^4*x^8*(3*A + 4*B*x^2) + 84
*a^2*b^3*x^6*(4*A + 5*B*x^2) + 56*a^3*b^2*x^4*(5*A + 6*B*x^2) + 2
0*a^4*b*x^2*(6*A + 7*B*x^2) + 3*a^5*(7*A + 8*B*x^2))/(336*x^16)
```

Maple [A] time = 0.009, size = 104, normalized size = 1.4

$$-\frac{5\,a^3b\,(2\,Ab+Ba)}{12\,x^{12}}-\frac{Aa^5}{16\,x^{16}}-\frac{b^4\,(Ab+5\,Ba)}{6\,x^6}-\frac{Bb^5}{4\,x^4}-\frac{a^2b^2\,(Ab+Ba)}{x^{10}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{8\,x^8}-\frac{a^4\,(5\,Ab+Ba)}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^17,x)
```

```
[Out] -5/12*a^3*b*(2*A*b+B*a)/x^12-1/16*A*a^5/x^16-1/6*b^4*(A*b+5*B*a)/
x^6-1/4*B*b^5/x^4-a^2*b^2*(A*b+B*a)/x^10-5/8*a*b^3*(A*b+2*B*a)/x^
8-1/14*a^4*(5*A*b+B*a)/x^14
```

Maxima [A] time = 1.33512, size = 163, normalized size = 2.14

$$-\frac{84\,Bb^5x^{12} + 56\left(5\,Bab^4 + Ab^5\right)x^{10} + 210\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 336\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 21\,Aa^5 + 140\left(Ba^4b + 2\,Aa^3b^2\right)x^4}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^
2*b^3 + A*a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5
 + 140*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/
x^16
```

Fricas [A] time = 0.221645, size = 163, normalized size = 2.14

$$-\frac{84\,Bb^5x^{12} + 56\left(5\,Bab^4 + Ab^5\right)x^{10} + 210\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 336\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 21\,Aa^5 + 140\left(Ba^4b + 2\,Aa^3b^2\right)x^4}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^
2*b^3 + A*a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5
 + 140*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/
x^16
```

Sympy [A] time = 152.951, size = 128, normalized size = 1.68

$$-\frac{21Aa^5 + 84Bb^5x^{12} + x^{10}\left(56Ab^5 + 280Bab^4\right) + x^8\left(210Aab^4 + 420Ba^2b^3\right) + x^6\left(336Aa^2b^3 + 336Ba^3b^2\right) + x^4\left(280Aa^3b^2 + 1\right.}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**17,x)
```

```
[Out] -(21*A*a**5 + 84*B*b**5*x**12 + x**10*(56*A*b**5 + 280*B*a*b**4)
+ x**8*(210*A*a*b**4 + 420*B*a**2*b**3) + x**6*(336*A*a**2*b**3 +
 336*B*a**3*b**2) + x**4*(280*A*a**3*b**2 + 140*B*a**4*b) + x**2*
(120*A*a**4*b + 24*B*a**5))/(336*x**16)
```

GIAC/XCAS [A] time = 0.230495, size = 171, normalized size = 2.25

$$-\frac{84\,Bb^5x^{12} + 280\,Bab^4x^{10} + 56\,Ab^5x^{10} + 420\,Ba^2b^3x^8 + 210\,Aab^4x^8 + 336\,Ba^3b^2x^6 + 336\,Aa^2b^3x^6 + 140\,Ba^4bx^4 + 280\,Aa^3b}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 280*B*a*b^4*x^10 + 56*A*b^5*x^10 + 420*B*
a^2*b^3*x^8 + 210*A*a*b^4*x^8 + 336*B*a^3*b^2*x^6 + 336*A*a^2*b^3
*x^6 + 140*B*a^4*b*x^4 + 280*A*a^3*b^2*x^4 + 24*B*a^5*x^2 + 120*A
*a^4*b*x^2 + 21*A*a^5)/x^16
```

3.50 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*
A*b + a*B))/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a
*b^3*(A*b + 2*a*B))/(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*
B)/(5*x^5)
```

Rubi [A] time = 0.19153, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*
A*b + a*B))/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a
*b^3*(A*b + 2*a*B))/(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*
B)/(5*x^5)
```

Rubi in Sympy [A] time = 24.4289, size = 116, normalized size = 0.99

$$-\frac{Aa^5}{17x^{17}}-\frac{Bb^5}{5x^5}-\frac{a^4\left(5Ab+Ba\right)}{15x^{15}}-\frac{5a^3b\left(2Ab+Ba\right)}{13x^{13}}-\frac{10a^2b^2\left(Ab+Ba\right)}{11x^{11}}-\frac{5ab^3\left(Ab+2Ba\right)}{9x^9}-\frac{b^4\left(Ab+5Ba\right)}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

```
[Out] -A*a**5/(17*x**17) - B*b**5/(5*x**5) - a**4*(5*A*b + B*a)/(15*x**
15) - 5*a**3*b*(2*A*b + B*a)/(13*x**13) - 10*a**2*b**2*(A*b + B*a
)/(11*x**11) - 5*a*b**3*(A*b + 2*B*a)/(9*x**9) - b**4*(A*b + 5*B*
a)/(7*x**7)
```

Mathematica [A] time = 0.0839226, size = 117, normalized size = 1.

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*
A*b + a*B))/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a
*b^3*(A*b + 2*a*B))/(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*
B)/(5*x^5)
```

Maple [A] time = 0.009, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{17\,x^{17}}-\frac{a^4\,(5\,Ab+Ba)}{15\,x^{15}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{13\,x^{13}}-\frac{10\,a^2b^2\,(Ab+Ba)}{11\,x^{11}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{9\,x^9}-\frac{b^4\,(Ab+5\,Ba)}{7\,x^7}-\frac{Bb^5}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^18,x)
```

```
[Out] -1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)
/x^13-10/11*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*
b^4*(A*b+5*B*a)/x^7-1/5*b^5*B/x^5
```

Maxima [A] time = 1.35182, size = 163, normalized size = 1.39

$$-\frac{153153\,Bb^5x^{12}+109395\,\left(5\,Bab^4+Ab^5\right)x^{10}+425425\,\left(2\,Ba^2b^3+Aab^4\right)x^8+696150\,\left(Ba^3b^2+Aa^2b^3\right)x^6+45045\,Aa^5+294}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^18,x, algorithm="maxima")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 +
425425*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^
3)*x^6 + 45045*A*a^5 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051
*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

Fricas [A] time = 0.21213, size = 163, normalized size = 1.39

$$-\frac{153153\,Bb^5x^{12}+109395\,\left(5\,Bab^4+Ab^5\right)x^{10}+425425\,\left(2\,Ba^2b^3+Aab^4\right)x^8+696150\,\left(Ba^3b^2+Aa^2b^3\right)x^6+45045\,Aa^5+294}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^18,x, algorithm="fricas")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 +
425425*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^
3)*x^6 + 45045*A*a^5 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051
*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.219056, size = 171, normalized size = 1.46

$$-\frac{153153\,Bb^5x^{12}+546975\,Bab^4x^{10}+109395\,Ab^5x^{10}+850850\,Ba^2b^3x^8+425425\,Aab^4x^8+696150\,Ba^3b^2x^6+696150\,Aa^2b^3x}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^18,x, algorithm="giac")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 546975*B*a*b^4*x^10 + 109395*A*b^5
*x^10 + 850850*B*a^2*b^3*x^8 + 425425*A*a*b^4*x^8 + 696150*B*a^3*
b^2*x^6 + 696150*A*a^2*b^3*x^6 + 294525*B*a^4*b*x^4 + 589050*A*a^
3*b^2*x^4 + 51051*B*a^5*x^2 + 255255*A*a^4*b*x^2 + 45045*A*a^5)/x
^17
```

3.51 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{18x^{18}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{6x^6}$$

```
[Out] -(a^5*A)/(18*x^18) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*
A*b + a*B))/(14*x^14) - (5*a^2*b^2*(A*b + a*B))/(6*x^12) - (a*b^3
*(A*b + 2*a*B))/(2*x^10) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/
(6*x^6)
```

Rubi [A] time = 0.255079, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^5A}{18x^{18}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -(a^5*A)/(18*x^18) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*
A*b + a*B))/(14*x^14) - (5*a^2*b^2*(A*b + a*B))/(6*x^12) - (a*b^3
*(A*b + 2*a*B))/(2*x^10) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/
(6*x^6)
```

Rubi in Sympy [A] time = 29.7642, size = 114, normalized size = 0.97

$$-\frac{Aa^5}{18x^{18}}-\frac{Bb^5}{6x^6}-\frac{a^4\left(5Ab+Ba\right)}{16x^{16}}-\frac{5a^3b\left(2Ab+Ba\right)}{14x^{14}}-\frac{5a^2b^2\left(Ab+Ba\right)}{6x^{12}}-\frac{ab^3\left(Ab+2Ba\right)}{2x^{10}}-\frac{b^4\left(Ab+5Ba\right)}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

```
[Out] -A*a**5/(18*x**18) - B*b**5/(6*x**6) - a**4*(5*A*b + B*a)/(16*x**
16) - 5*a**3*b*(2*A*b + B*a)/(14*x**14) - 5*a**2*b**2*(A*b + B*a)
/(6*x**12) - a*b**3*(A*b + 2*B*a)/(2*x**10) - b**4*(A*b + 5*B*a)/
(8*x**8)
```

Mathematica [A] time = 0.059305, size = 121, normalized size = 1.03

$$-\frac{7a^5\left(8A+9Bx^2\right)+45a^4bx^2\left(7A+8Bx^2\right)+120a^3b^2x^4\left(6A+7Bx^2\right)+168a^2b^3x^6\left(5A+6Bx^2\right)+126ab^4x^8\left(4A+5Bx^2\right)+4}{1008x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -(42*b^5*x^10*(3*A + 4*B*x^2) + 126*a*b^4*x^8*(4*A + 5*B*x^2) + 1
68*a^2*b^3*x^6*(5*A + 6*B*x^2) + 120*a^3*b^2*x^4*(6*A + 7*B*x^2)
+ 45*a^4*b*x^2*(7*A + 8*B*x^2) + 7*a^5*(8*A + 9*B*x^2))/(1008*x^1
8)
```

Maple [A] time = 0.01, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{18\,x^{18}}-\frac{a^4\,(5\,Ab+Ba)}{16\,x^{16}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{14\,x^{14}}-\frac{5\,a^2b^2\,(Ab+Ba)}{6\,x^{12}}-\frac{ab^3\,(Ab+2\,Ba)}{2\,x^{10}}-\frac{b^4\,(Ab+5\,Ba)}{8\,x^8}-\frac{Bb^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^19,x)
```

```
[Out] -1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)
/x^14-5/6*a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b
^4*(A*b+5*B*a)/x^8-1/6*b^5*B/x^6
```

Maxima [A] time = 1.38, size = 163, normalized size = 1.39

$$-\frac{168\,Bb^5x^{12}+126\,\left(5\,Bab^4+Ab^5\right)x^{10}+504\,\left(2\,Ba^2b^3+Aab^4\right)x^8+840\,\left(Ba^3b^2+Aa^2b^3\right)x^6+56\,Aa^5+360\,\left(Ba^4b+2\,Aa^3b^2\right)}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B
*a^2*b^3 + A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*
a^5 + 360*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^
2)/x^18
```

Fricas [A] time = 0.218908, size = 163, normalized size = 1.39

$$-\frac{168\,Bb^5x^{12}+126\,\left(5\,Bab^4+Ab^5\right)x^{10}+504\,\left(2\,Ba^2b^3+Aab^4\right)x^8+840\,\left(Ba^3b^2+Aa^2b^3\right)x^6+56\,Aa^5+360\,\left(Ba^4b+2\,Aa^3b^2\right)}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B
*a^2*b^3 + A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*
a^5 + 360*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^
2)/x^18
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224814, size = 171, normalized size = 1.46

$$-\frac{168\,Bb^5x^{12}+630\,Bab^4x^{10}+126\,Ab^5x^{10}+1008\,Ba^2b^3x^8+504\,Aab^4x^8+840\,Ba^3b^2x^6+840\,Aa^2b^3x^6+360\,Ba^4bx^4+720\,A}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 630*B*a*b^4*x^10 + 126*A*b^5*x^10 + 100
8*B*a^2*b^3*x^8 + 504*A*a*b^4*x^8 + 840*B*a^3*b^2*x^6 + 840*A*a^2
*b^3*x^6 + 360*B*a^4*b*x^4 + 720*A*a^3*b^2*x^4 + 63*B*a^5*x^2 + 3
15*A*a^4*b*x^2 + 56*A*a^5)/x^18
```

3.52 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*
b + a*B))/(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^
3*(A*b + 2*a*B))/(11*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B
)/(7*x^7)
```

Rubi [A] time = 0.196816, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*
b + a*B))/(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^
3*(A*b + 2*a*B))/(11*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B
)/(7*x^7)
```

Rubi in Sympy [A] time = 24.5347, size = 114, normalized size = 0.97

$$-\frac{Aa^5}{19x^{19}}-\frac{Bb^5}{7x^7}-\frac{a^4\,(5Ab+Ba)}{17x^{17}}-\frac{a^3b\,(2Ab+Ba)}{3x^{15}}-\frac{10a^2b^2\,(Ab+Ba)}{13x^{13}}-\frac{5ab^3\,(Ab+2Ba)}{11x^{11}}-\frac{b^4\,(Ab+5Ba)}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**20,x)
```

```
[Out] -A*a**5/(19*x**19) - B*b**5/(7*x**7) - a**4*(5*A*b + B*a)/(17*x**
17) - a**3*b*(2*A*b + B*a)/(3*x**15) - 10*a**2*b**2*(A*b + B*a)/(
13*x**13) - 5*a*b**3*(A*b + 2*B*a)/(11*x**11) - b**4*(A*b + 5*B*a
)/(9*x**9)
```

Mathematica [A] time = 0.0951047, size = 117, normalized size = 1.

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*
b + a*B))/(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^
3*(A*b + 2*a*B))/(11*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B
)/(7*x^7)
```

Maple [A] time = 0.008, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{19\,x^{19}}-\frac{a^4\,(5\,Ab+Ba)}{17\,x^{17}}-\frac{a^3b\,(2\,Ab+Ba)}{3\,x^{15}}-\frac{10\,a^2b^2\,(Ab+Ba)}{13\,x^{13}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{11\,x^{11}}-\frac{b^4\,(Ab+5\,Ba)}{9\,x^9}-\frac{Bb^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^20,x)
```

```
[Out] -1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/
x^15-10/13*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9
*b^4*(A*b+5*B*a)/x^9-1/7*b^5*B/x^7
```

Maxima [A] time = 1.37749, size = 163, normalized size = 1.39

$$-\frac{415701\,Bb^5x^{12}+323323\left(5\,Bab^4+Ab^5\right)x^{10}+1322685\left(2\,Ba^2b^3+Aab^4\right)x^8+2238390\left(Ba^3b^2+Aa^2b^3\right)x^6+153153\,Aa^5+}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^20,x, algorithm="maxima")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 +
 1322685*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2
*b^3)*x^6 + 153153*A*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1
71171*(B*a^5 + 5*A*a^4*b)*x^2)/x^19
```

Fricas [A] time = 0.203638, size = 163, normalized size = 1.39

$$-\frac{415701\,Bb^5x^{12}+323323\left(5\,Bab^4+Ab^5\right)x^{10}+1322685\left(2\,Ba^2b^3+Aab^4\right)x^8+2238390\left(Ba^3b^2+Aa^2b^3\right)x^6+153153\,Aa^5+}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^20,x, algorithm="fricas")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 +
 1322685*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2
*b^3)*x^6 + 153153*A*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1
71171*(B*a^5 + 5*A*a^4*b)*x^2)/x^19
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**20,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.238862, size = 171, normalized size = 1.46

$$-\frac{415701\,Bb^5x^{12}+1616615\,Bab^4x^{10}+323323\,Ab^5x^{10}+2645370\,Ba^2b^3x^8+1322685\,Aab^4x^8+2238390\,Ba^3b^2x^6+2238390\,A}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^20,x, algorithm="giac")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 1616615*B*a*b^4*x^10 + 323323*A*b
^5*x^10 + 2645370*B*a^2*b^3*x^8 + 1322685*A*a*b^4*x^8 + 2238390*B
*a^3*b^2*x^6 + 2238390*A*a^2*b^3*x^6 + 969969*B*a^4*b*x^4 + 19399
38*A*a^3*b^2*x^4 + 171171*B*a^5*x^2 + 855855*A*a^4*b*x^2 + 153153
*A*a^5)/x^19
```

3.53 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{21}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^5B}{8x^8}$$

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(18*x^18) - (5*a^3*b*(2*
A*b + a*B))/(16*x^16) - (5*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b
^3*(A*b + 2*a*B))/(12*x^12) - (b^4*(A*b + 5*a*B))/(10*x^10) - (b^
5*B)/(8*x^8)
```

Rubi [A] time = 0.24972, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^5B}{8x^8}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(18*x^18) - (5*a^3*b*(2*
A*b + a*B))/(16*x^16) - (5*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b
^3*(A*b + 2*a*B))/(12*x^12) - (b^4*(A*b + 5*a*B))/(10*x^10) - (b^
5*B)/(8*x^8)
```

Rubi in Sympy [A] time = 29.7298, size = 116, normalized size = 0.99

$$-\frac{Aa^5}{20x^{20}}-\frac{Bb^5}{8x^8}-\frac{a^4\left(5Ab+Ba\right)}{18x^{18}}-\frac{5a^3b\left(2Ab+Ba\right)}{16x^{16}}-\frac{5a^2b^2\left(Ab+Ba\right)}{7x^{14}}-\frac{5ab^3\left(Ab+2Ba\right)}{12x^{12}}-\frac{b^4\left(Ab+5Ba\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

```
[Out] -A*a**5/(20*x**20) - B*b**5/(8*x**8) - a**4*(5*A*b + B*a)/(18*x**
18) - 5*a**3*b*(2*A*b + B*a)/(16*x**16) - 5*a**2*b**2*(A*b + B*a)
/(7*x**14) - 5*a*b**3*(A*b + 2*B*a)/(12*x**12) - b**4*(A*b + 5*B*
a)/(10*x**10)
```

Mathematica [A] time = 0.057037, size = 121, normalized size = 1.03

$$-\frac{28a^5\left(9A+10Bx^2\right)+175a^4bx^2\left(8A+9Bx^2\right)+450a^3b^2x^4\left(7A+8Bx^2\right)+600a^2b^3x^6\left(6A+7Bx^2\right)+420ab^4x^8\left(5A+6Bx^2\right)}{5040x^{20}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -(126*b^5*x^10*(4*A + 5*B*x^2) + 420*a*b^4*x^8*(5*A + 6*B*x^2) +
600*a^2*b^3*x^6*(6*A + 7*B*x^2) + 450*a^3*b^2*x^4*(7*A + 8*B*x^2)
 + 175*a^4*b*x^2*(8*A + 9*B*x^2) + 28*a^5*(9*A + 10*B*x^2))/(5040
*x^20)
```

Maple [A] time = 0.009, size = 104, normalized size = 0.9

$$-\frac{Aa^{5}}{20\,x^{20}}-\frac{a^{4}\,(5\,Ab+Ba)}{18\,x^{18}}-\frac{5\,a^{3}b\,(2\,Ab+Ba)}{16\,x^{16}}-\frac{5\,a^{2}b^{2}\,(Ab+Ba)}{7\,x^{14}}-\frac{5\,ab^{3}\,(Ab+2\,Ba)}{12\,x^{12}}-\frac{b^{4}\,(Ab+5\,Ba)}{10\,x^{10}}-\frac{Bb^{5}}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^21,x)
```

```
[Out] -1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)
/x^16-5/7*a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10
*b^4*(A*b+5*B*a)/x^10-1/8*b^5*B/x^8
```

Maxima [A] time = 1.33923, size = 163, normalized size = 1.39

$$-\frac{630\,Bb^{5}x^{12}+504\,\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+2100\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+3600\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+252\,Aa^{5}+1575\,\left(Ba^{4}b+2\,Aa\right.}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^21,x, algorithm="maxima")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252
*A*a^5 + 1575*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*
b)*x^2)/x^20
```

Fricas [A] time = 0.21964, size = 163, normalized size = 1.39

$$-\frac{630\,Bb^{5}x^{12}+504\,\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+2100\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+3600\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+252\,Aa^{5}+1575\,\left(Ba^{4}b+2\,Aa\right.}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^21,x, algorithm="fricas")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252
*A*a^5 + 1575*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*
b)*x^2)/x^20
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.234423, size = 171, normalized size = 1.46

$$-\frac{630\,Bb^{5}x^{12}+2520\,Bab^{4}x^{10}+504\,Ab^{5}x^{10}+4200\,Ba^{2}b^{3}x^{8}+2100\,Aab^{4}x^{8}+3600\,Ba^{3}b^{2}x^{6}+3600\,Aa^{2}b^{3}x^{6}+1575\,Ba^{4}bx^{4}+}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^21,x, algorithm="giac")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 2520*B*a*b^4*x^10 + 504*A*b^5*x^10 + 42
00*B*a^2*b^3*x^8 + 2100*A*a*b^4*x^8 + 3600*B*a^3*b^2*x^6 + 3600*A
*a^2*b^3*x^6 + 1575*B*a^4*b*x^4 + 3150*A*a^3*b^2*x^4 + 280*B*a^5*
x^2 + 1400*A*a^4*b*x^2 + 252*A*a^5)/x^20
```

3.54 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*
A*b + a*B))/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b
^3*(A*b + 2*a*B))/(13*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^
5*B)/(9*x^9)
```

Rubi [A] time = 0.195954, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*
A*b + a*B))/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b
^3*(A*b + 2*a*B))/(13*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^
5*B)/(9*x^9)
```

Rubi in Sympy [A] time = 25.7328, size = 116, normalized size = 0.99

$$-\frac{Aa^5}{21x^{21}}-\frac{Bb^5}{9x^9}-\frac{a^4\left(5Ab+Ba\right)}{19x^{19}}-\frac{5a^3b\left(2Ab+Ba\right)}{17x^{17}}-\frac{2a^2b^2\left(Ab+Ba\right)}{3x^{15}}-\frac{5ab^3\left(Ab+2Ba\right)}{13x^{13}}-\frac{b^4\left(Ab+5Ba\right)}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**22,x)
```

```
[Out] -A*a**5/(21*x**21) - B*b**5/(9*x**9) - a**4*(5*A*b + B*a)/(19*x**
19) - 5*a**3*b*(2*A*b + B*a)/(17*x**17) - 2*a**2*b**2*(A*b + B*a)
/(3*x**15) - 5*a*b**3*(A*b + 2*B*a)/(13*x**13) - b**4*(A*b + 5*B*
a)/(11*x**11)
```

Mathematica [A] time = 0.08412, size = 117, normalized size = 1.

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*
A*b + a*B))/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b
^3*(A*b + 2*a*B))/(13*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^
5*B)/(9*x^9)
```

Maple [A] time = 0.008, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{21\,x^{21}}-\frac{a^4\,(5\,Ab+Ba)}{19\,x^{19}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{17\,x^{17}}-\frac{2\,a^2b^2\,(Ab+Ba)}{3\,x^{15}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{13\,x^{13}}-\frac{b^4\,(Ab+5\,Ba)}{11\,x^{11}}-\frac{Bb^5}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^22,x)
```

```
[Out] -1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)
/x^17-2/3*a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11
*b^4*(A*b+5*B*a)/x^11-1/9*b^5*B/x^9
```

Maxima [A] time = 1.33905, size = 163, normalized size = 1.39

$$-\frac{323323\,Bb^5x^{12}+264537\,\left(5\,Bab^4+Ab^5\right)x^{10}+1119195\,\left(2\,Ba^2b^3+Aab^4\right)x^8+1939938\,\left(Ba^3b^2+Aa^2b^3\right)x^6+138567\,Aa^5+}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 +
 1119195*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2
*b^3)*x^6 + 138567*A*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1
53153*(B*a^5 + 5*A*a^4*b)*x^2)/x^21
```

Fricas [A] time = 0.218186, size = 163, normalized size = 1.39

$$-\frac{323323\,Bb^5x^{12}+264537\,\left(5\,Bab^4+Ab^5\right)x^{10}+1119195\,\left(2\,Ba^2b^3+Aab^4\right)x^8+1939938\,\left(Ba^3b^2+Aa^2b^3\right)x^6+138567\,Aa^5+}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 +
 1119195*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2
*b^3)*x^6 + 138567*A*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1
53153*(B*a^5 + 5*A*a^4*b)*x^2)/x^21
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**22,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.233992, size = 171, normalized size = 1.46

$$-\frac{323323\,Bb^5x^{12}+1322685\,Bab^4x^{10}+264537\,Ab^5x^{10}+2238390\,Ba^2b^3x^8+1119195\,Aab^4x^8+1939938\,Ba^3b^2x^6+1939938\,A}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 1322685*B*a*b^4*x^10 + 264537*A*b
^5*x^10 + 2238390*B*a^2*b^3*x^8 + 1119195*A*a*b^4*x^8 + 1939938*B
*a^3*b^2*x^6 + 1939938*A*a^2*b^3*x^6 + 855855*B*a^4*b*x^4 + 17117
10*A*a^3*b^2*x^4 + 153153*B*a^5*x^2 + 765765*A*a^4*b*x^2 + 138567
*A*a^5)/x^21
```

3.55 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{23}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^5B}{10x^{10}}$$

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(20*x^20) - (5*a^3*b*(2*
A*b + a*B))/(18*x^18) - (5*a^2*b^2*(A*b + a*B))/(8*x^16) - (5*a*b
^3*(A*b + 2*a*B))/(14*x^14) - (b^4*(A*b + 5*a*B))/(12*x^12) - (b^
5*B)/(10*x^10)
```

Rubi [A] time = 0.241734, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^5B}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(20*x^20) - (5*a^3*b*(2*
A*b + a*B))/(18*x^18) - (5*a^2*b^2*(A*b + a*B))/(8*x^16) - (5*a*b
^3*(A*b + 2*a*B))/(14*x^14) - (b^4*(A*b + 5*a*B))/(12*x^12) - (b^
5*B)/(10*x^10)
```

Rubi in Sympy [A] time = 30.111, size = 116, normalized size = 0.99

$$-\frac{Aa^5}{22x^{22}}-\frac{Bb^5}{10x^{10}}-\frac{a^4\left(5Ab+Ba\right)}{20x^{20}}-\frac{5a^3b\left(2Ab+Ba\right)}{18x^{18}}-\frac{5a^2b^2\left(Ab+Ba\right)}{8x^{16}}-\frac{5ab^3\left(Ab+2Ba\right)}{14x^{14}}-\frac{b^4\left(Ab+5Ba\right)}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**5*(B*x**2+A)/x**23,x)
```

```
[Out] -A*a**5/(22*x**22) - B*b**5/(10*x**10) - a**4*(5*A*b + B*a)/(20*x
**20) - 5*a**3*b*(2*A*b + B*a)/(18*x**18) - 5*a**2*b**2*(A*b + B*
a)/(8*x**16) - 5*a*b**3*(A*b + 2*B*a)/(14*x**14) - b**4*(A*b + 5*
B*a)/(12*x**12)
```

Mathematica [A] time = 0.0611542, size = 121, normalized size = 1.03

$$-\frac{126a^5\left(10A+11Bx^2\right)+770a^4bx^2\left(9A+10Bx^2\right)+1925a^3b^2x^4\left(8A+9Bx^2\right)+2475a^2b^3x^6\left(7A+8Bx^2\right)+1650ab^4x^8\left(6A+\right.}{27720x^{22}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -(462*b^5*x^10*(5*A + 6*B*x^2) + 1650*a*b^4*x^8*(6*A + 7*B*x^2) +
 2475*a^2*b^3*x^6*(7*A + 8*B*x^2) + 1925*a^3*b^2*x^4*(8*A + 9*B*x
^2) + 770*a^4*b*x^2*(9*A + 10*B*x^2) + 126*a^5*(10*A + 11*B*x^2))
/(27720*x^22)
```

Maple [A] time = 0.01, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{22\,x^{22}}-\frac{a^4\,(5\,Ab+Ba)}{20\,x^{20}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{18\,x^{18}}-\frac{5\,a^2b^2\,(Ab+Ba)}{8\,x^{16}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{14\,x^{14}}-\frac{b^4\,(Ab+5\,Ba)}{12\,x^{12}}-\frac{Bb^5}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^5*(B*x^2+A)/x^23,x)
```

```
[Out] -1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)
/x^18-5/8*a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12
*b^4*(A*b+5*B*a)/x^12-1/10*b^5*B/x^10
```

Maxima [A] time = 1.35666, size = 163, normalized size = 1.39

$$-\frac{2772\,Bb^5x^{12}+2310\,\left(5\,Bab^4+Ab^5\right)x^{10}+9900\,\left(2\,Ba^2b^3+Aab^4\right)x^8+17325\,\left(Ba^3b^2+Aa^2b^3\right)x^6+1260\,Aa^5+7700\,\left(Ba^4b+\right.}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^23,x, algorithm="maxima")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*
(2*B*a^2*b^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 +
 1260*A*a^5 + 7700*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*
A*a^4*b)*x^2)/x^22
```

Fricas [A] time = 0.218847, size = 163, normalized size = 1.39

$$-\frac{2772\,Bb^5x^{12}+2310\,\left(5\,Bab^4+Ab^5\right)x^{10}+9900\,\left(2\,Ba^2b^3+Aab^4\right)x^8+17325\,\left(Ba^3b^2+Aa^2b^3\right)x^6+1260\,Aa^5+7700\,\left(Ba^4b+\right.}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^23,x, algorithm="fricas")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*
(2*B*a^2*b^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 +
 1260*A*a^5 + 7700*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*
A*a^4*b)*x^2)/x^22
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**5*(B*x**2+A)/x**23,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224371, size = 171, normalized size = 1.46

$$-\frac{2772\,Bb^5x^{12}+11550\,Bab^4x^{10}+2310\,Ab^5x^{10}+19800\,Ba^2b^3x^8+9900\,Aab^4x^8+17325\,Ba^3b^2x^6+17325\,Aa^2b^3x^6+7700\,Ba}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^5/x^23,x, algorithm="giac")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 11550*B*a*b^4*x^10 + 2310*A*b^5*x^10
+ 19800*B*a^2*b^3*x^8 + 9900*A*a*b^4*x^8 + 17325*B*a^3*b^2*x^6 +
17325*A*a^2*b^3*x^6 + 7700*B*a^4*b*x^4 + 15400*A*a^3*b^2*x^4 + 13
86*B*a^5*x^2 + 6930*A*a^4*b*x^2 + 1260*A*a^5)/x^22
```

3.56 $\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{a^2x(Ab-aB)}{b^4}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{x^5(Ab-aB)}{5b^2}+\frac{Bx^7}{7b}$$

```
[Out] (a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a
*B)*x^5)/(5*b^2) + (B*x^7)/(7*b) - (a^(5/2)*(A*b - a*B)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.170725, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{a^2x(Ab-aB)}{b^4}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{x^5(Ab-aB)}{5b^2}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a
*B)*x^5)/(5*b^2) + (B*x^7)/(7*b) - (a^(5/2)*(A*b - a*B)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^7}{7b}-\frac{a^{\frac{5}{2}}\left(Ab-Ba\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{9}{2}}}-\frac{ax^3\left(Ab-Ba\right)}{3b^3}+\frac{x^5\left(Ab-Ba\right)}{5b^2}+\frac{\left(Ab-Ba\right)\int a^2\,dx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**7/(7*b) - a**(5/2)*(A*b - B*a)*atan(sqrt(b)*x/sqrt(a))/b**(9
/2) - a*x**3*(A*b - B*a)/(3*b**3) + x**5*(A*b - B*a)/(5*b**2) + (
A*b - B*a)*Integral(a**2, x)/b**4
```

Mathematica [A] time = 0.113186, size = 98, normalized size = 1.

$$\frac{a^{5/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^2x(aB-Ab)}{b^4}+\frac{ax^3(aB-Ab)}{3b^3}+\frac{x^5(Ab-aB)}{5b^2}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(A*b) + a*B)*x)/b^4) + (a*(-(A*b) + a*B)*x^3)/(3*b^3) +
((A*b - a*B)*x^5)/(5*b^2) + (B*x^7)/(7*b) + (a^(5/2)*(-(A*b) + a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.005, size = 116, normalized size = 1.2

$$\frac{Bx^7}{7\,b}+\frac{Ax^5}{5\,b}-\frac{Bx^5a}{5\,b^2}-\frac{aAx^3}{3\,b^2}+\frac{Bx^3a^2}{3\,b^3}+\frac{a^2Ax}{b^3}-\frac{Ba^3x}{b^4}-\frac{a^3A}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{Ba^4}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/7*B*x^7/b+1/5/b*A*x^5-1/5/b^2*B*x^5*a-1/3/b^2*A*x^3*a+1/3/b^3*B
*x^3*a^2+1/b^3*A*a^2*x-1/b^4*B*a^3*x-a^3/b^3/(a*b)^(1/2)*arctan(x
*b/(a*b)^(1/2))*A+a^4/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^6/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238728, size = 1, normalized size = 0.01

$$\left[\frac{30\,Bb^3x^7-42\,\left(Bab^2-Ab^3\right)x^5+70\,\left(Ba^2b-Aab^2\right)x^3-105\,\left(Ba^3-Aa^2b\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,\left(Ba^3-Aa^2b\right)}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^6/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*b^3*x^7 - 42*(B*a*b^2 - A*b^3)*x^5 + 70*(B*a^2*b - A
*a*b^2)*x^3 - 105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x
*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(B*a^3 - A*a^2*b)*x)/b^4, 1/1
05*(15*B*b^3*x^7 - 21*(B*a*b^2 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b
^2)*x^3 + 105*(B*a^3 - A*a^2*b)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 1
05*(B*a^3 - A*a^2*b)*x)/b^4]
```

Sympy [A] time = 2.02436, size = 173, normalized size = 1.77

$$\frac{Bx^7}{7b}-\frac{\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^5}{b^9}}(-Ab+Ba)}{-Aa^2b+Ba^3}+x\right)}{2}+\frac{\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)\log\left(\frac{b^4\sqrt{-\frac{a^5}{b^9}}(-Ab+Ba)}{-Aa^2b+Ba^3}+x\right)}{2}-\frac{x^5\,(-Ab+Ba)}{5b^2}+\frac{x^3\,(-Aab+Ba^2)}{3b^3}-\frac{x\,(-Aa^2b+Ba^3)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**6*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**7/(7*b) - sqrt(-a**5/b**9)*(-A*b + B*a)*log(-b**4*sqrt(-a**5
/b**9)*(-A*b + B*a)/(-A*a**2*b + B*a**3) + x)/2 + sqrt(-a**5/b**9
)*(-A*b + B*a)*log(b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b
+ B*a**3) + x)/2 - x**5*(-A*b + B*a)/(5*b**2) + x**3*(-A*a*b + B*
a**2)/(3*b**3) - x*(-A*a**2*b + B*a**3)/b**4
```

GIAC/XCAS [A] time = 0.229673, size = 146, normalized size = 1.49

$$\frac{\left(Ba^4 - Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,Bb^6x^7 - 21\,Bab^5x^5 + 21\,Ab^6x^5 + 35\,Ba^2b^4x^3 - 35\,Aab^5x^3 - 105\,Ba^3b^3x + 105\,Aa^2b^4x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(
15*B*b^6*x^7 - 21*B*a*b^5*x^5 + 21*A*b^6*x^5 + 35*B*a^2*b^4*x^3 -
 35*A*a*b^5*x^3 - 105*B*a^3*b^3*x + 105*A*a^2*b^4*x)/b^7
```

3.57 $\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^6}{6b}$$

```
[Out] -(a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^6
)/(6*b) + (a^2*(A*b - a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.194155, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^6
)/(6*b) + (a^2*(A*b - a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^6}{6b}+\frac{a^2\,(Ab-Ba)\log\left(a+bx^2\right)}{2b^4}+\frac{(Ab-Ba)\int^{x^2} x\,dx}{2b^2}-\frac{(Ab-Ba)\int^{x^2} a\,dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**6/(6*b) + a**2*(A*b - B*a)*log(a + b*x**2)/(2*b**4) + (A*b -
 B*a)*Integral(x, (x, x**2))/(2*b**2) - (A*b - B*a)*Integral(a, (
x, x**2))/(2*b**3)
```

Mathematica [A] time = 0.0537712, size = 71, normalized size = 0.95

$$\frac{bx^2\left(6a^2B-3ab\left(2A+Bx^2\right)+b^2x^2\left(3A+2Bx^2\right)\right)+6a^2(Ab-aB)\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*B - 3*a*b*(2*A + B*x^2) + b^2*x^2*(3*A + 2*B*x^2))
+ 6*a^2*(A*b - a*B)*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.005, size = 86, normalized size = 1.2

$$\frac{Bx^6}{6\,b}+\frac{Ax^4}{4\,b}-\frac{Bx^4a}{4\,b^2}-\frac{aAx^2}{2\,b^2}+\frac{Bx^2a^2}{2\,b^3}+\frac{a^2\ln\left(bx^2+a\right)A}{2\,b^3}-\frac{a^3\ln\left(bx^2+a\right)B}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/6*B*x^6/b+1/4/b*A*x^4-1/4/b^2*B*x^4*a-1/2/b^2*A*x^2*a+1/2/b^3*B
*x^2*a^2+1/2*a^2/b^3*ln(b*x^2+a)*A-1/2*a^3/b^4*ln(b*x^2+a)*B
```

Maxima [A] time = 1.35327, size = 100, normalized size = 1.33

$$\frac{2\,Bb^2x^6 - 3\,\left(Bab - Ab^2\right)x^4 + 6\,\left(Ba^2 - Aab\right)x^2}{12\,b^3} - \frac{\left(Ba^3 - Aa^2b\right)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*(B*a*b - A*b^2)*x^4 + 6*(B*a^2 - A*a*b)*x^2
)/b^3 - 1/2*(B*a^3 - A*a^2*b)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.227649, size = 101, normalized size = 1.35

$$\frac{2\,Bb^3x^6 - 3\,\left(Bab^2 - Ab^3\right)x^4 + 6\,\left(Ba^2b - Aab^2\right)x^2 - 6\,\left(Ba^3 - Aa^2b\right)\log\left(bx^2 + a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^3*x^6 - 3*(B*a*b^2 - A*b^3)*x^4 + 6*(B*a^2*b - A*a*b^
2)*x^2 - 6*(B*a^3 - A*a^2*b)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 1.81479, size = 65, normalized size = 0.87

$$\frac{Bx^6}{6b} - \frac{a^2\left(-Ab + Ba\right)\log\left(a + bx^2\right)}{2b^4} - \frac{x^4\left(-Ab + Ba\right)}{4b^2} + \frac{x^2\left(-Aab + Ba^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**6/(6*b) - a**2*(-A*b + B*a)*log(a + b*x**2)/(2*b**4) - x**4*
(-A*b + B*a)/(4*b**2) + x**2*(-A*a*b + B*a**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.244918, size = 104, normalized size = 1.39

$$\frac{2\,Bb^2x^6 - 3\,Babx^4 + 3\,Ab^2x^4 + 6\,Ba^2x^2 - 6\,Aabx^2}{12\,b^3} - \frac{\left(Ba^3 - Aa^2b\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*B*a*b*x^4 + 3*A*b^2*x^4 + 6*B*a^2*x^2 - 6*A
*a*b*x^2)/b^3 - 1/2*(B*a^3 - A*a^2*b)*ln(abs(b*x^2 + a))/b^4
```

3.58 $\int \frac{x^4\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(Ab-aB)}{b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^5}{5b}$$

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5
*b) + (a^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.137078, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(Ab-aB)}{b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5
*b) + (a^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^5}{5b}+\frac{a^{\frac{3}{2}}\left(Ab-Ba\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{7}{2}}}+\frac{x^3\left(Ab-Ba\right)}{3b^2}-\frac{\left(Ab-Ba\right)\int a\,dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**5/(5*b) + a**(3/2)*(A*b - B*a)*atan(sqrt(b)*x/sqrt(a))/b**(7
/2) + x**3*(A*b - B*a)/(3*b**2) - (A*b - B*a)*Integral(a, x)/b**3
```

Mathematica [A] time = 0.0871806, size = 77, normalized size = 1.

$$-\frac{a^{3/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{ax(aB-Ab)}{b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(A*b) + a*B)*x)/b^3 + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5
*b) - (a^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2
)
```

Maple [A] time = 0.004, size = 92, normalized size = 1.2

$$\frac{Bx^5}{5\,b}+\frac{Ax^3}{3\,b}-\frac{Bx^3a}{3\,b^2}-\frac{aAx}{b^2}+\frac{Bxa^2}{b^3}+\frac{a^2A}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{Ba^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/5*B*x^5/b+1/3/b*A*x^3-1/3/b^2*B*x^3*a-1/b^2*A*x*a+1/b^3*B*x*a^2
+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A-a^3/b^3/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233398, size = 1, normalized size = 0.01

$$\left[\frac{6\,Bb^2x^5 - 10\left(Bab - Ab^2\right)x^3 - 15\left(Ba^2 - Aab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 30\left(Ba^2 - Aab\right)x}{30\,b^3}, \frac{3\,Bb^2x^5 - 5\left(Bab - Ab^2\right)x^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*b^2*x^5 - 10*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*
sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(
B*a^2 - A*a*b)*x)/b^3, 1/15*(3*B*b^2*x^5 - 5*(B*a*b - A*b^2)*x^3
- 15*(B*a^2 - A*a*b)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 15*(B*a^2 -
A*a*b)*x)/b^3]
```

Sympy [A] time = 1.94384, size = 150, normalized size = 1.95

$$\frac{Bx^5}{5b} + \frac{\sqrt{-\frac{a^3}{b^7}}\left(-Ab + Ba\right)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^7}}\left(-Ab + Ba\right)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2} - \frac{x^3\left(-Ab + Ba\right)}{3b^2} + \frac{x\left(-Aab + Ba^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**5/(5*b) + sqrt(-a**3/b**7)*(-A*b + B*a)*log(-b**3*sqrt(-a**3
/b**7)*(-A*b + B*a)/(-A*a*b + B*a**2) + x)/2 - sqrt(-a**3/b**7)*(
-A*b + B*a)*log(b**3*sqrt(-a**3/b**7)*(-A*b + B*a)/(-A*a*b + B*a*
*2) + x)/2 - x**3*(-A*b + B*a)/(3*b**2) + x*(-A*a*b + B*a**2)/b**
3
```

GIAC/XCAS [A] time = 0.242373, size = 115, normalized size = 1.49

$$-\frac{\left(Ba^{3}-Aa^{2}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^{3}}+\frac{3\,Bb^{4}x^{5}-5\,Bab^{3}x^{3}+5\,Ab^{4}x^{3}+15\,Ba^{2}b^{2}x-15\,Aab^{3}x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(
3*B*b^4*x^5 - 5*B*a*b^3*x^3 + 5*A*b^4*x^3 + 15*B*a^2*b^2*x - 15*A
*a*b^3*x)/b^5
```

3.59 $\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=54

$$-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^4}{4b}$$

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^4)/(4*b) - (a*(A*b - a*B)*Log[a
+ b*x^2])/(2*b^3)
```

Rubi [A] time = 0.133595, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^4)/(4*b) - (a*(A*b - a*B)*Log[a
+ b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\int^{x^2} x\,dx}{2b}-\frac{a\,(Ab-Ba)\log\left(a+bx^2\right)}{2b^3}+\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\int^{x^2}\frac{1}{b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*Integral(x, (x, x**2))/(2*b) - a*(A*b - B*a)*log(a + b*x**2)/(2
*b**3) + (A*b/2 - B*a/2)*Integral(b**(-2), (x, x**2))
```

Mathematica [A] time = 0.0311052, size = 47, normalized size = 0.87

$$\frac{bx^2\left(-2aB+2Ab+bBx^2\right)+2a(aB-Ab)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*A*b - 2*a*B + b*B*x^2) + 2*a*(-(A*b) + a*B)*Log[a + b*x
^2])/(4*b^3)
```

Maple [A] time = 0.003, size = 62, normalized size = 1.2

$$\frac{Bx^4}{4\,b}+\frac{Ax^2}{2\,b}-\frac{Bx^2a}{2\,b^2}-\frac{a\ln\left(bx^2+a\right)A}{2\,b^2}+\frac{a^2\ln\left(bx^2+a\right)B}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a-1/2*a/b^2*ln(b*x^2+a)*A+1
/2*a^2/b^3*ln(b*x^2+a)*B
```

Maxima [A] time = 1.35021, size = 68, normalized size = 1.26

$$\frac{Bbx^4 - 2(Ba - Ab)x^2}{4b^2} + \frac{(Ba^2 - Aab)\log(bx^2 + a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 2*(B*a - A*b)*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(b
*x^2 + a)/b^3
```

Fricas [A] time = 0.22928, size = 69, normalized size = 1.28

$$\frac{Bb^2x^4 - 2(Bab - Ab^2)x^2 + 2(Ba^2 - Aab)\log(bx^2 + a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^4 - 2*(B*a*b - A*b^2)*x^2 + 2*(B*a^2 - A*a*b)*log(b*
x^2 + a))/b^3
```

Sympy [A] time = 1.69101, size = 44, normalized size = 0.81

$$\frac{Bx^4}{4b} + \frac{a(-Ab + Ba)\log(a + bx^2)}{2b^3} - \frac{x^2(-Ab + Ba)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**4/(4*b) + a*(-A*b + B*a)*log(a + b*x**2)/(2*b**3) - x**2*(-A
*b + B*a)/(2*b**2)
```

GIAC/XCAS [A] time = 0.229871, size = 70, normalized size = 1.3

$$\frac{Bbx^4 - 2Bax^2 + 2Abx^2}{4b^2} + \frac{(Ba^2 - Aab)\ln(|bx^2 + a|)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(B*b*x^4 - 2*B*a*x^2 + 2*A*b*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*l
n(abs(b*x^2 + a))/b^3
```

3.60 $\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^3}{3b}$$

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) - (Sqrt[a]*(A*b - a*B)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.10151, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) - (Sqrt[a]*(A*b - a*B)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rubi in Sympy [A] time = 15.7168, size = 49, normalized size = 0.84

$$\frac{Bx^3}{3b}-\frac{\sqrt{a}\,(Ab-Ba)\,\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{x\,(Ab-Ba)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**3/(3*b) - sqrt(a)*(A*b - B*a)*atan(sqrt(b)*x/sqrt(a))/b**(5/
2) + x*(A*b - B*a)/b**2
```

Mathematica [A] time = 0.062653, size = 57, normalized size = 0.98

$$\frac{\sqrt{a}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) + (Sqrt[a]*(-(A*b) + a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A] time = 0.003, size = 68, normalized size = 1.2

$$\frac{Bx^3}{3\,b}+\frac{Ax}{b}-\frac{Bxa}{b^2}-\frac{aA}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2B}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/b+1/b*A*x-1/b^2*B*x*a-a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*A+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250151, size = 1, normalized size = 0.02

$$\left[\frac{2\,Bbx^3 - 3\,(Ba - Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 6\,(Ba - Ab)x}{6\,b^2},\ \frac{Bbx^3 + 3\,(Ba - Ab)\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) - 3\,(Ba - Ab)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*b*x^3 - 3*(B*a - A*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqr
t(-a/b) - a)/(b*x^2 + a)) - 6*(B*a - A*b)*x)/b^2, 1/3*(B*b*x^3 +
3*(B*a - A*b)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 3*(B*a - A*b)*x)/b^
2]
```

Sympy [A] time = 1.79807, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3b} - \frac{\sqrt{-\frac{a}{b^5}}\,(-Ab + Ba)\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}\,(-Ab + Ba)\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} - \frac{x\,(-Ab + Ba)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**3/(3*b) - sqrt(-a/b**5)*(-A*b + B*a)*log(-b**2*sqrt(-a/b**5)
 + x)/2 + sqrt(-a/b**5)*(-A*b + B*a)*log(b**2*sqrt(-a/b**5) + x)/
2 - x*(-A*b + B*a)/b**2
```

GIAC/XCAS [A] time = 0.230644, size = 77, normalized size = 1.33

$$\frac{\left(Ba^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Bb^2x^3 - 3\,Babx + 3\,Ab^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b^
2*x^3 - 3*B*a*b*x + 3*A*b^2*x)/b^3
```

3.61 $\int \frac{x(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=35

$$\frac{(Ab-aB)\log(a+bx^2)}{2b^2}+\frac{Bx^2}{2b}$$

```
[Out] (B*x^2)/(2*b) + ((A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.076657, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{(Ab-aB)\log(a+bx^2)}{2b^2}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^2)/(2*b) + ((A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^2} B\,dx}{2b}+\frac{(Ab-Ba)\log(a+bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Integral(B, (x, x**2))/(2*b) + (A*b - B*a)*log(a + b*x**2)/(2*b**
2)
```

Mathematica [A] time = 0.0182298, size = 31, normalized size = 0.89

$$\frac{(Ab-aB)\log(a+bx^2)+bBx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*B*x^2 + (A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.005, size = 40, normalized size = 1.1

$$\frac{Bx^2}{2\,b}+\frac{\ln(bx^2+a)\,A}{2\,b}-\frac{\ln(bx^2+a)\,Ba}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/2*B*x^2/b+1/2/b*ln(b*x^2+a)*A-1/2/b^2*ln(b*x^2+a)*B*a
```

Maxima [A] time = 1.35526, size = 42, normalized size = 1.2

$$\frac{Bx^2}{2\,b} - \frac{(Ba - Ab)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.228992, size = 41, normalized size = 1.17

$$\frac{Bbx^2 - (Ba - Ab)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*(B*b*x^2 - (B*a - A*b)*log(b*x^2 + a))/b^2
```

Sympy [A] time = 1.52729, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2b} - \frac{(-Ab + Ba)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**2/(2*b) - (-A*b + B*a)*log(a + b*x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.245061, size = 43, normalized size = 1.23

$$\frac{Bx^2}{2\,b} - \frac{(Ba - Ab)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*ln(abs(b*x^2 + a))/b^2
```

3.62 $\int \frac{A+Bx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Bx}{b}$$

```
[Out] (B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2))
```

Rubi [A] time = 0.048688, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2))
```

Rubi in Sympy [A] time = 8.51514, size = 34, normalized size = 0.87

$$\frac{Bx}{b}+\frac{(Ab-Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x/b + (A*b - B*a)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(3/2))
```

Mathematica [A] time = 0.0474816, size = 40, normalized size = 1.03

$$\frac{Bx}{b}-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b
^(3/2))
```

Maple [A] time = 0.003, size = 45, normalized size = 1.2

$$\frac{Bx}{b}+A\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{Ba}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a),x)
```

```
[Out] B*x/b+1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A-1/b/(a*b)^(1/2)*arc
tan(x*b/(a*b)^(1/2))*B*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235843, size = 1, normalized size = 0.03

$$\left[\frac{2\sqrt{-ab}Bx - (Ba - Ab)\log\left(\frac{2\,abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right)}{2\sqrt{-ab}b}, \frac{\sqrt{ab}Bx - (Ba - Ab)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*b)*B*x - (B*a - A*b)*log((2*a*b*x + (b*x^2 - a)*s
qrt(-a*b))/(b*x^2 + a)))/(sqrt(-a*b)*b), (sqrt(a*b)*B*x - (B*a -
A*b)*arctan(sqrt(a*b)*x/a))/(sqrt(a*b)*b)]
```

Sympy [A] time = 1.62372, size = 82, normalized size = 2.1

$$\frac{Bx}{b} + \frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab + Ba)\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab + Ba)\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x/b + sqrt(-1/(a*b**3))*(-A*b + B*a)*log(-a*b*sqrt(-1/(a*b**3))
 + x)/2 - sqrt(-1/(a*b**3))*(-A*b + B*a)*log(a*b*sqrt(-1/(a*b**3)
) + x)/2
```

GIAC/XCAS [A] time = 0.231714, size = 46, normalized size = 1.18

$$\frac{Bx}{b} - \frac{(Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.63 $\int \frac{A+Bx^2}{x(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.0895972, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2)),x]

[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^2])/(2*a*b)
```

Rubi in Sympy [A] time = 13.5099, size = 29, normalized size = 0.85

$$\frac{A\log\left(x^2\right)}{2a}-\frac{(Ab-Ba)\log\left(a+bx^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(b*x**2+a),x)

[Out] A*log(x**2)/(2*a) - (A*b - B*a)*log(a + b*x**2)/(2*a*b)
```

Mathematica [A] time = 0.0207135, size = 34, normalized size = 1.

$$\frac{(aB-Ab)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2)),x]

[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] time = 0.007, size = 37, normalized size = 1.1

$$\frac{A\ln(x)}{a}-\frac{\ln\left(bx^2+a\right)A}{2a}+\frac{\ln\left(bx^2+a\right)B}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(b*x^2+a),x)

[Out] A*ln(x)/a-1/2/a*ln(b*x^2+a)*A+1/2/b*ln(b*x^2+a)*B
```

Maxima [A] time = 1.35027, size = 47, normalized size = 1.38

$$\frac{A\log\left(x^2\right)}{2\,a}+\frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(b*x^2 + a)/(a*b)
```

Fricas [A] time = 0.215173, size = 43, normalized size = 1.26

$$\frac{2\,Ab\log\left(x\right)+(Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*b*log(x) + (B*a - A*b)*log(b*x^2 + a))/(a*b)
```

Sympy [A] time = 2.25521, size = 26, normalized size = 0.76

$$\frac{A\log\left(x\right)}{a}+\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(b*x**2+a),x)
```

```
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**2)/(2*a*b)
```

GIAC/XCAS [A] time = 0.230616, size = 49, normalized size = 1.44

$$\frac{A\ln\left(x^2\right)}{2\,a}+\frac{(Ba-Ab)\ln\left(\left|bx^2+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*A*ln(x^2)/a + 1/2*(B*a - A*b)*ln(abs(b*x^2 + a))/(a*b)
```

3.64 $\int \frac{A+Bx^2}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=43

$$-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

```
[Out] -(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*S
qrt[b])
```

Rubi [A] time = 0.0652404, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*S
qrt[b])
```

Rubi in Sympy [A] time = 9.8628, size = 36, normalized size = 0.84

$$-\frac{A}{ax}-\frac{(Ab-Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) - (A*b - B*a)*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*sqrt(b))
```

Mathematica [A] time = 0.0435932, size = 42, normalized size = 0.98

$$\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2
)*Sqrt[b])
```

Maple [A] time = 0.005, size = 48, normalized size = 1.1

$$-\frac{Ab}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+B\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A*b+1/(a*b)^(1/2)*arctan
(x*b/(a*b)^(1/2))*B-A/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236248, size = 1, normalized size = 0.02

$$\left[-\frac{(Ba-Ab)x\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\sqrt{-ab}A}{2\sqrt{-ab}ax},\ \frac{(Ba-Ab)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\sqrt{ab}A}{\sqrt{ab}ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a - A*b)*x*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x
^2 + a)) + 2*sqrt(-a*b)*A)/(sqrt(-a*b)*a*x), ((B*a - A*b)*x*arcta
n(sqrt(a*b)*x/a) - sqrt(a*b)*A)/(sqrt(a*b)*a*x)]
```

Sympy [A] time = 1.85375, size = 82, normalized size = 1.91

$$-\frac{A}{ax}-\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) - sqrt(-1/(a**3*b))*(-A*b + B*a)*log(-a**2*sqrt(-1/(a**3
*b)) + x)/2 + sqrt(-1/(a**3*b))*(-A*b + B*a)*log(a**2*sqrt(-1/(a*
*3*b)) + x)/2
```

GIAC/XCAS [A] time = 0.22596, size = 49, normalized size = 1.14

$$\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

3.65 $\int \frac{A+Bx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

```
[Out] -A/(2*a*x^2) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*
x^2])/(2*a^2)
```

Rubi [A] time = 0.120778, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*
x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 16.0316, size = 44, normalized size = 0.88

$$-\frac{A}{2ax^2}-\frac{(Ab-Ba)\log\left(x^2\right)}{2a^2}+\frac{(Ab-Ba)\log\left(a+bx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) - (A*b - B*a)*log(x**2)/(2*a**2) + (A*b - B*a)*log(
a + b*x**2)/(2*a**2)
```

Mathematica [A] time = 0.0343761, size = 49, normalized size = 0.98

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)(aB-Ab)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a +
 b*x^2])/(2*a^2)
```

Maple [A] time = 0.01, size = 56, normalized size = 1.1

$$-\frac{A}{2\,ax^2}-\frac{A\ln(x)\,b}{a^2}+\frac{\ln(x)\,B}{a}+\frac{\ln\left(bx^2+a\right)\,Ab}{2\,a^2}-\frac{\ln\left(bx^2+a\right)\,B}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*B+1/2/a^2*ln(b*x^2+a)*A*b-
1/2/a*ln(b*x^2+a)*B
```

Maxima [A] time = 1.34712, size = 65, normalized size = 1.3

$$-\frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,a^2}+\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*log(b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^
2 - 1/2*A/(a*x^2)
```

Fricas [A] time = 0.227302, size = 63, normalized size = 1.26

$$-\frac{(Ba-Ab)x^2\log\left(bx^2+a\right)-2\,(Ba-Ab)x^2\log\left(x\right)+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*((B*a - A*b)*x^2*log(b*x^2 + a) - 2*(B*a - A*b)*x^2*log(x) +
 A*a)/(a^2*x^2)
```

Sympy [A] time = 3.0413, size = 41, normalized size = 0.82

$$-\frac{A}{2ax^2}+\frac{(-Ab+Ba)\log\left(x\right)}{a^2}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b +
 x**2)/(2*a**2)
```

GIAC/XCAS [A] time = 0.228604, size = 96, normalized size = 1.92

$$\frac{(Ba-Ab)\ln\left(x^2\right)}{2\,a^2}-\frac{\left(Bab-Ab^2\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^2b}-\frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - A*b)*ln(x^2)/a^2 - 1/2*(B*a*b - A*b^2)*ln(abs(b*x^2 +
a))/(a^2*b) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

3.66 $\int \frac{A+Bx^2}{x^4(a+bx^2)} dx$

Optimal. Leaf size=59

$$\frac{\sqrt{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) + (Sqrt[b]*(A*b - a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.104534, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\sqrt{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) + (Sqrt[b]*(A*b - a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi in Sympy [A] time = 14.6906, size = 49, normalized size = 0.83

$$-\frac{A}{3ax^3}+\frac{Ab-Ba}{a^2x}+\frac{\sqrt{b}\,(Ab-Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(b*x**2+a),x)
```

```
[Out] -A/(3*a*x**3) + (A*b - B*a)/(a**2*x) + sqrt(b)*(A*b - B*a)*atan(s
qrt(b)*x/sqrt(a))/a**(5/2)
```

Mathematica [A] time = 0.0912105, size = 60, normalized size = 1.02

$$-\frac{\sqrt{b}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[b]*(-(A*b) + a*B)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A] time = 0.009, size = 72, normalized size = 1.2

$$-\frac{A}{3\,ax^3}+\frac{Ab}{a^2x}-\frac{B}{ax}+\frac{b^2A}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{Bb}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(b*x^2+a),x)
```

```
[Out] -1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B+b^2/a^2/(a*b)^(1/2)*arctan(x*b/(
a*b)^(1/2))*A-b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23766, size = 1, normalized size = 0.02

$$\left[-\frac{3\,(Ba-Ab)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,(Ba-Ab)x^2+2\,Aa}{6\,a^2x^3},\ -\frac{3\,(Ba-Ab)x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*a - A*b)*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) + 6*(B*a - A*b)*x^2 + 2*A*a)/(a^2*x^3), -1/3*(
3*(B*a - A*b)*x^3*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 3*(B*a -
A*b)*x^2 + A*a)/(a^2*x^3)]
```

Sympy [A] time = 2.26094, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}-\frac{Aa+x^2\,(-3Ab+3Ba)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(-A*b + B*a)*log(-a**3*sqrt(-b/a**5)*(-A*b + B*a)/(
-A*b**2 + B*a*b) + x)/2 - sqrt(-b/a**5)*(-A*b + B*a)*log(a**3*sqr
t(-b/a**5)*(-A*b + B*a)/(-A*b**2 + B*a*b) + x)/2 - (A*a + x**2*(-
3*A*b + 3*B*a))/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 0.225285, size = 77, normalized size = 1.31

$$-\frac{\left(Bab - Ab^{2}\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^{2}} - \frac{3 \, Bax^{2} - 3 \, Abx^{2} + Aa}{3 \, a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*B
*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)
```

3.67 $\int \frac{A+Bx^2}{x^5(a+bx^2)}\,dx$

Optimal. Leaf size=69

$$-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{4ax^4}$$

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(2*a^2*x^2) + (b*(A*b - a*B)*Log[x])/a
^3 - (b*(A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.152458, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(2*a^2*x^2) + (b*(A*b - a*B)*Log[x])/a
^3 - (b*(A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 20.0104, size = 63, normalized size = 0.91

$$-\frac{A}{4ax^4}+\frac{Ab-Ba}{2a^2x^2}+\frac{b\left(Ab-Ba\right)\log\left(x^2\right)}{2a^3}-\frac{b\left(Ab-Ba\right)\log\left(a+bx^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(b*x**2+a),x)
```

```
[Out] -A/(4*a*x**4) + (A*b - B*a)/(2*a**2*x**2) + b*(A*b - B*a)*log(x**
2)/(2*a**3) - b*(A*b - B*a)*log(a + b*x**2)/(2*a**3)
```

Mathematica [A] time = 0.0478807, size = 70, normalized size = 1.01

$$\frac{4bx^4\log(x)(Ab-aB)-a\left(aA+2aBx^2-2Abx^2\right)+2bx^4(aB-Ab)\log\left(a+bx^2\right)}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^2 + 2*a*B*x^2)) + 4*b*(A*b - a*B)*x^4*Log[x]
+ 2*b*(-(A*b) + a*B)*x^4*Log[a + b*x^2])/(4*a^3*x^4)
```

Maple [A] time = 0.01, size = 81, normalized size = 1.2

$$-\frac{A}{4\,ax^4}+\frac{Ab}{2\,a^2x^2}-\frac{B}{2\,ax^2}+\frac{A\ln(x)\,b^2}{a^3}-\frac{bB\ln(x)}{a^2}-\frac{b^2\ln\left(bx^2+a\right)A}{2\,a^3}+\frac{b\ln\left(bx^2+a\right)B}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(b*x^2+a),x)
```

```
[Out] -1/4*A/a/x^4+1/2/a^2/x^2*A*b-1/2/a/x^2*B+1/a^3*b^2*ln(x)*A-1/a^2*
b*ln(x)*B-1/2*b^2/a^3*ln(b*x^2+a)*A+1/2*b/a^2*ln(b*x^2+a)*B
```

Maxima [A] time = 1.35597, size = 95, normalized size = 1.38

$$\frac{\left(Bab - Ab^2\right)\log\left(bx^2 + a\right)}{2\,a^3} - \frac{\left(Bab - Ab^2\right)\log\left(x^2\right)}{2\,a^3} - \frac{2\,(Ba - Ab)x^2 + Aa}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b - A*b^2)*log(b*x^2 + a)/a^3 - 1/2*(B*a*b - A*b^2)*log(
x^2)/a^3 - 1/4*(2*(B*a - A*b)*x^2 + A*a)/(a^2*x^4)
```

Fricas [A] time = 0.236855, size = 99, normalized size = 1.43

$$\frac{2\left(Bab - Ab^2\right)x^4\log\left(bx^2 + a\right) - 4\left(Bab - Ab^2\right)x^4\log\left(x\right) - Aa^2 - 2\left(Ba^2 - Aab\right)x^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a*b - A*b^2)*x^4*log(b*x^2 + a) - 4*(B*a*b - A*b^2)*x^4
*log(x) - A*a^2 - 2*(B*a^2 - A*a*b)*x^2)/(a^3*x^4)
```

Sympy [A] time = 3.80495, size = 61, normalized size = 0.88

$$-\frac{Aa + x^2\left(-2Ab + 2Ba\right)}{4a^2x^4} - \frac{b\left(-Ab + Ba\right)\log\left(x\right)}{a^3} + \frac{b\left(-Ab + Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(b*x**2+a),x)
```

```
[Out] -(A*a + x**2*(-2*A*b + 2*B*a))/(4*a**2*x**4) - b*(-A*b + B*a)*log
(x)/a**3 + b*(-A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

GIAC/XCAS [A] time = 0.229093, size = 135, normalized size = 1.96

$$-\frac{\left(Bab - Ab^2\right)\ln\left(x^2\right)}{2\,a^3} + \frac{\left(Bab^2 - Ab^3\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,a^3b} + \frac{3\,Babx^4 - 3\,Ab^2x^4 - 2\,Ba^2x^2 + 2\,Aabx^2 - Aa^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b - A*b^2)*ln(x^2)/a^3 + 1/2*(B*a*b^2 - A*b^3)*ln(abs(b
*x^2 + a))/(a^3*b) + 1/4*(3*B*a*b*x^4 - 3*A*b^2*x^4 - 2*B*a^2*x^2
 + 2*A*a*b*x^2 - A*a^2)/(a^3*x^4)
```

3.68 $\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=80

$$-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{5ax^5}$$

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(a^3*x)
- (b^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.131393, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(a^3*x)
- (b^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Rubi in Sympy [A] time = 18.5684, size = 66, normalized size = 0.82

$$-\frac{A}{5ax^5}+\frac{Ab-Ba}{3a^2x^3}-\frac{b\,(Ab-Ba)}{a^3x}-\frac{b^{\frac{3}{2}}\,(Ab-Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**6/(b*x**2+a),x)
```

```
[Out] -A/(5*a*x**5) + (A*b - B*a)/(3*a**2*x**3) - b*(A*b - B*a)/(a**3*x
) - b**(3/2)*(A*b - B*a)*atan(sqrt(b)*x/sqrt(a))/a**(7/2)
```

Mathematica [A] time = 0.0868997, size = 78, normalized size = 0.98

$$\frac{b^{3/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b(aB-Ab)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) + (b*(-(A*b) + a*B))/(a^3*
x) + (b^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Maple [A] time = 0.009, size = 96, normalized size = 1.2

$$-\frac{A}{5\,ax^5}+\frac{Ab}{3\,a^2x^3}-\frac{B}{3\,ax^3}-\frac{b^2A}{a^3x}+\frac{Bb}{a^2x}-\frac{Ab^3}{a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{Bb^2}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^6/(b*x^2+a),x)
```

```
[Out] -1/5*A/a/x^5+1/3/a^2/x^3*A*b-1/3/a/x^3*B-1/a^3*b^2/x*A+1/a^2*b/x*
B-b^3/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+b^2/a^2/(a*b)^(1/
2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24022, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(Bab-Ab^{2}\right)x^{5}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)-30\left(Bab-Ab^{2}\right)x^{4}+6\,Aa^{2}+10\left(Ba^{2}-Aab\right)x^{2}}{30\,a^{3}x^{5}},\ \frac{15\left(Bab-Ab^{2}\right)x^{5}\sqrt{\frac{b}{a}}\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^6),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*a*b - A*b^2)*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt
(-b/a) - a)/(b*x^2 + a)) - 30*(B*a*b - A*b^2)*x^4 + 6*A*a^2 + 10*
(B*a^2 - A*a*b)*x^2)/(a^3*x^5), 1/15*(15*(B*a*b - A*b^2)*x^5*sqrt
(b/a)*arctan(b*x/(a*sqrt(b/a))) + 15*(B*a*b - A*b^2)*x^4 - 3*A*a^
2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)]
```

Sympy [A] time = 2.74752, size = 163, normalized size = 2.04

$$-\frac{\sqrt{-\frac{b^{3}}{a^{7}}}\left(-Ab+Ba\right)\log\left(-\frac{a^{4}\sqrt{-\frac{b^{3}}{a^{7}}}\left(-Ab+Ba\right)}{-Ab^{3}+Bab^{2}}+x\right)}{2}+\frac{\sqrt{-\frac{b^{3}}{a^{7}}}\left(-Ab+Ba\right)\log\left(\frac{a^{4}\sqrt{-\frac{b^{3}}{a^{7}}}\left(-Ab+Ba\right)}{-Ab^{3}+Bab^{2}}+x\right)}{2}+\frac{-3Aa^{2}+x^{4}\left(-15Ab^{2}+15Bab\right)+x^{2}\left(5Aab-5Ba^{2}\right)}{15a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**7)*(-A*b + B*a)*log(-a**4*sqrt(-b**3/a**7)*(-A*b +
 B*a)/(-A*b**3 + B*a*b**2) + x)/2 + sqrt(-b**3/a**7)*(-A*b + B*a)
*log(a**4*sqrt(-b**3/a**7)*(-A*b + B*a)/(-A*b**3 + B*a*b**2) + x)
/2 + (-3*A*a**2 + x**4*(-15*A*b**2 + 15*B*a*b) + x**2*(5*A*a*b -
5*B*a**2))/(15*a**3*x**5)
```

GIAC/XCAS [A] time = 0.226521, size = 109, normalized size = 1.36

$$\frac{\left(Bab^2 - Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{15\,Babx^4 - 15\,Ab^2x^4 - 5\,Ba^2x^2 + 5\,Aabx^2 - 3\,Aa^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(1
5*B*a*b*x^4 - 15*A*b^2*x^4 - 5*B*a^2*x^2 + 5*A*a*b*x^2 - 3*A*a^2)
/(a^3*x^5)
```

3.69 $\int \frac{A+Bx^2}{x^7(a+bx^2)} dx$

Optimal. Leaf size=93

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(2*a^3*x
^2) - (b^2*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x
^2])/(2*a^4)
```

Rubi [A] time = 0.191651, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(2*a^3*x
^2) - (b^2*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x
^2])/(2*a^4)
```

Rubi in Sympy [A] time = 24.2299, size = 83, normalized size = 0.89

$$-\frac{A}{6ax^6}+\frac{Ab-Ba}{4a^2x^4}-\frac{b\,(Ab-Ba)}{2a^3x^2}-\frac{b^2\,(Ab-Ba)\log\left(x^2\right)}{2a^4}+\frac{b^2\,(Ab-Ba)\log\left(a+bx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**7/(b*x**2+a),x)
```

```
[Out] -A/(6*a*x**6) + (A*b - B*a)/(4*a**2*x**4) - b*(A*b - B*a)/(2*a**3
*x**2) - b**2*(A*b - B*a)*log(x**2)/(2*a**4) + b**2*(A*b - B*a)*l
og(a + b*x**2)/(2*a**4)
```

Mathematica [A] time = 0.0645537, size = 96, normalized size = 1.03

$$\frac{\left(Ab^3-ab^2B\right)\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)\left(ab^2B-Ab^3\right)}{a^4}+\frac{b(aB-Ab)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) + (b*(-(A*b) + a*B))/(2*a^
3*x^2) + ((-(A*b^3) + a*b^2*B)*Log[x])/a^4 + ((A*b^3 - a*b^2*B)*L
og[a + b*x^2])/(2*a^4)
```

Maple [A] time = 0.011, size = 107, normalized size = 1.2

$$-\frac{A}{6\,ax^6}+\frac{Ab}{4\,a^2x^4}-\frac{B}{4\,ax^4}-\frac{b^2A}{2\,a^3x^2}+\frac{Bb}{2\,a^2x^2}-\frac{b^3\ln(x)\,A}{a^4}+\frac{b^2B\ln(x)}{a^3}+\frac{b^3\ln\left(bx^2+a\right)A}{2\,a^4}-\frac{b^2\ln\left(bx^2+a\right)B}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^7/(b*x^2+a),x)
```

```
[Out] -1/6*A/a/x^6+1/4/a^2/x^4*A*b-1/4/a/x^4*B-1/2/a^3*b^2/x^2*A+1/2/a^
2*b/x^2*B-1/a^4*b^3*ln(x)*A+1/a^3*b^2*ln(x)*B+1/2*b^3/a^4*ln(b*x^
2+a)*A-1/2*b^2/a^3*ln(b*x^2+a)*B
```

Maxima [A] time = 1.34099, size = 130, normalized size = 1.4

$$-\frac{\left(Bab^2 - Ab^3\right)\log\left(bx^2 + a\right)}{2\,a^4} + \frac{\left(Bab^2 - Ab^3\right)\log\left(x^2\right)}{2\,a^4} + \frac{6\left(Bab - Ab^2\right)x^4 - 2\,Aa^2 - 3\left(Ba^2 - Aab\right)x^2}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^7),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*b^2 - A*b^3)*log(b*x^2 + a)/a^4 + 1/2*(B*a*b^2 - A*b^3)
*log(x^2)/a^4 + 1/12*(6*(B*a*b - A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2
- A*a*b)*x^2)/(a^3*x^6)
```

Fricas [A] time = 0.237238, size = 132, normalized size = 1.42

$$-\frac{6\left(Bab^2 - Ab^3\right)x^6\log\left(bx^2 + a\right) - 12\left(Bab^2 - Ab^3\right)x^6\log\left(x\right) - 6\left(Ba^2b - Aab^2\right)x^4 + 2\,Aa^3 + 3\left(Ba^3 - Aa^2b\right)x^2}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^7),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*a*b^2 - A*b^3)*x^6*log(b*x^2 + a) - 12*(B*a*b^2 - A*b
^3)*x^6*log(x) - 6*(B*a^2*b - A*a*b^2)*x^4 + 2*A*a^3 + 3*(B*a^3 -
 A*a^2*b)*x^2)/(a^4*x^6)
```

Sympy [A] time = 4.39526, size = 88, normalized size = 0.95

$$\frac{-2Aa^2 + x^4\left(-6Ab^2 + 6Bab\right) + x^2\left(3Aab - 3Ba^2\right)}{12a^3x^6} + \frac{b^2\left(-Ab + Ba\right)\log\left(x\right)}{a^4} - \frac{b^2\left(-Ab + Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**7/(b*x**2+a),x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 6*B*a*b) + x**2*(3*A*a*b - 3*B*a**
2))/(12*a**3*x**6) + b**2*(-A*b + B*a)*log(x)/a**4 - b**2*(-A*b +
 B*a)*log(a/b + x**2)/(2*a**4)
```

GIAC/XCAS [A] time = 0.230746, size = 170, normalized size = 1.83

$$\frac{\left(Bab^2 - Ab^3\right)\ln\left(x^2\right)}{2\,a^4} - \frac{\left(Bab^3 - Ab^4\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,a^4b} - \frac{11\,Bab^2x^6 - 11\,Ab^3x^6 - 6\,Ba^2bx^4 + 6\,Aab^2x^4 + 3\,Ba^3x^2 - 3\,Aa^2bx^2 + 2\,Aa^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^7),x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b^2 - A*b^3)*ln(x^2)/a^4 - 1/2*(B*a*b^3 - A*b^4)*ln(abs(
b*x^2 + a))/(a^4*b) - 1/12*(11*B*a*b^2*x^6 - 11*A*b^3*x^6 - 6*B*a
^2*b*x^4 + 6*A*a*b^2*x^4 + 3*B*a^3*x^2 - 3*A*a^2*b*x^2 + 2*A*a^3)
/(a^4*x^6)
```

3.70 $\int \frac{A+Bx^2}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=99

$$\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^2(Ab-aB)}{a^4x}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (b*(A*b - a*B))/(3*a^3*x
^3) + (b^2*(A*b - a*B))/(a^4*x) + (b^(5/2)*(A*b - a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rubi [A] time = 0.167147, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^2(Ab-aB)}{a^4x}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (b*(A*b - a*B))/(3*a^3*x
^3) + (b^2*(A*b - a*B))/(a^4*x) + (b^(5/2)*(A*b - a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rubi in Sympy [A] time = 23.9656, size = 85, normalized size = 0.86

$$-\frac{A}{7ax^7}+\frac{Ab-Ba}{5a^2x^5}-\frac{b\,(Ab-Ba)}{3a^3x^3}+\frac{b^2\,(Ab-Ba)}{a^4x}+\frac{b^{\frac{5}{2}}\,(Ab-Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**8/(b*x**2+a),x)
```

```
[Out] -A/(7*a*x**7) + (A*b - B*a)/(5*a**2*x**5) - b*(A*b - B*a)/(3*a**3
*x**3) + b**2*(A*b - B*a)/(a**4*x) + b**(5/2)*(A*b - B*a)*atan(sq
rt(b)*x/sqrt(a))/a**(9/2)
```

Mathematica [A] time = 0.114859, size = 101, normalized size = 1.02

$$-\frac{b^{5/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{b^2(aB-Ab)}{a^4x}+\frac{b(aB-Ab)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) + (b*(-(A*b) + a*B))/(3*a^
3*x^3) - (b^2*(-(A*b) + a*B))/(a^4*x) - (b^(5/2)*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Maple [A] time = 0.01, size = 120, normalized size = 1.2

$$-\frac{A}{7\,a x^{7}}+\frac{A b}{5\,a^{2} x^{5}}-\frac{B}{5\,a x^{5}}-\frac{b^{2} A}{3\,a^{3} x^{3}}+\frac{B b}{3\,a^{2} x^{3}}+\frac{b^{3} A}{a^{4} x}-\frac{B b^{2}}{a^{3} x}+\frac{A b^{4}}{a^{4}}\arctan\left(b x\frac{1}{\sqrt{a b}}\right)\frac{1}{\sqrt{a b}}-\frac{b^{3} B}{a^{3}}\arctan\left(b x\frac{1}{\sqrt{a b}}\right)\frac{1}{\sqrt{a b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^8/(b*x^2+a),x)
```

```
[Out] -1/7*A/a/x^7+1/5/a^2/x^5*A*b-1/5/a/x^5*B-1/3/a^3*b^2/x^3*A+1/3/a^
2*b/x^3*B+1/a^4*b^3/x*A-1/a^3*b^2/x*B+b^4/a^4/(a*b)^(1/2)*arctan(
x*b/(a*b)^(1/2))*A-b^3/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247391, size = 1, normalized size = 0.01

$$\left[-\frac{105\left(B a b^{2}-A b^{3}\right) x^{7} \sqrt{-\frac{b}{a}} \log\left(\frac{b x^{2}+2\,a x \sqrt{-\frac{b}{a}}-a}{b x^{2}+a}\right)+210\left(B a b^{2}-A b^{3}\right) x^{6}-70\left(B a^{2} b-A a b^{2}\right) x^{4}+30\,A a^{3}+42\,(B a^{3}-A a^{2} b}{210\,a^{4} x^{7}},\; -\frac{105\left(B a b^{2}-A b^{3}\right) x^{7} \sqrt{\frac{b}{a}} \arctan\left(\frac{b x}{a \sqrt{\frac{b}{a}}}\right)+105\left(B a b^{2}-A b^{3}\right) x^{6}-35\left(B a^{2} b-A a b^{2}\right) x^{4}+15\,A a^{3}+21\left(B a^{3}-A a^{2} b\right) x^{2}}{105\,a^{4} x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^8),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(B*a*b^2 - A*b^3)*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*
sqrt(-b/a) - a)/(b*x^2 + a)) + 210*(B*a*b^2 - A*b^3)*x^6 - 70*(B*
a^2*b - A*a*b^2)*x^4 + 30*A*a^3 + 42*(B*a^3 - A*a^2*b)*x^2)/(a^4*
x^7), -1/105*(105*(B*a*b^2 - A*b^3)*x^7*sqrt(b/a)*arctan(b*x/(a*s
qrt(b/a))) + 105*(B*a*b^2 - A*b^3)*x^6 - 35*(B*a^2*b - A*a*b^2)*x
^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7)]
```

Sympy [A] time = 3.66796, size = 187, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^{5}}{a^{9}}}\left(-A b+B a\right)\log\left(-\frac{a^{5}\sqrt{-\frac{b^{5}}{a^{9}}}\left(-A b+B a\right)}{-A b^{4}+B a b^{3}}+x\right)}{2}-\frac{\sqrt{-\frac{b^{5}}{a^{9}}}\left(-A b+B a\right)\log\left(\frac{a^{5}\sqrt{-\frac{b^{5}}{a^{9}}}\left(-A b+B a\right)}{-A b^{4}+B a b^{3}}+x\right)}{2}-\frac{15 A a^{3}+x^{6}\left(-105 A b^{3}+105 B a b^{2}\right)+x^{4}\left(35 A a b^{2}-35 B a^{2} b\right)+x^{2}\left(-21 A a^{2} b+21 B a^{3}\right)}{105 a^{4} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**9)*(-A*b + B*a)*log(-a**5*sqrt(-b**5/a**9)*(-A*b +
B*a)/(-A*b**4 + B*a*b**3) + x)/2 - sqrt(-b**5/a**9)*(-A*b + B*a)*
log(a**5*sqrt(-b**5/a**9)*(-A*b + B*a)/(-A*b**4 + B*a*b**3) + x)/
2 - (15*A*a**3 + x**6*(-105*A*b**3 + 105*B*a*b**2) + x**4*(35*A*a
*b**2 - 35*B*a**2*b) + x**2*(-21*A*a**2*b + 21*B*a**3))/(105*a**4
*x**7)
```

GIAC/XCAS [A] time = 0.222384, size = 143, normalized size = 1.44

$$-\frac{\left(Bab^3 - Ab^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{105\,Bab^2x^6 - 105\,Ab^3x^6 - 35\,Ba^2bx^4 + 35\,Aab^2x^4 + 21\,Ba^3x^2 - 21\,Aa^2bx^2 + 15\,Aa^3}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^8),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*
(105*B*a*b^2*x^6 - 105*A*b^3*x^6 - 35*B*a^2*b*x^4 + 35*A*a*b^2*x^
4 + 21*B*a^3*x^2 - 21*A*a^2*b*x^2 + 15*A*a^3)/(a^4*x^7)
```

3.71 $\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{a^4(Ab-aB)}{2b^6(a+bx^2)}-\frac{a^3(4Ab-5aB)\log(a+bx^2)}{2b^6}+\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{x^6(Ab-2aB)}{6b^3}+\frac{Bx^8}{8b^2}$$

```
[Out] (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2*A*b - 3*a*B)*x^4)/(4*b^
4) + ((A*b - 2*a*B)*x^6)/(6*b^3) + (B*x^8)/(8*b^2) - (a^4*(A*b -
a*B))/(2*b^6*(a + b*x^2)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/
(2*b^6)
```

Rubi [A] time = 0.373282, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^4(Ab-aB)}{2b^6(a+bx^2)}-\frac{a^3(4Ab-5aB)\log(a+bx^2)}{2b^6}+\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{x^6(Ab-2aB)}{6b^3}+\frac{Bx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2*A*b - 3*a*B)*x^4)/(4*b^
4) + ((A*b - 2*a*B)*x^6)/(6*b^3) + (B*x^8)/(8*b^2) - (a^4*(A*b -
a*B))/(2*b^6*(a + b*x^2)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/
(2*b^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^8}{8b^2}-\frac{a^4(Ab-Ba)}{2b^6(a+bx^2)}-\frac{a^3(4Ab-5Ba)\log(a+bx^2)}{2b^6}-\frac{a(2Ab-3Ba)\int^{x^2}x\,dx}{2b^4}+\frac{x^6(Ab-2Ba)}{6b^3}+\frac{(3Ab-4Ba)\int^{x^2}a^2\,dx}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**8/(8*b**2) - a**4*(A*b - B*a)/(2*b**6*(a + b*x**2)) - a**3*(
4*A*b - 5*B*a)*log(a + b*x**2)/(2*b**6) - a*(2*A*b - 3*B*a)*Integ
ral(x, (x, x**2))/(2*b**4) + x**6*(A*b - 2*B*a)/(6*b**3) + (3*A*b
 - 4*B*a)*Integral(a**2, (x, x**2))/(2*b**5)
```

Mathematica [A] time = 0.135669, size = 113, normalized size = 0.9

$$\frac{\frac{12a^4(aB-Ab)}{a+bx^2}+12a^3(5aB-4Ab)\log(a+bx^2)-12a^2bx^2(4aB-3Ab)+4b^3x^6(Ab-2aB)+6ab^2x^4(3aB-2Ab)+3b^4Bx^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-12*a^2*b*(-3*A*b + 4*a*B)*x^2 + 6*a*b^2*(-2*A*b + 3*a*B)*x^4 +
4*b^3*(A*b - 2*a*B)*x^6 + 3*b^4*B*x^8 + (12*a^4*(-(A*b) + a*B))/(
a + b*x^2) + 12*a^3*(-4*A*b + 5*a*B)*Log[a + b*x^2])/(24*b^6)
```

Maple [A] time = 0.016, size = 146, normalized size = 1.2

$$\frac{Bx^8}{8\,b^2}+\frac{x^6A}{6\,b^2}-\frac{x^6Ba}{3\,b^3}-\frac{x^4Aa}{2\,b^3}+\frac{3\,x^4Ba^2}{4\,b^4}+\frac{3\,a^2Ax^2}{2\,b^4}-2\,\frac{Bx^2a^3}{b^5}-2\,\frac{a^3\ln\left(bx^2+a\right)A}{b^5}+\frac{5\,a^4\ln\left(bx^2+a\right)B}{2\,b^6}-\frac{a^4A}{2\,b^5\left(bx^2+a\right)}+\frac{a^5B}{2\,b^6\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^9*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/8*B*x^8/b^2+1/6/b^2*x^6*A-1/3/b^3*x^6*B*a-1/2/b^3*x^4*A*a+3/4/b
^4*x^4*B*a^2+3/2/b^4*x^2*A*a^2-2/b^5*x^2*B*a^3-2*a^3/b^5*ln(b*x^2
+a)*A+5/2*a^4/b^6*ln(b*x^2+a)*B-1/2*a^4/b^5/(b*x^2+a)*A+1/2*a^5/b
^6/(b*x^2+a)*B
```

Maxima [A] time = 1.35179, size = 177, normalized size = 1.4

$$\frac{Ba^5-Aa^4b}{2\left(b^7x^2+ab^6\right)}+\frac{3\,Bb^3x^8-4\left(2\,Bab^2-Ab^3\right)x^6+6\left(3\,Ba^2b-2\,Aab^2\right)x^4-12\left(4\,Ba^3-3\,Aa^2b\right)x^2}{24\,b^5}+\frac{\left(5\,Ba^4-4\,Aa^3b\right)\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^9/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^5 - A*a^4*b)/(b^7*x^2 + a*b^6) + 1/24*(3*B*b^3*x^8 - 4*(
2*B*a*b^2 - A*b^3)*x^6 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^4 - 12*(4*B*
a^3 - 3*A*a^2*b)*x^2)/b^5 + 1/2*(5*B*a^4 - 4*A*a^3*b)*log(b*x^2 +
 a)/b^6
```

Fricas [A] time = 0.233262, size = 232, normalized size = 1.84

$$\frac{3\,Bb^5x^{10}-\left(5\,Bab^4-4\,Ab^5\right)x^8+2\left(5\,Ba^2b^3-4\,Aab^4\right)x^6+12\,Ba^5-12\,Aa^4b-6\left(5\,Ba^3b^2-4\,Aa^2b^3\right)x^4-12\left(4\,Ba^4b-3\,A\right.}{24\left(b^7x^2+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^9/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^10 - (5*B*a*b^4 - 4*A*b^5)*x^8 + 2*(5*B*a^2*b^3 -
 4*A*a*b^4)*x^6 + 12*B*a^5 - 12*A*a^4*b - 6*(5*B*a^3*b^2 - 4*A*a^
2*b^3)*x^4 - 12*(4*B*a^4*b - 3*A*a^3*b^2)*x^2 + 12*(5*B*a^5 - 4*A
*a^4*b + (5*B*a^4*b - 4*A*a^3*b^2)*x^2)*log(b*x^2 + a))/(b^7*x^2
+ a*b^6)
```

Sympy [A] time = 3.82095, size = 126, normalized size = 1.

$$\frac{Bx^8}{8b^2}+\frac{a^3\left(-4Ab+5Ba\right)\log\left(a+bx^2\right)}{2b^6}+\frac{-Aa^4b+Ba^5}{2ab^6+2b^7x^2}-\frac{x^6\left(-Ab+2Ba\right)}{6b^3}+\frac{x^4\left(-2Aab+3Ba^2\right)}{4b^4}-\frac{x^2\left(-3Aa^2b+4Ba^3\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**8/(8*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x**2)/(2*b**6)
+ (-A*a**4*b + B*a**5)/(2*a*b**6 + 2*b**7*x**2) - x**6*(-A*b + 2*
B*a)/(6*b**3) + x**4*(-2*A*a*b + 3*B*a**2)/(4*b**4) - x**2*(-3*A*
a**2*b + 4*B*a**3)/(2*b**5)
```

GIAC/XCAS [A] time = 0.240052, size = 215, normalized size = 1.71

$$\frac{\left(5\,Ba^{4} - 4\,Aa^{3}b\right)\ln\left(\left|bx^{2} + a\right|\right)}{2\,b^{6}} - \frac{5\,Ba^{4}bx^{2} - 4\,Aa^{3}b^{2}x^{2} + 4\,Ba^{5} - 3\,Aa^{4}b}{2\,(bx^{2} + a)b^{6}} + \frac{3\,Bb^{6}x^{8} - 8\,Bab^{5}x^{6} + 4\,Ab^{6}x^{6} + 18\,Ba^{2}b^{4}x^{4} - 12\,Aab^{5}x^{4} - 48\,Ba^{3}b^{3}x^{2} + 36\,Aa^{2}b^{4}x^{2}}{24\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^9/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^4 - 4*A*a^3*b)*ln(abs(b*x^2 + a))/b^6 - 1/2*(5*B*a^4*b
*x^2 - 4*A*a^3*b^2*x^2 + 4*B*a^5 - 3*A*a^4*b)/((b*x^2 + a)*b^6) +
 1/24*(3*B*b^6*x^8 - 8*B*a*b^5*x^6 + 4*A*b^6*x^6 + 18*B*a^2*b^4*x
^4 - 12*A*a*b^5*x^4 - 48*B*a^3*b^3*x^2 + 36*A*a^2*b^4*x^2)/b^8
```

3.72 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=131

$$-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{a^3x(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{x^5(Ab-2aB)}{5b^3}+\frac{Bx^7}{7b^2}$$

```
[Out] (a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^3)/(3*b^4) + (
(A*b - 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + (a^3*(A*b - a*B)*x
)/(2*b^5*(a + b*x^2)) - (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] time = 0.287295, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{a^3x(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{x^5(Ab-2aB)}{5b^3}+\frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^3)/(3*b^4) + (
(A*b - 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + (a^3*(A*b - a*B)*x
)/(2*b^5*(a + b*x^2)) - (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^7}{7b^2}-\frac{a^{\frac{5}{2}}(7Ab-9Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{11}{2}}}+\frac{a^3x(Ab-Ba)}{2b^5(a+bx^2)}-\frac{ax^3(2Ab-3Ba)}{3b^4}+\frac{x^5(Ab-2Ba)}{5b^3}+\frac{(3Ab-4Ba)\int a^2\,dx}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) - a**(5/2)*(7*A*b - 9*B*a)*atan(sqrt(b)*x/sqrt(a)
)/(2*b**(11/2)) + a**3*x*(A*b - B*a)/(2*b**5*(a + b*x**2)) - a*x*
*3*(2*A*b - 3*B*a)/(3*b**4) + x**5*(A*b - 2*B*a)/(5*b**3) + (3*A*
b - 4*B*a)*Integral(a**2, x)/b**5
```

Mathematica [A] time = 0.179153, size = 134, normalized size = 1.02

$$\frac{a^{5/2}(9aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{a^2x(4aB-3Ab)}{b^5}+\frac{x\left(a^3Ab-a^4B\right)}{2b^5(a+bx^2)}+\frac{ax^3(3aB-2Ab)}{3b^4}+\frac{x^5(Ab-2aB)}{5b^3}+\frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a^2*(-3*A*b + 4*a*B)*x)/b^5) + (a*(-2*A*b + 3*a*B)*x^3)/(3*b^4
) + ((A*b - 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + ((a^3*A*b - a
```

```
^4*B)*x)/(2*b^5*(a + b*x^2)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [A] time = 0.013, size = 155, normalized size = 1.2

$$\frac{Bx^7}{7\,b^2}+\frac{Ax^5}{5\,b^2}-\frac{2\,Bx^5a}{5\,b^3}-\frac{2\,aAx^3}{3\,b^3}+\frac{Bx^3a^2}{b^4}+3\,\frac{a^2Ax}{b^4}-4\,\frac{Ba^3x}{b^5}+\frac{a^3xA}{2\,b^4\,(bx^2+a)}-\frac{a^4xB}{2\,b^5\,(bx^2+a)}-\frac{7\,Aa^3}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9\,Ba^4}{2\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*B*x^7/b^2+1/5/b^2*A*x^5-2/5/b^3*B*x^5*a-2/3/b^3*A*x^3*a+1/b^4
*B*x^3*a^2+3/b^4*A*a^2*x-4/b^5*B*a^3*x+1/2*a^3/b^4*x/(b*x^2+a)*A-
1/2*a^4/b^5*x/(b*x^2+a)*B-7/2*a^3/b^4/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))*A+9/2*a^4/b^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^8/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248265, size = 1, normalized size = 0.01

$$\left[\frac{60\,Bb^4x^9-12\left(9\,Bab^3-7\,Ab^4\right)x^7+28\left(9\,Ba^2b^2-7\,Aab^3\right)x^5-140\left(9\,Ba^3b-7\,Aa^2b^2\right)x^3-105\left(9\,Ba^4-7\,Aa^3b+\left(9\,Ba^3b\right.\right.}{420\,(b^6x^2+ab^5)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^8/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*b^4*x^9 - 12*(9*B*a*b^3 - 7*A*b^4)*x^7 + 28*(9*B*a^2
*b^2 - 7*A*a*b^3)*x^5 - 140*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 - 105*(
9*B*a^4 - 7*A*a^3*b + (9*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(-a/b)*l
og((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(9*B*a^4 - 7
*A*a^3*b)*x)/(b^6*x^2 + a*b^5), 1/210*(30*B*b^4*x^9 - 6*(9*B*a*b^
3 - 7*A*b^4)*x^7 + 14*(9*B*a^2*b^2 - 7*A*a*b^3)*x^5 - 70*(9*B*a^3
*b - 7*A*a^2*b^2)*x^3 + 105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*b - 7
*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 105*(9*B*a^4 - 7
*A*a^3*b)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [A] time = 3.52406, size = 233, normalized size = 1.78

$$\frac{Bx^7}{7b^2} - \frac{x\left(-Aa^3b + Ba^4\right)}{2ab^5 + 2b^6x^2} - \frac{\sqrt{-\frac{a^5}{b^{11}}}\left(-7Ab + 9Ba\right)\log\left(-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}(-7Ab+9Ba)}{-7Aa^2b+9Ba^3} + x\right)}{4} + \frac{\sqrt{-\frac{a^5}{b^{11}}}\left(-7Ab + 9Ba\right)\log\left(\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}(-7Ab+9Ba)}{-7Aa^2b+9Ba^3} + x\right)}{4} - \frac{x^5\left(-Ab + 2Ba\right)}{5b^3} + \frac{x^3\left(-2Aab + 3Ba^2\right)}{3b^4} - \frac{x\left(-3Aa^2b + 4Ba^3\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**8*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) - x*(-A*a**3*b + B*a**4)/(2*a*b**5 + 2*b**6*x**2)
 - sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)*log(-b**5*sqrt(-a**5/b**11)
*(-7*A*b + 9*B*a)/(-7*A*a**2*b + 9*B*a**3) + x)/4 + sqrt(-a**5/b*
*11)*(-7*A*b + 9*B*a)*log(b**5*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)
/(-7*A*a**2*b + 9*B*a**3) + x)/4 - x**5*(-A*b + 2*B*a)/(5*b**3) +
 x**3*(-2*A*a*b + 3*B*a**2)/(3*b**4) - x*(-3*A*a**2*b + 4*B*a**3)
/b**5
```

GIAC/XCAS [A] time = 0.231291, size = 188, normalized size = 1.44

$$\frac{\left(9\,Ba^4 - 7\,Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} - \frac{Ba^4x - Aa^3bx}{2\,(bx^2 + a)b^5} + \frac{15\,Bb^{12}x^7 - 42\,Bab^{11}x^5 + 21\,Ab^{12}x^5 + 105\,Ba^2b^{10}x^3 - 70\,Aab^{11}x^3 - 420\,Ba^3b^9x + 315\,Aa^2b^{10}x}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^8/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) -
 1/2*(B*a^4*x - A*a^3*b*x)/((b*x^2 + a)*b^5) + 1/105*(15*B*b^12*x
^7 - 42*B*a*b^11*x^5 + 21*A*b^12*x^5 + 105*B*a^2*b^10*x^3 - 70*A*
a*b^11*x^3 - 420*B*a^3*b^9*x + 315*A*a^2*b^10*x)/b^14
```

3.73 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=104

$$\frac{a^3(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2(3Ab-4aB)\log(a+bx^2)}{2b^5}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{x^4(Ab-2aB)}{4b^3}+\frac{Bx^6}{6b^2}$$

```
[Out] -(a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*a*B)*x^4)/(4*b^3) +
(B*x^6)/(6*b^2) + (a^3*(A*b - a*B))/(2*b^5*(a + b*x^2)) + (a^2*(3
*A*b - 4*a*B)*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.281264, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^3(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2(3Ab-4aB)\log(a+bx^2)}{2b^5}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{x^4(Ab-2aB)}{4b^3}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*a*B)*x^4)/(4*b^3) +
(B*x^6)/(6*b^2) + (a^3*(A*b - a*B))/(2*b^5*(a + b*x^2)) + (a^2*(3
*A*b - 4*a*B)*Log[a + b*x^2])/(2*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^6}{6b^2}+\frac{a^3(Ab-Ba)}{2b^5(a+bx^2)}+\frac{a^2(3Ab-4Ba)\log(a+bx^2)}{2b^5}+\frac{(Ab-2Ba)\int^{x^2}x\,dx}{2b^3}-\frac{(2Ab-3Ba)\int^{x^2}a\,dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) + a**3*(A*b - B*a)/(2*b**5*(a + b*x**2)) + a**2*(
3*A*b - 4*B*a)*log(a + b*x**2)/(2*b**5) + (A*b - 2*B*a)*Integral(
x, (x, x**2))/(2*b**3) - (2*A*b - 3*B*a)*Integral(a, (x, x**2))/(
2*b**4)
```

Mathematica [A] time = 0.121785, size = 93, normalized size = 0.89

$$\frac{\frac{6a^3(Ab-aB)}{a+bx^2}+6a^2(3Ab-4aB)\log(a+bx^2)+3b^2x^4(Ab-2aB)+6abx^2(3aB-2Ab)+2b^3Bx^6}{12b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (6*a*b*(-2*A*b + 3*a*B)*x^2 + 3*b^2*(A*b - 2*a*B)*x^4 + 2*b^3*B*x
^6 + (6*a^3*(A*b - a*B))/(a + b*x^2) + 6*a^2*(3*A*b - 4*a*B)*Log[
a + b*x^2])/(12*b^5)
```

Maple [A] time = 0.017, size = 122, normalized size = 1.2

$$\frac{Bx^6}{6\,b^2}+\frac{Ax^4}{4\,b^2}-\frac{Bx^4a}{2\,b^3}-\frac{aAx^2}{b^3}+\frac{3\,Bx^2a^2}{2\,b^4}+\frac{3\,a^2\ln\left(bx^2+a\right)A}{2\,b^4}-2\,\frac{a^3\ln\left(bx^2+a\right)B}{b^5}+\frac{a^3A}{2\,b^4\left(bx^2+a\right)}-\frac{Ba^4}{2\,b^5\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^7*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/6*B*x^6/b^2+1/4/b^2*A*x^4-1/2/b^3*B*x^4*a-1/b^3*A*x^2*a+3/2/b^4
*B*x^2*a^2+3/2*a^2/b^4*ln(b*x^2+a)*A-2*a^3/b^5*ln(b*x^2+a)*B+1/2*
a^3/b^4/(b*x^2+a)*A-1/2*a^4/b^5/(b*x^2+a)*B
```

Maxima [A] time = 1.34831, size = 144, normalized size = 1.38

$$-\frac{Ba^4-Aa^3b}{2\left(b^6x^2+ab^5\right)}+\frac{2\,Bb^2x^6-3\left(2\,Bab-Ab^2\right)x^4+6\left(3\,Ba^2-2\,Aab\right)x^2}{12\,b^4}-\frac{\left(4\,Ba^3-3\,Aa^2b\right)\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^7/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^4 - A*a^3*b)/(b^6*x^2 + a*b^5) + 1/12*(2*B*b^2*x^6 - 3*
(2*B*a*b - A*b^2)*x^4 + 6*(3*B*a^2 - 2*A*a*b)*x^2)/b^4 - 1/2*(4*B
*a^3 - 3*A*a^2*b)*log(b*x^2 + a)/b^5
```

Fricas [A] time = 0.231588, size = 200, normalized size = 1.92

$$\frac{2\,Bb^4x^8-\left(4\,Bab^3-3\,Ab^4\right)x^6-6\,Ba^4+6\,Aa^3b+3\left(4\,Ba^2b^2-3\,Aab^3\right)x^4+6\left(3\,Ba^3b-2\,Aa^2b^2\right)x^2-6\left(4\,Ba^4-3\,Aa^3b+(\cdots\right.}{12\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^7/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^4*x^8 - (4*B*a*b^3 - 3*A*b^4)*x^6 - 6*B*a^4 + 6*A*a^3
*b + 3*(4*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 6*(3*B*a^3*b - 2*A*a^2*b^2
)*x^2 - 6*(4*B*a^4 - 3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x^2)*l
og(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] time = 3.40407, size = 102, normalized size = 0.98

$$\frac{Bx^6}{6b^2}-\frac{a^2\left(-3Ab+4Ba\right)\log\left(a+bx^2\right)}{2b^5}-\frac{-Aa^3b+Ba^4}{2ab^5+2b^6x^2}-\frac{x^4\left(-Ab+2Ba\right)}{4b^3}+\frac{x^2\left(-2Aab+3Ba^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**7*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x**2)/(2*b**5)
- (-A*a**3*b + B*a**4)/(2*a*b**5 + 2*b**6*x**2) - x**4*(-A*b + 2*
B*a)/(4*b**3) + x**2*(-2*A*a*b + 3*B*a**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.224694, size = 182, normalized size = 1.75

$$-\frac{\left(4\,Ba^{3}-3\,Aa^{2}b\right)\ln\left(\left|bx^{2}+a\right|\right)}{2\,b^{5}}+\frac{2\,Bb^{4}x^{6}-6\,Bab^{3}x^{4}+3\,Ab^{4}x^{4}+18\,Ba^{2}b^{2}x^{2}-12\,Aab^{3}x^{2}}{12\,b^{6}}+\frac{4\,Ba^{3}bx^{2}-3\,Aa^{2}b^{2}x^{2}+3\,Ba^{4}-2\,Aa^{3}b}{2\,(bx^{2}+a)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*a^3 - 3*A*a^2*b)*ln(abs(b*x^2 + a))/b^5 + 1/12*(2*B*b^4
*x^6 - 6*B*a*b^3*x^4 + 3*A*b^4*x^4 + 18*B*a^2*b^2*x^2 - 12*A*a*b^
3*x^2)/b^6 + 1/2*(4*B*a^3*b*x^2 - 3*A*a^2*b^2*x^2 + 3*B*a^4 - 2*A
*a^3*b)/((b*x^2 + a)*b^5)
```

3.74 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=110

$$\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}-\frac{ax(2Ab-3aB)}{b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^5}{5b^2}$$

```
[Out] -((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x
^5)/(5*b^2) - (a^2*(A*b - a*B)*x)/(2*b^4*(a + b*x^2)) + (a^(3/2)*
(5*A*b - 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] time = 0.231389, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}-\frac{ax(2Ab-3aB)}{b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x
^5)/(5*b^2) - (a^2*(A*b - a*B)*x)/(2*b^4*(a + b*x^2)) + (a^(3/2)*
(5*A*b - 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi in Sympy [A] time = 59.4279, size = 102, normalized size = 0.93

$$\frac{Bx^5}{5b^2}+\frac{a^{\frac{3}{2}}(5Ab-7Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{9}{2}}}-\frac{a^2x(Ab-Ba)}{2b^4(a+bx^2)}-\frac{ax(2Ab-3Ba)}{b^4}+\frac{x^3(Ab-2Ba)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + a**(3/2)*(5*A*b - 7*B*a)*atan(sqrt(b)*x/sqrt(a)
)/(2*b**(9/2)) - a**2*x*(A*b - B*a)/(2*b**4*(a + b*x**2)) - a*x*(
2*A*b - 3*B*a)/b**4 + x**3*(A*b - 2*B*a)/(3*b**3)
```

Mathematica [A] time = 0.148634, size = 111, normalized size = 1.01

$$-\frac{a^{3/2}(7aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{x\left(a^2Ab-a^3B\right)}{2b^4(a+bx^2)}+\frac{ax(3aB-2Ab)}{b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*A*b + 3*a*B)*x)/b^4 + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5
)/(5*b^2) - ((a^2*A*b - a^3*B)*x)/(2*b^4*(a + b*x^2)) - (a^(3/2)*
(-5*A*b + 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.012, size = 132, normalized size = 1.2

$$\frac{Bx^5}{5\,b^2}+\frac{Ax^3}{3\,b^2}-\frac{2\,Bx^3a}{3\,b^3}-2\,\frac{aAx}{b^3}+3\,\frac{Bxa^2}{b^4}-\frac{a^2Ax}{2\,b^3\,(bx^2+a)}+\frac{a^3xB}{2\,b^4\,(bx^2+a)}+\frac{5\,Aa^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7\,Ba^3}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*B*x^5/b^2+1/3/b^2*A*x^3-2/3/b^3*B*x^3*a-2/b^3*A*x*a+3/b^4*B*x
*a^2-1/2*a^2/b^3*x/(b*x^2+a)*A+1/2*a^3/b^4*x/(b*x^2+a)*B+5/2*a^2/
b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A-7/2*a^3/b^4/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237218, size = 1, normalized size = 0.01

$$\left[\frac{12\,Bb^3x^7-4\left(7\,Bab^2-5\,Ab^3\right)x^5+20\left(7\,Ba^2b-5\,Aab^2\right)x^3-15\left(7\,Ba^3-5\,Aa^2b+\left(7\,Ba^2b-5\,Aab^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,b}{bx}\right.}{60\,(b^5x^2+ab^4)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*b^3*x^7 - 4*(7*B*a*b^2 - 5*A*b^3)*x^5 + 20*(7*B*a^2*b
 - 5*A*a*b^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*
b^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) + 30*(7*B*a^3 - 5*A*a^2*b)*x)/(b^5*x^2 + a*b^4), 1/30*(6*B*b^
3*x^7 - 2*(7*B*a*b^2 - 5*A*b^3)*x^5 + 10*(7*B*a^2*b - 5*A*a*b^2)*
x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt
(a/b)*arctan(x/sqrt(a/b)) + 15*(7*B*a^3 - 5*A*a^2*b)*x)/(b^5*x^2
+ a*b^4)]
```

Sympy [A] time = 3.32524, size = 206, normalized size = 1.87

$$\frac{Bx^5}{5b^2}+\frac{x\left(-Aa^2b+Ba^3\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{a^3}{b^9}}\left(-5Ab+7Ba\right)\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}(-5Ab+7Ba)}{-5Aab+7Ba^2}+x\right)}{4}-\frac{\sqrt{-\frac{a^3}{b^9}}\left(-5Ab+7Ba\right)\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}(-5Ab+7Ba)}{-5Aab+7Ba^2}+x\right)}{4}-\frac{x^3\left(-Ab+2Ba\right)}{3b^3}+\frac{x\left(-2Aab+3Ba^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x*(-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2)
 + sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)*log(-b**4*sqrt(-a**3/b**9)*(
-5*A*b + 7*B*a)/(-5*A*a*b + 7*B*a**2) + x)/4 - sqrt(-a**3/b**9)*(
-5*A*b + 7*B*a)*log(b**4*sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*A*
a*b + 7*B*a**2) + x)/4 - x**3*(-A*b + 2*B*a)/(3*b**3) + x*(-2*A*a
*b + 3*B*a**2)/b**4
```

GIAC/XCAS [A] time = 0.226564, size = 155, normalized size = 1.41

$$-\frac{\left(7\,Ba^3 - 5\,Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{Ba^3x - Aa^2bx}{2\,(bx^2 + a)b^4} + \frac{3\,Bb^8x^5 - 10\,Bab^7x^3 + 5\,Ab^8x^3 + 45\,Ba^2b^6x - 30\,Aab^7x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
+ 1/2*(B*a^3*x - A*a^2*b*x)/((b*x^2 + a)*b^4) + 1/15*(3*B*b^8*x^5
 - 10*B*a*b^7*x^3 + 5*A*b^8*x^3 + 45*B*a^2*b^6*x - 30*A*a*b^7*x)/
b^10
```

3.75 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2(Ab-aB)}{2b^4(a+bx^2)}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(Ab-2aB)}{2b^3}+\frac{Bx^4}{4b^2}$$

```
[Out] ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^4)/(4*b^2) - (a^2*(A*b - a*B))
/(2*b^4*(a + b*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.214515, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a^2(Ab-aB)}{2b^4(a+bx^2)}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(Ab-2aB)}{2b^3}+\frac{Bx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^4)/(4*b^2) - (a^2*(A*b - a*B))
/(2*b^4*(a + b*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\int^{x^2} x\,dx}{2b^2}-\frac{a^2(Ab-Ba)}{2b^4(a+bx^2)}-\frac{a(2Ab-3Ba)\log\left(a+bx^2\right)}{2b^4}+\left(\frac{Ab}{2}-Ba\right)\int^{x^2}\frac{1}{b^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*Integral(x, (x, x**2))/(2*b**2) - a**2*(A*b - B*a)/(2*b**4*(a +
 b*x**2)) - a*(2*A*b - 3*B*a)*log(a + b*x**2)/(2*b**4) + (A*b/2 -
 B*a)*Integral(b**(-3), (x, x**2))
```

Mathematica [A] time = 0.113879, size = 72, normalized size = 0.88

$$\frac{\frac{2a^2(aB-Ab)}{a+bx^2}+2bx^2(Ab-2aB)+2a(3aB-2Ab)\log\left(a+bx^2\right)+b^2Bx^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^2 + b^2*B*x^4 + (2*a^2*(-(A*b) + a*B))/(a +
b*x^2) + 2*a*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(4*b^4)
```

Maple [A] time = 0.015, size = 98, normalized size = 1.2

$$\frac{Bx^4}{4b^2}+\frac{Ax^2}{2b^2}-\frac{Bx^2a}{b^3}-\frac{a\ln\left(bx^2+a\right)A}{b^3}+\frac{3a^2\ln\left(bx^2+a\right)B}{2b^4}-\frac{a^2A}{2b^3(bx^2+a)}+\frac{Ba^3}{2b^4(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*B*x^4/b^2+1/2/b^2*A*x^2-1/b^3*B*x^2*a-a/b^3*ln(b*x^2+a)*A+3/2
*a^2/b^4*ln(b*x^2+a)*B-1/2*a^2/b^3/(b*x^2+a)*A+1/2*a^3/b^4/(b*x^2
+a)*B
```

Maxima [A] time = 1.351, size = 111, normalized size = 1.35

$$\frac{Ba^3 - Aa^2b}{2\,(b^5x^2 + ab^4)} + \frac{Bbx^4 - 2\,(2\,Ba - Ab)x^2}{4\,b^3} + \frac{\left(3\,Ba^2 - 2\,Aab\right)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^3 - A*a^2*b)/(b^5*x^2 + a*b^4) + 1/4*(B*b*x^4 - 2*(2*B*a
 - A*b)*x^2)/b^3 + 1/2*(3*B*a^2 - 2*A*a*b)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.233355, size = 163, normalized size = 1.99

$$\frac{Bb^3x^6 - \left(3\,Bab^2 - 2\,Ab^3\right)x^4 + 2\,Ba^3 - 2\,Aa^2b - 2\,\left(2\,Ba^2b - Aab^2\right)x^2 + 2\,\left(3\,Ba^3 - 2\,Aa^2b + \left(3\,Ba^2b - 2\,Aab^2\right)x^2\right)\log\,(bx^2}{4\,(b^5x^2 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^3*x^6 - (3*B*a*b^2 - 2*A*b^3)*x^4 + 2*B*a^3 - 2*A*a^2*b
- 2*(2*B*a^2*b - A*a*b^2)*x^2 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2
*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 3.20477, size = 78, normalized size = 0.95

$$\frac{Bx^4}{4b^2} + \frac{a\,(-2Ab + 3Ba)\log\left(a + bx^2\right)}{2b^4} + \frac{-Aa^2b + Ba^3}{2ab^4 + 2b^5x^2} - \frac{x^2\,(-Ab + 2Ba)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**4) + (
-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2) - x**2*(-A*b + 2*B*a
)/(2*b**3)
```

GIAC/XCAS [A] time = 0.240037, size = 143, normalized size = 1.74

$$\frac{\left(3\,Ba^2 - 2\,Aab\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{Bb^2x^4 - 4\,Babx^2 + 2\,Ab^2x^2}{4\,b^4} - \frac{3\,Ba^2bx^2 - 2\,Aab^2x^2 + 2\,Ba^3 - Aa^2b}{2\,(bx^2 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a^2 - 2*A*a*b)*ln(abs(b*x^2 + a))/b^4 + 1/4*(B*b^2*x^4 -
 4*B*a*b*x^2 + 2*A*b^2*x^2)/b^4 - 1/2*(3*B*a^2*b*x^2 - 2*A*a*b^2*
x^2 + 2*B*a^3 - A*a^2*b)/((b*x^2 + a)*b^4)
```

3.76 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(Ab-aB)}{2b^3(a+bx^2)}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^3}{3b^2}$$

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + (a*(A*b - a*B)*x)/(2*b^
3*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.174415, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\sqrt{a}(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(Ab-aB)}{2b^3(a+bx^2)}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + (a*(A*b - a*B)*x)/(2*b^
3*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*b^(7/2))
```

Rubi in Sympy [A] time = 43.9716, size = 80, normalized size = 0.92

$$\frac{Bx^3}{3b^2}-\frac{\sqrt{a}\,(3Ab-5Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{7}{2}}}+\frac{ax\,(Ab-Ba)}{2b^3\,(a+bx^2)}+\frac{x\,(Ab-2Ba)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) - sqrt(a)*(3*A*b - 5*B*a)*atan(sqrt(b)*x/sqrt(a))
/(2*b**(7/2)) + a*x*(A*b - B*a)/(2*b**3*(a + b*x**2)) + x*(A*b -
2*B*a)/b**3
```

Mathematica [A] time = 0.118742, size = 89, normalized size = 1.02

$$\frac{x\left(aAb-a^2B\right)}{2b^3(a+bx^2)}+\frac{\sqrt{a}(5aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + ((a*A*b - a^2*B)*x)/(2*
b^3*(a + b*x^2)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.012, size = 105, normalized size = 1.2

$$\frac{Bx^3}{3\,b^2}+\frac{Ax}{b^2}-2\,\frac{Bxa}{b^3}+\frac{aAx}{2\,b^2\,(bx^2+a)}-\frac{Bxa^2}{2\,b^3\,(bx^2+a)}-\frac{3\,Aa}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,a^2B}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*B*x^3/b^2+1/b^2*A*x-2/b^3*B*x*a+1/2*a/b^2*x/(b*x^2+a)*A-1/2*a
^2/b^3*x/(b*x^2+a)*B-3/2*a/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2)
)*A+5/2*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237634, size = 1, normalized size = 0.01

$$\left[\frac{4\,Bb^2x^5-4\,(5\,Bab-3\,Ab^2)\,x^3-3\,(5\,Ba^2-3\,Aab+(5\,Bab-3\,Ab^2)\,x^2)\,\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(5\,Ba^2-3\,Aab)\,x}{12\,(b^4x^2+ab^3)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*b^2*x^5 - 4*(5*B*a*b - 3*A*b^2)*x^3 - 3*(5*B*a^2 - 3*A
*a*b + (5*B*a*b - 3*A*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqr
t(-a/b) - a)/(b*x^2 + a)) - 6*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a
*b^3), 1/6*(2*B*b^2*x^5 - 2*(5*B*a*b - 3*A*b^2)*x^3 + 3*(5*B*a^2
- 3*A*a*b + (5*B*a*b - 3*A*b^2)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)
) - 3*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 3.07614, size = 128, normalized size = 1.47

$$\frac{Bx^3}{3b^2}-\frac{x\,(-Aab+Ba^2)}{2ab^3+2b^4x^2}-\frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab+5Ba)\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab+5Ba)\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}-\frac{x\,(-Ab+2Ba)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) - x*(-A*a*b + B*a**2)/(2*a*b**3 + 2*b**4*x**2) -
sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(-b**3*sqrt(-a/b**7) + x)/4 + s
qrt(-a/b**7)*(-3*A*b + 5*B*a)*log(b**3*sqrt(-a/b**7) + x)/4 - x*(
-A*b + 2*B*a)/b**3
```

GIAC/XCAS [A] time = 0.223948, size = 119, normalized size = 1.37

$$\frac{\left(5 Ba^2 - 3 Aab\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} b^3} - \frac{Ba^2 x - Aabx}{2 \left(bx^2 + a\right) b^3} + \frac{Bb^4 x^3 - 6 Bab^3 x + 3 Ab^4 x}{3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1
/2*(B*a^2*x - A*a*b*x)/((b*x^2 + a)*b^3) + 1/3*(B*b^4*x^3 - 6*B*a
*b^3*x + 3*A*b^4*x)/b^6
```

3.77 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=60

$$\frac{a(Ab-aB)}{2b^3(a+bx^2)}+\frac{(Ab-2aB)\log\left(a+bx^2\right)}{2b^3}+\frac{Bx^2}{2b^2}$$

```
[Out] (B*x^2)/(2*b^2) + (a*(A*b - a*B))/(2*b^3*(a + b*x^2)) + ((A*b - 2
*a*B)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.15111, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a(Ab-aB)}{2b^3(a+bx^2)}+\frac{(Ab-2aB)\log\left(a+bx^2\right)}{2b^3}+\frac{Bx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x^2)/(2*b^2) + (a*(A*b - a*B))/(2*b^3*(a + b*x^2)) + ((A*b - 2
*a*B)*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\,(Ab-Ba)}{2b^3\,(a+bx^2)}+\frac{\int^{x^2} B\,dx}{2b^2}+\frac{(Ab-2Ba)\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] a*(A*b - B*a)/(2*b**3*(a + b*x**2)) + Integral(B, (x, x**2))/(2*b
**2) + (A*b - 2*B*a)*log(a + b*x**2)/(2*b**3)
```

Mathematica [A] time = 0.057108, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab-aB)}{a+bx^2}+(Ab-2aB)\log\left(a+bx^2\right)+bBx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*B*x^2 + (a*(A*b - a*B))/(a + b*x^2) + (A*b - 2*a*B)*Log[a + b*
x^2])/(2*b^3)
```

Maple [A] time = 0.015, size = 74, normalized size = 1.2

$$\frac{Bx^2}{2\,b^2}+\frac{\ln\left(bx^2+a\right)A}{2\,b^2}-\frac{\ln\left(bx^2+a\right)Ba}{b^3}+\frac{aA}{2\,b^2\,(bx^2+a)}-\frac{a^2B}{2\,b^3\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/b^2+1/2/b^2*ln(b*x^2+a)*A-1/b^3*ln(b*x^2+a)*B*a+1/2/b^2
*a/(b*x^2+a)*A-1/2/b^3*a^2/(b*x^2+a)*B
```

Maxima [A] time = 1.35179, size = 81, normalized size = 1.35

$$\frac{Bx^2}{2\,b^2} - \frac{Ba^2 - Aab}{2\,(b^4x^2 + ab^3)} - \frac{(2\,Ba - Ab)\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b^2 - 1/2*(B*a^2 - A*a*b)/(b^4*x^2 + a*b^3) - 1/2*(2*B*
a - A*b)*log(b*x^2 + a)/b^3
```

Fricas [A] time = 0.227237, size = 109, normalized size = 1.82

$$\frac{Bb^2x^4 + Babx^2 - Ba^2 + Aab - \left(2\,Ba^2 - Aab + \left(2\,Bab - Ab^2\right)x^2\right)\log\left(bx^2 + a\right)}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b^2*x^4 + B*a*b*x^2 - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (
2*B*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 2.77839, size = 56, normalized size = 0.93

$$\frac{Bx^2}{2b^2} - \frac{-Aab + Ba^2}{2ab^3 + 2b^4x^2} - \frac{(-Ab + 2Ba)\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) - (-A*a*b + B*a**2)/(2*a*b**3 + 2*b**4*x**2) - (-
A*b + 2*B*a)*log(a + b*x**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.223952, size = 123, normalized size = 2.05

$$\frac{\frac{(bx^2+a)B}{b^2} + \frac{(2\,Ba-Ab)\ln\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^2+a} - \frac{Aab^2}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*B/b^2 + (2*B*a - A*b)*ln(abs(b*x^2 + a)/((b*x^2
+ a)^2*abs(b)))/b^2 - (B*a^2*b/(b*x^2 + a) - A*a*b^2/(b*x^2 + a))
/b^3)/b
```

3.78 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=67

$$\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{Bx}{b^2}$$

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) + ((A*b - 3*a*B)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.132068, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) + ((A*b - 3*a*B)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rubi in Sympy [A] time = 23.3748, size = 60, normalized size = 0.9

$$\frac{Bx}{b^2}-\frac{x\,(Ab-Ba)}{2b^2\left(a+bx^2\right)}+\frac{(Ab-3Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x/b**2 - x*(A*b - B*a)/(2*b**2*(a + b*x**2)) + (A*b - 3*B*a)*at
an(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b**(5/2))
```

Mathematica [A] time = 0.114519, size = 68, normalized size = 1.01

$$-\frac{(3aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) - ((-(A*b) + 3*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.012, size = 82, normalized size = 1.2

$$\frac{Bx}{b^2}-\frac{xA}{2\,b\,(bx^2+a)}+\frac{Bxa}{2\,b^2\,(bx^2+a)}+\frac{A}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,Ba}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] B*x/b^2-1/2/b*x/(b*x^2+a)*A+1/2/b^2*x/(b*x^2+a)*B*a+1/2/b/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*A-3/2/b^2/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*B*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231736, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,Ba^2 - Aab + \left(3\,Bab - Ab^2\right)x^2\right)\log\left(\frac{2\,abx + \left(bx^2 - a\right)\sqrt{-ab}}{bx^2 + a}\right) - 2\left(2\,Bbx^3 + \left(3\,Ba - Ab\right)x\right)\sqrt{-ab}}{4\left(b^3x^2 + ab^2\right)\sqrt{-ab}},\right.$$
$$\left.-\frac{\left(3\,Ba^2 - Aab + \left(3\,Bab - Ab^2\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right) - \left(2\,Bbx^3 + \left(3\,Ba - Ab\right)x\right)\sqrt{ab}}{2\left(b^3x^2 + ab^2\right)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x^2)*log((2*a*b*x + (
b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(2*B*b*x^3 + (3*B*a - A*b
)*x)*sqrt(-a*b))/((b^3*x^2 + a*b^2)*sqrt(-a*b)), -1/2*((3*B*a^2 -
 A*a*b + (3*B*a*b - A*b^2)*x^2)*arctan(sqrt(a*b)*x/a) - (2*B*b*x^
3 + (3*B*a - A*b)*x)*sqrt(a*b))/((b^3*x^2 + a*b^2)*sqrt(a*b))]
```

Sympy [A] time = 2.63281, size = 114, normalized size = 1.7

$$\frac{Bx}{b^2} + \frac{x\left(-Ab + Ba\right)}{2ab^2 + 2b^3x^2} + \frac{\sqrt{-\frac{1}{ab^5}}\left(-Ab + 3Ba\right)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^5}}\left(-Ab + 3Ba\right)\log\left(ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b
**5))*(-A*b + 3*B*a)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(
-1/(a*b**5))*(-A*b + 3*B*a)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/4
```

GIAC/XCAS [A] time = 0.222282, size = 80, normalized size = 1.19

$$\frac{Bx}{b^2} - \frac{\left(3\,Ba - Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Bax - Abx}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
 + 1/2*(B*a*x - A*b*x)/((b*x^2 + a)*b^2)
```

3.79 $\int \frac{x(A+Bx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=41

$$\frac{B\log(a+bx^2)}{2b^2} - \frac{Ab-aB}{2b^2(a+bx^2)}$$

```
[Out] -(A*b - a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0869263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{B\log(a+bx^2)}{2b^2} - \frac{Ab-aB}{2b^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2)^2,x]

[Out] -(A*b - a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [A] time = 13.3832, size = 32, normalized size = 0.78

$$\frac{B\log(a+bx^2)}{2b^2} - \frac{Ab-Ba}{2b^2(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(b*x**2+a)**2,x)

[Out] B*log(a + b*x**2)/(2*b**2) - (A*b - B*a)/(2*b**2*(a + b*x**2))
```

Mathematica [A] time = 0.0201065, size = 41, normalized size = 1.

$$\frac{aB-Ab}{2b^2(a+bx^2)} + \frac{B\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2)^2,x]

[Out] (-(A*b) + a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.013, size = 47, normalized size = 1.2

$$\frac{B\ln(bx^2+a)}{2b^2} - \frac{A}{(2bx^2+2a)b} + \frac{Ba}{(2bx^2+2a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*B*ln(b*x^2+a)/b^2-1/2/(b*x^2+a)/b*A+1/2/(b*x^2+a)/b^2*B*a
```

Maxima [A] time = 1.3521, size = 54, normalized size = 1.32

$$\frac{Ba-Ab}{2\left(b^3x^2+ab^2\right)}+\frac{B\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)/(b^3*x^2 + a*b^2) + 1/2*B*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.214756, size = 59, normalized size = 1.44

$$\frac{Ba-Ab+\left(Bbx^2+Ba\right)\log\left(bx^2+a\right)}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*a - A*b + (B*b*x^2 + B*a)*log(b*x^2 + a))/(b^3*x^2 + a*b^2
)
```

Sympy [A] time = 2.05519, size = 36, normalized size = 0.88

$$\frac{B\log\left(a+bx^2\right)}{2b^2}+\frac{-Ab+Ba}{2ab^2+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**2) + (-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2
)
```

GIAC/XCAS [A] time = 0.231593, size = 88, normalized size = 2.15

$$-\frac{B\left(\frac{\ln\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2\,b}-\frac{A}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*(ln(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 +
 a)*b))/b - 1/2*A/((b*x^2 + a)*b)
```

3.80 $\int \frac{A+Bx^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=63

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab(a+bx^2)}$$

```
[Out] ((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.061669, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi in Sympy [A] time = 9.44616, size = 51, normalized size = 0.81

$$\frac{x(Ab-Ba)}{2ab(a+bx^2)}+\frac{(Ab+Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(2*a*b*(a + b*x**2)) + (A*b + B*a)*atan(sqrt(b)*x/s
qrt(a))/(2*a**(3/2)*b**(3/2))
```

Mathematica [A] time = 0.0812478, size = 63, normalized size = 1.

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(aB-Ab)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -((-(A*b) + a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.01, size = 68, normalized size = 1.1

$$\frac{(Ab-Ba)x}{2ab(bx^2+a)}+\frac{A}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*A+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23813, size = 1, normalized size = 0.02

$$\left[-\frac{2\,(Ba-Ab)\sqrt{-ab}x-\left(Ba^2+Aab+\left(Bab+Ab^2\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)}{4\,(ab^2x^2+a^2b)\sqrt{-ab}},\right.$$
$$\left.-\frac{(Ba-Ab)\sqrt{ab}x-\left(Ba^2+Aab+\left(Bab+Ab^2\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(ab^2x^2+a^2b)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(B*a - A*b)*sqrt(-a*b)*x - (B*a^2 + A*a*b + (B*a*b + A*b
^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)))/((a
*b^2*x^2 + a^2*b)*sqrt(-a*b)), -1/2*((B*a - A*b)*sqrt(a*b)*x - (B
*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*arctan(sqrt(a*b)*x/a))/((a*b^
2*x^2 + a^2*b)*sqrt(a*b))]
```

Sympy [A] time = 2.11941, size = 112, normalized size = 1.78

$$-\frac{x\,(-Ab+Ba)}{2a^2b+2ab^2x^2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\,(Ab+Ba)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^3}}\,(Ab+Ba)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] -x*(-A*b + B*a)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))
*(A*b + B*a)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a
**3*b**3))*(A*b + B*a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

GIAC/XCAS [A] time = 0.225755, size = 77, normalized size = 1.22

$$\frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}-\frac{Bax-Abx}{2\,(bx^2+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(B*a*
x - A*b*x)/((b*x^2 + a)*a*b)
```

3.81 $\int \frac{A+Bx^2}{x(a+bx^2)^2} dx$

Optimal. Leaf size=51

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

```
[Out] (A*b - a*B)/(2*a*b*(a + b*x^2)) + (A*Log[x])/a^2 - (A*Log[a + b*x
^2])/(2*a^2)
```

Rubi [A] time = 0.113601, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*B)/(2*a*b*(a + b*x^2)) + (A*Log[x])/a^2 - (A*Log[a + b*x
^2])/(2*a^2)
```

Rubi in Sympy [A] time = 16.503, size = 44, normalized size = 0.86

$$\frac{A\log\left(x^2\right)}{2a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{Ab-Ba}{2ab\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x**2)/(2*a**2) - A*log(a + b*x**2)/(2*a**2) + (A*b - B*a)/(
2*a*b*(a + b*x**2))
```

Mathematica [A] time = 0.0500623, size = 46, normalized size = 0.9

$$\frac{\frac{a(Ab-aB)}{b(a+bx^2)}-A\log\left(a+bx^2\right)+2A\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^2)) + 2*A*Log[x] - A*Log[a + b*x^2])
/(2*a^2)
```

Maple [A] time = 0.017, size = 53, normalized size = 1.

$$\frac{A\ln(x)}{a^2}-\frac{A\ln\left(bx^2+a\right)}{2\,a^2}+\frac{A}{2\,a\left(bx^2+a\right)}-\frac{B}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(b*x^2+a)^2,x)
```

```
[Out] A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2+1/2/a/(b*x^2+a)*A-1/2/b/(b*x^2+
a)*B
```

Maxima [A] time = 1.34773, size = 69, normalized size = 1.35

$$-\frac{Ba - Ab}{2\,(ab^2x^2 + a^2b)} - \frac{A\log\left(bx^2 + a\right)}{2\,a^2} + \frac{A\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)/a^2 +
 1/2*A*log(x^2)/a^2
```

Fricas [A] time = 0.233235, size = 95, normalized size = 1.86

$$-\frac{Ba^2 - Aab + \left(Ab^2x^2 + Aab\right)\log\left(bx^2 + a\right) - 2\left(Ab^2x^2 + Aab\right)\log\left(x\right)}{2\,(a^2b^2x^2 + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*(B*a^2 - A*a*b + (A*b^2*x^2 + A*a*b)*log(b*x^2 + a) - 2*(A*b
^2*x^2 + A*a*b)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

Sympy [A] time = 2.19838, size = 46, normalized size = 0.9

$$\frac{A\log\left(x\right)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^2\right)}{2a^2} - \frac{-Ab + Ba}{2a^2b + 2ab^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**2)/(2*a**2) - (-A*b + B*a)/(2*a**2
*b + 2*a*b**2*x**2)
```

GIAC/XCAS [A] time = 0.22716, size = 85, normalized size = 1.67

$$\frac{A\ln\left(x^2\right)}{2\,a^2} - \frac{A\ln\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{Ab^2x^2 - Ba^2 + 2\,Aab}{2\,(bx^2 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*A*ln(x^2)/a^2 - 1/2*A*ln(abs(b*x^2 + a))/a^2 + 1/2*(A*b^2*x^2
 - B*a^2 + 2*A*a*b)/((b*x^2 + a)*a^2*b)
```

3.82 $\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(Ab-aB)}{2a^2\left(a+bx^2\right)}-\frac{A}{a^2x}$$

```
[Out] -(A/(a^2*x)) - ((A*b - a*B)*x)/(2*a^2*(a + b*x^2)) - ((3*A*b - a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.148648, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(Ab-aB)}{2a^2\left(a+bx^2\right)}-\frac{A}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) - ((A*b - a*B)*x)/(2*a^2*(a + b*x^2)) - ((3*A*b - a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 19.3615, size = 61, normalized size = 0.86

$$-\frac{A}{a^2x}-\frac{x\left(Ab-Ba\right)}{2a^2\left(a+bx^2\right)}-\frac{\left(3Ab-Ba\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x**2+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -A/(a**2*x) - x*(A*b - B*a)/(2*a**2*(a + b*x**2)) - (3*A*b - B*a)
*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/2)*sqrt(b))
```

Mathematica [A] time = 0.0556774, size = 70, normalized size = 0.99

$$\frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{x(aB-Ab)}{2a^2\left(a+bx^2\right)}-\frac{A}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + ((-(A*b) + a*B)*x)/(2*a^2*(a + b*x^2)) + ((-3*A*b
+ a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.014, size = 85, normalized size = 1.2

$$-\frac{A}{a^2x}-\frac{Axb}{2\,a^2\left(bx^2+a\right)}+\frac{xB}{2\,a\left(bx^2+a\right)}-\frac{3\,Ab}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -A/a^2/x-1/2/a^2*x/(b*x^2+a)*A*b+1/2/a*x/(b*x^2+a)*B-3/2/a^2/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*A*b+1/2/a/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243319, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(Bab-3\,Ab^2\right)x^3+\left(Ba^2-3\,Aab\right)x\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\,\left((Ba-3\,Ab)x^2-2\,Aa\right)\sqrt{-ab}}{4\,(a^2bx^3+a^3x)\sqrt{-ab}},\ \frac{\left(\left(Bab-3\,Ab^2\right)x^3+\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*log(-(2*a*b*
x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*((B*a - 3*A*b)*x^2 -
 2*A*a)*sqrt(-a*b))/((a^2*b*x^3 + a^3*x)*sqrt(-a*b)), 1/2*(((B*a*
b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*arctan(sqrt(a*b)*x/a) + (
(B*a - 3*A*b)*x^2 - 2*A*a)*sqrt(a*b))/((a^2*b*x^3 + a^3*x)*sqrt(a
*b))]
```

Sympy [A] time = 2.50535, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(-3Ab+Ba)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(-3Ab+Ba)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2Aa+x^2\,(-3Ab+Ba)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(-3*A*b + B*a)*log(-a**3*sqrt(-1/(a**5*b)) + x
)/4 + sqrt(-1/(a**5*b))*(-3*A*b + B*a)*log(a**3*sqrt(-1/(a**5*b))
 + x)/4 + (-2*A*a + x**2*(-3*A*b + B*a))/(2*a**3*x + 2*a**2*b*x**
3)
```

GIAC/XCAS [A] time = 0.222502, size = 84, normalized size = 1.18

$$\frac{(Ba-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}+\frac{Bax^2-3\,Abx^2-2\,Aa}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(B*
a*x^2 - 3*A*b*x^2 - 2*A*a)/((b*x^3 + a*x)*a^2)
```

3.83 $\int \frac{A+Bx^2}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=76

$$\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{A}{2a^2x^2}$$

```
[Out] -A/(2*a^2*x^2) - (A*b - a*B)/(2*a^2*(a + b*x^2)) - ((2*A*b - a*B)
*Log[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.177076, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{A}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -A/(2*a^2*x^2) - (A*b - a*B)/(2*a^2*(a + b*x^2)) - ((2*A*b - a*B)
*Log[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 21.2272, size = 68, normalized size = 0.89

$$-\frac{A}{2a^2x^2}-\frac{Ab-Ba}{2a^2\left(a+bx^2\right)}-\frac{(2Ab-Ba)\log\left(x^2\right)}{2a^3}+\frac{(2Ab-Ba)\log\left(a+bx^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x**2+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] -A/(2*a**2*x**2) - (A*b - B*a)/(2*a**2*(a + b*x**2)) - (2*A*b - B
*a)*log(x**2)/(2*a**3) + (2*A*b - B*a)*log(a + b*x**2)/(2*a**3)
```

Mathematica [A] time = 0.0772237, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB-Ab)}{a+bx^2}+(2Ab-aB)\log\left(a+bx^2\right)+2\log(x)(aB-2Ab)-\frac{aA}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) + (a*(-(A*b) + a*B))/(a + b*x^2) + 2*(-2*A*b + a*B)
*Log[x] + (2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.021, size = 86, normalized size = 1.1

$$-\frac{A}{2\,a^2x^2}-2\,\frac{A\ln(x)\,b}{a^3}+\frac{\ln(x)\,B}{a^2}+\frac{b\ln\left(bx^2+a\right)A}{a^3}-\frac{\ln\left(bx^2+a\right)B}{2\,a^2}-\frac{Ab}{2\,a^2\left(bx^2+a\right)}+\frac{B}{2\,a\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*A/a^2/x^2-2/a^3*ln(x)*A*b+1/a^2*ln(x)*B+1/a^3*b*ln(b*x^2+a)*
A-1/2/a^2*ln(b*x^2+a)*B-1/2/a^2*b/(b*x^2+a)*A+1/2/a/(b*x^2+a)*B
```

Maxima [A] time = 1.33573, size = 103, normalized size = 1.36

$$\frac{(Ba-2\,Ab)x^2-Aa}{2\,(a^2bx^4+a^3x^2)}-\frac{(Ba-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^3*x^2) - 1/2*(B*a -
2*A*b)*log(b*x^2 + a)/a^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

Fricas [A] time = 0.233238, size = 158, normalized size = 2.08

$$-\frac{Aa^2-\left(Ba^2-2\,Aab\right)x^2+\left(\left(Bab-2\,Ab^2\right)x^4+\left(Ba^2-2\,Aab\right)x^2\right)\log\left(bx^2+a\right)-2\left(\left(Bab-2\,Ab^2\right)x^4+\left(Ba^2-2\,Aab\right)x^2\right)}{2\,(a^3bx^4+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(A*a^2 - (B*a^2 - 2*A*a*b)*x^2 + ((B*a*b - 2*A*b^2)*x^4 + (B
*a^2 - 2*A*a*b)*x^2)*log(b*x^2 + a) - 2*((B*a*b - 2*A*b^2)*x^4 +
(B*a^2 - 2*A*a*b)*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 3.86799, size = 70, normalized size = 0.92

$$\frac{-Aa+x^2\left(-2Ab+Ba\right)}{2a^3x^2+2a^2bx^4}+\frac{(-2Ab+Ba)\log\left(x\right)}{a^3}-\frac{(-2Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-A*a + x**2*(-2*A*b + B*a))/(2*a**3*x**2 + 2*a**2*b*x**4) + (-2*
A*b + B*a)*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

GIAC/XCAS [A] time = 0.23442, size = 111, normalized size = 1.46

$$\frac{(Ba-2\,Ab)\ln\left(x^2\right)}{2\,a^3}+\frac{Bax^2-2\,Abx^2-Aa}{2\,(bx^4+ax^2)a^2}-\frac{\left(Bab-2\,Ab^2\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 2*A*b)*ln(x^2)/a^3 + 1/2*(B*a*x^2 - 2*A*b*x^2 - A*a)/(
(b*x^4 + a*x^2)*a^2) - 1/2*(B*a*b - 2*A*b^2)*ln(abs(b*x^2 + a))/(
a^3*b)
```

3.84 $\int \frac{A+Bx^2}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{b}(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}-\frac{A}{3a^2x^3}$$

```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B)*x)/(2*a^3
*(a + b*x^2)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2))
```

Rubi [A] time = 0.262274, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\sqrt{b}(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B)*x)/(2*a^3
*(a + b*x^2)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2))
```

Rubi in Sympy [A] time = 40.404, size = 80, normalized size = 0.89

$$-\frac{A}{3a^2x^3}+\frac{bx\left(Ab-Ba\right)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-Ba}{a^3x}+\frac{\sqrt{b}\left(5Ab-3Ba\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -A/(3*a**2*x**3) + b*x*(A*b - B*a)/(2*a**3*(a + b*x**2)) + (2*A*b
 - B*a)/(a**3*x) + sqrt(b)*(5*A*b - 3*B*a)*atan(sqrt(b)*x/sqrt(a)
)/(2*a**(7/2))
```

Mathematica [A] time = 0.132887, size = 90, normalized size = 1.

$$-\frac{\sqrt{b}(3aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{bx(aB-Ab)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) - (b*(-(A*b) + a*B)*x)/(2*
a^3*(a + b*x^2)) - (Sqrt[b]*(-5*A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.017, size = 110, normalized size = 1.2

$$-\frac{A}{3\,a^{2}x^{3}}+2\,\frac{Ab}{a^{3}x}-\frac{B}{a^{2}x}+\frac{Axb^{2}}{2\,a^{3}\,(bx^{2}+a)}-\frac{bBx}{2\,a^{2}\,(bx^{2}+a)}+\frac{5\,b^{2}A}{2\,a^{3}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,Bb}{2\,a^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*A/a^2/x^3+2/a^3/x*A*b-1/a^2/x*B+1/2/a^3*b^2*x/(b*x^2+a)*A-1/
2/a^2*b*x/(b*x^2+a)*B+5/2/a^3*b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))*A-3/2/a^2*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222186, size = 1, normalized size = 0.01

$$\left[-\frac{6\left(3\,Bab-5\,Ab^{2}\right)x^{4}+4\,Aa^{2}+4\left(3\,Ba^{2}-5\,Aab\right)x^{2}+3\left(\left(3\,Bab-5\,Ab^{2}\right)x^{5}+\left(3\,Ba^{2}-5\,Aab\right)x^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-}{bx^{2}+a}\right)}{12\,(a^{3}bx^{5}+a^{4}x^{3})},\ -\frac{3\left(3\,Bab-5\,Ab^{2}\right)x^{4}+2\,Aa^{2}+2\left(3\,Ba^{2}-5\,Aab\right)x^{2}+3\left(\left(3\,Bab-5\,Ab^{2}\right)x^{5}+\left(3\,Ba^{2}-5\,Aab\right)x^{3}\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{6\,(a^{3}bx^{5}+a^{4}x^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*a*b - 5*A*b^2)*x^4 + 4*A*a^2 + 4*(3*B*a^2 - 5*A*a*
b)*x^2 + 3*((3*B*a*b - 5*A*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*
x^5 + a^4*x^3), -1/6*(3*(3*B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*
B*a^2 - 5*A*a*b)*x^2 + 3*((3*B*a*b - 5*A*b^2)*x^5 + (3*B*a^2 - 5*
A*a*b)*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^3*b*x^5 + a^4
*x^3)]
```

Sympy [A] time = 3.17059, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\left(-5Ab+3Ba\right)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^7}}\left(-5Ab+3Ba\right)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}{4}-\frac{2Aa^2+x^4\left(-15Ab^2+9Bab\right)+x^2\left(-10Aab+6Ba^2\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(-a**4*sqrt(-b/a**7)*(-5*A*b +
3*B*a)/(-5*A*b**2 + 3*B*a*b) + x)/4 - sqrt(-b/a**7)*(-5*A*b + 3*B
*a)*log(a**4*sqrt(-b/a**7)*(-5*A*b + 3*B*a)/(-5*A*b**2 + 3*B*a*b)
 + x)/4 - (2*A*a**2 + x**4*(-15*A*b**2 + 9*B*a*b) + x**2*(-10*A*a
*b + 6*B*a**2))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

GIAC/XCAS [A] time = 0.230397, size = 115, normalized size = 1.28

$$-\frac{\left(3\,Bab-5\,Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}-\frac{Babx-Ab^2x}{2\,(bx^2+a)a^3}-\frac{3\,Bax^2-6\,Abx^2+Aa}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] -1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) -
1/2*(B*a*b*x - A*b^2*x)/((b*x^2 + a)*a^3) - 1/3*(3*B*a*x^2 - 6*A*
b*x^2 + A*a)/(a^3*x^3)
```

3.85 $\int \frac{A+Bx^2}{x^5(a+bx^2)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{A}{4a^2x^4}$$

```
[Out] -A/(4*a^2*x^4) + (2*A*b - a*B)/(2*a^3*x^2) + (b*(A*b - a*B))/(2*a
^3*(a + b*x^2)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*
a*B)*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.228237, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{A}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -A/(4*a^2*x^4) + (2*A*b - a*B)/(2*a^3*x^2) + (b*(A*b - a*B))/(2*a
^3*(a + b*x^2)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*
a*B)*Log[a + b*x^2])/(2*a^4)
```

Rubi in Sympy [A] time = 26.7476, size = 94, normalized size = 0.97

$$-\frac{A}{4a^2x^4}+\frac{b\left(Ab-Ba\right)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-Ba}{2a^3x^2}+\frac{b\left(3Ab-2Ba\right)\log\left(x^2\right)}{2a^4}-\frac{b\left(3Ab-2Ba\right)\log\left(a+bx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(b*x**2+a)**2,x)
```

```
[Out] -A/(4*a**2*x**4) + b*(A*b - B*a)/(2*a**3*(a + b*x**2)) + (2*A*b -
 B*a)/(2*a**3*x**2) + b*(3*A*b - 2*B*a)*log(x**2)/(2*a**4) - b*(3
*A*b - 2*B*a)*log(a + b*x**2)/(2*a**4)
```

Mathematica [A] time = 0.160471, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^4}+\frac{2ab(aB-Ab)}{a+bx^2}+\frac{2a(aB-2Ab)}{x^2}+2b(3Ab-2aB)\log\left(a+bx^2\right)-4b\log(x)(3Ab-2aB)}{4a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -((a^2*A)/x^4 + (2*a*(-2*A*b + a*B))/x^2 + (2*a*b*(-(A*b) + a*B))
/(a + b*x^2) - 4*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*L
og[a + b*x^2])/(4*a^4)
```

Maple [A] time = 0.022, size = 114, normalized size = 1.2

$$-\frac{A}{4\,a^2x^4}+\frac{Ab}{a^3x^2}-\frac{B}{2\,a^2x^2}+3\,\frac{A\ln(x)\,b^2}{a^4}-2\,\frac{bB\ln(x)}{a^3}-\frac{3\,b^2\ln\left(bx^2+a\right)A}{2\,a^4}+\frac{b\ln\left(bx^2+a\right)B}{a^3}+\frac{b^2A}{2\,a^3\left(bx^2+a\right)}-\frac{Bb}{2\,a^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(b*x^2+a)^2,x)
```

```
[Out] -1/4*A/a^2/x^4+1/a^3/x^2*A*b-1/2/a^2/x^2*B+3*b^2/a^4*ln(x)*A-2*b/
a^3*ln(x)*B-3/2/a^4*b^2*ln(b*x^2+a)*A+1/a^3*b*ln(b*x^2+a)*B+1/2/a
^3*b^2/(b*x^2+a)*A-1/2/a^2*b/(b*x^2+a)*B
```

Maxima [A] time = 1.36033, size = 143, normalized size = 1.47

$$-\frac{2\left(2\,Bab-3\,Ab^2\right)x^4+Aa^2+\left(2\,Ba^2-3\,Aab\right)x^2}{4\left(a^3bx^6+a^4x^4\right)}+\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(bx^2+a\right)}{2\,a^4}-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^5),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*a*b - 3*A*b^2)*x^4 + A*a^2 + (2*B*a^2 - 3*A*a*b)*x^2
)/(a^3*b*x^6 + a^4*x^4) + 1/2*(2*B*a*b - 3*A*b^2)*log(b*x^2 + a)/
a^4 - 1/2*(2*B*a*b - 3*A*b^2)*log(x^2)/a^4
```

Fricas [A] time = 0.228635, size = 208, normalized size = 2.14

$$-\frac{2\left(2\,Ba^2b-3\,Aab^2\right)x^4+Aa^3+\left(2\,Ba^3-3\,Aa^2b\right)x^2-2\left(\left(2\,Bab^2-3\,Ab^3\right)x^6+\left(2\,Ba^2b-3\,Aab^2\right)x^4\right)\log\left(bx^2+a\right)+4\left(\left(2\right.\right.}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^5),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a^2*b - 3*A*a*b^2)*x^4 + A*a^3 + (2*B*a^3 - 3*A*a^2*
b)*x^2 - 2*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x
^4)*log(b*x^2 + a) + 4*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b -
3*A*a*b^2)*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A] time = 4.97154, size = 100, normalized size = 1.03

$$-\frac{Aa^2+x^4\left(-6Ab^2+4Bab\right)+x^2\left(-3Aab+2Ba^2\right)}{4a^4x^4+4a^3bx^6}-\frac{b\left(-3Ab+2Ba\right)\log\left(x\right)}{a^4}+\frac{b\left(-3Ab+2Ba\right)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(b*x**2+a)**2,x)
```

```
[Out] -(A*a**2 + x**4*(-6*A*b**2 + 4*B*a*b) + x**2*(-3*A*a*b + 2*B*a**2
))/(4*a**4*x**4 + 4*a**3*b*x**6) - b*(-3*A*b + 2*B*a)*log(x)/a**4
 + b*(-3*A*b + 2*B*a)*log(a/b + x**2)/(2*a**4)
```

GIAC/XCAS [A] time = 0.234286, size = 203, normalized size = 2.09

$$-\frac{\left(2\,Bab-3\,Ab^2\right)\ln\left(x^2\right)}{2\,a^4}+\frac{\left(2\,Bab^2-3\,Ab^3\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^4b}-\frac{2\,Bab^2x^2-3\,Ab^3x^2+3\,Ba^2b-4\,Aab^2}{2\left(bx^2+a\right)a^4}+\frac{6\,Babx^4-9\,Ab^2x^4-2\,Ba^2x^2+4\,Aabx^2-Aa^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^5),x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*b - 3*A*b^2)*ln(x^2)/a^4 + 1/2*(2*B*a*b^2 - 3*A*b^3)*
ln(abs(b*x^2 + a))/(a^4*b) - 1/2*(2*B*a*b^2*x^2 - 3*A*b^3*x^2 + 3
*B*a^2*b - 4*A*a*b^2)/((b*x^2 + a)*a^4) + 1/4*(6*B*a*b*x^4 - 9*A*
b^2*x^4 - 2*B*a^2*x^2 + 4*A*a*b*x^2 - A*a^2)/(a^4*x^4)
```

3.86 $\int \frac{A+Bx^2}{x^6(a+bx^2)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^2x(Ab-aB)}{2a^4(a+bx^2)}-\frac{b(3Ab-2aB)}{a^4x}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{5a^2x^5}$$

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/
(a^4*x) - (b^2*(A*b - a*B)*x)/(2*a^4*(a + b*x^2)) - (b^(3/2)*(7*A
*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi [A] time = 0.341636, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^2x(Ab-aB)}{2a^4(a+bx^2)}-\frac{b(3Ab-2aB)}{a^4x}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/
(a^4*x) - (b^2*(A*b - a*B)*x)/(2*a^4*(a + b*x^2)) - (b^(3/2)*(7*A
*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi in Sympy [A] time = 74.3224, size = 102, normalized size = 0.9

$$-\frac{A}{5a^2x^5}+\frac{2Ab-Ba}{3a^3x^3}-\frac{b^2x\,(Ab-Ba)}{2a^4\,(a+bx^2)}-\frac{b\,(3Ab-2Ba)}{a^4x}-\frac{b^{\frac{3}{2}}\,(7Ab-5Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -A/(5*a**2*x**5) + (2*A*b - B*a)/(3*a**3*x**3) - b**2*x*(A*b - B*
a)/(2*a**4*(a + b*x**2)) - b*(3*A*b - 2*B*a)/(a**4*x) - b**(3/2)*
(7*A*b - 5*B*a)*atan(sqrt(b)*x/sqrt(a))/(2*a**(9/2))
```

Mathematica [A] time = 0.133975, size = 112, normalized size = 0.99

$$\frac{b^{3/2}(5aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{b^2x(aB-Ab)}{2a^4(a+bx^2)}+\frac{b(2aB-3Ab)}{a^4x}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(-3*A*b + 2*a*B))
/(a^4*x) + (b^2*(-(A*b) + a*B)*x)/(2*a^4*(a + b*x^2)) + (b^(3/2)*
(-7*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A] time = 0.017, size = 136, normalized size = 1.2

$$-\frac{A}{5\,a^{2}x^{5}}+\frac{2\,Ab}{3\,a^{3}x^{3}}-\frac{B}{3\,a^{2}x^{3}}-3\,\frac{b^{2}A}{a^{4}x}+2\,\frac{Bb}{a^{3}x}-\frac{b^{3}xA}{2\,a^{4}\,(bx^{2}+a)}+\frac{b^{2}Bx}{2\,a^{3}\,(bx^{2}+a)}-\frac{7\,Ab^{3}}{2\,a^{4}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,Bb^{2}}{2\,a^{3}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5*A/a^2/x^5+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B-3/a^4*b^2/x*A+2/a^3*
b/x*B-1/2/a^4*b^3*x/(b*x^2+a)*A+1/2/a^3*b^2*x/(b*x^2+a)*B-7/2/a^4
*b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+5/2/a^3*b^2/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240682, size = 1, normalized size = 0.01

$$\left[\frac{30\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{6}+20\left(5\,Ba^{2}b-7\,Aab^{2}\right)x^{4}-12\,Aa^{3}-4\left(5\,Ba^{3}-7\,Aa^{2}b\right)x^{2}-15\left(\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{7}+\left(5\,Ba^{2}b-7\right.\right.}{60\left(a^{4}bx^{7}+a^{5}x^{5}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^6),x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*a*b^2 - 7*A*b^3)*x^6 + 20*(5*B*a^2*b - 7*A*a*b^2)*
x^4 - 12*A*a^3 - 4*(5*B*a^3 - 7*A*a^2*b)*x^2 - 15*((5*B*a*b^2 - 7
*A*b^3)*x^7 + (5*B*a^2*b - 7*A*a*b^2)*x^5)*sqrt(-b/a)*log((b*x^2
- 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b*x^7 + a^5*x^5), 1/30
*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B*a^2*b - 7*A*a*b^2)*x^4 -
 6*A*a^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2 + 15*((5*B*a*b^2 - 7*A*b^3
)*x^7 + (5*B*a^2*b - 7*A*a*b^2)*x^5)*sqrt(b/a)*arctan(b*x/(a*sqrt
(b/a))))/(a^4*b*x^7 + a^5*x^5)]
```

Sympy [A] time = 4.05275, size = 218, normalized size = 1.93

$$-\frac{\sqrt{-\frac{b^{3}}{a^{9}}}\left(-7Ab+5Ba\right)\log\left(-\frac{a^{5}\sqrt{-\frac{b^{3}}{a^{9}}}\left(-7Ab+5Ba\right)}{-7Ab^{3}+5Bab^{2}}+x\right)}{4}+\frac{\sqrt{-\frac{b^{3}}{a^{9}}}\left(-7Ab+5Ba\right)\log\left(\frac{a^{5}\sqrt{-\frac{b^{3}}{a^{9}}}\left(-7Ab+5Ba\right)}{-7Ab^{3}+5Bab^{2}}+x\right)}{4}+\frac{-6Aa^{3}+x^{6}\left(-105Ab^{3}+75Bab^{2}\right)+x^{4}\left(-70Aab^{2}+50Ba^{2}b\right)+x^{2}\left(14Aa^{2}b-10Ba^{3}\right)}{30a^{5}x^{5}+30a^{4}bx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log(-a**5*sqrt(-b**3/a**9)*(-7
*A*b + 5*B*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + sqrt(-b**3/a**9)*
(-7*A*b + 5*B*a)*log(a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)/(-7*A
*b**3 + 5*B*a*b**2) + x)/4 + (-6*A*a**3 + x**6*(-105*A*b**3 + 75*
B*a*b**2) + x**4*(-70*A*a*b**2 + 50*B*a**2*b) + x**2*(14*A*a**2*b
 - 10*B*a**3))/(30*a**5*x**5 + 30*a**4*b*x**7)
```

GIAC/XCAS [A] time = 0.229561, size = 151, normalized size = 1.34

$$\frac{\left(5\,Bab^2 - 7\,Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4} + \frac{Bab^2x - Ab^3x}{2\,(bx^2 + a)a^4} + \frac{30\,Babx^4 - 45\,Ab^2x^4 - 5\,Ba^2x^2 + 10\,Aabx^2 - 3\,Aa^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^6),x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a*b^2 - 7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) +
 1/2*(B*a*b^2*x - A*b^3*x)/((b*x^2 + a)*a^4) + 1/15*(30*B*a*b*x^4
 - 45*A*b^2*x^4 - 5*B*a^2*x^2 + 10*A*a*b*x^2 - 3*A*a^2)/(a^4*x^5)
```

3.87 $\int \frac{A+Bx^2}{x^7(a+bx^2)^2} dx$

Optimal. Leaf size=124

$$\frac{b^2(4Ab-3aB)\log(a+bx^2)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b^2(Ab-aB)}{2a^4(a+bx^2)}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(4*a^3*x^4) - (b*(3*A*b - 2*a*B))/
(2*a^4*x^2) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)) - (b^2*(4*A*b
 - 3*a*B)*Log[x])/a^5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(2*a
^5)
```

Rubi [A] time = 0.298546, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{b^2(4Ab-3aB)\log(a+bx^2)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b^2(Ab-aB)}{2a^4(a+bx^2)}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(4*a^3*x^4) - (b*(3*A*b - 2*a*B))/
(2*a^4*x^2) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)) - (b^2*(4*A*b
 - 3*a*B)*Log[x])/a^5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(2*a
^5)
```

Rubi in Sympy [A] time = 34.9356, size = 119, normalized size = 0.96

$$-\frac{A}{6a^2x^6}+\frac{2Ab-Ba}{4a^3x^4}-\frac{b^2(Ab-Ba)}{2a^4(a+bx^2)}-\frac{b(3Ab-2Ba)}{2a^4x^2}-\frac{b^2(4Ab-3Ba)\log(x^2)}{2a^5}+\frac{b^2(4Ab-3Ba)\log(a+bx^2)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**7/(b*x**2+a)**2,x)
```

```
[Out] -A/(6*a**2*x**6) + (2*A*b - B*a)/(4*a**3*x**4) - b**2*(A*b - B*a)
/(2*a**4*(a + b*x**2)) - b*(3*A*b - 2*B*a)/(2*a**4*x**2) - b**2*(
4*A*b - 3*B*a)*log(x**2)/(2*a**5) + b**2*(4*A*b - 3*B*a)*log(a +
b*x**2)/(2*a**5)
```

Mathematica [A] time = 0.186201, size = 110, normalized size = 0.89

$$\frac{-\frac{2a^3A}{x^6}-\frac{3a^2(aB-2Ab)}{x^4}+\frac{6ab^2(aB-Ab)}{a+bx^2}+6b^2(4Ab-3aB)\log(a+bx^2)+12b^2\log(x)(3aB-4Ab)+\frac{6ab(2aB-3Ab)}{x^2}}{12a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-2*A*b + a*B))/x^4 + (6*a*b*(-3*A*b + 2
*a*B))/x^2 + (6*a*b^2*(-(A*b) + a*B))/(a + b*x^2) + 12*b^2*(-4*A*
b + 3*a*B)*Log[x] + 6*b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(12*a^5
)
```

Maple [A] time = 0.023, size = 143, normalized size = 1.2

$$-\frac{A}{6\,a^{2}x^{6}}+\frac{Ab}{2\,a^{3}x^{4}}-\frac{B}{4\,a^{2}x^{4}}-\frac{3\,b^{2}A}{2\,a^{4}x^{2}}+\frac{Bb}{a^{3}x^{2}}-4\,\frac{b^{3}\ln(x)\,A}{a^{5}}+3\,\frac{b^{2}B\ln(x)}{a^{4}}+2\,\frac{b^{3}\ln\left(bx^{2}+a\right)\,A}{a^{5}}-\frac{3\,b^{2}\ln\left(bx^{2}+a\right)\,B}{2\,a^{4}}-\frac{Ab^{3}}{2\,a^{4}\left(bx^{2}+a\right)}+\frac{Bb^{2}}{2\,a^{3}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^7/(b*x^2+a)^2,x)
```

```
[Out] -1/6*A/a^2/x^6+1/2/a^3/x^4*A*b-1/4/a^2/x^4*B-3/2/a^4*b^2/x^2*A+1/
a^3*b/x^2*B-4*b^3/a^5*ln(x)*A+3*b^2/a^4*ln(x)*B+2/a^5*b^3*ln(b*x^
2+a)*A-3/2/a^4*b^2*ln(b*x^2+a)*B-1/2/a^4*b^3/(b*x^2+a)*A+1/2/a^3*
b^2/(b*x^2+a)*B
```

Maxima [A] time = 1.35873, size = 184, normalized size = 1.48

$$\frac{6\left(3\,Bab^{2}-4\,Ab^{3}\right)x^{6}+3\left(3\,Ba^{2}b-4\,Aab^{2}\right)x^{4}-2\,Aa^{3}-\left(3\,Ba^{3}-4\,Aa^{2}b\right)x^{2}}{12\left(a^{4}bx^{8}+a^{5}x^{6}\right)}-\frac{\left(3\,Bab^{2}-4\,Ab^{3}\right)\log\left(bx^{2}+a\right)}{2\,a^{5}}+\frac{\left(3\,Bab^{2}-4\,Ab^{3}\right)\log\left(x^{2}\right)}{2\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^7),x, algorithm="maxima")
```

```
[Out] 1/12*(6*(3*B*a*b^2 - 4*A*b^3)*x^6 + 3*(3*B*a^2*b - 4*A*a*b^2)*x^4
 - 2*A*a^3 - (3*B*a^3 - 4*A*a^2*b)*x^2)/(a^4*b*x^8 + a^5*x^6) - 1
/2*(3*B*a*b^2 - 4*A*b^3)*log(b*x^2 + a)/a^5 + 1/2*(3*B*a*b^2 - 4*
A*b^3)*log(x^2)/a^5
```

Fricas [A] time = 0.232668, size = 248, normalized size = 2.

$$\frac{6\left(3\,Ba^{2}b^{2}-4\,Aab^{3}\right)x^{6}-2\,Aa^{4}+3\left(3\,Ba^{3}b-4\,Aa^{2}b^{2}\right)x^{4}-\left(3\,Ba^{4}-4\,Aa^{3}b\right)x^{2}-6\left(\left(3\,Bab^{3}-4\,Ab^{4}\right)x^{8}+\left(3\,Ba^{2}b^{2}-4\,Aa\right.\right.}{12\left(a^{5}bx^{8}+a^{6}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^7),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 2*A*a^4 + 3*(3*B*a^3*b -
4*A*a^2*b^2)*x^4 - (3*B*a^4 - 4*A*a^3*b)*x^2 - 6*((3*B*a*b^3 - 4*
A*b^4)*x^8 + (3*B*a^2*b^2 - 4*A*a*b^3)*x^6)*log(b*x^2 + a) + 12*(
(3*B*a*b^3 - 4*A*b^4)*x^8 + (3*B*a^2*b^2 - 4*A*a*b^3)*x^6)*log(x)
)/(a^5*b*x^8 + a^6*x^6)
```

Sympy [A] time = 6.21963, size = 129, normalized size = 1.04

$$\frac{-2Aa^{3}+x^{6}\left(-24Ab^{3}+18Bab^{2}\right)+x^{4}\left(-12Aab^{2}+9Ba^{2}b\right)+x^{2}\left(4Aa^{2}b-3Ba^{3}\right)}{12a^{5}x^{6}+12a^{4}bx^{8}}+\frac{b^{2}\left(-4Ab+3Ba\right)\log\left(x\right)}{a^{5}}-\frac{b^{2}\left(-4Ab+3Ba\right)\log\left(\frac{a}{b}+x^{2}\right)}{2a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-2*A*a**3 + x**6*(-24*A*b**3 + 18*B*a*b**2) + x**4*(-12*A*a*b**2
 + 9*B*a**2*b) + x**2*(4*A*a**2*b - 3*B*a**3))/(12*a**5*x**6 + 12
*a**4*b*x**8) + b**2*(-4*A*b + 3*B*a)*log(x)/a**5 - b**2*(-4*A*b
+ 3*B*a)*log(a/b + x**2)/(2*a**5)
```

GIAC/XCAS [A] time = 0.233154, size = 240, normalized size = 1.94

$$\frac{\left(3\,Bab^2-4\,Ab^3\right)\ln\left(x^2\right)}{2\,a^5}-\frac{\left(3\,Bab^3-4\,Ab^4\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^5b}+\frac{3\,Bab^3x^2-4\,Ab^4x^2+4\,Ba^2b^2-5\,Aab^3}{2\,(bx^2+a)a^5}-\frac{33\,Bab^2x^6-44\,Ab^3x^6-12\,Ba^2bx^4+18\,Aab^2x^4+3\,Ba^3x^2-6\,Aa^2bx^2+2\,Aa^3}{12\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^7),x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a*b^2 - 4*A*b^3)*ln(x^2)/a^5 - 1/2*(3*B*a*b^3 - 4*A*b^4)
*ln(abs(b*x^2 + a))/(a^5*b) + 1/2*(3*B*a*b^3*x^2 - 4*A*b^4*x^2 +
4*B*a^2*b^2 - 5*A*a*b^3)/((b*x^2 + a)*a^5) - 1/12*(33*B*a*b^2*x^6
 - 44*A*b^3*x^6 - 12*B*a^2*b*x^4 + 18*A*a*b^2*x^4 + 3*B*a^3*x^2 -
 6*A*a^2*b*x^2 + 2*A*a^3)/(a^5*x^6)
```

3.88 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=150

$$\frac{a^5(Ab-aB)}{4b^7(a+bx^2)^2}-\frac{a^4(5Ab-6aB)}{2b^7(a+bx^2)}-\frac{5a^3(2Ab-3aB)\log(a+bx^2)}{2b^7}+\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{3ax^4(Ab-2aB)}{4b^5}+\frac{x^6(Ab-3aB)}{6b^4}+\frac{Bx^8}{8b^3}$$

```
[Out] (a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(4*b^5) +
 ((A*b - 3*a*B)*x^6)/(6*b^4) + (B*x^8)/(8*b^3) + (a^5*(A*b - a*B)
)/(4*b^7*(a + b*x^2)^2) - (a^4*(5*A*b - 6*a*B))/(2*b^7*(a + b*x^2
)) - (5*a^3*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^7)
```

Rubi [A] time = 0.474172, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^5(Ab-aB)}{4b^7(a+bx^2)^2}-\frac{a^4(5Ab-6aB)}{2b^7(a+bx^2)}-\frac{5a^3(2Ab-3aB)\log(a+bx^2)}{2b^7}+\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{3ax^4(Ab-2aB)}{4b^5}+\frac{x^6(Ab-3aB)}{6b^4}+\frac{Bx^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(4*b^5) +
 ((A*b - 3*a*B)*x^6)/(6*b^4) + (B*x^8)/(8*b^3) + (a^5*(A*b - a*B)
)/(4*b^7*(a + b*x^2)^2) - (a^4*(5*A*b - 6*a*B))/(2*b^7*(a + b*x^2
)) - (5*a^3*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^7)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^8}{8b^3}+\frac{a^5(Ab-Ba)}{4b^7(a+bx^2)^2}-\frac{a^4(5Ab-6Ba)}{2b^7(a+bx^2)}-\frac{5a^3(2Ab-3Ba)\log(a+bx^2)}{2b^7}-\frac{3a(Ab-2Ba)\int^{x^2}x\,dx}{2b^5}+\frac{x^6(Ab-3Ba)}{6b^4}+\frac{(3Ab-5Ba)\int^{x^2}a^2\,dx}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**8/(8*b**3) + a**5*(A*b - B*a)/(4*b**7*(a + b*x**2)**2) - a**
4*(5*A*b - 6*B*a)/(2*b**7*(a + b*x**2)) - 5*a**3*(2*A*b - 3*B*a)*
log(a + b*x**2)/(2*b**7) - 3*a*(A*b - 2*B*a)*Integral(x, (x, x**2
))/(2*b**5) + x**6*(A*b - 3*B*a)/(6*b**4) + (3*A*b - 5*B*a)*Integ
ral(a**2, (x, x**2))/b**6
```

Mathematica [A] time = 0.146372, size = 136, normalized size = 0.91

$$\frac{\frac{6a^5(Ab-aB)}{(a+bx^2)^2}+\frac{12a^4(6aB-5Ab)}{a+bx^2}+60a^3(3aB-2Ab)\log(a+bx^2)-24a^2bx^2(5aB-3Ab)+4b^3x^6(Ab-3aB)+18ab^2x^4(2aB-Ab}{24b^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-24*a^2*b*(-3*A*b + 5*a*B)*x^2 + 18*a*b^2*(-(A*b) + 2*a*B)*x^4 +
 4*b^3*(A*b - 3*a*B)*x^6 + 3*b^4*B*x^8 + (6*a^5*(A*b - a*B))/(a +
 b*x^2)^2 + (12*a^4*(-5*A*b + 6*a*B))/(a + b*x^2) + 60*a^3*(-2*A*
b + 3*a*B)*Log[a + b*x^2])/(24*b^7)
```

Maple [A] time = 0.02, size = 182, normalized size = 1.2

$$\frac{Bx^8}{8\,b^3}+\frac{x^6A}{6\,b^3}-\frac{x^6Ba}{2\,b^4}-\frac{3\,x^4Aa}{4\,b^4}+\frac{3\,x^4Ba^2}{2\,b^5}+3\,\frac{a^2Ax^2}{b^5}-5\,\frac{Bx^2a^3}{b^6}+\frac{a^5A}{4\,b^6\,(bx^2+a)^2}-\frac{Ba^6}{4\,b^7\,(bx^2+a)^2}-5\,\frac{a^3\ln\left(bx^2+a\right)A}{b^6}+\frac{15\,a^4\ln\left(bx^2+a\right)B}{2\,b^7}-\frac{5\,a^4A}{2\,b^6\,(bx^2+a)}+3\,\frac{a^5B}{b^7\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/8*B*x^8/b^3+1/6/b^3*x^6*A-1/2/b^4*x^6*B*a-3/4/b^4*x^4*A*a+3/2/b
^5*x^4*B*a^2+3/b^5*A*x^2*a^2-5/b^6*B*x^2*a^3+1/4*a^5/b^6/(b*x^2+a
)^2*A-1/4*a^6/b^7/(b*x^2+a)^2*B-5*a^3/b^6*ln(b*x^2+a)*A+15/2*a^4/
b^7*ln(b*x^2+a)*B-5/2*a^4/b^6/(b*x^2+a)*A+3*a^5/b^7/(b*x^2+a)*B
```

Maxima [A] time = 1.36071, size = 223, normalized size = 1.49

$$\frac{11\,Ba^6-9\,Aa^5b+2\left(6\,Ba^5b-5\,Aa^4b^2\right)x^2}{4\,(b^9x^4+2\,ab^8x^2+a^2b^7)}+\frac{3\,Bb^3x^8-4\left(3\,Bab^2-Ab^3\right)x^6+18\left(2\,Ba^2b-Aab^2\right)x^4-24\left(5\,Ba^3-3\,Aa^2b\right)x^2}{24\,b^6}+\frac{5\left(3\,Ba^4-2\,Aa^3b\right)\log\left(bx^2+a\right)}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^11/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(11*B*a^6 - 9*A*a^5*b + 2*(6*B*a^5*b - 5*A*a^4*b^2)*x^2)/(b^9
*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 1/24*(3*B*b^3*x^8 - 4*(3*B*a*b^2
- A*b^3)*x^6 + 18*(2*B*a^2*b - A*a*b^2)*x^4 - 24*(5*B*a^3 - 3*A*a
^2*b)*x^2)/b^6 + 5/2*(3*B*a^4 - 2*A*a^3*b)*log(b*x^2 + a)/b^7
```

Fricas [A] time = 0.227804, size = 312, normalized size = 2.08

$$\frac{3\,Bb^6x^{12}-2\left(3\,Bab^5-2\,Ab^6\right)x^{10}+5\left(3\,Ba^2b^4-2\,Aab^5\right)x^8+66\,Ba^6-54\,Aa^5b-20\left(3\,Ba^3b^3-2\,Aa^2b^4\right)x^6-6\left(34\,Ba^4b^2-\right.}{24\,(b^9x^4+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^11/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^6*x^12 - 2*(3*B*a*b^5 - 2*A*b^6)*x^10 + 5*(3*B*a^2*b^
4 - 2*A*a*b^5)*x^8 + 66*B*a^6 - 54*A*a^5*b - 20*(3*B*a^3*b^3 - 2*
A*a^2*b^4)*x^6 - 6*(34*B*a^4*b^2 - 21*A*a^3*b^3)*x^4 - 12*(4*B*a^
5*b - A*a^4*b^2)*x^2 + 60*(3*B*a^6 - 2*A*a^5*b + (3*B*a^4*b^2 - 2
*A*a^3*b^3)*x^4 + 2*(3*B*a^5*b - 2*A*a^4*b^2)*x^2)*log(b*x^2 + a)
)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

Sympy [A] time = 7.2091, size = 163, normalized size = 1.09

$$\frac{Bx^8}{8b^3} + \frac{5a^3 (-2Ab + 3Ba) \log{(a + bx^2)}}{2b^7} + \frac{-9Aa^5b + 11Ba^6 + x^2 (-10Aa^4b^2 + 12Ba^5b)}{4a^2b^7 + 8ab^8x^2 + 4b^9x^4} - \frac{x^6 (-Ab + 3Ba)}{6b^4} + \frac{x^4 (-3Aab + 6Ba^2)}{4b^5} - \frac{x^2 (-3Aa^2b + 5Ba^3)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**8/(8*b**3) + 5*a**3*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**7
) + (-9*A*a**5*b + 11*B*a**6 + x**2*(-10*A*a**4*b**2 + 12*B*a**5*
b))/(4*a**2*b**7 + 8*a*b**8*x**2 + 4*b**9*x**4) - x**6*(-A*b + 3*
B*a)/(6*b**4) + x**4*(-3*A*a*b + 6*B*a**2)/(4*b**5) - x**2*(-3*A*
a**2*b + 5*B*a**3)/b**6
```

GIAC/XCAS [A] time = 0.246336, size = 247, normalized size = 1.65

$$\frac{5 (3 Ba^4 - 2 Aa^3b) \ln (|bx^2 + a|)}{2 b^7} - \frac{45 Ba^4b^2x^4 - 30 Aa^3b^3x^4 + 78 Ba^5bx^2 - 50 Aa^4b^2x^2 + 34 Ba^6 - 21 Aa^5b}{4 (bx^2 + a)^2b^7} + \frac{3 Bb^9x^8 - 12 Bab^8x^6 + 4 Ab^9x^6 + 36 Ba^2b^7x^4 - 18 Aab^8x^4 - 120 Ba^3b^6x^2 + 72 Aa^2b^7x^2}{24 b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^11/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 5/2*(3*B*a^4 - 2*A*a^3*b)*ln(abs(b*x^2 + a))/b^7 - 1/4*(45*B*a^4*
b^2*x^4 - 30*A*a^3*b^3*x^4 + 78*B*a^5*b*x^2 - 50*A*a^4*b^2*x^2 +
34*B*a^6 - 21*A*a^5*b)/((b*x^2 + a)^2*b^7) + 1/24*(3*B*b^9*x^8 -
12*B*a*b^8*x^6 + 4*A*b^9*x^6 + 36*B*a^2*b^7*x^4 - 18*A*a*b^8*x^4
- 120*B*a^3*b^6*x^2 + 72*A*a^2*b^7*x^2)/b^12
```

3.89 $\int \frac{x^9 (A+Bx^2)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=128

$$-\frac{a^4(Ab-aB)}{4b^6(a+bx^2)^2}+\frac{a^3(4Ab-5aB)}{2b^6(a+bx^2)}+\frac{a^2(3Ab-5aB)\log(a+bx^2)}{b^6}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{x^4(Ab-3aB)}{4b^4}+\frac{Bx^6}{6b^3}$$

```
[Out] (-3*a*(A*b - 2*a*B)*x^2)/(2*b^5) + ((A*b - 3*a*B)*x^4)/(4*b^4) +
(B*x^6)/(6*b^3) - (a^4*(A*b - a*B))/(4*b^6*(a + b*x^2)^2) + (a^3*
(4*A*b - 5*a*B))/(2*b^6*(a + b*x^2)) + (a^2*(3*A*b - 5*a*B)*Log[a
 + b*x^2])/b^6
```

Rubi [A] time = 0.371073, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a^4(Ab-aB)}{4b^6(a+bx^2)^2}+\frac{a^3(4Ab-5aB)}{2b^6(a+bx^2)}+\frac{a^2(3Ab-5aB)\log(a+bx^2)}{b^6}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{x^4(Ab-3aB)}{4b^4}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x^2)/(2*b^5) + ((A*b - 3*a*B)*x^4)/(4*b^4) +
(B*x^6)/(6*b^3) - (a^4*(A*b - a*B))/(4*b^6*(a + b*x^2)^2) + (a^3*
(4*A*b - 5*a*B))/(2*b^6*(a + b*x^2)) + (a^2*(3*A*b - 5*a*B)*Log[a
 + b*x^2])/b^6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx^6}{6b^3}-\frac{a^4(Ab-Ba)}{4b^6(a+bx^2)^2}+\frac{a^3(4Ab-5Ba)}{2b^6(a+bx^2)}+\frac{a^2(3Ab-5Ba)\log(a+bx^2)}{b^6}-\frac{3ax^2(Ab-2Ba)}{2b^5}+\frac{(Ab-3Ba)\int^{x^2} x\,dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) - a**4*(A*b - B*a)/(4*b**6*(a + b*x**2)**2) + a**
3*(4*A*b - 5*B*a)/(2*b**6*(a + b*x**2)) + a**2*(3*A*b - 5*B*a)*lo
g(a + b*x**2)/b**6 - 3*a*x**2*(A*b - 2*B*a)/(2*b**5) + (A*b - 3*B
*a)*Integral(x, (x, x**2))/(2*b**4)
```

Mathematica [A] time = 0.136892, size = 116, normalized size = 0.91

$$\frac{\frac{3a^4(aB-Ab)}{(a+bx^2)^2}+\frac{6a^3(4Ab-5aB)}{a+bx^2}+12a^2(3Ab-5aB)\log(a+bx^2)+3b^2x^4(Ab-3aB)+18abx^2(2aB-Ab)+2b^3Bx^6}{12b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (18*a*b*(-(A*b) + 2*a*B)*x^2 + 3*b^2*(A*b - 3*a*B)*x^4 + 2*b^3*B*
x^6 + (3*a^4*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a^3*(4*A*b - 5*a*
```

```
B))/(a + b*x^2) + 12*a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/(12*b^6)
```

Maple [A] time = 0.018, size = 158, normalized size = 1.2

$$\frac{Bx^{6}}{6\,b^{3}}+\frac{Ax^{4}}{4\,b^{3}}-\frac{3\,Bx^{4}a}{4\,b^{4}}-\frac{3\,aAx^{2}}{2\,b^{4}}+3\,\frac{Bx^{2}a^{2}}{b^{5}}-\frac{a^{4}A}{4\,b^{5}\left(bx^{2}+a\right)^{2}}+\frac{a^{5}B}{4\,b^{6}\left(bx^{2}+a\right)^{2}}+3\,\frac{a^{2}\ln\left(bx^{2}+a\right)A}{b^{5}}-5\,\frac{a^{3}\ln\left(bx^{2}+a\right)B}{b^{6}}+2\,\frac{a^{3}A}{b^{5}\left(bx^{2}+a\right)}-\frac{5\,a^{4}B}{2\,b^{6}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/6*B*x^6/b^3+1/4/b^3*A*x^4-3/4/b^4*B*x^4*a-3/2/b^4*A*x^2*a+3/b^5
*B*x^2*a^2-1/4*a^4/b^5/(b*x^2+a)^2*A+1/4*a^5/b^6/(b*x^2+a)^2*B+3*
a^2/b^5*ln(b*x^2+a)*A-5*a^3/b^6*ln(b*x^2+a)*B+2*a^3/b^5/(b*x^2+a)
*A-5/2*a^4/b^6/(b*x^2+a)*B
```

Maxima [A] time = 1.3449, size = 190, normalized size = 1.48

$$-\frac{9\,Ba^{5}-7\,Aa^{4}b+2\left(5\,Ba^{4}b-4\,Aa^{3}b^{2}\right)x^{2}}{4\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}+\frac{2\,Bb^{2}x^{6}-3\left(3\,Bab-Ab^{2}\right)x^{4}+18\left(2\,Ba^{2}-Aab\right)x^{2}}{12\,b^{5}}-\frac{\left(5\,Ba^{3}-3\,Aa^{2}b\right)\log\left(bx^{2}+a\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^9/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*B*a^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x^2)/(b^8
*x^4 + 2*a*b^7*x^2 + a^2*b^6) + 1/12*(2*B*b^2*x^6 - 3*(3*B*a*b -
A*b^2)*x^4 + 18*(2*B*a^2 - A*a*b)*x^2)/b^5 - (5*B*a^3 - 3*A*a^2*b
)*log(b*x^2 + a)/b^6
```

Fricas [A] time = 0.220257, size = 277, normalized size = 2.16

$$\frac{2\,Bb^{5}x^{10}-\left(5\,Bab^{4}-3\,Ab^{5}\right)x^{8}+4\left(5\,Ba^{2}b^{3}-3\,Aab^{4}\right)x^{6}-27\,Ba^{5}+21\,Aa^{4}b+3\left(21\,Ba^{3}b^{2}-11\,Aa^{2}b^{3}\right)x^{4}+6\left(Ba^{4}b+Aa^{3}b\right.}{12\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^9/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^10 - (5*B*a*b^4 - 3*A*b^5)*x^8 + 4*(5*B*a^2*b^3 -
 3*A*a*b^4)*x^6 - 27*B*a^5 + 21*A*a^4*b + 3*(21*B*a^3*b^2 - 11*A*
a^2*b^3)*x^4 + 6*(B*a^4*b + A*a^3*b^2)*x^2 - 12*(5*B*a^5 - 3*A*a^
4*b + (5*B*a^3*b^2 - 3*A*a^2*b^3)*x^4 + 2*(5*B*a^4*b - 3*A*a^3*b^
2)*x^2)*log(b*x^2 + a))/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)
```

Sympy [A] time = 6.70035, size = 138, normalized size = 1.08

$$\frac{Bx^{6}}{6b^{3}}-\frac{a^{2}\left(-3Ab+5Ba\right)\log\left(a+bx^{2}\right)}{b^{6}}-\frac{-7Aa^{4}b+9Ba^{5}+x^{2}\left(-8Aa^{3}b^{2}+10Ba^{4}b\right)}{4a^{2}b^{6}+8ab^{7}x^{2}+4b^{8}x^{4}}-\frac{x^{4}\left(-Ab+3Ba\right)}{4b^{4}}+\frac{x^{2}\left(-3Aab+6Ba^{2}\right)}{2b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) - a**2*(-3*A*b + 5*B*a)*log(a + b*x**2)/b**6 - (-
7*A*a**4*b + 9*B*a**5 + x**2*(-8*A*a**3*b**2 + 10*B*a**4*b))/(4*a
**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) - x**4*(-A*b + 3*B*a)/(4*
b**4) + x**2*(-3*A*a*b + 6*B*a**2)/(2*b**5)
```

GIAC/XCAS [A] time = 0.233176, size = 215, normalized size = 1.68

$$-\frac{\left(5\,Ba^3-3\,Aa^2b\right)\ln\left(\left|bx^2+a\right|\right)}{b^6}+\frac{30\,Ba^3b^2x^4-18\,Aa^2b^3x^4+50\,Ba^4bx^2-28\,Aa^3b^2x^2+21\,Ba^5-11\,Aa^4b}{4\,\left(bx^2+a\right)^2b^6}+\frac{2\,Bb^6x^6-9\,Bab^5x^4+3\,Ab^6x^4+36\,Ba^2b^4x^2-18\,Aab^5x^2}{12\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^9/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -(5*B*a^3 - 3*A*a^2*b)*ln(abs(b*x^2 + a))/b^6 + 1/4*(30*B*a^3*b^2
*x^4 - 18*A*a^2*b^3*x^4 + 50*B*a^4*b*x^2 - 28*A*a^3*b^2*x^2 + 21*
B*a^5 - 11*A*a^4*b)/((b*x^2 + a)^2*b^6) + 1/12*(2*B*b^6*x^6 - 9*B
*a*b^5*x^4 + 3*A*b^6*x^4 + 36*B*a^2*b^4*x^2 - 18*A*a*b^5*x^2)/b^9
```

3.90 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=109

$$\frac{a^3(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2(3Ab-4aB)}{2b^5(a+bx^2)}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}+\frac{x^2(Ab-3aB)}{2b^4}+\frac{Bx^4}{4b^3}$$

```
[Out] ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^4)/(4*b^3) + (a^3*(A*b - a*B))
/(4*b^5*(a + b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(2*b^5*(a + b*x^2)
) - (3*a*(A*b - 2*a*B)*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.29629, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^3(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2(3Ab-4aB)}{2b^5(a+bx^2)}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}+\frac{x^2(Ab-3aB)}{2b^4}+\frac{Bx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^4)/(4*b^3) + (a^3*(A*b - a*B))
/(4*b^5*(a + b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(2*b^5*(a + b*x^2)
) - (3*a*(A*b - 2*a*B)*Log[a + b*x^2])/(2*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\int^{x^2} x\,dx}{2b^3}+\frac{a^3(Ab-Ba)}{4b^5(a+bx^2)^2}-\frac{a^2(3Ab-4Ba)}{2b^5(a+bx^2)}-\frac{3a(Ab-2Ba)\log\left(a+bx^2\right)}{2b^5}+\left(\frac{Ab}{2}-\frac{3Ba}{2}\right)\int^{x^2}\frac{1}{b^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*Integral(x, (x, x**2))/(2*b**3) + a**3*(A*b - B*a)/(4*b**5*(a +
 b*x**2)**2) - a**2*(3*A*b - 4*B*a)/(2*b**5*(a + b*x**2)) - 3*a*(
A*b - 2*B*a)*log(a + b*x**2)/(2*b**5) + (A*b/2 - 3*B*a/2)*Integra
l(b**(-4), (x, x**2))
```

Mathematica [A] time = 0.108668, size = 94, normalized size = 0.86

$$\frac{\frac{a^3(Ab-aB)}{(a+bx^2)^2}+\frac{2a^2(4aB-3Ab)}{a+bx^2}+2bx^2(Ab-3aB)+6a(2aB-Ab)\log\left(a+bx^2\right)+b^2Bx^4}{4b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^2 + b^2*B*x^4 + (a^3*(A*b - a*B))/(a + b*x^2
)^2 + (2*a^2*(-3*A*b + 4*a*B))/(a + b*x^2) + 6*a*(-(A*b) + 2*a*B)
*Log[a + b*x^2])/(4*b^5)
```

Maple [A] time = 0.016, size = 134, normalized size = 1.2

$$\frac{Bx^4}{4\,b^3} - \frac{3\,Bx^2a}{2\,b^4} + \frac{Ax^2}{2\,b^3} + \frac{a^3A}{4\,b^4\,(bx^2+a)^2} - \frac{Ba^4}{4\,b^5\,(bx^2+a)^2} - \frac{3\,a\ln\left(bx^2+a\right)\,A}{2\,b^4} + 3\,\frac{a^2\ln\left(bx^2+a\right)\,B}{b^5} - \frac{3\,Aa^2}{2\,b^4\,(bx^2+a)} + 2\,\frac{Ba^3}{b^5\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^7*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*B*x^4/b^3-3/2/b^4*B*x^2*a+1/2/b^3*A*x^2+1/4*a^3/b^4/(b*x^2+a)
^2*A-1/4*a^4/b^5/(b*x^2+a)^2*B-3/2*a/b^4*ln(b*x^2+a)*A+3*a^2/b^5*
ln(b*x^2+a)*B-3/2*a^2/b^4/(b*x^2+a)*A+2*a^3/b^5/(b*x^2+a)*B
```

Maxima [A] time = 1.35077, size = 157, normalized size = 1.44

$$\frac{7\,Ba^4 - 5\,Aa^3b + 2\,\left(4\,Ba^3b - 3\,Aa^2b^2\right)x^2}{4\,(b^7x^4 + 2\,ab^6x^2 + a^2b^5)} + \frac{Bbx^4 - 2\,(3\,Ba - Ab)x^2}{4\,b^4} + \frac{3\,\left(2\,Ba^2 - Aab\right)\,\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^7/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^2)/(b^7*
x^4 + 2*a*b^6*x^2 + a^2*b^5) + 1/4*(B*b*x^4 - 2*(3*B*a - A*b)*x^2
)/b^4 + 3/2*(2*B*a^2 - A*a*b)*log(b*x^2 + a)/b^5
```

Fricas [A] time = 0.216152, size = 242, normalized size = 2.22

$$\frac{Bb^4x^8 - 2\,\left(2\,Bab^3 - Ab^4\right)x^6 + 7\,Ba^4 - 5\,Aa^3b - \left(11\,Ba^2b^2 - 4\,Aab^3\right)x^4 + 2\,\left(Ba^3b - 2\,Aa^2b^2\right)x^2 + 6\,\left(2\,Ba^4 - Aa^3b + \left(2\,Ba^2\right.\right.}{4\,(b^7x^4 + 2\,ab^6x^2 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^7/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^4*x^8 - 2*(2*B*a*b^3 - A*b^4)*x^6 + 7*B*a^4 - 5*A*a^3*b
- (11*B*a^2*b^2 - 4*A*a*b^3)*x^4 + 2*(B*a^3*b - 2*A*a^2*b^2)*x^2
+ 6*(2*B*a^4 - A*a^3*b + (2*B*a^2*b^2 - A*a*b^3)*x^4 + 2*(2*B*a^3
*b - A*a^2*b^2)*x^2)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2
*b^5)
```

Sympy [A] time = 6.31504, size = 116, normalized size = 1.06

$$\frac{Bx^4}{4b^3} + \frac{3a\,(-Ab + 2Ba)\log\left(a + bx^2\right)}{2b^5} + \frac{-5Aa^3b + 7Ba^4 + x^2\left(-6Aa^2b^2 + 8Ba^3b\right)}{4a^2b^5 + 8ab^6x^2 + 4b^7x^4} - \frac{x^2\,(-Ab + 3Ba)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**7*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + 3*a*(-A*b + 2*B*a)*log(a + b*x**2)/(2*b**5) + (
-5*A*a**3*b + 7*B*a**4 + x**2*(-6*A*a**2*b**2 + 8*B*a**3*b))/(4*a
**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) - x**2*(-A*b + 3*B*a)/(2*
b**4)
```

GIAC/XCAS [A] time = 0.238793, size = 178, normalized size = 1.63

$$\frac{3\left(2\,Ba^{2}-Aab\right)\ln\left(\left|bx^{2}+a\right|\right)}{2\,b^{5}}+\frac{Bb^{3}x^{4}-6\,Bab^{2}x^{2}+2\,Ab^{3}x^{2}}{4\,b^{6}}-\frac{18\,Ba^{2}b^{2}x^{4}-9\,Aab^{3}x^{4}+28\,Ba^{3}bx^{2}-12\,Aa^{2}b^{2}x^{2}+11\,Ba^{4}-4\,Aa^{3}b}{4\,\left(bx^{2}+a\right)^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*a^2 - A*a*b)*ln(abs(b*x^2 + a))/b^5 + 1/4*(B*b^3*x^4 - 6
*B*a*b^2*x^2 + 2*A*b^3*x^2)/b^6 - 1/4*(18*B*a^2*b^2*x^4 - 9*A*a*b
^3*x^4 + 28*B*a^3*b*x^2 - 12*A*a^2*b^2*x^2 + 11*B*a^4 - 4*A*a^3*b
)/((b*x^2 + a)^2*b^5)
```

3.91 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=88

$$-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{Bx^2}{2b^3}$$

```
[Out] (B*x^2)/(2*b^3) - (a^2*(A*b - a*B))/(4*b^4*(a + b*x^2)^2) + (a*(2
*A*b - 3*a*B))/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2
])/(2*b^4)
```

Rubi [A] time = 0.224423, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{Bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) - (a^2*(A*b - a*B))/(4*b^4*(a + b*x^2)^2) + (a*(2
*A*b - 3*a*B))/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2
])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2(Ab-Ba)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3Ba)}{2b^4(a+bx^2)}+\frac{\int^{x^2} B\,dx}{2b^3}+\frac{(Ab-3Ba)\log(a+bx^2)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -a**2*(A*b - B*a)/(4*b**4*(a + b*x**2)**2) + a*(2*A*b - 3*B*a)/(2
*b**4*(a + b*x**2)) + Integral(B, (x, x**2))/(2*b**3) + (A*b - 3*
B*a)*log(a + b*x**2)/(2*b**4)
```

Mathematica [A] time = 0.0621631, size = 92, normalized size = 1.05

$$\frac{2aAb-3a^2B}{2b^4(a+bx^2)}+\frac{a^3B-a^2Ab}{4b^4(a+bx^2)^2}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{Bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) + (-(a^2*A*b) + a^3*B)/(4*b^4*(a + b*x^2)^2) + (2
*a*A*b - 3*a^2*B)/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*
x^2])/(2*b^4)
```

Maple [A] time = 0.017, size = 109, normalized size = 1.2

$$\frac{Bx^2}{2\,b^3}-\frac{a^2A}{4\,b^3(bx^2+a)^2}+\frac{Ba^3}{4\,b^4(bx^2+a)^2}+\frac{\ln(bx^2+a)\,A}{2\,b^3}-\frac{3\,\ln(bx^2+a)\,Ba}{2\,b^4}+\frac{aA}{b^3(bx^2+a)}-\frac{3\,a^2B}{2\,b^4(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*B*x^2/b^3-1/4/b^3*a^2/(b*x^2+a)^2*A+1/4/b^4*a^3/(b*x^2+a)^2*B
+1/2/b^3*ln(b*x^2+a)*A-3/2/b^4*ln(b*x^2+a)*B*a+1/b^3*a/(b*x^2+a)*
A-3/2/b^4*a^2/(b*x^2+a)*B
```

Maxima [A] time = 1.33848, size = 127, normalized size = 1.44

$$-\frac{5\,Ba^3 - 3\,Aa^2b + 2\,\left(3\,Ba^2b - 2\,Aab^2\right)x^2}{4\,\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)} + \frac{Bx^2}{2\,b^3} - \frac{(3\,Ba - Ab)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^2)/(b^6*x
^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*l
og(b*x^2 + a)/b^4
```

Fricas [A] time = 0.223146, size = 192, normalized size = 2.18

$$\frac{2\,Bb^3x^6 + 4\,Bab^2x^4 - 5\,Ba^3 + 3\,Aa^2b - 4\,\left(Ba^2b - Aab^2\right)x^2 - 2\,\left(\left(3\,Bab^2 - Ab^3\right)x^4 + 3\,Ba^3 - Aa^2b + 2\,\left(3\,Ba^2b - Aab^2\right)x^2\right)\mathrm{l}}{4\,\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^3*x^6 + 4*B*a*b^2*x^4 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2
*b - A*a*b^2)*x^2 - 2*((3*B*a*b^2 - A*b^3)*x^4 + 3*B*a^3 - A*a^2*
b + 2*(3*B*a^2*b - A*a*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4)
```

Sympy [A] time = 5.53614, size = 94, normalized size = 1.07

$$\frac{Bx^2}{2b^3} - \frac{-3Aa^2b + 5Ba^3 + x^2\left(-4Aab^2 + 6Ba^2b\right)}{4a^2b^4 + 8ab^5x^2 + 4b^6x^4} - \frac{(-Ab + 3Ba)\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) - (-3*A*a**2*b + 5*B*a**3 + x**2*(-4*A*a*b**2 + 6
*B*a**2*b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - (-A*b +
 3*B*a)*log(a + b*x**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.226637, size = 126, normalized size = 1.43

$$\frac{Bx^2}{2\,b^3} - \frac{(3\,Ba - Ab)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{9\,Bab^2x^4 - 3\,Ab^3x^4 + 12\,Ba^2bx^2 - 2\,Aab^2x^2 + 4\,Ba^3}{4\,\left(bx^2 + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*ln(abs(b*x^2 + a))/b^4 + 1/4*(9
*B*a*b^2*x^4 - 3*A*b^3*x^4 + 12*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 4*B
*a^3)/((b*x^2 + a)^2*b^4)
```

3.92 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=66

$$-\frac{Ab-2aB}{2b^3(a+bx^2)}+\frac{a(Ab-aB)}{4b^3(a+bx^2)^2}+\frac{B\log(a+bx^2)}{2b^3}$$

```
[Out] (a*(A*b - a*B))/(4*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(2*b^3*(a +
 b*x^2)) + (B*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.167587, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{Ab-2aB}{2b^3(a+bx^2)}+\frac{a(Ab-aB)}{4b^3(a+bx^2)^2}+\frac{B\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*(A*b - a*B))/(4*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(2*b^3*(a +
 b*x^2)) + (B*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [A] time = 20.8252, size = 56, normalized size = 0.85

$$\frac{B\log(a+bx^2)}{2b^3}+\frac{a(Ab-Ba)}{4b^3(a+bx^2)^2}-\frac{Ab-2Ba}{2b^3(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**3) + a*(A*b - B*a)/(4*b**3*(a + b*x**2)**
2) - (A*b - 2*B*a)/(2*b**3*(a + b*x**2))
```

Mathematica [A] time = 0.0413053, size = 64, normalized size = 0.97

$$\frac{3a^2B-ab(A-4Bx^2)+2B(a+bx^2)^2\log(a+bx^2)-2Ab^2x^2}{4b^3(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^2 - a*b*(A - 4*B*x^2) + 2*B*(a + b*x^2)^2*Lo
g[a + b*x^2])/(4*b^3*(a + b*x^2)^2)
```

Maple [A] time = 0.012, size = 80, normalized size = 1.2

$$\frac{Aa}{4\,b^2(bx^2+a)^2}-\frac{a^2B}{4\,b^3(bx^2+a)^2}+\frac{B\ln(bx^2+a)}{2\,b^3}-\frac{A}{2\,b^2(bx^2+a)}+\frac{Ba}{b^3(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*a/b^2/(b*x^2+a)^2*A-1/4*a^2/b^3/(b*x^2+a)^2*B+1/2*B*ln(b*x^2+
a)/b^3-1/2/b^2/(b*x^2+a)*A+1/b^3/(b*x^2+a)*B*a
```

Maxima [A] time = 1.34407, size = 97, normalized size = 1.47

$$\frac{3\,Ba^2 - Aab + 2\left(2\,Bab - Ab^2\right)x^2}{4\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)} + \frac{B\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2)/(b^5*x^4 + 2*a*b^
4*x^2 + a^2*b^3) + 1/2*B*log(b*x^2 + a)/b^3
```

Fricas [A] time = 0.218774, size = 120, normalized size = 1.82

$$\frac{3\,Ba^2 - Aab + 2\left(2\,Bab - Ab^2\right)x^2 + 2\left(Bb^2x^4 + 2\,Babx^2 + Ba^2\right)\log\left(bx^2 + a\right)}{4\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2 + 2*(B*b^2*x^4 + 2
*B*a*b*x^2 + B*a^2)*log(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*
b^3)
```

Sympy [A] time = 4.21695, size = 70, normalized size = 1.06

$$\frac{B\log\left(a + bx^2\right)}{2b^3} + \frac{-Aab + 3Ba^2 + x^2\left(-2Ab^2 + 4Bab\right)}{4a^2b^3 + 8ab^4x^2 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**3) + (-A*a*b + 3*B*a**2 + x**2*(-2*A*b**2
 + 4*B*a*b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
```

GIAC/XCAS [A] time = 0.233296, size = 82, normalized size = 1.24

$$\frac{B\ln\left(\left|bx^2 + a\right|\right)}{2\,b^3} + \frac{2\,(2\,Ba - Ab)x^2 + \frac{3\,Ba^2 - Aab}{b}}{4\left(bx^2 + a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*ln(abs(b*x^2 + a))/b^3 + 1/4*(2*(2*B*a - A*b)*x^2 + (3*B*a^
2 - A*a*b)/b)/((b*x^2 + a)^2*b^2)
```

3.93 $\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{(A+Bx^2)^2}{4(a+bx^2)^2(Ab-aB)}$$

```
[Out] -(A + B*x^2)^2/(4*(A*b - a*B)*(a + b*x^2)^2)
```

Rubi [A] time = 0.0621845, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{(A+Bx^2)^2}{4(a+bx^2)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2)^3,x]

[Out] -(A + B*x^2)^2/(4*(A*b - a*B)*(a + b*x^2)^2)
```

Rubi in Sympy [A] time = 8.50241, size = 26, normalized size = 0.81

$$-\frac{(A+Bx^2)^2}{4(a+bx^2)^2(Ab-Ba)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(b*x**2+a)**3,x)

[Out] -(A + B*x**2)**2/(4*(a + b*x**2)**2*(A*b - B*a))
```

Mathematica [A] time = 0.02174, size = 30, normalized size = 0.94

$$-\frac{B(a+2bx^2)+Ab}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2)^3,x]

[Out] -(A*b + B*(a + 2*b*x^2))/(4*b^2*(a + b*x^2)^2)
```

Maple [A] time = 0.01, size = 39, normalized size = 1.2

$$-\frac{Ab-Ba}{4\,b^2(bx^2+a)^2}-\frac{B}{(2\,bx^2+2\,a)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] -1/4*(A*b-B*a)/b^2/(b*x^2+a)^2-1/2*B/(b*x^2+a)/b^2
```

Maxima [A] time = 1.34172, size = 57, normalized size = 1.78

$$-\frac{2\,Bbx^{2}+Ba+Ab}{4\,(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A] time = 0.219274, size = 57, normalized size = 1.78

$$-\frac{2\,Bbx^{2}+Ba+Ab}{4\,(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [A] time = 2.70544, size = 42, normalized size = 1.31

$$-\frac{Ab+Ba+2Bbx^{2}}{4a^{2}b^{2}+8ab^{3}x^{2}+4b^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -(A*b + B*a + 2*B*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x
**4)
```

GIAC/XCAS [A] time = 0.222838, size = 38, normalized size = 1.19

$$-\frac{2\,Bbx^{2}+Ba+Ab}{4\,(bx^{2}+a)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/((b*x^2 + a)^2*b^2)
```

3.94 $\int \frac{A+Bx^2}{x(a+bx^2)^3} dx$

Optimal. Leaf size=68

$$-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{A}{2a^2\left(a+bx^2\right)}+\frac{Ab-aB}{4ab\left(a+bx^2\right)^2}$$

```
[Out] (A*b - a*B)/(4*a*b*(a + b*x^2)^2) + A/(2*a^2*(a + b*x^2)) + (A*Lo
g[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.143386, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{A}{2a^2\left(a+bx^2\right)}+\frac{Ab-aB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*B)/(4*a*b*(a + b*x^2)^2) + A/(2*a^2*(a + b*x^2)) + (A*Lo
g[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 20.1948, size = 60, normalized size = 0.88

$$\frac{A}{2a^2\left(a+bx^2\right)}+\frac{A\log\left(x^2\right)}{2a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{Ab-Ba}{4ab\left(a+bx^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A/(2*a**2*(a + b*x**2)) + A*log(x**2)/(2*a**3) - A*log(a + b*x**2
)/(2*a**3) + (A*b - B*a)/(4*a*b*(a + b*x**2)**2)
```

Mathematica [A] time = 0.0773885, size = 59, normalized size = 0.87

$$\frac{\frac{a\left(a^2(-B)+3aAb+2Ab^2x^2\right)}{b\left(a+bx^2\right)^2}-2A\log\left(a+bx^2\right)+4A\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^2))/(b*(a + b*x^2)^2) + 4*A*Log[
x] - 2*A*Log[a + b*x^2])/(4*a^3)
```

Maple [A] time = 0.018, size = 68, normalized size = 1.

$$\frac{A\ln(x)}{a^3}+\frac{A}{4\,a\left(bx^2+a\right)^2}-\frac{B}{4\,b\left(bx^2+a\right)^2}-\frac{A\ln\left(bx^2+a\right)}{2\,a^3}+\frac{A}{2\,a^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(b*x^2+a)^3,x)
```

```
[Out] A*ln(x)/a^3+1/4/a/(b*x^2+a)^2*A-1/4/b/(b*x^2+a)^2*B-1/2*A*ln(b*x^
2+a)/a^3+1/2*A/a^2/(b*x^2+a)
```

Maxima [A] time = 1.34198, size = 104, normalized size = 1.53

$$\frac{2\,Ab^2x^2 - Ba^2 + 3\,Aab}{4\,(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b)} - \frac{A\log\left(bx^2 + a\right)}{2\,a^3} + \frac{A\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*b^2*x^2 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2
+ a^4*b) - 1/2*A*log(b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3
```

Fricas [A] time = 0.23246, size = 161, normalized size = 2.37

$$\frac{2\,Aab^2x^2 - Ba^3 + 3\,Aa^2b - 2\left(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b\right)\log\left(bx^2 + a\right) + 4\left(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b\right)\log\left(x\right)}{4\,(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*a*b^2*x^2 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^4 + 2*A*a*b^2
*x^2 + A*a^2*b)*log(b*x^2 + a) + 4*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A
*a^2*b)*log(x))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [A] time = 2.94906, size = 75, normalized size = 1.1

$$\frac{A\log\left(x\right)}{a^3} - \frac{A\log\left(\frac{a}{b} + x^2\right)}{2a^3} + \frac{3Aab + 2Ab^2x^2 - Ba^2}{4a^4b + 8a^3b^2x^2 + 4a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**2)/(2*a**3) + (3*A*a*b + 2*A*b**2*
x**2 - B*a**2)/(4*a**4*b + 8*a**3*b**2*x**2 + 4*a**2*b**3*x**4)
```

GIAC/XCAS [A] time = 0.225429, size = 103, normalized size = 1.51

$$\frac{A\ln\left(x^2\right)}{2\,a^3} - \frac{A\ln\left(\left|bx^2 + a\right|\right)}{2\,a^3} + \frac{3\,Ab^3x^4 + 8\,Aab^2x^2 - Ba^3 + 6\,Aa^2b}{4\,(bx^2 + a)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x),x, algorithm="giac")
```

```
[Out] 1/2*A*ln(x^2)/a^3 - 1/2*A*ln(abs(b*x^2 + a))/a^3 + 1/4*(3*A*b^3*x
^4 + 8*A*a*b^2*x^2 - B*a^3 + 6*A*a^2*b)/((b*x^2 + a)^2*a^3*b)
```

3.95 $\int \frac{A+Bx^2}{x^3(a+bx^2)^3}\,dx$

Optimal. Leaf size=101

$$\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}$$

```
[Out] -A/(2*a^3*x^2) - (A*b - a*B)/(4*a^2*(a + b*x^2)^2) - (2*A*b - a*B
)/(2*a^3*(a + b*x^2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*
B)*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.245005, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - (A*b - a*B)/(4*a^2*(a + b*x^2)^2) - (2*A*b - a*B
)/(2*a^3*(a + b*x^2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*
B)*Log[a + b*x^2])/(2*a^4)
```

Rubi in Sympy [A] time = 26.6393, size = 90, normalized size = 0.89

$$-\frac{A}{2a^3x^2}-\frac{Ab-Ba}{4a^2\left(a+bx^2\right)^2}-\frac{2Ab-Ba}{2a^3\left(a+bx^2\right)}-\frac{(3Ab-Ba)\log\left(x^2\right)}{2a^4}+\frac{(3Ab-Ba)\log\left(a+bx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] -A/(2*a**3*x**2) - (A*b - B*a)/(4*a**2*(a + b*x**2)**2) - (2*A*b
- B*a)/(2*a**3*(a + b*x**2)) - (3*A*b - B*a)*log(x**2)/(2*a**4) +
 (3*A*b - B*a)*log(a + b*x**2)/(2*a**4)
```

Mathematica [A] time = 0.0955015, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{\left(a+bx^2\right)^2}+\frac{2a(aB-2Ab)}{a+bx^2}+2(3Ab-aB)\log\left(a+bx^2\right)+4\log(x)(aB-3Ab)-\frac{2aA}{x^2}}{4a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a*A)/x^2 + (a^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (2*a*(-2*A*b
 + a*B))/(a + b*x^2) + 4*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*
Log[a + b*x^2])/(4*a^4)
```

Maple [A] time = 0.022, size = 118, normalized size = 1.2

$$-\frac{A}{2\,a^{3}x^{2}}-3\,\frac{A\ln\left(x\right)b}{a^{4}}+\frac{\ln\left(x\right)B}{a^{3}}-\frac{Ab}{4\,a^{2}\left(bx^{2}+a\right)^{2}}+\frac{B}{4\,a\left(bx^{2}+a\right)^{2}}+\frac{3\,b\ln\left(bx^{2}+a\right)A}{2\,a^{4}}-\frac{\ln\left(bx^{2}+a\right)B}{2\,a^{3}}-\frac{Ab}{a^{3}\left(bx^{2}+a\right)}+\frac{B}{2\,a^{2}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*A/a^3/x^2-3/a^4*ln(x)*A*b+1/a^3*ln(x)*B-1/4/a^2*b/(b*x^2+a)^
2*A+1/4/a/(b*x^2+a)^2*B+3/2/a^4*b*ln(b*x^2+a)*A-1/2/a^3*ln(b*x^2+
a)*B-1/a^3*b*A/(b*x^2+a)+1/2/a^2/(b*x^2+a)*B
```

Maxima [A] time = 1.34389, size = 147, normalized size = 1.46

$$\frac{2\left(Bab-3\,Ab^{2}\right)x^{4}-2\,Aa^{2}+3\left(Ba^{2}-3\,Aab\right)x^{2}}{4\left(a^{3}b^{2}x^{6}+2\,a^{4}bx^{4}+a^{5}x^{2}\right)}-\frac{\left(Ba-3\,Ab\right)\log\left(bx^{2}+a\right)}{2\,a^{4}}+\frac{\left(Ba-3\,Ab\right)\log\left(x^{2}\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*a*b - 3*A*b^2)*x^4 - 2*A*a^2 + 3*(B*a^2 - 3*A*a*b)*x^2)
/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2) - 1/2*(B*a - 3*A*b)*log(b*
x^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4
```

Fricas [A] time = 0.220231, size = 266, normalized size = 2.63

$$\frac{2\left(Ba^{2}b-3\,Aab^{2}\right)x^{4}-2\,Aa^{3}+3\left(Ba^{3}-3\,Aa^{2}b\right)x^{2}-2\left(\left(Bab^{2}-3\,Ab^{3}\right)x^{6}+2\left(Ba^{2}b-3\,Aab^{2}\right)x^{4}+\left(Ba^{3}-3\,Aa^{2}b\right)x^{2}\right)\log}{4\left(a^{4}b^{2}x^{6}+2\,a^{5}bx^{4}+a^{6}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a^2*b - 3*A*a*b^2)*x^4 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b
)*x^2 - 2*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4
+ (B*a^3 - 3*A*a^2*b)*x^2)*log(b*x^2 + a) + 4*((B*a*b^2 - 3*A*b^3
)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3*A*a^2*b)*x^2)*lo
g(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A] time = 5.1195, size = 107, normalized size = 1.06

$$\frac{-2Aa^{2}+x^{4}\left(-6Ab^{2}+2Bab\right)+x^{2}\left(-9Aab+3Ba^{2}\right)}{4a^{5}x^{2}+8a^{4}bx^{4}+4a^{3}b^{2}x^{6}}+\frac{\left(-3Ab+Ba\right)\log\left(x\right)}{a^{4}}-\frac{\left(-3Ab+Ba\right)\log\left(\frac{a}{b}+x^{2}\right)}{2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 2*B*a*b) + x**2*(-9*A*a*b + 3*B*a*
*2))/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3*b**2*x**6) + (-3*A*b +
 B*a)*log(x)/a**4 - (-3*A*b + B*a)*log(a/b + x**2)/(2*a**4)
```

GIAC/XCAS [A] time = 0.224067, size = 186, normalized size = 1.84

$$\frac{(Ba - 3\,Ab)\ln\left(x^2\right)}{2\,a^4} - \frac{\left(Bab - 3\,Ab^2\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,a^4 b} + \frac{3\,Bab^2x^4 - 9\,Ab^3x^4 + 8\,Ba^2bx^2 - 22\,Aab^2x^2 + 6\,Ba^3 - 14\,Aa^2b}{4\,(bx^2 + a)^2 a^4} - \frac{Bax^2 - 3\,Abx^2 + Aa}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*ln(x^2)/a^4 - 1/2*(B*a*b - 3*A*b^2)*ln(abs(b*x^
2 + a))/(a^4*b) + 1/4*(3*B*a*b^2*x^4 - 9*A*b^3*x^4 + 8*B*a^2*b*x^
2 - 22*A*a*b^2*x^2 + 6*B*a^3 - 14*A*a^2*b)/((b*x^2 + a)^2*a^4) -
1/2*(B*a*x^2 - 3*A*b*x^2 + A*a)/(a^4*x^2)
```

3.96 $\int \frac{A+Bx^2}{x^5(a+bx^2)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{4a^3x^4}$$

```
[Out] -A/(4*a^3*x^4) + (3*A*b - a*B)/(2*a^4*x^2) + (b*(A*b - a*B))/(4*a
^3*(a + b*x^2)^2) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x^2)) + (3*
b*(2*A*b - a*B)*Log[x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/
(2*a^5)
```

Rubi [A] time = 0.297223, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -A/(4*a^3*x^4) + (3*A*b - a*B)/(2*a^4*x^2) + (b*(A*b - a*B))/(4*a
^3*(a + b*x^2)^2) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x^2)) + (3*
b*(2*A*b - a*B)*Log[x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/
(2*a^5)
```

Rubi in Sympy [A] time = 32.9892, size = 119, normalized size = 0.96

$$-\frac{A}{4a^3x^4}+\frac{b\left(Ab-Ba\right)}{4a^3\left(a+bx^2\right)^2}+\frac{b\left(3Ab-2Ba\right)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-Ba}{2a^4x^2}+\frac{3b\left(2Ab-Ba\right)\log\left(x^2\right)}{2a^5}-\frac{3b\left(2Ab-Ba\right)\log\left(a+bx^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(b*x**2+a)**3,x)
```

```
[Out] -A/(4*a**3*x**4) + b*(A*b - B*a)/(4*a**3*(a + b*x**2)**2) + b*(3*
A*b - 2*B*a)/(2*a**4*(a + b*x**2)) + (3*A*b - B*a)/(2*a**4*x**2)
+ 3*b*(2*A*b - B*a)*log(x**2)/(2*a**5) - 3*b*(2*A*b - B*a)*log(a
+ b*x**2)/(2*a**5)
```

Mathematica [A] time = 0.13127, size = 108, normalized size = 0.87

$$\frac{\frac{a^2b(Ab-aB)}{\left(a+bx^2\right)^2}-\frac{a^2A}{x^4}+\frac{2ab(3Ab-2aB)}{a+bx^2}-\frac{2a(aB-3Ab)}{x^2}+6b(aB-2Ab)\log\left(a+bx^2\right)+12b\log(x)(2Ab-aB)}{4a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] (-((a^2*A)/x^4) - (2*a*(-3*A*b + a*B))/x^2 + (a^2*b*(A*b - a*B))/
(a + b*x^2)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^2) + 12*b*(2*A*b
 - a*B)*Log[x] + 6*b*(-2*A*b + a*B)*Log[a + b*x^2])/(4*a^5)
```

Maple [A] time = 0.023, size = 150, normalized size = 1.2

$$-\frac{A}{4a^3x^4}+\frac{3Ab}{2a^4x^2}-\frac{B}{2a^3x^2}+6\frac{A\ln(x)b^2}{a^5}-3\frac{bB\ln(x)}{a^4}+\frac{b^2A}{4a^3(bx^2+a)^2}-\frac{Bb}{4a^2(bx^2+a)^2}-3\frac{b^2\ln(bx^2+a)A}{a^5}+\frac{3b\ln(bx^2+a)B}{2a^4}+\frac{3b^2A}{2a^4(bx^2+a)}-\frac{Bb}{a^3(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(b*x^2+a)^3,x)
```

```
[Out] -1/4*A/a^3/x^4+3/2/a^4/x^2*A*b-1/2/a^3/x^2*B+6*b^2/a^5*ln(x)*A-3*
b/a^4*ln(x)*B+1/4/a^3*b^2/(b*x^2+a)^2*A-1/4/a^2*b/(b*x^2+a)^2*B-3
/a^5*b^2*ln(b*x^2+a)*A+3/2/a^4*b*ln(b*x^2+a)*B+3/2/a^4*b^2*A/(b*x
^2+a)-1/a^3*b/(b*x^2+a)*B
```

Maxima [A] time = 1.35134, size = 185, normalized size = 1.49

$$-\frac{6\left(Bab^2-2Ab^3\right)x^6+9\left(Ba^2b-2Aab^2\right)x^4+Aa^3+2\left(Ba^3-2Aa^2b\right)x^2}{4\left(a^4b^2x^8+2a^5bx^6+a^6x^4\right)}+\frac{3\left(Bab-2Ab^2\right)\log\left(bx^2+a\right)}{2a^5}-\frac{3\left(Bab-2Ab^2\right)\log\left(x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^5),x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*a*b^2 - 2*A*b^3)*x^6 + 9*(B*a^2*b - 2*A*a*b^2)*x^4 + A
*a^3 + 2*(B*a^3 - 2*A*a^2*b)*x^2)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^
6*x^4) + 3/2*(B*a*b - 2*A*b^2)*log(b*x^2 + a)/a^5 - 3/2*(B*a*b -
2*A*b^2)*log(x^2)/a^5
```

Fricas [A] time = 0.231131, size = 309, normalized size = 2.49

$$-\frac{6\left(Ba^2b^2-2Aab^3\right)x^6+Aa^4+9\left(Ba^3b-2Aa^2b^2\right)x^4+2\left(Ba^4-2Aa^3b\right)x^2-6\left(\left(Bab^3-2Ab^4\right)x^8+2\left(Ba^2b^2-2Aab^3\right)x^6\right.}{4\left(a^5b^2x^8+2a^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^5),x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + A*a^4 + 9*(B*a^3*b - 2*A*a^
2*b^2)*x^4 + 2*(B*a^4 - 2*A*a^3*b)*x^2 - 6*((B*a*b^3 - 2*A*b^4)*x
^8 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x^4)
*log(b*x^2 + a) + 12*((B*a*b^3 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 - 2*
A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x^4)*log(x))/(a^5*b^2*x^8
+ 2*a^6*b*x^6 + a^7*x^4)
```

Sympy [A] time = 7.30592, size = 136, normalized size = 1.1

$$-\frac{Aa^3+x^6\left(-12Ab^3+6Bab^2\right)+x^4\left(-18Aab^2+9Ba^2b\right)+x^2\left(-4Aa^2b+2Ba^3\right)}{4a^6x^4+8a^5bx^6+4a^4b^2x^8}-\frac{3b\left(-2Ab+Ba\right)\log(x)}{a^5}+\frac{3b\left(-2Ab+Ba\right)\log\left(\frac{a}{b}+x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(b*x**2+a)**3,x)
```

```
[Out] -(A*a**3 + x**6*(-12*A*b**3 + 6*B*a*b**2) + x**4*(-18*A*a*b**2 +
9*B*a**2*b) + x**2*(-4*A*a**2*b + 2*B*a**3))/(4*a**6*x**4 + 8*a**
5*b*x**6 + 4*a**4*b**2*x**8) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + 3
*b*(-2*A*b + B*a)*log(a/b + x**2)/(2*a**5)
```

GIAC/XCAS [A] time = 0.237997, size = 180, normalized size = 1.45

$$-\frac{3\left(Bab-2\,Ab^2\right)\ln\left(x^2\right)}{2\,a^5}+\frac{3\left(Bab^2-2\,Ab^3\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^5b}-\frac{6\,Bab^2x^6-12\,Ab^3x^6+9\,Ba^2bx^4-18\,Aab^2x^4+2\,Ba^3x^2-4\,Aa^2bx^2+Aa^3}{4\left(bx^4+ax^2\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^5),x, algorithm="giac")
```

```
[Out] -3/2*(B*a*b - 2*A*b^2)*ln(x^2)/a^5 + 3/2*(B*a*b^2 - 2*A*b^3)*ln(a
bs(b*x^2 + a))/(a^5*b) - 1/4*(6*B*a*b^2*x^6 - 12*A*b^3*x^6 + 9*B*
a^2*b*x^4 - 18*A*a*b^2*x^4 + 2*B*a^3*x^2 - 4*A*a^2*b*x^2 + A*a^3)
/((b*x^4 + a*x^2)^2*a^4)
```

3.97 $\int \frac{A+Bx^2}{x^7(a+bx^2)^3}\,dx$

Optimal. Leaf size=149

$$\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{2a^5x^2}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-aB}{4a^4x^4}-\frac{A}{6a^3x^6}$$

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(4*a^4*x^4) - (3*b*(2*A*b - a*B))/
(2*a^5*x^2) - (b^2*(A*b - a*B))/(4*a^4*(a + b*x^2)^2) - (b^2*(4*A
*b - 3*a*B))/(2*a^5*(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])
/a^6 + (b^2*(5*A*b - 3*a*B)*Log[a + b*x^2])/a^6
```

Rubi [A] time = 0.377389, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{2a^5x^2}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-aB}{4a^4x^4}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(4*a^4*x^4) - (3*b*(2*A*b - a*B))/
(2*a^5*x^2) - (b^2*(A*b - a*B))/(4*a^4*(a + b*x^2)^2) - (b^2*(4*A
*b - 3*a*B))/(2*a^5*(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])
/a^6 + (b^2*(5*A*b - 3*a*B)*Log[a + b*x^2])/a^6
```

Rubi in Sympy [A] time = 43.4647, size = 143, normalized size = 0.96

$$-\frac{A}{6a^3x^6}-\frac{b^2\left(Ab-Ba\right)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-Ba}{4a^4x^4}-\frac{b^2\left(4Ab-3Ba\right)}{2a^5\left(a+bx^2\right)}-\frac{3b\left(2Ab-Ba\right)}{2a^5x^2}-\frac{b^2\left(5Ab-3Ba\right)\log\left(x^2\right)}{a^6}+\frac{b^2\left(5Ab-3Ba\right)\log\left(a+bx^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**7/(b*x**2+a)**3,x)
```

```
[Out] -A/(6*a**3*x**6) - b**2*(A*b - B*a)/(4*a**4*(a + b*x**2)**2) + (3
*A*b - B*a)/(4*a**4*x**4) - b**2*(4*A*b - 3*B*a)/(2*a**5*(a + b*x
**2)) - 3*b*(2*A*b - B*a)/(2*a**5*x**2) - b**2*(5*A*b - 3*B*a)*lo
g(x**2)/a**6 + b**2*(5*A*b - 3*B*a)*log(a + b*x**2)/a**6
```

Mathematica [A] time = 0.223029, size = 135, normalized size = 0.91

$$\frac{-\frac{2a^3A}{x^6}+\frac{3a^2b^2(aB-Ab)}{(a+bx^2)^2}-\frac{3a^2(aB-3Ab)}{x^4}+\frac{6ab^2(3aB-4Ab)}{a+bx^2}+12b^2(5Ab-3aB)\log\left(a+bx^2\right)+24b^2\log(x)(3aB-5Ab)+\frac{18ab(aB-2A}{x^2}}{12a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-3*A*b + a*B))/x^4 + (18*a*b*(-2*A*b +
a*B))/x^2 + (3*a^2*b^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a*b^2*(
-4*A*b + 3*a*B))/(a + b*x^2) + 24*b^2*(-5*A*b + 3*a*B)*Log[x] + 1
2*b^2*(5*A*b - 3*a*B)*Log[a + b*x^2])/(12*a^6)
```

Maple [A] time = 0.025, size = 180, normalized size = 1.2

$$-\frac{A}{6\,a^3x^6}+\frac{3\,Ab}{4\,a^4x^4}-\frac{B}{4\,a^3x^4}-3\,\frac{b^2A}{a^5x^2}+\frac{3\,Bb}{2\,a^4x^2}-10\,\frac{b^3\ln(x)\,A}{a^6}+6\,\frac{b^2B\ln(x)}{a^5}-\frac{Ab^3}{4\,a^4\,(bx^2+a)^2}+\frac{Bb^2}{4\,a^3\,(bx^2+a)^2}+5\,\frac{b^3\ln\left(bx^2+a\right)\,A}{a^6}-3\,\frac{b^2\ln\left(bx^2+a\right)\,B}{a^5}-2\,\frac{Ab^3}{a^5\,(bx^2+a)}+\frac{3\,Bb^2}{2\,a^4\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^7/(b*x^2+a)^3,x)
```

```
[Out] -1/6*A/a^3/x^6+3/4/a^4/x^4*A*b-1/4/a^3/x^4*B-3*b^2/a^5/x^2*A+3/2*
b/a^4/x^2*B-10*b^3/a^6*ln(x)*A+6*b^2/a^5*ln(x)*B-1/4/a^4*b^3/(b*x
^2+a)^2*A+1/4/a^3*b^2/(b*x^2+a)^2*B+5/a^6*b^3*ln(b*x^2+a)*A-3/a^5
*b^2*ln(b*x^2+a)*B-2/a^5*b^3*A/(b*x^2+a)+3/2/a^4*b^2/(b*x^2+a)*B
```

Maxima [A] time = 1.35314, size = 230, normalized size = 1.54

$$\frac{12\left(3\,Bab^3-5\,Ab^4\right)x^8+18\left(3\,Ba^2b^2-5\,Aab^3\right)x^6-2\,Aa^4+4\left(3\,Ba^3b-5\,Aa^2b^2\right)x^4-\left(3\,Ba^4-5\,Aa^3b\right)x^2}{12\left(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6\right)}-\frac{\left(3\,Bab^2-5\,Ab^3\right)\log\left(bx^2+a\right)}{a^6}+\frac{\left(3\,Bab^2-5\,Ab^3\right)\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^7),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*a*b^3 - 5*A*b^4)*x^8 + 18*(3*B*a^2*b^2 - 5*A*a*b^3)
*x^6 - 2*A*a^4 + 4*(3*B*a^3*b - 5*A*a^2*b^2)*x^4 - (3*B*a^4 - 5*A
*a^3*b)*x^2)/(a^5*b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) - (3*B*a*b^2
- 5*A*b^3)*log(b*x^2 + a)/a^6 + (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^
6
```

Fricas [A] time = 0.233327, size = 360, normalized size = 2.42

$$\frac{12\left(3\,Ba^2b^3-5\,Aab^4\right)x^8+18\left(3\,Ba^3b^2-5\,Aa^2b^3\right)x^6-2\,Aa^5+4\left(3\,Ba^4b-5\,Aa^3b^2\right)x^4-\left(3\,Ba^5-5\,Aa^4b\right)x^2-12\left(\left(3\,Bab^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^7),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + 18*(3*B*a^3*b^2 - 5*A*a^
2*b^3)*x^6 - 2*A*a^5 + 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^4 - (3*B*a^5
 - 5*A*a^4*b)*x^2 - 12*((3*B*a*b^4 - 5*A*b^5)*x^10 + 2*(3*B*a^2*b
^3 - 5*A*a*b^4)*x^8 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6)*log(b*x^2
+ a) + 24*((3*B*a*b^4 - 5*A*b^5)*x^10 + 2*(3*B*a^2*b^3 - 5*A*a*b^
4)*x^8 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6)*log(x))/(a^6*b^2*x^10 +
 2*a^7*b*x^8 + a^8*x^6)
```

Sympy [A] time = 10.577, size = 165, normalized size = 1.11

$$\frac{-2Aa^4 + x^8\left(-60Ab^4 + 36Bab^3\right) + x^6\left(-90Aab^3 + 54Ba^2b^2\right) + x^4\left(-20Aa^2b^2 + 12Ba^3b\right) + x^2\left(5Aa^3b - 3Ba^4\right)}{12a^7x^6 + 24a^6bx^8 + 12a^5b^2x^{10}} + \frac{2b^2\left(-5Ab + 3Ba\right)\log\left(x\right)}{a^6} - \frac{b^2\left(-5Ab + 3Ba\right)\log\left(\frac{a}{b} + x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**4 + x**8*(-60*A*b**4 + 36*B*a*b**3) + x**6*(-90*A*a*b**3
 + 54*B*a**2*b**2) + x**4*(-20*A*a**2*b**2 + 12*B*a**3*b) + x**2*
(5*A*a**3*b - 3*B*a**4))/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5
*b**2*x**10) + 2*b**2*(-5*A*b + 3*B*a)*log(x)/a**6 - b**2*(-5*A*b
 + 3*B*a)*log(a/b + x**2)/a**6
```

GIAC/XCAS [A] time = 0.238064, size = 271, normalized size = 1.82

$$\frac{\left(3\,Bab^2 - 5\,Ab^3\right)\ln\left(x^2\right)}{a^6} - \frac{\left(3\,Bab^3 - 5\,Ab^4\right)\ln\left(\left|bx^2 + a\right|\right)}{a^6b} + \frac{18\,Bab^4x^4 - 30\,Ab^5x^4 + 42\,Ba^2b^3x^2 - 68\,Aab^4x^2 + 25\,Ba^3b^2 - 39\,Aa^2b^3}{4\,(bx^2 + a)^2a^6} - \frac{66\,Bab^2x^6 - 110\,Ab^3x^6 - 18\,Ba^2bx^4 + 36\,Aab^2x^4 + 3\,Ba^3x^2 - 9\,Aa^2bx^2 + 2\,Aa^3}{12\,a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^7),x, algorithm="giac")
```

```
[Out] (3*B*a*b^2 - 5*A*b^3)*ln(x^2)/a^6 - (3*B*a*b^3 - 5*A*b^4)*ln(abs(
b*x^2 + a))/(a^6*b) + 1/4*(18*B*a*b^4*x^4 - 30*A*b^5*x^4 + 42*B*a
^2*b^3*x^2 - 68*A*a*b^4*x^2 + 25*B*a^3*b^2 - 39*A*a^2*b^3)/((b*x^
2 + a)^2*a^6) - 1/12*(66*B*a*b^2*x^6 - 110*A*b^3*x^6 - 18*B*a^2*b
*x^4 + 36*A*a*b^2*x^4 + 3*B*a^3*x^2 - 9*A*a^2*b*x^2 + 2*A*a^3)/(a
^6*x^6)
```

3.98 $\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=158

$$-\frac{9a^{5/2}(7Ab-11aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{ax^3(Ab-2aB)}{b^5}+\frac{x^5(Ab-3aB)}{5b^4}+\frac{Bx^7}{7b^3}$$

```
[Out] (2*a^2*(3*A*b - 5*a*B)*x)/b^6 - (a*(A*b - 2*a*B)*x^3)/b^5 + ((A*b
 - 3*a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) - (a^4*(A*b - a*B)*x)/(4
*b^6*(a + b*x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2
)) - (9*a^(5/2)*(7*A*b - 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*
b^(13/2))
```

Rubi [A] time = 0.518603, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{9a^{5/2}(7Ab-11aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{ax^3(Ab-2aB)}{b^5}+\frac{x^5(Ab-3aB)}{5b^4}+\frac{Bx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*a^2*(3*A*b - 5*a*B)*x)/b^6 - (a*(A*b - 2*a*B)*x^3)/b^5 + ((A*b
 - 3*a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) - (a^4*(A*b - a*B)*x)/(4
*b^6*(a + b*x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2
)) - (9*a^(5/2)*(7*A*b - 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*
b^(13/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.15021, size = 158, normalized size = 1.

$$\frac{9a^{5/2}(11aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}+\frac{a^4x(aB-Ab)}{4b^6\left(a+bx^2\right)^2}+\frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)}-\frac{2a^2x(5aB-3Ab)}{b^6}+\frac{ax^3(2aB-Ab)}{b^5}+\frac{x^5(Ab-3aB)}{5b^4}+\frac{Bx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-2*a^2*(-3*A*b + 5*a*B)*x)/b^6 + (a*(-(A*b) + 2*a*B)*x^3)/b^5 +
((A*b - 3*a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) + (a^4*(-(A*b) + a*
B)*x)/(4*b^6*(a + b*x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a
 + b*x^2)) + (9*a^(5/2)*(-7*A*b + 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(8*b^(13/2))
```

Maple [A] time = 0.016, size = 198, normalized size = 1.3

$$\frac{Bx^7}{7\,b^3}+\frac{Ax^5}{5\,b^3}-\frac{3\,Bx^5a}{5\,b^4}-\frac{aAx^3}{b^4}+2\,\frac{Bx^3a^2}{b^5}+6\,\frac{a^2Ax}{b^5}-10\,\frac{Ba^3x}{b^6}+\frac{17\,Aa^3x^3}{8\,b^4\,(bx^2+a)^2}-\frac{21\,Ba^4x^3}{8\,b^5\,(bx^2+a)^2}+\frac{15\,Aa^4x}{8\,b^5\,(bx^2+a)^2}-\frac{19\,Bxa^5}{8\,b^6\,(bx^2+a)^2}-\frac{63\,Aa^3}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{99\,Ba^4}{8\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*B*x^7/b^3+1/5/b^3*A*x^5-3/5/b^4*B*x^5*a-1/b^4*A*x^3*a+2/b^5*B
*x^3*a^2+6/b^5*A*a^2*x-10/b^6*B*a^3*x+17/8*a^3/b^4/(b*x^2+a)^2*A*
x^3-21/8*a^4/b^5/(b*x^2+a)^2*B*x^3+15/8*a^4/b^5/(b*x^2+a)^2*A*x-1
9/8*a^5/b^6/(b*x^2+a)^2*B*x-63/8*a^3/b^5/(a*b)^(1/2)*arctan(x*b/(
a*b)^(1/2))*A+99/8*a^4/b^6/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^10/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235132, size = 1, normalized size = 0.01

$$\left[\frac{80\,Bb^5x^{11}-16\left(11\,Bab^4-7\,Ab^5\right)x^9+48\left(11\,Ba^2b^3-7\,Aab^4\right)x^7-336\left(11\,Ba^3b^2-7\,Aa^2b^3\right)x^5-1050\left(11\,Ba^4b-7\,Aa^3b^2\right.}{560\,(b^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^10/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*B*b^5*x^11 - 16*(11*B*a*b^4 - 7*A*b^5)*x^9 + 48*(11*B*
a^2*b^3 - 7*A*a*b^4)*x^7 - 336*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 -
 1050*(11*B*a^4*b - 7*A*a^3*b^2)*x^3 - 315*(11*B*a^5 - 7*A*a^4*b
+ (11*B*a^3*b^2 - 7*A*a^2*b^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)
*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
- 630*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)
, 1/280*(40*B*b^5*x^11 - 8*(11*B*a*b^4 - 7*A*b^5)*x^9 + 24*(11*B*
a^2*b^3 - 7*A*a*b^4)*x^7 - 168*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 -
 525*(11*B*a^4*b - 7*A*a^3*b^2)*x^3 + 315*(11*B*a^5 - 7*A*a^4*b +
 (11*B*a^3*b^2 - 7*A*a^2*b^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*
x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 315*(11*B*a^5 - 7*A*a^4*b)*x
)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)]
```

Sympy [A] time = 6.63615, size = 274, normalized size = 1.73

$$\frac{Bx^7}{7b^3} - \frac{9\sqrt{-\frac{a^5}{b^{13}}}\left(-7Ab+11Ba\right)\log\left(-\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16} + \frac{9\sqrt{-\frac{a^5}{b^{13}}}\left(-7Ab+11Ba\right)\log\left(\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16} - \frac{x^3\left(-17Aa^3b^2+21Ba^4b\right)+x\left(-15Aa^4b+19Ba^5\right)}{8a^2b^6+16ab^7x^2+8b^8x^4} - \frac{x^5\left(-Ab+3Ba\right)}{5b^4} + \frac{x^3\left(-Aab+2Ba^2\right)}{b^5} - \frac{x\left(-6Aa^2b+10Ba^3\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**7/(7*b**3) - 9*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)*log(-9*b*
*6*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*b + 99*B*a**3)
 + x)/16 + 9*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)*log(9*b**6*sqrt(
-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*b + 99*B*a**3) + x)/16
 - (x**3*(-17*A*a**3*b**2 + 21*B*a**4*b) + x*(-15*A*a**4*b + 19*B
*a**5))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4) - x**5*(-A*b
 + 3*B*a)/(5*b**4) + x**3*(-A*a*b + 2*B*a**2)/b**5 - x*(-6*A*a**2
*b + 10*B*a**3)/b**6
```

GIAC/XCAS [A] time = 0.229374, size = 219, normalized size = 1.39

$$\frac{9\left(11\,Ba^4-7\,Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6} - \frac{21\,Ba^4bx^3-17\,Aa^3b^2x^3+19\,Ba^5x-15\,Aa^4bx}{8\,(bx^2+a)^2b^6} + \frac{5\,Bb^{18}x^7-21\,Bab^{17}x^5+7\,Ab^{18}x^5+70\,Ba^2b^{16}x^3-35\,Aab^{17}x^3-350\,Ba^3b^{15}x+210\,Aa^2b^{16}x}{35\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^10/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6)
- 1/8*(21*B*a^4*b*x^3 - 17*A*a^3*b^2*x^3 + 19*B*a^5*x - 15*A*a^4*
b*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*B*b^18*x^7 - 21*B*a*b^17*x^5 +
 7*A*b^18*x^5 + 70*B*a^2*b^16*x^3 - 35*A*a*b^17*x^3 - 350*B*a^3*b
^15*x + 210*A*a^2*b^16*x)/b^21
```

3.99 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=138

$$\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{a^3x(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2x(13Ab-17aB)}{8b^5(a+bx^2)}-\frac{3ax(Ab-2aB)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^5}{5b^3}$$

```
[Out] (-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^5
)/(5*b^3) + (a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - (a^2*(13*
A*b - 17*a*B)*x)/(8*b^5*(a + b*x^2)) + (7*a^(3/2)*(5*A*b - 9*a*B)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2))
```

Rubi [A] time = 0.403303, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{a^3x(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2x(13Ab-17aB)}{8b^5(a+bx^2)}-\frac{3ax(Ab-2aB)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^5
)/(5*b^3) + (a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - (a^2*(13*
A*b - 17*a*B)*x)/(8*b^5*(a + b*x^2)) + (7*a^(3/2)*(5*A*b - 9*a*B)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2))
```

Rubi in Sympy [A] time = 129.032, size = 133, normalized size = 0.96

$$\frac{Bx^5}{5b^3}+\frac{7a^{\frac{3}{2}}\left(5Ab-9Ba\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{11}{2}}}+\frac{a^3x\left(Ab-Ba\right)}{4b^5\left(a+bx^2\right)^2}-\frac{a^2x\left(13Ab-17Ba\right)}{8b^5\left(a+bx^2\right)}-\frac{3ax\left(Ab-2Ba\right)}{b^5}+\frac{x^3\left(Ab-3Ba\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + 7*a**(3/2)*(5*A*b - 9*B*a)*atan(sqrt(b)*x/sqrt(
a))/(8*b**(11/2)) + a**3*x*(A*b - B*a)/(4*b**5*(a + b*x**2)**2) -
 a**2*x*(13*A*b - 17*B*a)/(8*b**5*(a + b*x**2)) - 3*a*x*(A*b - 2*
B*a)/b**5 + x**3*(A*b - 3*B*a)/(3*b**4)
```

Mathematica [A] time = 0.199172, size = 133, normalized size = 0.96

$$\frac{x\left(945a^4B-525a^3b\left(A-3Bx^2\right)+7a^2b^2x^2\left(72Bx^2-125A\right)-8ab^3x^4\left(35A+9Bx^2\right)+8b^4x^6\left(5A+3Bx^2\right)\right)}{120b^5\left(a+bx^2\right)^2}-\frac{7a^{3/2}(9aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*B - 525*a^3*b*(A - 3*B*x^2) + 8*b^4*x^6*(5*A + 3*B*x^
2) - 8*a*b^3*x^4*(35*A + 9*B*x^2) + 7*a^2*b^2*x^2*(-125*A + 72*B*
x^2)))/(120*b^5*(a + b*x^2)^2) - (7*a^(3/2)*(-5*A*b + 9*a*B)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2))
```

Maple [A] time = 0.015, size = 174, normalized size = 1.3

$$\frac{Bx^5}{5\,b^3}+\frac{Ax^3}{3\,b^3}-\frac{Bx^3a}{b^4}-3\,\frac{aAx}{b^4}+6\,\frac{Bxa^2}{b^5}-\frac{13\,a^2Ax^3}{8\,b^3\,(bx^2+a)^2}+\frac{17\,Ba^3x^3}{8\,b^4\,(bx^2+a)^2}-\frac{11\,Aa^3x}{8\,b^4\,(bx^2+a)^2}+\frac{15\,Ba^4x}{8\,b^5\,(bx^2+a)^2}+\frac{35\,Aa^2}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{63\,Ba^3}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*B*x^5/b^3+1/3/b^3*A*x^3-1/b^4*B*x^3*a-3/b^4*A*x*a+6/b^5*B*x*a
^2-13/8*a^2/b^3/(b*x^2+a)^2*A*x^3+17/8*a^3/b^4/(b*x^2+a)^2*B*x^3-
11/8*a^3/b^4/(b*x^2+a)^2*A*x+15/8*a^4/b^5/(b*x^2+a)^2*B*x+35/8*a^
2/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A-63/8*a^3/b^5/(a*b)^(1
/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^8/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239624, size = 1, normalized size = 0.01

$$\left[\frac{48\,Bb^4x^9-16\left(9\,Bab^3-5\,Ab^4\right)x^7+112\left(9\,Ba^2b^2-5\,Aab^3\right)x^5+350\left(9\,Ba^3b-5\,Aa^2b^2\right)x^3-105\left(9\,Ba^4-5\,Aa^3b+\left(9\,Ba^2\right.\right.}{240\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^8/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*b^4*x^9 - 16*(9*B*a*b^3 - 5*A*b^4)*x^7 + 112*(9*B*a^
2*b^2 - 5*A*a*b^3)*x^5 + 350*(9*B*a^3*b - 5*A*a^2*b^2)*x^3 - 105*
(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 2*(9*B*a^3
*b - 5*A*a^2*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 210*(9*B*a^4 - 5*A*a^3*b)*x)/(b^7*x^4 + 2*a*b^
6*x^2 + a^2*b^5), 1/120*(24*B*b^4*x^9 - 8*(9*B*a*b^3 - 5*A*b^4)*x
^7 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^5 + 175*(9*B*a^3*b - 5*A*a^2*
b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x
^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b
)) + 105*(9*B*a^4 - 5*A*a^3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^
5)]
```

Sympy [A] time = 6.232, size = 250, normalized size = 1.81

$$\frac{Bx^5}{5b^3} + \frac{7\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)\log\left(-\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16} - \frac{7\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)\log\left(\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16} + \frac{x^3\left(-13Aa^2b^2+17Ba^3b\right)+x\left(-11Aa^3b+15Ba^4\right)}{8a^2b^5+16ab^6x^2+8b^7x^4} - \frac{x^3(-Ab+3Ba)}{3b^4} + \frac{x\left(-3Aab+6Ba^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + 7*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)*log(-7*b**
5*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)/(-35*A*a*b + 63*B*a**2) + x)
/16 - 7*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)*log(7*b**5*sqrt(-a**3/
b**11)*(-5*A*b + 9*B*a)/(-35*A*a*b + 63*B*a**2) + x)/16 + (x**3*(
-13*A*a**2*b**2 + 17*B*a**3*b) + x*(-11*A*a**3*b + 15*B*a**4))/(8
*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4) - x**3*(-A*b + 3*B*a)/
(3*b**4) + x*(-3*A*a*b + 6*B*a**2)/b**5
```

GIAC/XCAS [A] time = 0.2287, size = 186, normalized size = 1.35

$$-\frac{7\left(9\,Ba^3-5\,Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}b^5} + \frac{17\,Ba^3bx^3-13\,Aa^2b^2x^3+15\,Ba^4x-11\,Aa^3bx}{8\left(bx^2+a\right)^2b^5} + \frac{3\,Bb^{12}x^5-15\,Bab^{11}x^3+5\,Ab^{12}x^3+90\,Ba^2b^{10}x-45\,Aab^{11}x}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^8/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5)
+ 1/8*(17*B*a^3*b*x^3 - 13*A*a^2*b^2*x^3 + 15*B*a^4*x - 11*A*a^3*
b*x)/((b*x^2 + a)^2*b^5) + 1/15*(3*B*b^12*x^5 - 15*B*a*b^11*x^3 +
 5*A*b^12*x^3 + 90*B*a^2*b^10*x - 45*A*a*b^11*x)/b^15
```

3.100 $\int \frac{x^6 (A+Bx^2)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=116

$$-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}-\frac{5\sqrt{a}(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{ax(9Ab-13aB)}{8b^4(a+bx^2)}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^3}{3b^3}$$

```
[Out] ((A*b - 3*a*B)*x)/b^4 + (B*x^3)/(3*b^3) - (a^2*(A*b - a*B)*x)/(4*
b^4*(a + b*x^2)^2) + (a*(9*A*b - 13*a*B)*x)/(8*b^4*(a + b*x^2)) -
 (5*Sqrt[a]*(3*A*b - 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/
2))
```

Rubi [A] time = 0.312935, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}-\frac{5\sqrt{a}(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{ax(9Ab-13aB)}{8b^4(a+bx^2)}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 + (B*x^3)/(3*b^3) - (a^2*(A*b - a*B)*x)/(4*
b^4*(a + b*x^2)^2) + (a*(9*A*b - 13*a*B)*x)/(8*b^4*(a + b*x^2)) -
 (5*Sqrt[a]*(3*A*b - 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/
2))
```

Rubi in Sympy [A] time = 88.7129, size = 110, normalized size = 0.95

$$\frac{Bx^3}{3b^3}-\frac{5\sqrt{a}\,(3Ab-7Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{9}{2}}}-\frac{a^2x\,(Ab-Ba)}{4b^4\,(a+bx^2)^2}+\frac{ax\,(9Ab-13Ba)}{8b^4\,(a+bx^2)}+\frac{x\,(Ab-3Ba)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) - 5*sqrt(a)*(3*A*b - 7*B*a)*atan(sqrt(b)*x/sqrt(a
))/(8*b**(9/2)) - a**2*x*(A*b - B*a)/(4*b**4*(a + b*x**2)**2) + a
*x*(9*A*b - 13*B*a)/(8*b**4*(a + b*x**2)) + x*(A*b - 3*B*a)/b**4
```

Mathematica [A] time = 0.150343, size = 113, normalized size = 0.97

$$\frac{-105a^3Bx+5a^2bx\left(9A-35Bx^2\right)+ab^2x^3\left(75A-56Bx^2\right)+8b^3x^5\left(3A+Bx^2\right)}{24b^4(a+bx^2)^2}+\frac{5\sqrt{a}(7aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-105*a^3*B*x + a*b^2*x^3*(75*A - 56*B*x^2) + 5*a^2*b*x*(9*A - 35
*B*x^2) + 8*b^3*x^5*(3*A + B*x^2))/(24*b^4*(a + b*x^2)^2) + (5*Sq
rt[a]*(-3*A*b + 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Maple [A] time = 0.016, size = 147, normalized size = 1.3

$$\frac{Bx^3}{3\,b^3}+\frac{Ax}{b^3}-3\,\frac{Bxa}{b^4}+\frac{9\,aAx^3}{8\,b^2\,(bx^2+a)^2}-\frac{13\,a^2Bx^3}{8\,b^3\,(bx^2+a)^2}+\frac{7\,a^2Ax}{8\,b^3\,(bx^2+a)^2}-\frac{11\,Ba^3x}{8\,b^4\,(bx^2+a)^2}-\frac{15\,Aa}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,a^2B}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*B*x^3/b^3+1/b^3*A*x-3/b^4*B*x*a+9/8*a/b^2/(b*x^2+a)^2*A*x^3-1
3/8*a^2/b^3/(b*x^2+a)^2*B*x^3+7/8*a^2/b^3/(b*x^2+a)^2*A*x-11/8*a^
3/b^4/(b*x^2+a)^2*B*x-15/8*a/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*A+35/8*a^2/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233512, size = 1, normalized size = 0.01

$$\left[\frac{16\,Bb^3x^7-16\left(7\,Bab^2-3\,Ab^3\right)x^5-50\left(7\,Ba^2b-3\,Aab^2\right)x^3-15\left(\left(7\,Bab^2-3\,Ab^3\right)x^4+7\,Ba^3-3\,Aa^2b+2\left(7\,Ba^2b-3\,Ac\right.\right.}{48\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*b^3*x^7 - 16*(7*B*a*b^2 - 3*A*b^3)*x^5 - 50*(7*B*a^2*
b - 3*A*a*b^2)*x^3 - 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*
A*a^2*b + 2*(7*B*a^2*b - 3*A*a*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 -
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 30*(7*B*a^3 - 3*A*a^2*b)*x)/
(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*(8*B*b^3*x^7 - 8*(7*B*a*b
^2 - 3*A*b^3)*x^5 - 25*(7*B*a^2*b - 3*A*a*b^2)*x^3 + 15*((7*B*a*b
^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2*b - 3*A*a*b^
2)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 15*(7*B*a^3 - 3*A*a^2*b)*
x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [A] time = 5.87337, size = 212, normalized size = 1.83

$$\frac{Bx^3}{3b^3} - \frac{5\sqrt{-\frac{a}{b^9}}\left(-3Ab + 7Ba\right)\log\left(-\frac{5b^4\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)}{-15Ab+35Ba} + x\right)}{16} + \frac{5\sqrt{-\frac{a}{b^9}}\left(-3Ab + 7Ba\right)\log\left(\frac{5b^4\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)}{-15Ab+35Ba} + x\right)}{16} - \frac{x^3\left(-9Aab^2 + 13Ba^2b\right) + x\left(-7Aa^2b + 11Ba^3\right)}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4} - \frac{x\left(-Ab + 3Ba\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) - 5*sqrt(-a/b**9)*(-3*A*b + 7*B*a)*log(-5*b**4*sq
rt(-a/b**9)*(-3*A*b + 7*B*a)/(-15*A*b + 35*B*a) + x)/16 + 5*sqrt(
-a/b**9)*(-3*A*b + 7*B*a)*log(5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*
a)/(-15*A*b + 35*B*a) + x)/16 - (x**3*(-9*A*a*b**2 + 13*B*a**2*b)
 + x*(-7*A*a**2*b + 11*B*a**3))/(8*a**2*b**4 + 16*a*b**5*x**2 + 8
*b**6*x**4) - x*(-A*b + 3*B*a)/b**4
```

GIAC/XCAS [A] time = 0.224981, size = 150, normalized size = 1.29

$$\frac{5\left(7\,Ba^2 - 3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} - \frac{13\,Ba^2bx^3 - 9\,Aab^2x^3 + 11\,Ba^3x - 7\,Aa^2bx}{8\,(bx^2 + a)^2b^4} + \frac{Bb^6x^3 - 9\,Bab^5x + 3\,Ab^6x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1
/8*(13*B*a^2*b*x^3 - 9*A*a*b^2*x^3 + 11*B*a^3*x - 7*A*a^2*b*x)/((
b*x^2 + a)^2*b^4) + 1/3*(B*b^6*x^3 - 9*B*a*b^5*x + 3*A*b^6*x)/b^9
```

3.101 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=94

$$\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{x(5Ab-9aB)}{8b^3\left(a+bx^2\right)}+\frac{ax(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{Bx}{b^3}$$

```
[Out] (B*x)/b^3 + (a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - ((5*A*b - 9
*a*B)*x)/(8*b^3*(a + b*x^2)) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.208867, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}-\frac{x(5Ab-9aB)}{8b^3\left(a+bx^2\right)}+\frac{ax(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{Bx}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x)/b^3 + (a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - ((5*A*b - 9
*a*B)*x)/(8*b^3*(a + b*x^2)) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2))
```

Rubi in Sympy [A] time = 44.7878, size = 88, normalized size = 0.94

$$\frac{Bx}{b^3}+\frac{ax\left(Ab-Ba\right)}{4b^3\left(a+bx^2\right)^2}-\frac{x\left(5Ab-9Ba\right)}{8b^3\left(a+bx^2\right)}+\frac{3\left(Ab-5Ba\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x/b**3 + a*x*(A*b - B*a)/(4*b**3*(a + b*x**2)**2) - x*(5*A*b -
9*B*a)/(8*b**3*(a + b*x**2)) + 3*(A*b - 5*B*a)*atan(sqrt(b)*x/sqr
t(a))/(8*sqrt(a)*b**(7/2))
```

Mathematica [A] time = 0.121026, size = 91, normalized size = 0.97

$$\frac{x\left(15a^2B+a\left(25bBx^2-3Ab\right)+b^2x^2\left(8Bx^2-5A\right)\right)}{8b^3\left(a+bx^2\right)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^2*B + b^2*x^2*(-5*A + 8*B*x^2) + a*(-3*A*b + 25*b*B*x^2)
))/(8*b^3*(a + b*x^2)^2) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.014, size = 122, normalized size = 1.3

$$\frac{Bx}{b^3}-\frac{5\,Ax^3}{8\,b\,(bx^2+a)^2}+\frac{9\,Bx^3a}{8\,b^2\,(bx^2+a)^2}-\frac{3\,aAx}{8\,b^2\,(bx^2+a)^2}+\frac{7\,Bxa^2}{8\,b^3\,(bx^2+a)^2}+\frac{3\,A}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,Ba}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] B*x/b^3-5/8/b/(b*x^2+a)^2*A*x^3+9/8/b^2/(b*x^2+a)^2*B*x^3*a-3/8/b
^2/(b*x^2+a)^2*A*x*a+7/8/b^3/(b*x^2+a)^2*B*x*a^2+3/8/b^2/(a*b)^(1
/2)*arctan(x*b/(a*b)^(1/2))*A-15/8/b^3/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*B*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228148, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(\left(5\,Bab^2-Ab^3\right)x^4+5\,Ba^3-Aa^2b+2\left(5\,Ba^2b-Aab^2\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(8\,Bb^2x^5+5\left(5\,Bab-Ab^2\right)x^3\right.}{16\,(b^5x^4+2\,ab^4x^2+a^2b^3)\sqrt{-ab}}\right.$$

$$-\frac{3\left(\left(5\,Bab^2-Ab^3\right)x^4+5\,Ba^3-Aa^2b+2\left(5\,Ba^2b-Aab^2\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(8\,Bb^2x^5+5\left(5\,Bab-Ab^2\right)x^3+3\left(5\,Ba^2-\right.\right.}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*B*a*b^2 - A*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a
^2*b - A*a*b^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^
2 + a)) - 2*(8*B*b^2*x^5 + 5*(5*B*a*b - A*b^2)*x^3 + 3*(5*B*a^2 -
 A*a*b)*x)*sqrt(-a*b))/((b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*sqrt(-a
*b)), -1/8*(3*((5*B*a*b^2 - A*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5
*B*a^2*b - A*a*b^2)*x^2)*arctan(sqrt(a*b)*x/a) - (8*B*b^2*x^5 + 5
*(5*B*a*b - A*b^2)*x^3 + 3*(5*B*a^2 - A*a*b)*x)*sqrt(a*b))/((b^5*
x^4 + 2*a*b^4*x^2 + a^2*b^3)*sqrt(a*b))]
```

Sympy [A] time = 4.97609, size = 194, normalized size = 2.06

$$\frac{Bx}{b^3}+\frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(-\frac{3ab^3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(\frac{3ab^3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16}+\frac{x^3\left(-5Ab^2+9Bab\right)+x\left(-3Aab+7Ba^2\right)}{8a^2b^3+16ab^4x^2+8b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x/b**3 + 3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)*log(-3*a*b**3*sqrt(
-1/(a*b**7))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/16 - 3*sqrt(-1
/(a*b**7))*(-A*b + 5*B*a)*log(3*a*b**3*sqrt(-1/(a*b**7))*(-A*b +
5*B*a)/(-3*A*b + 15*B*a) + x)/16 + (x**3*(-5*A*b**2 + 9*B*a*b) +
x*(-3*A*a*b + 7*B*a**2))/(8*a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x
**4)
```

GIAC/XCAS [A] time = 0.227916, size = 108, normalized size = 1.15

$$\frac{Bx}{b^3} - \frac{3\,(5\,Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{9\,Babx^3 - 5\,Ab^2x^3 + 7\,Ba^2x - 3\,Aabx}{8\,(bx^2 + a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/8*(9*B*a*b*x^3 - 5*A*b^2*x^3 + 7*B*a^2*x - 3*A*a*b*x)/((b*x^
2 + a)^2*b^3)
```

3.102 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=89

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2(a+bx^2)}-\frac{x(Ab-aB)}{4b^2(a+bx^2)^2}$$

```
[Out] -((A*b - a*B)*x)/(4*b^2*(a + b*x^2)^2) + ((A*b - 5*a*B)*x)/(8*a*b
^2*(a + b*x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*
a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.162397, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2(a+bx^2)}-\frac{x(Ab-aB)}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -((A*b - a*B)*x)/(4*b^2*(a + b*x^2)^2) + ((A*b - 5*a*B)*x)/(8*a*b
^2*(a + b*x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*
a^(3/2)*b^(5/2))
```

Rubi in Sympy [A] time = 23.9085, size = 78, normalized size = 0.88

$$-\frac{x(Ab-Ba)}{4b^2(a+bx^2)^2}+\frac{x(Ab-5Ba)}{8ab^2(a+bx^2)}+\frac{(Ab+3Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -x*(A*b - B*a)/(4*b**2*(a + b*x**2)**2) + x*(A*b - 5*B*a)/(8*a*b*
*2*(a + b*x**2)) + (A*b + 3*B*a)*atan(sqrt(b)*x/sqrt(a))/(8*a**(3
/2)*b**(5/2))
```

Mathematica [A] time = 0.140825, size = 83, normalized size = 0.93

$$\frac{\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}x\left(-3a^2B-ab\left(A+5Bx^2\right)+Ab^2x^2\right)}{a(a+bx^2)^2}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - a*b*(A + 5*B*x^2)))/(a*(a + b
*x^2)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))/(
8*b^(5/2))
```

Maple [A] time = 0.012, size = 89, normalized size = 1.

$$\frac{1}{(bx^2+a)^2}\left(\frac{(Ab-5Ba)\,x^3}{8\,ab}-\frac{(Ab+3\,Ba)\,x}{8\,b^2}\right)+\frac{A}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,B}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(A*b-5*B*a)/a/b*x^3-1/8*(A*b+3*B*a)/b^2*x)/(b*x^2+a)^2+1/8/b
/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+3/8/b^2/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247236, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(3\,Bab^2+Ab^3\right)x^4+3\,Ba^3+Aa^2b+2\left(3\,Ba^2b+Aab^2\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(\left(5\,Bab-Ab^2\right)x^3+\left(3\,Ba^2+Aab\right)\right.}{16\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((3*B*a*b^2 + A*b^3)*x^4 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*
b + A*a*b^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) - 2*((5*B*a*b - A*b^2)*x^3 + (3*B*a^2 + A*a*b)*x)*sqrt(-a*b)
)/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*sqrt(-a*b)), 1/8*(((3*B*
a*b^2 + A*b^3)*x^4 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*
x^2)*arctan(sqrt(a*b)*x/a) - ((5*B*a*b - A*b^2)*x^3 + (3*B*a^2 +
A*a*b)*x)*sqrt(a*b))/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*sqrt(
a*b))]
```

Sympy [A] time = 3.54251, size = 153, normalized size = 1.72

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}-\frac{x^3\left(-Ab^2+5Bab\right)+x\left(Aab+3Ba^2\right)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(-a**2*b**2*sqrt(-1/(a**3*
b**5)) + x)/16 + sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(a**2*b**2
```

```
*sqrt(-1/(a**3*b**5)) + x)/16 - (x**3*(-A*b**2 + 5*B*a*b) + x*(A*
a*b + 3*B*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4
)
```

GIAC/XCAS [A] time = 0.224734, size = 105, normalized size = 1.18

$$\frac{(3\,Ba + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{5\,Babx^3 - Ab^2x^3 + 3\,Ba^2x + Aabx}{8\,(bx^2 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(
5*B*a*b*x^3 - A*b^2*x^3 + 3*B*a^2*x + A*a*b*x)/((b*x^2 + a)^2*a*b
^2)
```

3.103 $\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=92

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\,(a+bx^2)}+\frac{x(Ab-aB)}{4ab\,(a+bx^2)^2}$$

```
[Out] ((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*x)/(8*a^2*
b*(a + b*x^2)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0908262, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\,(a+bx^2)}+\frac{x(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*x)/(8*a^2*
b*(a + b*x^2)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 12.7736, size = 78, normalized size = 0.85

$$\frac{x\,(Ab-Ba)}{4ab\,(a+bx^2)^2}+\frac{x\,(3Ab+Ba)}{8a^2b\,(a+bx^2)}+\frac{(3Ab+Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] x*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) + x*(3*A*b + B*a)/(8*a**2*b
*(a + b*x**2)) + (3*A*b + B*a)*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/2
)*b**(3/2))
```

Mathematica [A] time = 0.106749, size = 84, normalized size = 0.91

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x\left(a^2(-B)+ab\left(5A+Bx^2\right)+3Ab^2x^2\right)}{8a^2b\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (x*(-(a^2*B) + 3*A*b^2*x^2 + a*b*(5*A + B*x^2)))/(8*a^2*b*(a + b*
x^2)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*
b^(3/2))
```

Maple [A] time = 0.012, size = 90, normalized size = 1.

$$\frac{1}{\left(bx^{2}+a\right)^{2}}\left(\frac{\left(3\,Ab+Ba\right)x^{3}}{8\,a^{2}}+\frac{\left(5\,Ab-Ba\right)x}{8\,ab}\right)+\frac{3\,A}{8\,a^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+B*a)/a^2*x^3+1/8*(5*A*b-B*a)/a/b*x)/(b*x^2+a)^2+3/8/a
^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+1/8/a/b/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242552, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(Bab^{2}+3\,Ab^{3}\right)x^{4}+Ba^{3}+3\,Aa^{2}b+2\,\left(Ba^{2}b+3\,Aab^{2}\right)x^{2}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\,\left(\left(Bab+3\,Ab^{2}\right)x^{3}-\left(Ba^{2}-5\,Aab\right)\right.}{16\,\left(a^{2}b^{3}x^{4}+2\,a^{3}b^{2}x^{2}+a^{4}b\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((B*a*b^2 + 3*A*b^3)*x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b
+ 3*A*a*b^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) + 2*((B*a*b + 3*A*b^2)*x^3 - (B*a^2 - 5*A*a*b)*x)*sqrt(-a*b)
)/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*sqrt(-a*b)), 1/8*(((B*a*
b^2 + 3*A*b^3)*x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*
x^2)*arctan(sqrt(a*b)*x/a) + ((B*a*b + 3*A*b^2)*x^3 - (B*a^2 - 5*
A*a*b)*x)*sqrt(a*b))/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*sqrt(
a*b))]
```

Sympy [A] time = 2.82895, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\left(3Ab+Ba\right)\log\left(-a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\left(3Ab+Ba\right)\log\left(a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x\right)}{16}+\frac{x^{3}\left(3Ab^{2}+Bab\right)+x\left(5Aab-Ba^{2}\right)}{8a^{4}b+16a^{3}b^{2}x^{2}+8a^{2}b^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(-a**3*b*sqrt(-1/(a**5*b**
3)) + x)/16 + sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(a**3*b*sqrt(
```

```
-1/(a**5*b**3)) + x)/16 + (x**3*(3*A*b**2 + B*a*b) + x*(5*A*a*b -
 B*a**2))/(8*a**4*b + 16*a**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

GIAC/XCAS [A] time = 0.22289, size = 105, normalized size = 1.14

$$\frac{(Ba + 3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Babx^3 + 3\,Ab^2x^3 - Ba^2x + 5\,Aabx}{8\,(bx^2 + a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(
B*a*b*x^3 + 3*A*b^2*x^3 - B*a^2*x + 5*A*a*b*x)/((b*x^2 + a)^2*a^2
*b)
```

3.104 $\int \frac{A+Bx^2}{x^2(a+bx^2)^3} dx$

Optimal. Leaf size=97

$$-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{x(7Ab-3aB)}{8a^3(a+bx^2)}-\frac{A}{a^3x}-\frac{x(Ab-aB)}{4a^2(a+bx^2)^2}$$

```
[Out] -(A/(a^3*x)) - ((A*b - a*B)*x)/(4*a^2*(a + b*x^2)^2) - ((7*A*b -
3*a*B)*x)/(8*a^3*(a + b*x^2)) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.274091, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{x(7Ab-3aB)}{8a^3(a+bx^2)}-\frac{A}{a^3x}-\frac{x(Ab-aB)}{4a^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) - ((A*b - a*B)*x)/(4*a^2*(a + b*x^2)^2) - ((7*A*b -
3*a*B)*x)/(8*a^3*(a + b*x^2)) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 30.3122, size = 88, normalized size = 0.91

$$-\frac{A}{a^3x}-\frac{x(Ab-Ba)}{4a^2(a+bx^2)^2}-\frac{x(7Ab-3Ba)}{8a^3(a+bx^2)}-\frac{3(5Ab-Ba)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{7}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -A/(a**3*x) - x*(A*b - B*a)/(4*a**2*(a + b*x**2)**2) - x*(7*A*b -
 3*B*a)/(8*a**3*(a + b*x**2)) - 3*(5*A*b - B*a)*atan(sqrt(b)*x/sq
rt(a))/(8*a**(7/2)*sqrt(b))
```

Mathematica [A] time = 0.0979148, size = 96, normalized size = 0.99

$$\frac{3(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{x(3aB-7Ab)}{8a^3(a+bx^2)}-\frac{A}{a^3x}+\frac{x(aB-Ab)}{4a^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + ((-(A*b) + a*B)*x)/(4*a^2*(a + b*x^2)^2) + ((-7*A*
b + 3*a*B)*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*B)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.017, size = 125, normalized size = 1.3

$$-\frac{A}{a^3x}-\frac{7\,Ax^3b^2}{8\,a^3\,(bx^2+a)^2}+\frac{3\,bBx^3}{8\,a^2\,(bx^2+a)^2}-\frac{9\,Axb}{8\,a^2\,(bx^2+a)^2}+\frac{5\,Bx}{8\,a\,(bx^2+a)^2}-\frac{15\,Ab}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,B}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -A/a^3/x-7/8/a^3/(b*x^2+a)^2*A*x^3*b^2+3/8/a^2/(b*x^2+a)^2*B*x^3*
b-9/8/a^2/(b*x^2+a)^2*A*x*b+5/8/a/(b*x^2+a)^2*B*x-15/8/a^3/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))*A*b+3/8/a^2/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240613, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(\left(Bab^2-5\,Ab^3\right)x^5+2\left(Ba^2b-5\,Aab^2\right)x^3+\left(Ba^3-5\,Aa^2b\right)x\right)\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(3\left(Bab-5\,Ab^2\right)x^4-8\,A\right.}{16\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((B*a*b^2 - 5*A*b^3)*x^5 + 2*(B*a^2*b - 5*A*a*b^2)*x^3
+ (B*a^3 - 5*A*a^2*b)*x)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/
(b*x^2 + a)) - 2*(3*(B*a*b - 5*A*b^2)*x^4 - 8*A*a^2 + 5*(B*a^2 -
5*A*a*b)*x^2)*sqrt(-a*b))/((a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sq
rt(-a*b)), 1/8*(3*((B*a*b^2 - 5*A*b^3)*x^5 + 2*(B*a^2*b - 5*A*a*b
^2)*x^3 + (B*a^3 - 5*A*a^2*b)*x)*arctan(sqrt(a*b)*x/a) + (3*(B*a*
b - 5*A*b^2)*x^4 - 8*A*a^2 + 5*(B*a^2 - 5*A*a*b)*x^2)*sqrt(a*b))/
((a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqrt(a*b))]
```

Sympy [A] time = 3.54787, size = 194, normalized size = 2.

$$-\frac{3\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)\log\left(-\frac{3a^4\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)\log\left(\frac{3a^4\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{-8Aa^2+x^4\left(-15Ab^2+3Bab\right)+x^2\left(-25Aab+5Ba^2\right)}{8a^5x+16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(-3*a**4*sqrt(-1/(a**7*b))
*(-5*A*b + B*a)/(-15*A*b + 3*B*a) + x)/16 + 3*sqrt(-1/(a**7*b))*(
-5*A*b + B*a)*log(3*a**4*sqrt(-1/(a**7*b))*(-5*A*b + B*a)/(-15*A*
b + 3*B*a) + x)/16 + (-8*A*a**2 + x**4*(-15*A*b**2 + 3*B*a*b) + x
**2*(-25*A*a*b + 5*B*a**2))/(8*a**5*x + 16*a**4*b*x**3 + 8*a**3*b
**2*x**5)
```

GIAC/XCAS [A] time = 0.23221, size = 111, normalized size = 1.14

$$\frac{3\,(Ba - 5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3} - \frac{A}{a^3x} + \frac{3\,Babx^3 - 7\,Ab^2x^3 + 5\,Ba^2x - 9\,Aabx}{8\,(bx^2 + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)/((b*x^2 + a)^3*x^2),x, algorithm="giac")
```

```
[Out] 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - A/(a^3*
x) + 1/8*(3*B*a*b*x^3 - 7*A*b^2*x^3 + 5*B*a^2*x - 9*A*a*b*x)/((b*
x^2 + a)^2*a^3)
```

3.105 $\int \frac{A+Bx^2}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=117

$$\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{bx(11Ab-7aB)}{8a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{a^4x}+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{3a^3x^3}$$

```
[Out] -A/(3*a^3*x^3) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B)*x)/(4*a^3
*(a + b*x^2)^2) + (b*(11*A*b - 7*a*B)*x)/(8*a^4*(a + b*x^2)) + (5
*Sqrt[b]*(7*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] time = 0.432281, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{bx(11Ab-7aB)}{8a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{a^4x}+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -A/(3*a^3*x^3) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B)*x)/(4*a^3
*(a + b*x^2)^2) + (b*(11*A*b - 7*a*B)*x)/(8*a^4*(a + b*x^2)) + (5
*Sqrt[b]*(7*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rubi in Sympy [A] time = 63.2241, size = 109, normalized size = 0.93

$$-\frac{A}{3a^3x^3}+\frac{bx\,(Ab-Ba)}{4a^3\,(a+bx^2)^2}+\frac{bx\,(11Ab-7Ba)}{8a^4\,(a+bx^2)}+\frac{3Ab-Ba}{a^4x}+\frac{5\sqrt{b}\,(7Ab-3Ba)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

```
[Out] -A/(3*a**3*x**3) + b*x*(A*b - B*a)/(4*a**3*(a + b*x**2)**2) + b*x
*(11*A*b - 7*B*a)/(8*a**4*(a + b*x**2)) + (3*A*b - B*a)/(a**4*x)
+ 5*sqrt(b)*(7*A*b - 3*B*a)*atan(sqrt(b)*x/sqrt(a))/(8*a**(9/2))
```

Mathematica [A] time = 0.148157, size = 116, normalized size = 0.99

$$\frac{5\sqrt{b}(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{-8a^3\left(A+3Bx^2\right)+a^2bx^2\left(56A-75Bx^2\right)+5ab^2x^4\left(35A-9Bx^2\right)+105Ab^3x^6}{24a^4x^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (105*A*b^3*x^6 + a^2*b*x^2*(56*A - 75*B*x^2) + 5*a*b^2*x^4*(35*A
- 9*B*x^2) - 8*a^3*(A + 3*B*x^2))/(24*a^4*x^3*(a + b*x^2)^2) + (5
*Sqrt[b]*(7*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Maple [A] time = 0.02, size = 152, normalized size = 1.3

$$-\frac{A}{3\,a^3x^3}+3\,\frac{Ab}{xa^4}-\frac{B}{xa^3}+\frac{11\,b^3Ax^3}{8\,a^4\,(bx^2+a)^2}-\frac{7\,b^2Bx^3}{8\,a^3\,(bx^2+a)^2}+\frac{13\,Axb^2}{8\,a^3\,(bx^2+a)^2}-\frac{9\,bBx}{8\,a^2\,(bx^2+a)^2}+\frac{35\,b^2A}{8\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,Bb}{8\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3*A/a^3/x^3+3/x/a^4*A*b-1/x/a^3*B+11/8/a^4*b^3/(b*x^2+a)^2*A*x
^3-7/8/a^3*b^2/(b*x^2+a)^2*B*x^3+13/8/a^3*b^2/(b*x^2+a)^2*A*x-9/8
/a^2*b/(b*x^2+a)^2*B*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*A-15/8/a^3*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2464, size = 1, normalized size = 0.01

$$\left[-\frac{30\left(3\,Bab^2-7\,Ab^3\right)x^6+50\left(3\,Ba^2b-7\,Aab^2\right)x^4+16\,Aa^3+16\left(3\,Ba^3-7\,Aa^2b\right)x^2+15\left(\left(3\,Bab^2-7\,Ab^3\right)x^7+2\left(3\,Ba^2b\right.\right.}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}\right.$$

$$-\frac{15\left(3\,Bab^2-7\,Ab^3\right)x^6+25\left(3\,Ba^2b-7\,Aab^2\right)x^4+8\,Aa^3+8\left(3\,Ba^3-7\,Aa^2b\right)x^2+15\left(\left(3\,Bab^2-7\,Ab^3\right)x^7+2\left(3\,Ba^2b-7\right.\right.}{24\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^4),x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*a*b^2 - 7*A*b^3)*x^6 + 50*(3*B*a^2*b - 7*A*a*b^2)
*x^4 + 16*A*a^3 + 16*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 -
 7*A*b^3)*x^7 + 2*(3*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^
2*b)*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), -1/24*(15*(3*B*a*b^2
- 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 8*A*a^3 + 8*(3*
B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(3*B*a
^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(b/a)*arct
an(b*x/(a*sqrt(b/a))))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [A] time = 4.74115, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{b}{a^9}}\left(-7Ab+3Ba\right)\log\left(-\frac{5a^5\sqrt{-\frac{b}{a^9}}(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}-\frac{5\sqrt{-\frac{b}{a^9}}\left(-7Ab+3Ba\right)\log\left(\frac{5a^5\sqrt{-\frac{b}{a^9}}(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}-\frac{8Aa^3+x^6\left(-105Ab^3+45Bab^2\right)+x^4\left(-175Aab^2+75Ba^2b\right)+x^2\left(-56Aa^2b+24Ba^3\right)}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

```
[Out] 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(-5*a**5*sqrt(-b/a**9)*(-7*A*
b + 3*B*a)/(-35*A*b**2 + 15*B*a*b) + x)/16 - 5*sqrt(-b/a**9)*(-7*
A*b + 3*B*a)*log(5*a**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)/(-35*A*b**
2 + 15*B*a*b) + x)/16 - (8*A*a**3 + x**6*(-105*A*b**3 + 45*B*a*b*
*2) + x**4*(-175*A*a*b**2 + 75*B*a**2*b) + x**2*(-56*A*a**2*b + 2
4*B*a**3))/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4*b**2*x**7)
```

GIAC/XCAS [A] time = 0.250669, size = 146, normalized size = 1.25

$$-\frac{5\left(3\,Bab-7\,Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}-\frac{7\,Bab^2x^3-11\,Ab^3x^3+9\,Ba^2bx-13\,Aab^2x}{8\,(bx^2+a)^2a^4}-\frac{3\,Bax^2-9\,Abx^2+Aa}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^4),x, algorithm="giac")
```

```
[Out] -5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) -
1/8*(7*B*a*b^2*x^3 - 11*A*b^3*x^3 + 9*B*a^2*b*x - 13*A*a*b^2*x)/(
(b*x^2 + a)^2*a^4) - 1/3*(3*B*a*x^2 - 9*A*b*x^2 + A*a)/(a^4*x^3)
```

3.106 $\int \frac{A+Bx^2}{x^6(a+bx^2)^3} dx$

Optimal. Leaf size=142

$$-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{b^2x(15Ab-11aB)}{8a^5(a+bx^2)}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2x(Ab-aB)}{4a^4(a+bx^2)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{A}{5a^3x^5}$$

```
[Out] -A/(5*a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) - (3*b*(2*A*b - a*B))/
(a^5*x) - (b^2*(A*b - a*B)*x)/(4*a^4*(a + b*x^2)^2) - (b^2*(15*A*
b - 11*a*B)*x)/(8*a^5*(a + b*x^2)) - (7*b^(3/2)*(9*A*b - 5*a*B)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2))
```

Rubi [A] time = 0.590916, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{b^2x(15Ab-11aB)}{8a^5(a+bx^2)}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2x(Ab-aB)}{4a^4(a+bx^2)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{A}{5a^3x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -A/(5*a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) - (3*b*(2*A*b - a*B))/
(a^5*x) - (b^2*(A*b - a*B)*x)/(4*a^4*(a + b*x^2)^2) - (b^2*(15*A*
b - 11*a*B)*x)/(8*a^5*(a + b*x^2)) - (7*b^(3/2)*(9*A*b - 5*a*B)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.169057, size = 139, normalized size = 0.98

$$\frac{7b^{3/2}(5aB-9Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{-8a^4\left(3A+5Bx^2\right)+8a^3bx^2\left(9A+35Bx^2\right)+7a^2b^2x^4\left(125Bx^2-72A\right)+525ab^3x^6\left(Bx^2-3A\right)-945Ab^4x^8}{120a^5x^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] (-945*A*b^4*x^8 + 525*a*b^3*x^6*(-3*A + B*x^2) - 8*a^4*(3*A + 5*B
*x^2) + 8*a^3*b*x^2*(9*A + 35*B*x^2) + 7*a^2*b^2*x^4*(-72*A + 125
*B*x^2))/(120*a^5*x^5*(a + b*x^2)^2) + (7*b^(3/2)*(-9*A*b + 5*a*B
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2))
```

Maple [A] time = 0.02, size = 177, normalized size = 1.3

$$-\frac{A}{5\,a^3x^5}+\frac{Ab}{x^3a^4}-\frac{B}{3\,x^3a^3}-6\,\frac{b^2A}{a^5x}+3\,\frac{Bb}{a^4x}-\frac{15\,Ab^4x^3}{8\,a^5\,(bx^2+a)^2}+\frac{11\,Bb^3x^3}{8\,a^4\,(bx^2+a)^2}-\frac{17\,b^3Ax}{8\,a^4\,(bx^2+a)^2}+\frac{13\,b^2Bx}{8\,a^3\,(bx^2+a)^2}-\frac{63\,b^3A}{8\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{35\,Bb^2}{8\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5*A/a^3/x^5+1/x^3/a^4*A*b-1/3/x^3/a^3*B-6*b^2/a^5/x*A+3*b/a^4/
x*B-15/8/a^5*b^4/(b*x^2+a)^2*A*x^3+11/8/a^4*b^3/(b*x^2+a)^2*B*x^3
-17/8/a^4*b^3/(b*x^2+a)^2*A*x+13/8/a^3*b^2/(b*x^2+a)^2*B*x-63/8/a
^5*b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*A+35/8/a^4*b^2/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250722, size = 1, normalized size = 0.01

$$\left[\frac{210\left(5\,Bab^3-9\,Ab^4\right)x^8+350\left(5\,Ba^2b^2-9\,Aab^3\right)x^6-48\,Aa^4+112\left(5\,Ba^3b-9\,Aa^2b^2\right)x^4-16\left(5\,Ba^4-9\,Aa^3b\right)x^2-105\,(\ldots}{240\,(a^5b^2x^9+2\,a^6bx^7+a^7\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^6),x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*a*b^3 - 9*A*b^4)*x^8 + 350*(5*B*a^2*b^2 - 9*A*a*
b^3)*x^6 - 48*A*a^4 + 112*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 16*(5*B
*a^4 - 9*A*a^3*b)*x^2 - 105*((5*B*a*b^3 - 9*A*b^4)*x^9 + 2*(5*B*a
^2*b^2 - 9*A*a*b^3)*x^7 + (5*B*a^3*b - 9*A*a^2*b^2)*x^5)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^2*x^9
+ 2*a^6*b*x^7 + a^7*x^5), 1/120*(105*(5*B*a*b^3 - 9*A*b^4)*x^8 +
175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 - 24*A*a^4 + 56*(5*B*a^3*b - 9*
A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2 + 105*((5*B*a*b^3 -
9*A*b^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B*a^3*b - 9*A
*a^2*b^2)*x^5)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^5*b^2*x^9
+ 2*a^6*b*x^7 + a^7*x^5)]
```

Sympy [A] time = 7.11975, size = 260, normalized size = 1.83

$$-\frac{7\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)\log\left(-\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{7\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)\log\left(\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{-24Aa^4+x^8\left(-945Ab^4+525Bab^3\right)+x^6\left(-1575Aab^3+875Ba^2b^2\right)+x^4\left(-504Aa^2b^2+280Ba^3b\right)+x^2\left(72Aa^3b-40Ba^4\right)}{120a^7x^5+240a^6bx^7+120a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -7*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)*log(-7*a**6*sqrt(-b**3/a**1
1)*(-9*A*b + 5*B*a)/(-63*A*b**3 + 35*B*a*b**2) + x)/16 + 7*sqrt(-
b**3/a**11)*(-9*A*b + 5*B*a)*log(7*a**6*sqrt(-b**3/a**11)*(-9*A*b
 + 5*B*a)/(-63*A*b**3 + 35*B*a*b**2) + x)/16 + (-24*A*a**4 + x**8
*(-945*A*b**4 + 525*B*a*b**3) + x**6*(-1575*A*a*b**3 + 875*B*a**2
*b**2) + x**4*(-504*A*a**2*b**2 + 280*B*a**3*b) + x**2*(72*A*a**3
*b - 40*B*a**4))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5*b**2
*x**9)
```

GIAC/XCAS [A] time = 0.263786, size = 182, normalized size = 1.28

$$\frac{7\left(5\,Bab^2-9\,Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}+\frac{11\,Bab^3x^3-15\,Ab^4x^3+13\,Ba^2b^2x-17\,Aab^3x}{8\,(bx^2+a)^2a^5}+\frac{45\,Babx^4-90\,Ab^2x^4-5\,Ba^2x^2+15\,Aabx^2-3\,Aa^2}{15\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^6),x, algorithm="giac")
```

```
[Out] 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) +
 1/8*(11*B*a*b^3*x^3 - 15*A*b^4*x^3 + 13*B*a^2*b^2*x - 17*A*a*b^3
*x)/((b*x^2 + a)^2*a^5) + 1/15*(45*B*a*b*x^4 - 90*A*b^2*x^4 - 5*B
*a^2*x^2 + 15*A*a*b*x^2 - 3*A*a^2)/(a^5*x^5)
```

3.107 $\int \frac{a+bx^2}{1+x^2}\,dx$

Optimal. Leaf size=12

$$(a-b)\tan^{-1}(x)+bx$$

```
[Out] b*x + (a - b)*ArcTan[x]
```

Rubi [A] time = 0.0226548, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$(a-b)\tan^{-1}(x)+bx$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(1 + x^2),x]

[Out] b*x + (a - b)*ArcTan[x]
```

Rubi in Sympy [A] time = 5.06246, size = 8, normalized size = 0.67

$$bx+(a-b)\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(x**2+1),x)

[Out] b*x + (a - b)*atan(x)
```

Mathematica [A] time = 0.00878993, size = 12, normalized size = 1.

$$(a-b)\tan^{-1}(x)+bx$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(1 + x^2),x]

[Out] b*x + (a - b)*ArcTan[x]
```

Maple [A] time = 0.003, size = 14, normalized size = 1.2

$$bx+\arctan(x)\,a-\arctan(x)\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(x^2+1),x)

[Out] b*x+arctan(x)*a-arctan(x)*b
```

Maxima [A] time = 1.49925, size = 16, normalized size = 1.33

$$bx+(a-b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] b*x + (a - b)*arctan(x)
```

Fricas [A] time = 0.229946, size = 16, normalized size = 1.33

$$bx + (a - b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] b*x + (a - b)*arctan(x)
```

Sympy [A] time = 1.24099, size = 26, normalized size = 2.17

$$bx - \frac{i(a-b)\log(x-i)}{2} + \frac{i(a-b)\log(x+i)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(x**2+1),x)
```

```
[Out] b*x - I*(a - b)*log(x - I)/2 + I*(a - b)*log(x + I)/2
```

GIAC/XCAS [A] time = 0.251771, size = 16, normalized size = 1.33

$$bx + (a - b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] b*x + (a - b)*arctan(x)
```

3.108 $\int \frac{a+bx^2}{1-x^2}\,dx$

Optimal. Leaf size=11

$$(a+b)\tanh^{-1}(x) - bx$$

```
[Out] -(b*x) + (a + b)*ArcTanh[x]
```

Rubi [A] time = 0.0227556, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$(a+b)\tanh^{-1}(x) - bx$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(1 - x^2),x]

[Out] -(b*x) + (a + b)*ArcTanh[x]
```

Rubi in Sympy [A] time = 5.53314, size = 8, normalized size = 0.73

$$-bx + (a+b)\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(-x**2+1),x)

[Out] -b*x + (a + b)*atanh(x)
```

Mathematica [B] time = 0.0151253, size = 28, normalized size = 2.55

$$\frac{1}{2}(-(a+b)\log(1-x) + (a+b)\log(x+1) - 2bx)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(1 - x^2),x]

[Out] (-2*b*x - (a + b)*Log[1 - x] + (a + b)*Log[1 + x])/2
```

Maple [B] time = 0.005, size = 34, normalized size = 3.1

$$-bx - \frac{\ln(-1+x)a}{2} - \frac{\ln(-1+x)b}{2} + \frac{\ln(1+x)a}{2} + \frac{\ln(1+x)b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(-x^2+1),x)

[Out] -b*x-1/2*ln(-1+x)*a-1/2*ln(-1+x)*b+1/2*ln(1+x)*a+1/2*ln(1+x)*b
```

Maxima [A] time = 1.32702, size = 31, normalized size = 2.82

$$-bx + \frac{1}{2}(a+b)\log(x+1) - \frac{1}{2}(a+b)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 + a)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

Fricas [A] time = 0.233846, size = 31, normalized size = 2.82

$$-bx + \frac{1}{2}(a+b)\log(x+1) - \frac{1}{2}(a+b)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 + a)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

Sympy [A] time = 1.28211, size = 22, normalized size = 2.

$$-bx - \frac{(a+b)\log(x-1)}{2} + \frac{(a+b)\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(-x**2+1),x)
```

```
[Out] -b*x - (a + b)*log(x - 1)/2 + (a + b)*log(x + 1)/2
```

GIAC/XCAS [A] time = 0.232117, size = 34, normalized size = 3.09

$$-bx + \frac{1}{2}(a+b)\ln(|x+1|) - \frac{1}{2}(a+b)\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x^2 + a)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] -b*x + 1/2*(a + b)*ln(abs(x + 1)) - 1/2*(a + b)*ln(abs(x - 1))
```

3.109 $\int \frac{1+x^2}{(-1+x^2)^2} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(1 - x^2)
```

Rubi [A] time = 0.00728537, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(-1 + x^2)^2,x]

[Out] x/(1 - x^2)
```

Rubi in Sympy [A] time = 3.67251, size = 5, normalized size = 0.45

$$\frac{x}{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**2-1)**2,x)

[Out] x/(-x**2 + 1)
```

Mathematica [A] time = 0.00643358, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(-1 + x^2)^2,x]

[Out] -(x/(-1 + x^2))
```

Maple [A] time = 0.008, size = 16, normalized size = 1.5

$$-\frac{1}{-2+2x}-\frac{1}{2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^2-1)^2,x)

[Out] -1/2/(-1+x)-1/2/(1+x)
```

Maxima [A] time = 1.33919, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - 1)^2,x, algorithm="maxima")

[Out] -x/(x^2 - 1)
```

Fricas [A] time = 0.230401, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - 1)^2,x, algorithm="fricas")

[Out] -x/(x^2 - 1)
```

Sympy [A] time = 0.166185, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**2-1)**2,x)

[Out] -x/(x**2 - 1)
```

GIAC/XCAS [A] time = 0.218714, size = 15, normalized size = 1.36

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - 1)^2,x, algorithm="giac")

[Out] -1/(x - 1/x)
```

3.110 $\int \frac{1-x^2}{(1+x^2)^2} dx$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

```
[Out] x/(1 + x^2)
```

Rubi [A] time = 0.0071113, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2)
```

Rubi in Sympy [A] time = 4.16044, size = 5, normalized size = 0.56

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1)
```

Mathematica [A] time = 0.00642078, size = 9, normalized size = 1.

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2)
```

Maple [A] time = 0.009, size = 10, normalized size = 1.1

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)^2,x)
```

```
[Out] x/(x^2+1)
```

Maxima [A] time = 1.33772, size = 12, normalized size = 1.33

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

Fricas [A] time = 0.218094, size = 12, normalized size = 1.33

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

Sympy [A] time = 0.175413, size = 5, normalized size = 0.56

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1)
```

GIAC/XCAS [A] time = 0.240044, size = 9, normalized size = 1.

$$\frac{1}{x + \frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

3.111 $\int \frac{3+2x^2}{(1+x^2)^2}\,dx$

Optimal. Leaf size=19

$$\frac{x}{2(x^2+1)}+\frac{5}{2}\tan^{-1}(x)$$

```
[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Rubi [A] time = 0.0167511, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x}{2(x^2+1)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x^2)/(1 + x^2)^2,x]

[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Rubi in Sympy [A] time = 4.10227, size = 14, normalized size = 0.74

$$\frac{x}{2(x^2+1)}+\frac{5\,\text{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+3)/(x**2+1)**2,x)

[Out] x/(2*(x**2 + 1)) + 5*atan(x)/2
```

Mathematica [A] time = 0.0116605, size = 19, normalized size = 1.

$$\frac{x}{2(x^2+1)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x^2)/(1 + x^2)^2,x]

[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Maple [A] time = 0.008, size = 16, normalized size = 0.8

$$\frac{x}{2x^2+2}+\frac{5\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+3)/(x^2+1)^2,x)

[Out] 1/2*x/(x^2+1)+5/2*arctan(x)
```

Maxima [A] time = 1.48253, size = 20, normalized size = 1.05

$$\frac{x}{2\,(x^2+1)} + \frac{5}{2}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

Fricas [A] time = 0.236045, size = 27, normalized size = 1.42

$$\frac{5\left(x^2+1\right)\arctan\,(x)+x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.221471, size = 14, normalized size = 0.74

$$\frac{x}{2x^2+2} + \frac{5\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + 5*atan(x)/2
```

GIAC/XCAS [A] time = 0.221626, size = 20, normalized size = 1.05

$$\frac{x}{2\,(x^2+1)} + \frac{5}{2}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

3.112 $\int \frac{-2+x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$-\frac{3x}{2(x^2+1)} - \frac{1}{2}\tan^{-1}(x)$$

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Rubi [A] time = 0.0153595, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{3x}{2(x^2+1)} - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Rubi in Sympy [A] time = 3.87925, size = 15, normalized size = 0.79

$$-\frac{3x}{2(x^2+1)} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-2)/(x**2+1)**2,x)
```

```
[Out] -3*x/(2*(x**2 + 1)) - atan(x)/2
```

Mathematica [A] time = 0.0119485, size = 19, normalized size = 1.

$$-\frac{3x}{2(x^2+1)} - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Maple [A] time = 0.007, size = 16, normalized size = 0.8

$$-\frac{3x}{2x^2+2} - \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-2)/(x^2+1)^2,x)
```

```
[Out] -3/2*x/(x^2+1)-1/2*arctan(x)
```

Maxima [A] time = 1.49675, size = 20, normalized size = 1.05

$$-\frac{3\,x}{2\,(x^2+1)}-\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

Fricas [A] time = 0.223722, size = 28, normalized size = 1.47

$$-\frac{\left(x^2+1\right)\,\arctan\left(x\right)+3\,x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x) + 3*x)/(x^2 + 1)
```

Sympy [A] time = 0.219321, size = 15, normalized size = 0.79

$$-\frac{3x}{2x^2+2}-\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-2)/(x**2+1)**2,x)
```

```
[Out] -3*x/(2*x**2 + 2) - atan(x)/2
```

GIAC/XCAS [A] time = 0.223116, size = 20, normalized size = 1.05

$$-\frac{3\,x}{2\,(x^2+1)}-\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

3.113 $\int \frac{3+x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=14

$$\frac{x}{x^2+1} + 2\tan^{-1}(x)$$

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Rubi [A] time = 0.0153864, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x}{x^2+1} + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Rubi in Sympy [A] time = 3.87684, size = 10, normalized size = 0.71

$$\frac{x}{x^2+1} + 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3)/(x**2+1)**2,x)

[Out] x/(x**2 + 1) + 2*atan(x)
```

Mathematica [A] time = 0.00953869, size = 14, normalized size = 1.

$$\frac{x}{x^2+1} + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Maple [A] time = 0.01, size = 15, normalized size = 1.1

$$\frac{x}{x^2+1} + 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3)/(x^2+1)^2,x)

[Out] x/(x^2+1)+2*arctan(x)
```

Maxima [A] time = 1.4859, size = 19, normalized size = 1.36

$$\frac{x}{x^2+1} + 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

Fricas [A] time = 0.226713, size = 26, normalized size = 1.86

$$\frac{2\left(x^2+1\right)\arctan(x)+x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] (2*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.21219, size = 10, normalized size = 0.71

$$\frac{x}{x^2+1} + 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1) + 2*atan(x)
```

GIAC/XCAS [A] time = 0.221667, size = 19, normalized size = 1.36

$$\frac{x}{x^2+1} + 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

3.114 $\int \frac{a+bx^2}{(-a+bx^2)^2} dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

```
[Out] x/(a - b*x^2)
```

Rubi [A] time = 0.00976524, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(-a + b*x^2)^2,x]

[Out] x/(a - b*x^2)
```

Rubi in Sympy [A] time = 6.7902, size = 7, normalized size = 0.58

$$\frac{x}{a-bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(b*x**2-a)**2,x)

[Out] x/(a - b*x**2)
```

Mathematica [A] time = 0.0115159, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(-a + b*x^2)^2,x]

[Out] -(x/(-a + b*x^2))
```

Maple [A] time = 0.009, size = 15, normalized size = 1.3

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(b*x^2-a)^2,x)

[Out] -x/(b*x^2-a)
```

Maxima [A] time = 1.34745, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] -x/(b*x^2 - a)
```

Fricas [A] time = 0.223489, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] -x/(b*x^2 - a)
```

Sympy [A] time = 1.29411, size = 8, normalized size = 0.67

$$-\frac{x}{-a + bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(b*x**2-a)**2,x)
```

```
[Out] -x/(-a + b*x**2)
```

GIAC/XCAS [A] time = 0.228688, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] -x/(b*x^2 - a)
```

3.115 $\int \frac{a+bx^2}{(a-bx^2)^2} dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

```
[Out] x/(a - b*x^2)
```

Rubi [A] time = 0.00915279, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(a - b*x^2)^2,x]

[Out] x/(a - b*x^2)
```

Rubi in Sympy [A] time = 6.11514, size = 7, normalized size = 0.58

$$\frac{x}{a-bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(-b*x**2+a)**2,x)

[Out] x/(a - b*x**2)
```

Mathematica [A] time = 0.00900784, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(a - b*x^2)^2,x]

[Out] -(x/(-a + b*x^2))
```

Maple [A] time = 0.006, size = 15, normalized size = 1.3

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(-b*x^2+a)^2,x)

[Out] -x/(b*x^2-a)
```

Maxima [A] time = 1.34544, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b*x^2 - a)^2,x, algorithm="maxima")

[Out] -x/(b*x^2 - a)
```

Fricas [A] time = 0.214954, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b*x^2 - a)^2,x, algorithm="fricas")

[Out] -x/(b*x^2 - a)
```

Sympy [A] time = 1.30033, size = 8, normalized size = 0.67

$$-\frac{x}{-a + bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(-b*x**2+a)**2,x)

[Out] -x/(-a + b*x**2)
```

GIAC/XCAS [A] time = 0.219432, size = 19, normalized size = 1.58

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(b*x^2 - a)^2,x, algorithm="giac")

[Out] -x/(b*x^2 - a)
```

3.116 $\int \frac{A+Bx^2}{a-bx^2} \, dx$

Optimal. Leaf size=39

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*
b^(3/2))
```

Rubi [A] time = 0.0543235, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*
b^(3/2))
```

Rubi in Sympy [A] time = 9.5302, size = 34, normalized size = 0.87

$$-\frac{Bx}{b}+\frac{(Ab+Ba)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(-b*x**2+a),x)
```

```
[Out] -B*x/b + (A*b + B*a)*atanh(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(3/2))
```

Mathematica [A] time = 0.0376031, size = 39, normalized size = 1.

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*
b^(3/2))
```

Maple [A] time = 0.004, size = 37, normalized size = 1.

$$-\frac{Bx}{b}-\frac{-Ab-Ba}{b}Artanh\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(-b*x^2+a),x)
```

```
[Out] -B*x/b-(-A*b-B*a)/b/(a*b)^(1/2)*arctanh(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(B*x^2 + A)/(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235768, size = 1, normalized size = 0.03

$$\left[-\frac{2\sqrt{ab}Bx - (Ba + Ab)\log\left(\frac{2\,abx + (bx^2 + a)\sqrt{ab}}{bx^2 - a}\right)}{2\sqrt{ab}b}, -\frac{\sqrt{-ab}Bx - (Ba + Ab)\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{\sqrt{-ab}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(B*x^2 + A)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(a*b)*B*x - (B*a + A*b)*log((2*a*b*x + (b*x^2 + a)*s
qrt(a*b))/(b*x^2 - a)))/(sqrt(a*b)*b), -(sqrt(-a*b)*B*x - (B*a +
A*b)*arctan(sqrt(-a*b)*x/a))/(sqrt(-a*b)*b)]
```

Sympy [A] time = 1.63868, size = 75, normalized size = 1.92

$$-\frac{Bx}{b} - \frac{\sqrt{\frac{1}{ab^3}}(Ab + Ba)\log\left(-ab\sqrt{\frac{1}{ab^3}} + x\right)}{2} + \frac{\sqrt{\frac{1}{ab^3}}(Ab + Ba)\log\left(ab\sqrt{\frac{1}{ab^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(-b*x**2+a),x)
```

```
[Out] -B*x/b - sqrt(1/(a*b**3))*(A*b + B*a)*log(-a*b*sqrt(1/(a*b**3)) +
 x)/2 + sqrt(1/(a*b**3))*(A*b + B*a)*log(a*b*sqrt(1/(a*b**3)) + x
)/2
```

GIAC/XCAS [A] time = 0.227883, size = 49, normalized size = 1.26

$$-\frac{Bx}{b} - \frac{(Ba + Ab)\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(B*x^2 + A)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] -B*x/b - (B*a + A*b)*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b)
```

3.117 $\int \frac{1+x^2}{(16+x^2)^3}\,dx$

Optimal. Leaf size=35

$$\frac{19x}{2048\,(x^2+16)}-\frac{15x}{64\,(x^2+16)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan
[x/4])/8192
```

Rubi [A] time = 0.0234301, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{19x}{2048\,(x^2+16)}-\frac{15x}{64\,(x^2+16)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan
[x/4])/8192
```

Rubi in Sympy [A] time = 4.13407, size = 29, normalized size = 0.83

$$\frac{19x}{2048\,(x^2+16)}-\frac{15x}{64\,(x^2+16)^2}+\frac{19\,\mathrm{atan}\left(\frac{x}{4}\right)}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)/(x**2+16)**3,x)
```

```
[Out] 19*x/(2048*(x**2 + 16)) - 15*x/(64*(x**2 + 16)**2) + 19*atan(x/4)
/8192
```

Mathematica [A] time = 0.0153569, size = 35, normalized size = 1.

$$\frac{19x}{2048\,(x^2+16)}-\frac{15x}{64\,(x^2+16)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan
[x/4])/8192
```

Maple [A] time = 0.011, size = 25, normalized size = 0.7

$$\frac{1}{(x^2+16)^2}\left(\frac{19\,x^3}{2048}-\frac{11\,x}{128}\right)+\frac{19}{8192}\arctan\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^2+16)^3,x)
```

```
[Out] (19/2048*x^3-11/128*x)/(x^2+16)^2+19/8192*arctan(1/4*x)
```

Maxima [A] time = 1.50787, size = 41, normalized size = 1.17

$$\frac{19\,x^3-176\,x}{2048\,(x^4+32\,x^2+256)}+\frac{19}{8192}\,\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 + 16)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^4 + 32*x^2 + 256) + 19/8192*arctan(1/4
*x)
```

Fricas [A] time = 0.24074, size = 53, normalized size = 1.51

$$\frac{76\,x^3+19\,\left(x^4+32\,x^2+256\right)\arctan\left(\frac{1}{4}\,x\right)-704\,x}{8192\,(x^4+32\,x^2+256)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 + 16)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(76*x^3 + 19*(x^4 + 32*x^2 + 256)*arctan(1/4*x) - 704*x)/(
x^4 + 32*x^2 + 256)
```

Sympy [A] time = 0.311551, size = 27, normalized size = 0.77

$$\frac{19x^3-176x}{2048x^4+65536x^2+524288}+\frac{19\operatorname{atan}\left(\frac{x}{4}\right)}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**2+16)**3,x)
```

```
[Out] (19*x**3 - 176*x)/(2048*x**4 + 65536*x**2 + 524288) + 19*atan(x/4
)/8192
```

GIAC/XCAS [A] time = 0.227144, size = 34, normalized size = 0.97

$$\frac{19\,x^3-176\,x}{2048\,(x^2+16)^2}+\frac{19}{8192}\,\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 + 16)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^2 + 16)^2 + 19/8192*arctan(1/4*x)
```

3.118 $\int \frac{1+2x^2}{x^5(1+x^2)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4x^4(x^2+1)^2}$$

```
[Out] -1/(4*x^4*(1 + x^2)^2)
```

Rubi [A] time = 0.0097358, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{1}{4x^4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]

[Out] -1/(4*x^4*(1 + x^2)^2)
```

Rubi in Sympy [A] time = 6.69551, size = 14, normalized size = 1.

$$-\frac{1}{4x^4(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)/x**5/(x**2+1)**3,x)

[Out] -1/(4*x**4*(x**2 + 1)**2)
```

Mathematica [A] time = 0.00977804, size = 14, normalized size = 1.

$$-\frac{1}{4x^4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]

[Out] -1/(4*x^4*(1 + x^2)^2)
```

Maple [B] time = 0.016, size = 30, normalized size = 2.1

$$-\frac{1}{4\,x^4}+\frac{1}{2\,x^2}-\frac{1}{4\,(x^2+1)^2}-\frac{1}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)/x^5/(x^2+1)^3,x)

[Out] -1/4/x^4+1/2/x^2-1/4/(x^2+1)^2-1/2/(x^2+1)
```

Maxima [A] time = 1.34297, size = 22, normalized size = 1.57

$$-\frac{1}{4\left(x^{8}+2\,x^{6}+x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/((x^2 + 1)^3*x^5),x, algorithm="maxima")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Fricas [A] time = 0.203568, size = 22, normalized size = 1.57

$$-\frac{1}{4\left(x^{8}+2\,x^{6}+x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/((x^2 + 1)^3*x^5),x, algorithm="fricas")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A] time = 0.357549, size = 17, normalized size = 1.21

$$-\frac{1}{4x^{8}+8x^{6}+4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)/x**5/(x**2+1)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

GIAC/XCAS [A] time = 0.227022, size = 15, normalized size = 1.07

$$-\frac{1}{4\left(x^{4}+x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)/((x^2 + 1)^3*x^5),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

3.119 $\int \frac{(1-x^2)^2}{(-1+x^2)^2} dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.00318223, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)^2/(-1 + x^2)^2,x]

[Out] x
```

Rubi in Sympy [A] time = 0.041971, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**2/(x**2-1)**2,x)

[Out] x
```

Mathematica [A] time = 0.000409258, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)^2/(-1 + x^2)^2,x]

[Out] x
```

Maple [A] time = 0., size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^2/(x^2-1)^2,x)

[Out] x
```

Maxima [A] time = 1.34069, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 0.202327, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A] time = 0.071556, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**2/(x**2-1)**2,x)
```

```
[Out] x
```

GIAC/XCAS [A] time = 0.21752, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1,x, algorithm="giac")
```

```
[Out] x
```

3.120 $\int \frac{x^3\left(ac+bcx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=8

$$\frac{cx^4}{4}$$

```
[Out] (c*x^4)/4
```

Rubi [A] time = 0.00656093, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^4)/4
```

Rubi in Sympy [A] time = 3.56995, size = 5, normalized size = 0.62

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a),x)

[Out] c*x**4/4
```

Mathematica [A] time = 0.000807317, size = 8, normalized size = 1.

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^4)/4
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*c*x^2+a*c)/(b*x^2+a),x)

[Out] 1/4*c*x^4
```

Maxima [A] time = 1.34633, size = 8, normalized size = 1.

$$\frac{1}{4}cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4
```

Fricas [A] time = 0.216796, size = 8, normalized size = 1.

$$\frac{1}{4}cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4
```

Sympy [A] time = 0.104614, size = 5, normalized size = 0.62

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**4/4
```

GIAC/XCAS [A] time = 0.224878, size = 8, normalized size = 1.

$$\frac{1}{4}cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4
```

3.121 $\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=8

$$\frac{cx^3}{3}$$

```
[Out] (c*x^3)/3
```

Rubi [A] time = 0.00635038, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^3)/3
```

Rubi in Sympy [A] time = 3.58967, size = 5, normalized size = 0.62

$$\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a),x)

[Out] c*x**3/3
```

Mathematica [A] time = 0.000604448, size = 8, normalized size = 1.

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^3)/3
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*c*x^2+a*c)/(b*x^2+a),x)

[Out] 1/3*c*x^3
```

Maxima [A] time = 1.33462, size = 8, normalized size = 1.

$$\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a),x, algorithm="maxima")

[Out] 1/3*c*x^3
```

Fricas [A] time = 0.213022, size = 8, normalized size = 1.

$$\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a),x, algorithm="fricas")

[Out] 1/3*c*x^3
```

Sympy [A] time = 0.104626, size = 5, normalized size = 0.62

$$\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a),x)

[Out] c*x**3/3
```

GIAC/XCAS [A] time = 0.222676, size = 8, normalized size = 1.

$$\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a),x, algorithm="giac")

[Out] 1/3*c*x^3
```

3.122 $\int \frac{x(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^2}{2}$$

[Out] (c*x^2)/2

Rubi [A] time = 0.00598656, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^2)/2

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$c \int x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x*(b*c*x**2+a*c)/(b*x**2+a),x)

[Out] c*Integral(x, x)

Mathematica [A] time = 0.000429417, size = 8, normalized size = 1.

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^2)/2

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*c*x^2+a*c)/(b*x^2+a),x)

[Out] 1/2*c*x^2

Maxima [A] time = 1.3497, size = 8, normalized size = 1.

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a),x, algorithm="maxima")

[Out] 1/2*c*x^2
```

Fricas [A] time = 0.215096, size = 8, normalized size = 1.

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a),x, algorithm="fricas")

[Out] 1/2*c*x^2
```

Sympy [A] time = 0.098101, size = 5, normalized size = 0.62

$$\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*c*x**2+a*c)/(b*x**2+a),x)

[Out] c*x**2/2
```

GIAC/XCAS [A] time = 0.228116, size = 8, normalized size = 1.

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a),x, algorithm="giac")

[Out] 1/2*c*x^2
```

3.123 $\int \frac{ac+bcx^2}{a+bx^2} \, dx$

Optimal. Leaf size=3

$$cx$$

```
[Out] c*x
```

Rubi [A] time = 0.00466407, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$cx$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(a + b*x^2),x]

[Out] c*x
```

Rubi in Sympy [A] time = 3.14877, size = 2, normalized size = 0.67

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/(b*x**2+a),x)

[Out] c*x
```

Mathematica [A] time = 0.000350061, size = 3, normalized size = 1.

$$cx$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(a + b*x^2),x]

[Out] c*x
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/(b*x^2+a),x)

[Out] c*x
```

Maxima [A] time = 1.33213, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] c*x
```

Fricas [A] time = 0.219684, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] c*x
```

Sympy [A] time = 0.086926, size = 2, normalized size = 0.67

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x
```

GIAC/XCAS [A] time = 0.221594, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] c*x
```

3.124 $\int \frac{ac+bcx^2}{x(a+bx^2)}\,dx$

Optimal. Leaf size=4

$$c\log(x)$$

[Out] c*Log[x]

Rubi [A] time = 0.00530692, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$c\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]

[Out] c*Log[x]

Rubi in Sympy [A] time = 3.59288, size = 3, normalized size = 0.75

$$c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((b*c*x**2+a*c)/x/(b*x**2+a),x)

[Out] c*log(x)

Mathematica [A] time = 0.000593888, size = 4, normalized size = 1.

$$c\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]

[Out] c*Log[x]

Maple [A] time = 0.002, size = 5, normalized size = 1.3

$$c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*c*x^2+a*c)/x/(b*x^2+a),x)

[Out] c*ln(x)

Maxima [A] time = 1.3403, size = 9, normalized size = 2.25

$$\frac{1}{2}c\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)
```

Fricas [A] time = 0.220173, size = 5, normalized size = 1.25

$$c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] c*log(x)
```

Sympy [A] time = 0.115203, size = 3, normalized size = 0.75

$$c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)
```

GIAC/XCAS [A] time = 0.22222, size = 7, normalized size = 1.75

$$c\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] c*ln(abs(x))
```

3.125 $\int \frac{ac+bcx^2}{x^2(a+bx^2)} dx$

Optimal. Leaf size=6

$$-\frac{c}{x}$$

[Out] -(c/x)

Rubi [A] time = 0.00592865, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/x)
```

Rubi in Sympy [A] time = 3.59483, size = 3, normalized size = 0.5

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/x**2/(b*x**2+a),x)
```

```
[Out] -c/x
```

Mathematica [A] time = 0.000589409, size = 6, normalized size = 1.

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/x)
```

Maple [A] time = 0.001, size = 7, normalized size = 1.2

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x^2/(b*x^2+a),x)
```

```
[Out] -c/x
```

Maxima [A] time = 1.34327, size = 8, normalized size = 1.33

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -c/x
```

Fricas [A] time = 0.216027, size = 8, normalized size = 1.33

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -c/x
```

Sympy [A] time = 0.122779, size = 3, normalized size = 0.5

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x**2/(b*x**2+a),x)
```

```
[Out] -c/x
```

GIAC/XCAS [A] time = 0.222543, size = 8, normalized size = 1.33

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -c/x
```

3.126 $\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=8

$$-\frac{c}{2x^2}$$

[Out] -c/(2*x^2)

Rubi [A] time = 0.00625471, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]

[Out] -c/(2*x^2)

Rubi in Sympy [A] time = 3.57448, size = 7, normalized size = 0.88

$$-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((b*c*x**2+a*c)/x**3/(b*x**2+a),x)

[Out] -c/(2*x**2)

Mathematica [A] time = 0.000651485, size = 8, normalized size = 1.

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]

[Out] -c/(2*x^2)

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a),x)

[Out] -1/2*c/x^2

Maxima [A] time = 1.3463, size = 8, normalized size = 1.

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*c/x^2
```

Fricas [A] time = 0.217426, size = 8, normalized size = 1.

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*c/x^2
```

Sympy [A] time = 0.115672, size = 7, normalized size = 0.88

$$-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*x**2)
```

GIAC/XCAS [A] time = 0.224218, size = 8, normalized size = 1.

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*c/x^2
```

3.127 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=29

$$\frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

```
[Out] (c*x^2)/(2*b) - (a*c*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0531508, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]

[Out] (c*x^2)/(2*b) - (a*c*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ac\log(a+bx^2)}{2b^2} + \frac{c\int^{x^2}\frac{1}{b}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**2,x)

[Out] -a*c*log(a + b*x**2)/(2*b**2) + c*Integral(1/b, (x, x**2))/2
```

Mathematica [A] time = 0.00728249, size = 29, normalized size = 1.

$$c\left(\frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]

[Out] c*(x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.9

$$\frac{cx^2}{2\,b} - \frac{ac\ln(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*c*x^2/b-1/2*a*c*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.36612, size = 34, normalized size = 1.17

$$\frac{cx^2}{2\,b} - \frac{ac\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/b - 1/2*a*c*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.20791, size = 32, normalized size = 1.1

$$\frac{bcx^2 - ac\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2 - a*c*log(b*x^2 + a))/b^2
```

Sympy [A] time = 1.18829, size = 22, normalized size = 0.76

$$c\left(-\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b))
```

GIAC/XCAS [A] time = 0.243218, size = 63, normalized size = 2.17

$$\frac{\frac{ac\ln\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b} + \frac{\left(bx^2+a\right)c}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c*ln(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b + (b*x^2 + a
)*c/b)/b
```

3.128 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=33

$$\frac{cx}{b}-\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] (c*x)/b - (Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.0426848, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{cx}{b}-\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*x)/b - (Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi in Sympy [A] time = 10.0641, size = 29, normalized size = 0.88

$$-\frac{\sqrt{a}c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(a)*c*atan(sqrt(b)*x/sqrt(a))/b**(3/2) + c*x/b
```

Mathematica [A] time = 0.0153518, size = 33, normalized size = 1.

$$c\left(\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] c*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2))
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{cx}{b}-\frac{ac}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x)

[Out] c*x/b-c*a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238078, size = 1, normalized size = 0.03

$$\left[\frac{c\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,cx}{2\,b},-\frac{c\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-cx}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")

[Out] [1/2*(c*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)
) + 2*c*x)/b, -(c*sqrt(a/b)*arctan(x/sqrt(a/b)) - c*x)/b]
```

Sympy [A] time = 1.2243, size = 58, normalized size = 1.76

$$c\left(\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**2,x)

[Out] c*(sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(
b*sqrt(-a/b**3) + x)/2 + x/b)
```

GIAC/XCAS [A] time = 0.22312, size = 38, normalized size = 1.15

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a)^2,x, algorithm="giac")

[Out] -a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

3.129 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=16

$$\frac{c\log(a+bx^2)}{2b}$$

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

Rubi [A] time = 0.012239, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{c\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]

[Out] (c*Log[a + b*x^2])/(2*b)
```

Rubi in Sympy [A] time = 4.77827, size = 12, normalized size = 0.75

$$\frac{c\log(a+bx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**2,x)

[Out] c*log(a + b*x**2)/(2*b)
```

Mathematica [A] time = 0.00323439, size = 16, normalized size = 1.

$$\frac{c\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]

[Out] (c*Log[a + b*x^2])/(2*b)
```

Maple [A] time = 0.001, size = 15, normalized size = 0.9

$$\frac{c\ln(bx^2+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x)

[Out] 1/2*c*ln(b*x^2+a)/b
```

Maxima [A] time = 1.3302, size = 19, normalized size = 1.19

$$\frac{c\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

Fricas [A] time = 0.227455, size = 19, normalized size = 1.19

$$\frac{c\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

Sympy [A] time = 0.271459, size = 12, normalized size = 0.75

$$\frac{c\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*log(a + b*x**2)/(2*b)
```

GIAC/XCAS [A] time = 0.222329, size = 85, normalized size = 5.31

$$-\frac{1}{2}\,c\left(\frac{\ln\left(\frac{|bx^2+a|}{(bx^2+a)^2\,|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)-\frac{ac}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*(ln(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 +
 a)*b)) - 1/2*a*c/((b*x^2 + a)*b)
```

3.130 $\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=25

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0226602, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(a + b*x^2)^2,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 5.43016, size = 24, normalized size = 0.96

$$\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/(b*x**2+a)**2,x)

[Out] c*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.00701851, size = 25, normalized size = 1.

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(a + b*x^2)^2,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.003, size = 17, normalized size = 0.7

$$c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/(b*x^2+a)^2,x)

[Out] 1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229301, size = 1, normalized size = 0.04

$$\left[\frac{c\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{2\sqrt{-ab}}, \frac{c\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a)^2,x, algorithm="fricas")

[Out] [1/2*c*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a))/sqrt(-
a*b), c*arctan(sqrt(a*b)*x/a)/sqrt(a*b)]
```

Sympy [A] time = 0.327256, size = 54, normalized size = 2.16

$$c\left(-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/(b*x**2+a)**2,x)

[Out] c*(-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*
log(a*sqrt(-1/(a*b)) + x)/2)
```

GIAC/XCAS [A] time = 0.22193, size = 22, normalized size = 0.88

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a)^2,x, algorithm="giac")

[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.131 $\int \frac{ac+bcx^2}{x(a+bx^2)^2} \, dx$

Optimal. Leaf size=24

$$\frac{c\log(x)}{a} - \frac{c\log(a+bx^2)}{2a}$$

```
[Out] (c*Log[x])/a - (c*Log[a + b*x^2])/(2*a)
```

Rubi [A] time = 0.0368758, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{c\log(x)}{a} - \frac{c\log(a+bx^2)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]

[Out] (c*Log[x])/a - (c*Log[a + b*x^2])/(2*a)
```

Rubi in Sympy [A] time = 9.09932, size = 22, normalized size = 0.92

$$\frac{c\log(x^2)}{2a} - \frac{c\log(a+bx^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/x/(b*x**2+a)**2,x)

[Out] c*log(x**2)/(2*a) - c*log(a + b*x**2)/(2*a)
```

Mathematica [A] time = 0.00873298, size = 24, normalized size = 1.

$$c\left(\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]

[Out] c*(Log[x]/a - Log[a + b*x^2]/(2*a))
```

Maple [A] time = 0.006, size = 23, normalized size = 1.

$$\frac{c\ln(x)}{a} - \frac{c\ln(bx^2+a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x/(b*x^2+a)^2,x)

[Out] c*ln(x)/a-1/2*c*ln(b*x^2+a)/a
```

Maxima [A] time = 1.333, size = 34, normalized size = 1.42

$$-\frac{c\log\left(bx^2+a\right)}{2\,a}+\frac{c\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*c*log(b*x^2 + a)/a + 1/2*c*log(x^2)/a
```

Fricas [A] time = 0.231777, size = 28, normalized size = 1.17

$$-\frac{c\log\left(bx^2+a\right)-2\,c\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*(c*log(b*x^2 + a) - 2*c*log(x))/a
```

Sympy [A] time = 0.536742, size = 17, normalized size = 0.71

$$c\left(\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x/(b*x**2+a)**2,x)
```

```
[Out] c*(log(x)/a - log(a/b + x**2)/(2*a))
```

GIAC/XCAS [A] time = 0.223636, size = 35, normalized size = 1.46

$$\frac{c\ln\left(x^2\right)}{2\,a}-\frac{c\ln\left(\left|bx^2+a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*c*ln(x^2)/a - 1/2*c*ln(abs(b*x^2 + a))/a
```

3.132 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

```
[Out] -(c/(a*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0412215, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi in Sympy [A] time = 9.88082, size = 31, normalized size = 0.86

$$-\frac{c}{ax}-\frac{\sqrt{b}c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -c/(a*x) - sqrt(b)*c*atan(sqrt(b)*x/sqrt(a))/a**(3/2)
```

Mathematica [A] time = 0.0219419, size = 36, normalized size = 1.

$$c\left(-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] c*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))
```

Maple [A] time = 0.006, size = 32, normalized size = 0.9

$$-\frac{bc}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c*b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))-c/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248306, size = 1, normalized size = 0.03

$$\left[\frac{cx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,c}{2\,ax}, -\frac{cx\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+c}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(c*x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)) - 2*c)/(a*x), -(c*x*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + c)/
(a*x)]
```

Sympy [A] time = 1.32918, size = 66, normalized size = 1.83

$$c\left(\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)
*log(a**2*sqrt(-b/a**3)/b + x)/2 - 1/(a*x))
```

GIAC/XCAS [A] time = 0.221115, size = 42, normalized size = 1.17

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.133 $\int \frac{ac+bcx^2}{x^3(a+bx^2)^2} \, dx$

Optimal. Leaf size=38

$$\frac{bc\log\left(a+bx^2\right)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

```
[Out] -c/(2*a*x^2) - (b*c*Log[x])/a^2 + (b*c*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.0626028, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{bc\log\left(a+bx^2\right)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]

[Out] -c/(2*a*x^2) - (b*c*Log[x])/a^2 + (b*c*Log[a + b*x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 12.4026, size = 37, normalized size = 0.97

$$-\frac{c}{2ax^2}-\frac{bc\log\left(x^2\right)}{2a^2}+\frac{bc\log\left(a+bx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**2,x)

[Out] -c/(2*a*x**2) - b*c*log(x**2)/(2*a**2) + b*c*log(a + b*x**2)/(2*a
**2)
```

Mathematica [A] time = 0.0112903, size = 37, normalized size = 0.97

$$c\left(\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]

[Out] c*(-1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2))
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$-\frac{c}{2\,ax^2}-\frac{bc\ln(x)}{a^2}+\frac{bc\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c/a/x^2-b*c*ln(x)/a^2+1/2*b*c*ln(b*x^2+a)/a^2
```

Maxima [A] time = 1.32238, size = 49, normalized size = 1.29

$$\frac{bc\log\left(bx^2+a\right)}{2\,a^2}-\frac{bc\log\left(x^2\right)}{2\,a^2}-\frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*c*log(b*x^2 + a)/a^2 - 1/2*b*c*log(x^2)/a^2 - 1/2*c/(a*x^2)
```

Fricas [A] time = 0.230648, size = 49, normalized size = 1.29

$$\frac{bcx^2\log\left(bx^2+a\right)-2\,bcx^2\log\left(x\right)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log(b*x^2 + a) - 2*b*c*x^2*log(x) - a*c)/(a^2*x^2)
```

Sympy [A] time = 1.63474, size = 32, normalized size = 0.84

$$c\left(-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] c*(-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2))
```

GIAC/XCAS [A] time = 0.224729, size = 63, normalized size = 1.66

$$-\frac{bc\ln\left(x^2\right)}{2\,a^2}+\frac{bc\ln\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bcx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*c*ln(x^2)/a^2 + 1/2*b*c*ln(abs(b*x^2 + a))/a^2 + 1/2*(b*c*
x^2 - a*c)/(a^2*x^2)
```

3.134 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=35

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

```
[Out] (a*c)/(2*b^2*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0624562, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*c)/(2*b^2*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [A] time = 11.4785, size = 29, normalized size = 0.83

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] a*c/(2*b**2*(a + b*x**2)) + c*log(a + b*x**2)/(2*b**2)
```

Mathematica [A] time = 0.0157707, size = 28, normalized size = 0.8

$$\frac{c\left(\frac{a}{a+bx^2}+\log(a+bx^2)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (c*(a/(a + b*x^2) + Log[a + b*x^2]))/(2*b^2)
```

Maple [A] time = 0.007, size = 32, normalized size = 0.9

$$\frac{ac}{2\,b^2(bx^2+a)}+\frac{c\ln(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*a*c/b^2/(b*x^2+a)+1/2*c*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.3366, size = 46, normalized size = 1.31

$$\frac{ac}{2\,(b^3x^2+ab^2)}+\frac{c\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c/(b^3*x^2 + a*b^2) + 1/2*c*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.227442, size = 54, normalized size = 1.54

$$\frac{ac+\left(bcx^2+ac\right)\log\left(bx^2+a\right)}{2\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c + (b*c*x^2 + a*c)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 1.39611, size = 31, normalized size = 0.89

$$c\left(\frac{a}{2ab^2+2b^3x^2}+\frac{\log\left(a+bx^2\right)}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2))
```

GIAC/XCAS [A] time = 0.222955, size = 43, normalized size = 1.23

$$\frac{c\ln\left(\left|bx^2+a\right|\right)}{2\,b^2}+\frac{ac}{2\,(bx^2+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^3/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*ln(abs(b*x^2 + a))/b^2 + 1/2*a*c/((b*x^2 + a)*b^2)
```

3.135 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=47

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{cx}{2b(a+bx^2)}$$

```
[Out] -(c*x)/(2*b*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqr
t[a]*b^(3/2))
```

Rubi [A] time = 0.0452267, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{cx}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -(c*x)/(2*b*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqr
t[a]*b^(3/2))
```

Rubi in Sympy [A] time = 9.95755, size = 39, normalized size = 0.83

$$-\frac{cx}{2b(a+bx^2)}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] -c*x/(2*b*(a + b*x**2)) + c*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b*
*(3/2))
```

Mathematica [A] time = 0.03489, size = 47, normalized size = 1.

$$c\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(a+bx^2)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] c*(-x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*
b^(3/2)))
```

Maple [A] time = 0.009, size = 38, normalized size = 0.8

$$-\frac{cx}{2\,b\,(bx^2+a)}+\frac{c}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c*x/b/(b*x^2+a)+1/2*c/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236557, size = 1, normalized size = 0.02

$$\left[-\frac{2\sqrt{-ab}cx - (bcx^2 + ac)\log\left(\frac{2abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right)}{4(b^2x^2 + ab)\sqrt{-ab}}, -\frac{\sqrt{ab}cx - (bcx^2 + ac)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2(b^2x^2 + ab)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a*b)*c*x - (b*c*x^2 + a*c)*log((2*a*b*x + (b*x^2 -
 a)*sqrt(-a*b))/(b*x^2 + a)))/((b^2*x^2 + a*b)*sqrt(-a*b)), -1/2*
(sqrt(a*b)*c*x - (b*c*x^2 + a*c)*arctan(sqrt(a*b)*x/a))/((b^2*x^2
 + a*b)*sqrt(a*b))]
```

Sympy [A] time = 1.44071, size = 80, normalized size = 1.7

$$c\left(-\frac{x}{2ab + 2b^2x^2} - \frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/
(a*b**3)) + x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) +
x)/4)
```

GIAC/XCAS [A] time = 0.227216, size = 50, normalized size = 1.06

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b} - \frac{cx}{2(bx^2 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x^2/(b*x^2 + a)^3,x, algorithm="giac")

[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*c*x/((b*x^2 + a)*
b)
```

3.136 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=17

$$-\frac{c}{2b(a+bx^2)}$$

```
[Out] -c/(2*b*(a + b*x^2))
```

Rubi [A] time = 0.0131958, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$-\frac{c}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]

[Out] -c/(2*b*(a + b*x^2))
```

Rubi in Sympy [A] time = 4.76808, size = 12, normalized size = 0.71

$$-\frac{c}{2b(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**3,x)

[Out] -c/(2*b*(a + b*x**2))
```

Mathematica [A] time = 0.00378476, size = 17, normalized size = 1.

$$-\frac{c}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]

[Out] -c/(2*b*(a + b*x^2))
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$-\frac{c}{2\,b\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x)

[Out] -1/2*c/b/(b*x^2+a)
```

Maxima [A] time = 1.34255, size = 22, normalized size = 1.29

$$-\frac{c}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a)^3,x, algorithm="maxima")

[Out] -1/2*c/(b^2*x^2 + a*b)
```

Fricas [A] time = 0.211298, size = 22, normalized size = 1.29

$$-\frac{c}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a)^3,x, algorithm="fricas")

[Out] -1/2*c/(b^2*x^2 + a*b)
```

Sympy [A] time = 1.29719, size = 15, normalized size = 0.88

$$-\frac{c}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**3,x)

[Out] -c/(2*a*b + 2*b**2*x**2)
```

GIAC/XCAS [A] time = 0.230283, size = 20, normalized size = 1.18

$$-\frac{c}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)*x/(b*x^2 + a)^3,x, algorithm="giac")

[Out] -1/2*c/((b*x^2 + a)*b)
```

3.137 $\int \frac{ac+bcx^2}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=47

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{cx}{2a\left(a+bx^2\right)}$$

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3
/2)*Sqrt[b])
```

Rubi [A] time = 0.0373577, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{cx}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3
/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 7.2085, size = 39, normalized size = 0.83

$$\frac{cx}{2a\left(a+bx^2\right)} + \frac{c \operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*x/(2*a*(a + b*x**2)) + c*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*sq
rt(b))
```

Mathematica [A] time = 0.039475, size = 47, normalized size = 1.

$$c\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+bx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

```
[Out] c*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*S
qrt[b]))
```

Maple [A] time = 0.005, size = 38, normalized size = 0.8

$$\frac{cx}{2\,a\left(bx^2+a\right)} + \frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*c*x/a/(b*x^2+a)+1/2*c/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244999, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{-ab}cx + \left(bcx^2 + ac\right)\log\left(\frac{2\,abx + \left(bx^2 - a\right)\sqrt{-ab}}{bx^2 + a}\right)}{4\left(abx^2 + a^2\right)\sqrt{-ab}}, \frac{\sqrt{ab}cx + \left(bcx^2 + ac\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(abx^2 + a^2\right)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b)*c*x + (b*c*x^2 + a*c)*log((2*a*b*x + (b*x^2 -
a)*sqrt(-a*b))/(b*x^2 + a)))/((a*b*x^2 + a^2)*sqrt(-a*b)), 1/2*(s
qrt(a*b)*c*x + (b*c*x^2 + a*c)*arctan(sqrt(a*b)*x/a))/((a*b*x^2 +
 a^2)*sqrt(a*b))]
```

Sympy [A] time = 1.47462, size = 80, normalized size = 1.7

$$c\left(\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/
(a**3*b)) + x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) +
 x)/4)
```

GIAC/XCAS [A] time = 0.228685, size = 50, normalized size = 1.06

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{cx}{2\left(bx^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*c*x/((b*x^2 + a)*
a)
```

3.138 $\int \frac{ac+bcx^2}{x(a+bx^2)^3}\,dx$

Optimal. Leaf size=41

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a\left(a+bx^2\right)}$$

```
[Out] c/(2*a*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] time = 0.0658263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] c/(2*a*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 12.5625, size = 37, normalized size = 0.9

$$\frac{c}{2a\left(a+bx^2\right)}+\frac{c\log\left(x^2\right)}{2a^2}-\frac{c\log\left(a+bx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/x/(b*x**2+a)**3,x)
```

```
[Out] c/(2*a*(a + b*x**2)) + c*log(x**2)/(2*a**2) - c*log(a + b*x**2)/(
2*a**2)
```

Mathematica [A] time = 0.0219611, size = 34, normalized size = 0.83

$$\frac{c\left(\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (c*(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2]))/(2*a^2)
```

Maple [A] time = 0.011, size = 38, normalized size = 0.9

$$\frac{c}{2\,a\left(bx^2+a\right)}+\frac{c\ln(x)}{a^2}-\frac{c\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x/(b*x^2+a)^3,x)
```

```
[Out] 1/2*c/a/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

Maxima [A] time = 1.3293, size = 54, normalized size = 1.32

$$\frac{c}{2\,(abx^2+a^2)}-\frac{c\log\left(bx^2+a\right)}{2\,a^2}+\frac{c\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x),x, algorithm="maxima")
```

```
[Out] 1/2*c/(a*b*x^2 + a^2) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(x^2)
/a^2
```

Fricas [A] time = 0.226854, size = 73, normalized size = 1.78

$$\frac{ac-\left(bcx^2+ac\right)\log\left(bx^2+a\right)+2\left(bcx^2+ac\right)\log\left(x\right)}{2\,(a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c - (b*c*x^2 + a*c)*log(b*x^2 + a) + 2*(b*c*x^2 + a*c)*log
(x))/(a^2*b*x^2 + a^3)
```

Sympy [A] time = 1.72782, size = 36, normalized size = 0.88

$$c\left(\frac{1}{2a^2+2abx^2}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x/(b*x**2+a)**3,x)
```

```
[Out] c*(1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**
2))
```

GIAC/XCAS [A] time = 0.2273, size = 69, normalized size = 1.68

$$\frac{c\ln\left(x^2\right)}{2\,a^2}-\frac{c\ln\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bcx^2+2\,ac}{2\,(bx^2+a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x),x, algorithm="giac")
```

```
[Out] 1/2*c*ln(x^2)/a^2 - 1/2*c*ln(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 +
 2*a*c)/((b*x^2 + a)*a^2)
```

3.139 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=60

$$-\frac{3\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

```
[Out] (-3*c)/(2*a^2*x) + c/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*c*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.0599194, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$-\frac{3\sqrt{b}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (-3*c)/(2*a^2*x) + c/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*c*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 13.8788, size = 51, normalized size = 0.85

$$\frac{c}{2ax\left(a+bx^2\right)}-\frac{3c}{2a^2x}-\frac{3\sqrt{b}c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] c/(2*a*x*(a + b*x**2)) - 3*c/(2*a**2*x) - 3*sqrt(b)*c*atan(sqrt(b
)*x/sqrt(a))/(2*a**(5/2))
```

Mathematica [A] time = 0.0615778, size = 56, normalized size = 0.93

$$c\left(-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{1}{a^2x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] c*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)))
```

Maple [A] time = 0.012, size = 49, normalized size = 0.8

$$-\frac{c}{a^2x}-\frac{bcx}{2\,a^2\left(bx^2+a\right)}-\frac{3\,bc}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -c/a^2/x-1/2*c*b/a^2*x/(b*x^2+a)-3/2*c*b/a^2/(a*b)^(1/2)*arctan(x
*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239745, size = 1, normalized size = 0.02

$$\left[-\frac{6\,bcx^2 - 3\,\left(bcx^3 + acx\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + 4\,ac}{4\,\left(a^2bx^3 + a^3x\right)},\; -\frac{3\,bcx^2 + 3\,\left(bcx^3 + acx\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right) + 2\,ac}{2\,\left(a^2bx^3 + a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*c*x^2 - 3*(b*c*x^3 + a*c*x)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 4*a*c)/(a^2*b*x^3 + a^3*x), -1
/2*(3*b*c*x^2 + 3*(b*c*x^3 + a*c*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(
b/a))) + 2*a*c)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] time = 1.76036, size = 92, normalized size = 1.53

$$c\left(\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{2a + 3bx^2}{2a^3x + 2a^2bx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] c*(3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a
**5)*log(a**3*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x
 + 2*a**2*b*x**3))
```

GIAC/XCAS [A] time = 0.229111, size = 68, normalized size = 1.13

$$-\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bcx^2 + 2\,ac}{2\,(bx^3 + ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x^2),x, algorithm="giac")
```

```
[Out] -3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 +
 2*a*c)/((b*x^3 + a*x)*a^2)
```

3.140 $\int \frac{ac+bcx^2}{x^3(a+bx^2)^3} dx$

Optimal. Leaf size=53

$$\frac{bc\log(a+bx^2)}{a^3} - \frac{2bc\log(x)}{a^3} - \frac{bc}{2a^2(a+bx^2)} - \frac{c}{2a^2x^2}$$

```
[Out] -c/(2*a^2*x^2) - (b*c)/(2*a^2*(a + b*x^2)) - (2*b*c*Log[x])/a^3 +
 (b*c*Log[a + b*x^2])/a^3
```

Rubi [A] time = 0.088698, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{bc\log(a+bx^2)}{a^3} - \frac{2bc\log(x)}{a^3} - \frac{bc}{2a^2(a+bx^2)} - \frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -c/(2*a^2*x^2) - (b*c)/(2*a^2*(a + b*x^2)) - (2*b*c*Log[x])/a^3 +
 (b*c*Log[a + b*x^2])/a^3
```

Rubi in Sympy [A] time = 15.6278, size = 51, normalized size = 0.96

$$-\frac{bc}{2a^2(a+bx^2)} - \frac{c}{2a^2x^2} - \frac{bc\log(x^2)}{a^3} + \frac{bc\log(a+bx^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] -b*c/(2*a**2*(a + b*x**2)) - c/(2*a**2*x**2) - b*c*log(x**2)/a**3
 + b*c*log(a + b*x**2)/a**3
```

Mathematica [A] time = 0.0604297, size = 42, normalized size = 0.79

$$-\frac{c\left(a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log(a+bx^2)+4b\log(x)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -(c*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2]
))/(2*a^3)
```

Maple [A] time = 0.013, size = 50, normalized size = 0.9

$$-\frac{c}{2\,a^2x^2} - \frac{bc}{2\,a^2(bx^2+a)} - 2\,\frac{bc\ln(x)}{a^3} + \frac{bc\ln(bx^2+a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c/a^2/x^2-1/2*b*c/a^2/(b*x^2+a)-2*b*c*ln(x)/a^3+b*c*ln(b*x^2
+a)/a^3
```

Maxima [A] time = 1.35243, size = 77, normalized size = 1.45

$$-\frac{2\,bcx^{2}+ac}{2\,\left(a^{2}bx^{4}+a^{3}x^{2}\right)}+\frac{bc\log\left(bx^{2}+a\right)}{a^{3}}-\frac{bc\log\left(x^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*c*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + b*c*log(b*x^2 + a)
/a^3 - b*c*log(x^2)/a^3
```

Fricas [A] time = 0.228062, size = 108, normalized size = 2.04

$$-\frac{2\,abcx^{2}+a^{2}c-2\,\left(b^{2}cx^{4}+abcx^{2}\right)\log\left(bx^{2}+a\right)+4\,\left(b^{2}cx^{4}+abcx^{2}\right)\log\left(x\right)}{2\,\left(a^{3}bx^{4}+a^{4}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c*x^2 + a^2*c - 2*(b^2*c*x^4 + a*b*c*x^2)*log(b*x^2 +
 a) + 4*(b^2*c*x^4 + a*b*c*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 2.09321, size = 51, normalized size = 0.96

$$c\left(-\frac{a+2bx^{2}}{2a^{3}x^{2}+2a^{2}bx^{4}}-\frac{2b\log\left(x\right)}{a^{3}}+\frac{b\log\left(\frac{a}{b}+x^{2}\right)}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] c*(-(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**
3 + b*log(a/b + x**2)/a**3)
```

GIAC/XCAS [A] time = 0.24299, size = 76, normalized size = 1.43

$$-\frac{bc\ln\left(x^{2}\right)}{a^{3}}+\frac{bc\ln\left(\left|bx^{2}+a\right|\right)}{a^{3}}-\frac{2\,bcx^{2}+ac}{2\,\left(bx^{4}+ax^{2}\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*c*x^2 + a*c)/((b*x^2 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] -b*c*ln(x^2)/a^3 + b*c*ln(abs(b*x^2 + a))/a^3 - 1/2*(2*b*c*x^2 +
a*c)/((b*x^4 + a*x^2)*a^2)
```

3.141 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9
 + (b^2*d*x^11)/11
```

Rubi [A] time = 0.11334, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9
 + (b^2*d*x^11)/11
```

Rubi in Sympy [A] time = 13.9745, size = 49, normalized size = 0.89

$$\frac{a^2cx^5}{5}+\frac{ax^7\left(ad+2bc\right)}{7}+\frac{b^2dx^{11}}{11}+\frac{bx^9\left(2ad+bc\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + a*x**7*(a*d + 2*b*c)/7 + b**2*d*x**11/11 + b*x**9
*(2*a*d + b*c)/9
```

Mathematica [A] time = 0.0149234, size = 55, normalized size = 1.

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9
 + (b^2*d*x^11)/11
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{b^2dx^{11}}{11}+\frac{\left(2\,abd+b^2c\right)x^9}{9}+\frac{\left(a^2d+2\,abc\right)x^7}{7}+\frac{a^2cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/11*b^2*d*x^11+1/9*(2*a*b*d+b^2*c)*x^9+1/7*(a^2*d+2*a*b*c)*x^7+1
/5*a^2*c*x^5
```

Maxima [A] time = 1.34111, size = 69, normalized size = 1.25

$$\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\,a^2cx^5+\frac{1}{7}\left(2\,abc+a^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^4,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7
*(2*a*b*c + a^2*d)*x^7
```

Fricas [A] time = 0.19699, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}db^2+\frac{1}{9}x^9cb^2+\frac{2}{9}x^9dba+\frac{2}{7}x^7cba+\frac{1}{7}x^7da^2+\frac{1}{5}x^5ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^4,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/7*x^7*c*b*a +
 1/7*x^7*d*a^2 + 1/5*x^5*c*a^2
```

Sympy [A] time = 0.114571, size = 56, normalized size = 1.02

$$\frac{a^2cx^5}{5}+\frac{b^2dx^{11}}{11}+x^9\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^7\left(\frac{a^2d}{7}+\frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + b**2*d*x**11/11 + x**9*(2*a*b*d/9 + b**2*c/9) + x
**7*(a**2*d/7 + 2*a*b*c/7)
```

GIAC/XCAS [A] time = 0.221495, size = 72, normalized size = 1.31

$$\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\,b^2cx^9+\frac{2}{9}\,abdx^9+\frac{2}{7}\,abcx^7+\frac{1}{7}\,a^2dx^7+\frac{1}{5}\,a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^4,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/7*a*b*c*x^7 +
 1/7*a^2*d*x^7 + 1/5*a^2*c*x^5
```

3.142 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx$

Optimal. Leaf size=55

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8
 + (b^2*d*x^10)/10
```

Rubi [A] time = 0.182199, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8
 + (b^2*d*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2c\int^{x^2} x\,dx}{2}+\frac{ax^6\,(ad+2bc)}{6}+\frac{b^2dx^{10}}{10}+\frac{bx^8\,(2ad+bc)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*Integral(x, (x, x**2))/2 + a*x**6*(a*d + 2*b*c)/6 + b**2*d
*x**10/10 + b*x**8*(2*a*d + b*c)/8
```

Mathematica [A] time = 0.0131721, size = 55, normalized size = 1.

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8
 + (b^2*d*x^10)/10
```

Maple [A] time = 0.001, size = 52, normalized size = 1.

$$\frac{b^2dx^{10}}{10}+\frac{\left(2\,abd+b^2c\right)x^8}{8}+\frac{\left(a^2d+2\,abc\right)x^6}{6}+\frac{a^2cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/8*(2*a*b*d+b^2*c)*x^8+1/6*(a^2*d+2*a*b*c)*x^6+1
/4*a^2*c*x^4
```

Maxima [A] time = 1.32444, size = 69, normalized size = 1.25

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{8}\left(b^2c + 2\,abd\right)x^8 + \frac{1}{4}\,a^2cx^4 + \frac{1}{6}\left(2\,abc + a^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6
*(2*a*b*c + a^2*d)*x^6
```

Fricas [A] time = 0.199242, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}db^2 + \frac{1}{8}x^8cb^2 + \frac{1}{4}x^8dba + \frac{1}{3}x^6cba + \frac{1}{6}x^6da^2 + \frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/8*x^8*c*b^2 + 1/4*x^8*d*b*a + 1/3*x^6*c*b*a +
 1/6*x^6*d*a^2 + 1/4*x^4*c*a^2
```

Sympy [A] time = 0.115505, size = 53, normalized size = 0.96

$$\frac{a^2cx^4}{4} + \frac{b^2dx^{10}}{10} + x^8\left(\frac{abd}{4} + \frac{b^2c}{8}\right) + x^6\left(\frac{a^2d}{6} + \frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**4/4 + b**2*d*x**10/10 + x**8*(a*b*d/4 + b**2*c/8) + x**
6*(a**2*d/6 + a*b*c/3)
```

GIAC/XCAS [A] time = 0.222238, size = 72, normalized size = 1.31

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{8}\,b^2cx^8 + \frac{1}{4}\,abdx^8 + \frac{1}{3}\,abcx^6 + \frac{1}{6}\,a^2dx^6 + \frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^3,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*b^2*c*x^8 + 1/4*a*b*d*x^8 + 1/3*a*b*c*x^6 +
 1/6*a^2*d*x^6 + 1/4*a^2*c*x^4
```

3.143 $\int x^2 \left(a + bx^2\right)^2 \left(c + dx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2cx^3 + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{9}b^2dx^9$$

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7
 + (b^2*d*x^9)/9
```

Rubi [A] time = 0.106369, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{3}a^2cx^3 + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7
 + (b^2*d*x^9)/9
```

Rubi in Sympy [A] time = 16.0235, size = 49, normalized size = 0.89

$$\frac{a^2cx^3}{3} + \frac{ax^5\left(ad + 2bc\right)}{5} + \frac{b^2dx^9}{9} + \frac{bx^7\left(2ad + bc\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + a*x**5*(a*d + 2*b*c)/5 + b**2*d*x**9/9 + b*x**7*(
2*a*d + b*c)/7
```

Mathematica [A] time = 0.0125667, size = 55, normalized size = 1.

$$\frac{1}{3}a^2cx^3 + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7
 + (b^2*d*x^9)/9
```

Maple [A] time = 0.002, size = 52, normalized size = 1.

$$\frac{b^2dx^9}{9} + \frac{\left(2\,abd + b^2c\right)x^7}{7} + \frac{\left(a^2d + 2\,abc\right)x^5}{5} + \frac{a^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/9*b^2*d*x^9+1/7*(2*a*b*d+b^2*c)*x^7+1/5*(a^2*d+2*a*b*c)*x^5+1/3
*a^2*c*x^3
```

Maxima [A] time = 1.35105, size = 69, normalized size = 1.25

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\,\left(b^2c+2\,abd\right)x^7+\frac{1}{3}\,a^2cx^3+\frac{1}{5}\,\left(2\,abc+a^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(
2*a*b*c + a^2*d)*x^5
```

Fricas [A] time = 0.201392, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9db^2+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7dba+\frac{2}{5}x^5cba+\frac{1}{5}x^5da^2+\frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 2/5*x^5*c*b*a + 1
/5*x^5*d*a^2 + 1/3*x^3*c*a^2
```

Sympy [A] time = 0.114664, size = 56, normalized size = 1.02

$$\frac{a^2cx^3}{3}+\frac{b^2dx^9}{9}+x^7\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + b**2*d*x**9/9 + x**7*(2*a*b*d/7 + b**2*c/7) + x**
5*(a**2*d/5 + 2*a*b*c/5)
```

GIAC/XCAS [A] time = 0.246199, size = 72, normalized size = 1.31

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2dx^5+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 2/5*a*b*c*x^5 + 1
/5*a^2*d*x^5 + 1/3*a^2*c*x^3
```

3.144 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^3(bc-ad)}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x^2)^3)/(6*b^2) + (d*(a + b*x^2)^4)/(8*b^2)
```

Rubi [A] time = 0.151151, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\left(a+bx^2\right)^3(bc-ad)}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x^2)^3)/(6*b^2) + (d*(a + b*x^2)^4)/(8*b^2)
```

Rubi in Sympy [A] time = 15.6108, size = 34, normalized size = 0.81

$$\frac{d\left(a+bx^2\right)^4}{8b^2}-\frac{\left(a+bx^2\right)^3(ad-bc)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] d*(a + b*x**2)**4/(8*b**2) - (a + b*x**2)**3*(a*d - b*c)/(6*b**2)
```

Mathematica [A] time = 0.0207506, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2c+4bx^4(2ad+bc)+6ax^2(ad+2bc)+3b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (x^2*(12*a^2*c + 6*a*(2*b*c + a*d)*x^2 + 4*b*(b*c + 2*a*d)*x^4 +
3*b^2*d*x^6))/24
```

Maple [A] time = 0.001, size = 52, normalized size = 1.2

$$\frac{b^2dx^8}{8}+\frac{\left(2\,abd+b^2c\right)x^6}{6}+\frac{\left(a^2d+2\,abc\right)x^4}{4}+\frac{a^2cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/8*b^2*d*x^8+1/6*(2*a*b*d+b^2*c)*x^6+1/4*(a^2*d+2*a*b*c)*x^4+1/2
*a^2*c*x^2
```

Maxima [A] time = 1.32345, size = 69, normalized size = 1.64

$$\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,\left(b^2c+2\,abd\right)x^6+\frac{1}{2}\,a^2cx^2+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(
2*a*b*c + a^2*d)*x^4
```

Fricas [A] time = 0.194868, size = 1, normalized size = 0.02

$$\frac{1}{8}x^8db^2+\frac{1}{6}x^6cb^2+\frac{1}{3}x^6dba+\frac{1}{2}x^4cba+\frac{1}{4}x^4da^2+\frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d*b^2 + 1/6*x^6*c*b^2 + 1/3*x^6*d*b*a + 1/2*x^4*c*b*a + 1
/4*x^4*d*a^2 + 1/2*x^2*c*a^2
```

Sympy [A] time = 0.115064, size = 53, normalized size = 1.26

$$\frac{a^2cx^2}{2}+\frac{b^2dx^8}{8}+x^6\left(\frac{abd}{3}+\frac{b^2c}{6}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**2/2 + b**2*d*x**8/8 + x**6*(a*b*d/3 + b**2*c/6) + x**4*
(a**2*d/4 + a*b*c/2)
```

GIAC/XCAS [A] time = 0.220245, size = 72, normalized size = 1.71

$$\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,b^2cx^6+\frac{1}{3}\,abdx^6+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2dx^4+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*b^2*c*x^6 + 1/3*a*b*d*x^6 + 1/2*a*b*c*x^4 + 1
/4*a^2*d*x^4 + 1/2*a^2*c*x^2
```

3.145 $\int (a+bx^2)^2 (c+dx^2) \, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^
2*d*x^7)/7
```

Rubi [A] time = 0.0651005, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^
2*d*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\int c\,dx+\frac{ax^3\,(ad+2bc)}{3}+\frac{b^2dx^7}{7}+\frac{bx^5\,(2ad+bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*Integral(c, x) + a*x**3*(a*d + 2*b*c)/3 + b**2*d*x**7/7 + b*
x**5*(2*a*d + b*c)/5
```

Mathematica [A] time = 0.0124822, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^
2*d*x^7)/7
```

Maple [A] time = 0., size = 49, normalized size = 1.

$$\frac{b^2dx^7}{7}+\frac{\left(2\,abd+b^2c\right)x^5}{5}+\frac{\left(a^2d+2\,abc\right)x^3}{3}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2
*c*x
```

Maxima [A] time = 1.33252, size = 65, normalized size = 1.3

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,\left(b^2c + 2\,abd\right)x^5 + a^2cx + \frac{1}{3}\,\left(2\,abc + a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*
c + a^2*d)*x^3
```

Fricas [A] time = 0.187292, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7db^2 + \frac{1}{5}x^5cb^2 + \frac{2}{5}x^5dba + \frac{2}{3}x^3cba + \frac{1}{3}x^3da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b^2 + 1/5*x^5*c*b^2 + 2/5*x^5*d*b*a + 2/3*x^3*c*b*a + 1
/3*x^3*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.116671, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^7}{7} + x^5\left(\frac{2abd}{5} + \frac{b^2c}{5}\right) + x^3\left(\frac{a^2d}{3} + \frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a*
*2*d/3 + 2*a*b*c/3)
```

GIAC/XCAS [A] time = 0.247616, size = 68, normalized size = 1.36

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abdx^5 + \frac{2}{3}\,abcx^3 + \frac{1}{3}\,a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1
/3*a^2*d*x^3 + a^2*c*x
```

3.146 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x}\,dx$

Optimal. Leaf size=43

$$a^2c\log(x)+abcx^2+\frac{d\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2cx^4$$

```
[Out] a*b*c*x^2 + (b^2*c*x^4)/4 + (d*(a + b*x^2)^3)/(6*b) + a^2*c*Log[x
]
```

Rubi [A] time = 0.0796131, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$a^2c\log(x)+abcx^2+\frac{d\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2cx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] a*b*c*x^2 + (b^2*c*x^4)/4 + (d*(a + b*x^2)^3)/(6*b) + a^2*c*Log[x
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2c\log\left(x^2\right)}{2}+abcx^2+\frac{b^2c\int^{x^2}x\,dx}{2}+\frac{d\left(a+bx^2\right)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x,x)
```

```
[Out] a**2*c*log(x**2)/2 + a*b*c*x**2 + b**2*c*Integral(x, (x, x**2))/2
 + d*(a + b*x**2)**3/(6*b)
```

Mathematica [A] time = 0.0238551, size = 51, normalized size = 1.19

$$a^2c\log(x)+\frac{1}{4}bx^4(2ad+bc)+\frac{1}{2}ax^2(ad+2bc)+\frac{1}{6}b^2dx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] (a*(2*b*c + a*d)*x^2)/2 + (b*(b*c + 2*a*d)*x^4)/4 + (b^2*d*x^6)/6
 + a^2*c*Log[x]
```

Maple [A] time = 0.003, size = 51, normalized size = 1.2

$$\frac{b^2dx^6}{6}+\frac{x^4abd}{2}+\frac{b^2cx^4}{4}+\frac{x^2a^2d}{2}+abcx^2+a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)/x,x)
```

```
[Out] 1/6*b^2*d*x^6+1/2*x^4*a*b*d+1/4*b^2*c*x^4+1/2*x^2*a^2*d+a*b*c*x^2
+a^2*c*ln(x)
```

Maxima [A] time = 1.34861, size = 70, normalized size = 1.63

$$\frac{1}{6}b^2dx^6 + \frac{1}{4}\left(b^2c + 2\,abd\right)x^4 + \frac{1}{2}\,a^2c\log\left(x^2\right) + \frac{1}{2}\left(2\,abc + a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + 1/2*a^2*c*log(x^2) +
1/2*(2*a*b*c + a^2*d)*x^2
```

Fricas [A] time = 0.218032, size = 66, normalized size = 1.53

$$\frac{1}{6}b^2dx^6 + \frac{1}{4}\left(b^2c + 2\,abd\right)x^4 + a^2c\log\left(x\right) + \frac{1}{2}\left(2\,abc + a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + a^2*c*log(x) + 1/2*(2
*a*b*c + a^2*d)*x^2
```

Sympy [A] time = 1.22489, size = 49, normalized size = 1.14

$$a^2c\log\left(x\right) + \frac{b^2dx^6}{6} + x^4\left(\frac{abd}{2} + \frac{b^2c}{4}\right) + x^2\left(\frac{a^2d}{2} + abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**2*(d*x**2+c)/x,x)
```

```
[Out] a**2*c*log(x) + b**2*d*x**6/6 + x**4*(a*b*d/2 + b**2*c/4) + x**2*
(a**2*d/2 + a*b*c)
```

GIAC/XCAS [A] time = 0.231116, size = 72, normalized size = 1.67

$$\frac{1}{6}b^2dx^6 + \frac{1}{4}b^2cx^4 + \frac{1}{2}abdx^4 + abcx^2 + \frac{1}{2}a^2dx^2 + \frac{1}{2}a^2c\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*b^2*c*x^4 + 1/2*a*b*d*x^4 + a*b*c*x^2 + 1/2*a
^2*d*x^2 + 1/2*a^2*c*ln(x^2)
```

3.147 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2
*d*x^5)/5
```

Rubi [A] time = 0.0780941, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2
*d*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{x}+\frac{a\,(ad+2bc)\int d\,dx}{d}+\frac{b^2dx^5}{5}+\frac{bx^3\,(2ad+bc)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**2,x)
```

```
[Out] -a**2*c/x + a*(a*d + 2*b*c)*Integral(d, x)/d + b**2*d*x**5/5 + b*
x**3*(2*a*d + b*c)/3
```

Mathematica [A] time = 0.0273953, size = 48, normalized size = 1.

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2
*d*x^5)/5
```

Maple [A] time = 0.006, size = 49, normalized size = 1.

$$\frac{b^2dx^5}{5}+\frac{2\,x^3abd}{3}+\frac{x^3b^2c}{3}+xa^2d+2\,xabc-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)/x^2,x)
```

```
[Out] 1/5*b^2*d*x^5+2/3*x^3*a*b*d+1/3*x^3*b^2*c+x*a^2*d+2*x*a*b*c-a^2*c
/x
```

Maxima [A] time = 1.35448, size = 65, normalized size = 1.35

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\,\left(b^2c+2\,abd\right)x^3-\frac{a^2c}{x}+\left(2\,abc+a^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*(b^2*c + 2*a*b*d)*x^3 - a^2*c/x + (2*a*b*c +
a^2*d)*x
```

Fricas [A] time = 0.22208, size = 72, normalized size = 1.5

$$\frac{3\,b^2dx^6+5\,\left(b^2c+2\,abd\right)x^4-15\,a^2c+15\,\left(2\,abc+a^2d\right)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d*x^6 + 5*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 15*(2*a*
b*c + a^2*d)*x^2)/x
```

Sympy [A] time = 1.2204, size = 48, normalized size = 1.

$$-\frac{a^2c}{x}+\frac{b^2dx^5}{5}+x^3\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x\left(a^2d+2abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)/x**2,x)
```

```
[Out] -a**2*c/x + b**2*d*x**5/5 + x**3*(2*a*b*d/3 + b**2*c/3) + x*(a**2
*d + 2*a*b*c)
```

GIAC/XCAS [A] time = 0.221327, size = 65, normalized size = 1.35

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abdx^3+2\,abcx+a^2dx-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + 2*a*b*c*x + a^2*d
*x - a^2*c/x
```

3.148 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^3} dx$

Optimal. Leaf size=51

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

```
[Out] -(a^2*c)/(2*x^2) + (b*(b*c + 2*a*d)*x^2)/2 + (b^2*d*x^4)/4 + a*(2
*b*c + a*d)*Log[x]
```

Rubi [A] time = 0.125256, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) + (b*(b*c + 2*a*d)*x^2)/2 + (b^2*d*x^4)/4 + a*(2
*b*c + a*d)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{2x^2}+\frac{a\,(ad+2bc)\log\left(x^2\right)}{2}+\frac{b^2d\int^{x^2}x\,dx}{2}+\frac{b\,(2ad+bc)\int^{x^2}c\,dx}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**3,x)
```

```
[Out] -a**2*c/(2*x**2) + a*(a*d + 2*b*c)*log(x**2)/2 + b**2*d*Integral(
x, (x, x**2))/2 + b*(2*a*d + b*c)*Integral(c, (x, x**2))/(2*c)
```

Mathematica [A] time = 0.0418176, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2c}{x^2}+2bx^2(2ad+bc)+4a\log(x)(ad+2bc)+b^2dx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*c)/x^2 + 2*b*(b*c + 2*a*d)*x^2 + b^2*d*x^4 + 4*a*(2*b*c
+ a*d)*Log[x])/4
```

Maple [A] time = 0.009, size = 50, normalized size = 1.

$$\frac{b^2dx^4}{4}+x^2abd+\frac{b^2cx^2}{2}+\ln(x)\,a^2d+2\,\ln(x)\,abc-\frac{a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)/x^3,x)
```

```
[Out] 1/4*b^2*d*x^4+x^2*a*b*d+1/2*b^2*c*x^2+ln(x)*a^2*d+2*ln(x)*a*b*c-1
/2*a^2*c/x^2
```

Maxima [A] time = 1.35337, size = 70, normalized size = 1.37

$$\frac{1}{4}\,b^{2}dx^{4}+\frac{1}{2}\,\left(b^{2}c+2\,abd\right)x^{2}+\frac{1}{2}\,\left(2\,abc+a^{2}d\right)\,\log\left(x^{2}\right)-\frac{a^{2}c}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*(b^2*c + 2*a*b*d)*x^2 + 1/2*(2*a*b*c + a^2*d)
*log(x^2) - 1/2*a^2*c/x^2
```

Fricas [A] time = 0.22059, size = 73, normalized size = 1.43

$$\frac{b^{2}dx^{6}+2\,\left(b^{2}c+2\,abd\right)x^{4}+4\,\left(2\,abc+a^{2}d\right)x^{2}\log\left(x\right)-2\,a^{2}c}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^6 + 2*(b^2*c + 2*a*b*d)*x^4 + 4*(2*a*b*c + a^2*d)*x^
2*log(x) - 2*a^2*c)/x^2
```

Sympy [A] time = 1.61829, size = 48, normalized size = 0.94

$$-\frac{a^{2}c}{2x^{2}}+a\left(ad+2bc\right)\log\left(x\right)+\frac{b^{2}dx^{4}}{4}+x^{2}\left(abd+\frac{b^{2}c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**2*(d*x**2+c)/x**3,x)
```

```
[Out] -a**2*c/(2*x**2) + a*(a*d + 2*b*c)*log(x) + b**2*d*x**4/4 + x**2*
(a*b*d + b**2*c/2)
```

GIAC/XCAS [A] time = 0.223446, size = 95, normalized size = 1.86

$$\frac{1}{4}\,b^{2}dx^{4}+\frac{1}{2}\,b^{2}cx^{2}+abdx^{2}+\frac{1}{2}\,\left(2\,abc+a^{2}d\right)\ln\left(x^{2}\right)-\frac{2\,abcx^{2}+a^{2}dx^{2}+a^{2}c}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*b^2*c*x^2 + a*b*d*x^2 + 1/2*(2*a*b*c + a^2*d)
*ln(x^2) - 1/2*(2*a*b*c*x^2 + a^2*d*x^2 + a^2*c)/x^2
```

3.149 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

```
[Out] -(a^2*c)/(3*x^3) - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2
*d*x^3)/3
```

Rubi [A] time = 0.0861551, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2
*d*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c}{3x^3}-\frac{a(ad+2bc)}{x}+\frac{b^2dx^3}{3}+\frac{b(2ad+bc)\int c\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**4,x)
```

```
[Out] -a**2*c/(3*x**3) - a*(a*d + 2*b*c)/x + b**2*d*x**3/3 + b*(2*a*d +
 b*c)*Integral(c, x)/c
```

Mathematica [A] time = 0.0326024, size = 50, normalized size = 1.04

$$\frac{a^2(-d)-2abc}{x}-\frac{a^2c}{3x^3}+bx(2ad+bc)+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) + (-2*a*b*c - a^2*d)/x + b*(b*c + 2*a*d)*x + (b^
2*d*x^3)/3
```

Maple [A] time = 0.008, size = 46, normalized size = 1.

$$\frac{b^2dx^3}{3}+2\,xabd+xb^2c-\frac{a^2c}{3\,x^3}-\frac{a\,(ad+2\,bc)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)/x^4,x)
```

```
[Out] 1/3*b^2*d*x^3+2*x*a*b*d+x*b^2*c-1/3*a^2*c/x^3-a*(a*d+2*b*c)/x
```

Maxima [A] time = 1.34971, size = 68, normalized size = 1.42

$$\frac{1}{3}\,b^2dx^3 + \left(b^2c + 2\,abd\right)x - \frac{a^2c + 3\left(2\,abc + a^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*d*x^3 + (b^2*c + 2*a*b*d)*x - 1/3*(a^2*c + 3*(2*a*b*c + a
^2*d)*x^2)/x^3
```

Fricas [A] time = 0.225974, size = 70, normalized size = 1.46

$$\frac{b^2dx^6 + 3\left(b^2c + 2\,abd\right)x^4 - a^2c - 3\left(2\,abc + a^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d*x^6 + 3*(b^2*c + 2*a*b*d)*x^4 - a^2*c - 3*(2*a*b*c + a
^2*d)*x^2)/x^3
```

Sympy [A] time = 1.65417, size = 49, normalized size = 1.02

$$\frac{b^2dx^3}{3} + x\left(2abd + b^2c\right) - \frac{a^2c + x^2\left(3a^2d + 6abc\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)/x**4,x)
```

```
[Out] b**2*d*x**3/3 + x*(2*a*b*d + b**2*c) - (a**2*c + x**2*(3*a**2*d +
 6*a*b*c))/(3*x**3)
```

GIAC/XCAS [A] time = 0.219653, size = 68, normalized size = 1.42

$$\frac{1}{3}\,b^2dx^3 + b^2cx + 2\,abdx - \frac{6\,abcx^2 + 3\,a^2dx^2 + a^2c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^2*d*x^3 + b^2*c*x + 2*a*b*d*x - 1/3*(6*a*b*c*x^2 + 3*a^2*d*
x^2 + a^2*c)/x^3
```

3.150 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^1
3)/13
```

Rubi [A] time = 0.18699, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^1
3)/13
```

Rubi in Sympy [A] time = 24.1552, size = 87, normalized size = 1.

$$\frac{a^2c^2x^5}{5}+\frac{2acx^7\left(ad+bc\right)}{7}+\frac{b^2d^2x^{13}}{13}+\frac{2bdx^{11}\left(ad+bc\right)}{11}+x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**5/5 + 2*a*c*x**7*(a*d + b*c)/7 + b**2*d**2*x**13/13
+ 2*b*d*x**11*(a*d + b*c)/11 + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 +
b**2*c**2/9)
```

Mathematica [A] time = 0.0261577, size = 87, normalized size = 1.

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^1
3)/13
```

Maple [A] time = 0.002, size = 90, normalized size = 1.

$$\frac{b^2d^2x^{13}}{13}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{11}}{11}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^9}{9}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^7}{7}+\frac{a^2c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/11*(2*a*b*d^2+2*b^2*c*d)*x^11+1/9*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^9+1/7*(2*a^2*c*d+2*a*b*c^2)*x^7+1/5*a^2*c^2*x^
5
```

Maxima [A] time = 1.32575, size = 115, normalized size = 1.32

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\left(b^2cd+abd^2\right)x^{11}+\frac{1}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^9+\frac{1}{5}\,a^2c^2x^5+\frac{2}{7}\left(abc^2+a^2cd\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^4,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2
*c*d)*x^7
```

Fricas [A] time = 0.206324, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^2b^2+\frac{2}{11}x^{11}dcb^2+\frac{2}{11}x^{11}d^2ba+\frac{1}{9}x^9c^2b^2+\frac{4}{9}x^9dcba+\frac{1}{9}x^9d^2a^2+\frac{2}{7}x^7c^2ba+\frac{2}{7}x^7dca^2+\frac{1}{5}x^5c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 2/11*x^11*d*c*b^2 + 2/11*x^11*d^2*b*a + 1/9*x
^9*c^2*b^2 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/7*x^7*c^2*b*a
+ 2/7*x^7*d*c*a^2 + 1/5*x^5*c^2*a^2
```

Sympy [A] time = 0.151851, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^5}{5}+\frac{b^2d^2x^{13}}{13}+x^{11}\left(\frac{2abd^2}{11}+\frac{2b^2cd}{11}\right)+x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right)+x^7\left(\frac{2a^2cd}{7}+\frac{2abc^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**5/5 + b**2*d**2*x**13/13 + x**11*(2*a*b*d**2/11 + 2*
b**2*c*d/11) + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x
**7*(2*a**2*c*d/7 + 2*a*b*c**2/7)
```

GIAC/XCAS [A] time = 0.223804, size = 127, normalized size = 1.46

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\,b^2cdx^{11}+\frac{2}{11}\,abd^2x^{11}+\frac{1}{9}\,b^2c^2x^9+\frac{4}{9}\,abcdx^9+\frac{1}{9}\,a^2d^2x^9+\frac{2}{7}\,abc^2x^7+\frac{2}{7}\,a^2cdx^7+\frac{1}{5}\,a^2c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^4,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*b^2*c*d*x^11 + 2/11*a*b*d^2*x^11 + 1/9*b
^2*c^2*x^9 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/7*a*b*c^2*x^7
+ 2/7*a^2*c*d*x^7 + 1/5*a^2*c^2*x^5
```

3.151 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 dx$

Optimal. Leaf size=87

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

```
[Out] (a^2*c^2*x^4)/4 + (a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^8)/8 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/12
```

Rubi [A] time = 0.317733, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^4)/4 + (a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^8)/8 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2c^2\int^{x^2} x\,dx}{2}+\frac{acx^6(ad+bc)}{3}+\frac{b^2d^2x^{12}}{12}+\frac{bdx^{10}(ad+bc)}{5}+x^8\left(\frac{a^2d^2}{8}+\frac{abcd}{2}+\frac{b^2c^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*Integral(x, (x, x**2))/2 + a*c*x**6*(a*d + b*c)/3 + b**
2*d**2*x**12/12 + b*d*x**10*(a*d + b*c)/5 + x**8*(a**2*d**2/8 + a
*b*c*d/2 + b**2*c**2/8)
```

Mathematica [A] time = 0.044613, size = 81, normalized size = 0.93

$$\frac{1}{120}x^4\left(15x^4\left(a^2d^2+4abcd+b^2c^2\right)+30a^2c^2+24bdx^6(ad+bc)+40acx^2(ad+bc)+10b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^4*(30*a^2*c^2 + 40*a*c*(b*c + a*d)*x^2 + 15*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^4 + 24*b*d*(b*c + a*d)*x^6 + 10*b^2*d^2*x^8))/120
```

Maple [A] time = 0.001, size = 90, normalized size = 1.

$$\frac{b^2d^2x^{12}}{12}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{10}}{10}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^8}{8}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^6}{6}+\frac{a^2c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/12*b^2*d^2*x^12+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/8*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^8+1/6*(2*a^2*c*d+2*a*b*c^2)*x^6+1/4*a^2*c^2*x^
4
```

Maxima [A] time = 1.34998, size = 115, normalized size = 1.32

$$\frac{1}{12}\,b^2d^2x^{12} + \frac{1}{5}\left(b^2cd + abd^2\right)x^{10} + \frac{1}{8}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^8 + \frac{1}{4}\,a^2c^2x^4 + \frac{1}{3}\left(abc^2 + a^2cd\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*
c*d)*x^6
```

Fricas [A] time = 0.19179, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}d^2b^2 + \frac{1}{5}x^{10}dcb^2 + \frac{1}{5}x^{10}d^2ba + \frac{1}{8}x^8c^2b^2 + \frac{1}{2}x^8dcba + \frac{1}{8}x^8d^2a^2 + \frac{1}{3}x^6c^2ba + \frac{1}{3}x^6dca^2 + \frac{1}{4}x^4c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/8*x^8
*c^2*b^2 + 1/2*x^8*d*c*b*a + 1/8*x^8*d^2*a^2 + 1/3*x^6*c^2*b*a +
1/3*x^6*d*c*a^2 + 1/4*x^4*c^2*a^2
```

Sympy [A] time = 0.152069, size = 92, normalized size = 1.06

$$\frac{a^2c^2x^4}{4} + \frac{b^2d^2x^{12}}{12} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^8\left(\frac{a^2d^2}{8} + \frac{abcd}{2} + \frac{b^2c^2}{8}\right) + x^6\left(\frac{a^2cd}{3} + \frac{abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**4/4 + b**2*d**2*x**12/12 + x**10*(a*b*d**2/5 + b**2*
c*d/5) + x**8*(a**2*d**2/8 + a*b*c*d/2 + b**2*c**2/8) + x**6*(a**
2*c*d/3 + a*b*c**2/3)
```

GIAC/XCAS [A] time = 0.223375, size = 127, normalized size = 1.46

$$\frac{1}{12}\,b^2d^2x^{12} + \frac{1}{5}\,b^2cdx^{10} + \frac{1}{5}\,abd^2x^{10} + \frac{1}{8}\,b^2c^2x^8 + \frac{1}{2}\,abcdx^8 + \frac{1}{8}\,a^2d^2x^8 + \frac{1}{3}\,abc^2x^6 + \frac{1}{3}\,a^2cdx^6 + \frac{1}{4}\,a^2c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/8*b^2
*c^2*x^8 + 1/2*a*b*c*d*x^8 + 1/8*a^2*d^2*x^8 + 1/3*a*b*c^2*x^6 +
1/3*a^2*c*d*x^6 + 1/4*a^2*c^2*x^4
```

3.152 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)
/11
```

Rubi [A] time = 0.1741, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)
/11
```

Rubi in Sympy [A] time = 28.6622, size = 87, normalized size = 1.

$$\frac{a^2c^2x^3}{3}+\frac{2acx^5\left(ad+bc\right)}{5}+\frac{b^2d^2x^{11}}{11}+\frac{2bdx^9\left(ad+bc\right)}{9}+x^7\left(\frac{a^2d^2}{7}+\frac{4abcd}{7}+\frac{b^2c^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + 2*a*c*x**5*(a*d + b*c)/5 + b**2*d**2*x**11/11
+ 2*b*d*x**9*(a*d + b*c)/9 + x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b*
*2*c**2/7)
```

Mathematica [A] time = 0.0293511, size = 87, normalized size = 1.

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)
/11
```

Maple [A] time = 0., size = 90, normalized size = 1.

$$\frac{b^2d^2x^{11}}{11}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^9}{9}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^7}{7}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^5}{5}+\frac{a^2c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/11*b^2*d^2*x^11+1/9*(2*a*b*d^2+2*b^2*c*d)*x^9+1/7*(a^2*d^2+4*a*
b*c*d+b^2*c^2)*x^7+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+1/3*a^2*c^2*x^3
```

Maxima [A] time = 1.3458, size = 115, normalized size = 1.32

$$\frac{1}{11}\,b^2d^2x^{11} + \frac{2}{9}\left(b^2cd + abd^2\right)x^9 + \frac{1}{7}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^7 + \frac{1}{3}\,a^2c^2x^3 + \frac{2}{5}\left(abc^2 + a^2cd\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c
*d)*x^5
```

Fricas [A] time = 0.206938, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}d^2b^2 + \frac{2}{9}x^9dcb^2 + \frac{2}{9}x^9d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{2}{5}x^5c^2ba + \frac{2}{5}x^5dca^2 + \frac{1}{3}x^3c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^2*b^2 + 2/9*x^9*d*c*b^2 + 2/9*x^9*d^2*b*a + 1/7*x^7*c
^2*b^2 + 4/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/
5*x^5*d*c*a^2 + 1/3*x^3*c^2*a^2
```

Sympy [A] time = 0.149637, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^3}{3} + \frac{b^2d^2x^{11}}{11} + x^9\left(\frac{2abd^2}{9} + \frac{2b^2cd}{9}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^5\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + b**2*d**2*x**11/11 + x**9*(2*a*b*d**2/9 + 2*b*
*2*c*d/9) + x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**5
*(2*a**2*c*d/5 + 2*a*b*c**2/5)
```

GIAC/XCAS [A] time = 0.222938, size = 127, normalized size = 1.46

$$\frac{1}{11}\,b^2d^2x^{11} + \frac{2}{9}\,b^2cdx^9 + \frac{2}{9}\,abd^2x^9 + \frac{1}{7}\,b^2c^2x^7 + \frac{4}{7}\,abcdx^7 + \frac{1}{7}\,a^2d^2x^7 + \frac{2}{5}\,abc^2x^5 + \frac{2}{5}\,a^2cdx^5 + \frac{1}{3}\,a^2c^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*b^2*c*d*x^9 + 2/9*a*b*d^2*x^9 + 1/7*b^2*c
^2*x^7 + 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 2/5*a*b*c^2*x^5 + 2/
5*a^2*c*d*x^5 + 1/3*a^2*c^2*x^3
```

3.153 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=71

$$\frac{d\left(a+bx^2\right)^4(bc-ad)}{4b^3}+\frac{\left(a+bx^2\right)^3(bc-ad)^2}{6b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x^2)^3)/(6*b^3) + (d*(b*c - a*d)*(a + b*x^2
)^4)/(4*b^3) + (d^2*(a + b*x^2)^5)/(10*b^3)
```

Rubi [A] time = 0.27408, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{d\left(a+bx^2\right)^4(bc-ad)}{4b^3}+\frac{\left(a+bx^2\right)^3(bc-ad)^2}{6b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x^2)^3)/(6*b^3) + (d*(b*c - a*d)*(a + b*x^2
)^4)/(4*b^3) + (d^2*(a + b*x^2)^5)/(10*b^3)
```

Rubi in Sympy [A] time = 29.2518, size = 60, normalized size = 0.85

$$\frac{d^2\left(a+bx^2\right)^5}{10b^3}-\frac{d\left(a+bx^2\right)^4(ad-bc)}{4b^3}+\frac{\left(a+bx^2\right)^3(ad-bc)^2}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] d**2*(a + b*x**2)**5/(10*b**3) - d*(a + b*x**2)**4*(a*d - b*c)/(4
*b**3) + (a + b*x**2)**3*(a*d - b*c)**2/(6*b**3)
```

Mathematica [A] time = 0.0432828, size = 81, normalized size = 1.14

$$\frac{1}{60}x^2\left(10x^4\left(a^2d^2+4abcd+b^2c^2\right)+30a^2c^2+15bdx^6(ad+bc)+30acx^2(ad+bc)+6b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^2*(30*a^2*c^2 + 30*a*c*(b*c + a*d)*x^2 + 10*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^4 + 15*b*d*(b*c + a*d)*x^6 + 6*b^2*d^2*x^8))/60
```

Maple [A] time = 0.001, size = 90, normalized size = 1.3

$$\frac{b^2d^2x^{10}}{10}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^8}{8}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^6}{6}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^4}{4}+\frac{a^2c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/10*b^2*d^2*x^10+1/8*(2*a*b*d^2+2*b^2*c*d)*x^8+1/6*(a^2*d^2+4*a*
b*c*d+b^2*c^2)*x^6+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+1/2*a^2*c^2*x^2
```

Maxima [A] time = 1.35656, size = 115, normalized size = 1.62

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\left(b^2cd+abd^2\right)x^8+\frac{1}{6}\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\left(abc^2+a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c
*d)*x^4
```

Fricas [A] time = 0.20764, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}d^2b^2+\frac{1}{4}x^8dcb^2+\frac{1}{4}x^8d^2ba+\frac{1}{6}x^6c^2b^2+\frac{2}{3}x^6dcba+\frac{1}{6}x^6d^2a^2+\frac{1}{2}x^4c^2ba+\frac{1}{2}x^4dca^2+\frac{1}{2}x^2c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b^2 + 1/4*x^8*d*c*b^2 + 1/4*x^8*d^2*b*a + 1/6*x^6*c
^2*b^2 + 2/3*x^6*d*c*b*a + 1/6*x^6*d^2*a^2 + 1/2*x^4*c^2*b*a + 1/
2*x^4*d*c*a^2 + 1/2*x^2*c^2*a^2
```

Sympy [A] time = 0.152933, size = 94, normalized size = 1.32

$$\frac{a^2c^2x^2}{2}+\frac{b^2d^2x^{10}}{10}+x^8\left(\frac{abd^2}{4}+\frac{b^2cd}{4}\right)+x^6\left(\frac{a^2d^2}{6}+\frac{2abcd}{3}+\frac{b^2c^2}{6}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**2/2 + b**2*d**2*x**10/10 + x**8*(a*b*d**2/4 + b**2*c
*d/4) + x**6*(a**2*d**2/6 + 2*a*b*c*d/3 + b**2*c**2/6) + x**4*(a*
*2*c*d/2 + a*b*c**2/2)
```

GIAC/XCAS [A] time = 0.222479, size = 127, normalized size = 1.79

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,b^2cdx^8+\frac{1}{4}\,abd^2x^8+\frac{1}{6}\,b^2c^2x^6+\frac{2}{3}\,abcdx^6+\frac{1}{6}\,a^2d^2x^6+\frac{1}{2}\,abc^2x^4+\frac{1}{2}\,a^2cdx^4+\frac{1}{2}\,a^2c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*b^2*c*d*x^8 + 1/4*a*b*d^2*x^8 + 1/6*b^2*c
^2*x^6 + 2/3*a*b*c*d*x^6 + 1/6*a^2*d^2*x^6 + 1/2*a*b*c^2*x^4 + 1/
2*a^2*c*d*x^4 + 1/2*a^2*c^2*x^2
```

3.154 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rubi [A] time = 0.107183, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2acx^3\left(ad+bc\right)}{3}+\frac{b^2d^2x^9}{9}+\frac{2bdx^7\left(ad+bc\right)}{7}+c^2\int a^2\,dx+x^5\left(\frac{a^2d^2}{5}+\frac{4abcd}{5}+\frac{b^2c^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a*c*x**3*(a*d + b*c)/3 + b**2*d**2*x**9/9 + 2*b*d*x**7*(a*d + b
*c)/7 + c**2*Integral(a**2, x) + x**5*(a**2*d**2/5 + 4*a*b*c*d/5
+ b**2*c**2/5)
```

Mathematica [A] time = 0.0284644, size = 82, normalized size = 1.

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A] time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^9}{9}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^7}{7}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^5}{5}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^3}{3}+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*
c*d+b^2*c^2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

Maxima [A] time = 1.35207, size = 111, normalized size = 1.35

$$\frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\left(b^2cd + abd^2\right)x^7 + \frac{1}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^5 + a^2c^2x + \frac{2}{3}\left(abc^2 + a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Fricas [A] time = 0.205781, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9d^2b^2 + \frac{2}{7}x^7dcb^2 + \frac{2}{7}x^7d^2ba + \frac{1}{5}x^5c^2b^2 + \frac{4}{5}x^5dcba + \frac{1}{5}x^5d^2a^2 + \frac{2}{3}x^3c^2ba + \frac{2}{3}x^3dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^2*b^2 + 2/7*x^7*d*c*b^2 + 2/7*x^7*d^2*b*a + 1/5*x^5*c^2
*b^2 + 4/5*x^5*d*c*b*a + 1/5*x^5*d^2*a^2 + 2/3*x^3*c^2*b*a + 2/3*
x^3*d*c*a^2 + x*c^2*a^2
```

Sympy [A] time = 0.144716, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^9}{9} + x^7\left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3\left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/
7) + x**5*(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**
2*c*d/3 + 2*a*b*c**2/3)
```

GIAC/XCAS [A] time = 0.224075, size = 123, normalized size = 1.5

$$\frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\,b^2cdx^7 + \frac{2}{7}\,abd^2x^7 + \frac{1}{5}\,b^2c^2x^5 + \frac{4}{5}\,abcdx^5 + \frac{1}{5}\,a^2d^2x^5 + \frac{2}{3}\,abc^2x^3 + \frac{2}{3}\,a^2cdx^3 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2
*x^5 + 4/5*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*
a^2*c*d*x^3 + a^2*c^2*x
```

3.155 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x} \, dx$

Optimal. Leaf size=80

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (
b*d*(b*c + a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Rubi [A] time = 0.168724, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (
b*d*(b*c + a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2c^2\log\left(x^2\right)}{2}+acx^2\left(ad+bc\right)+\frac{b^2d^2x^8}{8}+\frac{bdx^6\left(ad+bc\right)}{3}+\left(\frac{a^2d^2}{2}+2abcd+\frac{b^2c^2}{2}\right)\int^{x^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x,x)
```

```
[Out] a**2*c**2*log(x**2)/2 + a*c*x**2*(a*d + b*c) + b**2*d**2*x**8/8 +
 b*d*x**6*(a*d + b*c)/3 + (a**2*d**2/2 + 2*a*b*c*d + b**2*c**2/2)
*Integral(x, (x, x**2))
```

Mathematica [A] time = 0.0431123, size = 80, normalized size = 1.

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (
b*d*(b*c + a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Maple [A] time = 0.004, size = 90, normalized size = 1.1

$$\frac{b^2d^2x^8}{8}+\frac{x^6abd^2}{3}+\frac{x^6b^2cd}{3}+\frac{x^4a^2d^2}{4}+x^4abcd+\frac{x^4b^2c^2}{4}+x^2a^2cd+ac^2bx^2+a^2c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x,x)
```

```
[Out] 1/8*b^2*d^2*x^8+1/3*x^6*a*b*d^2+1/3*x^6*b^2*c*d+1/4*x^4*a^2*d^2+x
^4*a*b*c*d+1/4*x^4*b^2*c^2+x^2*a^2*c*d+a*c^2*b*x^2+a^2*c^2*ln(x)
```

Maxima [A] time = 1.35938, size = 115, normalized size = 1.44

$$\frac{1}{8}b^2d^2x^8 + \frac{1}{3}\left(b^2cd + abd^2\right)x^6 + \frac{1}{4}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^4 + \frac{1}{2}a^2c^2\log\left(x^2\right) + \left(abc^2 + a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^4 + 1/2*a^2*c^2*log(x^2) + (a*b*c^2 + a^2*c*
d)*x^2
```

Fricas [A] time = 0.228289, size = 111, normalized size = 1.39

$$\frac{1}{8}b^2d^2x^8 + \frac{1}{3}\left(b^2cd + abd^2\right)x^6 + \frac{1}{4}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^4 + a^2c^2\log\left(x\right) + \left(abc^2 + a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^4 + a^2*c^2*log(x) + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A] time = 1.41373, size = 85, normalized size = 1.06

$$a^2c^2\log\left(x\right) + \frac{b^2d^2x^8}{8} + x^6\left(\frac{abd^2}{3} + \frac{b^2cd}{3}\right) + x^4\left(\frac{a^2d^2}{4} + abcd + \frac{b^2c^2}{4}\right) + x^2\left(a^2cd + abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x,x)
```

```
[Out] a**2*c**2*log(x) + b**2*d**2*x**8/8 + x**6*(a*b*d**2/3 + b**2*c*d
/3) + x**4*(a**2*d**2/4 + a*b*c*d + b**2*c**2/4) + x**2*(a**2*c*d
 + a*b*c**2)
```

GIAC/XCAS [A] time = 0.228196, size = 124, normalized size = 1.55

$$\frac{1}{8}b^2d^2x^8 + \frac{1}{3}b^2cdx^6 + \frac{1}{3}abd^2x^6 + \frac{1}{4}b^2c^2x^4 + abcdx^4 + \frac{1}{4}a^2d^2x^4 + abc^2x^2 + a^2cdx^2 + \frac{1}{2}a^2c^2\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*b^2*c*d*x^6 + 1/3*a*b*d^2*x^6 + 1/4*b^2*c^2
*x^4 + a*b*c*d*x^4 + 1/4*a^2*d^2*x^4 + a*b*c^2*x^2 + a^2*c*d*x^2
+ 1/2*a^2*c^2*ln(x^2)
```

3.156 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx$

Optimal. Leaf size=81

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^3)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Rubi [A] time = 0.127475, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^3)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Rubi in Sympy [A] time = 25.3281, size = 80, normalized size = 0.99

$$-\frac{a^2c^2}{x}+2acx\,(ad+bc)+\frac{b^2d^2x^7}{7}+\frac{2bdx^5\,(ad+bc)}{5}+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**2,x)
```

```
[Out] -a**2*c**2/x + 2*a*c*x*(a*d + b*c) + b**2*d**2*x**7/7 + 2*b*d*x**
5*(a*d + b*c)/5 + x**3*(a**2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3)
```

Mathematica [A] time = 0.0637022, size = 81, normalized size = 1.

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^3)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Maple [A] time = 0.006, size = 91, normalized size = 1.1

$$\frac{b^2d^2x^7}{7}+\frac{2\,x^5abd^2}{5}+\frac{2\,x^5b^2cd}{5}+\frac{x^3a^2d^2}{3}+\frac{4\,x^3abcd}{3}+\frac{x^3b^2c^2}{3}+2\,xa^2cd+2\,xabc^2-\frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x^2,x)
```

```
[Out] 1/7*b^2*d^2*x^7+2/5*x^5*a*b*d^2+2/5*x^5*b^2*c*d+1/3*x^3*a^2*d^2+4
/3*x^3*a*b*c*d+1/3*x^3*b^2*c^2+2*x*a^2*c*d+2*x*a*b*c^2-a^2*c^2/x
```

Maxima [A] time = 1.35274, size = 112, normalized size = 1.38

$$\frac{1}{7}\,b^2d^2x^7+\frac{2}{5}\left(b^2cd+abd^2\right)x^5+\frac{1}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^3-\frac{a^2c^2}{x}+2\left(abc^2+a^2cd\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*(b^2*c*d + a*b*d^2)*x^5 + 1/3*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^3 - a^2*c^2/x + 2*(a*b*c^2 + a^2*c*d)*x
```

Fricas [A] time = 0.215754, size = 117, normalized size = 1.44

$$\frac{15\,b^2d^2x^8+42\left(b^2cd+abd^2\right)x^6+35\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-105\,a^2c^2+210\left(abc^2+a^2cd\right)x^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^2*x^8 + 42*(b^2*c*d + a*b*d^2)*x^6 + 35*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^4 - 105*a^2*c^2 + 210*(a*b*c^2 + a^2*c*d
)*x^2)/x
```

Sympy [A] time = 1.38349, size = 92, normalized size = 1.14

$$-\frac{a^2c^2}{x}+\frac{b^2d^2x^7}{7}+x^5\left(\frac{2abd^2}{5}+\frac{2b^2cd}{5}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x\left(2a^2cd+2abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x**2,x)
```

```
[Out] -a**2*c**2/x + b**2*d**2*x**7/7 + x**5*(2*a*b*d**2/5 + 2*b**2*c*d
/5) + x**3*(a**2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x*(2*a**2*
c*d + 2*a*b*c**2)
```

GIAC/XCAS [A] time = 0.227856, size = 122, normalized size = 1.51

$$\frac{1}{7}\,b^2d^2x^7+\frac{2}{5}\,b^2cdx^5+\frac{2}{5}\,abd^2x^5+\frac{1}{3}\,b^2c^2x^3+\frac{4}{3}\,abcdx^3+\frac{1}{3}\,a^2d^2x^3+2\,abc^2x+2\,a^2cdx-\frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*b^2*c*d*x^5 + 2/5*a*b*d^2*x^5 + 1/3*b^2*c^2
*x^3 + 4/3*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + 2*a*b*c^2*x + 2*a^2*c*
d*x - a^2*c^2/x
```

3.157 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

```
[Out] -(a^2*c^2)/(2*x^2) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)/2 + (b
*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^6)/6 + 2*a*c*(b*c + a*d)*Log[x
]
```

Rubi [A] time = 0.213321, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] -(a^2*c^2)/(2*x^2) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)/2 + (b
*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^6)/6 + 2*a*c*(b*c + a*d)*Log[x
]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c^2}{2x^2}+ac\,(ad+bc)\log\left(x^2\right)+\frac{b^2d^2x^6}{6}+bd\,(ad+bc)\int^{x^2}x\,dx+\frac{\left(a^2d^2+bc\,(4ad+bc)\right)\int^{x^2}a^2\,dx}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**3,x)
```

```
[Out] -a**2*c**2/(2*x**2) + a*c*(a*d + b*c)*log(x**2) + b**2*d**2*x**6/
6 + b*d*(a*d + b*c)*Integral(x, (x, x**2)) + (a**2*d**2 + b*c*(4*
a*d + b*c))*Integral(a**2, (x, x**2))/(2*a**2)
```

Mathematica [A] time = 0.0786349, size = 83, normalized size = 0.99

$$\frac{1}{6}\left(\frac{3a^2\left(d^2x^4-c^2\right)}{x^2}+3abdx^2\left(4c+dx^2\right)+12ac\log(x)(ad+bc)+b^2x^2\left(3c^2+3cdx^2+d^2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] (3*a*b*d*x^2*(4*c + d*x^2) + (3*a^2*(-c^2 + d^2*x^4))/x^2 + b^2*x
^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4) + 12*a*c*(b*c + a*d)*Log[x])/6
```

Maple [A] time = 0.009, size = 93, normalized size = 1.1

$$\frac{b^2d^2x^6}{6}+\frac{x^4abd^2}{2}+\frac{x^4b^2cd}{2}+\frac{x^2a^2d^2}{2}+2\,x^2abcd+\frac{x^2b^2c^2}{2}+2\,\ln(x)\,a^2cd+2\,\ln(x)\,abc^2-\frac{a^2c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x^3,x)
```

```
[Out] 1/6*b^2*d^2*x^6+1/2*x^4*a*b*d^2+1/2*x^4*b^2*c*d+1/2*x^2*a^2*d^2+2
*x^2*a*b*c*d+1/2*x^2*b^2*c^2+2*ln(x)*a^2*c*d+2*ln(x)*a*b*c^2-1/2*
a^2*c^2/x^2
```

Maxima [A] time = 1.35796, size = 115, normalized size = 1.37

$$\frac{1}{6}b^2d^2x^6 + \frac{1}{2}\left(b^2cd + abd^2\right)x^4 + \frac{1}{2}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^2 - \frac{a^2c^2}{2\,x^2} + \left(abc^2 + a^2cd\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/2*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^2 - 1/2*a^2*c^2/x^2 + (a*b*c^2 + a^2*c*d)*lo
g(x^2)
```

Fricas [A] time = 0.211374, size = 119, normalized size = 1.42

$$\frac{b^2d^2x^8 + 3\left(b^2cd + abd^2\right)x^6 + 3\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^4 - 3\,a^2c^2 + 12\left(abc^2 + a^2cd\right)x^2\log\left(x\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*d^2*x^8 + 3*(b^2*c*d + a*b*d^2)*x^6 + 3*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 - 3*a^2*c^2 + 12*(a*b*c^2 + a^2*c*d)*x^2*log(
x))/x^2
```

Sympy [A] time = 1.74269, size = 87, normalized size = 1.04

$$-\frac{a^2c^2}{2x^2} + 2ac\left(ad + bc\right)\log\left(x\right) + \frac{b^2d^2x^6}{6} + x^4\left(\frac{abd^2}{2} + \frac{b^2cd}{2}\right) + x^2\left(\frac{a^2d^2}{2} + 2abcd + \frac{b^2c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x**3,x)
```

```
[Out] -a**2*c**2/(2*x**2) + 2*a*c*(a*d + b*c)*log(x) + b**2*d**2*x**6/6
 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**2*(a**2*d**2/2 + 2*a*b*c*d
 + b**2*c**2/2)
```

GIAC/XCAS [A] time = 0.227304, size = 154, normalized size = 1.83

$$\frac{1}{6}b^2d^2x^6 + \frac{1}{2}b^2cdx^4 + \frac{1}{2}abd^2x^4 + \frac{1}{2}b^2c^2x^2 + 2\,abcdx^2 + \frac{1}{2}a^2d^2x^2 + \left(abc^2 + a^2cd\right)\ln\left(x^2\right) - \frac{2\,abc^2x^2 + 2\,a^2cdx^2 + a^2c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/2*b^2*c^2
*x^2 + 2*a*b*c*d*x^2 + 1/2*a^2*d^2*x^2 + (a*b*c^2 + a^2*c*d)*ln(x
^2) - 1/2*(2*a*b*c^2*x^2 + 2*a^2*c*d*x^2 + a^2*c^2)/x^2
```

3.158 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\, dx$

Optimal. Leaf size=80

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Rubi [A] time = 0.145622, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c^2}{3x^3}-\frac{2ac\,(ad+bc)}{x}+\frac{b^2d^2x^5}{5}+\frac{2bdx^3\,(ad+bc)}{3}+\frac{\left(a^2d^2+bc\,(4ad+bc)\right)\int a^2\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**4,x)
```

```
[Out] -a**2*c**2/(3*x**3) - 2*a*c*(a*d + b*c)/x + b**2*d**2*x**5/5 + 2*
b*d*x**3*(a*d + b*c)/3 + (a**2*d**2 + b*c*(4*a*d + b*c))*Integral
(a**2, x)/a**2
```

Mathematica [A] time = 0.0725203, size = 80, normalized size = 1.

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Maple [A] time = 0.008, size = 81, normalized size = 1.

$$\frac{b^2d^2x^5}{5}+\frac{2\,x^3abd^2}{3}+\frac{2\,x^3b^2cd}{3}+a^2d^2x+4\,abcdx+b^2c^2x-\frac{a^2c^2}{3\,x^3}-2\,\frac{ac\,(ad+bc)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x^4,x)
```

```
[Out] 1/5*b^2*d^2*x^5+2/3*x^3*a*b*d^2+2/3*x^3*b^2*c*d+a^2*d^2*x+4*a*b*c
*d*x+b^2*c^2*x-1/3*a^2*c^2/x^3-2*a*c*(a*d+b*c)/x
```

Maxima [A] time = 1.34794, size = 113, normalized size = 1.41

$$\frac{1}{5}\,b^2d^2x^5 + \frac{2}{3}\left(b^2cd + abd^2\right)x^3 + \left(b^2c^2 + 4\,abcd + a^2d^2\right)x - \frac{a^2c^2 + 6\left(abc^2 + a^2cd\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*(b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x - 1/3*(a^2*c^2 + 6*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

Fricas [A] time = 0.222045, size = 117, normalized size = 1.46

$$\frac{3\,b^2d^2x^8 + 10\left(b^2cd + abd^2\right)x^6 + 15\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^4 - 5\,a^2c^2 - 30\left(abc^2 + a^2cd\right)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^8 + 10*(b^2*c*d + a*b*d^2)*x^6 + 15*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^4 - 5*a^2*c^2 - 30*(a*b*c^2 + a^2*c*d)*x^2
)/x^3
```

Sympy [A] time = 1.84747, size = 90, normalized size = 1.12

$$\frac{b^2d^2x^5}{5} + x^3\left(\frac{2abd^2}{3} + \frac{2b^2cd}{3}\right) + x\left(a^2d^2 + 4abcd + b^2c^2\right) - \frac{a^2c^2 + x^2\left(6a^2cd + 6abc^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x**4,x)
```

```
[Out] b**2*d**2*x**5/5 + x**3*(2*a*b*d**2/3 + 2*b**2*c*d/3) + x*(a**2*d
**2 + 4*a*b*c*d + b**2*c**2) - (a**2*c**2 + x**2*(6*a**2*c*d + 6*
a*b*c**2))/(3*x**3)
```

GIAC/XCAS [A] time = 0.222375, size = 119, normalized size = 1.49

$$\frac{1}{5}\,b^2d^2x^5 + \frac{2}{3}\,b^2cdx^3 + \frac{2}{3}\,abd^2x^3 + b^2c^2x + 4\,abcdx + a^2d^2x - \frac{6\,abc^2x^2 + 6\,a^2cdx^2 + a^2c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*b^2*c*d*x^3 + 2/3*a*b*d^2*x^3 + b^2*c^2*x +
 4*a*b*c*d*x + a^2*d^2*x - 1/3*(6*a*b*c^2*x^2 + 6*a^2*c*d*x^2 + a
^2*c^2)/x^3
```

3.159 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
2)*x^11)/11 + (b*d^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Rubi [A] time = 0.251225, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
2)*x^11)/11 + (b*d^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Rubi in Sympy [A] time = 34.2225, size = 124, normalized size = 0.98

$$\frac{a^2c^3x^5}{5}+\frac{ac^2x^7\left(3ad+2bc\right)}{7}+\frac{b^2d^3x^{15}}{15}+\frac{bd^2x^{13}\left(2ad+3bc\right)}{13}+\frac{cx^9\left(3a^2d^2+6abcd+b^2c^2\right)}{9}+\frac{dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**5/5 + a*c**2*x**7*(3*a*d + 2*b*c)/7 + b**2*d**3*x**1
5/15 + b*d**2*x**13*(2*a*d + 3*b*c)/13 + c*x**9*(3*a**2*d**2 + 6*
a*b*c*d + b**2*c**2)/9 + d*x**11*(a**2*d**2 + 6*a*b*c*d + 3*b**2*
c**2)/11
```

Mathematica [A] time = 0.0451218, size = 127, normalized size = 1.

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
```

```
2)*x^11)/11 + (b*d^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Maple [A] time = 0.001, size = 128, normalized size = 1.

$$\frac{b^2d^3x^{15}}{15}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{11}}{11}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^9}{9}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^7}{7}+\frac{a^2c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/15*b^2*d^3*x^15+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/11*(a^2*d^3
+6*a*b*c*d^2+3*b^2*c^2*d)*x^11+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c
^3)*x^9+1/7*(3*a^2*c^2*d+2*a*b*c^3)*x^7+1/5*a^2*c^3*x^5
```

Maxima [A] time = 1.3555, size = 171, normalized size = 1.35

$$\frac{1}{15}\,b^2d^3x^{15}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{11}+\frac{1}{5}\,a^2c^3x^5+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+\frac{1}{7}\left(2\,abc^3+3\,a^2c^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^4,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*
(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^
2*c^2*d)*x^7
```

Fricas [A] time = 0.205901, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}d^3b^2+\frac{3}{13}x^{13}d^2cb^2+\frac{2}{13}x^{13}d^3ba+\frac{3}{11}x^{11}dc^2b^2+\frac{6}{11}x^{11}d^2cba+\frac{1}{11}x^{11}d^3a^2+\frac{1}{9}x^9c^3b^2+\frac{2}{3}x^9dc^2ba+\frac{1}{3}x^9d^2ca^2+\frac{2}{7}x^7c^3ba+\frac{3}{7}x^7dc^2a^2+\frac{1}{5}x^5c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^4,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/1
1*x^11*d*c^2*b^2 + 6/11*x^11*d^2*c*b*a + 1/11*x^11*d^3*a^2 + 1/9*
x^9*c^3*b^2 + 2/3*x^9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/7*x^7*c^3
*b*a + 3/7*x^7*d*c^2*a^2 + 1/5*x^5*c^3*a^2
```

Sympy [A] time = 0.17853, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^5}{5}+\frac{b^2d^3x^{15}}{15}+x^{13}\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{11}\left(\frac{a^2d^3}{11}+\frac{6abcd^2}{11}+\frac{3b^2c^2d}{11}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^7\left(\frac{3a^2c^2d}{7}+\frac{2abc^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**5/5 + b**2*d**3*x**15/15 + x**13*(2*a*b*d**3/13 + 3*
b**2*c*d**2/13) + x**11*(a**2*d**3/11 + 6*a*b*c*d**2/11 + 3*b**2*
c**2*d/11) + x**9*(a**2*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9)
+ x**7*(3*a**2*c**2*d/7 + 2*a*b*c**3/7)
```

GIAC/XCAS [A] time = 0.222654, size = 182, normalized size = 1.43

$$\frac{1}{15}\,b^2d^3x^{15} + \frac{3}{13}\,b^2cd^2x^{13} + \frac{2}{13}\,abd^3x^{13} + \frac{3}{11}\,b^2c^2dx^{11} + \frac{6}{11}\,abcd^2x^{11} + \frac{1}{11}\,a^2d^3x^{11} + \frac{1}{9}\,b^2c^3x^9 + \frac{2}{3}\,abc^2dx^9 + \frac{1}{3}\,a^2cd^2x^9 + \frac{2}{7}\,abc^3x^7 + \frac{3}{7}\,a^2c^2dx^7 + \frac{1}{5}\,a^2c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^4,x, algorithm="giac")
```

```
[Out] 1/15*b^2*d^3*x^15 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/1
1*b^2*c^2*d*x^11 + 6/11*a*b*c*d^2*x^11 + 1/11*a^2*d^3*x^11 + 1/9*
b^2*c^3*x^9 + 2/3*a*b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/7*a*b*c^3
*x^7 + 3/7*a^2*c^2*d*x^7 + 1/5*a^2*c^3*x^5
```

3.160 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=106

$$-\frac{b\left(c+dx^2\right)^6(3bc-2ad)}{12d^4}+\frac{\left(c+dx^2\right)^5(bc-ad)(3bc-ad)}{10d^4}-\frac{c\left(c+dx^2\right)^4(bc-ad)^2}{8d^4}+\frac{b^2\left(c+dx^2\right)^7}{14d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^4)/(8*d^4) + ((b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^5)/(10*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/
(12*d^4) + (b^2*(c + d*x^2)^7)/(14*d^4)
```

Rubi [A] time = 0.538959, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{b\left(c+dx^2\right)^6(3bc-2ad)}{12d^4}+\frac{\left(c+dx^2\right)^5(bc-ad)(3bc-ad)}{10d^4}-\frac{c\left(c+dx^2\right)^4(bc-ad)^2}{8d^4}+\frac{b^2\left(c+dx^2\right)^7}{14d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^4)/(8*d^4) + ((b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^5)/(10*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/
(12*d^4) + (b^2*(c + d*x^2)^7)/(14*d^4)
```

Rubi in Sympy [A] time = 41.0761, size = 94, normalized size = 0.89

$$\frac{b^2\left(c+dx^2\right)^7}{14d^4}+\frac{b\left(c+dx^2\right)^6(2ad-3bc)}{12d^4}-\frac{c\left(c+dx^2\right)^4(ad-bc)^2}{8d^4}+\frac{\left(c+dx^2\right)^5(ad-3bc)(ad-bc)}{10d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] b**2*(c + d*x**2)**7/(14*d**4) + b*(c + d*x**2)**6*(2*a*d - 3*b*c
)/(12*d**4) - c*(c + d*x**2)**4*(a*d - b*c)**2/(8*d**4) + (c + d*
x**2)**5*(a*d - 3*b*c)*(a*d - b*c)/(10*d**4)
```

Mathematica [A] time = 0.0589492, size = 119, normalized size = 1.12

$$\frac{1}{840}x^4\left(84dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+105cx^4\left(3a^2d^2+6abcd+b^2c^2\right)\right.$$
$$\left.+210a^2c^3+140ac^2x^2(3ad+2bc)+70bd^2x^8(2ad+3bc)+60b^2d^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^4*(210*a^2*c^3 + 140*a*c^2*(2*b*c + 3*a*d)*x^2 + 105*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^4 + 84*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^6 + 70*b*d^2*(3*b*c + 2*a*d)*x^8 + 60*b^2*d^3*x^10))/840
```

Maple [A] time = 0.002, size = 128, normalized size = 1.2

$$\frac{b^2d^3x^{14}}{14}+\frac{(2\,abd^3+3\,b^2cd^2)\,x^{12}}{12}+\frac{(a^2d^3+6\,abcd^2+3\,b^2c^2d)\,x^{10}}{10}+\frac{(3\,a^2cd^2+6\,abc^2d+b^2c^3)\,x^8}{8}+\frac{(3\,a^2c^2d+2\,abc^3)\,x^6}{6}+\frac{a^2c^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/14*b^2*d^3*x^14+1/12*(2*a*b*d^3+3*b^2*c*d^2)*x^12+1/10*(a^2*d^3
+6*a*b*c*d^2+3*b^2*c^2*d)*x^10+1/8*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c
^3)*x^8+1/6*(3*a^2*c^2*d+2*a*b*c^3)*x^6+1/4*a^2*c^3*x^4
```

Maxima [A] time = 1.34878, size = 171, normalized size = 1.61

$$\frac{1}{14}\,b^2d^3x^{14}+\frac{1}{12}\,(3\,b^2cd^2+2\,abd^3)\,x^{12}+\frac{1}{10}\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)\,x^{10}+\frac{1}{4}\,a^2c^3x^4+\frac{1}{8}\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)\,x^8+\frac{1}{6}\,(2\,abc^3+3\,a^2c^2d)\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*
(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^
2*c^2*d)*x^6
```

Fricas [A] time = 0.209313, size = 1, normalized size = 0.01

$$\frac{1}{14}x^{14}d^3b^2+\frac{1}{4}x^{12}d^2cb^2+\frac{1}{6}x^{12}d^3ba+\frac{3}{10}x^{10}dc^2b^2+\frac{3}{5}x^{10}d^2cba+\frac{1}{10}x^{10}d^3a^2+\frac{1}{8}x^8c^3b^2+\frac{3}{4}x^8dc^2ba+\frac{3}{8}x^8d^2ca^2+\frac{1}{3}x^6c^3ba+\frac{1}{2}x^6dc^2a^2+\frac{1}{4}x^4c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^3*b^2 + 1/4*x^12*d^2*c*b^2 + 1/6*x^12*d^3*b*a + 3/10*
x^10*d*c^2*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/8*x^8
*c^3*b^2 + 3/4*x^8*d*c^2*b*a + 3/8*x^8*d^2*c*a^2 + 1/3*x^6*c^3*b*
a + 1/2*x^6*d*c^2*a^2 + 1/4*x^4*c^3*a^2
```

Sympy [A] time = 0.177449, size = 138, normalized size = 1.3

$$\frac{a^2c^3x^4}{4}+\frac{b^2d^3x^{14}}{14}+x^{12}\left(\frac{abd^3}{6}+\frac{b^2cd^2}{4}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^8\left(\frac{3a^2cd^2}{8}+\frac{3abc^2d}{4}+\frac{b^2c^3}{8}\right)+x^6\left(\frac{a^2c^2d}{2}+\frac{abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**4/4 + b**2*d**3*x**14/14 + x**12*(a*b*d**3/6 + b**2*
c*d**2/4) + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/
10) + x**8*(3*a**2*c*d**2/8 + 3*a*b*c**2*d/4 + b**2*c**3/8) + x**
6*(a**2*c**2*d/2 + a*b*c**3/3)
```

GIAC/XCAS [A] time = 0.228826, size = 182, normalized size = 1.72

$$\frac{1}{14}\,b^2d^3x^{14} + \frac{1}{4}\,b^2cd^2x^{12} + \frac{1}{6}\,abd^3x^{12} + \frac{3}{10}\,b^2c^2dx^{10} + \frac{3}{5}\,abcd^2x^{10} + \frac{1}{10}\,a^2d^3x^{10} + \frac{1}{8}\,b^2c^3x^8 + \frac{3}{4}\,abc^2dx^8 + \frac{3}{8}\,a^2cd^2x^8 + \frac{1}{3}\,abc^3x^6 + \frac{1}{2}\,a^2c^2dx^6 + \frac{1}{4}\,a^2c^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^3,x, algorithm="giac")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/4*b^2*c*d^2*x^12 + 1/6*a*b*d^3*x^12 + 3/10*
b^2*c^2*d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/8*b^2
*c^3*x^8 + 3/4*a*b*c^2*d*x^8 + 3/8*a^2*c*d^2*x^8 + 1/3*a*b*c^3*x^
6 + 1/2*a^2*c^2*d*x^6 + 1/4*a^2*c^3*x^4
```

3.161 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
2)*x^9)/9 + (b*d^2*(3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Rubi [A] time = 0.232067, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
2)*x^9)/9 + (b*d^2*(3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Rubi in Sympy [A] time = 36.8853, size = 124, normalized size = 0.98

$$\frac{a^2c^3x^3}{3}+\frac{ac^2x^5\left(3ad+2bc\right)}{5}+\frac{b^2d^3x^{13}}{13}+\frac{bd^2x^{11}\left(2ad+3bc\right)}{11}+\frac{cx^7\left(3a^2d^2+6abcd+b^2c^2\right)}{7}+\frac{dx^9\left(a^2d^2+6abcd+3b^2c^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**3/3 + a*c**2*x**5*(3*a*d + 2*b*c)/5 + b**2*d**3*x**1
3/13 + b*d**2*x**11*(2*a*d + 3*b*c)/11 + c*x**7*(3*a**2*d**2 + 6*
a*b*c*d + b**2*c**2)/7 + d*x**9*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c
**2)/9
```

Mathematica [A] time = 0.0401486, size = 127, normalized size = 1.

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
```

```
2)*x^9)/9 + (b*d^2*(3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Maple [A] time = 0.001, size = 128, normalized size = 1.

$$\frac{b^2d^3x^{13}}{13}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{11}}{11}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^9}{9}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^7}{7}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^5}{5}+\frac{a^2c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/13*b^2*d^3*x^13+1/11*(2*a*b*d^3+3*b^2*c*d^2)*x^11+1/9*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^9+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3
)*x^7+1/5*(3*a^2*c^2*d+2*a*b*c^3)*x^5+1/3*a^2*c^3*x^3
```

Maxima [A] time = 1.36089, size = 171, normalized size = 1.35

$$\frac{1}{13}\,b^2d^3x^{13}+\frac{1}{11}\left(3\,b^2cd^2+2\,abd^3\right)x^{11}+\frac{1}{9}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^9+\frac{1}{3}\,a^2c^3x^3+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+\frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b
^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*
c^2*d)*x^5
```

Fricas [A] time = 0.207578, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^3b^2+\frac{3}{11}x^{11}d^2cb^2+\frac{2}{11}x^{11}d^3ba+\frac{1}{3}x^9dc^2b^2+\frac{2}{3}x^9d^2cba+\frac{1}{9}x^9d^3a^2+\frac{1}{7}x^7c^3b^2+\frac{6}{7}x^7dc^2ba+\frac{3}{7}x^7d^2ca^2+\frac{2}{5}x^5c^3ba+\frac{3}{5}x^5dc^2a^2+\frac{1}{3}x^3c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b^2 + 3/11*x^11*d^2*c*b^2 + 2/11*x^11*d^3*b*a + 1/3
*x^9*d*c^2*b^2 + 2/3*x^9*d^2*c*b*a + 1/9*x^9*d^3*a^2 + 1/7*x^7*c^
3*b^2 + 6/7*x^7*d*c^2*b*a + 3/7*x^7*d^2*c*a^2 + 2/5*x^5*c^3*b*a +
 3/5*x^5*d*c^2*a^2 + 1/3*x^3*c^3*a^2
```

Sympy [A] time = 0.169082, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^3}{3}+\frac{b^2d^3x^{13}}{13}+x^{11}\left(\frac{2abd^3}{11}+\frac{3b^2cd^2}{11}\right)+x^9\left(\frac{a^2d^3}{9}+\frac{2abcd^2}{3}+\frac{b^2c^2d}{3}\right)+x^7\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^5\left(\frac{3a^2c^2d}{5}+\frac{2abc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**3/3 + b**2*d**3*x**13/13 + x**11*(2*a*b*d**3/11 + 3*
b**2*c*d**2/11) + x**9*(a**2*d**3/9 + 2*a*b*c*d**2/3 + b**2*c**2*
d/3) + x**7*(3*a**2*c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x*
*5*(3*a**2*c**2*d/5 + 2*a*b*c**3/5)
```

GIAC/XCAS [A] time = 0.221822, size = 182, normalized size = 1.43

$$\frac{1}{13}\,b^2d^3x^{13} + \frac{3}{11}\,b^2cd^2x^{11} + \frac{2}{11}\,abd^3x^{11} + \frac{1}{3}\,b^2c^2dx^9 + \frac{2}{3}\,abcd^2x^9 + \frac{1}{9}\,a^2d^3x^9 + \frac{1}{7}\,b^2c^3x^7 + \frac{6}{7}\,abc^2dx^7 + \frac{3}{7}\,a^2cd^2x^7 + \frac{2}{5}\,abc^3x^5 + \frac{3}{5}\,a^2c^2dx^5 + \frac{1}{3}\,a^2c^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^3*x^13 + 3/11*b^2*c*d^2*x^11 + 2/11*a*b*d^3*x^11 + 1/3
*b^2*c^2*d*x^9 + 2/3*a*b*c*d^2*x^9 + 1/9*a^2*d^3*x^9 + 1/7*b^2*c^
3*x^7 + 6/7*a*b*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 2/5*a*b*c^3*x^5 +
 3/5*a^2*c^2*d*x^5 + 1/3*a^2*c^3*x^3
```

3.162 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=71

$$-\frac{b\left(c+dx^2\right)^5(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^4(bc-ad)^2}{8d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^4)/(8*d^3) - (b*(b*c - a*d)*(c + d*x^2
)^5)/(5*d^3) + (b^2*(c + d*x^2)^6)/(12*d^3)
```

Rubi [A] time = 0.323018, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{b\left(c+dx^2\right)^5(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^4(bc-ad)^2}{8d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^4)/(8*d^3) - (b*(b*c - a*d)*(c + d*x^2
)^5)/(5*d^3) + (b^2*(c + d*x^2)^6)/(12*d^3)
```

Rubi in Sympy [A] time = 29.9885, size = 60, normalized size = 0.85

$$\frac{b^2\left(c+dx^2\right)^6}{12d^3}+\frac{b\left(c+dx^2\right)^5(ad-bc)}{5d^3}+\frac{\left(c+dx^2\right)^4(ad-bc)^2}{8d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] b**2*(c + d*x**2)**6/(12*d**3) + b*(c + d*x**2)**5*(a*d - b*c)/(5
*d**3) + (c + d*x**2)**4*(a*d - b*c)**2/(8*d**3)
```

Mathematica [A] time = 0.0533124, size = 119, normalized size = 1.68

$$\frac{1}{120}x^2\left(15dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+20cx^4\left(3a^2d^2+6abcd+b^2c^2\right)\right.$$
$$\left.+60a^2c^3+30ac^2x^2(3ad+2bc)+12bd^2x^8(2ad+3bc)+10b^2d^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^2*(60*a^2*c^3 + 30*a*c^2*(2*b*c + 3*a*d)*x^2 + 20*c*(b^2*c^2 +
 6*a*b*c*d + 3*a^2*d^2)*x^4 + 15*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^6 + 12*b*d^2*(3*b*c + 2*a*d)*x^8 + 10*b^2*d^3*x^10))/120
```

Maple [A] time = 0.001, size = 128, normalized size = 1.8

$$\frac{b^2d^3x^{12}}{12}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{10}}{10}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^8}{8}$$
$$+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^6}{6}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^4}{4}+\frac{a^2c^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/12*b^2*d^3*x^12+1/10*(2*a*b*d^3+3*b^2*c*d^2)*x^10+1/8*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^8+1/6*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3
)*x^6+1/4*(3*a^2*c^2*d+2*a*b*c^3)*x^4+1/2*a^2*c^3*x^2
```

Maxima [A] time = 1.34848, size = 171, normalized size = 2.41

$$\frac{1}{12}\,b^2d^3x^{12} + \frac{1}{10}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{10} + \frac{1}{8}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^8 + \frac{1}{2}\,a^2c^3x^2 + \frac{1}{6}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^6 + \frac{1}{4}\left(2\,abc^3 + 3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b
^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*
c^2*d)*x^4
```

Fricas [A] time = 0.200806, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}d^3b^2 + \frac{3}{10}x^{10}d^2cb^2 + \frac{1}{5}x^{10}d^3ba + \frac{3}{8}x^8dc^2b^2 + \frac{3}{4}x^8d^2cba + \frac{1}{8}x^8d^3a^2 + \frac{1}{6}x^6c^3b^2 + x^6dc^2ba + \frac{1}{2}x^6d^2ca^2 + \frac{1}{2}x^4c^3ba + \frac{3}{4}x^4dc^2a^2 + \frac{1}{2}x^2c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^3*b^2 + 3/10*x^10*d^2*c*b^2 + 1/5*x^10*d^3*b*a + 3/8*
x^8*d*c^2*b^2 + 3/4*x^8*d^2*c*b*a + 1/8*x^8*d^3*a^2 + 1/6*x^6*c^3
*b^2 + x^6*d*c^2*b*a + 1/2*x^6*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*
x^4*d*c^2*a^2 + 1/2*x^2*c^3*a^2
```

Sympy [A] time = 0.172237, size = 136, normalized size = 1.92

$$\frac{a^2c^3x^2}{2} + \frac{b^2d^3x^{12}}{12} + x^{10}\left(\frac{abd^3}{5} + \frac{3b^2cd^2}{10}\right) + x^8\left(\frac{a^2d^3}{8} + \frac{3abcd^2}{4} + \frac{3b^2c^2d}{8}\right) + x^6\left(\frac{a^2cd^2}{2} + abc^2d + \frac{b^2c^3}{6}\right) + x^4\left(\frac{3a^2c^2d}{4} + \frac{abc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**2/2 + b**2*d**3*x**12/12 + x**10*(a*b*d**3/5 + 3*b**
2*c*d**2/10) + x**8*(a**2*d**3/8 + 3*a*b*c*d**2/4 + 3*b**2*c**2*d
/8) + x**6*(a**2*c*d**2/2 + a*b*c**2*d + b**2*c**3/6) + x**4*(3*a
**2*c**2*d/4 + a*b*c**3/2)
```

GIAC/XCAS [A] time = 0.22499, size = 181, normalized size = 2.55

$$\frac{1}{12}\,b^2d^3x^{12} + \frac{3}{10}\,b^2cd^2x^{10} + \frac{1}{5}\,abd^3x^{10} + \frac{3}{8}\,b^2c^2dx^8 + \frac{3}{4}\,abcd^2x^8 + \frac{1}{8}\,a^2d^3x^8 + \frac{1}{6}\,b^2c^3x^6 + abc^2dx^6 + \frac{1}{2}\,a^2cd^2x^6 + \frac{1}{2}\,abc^3x^4 + \frac{3}{4}\,a^2c^2dx^4 + \frac{1}{2}\,a^2c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^3*x^12 + 3/10*b^2*c*d^2*x^10 + 1/5*a*b*d^3*x^10 + 3/8*
b^2*c^2*d*x^8 + 3/4*a*b*c*d^2*x^8 + 1/8*a^2*d^3*x^8 + 1/6*b^2*c^3
*x^6 + a*b*c^2*d*x^6 + 1/2*a^2*c*d^2*x^6 + 1/2*a*b*c^3*x^4 + 3/4*
a^2*c^2*d*x^4 + 1/2*a^2*c^3*x^2
```

3.163 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d^2*(3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rubi [A] time = 0.149301, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d^2*(3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ac^2x^3\left(3ad+2bc\right)}{3}+\frac{b^2d^3x^{11}}{11}+\frac{bd^2x^9\left(2ad+3bc\right)}{9}+c^3\int a^2\,dx+\frac{cx^5\left(3a^2d^2+6abcd+b^2c^2\right)}{5}+\frac{dx^7\left(a^2d^2+6abcd+3b^2c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a*c**2*x**3*(3*a*d + 2*b*c)/3 + b**2*d**3*x**11/11 + b*d**2*x**9*
(2*a*d + 3*b*c)/9 + c**3*Integral(a**2, x) + c*x**5*(3*a**2*d**2
+ 6*a*b*c*d + b**2*c**2)/5 + d*x**7*(a**2*d**2 + 6*a*b*c*d + 3*b*
*2*c**2)/7
```

Mathematica [A] time = 0.0382044, size = 122, normalized size = 1.

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7
```

```
)/7 + (b*d^2*(3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A] time = 0.002, size = 125, normalized size = 1.

$$\frac{b^2d^3x^{11}}{11}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^9}{9}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^7}{7}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^5}{5}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^3}{3}+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*
a*b*c*d^2+3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*
x^5+1/3*(3*a^2*c^2*d+2*a*b*c^3)*x^3+a^2*c^3*x
```

Maxima [A] time = 1.35045, size = 167, normalized size = 1.37

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\left(3\,b^2cd^2+2\,abd^3\right)x^9+\frac{1}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+\frac{1}{3}\left(2\,abc^3+3\,a^2c^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 +
 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x
^3
```

Fricas [A] time = 0.200968, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}d^3b^2+\frac{1}{3}x^9d^2cb^2+\frac{2}{9}x^9d^3ba+\frac{3}{7}x^7dc^2b^2+\frac{6}{7}x^7d^2cba+\frac{1}{7}x^7d^3a^2+\frac{1}{5}x^5c^3b^2+\frac{6}{5}x^5dc^2ba+\frac{3}{5}x^5d^2ca^2+\frac{2}{3}x^3c^3ba+x^3dc^2a^2+xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^3*b^2 + 1/3*x^9*d^2*c*b^2 + 2/9*x^9*d^3*b*a + 3/7*x^7
*d*c^2*b^2 + 6/7*x^7*d^2*c*b*a + 1/7*x^7*d^3*a^2 + 1/5*x^5*c^3*b^
2 + 6/5*x^5*d*c^2*b*a + 3/5*x^5*d^2*c*a^2 + 2/3*x^3*c^3*b*a + x^3
*d*c^2*a^2 + x*c^3*a^2
```

Sympy [A] time = 0.174673, size = 136, normalized size = 1.11

$$a^2c^3x+\frac{b^2d^3x^{11}}{11}+x^9\left(\frac{2abd^3}{9}+\frac{b^2cd^2}{3}\right)+x^7\left(\frac{a^2d^3}{7}+\frac{6abcd^2}{7}+\frac{3b^2c^2d}{7}\right)+x^5\left(\frac{3a^2cd^2}{5}+\frac{6abc^2d}{5}+\frac{b^2c^3}{5}\right)+x^3\left(a^2c^2d+\frac{2abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d*
*2/3) + x**7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x
**5*(3*a**2*c*d**2/5 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2
*c**2*d + 2*a*b*c**3/3)
```

GIAC/XCAS [A] time = 0.22267, size = 177, normalized size = 1.45

$$\frac{1}{11}\,b^2d^3x^{11} + \frac{1}{3}\,b^2cd^2x^9 + \frac{2}{9}\,abd^3x^9 + \frac{3}{7}\,b^2c^2dx^7 + \frac{6}{7}\,abcd^2x^7 + \frac{1}{7}\,a^2d^3x^7 + \frac{1}{5}\,b^2c^3x^5 + \frac{6}{5}\,abc^2dx^5 + \frac{3}{5}\,a^2cd^2x^5 + \frac{2}{3}\,abc^3x^3 + a^2c^2dx^3 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2
*c^2*d*x^7 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^
5 + 6/5*a*b*c^2*d*x^5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2
*c^2*d*x^3 + a^2*c^3*x
```

3.164 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x} \, dx$

Optimal. Leaf size=123

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+\frac{1}{10}b^2d^3x^{10}$$

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d
^2)*x^4)/4 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2
*(3*b*c + 2*a*d)*x^8)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Rubi [A] time = 0.224298, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+\frac{1}{10}b^2d^3x^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d
^2)*x^4)/4 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2
*(3*b*c + 2*a*d)*x^8)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2c^3\log\left(x^2\right)}{2}+\frac{b^2d^3x^{10}}{10}+\frac{bd^2x^8\left(2ad+3bc\right)}{8}+\frac{c^2\left(3ad+2bc\right)\int^{x^2}a\,dx}{2}+\frac{c\left(3a^2d^2+6abcd+b^2c^2\right)\int^{x^2}x\,dx}{2}+\frac{dx^6\left(a^2d^2+6abcd+3b^2c^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x,x)
```

```
[Out] a**2*c**3*log(x**2)/2 + b**2*d**3*x**10/10 + b*d**2*x**8*(2*a*d +
 3*b*c)/8 + c**2*(3*a*d + 2*b*c)*Integral(a, (x, x**2))/2 + c*(3*
a**2*d**2 + 6*a*b*c*d + b**2*c**2)*Integral(x, (x, x**2))/2 + d*x
**6*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/6
```

Mathematica [A] time = 0.0549936, size = 123, normalized size = 1.

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+\frac{1}{10}b^2d^3x^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d
^2)*x^4)/4 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2
```

```
*(3*b*c + 2*a*d)*x^8)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Maple [A] time = 0.004, size = 132, normalized size = 1.1

$$\frac{b^2d^3x^{10}}{10}+\frac{x^8abd^3}{4}+\frac{3\,x^8b^2cd^2}{8}+\frac{x^6a^2d^3}{6}+x^6abcd^2+\frac{x^6b^2c^2d}{2}+\frac{3\,x^4a^2cd^2}{4}+\frac{3\,x^4abc^2d}{2}+\frac{x^4b^2c^3}{4}+\frac{3\,x^2a^2c^2d}{2}+x^2abc^3+a^2c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)^3/x,x)
```

```
[Out] 1/10*b^2*d^3*x^10+1/4*x^8*a*b*d^3+3/8*x^8*b^2*c*d^2+1/6*x^6*a^2*d
^3+x^6*a*b*c*d^2+1/2*x^6*b^2*c^2*d+3/4*x^4*a^2*c*d^2+3/2*x^4*a*b*
c^2*d+1/4*x^4*b^2*c^3+3/2*x^2*a^2*c^2*d+x^2*a*b*c^3+a^2*c^3*ln(x)
```

Maxima [A] time = 1.35397, size = 173, normalized size = 1.41

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{1}{8}\left(3\,b^2cd^2+2\,abd^3\right)x^8+\frac{1}{6}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+\frac{1}{2}\,a^2c^3\log\left(x^2\right)+\frac{1}{4}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+\frac{1}{2}\left(2\,abc^3+3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + 1/2*a^2*c^3*log(x^2) + 1/4
*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a
^2*c^2*d)*x^2
```

Fricas [A] time = 0.234962, size = 169, normalized size = 1.37

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{1}{8}\left(3\,b^2cd^2+2\,abd^3\right)x^8+\frac{1}{6}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+a^2c^3\log(x)+\frac{1}{4}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+\frac{1}{2}\left(2\,abc^3+3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3*log(x) + 1/4*(b^2*
c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^2
```

Sympy [A] time = 1.53797, size = 133, normalized size = 1.08

$$a^2c^3\log(x)+\frac{b^2d^3x^{10}}{10}+x^8\left(\frac{abd^3}{4}+\frac{3b^2cd^2}{8}\right)+x^6\left(\frac{a^2d^3}{6}+abcd^2+\frac{b^2c^2d}{2}\right)+x^4\left(\frac{3a^2cd^2}{4}+\frac{3abc^2d}{2}+\frac{b^2c^3}{4}\right)+x^2\left(\frac{3a^2c^2d}{2}+abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x,x)
```

```
[Out] a**2*c**3*log(x) + b**2*d**3*x**10/10 + x**8*(a*b*d**3/4 + 3*b**2
*c*d**2/8) + x**6*(a**2*d**3/6 + a*b*c*d**2 + b**2*c**2*d/2) + x*
*4*(3*a**2*c*d**2/4 + 3*a*b*c**2*d/2 + b**2*c**3/4) + x**2*(3*a**
2*c**2*d/2 + a*b*c**3)
```

GIAC/XCAS [A] time = 0.229611, size = 181, normalized size = 1.47

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{3}{8}\,b^2cd^2x^8+\frac{1}{4}\,abd^3x^8+\frac{1}{2}\,b^2c^2dx^6+abcd^2x^6+\frac{1}{6}\,a^2d^3x^6+\frac{1}{4}\,b^2c^3x^4+\frac{3}{2}\,abc^2dx^4+\frac{3}{4}\,a^2cd^2x^4+abc^3x^2+\frac{3}{2}\,a^2c^2dx^2+\frac{1}{2}\,a^2c^3\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^3*x^10 + 3/8*b^2*c*d^2*x^8 + 1/4*a*b*d^3*x^8 + 1/2*b^2
*c^2*d*x^6 + a*b*c*d^2*x^6 + 1/6*a^2*d^3*x^6 + 1/4*b^2*c^3*x^4 +
3/2*a*b*c^2*d*x^4 + 3/4*a^2*c*d^2*x^4 + a*b*c^3*x^2 + 3/2*a^2*c^2
*d*x^2 + 1/2*a^2*c^3*ln(x^2)
```

3.165 $\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^2} \, dx$

Optimal. Leaf size=120

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*
d + 3*a^2*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)
/5 + (b*d^2*(3*b*c + 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Rubi [A] time = 0.155721, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*
d + 3*a^2*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)
/5 + (b*d^2*(3*b*c + 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Rubi in Sympy [A] time = 34.8776, size = 117, normalized size = 0.98

$$-\frac{a^2c^3}{x}+ac^2x\,(3ad+2bc)+\frac{b^2d^3x^9}{9}+\frac{bd^2x^7\,(2ad+3bc)}{7}+\frac{cx^3\left(3a^2d^2+6abcd+b^2c^2\right)}{3}+\frac{dx^5\left(a^2d^2+6abcd+3b^2c^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**2,x)
```

```
[Out] -a**2*c**3/x + a*c**2*x*(3*a*d + 2*b*c) + b**2*d**3*x**9/9 + b*d*
*2*x**7*(2*a*d + 3*b*c)/7 + c*x**3*(3*a**2*d**2 + 6*a*b*c*d + b**
2*c**2)/3 + d*x**5*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/5
```

Mathematica [A] time = 0.0719165, size = 120, normalized size = 1.

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*
d + 3*a^2*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)
```

```
/5 + (b*d^2*(3*b*c + 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Maple [A] time = 0.007, size = 131, normalized size = 1.1

$$\frac{b^2d^3x^9}{9}+\frac{2\,x^7abd^3}{7}+\frac{3\,x^7b^2cd^2}{7}+\frac{x^5a^2d^3}{5}+\frac{6\,x^5abcd^2}{5}+\frac{3\,x^5b^2c^2d}{5}+x^3a^2cd^2+2\,x^3abc^2d+\frac{x^3b^2c^3}{3}+3\,xa^2c^2d+2\,xabc^3-\frac{a^2c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^3/x^2,x)
```

```
[Out] 1/9*b^2*d^3*x^9+2/7*x^7*a*b*d^3+3/7*x^7*b^2*c*d^2+1/5*x^5*a^2*d^3
+6/5*x^5*a*b*c*d^2+3/5*x^5*b^2*c^2*d+x^3*a^2*c*d^2+2*x^3*a*b*c^2*
d+1/3*x^3*b^2*c^3+3*x*a^2*c^2*d+2*x*a*b*c^3-a^2*c^3/x
```

Maxima [A] time = 1.3534, size = 167, normalized size = 1.39

$$\frac{1}{9}\,b^2d^3x^9+\frac{1}{7}\left(3\,b^2cd^2+2\,abd^3\right)x^7+\frac{1}{5}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^5-\frac{a^2c^3}{x}+\frac{1}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^3+\left(2\,abc^3+3\,a^2c^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^3*x^9 + 1/7*(3*b^2*c*d^2 + 2*a*b*d^3)*x^7 + 1/5*(3*b^2*
c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^5 - a^2*c^3/x + 1/3*(b^2*c^3 + 6
*a*b*c^2*d + 3*a^2*c*d^2)*x^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*x
```

Fricas [A] time = 0.225515, size = 174, normalized size = 1.45

$$\frac{35\,b^2d^3x^{10}+45\left(3\,b^2cd^2+2\,abd^3\right)x^8+63\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-315\,a^2c^3+105\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+315}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^3*x^10 + 45*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 63*(3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 315*a^2*c^3 + 105*(b^2*
c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 315*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^2)/x
```

Sympy [A] time = 1.52237, size = 131, normalized size = 1.09

$$-\frac{a^2c^3}{x}+\frac{b^2d^3x^9}{9}+x^7\left(\frac{2abd^3}{7}+\frac{3b^2cd^2}{7}\right)+x^5\left(\frac{a^2d^3}{5}+\frac{6abcd^2}{5}+\frac{3b^2c^2d}{5}\right)+x^3\left(a^2cd^2+2abc^2d+\frac{b^2c^3}{3}\right)+x\left(3a^2c^2d+2abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**2,x)
```

```
[Out] -a**2*c**3/x + b**2*d**3*x**9/9 + x**7*(2*a*b*d**3/7 + 3*b**2*c*d
**2/7) + x**5*(a**2*d**3/5 + 6*a*b*c*d**2/5 + 3*b**2*c**2*d/5) +
x**3*(a**2*c*d**2 + 2*a*b*c**2*d + b**2*c**3/3) + x*(3*a**2*c**2*
d + 2*a*b*c**3)
```

GIAC/XCAS [A] time = 0.219856, size = 176, normalized size = 1.47

$$\frac{1}{9}\,b^2d^3x^9+\frac{3}{7}\,b^2cd^2x^7+\frac{2}{7}\,abd^3x^7+\frac{3}{5}\,b^2c^2dx^5+\frac{6}{5}\,abcd^2x^5+\frac{1}{5}\,a^2d^3x^5+\frac{1}{3}\,b^2c^3x^3+2\,abc^2dx^3+a^2cd^2x^3+2\,abc^3x+3\,a^2c^2dx-\frac{a^2c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^3*x^9 + 3/7*b^2*c*d^2*x^7 + 2/7*a*b*d^3*x^7 + 3/5*b^2*c
^2*d*x^5 + 6/5*a*b*c*d^2*x^5 + 1/5*a^2*d^3*x^5 + 1/3*b^2*c^3*x^3
+ 2*a*b*c^2*d*x^3 + a^2*c*d^2*x^3 + 2*a*b*c^3*x + 3*a^2*c^2*d*x -
 a^2*c^3/x
```

3.166 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}b^2d^3x^8$$

```
[Out] -(a^2*c^3)/(2*x^2) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2)/2
+ (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c + 2
*a*d)*x^6)/6 + (b^2*d^3*x^8)/8 + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Rubi [A] time = 0.267089, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}b^2d^3x^8$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] -(a^2*c^3)/(2*x^2) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2)/2
+ (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c + 2
*a*d)*x^6)/6 + (b^2*d^3*x^8)/8 + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c^3}{2x^2}+\frac{ac^2(3ad+2bc)\log\left(x^2\right)}{2}+\frac{b^2d^3x^8}{8}+\frac{bd^2x^6(2ad+3bc)}{6}+\frac{d\left(a^2d^2+6abcd+3b^2c^2\right)\int^{x^2}x\,dx}{2}+\frac{c\left(3ad(ad+2bc)+b^2c^2\right)\int^{x^2}b^2\,dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**3,x)
```

```
[Out] -a**2*c**3/(2*x**2) + a*c**2*(3*a*d + 2*b*c)*log(x**2)/2 + b**2*d
**3*x**8/8 + b*d**2*x**6*(2*a*d + 3*b*c)/6 + d*(a**2*d**2 + 6*a*b
*c*d + 3*b**2*c**2)*Integral(x, (x, x**2))/2 + c*(3*a*d*(a*d + 2*
b*c) + b**2*c**2)*Integral(b**2, (x, x**2))/(2*b**2)
```

Mathematica [A] time = 0.087429, size = 120, normalized size = 0.98

$$\frac{6a^2\left(-2c^3+6cd^2x^4+d^3x^6\right)+4abdx^4\left(18c^2+9cdx^2+2d^2x^4\right)+3b^2x^4\left(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6\right)}{24x^2}+ac^2\log(x)(3ad+2bc)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] (4*a*b*d*x^4*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 3*b^2*x^4*(4*c^3
+ 6*c^2*d*x^2 + 4*c*d^2*x^4 + d^3*x^6) + 6*a^2*(-2*c^3 + 6*c*d^2*
```

```
x^4 + d^3*x^6))/(24*x^2) + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Maple [A] time = 0.009, size = 134, normalized size = 1.1

$$\frac{b^2d^3x^8}{8}+\frac{x^6abd^3}{3}+\frac{x^6b^2cd^2}{2}+\frac{x^4a^2d^3}{4}+\frac{3\,x^4abcd^2}{2}+\frac{3\,x^4b^2c^2d}{4}+\frac{3\,x^2a^2cd^2}{2}+3\,x^2abc^2d+\frac{x^2b^2c^3}{2}+3\,\ln(x)\,a^2c^2d+2\,\ln(x)\,abc^3-\frac{a^2c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^3/x^3,x)
```

```
[Out] 1/8*b^2*d^3*x^8+1/3*x^6*a*b*d^3+1/2*x^6*b^2*c*d^2+1/4*x^4*a^2*d^3
+3/2*x^4*a*b*c*d^2+3/4*x^4*b^2*c^2*d+3/2*x^2*a^2*c*d^2+3*x^2*a*b*
c^2*d+1/2*x^2*b^2*c^3+3*ln(x)*a^2*c^2*d+2*ln(x)*a*b*c^3-1/2*a^2*c
^3/x^2
```

Maxima [A] time = 1.35486, size = 173, normalized size = 1.41

$$\frac{1}{8}\,b^2d^3x^8+\frac{1}{6}\left(3\,b^2cd^2+2\,abd^3\right)x^6+\frac{1}{4}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^4-\frac{a^2c^3}{2\,x^2}+\frac{1}{2}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^2+\frac{1}{2}\left(2\,abc^3+3\,a^2c^2d\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/6*(3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 1/4*(3*b^2*
c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 - 1/2*a^2*c^3/x^2 + 1/2*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*
d)*log(x^2)
```

Fricas [A] time = 0.23032, size = 177, normalized size = 1.44

$$\frac{3\,b^2d^3x^{10}+4\left(3\,b^2cd^2+2\,abd^3\right)x^8+6\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-12\,a^2c^3+12\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+24\left(2\,abc\right.}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*d^3*x^10 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 6*(3*b^2
*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 12*a^2*c^3 + 12*(b^2*c^3 +
6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 24*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
*log(x))/x^2
```

Sympy [A] time = 1.93563, size = 133, normalized size = 1.08

$$-\frac{a^2c^3}{2x^2}+ac^2\left(3ad+2bc\right)\log\left(x\right)+\frac{b^2d^3x^8}{8}+x^6\left(\frac{abd^3}{3}+\frac{b^2cd^2}{2}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{3b^2c^2d}{4}\right)+x^2\left(\frac{3a^2cd^2}{2}+3abc^2d+\frac{b^2c^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**3,x)
```

```
[Out] -a**2*c**3/(2*x**2) + a*c**2*(3*a*d + 2*b*c)*log(x) + b**2*d**3*x
**8/8 + x**6*(a*b*d**3/3 + b**2*c*d**2/2) + x**4*(a**2*d**3/4 + 3
*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**2*(3*a**2*c*d**2/2 + 3*a*b*
c**2*d + b**2*c**3/2)
```

GIAC/XCAS [A] time = 0.237696, size = 216, normalized size = 1.76

$$\frac{1}{8}\,b^2d^3x^8+\frac{1}{2}\,b^2cd^2x^6+\frac{1}{3}\,abd^3x^6+\frac{3}{4}\,b^2c^2dx^4+\frac{3}{2}\,abcd^2x^4+\frac{1}{4}\,a^2d^3x^4+\frac{1}{2}\,b^2c^3x^2+3\,abc^2dx^2+\frac{3}{2}\,a^2cd^2x^2+\frac{1}{2}\left(2\,abc^3+3\,a^2c^2d\right)\ln\left(x^2\right)-\frac{2\,abc^3x^2+3\,a^2c^2dx^2+a^2c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/2*b^2*c*d^2*x^6 + 1/3*a*b*d^3*x^6 + 3/4*b^2*c
^2*d*x^4 + 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/2*b^2*c^3*x^2
+ 3*a*b*c^2*d*x^2 + 3/2*a^2*c*d^2*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^
2*d)*ln(x^2) - 1/2*(2*a*b*c^3*x^2 + 3*a^2*c^2*d*x^2 + a^2*c^3)/x^
2
```

3.167 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a
*b*c*d + 3*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)
/3 + (b*d^2*(3*b*c + 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Rubi [A] time = 0.166695, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a
*b*c*d + 3*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)
/3 + (b*d^2*(3*b*c + 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2c^3}{3x^3}-\frac{ac^2\left(3ad+2bc\right)}{x}+\frac{b^2d^3x^7}{7}+\frac{bd^2x^5\left(2ad+3bc\right)}{5}+\frac{dx^3\left(a^2d^2+6abcd+3b^2c^2\right)}{3}+\frac{c\left(3ad\left(ad+2bc\right)+b^2c^2\right)\int b^2\,dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**4,x)
```

```
[Out] -a**2*c**3/(3*x**3) - a*c**2*(3*a*d + 2*b*c)/x + b**2*d**3*x**7/7
 + b*d**2*x**5*(2*a*d + 3*b*c)/5 + d*x**3*(a**2*d**2 + 6*a*b*c*d
+ 3*b**2*c**2)/3 + c*(3*a*d*(a*d + 2*b*c) + b**2*c**2)*Integral(b
**2, x)/b**2
```

Mathematica [A] time = 0.0808203, size = 120, normalized size = 1.

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a
*b*c*d + 3*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)
```

```
/3 + (b*d^2*(3*b*c + 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Maple [A] time = 0.008, size = 124, normalized size = 1.

$$\frac{b^2d^3x^7}{7}+\frac{2\,x^5abd^3}{5}+\frac{3\,x^5b^2cd^2}{5}+\frac{x^3a^2d^3}{3}+2\,x^3abcd^2+x^3b^2c^2d+3\,xa^2cd^2+6\,xabc^2d+xb^2c^3-\frac{a^2c^3}{3\,x^3}-\frac{ac^2\,(3\,ad+2\,bc)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^3/x^4,x)
```

```
[Out] 1/7*b^2*d^3*x^7+2/5*x^5*a*b*d^3+3/5*x^5*b^2*c*d^2+1/3*x^3*a^2*d^3
+2*x^3*a*b*c*d^2+x^3*b^2*c^2*d+3*x*a^2*c*d^2+6*x*a*b*c^2*d+x*b^2*
c^3-1/3*a^2*c^3/x^3-a*c^2*(3*a*d+2*b*c)/x
```

Maxima [A] time = 1.33104, size = 170, normalized size = 1.42

$$\frac{1}{7}\,b^2d^3x^7+\frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5+\frac{1}{3}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x-\frac{a^2c^3+3\left(2\,abc^3+3\,a^2c^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^3*x^7 + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/3*(3*b^2*
c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x - 1/3*(a^2*c^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

Fricas [A] time = 0.229272, size = 174, normalized size = 1.45

$$\frac{15\,b^2d^3x^{10}+21\left(3\,b^2cd^2+2\,abd^3\right)x^8+35\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-35\,a^2c^3+105\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4-105}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^10 + 21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 35*(3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 35*a^2*c^3 + 105*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 105*(2*a*b*c^3 + 3*a^2*c^2*
d)*x^2)/x^3
```

Sympy [A] time = 2.04625, size = 129, normalized size = 1.08

$$\frac{b^2d^3x^7}{7}+x^5\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}\right)+x^3\left(\frac{a^2d^3}{3}+2abcd^2+b^2c^2d\right)+x\left(3a^2cd^2+6abc^2d+b^2c^3\right)-\frac{a^2c^3+x^2\left(9a^2c^2d+6abc^3\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**4,x)
```

```
[Out] b**2*d**3*x**7/7 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x**3*(
a**2*d**3/3 + 2*a*b*c*d**2 + b**2*c**2*d) + x*(3*a**2*c*d**2 + 6*
a*b*c**2*d + b**2*c**3) - (a**2*c**3 + x**2*(9*a**2*c**2*d + 6*a*
b*c**3))/(3*x**3)
```

GIAC/XCAS [A] time = 0.221275, size = 174, normalized size = 1.45

$$\frac{1}{7}\,b^2d^3x^7+\frac{3}{5}\,b^2cd^2x^5+\frac{2}{5}\,abd^3x^5+b^2c^2dx^3+2\,abcd^2x^3+\frac{1}{3}\,a^2d^3x^3+b^2c^3x+6\,abc^2dx+3\,a^2cd^2x-\frac{6\,abc^3x^2+9\,a^2c^2dx^2+a^2c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^3*x^7 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + b^2*c^2*d
*x^3 + 2*a*b*c*d^2*x^3 + 1/3*a^2*d^3*x^3 + b^2*c^3*x + 6*a*b*c^2*
d*x + 3*a^2*c*d^2*x - 1/3*(6*a*b*c^3*x^2 + 9*a^2*c^2*d*x^2 + a^2*
c^3)/x^3
```

3.168 $\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=104

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*
c - 2*a*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^
2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(9/2)
```

Rubi [A] time = 0.177235, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*
c - 2*a*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^
2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(9/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^7}{7d}+\frac{bx^5(2ad-bc)}{5d^2}+\frac{c^{\frac{3}{2}}(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{\frac{9}{2}}}+\frac{x^3(ad-bc)^2}{3d^3}-\frac{(ad-bc)^2\int c\,dx}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**7/(7*d) + b*x**5*(2*a*d - b*c)/(5*d**2) + c**(3/2)*(a*d -
 b*c)**2*atan(sqrt(d)*x/sqrt(c))/d**(9/2) + x**3*(a*d - b*c)**2/(
3*d**3) - (a*d - b*c)**2*Integral(c, x)/d**4
```

Mathematica [A] time = 0.157526, size = 104, normalized size = 1.

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(ad-bc)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((-(b*c) + a*d)^2*x^3)/(3*d^3) - (b*
(b*c - 2*a*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*
d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(9/2)
```

Maple [A] time = 0.004, size = 176, normalized size = 1.7

$$\frac{b^2x^7}{7\,d}+\frac{2\,x^5ab}{5\,d}-\frac{b^2x^5c}{5\,d^2}+\frac{x^3a^2}{3\,d}-\frac{2\,abx^3c}{3\,d^2}+\frac{x^3b^2c^2}{3\,d^3}-\frac{a^2cx}{d^2}+2\,\frac{xabc^2}{d^3}-\frac{xb^2c^3}{d^4}+\frac{a^2c^2}{d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-2\,\frac{abc^3}{d^3\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\frac{b^2c^4}{d^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/7*b^2*x^7/d+2/5/d*x^5*a*b-1/5/d^2*x^5*b^2*c+1/3/d*x^3*a^2-2/3/d
^2*x^3*a*b*c+1/3/d^3*x^3*b^2*c^2-1/d^2*x*a^2*c+2/d^3*x*a*b*c^2-1/
d^4*x*b^2*c^3+c^2/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-2*c
^3/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+c^4/d^4/(c*d)^(1/2
)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^4/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244786, size = 1, normalized size = 0.01

$$\left[\frac{30\,b^2d^3x^7-42\left(b^2cd^2-2\,abd^3\right)x^5+70\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^3+105\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}}{dx^2+c}\right.}{210\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^4/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^2*d^3*x^7 - 42*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 70*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*c^2*d
+ a^2*c*d^2)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2
 + c)) - 210*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x)/d^4, 1/105*(1
5*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 35*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) - 105*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)*x)/d^4]
```

Sympy [A] time = 2.72646, size = 240, normalized size = 2.31

$$\frac{b^2x^7}{7d}-\frac{\sqrt{-\frac{c^3}{d^9}}\,(ad-bc)^2\log\left(-\frac{d^4\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2}{a^2cd^2-2abc^2d+b^2c^3}+x\right)}{2}+\frac{\sqrt{-\frac{c^3}{d^9}}\,(ad-bc)^2\log\left(\frac{d^4\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2}{a^2cd^2-2abc^2d+b^2c^3}+x\right)}{2}+\frac{x^5\left(2abd-b^2c\right)}{5d^2}+\frac{x^3\left(a^2d^2-2abcd+b^2c^2\right)}{3d^3}-\frac{x\left(a^2cd^2-2abc^2d+b^2c^3\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**7/(7*d) - sqrt(-c**3/d**9)*(a*d - b*c)**2*log(-d**4*sqrt(
-c**3/d**9)*(a*d - b*c)**2/(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**
3) + x)/2 + sqrt(-c**3/d**9)*(a*d - b*c)**2*log(d**4*sqrt(-c**3/d
**9)*(a*d - b*c)**2/(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)
/2 + x**5*(2*a*b*d - b**2*c)/(5*d**2) + x**3*(a**2*d**2 - 2*a*b*c
*d + b**2*c**2)/(3*d**3) - x*(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c
**3)/d**4
```

GIAC/XCAS [A] time = 0.221318, size = 207, normalized size = 1.99

$$\frac{\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^4} + \frac{15\,b^2d^6x^7 - 21\,b^2cd^5x^5 + 42\,abd^6x^5 + 35\,b^2c^2d^4x^3 - 70\,abcd^5x^3 + 35\,a^2d^6x^3 - 105\,b^2c^3d^3x + 210\,abc^2d^4x - 105\,a^2cd^5x}{105\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt
(c*d)*d^4) + 1/105*(15*b^2*d^6*x^7 - 21*b^2*c*d^5*x^5 + 42*a*b*d^
6*x^5 + 35*b^2*c^2*d^4*x^3 - 70*a*b*c*d^5*x^3 + 35*a^2*d^6*x^3 -
105*b^2*c^3*d^3*x + 210*a*b*c^2*d^4*x - 105*a^2*c*d^5*x)/d^7
```

3.169 $\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=79

$$-\frac{c(bc-ad)^2\log(c+dx^2)}{2d^4}+\frac{x^2(bc-ad)^2}{2d^3}-\frac{bx^4(bc-2ad)}{4d^2}+\frac{b^2x^6}{6d}$$

```
[Out] ((b*c - a*d)^2*x^2)/(2*d^3) - (b*(b*c - 2*a*d)*x^4)/(4*d^2) + (b^
2*x^6)/(6*d) - (c*(b*c - a*d)^2*Log[c + d*x^2])/(2*d^4)
```

Rubi [A] time = 0.202514, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{c(bc-ad)^2\log(c+dx^2)}{2d^4}+\frac{x^2(bc-ad)^2}{2d^3}-\frac{bx^4(bc-2ad)}{4d^2}+\frac{b^2x^6}{6d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```


```
[Out] ((b*c - a*d)^2*x^2)/(2*d^3) - (b*(b*c - 2*a*d)*x^4)/(4*d^2) + (b^
2*x^6)/(6*d) - (c*(b*c - a*d)^2*Log[c + d*x^2])/(2*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^6}{6d}+\frac{b(2ad-bc)\int^{x^2} x\,dx}{2d^2}-\frac{c(ad-bc)^2\log(c+dx^2)}{2d^4}+\frac{(ad-bc)^2\int^{x^2}\frac{1}{d^3}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**6/(6*d) + b*(2*a*d - b*c)*Integral(x, (x, x**2))/(2*d**2)
 - c*(a*d - b*c)**2*log(c + d*x**2)/(2*d**4) + (a*d - b*c)**2*Int
egral(d**(-3), (x, x**2))/2
```

Mathematica [A] time = 0.0688523, size = 82, normalized size = 1.04

$$\frac{dx^2\left(6a^2d^2+6abd\left(dx^2-2c\right)+b^2\left(6c^2-3cdx^2+2d^2x^4\right)\right)-6c(bc-ad)^2\log(c+dx^2)}{12d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (d*x^2*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^2) + b^2*(6*c^2 - 3*c*d*x
^2 + 2*d^2*x^4)) - 6*c*(b*c - a*d)^2*Log[c + d*x^2])/(12*d^4)
```

Maple [A] time = 0.006, size = 124, normalized size = 1.6

$$\frac{b^2x^6}{6\,d}+\frac{x^4ab}{2\,d}-\frac{b^2cx^4}{4\,d^2}+\frac{a^2x^2}{2\,d}-\frac{abcx^2}{d^2}+\frac{x^2b^2c^2}{2\,d^3}-\frac{c\ln(dx^2+c)\,a^2}{2\,d^2}+\frac{c^2\ln(dx^2+c)\,ab}{d^3}-\frac{c^3\ln(dx^2+c)\,b^2}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/6*b^2*x^6/d+1/2/d*x^4*a*b-1/4/d^2*x^4*b^2*c+1/2/d*x^2*a^2-1/d^2
*x^2*a*b*c+1/2/d^3*x^2*b^2*c^2-1/2*c/d^2*ln(d*x^2+c)*a^2+c^2/d^3*
ln(d*x^2+c)*a*b-1/2*c^3/d^4*ln(d*x^2+c)*b^2
```

Maxima [A] time = 1.34784, size = 135, normalized size = 1.71

$$\frac{2\,b^2d^2x^6 - 3\,\left(b^2cd - 2\,abd^2\right)x^4 + 6\,\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^2}{12\,d^3} - \frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\log\left(dx^2 + c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*(b^2*c*d - 2*a*b*d^2)*x^4 + 6*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*
c*d^2)*log(d*x^2 + c)/d^4
```

Fricas [A] time = 0.225583, size = 136, normalized size = 1.72

$$\frac{2\,b^2d^3x^6 - 3\,\left(b^2cd^2 - 2\,abd^3\right)x^4 + 6\,\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^2 - 6\,\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\log\left(dx^2 + c\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^4 + 6*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*x^2 - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2)*log(d*x^2 + c))/d^4
```

Sympy [A] time = 2.19881, size = 83, normalized size = 1.05

$$\frac{b^2x^6}{6d} - \frac{c\left(ad - bc\right)^2\log\left(c + dx^2\right)}{2d^4} + \frac{x^4\left(2abd - b^2c\right)}{4d^2} + \frac{x^2\left(a^2d^2 - 2abcd + b^2c^2\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**6/(6*d) - c*(a*d - b*c)**2*log(c + d*x**2)/(2*d**4) + x**
4*(2*a*b*d - b**2*c)/(4*d**2) + x**2*(a**2*d**2 - 2*a*b*c*d + b**
2*c**2)/(2*d**3)
```

GIAC/XCAS [A] time = 0.221766, size = 144, normalized size = 1.82

$$\frac{2\,b^2d^2x^6 - 3\,b^2cdx^4 + 6\,abd^2x^4 + 6\,b^2c^2x^2 - 12\,abcdx^2 + 6\,a^2d^2x^2}{12\,d^3} - \frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*b^2*c*d*x^4 + 6*a*b*d^2*x^4 + 6*b^2*c^2*x
^2 - 12*a*b*c*d*x^2 + 6*a^2*d^2*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)*ln(abs(d*x^2 + c))/d^4
```

3.170 $\int \frac{x^2(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(bc-ad)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}$$

```
[Out] ((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)
/(5*d) - (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7
/2)
```

Rubi [A] time = 0.14965, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(bc-ad)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)
/(5*d) - (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7
/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^5}{5d}+\frac{bx^3\left(2ad-bc\right)}{3d^2}-\frac{\sqrt{c}\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{\frac{7}{2}}}+\left(ad-bc\right)^2\int\frac{1}{d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**5/(5*d) + b*x**3*(2*a*d - b*c)/(3*d**2) - sqrt(c)*(a*d -
b*c)**2*atan(sqrt(d)*x/sqrt(c))/d**(7/2) + (a*d - b*c)**2*Integra
l(d**(-3), x)
```

Mathematica [A] time = 0.114923, size = 83, normalized size = 1.

$$-\frac{\sqrt{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(ad-bc)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((-(b*c) + a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x
^5)/(5*d) - (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d
^(7/2)
```

Maple [A] time = 0.005, size = 135, normalized size = 1.6

$$\frac{b^2x^5}{5d}+\frac{2\,abx^3}{3\,d}-\frac{x^3b^2c}{3\,d^2}+\frac{a^2x}{d}-2\,\frac{xabc}{d^2}+\frac{b^2c^2x}{d^3}-\frac{a^2c}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+2\,\frac{abc^2}{d^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{b^2c^3}{d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2/3/d*x^3*a*b-1/3/d^2*x^3*b^2*c+1/d*a^2*x-2/d^2*a*b
*c*x+1/d^3*b^2*c^2*x-c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+
2*c^2/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-c^3/d^3/(c*d)^(
1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243301, size = 1, normalized size = 0.01

$$\left[\frac{6\,b^2d^2x^5-10\left(b^2cd-2\,abd^2\right)x^3+15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+30\left(b^2c^2-2\,abcd+a^2d^2\right)x}{30\,d^3},\ \frac{3\,b^2 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 - 10*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d)
- c)/(d*x^2 + c)) + 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3, 1/
15*(3*b^2*d^2*x^5 - 5*(b^2*c*d - 2*a*b*d^2)*x^3 - 15*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) + 15*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*x)/d^3]
```

Sympy [A] time = 2.43984, size = 192, normalized size = 2.31

$$\frac{b^2x^5}{5d}+\frac{\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2\log\left(-\frac{d^3\sqrt{-\frac{c}{d^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2\log\left(\frac{d^3\sqrt{-\frac{c}{d^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{x^3\left(2abd-b^2c\right)}{3d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**5/(5*d) + sqrt(-c/d**7)*(a*d - b*c)**2*log(-d**3*sqrt(-c/
d**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 -
 sqrt(-c/d**7)*(a*d - b*c)**2*log(d**3*sqrt(-c/d**7)*(a*d - b*c)*
*2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + x**3*(2*a*b*d - b
**2*c)/(3*d**2) + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d**3
```

GIAC/XCAS [A] time = 0.231221, size = 153, normalized size = 1.84

$$-\frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^2d^4x^5 - 5\,b^2cd^3x^3 + 10\,abd^4x^3 + 15\,b^2c^2d^2x - 30\,abcd^3x + 15\,a^2d^4x}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(
c*d)*d^3) + 1/15*(3*b^2*d^4*x^5 - 5*b^2*c*d^3*x^3 + 10*a*b*d^4*x^
3 + 15*b^2*c^2*d^2*x - 30*a*b*c*d^3*x + 15*a^2*d^4*x)/d^5
```

3.171 $\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=61

$$\frac{(bc-ad)^2\log(c+dx^2)}{2d^3}-\frac{bx^2(bc-ad)}{2d^2}+\frac{(a+bx^2)^2}{4d}$$

```
[Out] -(b*(b*c - a*d)*x^2)/(2*d^2) + (a + b*x^2)^2/(4*d) + ((b*c - a*d)
^2*Log[c + d*x^2])/(2*d^3)
```

Rubi [A] time = 0.119995, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(bc-ad)^2\log(c+dx^2)}{2d^3}-\frac{bx^2(bc-ad)}{2d^2}+\frac{(a+bx^2)^2}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -(b*(b*c - a*d)*x^2)/(2*d^2) + (a + b*x^2)^2/(4*d) + ((b*c - a*d)
^2*Log[c + d*x^2])/(2*d^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(a+bx^2)^2}{4d}+\frac{(ad-bc)\int^{x^2} b\,dx}{2d^2}+\frac{(ad-bc)^2\log(c+dx^2)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] (a + b*x**2)**2/(4*d) + (a*d - b*c)*Integral(b, (x, x**2))/(2*d**
2) + (a*d - b*c)**2*log(c + d*x**2)/(2*d**3)
```

Mathematica [A] time = 0.0380741, size = 49, normalized size = 0.8

$$\frac{bdx^2(4ad-2bc+bdx^2)+2(bc-ad)^2\log(c+dx^2)}{4d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (b*d*x^2*(-2*b*c + 4*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[c + d*x
^2])/(4*d^3)
```

Maple [A] time = 0.005, size = 85, normalized size = 1.4

$$\frac{b^2x^4}{4d}+\frac{abx^2}{d}-\frac{b^2cx^2}{2d^2}+\frac{\ln(dx^2+c)a^2}{2d}-\frac{\ln(dx^2+c)cab}{d^2}+\frac{\ln(dx^2+c)b^2c^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/4*b^2/d*x^4+b/d*a*x^2-1/2*b^2/d^2*x^2*c+1/2/d*ln(d*x^2+c)*a^2-1
/d^2*ln(d*x^2+c)*c*a*b+1/2/d^3*ln(d*x^2+c)*b^2*c^2
```

Maxima [A] time = 1.33973, size = 88, normalized size = 1.44

$$\frac{b^2dx^4 - 2\left(b^2c - 2\,abd\right)x^2}{4\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx^2 + c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*(b^2*c - 2*a*b*d)*x^2)/d^2 + 1/2*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*log(d*x^2 + c)/d^3
```

Fricas [A] time = 0.214423, size = 89, normalized size = 1.46

$$\frac{b^2d^2x^4 - 2\left(b^2cd - 2\,abd^2\right)x^2 + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx^2 + c\right)}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 - 2*(b^2*c*d - 2*a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(d*x^2 + c))/d^3
```

Sympy [A] time = 1.9789, size = 51, normalized size = 0.84

$$\frac{b^2x^4}{4d} + \frac{x^2\left(2abd - b^2c\right)}{2d^2} + \frac{(ad - bc)^2\log\left(c + dx^2\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**4/(4*d) + x**2*(2*a*b*d - b**2*c)/(2*d**2) + (a*d - b*c)*
*2*log(c + d*x**2)/(2*d**3)
```

GIAC/XCAS [A] time = 0.223648, size = 90, normalized size = 1.48

$$\frac{b^2dx^4 - 2\,b^2cx^2 + 4\,abdx^2}{4\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*b^2*c*x^2 + 4*a*b*d*x^2)/d^2 + 1/2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*ln(abs(d*x^2 + c))/d^3
```

3.172 $\int \frac{(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^3}{3d}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rubi [A] time = 0.0923967, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^3}{3d} + \frac{(2ad-bc)\int b\,dx}{d^2} + \frac{(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + (2*a*d - b*c)*Integral(b, x)/d**2 + (a*d - b*c)
**2*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*d**(5/2))
```

Mathematica [A] time = 0.079031, size = 59, normalized size = 0.94

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} + \frac{bx\left(6ad-3bc+bdx^2\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[
(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Maple [A] time = 0.001, size = 95, normalized size = 1.5

$$\frac{b^2x^3}{3\,d} + 2\,\frac{abx}{d} - \frac{xb^2c}{d^2} + a^2 \arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - 2\,\frac{abc}{d\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right) + \frac{b^2c^2}{d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/3*b^2*x^3/d+2*b/d*a*x-b^2/d^2*x*c+1/(c*d)^(1/2)*arctan(x*d/(c*d
)^(1/2))*a^2-2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*c*a*b+1/d^2/
(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244299, size = 1, normalized size = 0.02

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)+2\left(b^2dx^3-3\left(b^2c-2\,abd\right)x\right)\sqrt{-cd}}{6\sqrt{-cd}d^2},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((2*c*d*x + (d*x^2 - c
)*sqrt(-c*d))/(d*x^2 + c)) + 2*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x
)*sqrt(-c*d))/(sqrt(-c*d)*d^2), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*arctan(sqrt(c*d)*x/c) + (b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)
*sqrt(c*d))/(sqrt(c*d)*d^2)]
```

Sympy [A] time = 2.12824, size = 172, normalized size = 2.73

$$\frac{b^2x^3}{3d}-\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{x\left(2abd-b^2c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) - sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(-c*d**2*sq
rt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2
) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c
*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
 + x*(2*a*b*d - b**2*c)/d**2
```

GIAC/XCAS [A] time = 0.226779, size = 97, normalized size = 1.54

$$\frac{\left(b^{2}c^{2} - 2\,abcd + a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^{2}} + \frac{b^{2}d^{2}x^{3} - 3\,b^{2}cdx + 6\,abd^{2}x}{3\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
d^2) + 1/3*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

3.173 $\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx$

Optimal. Leaf size=51

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}+\frac{b^2x^2}{2d}$$

```
[Out] (b^2*x^2)/(2*d) + (a^2*Log[x])/c - ((b*c - a*d)^2*Log[c + d*x^2])
/(2*c*d^2)
```

Rubi [A] time = 0.12011, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}+\frac{b^2x^2}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*x^2)/(2*d) + (a^2*Log[x])/c - ((b*c - a*d)^2*Log[c + d*x^2])
/(2*c*d^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\log\left(x^2\right)}{2c}+\frac{\int^{x^2} b^2\,dx}{2d}-\frac{(ad-bc)^2\log\left(c+dx^2\right)}{2cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x/(d*x**2+c),x)
```

```
[Out] a**2*log(x**2)/(2*c) + Integral(b**2, (x, x**2))/(2*d) - (a*d - b
*c)**2*log(c + d*x**2)/(2*c*d**2)
```

Mathematica [A] time = 0.036119, size = 50, normalized size = 0.98

$$\frac{2a^2d^2\log(x)-(bc-ad)^2\log\left(c+dx^2\right)+b^2cdx^2}{2cd^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*c*d*x^2 + 2*a^2*d^2*Log[x] - (b*c - a*d)^2*Log[c + d*x^2])/(
2*c*d^2)
```

Maple [A] time = 0.008, size = 69, normalized size = 1.4

$$\frac{b^2x^2}{2\,d}+\frac{a^2\ln(x)}{c}-\frac{\ln\left(dx^2+c\right)a^2}{2\,c}+\frac{\ln\left(dx^2+c\right)ab}{d}-\frac{c\ln\left(dx^2+c\right)b^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x/(d*x^2+c),x)
```

```
[Out] 1/2*b^2*x^2/d+a^2*ln(x)/c-1/2/c*ln(d*x^2+c)*a^2+1/d*ln(d*x^2+c)*a
*b-1/2*c/d^2*ln(d*x^2+c)*b^2
```

Maxima [A] time = 1.3582, size = 82, normalized size = 1.61

$$\frac{b^2x^2}{2\,d}+\frac{a^2\log\left(x^2\right)}{2\,c}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(d*x^2 + c)/(c*d^2)
```

Fricas [A] time = 0.235367, size = 80, normalized size = 1.57

$$\frac{b^2cdx^2+2\,a^2d^2\log\left(x\right)-\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*d*x^2 + 2*a^2*d^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*log(d*x^2 + c))/(c*d^2)
```

Sympy [A] time = 5.13265, size = 41, normalized size = 0.8

$$\frac{a^2\log\left(x\right)}{c}+\frac{b^2x^2}{2d}-\frac{(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x/(d*x**2+c),x)
```

```
[Out] a**2*log(x)/c + b**2*x**2/(2*d) - (a*d - b*c)**2*log(c/d + x**2)/
(2*c*d**2)
```

GIAC/XCAS [A] time = 0.229867, size = 84, normalized size = 1.65

$$\frac{b^2x^2}{2\,d}+\frac{a^2\ln\left(x^2\right)}{2\,c}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\ln\left(\left|dx^2+c\right|\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*ln(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*ln(abs(d*x^2 + c))/(c*d^2)
```

3.174 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(3/2)*d^(3/2))
```

Rubi [A] time = 0.115327, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(3/2)*d^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{cx}+\frac{\int b^2\,dx}{d}-\frac{(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**2/(d*x**2+c),x)
```

```
[Out] -a**2/(c*x) + Integral(b**2, x)/d - (a*d - b*c)**2*atan(sqrt(d)*x
/sqrt(c))/(c**(3/2)*d**(3/2))
```

Mathematica [A] time = 0.0739474, size = 55, normalized size = 1.

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(3/2)*d^(3/2))
```

Maple [A] time = 0.007, size = 85, normalized size = 1.6

$$\frac{b^2x}{d}-\frac{a^2d}{c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+2\frac{ab}{\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{b^2c}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^2/(d*x^2+c),x)
```

```
[Out] b^2*x/d-1/c*d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+2/(c*d)^(1/
2)*arctan(x*d/(c*d)^(1/2))*a*b-c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*b^2-a^2/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231899, size = 1, normalized size = 0.02

$$\left[\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x\log\left(-\frac{2\,cdx-(dx^2-c)\,\sqrt{-cd}}{dx^2+c}\right)+2\left(b^2cx^2-a^2d\right)\sqrt{-cd}}{2\,\sqrt{-cd}cdx},\ -\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x\arctan\left(\frac{\sqrt{cd}x}{c}\right)-\left(b^2cx^2-a^2d\right)\sqrt{cd}}{\sqrt{cd}cdx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*log(-(2*c*d*x - (d*x^2 -
c)*sqrt(-c*d))/(d*x^2 + c)) + 2*(b^2*c*x^2 - a^2*d)*sqrt(-c*d))/(
sqrt(-c*d)*c*d*x), -((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*arctan(sqr
t(c*d)*x/c) - (b^2*c*x^2 - a^2*d)*sqrt(c*d))/(sqrt(c*d)*c*d*x)]
```

Sympy [A] time = 2.88532, size = 165, normalized size = 3.

$$-\frac{a^2}{cx}+\frac{b^2x}{d}+\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2\log\left(-\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2\log\left(\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**2/(d*x**2+c),x)
```

```
[Out] -a**2/(c*x) + b**2*x/d + sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(
-c**2*d*sqrt(-1/(c**3*d**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*
d + b**2*c**2) + x)/2 - sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(c
**2*d*sqrt(-1/(c**3*d**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d
+ b**2*c**2) + x)/2
```

GIAC/XCAS [A] time = 0.234251, size = 85, normalized size = 1.55

$$\frac{b^2x}{d} - \frac{a^2}{cx} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/
sqrt(c*d))/(sqrt(c*d)*c*d)
```

3.175 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

```
[Out] -a^2/(2*c*x^2) + (a*(2*b*c - a*d)*Log[x])/c^2 + ((b*c - a*d)^2*Lo
g[c + d*x^2])/(2*c^2*d)
```

Rubi [A] time = 0.149173, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] -a^2/(2*c*x^2) + (a*(2*b*c - a*d)*Log[x])/c^2 + ((b*c - a*d)^2*Lo
g[c + d*x^2])/(2*c^2*d)
```

Rubi in Sympy [A] time = 22.9969, size = 53, normalized size = 0.91

$$-\frac{a^2}{2cx^2}-\frac{a\,(ad-2bc)\log\left(x^2\right)}{2c^2}+\frac{(ad-bc)^2\log\left(c+dx^2\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**3/(d*x**2+c),x)
```

```
[Out] -a**2/(2*c*x**2) - a*(a*d - 2*b*c)*log(x**2)/(2*c**2) + (a*d - b*
c)**2*log(c + d*x**2)/(2*c**2*d)
```

Mathematica [A] time = 0.0473834, size = 60, normalized size = 1.03

$$\frac{a^2(-c)d-2adx^2\log(x)(ad-2bc)+x^2(bc-ad)^2\log\left(c+dx^2\right)}{2c^2dx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] (-(a^2*c*d) - 2*a*d*(-2*b*c + a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2
*Log[c + d*x^2])/(2*c^2*d*x^2)
```

Maple [A] time = 0.01, size = 81, normalized size = 1.4

$$-\frac{a^2}{2\,cx^2}-\frac{\ln(x)\,a^2d}{c^2}+2\,\frac{a\ln(x)\,b}{c}+\frac{d\ln\left(dx^2+c\right)a^2}{2\,c^2}-\frac{\ln\left(dx^2+c\right)ab}{c}+\frac{\ln\left(dx^2+c\right)b^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^3/(d*x^2+c),x)
```

```
[Out] -1/2*a^2/c/x^2-a^2/c^2*ln(x)*d+2*a/c*ln(x)*b+1/2/c^2*d*ln(d*x^2+c
)*a^2-1/c*ln(d*x^2+c)*a*b+1/2/d*ln(d*x^2+c)*b^2
```

Maxima [A] time = 1.36137, size = 95, normalized size = 1.64

$$\frac{\left(2\,abc - a^2d\right)\,\log\left(x^2\right)}{2\,c^2} - \frac{a^2}{2\,cx^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\,\log\left(dx^2 + c\right)}{2\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 - 1/2*a^2/(c*x^2) + 1/2*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*log(d*x^2 + c)/(c^2*d)
```

Fricas [A] time = 0.239451, size = 100, normalized size = 1.72

$$-\frac{a^2cd - \left(b^2c^2 - 2\,abcd + a^2d^2\right)x^2\log\left(dx^2 + c\right) - 2\left(2\,abcd - a^2d^2\right)x^2\log\left(x\right)}{2\,c^2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*d - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(d*x^2 + c
) - 2*(2*a*b*c*d - a^2*d^2)*x^2*log(x))/(c^2*d*x^2)
```

Sympy [A] time = 5.63598, size = 49, normalized size = 0.84

$$-\frac{a^2}{2cx^2} - \frac{a\left(ad - 2bc\right)\log\left(x\right)}{c^2} + \frac{\left(ad - bc\right)^2\log\left(\frac{c}{d} + x^2\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**3/(d*x**2+c),x)
```

```
[Out] -a**2/(2*c*x**2) - a*(a*d - 2*b*c)*log(x)/c**2 + (a*d - b*c)**2*l
og(c/d + x**2)/(2*c**2*d)
```

GIAC/XCAS [A] time = 0.228465, size = 123, normalized size = 2.12

$$\frac{\left(2\,abc - a^2d\right)\ln\left(x^2\right)}{2\,c^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,c^2d} - \frac{2\,abcx^2 - a^2dx^2 + a^2c}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*ln(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*ln(abs(d*x^2 + c))/(c^2*d) - 1/2*(2*a*b*c*x^2 - a^2*d*x^2
+ a^2*c)/(c^2*x^2)
```

3.176 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}-\frac{a(2bc-ad)}{c^2x}$$

```
[Out] -a^2/(3*c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.12916, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}-\frac{a(2bc-ad)}{c^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -a^2/(3*c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Rubi in Sympy [A] time = 21.3941, size = 56, normalized size = 0.85

$$-\frac{a^2}{3cx^3}+\frac{a\,(ad-2bc)}{c^2x}+\frac{(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**4/(d*x**2+c),x)
```

```
[Out] -a**2/(3*c*x**3) + a*(a*d - 2*b*c)/(c**2*x) + (a*d - b*c)**2*atan
(sqrt(d)*x/sqrt(c))/(c**(5/2)*sqrt(d))
```

Mathematica [A] time = 0.0969104, size = 64, normalized size = 0.97

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}+\frac{a(ad-2bc)}{c^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -a^2/(3*c*x^3) + (a*(-2*b*c + a*d))/(c^2*x) + ((b*c - a*d)^2*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Maple [A] time = 0.01, size = 98, normalized size = 1.5

$$-\frac{a^2}{3\,cx^3}+\frac{a^2d}{c^2x}-2\,\frac{ab}{cx}+\frac{a^2d^2}{c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-2\,\frac{abd}{c\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+b^2\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^4/(d*x^2+c),x)
```

```
[Out] -1/3*a^2/c/x^3+a^2/c^2/x*d-2*a/c/x*b+1/c^2/(c*d)^(1/2)*arctan(x*d
/(c*d)^(1/2))*a^2*d^2-2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b
*d+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235958, size = 1, normalized size = 0.02

$$\left[\frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^3\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right) - 2\left(a^2c + 3\left(2\,abc - a^2d\right)x^2\right)\sqrt{-cd}}{6\sqrt{-cd}c^2x^3}, \frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^3 \arctan\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*log((2*c*d*x + (d*x^2
 - c)*sqrt(-c*d))/(d*x^2 + c)) - 2*(a^2*c + 3*(2*a*b*c - a^2*d)*x
^2)*sqrt(-c*d))/(sqrt(-c*d)*c^2*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*x^3*arctan(sqrt(c*d)*x/c) - (a^2*c + 3*(2*a*b*c - a^2
*d)*x^2)*sqrt(c*d))/(sqrt(c*d)*c^2*x^3)]
```

Sympy [A] time = 3.13898, size = 172, normalized size = 2.61

$$-\frac{\sqrt{-\frac{1}{c^5d}}\left(ad-bc\right)^2\log\left(-\frac{c^3\sqrt{-\frac{1}{c^5d}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{c^5d}}\left(ad-bc\right)^2\log\left(\frac{c^3\sqrt{-\frac{1}{c^5d}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{-a^2c + x^2\left(3a^2d - 6abc\right)}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**4/(d*x**2+c),x)
```

```
[Out] -sqrt(-1/(c**5*d))*(a*d - b*c)**2*log(-c**3*sqrt(-1/(c**5*d))*(a*
d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/
(c**5*d))*(a*d - b*c)**2*log(c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + (-a**2*c + x**2*(3
*a**2*d - 6*a*b*c))/(3*c**2*x**3)
```

GIAC/XCAS [A] time = 0.223582, size = 96, normalized size = 1.45

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^2} - \frac{6\,abcx^2 - 3\,a^2dx^2 + a^2c}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
c^2) - 1/3*(6*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^2*x^3)
```

3.177 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)} dx$

Optimal. Leaf size=75

$$-\frac{a^2}{4cx^4}-\frac{(bc-ad)^2\log(c+dx^2)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}-\frac{a(2bc-ad)}{2c^2x^2}$$

```
[Out] -a^2/(4*c*x^4) - (a*(2*b*c - a*d))/(2*c^2*x^2) + ((b*c - a*d)^2*L
og[x])/c^3 - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c^3)
```

Rubi [A] time = 0.165956, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{4cx^4}-\frac{(bc-ad)^2\log(c+dx^2)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}-\frac{a(2bc-ad)}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -a^2/(4*c*x^4) - (a*(2*b*c - a*d))/(2*c^2*x^2) + ((b*c - a*d)^2*L
og[x])/c^3 - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c^3)
```

Rubi in Sympy [A] time = 26.7758, size = 68, normalized size = 0.91

$$-\frac{a^2}{4cx^4}+\frac{a(ad-2bc)}{2c^2x^2}+\frac{(ad-bc)^2\log(x^2)}{2c^3}-\frac{(ad-bc)^2\log(c+dx^2)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**5/(d*x**2+c),x)
```

```
[Out] -a**2/(4*c*x**4) + a*(a*d - 2*b*c)/(2*c**2*x**2) + (a*d - b*c)**2
*log(x**2)/(2*c**3) - (a*d - b*c)**2*log(c + d*x**2)/(2*c**3)
```

Mathematica [A] time = 0.0752354, size = 72, normalized size = 0.96

$$-\frac{-4x^4\log(x)(bc-ad)^2+ac(ac-2adx^2+4bcx^2)+2x^4(bc-ad)^2\log(c+dx^2)}{4c^3x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -(a*c*(a*c + 4*b*c*x^2 - 2*a*d*x^2) - 4*(b*c - a*d)^2*x^4*Log[x]
+ 2*(b*c - a*d)^2*x^4*Log[c + d*x^2])/(4*c^3*x^4)
```

Maple [A] time = 0.011, size = 116, normalized size = 1.6

$$-\frac{a^2}{4\,cx^4}+\frac{\ln(x)\,a^2d^2}{c^3}-2\,\frac{\ln(x)\,abd}{c^2}+\frac{\ln(x)\,b^2}{c}+\frac{a^2d}{2\,c^2x^2}-\frac{ab}{cx^2}-\frac{\ln(dx^2+c)\,a^2d^2}{2\,c^3}+\frac{\ln(dx^2+c)\,abd}{c^2}-\frac{\ln(dx^2+c)\,b^2}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^5/(d*x^2+c),x)
```

```
[Out] -1/4*a^2/c/x^4+1/c^3*ln(x)*a^2*d^2-2/c^2*ln(x)*a*b*d+1/c*ln(x)*b^
2+1/2*a^2/c^2/x^2*d-a/c/x^2*b-1/2/c^3*ln(d*x^2+c)*a^2*d^2+1/c^2*l
n(d*x^2+c)*a*b*d-1/2/c*ln(d*x^2+c)*b^2
```

Maxima [A] time = 1.32746, size = 130, normalized size = 1.73

$$-\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx^2 + c\right)}{2\,c^3} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(x^2\right)}{2\,c^3} - \frac{a^2c + 2\left(2\,abc - a^2d\right)x^2}{4\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d*x^2 + c)/c^3 + 1/2*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/4*(a^2*c + 2*(2*a*b
*c - a^2*d)*x^2)/(c^2*x^4)
```

Fricas [A] time = 0.229185, size = 132, normalized size = 1.76

$$-\frac{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^4\log\left(dx^2 + c\right) - 4\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^4\log\left(x\right) + a^2c^2 + 2\left(2\,abc^2 - a^2cd\right)x^2}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(d*x^2 + c) - 4*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(x) + a^2*c^2 + 2*(2*a*b*c^2
 - a^2*c*d)*x^2)/(c^3*x^4)
```

Sympy [A] time = 5.30967, size = 66, normalized size = 0.88

$$\frac{-a^2c + x^2\left(2a^2d - 4abc\right)}{4c^2x^4} + \frac{(ad - bc)^2\log\left(x\right)}{c^3} - \frac{(ad - bc)^2\log\left(\frac{c}{d} + x^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**5/(d*x**2+c),x)
```

```
[Out] (-a**2*c + x**2*(2*a**2*d - 4*a*b*c))/(4*c**2*x**4) + (a*d - b*c)
**2*log(x)/c**3 - (a*d - b*c)**2*log(c/d + x**2)/(2*c**3)
```

GIAC/XCAS [A] time = 0.225543, size = 188, normalized size = 2.51

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\ln\left(x^2\right)}{2\,c^3} - \frac{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,c^3d} - \frac{3\,b^2c^2x^4 - 6\,abcdx^4 + 3\,a^2d^2x^4 + 4\,abc^2x^2 - 2\,a^2cdx^2 + a^2c^2}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ln(x^2)/c^3 - 1/2*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*ln(abs(d*x^2 + c))/(c^3*d) - 1/4*(3*b^2*
c^2*x^4 - 6*a*b*c*d*x^4 + 3*a^2*d^2*x^4 + 4*a*b*c^2*x^2 - 2*a^2*c
*d*x^2 + a^2*c^2)/(c^3*x^4)
```

3.178 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)} \, dx$

Optimal. Leaf size=87

$$-\frac{a^2}{5cx^5}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}-\frac{a(2bc-ad)}{3c^2x^3}$$

```
[Out] -a^2/(5*c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c
^3*x) - (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/
2)
```

Rubi [A] time = 0.151743, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{5cx^5}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}-\frac{a(2bc-ad)}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -a^2/(5*c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c
^3*x) - (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/
2)
```

Rubi in Sympy [A] time = 27.0113, size = 73, normalized size = 0.84

$$-\frac{a^2}{5cx^5}+\frac{a\left(ad-2bc\right)}{3c^2x^3}-\frac{\left(ad-bc\right)^2}{c^3x}-\frac{\sqrt{d}\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**6/(d*x**2+c),x)
```

```
[Out] -a**2/(5*c*x**5) + a*(a*d - 2*b*c)/(3*c**2*x**3) - (a*d - b*c)**2
/(c**3*x) - sqrt(d)*(a*d - b*c)**2*atan(sqrt(d)*x/sqrt(c))/c**(7/
2)
```

Mathematica [A] time = 0.119188, size = 86, normalized size = 0.99

$$-\frac{a^2}{5cx^5}-\frac{\sqrt{d}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}+\frac{a(ad-2bc)}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -a^2/(5*c*x^5) + (a*(-2*b*c + a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(
c^3*x) - (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7
/2)
```

Maple [A] time = 0.01, size = 143, normalized size = 1.6

$$-\frac{a^{2}}{5\,c x^{5}}-\frac{a^{2} d^{2}}{c^{3} x}+2\,\frac{abd}{c^{2} x}-\frac{b^{2}}{cx}+\frac{a^{2} d}{3\,c^{2} x^{3}}-\frac{2\,ab}{3\,c x^{3}}-\frac{a^{2} d^{3}}{c^{3}}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+2\,\frac{ab d^{2}}{c^{2}\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{d b^{2}}{c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^6/(d*x^2+c),x)
```

```
[Out] -1/5*a^2/c/x^5-1/c^3/x*a^2*d^2+2/c^2/x*a*b*d-1/c/x*b^2+1/3*a^2/c^
2/x^3*d-2/3*a/c/x^3*b-d^3/c^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
*a^2+2*d^2/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-d/c/(c*d)^
(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242336, size = 1, normalized size = 0.01

$$\left[\frac{15\,\left(b^{2} c^{2}-2\,abcd+a^{2} d^{2}\right) x^{5}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)-30\,\left(b^{2} c^{2}-2\,abcd+a^{2} d^{2}\right) x^{4}-6\,a^{2} c^{2}-10\,\left(2\,abc^{2}-a^{2} cd\right) x^{2}}{30\,c^{3} x^{5}},\; -\frac{15\,\left(b^{2} c^{2}-2\,abcd+a^{2} d^{2}\right) x^{5}\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)+15\,\left(b^{2} c^{2}-2\,abcd+a^{2} d^{2}\right) x^{4}+3\,a^{2} c^{2}+5\,\left(2\,abc^{2}-a^{2} cd\right) x^{2}}{15\,c^{3} x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^6),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^5*sqrt(-d/c)*log((d*x
^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 30*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*x^4 - 6*a^2*c^2 - 10*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*
x^5), -1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^5*sqrt(d/c)*arc
tan(d*x/(c*sqrt(d/c))) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 +
 3*a^2*c^2 + 5*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5)]
```

Sympy [A] time = 3.91206, size = 207, normalized size = 2.38

$$\frac{\sqrt{-\frac{d}{c^{7}}}\,(ad-bc)^{2}\log\left(-\frac{c^{4}\sqrt{-\frac{d}{c^{7}}}(ad-bc)^{2}}{a^{2}d^{3}-2abcd^{2}+b^{2}c^{2}d}+x\right)}{2}-\frac{\sqrt{-\frac{d}{c^{7}}}\,(ad-bc)^{2}\log\left(\frac{c^{4}\sqrt{-\frac{d}{c^{7}}}(ad-bc)^{2}}{a^{2}d^{3}-2abcd^{2}+b^{2}c^{2}d}+x\right)}{2}-\frac{3a^{2}c^{2}+x^{4}\left(15a^{2}d^{2}-30abcd+15b^{2}c^{2}\right)+x^{2}\left(-5a^{2}cd+10abc^{2}\right)}{15c^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**6/(d*x**2+c),x)
```

```
[Out] sqrt(-d/c**7)*(a*d - b*c)**2*log(-c**4*sqrt(-d/c**7)*(a*d - b*c)*
*2/(a**2*d**3 - 2*a*b*c*d**2 + b**2*c**2*d) + x)/2 - sqrt(-d/c**7
)*(a*d - b*c)**2*log(c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d**3
 - 2*a*b*c*d**2 + b**2*c**2*d) + x)/2 - (3*a**2*c**2 + x**4*(15*a
**2*d**2 - 30*a*b*c*d + 15*b**2*c**2) + x**2*(-5*a**2*c*d + 10*a*
b*c**2))/(15*c**3*x**5)
```

GIAC/XCAS [A] time = 0.221866, size = 151, normalized size = 1.74

$$-\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^3}-\frac{15\,b^2c^2x^4-30\,abcdx^4+15\,a^2d^2x^4+10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^6),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(
c*d)*c^3) - 1/15*(15*b^2*c^2*x^4 - 30*a*b*c*d*x^4 + 15*a^2*d^2*x^
4 + 10*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(c^3*x^5)
```

3.179 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2}{6cx^6}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{a(2bc-ad)}{4c^2x^4}$$

```
[Out] -a^2/(6*c*x^6) - (a*(2*b*c - a*d))/(4*c^2*x^4) - (b*c - a*d)^2/(2
*c^3*x^2) - (d*(b*c - a*d)^2*Log[x])/c^4 + (d*(b*c - a*d)^2*Log[c
 + d*x^2])/(2*c^4)
```

Rubi [A] time = 0.19714, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{6cx^6}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{a(2bc-ad)}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -a^2/(6*c*x^6) - (a*(2*b*c - a*d))/(4*c^2*x^4) - (b*c - a*d)^2/(2
*c^3*x^2) - (d*(b*c - a*d)^2*Log[x])/c^4 + (d*(b*c - a*d)^2*Log[c
 + d*x^2])/(2*c^4)
```

Rubi in Sympy [A] time = 32.1174, size = 88, normalized size = 0.9

$$-\frac{a^2}{6cx^6}+\frac{a\,(ad-2bc)}{4c^2x^4}-\frac{(ad-bc)^2}{2c^3x^2}-\frac{d\,(ad-bc)^2\log\left(x^2\right)}{2c^4}+\frac{d\,(ad-bc)^2\log\left(c+dx^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**7/(d*x**2+c),x)
```

```
[Out] -a**2/(6*c*x**6) + a*(a*d - 2*b*c)/(4*c**2*x**4) - (a*d - b*c)**2
/(2*c**3*x**2) - d*(a*d - b*c)**2*log(x**2)/(2*c**4) + d*(a*d - b
*c)**2*log(c + d*x**2)/(2*c**4)
```

Mathematica [A] time = 0.111767, size = 108, normalized size = 1.1

$$-\frac{c\left(a^2\left(2c^2-3cdx^2+6d^2x^4\right)+6abcx^2\left(c-2dx^2\right)+6b^2c^2x^4\right)+12dx^6\log(x)(bc-ad)^2-6dx^6(bc-ad)^2\log\left(c+dx^2\right)}{12c^4x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -(c*(6*b^2*c^2*x^4 + 6*a*b*c*x^2*(c - 2*d*x^2) + a^2*(2*c^2 - 3*c
*d*x^2 + 6*d^2*x^4)) + 12*d*(b*c - a*d)^2*x^6*Log[x] - 6*d*(b*c -
 a*d)^2*x^6*Log[c + d*x^2])/(12*c^4*x^6)
```

Maple [A] time = 0.012, size = 160, normalized size = 1.6

$$-\frac{a^2}{6\,cx^6}-\frac{a^2d^2}{2\,c^3x^2}+\frac{abd}{c^2x^2}-\frac{b^2}{2\,cx^2}+\frac{a^2d}{4\,c^2x^4}-\frac{ab}{2\,cx^4}-\frac{d^3\ln(x)\,a^2}{c^4}+2\,\frac{d^2\ln(x)\,ab}{c^3}$$
$$-\frac{d\ln(x)\,b^2}{c^2}+\frac{d^3\ln\left(dx^2+c\right)a^2}{2\,c^4}-\frac{d^2\ln\left(dx^2+c\right)ab}{c^3}+\frac{d\ln\left(dx^2+c\right)b^2}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^7/(d*x^2+c),x)
```

```
[Out] -1/6*a^2/c/x^6-1/2/c^3/x^2*a^2*d^2+1/c^2/x^2*a*b*d-1/2/c/x^2*b^2+
1/4*a^2/c^2/x^4*d-1/2*a/c/x^4*b-1/c^4*d^3*ln(x)*a^2+2/c^3*d^2*ln(
x)*a*b-1/c^2*d*ln(x)*b^2+1/2*d^3/c^4*ln(d*x^2+c)*a^2-d^2/c^3*ln(d
*x^2+c)*a*b+1/2*d/c^2*ln(d*x^2+c)*b^2
```

Maxima [A] time = 1.36392, size = 181, normalized size = 1.85

$$\frac{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(dx^2 + c\right)}{2\,c^4} - \frac{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(x^2\right)}{2\,c^4} - \frac{6\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^4 + 2\,a^2c^2 + 3\left(2\,abc^2 - a^2cd\right)x^2}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^7),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/c^4 - 1/2*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 - 1/12*(6*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + 3*(2*a*b*c^2 - a^2*c*
d)*x^2)/(c^3*x^6)
```

Fricas [A] time = 0.237639, size = 184, normalized size = 1.88

$$\frac{6\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^6\log\left(dx^2 + c\right) - 12\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^6\log\left(x\right) - 2\,a^2c^3 - 6\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)x^4}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^7),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(d*x^2 + c) -
12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(x) - 2*a^2*c^3 - 6
*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x^4 - 3*(2*a*b*c^3 - a^2*c^2
*d)*x^2)/(c^4*x^6)
```

Sympy [A] time = 6.42926, size = 105, normalized size = 1.07

$$-\frac{2a^2c^2 + x^4\left(6a^2d^2 - 12abcd + 6b^2c^2\right) + x^2\left(-3a^2cd + 6abc^2\right)}{12c^3x^6} - \frac{d\left(ad - bc\right)^2\log\left(x\right)}{c^4} + \frac{d\left(ad - bc\right)^2\log\left(\frac{c}{d} + x^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**7/(d*x**2+c),x)
```

```
[Out] -(2*a**2*c**2 + x**4*(6*a**2*d**2 - 12*a*b*c*d + 6*b**2*c**2) + x
**2*(-3*a**2*c*d + 6*a*b*c**2))/(12*c**3*x**6) - d*(a*d - b*c)**2
*log(x)/c**4 + d*(a*d - b*c)**2*log(c/d + x**2)/(2*c**4)
```

GIAC/XCAS [A] time = 0.231614, size = 248, normalized size = 2.53

$$-\frac{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\ln\left(x^2\right)}{2\,c^4} + \frac{\left(b^2c^2d^2 - 2\,abcd^3 + a^2d^4\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,c^4d} + \frac{11\,b^2c^2dx^6 - 22\,abcd^2x^6 + 11\,a^2d^3x^6 - 6\,b^2c^3x^4 + 12\,abc^2dx^4 - 6\,a^2cd^2x^4 - 6\,abc^3x^2 + 3\,a^2c^2dx^2 - 2\,a^2c^3}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^7),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*ln(x^2)/c^4 + 1/2*(b^2*c
^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*ln(abs(d*x^2 + c))/(c^4*d) + 1/12
*(11*b^2*c^2*d*x^6 - 22*a*b*c*d^2*x^6 + 11*a^2*d^3*x^6 - 6*b^2*c^
3*x^4 + 12*a*b*c^2*d*x^4 - 6*a^2*c*d^2*x^4 - 6*a*b*c^3*x^2 + 3*a^
2*c^2*d*x^2 - 2*a^2*c^3)/(c^4*x^6)
```

3.180 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{c}(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^5}{5d^2}$$

```
[Out] ((7*b*c - 3*a*d)*(b*c - a*d)*x)/(2*d^4) - ((7*b*c - 3*a*d)*(b*c - a*d)*x^3)/(6*c*d^3) + (b^2*x^5)/(5*d^2) + ((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2)) - (Sqrt[c]*(7*b*c - 3*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9/2))
```

Rubi [A] time = 0.334834, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{\sqrt{c}(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((7*b*c - 3*a*d)*(b*c - a*d)*x)/(2*d^4) - ((7*b*c - 3*a*d)*(b*c - a*d)*x^3)/(6*c*d^3) + (b^2*x^5)/(5*d^2) + ((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2)) - (Sqrt[c]*(7*b*c - 3*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^5}{5d^2}-\frac{\sqrt{c}\,(ad-bc)\,(3ad-7bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{\frac{9}{2}}}+\frac{x^5\,(ad-bc)^2}{2cd^2\,(c+dx^2)}-\frac{x^3\,(ad-bc)\,(3ad-7bc)}{6cd^3}+\frac{(ad-bc)\,(3ad-7bc)\int c\,dx}{2cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**5/(5*d**2) - sqrt(c)*(a*d - b*c)*(3*a*d - 7*b*c)*atan(sqrt(d)*x/sqrt(c))/(2*d**(9/2)) + x**5*(a*d - b*c)**2/(2*c*d**2*(c + d*x**2)) - x**3*(a*d - b*c)*(3*a*d - 7*b*c)/(6*c*d**3) + (a*d - b*c)*(3*a*d - 7*b*c)*Integral(c, x)/(2*c*d**4)
```

Mathematica [A] time = 0.152915, size = 138, normalized size = 0.95

$$-\frac{\sqrt{c}\left(3a^2d^2-10abcd+7b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{x\left(a^2d^2-4abcd+3b^2c^2\right)}{d^4}+\frac{cx(bc-ad)^2}{2d^4\left(c+dx^2\right)}-\frac{2bx^3(bc-ad)}{3d^3}+\frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x)/d^4 - (2*b*(b*c - a*d)*x^3)
/(3*d^3) + (b^2*x^5)/(5*d^2) + (c*(b*c - a*d)^2*x)/(2*d^4*(c + d*
x^2)) - (Sqrt[c]*(7*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(2*d^(9/2))
```

Maple [A] time = 0.014, size = 196, normalized size = 1.4

$$\frac{b^2x^5}{5\,d^2}+\frac{2\,abx^3}{3\,d^2}-\frac{2\,x^3b^2c}{3\,d^3}+\frac{a^2x}{d^2}-4\,\frac{xabc}{d^3}+3\,\frac{b^2c^2x}{d^4}+\frac{a^2cx}{2\,d^2\,(dx^2+c)}-\frac{xabc^2}{d^3\,(dx^2+c)}+\frac{xb^2c^3}{2\,d^4\,(dx^2+c)}-\frac{3\,a^2c}{2\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+5\,\frac{abc^2}{d^3\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{7\,b^2c^3}{2\,d^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/5*b^2*x^5/d^2+2/3/d^2*x^3*a*b-2/3/d^3*x^3*b^2*c+1/d^2*a^2*x-4/d
^3*a*b*c*x+3/d^4*b^2*c^2*x+1/2*c/d^2*x/(d*x^2+c)*a^2-c^2/d^3*x/(d
*x^2+c)*a*b+1/2*c^3/d^4*x/(d*x^2+c)*b^2-3/2*c/d^2/(c*d)^(1/2)*arc
tan(x*d/(c*d)^(1/2))*a^2+5*c^2/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*a*b-7/2*c^3/d^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244531, size = 1, normalized size = 0.01

$$\left[\frac{12\,b^2d^3x^7-4\left(7\,b^2cd^2-10\,abd^3\right)x^5+20\left(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3\right)x^3+15\left(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2+\left(7\,b^2c^2d-1\right.\right.}{60\left(d^5x^2+cd^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^2*d^3*x^7 - 4*(7*b^2*c*d^2 - 10*a*b*d^3)*x^5 + 20*(7*
b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 + 15*(7*b^2*c^3 - 10*a*
b*c^2*d + 3*a^2*c*d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*
x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) +
 30*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*x)/(d^5*x^2 + c*d^4)
, 1/30*(6*b^2*d^3*x^7 - 2*(7*b^2*c*d^2 - 10*a*b*d^3)*x^5 + 10*(7*
b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 15*(7*b^2*c^3 - 10*a*
b*c^2*d + 3*a^2*c*d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*
x^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) + 15*(7*b^2*c^3 - 10*a*b*c^2*d
 + 3*a^2*c*d^2)*x)/(d^5*x^2 + c*d^4)]
```

Sympy [A] time = 4.56937, size = 280, normalized size = 1.93

$$\frac{b^2x^5}{5d^2} + \frac{x\left(a^2cd^2 - 2abc^2d + b^2c^3\right)}{2cd^4 + 2d^5x^2} + \frac{\sqrt{-\frac{c}{d^9}}\,(ad - bc)\,(3ad - 7bc)\log\left(-\frac{d^4\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{c}{d^9}}\,(ad - bc)\,(3ad - 7bc)\log\left(\frac{d^4\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4} + \frac{x^3\left(2abd - 2b^2c\right)}{3d^3} + \frac{x\left(a^2d^2 - 4abcd + 3b^2c^2\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**5/(5*d**2) + x*(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3)/(
2*c*d**4 + 2*d**5*x**2) + sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*
c)*log(-d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d*
*2 - 10*a*b*c*d + 7*b**2*c**2) + x)/4 - sqrt(-c/d**9)*(a*d - b*c)
*(3*a*d - 7*b*c)*log(d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*
c)/(3*a**2*d**2 - 10*a*b*c*d + 7*b**2*c**2) + x)/4 + x**3*(2*a*b*
d - 2*b**2*c)/(3*d**3) + x*(a**2*d**2 - 4*a*b*c*d + 3*b**2*c**2)/
d**4
```

GIAC/XCAS [A] time = 0.220427, size = 211, normalized size = 1.46

$$-\frac{\left(7\,b^2c^3 - 10\,abc^2d + 3\,a^2cd^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^4} + \frac{b^2c^3x - 2\,abc^2dx + a^2cd^2x}{2\,(dx^2 + c)d^4} + \frac{3\,b^2d^8x^5 - 10\,b^2cd^7x^3 + 10\,abd^8x^3 + 45\,b^2c^2d^6x - 60\,abcd^7x + 15\,a^2d^8x}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d
))/(sqrt(c*d)*d^4) + 1/2*(b^2*c^3*x - 2*a*b*c^2*d*x + a^2*c*d^2*x
)/((d*x^2 + c)*d^4) + 1/15*(3*b^2*d^8*x^5 - 10*b^2*c*d^7*x^3 + 10
*a*b*d^8*x^3 + 45*b^2*c^2*d^6*x - 60*a*b*c*d^7*x + 15*a^2*d^8*x)/
d^10
```

3.181 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=90

$$\frac{c(bc-ad)^2}{2d^4(c+dx^2)} + \frac{(bc-ad)(3bc-ad)\log(c+dx^2)}{2d^4} - \frac{bx^2(bc-ad)}{d^3} + \frac{b^2x^4}{4d^2}$$

```
[Out] -((b*(b*c - a*d)*x^2)/d^3) + (b^2*x^4)/(4*d^2) + (c*(b*c - a*d)^2
)/(2*d^4*(c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2]
)/(2*d^4)
```

Rubi [A] time = 0.257606, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{c(bc-ad)^2}{2d^4(c+dx^2)} + \frac{(bc-ad)(3bc-ad)\log(c+dx^2)}{2d^4} - \frac{bx^2(bc-ad)}{d^3} + \frac{b^2x^4}{4d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*(b*c - a*d)*x^2)/d^3) + (b^2*x^4)/(4*d^2) + (c*(b*c - a*d)^2
)/(2*d^4*(c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2]
)/(2*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2 \int^{x^2} x\,dx}{2d^2} + \frac{bx^2(ad-bc)}{d^3} + \frac{c(ad-bc)^2}{2d^4(c+dx^2)} + \frac{(ad-3bc)(ad-bc)\log(c+dx^2)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*Integral(x, (x, x**2))/(2*d**2) + b*x**2*(a*d - b*c)/d**3 +
c*(a*d - b*c)**2/(2*d**4*(c + d*x**2)) + (a*d - 3*b*c)*(a*d - b*c
)*log(c + d*x**2)/(2*d**4)
```

Mathematica [A] time = 0.10544, size = 87, normalized size = 0.97

$$\frac{2(a^2d^2-4abcd+3b^2c^2)\log(c+dx^2) + 4bdx^2(ad-bc) + \frac{2c(bc-ad)^2}{c+dx^2} + b^2d^2x^4}{4d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (4*b*d*(-(b*c) + a*d)*x^2 + b^2*d^2*x^4 + (2*c*(b*c - a*d)^2)/(c
+ d*x^2) + 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(4
*d^4)
```

Maple [A] time = 0.016, size = 142, normalized size = 1.6

$$\frac{b^{2}x^{4}}{4\,d^{2}}+\frac{abx^{2}}{d^{2}}-\frac{b^{2}cx^{2}}{d^{3}}+\frac{a^{2}c}{2\,d^{2}\,(dx^{2}+c)}-\frac{abc^{2}}{d^{3}\,(dx^{2}+c)}+\frac{b^{2}c^{3}}{2\,d^{4}\,(dx^{2}+c)}+\frac{\ln\left(dx^{2}+c\right)a^{2}}{2\,d^{2}}-2\,\frac{\ln\left(dx^{2}+c\right)abc}{d^{3}}+\frac{3\,\ln\left(dx^{2}+c\right)b^{2}c^{2}}{2\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/4*b^2*x^4/d^2+b/d^2*a*x^2-b^2/d^3*x^2*c+1/2/d^2*c/(d*x^2+c)*a^2
-1/d^3*c^2/(d*x^2+c)*a*b+1/2/d^4*c^3/(d*x^2+c)*b^2+1/2/d^2*ln(d*x
^2+c)*a^2-2/d^3*ln(d*x^2+c)*a*b*c+3/2/d^4*ln(d*x^2+c)*b^2*c^2
```

Maxima [A] time = 1.3477, size = 144, normalized size = 1.6

$$\frac{b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}}{2\,(d^{5}x^{2}+cd^{4})}+\frac{b^{2}dx^{4}-4\,\left(b^{2}c-abd\right)x^{2}}{4\,d^{3}}+\frac{\left(3\,b^{2}c^{2}-4\,abcd+a^{2}d^{2}\right)\log\left(dx^{2}+c\right)}{2\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d^5*x^2 + c*d^4) + 1/4*(
b^2*d*x^4 - 4*(b^2*c - a*b*d)*x^2)/d^3 + 1/2*(3*b^2*c^2 - 4*a*b*c
*d + a^2*d^2)*log(d*x^2 + c)/d^4
```

Fricas [A] time = 0.229741, size = 217, normalized size = 2.41

$$\frac{b^{2}d^{3}x^{6}+2\,b^{2}c^{3}-4\,abc^{2}d+2\,a^{2}cd^{2}-\left(3\,b^{2}cd^{2}-4\,abd^{3}\right)x^{4}-4\,\left(b^{2}c^{2}d-abcd^{2}\right)x^{2}+2\,\left(3\,b^{2}c^{3}-4\,abc^{2}d+a^{2}cd^{2}+\left(3\,b^{2}c^{2}d\right.\right.}{4\,(d^{5}x^{2}+cd^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^3*x^6 + 2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 - (3*b^2
*c*d^2 - 4*a*b*d^3)*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*x^2 + 2*(3*b^
2*c^3 - 4*a*b*c^2*d + a^2*c*d^2 + (3*b^2*c^2*d - 4*a*b*c*d^2 + a^
2*d^3)*x^2)*log(d*x^2 + c))/(d^5*x^2 + c*d^4)
```

Sympy [A] time = 4.36309, size = 97, normalized size = 1.08

$$\frac{b^{2}x^{4}}{4d^{2}}+\frac{a^{2}cd^{2}-2abc^{2}d+b^{2}c^{3}}{2cd^{4}+2d^{5}x^{2}}+\frac{x^{2}\left(abd-b^{2}c\right)}{d^{3}}+\frac{(ad-3bc)(ad-bc)\log\left(c+dx^{2}\right)}{2d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**4/(4*d**2) + (a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3)/(2*
c*d**4 + 2*d**5*x**2) + x**2*(a*b*d - b**2*c)/d**3 + (a*d - 3*b*c
)*(a*d - b*c)*log(c + d*x**2)/(2*d**4)
```

GIAC/XCAS [A] time = 0.224205, size = 220, normalized size = 2.44

$$\frac{\frac{(dx^2+c)^2\left(b^2-\frac{2\left(3\,b^2cd-2\,abd^2\right)}{(dx^2+c)d}\right)}{d^3}-\frac{2\left(3\,b^2c^2-4\,abcd+a^2d^2\right)\ln\left(\frac{|dx^2+c|}{(dx^2+c)^2|d|}\right)}{d^3}+\frac{2\left(\frac{b^2c^3d^2}{dx^2+c}-\frac{2\,abc^2d^3}{dx^2+c}+\frac{a^2cd^4}{dx^2+c}\right)}{d^5}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((d*x^2 + c)^2*(b^2 - 2*(3*b^2*c*d - 2*a*b*d^2)/((d*x^2 + c)*
d))/d^3 - 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ln(abs(d*x^2 + c)/(
(d*x^2 + c)^2*abs(d)))/d^3 + 2*(b^2*c^3*d^2/(d*x^2 + c) - 2*a*b*c
^2*d^3/(d*x^2 + c) + a^2*c*d^4/(d*x^2 + c))/d^5)/d
```

3.182 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=118

$$\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{x^3(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^3}{3d^2}$$

```
[Out] -((b*c - a*d)*(5*b*c - a*d)*x)/(2*c*d^3) + (b^2*x^3)/(3*d^2) + ((
b*c - a*d)^2*x^3)/(2*c*d^2*(c + d*x^2)) + ((b*c - a*d)*(5*b*c - a
*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Rubi [A] time = 0.292757, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{x^3(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*(5*b*c - a*d)*x)/(2*c*d^3) + (b^2*x^3)/(3*d^2) + ((
b*c - a*d)^2*x^3)/(2*c*d^2*(c + d*x^2)) + ((b*c - a*d)*(5*b*c - a
*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Rubi in Sympy [A] time = 44.0846, size = 100, normalized size = 0.85

$$\frac{b^2x^3}{3d^2}+\frac{x^3(ad-bc)^2}{2cd^2(c+dx^2)}-\frac{x(ad-5bc)(ad-bc)}{2cd^3}+\frac{(ad-5bc)(ad-bc)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**3/(3*d**2) + x**3*(a*d - b*c)**2/(2*c*d**2*(c + d*x**2))
- x*(a*d - 5*b*c)*(a*d - b*c)/(2*c*d**3) + (a*d - 5*b*c)*(a*d - b
*c)*atan(sqrt(d)*x/sqrt(c))/(2*sqrt(c)*d**(7/2))
```

Mathematica [A] time = 0.120253, size = 105, normalized size = 0.89

$$\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}-\frac{x(bc-ad)^2}{2d^3(c+dx^2)}-\frac{2bx(bc-ad)}{d^3}+\frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-2*b*(b*c - a*d)*x)/d^3 + (b^2*x^3)/(3*d^2) - ((b*c - a*d)^2*x)/
(2*d^3*(c + d*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Maple [A] time = 0.013, size = 156, normalized size = 1.3

$$\frac{b^2x^3}{3\,d^2}+2\,\frac{abx}{d^2}-2\,\frac{xb^2c}{d^3}-\frac{a^2x}{2\,d\,(dx^2+c)}+\frac{xabc}{d^2\,(dx^2+c)}-\frac{b^2c^2x}{2\,d^3\,(dx^2+c)}+\frac{a^2}{2\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-3\,\frac{abc}{d^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\frac{5\,b^2c^2}{2\,d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/3*b^2*x^3/d^2+2/d^2*b*a*x-2/d^3*b^2*x*c-1/2/d*x/(d*x^2+c)*a^2+1
/d^2*x/(d*x^2+c)*c*a*b-1/2/d^3*x/(d*x^2+c)*b^2*c^2+1/2/d/(c*d)^(1
/2)*arctan(x*d/(c*d)^(1/2))*a^2-3/d^2/(c*d)^(1/2)*arctan(x*d/(c*d
)^(1/2))*c*a*b+5/2/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248417, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,b^2c^3-6\,abc^2d+a^2cd^2+\left(5\,b^2c^2d-6\,abcd^2+a^2d^3\right)x^2\right)\log\left(\frac{2\,cdx+(dx^2-c)\sqrt{-cd}}{dx^2+c}\right)+2\left(2\,b^2d^2x^5-2\left(5\,b^2cd-6\,abd^2\right)x^3\right.}{12\,(d^4x^2+cd^3)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + (5*b^2*c^2*d - 6*
a*b*c*d^2 + a^2*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/
(d*x^2 + c)) + 2*(2*b^2*d^2*x^5 - 2*(5*b^2*c*d - 6*a*b*d^2)*x^3 -
 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x)*sqrt(-c*d))/((d^4*x^2 + c
*d^3)*sqrt(-c*d)), 1/6*(3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 +
(5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*arctan(sqrt(c*d)*x/c)
+ (2*b^2*d^2*x^5 - 2*(5*b^2*c*d - 6*a*b*d^2)*x^3 - 3*(5*b^2*c^2 -
 6*a*b*c*d + a^2*d^2)*x)*sqrt(c*d))/((d^4*x^2 + c*d^3)*sqrt(c*d))
]
```

Sympy [A] time = 3.94537, size = 245, normalized size = 2.08

$$\frac{b^2x^3}{3d^2}-\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2cd^3+2d^4x^2}-\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-5bc)\,(ad-bc)\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-5bc)\,(ad-bc)\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{x\left(2abd-2b^2c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**3/(3*d**2) - x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c*d
**3 + 2*d**4*x**2) - sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)*
log(-c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**
2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt(-1/(c*d**7))*(a*d - 5*
b*c)*(a*d - b*c)*log(c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d
- b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + x*(2*a*b*d
- 2*b**2*c)/d**3
```

GIAC/XCAS [A] time = 0.227917, size = 154, normalized size = 1.31

$$\frac{\left(5\,b^2c^2 - 6\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^3} - \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2 + c)d^3} + \frac{b^2d^4x^3 - 6\,b^2cd^3x + 6\,abd^4x}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt
(c*d)*d^3) - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 +
c)*d^3) + 1/3*(b^2*d^4*x^3 - 6*b^2*c*d^3*x + 6*a*b*d^4*x)/d^6
```

3.183 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{(bc-ad)^2}{2d^3(c+dx^2)}-\frac{b(bc-ad)\log(c+dx^2)}{d^3}+\frac{b^2x^2}{2d^2}$$

```
[Out] (b^2*x^2)/(2*d^2) - (b*c - a*d)^2/(2*d^3*(c + d*x^2)) - (b*(b*c -
 a*d)*Log[c + d*x^2])/d^3
```

Rubi [A] time = 0.150019, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{(bc-ad)^2}{2d^3(c+dx^2)}-\frac{b(bc-ad)\log(c+dx^2)}{d^3}+\frac{b^2x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^2)/(2*d^2) - (b*c - a*d)^2/(2*d^3*(c + d*x^2)) - (b*(b*c -
 a*d)*Log[c + d*x^2])/d^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b(ad-bc)\log(c+dx^2)}{d^3}+\frac{\int^{x^2} b^2\,dx}{2d^2}-\frac{(ad-bc)^2}{2d^3(c+dx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b*(a*d - b*c)*log(c + d*x**2)/d**3 + Integral(b**2, (x, x**2))/(2
*d**2) - (a*d - b*c)**2/(2*d**3*(c + d*x**2))
```

Mathematica [A] time = 0.0759851, size = 56, normalized size = 0.9

$$\frac{-\frac{(bc-ad)^2}{c+dx^2}+2b(ad-bc)\log(c+dx^2)+b^2dx^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*d*x^2 - (b*c - a*d)^2/(c + d*x^2) + 2*b*(-(b*c) + a*d)*Log[c
 + d*x^2])/(2*d^3)
```

Maple [A] time = 0.015, size = 97, normalized size = 1.6

$$\frac{b^2x^2}{2\,d^2}-\frac{a^2}{2\,d\,(dx^2+c)}+\frac{abc}{d^2\,(dx^2+c)}-\frac{b^2c^2}{2\,d^3\,(dx^2+c)}+\frac{b\ln(dx^2+c)\,a}{d^2}-\frac{b^2\ln(dx^2+c)\,c}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^2*x^2/d^2-1/2/d/(d*x^2+c)*a^2+1/d^2/(d*x^2+c)*a*b*c-1/2/d^3
/(d*x^2+c)*b^2*c^2+1/d^2*b*ln(d*x^2+c)*a-1/d^3*b^2*ln(d*x^2+c)*c
```

Maxima [A] time = 1.33241, size = 100, normalized size = 1.61

$$\frac{b^2 x^2}{2\,d^2} - \frac{b^2 c^2 - 2\,abcd + a^2 d^2}{2\,(d^4 x^2 + cd^3)} - \frac{\left(b^2 c - abd\right)\log\left(dx^2 + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x^2 +
c*d^3) - (b^2*c - a*b*d)*log(d*x^2 + c)/d^3
```

Fricas [A] time = 0.221388, size = 136, normalized size = 2.19

$$\frac{b^2 d^2 x^4 + b^2 cdx^2 - b^2 c^2 + 2\,abcd - a^2 d^2 - 2\left(b^2 c^2 - abcd + \left(b^2 cd - abd^2\right)x^2\right)\log\left(dx^2 + c\right)}{2\,(d^4 x^2 + cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + b^2*c*d*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 -
2*(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)*log(d*x^2 + c))/(
d^4*x^2 + c*d^3)
```

Sympy [A] time = 3.6142, size = 68, normalized size = 1.1

$$\frac{b^2 x^2}{2d^2} + \frac{b\left(ad - bc\right)\log\left(c + dx^2\right)}{d^3} - \frac{a^2 d^2 - 2abcd + b^2 c^2}{2cd^3 + 2d^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**2/(2*d**2) + b*(a*d - b*c)*log(c + d*x**2)/d**3 - (a**2*d
**2 - 2*a*b*c*d + b**2*c**2)/(2*c*d**3 + 2*d**4*x**2)
```

GIAC/XCAS [A] time = 0.236121, size = 149, normalized size = 2.4

$$\frac{\left(dx^2 + c\right)b^2}{2\,d^3} + \frac{\left(b^2 c - abd\right)\ln\left(\frac{|dx^2+c|}{(dx^2+c)^2|d|}\right)}{d^3} - \frac{\frac{b^2 c^2 d}{dx^2+c} - \frac{2\,abcd^2}{dx^2+c} + \frac{a^2 d^3}{dx^2+c}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + c)*b^2/d^3 + (b^2*c - a*b*d)*ln(abs(d*x^2 + c)/((d*x
^2 + c)^2*abs(d)))/d^3 - 1/2*(b^2*c^2*d/(d*x^2 + c) - 2*a*b*c*d^2
/(d*x^2 + c) + a^2*d^3/(d*x^2 + c))/d^4
```

3.184 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=82

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a
*d)*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2)
)
```

Rubi [A] time = 0.221086, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a
*d)*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2)
)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int b^2\,dx}{d^2}+\frac{x\,(ad-bc)^2}{2cd^2\,(c+dx^2)}+\frac{(ad-bc)\,(ad+3bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Integral(b**2, x)/d**2 + x*(a*d - b*c)**2/(2*c*d**2*(c + d*x**2))
 + (a*d - b*c)*(a*d + 3*b*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(3/2)*
d**(5/2))
```

Mathematica [A] time = 0.0971772, size = 89, normalized size = 1.09

$$-\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c
^2 - 2*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)
*d^(5/2))
```

Maple [A] time = 0., size = 129, normalized size = 1.6

$$\frac{b^2x}{d^2}+\frac{a^2x}{2\,c\,(dx^2+c)}-\frac{abx}{d\,(dx^2+c)}+\frac{xb^2c}{2\,d^2\,(dx^2+c)}+\frac{a^2}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,b^2c}{2\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] b^2*x/d^2+1/2/c*x/(d*x^2+c)*a^2-1/d*x/(d*x^2+c)*a*b+1/2/d^2*c*x/(
d*x^2+c)*b^2+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+1/d/(c
*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-3/2/d^2*c/(c*d)^(1/2)*arcta
n(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239638, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^2c^3-2\,abc^2d-a^2cd^2+\left(3\,b^2c^2d-2\,abcd^2-a^2d^3\right)x^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)-2\left(2\,b^2cdx^3+\left(3\,b^2c^2-2\,abcd+a^2\right.\right.}{4\,(cd^3x^2+c^2d^2)\sqrt{-cd}}\right.$$

$$\left.-\frac{\left(3\,b^2c^3-2\,abc^2d-a^2cd^2+\left(3\,b^2c^2d-2\,abcd^2-a^2d^3\right)x^2\right)\arctan\left(\frac{\sqrt{cd}x}{c}\right)-\left(2\,b^2cdx^3+\left(3\,b^2c^2-2\,abcd+a^2d^2\right)x\right)\sqrt{cd}}{2\,(cd^3x^2+c^2d^2)\sqrt{cd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*
b*c*d^2 - a^2*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d
*x^2 + c)) - 2*(2*b^2*c*d*x^3 + (3*b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*x)*sqrt(-c*d))/((c*d^3*x^2 + c^2*d^2)*sqrt(-c*d)), -1/2*((3*b^2*
c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*
d^3)*x^2)*arctan(sqrt(c*d)*x/c) - (2*b^2*c*d*x^3 + (3*b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x)*sqrt(c*d))/((c*d^3*x^2 + c^2*d^2)*sqrt(c*d
))]
```

Sympy [A] time = 3.41042, size = 236, normalized size = 2.88

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2c^2d^2+2cd^3x^2}-\frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2
+ 2*c*d**3*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)
*log(-c**2*d**2*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a
**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)/4 + sqrt(-1/(c**3*d**5))
*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2*sqrt(-1/(c**3*d**5))*(a*
d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)
/4
```

GIAC/XCAS [A] time = 0.22602, size = 128, normalized size = 1.56

$$\frac{b^2x}{d^2} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2 + c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt
(c*d))/(sqrt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2
*x)/((d*x^2 + c)*c*d^2)
```

3.185 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log(x)}{c^2}+\frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

```
[Out] (b*c - a*d)^2/(2*c*d^2*(c + d*x^2)) + (a^2*Log[x])/c^2 - ((a^2/c^
2 - b^2/d^2)*Log[c + d*x^2])/2
```

Rubi [A] time = 0.154413, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log(x)}{c^2}+\frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*c*d^2*(c + d*x^2)) + (a^2*Log[x])/c^2 - ((a^2/c^
2 - b^2/d^2)*Log[c + d*x^2])/2
```

Rubi in Sympy [A] time = 29.7366, size = 60, normalized size = 0.9

$$\frac{a^2\log\left(x^2\right)}{2c^2}-\left(\frac{a^2}{2c^2}-\frac{b^2}{2d^2}\right)\log\left(c+dx^2\right)+\frac{(ad-bc)^2}{2cd^2\left(c+dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x/(d*x**2+c)**2,x)
```

```
[Out] a**2*log(x**2)/(2*c**2) - (a**2/(2*c**2) - b**2/(2*d**2))*log(c +
 d*x**2) + (a*d - b*c)**2/(2*c*d**2*(c + d*x**2))
```

Mathematica [A] time = 0.0716916, size = 70, normalized size = 1.04

$$\frac{2a^2\log(x)+\frac{(bc-ad)\left(\left(c+dx^2\right)(ad+bc)\log\left(c+dx^2\right)+c(bc-ad)\right)}{d^2\left(c+dx^2\right)}}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (2*a^2*Log[x] + ((b*c - a*d)*(c*(b*c - a*d) + (b*c + a*d)*(c + d*
x^2)*Log[c + d*x^2]))/(d^2*(c + d*x^2)))/(2*c^2)
```

Maple [A] time = 0.021, size = 94, normalized size = 1.4

$$\frac{a^2\ln(x)}{c^2}+\frac{a^2}{2\,c\,(dx^2+c)}-\frac{ab}{d\,(dx^2+c)}+\frac{b^2c}{2\,d^2\,(dx^2+c)}-\frac{\ln\left(dx^2+c\right)a^2}{2\,c^2}+\frac{\ln\left(dx^2+c\right)b^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x/(d*x^2+c)^2,x)
```

```
[Out] a^2*ln(x)/c^2+1/2/c/(d*x^2+c)*a^2-1/d/(d*x^2+c)*a*b+1/2*c/d^2/(d*
x^2+c)*b^2-1/2/c^2*ln(d*x^2+c)*a^2+1/2/d^2*ln(d*x^2+c)*b^2
```

Maxima [A] time = 1.34436, size = 116, normalized size = 1.73

$$\frac{a^2 \log\left(x^2\right)}{2\,c^2} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\left(cd^3x^2 + c^2d^2\right)} + \frac{\left(b^2c^2 - a^2d^2\right)\log\left(dx^2 + c\right)}{2\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(c*d^3
*x^2 + c^2*d^2) + 1/2*(b^2*c^2 - a^2*d^2)*log(d*x^2 + c)/(c^2*d^2
)
```

Fricas [A] time = 0.235894, size = 157, normalized size = 2.34

$$\frac{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^3 - a^2cd^2 + \left(b^2c^2d - a^2d^3\right)x^2\right)\log\left(dx^2 + c\right) + 2\left(a^2d^3x^2 + a^2cd^2\right)\log\left(x\right)}{2\left(c^2d^3x^2 + c^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^3 - a^2*c*d^2 + (
b^2*c^2*d - a^2*d^3)*x^2)*log(d*x^2 + c) + 2*(a^2*d^3*x^2 + a^2*c
*d^2)*log(x))/(c^2*d^3*x^2 + c^3*d^2)
```

Sympy [A] time = 5.23226, size = 80, normalized size = 1.19

$$\frac{a^2\log\left(x\right)}{c^2} + \frac{a^2d^2 - 2abcd + b^2c^2}{2c^2d^2 + 2cd^3x^2} - \frac{\left(ad - bc\right)\left(ad + bc\right)\log\left(\frac{c}{d} + x^2\right)}{2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x/(d*x**2+c)**2,x)
```

```
[Out] a**2*log(x)/c**2 + (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d*
*2 + 2*c*d**3*x**2) - (a*d - b*c)*(a*d + b*c)*log(c/d + x**2)/(2*
c**2*d**2)
```

GIAC/XCAS [A] time = 0.227297, size = 134, normalized size = 2.

$$\frac{a^2\ln\left(x^2\right)}{2\,c^2} + \frac{\left(b^2c^2 - a^2d^2\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,c^2d^2} - \frac{b^2c^2x^2 - a^2d^2x^2 + 2\,abc^2 - 2\,a^2cd}{2\left(dx^2 + c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*a^2*ln(x^2)/c^2 + 1/2*(b^2*c^2 - a^2*d^2)*ln(abs(d*x^2 + c))/
(c^2*d^2) - 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*
d)/((d*x^2 + c)*c^2*d)
```

3.186 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2} \, dx$

Optimal. Leaf size=106

$$-\frac{x\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{a^2}{cx\left(c+dx^2\right)}+\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

```
[Out] -(a^2/(c*x*(c + d*x^2))) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x)/
(2*c^2*d*(c + d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(2*c^(5/2)*d^(3/2))
```

Rubi [A] time = 0.190138, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{x\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{a^2}{cx\left(c+dx^2\right)}+\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2))) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x)/
(2*c^2*d*(c + d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(2*c^(5/2)*d^(3/2))
```

Rubi in Sympy [A] time = 24.3111, size = 90, normalized size = 0.85

$$-\frac{a^2}{cx\left(c+dx^2\right)}-\frac{x\left(ad\left(3ad-2bc\right)+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{\left(ad-bc\right)\left(3ad+bc\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{5}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**2/(d*x**2+c)**2,x)
```

```
[Out] -a**2/(c*x*(c + d*x**2)) - x*(a*d*(3*a*d - 2*b*c) + b**2*c**2)/(2
*c**2*d*(c + d*x**2)) - (a*d - b*c)*(3*a*d + b*c)*atan(sqrt(d)*x/
sqrt(c))/(2*c**(5/2)*d**(3/2))
```

Mathematica [A] time = 0.0984207, size = 91, normalized size = 0.86

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}-\frac{a^2}{c^2x}-\frac{x(bc-ad)^2}{2c^2d\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c^2*x)) - ((b*c - a*d)^2*x)/(2*c^2*d*(c + d*x^2)) + ((b^2*
c^2 + 2*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5
/2)*d^(3/2))
```

Maple [A] time = 0.016, size = 131, normalized size = 1.2

$$-\frac{a^2}{c^2x}-\frac{xa^2d}{2\,c^2\,(dx^2+c)}+\frac{abx}{c\,(dx^2+c)}-\frac{xb^2}{2\,d\,(dx^2+c)}-\frac{3\,a^2d}{2\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2}{2\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^2/(d*x^2+c)^2,x)
```

```
[Out] -a^2/c^2/x-1/2/c^2*d*x/(d*x^2+c)*a^2+1/c*x/(d*x^2+c)*a*b-1/2/d*x/
(d*x^2+c)*b^2-3/2/c^2*d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+1
/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+1/2/d/(c*d)^(1/2)*arct
an(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240258, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(b^2c^2d+2\,abcd^2-3\,a^2d^3\right)x^3+\left(b^2c^3+2\,abc^2d-3\,a^2cd^2\right)x\right)\log\left(-\frac{2\,cdx-(dx^2-c)\sqrt{-cd}}{dx^2+c}\right)+2\left(2\,a^2cd+\left(b^2c^2-2\,abcd+3\right.\right.}{4\,(c^2d^2x^3+c^3dx)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*
a*b*c^2*d - 3*a^2*c*d^2)*x)*log(-(2*c*d*x - (d*x^2 - c)*sqrt(-c*d
))/(d*x^2 + c)) + 2*(2*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2
)*x^2)*sqrt(-c*d))/((c^2*d^2*x^3 + c^3*d*x)*sqrt(-c*d)), 1/2*(((b
^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b*c^2*d
- 3*a^2*c*d^2)*x)*arctan(sqrt(c*d)*x/c) - (2*a^2*c*d + (b^2*c^2 -
 2*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c*d))/((c^2*d^2*x^3 + c^3*d*x)*
sqrt(c*d))]
```

Sympy [A] time = 3.97024, size = 238, normalized size = 2.25

$$\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{2a^2cd+x^2\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^3dx+2c^2d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(-c**3*d*sqrt(-
1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d
 - b**2*c**2) + x)/4 - sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d +
b*c)*log(c**3*d*sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)/(3
*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 - (2*a**2*c*d + x**2*(
3*a**2*d**2 - 2*a*b*c*d + b**2*c**2))/(2*c**3*d*x + 2*c**2*d**2*x
**3)
```

GIAC/XCAS [A] time = 0.224013, size = 138, normalized size = 1.3

$$\frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^2d} - \frac{b^2c^2x^2 - 2\,abcdx^2 + 3\,a^2d^2x^2 + 2\,a^2cd}{2\,(dx^3 + cx)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt
(c*d)*c^2*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 3*a^2*d^2*x^2 +
 2*a^2*c*d)/((d*x^3 + c*x)*c^2*d)
```

3.187 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{a^2}{2c^2x^2}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}$$

```
[Out] -a^2/(2*c^2*x^2) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)) + (2*a*(b*
c - a*d)*Log[x])/c^3 - (a*(b*c - a*d)*Log[c + d*x^2])/c^3
```

Rubi [A] time = 0.199521, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{2c^2x^2}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -a^2/(2*c^2*x^2) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)) + (2*a*(b*
c - a*d)*Log[x])/c^3 - (a*(b*c - a*d)*Log[c + d*x^2])/c^3
```

Rubi in Sympy [A] time = 27.8463, size = 70, normalized size = 0.86

$$-\frac{a^2}{2c^2x^2}-\frac{a\,(ad-bc)\log\left(x^2\right)}{c^3}+\frac{a\,(ad-bc)\log\left(c+dx^2\right)}{c^3}-\frac{(ad-bc)^2}{2c^2d\left(c+dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**3/(d*x**2+c)**2,x)
```

```
[Out] -a**2/(2*c**2*x**2) - a*(a*d - b*c)*log(x**2)/c**3 + a*(a*d - b*c
)*log(c + d*x**2)/c**3 - (a*d - b*c)**2/(2*c**2*d*(c + d*x**2))
```

Mathematica [A] time = 0.163344, size = 72, normalized size = 0.89

$$-\frac{\frac{a^2c}{x^2}+\frac{c(bc-ad)^2}{d(c+dx^2)}-2a(ad-bc)\log\left(c+dx^2\right)+4a\log(x)(ad-bc)}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -((a^2*c)/x^2 + (c*(b*c - a*d)^2)/(d*(c + d*x^2)) + 4*a*(-(b*c) +
 a*d)*Log[x] - 2*a*(-(b*c) + a*d)*Log[c + d*x^2])/(2*c^3)
```

Maple [A] time = 0.02, size = 114, normalized size = 1.4

$$-\frac{a^2}{2\,c^2x^2}-2\,\frac{\ln(x)\,a^2d}{c^3}+2\,\frac{a\ln(x)\,b}{c^2}-\frac{a^2d}{2\,c^2\left(dx^2+c\right)}+\frac{ab}{c\left(dx^2+c\right)}-\frac{b^2}{2\,d\left(dx^2+c\right)}+\frac{a^2\ln\left(dx^2+c\right)d}{c^3}-\frac{a\ln\left(dx^2+c\right)b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^3/(d*x^2+c)^2,x)
```

```
[Out] -1/2*a^2/c^2/x^2-2*a^2/c^3*ln(x)*d+2*a/c^2*ln(x)*b-1/2/c^2/(d*x^2
+c)*a^2*d+1/c/(d*x^2+c)*a*b-1/2/d/(d*x^2+c)*b^2+1/c^3*a^2*ln(d*x^
2+c)*d-1/c^2*a*ln(d*x^2+c)*b
```

Maxima [A] time = 1.32583, size = 135, normalized size = 1.67

$$-\frac{a^2cd + \left(b^2c^2 - 2\,abcd + 2\,a^2d^2\right)x^2}{2\left(c^2d^2x^4 + c^3dx^2\right)} - \frac{\left(abc - a^2d\right)\log\left(dx^2 + c\right)}{c^3} + \frac{\left(abc - a^2d\right)\log\left(x^2\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)/(c^2*d^2*x
^4 + c^3*d*x^2) - (a*b*c - a^2*d)*log(d*x^2 + c)/c^3 + (a*b*c - a
^2*d)*log(x^2)/c^3
```

Fricas [A] time = 0.235014, size = 215, normalized size = 2.65

$$-\frac{a^2c^2d + \left(b^2c^3 - 2\,abc^2d + 2\,a^2cd^2\right)x^2 + 2\left(\left(abcd^2 - a^2d^3\right)x^4 + \left(abc^2d - a^2cd^2\right)x^2\right)\log\left(dx^2 + c\right) - 4\left(\left(abcd^2 - a^2d^3\right)x^4 + \right.}{2\left(c^3d^2x^4 + c^4dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c^2*d + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x^2 + 2*(
(a*b*c*d^2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(d*x^
2 + c) - 4*((a*b*c*d^2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x
^2)*log(x))/(c^3*d^2*x^4 + c^4*d*x^2)
```

Sympy [A] time = 5.86392, size = 92, normalized size = 1.14

$$-\frac{2a\left(ad - bc\right)\log\left(x\right)}{c^3} + \frac{a\left(ad - bc\right)\log\left(\frac{c}{d} + x^2\right)}{c^3} - \frac{a^2cd + x^2\left(2a^2d^2 - 2abcd + b^2c^2\right)}{2c^3dx^2 + 2c^2d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**3/(d*x**2+c)**2,x)
```

```
[Out] -2*a*(a*d - b*c)*log(x)/c**3 + a*(a*d - b*c)*log(c/d + x**2)/c**3
 - (a**2*c*d + x**2*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2))/(2*c**
3*d*x**2 + 2*c**2*d**2*x**4)
```

GIAC/XCAS [A] time = 0.235126, size = 147, normalized size = 1.81

$$\frac{\left(abc - a^2d\right)\ln\left(x^2\right)}{c^3} - \frac{\left(abcd - a^2d^2\right)\ln\left(\left|dx^2 + c\right|\right)}{c^3d} - \frac{b^2c^2x^2 - 2\,abcdx^2 + 2\,a^2d^2x^2 + a^2cd}{2\left(dx^4 + cx^2\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^3),x, algorithm="giac")
```

```
[Out] (a*b*c - a^2*d)*ln(x^2)/c^3 - (a*b*c*d - a^2*d^2)*ln(abs(d*x^2 +
c))/(c^3*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 2*a^2*d^2*x^2 +
a^2*c*d)/((d*x^4 + c*x^2)*c^2*d)
```

3.188 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx$

Optimal. Leaf size=126

$$\frac{x\left(5a^2d^2-6abcd+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}-\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a(6bc-5ad)}{3c^3x}$$

```
[Out] -(a*(6*b*c - 5*a*d))/(3*c^3*x) - a^2/(3*c*x^3*(c + d*x^2)) + ((3*
b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x)/(6*c^3*(c + d*x^2)) + ((b*c -
 5*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[
d])
```

Rubi [A] time = 0.336312, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x\left(5a^2d^2-6abcd+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}-\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a(6bc-5ad)}{3c^3x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -(a*(6*b*c - 5*a*d))/(3*c^3*x) - a^2/(3*c*x^3*(c + d*x^2)) + ((3*
b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x)/(6*c^3*(c + d*x^2)) + ((b*c -
 5*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[
d])
```

Rubi in Sympy [A] time = 42.3322, size = 110, normalized size = 0.87

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{a\left(5ad-6bc\right)}{3c^3x}+\frac{x\left(ad\left(5ad-6bc\right)+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}+\frac{\left(ad-bc\right)\left(5ad-bc\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{7}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**4/(d*x**2+c)**2,x)
```

```
[Out] -a**2/(3*c*x**3*(c + d*x**2)) + a*(5*a*d - 6*b*c)/(3*c**3*x) + x*
(a*d*(5*a*d - 6*b*c) + 3*b**2*c**2)/(6*c**3*(c + d*x**2)) + (a*d
- b*c)*(5*a*d - b*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(7/2)*sqrt(d))
```

Mathematica [A] time = 0.105999, size = 107, normalized size = 0.85

$$\frac{\left(5a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a^2}{3c^2x^3}+\frac{x(bc-ad)^2}{2c^3\left(c+dx^2\right)}+\frac{2a(ad-bc)}{c^3x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -a^2/(3*c^2*x^3) + (2*a*(-(b*c) + a*d))/(c^3*x) + ((b*c - a*d)^2*
x)/(2*c^3*(c + d*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[d])
```

Maple [A] time = 0.016, size = 161, normalized size = 1.3

$$-\frac{a^2}{3\,c^2x^3}+2\,\frac{a^2d}{c^3x}-2\,\frac{ab}{c^2x}+\frac{a^2d^2x}{2\,c^3\,(dx^2+c)}-\frac{xabd}{c^2\,(dx^2+c)}+\frac{xb^2}{2\,c\,(dx^2+c)}+\frac{5\,a^2d^2}{2\,c^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-3\,\frac{abd}{c^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\frac{b^2}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^4/(d*x^2+c)^2,x)
```

```
[Out] -1/3*a^2/c^2/x^3+2*a^2/c^3/x*d-2*a/c^2/x*b+1/2/c^3*x/(d*x^2+c)*a^
2*d^2-1/c^2*x/(d*x^2+c)*a*b*d+1/2/c*x/(d*x^2+c)*b^2+5/2/c^3/(c*d)
^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d^2-3/c^2/(c*d)^(1/2)*arctan(x
*d/(c*d)^(1/2))*a*b*d+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228138, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(b^2c^2d-6\,abcd^2+5\,a^2d^3\right)x^5+\left(b^2c^3-6\,abc^2d+5\,a^2cd^2\right)x^3\right)\log\left(\frac{2\,cdx+(dx^2-c)\sqrt{-cd}}{dx^2+c}\right)+2\left(3\left(b^2c^2-6\,abcd+5\,a^2d^2\right)x^4\cdots\right.}{12\,(c^3dx^5+c^4x^3)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^5 + (b^2*c^3 -
6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*log((2*c*d*x + (d*x^2 - c)*sqrt(-
c*d))/(d*x^2 + c)) + 2*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4 -
 2*a^2*c^2 - 2*(6*a*b*c^2 - 5*a^2*c*d)*x^2)*sqrt(-c*d))/((c^3*d*x
^5 + c^4*x^3)*sqrt(-c*d)), 1/6*(3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a
^2*d^3)*x^5 + (b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*arctan(s
qrt(c*d)*x/c) + (3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4 - 2*a^2*
c^2 - 2*(6*a*b*c^2 - 5*a^2*c*d)*x^2)*sqrt(c*d))/((c^3*d*x^5 + c^4
*x^3)*sqrt(c*d))]
```

Sympy [A] time = 4.63194, size = 248, normalized size = 1.97

$$-\frac{\sqrt{-\frac{1}{c^7d}}\,(ad-bc)\,(5ad-bc)\log\left(-\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^7d}}\,(ad-bc)\,(5ad-bc)\log\left(\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{-2a^2c^2+x^4\left(15a^2d^2-18abcd+3b^2c^2\right)+x^2\left(10a^2cd-12abc^2\right)}{6c^4x^3+6c^3dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**4/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(-c**4*sqrt(-1/(c
**7*d))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2
*c**2) + x)/4 + sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(c
**4*sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*
a*b*c*d + b**2*c**2) + x)/4 + (-2*a**2*c**2 + x**4*(15*a**2*d**2
- 18*a*b*c*d + 3*b**2*c**2) + x**2*(10*a**2*c*d - 12*a*b*c**2))/(
6*c**4*x**3 + 6*c**3*d*x**5)
```

GIAC/XCAS [A] time = 0.226281, size = 150, normalized size = 1.19

$$\frac{\left(b^2c^2-6\,abcd+5\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^3}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(dx^2+c)c^3}-\frac{6\,abcx^2-6\,a^2dx^2+a^2c}{3\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt
(c*d)*c^3) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 +
c)*c^3) - 1/3*(6*a*b*c*x^2 - 6*a^2*d*x^2 + a^2*c)/(c^3*x^3)
```

3.189 $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}-\frac{x(bc-ad)(9bc-ad)}{8d^4\left(c+dx^2\right)}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{b^2x^3}{3d^3}$$

```
[Out] -((13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x)/(4*c*d^4) + (b^2*x^3)/(3
*d^3) + ((b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d
)*(9*b*c - a*d)*x)/(8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*
d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Rubi [A] time = 0.392188, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}-\frac{x(bc-ad)(9bc-ad)}{8d^4\left(c+dx^2\right)}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{b^2x^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x)/(4*c*d^4) + (b^2*x^3)/(3
*d^3) + ((b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d
)*(9*b*c - a*d)*x)/(8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*
d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Rubi in Sympy [A] time = 106.856, size = 150, normalized size = 0.92

$$\frac{b^2x^3}{3d^3}-\frac{x\left(ad-9bc\right)\left(ad-bc\right)}{8d^4\left(c+dx^2\right)}+\frac{x^5\left(ad-bc\right)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}+\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**3/(3*d**3) - x*(a*d - 9*b*c)*(a*d - b*c)/(8*d**4*(c + d*x
**2)) + x**5*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) - x*(a**2*
d**2 - 10*a*b*c*d + 13*b**2*c**2)/(4*c*d**4) + (3*a**2*d**2 - 30*
a*b*c*d + 35*b**2*c**2)*atan(sqrt(d)*x/sqrt(c))/(8*sqrt(c)*d**(9/
2))
```

Mathematica [A] time = 0.153554, size = 148, normalized size = 0.91

$$\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}-\frac{x\left(5a^2d^2-18abcd+13b^2c^2\right)}{8d^4\left(c+dx^2\right)}+\frac{cx(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{bx(3bc-2ad)}{d^4}+\frac{b^2x^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*(3*b*c - 2*a*d)*x)/d^4) + (b^2*x^3)/(3*d^3) + (c*(b*c - a*d)
^2*x)/(4*d^4*(c + d*x^2)^2) - ((13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*x)/(8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^
2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Maple [A] time = 0.016, size = 223, normalized size = 1.4

$$\frac{b^2x^3}{3\,d^3}+2\,\frac{abx}{d^3}-3\,\frac{xb^2c}{d^4}-\frac{5\,x^3a^2}{8\,d\,(dx^2+c)^2}+\frac{9\,abx^3c}{4\,d^2\,(dx^2+c)^2}-\frac{13\,x^3b^2c^2}{8\,d^3\,(dx^2+c)^2}-\frac{3\,a^2cx}{8\,d^2\,(dx^2+c)^2}+\frac{7\,xabc^2}{4\,d^3\,(dx^2+c)^2}-\frac{11\,xb^2c^3}{8\,d^4\,(dx^2+c)^2}+\frac{3\,a^2}{8\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{15\,abc}{4\,d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{35\,b^2c^2}{8\,d^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/3*b^2*x^3/d^3+2/d^3*b*a*x-3/d^4*b^2*x*c-5/8/d/(d*x^2+c)^2*x^3*a
^2+9/4/d^2/(d*x^2+c)^2*x^3*a*b*c-13/8/d^3/(d*x^2+c)^2*x^3*b^2*c^2
-3/8/d^2/(d*x^2+c)^2*x*a^2*c+7/4/d^3/(d*x^2+c)^2*x*a*b*c^2-11/8/d
^4/(d*x^2+c)^2*x*b^2*c^3+3/8/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/
2))*a^2-15/4/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*c*a*b+35/8/d
^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241915, size = 1, normalized size = 0.01

$$\left[\frac{3\left(35\,b^2c^4-30\,abc^3d+3\,a^2c^2d^2+\left(35\,b^2c^2d^2-30\,abcd^3+3\,a^2d^4\right)x^4+2\left(35\,b^2c^3d-30\,abc^2d^2+3\,a^2cd^3\right)x^2\right)\log\left(\frac{2\,cdx+(a}{d}\right.}{48\,(d^6x^4+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*b^2*c^4 - 30*a*b*c^3*d + 3*a^2*c^2*d^2 + (35*b^2*c^2
*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 30*a*b*c
^2*d^2 + 3*a^2*c*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))
/(d*x^2 + c)) + 2*(8*b^2*d^3*x^7 - 8*(7*b^2*c*d^2 - 6*a*b*d^3)*x^
5 - 5*(35*b^2*c^2*d - 30*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 3*(35*b^2*c
^3 - 30*a*b*c^2*d + 3*a^2*c*d^2)*x)*sqrt(-c*d))/((d^6*x^4 + 2*c*d
^5*x^2 + c^2*d^4)*sqrt(-c*d)), 1/24*(3*(35*b^2*c^4 - 30*a*b*c^3*d
```

```
 + 3*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^
4 + 2*(35*b^2*c^3*d - 30*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*arctan(s
qrt(c*d)*x/c) + (8*b^2*d^3*x^7 - 8*(7*b^2*c*d^2 - 6*a*b*d^3)*x^5
- 5*(35*b^2*c^2*d - 30*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 3*(35*b^2*c^3
 - 30*a*b*c^2*d + 3*a^2*c*d^2)*x)*sqrt(c*d))/((d^6*x^4 + 2*c*d^5*
x^2 + c^2*d^4)*sqrt(c*d))]
```

Sympy [A] time = 8.47404, size = 238, normalized size = 1.46

$$\frac{b^2x^3}{3d^3} - \frac{\sqrt{-\frac{1}{cd^9}}\left(3a^2d^2 - 30abcd + 35b^2c^2\right)\log\left(-cd^4\sqrt{-\frac{1}{cd^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{cd^9}}\left(3a^2d^2 - 30abcd + 35b^2c^2\right)\log\left(cd^4\sqrt{-\frac{1}{cd^9}} + x\right)}{16} - \frac{x^3\left(5a^2d^3 - 18abcd^2 + 13b^2c^2d\right) + x\left(3a^2cd^2 - 14abc^2d + 11b^2c^3\right)}{8c^2d^4 + 16cd^5x^2 + 8d^6x^4} + \frac{x\left(2abd - 3b^2c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**3/(3*d**3) - sqrt(-1/(c*d**9))*(3*a**2*d**2 - 30*a*b*c*d
+ 35*b**2*c**2)*log(-c*d**4*sqrt(-1/(c*d**9)) + x)/16 + sqrt(-1/(
c*d**9))*(3*a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(c*d**4*sqr
t(-1/(c*d**9)) + x)/16 - (x**3*(5*a**2*d**3 - 18*a*b*c*d**2 + 13*
b**2*c**2*d) + x*(3*a**2*c*d**2 - 14*a*b*c**2*d + 11*b**2*c**3))/
(8*c**2*d**4 + 16*c*d**5*x**2 + 8*d**6*x**4) + x*(2*a*b*d - 3*b**
2*c)/d**4
```

GIAC/XCAS [A] time = 0.227829, size = 208, normalized size = 1.28

$$\frac{\left(35\,b^2c^2 - 30\,abcd + 3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}d^4} - \frac{13\,b^2c^2dx^3 - 18\,abcd^2x^3 + 5\,a^2d^3x^3 + 11\,b^2c^3x - 14\,abc^2dx + 3\,a^2cd^2x}{8\,(dx^2 + c)^2d^4} + \frac{b^2d^6x^3 - 9\,b^2cd^5x + 6\,abd^6x}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(
sqrt(c*d)*d^4) - 1/8*(13*b^2*c^2*d*x^3 - 18*a*b*c*d^2*x^3 + 5*a^2
*d^3*x^3 + 11*b^2*c^3*x - 14*a*b*c^2*d*x + 3*a^2*c*d^2*x)/((d*x^2
 + c)^2*d^4) + 1/3*(b^2*d^6*x^3 - 9*b^2*c*d^5*x + 6*a*b*d^6*x)/d^
9
```

3.190 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=99

$$\frac{c(bc-ad)^2}{4d^4(c+dx^2)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4(c+dx^2)}-\frac{b(3bc-2ad)\log(c+dx^2)}{2d^4}+\frac{b^2x^2}{2d^3}$$

```
[Out] (b^2*x^2)/(2*d^3) + (c*(b*c - a*d)^2)/(4*d^4*(c + d*x^2)^2) - ((b
*c - a*d)*(3*b*c - a*d))/(2*d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)
*Log[c + d*x^2])/(2*d^4)
```

Rubi [A] time = 0.263154, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{c(bc-ad)^2}{4d^4(c+dx^2)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4(c+dx^2)}-\frac{b(3bc-2ad)\log(c+dx^2)}{2d^4}+\frac{b^2x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x^2)/(2*d^3) + (c*(b*c - a*d)^2)/(4*d^4*(c + d*x^2)^2) - ((b
*c - a*d)*(3*b*c - a*d))/(2*d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)
*Log[c + d*x^2])/(2*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b(2ad-3bc)\log(c+dx^2)}{2d^4}+\frac{c(ad-bc)^2}{4d^4(c+dx^2)^2}+\frac{\int^{x^2} b^2\,dx}{2d^3}-\frac{(ad-3bc)(ad-bc)}{2d^4(c+dx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b*(2*a*d - 3*b*c)*log(c + d*x**2)/(2*d**4) + c*(a*d - b*c)**2/(4*
d**4*(c + d*x**2)**2) + Integral(b**2, (x, x**2))/(2*d**3) - (a*d
 - 3*b*c)*(a*d - b*c)/(2*d**4*(c + d*x**2))
```

Mathematica [A] time = 0.0927432, size = 114, normalized size = 1.15

$$\frac{-a^2d^2(c+2dx^2)+2abcd(3c+4dx^2)-2b(c+dx^2)^2(3bc-2ad)\log(c+dx^2)+b^2(-5c^3-4c^2dx^2+4cd^2x^4+2d^3x^6)}{4d^4(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2*(c + 2*d*x^2)) + 2*a*b*c*d*(3*c + 4*d*x^2) + b^2*(-5*c
^3 - 4*c^2*d*x^2 + 4*c*d^2*x^4 + 2*d^3*x^6) - 2*b*(3*b*c - 2*a*d)
*(c + d*x^2)^2*Log[c + d*x^2])/(4*d^4*(c + d*x^2)^2)
```

Maple [A] time = 0.016, size = 155, normalized size = 1.6

$$\frac{b^2x^2}{2\,d^3}-\frac{a^2}{2\,d^2\,(dx^2+c)}+2\,\frac{abc}{d^3\,(dx^2+c)}-\frac{3\,b^2c^2}{2\,d^4\,(dx^2+c)}+\frac{a^2c}{4\,d^2\,(dx^2+c)^2}-\frac{abc^2}{2\,d^3\,(dx^2+c)^2}+\frac{b^2c^3}{4\,d^4\,(dx^2+c)^2}+\frac{b\ln\left(dx^2+c\right)a}{d^3}-\frac{3\,b^2\ln\left(dx^2+c\right)c}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2*x^2/d^3-1/2/d^2/(d*x^2+c)*a^2+2/d^3/(d*x^2+c)*c*a*b-3/2/d
^4/(d*x^2+c)*b^2*c^2+1/4/d^2*c/(d*x^2+c)^2*a^2-1/2/d^3*c^2/(d*x^2
+c)^2*a*b+1/4/d^4*c^3/(d*x^2+c)^2*b^2+1/d^3*b*ln(d*x^2+c)*a-3/2/d
^4*b^2*ln(d*x^2+c)*c
```

Maxima [A] time = 1.39111, size = 162, normalized size = 1.64

$$\frac{b^2x^2}{2\,d^3}-\frac{5\,b^2c^3-6\,abc^2d+a^2cd^2+2\left(3\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^2}{4\,(d^6x^4+2\,cd^5x^2+c^2d^4)}-\frac{\left(3\,b^2c-2\,abd\right)\log\left(dx^2+c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/4*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3
*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2)/(d^6*x^4 + 2*c*d^5*x^2 +
 c^2*d^4) - 1/2*(3*b^2*c - 2*a*b*d)*log(d*x^2 + c)/d^4
```

Fricas [A] time = 0.236008, size = 240, normalized size = 2.42

$$\frac{2\,b^2d^3x^6+4\,b^2cd^2x^4-5\,b^2c^3+6\,abc^2d-a^2cd^2-2\left(2\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^2-2\left(3\,b^2c^3-2\,abc^2d+\left(3\,b^2cd^2-2\,abd^3\right.\right.}{4\,(d^6x^4+2\,cd^5x^2+c^2d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^3*x^6 + 4*b^2*c*d^2*x^4 - 5*b^2*c^3 + 6*a*b*c^2*d -
a^2*c*d^2 - 2*(2*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2 - 2*(3*b^
2*c^3 - 2*a*b*c^2*d + (3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 2*(3*b^2*c^
2*d - 2*a*b*c*d^2)*x^2)*log(d*x^2 + c))/(d^6*x^4 + 2*c*d^5*x^2 +
c^2*d^4)
```

Sympy [A] time = 9.53764, size = 122, normalized size = 1.23

$$\frac{b^2x^2}{2d^3}+\frac{b\,(2ad-3bc)\log\left(c+dx^2\right)}{2d^4}-\frac{a^2cd^2-6abc^2d+5b^2c^3+x^2\left(2a^2d^3-8abcd^2+6b^2c^2d\right)}{4c^2d^4+8cd^5x^2+4d^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**2/(2*d**3) + b*(2*a*d - 3*b*c)*log(c + d*x**2)/(2*d**4) -
 (a**2*c*d**2 - 6*a*b*c**2*d + 5*b**2*c**3 + x**2*(2*a**2*d**3 -
8*a*b*c*d**2 + 6*b**2*c**2*d))/(4*c**2*d**4 + 8*c*d**5*x**2 + 4*d
```

```
**6*x**4)
```

GIAC/XCAS [A] time = 0.235082, size = 144, normalized size = 1.45

$$\frac{b^2x^2}{2\,d^3} - \frac{\left(3\,b^2c - 2\,abd\right)\ln\left(\left|dx^2 + c\right|\right)}{2\,d^4} - \frac{5\,b^2c^3 - 6\,abc^2d + a^2cd^2 + 2\left(3\,b^2c^2d - 4\,abcd^2 + a^2d^3\right)x^2}{4\left(dx^2 + c\right)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/2*(3*b^2*c - 2*a*b*d)*ln(abs(d*x^2 + c))/d^4
- 1/4*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d - 4*a
*b*c*d^2 + a^2*d^3)*x^2)/((d*x^2 + c)^2*d^4)
```

3.191 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(-a^2d^2-6abcd+15b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x(bc-ad)(ad+7bc)}{8cd^3(c+dx^2)}+\frac{x^3(bc-ad)^2}{4cd^2(c+dx^2)^2}+\frac{b^2x}{d^3}$$

```
[Out] (b^2*x)/d^3 + ((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) + ((b*c
 - a*d)*(7*b*c + a*d)*x)/(8*c*d^3*(c + d*x^2)) - ((15*b^2*c^2 - 6
*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/
2))
```

Rubi [A] time = 0.32306, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{(-a^2d^2-6abcd+15b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x(bc-ad)(ad+7bc)}{8cd^3(c+dx^2)}+\frac{x^3(bc-ad)^2}{4cd^2(c+dx^2)^2}+\frac{b^2x}{d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x)/d^3 + ((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) + ((b*c
 - a*d)*(7*b*c + a*d)*x)/(8*c*d^3*(c + d*x^2)) - ((15*b^2*c^2 - 6
*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/
2))
```

Rubi in Sympy [A] time = 70.5266, size = 114, normalized size = 0.9

$$\frac{b^2x}{d^3}+\frac{x^3(ad-bc)^2}{4cd^2(c+dx^2)^2}-\frac{x(ad-bc)(ad+7bc)}{8cd^3(c+dx^2)}+\frac{(a^2d^2+6abcd-15b^2c^2)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x/d**3 + x**3*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) - x*
(a*d - b*c)*(a*d + 7*b*c)/(8*c*d**3*(c + d*x**2)) + (a**2*d**2 +
6*a*b*c*d - 15*b**2*c**2)*atan(sqrt(d)*x/sqrt(c))/(8*c**(3/2)*d**
(7/2))
```

Mathematica [A] time = 0.178515, size = 130, normalized size = 1.02

$$\frac{x\left(a^2d^2(dx^2-c)-2abcd(3c+5dx^2)+b^2c(15c^2+25cdx^2+8d^2x^4)\right)}{8cd^3(c+dx^2)^2}-\frac{(-a^2d^2-6abcd+15b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(3*c + 5*d*x^2) + b^2*c*(15*
c^2 + 25*c*d*x^2 + 8*d^2*x^4)))/(8*c*d^3*(c + d*x^2)^2) - ((15*b^
```

```
2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3
/2)*d^(7/2))
```

Maple [A] time = 0.015, size = 196, normalized size = 1.5

$$\frac{b^2x}{d^3}+\frac{x^3a^2}{8\,(dx^2+c)^2\,c}-\frac{5\,abx^3}{4\,d\,(dx^2+c)^2}+\frac{9\,x^3b^2c}{8\,d^2\,(dx^2+c)^2}-\frac{a^2x}{8\,d\,(dx^2+c)^2}-\frac{3\,xabc}{4\,d^2\,(dx^2+c)^2}+\frac{7\,b^2c^2x}{8\,d^3\,(dx^2+c)^2}+\frac{a^2}{8\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,ab}{4\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{15\,b^2c}{8\,d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] b^2*x/d^3+1/8/(d*x^2+c)^2/c*x^3*a^2-5/4/d/(d*x^2+c)^2*x^3*a*b+9/8
/d^2/(d*x^2+c)^2*x^3*b^2*c-1/8/d/(d*x^2+c)^2*a^2*x-3/4/d^2/(d*x^2
+c)^2*a*b*c*x+7/8/d^3/(d*x^2+c)^2*b^2*c^2*x+1/8/d/c/(c*d)^(1/2)*a
rctan(x*d/(c*d)^(1/2))*a^2+3/4/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*a*b-15/8/d^3*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240286, size = 1, normalized size = 0.01

$$\left[-\frac{\left(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+\left(15\,b^2c^2d^2-6\,abcd^3-a^2d^4\right)x^4+2\left(15\,b^2c^3d-6\,abc^2d^2-a^2cd^3\right)x^2\right)\log\left(\frac{2\,cdx+(dx^2-c)\sqrt{-}}{dx^2+c}\right.}{16\,(cd^5x^4+2\,c^2d^4x^2+c^3d^3)\sqrt{-}}\right.$$

$$-\frac{\left(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+\left(15\,b^2c^2d^2-6\,abcd^3-a^2d^4\right)x^4+2\left(15\,b^2c^3d-6\,abc^2d^2-a^2cd^3\right)x^2\right)\arctan\left(\frac{\sqrt{cd}x}{c}\right)-(8\,b}{8\,(cd^5x^4+2\,c^2d^4x^2+c^3d^3)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((15*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2
 - 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 -
 a^2*c*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 +
c)) - 2*(8*b^2*c*d^2*x^5 + (25*b^2*c^2*d - 10*a*b*c*d^2 + a^2*d^3
)*x^3 + (15*b^2*c^3 - 6*a*b*c^2*d - a^2*c*d^2)*x)*sqrt(-c*d))/((c
*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*sqrt(-c*d)), -1/8*((15*b^2*c^
4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*d^3 - a
^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*a
rctan(sqrt(c*d)*x/c) - (8*b^2*c*d^2*x^5 + (25*b^2*c^2*d - 10*a*b*
c*d^2 + a^2*d^3)*x^3 + (15*b^2*c^3 - 6*a*b*c^2*d - a^2*c*d^2)*x)*
sqrt(c*d))/((c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*sqrt(c*d))]
```

Sympy [A] time = 6.68556, size = 223, normalized size = 1.76

$$\frac{b^2x}{d^3} - \frac{\sqrt{-\frac{1}{c^3d^7}}\left(a^2d^2+6abcd-15b^2c^2\right)\log\left(-c^2d^3\sqrt{-\frac{1}{c^3d^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^3d^7}}\left(a^2d^2+6abcd-15b^2c^2\right)\log\left(c^2d^3\sqrt{-\frac{1}{c^3d^7}}+x\right)}{16} + \frac{x^3\left(a^2d^3-10abcd^2+9b^2c^2d\right)+x\left(-a^2cd^2-6abc^2d+7b^2c^3\right)}{8c^3d^3+16c^2d^4x^2+8cd^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x/d**3 - sqrt(-1/(c**3*d**7))*(a**2*d**2 + 6*a*b*c*d - 15*b*
*2*c**2)*log(-c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 + sqrt(-1/(c
**3*d**7))*(a**2*d**2 + 6*a*b*c*d - 15*b**2*c**2)*log(c**2*d**3*s
qrt(-1/(c**3*d**7)) + x)/16 + (x**3*(a**2*d**3 - 10*a*b*c*d**2 +
9*b**2*c**2*d) + x*(-a**2*c*d**2 - 6*a*b*c**2*d + 7*b**2*c**3))/(
8*c**3*d**3 + 16*c**2*d**4*x**2 + 8*c*d**5*x**4)
```

GIAC/XCAS [A] time = 0.229788, size = 180, normalized size = 1.42

$$\frac{b^2x}{d^3} - \frac{\left(15\,b^2c^2-6\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}cd^3} + \frac{9\,b^2c^2dx^3-10\,abcd^2x^3+a^2d^3x^3+7\,b^2c^3x-6\,abc^2dx-a^2cd^2x}{8\,(dx^2+c)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] b^2*x/d^3 - 1/8*(15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqr
t(c*d))/(sqrt(c*d)*c*d^3) + 1/8*(9*b^2*c^2*d*x^3 - 10*a*b*c*d^2*x
^3 + a^2*d^3*x^3 + 7*b^2*c^3*x - 6*a*b*c^2*d*x - a^2*c*d^2*x)/((d
*x^2 + c)^2*c*d^3)
```

3.192 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=67

$$\frac{b(bc-ad)}{d^3(c+dx^2)} - \frac{(bc-ad)^2}{4d^3(c+dx^2)^2} + \frac{b^2\log(c+dx^2)}{2d^3}$$

```
[Out] -(b*c - a*d)^2/(4*d^3*(c + d*x^2)^2) + (b*(b*c - a*d))/(d^3*(c +
d*x^2)) + (b^2*Log[c + d*x^2])/(2*d^3)
```

Rubi [A] time = 0.151324, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{b(bc-ad)}{d^3(c+dx^2)} - \frac{(bc-ad)^2}{4d^3(c+dx^2)^2} + \frac{b^2\log(c+dx^2)}{2d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -(b*c - a*d)^2/(4*d^3*(c + d*x^2)^2) + (b*(b*c - a*d))/(d^3*(c +
d*x^2)) + (b^2*Log[c + d*x^2])/(2*d^3)
```

Rubi in Sympy [A] time = 25.5464, size = 56, normalized size = 0.84

$$\frac{b^2\log(c+dx^2)}{2d^3} - \frac{b(ad-bc)}{d^3(c+dx^2)} - \frac{(ad-bc)^2}{4d^3(c+dx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(c + d*x**2)/(2*d**3) - b*(a*d - b*c)/(d**3*(c + d*x**2))
 - (a*d - b*c)**2/(4*d**3*(c + d*x**2)**2)
```

Mathematica [A] time = 0.0503628, size = 75, normalized size = 1.12

$$\frac{-a^2d^2 - 2abd(c+2dx^2) + b^2c(3c+4dx^2) + 2b^2(c+dx^2)^2\log(c+dx^2)}{4d^3(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(c + 2*d*x^2) + b^2*c*(3*c + 4*d*x^2) + 2*b
^2*(c + d*x^2)^2*Log[c + d*x^2])/(4*d^3*(c + d*x^2)^2)
```

Maple [A] time = 0.014, size = 105, normalized size = 1.6

$$-\frac{ab}{d^2(dx^2+c)} + \frac{b^2c}{d^3(dx^2+c)} - \frac{a^2}{4d(dx^2+c)^2} + \frac{abc}{2d^2(dx^2+c)^2} - \frac{b^2c^2}{4d^3(dx^2+c)^2} + \frac{b^2\ln(dx^2+c)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -b/d^2/(d*x^2+c)*a+b^2/d^3/(d*x^2+c)*c-1/4/d/(d*x^2+c)^2*a^2+1/2/
d^2/(d*x^2+c)^2*c*a*b-1/4/d^3/(d*x^2+c)^2*b^2*c^2+1/2*b^2*ln(d*x^
2+c)/d^3
```

Maxima [A] time = 1.33405, size = 117, normalized size = 1.75

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\,\left(b^2cd - abd^2\right)x^2}{4\,(d^5x^4 + 2\,cd^4x^2 + c^2d^3)} + \frac{b^2\log\left(dx^2 + c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2)
/(d^5*x^4 + 2*c*d^4*x^2 + c^2*d^3) + 1/2*b^2*log(d*x^2 + c)/d^3
```

Fricas [A] time = 0.233684, size = 146, normalized size = 2.18

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\,\left(b^2cd - abd^2\right)x^2 + 2\,\left(b^2d^2x^4 + 2\,b^2cdx^2 + b^2c^2\right)\log\left(dx^2 + c\right)}{4\,(d^5x^4 + 2\,cd^4x^2 + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2
+ 2*(b^2*d^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(d^5*
x^4 + 2*c*d^4*x^2 + c^2*d^3)
```

Sympy [A] time = 5.96983, size = 87, normalized size = 1.3

$$\frac{b^2\log\left(c + dx^2\right)}{2d^3} - \frac{a^2d^2 + 2abcd - 3b^2c^2 + x^2\left(4abd^2 - 4b^2cd\right)}{4c^2d^3 + 8cd^4x^2 + 4d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(c + d*x**2)/(2*d**3) - (a**2*d**2 + 2*a*b*c*d - 3*b**2*c
**2 + x**2*(4*a*b*d**2 - 4*b**2*c*d))/(4*c**2*d**3 + 8*c*d**4*x**
2 + 4*d**5*x**4)
```

GIAC/XCAS [A] time = 0.231473, size = 103, normalized size = 1.54

$$\frac{b^2\ln\left(\left|dx^2 + c\right|\right)}{2\,d^3} + \frac{4\,\left(b^2c - abd\right)x^2 + \frac{3\,b^2c^2 - 2\,abcd - a^2d^2}{d}}{4\,(dx^2 + c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*ln(abs(d*x^2 + c))/d^3 + 1/4*(4*(b^2*c - a*b*d)*x^2 + (3*
b^2*c^2 - 2*a*b*c*d - a^2*d^2)/d)/((d*x^2 + c)^2*d^2)
```

3.193 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)\left(bc-ad\right)}{4cd\left(c+dx^2\right)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2
- b^2/d^2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.18291, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)\left(bc-ad\right)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2
- b^2/d^2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rubi in Sympy [A] time = 24.1516, size = 105, normalized size = 0.91

$$\frac{x\left(\frac{3a^2}{8c^2}-\frac{3b^2}{8d^2}\right)}{c+dx^2}+\frac{x\left(a+bx^2\right)\left(ad-bc\right)}{4cd\left(c+dx^2\right)^2}+\frac{\left(ad\left(3ad+bc\right)+bc\left(ad+3bc\right)\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] x*(3*a**2/(8*c**2) - 3*b**2/(8*d**2))/(c + d*x**2) + x*(a + b*x**
2)*(a*d - b*c)/(4*c*d*(c + d*x**2)**2) + (a*d*(3*a*d + b*c) + b*c
*(a*d + 3*b*c))*atan(sqrt(d)*x/sqrt(c))/(8*c**(5/2)*d**(5/2))
```

Mathematica [A] time = 0.175364, size = 121, normalized size = 1.04

$$\frac{x\left(a^2d^2\left(5c+3dx^2\right)-2abcd\left(c-dx^2\right)-b^2c^2\left(3c+5dx^2\right)\right)}{8c^2d^2\left(c+dx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3
*c + 5*d*x^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c
*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A] time = 0.001, size = 147, normalized size = 1.3

$$\frac{1}{\left(dx^2+c\right)^2}\left(\frac{\left(3\,a^2d^2+2\,cabd-5\,b^2c^2\right)x^3}{8\,c^2d}+\frac{\left(5\,a^2d^2-2\,cabd-3\,b^2c^2\right)x}{8\,d^2c}\right)+\frac{3\,a^2}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{4\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,b^2}{8\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a
*b*c*d-3*b^2*c^2)/d^2/c*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(1/2)*arctan
(x*d/(c*d)^(1/2))*a^2+1/4/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
*a*b+3/8/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244869, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,b^2c^4+2\,abc^3d+3\,a^2c^2d^2+\left(3\,b^2c^2d^2+2\,abcd^3+3\,a^2d^4\right)x^4+2\left(3\,b^2c^3d+2\,abc^2d^2+3\,a^2cd^3\right)x^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)}{16\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2
+ 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 +
 3*a^2*c*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2
+ c)) - 2*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c
^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x)*sqrt(-c*d))/((c^2*d^4*x^4 + 2*
c^3*d^3*x^2 + c^4*d^2)*sqrt(-c*d)), 1/8*((3*b^2*c^4 + 2*a*b*c^3*d
 + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
+ 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*arctan(sqrt(
c*d)*x/c) - ((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2
*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x)*sqrt(c*d))/((c^2*d^4*x^4 + 2
*c^3*d^3*x^2 + c^4*d^2)*sqrt(c*d))]
```

Sympy [A] time = 4.59379, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{x^3\left(3a^2d^3+2abcd^2-5b^2c^2d\right)+x\left(5a^2cd^2-2abc^2d-3b^2c^3\right)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log
(-c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(
3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**
5*d**5)) + x)/16 + (x**3*(3*a**2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**
2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d - 3*b**2*c**3))/(8*c**4*d*
*2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

GIAC/XCAS [A] time = 0.225045, size = 170, normalized size = 1.47

$$\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2}-\frac{5\,b^2c^2dx^3-2\,abcd^2x^3-3\,a^2d^3x^3+3\,b^2c^3x+2\,abc^2dx-5\,a^2cd^2x}{8\,(dx^2+c)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c^2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2
*d^3*x^3 + 3*b^2*c^3*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 +
 c)^2*c^2*d^2)
```

3.194 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^3} dx$

Optimal. Leaf size=86

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2(c+dx^2)}-\frac{a^2\log(c+dx^2)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

```
[Out] (b*c - a*d)^2/(4*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(2*(c
 + d*x^2)) + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*c^3)
```

Rubi [A] time = 0.191577, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2(c+dx^2)}-\frac{a^2\log(c+dx^2)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b*c - a*d)^2/(4*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(2*(c
 + d*x^2)) + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*c^3)
```

Rubi in Sympy [A] time = 34.2807, size = 76, normalized size = 0.88

$$\frac{a^2\log(x^2)}{2c^3}-\frac{a^2\log(c+dx^2)}{2c^3}+\frac{\frac{a^2}{2c^2}-\frac{b^2}{2d^2}}{c+dx^2}+\frac{(ad-bc)^2}{4cd^2(c+dx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x/(d*x**2+c)**3,x)
```

```
[Out] a**2*log(x**2)/(2*c**3) - a**2*log(c + d*x**2)/(2*c**3) + (a**2/(
2*c**2) - b**2/(2*d**2))/(c + d*x**2) + (a*d - b*c)**2/(4*c*d**2*
(c + d*x**2)**2)
```

Mathematica [A] time = 0.0742303, size = 103, normalized size = 1.2

$$\frac{a^2d^2-b^2c^2}{2c^2d^2(c+dx^2)}+\frac{a^2d^2-2abcd+b^2c^2}{4cd^2(c+dx^2)^2}-\frac{a^2\log(c+dx^2)}{2c^3}+\frac{a^2\log(x)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(4*c*d^2*(c + d*x^2)^2) + (-(b^2*
c^2) + a^2*d^2)/(2*c^2*d^2*(c + d*x^2)) + (a^2*Log[x])/c^3 - (a^2
*Log[c + d*x^2])/(2*c^3)
```

Maple [A] time = 0.019, size = 112, normalized size = 1.3

$$\frac{a^2\ln(x)}{c^3}+\frac{a^2}{2c^2(dx^2+c)}-\frac{b^2}{2d^2(dx^2+c)}+\frac{a^2}{4c(dx^2+c)^2}-\frac{ab}{2d(dx^2+c)^2}+\frac{b^2c}{4d^2(dx^2+c)^2}-\frac{a^2\ln(dx^2+c)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x/(d*x^2+c)^3,x)
```

```
[Out] a^2*ln(x)/c^3+1/2/c^2/(d*x^2+c)*a^2-1/2/d^2/(d*x^2+c)*b^2+1/4/c/(
d*x^2+c)^2*a^2-1/2/d/(d*x^2+c)^2*a*b+1/4*c/d^2/(d*x^2+c)^2*b^2-1/
2*a^2*ln(d*x^2+c)/c^3
```

Maxima [A] time = 1.35135, size = 147, normalized size = 1.71

$$-\frac{b^2c^3 + 2\,abc^2d - 3\,a^2cd^2 + 2\left(b^2c^2d - a^2d^3\right)x^2}{4\left(c^2d^4x^4 + 2\,c^3d^3x^2 + c^4d^2\right)} - \frac{a^2\log\left(dx^2 + c\right)}{2\,c^3} + \frac{a^2\log\left(x^2\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x),x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2 + 2*(b^2*c^2*d - a^2*d^
3)*x^2)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) - 1/2*a^2*log(d*x
^2 + c)/c^3 + 1/2*a^2*log(x^2)/c^3
```

Fricas [A] time = 0.230296, size = 220, normalized size = 2.56

$$-\frac{b^2c^4 + 2\,abc^3d - 3\,a^2c^2d^2 + 2\left(b^2c^3d - a^2cd^3\right)x^2 + 2\left(a^2d^4x^4 + 2\,a^2cd^3x^2 + a^2c^2d^2\right)\log\left(dx^2 + c\right) - 4\left(a^2d^4x^4 + 2\,a^2cd^3x^2\right.}{4\left(c^3d^4x^4 + 2\,c^4d^3x^2 + c^5d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^4 + 2*a*b*c^3*d - 3*a^2*c^2*d^2 + 2*(b^2*c^3*d - a^2*
c*d^3)*x^2 + 2*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(
d*x^2 + c) - 4*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(
x))/(c^3*d^4*x^4 + 2*c^4*d^3*x^2 + c^5*d^2)
```

Sympy [A] time = 5.56854, size = 107, normalized size = 1.24

$$\frac{a^2\log(x)}{c^3} - \frac{a^2\log\left(\frac{c}{d} + x^2\right)}{2c^3} + \frac{3a^2cd^2 - 2abc^2d - b^2c^3 + x^2\left(2a^2d^3 - 2b^2c^2d\right)}{4c^4d^2 + 8c^3d^3x^2 + 4c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x/(d*x**2+c)**3,x)
```

```
[Out] a**2*log(x)/c**3 - a**2*log(c/d + x**2)/(2*c**3) + (3*a**2*c*d**2
 - 2*a*b*c**2*d - b**2*c**3 + x**2*(2*a**2*d**3 - 2*b**2*c**2*d))
/(4*c**4*d**2 + 8*c**3*d**3*x**2 + 4*c**2*d**4*x**4)
```

GIAC/XCAS [A] time = 0.228911, size = 149, normalized size = 1.73

$$\frac{a^2\ln\left(x^2\right)}{2\,c^3} - \frac{a^2\ln\left(\left|dx^2 + c\right|\right)}{2\,c^3} + \frac{3\,a^2d^4x^4 - 2\,b^2c^3dx^2 + 8\,a^2cd^3x^2 - b^2c^4 - 2\,abc^3d + 6\,a^2c^2d^2}{4\left(dx^2 + c\right)^2c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x),x, algorithm="giac")
```

```
[Out] 1/2*a^2*ln(x^2)/c^3 - 1/2*a^2*ln(abs(d*x^2 + c))/c^3 + 1/4*(3*a^2
*d^4*x^4 - 2*b^2*c^3*d*x^2 + 8*a^2*c*d^3*x^2 - b^2*c^4 - 2*a*b*c^
3*d + 6*a^2*c^2*d^2)/((d*x^2 + c)^2*c^3*d^2)
```

3.195 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{x\left(5a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(3ad(2bc-5ad)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(3ad(2bc-5ad)+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}$$

```
[Out] -(a^2/(c*x*(c + d*x^2)^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x
)/(4*c^2*d*(c + d*x^2)^2) + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*x)
/(8*c^3*d*(c + d*x^2)) + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d^(3/2))
```

Rubi [A] time = 0.269135, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{x\left(5a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{x\left(\frac{3a(2bc-5ad)}{c^2}+\frac{b^2}{d}\right)}{8c\left(c+dx^2\right)}+\frac{\left(3ad(2bc-5ad)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x
)/(4*c^2*d*(c + d*x^2)^2) + ((b^2/d + (3*a*(2*b*c - 5*a*d))/c^2)*
x)/(8*c*(c + d*x^2)) + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*ArcTan[
(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d^(3/2))
```

Rubi in Sympy [A] time = 29.9501, size = 136, normalized size = 0.89

$$-\frac{a^2}{cx\left(c+dx^2\right)^2}-\frac{x\left(ad\left(5ad-2bc\right)+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}+\frac{x\left(-3ad\left(5ad-2bc\right)+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}+\frac{\left(-3ad\left(5ad-2bc\right)+b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{7}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**2/(d*x**2+c)**3,x)
```

```
[Out] -a**2/(c*x*(c + d*x**2)**2) - x*(a*d*(5*a*d - 2*b*c) + b**2*c**2)
/(4*c**2*d*(c + d*x**2)**2) + x*(-3*a*d*(5*a*d - 2*b*c) + b**2*c*
*2)/(8*c**3*d*(c + d*x**2)) + (-3*a*d*(5*a*d - 2*b*c) + b**2*c**2
)*atan(sqrt(d)*x/sqrt(c))/(8*c**(7/2)*d**(3/2))
```

Mathematica [A] time = 0.147895, size = 133, normalized size = 0.88

$$\frac{\left(-15a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(-7a^2d^2+6abcd+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}-\frac{a^2}{c^3x}-\frac{x(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c^3*x)) - ((b*c - a*d)^2*x)/(4*c^2*d*(c + d*x^2)^2) + ((b^
2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*x)/(8*c^3*d*(c + d*x^2)) + ((b^2*c
^2 + 6*a*b*c*d - 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7
/2)*d^(3/2))
```

Maple [A] time = 0.017, size = 199, normalized size = 1.3

$$-\frac{a^2}{c^3x}-\frac{7\,x^3a^2d^2}{8\,c^3\,(dx^2+c)^2}+\frac{3\,x^3abd}{4\,c^2\,(dx^2+c)^2}+\frac{b^2x^3}{8\,c\,(dx^2+c)^2}-\frac{9\,xa^2d}{8\,c^2\,(dx^2+c)^2}+\frac{5\,abx}{4\,c\,(dx^2+c)^2}-\frac{xb^2}{8\,(dx^2+c)^2\,d}-\frac{15\,a^2d}{8\,c^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,ab}{4\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2}{8\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^2/(d*x^2+c)^3,x)
```

```
[Out] -a^2/c^3/x-7/8/c^3/(d*x^2+c)^2*x^3*a^2*d^2+3/4/c^2/(d*x^2+c)^2*x^
3*a*b*d+1/8/c/(d*x^2+c)^2*x^3*b^2-9/8/c^2/(d*x^2+c)^2*x*a^2*d+5/4
/c/(d*x^2+c)^2*x*a*b-1/8/(d*x^2+c)^2/d*x*b^2-15/8/c^3*d/(c*d)^(1/
2)*arctan(x*d/(c*d)^(1/2))*a^2+3/4/c^2/(c*d)^(1/2)*arctan(x*d/(c*
d)^(1/2))*a*b+1/8/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.252564, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(b^2c^2d^2+6\,abcd^3-15\,a^2d^4\right)x^5+2\left(b^2c^3d+6\,abc^2d^2-15\,a^2cd^3\right)x^3+\left(b^2c^4+6\,abc^3d-15\,a^2c^2d^2\right)x\right)\log\left(-\frac{2\,cdx-(dx^2-}{dx^2+}\right.}{16\,(c^3d^3x^5+2\,c^4d^2x^3+c^5dx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((b^2*c^2*d^2 + 6*a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^
3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*x^3 + (b^2*c^4 + 6*a*b*c^3*d
- 15*a^2*c^2*d^2)*x)*log(-(2*c*d*x - (d*x^2 - c)*sqrt(-c*d))/(d*x
^2 + c)) + 2*(8*a^2*c^2*d - (b^2*c^2*d + 6*a*b*c*d^2 - 15*a^2*d^3
)*x^4 + (b^2*c^3 - 10*a*b*c^2*d + 25*a^2*c*d^2)*x^2)*sqrt(-c*d))/
((c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*sqrt(-c*d)), 1/8*(((b^2*
c^2*d^2 + 6*a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^
2*d^2 - 15*a^2*c*d^3)*x^3 + (b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d
^2)*x)*arctan(sqrt(c*d)*x/c) - (8*a^2*c^2*d - (b^2*c^2*d + 6*a*b*
c*d^2 - 15*a^2*d^3)*x^4 + (b^2*c^3 - 10*a*b*c^2*d + 25*a^2*c*d^2)
*x^2)*sqrt(c*d))/((c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*sqrt(c*
d))]
```

Sympy [A] time = 5.60954, size = 224, normalized size = 1.47

$$\frac{\sqrt{-\frac{1}{c^7d^3}}\left(15a^2d^2-6abcd-b^2c^2\right)\log\left(-c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{c^7d^3}}\left(15a^2d^2-6abcd-b^2c^2\right)\log\left(c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16}-\frac{8a^2c^2d+x^4\left(15a^2d^3-6abcd^2-b^2c^2d\right)+x^2\left(25a^2cd^2-10abc^2d+b^2c^3\right)}{8c^5dx+16c^4d^2x^3+8c^3d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**2/(d*x**2+c)**3,x)
```

```
[Out] sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d - b**2*c**2)*log(-
c**4*d*sqrt(-1/(c**7*d**3)) + x)/16 - sqrt(-1/(c**7*d**3))*(15*a*
*2*d**2 - 6*a*b*c*d - b**2*c**2)*log(c**4*d*sqrt(-1/(c**7*d**3))
+ x)/16 - (8*a**2*c**2*d + x**4*(15*a**2*d**3 - 6*a*b*c*d**2 - b*
*2*c**2*d) + x**2*(25*a**2*c*d**2 - 10*a*b*c**2*d + b**2*c**3))/(
8*c**5*d*x + 16*c**4*d**2*x**3 + 8*c**3*d**3*x**5)
```

GIAC/XCAS [A] time = 0.222535, size = 182, normalized size = 1.2

$$-\frac{a^2}{c^3x}+\frac{\left(b^2c^2+6\,abcd-15\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^3d}+\frac{b^2c^2dx^3+6\,abcd^2x^3-7\,a^2d^3x^3-b^2c^3x+10\,abc^2dx-9\,a^2cd^2x}{8\,(dx^2+c)^2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^2),x, algorithm="giac")
```

```
[Out] -a^2/(c^3*x) + 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/
sqrt(c*d))/(sqrt(c*d)*c^3*d) + 1/8*(b^2*c^2*d*x^3 + 6*a*b*c*d^2*x
^3 - 7*a^2*d^3*x^3 - b^2*c^3*x + 10*a*b*c^2*d*x - 9*a^2*c*d^2*x)/
((d*x^2 + c)^2*c^3*d)
```

3.196 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3} \, dx$

Optimal. Leaf size=106

$$-\frac{a^2}{2c^3x^2} - \frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4} + \frac{a\log(x)(2bc-3ad)}{c^4} + \frac{a(bc-ad)}{c^3\left(c+dx^2\right)} - \frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

```
[Out] -a^2/(2*c^3*x^2) - (b*c - a*d)^2/(4*c^2*d*(c + d*x^2)^2) + (a*(b*
c - a*d))/(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x])/c^4 - (a
*(2*b*c - 3*a*d)*Log[c + d*x^2])/(2*c^4)
```

Rubi [A] time = 0.276507, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a^2}{2c^3x^2} - \frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4} + \frac{a\log(x)(2bc-3ad)}{c^4} + \frac{a(bc-ad)}{c^3\left(c+dx^2\right)} - \frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] -a^2/(2*c^3*x^2) - (b*c - a*d)^2/(4*c^2*d*(c + d*x^2)^2) + (a*(b*
c - a*d))/(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x])/c^4 - (a
*(2*b*c - 3*a*d)*Log[c + d*x^2])/(2*c^4)
```

Rubi in Sympy [A] time = 36.5827, size = 100, normalized size = 0.94

$$-\frac{a^2}{2c^3x^2} - \frac{a\,(ad-bc)}{c^3\left(c+dx^2\right)} - \frac{a\,(3ad-2bc)\log\left(x^2\right)}{2c^4} + \frac{a\,(3ad-2bc)\log\left(c+dx^2\right)}{2c^4} - \frac{(ad-bc)^2}{4c^2d\left(c+dx^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**3/(d*x**2+c)**3,x)
```

```
[Out] -a**2/(2*c**3*x**2) - a*(a*d - b*c)/(c**3*(c + d*x**2)) - a*(3*a*
d - 2*b*c)*log(x**2)/(2*c**4) + a*(3*a*d - 2*b*c)*log(c + d*x**2)
/(2*c**4) - (a*d - b*c)**2/(4*c**2*d*(c + d*x**2)**2)
```

Mathematica [A] time = 0.159818, size = 99, normalized size = 0.93

$$\frac{-\frac{2a^2c}{x^2} - \frac{c^2(bc-ad)^2}{d(c+dx^2)^2} + \frac{4ac(bc-ad)}{c+dx^2} + 2a(3ad-2bc)\log\left(c+dx^2\right) + 4a\log(x)(2bc-3ad)}{4c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] ((-2*a^2*c)/x^2 - (c^2*(b*c - a*d)^2)/(d*(c + d*x^2)^2) + (4*a*c*
(b*c - a*d))/(c + d*x^2) + 4*a*(2*b*c - 3*a*d)*Log[x] + 2*a*(-2*b
*c + 3*a*d)*Log[c + d*x^2])/(4*c^4)
```

Maple [A] time = 0.023, size = 149, normalized size = 1.4

$$-\frac{a^2}{2\,c^3x^2}-3\,\frac{\ln(x)\,a^2d}{c^4}+2\,\frac{a\ln(x)\,b}{c^3}-\frac{a^2d}{c^3\,(dx^2+c)}+\frac{ab}{c^2\,(dx^2+c)}-\frac{a^2d}{4\,c^2\,(dx^2+c)^2}+\frac{ab}{2\,c\,(dx^2+c)^2}-\frac{b^2}{4\,d\,(dx^2+c)^2}+\frac{3\,\ln\left(dx^2+c\right)\,a^2d}{2\,c^4}-\frac{\ln\left(dx^2+c\right)\,ab}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^3/(d*x^2+c)^3,x)
```

```
[Out] -1/2*a^2/c^3/x^2-3*a^2/c^4*ln(x)*d+2*a/c^3*ln(x)*b-1/c^3*a^2/(d*x
^2+c)*d+1/c^2*a/(d*x^2+c)*b-1/4/c^2*d/(d*x^2+c)^2*a^2+1/2/c/(d*x^
2+c)^2*a*b-1/4/d/(d*x^2+c)^2*b^2+3/2/c^4*ln(d*x^2+c)*a^2*d-1/c^3*
ln(d*x^2+c)*a*b
```

Maxima [A] time = 1.36245, size = 192, normalized size = 1.81

$$-\frac{2\,a^2c^2d-2\left(2\,abcd^2-3\,a^2d^3\right)x^4+\left(b^2c^3-6\,abc^2d+9\,a^2cd^2\right)x^2}{4\,(c^3d^3x^6+2\,c^4d^2x^4+c^5dx^2)}-\frac{\left(2\,abc-3\,a^2d\right)\,\log\left(dx^2+c\right)}{2\,c^4}+\frac{\left(2\,abc-3\,a^2d\right)\,\log\left(x^2\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^3),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*c^2*d - 2*(2*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (b^2*c^3 -
6*a*b*c^2*d + 9*a^2*c*d^2)*x^2)/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^
5*d*x^2) - 1/2*(2*a*b*c - 3*a^2*d)*log(d*x^2 + c)/c^4 + 1/2*(2*a*
b*c - 3*a^2*d)*log(x^2)/c^4
```

Fricas [A] time = 0.227512, size = 346, normalized size = 3.26

$$-\frac{2\,a^2c^3d-2\left(2\,abc^2d^2-3\,a^2cd^3\right)x^4+\left(b^2c^4-6\,abc^3d+9\,a^2c^2d^2\right)x^2+2\left(\left(2\,abcd^3-3\,a^2d^4\right)x^6+2\left(2\,abc^2d^2-3\,a^2cd^3\right)x^4\right.}{4\,(c^4d^3x^6+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*c^3*d - 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (b^2*c^
4 - 6*a*b*c^3*d + 9*a^2*c^2*d^2)*x^2 + 2*((2*a*b*c*d^3 - 3*a^2*d^
4)*x^6 + 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (2*a*b*c^3*d - 3*a
^2*c^2*d^2)*x^2)*log(d*x^2 + c) - 4*((2*a*b*c*d^3 - 3*a^2*d^4)*x^
6 + 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (2*a*b*c^3*d - 3*a^2*c^
2*d^2)*x^2)*log(x))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2)
```

Sympy [A] time = 8.21869, size = 139, normalized size = 1.31

$$-\frac{a\,(3ad-2bc)\log{(x)}}{c^4}+\frac{a\,(3ad-2bc)\log{\left(\frac{c}{d}+x^2\right)}}{2c^4}-\frac{2a^2c^2d+x^4\left(6a^2d^3-4abcd^2\right)+x^2\left(9a^2cd^2-6abc^2d+b^2c^3\right)}{4c^5dx^2+8c^4d^2x^4+4c^3d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**3/(d*x**2+c)**3,x)
```

```
[Out] -a*(3*a*d - 2*b*c)*log(x)/c**4 + a*(3*a*d - 2*b*c)*log(c/d + x**2
)/(2*c**4) - (2*a**2*c**2*d + x**4*(6*a**2*d**3 - 4*a*b*c*d**2) +
 x**2*(9*a**2*c*d**2 - 6*a*b*c**2*d + b**2*c**3))/(4*c**5*d*x**2
+ 8*c**4*d**2*x**4 + 4*c**3*d**3*x**6)
```

GIAC/XCAS [A] time = 0.227098, size = 239, normalized size = 2.25

$$\frac{\left(2\,abc-3\,a^2d\right)\ln\left(x^2\right)}{2\,c^4}-\frac{\left(2\,abcd-3\,a^2d^2\right)\ln\left(\left|dx^2+c\right|\right)}{2\,c^4d}-\frac{2\,abcx^2-3\,a^2dx^2+a^2c}{2\,c^4x^2}+\frac{6\,abcd^3x^4-9\,a^2d^4x^4+16\,abc^2d^2x^2-22\,a^2cd^3x^2-b^2c^4+12\,abc^3d-14\,a^2c^2d^2}{4\,(dx^2+c)^2c^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - 3*a^2*d)*ln(x^2)/c^4 - 1/2*(2*a*b*c*d - 3*a^2*d^2)
*ln(abs(d*x^2 + c))/(c^4*d) - 1/2*(2*a*b*c*x^2 - 3*a^2*d*x^2 + a^
2*c)/(c^4*x^2) + 1/4*(6*a*b*c*d^3*x^4 - 9*a^2*d^4*x^4 + 16*a*b*c^
2*d^2*x^2 - 22*a^2*c*d^3*x^2 - b^2*c^4 + 12*a*b*c^3*d - 14*a^2*c^
2*d^2)/((d*x^2 + c)^2*c^4*d)
```

3.197 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx$

Optimal. Leaf size=161

$$\frac{(35a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}+\frac{x(7a^2d^2-6abcd+3b^2c^2)}{12c^3(c+dx^2)^2}-\frac{a^2}{3cx^3(c+dx^2)^2}+\frac{x(3bc-7ad)^2}{24c^4(c+dx^2)}-\frac{a(6bc-7ad)}{3c^4x}$$

```
[Out] -(a*(6*b*c - 7*a*d))/(3*c^4*x) - a^2/(3*c*x^3*(c + d*x^2)^2) + ((
3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x)/(12*c^3*(c + d*x^2)^2) + ((
3*b*c - 7*a*d)^2*x)/(24*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c
*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Rubi [A] time = 0.474135, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{(35a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}+\frac{x(7a^2d^2-6abcd+3b^2c^2)}{12c^3(c+dx^2)^2}-\frac{a^2}{3cx^3(c+dx^2)^2}+\frac{x(3bc-7ad)^2}{24c^4(c+dx^2)}-\frac{a(6bc-7ad)}{3c^4x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -(a*(6*b*c - 7*a*d))/(3*c^4*x) - a^2/(3*c*x^3*(c + d*x^2)^2) + ((
3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x)/(12*c^3*(c + d*x^2)^2) + ((
3*b*c - 7*a*d)^2*x)/(24*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c
*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Rubi in Sympy [A] time = 68.002, size = 150, normalized size = 0.93

$$-\frac{a^2}{3cx^3(c+dx^2)^2}+\frac{a(7ad-6bc)}{3c^4x}+\frac{x(ad(7ad-6bc)+3b^2c^2)}{12c^3(c+dx^2)^2}+\frac{x(7ad-3bc)^2}{24c^4(c+dx^2)}+\frac{(35a^2d^2-30abcd+3b^2c^2)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{9}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**4/(d*x**2+c)**3,x)
```

```
[Out] -a**2/(3*c*x**3*(c + d*x**2)**2) + a*(7*a*d - 6*b*c)/(3*c**4*x) +
 x*(a*d*(7*a*d - 6*b*c) + 3*b**2*c**2)/(12*c**3*(c + d*x**2)**2)
+ x*(7*a*d - 3*b*c)**2/(24*c**4*(c + d*x**2)) + (35*a**2*d**2 - 3
0*a*b*c*d + 3*b**2*c**2)*atan(sqrt(d)*x/sqrt(c))/(8*c**(9/2)*sqrt
(d))
```

Mathematica [A] time = 0.119381, size = 148, normalized size = 0.92

$$\frac{(35a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}+\frac{x(11a^2d^2-14abcd+3b^2c^2)}{8c^4(c+dx^2)}-\frac{a^2}{3c^3x^3}+\frac{a(3ad-2bc)}{c^4x}+\frac{x(bc-ad)^2}{4c^3(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -a^2/(3*c^3*x^3) + (a*(-2*b*c + 3*a*d))/(c^4*x) + ((b*c - a*d)^2*
x)/(4*c^3*(c + d*x^2)^2) + ((3*b^2*c^2 - 14*a*b*c*d + 11*a^2*d^2)
*x)/(8*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Maple [A] time = 0.02, size = 227, normalized size = 1.4

$$-\frac{a^2}{3\,c^3x^3}+3\,\frac{a^2d}{c^4x}-2\,\frac{ab}{c^3x}+\frac{11\,x^3a^2d^3}{8\,c^4\,(dx^2+c)^2}-\frac{7\,x^3abd^2}{4\,c^3\,(dx^2+c)^2}+\frac{3\,b^2dx^3}{8\,c^2\,(dx^2+c)^2}+\frac{13\,a^2d^2x}{8\,c^3\,(dx^2+c)^2}-\frac{9\,xabd}{4\,c^2\,(dx^2+c)^2}+\frac{5\,xb^2}{8\,c\,(dx^2+c)^2}+\frac{35\,a^2d^2}{8\,c^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{15\,abd}{4\,c^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,b^2}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2/x^4/(d*x^2+c)^3,x)
```

```
[Out] -1/3*a^2/c^3/x^3+3*a^2/c^4/x*d-2*a/c^3/x*b+11/8/c^4/(d*x^2+c)^2*x
^3*a^2*d^3-7/4/c^3/(d*x^2+c)^2*x^3*a*b*d^2+3/8/c^2/(d*x^2+c)^2*x^
3*b^2*d+13/8/c^3/(d*x^2+c)^2*x*a^2*d^2-9/4/c^2/(d*x^2+c)^2*x*a*b*
d+5/8/c/(d*x^2+c)^2*x*b^2+35/8/c^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*a^2*d^2-15/4/c^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b*d+
3/8/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248185, size = 1, normalized size = 0.01

$$\left[\frac{3\,\left(\left(3\,b^2c^2d^2-30\,abcd^3+35\,a^2d^4\right)x^7+2\,\left(3\,b^2c^3d-30\,abc^2d^2+35\,a^2cd^3\right)x^5+\left(3\,b^2c^4-30\,abc^3d+35\,a^2c^2d^2\right)x^3\right)\log\left(\frac{2\,c\cdots}{\cdots}\right.}{48\,(c^4d^2x^7\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^4),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7 + 2*(3*
b^2*c^3*d - 30*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*
a*b*c^3*d + 35*a^2*c^2*d^2)*x^3)*log((2*c*d*x + (d*x^2 - c)*sqrt(
-c*d))/(d*x^2 + c)) + 2*(3*(3*b^2*c^2*d - 30*a*b*c*d^2 + 35*a^2*d
^3)*x^6 - 8*a^2*c^3 + 5*(3*b^2*c^3 - 30*a*b*c^2*d + 35*a^2*c*d^2)
*x^4 - 8*(6*a*b*c^3 - 7*a^2*c^2*d)*x^2)*sqrt(-c*d))/((c^4*d^2*x^7
 + 2*c^5*d*x^5 + c^6*x^3)*sqrt(-c*d)), 1/24*(3*((3*b^2*c^2*d^2 -
```

```
30*a*b*c*d^3 + 35*a^2*d^4)*x^7 + 2*(3*b^2*c^3*d - 30*a*b*c^2*d^2
+ 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*b*c^3*d + 35*a^2*c^2*d^2)
*x^3)*arctan(sqrt(c*d)*x/c) + (3*(3*b^2*c^2*d - 30*a*b*c*d^2 + 35
*a^2*d^3)*x^6 - 8*a^2*c^3 + 5*(3*b^2*c^3 - 30*a*b*c^2*d + 35*a^2*
c*d^2)*x^4 - 8*(6*a*b*c^3 - 7*a^2*c^2*d)*x^2)*sqrt(c*d))/((c^4*d^
2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*sqrt(c*d))]
```

Sympy [A] time = 6.92507, size = 240, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{c^9 d}}\left(35a^2d^2-30abcd+3b^2c^2\right)\log\left(-c^5\sqrt{-\frac{1}{c^9 d}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^9 d}}\left(35a^2d^2-30abcd+3b^2c^2\right)\log\left(c^5\sqrt{-\frac{1}{c^9 d}}+x\right)}{16}+\frac{-8a^2c^3+x^6\left(105a^2d^3-90abcd^2+9b^2c^2d\right)+x^4\left(175a^2cd^2-150abc^2d+15b^2c^3\right)+x^2\left(56a^2c^2d-48abc^3\right)}{24c^6x^3+48c^5dx^5+24c^4d^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**4/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b**2*c**2)*log(
-c**5*sqrt(-1/(c**9*d)) + x)/16 + sqrt(-1/(c**9*d))*(35*a**2*d**2
 - 30*a*b*c*d + 3*b**2*c**2)*log(c**5*sqrt(-1/(c**9*d)) + x)/16 +
 (-8*a**2*c**3 + x**6*(105*a**2*d**3 - 90*a*b*c*d**2 + 9*b**2*c**
2*d) + x**4*(175*a**2*c*d**2 - 150*a*b*c**2*d + 15*b**2*c**3) + x
**2*(56*a**2*c**2*d - 48*a*b*c**3))/(24*c**6*x**3 + 48*c**5*d*x**
5 + 24*c**4*d**2*x**7)
```

GIAC/XCAS [A] time = 0.226923, size = 204, normalized size = 1.27

$$\frac{\left(3\,b^2c^2-30\,abcd+35\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^4}+\frac{3\,b^2c^2dx^3-14\,abcd^2x^3+11\,a^2d^3x^3+5\,b^2c^3x-18\,abc^2dx+13\,a^2cd^2x}{8\,(dx^2+c)^2c^4}-\frac{6\,abcx^2-9\,a^2dx^2+a^2c}{3\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^4),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*arctan(d*x/sqrt(c*d))/(
sqrt(c*d)*c^4) + 1/8*(3*b^2*c^2*d*x^3 - 14*a*b*c*d^2*x^3 + 11*a^2
*d^3*x^3 + 5*b^2*c^3*x - 18*a*b*c^2*d*x + 13*a^2*c*d^2*x)/((d*x^2
 + c)^2*c^4) - 1/3*(6*a*b*c*x^2 - 9*a^2*d*x^2 + a^2*c)/(c^4*x^3)
```

3.198 $\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(bc-ad)}{2b^3}+\frac{x^4(bc-ad)}{4b^2}+\frac{dx^6}{6b}$$

```
[Out] -(a*(b*c - a*d)*x^2)/(2*b^3) + ((b*c - a*d)*x^4)/(4*b^2) + (d*x^6
)/(6*b) + (a^2*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.200813, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(bc-ad)}{2b^3}+\frac{x^4(bc-ad)}{4b^2}+\frac{dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)*x^2)/(2*b^3) + ((b*c - a*d)*x^4)/(4*b^2) + (d*x^6
)/(6*b) + (a^2*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(ad-bc\right)\log\left(a+bx^2\right)}{2b^4}+\frac{dx^6}{6b}-\frac{(ad-bc)\int^{x^2}x\,dx}{2b^2}+\frac{(ad-bc)\int^{x^2}a\,dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)*log(a + b*x**2)/(2*b**4) + d*x**6/(6*b) - (a*d
- b*c)*Integral(x, (x, x**2))/(2*b**2) + (a*d - b*c)*Integral(a,
(x, x**2))/(2*b**3)
```

Mathematica [A] time = 0.05094, size = 71, normalized size = 0.95

$$\frac{bx^2\left(6a^2d-3ab\left(2c+dx^2\right)+b^2x^2\left(3c+2dx^2\right)\right)+6a^2(bc-ad)\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d - 3*a*b*(2*c + d*x^2) + b^2*x^2*(3*c + 2*d*x^2))
+ 6*a^2*(b*c - a*d)*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.004, size = 86, normalized size = 1.2

$$\frac{dx^6}{6\,b}-\frac{x^4ad}{4\,b^2}+\frac{cx^4}{4\,b}+\frac{x^2a^2d}{2\,b^3}-\frac{ax^2c}{2\,b^2}-\frac{a^3\ln\left(bx^2+a\right)d}{2\,b^4}+\frac{a^2\ln\left(bx^2+a\right)c}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/6*d*x^6/b-1/4/b^2*x^4*a*d+1/4/b*x^4*c+1/2/b^3*x^2*a^2*d-1/2/b^2
*x^2*a*c-1/2*a^3/b^4*ln(b*x^2+a)*d+1/2*a^2/b^3*ln(b*x^2+a)*c
```

Maxima [A] time = 1.33729, size = 100, normalized size = 1.33

$$\frac{2\,b^{2}dx^{6}+3\,\left(b^{2}c-abd\right)x^{4}-6\,\left(abc-a^{2}d\right)x^{2}}{12\,b^{3}}+\frac{\left(a^{2}bc-a^{3}d\right)\log\left(bx^{2}+a\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^5/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*(b^2*c - a*b*d)*x^4 - 6*(a*b*c - a^2*d)*x^2
)/b^3 + 1/2*(a^2*b*c - a^3*d)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.225877, size = 101, normalized size = 1.35

$$\frac{2\,b^{3}dx^{6}+3\,\left(b^{3}c-ab^{2}d\right)x^{4}-6\,\left(ab^{2}c-a^{2}bd\right)x^{2}+6\,\left(a^{2}bc-a^{3}d\right)\log\left(bx^{2}+a\right)}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^5/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d*x^6 + 3*(b^3*c - a*b^2*d)*x^4 - 6*(a*b^2*c - a^2*b*
d)*x^2 + 6*(a^2*b*c - a^3*d)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 1.76957, size = 65, normalized size = 0.87

$$-\frac{a^{2}\left(ad-bc\right)\log\left(a+bx^{2}\right)}{2b^{4}}+\frac{dx^{6}}{6b}-\frac{x^{4}\left(ad-bc\right)}{4b^{2}}+\frac{x^{2}\left(a^{2}d-abc\right)}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)*log(a + b*x**2)/(2*b**4) + d*x**6/(6*b) - x**4*
(a*d - b*c)/(4*b**2) + x**2*(a**2*d - a*b*c)/(2*b**3)
```

GIAC/XCAS [A] time = 0.229953, size = 104, normalized size = 1.39

$$\frac{2\,b^{2}dx^{6}+3\,b^{2}cx^{4}-3\,abdx^{4}-6\,abcx^{2}+6\,a^{2}dx^{2}}{12\,b^{3}}+\frac{\left(a^{2}bc-a^{3}d\right)\ln\left(\left|bx^{2}+a\right|\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^5/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*b^2*c*x^4 - 3*a*b*d*x^4 - 6*a*b*c*x^2 + 6*a
^2*d*x^2)/b^3 + 1/2*(a^2*b*c - a^3*d)*ln(abs(b*x^2 + a))/b^4
```

3.199 $\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(bc-ad)}{b^3}+\frac{x^3(bc-ad)}{3b^2}+\frac{dx^5}{5b}$$

```
[Out] -((a*(b*c - a*d)*x)/b^3) + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5
*b) + (a^(3/2)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.138298, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(bc-ad)}{b^3}+\frac{x^3(bc-ad)}{3b^2}+\frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*x)/b^3) + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5
*b) + (a^(3/2)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{\frac{3}{2}}(ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{7}{2}}}+\frac{dx^5}{5b}-\frac{x^3(ad-bc)}{3b^2}+\frac{(ad-bc)\int a\,dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**(3/2)*(a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/b**(7/2) + d*x**5/(
5*b) - x**3*(a*d - b*c)/(3*b**2) + (a*d - b*c)*Integral(a, x)/b**
3
```

Mathematica [A] time = 0.086598, size = 77, normalized size = 1.

$$-\frac{a^{3/2}(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{ax(ad-bc)}{b^3}+\frac{x^3(bc-ad)}{3b^2}+\frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)*x)/b^3 + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5
*b) - (a^(3/2)*(-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2
)
```

Maple [A] time = 0.005, size = 92, normalized size = 1.2

$$\frac{dx^5}{5\,b}-\frac{x^3ad}{3\,b^2}+\frac{cx^3}{3\,b}+\frac{xa^2d}{b^3}-\frac{acx}{b^2}-\frac{a^3d}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{a^2c}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*d*x^5/b-1/3/b^2*x^3*a*d+1/3/b*x^3*c+1/b^3*x*a^2*d-1/b^2*x*a*c
-a^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+a^2/b^2/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23309, size = 1, normalized size = 0.01

$$\left[\frac{6\,b^2dx^5+10\,\left(b^2c-abd\right)x^3-15\,\left(abc-a^2d\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,\left(abc-a^2d\right)x}{30\,b^3},\frac{3\,b^2dx^5+5\,\left(b^2c-abd\right)x^3+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d*x^5 + 10*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*
sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 30*(
a*b*c - a^2*d)*x)/b^3, 1/15*(3*b^2*d*x^5 + 5*(b^2*c - a*b*d)*x^3
+ 15*(a*b*c - a^2*d)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 15*(a*b*c -
a^2*d)*x)/b^3]
```

Sympy [A] time = 1.91, size = 150, normalized size = 1.95

$$\frac{\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}(ad-bc)}{a^2d-abc}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}(ad-bc)}{a^2d-abc}+x\right)}{2}+\frac{dx^5}{5b}-\frac{x^3\,(ad-bc)}{3b^2}+\frac{x\left(a^2d-abc\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-a**3/b**7)*(a*d - b*c)*log(-b**3*sqrt(-a**3/b**7)*(a*d - b*
c)/(a**2*d - a*b*c) + x)/2 - sqrt(-a**3/b**7)*(a*d - b*c)*log(b**
3*sqrt(-a**3/b**7)*(a*d - b*c)/(a**2*d - a*b*c) + x)/2 + d*x**5/(
5*b) - x**3*(a*d - b*c)/(3*b**2) + x*(a**2*d - a*b*c)/b**3
```

<u>GIAC/XCAS</u> [A] time = 0.218848, size = 113, normalized size = 1.47

$$\frac{\left(a^2bc - a^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4dx^5 + 5\,b^4cx^3 - 5\,ab^3dx^3 - 15\,ab^3cx + 15\,a^2b^2dx}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3
*b^4*d*x^5 + 5*b^4*c*x^3 - 5*a*b^3*d*x^3 - 15*a*b^3*c*x + 15*a^2*
b^2*d*x)/b^5
```

3.200 $\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=54

$$-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bc-ad)}{2b^2}+\frac{dx^4}{4b}$$

```
[Out] ((b*c - a*d)*x^2)/(2*b^2) + (d*x^4)/(4*b) - (a*(b*c - a*d)*Log[a
+ b*x^2])/(2*b^3)
```

Rubi [A] time = 0.131333, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bc-ad)}{2b^2}+\frac{dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x^2)/(2*b^2) + (d*x^4)/(4*b) - (a*(b*c - a*d)*Log[a
+ b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a(ad-bc)\log\left(a+bx^2\right)}{2b^3}-\left(\frac{ad}{2}-\frac{bc}{2}\right)\int^{x^2}\frac{1}{b^2}\,dx+\frac{d\int^{x^2}x\,dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)*log(a + b*x**2)/(2*b**3) - (a*d/2 - b*c/2)*Integral
(b**(-2), (x, x**2)) + d*Integral(x, (x, x**2))/(2*b)
```

Mathematica [A] time = 0.0319273, size = 47, normalized size = 0.87

$$\frac{bx^2\left(-2ad+2bc+bdx^2\right)+2a(ad-bc)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*b*c - 2*a*d + b*d*x^2) + 2*a*(-(b*c) + a*d)*Log[a + b*x
^2])/(4*b^3)
```

Maple [A] time = 0.004, size = 62, normalized size = 1.2

$$\frac{dx^4}{4\,b}-\frac{adx^2}{2\,b^2}+\frac{cx^2}{2\,b}+\frac{a^2\ln\left(bx^2+a\right)d}{2\,b^3}-\frac{ac\ln\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/4*d*x^4/b-1/2/b^2*a*d*x^2+1/2*c*x^2/b+1/2*a^2/b^3*ln(b*x^2+a)*d
-1/2*a*c*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.34607, size = 68, normalized size = 1.26

$$\frac{bdx^4 + 2\,(bc - ad)x^2}{4\,b^2} - \frac{\left(abc - a^2 d\right)\,\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d*x^4 + 2*(b*c - a*d)*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(b
*x^2 + a)/b^3
```

Fricas [A] time = 0.232231, size = 69, normalized size = 1.28

$$\frac{b^2 dx^4 + 2\,\left(b^2 c - abd\right)x^2 - 2\,\left(abc - a^2 d\right)\,\log\left(bx^2 + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^4 + 2*(b^2*c - a*b*d)*x^2 - 2*(a*b*c - a^2*d)*log(b*
x^2 + a))/b^3
```

Sympy [A] time = 1.65353, size = 44, normalized size = 0.81

$$\frac{a\,(ad - bc)\log\left(a + bx^2\right)}{2b^3} + \frac{dx^4}{4b} - \frac{x^2\,(ad - bc)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)*log(a + b*x**2)/(2*b**3) + d*x**4/(4*b) - x**2*(a*d
 - b*c)/(2*b**2)
```

GIAC/XCAS [A] time = 0.233297, size = 70, normalized size = 1.3

$$\frac{bdx^4 + 2\,bcx^2 - 2\,adx^2}{4\,b^2} - \frac{\left(abc - a^2 d\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x^4 + 2*b*c*x^2 - 2*a*d*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*l
n(abs(b*x^2 + a))/b^3
```

3.201 $\int \frac{x^2(c+dx^2)}{a+bx^2} dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{a}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bc-ad)}{b^2}+\frac{dx^3}{3b}$$

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) - (Sqrt[a]*(b*c - a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.101515, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\sqrt{a}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bc-ad)}{b^2}+\frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) - (Sqrt[a]*(b*c - a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rubi in Sympy [A] time = 16.1229, size = 49, normalized size = 0.84

$$\frac{\sqrt{a}\,(ad-bc)\,\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{dx^3}{3b}-\frac{x\,(ad-bc)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(a)*(a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/b**(5/2) + d*x**3/(3*
b) - x*(a*d - b*c)/b**2
```

Mathematica [A] time = 0.0703124, size = 57, normalized size = 0.98

$$\frac{\sqrt{a}(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bc-ad)}{b^2}+\frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) + (Sqrt[a]*(-(b*c) + a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A] time = 0.005, size = 68, normalized size = 1.2

$$\frac{dx^3}{3\,b}-\frac{adx}{b^2}+\frac{cx}{b}+\frac{a^2d}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{ac}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/3*d*x^3/b-1/b^2*a*d*x+c*x/b+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))*d-c*a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.246286, size = 1, normalized size = 0.02

$$\left[\frac{2\,bdx^3 - 3\,(bc - ad)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,(bc - ad)x}{6\,b^2}, \frac{bdx^3 - 3\,(bc - ad)\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) + 3\,(bc - ad)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*d*x^3 - 3*(b*c - a*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqr
t(-a/b) - a)/(b*x^2 + a)) + 6*(b*c - a*d)*x)/b^2, 1/3*(b*d*x^3 -
3*(b*c - a*d)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 3*(b*c - a*d)*x)/b^
2]
```

Sympy [A] time = 1.78137, size = 90, normalized size = 1.55

$$-\frac{\sqrt{-\frac{a}{b^5}}\,(ad - bc)\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}\,(ad - bc)\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{dx^3}{3b} - \frac{x\,(ad - bc)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(-a/b**5)*(a*d - b*c)*log(-b**2*sqrt(-a/b**5) + x)/2 + sqrt(
-a/b**5)*(a*d - b*c)*log(b**2*sqrt(-a/b**5) + x)/2 + d*x**3/(3*b)
 - x*(a*d - b*c)/b**2
```

GIAC/XCAS [A] time = 0.227073, size = 78, normalized size = 1.34

$$-\frac{\left(abc - a^2 d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2dx^3 + 3\,b^2cx - 3\,abdx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2
*d*x^3 + 3*b^2*c*x - 3*a*b*d*x)/b^3
```

3.202 $\int \frac{x(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=35

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2b^2}+\frac{dx^2}{2b}$$

```
[Out] (d*x^2)/(2*b) + ((b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0770557, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2b^2}+\frac{dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (d*x^2)/(2*b) + ((b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^2} d\,dx}{2b}-\frac{(ad-bc)\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] Integral(d, (x, x**2))/(2*b) - (a*d - b*c)*log(a + b*x**2)/(2*b**
2)
```

Mathematica [A] time = 0.016691, size = 31, normalized size = 0.89

$$\frac{(bc-ad)\log\left(a+bx^2\right)+bdx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*d*x^2 + (b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.004, size = 40, normalized size = 1.1

$$\frac{dx^2}{2\,b}-\frac{\ln\left(bx^2+a\right)ad}{2\,b^2}+\frac{c\ln\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/2*d*x^2/b-1/2/b^2*ln(b*x^2+a)*a*d+1/2*c*ln(b*x^2+a)/b
```

Maxima [A] time = 1.34493, size = 42, normalized size = 1.2

$$\frac{dx^2}{2\,b} + \frac{(bc - ad)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.22413, size = 39, normalized size = 1.11

$$\frac{bdx^2 + (bc - ad)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x^2 + (b*c - a*d)*log(b*x^2 + a))/b^2
```

Sympy [A] time = 1.5444, size = 27, normalized size = 0.77

$$\frac{dx^2}{2b} - \frac{(ad - bc)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] d*x**2/(2*b) - (a*d - b*c)*log(a + b*x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.228133, size = 43, normalized size = 1.23

$$\frac{dx^2}{2\,b} + \frac{(bc - ad)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*ln(abs(b*x^2 + a))/b^2
```

3.203 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2))
```

Rubi [A] time = 0.0472215, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2))
```

Rubi in Sympy [A] time = 8.63362, size = 34, normalized size = 0.87

$$\frac{dx}{b}-\frac{(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] d*x/b - (a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(3/2))
```

Mathematica [A] time = 0.0416314, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b
^(3/2))
```

Maple [A] time = 0., size = 45, normalized size = 1.2

$$\frac{dx}{b}-\frac{ad}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a),x)
```

```
[Out] d*x/b-1/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+1/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243014, size = 1, normalized size = 0.03

$$\left[\frac{2\,\sqrt{-ab}dx - (bc - ad)\log\left(-\frac{2\,abx - (bx^2 - a)\,\sqrt{-ab}}{bx^2 + a}\right)}{2\,\sqrt{-ab}b}, \frac{\sqrt{ab}dx + (bc - ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*b)*d*x - (b*c - a*d)*log(-(2*a*b*x - (b*x^2 - a)*
sqrt(-a*b))/(b*x^2 + a)))/(sqrt(-a*b)*b), (sqrt(a*b)*d*x + (b*c -
 a*d)*arctan(sqrt(a*b)*x/a))/(sqrt(a*b)*b)]
```

Sympy [A] time = 1.60859, size = 82, normalized size = 2.1

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad - bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}\,(ad - bc)\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
 sqrt(-1/(a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 +
 d*x/b
```

GIAC/XCAS [A] time = 0.223809, size = 45, normalized size = 1.15

$$\frac{dx}{b} + \frac{(bc - ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.204 $\int \frac{c+dx^2}{x(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$\frac{c\log(x)}{a}-\frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}$$

```
[Out] (c*Log[x])/a - ((b*c - a*d)*Log[a + b*x^2])/(2*a*b)
```

Rubi [A] time = 0.0895671, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{c\log(x)}{a}-\frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x*(a + b*x^2)),x]

[Out] (c*Log[x])/a - ((b*c - a*d)*Log[a + b*x^2])/(2*a*b)
```

Rubi in Sympy [A] time = 13.987, size = 29, normalized size = 0.85

$$\frac{c\log\left(x^2\right)}{2a}+\frac{(ad-bc)\log\left(a+bx^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/x/(b*x**2+a),x)

[Out] c*log(x**2)/(2*a) + (a*d - b*c)*log(a + b*x**2)/(2*a*b)
```

Mathematica [A] time = 0.019751, size = 34, normalized size = 1.

$$\frac{(ad-bc)\log\left(a+bx^2\right)}{2ab}+\frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(x*(a + b*x^2)),x]

[Out] (c*Log[x])/a + ((-(b*c) + a*d)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] time = 0.007, size = 37, normalized size = 1.1

$$\frac{c\ln(x)}{a}+\frac{\ln\left(bx^2+a\right)d}{2\,b}-\frac{c\ln\left(bx^2+a\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x/(b*x^2+a),x)

[Out] c*ln(x)/a+1/2/b*ln(b*x^2+a)*d-1/2*c*ln(b*x^2+a)/a
```

Maxima [A] time = 1.34795, size = 47, normalized size = 1.38

$$\frac{c \log\left(x^2\right)}{2\,a} - \frac{(bc - ad)\log\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(b*x^2 + a)/(a*b)
```

Fricas [A] time = 0.237207, size = 45, normalized size = 1.32

$$\frac{2\,bc\log(x) - (bc - ad)\log\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*log(x) - (b*c - a*d)*log(b*x^2 + a))/(a*b)
```

Sympy [A] time = 2.23361, size = 26, normalized size = 0.76

$$\frac{c\log(x)}{a} + \frac{(ad - bc)\log\left(\frac{a}{b} + x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)/a + (a*d - b*c)*log(a/b + x**2)/(2*a*b)
```

GIAC/XCAS [A] time = 0.245467, size = 49, normalized size = 1.44

$$\frac{c\ln\left(x^2\right)}{2\,a} - \frac{(bc - ad)\ln\left(\left|bx^2 + a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*c*ln(x^2)/a - 1/2*(b*c - a*d)*ln(abs(b*x^2 + a))/(a*b)
```

3.205 $\int \frac{c+dx^2}{x^2(a+bx^2)} \, dx$

Optimal. Leaf size=43

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

```
[Out] -(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*S
qrt[b])
```

Rubi [A] time = 0.0643476, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*S
qrt[b])
```

Rubi in Sympy [A] time = 9.92165, size = 34, normalized size = 0.79

$$-\frac{c}{ax}+\frac{(ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -c/(a*x) + (a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*sqrt(b))
```

Mathematica [A] time = 0.0424333, size = 42, normalized size = 0.98

$$\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2
)*Sqrt[b])
```

Maple [A] time = 0.005, size = 48, normalized size = 1.1

$$d\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{bc}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-c*b/a/(a*b)^(1/2)*arctan(
x*b/(a*b)^(1/2))-c/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.246497, size = 1, normalized size = 0.02

$$\left[-\frac{(bc-ad)x\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\sqrt{-ab}c}{2\sqrt{-ab}ax}, -\frac{(bc-ad)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\sqrt{ab}c}{\sqrt{ab}ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c - a*d)*x*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^
2 + a)) + 2*sqrt(-a*b)*c)/(sqrt(-a*b)*a*x), -((b*c - a*d)*x*arcta
n(sqrt(a*b)*x/a) + sqrt(a*b)*c)/(sqrt(a*b)*a*x)]
```

Sympy [A] time = 1.82574, size = 82, normalized size = 1.91

$$-\frac{\sqrt{-\frac{1}{a^3b}}(ad-bc)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}(ad-bc)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*d - b*c)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2
 + sqrt(-1/(a**3*b))*(a*d - b*c)*log(a**2*sqrt(-1/(a**3*b)) + x)/
2 - c/(a*x)
```

GIAC/XCAS [A] time = 0.230791, size = 50, normalized size = 1.16

$$-\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.206 $\int \frac{c+dx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{2ax^2}$$

```
[Out] -c/(2*a*x^2) - ((b*c - a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*
x^2])/(2*a^2)
```

Rubi [A] time = 0.120203, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -c/(2*a*x^2) - ((b*c - a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*
x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 16.0644, size = 44, normalized size = 0.88

$$-\frac{c}{2ax^2}+\frac{(ad-bc)\log\left(x^2\right)}{2a^2}-\frac{(ad-bc)\log\left(a+bx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*a*x**2) + (a*d - b*c)*log(x**2)/(2*a**2) - (a*d - b*c)*log(
a + b*x**2)/(2*a**2)
```

Mathematica [A] time = 0.0423776, size = 49, normalized size = 0.98

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)(ad-bc)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -c/(2*a*x^2) + ((-(b*c) + a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a +
 b*x^2])/(2*a^2)
```

Maple [A] time = 0.01, size = 56, normalized size = 1.1

$$-\frac{c}{2\,ax^2}+\frac{\ln(x)\,d}{a}-\frac{bc\ln(x)}{a^2}-\frac{\ln\left(bx^2+a\right)d}{2\,a}+\frac{bc\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*c/a/x^2+1/a*ln(x)*d-b*c*ln(x)/a^2-1/2/a*ln(b*x^2+a)*d+1/2*b*
c*ln(b*x^2+a)/a^2
```

Maxima [A] time = 1.35141, size = 65, normalized size = 1.3

$$\frac{(bc-ad)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}-\frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*log(b*x^2 + a)/a^2 - 1/2*(b*c - a*d)*log(x^2)/a^2
 - 1/2*c/(a*x^2)
```

Fricas [A] time = 0.242194, size = 65, normalized size = 1.3

$$\frac{(bc-ad)x^2\log\left(bx^2+a\right)-2\,(bc-ad)x^2\log\left(x\right)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c - a*d)*x^2*log(b*x^2 + a) - 2*(b*c - a*d)*x^2*log(x) -
a*c)/(a^2*x^2)
```

Sympy [A] time = 3.01857, size = 41, normalized size = 0.82

$$-\frac{c}{2ax^2}+\frac{(ad-bc)\log\left(x\right)}{a^2}-\frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*a*x**2) + (a*d - b*c)*log(x)/a**2 - (a*d - b*c)*log(a/b + x
**2)/(2*a**2)
```

GIAC/XCAS [A] time = 0.224113, size = 97, normalized size = 1.94

$$-\frac{(bc-ad)\ln\left(x^2\right)}{2\,a^2}+\frac{\left(b^2c-abd\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bcx^2-adx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(b*c - a*d)*ln(x^2)/a^2 + 1/2*(b^2*c - a*b*d)*ln(abs(b*x^2 +
 a))/(a^2*b) + 1/2*(b*c*x^2 - a*d*x^2 - a*c)/(a^2*x^2)
```

3.207 $\int \frac{c+dx^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}$$

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (Sqrt[b]*(b*c - a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.101354, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\sqrt{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (Sqrt[b]*(b*c - a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi in Sympy [A] time = 14.0667, size = 51, normalized size = 0.86

$$-\frac{c}{3ax^3}-\frac{ad-bc}{a^2x}-\frac{\sqrt{b}\,(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] -c/(3*a*x**3) - (a*d - b*c)/(a**2*x) - sqrt(b)*(a*d - b*c)*atan(s
qrt(b)*x/sqrt(a))/a**(5/2)
```

Mathematica [A] time = 0.0949335, size = 60, normalized size = 1.02

$$-\frac{\sqrt{b}(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) - (Sqrt[b]*(-(b*c) + a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A] time = 0.008, size = 72, normalized size = 1.2

$$-\frac{c}{3\,ax^3}-\frac{d}{ax}+\frac{bc}{a^2x}-\frac{bd}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^2c}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x^4/(b*x^2+a),x)

[Out] -1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c-b/a/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*d+b^2/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236148, size = 1, normalized size = 0.02

$$\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6\,(bc-ad)x^2+2\,ac}{6\,a^2x^3},\ \frac{3\,(bc-ad)x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+3\,(bc-ad)x^2-ac}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="fricas")

[Out] [-1/6*(3*(b*c - a*d)*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) - 6*(b*c - a*d)*x^2 + 2*a*c)/(a^2*x^3), 1/3*(3
*(b*c - a*d)*x^3*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 3*(b*c - a
*d)*x^2 - a*c)/(a^2*x^3)]
```

Sympy [A] time = 2.2177, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\,(ad-bc)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}(ad-bc)}{abd-b^2c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\,(ad-bc)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}(ad-bc)}{abd-b^2c}+x\right)}{2}-\frac{ac+x^2\,(3ad-3bc)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x**4/(b*x**2+a),x)

[Out] sqrt(-b/a**5)*(a*d - b*c)*log(-a**3*sqrt(-b/a**5)*(a*d - b*c)/(a*
b*d - b**2*c) + x)/2 - sqrt(-b/a**5)*(a*d - b*c)*log(a**3*sqrt(-b
/a**5)*(a*d - b*c)/(a*b*d - b**2*c) + x)/2 - (a*c + x**2*(3*a*d -
 3*b*c))/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 0.220508, size = 77, normalized size = 1.31

$$\frac{\left(b^2c-abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*
c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

3.208 $\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=103

$$\frac{a^2(bc-ad)^2\log(a+bx^2)}{2b^5}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{x^4(bc-ad)^2}{4b^3}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{d^2x^8}{8b}$$

```
[Out] -(a*(b*c - a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) + (d
*(2*b*c - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + (a^2*(b*c - a*d)^
2*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.274379, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2(bc-ad)^2\log(a+bx^2)}{2b^5}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{x^4(bc-ad)^2}{4b^3}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{d^2x^8}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) + (d
*(2*b*c - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + (a^2*(b*c - a*d)^
2*Log[a + b*x^2])/(2*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2(ad-bc)^2\log(a+bx^2)}{2b^5}+\frac{d^2x^8}{8b}-\frac{dx^6(ad-2bc)}{6b^2}+\frac{(ad-bc)^2\int^{x^2}x\,dx}{2b^3}-\frac{(ad-bc)^2\int^{x^2}a\,dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] a**2*(a*d - b*c)**2*log(a + b*x**2)/(2*b**5) + d**2*x**8/(8*b) -
d*x**6*(a*d - 2*b*c)/(6*b**2) + (a*d - b*c)**2*Integral(x, (x, x*
*2))/(2*b**3) - (a*d - b*c)**2*Integral(a, (x, x**2))/(2*b**4)
```

Mathematica [A] time = 0.0873909, size = 116, normalized size = 1.13

$$\frac{(a^4d^2-2a^3bcd+a^2b^2c^2)\log(a+bx^2)}{2b^5}-\frac{ax^2(ad-bc)^2}{2b^4}+\frac{x^4(bc-ad)^2}{4b^3}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{d^2x^8}{8b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -(a*(-(b*c) + a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) +
 (d*(2*b*c - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + ((a^2*b^2*c^2
- 2*a^3*b*c*d + a^4*d^2)*Log[a + b*x^2])/(2*b^5)
```

Maple [A] time = 0.006, size = 165, normalized size = 1.6

$$\frac{d^2x^8}{8\,b}-\frac{x^6ad^2}{6\,b^2}+\frac{x^6cd}{3\,b}+\frac{x^4a^2d^2}{4\,b^3}-\frac{x^4acd}{2\,b^2}+\frac{x^4c^2}{4\,b}-\frac{a^3d^2x^2}{2\,b^4}+\frac{x^2a^2cd}{b^3}-\frac{ac^2x^2}{2\,b^2}+\frac{a^4\ln(bx^2+a)\,d^2}{2\,b^5}-\frac{a^3\ln(bx^2+a)\,cd}{b^4}+\frac{a^2\ln(bx^2+a)\,c^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/8*d^2*x^8/b-1/6/b^2*x^6*a*d^2+1/3/b*x^6*c*d+1/4/b^3*x^4*a^2*d^2
-1/2/b^2*x^4*a*c*d+1/4/b*x^4*c^2-1/2/b^4*a^3*d^2*x^2+1/b^3*a^2*c*
d*x^2-1/2/b^2*a*c^2*x^2+1/2*a^4/b^5*ln(b*x^2+a)*d^2-a^3/b^4*ln(b*
x^2+a)*c*d+1/2*a^2/b^3*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.34727, size = 185, normalized size = 1.8

$$\frac{3\,b^3d^2x^8 + 4\left(2\,b^3cd - ab^2d^2\right)x^6 + 6\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)x^4 - 12\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)x^2}{24\,b^4} + \frac{\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\log\left(bx^2 + a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^5/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 4*(2*b^3*c*d - a*b^2*d^2)*x^6 + 6*(b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2)*x^4 - 12*(a*b^2*c^2 - 2*a^2*b*c*d + a^
3*d^2)*x^2)/b^4 + 1/2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b
*x^2 + a)/b^5
```

Fricas [A] time = 0.23453, size = 186, normalized size = 1.81

$$\frac{3\,b^4d^2x^8 + 4\left(2\,b^4cd - ab^3d^2\right)x^6 + 6\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)x^4 - 12\left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x^2 + 12\left(a^2b^2c^2 - 2\,a^3bcd\right.}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^5/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^2*x^8 + 4*(2*b^4*c*d - a*b^3*d^2)*x^6 + 6*(b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 - 12*(a*b^3*c^2 - 2*a^2*b^2*c*d
+ a^3*b*d^2)*x^2 + 12*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b
*x^2 + a))/b^5
```

Sympy [A] time = 2.28162, size = 119, normalized size = 1.16

$$\frac{a^2\left(ad - bc\right)^2\log\left(a + bx^2\right)}{2b^5} + \frac{d^2x^8}{8b} - \frac{x^6\left(ad^2 - 2bcd\right)}{6b^2} + \frac{x^4\left(a^2d^2 - 2abcd + b^2c^2\right)}{4b^3} - \frac{x^2\left(a^3d^2 - 2a^2bcd + ab^2c^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] a**2*(a*d - b*c)**2*log(a + b*x**2)/(2*b**5) + d**2*x**8/(8*b) -
x**6*(a*d**2 - 2*b*c*d)/(6*b**2) + x**4*(a**2*d**2 - 2*a*b*c*d +
b**2*c**2)/(4*b**3) - x**2*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**
2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.225993, size = 200, normalized size = 1.94

$$\frac{3\,b^3d^2x^8+8\,b^3cdx^6-4\,ab^2d^2x^6+6\,b^3c^2x^4-12\,ab^2cdx^4+6\,a^2bd^2x^4-12\,ab^2c^2x^2+24\,a^2bcdx^2-12\,a^3d^2x^2}{24\,b^4}+\frac{\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\ln\left(\left|bx^2+a\right|\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^5/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 8*b^3*c*d*x^6 - 4*a*b^2*d^2*x^6 + 6*b^3*c^2
*x^4 - 12*a*b^2*c*d*x^4 + 6*a^2*b*d^2*x^4 - 12*a*b^2*c^2*x^2 + 24
*a^2*b*c*d*x^2 - 12*a^3*d^2*x^2)/b^4 + 1/2*(a^2*b^2*c^2 - 2*a^3*b
*c*d + a^4*d^2)*ln(abs(b*x^2 + a))/b^5
```

3.209 $\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=105

$$\frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax(bc-ad)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

```
[Out] -((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*
b*c - a*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^
2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.165961, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax(bc-ad)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*
b*c - a*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^
2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{\frac{3}{2}}(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{9}{2}}}+\frac{d^2x^7}{7b}-\frac{dx^5(ad-2bc)}{5b^2}+\frac{x^3(ad-bc)^2}{3b^3}-\frac{(ad-bc)^2\int a\,dx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] a**(3/2)*(a*d - b*c)**2*atan(sqrt(b)*x/sqrt(a))/b**(9/2) + d**2*x
**7/(7*b) - d*x**5*(a*d - 2*b*c)/(5*b**2) + x**3*(a*d - b*c)**2/(
3*b**3) - (a*d - b*c)**2*Integral(a, x)/b**4
```

Mathematica [A] time = 0.199298, size = 105, normalized size = 1.

$$\frac{a^{3/2}(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax(ad-bc)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(-(b*c) + a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*
(2*b*c - a*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(-(b*c) +
 a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] time = 0.004, size = 176, normalized size = 1.7

$$\frac{d^2x^7}{7\,b}-\frac{x^5ad^2}{5\,b^2}+\frac{2\,x^5cd}{5\,b}+\frac{x^3a^2d^2}{3\,b^3}-\frac{2\,x^3acd}{3\,b^2}+\frac{x^3c^2}{3\,b}-\frac{a^3d^2x}{b^4}+2\,\frac{xa^2cd}{b^3}-\frac{ac^2x}{b^2}+\frac{a^4d^2}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{a^3cd}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{a^2c^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/7*d^2*x^7/b-1/5/b^2*x^5*a*d^2+2/5/b*x^5*c*d+1/3/b^3*x^3*a^2*d^2
-2/3/b^2*x^3*a*c*d+1/3/b*x^3*c^2-1/b^4*a^3*d^2*x+2/b^3*a^2*c*d*x-
1/b^2*a*c^2*x+a^4/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^2-2*a
^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d+a^2/b^2/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248449, size = 1, normalized size = 0.01

$$\left[\frac{30\,b^3d^2x^7+42\left(2\,b^3cd-ab^2d^2\right)x^5+70\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^3+105\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}}{bx^2+a}\right.}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^2*x^7 + 42*(2*b^3*c*d - a*b^2*d^2)*x^5 + 70*(b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^2*b*c*
d + a^3*d^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2
 + a)) - 210*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x)/b^4, 1/105*(1
5*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^2)*x^5 + 35*(b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d
^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 105*(a*b^2*c^2 - 2*a^2*b*c*d
+ a^3*d^2)*x)/b^4]
```

Sympy [A] time = 2.50455, size = 240, normalized size = 2.29

$$-\frac{\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{d^2x^7}{7b}-\frac{x^5\left(ad^2-2bcd\right)}{5b^2}+\frac{x^3\left(a^2d^2-2abcd+b^2c^2\right)}{3b^3}-\frac{x\left(a^3d^2-2a^2bcd+ab^2c^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-a**3/b**9)*(a*d - b*c)**2*log(-b**4*sqrt(-a**3/b**9)*(a*d
- b*c)**2/(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2) + x)/2 + sqrt(
-a**3/b**9)*(a*d - b*c)**2*log(b**4*sqrt(-a**3/b**9)*(a*d - b*c)*
*2/(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2) + x)/2 + d**2*x**7/(7
*b) - x**5*(a*d**2 - 2*b*c*d)/(5*b**2) + x**3*(a**2*d**2 - 2*a*b*
c*d + b**2*c**2)/(3*b**3) - x*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*
c**2)/b**4
```

GIAC/XCAS [A] time = 0.234725, size = 207, normalized size = 1.97

$$\frac{\left(a^{2} b^{2} c^{2} - 2 \, a^{3} b c d + a^{4} d^{2}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{4}} + \frac{15 \, b^{6} d^{2} x^{7} + 42 \, b^{6} c d x^{5} - 21 \, a b^{5} d^{2} x^{5} + 35 \, b^{6} c^{2} x^{3} - 70 \, a b^{5} c d x^{3} + 35 \, a^{2} b^{4} d^{2} x^{3} - 105 \, a b^{5} c^{2} x + 210 \, a^{2} b^{4} c d x - 105 \, a^{3} b^{3} d^{2} x}{105 \, b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^4) + 1/105*(15*b^6*d^2*x^7 + 42*b^6*c*d*x^5 - 21*a*b^5*d^
2*x^5 + 35*b^6*c^2*x^3 - 70*a*b^5*c*d*x^3 + 35*a^2*b^4*d^2*x^3 -
105*a*b^5*c^2*x + 210*a^2*b^4*c*d*x - 105*a^3*b^3*d^2*x)/b^7
```

3.210 $\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(bc-ad)^2}{2b^3}+\frac{dx^4(2bc-ad)}{4b^2}+\frac{d^2x^6}{6b}$$

```
[Out] ((b*c - a*d)^2*x^2)/(2*b^3) + (d*(2*b*c - a*d)*x^4)/(4*b^2) + (d^
2*x^6)/(6*b) - (a*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.186262, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(bc-ad)^2}{2b^3}+\frac{dx^4(2bc-ad)}{4b^2}+\frac{d^2x^6}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*b^3) + (d*(2*b*c - a*d)*x^4)/(4*b^2) + (d^
2*x^6)/(6*b) - (a*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\left(ad-bc\right)^2\log\left(a+bx^2\right)}{2b^4}+\frac{\left(ad-bc\right)^2\int^{x^2}\frac{1}{b^3}\,dx}{2}+\frac{d^2x^6}{6b}-\frac{d\left(ad-2bc\right)\int^{x^2}x\,dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -a*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4) + (a*d - b*c)**2*Integ
ral(b**(-3), (x, x**2))/2 + d**2*x**6/(6*b) - d*(a*d - 2*b*c)*Int
egral(x, (x, x**2))/(2*b**2)
```

Mathematica [A] time = 0.0632728, size = 82, normalized size = 1.02

$$\frac{bx^2\left(6a^2d^2-3abd\left(4c+dx^2\right)+2b^2\left(3c^2+3cdx^2+d^2x^4\right)\right)-6a(bc-ad)^2\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d^2 - 3*a*b*d*(4*c + d*x^2) + 2*b^2*(3*c^2 + 3*c*d*
x^2 + d^2*x^4)) - 6*a*(b*c - a*d)^2*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.003, size = 124, normalized size = 1.6

$$\frac{d^2x^6}{6\,b}-\frac{x^4ad^2}{4\,b^2}+\frac{cx^4d}{2\,b}+\frac{x^2a^2d^2}{2\,b^3}-\frac{ax^2cd}{b^2}+\frac{x^2c^2}{2\,b}-\frac{a^3\ln\left(bx^2+a\right)d^2}{2\,b^4}+\frac{a^2\ln\left(bx^2+a\right)cd}{b^3}-\frac{a\ln\left(bx^2+a\right)c^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/6*d^2*x^6/b-1/4/b^2*x^4*a*d^2+1/2/b*x^4*c*d+1/2/b^3*x^2*a^2*d^2
-1/b^2*x^2*a*c*d+1/2/b*x^2*c^2-1/2*a^3/b^4*ln(b*x^2+a)*d^2+a^2/b^
3*ln(b*x^2+a)*c*d-1/2*a/b^2*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.35213, size = 136, normalized size = 1.7

$$\frac{2\,b^2d^2x^6+3\,\left(2\,b^2cd-abd^2\right)x^4+6\,\left(b^2c^2-2\,abcd+a^2d^2\right)x^2}{12\,b^3}-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 3*(2*b^2*c*d - a*b*d^2)*x^4 + 6*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^
3*d^2)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.227416, size = 138, normalized size = 1.72

$$\frac{2\,b^3d^2x^6+3\,\left(2\,b^3cd-ab^2d^2\right)x^4+6\,\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^2-6\,\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^6 + 3*(2*b^3*c*d - a*b^2*d^2)*x^4 + 6*(b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2)*x^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d + a^3
*d^2)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 2.16224, size = 83, normalized size = 1.04

$$-\frac{a\left(ad-bc\right)^2\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^6}{6b}-\frac{x^4\left(ad^2-2bcd\right)}{4b^2}+\frac{x^2\left(a^2d^2-2abcd+b^2c^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -a*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4) + d**2*x**6/(6*b) - x*
*4*(a*d**2 - 2*b*c*d)/(4*b**2) + x**2*(a**2*d**2 - 2*a*b*c*d + b*
*2*c**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.221629, size = 144, normalized size = 1.8

$$\frac{2\,b^2d^2x^6+6\,b^2cdx^4-3\,abd^2x^4+6\,b^2c^2x^2-12\,abcdx^2+6\,a^2d^2x^2}{12\,b^3}-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\ln\left(\left|bx^2+a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 6*b^2*c*d*x^4 - 3*a*b*d^2*x^4 + 6*b^2*c^2*x
^2 - 12*a*b*c*d*x^2 + 6*a^2*d^2*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2)*ln(abs(b*x^2 + a))/b^4
```

3.211 $\int \frac{x^2(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{a}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)
/(5*b) - (Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7
/2)
```

Rubi [A] time = 0.145481, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\sqrt{a}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)
/(5*b) - (Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7
/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{a}\,(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{7}{2}}}+(ad-bc)^2\int\frac{1}{b^3}\,dx+\frac{d^2x^5}{5b}-\frac{dx^3\,(ad-2bc)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(a)*(a*d - b*c)**2*atan(sqrt(b)*x/sqrt(a))/b**(7/2) + (a*d -
 b*c)**2*Integral(b**(-3), x) + d**2*x**5/(5*b) - d*x**3*(a*d - 2
*b*c)/(3*b**2)
```

Mathematica [A] time = 0.124637, size = 84, normalized size = 1.

$$-\frac{\sqrt{a}(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)
/(5*b) - (Sqrt[a]*(-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b
^(7/2)
```

Maple [A] time = 0.005, size = 135, normalized size = 1.6

$$\frac{d^2x^5}{5\,b}-\frac{x^3ad^2}{3\,b^2}+\frac{2\,cx^3d}{3\,b}+\frac{a^2d^2x}{b^3}-2\,\frac{acdx}{b^2}+\frac{c^2x}{b}-\frac{a^3d^2}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+2\,\frac{a^2cd}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\frac{ac^2}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/5*d^2*x^5/b-1/3/b^2*x^3*a*d^2+2/3/b*x^3*c*d+1/b^3*a^2*d^2*x-2/b
^2*a*c*d*x+1/b*c^2*x-a^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*
d^2+2*a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d-a/b/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244503, size = 1, normalized size = 0.01

$$\left[\frac{6\,b^2d^2x^5+10\,\left(2\,b^2cd-abd^2\right)x^3+15\,\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,\left(b^2c^2-2\,abcd+a^2d^2\right)x}{30\,b^3},\ \frac{3\,b^2\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 + 10*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
- a)/(b*x^2 + a)) + 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3, 1/
15*(3*b^2*d^2*x^5 + 5*(2*b^2*c*d - a*b*d^2)*x^3 - 15*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 15*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*x)/b^3]
```

Sympy [A] time = 2.36052, size = 192, normalized size = 2.29

$$\frac{\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2\log\left(-\frac{b^3\sqrt{-\frac{a}{b^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2\log\left(\frac{b^3\sqrt{-\frac{a}{b^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^5}{5b}-\frac{x^3\left(ad^2-2bcd\right)}{3b^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] sqrt(-a/b**7)*(a*d - b*c)**2*log(-b**3*sqrt(-a/b**7)*(a*d - b*c)*
*2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-a/b**7)*(a*
d - b*c)**2*log(b**3*sqrt(-a/b**7)*(a*d - b*c)**2/(a**2*d**2 - 2*
a*b*c*d + b**2*c**2) + x)/2 + d**2*x**5/(5*b) - x**3*(a*d**2 - 2*
b*c*d)/(3*b**2) + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/b**3
```

GIAC/XCAS [A] time = 0.221803, size = 153, normalized size = 1.82

$$-\frac{\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4d^2x^5 + 10\,b^4cdx^3 - 5\,ab^3d^2x^3 + 15\,b^4c^2x - 30\,ab^3cdx + 15\,a^2b^2d^2x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^3) + 1/15*(3*b^4*d^2*x^5 + 10*b^4*c*d*x^3 - 5*a*b^3*d^2*x^
3 + 15*b^4*c^2*x - 30*a*b^3*c*d*x + 15*a^2*b^2*d^2*x)/b^5
```

3.212 $\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=61

$$\frac{(bc-ad)^2\log(a+bx^2)}{2b^3}+\frac{dx^2(bc-ad)}{2b^2}+\frac{(c+dx^2)^2}{4b}$$

```
[Out] (d*(b*c - a*d)*x^2)/(2*b^2) + (c + d*x^2)^2/(4*b) + ((b*c - a*d)^
2*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.114315, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(bc-ad)^2\log(a+bx^2)}{2b^3}+\frac{dx^2(bc-ad)}{2b^2}+\frac{(c+dx^2)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)*x^2)/(2*b^2) + (c + d*x^2)^2/(4*b) + ((b*c - a*d)^
2*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(c+dx^2)^2}{4b}-\frac{(ad-bc)\int^{x^2} d\,dx}{2b^2}+\frac{(ad-bc)^2\log(a+bx^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**2/(4*b) - (a*d - b*c)*Integral(d, (x, x**2))/(2*b**
2) + (a*d - b*c)**2*log(a + b*x**2)/(2*b**3)
```

Mathematica [A] time = 0.0385013, size = 49, normalized size = 0.8

$$\frac{bdx^2(-2ad+4bc+bdx^2)+2(bc-ad)^2\log(a+bx^2)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(4*b*c - 2*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[a + b*x^
2])/(4*b^3)
```

Maple [A] time = 0.004, size = 85, normalized size = 1.4

$$\frac{d^2x^4}{4b}-\frac{ad^2x^2}{2b^2}+\frac{dx^2c}{b}+\frac{\ln(bx^2+a)a^2d^2}{2b^3}-\frac{\ln(bx^2+a)cad}{b^2}+\frac{\ln(bx^2+a)c^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/4*d^2/b*x^4-1/2*d^2/b^2*a*x^2+d/b*x^2*c+1/2/b^3*ln(b*x^2+a)*a^2
*d^2-1/b^2*ln(b*x^2+a)*c*a*d+1/2/b*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.34031, size = 89, normalized size = 1.46

$$\frac{bd^2x^4 + 2\left(2\,bcd - ad^2\right)x^2}{4\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 2*(2*b*c*d - a*d^2)*x^2)/b^2 + 1/2*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*log(b*x^2 + a)/b^3
```

Fricas [A] time = 0.234495, size = 90, normalized size = 1.48

$$\frac{b^2d^2x^4 + 2\left(2\,b^2cd - abd^2\right)x^2 + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx^2 + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 + 2*(2*b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(b*x^2 + a))/b^3
```

Sympy [A] time = 1.97639, size = 51, normalized size = 0.84

$$\frac{d^2x^4}{4b} - \frac{x^2\left(ad^2 - 2bcd\right)}{2b^2} + \frac{(ad - bc)^2\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] d**2*x**4/(4*b) - x**2*(a*d**2 - 2*b*c*d)/(2*b**2) + (a*d - b*c)*
*2*log(a + b*x**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.225176, size = 90, normalized size = 1.48

$$\frac{bd^2x^4 + 4\,bcdx^2 - 2\,ad^2x^2}{4\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*b*c*d*x^2 - 2*a*d^2*x^2)/b^2 + 1/2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*ln(abs(b*x^2 + a))/b^3
```

3.213 $\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0918982, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^2x^3}{3b}-\frac{(ad-2bc)\int d\,dx}{b^2}+\frac{(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] d**2*x**3/(3*b) - (a*d - 2*b*c)*Integral(d, x)/b**2 + (a*d - b*c)
**2*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(5/2))
```

Mathematica [A] time = 0.0829892, size = 59, normalized size = 0.94

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{dx\left(-3ad+6bc+bdx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A] time = 0., size = 95, normalized size = 1.5

$$\frac{d^2x^3}{3\,b}-\frac{ad^2x}{b^2}+2\,\frac{dxc}{b}+\frac{a^2d^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{acd}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+c^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^2*x^3/b-d^2/b^2*a*x+2*d/b*x*c+1/b^2/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*a^2*d^2-2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*c*
d+1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232564, size = 1, normalized size = 0.02

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(bd^2x^3+3\left(2\,bcd-ad^2\right)x\right)\sqrt{-ab}}{6\,\sqrt{-ab}b^2},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\cdot\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((2*a*b*x + (b*x^2 - a
)*sqrt(-a*b))/(b*x^2 + a)) + 2*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x
)*sqrt(-a*b))/(sqrt(-a*b)*b^2), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*arctan(sqrt(a*b)*x/a) + (b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)
*sqrt(a*b))/(sqrt(a*b)*b^2)]
```

Sympy [A] time = 2.18975, size = 172, normalized size = 2.73

$$-\frac{\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}\left(ad-bc\right)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^3}{3b}-\frac{x\left(ad^2-2bcd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*sqrt(-1/(a*b**5))*(
a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-
1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*
c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b
) - x*(a*d**2 - 2*b*c*d)/b**2
```

GIAC/XCAS [A] time = 0.221278, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2d^2x^3 + 6\,b^2cdx - 3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^2) + 1/3*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

3.214 $\int \frac{(c+dx^2)^2}{x(a+bx^2)} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2\log(a+bx^2)}{2ab^2}+\frac{c^2\log(x)}{a}+\frac{d^2x^2}{2b}$$

```
[Out] (d^2*x^2)/(2*b) + (c^2*Log[x])/a - ((b*c - a*d)^2*Log[a + b*x^2])
/(2*a*b^2)
```

Rubi [A] time = 0.125849, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{(bc-ad)^2\log(a+bx^2)}{2ab^2}+\frac{c^2\log(x)}{a}+\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*x^2)/(2*b) + (c^2*Log[x])/a - ((b*c - a*d)^2*Log[a + b*x^2])
/(2*a*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^2\int^{x^2}\frac{1}{b}\,dx}{2}+\frac{c^2\log(x^2)}{2a}-\frac{(ad-bc)^2\log(a+bx^2)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x/(b*x**2+a),x)
```

```
[Out] d**2*Integral(1/b, (x, x**2))/2 + c**2*log(x**2)/(2*a) - (a*d - b
*c)**2*log(a + b*x**2)/(2*a*b**2)
```

Mathematica [A] time = 0.0355104, size = 50, normalized size = 0.98

$$\frac{-(bc-ad)^2\log(a+bx^2)+abd^2x^2+2b^2c^2\log(x)}{2ab^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2 + 2*b^2*c^2*Log[x] - (b*c - a*d)^2*Log[a + b*x^2])/(
2*a*b^2)
```

Maple [A] time = 0.006, size = 69, normalized size = 1.4

$$\frac{d^2x^2}{2b}+\frac{c^2\ln(x)}{a}-\frac{a\ln(bx^2+a)\,d^2}{2b^2}+\frac{\ln(bx^2+a)\,cd}{b}-\frac{\ln(bx^2+a)\,c^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x/(b*x^2+a),x)
```

```
[Out] 1/2*d^2*x^2/b+c^2*ln(x)/a-1/2*a/b^2*ln(b*x^2+a)*d^2+1/b*ln(b*x^2+
a)*c*d-1/2/a*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.35602, size = 82, normalized size = 1.61

$$\frac{d^2x^2}{2\,b} + \frac{c^2\log\left(x^2\right)}{2\,a} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx^2 + a\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(b*x^2 + a)/(a*b^2)
```

Fricas [A] time = 0.239131, size = 80, normalized size = 1.57

$$\frac{abd^2x^2 + 2\,b^2c^2\log\left(x\right) - \left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx^2 + a\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*d^2*x^2 + 2*b^2*c^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*log(b*x^2 + a))/(a*b^2)
```

Sympy [A] time = 5.21017, size = 41, normalized size = 0.8

$$\frac{d^2x^2}{2b} + \frac{c^2\log\left(x\right)}{a} - \frac{\left(ad - bc\right)^2\log\left(\frac{a}{b} + x^2\right)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x/(b*x**2+a),x)
```

```
[Out] d**2*x**2/(2*b) + c**2*log(x)/a - (a*d - b*c)**2*log(a/b + x**2)/
(2*a*b**2)
```

GIAC/XCAS [A] time = 0.224925, size = 84, normalized size = 1.65

$$\frac{d^2x^2}{2\,b} + \frac{c^2\ln\left(x^2\right)}{2\,a} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*ln(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*ln(abs(b*x^2 + a))/(a*b^2)
```

3.215 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=55

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.11726, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(3/2)*b^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\int\frac{1}{b}\,dx-\frac{c^2}{ax}-\frac{(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x**2/(b*x**2+a),x)
```

```
[Out] d**2*Integral(1/b, x) - c**2/(a*x) - (a*d - b*c)**2*atan(sqrt(b)*
x/sqrt(a))/(a**(3/2)*b**(3/2))
```

Mathematica [A] time = 0.0937019, size = 55, normalized size = 1.

$$-\frac{(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(3/2)*b^(3/2))
```

Maple [A] time = 0.007, size = 85, normalized size = 1.6

$$\frac{d^2x}{b}-\frac{ad^2}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+2\frac{cd}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\frac{bc^2}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{c^2}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^2/(b*x^2+a),x)
```

```
[Out] d^2*x/b-1/b*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^2+2/(a*b)^(1/
2)*arctan(x*b/(a*b)^(1/2))*c*d-b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))*c^2-c^2/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24246, size = 1, normalized size = 0.02

$$\left[\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\log\left(-\frac{2\,abx-\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\left(ad^{2}x^{2}-bc^{2}\right)\sqrt{-ab}}{2\sqrt{-ab}abx},\right.$$
$$\left.-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(ad^{2}x^{2}-bc^{2}\right)\sqrt{ab}}{\sqrt{ab}abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*log(-(2*a*b*x - (b*x^2 -
a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(a*d^2*x^2 - b*c^2)*sqrt(-a*b))/(
sqrt(-a*b)*a*b*x), -((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*arctan(sqr
t(a*b)*x/a) - (a*d^2*x^2 - b*c^2)*sqrt(a*b))/(sqrt(a*b)*a*b*x)]
```

Sympy [A] time = 2.76457, size = 165, normalized size = 3.

$$\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\left(ad-bc\right)^{2}\log\left(-\frac{a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}\left(ad-bc\right)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\left(ad-bc\right)^{2}\log\left(\frac{a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}\left(ad-bc\right)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+x\right)}{2}+\frac{d^{2}x}{b}-\frac{c^{2}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(-a**2*b*sqrt(-1/(a**3*b**
3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - s
qrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(a**2*b*sqrt(-1/(a**3*b**3)
)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**
2*x/b - c**2/(a*x)
```

GIAC/XCAS [A] time = 0.223369, size = 85, normalized size = 1.55

$$\frac{d^2x}{b} - \frac{c^2}{ax} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a*b)
```

3.216 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

```
[Out] -c^2/(2*a*x^2) - (c*(b*c - 2*a*d)*Log[x])/a^2 + ((b*c - a*d)^2*Lo
g[a + b*x^2])/(2*a^2*b)
```

Rubi [A] time = 0.148687, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] -c^2/(2*a*x^2) - (c*(b*c - 2*a*d)*Log[x])/a^2 + ((b*c - a*d)^2*Lo
g[a + b*x^2])/(2*a^2*b)
```

Rubi in Sympy [A] time = 22.9183, size = 53, normalized size = 0.91

$$-\frac{c^2}{2ax^2}+\frac{c\,(2ad-bc)\log\left(x^2\right)}{2a^2}+\frac{(ad-bc)^2\log\left(a+bx^2\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**2/x**3/(b*x**2+a),x)
```

```
[Out] -c**2/(2*a*x**2) + c*(2*a*d - b*c)*log(x**2)/(2*a**2) + (a*d - b*
c)**2*log(a + b*x**2)/(2*a**2*b)
```

Mathematica [A] time = 0.046068, size = 60, normalized size = 1.03

$$\frac{-abc^2-2bcx^2\log(x)(bc-2ad)+x^2(bc-ad)^2\log\left(a+bx^2\right)}{2a^2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] (-(a*b*c^2) - 2*b*c*(b*c - 2*a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*
Log[a + b*x^2])/(2*a^2*b*x^2)
```

Maple [A] time = 0.01, size = 81, normalized size = 1.4

$$-\frac{c^2}{2\,ax^2}+2\,\frac{c\ln(x)\,d}{a}-\frac{c^2\ln(x)\,b}{a^2}+\frac{\ln\left(bx^2+a\right)d^2}{2\,b}-\frac{\ln\left(bx^2+a\right)cd}{a}+\frac{b\ln\left(bx^2+a\right)c^2}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^3/(b*x^2+a),x)
```

```
[Out] -1/2*c^2/a/x^2+2*c/a*ln(x)*d-c^2/a^2*ln(x)*b+1/2/b*ln(b*x^2+a)*d^
2-1/a*ln(b*x^2+a)*c*d+1/2/a^2*b*ln(b*x^2+a)*c^2
```

Maxima [A] time = 1.35515, size = 93, normalized size = 1.6

$$-\frac{\left(bc^2-2\,acd\right)\log\left(x^2\right)}{2\,a^2}-\frac{c^2}{2\,ax^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx^2+a\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 - 1/2*c^2/(a*x^2) + 1/2*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*log(b*x^2 + a)/(a^2*b)
```

Fricas [A] time = 0.237571, size = 99, normalized size = 1.71

$$-\frac{abc^2-\left(b^2c^2-2\,abcd+a^2d^2\right)x^2\log\left(bx^2+a\right)+2\left(b^2c^2-2\,abcd\right)x^2\log\left(x\right)}{2\,a^2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(b*x^2 + a
) + 2*(b^2*c^2 - 2*a*b*c*d)*x^2*log(x))/(a^2*b*x^2)
```

Sympy [A] time = 5.81095, size = 49, normalized size = 0.84

$$-\frac{c^2}{2ax^2}+\frac{c\left(2ad-bc\right)\log\left(x\right)}{a^2}+\frac{\left(ad-bc\right)^2\log\left(\frac{a}{b}+x^2\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x**3/(b*x**2+a),x)
```

```
[Out] -c**2/(2*a*x**2) + c*(2*a*d - b*c)*log(x)/a**2 + (a*d - b*c)**2*l
og(a/b + x**2)/(2*a**2*b)
```

GIAC/XCAS [A] time = 0.226419, size = 122, normalized size = 2.1

$$-\frac{\left(bc^2-2\,acd\right)\ln\left(x^2\right)}{2\,a^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bc^2x^2-2\,acdx^2-ac^2}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*ln(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*ln(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*c^2*x^2 - 2*a*c*d*x^2
 - a*c^2)/(a^2*x^2)
```

3.217 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=64

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}+\frac{c(bc-2ad)}{a^2x}-\frac{c^2}{3ax^3}$$

```
[Out] -c^2/(3*a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.133459, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}+\frac{c(bc-2ad)}{a^2x}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^2/(3*a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 21.0205, size = 56, normalized size = 0.88

$$-\frac{c^2}{3ax^3}-\frac{c\,(2ad-bc)}{a^2x}+\frac{(ad-bc)^2\,\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x**4/(b*x**2+a),x)
```

```
[Out] -c**2/(3*a*x**3) - c*(2*a*d - b*c)/(a**2*x) + (a*d - b*c)**2*atan
(sqrt(b)*x/sqrt(a))/(a**(5/2)*sqrt(b))
```

Mathematica [A] time = 0.112273, size = 66, normalized size = 1.03

$$\frac{(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}-\frac{c(2ad-bc)}{a^2x}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^2/(3*a*x^3) - (c*(-(b*c) + 2*a*d))/(a^2*x) + ((-(b*c) + a*d)^2
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.01, size = 98, normalized size = 1.5

$$-\frac{c^2}{3\,ax^3}-2\,\frac{cd}{ax}+\frac{bc^2}{a^2x}+d^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{bcd}{a\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{b^2c^2}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^4/(b*x^2+a),x)
```

```
[Out] -1/3*c^2/a/x^3-2*c/a/x*d+c^2/a^2/x*b+1/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*d^2-2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*b*d+1/a^2
/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241347, size = 1, normalized size = 0.02

$$\left[\frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^3\log\left(\frac{2\,abx + \left(bx^2 - a\right)\sqrt{-ab}}{bx^2 + a}\right) - 2\left(ac^2 - 3\left(bc^2 - 2\,acd\right)x^2\right)\sqrt{-ab}}{6\,\sqrt{-ab}a^2x^3}, \frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^3\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*log((2*a*b*x + (b*x^2
 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(a*c^2 - 3*(b*c^2 - 2*a*c*d)*x
^2)*sqrt(-a*b))/(sqrt(-a*b)*a^2*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*x^3*arctan(sqrt(a*b)*x/a) - (a*c^2 - 3*(b*c^2 - 2*a*c
*d)*x^2)*sqrt(a*b))/(sqrt(a*b)*a^2*x^3)]
```

Sympy [A] time = 3.10921, size = 172, normalized size = 2.69

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad - bc)^2\log\left(-\frac{a^3\sqrt{-\frac{1}{a^5b}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^5b}}\,(ad - bc)^2\log\left(\frac{a^3\sqrt{-\frac{1}{a^5b}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2} + x\right)}{2} - \frac{ac^2 + x^2\left(6acd - 3bc^2\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - b*c)**2*log(-a**3*sqrt(-1/(a**5*b))*(a*
d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/
(a**5*b))*(a*d - b*c)**2*log(a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - (a*c**2 + x**2*(6*
a*c*d - 3*b*c**2))/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 0.224712, size = 97, normalized size = 1.52

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bc^2x^2 - 6\,acdx^2 - ac^2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^2) + 1/3*(3*b*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^2*x^3)
```

3.218 $\int \frac{x^5(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=138

$$\frac{a^2(bc-ad)^3\log\left(a+bx^2\right)}{2b^6}+\frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{6b^3}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{x^4(bc-ad)^3}{4b^4}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{d^3x^{10}}{10b}$$

```
[Out] -(a*(b*c - a*d)^3*x^2)/(2*b^5) + ((b*c - a*d)^3*x^4)/(4*b^4) + (d
*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^6)/(6*b^3) + (d^2*(3*b*c - a
*d)*x^8)/(8*b^2) + (d^3*x^10)/(10*b) + (a^2*(b*c - a*d)^3*Log[a +
 b*x^2])/(2*b^6)
```

Rubi [A] time = 0.377444, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^2(bc-ad)^3\log\left(a+bx^2\right)}{2b^6}+\frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{6b^3}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{x^4(bc-ad)^3}{4b^4}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{d^3x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)^3*x^2)/(2*b^5) + ((b*c - a*d)^3*x^4)/(4*b^4) + (d
*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^6)/(6*b^3) + (d^2*(3*b*c - a
*d)*x^8)/(8*b^2) + (d^3*x^10)/(10*b) + (a^2*(b*c - a*d)^3*Log[a +
 b*x^2])/(2*b^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(ad-bc\right)^3\log\left(a+bx^2\right)}{2b^6}+\frac{d^3x^{10}}{10b}-\frac{d^2x^8\left(ad-3bc\right)}{8b^2}+\frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{6b^3}-\frac{\left(ad-bc\right)^3\int^{x^2}x\,dx}{2b^4}+\frac{\left(ad-bc\right)^3\int^{x^2}a\,dx}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)**3*log(a + b*x**2)/(2*b**6) + d**3*x**10/(10*b)
 - d**2*x**8*(a*d - 3*b*c)/(8*b**2) + d*x**6*(a**2*d**2 - 3*a*b*c
*d + 3*b**2*c**2)/(6*b**3) - (a*d - b*c)**3*Integral(x, (x, x**2)
)/(2*b**4) + (a*d - b*c)**3*Integral(a, (x, x**2))/(2*b**5)
```

Mathematica [A] time = 0.139149, size = 128, normalized size = 0.93

$$\frac{20b^3dx^6\left(a^2d^2-3abcd+3b^2c^2\right)+60a^2(bc-ad)^3\log\left(a+bx^2\right)+15b^4d^2x^8(3bc-ad)+30b^2x^4(bc-ad)^3+60abx^2(ad-bc}{120b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (60*a*b*(-(b*c) + a*d)^3*x^2 + 30*b^2*(b*c - a*d)^3*x^4 + 20*b^3*
d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^6 + 15*b^4*d^2*(3*b*c - a*d
)*x^8 + 12*b^5*d^3*x^10 + 60*a^2*(b*c - a*d)^3*Log[a + b*x^2])/(1
20*b^6)
```

Maple [B] time = 0.006, size = 263, normalized size = 1.9

$$\frac{d^3x^{10}}{10\,b}-\frac{x^8ad^3}{8\,b^2}+\frac{3\,x^8cd^2}{8\,b}+\frac{x^6a^2d^3}{6\,b^3}-\frac{x^6acd^2}{2\,b^2}+\frac{x^6c^2d}{2\,b}-\frac{x^4a^3d^3}{4\,b^4}+\frac{3\,x^4a^2cd^2}{4\,b^3}-\frac{3\,x^4ac^2d}{4\,b^2}+\frac{x^4c^3}{4\,b}+\frac{a^4d^3x^2}{2\,b^5}-\frac{3\,a^3cd^2x^2}{2\,b^4}+\frac{3\,x^2a^2c^2d}{2\,b^3}-\frac{ac^3x^2}{2\,b^2}-\frac{a^5\ln\left(bx^2+a\right)d^3}{2\,b^6}+\frac{3\,a^4\ln\left(bx^2+a\right)cd^2}{2\,b^5}-\frac{3\,a^3\ln\left(bx^2+a\right)c^2d}{2\,b^4}+\frac{a^2\ln\left(bx^2+a\right)c^3}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/10*d^3*x^10/b-1/8/b^2*x^8*a*d^3+3/8/b*x^8*c*d^2+1/6/b^3*x^6*a^2
*d^3-1/2/b^2*x^6*a*c*d^2+1/2/b*x^6*c^2*d-1/4/b^4*x^4*a^3*d^3+3/4/
b^3*x^4*a^2*c*d^2-3/4/b^2*x^4*a*c^2*d+1/4/b*x^4*c^3+1/2/b^5*a^4*d
^3*x^2-3/2/b^4*a^3*c*d^2*x^2+3/2/b^3*a^2*c^2*d*x^2-1/2/b^2*a*c^3*
x^2-1/2*a^5/b^6*ln(b*x^2+a)*d^3+3/2*a^4/b^5*ln(b*x^2+a)*c*d^2-3/2
*a^3/b^4*ln(b*x^2+a)*c^2*d+1/2*a^2/b^3*ln(b*x^2+a)*c^3
```

Maxima [A] time = 1.34387, size = 296, normalized size = 2.14

$$\frac{12\,b^4d^3x^{10}+15\left(3\,b^4cd^2-ab^3d^3\right)x^8+20\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^6+30\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^4-}{120\,b^5}+\frac{\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^5/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 15*(3*b^4*c*d^2 - a*b^3*d^3)*x^8 + 20*(3
*b^4*c^2*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^6 + 30*(b^4*c^3 - 3*a
*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^4 - 60*(a*b^3*c^3 - 3
*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x^2)/b^5 + 1/2*(a^2*b^3
*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*log(b*x^2 + a)/
b^6
```

Fricas [A] time = 0.212227, size = 297, normalized size = 2.15

$$\frac{12\,b^5d^3x^{10}+15\left(3\,b^5cd^2-ab^4d^3\right)x^8+20\left(3\,b^5c^2d-3\,ab^4cd^2+a^2b^3d^3\right)x^6+30\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)x^4-}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^5/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*d^3*x^10 + 15*(3*b^5*c*d^2 - a*b^4*d^3)*x^8 + 20*(3
*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^6 + 30*(b^5*c^3 - 3*a
*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 60*(a*b^4*c^3 -
 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x^2 + 60*(a^2*b^3
*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*log(b*x^2 + a))
/b^6
```

Sympy [A] time = 2.85442, size = 187, normalized size = 1.36

$$-\frac{a^{2}\left(ad-bc\right)^{3}\log\left(a+bx^{2}\right)}{2b^{6}}+\frac{d^{3}x^{10}}{10b}-\frac{x^{8}\left(ad^{3}-3bcd^{2}\right)}{8b^{2}}+\frac{x^{6}\left(a^{2}d^{3}-3abcd^{2}+3b^{2}c^{2}d\right)}{6b^{3}}-\frac{x^{4}\left(a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}\right)}{4b^{4}}+\frac{x^{2}\left(a^{4}d^{3}-3a^{3}bcd^{2}+3a^{2}b^{2}c^{2}d-ab^{3}c^{3}\right)}{2b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)**3*log(a + b*x**2)/(2*b**6) + d**3*x**10/(10*b)
 - x**8*(a*d**3 - 3*b*c*d**2)/(8*b**2) + x**6*(a**2*d**3 - 3*a*b*
c*d**2 + 3*b**2*c**2*d)/(6*b**3) - x**4*(a**3*d**3 - 3*a**2*b*c*d
**2 + 3*a*b**2*c**2*d - b**3*c**3)/(4*b**4) + x**2*(a**4*d**3 - 3
*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**3)/(2*b**5)
```

GIAC/XCAS [A] time = 0.227672, size = 321, normalized size = 2.33

$$\frac{12\,b^{4}d^{3}x^{10}+45\,b^{4}cd^{2}x^{8}-15\,ab^{3}d^{3}x^{8}+60\,b^{4}c^{2}dx^{6}-60\,ab^{3}cd^{2}x^{6}+20\,a^{2}b^{2}d^{3}x^{6}+30\,b^{4}c^{3}x^{4}-90\,ab^{3}c^{2}dx^{4}+90\,a^{2}b^{2}cd^{2}x^{4}}{120\,b^{5}}+\frac{\left(a^{2}b^{3}c^{3}-3\,a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-a^{5}d^{3}\right)\ln\left(\left|bx^{2}+a\right|\right)}{2\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^5/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 45*b^4*c*d^2*x^8 - 15*a*b^3*d^3*x^8 + 60
*b^4*c^2*d*x^6 - 60*a*b^3*c*d^2*x^6 + 20*a^2*b^2*d^3*x^6 + 30*b^4
*c^3*x^4 - 90*a*b^3*c^2*d*x^4 + 90*a^2*b^2*c*d^2*x^4 - 30*a^3*b*d
^3*x^4 - 60*a*b^3*c^3*x^2 + 180*a^2*b^2*c^2*d*x^2 - 180*a^3*b*c*d
^2*x^2 + 60*a^4*d^3*x^2)/b^5 + 1/2*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d
 + 3*a^4*b*c*d^2 - a^5*d^3)*ln(abs(b*x^2 + a))/b^6
```

3.219 $\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=140

$$\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}-\frac{ax(bc-ad)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

```
[Out] -((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*
b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*
x^7)/(7*b^2) + (d^3*x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rubi [A] time = 0.217926, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}-\frac{ax(bc-ad)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*
b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*
x^7)/(7*b^2) + (d^3*x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{\frac{3}{2}}\left(ad-bc\right)^3\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{11}{2}}}+\frac{d^3x^9}{9b}-\frac{d^2x^7\left(ad-3bc\right)}{7b^2}+\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}-\frac{x^3\left(ad-bc\right)^3}{3b^4}+\frac{\left(ad-bc\right)^3\int a\,dx}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -a**(3/2)*(a*d - b*c)**3*atan(sqrt(b)*x/sqrt(a))/b**(11/2) + d**3
*x**9/(9*b) - d**2*x**7*(a*d - 3*b*c)/(7*b**2) + d*x**5*(a**2*d**
2 - 3*a*b*c*d + 3*b**2*c**2)/(5*b**3) - x**3*(a*d - b*c)**3/(3*b*
*4) + (a*d - b*c)**3*Integral(a, x)/b**5
```

Mathematica [A] time = 0.078615, size = 140, normalized size = 1.

$$-\frac{a^{3/2}(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{ax(ad-bc)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)^3*x)/b^5 + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*
b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*
x^7)/(7*b^2) + (d^3*x^9)/(9*b) - (a^(3/2)*(-(b*c) + a*d)^3*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Maple [B] time = 0.006, size = 276, normalized size = 2.

$$\frac{d^3x^9}{9\,b} - \frac{x^7ad^3}{7\,b^2} + \frac{3\,x^7cd^2}{7\,b} + \frac{x^5a^2d^3}{5\,b^3} - \frac{3\,x^5acd^2}{5\,b^2} + \frac{3\,x^5c^2d}{5\,b} - \frac{x^3a^3d^3}{3\,b^4} + \frac{x^3a^2cd^2}{b^3} - \frac{x^3ac^2d}{b^2} + \frac{x^3c^3}{3\,b} + \frac{a^4d^3x}{b^5} - 3\,\frac{a^3cd^2x}{b^4} + 3\,\frac{xa^2c^2d}{b^3} - \frac{ac^3x}{b^2} - \frac{a^5d^3}{b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + 3\,\frac{a^4cd^2}{b^4\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) - 3\,\frac{a^3c^2d}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) + \frac{a^2c^3}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/7/b^2*x^7*a*d^3+3/7/b*x^7*c*d^2+1/5/b^3*x^5*a^2*d
^3-3/5/b^2*x^5*a*c*d^2+3/5/b*x^5*c^2*d-1/3/b^4*x^3*a^3*d^3+1/b^3*
x^3*a^2*c*d^2-1/b^2*x^3*a*c^2*d+1/3/b*x^3*c^3+1/b^5*a^4*d^3*x-3/b
^4*a^3*c*d^2*x+3/b^3*a^2*c^2*d*x-1/b^2*a*c^3*x-a^5/b^5/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*d^3+3*a^4/b^4/(a*b)^(1/2)*arctan(x*b/(a
*b)^(1/2))*c*d^2-3*a^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^
2*d+a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239898, size = 1, normalized size = 0.01

$$\left[\frac{70\,b^4d^3x^9 + 90\left(3\,b^4cd^2 - ab^3d^3\right)x^7 + 126\left(3\,b^4c^2d - 3\,ab^3cd^2 + a^2b^2d^3\right)x^5 + 210\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)x^3}{630\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*d^3*x^9 + 90*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 126*(
3*b^4*c^2*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 210*(b^4*c^3 - 3
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3
- 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(-a/b)*log((b*x^
2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(a*b^3*c^3 - 3*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5, 1/315*(35*b^4*d^3*x^9
 + 45*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 63*(3*b^4*c^2*d - 3*a*b^3*c
*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^
```

```
2*c*d^2 - a^3*b*d^3)*x^3 + 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a
^3*b*c*d^2 - a^4*d^3)*sqrt(a/b)*arctan(x/sqrt(a/b)) - 315*(a*b^3*
c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5]
```

Sympy [A] time = 3.18588, size = 338, normalized size = 2.41

$$\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}+\frac{d^3x^9}{9b}-\frac{x^7\left(ad^3-3bcd^2\right)}{7b^2}+\frac{x^5\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{5b^3}-\frac{x^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3b^4}+\frac{x\left(a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(-a**3/b**11)*(a*d - b*c)**3*log(-b**5*sqrt(-a**3/b**11)*(a*d
 - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*
b**3*c**3) + x)/2 - sqrt(-a**3/b**11)*(a*d - b*c)**3*log(b**5*sqr
t(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a*
*2*b**2*c**2*d - a*b**3*c**3) + x)/2 + d**3*x**9/(9*b) - x**7*(a*
d**3 - 3*b*c*d**2)/(7*b**2) + x**5*(a**2*d**3 - 3*a*b*c*d**2 + 3*
b**2*c**2*d)/(5*b**3) - x**3*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b
**2*c**2*d - b**3*c**3)/(3*b**4) + x*(a**4*d**3 - 3*a**3*b*c*d**2
 + 3*a**2*b**2*c**2*d - a*b**3*c**3)/b**5
```

GIAC/XCAS [A] time = 0.223493, size = 325, normalized size = 2.32

$$\frac{\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}+\frac{35\,b^8d^3x^9+135\,b^8cd^2x^7-45\,ab^7d^3x^7+189\,b^8c^2dx^5-189\,ab^7cd^2x^5+63\,a^2b^6d^3x^5+105\,b^8c^3x^3-315\,ab^7c^2dx^3+315\,a^2b}{315\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*d^3*x^9 + 135*b^8*
c*d^2*x^7 - 45*a*b^7*d^3*x^7 + 189*b^8*c^2*d*x^5 - 189*a*b^7*c*d^
2*x^5 + 63*a^2*b^6*d^3*x^5 + 105*b^8*c^3*x^3 - 315*a*b^7*c^2*d*x^
3 + 315*a^2*b^6*c*d^2*x^3 - 105*a^3*b^5*d^3*x^3 - 315*a*b^7*c^3*x
 + 945*a^2*b^6*c^2*d*x - 945*a^3*b^5*c*d^2*x + 315*a^4*b^4*d^3*x)
/b^9
```

3.220 $\int \frac{x^3(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=115

$$\frac{dx^4(a^2d^2-3abcd+3b^2c^2)}{4b^3}-\frac{a(bc-ad)^3\log(a+bx^2)}{2b^5}+\frac{x^2(bc-ad)^3}{2b^4}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{d^3x^8}{8b}$$

```
[Out] ((b*c - a*d)^3*x^2)/(2*b^4) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2
)*x^4)/(4*b^3) + (d^2*(3*b*c - a*d)*x^6)/(6*b^2) + (d^3*x^8)/(8*b
) - (a*(b*c - a*d)^3*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.276325, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{dx^4(a^2d^2-3abcd+3b^2c^2)}{4b^3}-\frac{a(bc-ad)^3\log(a+bx^2)}{2b^5}+\frac{x^2(bc-ad)^3}{2b^4}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{d^3x^8}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x^2)/(2*b^4) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2
)*x^4)/(4*b^3) + (d^2*(3*b*c - a*d)*x^6)/(6*b^2) + (d^3*x^8)/(8*b
) - (a*(b*c - a*d)^3*Log[a + b*x^2])/(2*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a(ad-bc)^3\log(a+bx^2)}{2b^5}-\frac{(ad-bc)^3\int^{x^2}\frac{1}{b^4}\,dx}{2}+\frac{d^3x^8}{8b}-\frac{d^2x^6(ad-3bc)}{6b^2}+\frac{d(a^2d^2-3abcd+3b^2c^2)\int^{x^2}x\,dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)**3*log(a + b*x**2)/(2*b**5) - (a*d - b*c)**3*Integr
al(b**(-4), (x, x**2))/2 + d**3*x**8/(8*b) - d**2*x**6*(a*d - 3*b
*c)/(6*b**2) + d*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)*Integral(x
, (x, x**2))/(2*b**3)
```

Mathematica [A] time = 0.093547, size = 125, normalized size = 1.09

$$\frac{bx^2(-12a^3d^3+6a^2bd^2(6c+dx^2)-2ab^2d(18c^2+9cdx^2+2d^2x^4)+3b^3(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6))+12a(ad-bc)^3}{24b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(-12*a^3*d^3 + 6*a^2*b*d^2*(6*c + d*x^2) - 2*a*b^2*d*(18*c
^2 + 9*c*d*x^2 + 2*d^2*x^4) + 3*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d^
2*x^4 + d^3*x^6)) + 12*a*(-(b*c) + a*d)^3*Log[a + b*x^2])/(24*b^5
)
```

Maple [A] time = 0.005, size = 205, normalized size = 1.8

$$\frac{d^3x^8}{8\,b}-\frac{x^6ad^3}{6\,b^2}+\frac{x^6cd^2}{2\,b}+\frac{x^4a^2d^3}{4\,b^3}-\frac{3\,x^4acd^2}{4\,b^2}+\frac{3\,x^4c^2d}{4\,b}-\frac{a^3d^3x^2}{2\,b^4}+\frac{3\,x^2a^2cd^2}{2\,b^3}-\frac{3\,ac^2dx^2}{2\,b^2}+\frac{c^3x^2}{2\,b}+\frac{a^4\ln\left(bx^2+a\right)d^3}{2\,b^5}-\frac{3\,a^3\ln\left(bx^2+a\right)cd^2}{2\,b^4}+\frac{3\,a^2\ln\left(bx^2+a\right)c^2d}{2\,b^3}-\frac{a\ln\left(bx^2+a\right)c^3}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/8*d^3*x^8/b-1/6/b^2*x^6*a*d^3+1/2/b*x^6*c*d^2+1/4/b^3*x^4*a^2*d
^3-3/4/b^2*x^4*a*c*d^2+3/4/b*x^4*c^2*d-1/2/b^4*a^3*d^3*x^2+3/2/b^
3*a^2*c*d^2*x^2-3/2/b^2*a*c^2*d*x^2+1/2/b*c^3*x^2+1/2*a^4/b^5*ln(
b*x^2+a)*d^3-3/2*a^3/b^4*ln(b*x^2+a)*c*d^2+3/2*a^2/b^3*ln(b*x^2+a
)*c^2*d-1/2*a/b^2*ln(b*x^2+a)*c^3
```

Maxima [A] time = 1.34921, size = 227, normalized size = 1.97

$$\frac{3\,b^3d^3x^8+4\left(3\,b^3cd^2-ab^2d^3\right)x^6+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^4+12\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^2}{24\,b^4}-\frac{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 4*(3*b^3*c*d^2 - a*b^2*d^3)*x^6 + 6*(3*b^3*
c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + 12*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*
b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*log(b*x^2 + a)/b^5
```

Fricas [A] time = 0.221415, size = 228, normalized size = 1.98

$$\frac{3\,b^4d^3x^8+4\left(3\,b^4cd^2-ab^3d^3\right)x^6+6\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^4+12\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^2-12\,(}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^3*x^8 + 4*(3*b^4*c*d^2 - a*b^3*d^3)*x^6 + 6*(3*b^4*
c^2*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^4 + 12*(b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2 - 12*(a*b^3*c^3 - 3*a^2*
b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*log(b*x^2 + a))/b^5
```

Sympy [A] time = 2.65603, size = 136, normalized size = 1.18

$$\frac{a\left(ad-bc\right)^3\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^8}{8b}-\frac{x^6\left(ad^3-3bcd^2\right)}{6b^2}+\frac{x^4\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{4b^3}-\frac{x^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)**3*log(a + b*x**2)/(2*b**5) + d**3*x**8/(8*b) - x**
6*(a*d**3 - 3*b*c*d**2)/(6*b**2) + x**4*(a**2*d**3 - 3*a*b*c*d**2
 + 3*b**2*c**2*d)/(4*b**3) - x**2*(a**3*d**3 - 3*a**2*b*c*d**2 +
3*a*b**2*c**2*d - b**3*c**3)/(2*b**4)
```

GIAC/XCAS [A] time = 0.225778, size = 243, normalized size = 2.11

$$\frac{3\,b^3d^3x^8 + 12\,b^3cd^2x^6 - 4\,ab^2d^3x^6 + 18\,b^3c^2dx^4 - 18\,ab^2cd^2x^4 + 6\,a^2bd^3x^4 + 12\,b^3c^3x^2 - 36\,ab^2c^2dx^2 + 36\,a^2bcd^2x^2 - 12\,a}{24\,b^4} - \frac{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 12*b^3*c*d^2*x^6 - 4*a*b^2*d^3*x^6 + 18*b^3
*c^2*d*x^4 - 18*a*b^2*c*d^2*x^4 + 6*a^2*b*d^3*x^4 + 12*b^3*c^3*x^
2 - 36*a*b^2*c^2*d*x^2 + 36*a^2*b*c*d^2*x^2 - 12*a^3*d^3*x^2)/b^4
 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*ln
(abs(b*x^2 + a))/b^5
```

3.221 $\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=119

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}-\frac{\sqrt{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}$$

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)
/(3*b^3) + (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (S
qrt[a]*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] time = 0.191106, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}-\frac{\sqrt{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)
/(3*b^3) + (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (S
qrt[a]*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{a}\,(ad-bc)^3\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{9}{2}}}-(ad-bc)^3\int\frac{1}{b^4}\,dx+\frac{d^3x^7}{7b}-\frac{d^2x^5\,(ad-3bc)}{5b^2}+\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(a)*(a*d - b*c)**3*atan(sqrt(b)*x/sqrt(a))/b**(9/2) - (a*d -
b*c)**3*Integral(b**(-4), x) + d**3*x**7/(7*b) - d**2*x**5*(a*d -
 3*b*c)/(5*b**2) + d*x**3*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)/(
3*b**3)
```

Mathematica [A] time = 0.0665046, size = 118, normalized size = 0.99

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{\sqrt{a}(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)
/(3*b^3) + (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) + (S
qrt[a]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [B] time = 0.005, size = 218, normalized size = 1.8

$$\frac{d^3x^7}{7b}-\frac{x^5ad^3}{5b^2}+\frac{3x^5cd^2}{5b}+\frac{x^3a^2d^3}{3b^3}-\frac{x^3acd^2}{b^2}+\frac{x^3c^2d}{b}-\frac{a^3d^3x}{b^4}+3\frac{xa^2cd^2}{b^3}-3\frac{ac^2dx}{b^2}+\frac{c^3x}{b}+\frac{a^4d^3}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-3\frac{a^3cd^2}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\frac{a^2c^2d}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\frac{ac^3}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/5/b^2*x^5*a*d^3+3/5/b*x^5*c*d^2+1/3/b^3*x^3*a^2*d
^3-1/b^2*x^3*a*c*d^2+1/b*x^3*c^2*d-1/b^4*a^3*d^3*x+3/b^3*a^2*c*d^
2*x-3/b^2*a*c^2*d*x+1/b*c^3*x+a^4/b^4/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))*d^3-3*a^3/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^2+
3*a^2/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d-a/b/(a*b)^(1/
2)*arctan(x*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23907, size = 1, normalized size = 0.01

$$\left[\frac{30\,b^3d^3x^7+42\left(3\,b^3cd^2-ab^2d^3\right)x^5+70\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^3-105\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-\frac{a}{b}}\,\mathrm{lo}}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^3*x^7 + 42*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 70*(3
*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a/b)*log((b*x^2 + 2*b*
x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*x)/b^4, 1/105*(15*b^3*d^3*x^7 + 21*(3*b^3
*c*d^2 - a*b^2*d^3)*x^5 + 35*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b
*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 105*(b^3*c^3 - 3*a*b^2*c^2*d +
 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4]
```

Sympy [A] time = 2.94718, size = 275, normalized size = 2.31

$$
-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^7}{7b}-\frac{x^5\left(ad^3-3bcd^2\right)}{5b^2}+\frac{x^3\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{3b^3}-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -sqrt(-a/b**9)*(a*d - b*c)**3*log(-b**4*sqrt(-a/b**9)*(a*d - b*c)
**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
 x)/2 + sqrt(-a/b**9)*(a*d - b*c)**3*log(b**4*sqrt(-a/b**9)*(a*d
- b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c
**3) + x)/2 + d**3*x**7/(7*b) - x**5*(a*d**3 - 3*b*c*d**2)/(5*b**
2) + x**3*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2*d)/(3*b**3) - x
*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/b**4
```

GIAC/XCAS [A] time = 0.229567, size = 248, normalized size = 2.08

$$
-\frac{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{15\,b^6d^3x^7+63\,b^6cd^2x^5-21\,ab^5d^3x^5+105\,b^6c^2dx^3-105\,ab^5cd^2x^3+35\,a^2b^4d^3x^3+105\,b^6c^3x-315\,ab^5c^2dx+315\,a^2b^4cd}{105\,b^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^3*x^7 + 63*b^6*c*
d^2*x^5 - 21*a*b^5*d^3*x^5 + 105*b^6*c^2*d*x^3 - 105*a*b^5*c*d^2*
x^3 + 35*a^2*b^4*d^3*x^3 + 105*b^6*c^3*x - 315*a*b^5*c^2*d*x + 31
5*a^2*b^4*c*d^2*x - 105*a^3*b^3*d^3*x)/b^7
```

3.222 $\int \frac{x(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=87

$$\frac{(bc-ad)^3\log(a+bx^2)}{2b^4}+\frac{dx^2(bc-ad)^2}{2b^3}+\frac{(c+dx^2)^2(bc-ad)}{4b^2}+\frac{(c+dx^2)^3}{6b}$$

```
[Out] (d*(b*c - a*d)^2*x^2)/(2*b^3) + ((b*c - a*d)*(c + d*x^2)^2)/(4*b^
2) + (c + d*x^2)^3/(6*b) + ((b*c - a*d)^3*Log[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.168154, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(bc-ad)^3\log(a+bx^2)}{2b^4}+\frac{dx^2(bc-ad)^2}{2b^3}+\frac{(c+dx^2)^2(bc-ad)}{4b^2}+\frac{(c+dx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)^2*x^2)/(2*b^3) + ((b*c - a*d)*(c + d*x^2)^2)/(4*b^
2) + (c + d*x^2)^3/(6*b) + ((b*c - a*d)^3*Log[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{(c+dx^2)^3}{6b}-\frac{(c+dx^2)^2(ad-bc)}{4b^2}+\frac{(ad-bc)^2\int^{x^2} d\,dx}{2b^3}-\frac{(ad-bc)^3\log(a+bx^2)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**3/(6*b) - (c + d*x**2)**2*(a*d - b*c)/(4*b**2) + (a
*d - b*c)**2*Integral(d, (x, x**2))/(2*b**3) - (a*d - b*c)**3*log
(a + b*x**2)/(2*b**4)
```

Mathematica [A] time = 0.0537773, size = 82, normalized size = 0.94

$$\frac{bdx^2\left(6a^2d^2-3abd(6c+dx^2)+b^2(18c^2+9cdx^2+2d^2x^4)\right)+6(bc-ad)^3\log(a+bx^2)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x^2) + b^2*(18*c^2 + 9*c*d
*x^2 + 2*d^2*x^4)) + 6*(b*c - a*d)^3*Log[a + b*x^2])/(12*b^4)
```

Maple [A] time = 0.005, size = 149, normalized size = 1.7

$$\frac{d^3x^6}{6\,b}-\frac{d^3x^4a}{4\,b^2}+\frac{3\,d^2x^4c}{4\,b}+\frac{d^3x^2a^2}{2\,b^3}-\frac{3\,d^2x^2ac}{2\,b^2}+\frac{3\,dx^2c^2}{2\,b}-\frac{\ln(bx^2+a)\,a^3d^3}{2\,b^4}+\frac{3\,\ln(bx^2+a)\,a^2cd^2}{2\,b^3}-\frac{3\,\ln(bx^2+a)\,ac^2d}{2\,b^2}+\frac{\ln(bx^2+a)\,c^3}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/6*d^3/b*x^6-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+1/2*d^3/b^3*x^2*a
^2-3/2*d^2/b^2*x^2*a*c+3/2*d/b*x^2*c^2-1/2/b^4*ln(b*x^2+a)*a^3*d^
3+3/2/b^3*ln(b*x^2+a)*a^2*c*d^2-3/2/b^2*ln(b*x^2+a)*a*c^2*d+1/2/b
*ln(b*x^2+a)*c^3
```

Maxima [A] time = 1.32954, size = 161, normalized size = 1.85

$$\frac{2\,b^2d^3x^6 + 3\left(3\,b^2cd^2 - abd^3\right)x^4 + 6\left(3\,b^2c^2d - 3\,abcd^2 + a^2d^3\right)x^2}{12\,b^3} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 6*(3*b^2*c^
2*d - 3*a*b*c*d^2 + a^2*d^3)*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.226952, size = 162, normalized size = 1.86

$$\frac{2\,b^3d^3x^6 + 3\left(3\,b^3cd^2 - ab^2d^3\right)x^4 + 6\left(3\,b^3c^2d - 3\,ab^2cd^2 + a^2bd^3\right)x^2 + 6\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(bx^2 + a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^3*x^6 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 6*(3*b^3*
c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + 6*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a))/b^4
```

Sympy [A] time = 2.4438, size = 88, normalized size = 1.01

$$\frac{d^3x^6}{6b} - \frac{x^4\left(ad^3 - 3bcd^2\right)}{4b^2} + \frac{x^2\left(a^2d^3 - 3abcd^2 + 3b^2c^2d\right)}{2b^3} - \frac{(ad - bc)^3\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] d**3*x**6/(6*b) - x**4*(a*d**3 - 3*b*c*d**2)/(4*b**2) + x**2*(a**
2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2*d)/(2*b**3) - (a*d - b*c)**3*
log(a + b*x**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.222008, size = 167, normalized size = 1.92

$$\frac{2\,b^2d^3x^6 + 9\,b^2cd^2x^4 - 3\,abd^3x^4 + 18\,b^2c^2dx^2 - 18\,abcd^2x^2 + 6\,a^2d^3x^2}{12\,b^3} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 9*b^2*c*d^2*x^4 - 3*a*b*d^3*x^4 + 18*b^2*c^
2*d*x^2 - 18*a*b*c*d^2*x^2 + 6*a^2*d^3*x^2)/b^3 + 1/2*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*ln(abs(b*x^2 + a))/b^4
```

3.223 $\int \frac{(c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=98

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^3)/(3*b^2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.130333, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^3)/(3*b^2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^3x^5}{5b}-\frac{d^2x^3\,(ad-3bc)}{3b^2}+\frac{\left(a^2d^2-3abcd+3b^2c^2\right)\int d\,dx}{b^3}-\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] d**3*x**5/(5*b) - d**2*x**3*(a*d - 3*b*c)/(3*b**2) + (a**2*d**2 -
 3*a*b*c*d + 3*b**2*c**2)*Integral(d, x)/b**3 - (a*d - b*c)**3*at
an(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(7/2))
```

Mathematica [A] time = 0.114231, size = 92, normalized size = 0.94

$$\frac{dx\left(15a^2d^2-5abd\left(9c+dx^2\right)+3b^2\left(15c^2+5cdx^2+d^2x^4\right)\right)}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*
x^2 + d^2*x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*b^(7/2))
```

Maple [A] time = 0., size = 161, normalized size = 1.6

$$\frac{d^3x^5}{5\,b} - \frac{d^3x^3a}{3\,b^2} + \frac{d^2x^3c}{b} + \frac{a^2d^3x}{b^3} - 3\,\frac{ad^2cx}{b^2} + 3\,\frac{dc^2x}{b} - \frac{a^3d^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + 3\,\frac{a^2cd^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) - 3\,\frac{ac^2d}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right) + c^3\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/5*d^3*x^5/b-1/3*d^3/b^2*x^3*a+d^2/b*x^3*c+d^3/b^3*a^2*x-3*d^2/b
^2*a*c*x+3*d/b*c^2*x-1/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^
3*d^3+3/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*c*d^2-3/b/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*c^2*d+1/(a*b)^(1/2)*arctan(x*b
/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233497, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right) - 2\left(3\,b^2d^3x^5 + 5\left(3\,b^2cd^2 - abd^3\right)x^3 + 15\left(3\,b^2c^2d - 3\right.\right.}{30\sqrt{-ab}b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(3*b^2*d^3
*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(3*b^2*c^2*d - 3*a*b*c*
d^2 + a^2*d^3)*x)*sqrt(-a*b))/(sqrt(-a*b)*b^3), 1/15*(15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(a*b)*x/a)
 + (3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(3*b^2*c^2
*d - 3*a*b*c*d^2 + a^2*d^3)*x)*sqrt(a*b))/(sqrt(a*b)*b^3)]
```

Sympy [A] time = 2.78379, size = 240, normalized size = 2.45

$$\frac{\sqrt{-\frac{1}{ab^7}}\left(ad - bc\right)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}\left(ad - bc\right)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} + \frac{d^3x^5}{5b} - \frac{x^3\left(ad^3 - 3bcd^2\right)}{3b^2} + \frac{x\left(a^2d^3 - 3abcd^2 + 3b^2c^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**
3*c**3) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt
(-1/(a*b**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b
**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**5/(5*b) - x**3*(a*d**3 -
 3*b*c*d**2)/(3*b**2) + x*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2
*d)/b**3
```

GIAC/XCAS [A] time = 0.226119, size = 174, normalized size = 1.78

$$\frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4d^3x^5 + 15\,b^4cd^2x^3 - 5\,ab^3d^3x^3 + 45\,b^4c^2dx - 45\,ab^3cd^2x + 15\,a^2b^2d^3x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3
 - 5*a*b^3*d^3*x^3 + 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b
^2*d^3*x)/b^5
```

3.224 $\int \frac{(c+dx^2)^3}{x(a+bx^2)} dx$

Optimal. Leaf size=73

$$-\frac{(bc-ad)^3 \log(a+bx^2)}{2ab^3}+\frac{d^2x^2(3bc-ad)}{2b^2}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

```
[Out] (d^2*(3*b*c - a*d)*x^2)/(2*b^2) + (d^3*x^4)/(4*b) + (c^3*Log[x])/
a - ((b*c - a*d)^3*Log[a + b*x^2])/(2*a*b^3)
```

Rubi [A] time = 0.178027, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{(bc-ad)^3 \log(a+bx^2)}{2ab^3}+\frac{d^2x^2(3bc-ad)}{2b^2}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*(3*b*c - a*d)*x^2)/(2*b^2) + (d^3*x^4)/(4*b) + (c^3*Log[x])/
a - ((b*c - a*d)^3*Log[a + b*x^2])/(2*a*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^2(ad-3bc)\int^{x^2}\frac{1}{b^2}\,dx}{2}+\frac{d^3\int^{x^2}x\,dx}{2b}+\frac{c^3\log(x^2)}{2a}+\frac{(ad-bc)^3\log(a+bx^2)}{2ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**3/x/(b*x**2+a),x)
```

```
[Out] -d**2*(a*d - 3*b*c)*Integral(b**(-2), (x, x**2))/2 + d**3*Integra
l(x, (x, x**2))/(2*b) + c**3*log(x**2)/(2*a) + (a*d - b*c)**3*log
(a + b*x**2)/(2*a*b**3)
```

Mathematica [A] time = 0.0510562, size = 65, normalized size = 0.89

$$\frac{abd^2x^2(-2ad+6bc+bdx^2)-2(bc-ad)^3\log(a+bx^2)+4b^3c^3\log(x)}{4ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2*(6*b*c - 2*a*d + b*d*x^2) + 4*b^3*c^3*Log[x] - 2*(b*
c - a*d)^3*Log[a + b*x^2])/(4*a*b^3)
```

Maple [A] time = 0.008, size = 116, normalized size = 1.6

$$\frac{d^3x^4}{4\,b}-\frac{d^3ax^2}{2\,b^2}+\frac{3\,d^2x^2c}{2\,b}+\frac{c^3\ln(x)}{a}+\frac{a^2\ln(bx^2+a)\,d^3}{2\,b^3}-\frac{3\,a\ln(bx^2+a)\,cd^2}{2\,b^2}+\frac{3\,\ln(bx^2+a)\,c^2d}{2\,b}-\frac{\ln(bx^2+a)\,c^3}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x/(b*x^2+a),x)
```

```
[Out] 1/4*d^3*x^4/b-1/2*d^3/b^2*a*x^2+3/2*d^2/b*x^2*c+c^3*ln(x)/a+1/2*a
^2/b^3*ln(b*x^2+a)*d^3-3/2*a/b^2*ln(b*x^2+a)*c*d^2+3/2/b*ln(b*x^2
+a)*c^2*d-1/2/a*ln(b*x^2+a)*c^3
```

Maxima [A] time = 1.34101, size = 132, normalized size = 1.81

$$\frac{c^3 \log\left(x^2\right)}{2\,a} + \frac{bd^3x^4 + 2\left(3\,bcd^2 - ad^3\right)x^2}{4\,b^2} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(bx^2 + a\right)}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - a*d^3)*x^2)/
b^2 - 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(b*x^2 + a)/(a*b^3)
```

Fricas [A] time = 0.226587, size = 136, normalized size = 1.86

$$\frac{ab^2d^3x^4 + 4\,b^3c^3\log(x) + 2\left(3\,ab^2cd^2 - a^2bd^3\right)x^2 - 2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(bx^2 + a\right)}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] 1/4*(a*b^2*d^3*x^4 + 4*b^3*c^3*log(x) + 2*(3*a*b^2*c*d^2 - a^2*b*
d^3)*x^2 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(b*x^2 + a))/(a*b^3)
```

Sympy [A] time = 7.28784, size = 63, normalized size = 0.86

$$\frac{d^3x^4}{4b} - \frac{x^2\left(ad^3 - 3bcd^2\right)}{2b^2} + \frac{c^3\log(x)}{a} + \frac{(ad - bc)^3\log\left(\frac{a}{b} + x^2\right)}{2ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x/(b*x**2+a),x)
```

```
[Out] d**3*x**4/(4*b) - x**2*(a*d**3 - 3*b*c*d**2)/(2*b**2) + c**3*log(
x)/a + (a*d - b*c)**3*log(a/b + x**2)/(2*a*b**3)
```

GIAC/XCAS [A] time = 0.240043, size = 134, normalized size = 1.84

$$\frac{c^3\ln\left(x^2\right)}{2\,a} + \frac{bd^3x^4 + 6\,bcd^2x^2 - 2\,ad^3x^2}{4\,b^2} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*c^3*ln(x^2)/a + 1/4*(b*d^3*x^4 + 6*b*c*d^2*x^2 - 2*a*d^3*x^2)
/b^2 - 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*ln
(abs(b*x^2 + a))/(a*b^3)
```

3.225 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=77

$$-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*
c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.153554, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*
c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^2(ad-3bc)\int\frac{1}{b^2}\,dx+\frac{d^3x^3}{3b}-\frac{c^3}{ax}+\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**2/(b*x**2+a),x)
```

```
[Out] -d**2*(a*d - 3*b*c)*Integral(b**(-2), x) + d**3*x**3/(3*b) - c**3
/(a*x) + (a*d - b*c)**3*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*b**(5/2
))
```

Mathematica [A] time = 0.0544016, size = 76, normalized size = 0.99

$$\frac{(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) + ((-(
b*c) + a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Maple [A] time = 0.008, size = 135, normalized size = 1.8

$$\frac{d^3x^3}{3b}-\frac{d^3ax}{b^2}+3\frac{d^2xc}{b}+\frac{a^2d^3}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-3\frac{acd^2}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)$$
$$+3\frac{c^2d}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\frac{bc^3}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{c^3}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^3*x^3/b-d^3/b^2*a*x+3*d^2/b*x*c+a^2/b^2/(a*b)^(1/2)*arctan(
x*b/(a*b)^(1/2))*d^3-3*a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*
d^2+3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d-1/a*b/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))*c^3-c^3/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236956, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(abd^3x^4-3\,b^2c^3+3\left(3\,abcd^2-a^2d^3\right)x^2\right)\sqrt{-ab}}{6\sqrt{-ab}ab^2x},\right.$$
$$\left.-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(abd^3x^4-3\,b^2c^3+3\left(3\,abcd^2-a^2d^3\right)x^2\right)\sqrt{ab}}{3\sqrt{ab}ab^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x*lo
g((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(a*b*d^3*x^
4 - 3*b^2*c^3 + 3*(3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-a*b))/(sqrt(
-a*b)*a*b^2*x), -1/3*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*x*arctan(sqrt(a*b)*x/a) - (a*b*d^3*x^4 - 3*b^2*c^3 + 3
*(3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(a*b))/(sqrt(a*b)*a*b^2*x)]
```

Sympy [A] time = 3.65107, size = 221, normalized size = 2.87

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^3}{3b}-\frac{x\left(ad^3-3bcd^2\right)}{b^2}-\frac{c^3}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*log(-a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c*
```

```
*2*d - b**3*c**3) + x)/2 + sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*lo
g(a**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a*
*2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**3/(3*
b) - x*(a*d**3 - 3*b*c*d**2)/b**2 - c**3/(a*x)
```

GIAC/XCAS [A] time = 0.234263, size = 140, normalized size = 1.82

$$-\frac{c^3}{ax} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2} + \frac{b^2d^3x^3 + 9\,b^2cd^2x - 3\,abd^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -c^3/(a*x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/3*(b^2*d^3*x^3 + 9*b^
2*c*d^2*x - 3*a*b*d^3*x)/b^3
```

3.226 $\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=73

$$\frac{(bc-ad)^3\log(a+bx^2)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

```
[Out] -c^3/(2*a*x^2) + (d^3*x^2)/(2*b) - (c^2*(b*c - 3*a*d)*Log[x])/a^2
 + ((b*c - a*d)^3*Log[a + b*x^2])/(2*a^2*b^2)
```

Rubi [A] time = 0.181806, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(bc-ad)^3\log(a+bx^2)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] -c^3/(2*a*x^2) + (d^3*x^2)/(2*b) - (c^2*(b*c - 3*a*d)*Log[x])/a^2
 + ((b*c - a*d)^3*Log[a + b*x^2])/(2*a^2*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^3\int^{x^2}\frac{1}{b}\,dx}{2}-\frac{c^3}{2ax^2}+\frac{c^2(3ad-bc)\log(x^2)}{2a^2}-\frac{(ad-bc)^3\log(a+bx^2)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**3/(b*x**2+a),x)
```

```
[Out] d**3*Integral(1/b, (x, x**2))/2 - c**3/(2*a*x**2) + c**2*(3*a*d -
 b*c)*log(x**2)/(2*a**2) - (a*d - b*c)**3*log(a + b*x**2)/(2*a**2
*b**2)
```

Mathematica [A] time = 0.060464, size = 75, normalized size = 1.03

$$\frac{-2b^2c^2x^2\log(x)(bc-3ad)+ab(ad^3x^4-bc^3)+x^2(bc-ad)^3\log(a+bx^2)}{2a^2b^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] (a*b*(-(b*c^3) + a*d^3*x^4) - 2*b^2*c^2*(b*c - 3*a*d)*x^2*Log[x]
+ (b*c - a*d)^3*x^2*Log[a + b*x^2])/(2*a^2*b^2*x^2)
```

Maple [A] time = 0.011, size = 114, normalized size = 1.6

$$\frac{d^3x^2}{2b}-\frac{c^3}{2ax^2}+3\frac{c^2\ln(x)d}{a}-\frac{c^3\ln(x)b}{a^2}-\frac{a\ln(bx^2+a)d^3}{2b^2}+\frac{3\ln(bx^2+a)cd^2}{2b}-\frac{3\ln(bx^2+a)c^2d}{2a}+\frac{b\ln(bx^2+a)c^3}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^3/(b*x^2+a),x)
```

```
[Out] 1/2*d^3*x^2/b-1/2*c^3/a/x^2+3*c^2/a*ln(x)*d-c^3/a^2*ln(x)*b-1/2*a
/b^2*ln(b*x^2+a)*d^3+3/2/b*ln(b*x^2+a)*c*d^2-3/2/a*ln(b*x^2+a)*c^
2*d+1/2/a^2*b*ln(b*x^2+a)*c^3
```

Maxima [A] time = 1.33891, size = 131, normalized size = 1.79

$$\frac{d^3x^2}{2\,b} - \frac{c^3}{2\,ax^2} - \frac{\left(bc^3 - 3\,ac^2d\right)\log\left(x^2\right)}{2\,a^2} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(bx^2 + a\right)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*c^3/(a*x^2) - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2
)/a^2 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*l
og(b*x^2 + a)/(a^2*b^2)
```

Fricas [A] time = 0.232795, size = 142, normalized size = 1.95

$$\frac{a^2bd^3x^4 - ab^2c^3 + \left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x^2\log\left(bx^2 + a\right) - 2\left(b^3c^3 - 3\,ab^2c^2d\right)x^2\log\left(x\right)}{2\,a^2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*d^3*x^4 - a*b^2*c^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2*log(b*x^2 + a) - 2*(b^3*c^3 - 3*a*b^2*c^2
*d)*x^2*log(x))/(a^2*b^2*x^2)
```

Sympy [A] time = 8.89395, size = 63, normalized size = 0.86

$$\frac{d^3x^2}{2b} - \frac{c^3}{2ax^2} + \frac{c^2\left(3ad - bc\right)\log\left(x\right)}{a^2} - \frac{\left(ad - bc\right)^3\log\left(\frac{a}{b} + x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**3/(b*x**2+a),x)
```

```
[Out] d**3*x**2/(2*b) - c**3/(2*a*x**2) + c**2*(3*a*d - b*c)*log(x)/a**
2 - (a*d - b*c)**3*log(a/b + x**2)/(2*a**2*b**2)
```

GIAC/XCAS [A] time = 0.224075, size = 162, normalized size = 2.22

$$\frac{d^3x^2}{2\,b} - \frac{\left(bc^3 - 3\,ac^2d\right)\ln\left(x^2\right)}{2\,a^2} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,a^2b^2} + \frac{bc^3x^2 - 3\,ac^2dx^2 - ac^3}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*(b*c^3 - 3*a*c^2*d)*ln(x^2)/a^2 + 1/2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*ln(abs(b*x^2 + a))/(
a^2*b^2) + 1/2*(b*c^3*x^2 - 3*a*c^2*d*x^2 - a*c^3)/(a^2*x^2)
```

3.227 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)} \, dx$

Optimal. Leaf size=74

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c
- a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.157758, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c
- a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\int\frac{1}{b}\,dx-\frac{c^3}{3ax^3}-\frac{c^2\,(3ad-bc)}{a^2x}-\frac{(ad-bc)^3\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**4/(b*x**2+a),x)
```

```
[Out] d**3*Integral(1/b, x) - c**3/(3*a*x**3) - c**2*(3*a*d - b*c)/(a**
2*x) - (a*d - b*c)**3*atan(sqrt(b)*x/sqrt(a))/(a**(5/2)*b**(3/2))
```

Mathematica [A] time = 0.0633176, size = 74, normalized size = 1.

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c
- a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Maple [B] time = 0.01, size = 135, normalized size = 1.8

$$\frac{d^3x}{b}-\frac{c^3}{3\,ax^3}-3\,\frac{c^2d}{ax}+\frac{bc^3}{a^2x}-\frac{ad^3}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+3\,\frac{cd^2}{\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)$$
$$-3\,\frac{bc^2d}{a\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{b^2c^3}{a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^4/(b*x^2+a),x)
```

```
[Out] d^3*x/b-1/3*c^3/a/x^3-3*c^2/x/a*d+c^3/x/a^2*b-a/b/(a*b)^(1/2)*arc
tan(x*b/(a*b)^(1/2))*d^3+3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*
d^2-3/a*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d+1/a^2*b^2/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242014, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^3\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(3\,a^2d^3x^4-abc^3+3\left(b^2c^3-3\,abc^2d\right)x^2\right)\sqrt{-ab}}{6\sqrt{-ab}a^2bx^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3*
log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(3*a^2*d
^3*x^4 - a*b*c^3 + 3*(b^2*c^3 - 3*a*b*c^2*d)*x^2)*sqrt(-a*b))/(sq
rt(-a*b)*a^2*b*x^3), 1/3*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*x^3*arctan(sqrt(a*b)*x/a) + (3*a^2*d^3*x^4 - a*b*c
^3 + 3*(b^2*c^3 - 3*a*b*c^2*d)*x^2)*sqrt(a*b))/(sqrt(a*b)*a^2*b*x
^3)]
```

Sympy [A] time = 4.92188, size = 221, normalized size = 2.99

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x}{b}-\frac{ac^3+x^2\left(9ac^2d-3bc^3\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(-a**3*b*sqrt(-1/(a**5*b**
3))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
 - b**3*c**3) + x)/2 - sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(a*
*3*b*sqrt(-1/(a**5*b**3))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*
d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x/b - (a*c**3 +
```

```
x**2*(9*a*c**2*d - 3*b*c**3))/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 0.230552, size = 135, normalized size = 1.82

$$\frac{d^3x}{b}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b}+\frac{3\,bc^3x^2-9\,ac^2dx^2-ac^3}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/3*(3*b*c^3*x^2 - 9*a*c^2
*d*x^2 - a*c^3)/(a^2*x^3)
```

3.228 $\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2\log\left(a+bx^2\right)}{2b^2(bc-ad)}-\frac{c^2\log\left(c+dx^2\right)}{2d^2(bc-ad)}+\frac{x^2}{2bd}$$

```
[Out] x^2/(2*b*d) + (a^2*Log[a + b*x^2])/(2*b^2*(b*c - a*d)) - (c^2*Log
[c + d*x^2])/(2*d^2*(b*c - a*d))
```

Rubi [A] time = 0.17453, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2\log\left(a+bx^2\right)}{2b^2(bc-ad)}-\frac{c^2\log\left(c+dx^2\right)}{2d^2(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x^2/(2*b*d) + (a^2*Log[a + b*x^2])/(2*b^2*(b*c - a*d)) - (c^2*Log
[c + d*x^2])/(2*d^2*(b*c - a*d))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\log\left(a+bx^2\right)}{2b^2\left(ad-bc\right)}+\frac{c^2\log\left(c+dx^2\right)}{2d^2\left(ad-bc\right)}+\frac{\int^{x^2}\frac{1}{b}\,dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -a**2*log(a + b*x**2)/(2*b**2*(a*d - b*c)) + c**2*log(c + d*x**2)
/(2*d**2*(a*d - b*c)) + Integral(1/b, (x, x**2))/(2*d)
```

Mathematica [A] time = 0.0500274, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^2\right)-b\left(dx^2(ad-bc)+bc^2\log\left(c+dx^2\right)\right)}{2b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^2] - b*(d*(-(b*c) + a*d)*x^2 + b*c^2*Log[c +
 d*x^2]))/(2*b^2*d^2*(b*c - a*d))
```

Maple [A] time = 0.01, size = 65, normalized size = 0.9

$$\frac{x^2}{2\,bd}+\frac{c^2\ln\left(dx^2+c\right)}{\left(2\,ad-2\,bc\right)d^2}-\frac{a^2\ln\left(bx^2+a\right)}{\left(2\,ad-2\,bc\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*x^2/b/d+1/2*c^2/(a*d-b*c)/d^2*ln(d*x^2+c)-1/2*a^2/(a*d-b*c)/b
^2*ln(b*x^2+a)
```

Maxima [A] time = 1.33316, size = 92, normalized size = 1.31

$$\frac{a^2 \log \left(bx^2 + a\right)}{2 \left(b^3 c - ab^2 d\right)} - \frac{c^2 \log \left(dx^2 + c\right)}{2 \left(bcd^2 - ad^3\right)} + \frac{x^2}{2 \, bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/(b^3*c - a*b^2*d) - 1/2*c^2*log(d*x^2 + c)
/(b*c*d^2 - a*d^3) + 1/2*x^2/(b*d)
```

Fricas [A] time = 0.247015, size = 97, normalized size = 1.39

$$\frac{a^2 d^2 \log \left(bx^2 + a\right) - b^2 c^2 \log \left(dx^2 + c\right) + \left(b^2 cd - abd^2\right) x^2}{2 \left(b^3 cd^2 - ab^2 d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*log(b*x^2 + a) - b^2*c^2*log(d*x^2 + c) + (b^2*c*d -
 a*b*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [A] time = 11.7071, size = 201, normalized size = 2.87

$$-\frac{a^2 \log \left(x^2 + \frac{\frac{a^4 d^3}{b(ad-bc)} - \frac{2a^3 cd^2}{ad-bc} + \frac{a^2 bc^2 d}{ad-bc} + a^2 cd + abc^2}{a^2 d^2 + b^2 c^2}\right)}{2b^2 \left(ad - bc\right)} + \frac{c^2 \log \left(x^2 + \frac{-\frac{a^2 bc^2 d}{ad-bc} + a^2 cd + \frac{2ab^2 c^3}{ad-bc} + abc^2 - \frac{b^3 c^4}{d(ad-bc)}}{a^2 d^2 + b^2 c^2}\right)}{2d^2 \left(ad - bc\right)} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -a**2*log(x**2 + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d
- b*c) + a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d
**2 + b**2*c**2))/(2*b**2*(a*d - b*c)) + c**2*log(x**2 + (-a**2*b
*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b**2*c**3/(a*d - b*c) + a*b*
c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b**2*c**2))/(2*d**
2*(a*d - b*c)) + x**2/(2*b*d)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.229 $\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=78

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

```
[Out] x/(b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*(b*c - a
*d)) - (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(d^(3/2)*(b*c - a*d)
)
```

Rubi [A] time = 0.222778, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x/(b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*(b*c - a
*d)) - (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(d^(3/2)*(b*c - a*d)
)
```

Rubi in Sympy [A] time = 35.5842, size = 65, normalized size = 0.83

$$-\frac{a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}(ad-bc)}+\frac{c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{\frac{3}{2}}(ad-bc)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -a**(3/2)*atan(sqrt(b)*x/sqrt(a))/(b**(3/2)*(a*d - b*c)) + c**(3/
2)*atan(sqrt(d)*x/sqrt(c))/(d**(3/2)*(a*d - b*c)) + x/(b*d)
```

Mathematica [A] time = 0.148814, size = 74, normalized size = 0.95

$$\frac{\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{ax}{b}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}}+\frac{cx}{d}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((a*x)/b) + (c*x)/d + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(
3/2) - (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(3/2))/(b*c - a*d)
```

Maple [A] time = 0.012, size = 73, normalized size = 0.9

$$\frac{x}{bd}+\frac{c^2}{(ad-bc)\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2}{(ad-bc)\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] x/b/d+1/d*c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/b*a
^2/(a*d-b*c)/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248432, size = 1, normalized size = 0.01

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\,(bc-ad)x}{2\,(b^2cd-abd^2)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\cdots}{d\cdots}\right)\cdots}{2\,(b^2cd-abd^2)}\right.$$

$$-\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{x}{\sqrt{\frac{c}{d}}}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,(bc-ad)x}{2\,(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-bc\sqrt{\frac{c}{d}}\arctan\left(\frac{x}{\sqrt{\frac{c}{d}}}\right)+(bc-a\cdots}{b^2cd-abd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) + b*c*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 +
 c)) - 2*(b*c - a*d)*x)/(b^2*c*d - a*b*d^2), 1/2*(2*a*d*sqrt(a/b)
*arctan(x/sqrt(a/b)) - b*c*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/
d) - c)/(d*x^2 + c)) + 2*(b*c - a*d)*x)/(b^2*c*d - a*b*d^2), -1/2
*(2*b*c*sqrt(c/d)*arctan(x/sqrt(c/d)) + a*d*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b*c - a*d)*x)/(b^2*c*d
 - a*b*d^2), (a*d*sqrt(a/b)*arctan(x/sqrt(a/b)) - b*c*sqrt(c/d)*a
rctan(x/sqrt(c/d)) + (b*c - a*d)*x)/(b^2*c*d - a*b*d^2)]
```

Sympy [A] time = 15.7113, size = 921, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c
) - a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4
*c*d**5*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a
**3/b**3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3
/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**
3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2*(a*d - b*c)) + sqrt(-
```

```
a**3/b**3)*log(x + (a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) + a**3
*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5*
(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3
)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d
 - b*c)**3 + b**4*c**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2
 + a**2*b*c**2*d + a*b**2*c**3))/(2*(a*d - b*c)) - sqrt(-c**3/d**
3)*log(x + (-a**4*d**4*sqrt(-c**3/d**3)/(a*d - b*c) - a**3*b**3*d
**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-c**3/
d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d**3)**(3/2
)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)
**3 - b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2
*b*c**2*d + a*b**2*c**3))/(2*(a*d - b*c)) + sqrt(-c**3/d**3)*log(
x + (a**4*d**4*sqrt(-c**3/d**3)/(a*d - b*c) + a**3*b**3*d**6*(-c*
*3/d**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5*(-c**3/d**3)**(
3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-c**3/d**3)**(3/2)/(a*d -
 b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b*
*4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*
d + a*b**2*c**3))/(2*(a*d - b*c)) + x/(b*d)
```

GIAC/XCAS [A] time = 0.292039, size = 576, normalized size = 7.38

$$-\frac{\left(\sqrt{cd}b^3c^2d|d| + \sqrt{cd}a^2bd^3|d| + \sqrt{cd}bc\left|b^2cd - abd^2\right||d| + \sqrt{cd}ad\left|b^2cd - abd^2\right||d|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b^2cd+abd^2+\sqrt{-4ab^3cd^3+\left(b^2cd+abd^2\right)^2}}{b^2d^2}}}\right)}{b^2cd^3|b^2cd - abd^2| + abd^4|b^2cd - abd^2| + (b^2cd - abd^2)^2d^2}$$
$$+\frac{\left(\sqrt{ab}b^3c^2d|b| + \sqrt{ab}a^2bd^3|b| - \sqrt{ab}bc\left|b^2cd - abd^2\right||b| - \sqrt{ab}ad\left|b^2cd - abd^2\right||b|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b^2cd+abd^2-\sqrt{-4ab^3cd^3+\left(b^2cd+abd^2\right)^2}}{b^2d^2}}}\right)}{b^4cd|b^2cd - abd^2| + ab^3d^2|b^2cd - abd^2| - (b^2cd - abd^2)^2b^2}$$
$$+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -(sqrt(c*d)*b^3*c^2*d*abs(d) + sqrt(c*d)*a^2*b*d^3*abs(d) + sqrt(
c*d)*b*c*abs(b^2*c*d - a*b*d^2)*abs(d) + sqrt(c*d)*a*d*abs(b^2*c*
d - a*b*d^2)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt((b^2*c*d + a*b*d^2
 + sqrt(-4*a*b^3*c*d^3 + (b^2*c*d + a*b*d^2)^2))/(b^2*d^2)))/(b^2
*c*d^3*abs(b^2*c*d - a*b*d^2) + a*b*d^4*abs(b^2*c*d - a*b*d^2) +
(b^2*c*d - a*b*d^2)^2*d^2) + (sqrt(a*b)*b^3*c^2*d*abs(b) + sqrt(a
*b)*a^2*b*d^3*abs(b) - sqrt(a*b)*b*c*abs(b^2*c*d - a*b*d^2)*abs(b
) - sqrt(a*b)*a*d*abs(b^2*c*d - a*b*d^2)*abs(b))*arctan(2*sqrt(1/
2)*x/sqrt((b^2*c*d + a*b*d^2 - sqrt(-4*a*b^3*c*d^3 + (b^2*c*d + a
*b*d^2)^2))/(b^2*d^2)))/(b^4*c*d*abs(b^2*c*d - a*b*d^2) + a*b^3*d
^2*abs(b^2*c*d - a*b*d^2) - (b^2*c*d - a*b*d^2)^2*b^2) + x/(b*d)
```

3.230 $\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=53

$$\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}$$

```
[Out] -(a*Log[a + b*x^2])/(2*b*(b*c - a*d)) + (c*Log[c + d*x^2])/(2*d*(
b*c - a*d))
```

Rubi [A] time = 0.129768, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(a*Log[a + b*x^2])/(2*b*(b*c - a*d)) + (c*Log[c + d*x^2])/(2*d*(
b*c - a*d))
```

Rubi in Sympy [A] time = 20.8976, size = 39, normalized size = 0.74

$$\frac{a\log\left(a+bx^2\right)}{2b\left(ad-bc\right)}-\frac{c\log\left(c+dx^2\right)}{2d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*log(a + b*x**2)/(2*b*(a*d - b*c)) - c*log(c + d*x**2)/(2*d*(a*d
 - b*c))
```

Mathematica [A] time = 0.0312515, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^2\right)-bc\log\left(c+dx^2\right)}{2b^2cd-2abd^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((a*d*Log[a + b*x^2] - b*c*Log[c + d*x^2])/(2*b^2*c*d - 2*a*b*d^
2))
```

Maple [A] time = 0.01, size = 50, normalized size = 0.9

$$-\frac{c\ln\left(dx^2+c\right)}{\left(2\,ad-2\,bc\right)d}+\frac{a\ln\left(bx^2+a\right)}{\left(2\,ad-2\,bc\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2+c),x)

[Out] -1/2*c/(a*d-b*c)/d*ln(d*x^2+c)+1/2*a/(a*d-b*c)/b*ln(b*x^2+a)
```

Maxima [A] time = 1.34683, size = 66, normalized size = 1.25

$$-\frac{a\log\left(bx^2+a\right)}{2\left(b^2c-abd\right)}+\frac{c\log\left(dx^2+c\right)}{2\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")

[Out] -1/2*a*log(b*x^2 + a)/(b^2*c - a*b*d) + 1/2*c*log(d*x^2 + c)/(b*c
*d - a*d^2)
```

Fricas [A] time = 0.231717, size = 57, normalized size = 1.08

$$-\frac{ad\log\left(bx^2+a\right)-bc\log\left(dx^2+c\right)}{2\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")

[Out] -1/2*(a*d*log(b*x^2 + a) - b*c*log(d*x^2 + c))/(b^2*c*d - a*b*d^2
)
```

Sympy [A] time = 6.39369, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^2+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{2b\left(ad-bc\right)}-\frac{c\log\left(x^2+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{2d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)/(d*x**2+c),x)

[Out] a*log(x**2 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c)
+ a*b*c**2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(2*b*(a*d - b*c)) -
c*log(x**2 + (-a**2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*
a*c - b**2*c**3/(d*(a*d - b*c)))/(a*d + b*c))/(2*d*(a*d - b*c))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.231 $\int \frac{x^2}{(a+bx^2)(c+dx^2)} dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) +
(Sqrt[c]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

Rubi [A] time = 0.0979148, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) +
(Sqrt[c]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

Rubi in Sympy [A] time = 19.0047, size = 60, normalized size = 0.86

$$\frac{\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad-bc\right)}-\frac{\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(a)*atan(sqrt(b)*x/sqrt(a))/(sqrt(b)*(a*d - b*c)) - sqrt(c)*a
tan(sqrt(d)*x/sqrt(c))/(sqrt(d)*(a*d - b*c))
```

Mathematica [A] time = 0.0688101, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])/(b*c - a*d)
```

Maple [A] time = 0.008, size = 55, normalized size = 0.8

$$-\frac{c}{ad-bc}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{a}{ad-bc}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -c/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))+a/(a*d-b*c)/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256699, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$
$$-\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\frac{2\sqrt{\frac{c}{d}}\arctan\left(\frac{x}{\sqrt{\frac{c}{d}}}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2\,(bc-ad)},$$
$$\left.-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{x}{\sqrt{\frac{c}{d}}}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
 + sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(b
*c - a*d), -1/2*(2*sqrt(a/b)*arctan(x/sqrt(a/b)) + sqrt(-c/d)*log
((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(b*c - a*d), 1/2*(2
*sqrt(c/d)*arctan(x/sqrt(c/d)) - sqrt(-a/b)*log((b*x^2 + 2*b*x*sq
rt(-a/b) - a)/(b*x^2 + a)))/(b*c - a*d), -(sqrt(a/b)*arctan(x/sqr
t(a/b)) - sqrt(c/d)*arctan(x/sqrt(c/d)))/(b*c - a*d)]
```

Sympy [A] time = 7.46214, size = 570, normalized size = 8.14

$$\frac{\sqrt{-\frac{a}{b}}\log\left(-\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{c}{d}}\log\left(-\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{c}{d}}\log\left(\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-a/b)*log(-2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 + 4*a*
b**2*c*d**2*(-a/b)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-a/b)/(a*d -
b*c) - 2*b**3*c**2*d*(-a/b)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-a/b
)/(a*d - b*c) + x)/(2*(a*d - b*c)) - sqrt(-a/b)*log(2*a**2*b*d**3
*(-a/b)**(3/2)/(a*d - b*c)**3 - 4*a*b**2*c*d**2*(-a/b)**(3/2)/(a*
d - b*c)**3 + a*d*sqrt(-a/b)/(a*d - b*c) + 2*b**3*c**2*d*(-a/b)**
(3/2)/(a*d - b*c)**3 + b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a*d -
b*c)) + sqrt(-c/d)*log(-2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)**
3 + 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-c/d)
/(a*d - b*c) - 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 - b*c*s
qrt(-c/d)/(a*d - b*c) + x)/(2*(a*d - b*c)) - sqrt(-c/d)*log(2*a**
2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)**3 - 4*a*b**2*c*d**2*(-c/d)**(
3/2)/(a*d - b*c)**3 + a*d*sqrt(-c/d)/(a*d - b*c) + 2*b**3*c**2*d*
(-c/d)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-c/d)/(a*d - b*c) + x)/(2
*(a*d - b*c))
```

GIAC/XCAS [A] time = 0.259343, size = 176, normalized size = 2.51

$$\frac{\sqrt{cd}|d|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad+\sqrt{-4abcd+(bc+ad)^2}}{bd}}}\right)}{d^2|bc-ad|}-\frac{\sqrt{ab}|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad-\sqrt{-4abcd+(bc+ad)^2}}{bd}}}\right)}{b^2|bc-ad|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sqrt(c*d)*abs(d)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d + sqrt(-4*a
*b*c*d + (b*c + a*d)^2))/(b*d)))/(d^2*abs(b*c - a*d)) - sqrt(a*b)
*abs(b)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d - sqrt(-4*a*b*c*d +
(b*c + a*d)^2))/(b*d)))/(b^2*abs(b*c - a*d))
```

3.232 $\int \frac{x}{(a+bx^2)(c+dx^2)} dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^2)}{2(bc-ad)} - \frac{\log(c+dx^2)}{2(bc-ad)}$$

```
[Out] Log[a + b*x^2]/(2*(b*c - a*d)) - Log[c + d*x^2]/(2*(b*c - a*d))
```

Rubi [A] time = 0.0725536, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{\log(a+bx^2)}{2(bc-ad)} - \frac{\log(c+dx^2)}{2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)*(c + d*x^2)),x]

[Out] Log[a + b*x^2]/(2*(b*c - a*d)) - Log[c + d*x^2]/(2*(b*c - a*d))
```

Rubi in Sympy [A] time = 12.8105, size = 36, normalized size = 0.8

$$-\frac{\log(a+bx^2)}{2(ad-bc)} + \frac{\log(c+dx^2)}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2+c),x)

[Out] -log(a + b*x**2)/(2*(a*d - b*c)) + log(c + d*x**2)/(2*(a*d - b*c)
)
```

Mathematica [A] time = 0.0266405, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^2) - \log(c+dx^2)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)*(c + d*x^2)),x]

[Out] (Log[a + b*x^2] - Log[c + d*x^2])/(2*b*c - 2*a*d)
```

Maple [A] time = 0.009, size = 42, normalized size = 0.9

$$\frac{\ln(dx^2+c)}{2ad-2bc} - \frac{\ln(bx^2+a)}{2ad-2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2/(a*d-b*c)*ln(d*x^2+c)-1/2/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.33834, size = 55, normalized size = 1.22

$$\frac{\log\left(bx^2+a\right)}{2\left(bc-ad\right)}-\frac{\log\left(dx^2+c\right)}{2\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/(b*c - a*d) - 1/2*log(d*x^2 + c)/(b*c - a*d)
```

Fricas [A] time = 0.232263, size = 42, normalized size = 0.93

$$\frac{\log\left(bx^2+a\right)-\log\left(dx^2+c\right)}{2\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x^2 + a) - log(d*x^2 + c))/(b*c - a*d)
```

Sympy [A] time = 3.24171, size = 138, normalized size = 3.07

$$\frac{\log\left(x^2+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2\left(ad-bc\right)}-\frac{\log\left(x^2+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] log(x**2 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d
- b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(2*(a*d - b*c)) - log(x**
2 + (a**2*d**2/(a*d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c
**2/(a*d - b*c) + b*c)/(2*b*d))/(2*(a*d - b*c))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.233 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sq
rt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rubi [A] time = 0.0685471, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sq
rt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rubi in Sympy [A] time = 15.6197, size = 60, normalized size = 0.86

$$\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\,(ad-bc)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(d)*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*(a*d - b*c)) - sqrt(b)*a
tan(sqrt(b)*x/sqrt(a))/(sqrt(a)*(a*d - b*c))
```

Mathematica [A] time = 0.0718551, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[
(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A] time = 0., size = 55, normalized size = 0.8

$$\frac{d}{ad-bc}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b}{ad-bc}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] d/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-b/(a*d-b*c)/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.251557, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$
$$-\frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)},\frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
 + sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b
*c - a*d), -1/2*(2*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(b*c - a*d),
1/2*(2*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - sqrt(-d/c)*log((d*x^
2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a)*a
rctan(b*x/(a*sqrt(b/a))) - sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/(
b*c - a*d)]
```

Sympy [A] time = 8.12987, size = 712, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a
**3*b*c**2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(
-b/a)**(3/2)/(a*d - b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) -
a*b**3*c**4*(-b/a)**(3/2)/(a*d - b*c)**3 - b**2*c**2*sqrt(-b/a)/(
a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-b/a)*log(x + (a**4*c*d
**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-b/a)**(3/2)
/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)**3 +
```

```
 a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*
d - b*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d -
b*c)) + sqrt(-d/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c
)**3 + a**3*b*c**2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*
c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*d**2*sqrt(-d/c)/(a*d -
 b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3 - b**2*c**2*sqrt
(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)*log(x + (
a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b
*c)**3 + a**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(
3/2)/(a*d - b*c)**3 + b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2
*(a*d - b*c))
```

GIAC/XCAS [A] time = 0.25289, size = 257, normalized size = 3.67

$$-\frac{2\,\sqrt{cd}b|d|\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad+\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{bcd|bc-ad|+ad^2|bc-ad|+(bc-ad)^2d}+\frac{2\,\sqrt{ab}d|b|\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad-\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{b^2c|bc-ad|+abd|bc-ad|-(bc-ad)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -2*sqrt(c*d)*b*abs(d)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d + sqrt
(-4*a*b*c*d + (b*c + a*d)^2))/(b*d)))/(b*c*d*abs(b*c - a*d) + a*d
^2*abs(b*c - a*d) + (b*c - a*d)^2*d) + 2*sqrt(a*b)*d*abs(b)*arcta
n(2*sqrt(1/2)*x/sqrt((b*c + a*d - sqrt(-4*a*b*c*d + (b*c + a*d)^2
))/(b*d)))/(b^2*c*abs(b*c - a*d) + a*b*d*abs(b*c - a*d) - (b*c -
a*d)^2*b)
```

3.234 $\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^2])/(2*a*(b*c - a*d)) + (d*Log[c +
d*x^2])/(2*c*(b*c - a*d))
```

Rubi [A] time = 0.155338, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^2])/(2*a*(b*c - a*d)) + (d*Log[c +
d*x^2])/(2*c*(b*c - a*d))
```

Rubi in Sympy [A] time = 25.999, size = 49, normalized size = 0.79

$$-\frac{d\log\left(c+dx^2\right)}{2c\left(ad-bc\right)}+\frac{b\log\left(a+bx^2\right)}{2a\left(ad-bc\right)}+\frac{\log\left(x^2\right)}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -d*log(c + d*x**2)/(2*c*(a*d - b*c)) + b*log(a + b*x**2)/(2*a*(a*
d - b*c)) + log(x**2)/(2*a*c)
```

Mathematica [A] time = 0.0441077, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^2\right)+ad\log\left(c+dx^2\right)-2ad\log(x)+2bc\log(x)}{2abc^2-2a^2cd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*b*c*Log[x] - 2*a*d*Log[x] - b*c*Log[a + b*x^2] + a*d*Log[c + d
*x^2])/(2*a*b*c^2 - 2*a^2*c*d)
```

Maple [A] time = 0.013, size = 59, normalized size = 1.

$$\frac{\ln\left(x\right)}{ac}-\frac{d\ln\left(dx^2+c\right)}{2\,c\left(ad-bc\right)}+\frac{b\ln\left(bx^2+a\right)}{2\,a\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] ln(x)/a/c-1/2*d/c/(a*d-b*c)*ln(d*x^2+c)+1/2*b/a/(a*d-b*c)*ln(b*x^
2+a)
```

Maxima [A] time = 1.34522, size = 82, normalized size = 1.32

$$-\frac{b\log\left(bx^2+a\right)}{2\left(abc-a^2d\right)}+\frac{d\log\left(dx^2+c\right)}{2\left(bc^2-acd\right)}+\frac{\log\left(x^2\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(b*x^2 + a)/(a*b*c - a^2*d) + 1/2*d*log(d*x^2 + c)/(b*c
^2 - a*c*d) + 1/2*log(x^2)/(a*c)
```

Fricas [A] time = 0.283252, size = 73, normalized size = 1.18

$$-\frac{bc\log\left(bx^2+a\right)-ad\log\left(dx^2+c\right)-2\left(bc-ad\right)\log\left(x\right)}{2\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*log(b*x^2 + a) - a*d*log(d*x^2 + c) - 2*(b*c - a*d)*log
(x))/(a*b*c^2 - a^2*c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.235 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)} \, dx$

Optimal. Leaf size=81

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

```
[Out] -(1/(a*c*x)) - (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*
c - a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c -
 a*d))
```

Rubi [A] time = 0.214359, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(1/(a*c*x)) - (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*
c - a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c -
 a*d))
```

Rubi in Sympy [A] time = 42.5972, size = 66, normalized size = 0.81

$$-\frac{d^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}(ad-bc)}-\frac{1}{acx}+\frac{b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -d**(3/2)*atan(sqrt(d)*x/sqrt(c))/(c**(3/2)*(a*d - b*c)) - 1/(a*c
*x) + b**(3/2)*atan(sqrt(b)*x/sqrt(a))/(a**(3/2)*(a*d - b*c))
```

Mathematica [A] time = 0.143093, size = 76, normalized size = 0.94

$$\frac{-\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a}+\frac{d^{3/2}x\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{d}{c}}{bcx-adx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-(b/a) + d/c - (b^(3/2)*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) +
 (d^(3/2)*x*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c*x - a*d*x)
```

Maple [A] time = 0.012, size = 76, normalized size = 0.9

$$-\frac{d^2}{c(ad-bc)}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2}{a(ad-bc)}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/c*d^2/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))+1/a*b^2/(a
*d-b*c)/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))-1/a/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257627, size = 1, normalized size = 0.01

$$\left[-\frac{bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\,(abc^2-a^2cd)x}, \frac{2\,adx\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)-bcx\sqrt{-\frac{b}{a}}\log\left(\right.}{2\,(abc^2-a^2cd}\right.$$

$$-\frac{2\,bcx\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\,(abc^2-a^2cd)x},$$

$$\left.-\frac{bcx\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)-adx\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)+bc-ad}{(abc^2-a^2cd)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x
^2 + c)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x), 1/2*(2*a*d*x*s
qrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - b*c*x*sqrt(-b/a)*log((b*x^2
+ 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*b*c + 2*a*d)/((a*b*c^2 -
 a^2*c*d)*x), -1/2*(2*b*c*x*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) +
 a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
 + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -(b*c*x*sqrt(b/a)*arct
an(b*x/(a*sqrt(b/a))) - a*d*x*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c)))
 + b*c - a*d)/((a*b*c^2 - a^2*c*d)*x)]
```

Sympy [A] time = 17.0518, size = 1093, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a
*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)
**3 - 2*a**5*b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 -
a**5*d**5*sqrt(-b**3/a**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**
3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*b**4*c**7*(-b**3/a**3)**(3/2
)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(a*d - b*c))/(a**2*
b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)) + sq
rt(-b**3/a**3)*log(x + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d -
 b*c)**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3
+ 2*a**5*b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5
*d**5*sqrt(-b**3/a**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a*
*3)**(3/2)/(a*d - b*c)**3 + a**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a
*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**3)/(a*d - b*c))/(a**2*b**2
*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)) - sqrt(-
d**3/c**3)*log(x + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2
*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d*
*5*sqrt(-d**3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)
**(3/2)/(a*d - b*c)**3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d
- b*c)**3 - b**5*c**5*sqrt(-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d*
*4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)) + sqrt(-d**
3/c**3)*log(x + (a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b*c)**
3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + 2*a**
5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*s
qrt(-d**3/c**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3
/2)/(a*d - b*c)**3 + a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + b**5*c**5*sqrt(-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 +
 a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)) - 1/(a*c*x)
```

GIAC/XCAS [A] time = 0.299003, size = 520, normalized size = 6.42

$$\frac{\left(\sqrt{cd}ab^2c^2|d| + \sqrt{cd}a^2bcd|d| - \sqrt{cd}b\left|abc^2 - a^2cd\right||d|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{abc^2+a^2cd+\sqrt{-4a^3bc^3d+\left(abc^2+a^2cd\right)^2}}{abcd}}}\right)}{abc^2d\left|abc^2 - a^2cd\right| + a^2cd^2\left|abc^2 - a^2cd\right| + \left(abc^2 - a^2cd\right)^2d} - \frac{\left(\sqrt{ab}abc^2d|b| + \sqrt{ab}a^2cd^2|b| + \sqrt{ab}d\left|abc^2 - a^2cd\right||b|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{abc^2+a^2cd-\sqrt{-4a^3bc^3d+\left(abc^2+a^2cd\right)^2}}{abcd}}}\right)}{ab^2c^2\left|abc^2 - a^2cd\right| + a^2bcd\left|abc^2 - a^2cd\right| - \left(abc^2 - a^2cd\right)^2b} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] (sqrt(c*d)*a*b^2*c^2*abs(d) + sqrt(c*d)*a^2*b*c*d*abs(d) - sqrt(c
*d)*b*abs(a*b*c^2 - a^2*c*d)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt((a
*b*c^2 + a^2*c*d + sqrt(-4*a^3*b*c^3*d + (a*b*c^2 + a^2*c*d)^2))/
(a*b*c*d)))/(a*b*c^2*d*abs(a*b*c^2 - a^2*c*d) + a^2*c*d^2*abs(a*b
*c^2 - a^2*c*d) + (a*b*c^2 - a^2*c*d)^2*d) - (sqrt(a*b)*a*b*c^2*d
*abs(b) + sqrt(a*b)*a^2*c*d^2*abs(b) + sqrt(a*b)*d*abs(a*b*c^2 -
a^2*c*d)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a*b*c^2 + a^2*c*d - s
qrt(-4*a^3*b*c^3*d + (a*b*c^2 + a^2*c*d)^2))/(a*b*c*d)))/(a*b^2*c
^2*abs(a*b*c^2 - a^2*c*d) + a^2*b*c*d*abs(a*b*c^2 - a^2*c*d) - (a
*b*c^2 - a^2*c*d)^2*b) - 1/(a*c*x)
```

3.236 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

```
[Out] -1/(2*a*c*x^2) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*
x^2])/(2*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*
d))
```

Rubi [A] time = 0.226018, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(2*a*c*x^2) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*
x^2])/(2*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*
d))
```

Rubi in Sympy [A] time = 33.7355, size = 76, normalized size = 0.87

$$\frac{d^2\log\left(c+dx^2\right)}{2c^2\left(ad-bc\right)}-\frac{1}{2acx^2}-\frac{b^2\log\left(a+bx^2\right)}{2a^2\left(ad-bc\right)}-\frac{(ad+bc)\log\left(x^2\right)}{2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] d**2*log(c + d*x**2)/(2*c**2*(a*d - b*c)) - 1/(2*a*c*x**2) - b**2
*log(a + b*x**2)/(2*a**2*(a*d - b*c)) - (a*d + b*c)*log(x**2)/(2*
a**2*c**2)
```

Mathematica [A] time = 0.0658263, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(2*a*c*x^2) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a +
 b*x^2])/(2*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*
c - a*d))
```

Maple [A] time = 0.016, size = 87, normalized size = 1.

$$-\frac{1}{2\,ac{x}^{2}}-\frac{\ln\left(x\right)d}{a{c}^{2}}-\frac{b\ln\left(x\right)}{{a}^{2}c}+\frac{{d}^{2}\ln\left(d{x}^{2}+c\right)}{2\,{c}^{2}\left(ad-bc\right)}-\frac{{b}^{2}\ln\left(b{x}^{2}+a\right)}{2\,{a}^{2}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2/a/c/x^2-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b+1/2*d^2/c^2/(a*d-b*c
)*ln(d*x^2+c)-1/2*b^2/a^2/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.38471, size = 117, normalized size = 1.34

$$\frac{b^2\log\left(bx^2+a\right)}{2\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^2+c\right)}{2\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^2\right)}{2\,a^2c^2}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(a^2*b*c - a^3*d) - 1/2*d^2*log(d*x^2 + c)
/(b*c^3 - a*c^2*d) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) - 1/2/(a*
c*x^2)
```

Fricas [A] time = 0.439202, size = 134, normalized size = 1.54

$$\frac{b^2c^2x^2\log\left(bx^2+a\right)-a^2d^2x^2\log\left(dx^2+c\right)-abc^2+a^2cd-2\left(b^2c^2-a^2d^2\right)x^2\log\left(x\right)}{2\left(a^2bc^3-a^3c^2d\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log(b*x^2 + a) - a^2*d^2*x^2*log(d*x^2 + c) - a*
b*c^2 + a^2*c*d - 2*(b^2*c^2 - a^2*d^2)*x^2*log(x))/((a^2*b*c^3 -
 a^3*c^2*d)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.237 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=100

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) + (b^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d))
```

Rubi [A] time = 0.455182, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) + (b^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d))
```

Rubi in Sympy [A] time = 86.3471, size = 85, normalized size = 0.85

$$\frac{d^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}(ad-bc)}-\frac{1}{3acx^3}+\frac{ad+bc}{a^2c^2x}-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{5}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] d**(5/2)*atan(sqrt(d)*x/sqrt(c))/(c**(5/2)*(a*d - b*c)) - 1/(3*a*
c*x**3) + (a*d + b*c)/(a**2*c**2*x) - b**(5/2)*atan(sqrt(b)*x/sqr
t(a))/(a**(5/2)*(a*d - b*c))
```

Mathematica [A] time = 0.252064, size = 101, normalized size = 1.01

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) - (b^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)) - (d^(5/2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d))
```

Maple [A] time = 0.014, size = 98, normalized size = 1.

$$-\frac{1}{3\,ac x^{3}}+\frac{d}{axc^{2}}+\frac{b}{a^{2}cx}+\frac{d^{3}}{c^{2}\left(ad-bc\right)}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^{3}}{a^{2}\left(ad-bc\right)}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/3/a/c/x^3+1/x/a/c^2*d+1/x/a^2/c*b+1/c^2*d^3/(a*d-b*c)/(c*d)^(1
/2)*arctan(x*d/(c*d)^(1/2))-1/a^2*b^3/(a*d-b*c)/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278437, size = 1, normalized size = 0.01

$$\left[-\frac{3\,b^{2}c^{2}x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+3\,a^{2}d^{2}x^{3}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)+2\,abc^{2}-2\,a^{2}cd-6\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}}{6\left(a^{2}bc^{3}-a^{3}c^{2}d\right)x^{3}},\right.$$
$$-\frac{6\,a^{2}d^{2}x^{3}\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)+3\,b^{2}c^{2}x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+2\,abc^{2}-2\,a^{2}cd-6\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}}{6\left(a^{2}bc^{3}-a^{3}c^{2}d\right)x^{3}},\ \frac{6\,b^{2}c^{2}x^{3}\sqrt{\frac{b}{a}}\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*b^2*c^2*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) + 3*a^2*d^2*x^3*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt
(-d/c) - c)/(d*x^2 + c)) + 2*a*b*c^2 - 2*a^2*c*d - 6*(b^2*c^2 - a
^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), -1/6*(6*a^2*d^2*x^3*s
qrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + 3*b^2*c^2*x^3*sqrt(-b/a)*log
((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*a*b*c^2 - 2*a^2*
c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1
/6*(6*b^2*c^2*x^3*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 3*a^2*d^2
*x^3*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) -
 2*a*b*c^2 + 2*a^2*c*d + 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 -
 a^3*c^2*d)*x^3), 1/3*(3*b^2*c^2*x^3*sqrt(b/a)*arctan(b*x/(a*sqrt
(b/a))) - 3*a^2*d^2*x^3*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - a*b
*c^2 + a^2*c*d + 3*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2
*d)*x^3)]
```

Sympy [A] time = 30.8518, size = 1353, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b**5/a**5)*log(x + (-a**10*c**5*d**5*(-b**5/a**5)**(3/2)/(a
*d - b*c)**3 + 2*a**9*b*c**6*d**4*(-b**5/a**5)**(3/2)/(a*d - b*c)
**3 - a**8*b**2*c**7*d**3*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a*
*8*d**8*sqrt(-b**5/a**5)/(a*d - b*c) - a**7*b**3*c**8*d**2*(-b**5
/a**5)**(3/2)/(a*d - b*c)**3 + 2*a**6*b**4*c**9*d*(-b**5/a**5)**(
3/2)/(a*d - b*c)**3 - a**5*b**5*c**10*(-b**5/a**5)**(3/2)/(a*d -
b*c)**3 - b**8*c**8*sqrt(-b**5/a**5)/(a*d - b*c))/(a**4*b**3*d**7
 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 + b*
*7*c**4*d**3))/(2*(a*d - b*c)) - sqrt(-b**5/a**5)*log(x + (a**10*
c**5*d**5*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - 2*a**9*b*c**6*d**4
*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + a**8*b**2*c**7*d**3*(-b**5/
a**5)**(3/2)/(a*d - b*c)**3 + a**8*d**8*sqrt(-b**5/a**5)/(a*d - b
*c) + a**7*b**3*c**8*d**2*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - 2*
a**6*b**4*c**9*d*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + a**5*b**5*c
**10*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + b**8*c**8*sqrt(-b**5/a*
*5)/(a*d - b*c))/(a**4*b**3*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c
**2*d**5 + a*b**6*c**3*d**4 + b**7*c**4*d**3))/(2*(a*d - b*c)) +
sqrt(-d**5/c**5)*log(x + (-a**10*c**5*d**5*(-d**5/c**5)**(3/2)/(a
*d - b*c)**3 + 2*a**9*b*c**6*d**4*(-d**5/c**5)**(3/2)/(a*d - b*c)
**3 - a**8*b**2*c**7*d**3*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - a*
*8*d**8*sqrt(-d**5/c**5)/(a*d - b*c) - a**7*b**3*c**8*d**2*(-d**5
/c**5)**(3/2)/(a*d - b*c)**3 + 2*a**6*b**4*c**9*d*(-d**5/c**5)**(
3/2)/(a*d - b*c)**3 - a**5*b**5*c**10*(-d**5/c**5)**(3/2)/(a*d -
b*c)**3 - b**8*c**8*sqrt(-d**5/c**5)/(a*d - b*c))/(a**4*b**3*d**7
 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 + b*
*7*c**4*d**3))/(2*(a*d - b*c)) - sqrt(-d**5/c**5)*log(x + (a**10*
c**5*d**5*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - 2*a**9*b*c**6*d**4
*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**8*b**2*c**7*d**3*(-d**5/
c**5)**(3/2)/(a*d - b*c)**3 + a**8*d**8*sqrt(-d**5/c**5)/(a*d - b
*c) + a**7*b**3*c**8*d**2*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - 2*
a**6*b**4*c**9*d*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**5*b**5*c
**10*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + b**8*c**8*sqrt(-d**5/c*
*5)/(a*d - b*c))/(a**4*b**3*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c
**2*d**5 + a*b**6*c**3*d**4 + b**7*c**4*d**3))/(2*(a*d - b*c)) +
(-a*c + x**2*(3*a*d + 3*b*c))/(3*a**2*c**2*x**3)
```

GIAC/XCAS [A] time = 0.29586, size = 728, normalized size = 7.28

$$-\frac{\left(\sqrt{cd}a^2b^3c^4|d| + \sqrt{cd}a^4bc^2d^2|d| - \sqrt{cd}b^2c\left|a^2bc^3 - a^3c^2d\right||d| - \sqrt{cd}abd\left|a^2bc^3 - a^3c^2d\right||d|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{a^2bc^3+a^3c^2d+\sqrt{-4a^5bc^5d+}}{a^2bc^2d}}}\right.}{a^2bc^3d|a^2bc^3 - a^3c^2d| + a^3c^2d^2|a^2bc^3 - a^3c^2d| + (a^2bc^3 - a^3c^2d)^2d}$$
$$+\frac{\left(\sqrt{ab}a^2b^2c^4d|b| + \sqrt{ab}a^4c^2d^3|b| + \sqrt{ab}bcd\left|a^2bc^3 - a^3c^2d\right||b| + \sqrt{ab}ad^2\left|a^2bc^3 - a^3c^2d\right||b|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{a^2bc^3+a^3c^2d-\sqrt{-4a^5bc^5d+(}}{a^2bc^2d}}}\right.}{a^2b^2c^3|a^2bc^3 - a^3c^2d| + a^3bc^2d|a^2bc^3 - a^3c^2d| - (a^2bc^3 - a^3c^2d)^2b}$$
$$+\frac{3\,bcx^2 + 3\,adx^2 - ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] -(sqrt(c*d)*a^2*b^3*c^4*abs(d) + sqrt(c*d)*a^4*b*c^2*d^2*abs(d) -
 sqrt(c*d)*b^2*c*abs(a^2*b*c^3 - a^3*c^2*d)*abs(d) - sqrt(c*d)*a*
b*d*abs(a^2*b*c^3 - a^3*c^2*d)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt(
(a^2*b*c^3 + a^3*c^2*d + sqrt(-4*a^5*b*c^5*d + (a^2*b*c^3 + a^3*c
^2*d)^2))/(a^2*b*c^2*d)))/(a^2*b*c^3*d*abs(a^2*b*c^3 - a^3*c^2*d)
 + a^3*c^2*d^2*abs(a^2*b*c^3 - a^3*c^2*d) + (a^2*b*c^3 - a^3*c^2*
d)^2*d) + (sqrt(a*b)*a^2*b^2*c^4*d*abs(b) + sqrt(a*b)*a^4*c^2*d^3
*abs(b) + sqrt(a*b)*b*c*d*abs(a^2*b*c^3 - a^3*c^2*d)*abs(b) + sqr
```

```
t(a*b)*a*d^2*abs(a^2*b*c^3 - a^3*c^2*d)*abs(b))*arctan(2*sqrt(1/2
)*x/sqrt((a^2*b*c^3 + a^3*c^2*d - sqrt(-4*a^5*b*c^5*d + (a^2*b*c^
3 + a^3*c^2*d)^2))/(a^2*b*c^2*d)))/(a^2*b^2*c^3*abs(a^2*b*c^3 - a
^3*c^2*d) + a^3*b*c^2*d*abs(a^2*b*c^3 - a^3*c^2*d) - (a^2*b*c^3 -
 a^3*c^2*d)^2*b) + 1/3*(3*b*c*x^2 + 3*a*d*x^2 - a*c)/(a^2*c^2*x^3
)
```

3.238 $\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=119

$$-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*
d + a^2*d^2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(2*a^3*(b*c
 - a*d)) + (d^3*Log[c + d*x^2])/(2*c^3*(b*c - a*d))
```

Rubi [A] time = 0.293266, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*
d + a^2*d^2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(2*a^3*(b*c
 - a*d)) + (d^3*Log[c + d*x^2])/(2*c^3*(b*c - a*d))
```

Rubi in Sympy [A] time = 43.685, size = 109, normalized size = 0.92

$$-\frac{d^3\log\left(c+dx^2\right)}{2c^3\left(ad-bc\right)}-\frac{1}{4acx^4}+\frac{ad+bc}{2a^2c^2x^2}+\frac{b^3\log\left(a+bx^2\right)}{2a^3\left(ad-bc\right)}+\frac{\left(a^2d^2+abcd+b^2c^2\right)\log\left(x^2\right)}{2a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -d**3*log(c + d*x**2)/(2*c**3*(a*d - b*c)) - 1/(4*a*c*x**4) + (a*
d + b*c)/(2*a**2*c**2*x**2) + b**3*log(a + b*x**2)/(2*a**3*(a*d -
 b*c)) + (a**2*d**2 + a*b*c*d + b**2*c**2)*log(x**2)/(2*a**3*c**3
)
```

Mathematica [A] time = 0.0964288, size = 119, normalized size = 1.

$$\frac{b^3\log\left(a+bx^2\right)}{2a^3(ad-bc)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*
d + a^2*d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^2])/(2*a^3*(-(b
*c) + a*d)) + (d^3*Log[c + d*x^2])/(2*c^3*(b*c - a*d))
```

Maple [A] time = 0.02, size = 124, normalized size = 1.

$$-\frac{1}{4\,ac{x}^{4}}+\frac{d}{2\,a{x}^{2}{c}^{2}}+\frac{b}{2\,{a}^{2}c{x}^{2}}+\frac{\ln\left(x\right){d}^{2}}{a{c}^{3}}+\frac{b\ln\left(x\right)d}{{a}^{2}{c}^{2}}+\frac{\ln\left(x\right){b}^{2}}{{a}^{3}c}-\frac{{d}^{3}\ln\left(d{x}^{2}+c\right)}{2\,{c}^{3}\left(ad-bc\right)}+\frac{{b}^{3}\ln\left(b{x}^{2}+a\right)}{2\,{a}^{3}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4/a/c/x^4+1/2/x^2/a/c^2*d+1/2/x^2/a^2/c*b+1/a/c^3*ln(x)*d^2+1/
a^2/c^2*ln(x)*b*d+1/a^3/c*ln(x)*b^2-1/2*d^3/c^3/(a*d-b*c)*ln(d*x^
2+c)+1/2*b^3/a^3/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.35002, size = 158, normalized size = 1.33

$$-\frac{b^3\log\left(bx^2+a\right)}{2\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^2+c\right)}{2\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^3c^3}+\frac{2\left(bc+ad\right)x^2-ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a^3*b*c - a^4*d) + 1/2*d^3*log(d*x^2 + c
)/(b*c^4 - a*c^3*d) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/
(a^3*c^3) + 1/4*(2*(b*c + a*d)*x^2 - a*c)/(a^2*c^2*x^4)
```

Fricas [A] time = 1.25235, size = 171, normalized size = 1.44

$$-\frac{2\,b^3c^3x^4\log\left(bx^2+a\right)-2\,a^3d^3x^4\log\left(dx^2+c\right)+a^2bc^3-a^3c^2d-4\left(b^3c^3-a^3d^3\right)x^4\log\left(x\right)-2\left(ab^2c^3-a^3cd^2\right)x^2}{4\left(a^3bc^4-a^4c^3d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3*x^4*log(b*x^2 + a) - 2*a^3*d^3*x^4*log(d*x^2 + c)
 + a^2*b*c^3 - a^3*c^2*d - 4*(b^3*c^3 - a^3*d^3)*x^4*log(x) - 2*(
a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.239 $\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=134

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) - (b^2*c^2 + a*b*c*d
 + a^2*d^2)/(a^3*c^3*x) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(7/2)*(b*c - a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
7/2)*(b*c - a*d))
```

Rubi [A] time = 0.65961, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) - (b^2*c^2 + a*b*c*d
 + a^2*d^2)/(a^3*c^3*x) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(7/2)*(b*c - a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
7/2)*(b*c - a*d))
```

Rubi in Sympy [A] time = 136.503, size = 112, normalized size = 0.84

$$-\frac{d^{\frac{7}{2}}\,\text{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{7}{2}}\,(ad-bc)}-\frac{1}{5acx^5}+\frac{ad+bc}{3a^2c^2x^3}+\frac{abcd-(ad+bc)^2}{a^3c^3x}+\frac{b^{\frac{7}{2}}\,\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{7}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -d**(7/2)*atan(sqrt(d)*x/sqrt(c))/(c**(7/2)*(a*d - b*c)) - 1/(5*a
*c*x**5) + (a*d + b*c)/(3*a**2*c**2*x**3) + (a*b*c*d - (a*d + b*c
)**2)/(a**3*c**3*x) + b**(7/2)*atan(sqrt(b)*x/sqrt(a))/(a**(7/2)*
(a*d - b*c))
```

Mathematica [A] time = 0.241922, size = 135, normalized size = 1.01

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(ad-bc)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{-a^2d^2-abcd-b^2c^2}{a^3c^3x}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) + (-(b^2*c^2) - a*b*
c*d - a^2*d^2)/(a^3*c^3*x) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(7/2)*(-(b*c) + a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(c^(7/2)*(b*c - a*d))
```

Maple [A] time = 0.016, size = 141, normalized size = 1.1

$$-\frac{1}{5\,acx^5}+\frac{d}{3\,x^3ac^2}+\frac{b}{3\,a^2cx^3}-\frac{d^2}{ac^3x}-\frac{bd}{a^2c^2x}-\frac{b^2}{a^3cx}-\frac{d^4}{c^3\,(ad-bc)}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^4}{a^3\,(ad-bc)}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/5/a/c/x^5+1/3/x^3/a/c^2*d+1/3/x^3/a^2/c*b-1/a/c^3/x*d^2-1/a^2/
c^2/x*b*d-1/a^3/c/x*b^2-1/c^3*d^4/(a*d-b*c)/(c*d)^(1/2)*arctan(x*
d/(c*d)^(1/2))+1/a^3*b^4/(a*d-b*c)/(a*b)^(1/2)*arctan(x*b/(a*b)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.313368, size = 1, normalized size = 0.01

$$\left[-\frac{15\,b^3c^3x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+15\,a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\,a^2bc^3-6\,a^3c^2d+30\left(b^3c^3-a^3d^3\right)x^4-1}{30\,(a^3bc^4-a^4c^3d)x^5}\right.$$

$$-\frac{30\,b^3c^3x^5\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+15\,a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\,a^2bc^3-6\,a^3c^2d+30\left(b^3c^3-a^3d^3\right)x^4-10\left(ab^2c^3\right.}{30\,(a^3bc^4-a^4c^3d)x^5}$$

$$-\frac{15\,b^3c^3x^5\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)-15\,a^3d^3x^5\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)+3\,a^2bc^3-3\,a^3c^2d+15\left(b^3c^3-a^3d^3\right)x^4-5\left(ab^2c^3-a^3cd^2\right)x}{15\,(a^3bc^4-a^4c^3d)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^6),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 15*a^3*d^3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*s
qrt(-d/c) - c)/(d*x^2 + c)) + 6*a^2*b*c^3 - 6*a^3*c^2*d + 30*(b^3
*c^3 - a^3*d^3)*x^4 - 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4
 - a^4*c^3*d)*x^5), 1/30*(30*a^3*d^3*x^5*sqrt(d/c)*arctan(d*x/(c*
sqrt(d/c))) - 15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-
b/a) - a)/(b*x^2 + a)) - 6*a^2*b*c^3 + 6*a^3*c^2*d - 30*(b^3*c^3
- a^3*d^3)*x^4 + 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^
4*c^3*d)*x^5), -1/30*(30*b^3*c^3*x^5*sqrt(b/a)*arctan(b*x/(a*sqrt
(b/a))) + 15*a^3*d^3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c)
 - c)/(d*x^2 + c)) + 6*a^2*b*c^3 - 6*a^3*c^2*d + 30*(b^3*c^3 - a^
3*d^3)*x^4 - 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^
```

```
3*d)*x^5), -1/15*(15*b^3*c^3*x^5*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a
))) - 15*a^3*d^3*x^5*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + 3*a^2*
b*c^3 - 3*a^3*c^2*d + 15*(b^3*c^3 - a^3*d^3)*x^4 - 5*(a*b^2*c^3 -
 a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5)]
```

Sympy [A] time = 63.8952, size = 1504, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**7/a**7)*log(x + (-a**13*c**7*d**6*(-b**7/a**7)**(3/2)/(
a*d - b*c)**3 + 2*a**12*b*c**8*d**5*(-b**7/a**7)**(3/2)/(a*d - b*
c)**3 - a**11*b**2*c**9*d**4*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 -
 a**11*d**11*sqrt(-b**7/a**7)/(a*d - b*c) - a**9*b**4*c**11*d**2*
(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + 2*a**8*b**5*c**12*d*(-b**7/a
**7)**(3/2)/(a*d - b*c)**3 - a**7*b**6*c**13*(-b**7/a**7)**(3/2)/
(a*d - b*c)**3 - b**11*c**11*sqrt(-b**7/a**7)/(a*d - b*c))/(a**6*
b**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c**2*d**8 + a**3*b**7*c
**3*d**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5*d**5 + b**10*c**6*d*
*4))/(2*(a*d - b*c)) + sqrt(-b**7/a**7)*log(x + (a**13*c**7*d**6*
(-b**7/a**7)**(3/2)/(a*d - b*c)**3 - 2*a**12*b*c**8*d**5*(-b**7/a
**7)**(3/2)/(a*d - b*c)**3 + a**11*b**2*c**9*d**4*(-b**7/a**7)**(
3/2)/(a*d - b*c)**3 + a**11*d**11*sqrt(-b**7/a**7)/(a*d - b*c) +
a**9*b**4*c**11*d**2*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 - 2*a**8*
b**5*c**12*d*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + a**7*b**6*c**13
*(-b**7/a**7)**(3/2)/(a*d - b*c)**3 + b**11*c**11*sqrt(-b**7/a**7
)/(a*d - b*c))/(a**6*b**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c*
*2*d**8 + a**3*b**7*c**3*d**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5
*d**5 + b**10*c**6*d**4))/(2*(a*d - b*c)) - sqrt(-d**7/c**7)*log(
x + (-a**13*c**7*d**6*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 + 2*a**1
2*b*c**8*d**5*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 - a**11*b**2*c**
9*d**4*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 - a**11*d**11*sqrt(-d**
7/c**7)/(a*d - b*c) - a**9*b**4*c**11*d**2*(-d**7/c**7)**(3/2)/(a
*d - b*c)**3 + 2*a**8*b**5*c**12*d*(-d**7/c**7)**(3/2)/(a*d - b*c
)**3 - a**7*b**6*c**13*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 - b**11
*c**11*sqrt(-d**7/c**7)/(a*d - b*c))/(a**6*b**4*d**10 + a**5*b**5
*c*d**9 + a**4*b**6*c**2*d**8 + a**3*b**7*c**3*d**7 + a**2*b**8*c
**4*d**6 + a*b**9*c**5*d**5 + b**10*c**6*d**4))/(2*(a*d - b*c)) +
 sqrt(-d**7/c**7)*log(x + (a**13*c**7*d**6*(-d**7/c**7)**(3/2)/(a
*d - b*c)**3 - 2*a**12*b*c**8*d**5*(-d**7/c**7)**(3/2)/(a*d - b*c
)**3 + a**11*b**2*c**9*d**4*(-d**7/c**7)**(3/2)/(a*d - b*c)**3 +
a**11*d**11*sqrt(-d**7/c**7)/(a*d - b*c) + a**9*b**4*c**11*d**2*(
-d**7/c**7)**(3/2)/(a*d - b*c)**3 - 2*a**8*b**5*c**12*d*(-d**7/c*
*7)**(3/2)/(a*d - b*c)**3 + a**7*b**6*c**13*(-d**7/c**7)**(3/2)/(
a*d - b*c)**3 + b**11*c**11*sqrt(-d**7/c**7)/(a*d - b*c))/(a**6*b
**4*d**10 + a**5*b**5*c*d**9 + a**4*b**6*c**2*d**8 + a**3*b**7*c*
*3*d**7 + a**2*b**8*c**4*d**6 + a*b**9*c**5*d**5 + b**10*c**6*d**
4))/(2*(a*d - b*c)) - (3*a**2*c**2 + x**4*(15*a**2*d**2 + 15*a*b*
c*d + 15*b**2*c**2) + x**2*(-5*a**2*c*d - 5*a*b*c**2))/(15*a**3*c
**3*x**5)
```

GIAC/XCAS [A] time = 0.313644, size = 890, normalized size = 6.64

$$\frac{\left(\sqrt{cd}a^3b^4c^6|d| + \sqrt{cd}a^6bc^3d^3|d| - \sqrt{cd}b^3c^2\left|a^3bc^4 - a^4c^3d\right||d| - \sqrt{cd}ab^2cd\left|a^3bc^4 - a^4c^3d\right||d| - \sqrt{cd}a^2bd^2\left|a^3bc^4 - a^4c^3d\right||d|\right.}{a^3bc^4d|a^3bc^4 - a^4c^3d| + a^4c^3d^2|a^3bc^4 - a^4c^3d| + (a^3bc^4 - a^4c^3d)^2d} - \frac{\left(\sqrt{ab}a^3b^3c^6d|b| + \sqrt{ab}a^6c^3d^4|b| + \sqrt{ab}b^2c^2d\left|a^3bc^4 - a^4c^3d\right||b| + \sqrt{ab}abcd^2\left|a^3bc^4 - a^4c^3d\right||b| + \sqrt{ab}a^2d^3\left|a^3bc^4 - a^4c^3d\right||b|\right.}{a^3b^2c^4|a^3bc^4 - a^4c^3d| + a^4bc^3d|a^3bc^4 - a^4c^3d| - (a^3bc^4 - a^4c^3d)^2b} - \frac{15\,b^2c^2x^4 + 15\,abcdx^4 + 15\,a^2d^2x^4 - 5\,abc^2x^2 - 5\,a^2cdx^2 + 3\,a^2c^2}{15\,a^3c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^6),x, algorithm="giac")
```

```
[Out] (sqrt(c*d)*a^3*b^4*c^6*abs(d) + sqrt(c*d)*a^6*b*c^3*d^3*abs(d) -
sqrt(c*d)*b^3*c^2*abs(a^3*b*c^4 - a^4*c^3*d)*abs(d) - sqrt(c*d)*a
*b^2*c*d*abs(a^3*b*c^4 - a^4*c^3*d)*abs(d) - sqrt(c*d)*a^2*b*d^2*
abs(a^3*b*c^4 - a^4*c^3*d)*abs(d))*arctan(2*sqrt(1/2)*x/sqrt((a^3
*b*c^4 + a^4*c^3*d + sqrt(-4*a^7*b*c^7*d + (a^3*b*c^4 + a^4*c^3*d
)^2))/(a^3*b*c^3*d)))/(a^3*b*c^4*d*abs(a^3*b*c^4 - a^4*c^3*d) + a
^4*c^3*d^2*abs(a^3*b*c^4 - a^4*c^3*d) + (a^3*b*c^4 - a^4*c^3*d)^2
*d) - (sqrt(a*b)*a^3*b^3*c^6*d*abs(b) + sqrt(a*b)*a^6*c^3*d^4*abs
(b) + sqrt(a*b)*b^2*c^2*d*abs(a^3*b*c^4 - a^4*c^3*d)*abs(b) + sqr
t(a*b)*a*b*c*d^2*abs(a^3*b*c^4 - a^4*c^3*d)*abs(b) + sqrt(a*b)*a^
2*d^3*abs(a^3*b*c^4 - a^4*c^3*d)*abs(b))*arctan(2*sqrt(1/2)*x/sqr
t((a^3*b*c^4 + a^4*c^3*d - sqrt(-4*a^7*b*c^7*d + (a^3*b*c^4 + a^4
*c^3*d)^2))/(a^3*b*c^3*d)))/(a^3*b^2*c^4*abs(a^3*b*c^4 - a^4*c^3*
d) + a^4*b*c^3*d*abs(a^3*b*c^4 - a^4*c^3*d) - (a^3*b*c^4 - a^4*c^
3*d)^2*b) - 1/15*(15*b^2*c^2*x^4 + 15*a*b*c*d*x^4 + 15*a^2*d^2*x^
4 - 5*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(a^3*c^3*x^5)
```

3.240 $\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=155

$$\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d
 + a^2*d^2)/(2*a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Lo
g[x])/(a^4*c^4) + (b^4*Log[a + b*x^2])/(2*a^4*(b*c - a*d)) - (d^4
*Log[c + d*x^2])/(2*c^4*(b*c - a*d))
```

Rubi [A] time = 0.386154, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d
 + a^2*d^2)/(2*a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Lo
g[x])/(a^4*c^4) + (b^4*Log[a + b*x^2])/(2*a^4*(b*c - a*d)) - (d^4
*Log[c + d*x^2])/(2*c^4*(b*c - a*d))
```

Rubi in Sympy [A] time = 53.3115, size = 141, normalized size = 0.91

$$\frac{d^4\log\left(c+dx^2\right)}{2c^4\left(ad-bc\right)}-\frac{1}{6acx^6}+\frac{ad+bc}{4a^2c^2x^4}-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{b^4\log\left(a+bx^2\right)}{2a^4\left(ad-bc\right)}-\frac{\left(ad+bc\right)\left(a^2d^2+b^2c^2\right)\log\left(x^2\right)}{2a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] d**4*log(c + d*x**2)/(2*c**4*(a*d - b*c)) - 1/(6*a*c*x**6) + (a*d
 + b*c)/(4*a**2*c**2*x**4) - (a**2*d**2 + a*b*c*d + b**2*c**2)/(2
*a**3*c**3*x**2) - b**4*log(a + b*x**2)/(2*a**4*(a*d - b*c)) - (a
*d + b*c)*(a**2*d**2 + b**2*c**2)*log(x**2)/(2*a**4*c**4)
```

Mathematica [A] time = 0.106573, size = 147, normalized size = 0.95

$$\frac{12x^6\log(x)\left(b^4c^4-a^4d^4\right)+a\left(a^3cd\left(-2c^2+3cdx^2-6d^2x^4\right)+6a^3d^4x^6\log\left(c+dx^2\right)+2a^2bc^4-3ab^2c^4x^2+6b^3c^4x^4\right)-6b^4}{12a^4c^4x^6(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (12*(b^4*c^4 - a^4*d^4)*x^6*Log[x] - 6*b^4*c^4*x^6*Log[a + b*x^2]
 + a*(2*a^2*b*c^4 - 3*a*b^2*c^4*x^2 + 6*b^3*c^4*x^4 + a^3*c*d*(-2
*c^2 + 3*c*d*x^2 - 6*d^2*x^4) + 6*a^3*d^4*x^6*Log[c + d*x^2]))/(1
2*a^4*c^4*(-(b*c) + a*d)*x^6)
```

Maple [A] time = 0.022, size = 184, normalized size = 1.2

$$-\frac{1}{6\,ac x^{6}}+\frac{d}{4\,x^{4}ac^{2}}+\frac{b}{4\,a^{2}c x^{4}}-\frac{d^{2}}{2\,ac^{3}x^{2}}-\frac{bd}{2\,a^{2}c^{2}x^{2}}-\frac{b^{2}}{2\,a^{3}c x^{2}}-\frac{\ln \left(x\right)d^{3}}{ac^{4}} -\frac{b\ln \left(x\right)d^{2}}{a^{2}c^{3}}-\frac{\ln \left(x\right)b^{2}d}{a^{3}c^{2}}-\frac{\ln \left(x\right)b^{3}}{a^{4}c}+\frac{d^{4}\ln \left(dx^{2}+c\right)}{2\,c^{4}\left(ad-bc\right)}-\frac{b^{4}\ln \left(bx^{2}+a\right)}{2\,a^{4}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/6/a/c/x^6+1/4/x^4/a/c^2*d+1/4/x^4/a^2/c*b-1/2/a/c^3/x^2*d^2-1/
2/a^2/c^2/x^2*b*d-1/2/a^3/c/x^2*b^2-1/a/c^4*ln(x)*d^3-1/a^2/c^3*l
n(x)*b*d^2-1/a^3/c^2*ln(x)*b^2*d-1/a^4/c*ln(x)*b^3+1/2*d^4/c^4/(a
*d-b*c)*ln(d*x^2+c)-1/2*b^4/a^4/(a*d-b*c)*ln(b*x^2+a)
```

Maxima [A] time = 1.36096, size = 223, normalized size = 1.44

$$\frac{b^{4}\log \left(bx^{2}+a\right)}{2\left(a^{4}bc-a^{5}d\right)}-\frac{d^{4}\log \left(dx^{2}+c\right)}{2\left(bc^{5}-ac^{4}d\right)}-\frac{\left(b^{3}c^{3}+ab^{2}c^{2}d+a^{2}bcd^{2}+a^{3}d^{3}\right)\log \left(x^{2}\right)}{2\,a^{4}c^{4}} -\frac{6\left(b^{2}c^{2}+abcd+a^{2}d^{2}\right)x^{4}+2\,a^{2}c^{2}-3\left(abc^{2}+a^{2}cd\right)x^{2}}{12\,a^{3}c^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^7),x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^4*b*c - a^5*d) - 1/2*d^4*log(d*x^2 + c)
/(b*c^5 - a*c^4*d) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a
^3*d^3)*log(x^2)/(a^4*c^4) - 1/12*(6*(b^2*c^2 + a*b*c*d + a^2*d^2
)*x^4 + 2*a^2*c^2 - 3*(a*b*c^2 + a^2*c*d)*x^2)/(a^3*c^3*x^6)
```

Fricas [A] time = 1.69139, size = 209, normalized size = 1.35

$$\frac{6\,b^{4}c^{4}x^{6}\log \left(bx^{2}+a\right)-6\,a^{4}d^{4}x^{6}\log \left(dx^{2}+c\right)-2\,a^{3}bc^{4}+2\,a^{4}c^{3}d-12\left(b^{4}c^{4}-a^{4}d^{4}\right)x^{6}\log \left(x\right)-6\left(ab^{3}c^{4}-a^{4}cd^{3}\right)x^{4}+3\,(}{12\left(a^{4}bc^{5}-a^{5}c^{4}d\right)x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^7),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^4*c^4*x^6*log(b*x^2 + a) - 6*a^4*d^4*x^6*log(d*x^2 + c)
 - 2*a^3*b*c^4 + 2*a^4*c^3*d - 12*(b^4*c^4 - a^4*d^4)*x^6*log(x)
- 6*(a*b^3*c^4 - a^4*c*d^3)*x^4 + 3*(a^2*b^2*c^4 - a^4*c^2*d^2)*x
^2)/((a^4*b*c^5 - a^5*c^4*d)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^7),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.241 $\int \frac{x^5}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

```
[Out] -a^2/(2*b^2*(b*c - a*d)*(a + b*x^2)) - (a*(2*b*c - a*d)*Log[a + b
*x^2])/(2*b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(2*d*(b*c - a
*d)^2)
```

Rubi [A] time = 0.214763, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -a^2/(2*b^2*(b*c - a*d)*(a + b*x^2)) - (a*(2*b*c - a*d)*Log[a + b
*x^2])/(2*b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(2*d*(b*c - a
*d)^2)
```

Rubi in Sympy [A] time = 35.8984, size = 76, normalized size = 0.82

$$\frac{a^2}{2b^2\left(a+bx^2\right)(ad-bc)}+\frac{a\,(ad-2bc)\log\left(a+bx^2\right)}{2b^2\,(ad-bc)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2/(2*b**2*(a + b*x**2)*(a*d - b*c)) + a*(a*d - 2*b*c)*log(a +
b*x**2)/(2*b**2*(a*d - b*c)**2) + c**2*log(c + d*x**2)/(2*d*(a*d
- b*c)**2)
```

Mathematica [A] time = 0.0769028, size = 91, normalized size = 0.98

$$\frac{a^2d(ad-bc)+b^2c^2\left(a+bx^2\right)\log\left(c+dx^2\right)+ad\left(a+bx^2\right)(ad-2bc)\log\left(a+bx^2\right)}{2b^2d\left(a+bx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a^2*d*(-(b*c) + a*d) + a*d*(-2*b*c + a*d)*(a + b*x^2)*Log[a + b*
x^2] + b^2*c^2*(a + b*x^2)*Log[c + d*x^2])/(2*b^2*d*(b*c - a*d)^2
*(a + b*x^2))
```

Maple [A] time = 0.019, size = 136, normalized size = 1.5

$$\frac{c^2\ln\left(dx^2+c\right)}{2\,(ad-bc)^2\,d}+\frac{a^2\ln\left(bx^2+a\right)d}{2\,(ad-bc)^2\,b^2}-\frac{a\ln\left(bx^2+a\right)c}{(ad-bc)^2\,b}+\frac{a^3d}{2\,(ad-bc)^2\,b^2\,(bx^2+a)}-\frac{a^2c}{2\,(ad-bc)^2\,b\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*c^2/(a*d-b*c)^2/d*ln(d*x^2+c)+1/2*a^2/(a*d-b*c)^2/b^2*ln(b*x^
2+a)*d-a/(a*d-b*c)^2/b*ln(b*x^2+a)*c+1/2*a^3/(a*d-b*c)^2/b^2/(b*x
^2+a)*d-1/2*a^2/(a*d-b*c)^2/b/(b*x^2+a)*c
```

Maxima [A] time = 1.34266, size = 176, normalized size = 1.89

$$\frac{c^2 \log \left(dx^2 + c\right)}{2 \left(b^2c^2d - 2 \, abcd^2 + a^2d^3\right)} - \frac{a^2}{2 \left(ab^3c - a^2b^2d + (b^4c - ab^3d)x^2\right)} - \frac{\left(2 \, abc - a^2d\right) \log \left(bx^2 + a\right)}{2 \left(b^4c^2 - 2 \, ab^3cd + a^2b^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(d*x^2 + c)/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*
a^2/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2) - 1/2*(2*a*b*c
- a^2*d)*log(b*x^2 + a)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

Fricas [A] time = 0.259955, size = 219, normalized size = 2.35

$$-\frac{a^2bcd - a^3d^2 + \left(2 \, a^2bcd - a^3d^2 + \left(2 \, ab^2cd - a^2bd^2\right)x^2\right) \log \left(bx^2 + a\right) - \left(b^3c^2x^2 + ab^2c^2\right) \log \left(dx^2 + c\right)}{2 \left(ab^4c^2d - 2 \, a^2b^3cd^2 + a^3b^2d^3 + (b^5c^2d - 2 \, ab^4cd^2 + a^2b^3d^3)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d
 - a^2*b*d^2)*x^2)*log(b*x^2 + a) - (b^3*c^2*x^2 + a*b^2*c^2)*log
(d*x^2 + c))/(a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3 + (b^5*
c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*x^2)
```

Sympy [A] time = 20.7442, size = 348, normalized size = 3.74

$$\frac{a^2}{2a^2b^2d - 2ab^3c + x^2 \left(2ab^3d - 2b^4c\right)} + \frac{a \left(ad - 2bc\right) \log \left(x^2 + \frac{\frac{a^4d^3(ad-2bc)}{b(ad-bc)^2} - \frac{3a^3cd^2(ad-2bc)}{(ad-bc)^2} + \frac{3a^2bc^2d(ad-2bc)}{(ad-bc)^2} + a^2cd - \frac{ab^2c^3(ad-2bc)}{(ad-bc)^2} - 3abc^2}{a^2d^2-2abcd-b^2c^2}\right)}{2b^2 \left(ad - bc\right)^2} + \frac{c^2 \log \left(x^2 + \frac{\frac{a^3bc^2d^2}{(ad-bc)^2} - \frac{3a^2b^2c^3d}{(ad-bc)^2} + a^2cd + \frac{3ab^3c^4}{(ad-bc)^2} - 3abc^2 - \frac{b^4c^5}{d(ad-bc)^2}}{a^2d^2-2abcd-b^2c^2}\right)}{2d \left(ad - bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2/(2*a**2*b**2*d - 2*a*b**3*c + x**2*(2*a*b**3*d - 2*b**4*c))
+ a*(a*d - 2*b*c)*log(x**2 + (a**4*d**3*(a*d - 2*b*c)/(b*(a*d - b
*c)**2) - 3*a**3*c*d**2*(a*d - 2*b*c)/(a*d - b*c)**2 + 3*a**2*b*c
**2*d*(a*d - 2*b*c)/(a*d - b*c)**2 + a**2*c*d - a*b**2*c**3*(a*d
- 2*b*c)/(a*d - b*c)**2 - 3*a*b*c**2)/(a**2*d**2 - 2*a*b*c*d - b*
*2*c**2))/(2*b**2*(a*d - b*c)**2) + c**2*log(x**2 + (a**3*b*c**2*
```

```
d**2/(a*d - b*c)**2 - 3*a**2*b**2*c**3*d/(a*d - b*c)**2 + a**2*c*
d + 3*a*b**3*c**4/(a*d - b*c)**2 - 3*a*b*c**2 - b**4*c**5/(d*(a*d
 - b*c)**2))/(a**2*d**2 - 2*a*b*c*d - b**2*c**2))/(2*d*(a*d - b*c
)**2)
```

GIAC/XCAS [A] time = 0.253664, size = 205, normalized size = 2.2

$$\frac{c^2\ln\left(\left|dx^2+c\right|\right)}{2\,(b^2c^2d-2\,abcd^2+a^2d^3)}-\frac{\left(2\,abc-a^2d\right)\ln\left(\left|bx^2+a\right|\right)}{2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)}+\frac{2\,abcx^2-a^2dx^2+a^2c}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 1/2*c^2*ln(abs(d*x^2 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) -
1/2*(2*a*b*c - a^2*d)*ln(abs(b*x^2 + a))/(b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2) + 1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*(b*x^2 + a))
```

3.242 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=108

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)^2}+\frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(c*x)/(2*d*(b*c - a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.243666, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)^2}+\frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*x)/(2*d*(b*c - a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 38.5092, size = 94, normalized size = 0.87

$$\frac{a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad-bc\right)^2}-\frac{\sqrt{c}\left(3ad-bc\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}\left(ad-bc\right)^2}+\frac{cx}{2d\left(c+dx^2\right)(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] a**(3/2)*atan(sqrt(b)*x/sqrt(a))/(sqrt(b)*(a*d - b*c)**2) - sqrt(
c)*(3*a*d - b*c)*atan(sqrt(d)*x/sqrt(c))/(2*d**(3/2)*(a*d - b*c)*
*2) + c*x/(2*d*(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.218645, size = 108, normalized size = 1.

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad-bc)^2}+\frac{\sqrt{c}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}+\frac{cx}{2d\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (c*x)/(2*d*(-(b*c) + a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(Sqrt[b]*(-(b*c) + a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.014, size = 144, normalized size = 1.3

$$\frac{acx}{2\left(ad-bc\right)^{2}\left(dx^{2}+c\right)}-\frac{c^{2}xb}{2\left(ad-bc\right)^{2}d\left(dx^{2}+c\right)}-\frac{3\,ac}{2\left(ad-bc\right)^{2}}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{c^{2}b}{2\left(ad-bc\right)^{2}d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{a^{2}}{\left(ad-bc\right)^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^2*x/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x/(d*x^2+c)
*b-3/2*c/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+1/2*c^
2/(a*d-b*c)^2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+a^2/(a*d-b*
c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.306612, size = 1, normalized size = 0.01

$$\left[\frac{2\left(ad^{2}x^{2}+acd\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-\left(bc^{2}-3\,acd+\left(bcd-3\,ad^{2}\right)x^{2}\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^{2}-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^{2}+c}\right)-2\left(bc^{2}-acd\right)x}{4\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}+\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*d^2*x^2 + a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a
/b) - a)/(b*x^2 + a)) - (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)
*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(
b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c
^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/4*(4*(a*d^2*x^2 + a*c*d)*
sqrt(a/b)*arctan(x/sqrt(a/b)) - (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d
^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c
)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3
+ (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/2*((b*c^2 - 3*a*c
*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) + (a*d^
2*x^2 + a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)) - (b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/2*(2*(a*d^2*x
^2 + a*c*d)*sqrt(a/b)*arctan(x/sqrt(a/b)) + (b*c^2 - 3*a*c*d + (b
*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) - (b*c^2 - a*c
*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*x^2)]
```

Sympy [A] time = 42.6211, size = 1850, normalized size = 17.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] c*x/(2*a*c*d**2 - 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2)) + sq
rt(-a**3/b)*log(x + (-20*a**5*b*d**8*(-a**3/b)**(3/2)/(a*d - b*c)
**6 + 84*a**4*b**2*c*d**7*(-a**3/b)**(3/2)/(a*d - b*c)**6 - 8*a**
4*d**4*sqrt(-a**3/b)/(a*d - b*c)**2 - 136*a**3*b**3*c**2*d**6*(-a
**3/b)**(3/2)/(a*d - b*c)**6 - 27*a**3*b*c*d**3*sqrt(-a**3/b)/(a*
d - b*c)**2 + 104*a**2*b**4*c**3*d**5*(-a**3/b)**(3/2)/(a*d - b*c
)**6 + 27*a**2*b**2*c**2*d**2*sqrt(-a**3/b)/(a*d - b*c)**2 - 36*a
*b**5*c**4*d**4*(-a**3/b)**(3/2)/(a*d - b*c)**6 - 9*a*b**3*c**3*d
*sqrt(-a**3/b)/(a*d - b*c)**2 + 4*b**6*c**5*d**3*(-a**3/b)**(3/2)
/(a*d - b*c)**6 + b**4*c**4*sqrt(-a**3/b)/(a*d - b*c)**2)/(12*a**
3*d**2 - 7*a**2*b*c*d + a*b**2*c**2))/(2*(a*d - b*c)**2) - sqrt(-
a**3/b)*log(x + (20*a**5*b*d**8*(-a**3/b)**(3/2)/(a*d - b*c)**6 -
 84*a**4*b**2*c*d**7*(-a**3/b)**(3/2)/(a*d - b*c)**6 + 8*a**4*d**
4*sqrt(-a**3/b)/(a*d - b*c)**2 + 136*a**3*b**3*c**2*d**6*(-a**3/b
)**(3/2)/(a*d - b*c)**6 + 27*a**3*b*c*d**3*sqrt(-a**3/b)/(a*d - b
*c)**2 - 104*a**2*b**4*c**3*d**5*(-a**3/b)**(3/2)/(a*d - b*c)**6
- 27*a**2*b**2*c**2*d**2*sqrt(-a**3/b)/(a*d - b*c)**2 + 36*a*b**5
*c**4*d**4*(-a**3/b)**(3/2)/(a*d - b*c)**6 + 9*a*b**3*c**3*d*sqrt
(-a**3/b)/(a*d - b*c)**2 - 4*b**6*c**5*d**3*(-a**3/b)**(3/2)/(a*d
 - b*c)**6 - b**4*c**4*sqrt(-a**3/b)/(a*d - b*c)**2)/(12*a**3*d**
2 - 7*a**2*b*c*d + a*b**2*c**2))/(2*(a*d - b*c)**2) + sqrt(-c/d**
3)*(3*a*d - b*c)*log(x + (-5*a**5*b*d**8*(-c/d**3)**(3/2)*(3*a*d
- b*c)**3/(2*(a*d - b*c)**6) + 21*a**4*b**2*c*d**7*(-c/d**3)**(3/
2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 4*a**4*d**4*sqrt(-c/d**3
)*(3*a*d - b*c)/(a*d - b*c)**2 - 17*a**3*b**3*c**2*d**6*(-c/d**3)
**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 27*a**3*b*c*d**3*sqrt(-
c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + 13*a**2*b**4*c**3*d**5
*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 27*a**2*b**2*
c**2*d**2*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 9*a*b*
*5*c**4*d**4*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6)
 - 9*a*b**3*c**3*d*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2)
 + b**6*c**5*d**3*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c
)**6) + b**4*c**4*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2))
/(12*a**3*d**2 - 7*a**2*b*c*d + a*b**2*c**2))/(4*(a*d - b*c)**2)
- sqrt(-c/d**3)*(3*a*d - b*c)*log(x + (5*a**5*b*d**8*(-c/d**3)**(
3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 21*a**4*b**2*c*d**7*(-
c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + 4*a**4*d**4*
sqrt(-c/d**3)*(3*a*d - b*c)/(a*d - b*c)**2 + 17*a**3*b**3*c**2*d*
*6*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 27*a**3*b*c
*d**3*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 13*a**2*b*
*4*c**3*d**5*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 2
7*a**2*b**2*c**2*d**2*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d - b*c)*
*2) + 9*a*b**5*c**4*d**4*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*
d - b*c)**6) + 9*a*b**3*c**3*d*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*
d - b*c)**2) - b**6*c**5*d**3*(-c/d**3)**(3/2)*(3*a*d - b*c)**3/(
2*(a*d - b*c)**6) - b**4*c**4*sqrt(-c/d**3)*(3*a*d - b*c)/(2*(a*d
 - b*c)**2))/(12*a**3*d**2 - 7*a**2*b*c*d + a*b**2*c**2))/(4*(a*d
 - b*c)**2)
```

GIAC/XCAS [A] time = 0.288677, size = 163, normalized size = 1.51

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} + \frac{\left(bc^2 - 3\,acd\right) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{cd}} - \frac{cx}{2\,(bcd - ad^2)(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) + 1/2*(b*c^2 - 3*a*c*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*sqrt(c*d)) - 1/2*c*x/((b*c*d - a*d^2)*(d*x
^2 + c))
```

3.243 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^2} \, dx$

Optimal. Leaf size=74

$$-\frac{c}{2d\left(c+dx^2\right)(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] -c/(2*d*(b*c - a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a
*d)^2) + (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.160293, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{c}{2d\left(c+dx^2\right)(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -c/(2*d*(b*c - a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a
*d)^2) + (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 25.5599, size = 58, normalized size = 0.78

$$-\frac{a\log\left(a+bx^2\right)}{2\left(ad-bc\right)^2}+\frac{a\log\left(c+dx^2\right)}{2\left(ad-bc\right)^2}+\frac{c}{2d\left(c+dx^2\right)(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -a*log(a + b*x**2)/(2*(a*d - b*c)**2) + a*log(c + d*x**2)/(2*(a*d
 - b*c)**2) + c/(2*d*(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.0556604, size = 74, normalized size = 1.

$$\frac{c}{2d\left(c+dx^2\right)(ad-bc)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] c/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c -
 a*d)^2) + (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.017, size = 95, normalized size = 1.3

$$\frac{ac}{2\left(ad-bc\right)^2\left(dx^2+c\right)}-\frac{bc^2}{2\left(ad-bc\right)^2d\left(dx^2+c\right)}+\frac{a\ln\left(dx^2+c\right)}{2\left(ad-bc\right)^2}-\frac{a\ln\left(bx^2+a\right)}{2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2/(a*d-b*c)^2*c/(d*x^2+c)*a-1/2/(a*d-b*c)^2*c^2/d/(d*x^2+c)*b+1
/2/(a*d-b*c)^2*a*ln(d*x^2+c)-1/2*a/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.35796, size = 142, normalized size = 1.92

$$-\frac{a\log\left(bx^2+a\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)}+\frac{a\log\left(dx^2+c\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)}-\frac{c}{2\,(bc^2d-acd^2+(bcd^2-ad^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a*log
(d*x^2 + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c/(b*c^2*d - a*
c*d^2 + (b*c*d^2 - a*d^3)*x^2)
```

Fricas [A] time = 0.224049, size = 158, normalized size = 2.14

$$-\frac{bc^2-acd+\left(ad^2x^2+acd\right)\log\left(bx^2+a\right)-\left(ad^2x^2+acd\right)\log\left(dx^2+c\right)}{2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c^2 - a*c*d + (a*d^2*x^2 + a*c*d)*log(b*x^2 + a) - (a*d^2
*x^2 + a*c*d)*log(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2)
```

Sympy [A] time = 8.44839, size = 253, normalized size = 3.42

$$\frac{a\log\left(x^2+\frac{-\frac{a^4d^3}{(ad-bc)^2}+\frac{3a^3bcd^2}{(ad-bc)^2}-\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d+\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2\,(ad-bc)^2}-\frac{a\log\left(x^2+\frac{\frac{a^4d^3}{(ad-bc)^2}-\frac{3a^3bcd^2}{(ad-bc)^2}+\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d-\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2\,(ad-bc)^2}+\frac{c}{2acd^2-2bc^2d+x^2\,(2ad^3-2bcd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] a*log(x**2 + (-a**4*d**3/(a*d - b*c)**2 + 3*a**3*b*c*d**2/(a*d -
b*c)**2 - 3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a**2*d + a*b**3*c**
3/(a*d - b*c)**2 + a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) - a*log(x
**2 + (a**4*d**3/(a*d - b*c)**2 - 3*a**3*b*c*d**2/(a*d - b*c)**2
+ 3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a**2*d - a*b**3*c**3/(a*d -
 b*c)**2 + a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) + c/(2*a*c*d**2 -
 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2))
```

GIAC/XCAS [A] time = 0.295666, size = 123, normalized size = 1.66

$$-\frac{\frac{ad^2\ln\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{cd}{(bcd-ad^2)(dx^2+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*ln(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3) + c*d/((b*c*d - a*d^2)*(d*x^2 + c))
)/d
```

3.244 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=104

$$\frac{x}{2(c+dx^2)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)^2}$$

```
[Out] x/(2*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(b*c - a*d)^2 + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*Sqrt[c]*Sqrt[d]*(b*c - a*d)^2)
```

Rubi [A] time = 0.182265, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{x}{2(c+dx^2)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x/(2*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(b*c - a*d)^2 + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*Sqrt[c]*Sqrt[d]*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 34.8828, size = 88, normalized size = 0.85

$$-\frac{\sqrt{a}\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(ad-bc)^2}-\frac{x}{2(c+dx^2)(ad-bc)}+\frac{(ad+bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(a)*sqrt(b)*atan(sqrt(b)*x/sqrt(a))/(a*d - b*c)**2 - x/(2*(c
 + d*x**2)*(a*d - b*c)) + (a*d + b*c)*atan(sqrt(d)*x/sqrt(c))/(2*
sqrt(c)*sqrt(d)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.213511, size = 90, normalized size = 0.87

$$\frac{\frac{x(bc-ad)}{c+dx^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-2\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (((b*c - a*d)*x)/(c + d*x^2) - 2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*
x)/Sqrt[a]] + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*
Sqrt[d]))/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.014, size = 134, normalized size = 1.3

$$-\frac{axd}{2\,(ad-bc)^2\,(dx^2+c)}+\frac{bxc}{2\,(ad-bc)^2\,(dx^2+c)}+\frac{ad}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{bc}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{ab}{(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*x/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x/(d*x^2+c)*b*c+
1/2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*d+1/2/(a*d-
b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b*c-a*b/(a*d-b*c)^2/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280536, size = 1, normalized size = 0.01

$$\left[\frac{2\left(dx^2+c\right)\sqrt{-ab}\sqrt{-cd}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(bc-ad)\sqrt{-cd}x+\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\log\left(\frac{2\,cdx+(dx^2-c)\sqrt{-cd}}{dx^2+c}\right)}{4\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x^2)\sqrt{-cd}},\ -\frac{4\left(dx^2+c\right)\sqrt{ab}\sqrt{-cd}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-2\,(bc-ad)\sqrt{-cd}x-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\log\left(\frac{2\,cdx+(dx^2-c)\sqrt{-cd}}{dx^2+c}\right)}{4\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x^2)\sqrt{-cd}},\ -\frac{2\left(dx^2+c\right)\sqrt{ab}\sqrt{cd}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-(bc-ad)\sqrt{cd}x-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\arctan\left(\frac{\sqrt{cd}x}{c}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x^2)\sqrt{cd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(d*x^2 + c)*sqrt(-a*b)*sqrt(-c*d)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)) + 2*(b*c - a*d)*sqrt(-c*d)*x + (b*c^2 + a*
c*d + (b*c*d + a*d^2)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))
/(d*x^2 + c)))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(-c*d)), 1/2*((d*x^2 + c)*sqrt(-
a*b)*sqrt(c*d)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + (b
*c - a*d)*sqrt(c*d)*x + (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*arc
tan(sqrt(c*d)*x/c))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(c*d)), -1/4*(4*(d*x^2 + c)
*sqrt(a*b)*sqrt(-c*d)*arctan(b*x/sqrt(a*b)) - 2*(b*c - a*d)*sqrt(
-c*d)*x - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*log((2*c*d*x + (d
*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)))/((b^2*c^3 - 2*a*b*c^2*d + a^2
*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(-c*d)), -1
/2*(2*(d*x^2 + c)*sqrt(a*b)*sqrt(c*d)*arctan(b*x/sqrt(a*b)) - (b*
c - a*d)*sqrt(c*d)*x - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*arct
an(sqrt(c*d)*x/c))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2
```

```
*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(c*d))]
```

Sympy [A] time = 23.4487, size = 1530, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -x/(2*a*c*d - 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d)) + sqrt(-a*b)*
log(x + (-4*a**5*c*d**6*(-a*b)**(3/2)/(a*d - b*c)**6 + 4*a**4*b*c
**2*d**5*(-a*b)**(3/2)/(a*d - b*c)**6 + 24*a**3*b**2*c**3*d**4*(-
a*b)**(3/2)/(a*d - b*c)**6 - a**3*d**3*sqrt(-a*b)/(a*d - b*c)**2
- 56*a**2*b**3*c**4*d**3*(-a*b)**(3/2)/(a*d - b*c)**6 - 3*a**2*b*
c*d**2*sqrt(-a*b)/(a*d - b*c)**2 + 44*a*b**4*c**5*d**2*(-a*b)**(3
/2)/(a*d - b*c)**6 - 11*a*b**2*c**2*d*sqrt(-a*b)/(a*d - b*c)**2 -
 12*b**5*c**6*d*(-a*b)**(3/2)/(a*d - b*c)**6 - b**3*c**3*sqrt(-a*
b)/(a*d - b*c)**2)/(a*b*d + b**2*c))/(2*(a*d - b*c)**2) - sqrt(-a
*b)*log(x + (4*a**5*c*d**6*(-a*b)**(3/2)/(a*d - b*c)**6 - 4*a**4*
b*c**2*d**5*(-a*b)**(3/2)/(a*d - b*c)**6 - 24*a**3*b**2*c**3*d**4
*(-a*b)**(3/2)/(a*d - b*c)**6 + a**3*d**3*sqrt(-a*b)/(a*d - b*c)*
*2 + 56*a**2*b**3*c**4*d**3*(-a*b)**(3/2)/(a*d - b*c)**6 + 3*a**2
*b*c*d**2*sqrt(-a*b)/(a*d - b*c)**2 - 44*a*b**4*c**5*d**2*(-a*b)*
*(3/2)/(a*d - b*c)**6 + 11*a*b**2*c**2*d*sqrt(-a*b)/(a*d - b*c)**
2 + 12*b**5*c**6*d*(-a*b)**(3/2)/(a*d - b*c)**6 + b**3*c**3*sqrt(
-a*b)/(a*d - b*c)**2)/(a*b*d + b**2*c))/(2*(a*d - b*c)**2) + sqrt
(-1/(c*d))*(a*d + b*c)*log(x + (-a**5*c*d**6*(-1/(c*d))**(3/2)*(a
*d + b*c)**3/(2*(a*d - b*c)**6) + a**4*b*c**2*d**5*(-1/(c*d))**(3
/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) + 3*a**3*b**2*c**3*d**4*(-1
/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 - a**3*d**3*sqrt(-1/
(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2) - 7*a**2*b**3*c**4*d**3*(-1
/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 - 3*a**2*b*c*d**2*sq
rt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2) + 11*a*b**4*c**5*d**2
*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 11*a*b**2*
c**2*d*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2) - 3*b**5*c**
6*d*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - b**3*c*
*3*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2))/(a*b*d + b**2*c
))/(4*(a*d - b*c)**2) - sqrt(-1/(c*d))*(a*d + b*c)*log(x + (a**5*
c*d**6*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - a**4
*b*c**2*d**5*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6)
- 3*a**3*b**2*c**3*d**4*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b
*c)**6 + a**3*d**3*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c)**2)
+ 7*a**2*b**3*c**4*d**3*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(a*d - b
*c)**6 + 3*a**2*b*c*d**2*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d - b*c
)**2) - 11*a*b**4*c**5*d**2*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(2*(
a*d - b*c)**6) + 11*a*b**2*c**2*d*sqrt(-1/(c*d))*(a*d + b*c)/(2*(
a*d - b*c)**2) + 3*b**5*c**6*d*(-1/(c*d))**(3/2)*(a*d + b*c)**3/(
2*(a*d - b*c)**6) + b**3*c**3*sqrt(-1/(c*d))*(a*d + b*c)/(2*(a*d
- b*c)**2))/(a*b*d + b**2*c))/(4*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.270926, size = 149, normalized size = 1.43

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} + \frac{(bc + ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{x}{2\,(dx^2 + c)(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] -a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
a*b)) + 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(c*d)) + 1/2*x/((d*x^2 + c)*(b*c - a*d))
```

3.245 $\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2\left(c+dx^2\right)(bc-ad)}+\frac{b\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{b\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] 1/(2*(b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(2*(b*c - a*d)
^2) - (b*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.121814, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{1}{2\left(c+dx^2\right)(bc-ad)}+\frac{b\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{b\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(2*(b*c - a*d)
^2) - (b*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 21.9376, size = 56, normalized size = 0.8

$$\frac{b\log\left(a+bx^2\right)}{2\,(ad-bc)^2}-\frac{b\log\left(c+dx^2\right)}{2\,(ad-bc)^2}-\frac{1}{2\left(c+dx^2\right)(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] b*log(a + b*x**2)/(2*(a*d - b*c)**2) - b*log(c + d*x**2)/(2*(a*d
- b*c)**2) - 1/(2*(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.0477443, size = 66, normalized size = 0.94

$$\frac{b\left(c+dx^2\right)\log\left(a+bx^2\right)-ad-b\left(c+dx^2\right)\log\left(c+dx^2\right)+bc}{2\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x^2)*Log[a + b*x^2] - b*(c + d*x^2)*Log[c +
 d*x^2])/(2*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.016, size = 90, normalized size = 1.3

$$-\frac{ad}{2\,(ad-bc)^2\,(dx^2+c)}+\frac{bc}{2\,(ad-bc)^2\,(dx^2+c)}-\frac{b\ln\left(dx^2+c\right)}{2\,(ad-bc)^2}+\frac{b\ln\left(bx^2+a\right)}{2\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x^2+c)*a+1/2/(a*d-b*c)^2/(d*x^2+c)*b*c-1/2/
(a*d-b*c)^2*b*ln(d*x^2+c)+1/2*b/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.35508, size = 134, normalized size = 1.91

$$\frac{b \log\left(bx^2 + a\right)}{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)} - \frac{b \log\left(dx^2 + c\right)}{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)} + \frac{1}{2\left(bc^2 - acd + (bcd - ad^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*b*log(
d*x^2 + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2/(b*c^2 - a*c*d +
 (b*c*d - a*d^2)*x^2)
```

Fricas [A] time = 0.235126, size = 139, normalized size = 1.99

$$\frac{bc - ad + \left(bdx^2 + bc\right)\log\left(bx^2 + a\right) - \left(bdx^2 + bc\right)\log\left(dx^2 + c\right)}{2\left(b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c - a*d + (b*d*x^2 + b*c)*log(b*x^2 + a) - (b*d*x^2 + b*c)
*log(d*x^2 + c))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*x^2)
```

Sympy [A] time = 9.39844, size = 248, normalized size = 3.54

$$-\frac{b \log\left(x^2 + \frac{-\frac{a^3bd^3}{(ad-bc)^2} + \frac{3a^2b^2cd^2}{(ad-bc)^2} - \frac{3ab^3c^2d}{(ad-bc)^2} + abd + \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{2\left(ad - bc\right)^2} + \frac{b \log\left(x^2 + \frac{\frac{a^3bd^3}{(ad-bc)^2} - \frac{3a^2b^2cd^2}{(ad-bc)^2} + \frac{3ab^3c^2d}{(ad-bc)^2} + abd - \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{2\left(ad - bc\right)^2} - \frac{1}{2acd - 2bc^2 + x^2\left(2ad^2 - 2bcd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -b*log(x**2 + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(
a*d - b*c)**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**
3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2) + b*log
(x**2 + (a**3*b*d**3/(a*d - b*c)**2 - 3*a**2*b**2*c*d**2/(a*d - b
*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d - b**4*c**3/(a*d
- b*c)**2 + b**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2) - 1/(2*a*c*d -
 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d))
```

GIAC/XCAS [A] time = 0.270448, size = 115, normalized size = 1.64

$$\frac{bd\ln\left(\left|b - \frac{bc}{dx^2+c} + \frac{ad}{dx^2+c}\right|\right)}{2\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)} + \frac{d}{2\left(bcd - ad^2\right)(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] 1/2*b*d*ln(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3) + 1/2*d/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.246 $\int \frac{1}{(a+bx^2)(c+dx^2)^2} dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.212538, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 42.7351, size = 94, normalized size = 0.86

$$\frac{dx}{2c\left(c+dx^2\right)(ad-bc)}+\frac{\sqrt{d}\,(ad-3bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}\,(ad-bc)^2}+\frac{b^{\frac{3}{2}}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*c*(c + d*x**2)*(a*d - b*c)) + sqrt(d)*(a*d - 3*b*c)*atan(s
qrt(d)*x/sqrt(c))/(2*c**(3/2)*(a*d - b*c)**2) + b**(3/2)*atan(sqr
t(b)*x/sqrt(a))/(sqrt(a)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.307808, size = 95, normalized size = 0.87

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}(ad-3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{dx(ad-bc)}{c(c+dx^2)}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/c^(3/2))/(2*(b*c - a*d)^2)
```

Maple [A] time = 0., size = 144, normalized size = 1.3

$$\frac{d^2xa}{2\left(ad-bc\right)^2 c\left(dx^2+c\right)}-\frac{dxb}{2\left(ad-bc\right)^2\left(dx^2+c\right)}+\frac{ad^2}{2\left(ad-bc\right)^2 c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,bd}{2\left(ad-bc\right)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2}{\left(ad-bc\right)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x/(d*x^2+c)
*b+1/2*d^2/(a*d-b*c)^2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-3/
2*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+b^2/(a*d-b*
c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.348928, size = 1, normalized size = 0.01

$$\left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)}\right.$$
$$-\frac{\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)-\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\left(bcd-ad^2\right)x}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)},\ \frac{4\left(bcdx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)
*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(
b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1/2*((3*b*c^2 - a*c*d + (3
*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - (b*c*d
*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^
2 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2
 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^
2 + b*c^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - (3*b*c^2 - a*c*d
 + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c
) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/
2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - (3
*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(d*x/(c*s
qrt(d/c))) - (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*
d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

Sympy [A] time = 46.4138, size = 2033, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*a*c**2*d - 2*b*c**3 + x**2*(2*a*c*d**2 - 2*b*c**2*d)) - sq
rt(-b**3/a)*log(x + (-4*a**7*c**3*d**6*(-b**3/a)**(3/2)/(a*d - b*
c)**6 + 28*a**6*b*c**4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 64*
a**5*b**2*c**5*d**4*(-b**3/a)**(3/2)/(a*d - b*c)**6 - a**5*d**5*s
qrt(-b**3/a)/(a*d - b*c)**2 + 56*a**4*b**3*c**6*d**3*(-b**3/a)**(
3/2)/(a*d - b*c)**6 + 9*a**4*b*c*d**4*sqrt(-b**3/a)/(a*d - b*c)**
2 - 4*a**3*b**4*c**7*d**2*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 27*a*
*3*b**2*c**2*d**3*sqrt(-b**3/a)/(a*d - b*c)**2 - 20*a**2*b**5*c**
8*d*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**2*b**3*c**3*d**2*sqrt
(-b**3/a)/(a*d - b*c)**2 + 8*a*b**6*c**9*(-b**3/a)**(3/2)/(a*d -
b*c)**6 + 8*b**5*c**5*sqrt(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*d*
*3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(2*(a*d - b*c)**2) + sqrt
(-b**3/a)*log(x + (4*a**7*c**3*d**6*(-b**3/a)**(3/2)/(a*d - b*c)*
*6 - 28*a**6*b*c**4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 64*a**
5*b**2*c**5*d**4*(-b**3/a)**(3/2)/(a*d - b*c)**6 + a**5*d**5*sqrt
(-b**3/a)/(a*d - b*c)**2 - 56*a**4*b**3*c**6*d**3*(-b**3/a)**(3/2
)/(a*d - b*c)**6 - 9*a**4*b*c*d**4*sqrt(-b**3/a)/(a*d - b*c)**2 +
 4*a**3*b**4*c**7*d**2*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**3*
b**2*c**2*d**3*sqrt(-b**3/a)/(a*d - b*c)**2 + 20*a**2*b**5*c**8*d
*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 27*a**2*b**3*c**3*d**2*sqrt(-b
**3/a)/(a*d - b*c)**2 - 8*a*b**6*c**9*(-b**3/a)**(3/2)/(a*d - b*c
)**6 - 8*b**5*c**5*sqrt(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*d**3
- 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(2*(a*d - b*c)**2) - sqrt(-d
/c**3)*(a*d - 3*b*c)*log(x + (-a**7*c**3*d**6*(-d/c**3)**(3/2)*(a
*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 7*a**6*b*c**4*d**5*(-d/c**3)*
*(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 8*a**5*b**2*c**5*d**
4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - a**5*d**5*sq
rt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 7*a**4*b**3*c**6*d
**3*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + 9*a**4*b*c
*d**4*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - a**3*b**4*
c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) -
27*a**3*b**2*c**2*d**3*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)
**2) - 5*a**2*b**5*c**8*d*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a
*d - b*c)**6) + 27*a**2*b**3*c**3*d**2*sqrt(-d/c**3)*(a*d - 3*b*c
)/(2*(a*d - b*c)**2) + a*b**6*c**9*(-d/c**3)**(3/2)*(a*d - 3*b*c)
**3/(a*d - b*c)**6 + 4*b**5*c**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(a*d
 - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/
(4*(a*d - b*c)**2) + sqrt(-d/c**3)*(a*d - 3*b*c)*log(x + (a**7*c*
*3*d**6*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 7*
a**6*b*c**4*d**5*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)
**6) + 8*a**5*b**2*c**5*d**4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a
*d - b*c)**6 + a**5*d**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*
c)**2) - 7*a**4*b**3*c**6*d**3*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/
(a*d - b*c)**6 - 9*a**4*b*c*d**4*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(
a*d - b*c)**2) + a**3*b**4*c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*
c)**3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt(-d/c**3)*(
a*d - 3*b*c)/(2*(a*d - b*c)**2) + 5*a**2*b**5*c**8*d*(-d/c**3)**(
3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 27*a**2*b**3*c**3*d**2
*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - a*b**6*c**9*(-d
/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 4*b**5*c**5*sqrt(
-d/c**3)*(a*d - 3*b*c)/(a*d - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3
*c*d**2 + 12*b**4*c**2*d))/(4*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.296325, size = 165, normalized size = 1.51

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}}-\frac{\left(3\,bcd-ad^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}}-\frac{dx}{2\,(bc^2-acd)(dx^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*
a*b*c^2*d + a^2*c*d^2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x
^2 + c))
```

3.247 $\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}-\frac{d}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\log(x)}{ac^2}$$

```
[Out] -d/(2*c*(b*c - a*d)*(c + d*x^2)) + Log[x]/(a*c^2) - (b^2*Log[a +
b*x^2])/(2*a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(2
*c^2*(b*c - a*d)^2)
```

Rubi [A] time = 0.229838, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}-\frac{d}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\log(x)}{ac^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -d/(2*c*(b*c - a*d)*(c + d*x^2)) + Log[x]/(a*c^2) - (b^2*Log[a +
b*x^2])/(2*a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(2
*c^2*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 38.8441, size = 85, normalized size = 0.85

$$\frac{d}{2c\left(c+dx^2\right)(ad-bc)}-\frac{d\,(ad-2bc)\log\left(c+dx^2\right)}{2c^2\,(ad-bc)^2}-\frac{b^2\log\left(a+bx^2\right)}{2a\,(ad-bc)^2}+\frac{\log\left(x^2\right)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d/(2*c*(c + d*x**2)*(a*d - b*c)) - d*(a*d - 2*b*c)*log(c + d*x**2
)/(2*c**2*(a*d - b*c)**2) - b**2*log(a + b*x**2)/(2*a*(a*d - b*c)
**2) + log(x**2)/(2*a*c**2)
```

Mathematica [A] time = 0.16313, size = 98, normalized size = 0.98

$$\frac{1}{2}\left(-\frac{b^2\log\left(a+bx^2\right)}{a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{c^2(bc-ad)^2}-\frac{d}{c\left(c+dx^2\right)(bc-ad)}+\frac{2\log(x)}{ac^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-(d/(c*(b*c - a*d)*(c + d*x^2))) + (2*Log[x])/(a*c^2) - (b^2*Log
[a + b*x^2])/(a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])
/(c^2*(b*c - a*d)^2))/2
```

Maple [A] time = 0.023, size = 139, normalized size = 1.4

$$\frac{\ln(x)}{ac^2}+\frac{ad^2}{2c(ad-bc)^2(dx^2+c)}-\frac{bd}{2(ad-bc)^2(dx^2+c)}-\frac{d^2\ln\left(dx^2+c\right)a}{2c^2(ad-bc)^2}+\frac{d\ln\left(dx^2+c\right)b}{c(ad-bc)^2}-\frac{b^2\ln\left(bx^2+a\right)}{2a(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] ln(x)/a/c^2+1/2*d^2/c/(a*d-b*c)^2/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2/(
d*x^2+c)*b-1/2*d^2/c^2/(a*d-b*c)^2*ln(d*x^2+c)*a+d/c/(a*d-b*c)^2*
ln(d*x^2+c)*b-1/2*b^2/a/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.35437, size = 186, normalized size = 1.86

$$-\frac{b^2\log\left(bx^2+a\right)}{2\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(2bcd-ad^2\right)\log\left(dx^2+c\right)}{2\left(b^2c^4-2abc^3d+a^2c^2d^2\right)}-\frac{d}{2\left(bc^3-ac^2d+(bc^2d-acd^2)x^2\right)}+\frac{\log\left(x^2\right)}{2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/2
*(2*b*c*d - a*d^2)*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^
2*d^2) - 1/2*d/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/2*
log(x^2)/(a*c^2)
```

Fricas [A] time = 1.00962, size = 296, normalized size = 2.96

$$-\frac{abc^2d-a^2cd^2+\left(b^2c^2dx^2+b^2c^3\right)\log\left(bx^2+a\right)-\left(2abc^2d-a^2cd^2+\left(2abcd^2-a^2d^3\right)x^2\right)\log\left(dx^2+c\right)-2\left(b^2c^3-2abc\right.}{2\left(ab^2c^5-2a^2bc^4d+a^3c^3d^2+\left(ab^2c^4d-2a^2bc^3d^2+a^3c^2d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2*d - a^2*c*d^2 + (b^2*c^2*d*x^2 + b^2*c^3)*log(b*x^2
 + a) - (2*a*b*c^2*d - a^2*c*d^2 + (2*a*b*c*d^2 - a^2*d^3)*x^2)*l
og(d*x^2 + c) - 2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*x^2)*log(x))/(a*b^2*c^5 - 2*a^2*b*c^4*d
 + a^3*c^3*d^2 + (a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282932, size = 250, normalized size = 2.5

$$-\frac{b^3\ln\left(\left|bx^2+a\right|\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}+\frac{\left(2\,bcd^2-ad^3\right)\ln\left(\left|dx^2+c\right|\right)}{2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)}-\frac{2\,bcd^2x^2-ad^3x^2+3\,bc^2d-2\,acd^2}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\left(dx^2+c\right)}+\frac{\ln\left(x^2\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x),x, algorithm="giac")
```

```
[Out] -1/2*b^3*ln(abs(b*x^2 + a))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^
2) + 1/2*(2*b*c*d^2 - a*d^3)*ln(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3) - 1/2*(2*b*c*d^2*x^2 - a*d^3*x^2 + 3*b*c
^2*d - 2*a*c*d^2)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 +
 c)) + 1/2*ln(x^2)/(a*c^2)
```

3.248 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=144

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(2*b*c - 3*a*d)/(2*a*c^2*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x*(
c + d*x^2)) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c
 - a*d)^2) + (d^(3/2)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(2*c^(5/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.530002, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(2*b*c - 3*a*d)/(2*a*c^2*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x*(
c + d*x^2)) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c
 - a*d)^2) + (d^(3/2)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(2*c^(5/2)*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 102.365, size = 121, normalized size = 0.84

$$\frac{d}{2cx\left(c+dx^2\right)(ad-bc)}-\frac{d^{\frac{3}{2}}\left(3ad-5bc\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{5}{2}}\left(ad-bc\right)^2}-\frac{3ad-2bc}{2ac^2x\left(ad-bc\right)}-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d/(2*c*x*(c + d*x**2)*(a*d - b*c)) - d**(3/2)*(3*a*d - 5*b*c)*ata
n(sqrt(d)*x/sqrt(c))/(2*c**(5/2)*(a*d - b*c)**2) - (3*a*d - 2*b*c
)/(2*a*c**2*x*(a*d - b*c)) - b**(5/2)*atan(sqrt(b)*x/sqrt(a))/(a*
*(3/2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.480483, size = 123, normalized size = 0.85

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}+\frac{d^2x}{2c^2\left(c+dx^2\right)(bc-ad)}-\frac{1}{ac^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(1/(a*c^2*x)) + (d^2*x)/(2*c^2*(b*c - a*d)*(c + d*x^2)) - (b^(5/
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^2) + (d^(
3/2)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c
 - a*d)^2)
```

Maple [A] time = 0.02, size = 169, normalized size = 1.2

$$-\frac{1}{ac^{2}x}-\frac{d^{3}xa}{2\,c^{2}\,(ad-bc)^{2}\,(dx^{2}+c)}+\frac{d^{2}xb}{2\,c\,(ad-bc)^{2}\,(dx^{2}+c)}-\frac{3\,ad^{3}}{2\,c^{2}\,(ad-bc)^{2}}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{5\,bd^{2}}{2\,c\,(ad-bc)^{2}}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^{3}}{a\,(ad-bc)^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/a/c^2/x-1/2*d^3/c^2/(a*d-b*c)^2*x/(d*x^2+c)*a+1/2*d^2/c/(a*d-b
*c)^2*x/(d*x^2+c)*b-3/2*d^3/c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*
d/(c*d)^(1/2))*a+5/2*d^2/c/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*
d)^(1/2))*b-1/a*b^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.614686, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d^2 + 2*(2*b^2*c^2*d - 5
*a*b*c*d^2 + 3*a^2*d^3)*x^2 - 2*(b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((5*a*b*c
*d^2 - 3*a^2*d^3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)
*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^2*c^4*d -
 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d
+ a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 +
(2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 - ((5*a*b*c*d^2 - 3*a
^2*d^3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(d*x
/(c*sqrt(d/c))) - (b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x
^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a*b^2*c^4*d - 2*a^2*b*
c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3
*d^2)*x), -1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d^2 + 2*(2*b^2*
c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 4*(b^2*c^2*d*x^3 + b^2*c^3
*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + ((5*a*b*c*d^2 - 3*a^2*d
^3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 -
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d
^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)
*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 + (2*b^2*c^2*d -
 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 2*(b^2*c^2*d*x^3 + b^2*c^3*x)*sqr
t(b/a)*arctan(b*x/(a*sqrt(b/a))) - ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3
 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/
c))))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2
*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283047, size = 221, normalized size = 1.53

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}} + \frac{\left(5\,bcd^2 - 3\,ad^3\right) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\sqrt{cd}} - \frac{2\,bcdx^2 - 3\,ad^2x^2 + 2\,bc^2 - 2\,acd}{2\,(abc^3 - a^2c^2d)(dx^3 + cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*s
qrt(a*b)) + 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c*d*x^2 -
 3*a*d^2*x^2 + 2*b*c^2 - 2*a*c*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^3 +
 c*x))
```

3.249 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=126

$$\frac{b^3\log(a+bx^2)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}-\frac{d^2(3bc-2ad)\log(c+dx^2)}{2c^3(bc-ad)^2}+\frac{d^2}{2c^2(c+dx^2)(bc-ad)}-\frac{1}{2ac^2x^2}$$

```
[Out] -1/(2*a*c^2*x^2) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c +
2*a*d)*Log[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(2*a^2*(b*c - a*d
)^2) - (d^2*(3*b*c - 2*a*d)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rubi [A] time = 0.329082, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^3\log(a+bx^2)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}-\frac{d^2(3bc-2ad)\log(c+dx^2)}{2c^3(bc-ad)^2}+\frac{d^2}{2c^2(c+dx^2)(bc-ad)}-\frac{1}{2ac^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/(2*a*c^2*x^2) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c +
2*a*d)*Log[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(2*a^2*(b*c - a*d
)^2) - (d^2*(3*b*c - 2*a*d)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 51.5114, size = 116, normalized size = 0.92

$$-\frac{d^2}{2c^2(c+dx^2)(ad-bc)}+\frac{d^2(2ad-3bc)\log(c+dx^2)}{2c^3(ad-bc)^2}-\frac{1}{2ac^2x^2}+\frac{b^3\log(a+bx^2)}{2a^2(ad-bc)^2}-\frac{(2ad+bc)\log(x^2)}{2a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -d**2/(2*c**2*(c + d*x**2)*(a*d - b*c)) + d**2*(2*a*d - 3*b*c)*lo
g(c + d*x**2)/(2*c**3*(a*d - b*c)**2) - 1/(2*a*c**2*x**2) + b**3*
log(a + b*x**2)/(2*a**2*(a*d - b*c)**2) - (2*a*d + b*c)*log(x**2)
/(2*a**2*c**3)
```

Mathematica [A] time = 0.549605, size = 117, normalized size = 0.93

$$\frac{1}{2}\left(\frac{b^3\log(a+bx^2)}{a^2(bc-ad)^2}-\frac{2\log(x)(2ad+bc)}{a^2c^3}+\frac{\frac{cd^2}{(c+dx^2)(bc-ad)}+\frac{d^2(2ad-3bc)\log(c+dx^2)}{(bc-ad)^2}-\frac{c}{ax^2}}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((-2*(b*c + 2*a*d)*Log[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(a^2*
(b*c - a*d)^2) + (-(c/(a*x^2)) + (c*d^2)/((b*c - a*d)*(c + d*x^2)
) + (d^2*(-3*b*c + 2*a*d)*Log[c + d*x^2])/(b*c - a*d)^2)/c^3)/2
```

Maple [A] time = 0.026, size = 170, normalized size = 1.4

$$-\frac{1}{2\,ac^{2}x^{2}}-2\,\frac{\ln\left(x\right)d}{ac^{3}}-\frac{b\ln\left(x\right)}{a^{2}c^{2}}-\frac{d^{3}a}{2\,c^{2}\left(ad-bc\right)^{2}\left(dx^{2}+c\right)}+\frac{bd^{2}}{2\,c\left(ad-bc\right)^{2}\left(dx^{2}+c\right)}+\frac{d^{3}\ln\left(dx^{2}+c\right)a}{c^{3}\left(ad-bc\right)^{2}}-\frac{3\,d^{2}\ln\left(dx^{2}+c\right)b}{2\,c^{2}\left(ad-bc\right)^{2}}+\frac{b^{3}\ln\left(bx^{2}+a\right)}{2\,a^{2}\left(ad-bc\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/a/c^2/x^2-2/a/c^3*ln(x)*d-1/a^2/c^2*ln(x)*b-1/2*d^3/c^2/(a*d
-b*c)^2/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2/(d*x^2+c)*b+d^3/c^3/(a*
d-b*c)^2*ln(d*x^2+c)*a-3/2*d^2/c^2/(a*d-b*c)^2*ln(d*x^2+c)*b+1/2*
b^3/a^2/(a*d-b*c)^2*ln(b*x^2+a)
```

Maxima [A] time = 1.36354, size = 254, normalized size = 2.02

$$\frac{b^{3}\log\left(bx^{2}+a\right)}{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)}-\frac{\left(3\,bcd^{2}-2\,ad^{3}\right)\log\left(dx^{2}+c\right)}{2\left(b^{2}c^{5}-2\,abc^{4}d+a^{2}c^{3}d^{2}\right)}-\frac{bc^{2}-acd+\left(bcd-2\,ad^{2}\right)x^{2}}{2\left(\left(abc^{3}d-a^{2}c^{2}d^{2}\right)x^{4}+\left(abc^{4}-a^{2}c^{3}d\right)x^{2}\right)}-\frac{\left(bc+2\,ad\right)\log\left(x^{2}\right)}{2\,a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/
2*(3*b*c*d^2 - 2*a*d^3)*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a
^2*c^3*d^2) - 1/2*(b*c^2 - a*c*d + (b*c*d - 2*a*d^2)*x^2)/((a*b*c
^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2) - 1/2*(b*c +
 2*a*d)*log(x^2)/(a^2*c^3)
```

Fricas [A] time = 2.36857, size = 408, normalized size = 3.24

$$-\frac{ab^{2}c^{4}-2\,a^{2}bc^{3}d+a^{3}c^{2}d^{2}+\left(ab^{2}c^{3}d-3\,a^{2}bc^{2}d^{2}+2\,a^{3}cd^{3}\right)x^{2}-\left(b^{3}c^{3}dx^{4}+b^{3}c^{4}x^{2}\right)\log\left(bx^{2}+a\right)+\left(\left(3\,a^{2}bcd^{3}-2\,a^{3}d^{4}\right)\cdots\right.}{2\left(\left(a^{2}b^{2}c^{5}d-2\,a^{3}bc^{4}d^{2}+a^{4}c^{3}d^{3}\right)x^{4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 3*
a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2 - (b^3*c^3*d*x^4 + b^3*c^4*x^2)*
log(b*x^2 + a) + ((3*a^2*b*c*d^3 - 2*a^3*d^4)*x^4 + (3*a^2*b*c^2*
d^2 - 2*a^3*c*d^3)*x^2)*log(d*x^2 + c) + 2*((b^3*c^3*d - 3*a^2*b*
c*d^3 + 2*a^3*d^4)*x^4 + (b^3*c^4 - 3*a^2*b*c^2*d^2 + 2*a^3*c*d^3
)*x^2)*log(x))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x
^4 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291994, size = 347, normalized size = 2.75

$$\frac{b^4\ln\left(\left|bx^2+a\right|\right)}{2\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)}-\frac{\left(3\,bcd^3-2\,ad^4\right)\ln\left(\left|dx^2+c\right|\right)}{2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)}+\frac{b^3c^2dx^4+b^3c^3x^2-2\,ab^2c^2dx^2+6\,a^2bcd^2x^2-4\,a^3d^3x^2-2\,ab^2c^3+4\,a^2bc^2d-2\,a^3cd^2}{4\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2)(dx^4+cx^2)}-\frac{(bc+2\,ad)\ln\left(x^2\right)}{2\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^3),x, algorithm="giac")
```

```
[Out] 1/2*b^4*ln(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d
^2) - 1/2*(3*b*c*d^3 - 2*a*d^4)*ln(abs(d*x^2 + c))/(b^2*c^5*d - 2
*a*b*c^4*d^2 + a^2*c^3*d^3) + 1/4*(b^3*c^2*d*x^4 + b^3*c^3*x^2 -
2*a*b^2*c^2*d*x^2 + 6*a^2*b*c*d^2*x^2 - 4*a^3*d^3*x^2 - 2*a*b^2*c
^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d +
 a^4*c^2*d^2)*(d*x^4 + c*x^2)) - 1/2*(b*c + 2*a*d)*ln(x^2)/(a^2*c
^3)
```

3.250 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=189

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}+\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(2*b*c - 5*a*d)/(6*a*c^2*(b*c - a*d)*x^3) + (2*b^2*c^2 + 2*a*b*c
*d - 5*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x^
3*(c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(
b*c - a*d)^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.754436, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}+\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(2*b*c - 5*a*d)/(6*a*c^2*(b*c - a*d)*x^3) + (2*b^2*c^2 + 2*a*b*c
*d - 5*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x^
3*(c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(
b*c - a*d)^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.904535, size = 142, normalized size = 0.75

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^2}+\frac{2ad+bc}{a^2c^3x}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{d^3x}{2c^3\left(c+dx^2\right)(bc-ad)}-\frac{1}{3ac^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/(3*a*c^2*x^3) + (b*c + 2*a*d)/(a^2*c^3*x) - (d^3*x)/(2*c^3*(b*
c - a*d)*(c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
```

```
(5/2)*(-(b*c) + a*d)^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.022, size = 191, normalized size = 1.

$$-\frac{1}{3\,ac^2x^3}+2\,\frac{d}{axc^3}+\frac{b}{a^2c^2x}+\frac{d^4xa}{2\,c^3\,(ad-bc)^2\,(dx^2+c)}-\frac{d^3xb}{2\,c^2\,(ad-bc)^2\,(dx^2+c)}+\frac{5\,d^4a}{2\,c^3\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,d^3b}{2\,c^2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^4}{a^2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/3/a/c^2/x^3+2/x/a/c^3*d+1/x/a^2/c^2*b+1/2*d^4/c^3/(a*d-b*c)^2*
x/(d*x^2+c)*a-1/2*d^3/c^2/(a*d-b*c)^2*x/(d*x^2+c)*b+5/2*d^4/c^3/(
a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-7/2*d^3/c^2/(a*d
-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/a^2*b^4/(a*d-b*c)
^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3871, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^
3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d
 - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^
4*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
 + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*
c*d^3)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2
+ c)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^
2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4
- 4*a^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*(2*b^3*c^3*d - 7*a^2*b*c*d^3
+ 5*a^3*d^4)*x^4 - 2*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 +
 5*a^3*c*d^3)*x^2 + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b
*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c)))
- 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*s
qrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 +
a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^
3), -1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3
*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^
3*d - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 - 12*(b^3*c^3*d*x^5 + b^
3*c^4*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 3*((7*a^2*b*c*d^
```

```
3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^2*c^
5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b
*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a^2*b*c^3*d + 2
*a^3*c^2*d^2 - 3*(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 -
2*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 -
 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a
))) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a
^3*c*d^3)*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/((a^2*b^2*c^5
*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*
c^5*d + a^4*c^4*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273704, size = 223, normalized size = 1.18

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^2c^2 - 2\,a^3bcd + a^4d^2)\sqrt{ab}} - \frac{d^3x}{2\,(bc^4 - ac^3d)(dx^2 + c)} - \frac{\left(7\,bcd^3 - 5\,ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2)\sqrt{cd}} + \frac{3\,bcx^2 + 6\,adx^2 - ac}{3\,a^2c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^4),x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*
sqrt(a*b)) - 1/2*d^3*x/((b*c^4 - a*c^3*d)*(d*x^2 + c)) - 1/2*(7*b
*c*d^3 - 5*a*d^4)*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d +
 a^2*c^3*d^2)*sqrt(c*d)) + 1/3*(3*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^2
*c^3*x^3)
```

3.251 $\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx$

Optimal. Leaf size=116

$$-\frac{a^2}{4b^2\left(a+bx^2\right)^2(bc-ad)}+\frac{a(2bc-ad)}{2b^2\left(a+bx^2\right)(bc-ad)^2}+\frac{c^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{c^2\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

```
[Out] -a^2/(4*b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(2*b^2
*(b*c - a*d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(2*(b*c - a*d)
^3) - (c^2*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.265235, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2}{4b^2\left(a+bx^2\right)^2(bc-ad)}+\frac{a(2bc-ad)}{2b^2\left(a+bx^2\right)(bc-ad)^2}+\frac{c^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{c^2\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] -a^2/(4*b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(2*b^2
*(b*c - a*d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(2*(b*c - a*d)
^3) - (c^2*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 43.688, size = 97, normalized size = 0.84

$$\frac{a^2}{4b^2\left(a+bx^2\right)^2(ad-bc)}-\frac{a\,(ad-2bc)}{2b^2\left(a+bx^2\right)(ad-bc)^2}-\frac{c^2\log\left(a+bx^2\right)}{2\,(ad-bc)^3}+\frac{c^2\log\left(c+dx^2\right)}{2\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] a**2/(4*b**2*(a + b*x**2)**2*(a*d - b*c)) - a*(a*d - 2*b*c)/(2*b*
*2*(a + b*x**2)*(a*d - b*c)**2) - c**2*log(a + b*x**2)/(2*(a*d -
b*c)**3) + c**2*log(c + d*x**2)/(2*(a*d - b*c)**3)
```

Mathematica [A] time = 0.197152, size = 99, normalized size = 0.85

$$\frac{-\frac{a^2(bc-ad)^2}{b^2(a+bx^2)^2}+\frac{2a(ad-2bc)(ad-bc)}{b^2(a+bx^2)}+2c^2\log\left(a+bx^2\right)-2c^2\log\left(c+dx^2\right)}{4(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (-((a^2*(b*c - a*d)^2)/(b^2*(a + b*x^2)^2)) + (2*a*(-2*b*c + a*d)
*(-(b*c) + a*d))/(b^2*(a + b*x^2)) + 2*c^2*Log[a + b*x^2] - 2*c^2
*Log[c + d*x^2])/(4*(b*c - a*d)^3)
```

Maple [B] time = 0.019, size = 218, normalized size = 1.9

$$\frac{c^2 \ln\left(dx^2+c\right)}{2\,(ad-bc)^3}+\frac{a^4 d^2}{4\,(ad-bc)^3\,b^2\,(bx^2+a)^2}-\frac{a^3 cd}{2\,(ad-bc)^3\,b\,(bx^2+a)^2}+\frac{a^2c^2}{4\,(ad-bc)^3\,(bx^2+a)^2}-\frac{c^2\ln\left(bx^2+a\right)}{2\,(ad-bc)^3}-\frac{a^3d^2}{2\,(ad-bc)^3\,b^2\,(bx^2+a)}+\frac{3\,a^2cd}{2\,(ad-bc)^3\,b\,(bx^2+a)}-\frac{ac^2}{(ad-bc)^3\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] 1/2*c^2/(a*d-b*c)^3*ln(d*x^2+c)+1/4/(a*d-b*c)^3*a^4/b^2/(b*x^2+a)
^2*d^2-1/2/(a*d-b*c)^3*a^3/b/(b*x^2+a)^2*c*d+1/4/(a*d-b*c)^3*a^2/
(b*x^2+a)^2*c^2-1/2/(a*d-b*c)^3*c^2*ln(b*x^2+a)-1/2/(a*d-b*c)^3*a
^3/b^2/(b*x^2+a)*d^2+3/2/(a*d-b*c)^3*a^2/b/(b*x^2+a)*c*d-1/(a*d-b
*c)^3*a/(b*x^2+a)*c^2
```

Maxima [A] time = 1.36257, size = 319, normalized size = 2.75

$$\frac{c^2\log\left(bx^2+a\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}-\frac{c^2\log\left(dx^2+c\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}+\frac{3\,a^2bc-a^3d+2\left(2\,ab^2c-a^2bd\right)x^2}{4\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^4+2\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3) - 1/2*c^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3) + 1/4*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a
^2*b*d)*x^2)/(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^
2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d
 + a^3*b^3*d^2)*x^2)
```

Fricas [A] time = 0.243326, size = 392, normalized size = 3.38

$$\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\left(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2\right)x^2+2\left(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2\right)\log\left(bx^2+a\right)-2\left(b^4c^2x^4+\ldots\right.}{4\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3+(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)x^4+2\,(ab^6c^3-3\,a^2b^5c^2d+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a
^2*b^2*c*d + a^3*b*d^2)*x^2 + 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 +
a^2*b^2*c^2)*log(b*x^2 + a) - 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 +
a^2*b^2*c^2)*log(d*x^2 + c))/(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a
^4*b^3*c*d^2 - a^5*b^2*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5
*c*d^2 - a^3*b^4*d^3)*x^4 + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^
3*b^4*c*d^2 - a^4*b^3*d^3)*x^2)
```

Sympy [A] time = 17.3758, size = 418, normalized size = 3.6

$$\frac{c^2\log\left(x^2+\frac{-\frac{a^4c^2d^4}{(ad-bc)^3}+\frac{4a^3bc^3d^3}{(ad-bc)^3}-\frac{6a^2b^2c^4d^2}{(ad-bc)^3}+\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d-\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2(ad-bc)^3}-\frac{c^2\log\left(x^2+\frac{\frac{a^4c^2d^4}{(ad-bc)^3}-\frac{4a^3bc^3d^3}{(ad-bc)^3}+\frac{6a^2b^2c^4d^2}{(ad-bc)^3}-\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d+\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2(ad-bc)^3}-\frac{a^3d-3a^2bc+x^2\left(2a^2bd-4ab^2c\right)}{4a^4b^2d^2-8a^3b^3cd+4a^2b^4c^2+x^4\left(4a^2b^4d^2-8ab^5cd+4b^6c^2\right)+x^2\left(8a^3b^3d^2-16a^2b^4cd+8ab^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] c**2*log(x**2 + (-a**4*c**2*d**4/(a*d - b*c)**3 + 4*a**3*b*c**3*d
**3/(a*d - b*c)**3 - 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 + 4*a*b
**3*c**5*d/(a*d - b*c)**3 + a*c**2*d - b**4*c**6/(a*d - b*c)**3 +
 b*c**3)/(2*b*c**2*d))/(2*(a*d - b*c)**3) - c**2*log(x**2 + (a**4
*c**2*d**4/(a*d - b*c)**3 - 4*a**3*b*c**3*d**3/(a*d - b*c)**3 + 6
*a**2*b**2*c**4*d**2/(a*d - b*c)**3 - 4*a*b**3*c**5*d/(a*d - b*c)
**3 + a*c**2*d + b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**2*d))
/(2*(a*d - b*c)**3) - (a**3*d - 3*a**2*b*c + x**2*(2*a**2*b*d - 4
*a*b**2*c))/(4*a**4*b**2*d**2 - 8*a**3*b**3*c*d + 4*a**2*b**4*c**
2 + x**4*(4*a**2*b**4*d**2 - 8*a*b**5*c*d + 4*b**6*c**2) + x**2*(
8*a**3*b**3*d**2 - 16*a**2*b**4*c*d + 8*a*b**5*c**2))
```

GIAC/XCAS [A] time = 0.290154, size = 313, normalized size = 2.7

$$\frac{bc^2\ln\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{c^2d\ln\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{3\,b^4c^2x^4+2\,ab^3c^2x^2+6\,a^2b^2cdx^2-2\,a^3bd^2x^2+4\,a^3bcd-a^4d^2}{4\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^2*ln(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2
*c*d^2 - a^3*b*d^3) - 1/2*c^2*d*ln(abs(d*x^2 + c))/(b^3*c^3*d - 3
*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(3*b^4*c^2*x^4 +
2*a*b^3*c^2*x^2 + 6*a^2*b^2*c*d*x^2 - 2*a^3*b*d^2*x^2 + 4*a^3*b*c
*d - a^4*d^2)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b
^2*d^3)*(b*x^2 + a)^2)
```

3.252 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=157

$$\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{x(bc-5ad)}{8d\left(c+dx^2\right)(bc-ad)^2}-\frac{cx}{4d\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(c*x)/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(8*d*(
b*c - a*d)^2*(c + d*x^2)) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(b*c - a*d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*d^(3/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.434987, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{x(bc-5ad)}{8d\left(c+dx^2\right)(bc-ad)^2}-\frac{cx}{4d\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(c*x)/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(8*d*(
b*c - a*d)^2*(c + d*x^2)) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(b*c - a*d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*d^(3/2)*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 73.4853, size = 139, normalized size = 0.89

$$-\frac{a^{\frac{3}{2}}\sqrt{b}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(ad-bc)^3}+\frac{cx}{4d\left(c+dx^2\right)^2(ad-bc)}-\frac{x\,(5ad-bc)}{8d\left(c+dx^2\right)(ad-bc)^2}+\frac{\left(3a^2d^2+6abcd-b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{\frac{3}{2}}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -a**(3/2)*sqrt(b)*atan(sqrt(b)*x/sqrt(a))/(a*d - b*c)**3 + c*x/(4
*d*(c + d*x**2)**2*(a*d - b*c)) - x*(5*a*d - b*c)/(8*d*(c + d*x**
2)*(a*d - b*c)**2) + (3*a**2*d**2 + 6*a*b*c*d - b**2*c**2)*atan(s
qrt(d)*x/sqrt(c))/(8*sqrt(c)*d**(3/2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.424337, size = 154, normalized size = 0.98

$$\frac{1}{8}\left(\frac{8a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{x(bc-5ad)}{d\left(c+dx^2\right)(bc-ad)^2}+\frac{2cx}{d\left(c+dx^2\right)^2(ad-bc)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*c*x)/(d*(-(b*c) + a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(d*
(b*c - a*d)^2*(c + d*x^2)) + (8*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(b*c - a*d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*Ar
cTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2)*(b*c - a*d)^3))/8
```

Maple [B] time = 0.017, size = 299, normalized size = 1.9

$$-\frac{5\,x^3a^2d^2}{8\,(ad-bc)^3\,(dx^2+c)^2}+\frac{3\,x^3abcd}{4\,(ad-bc)^3\,(dx^2+c)^2}-\frac{x^3b^2c^2}{8\,(ad-bc)^3\,(dx^2+c)^2}-\frac{3\,xa^2cd}{8\,(ad-bc)^3\,(dx^2+c)^2}+\frac{xabc^2}{4\,(ad-bc)^3\,(dx^2+c)^2}+\frac{xb^2c^3}{8\,(ad-bc)^3\,(dx^2+c)^2\,d}+\frac{3\,a^2d}{8\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,acb}{4\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^2c^2}{8\,(ad-bc)^3\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{ba^2}{(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -5/8/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a^2*d^2+3/4/(a*d-b*c)^3/(d*x^2+c
)^2*x^3*a*b*c*d-1/8/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2*c^2-3/8/(a*d-
b*c)^3/(d*x^2+c)^2*x*a^2*c*d+1/4/(a*d-b*c)^3/(d*x^2+c)^2*x*a*b*c^
2+1/8/(a*d-b*c)^3/(d*x^2+c)^2*c^3/d*x*b^2+3/8/(a*d-b*c)^3*d/(c*d)
^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+3/4/(a*d-b*c)^3/(c*d)^(1/2)*ar
ctan(x*d/(c*d)^(1/2))*c*a*b-1/8/(a*d-b*c)^3/d/(c*d)^(1/2)*arctan(
x*d/(c*d)^(1/2))*b^2*c^2-b/(a*d-b*c)^3*a^2/(a*b)^(1/2)*arctan(x*b
/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.596803, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(-a*b)*sqrt(-
c*d)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - (b^2*c^4 - 6
*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d
^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*log((2
*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) - 2*((b^2*c^2*d - 6
*a*b*c*d^2 + 5*a^2*d^3)*x^3 - (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^
```

```
2)*x)*sqrt(-c*d))/((b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3
 - a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 -
 a^3*d^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^
4 - a^3*c*d^5)*x^2)*sqrt(-c*d)), -1/8*(4*(a*d^3*x^4 + 2*a*c*d^2*x
^2 + a*c^2*d)*sqrt(-a*b)*sqrt(c*d)*log((b*x^2 - 2*sqrt(-a*b)*x -
a)/(b*x^2 + a)) - (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c
^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*
d^2 - 3*a^2*c*d^3)*x^2)*arctan(sqrt(c*d)*x/c) - ((b^2*c^2*d - 6*a
*b*c*d^2 + 5*a^2*d^3)*x^3 - (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2)
*x)*sqrt(c*d))/((b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 -
a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^
3*d^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 -
 a^3*c*d^5)*x^2)*sqrt(c*d)), 1/16*(16*(a*d^3*x^4 + 2*a*c*d^2*x^2
+ a*c^2*d)*sqrt(a*b)*sqrt(-c*d)*arctan(b*x/sqrt(a*b)) + (b^2*c^4
- 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^
2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*log
((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) + 2*((b^2*c^2*d
- 6*a*b*c*d^2 + 5*a^2*d^3)*x^3 - (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c
*d^2)*x)*sqrt(-c*d))/((b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*
d^3 - a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^
5 - a^3*d^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2
*d^4 - a^3*c*d^5)*x^2)*sqrt(-c*d)), 1/8*(8*(a*d^3*x^4 + 2*a*c*d^2
*x^2 + a*c^2*d)*sqrt(a*b)*sqrt(c*d)*arctan(b*x/sqrt(a*b)) + (b^2*
c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 -
3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)
*arctan(sqrt(c*d)*x/c) + ((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x
^3 - (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(c*d))/((b^3*c^
5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4 + (b^3*c^3*
d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*x^4 + 2*(b^3*c^4
*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x^2)*sqrt(c
*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.258904, size = 275, normalized size = 1.75

$$\frac{a^2 b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} + \frac{\left(b^2c^2 - 6\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4\right)\sqrt{cd}} + \frac{bcdx^3 - 5\,ad^2x^3 - bc^2x - 3\,acdx}{8\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\left(dx^2 + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2
)*arctan(d*x/sqrt(c*d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c
*d^3 - a^3*d^4)*sqrt(c*d)) + 1/8*(b*c*d*x^3 - 5*a*d^2*x^3 - b*c^2
*x - 3*a*c*d*x)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(d*x^2 + c)^
2)
```

3.253 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{a}{2(c+dx^2)(bc-ad)^2}-\frac{c}{4d(c+dx^2)^2(bc-ad)}-\frac{ab\log(a+bx^2)}{2(bc-ad)^3}+\frac{ab\log(c+dx^2)}{2(bc-ad)^3}$$

```
[Out] -c/(4*d*(b*c - a*d)*(c + d*x^2)^2) - a/(2*(b*c - a*d)^2*(c + d*x^
2)) - (a*b*Log[a + b*x^2])/(2*(b*c - a*d)^3) + (a*b*Log[c + d*x^2
])/(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.217837, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a}{2(c+dx^2)(bc-ad)^2}-\frac{c}{4d(c+dx^2)^2(bc-ad)}-\frac{ab\log(a+bx^2)}{2(bc-ad)^3}+\frac{ab\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -c/(4*d*(b*c - a*d)*(c + d*x^2)^2) - a/(2*(b*c - a*d)^2*(c + d*x^
2)) - (a*b*Log[a + b*x^2])/(2*(b*c - a*d)^3) + (a*b*Log[c + d*x^2
])/(2*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 35.0775, size = 82, normalized size = 0.82

$$\frac{ab\log(a+bx^2)}{2(ad-bc)^3}-\frac{ab\log(c+dx^2)}{2(ad-bc)^3}-\frac{a}{2(c+dx^2)(ad-bc)^2}+\frac{c}{4d(c+dx^2)^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] a*b*log(a + b*x**2)/(2*(a*d - b*c)**3) - a*b*log(c + d*x**2)/(2*(
a*d - b*c)**3) - a/(2*(c + d*x**2)*(a*d - b*c)**2) + c/(4*d*(c +
d*x**2)**2*(a*d - b*c))
```

Mathematica [A] time = 0.217999, size = 77, normalized size = 0.77

$$\frac{\frac{(ad-bc)(ad(c+2dx^2)+bc^2)}{d(c+dx^2)^2}+2ab\log(c+dx^2)-2ab\log(a+bx^2)}{4(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (((-(b*c) + a*d)*(b*c^2 + a*d*(c + 2*d*x^2)))/(d*(c + d*x^2)^2) -
 2*a*b*Log[a + b*x^2] + 2*a*b*Log[c + d*x^2])/(4*(b*c - a*d)^3)
```

Maple [A] time = 0.019, size = 177, normalized size = 1.8

$$-\frac{a^2 d}{2\,(ad-bc)^3\,(dx^2+c)}+\frac{abc}{2\,(ad-bc)^3\,(dx^2+c)}+\frac{a^2 cd}{4\,(ad-bc)^3\,(dx^2+c)^2} -\frac{abc^2}{2\,(ad-bc)^3\,(dx^2+c)^2}+\frac{b^2c^3}{4\,(ad-bc)^3\,d\,(dx^2+c)^2}-\frac{ab\ln\left(dx^2+c\right)}{2\,(ad-bc)^3}+\frac{ab\ln\left(bx^2+a\right)}{2\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/2/(a*d-b*c)^3*a^2/(d*x^2+c)*d+1/2/(a*d-b*c)^3*a/(d*x^2+c)*b*c+
1/4/(a*d-b*c)^3*c*d/(d*x^2+c)^2*a^2-1/2/(a*d-b*c)^3*c^2/(d*x^2+c)
^2*a*b+1/4/(a*d-b*c)^3*c^3/d/(d*x^2+c)^2*b^2-1/2/(a*d-b*c)^3*a*b*
ln(d*x^2+c)+1/2*a*b/(a*d-b*c)^3*ln(b*x^2+a)
```

Maxima [A] time = 1.35306, size = 293, normalized size = 2.93

$$-\frac{ab\log\left(bx^2+a\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}+\frac{ab\log\left(dx^2+c\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)} -\frac{2\,ad^2x^2+bc^2+acd}{4\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^2d^3-2\,abcd^4+a^2d^5)x^4+2\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] -1/2*a*b*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3) + 1/2*a*b*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3) - 1/4*(2*a*d^2*x^2 + b*c^2 + a*c*d)/(b^2*c
^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)
```

Fricas [A] time = 0.244985, size = 346, normalized size = 3.46

$$-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2+2\left(abd^3x^4+2\,abcd^2x^2+abc^2d\right)\log\left(bx^2+a\right)-2\left(abd^3x^4+2\,abcd^2x^2+abc^2d\right)\log}{4\,(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4+(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)x^4+2\,(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^3)*x^2 + 2*(a*b*
d^3*x^4 + 2*a*b*c*d^2*x^2 + a*b*c^2*d)*log(b*x^2 + a) - 2*(a*b*d^
3*x^4 + 2*a*b*c*d^2*x^2 + a*b*c^2*d)*log(d*x^2 + c))/(b^3*c^5*d -
 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4 + (b^3*c^3*d^3 -
 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*x^4 + 2*(b^3*c^4*d^2
- 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x^2)
```

Sympy [A] time = 16.3175, size = 410, normalized size = 4.1

$$-\frac{ab\log\left(x^2+\frac{-\frac{a^5bd^4}{(ad-bc)^3}+\frac{4a^4b^2cd^3}{(ad-bc)^3}-\frac{6a^3b^3c^2d^2}{(ad-bc)^3}+\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd-\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2(ad-bc)^3}+\frac{ab\log\left(x^2+\frac{\frac{a^5bd^4}{(ad-bc)^3}-\frac{4a^4b^2cd^3}{(ad-bc)^3}+\frac{6a^3b^3c^2d^2}{(ad-bc)^3}-\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd+\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2(ad-bc)^3}-\frac{acd+2ad^2x^2+bc^2}{4a^2c^2d^3-8abc^3d^2+4b^2c^4d+x^4(4a^2d^5-8abcd^4+4b^2c^2d^3)+x^2(8a^2cd^4-16abc^2d^3+8b^2c^3d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -a*b*log(x**2 + (-a**5*b*d**4/(a*d - b*c)**3 + 4*a**4*b**2*c*d**3
/(a*d - b*c)**3 - 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 + 4*a**2*b
**4*c**3*d/(a*d - b*c)**3 + a**2*b*d - a*b**5*c**4/(a*d - b*c)**3
 + a*b**2*c)/(2*a*b**2*d))/(2*(a*d - b*c)**3) + a*b*log(x**2 + (a
**5*b*d**4/(a*d - b*c)**3 - 4*a**4*b**2*c*d**3/(a*d - b*c)**3 + 6
*a**3*b**3*c**2*d**2/(a*d - b*c)**3 - 4*a**2*b**4*c**3*d/(a*d - b
*c)**3 + a**2*b*d + a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(2*a*b
**2*d))/(2*(a*d - b*c)**3) - (a*c*d + 2*a*d**2*x**2 + b*c**2)/(4*
a**2*c**2*d**3 - 8*a*b*c**3*d**2 + 4*b**2*c**4*d + x**4*(4*a**2*d
**5 - 8*a*b*c*d**4 + 4*b**2*c**2*d**3) + x**2*(8*a**2*c*d**4 - 16
*a*b*c**2*d**3 + 8*b**2*c**3*d**2))
```

GIAC/XCAS [A] time = 0.275597, size = 235, normalized size = 2.35

$$-\frac{ab^2\ln\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}+\frac{abd\ln\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2}{4\left(dx^2+c\right)^2(bc-ad)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] -1/2*a*b^2*ln(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^
2*c*d^2 - a^3*b*d^3) + 1/2*a*b*d*ln(abs(d*x^2 + c))/(b^3*c^3*d -
3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(b^2*c^3 - a^2*c
*d^2 + 2*(a*b*c*d^2 - a^2*d^3)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3*
d)
```

3.254 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=155

$$\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}(bc-ad)^3}-\frac{\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(bc-ad)^2}+\frac{x}{4\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] x/(4*(b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(8*c*(b*c - a
*d)^2*(c + d*x^2)) - (Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(b*c - a*d)^3 + ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(8*c^(3/2)*Sqrt[d]*(b*c - a*d)^3)
```

Rubi [A] time = 0.360506, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}(bc-ad)^3}-\frac{\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(bc-ad)^2}+\frac{x}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x/(4*(b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(8*c*(b*c - a
*d)^2*(c + d*x^2)) - (Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(b*c - a*d)^3 + ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(8*c^(3/2)*Sqrt[d]*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 70.5863, size = 136, normalized size = 0.88

$$\frac{\sqrt{a}b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(ad-bc)^3}-\frac{x}{4\left(c+dx^2\right)^2(ad-bc)}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(ad-bc)^2}+\frac{\left(a^2d^2-6abcd-3b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}\sqrt{d}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] sqrt(a)*b**(3/2)*atan(sqrt(b)*x/sqrt(a))/(a*d - b*c)**3 - x/(4*(c
 + d*x**2)**2*(a*d - b*c)) + x*(a*d + 3*b*c)/(8*c*(c + d*x**2)*(a
*d - b*c)**2) + (a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)*atan(sqrt(d
)*x/sqrt(c))/(8*c**(3/2)*sqrt(d)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.397621, size = 151, normalized size = 0.97

$$\frac{1}{8}\left(\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}(bc-ad)^3}+\frac{8\sqrt{a}b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(ad-bc)^3}+\frac{x(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)^2}+\frac{2x}{\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*x)/((b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(c*(b*c -
a*d)^2*(c + d*x^2)) + (8*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(-(b*c) + a*d)^3 + ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]*(b*c - a*d)^3))/8
```

Maple [B] time = 0.017, size = 298, normalized size = 1.9

$$\frac{x^3a^2d^3}{8\,(ad-bc)^3\,(dx^2+c)^2\,c}+\frac{x^3abd^2}{4\,(ad-bc)^3\,(dx^2+c)^2}-\frac{3\,x^3b^2cd}{8\,(ad-bc)^3\,(dx^2+c)^2}+\frac{3\,abcdx}{4\,(ad-bc)^3\,(dx^2+c)^2}-\frac{5\,b^2c^2x}{8\,(ad-bc)^3\,(dx^2+c)^2}-\frac{a^2d^2x}{8\,(ad-bc)^3\,(dx^2+c)^2}+\frac{a^2d^2}{8\,(ad-bc)^3\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,abd}{4\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,b^2c}{8\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab^2}{(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/8/(a*d-b*c)^3/(d*x^2+c)^2*d^3/c*x^3*a^2+1/4/(a*d-b*c)^3/(d*x^2+
c)^2*x^3*a*b*d^2-3/8/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2*c*d+3/4/(a*d
-b*c)^3/(d*x^2+c)^2*a*b*c*d*x-5/8/(a*d-b*c)^3/(d*x^2+c)^2*b^2*c^2
*x-1/8/(a*d-b*c)^3/(d*x^2+c)^2*a^2*d^2*x+1/8/(a*d-b*c)^3/c/(c*d)^
(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d^2-3/4/(a*d-b*c)^3/(c*d)^(1/2)
*arctan(x*d/(c*d)^(1/2))*a*b*d-3/8/(a*d-b*c)^3*c/(c*d)^(1/2)*arct
an(x*d/(c*d)^(1/2))*b^2+a*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.616961, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3)*sqrt(-a*b)*sqrt(-
c*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - (3*b^2*c^4 +
 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d
^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*log((2
*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) - 2*((3*b^2*c^2*d -
 2*a*b*c*d^2 - a^2*d^3)*x^3 + (5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^
```

```
2)*x)*sqrt(-c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a
^3*c^3*d^3 + (b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a
^3*c*d^5)*x^4 + 2*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3
- a^3*c^2*d^4)*x^2)*sqrt(-c*d)), -1/8*(4*(b*c*d^2*x^4 + 2*b*c^2*d
*x^2 + b*c^3)*sqrt(-a*b)*sqrt(c*d)*log((b*x^2 + 2*sqrt(-a*b)*x -
a)/(b*x^2 + a)) - (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2
*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^
2*d^2 - a^2*c*d^3)*x^2)*arctan(sqrt(c*d)*x/c) - ((3*b^2*c^2*d - 2
*a*b*c*d^2 - a^2*d^3)*x^3 + (5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2)
*x)*sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*
c^3*d^3 + (b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*
c*d^5)*x^4 + 2*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a
^3*c^2*d^4)*x^2)*sqrt(c*d)), -1/16*(16*(b*c*d^2*x^4 + 2*b*c^2*d*x
^2 + b*c^3)*sqrt(a*b)*sqrt(-c*d)*arctan(b*x/sqrt(a*b)) - (3*b^2*c
^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a
^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*lo
g((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) - 2*((3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^3 + (5*b^2*c^3 - 6*a*b*c^2*d + a^2*
c*d^2)*x)*sqrt(-c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2
 - a^3*c^3*d^3 + (b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4
 - a^3*c*d^5)*x^4 + 2*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*
d^3 - a^3*c^2*d^4)*x^2)*sqrt(-c*d)), -1/8*(8*(b*c*d^2*x^4 + 2*b*c
^2*d*x^2 + b*c^3)*sqrt(a*b)*sqrt(c*d)*arctan(b*x/sqrt(a*b)) - (3*
b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^
3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^
2)*arctan(sqrt(c*d)*x/c) - ((3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)
*x^3 + (5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2)*x)*sqrt(c*d))/((b^3*
c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^4*d^
2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x^4 + 2*(b^3*c
^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*x^2)*sqrt
(c*d))]
```

Sympy [A] time = 164.665, size = 3386, normalized size = 21.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-a*b**3)*log(x + (-64*a**8*c**3*d**9*(-a*b**3)**(3/2)/(a*d
- b*c)**9 + 768*a**7*b*c**4*d**8*(-a*b**3)**(3/2)/(a*d - b*c)**9
- 2560*a**6*b**2*c**5*d**7*(-a*b**3)**(3/2)/(a*d - b*c)**9 - a**6
*d**6*sqrt(-a*b**3)/(a*d - b*c)**3 + 2816*a**5*b**3*c**6*d**6*(-a
*b**3)**(3/2)/(a*d - b*c)**9 + 18*a**5*b*c*d**5*sqrt(-a*b**3)/(a*
d - b*c)**3 + 1920*a**4*b**4*c**7*d**5*(-a*b**3)**(3/2)/(a*d - b*
c)**9 - 99*a**4*b**2*c**2*d**4*sqrt(-a*b**3)/(a*d - b*c)**3 - 793
6*a**3*b**5*c**8*d**4*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 108*a**3*
b**3*c**3*d**3*sqrt(-a*b**3)/(a*d - b*c)**3 + 8192*a**2*b**6*c**9
*d**3*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 297*a**2*b**4*c**4*d**2*s
qrt(-a*b**3)/(a*d - b*c)**3 - 3840*a*b**7*c**10*d**2*(-a*b**3)**(
3/2)/(a*d - b*c)**9 + 674*a*b**5*c**5*d*sqrt(-a*b**3)/(a*d - b*c)
**3 + 704*b**8*c**11*d*(-a*b**3)**(3/2)/(a*d - b*c)**9 + 27*b**6*
c**6*sqrt(-a*b**3)/(a*d - b*c)**3)/(a**3*b**2*d**3 - 15*a**2*b**3
*c*d**2 + 51*a*b**4*c**2*d + 27*b**5*c**3))/(2*(a*d - b*c)**3) +
sqrt(-a*b**3)*log(x + (64*a**8*c**3*d**9*(-a*b**3)**(3/2)/(a*d -
b*c)**9 - 768*a**7*b*c**4*d**8*(-a*b**3)**(3/2)/(a*d - b*c)**9 +
2560*a**6*b**2*c**5*d**7*(-a*b**3)**(3/2)/(a*d - b*c)**9 + a**6*d
**6*sqrt(-a*b**3)/(a*d - b*c)**3 - 2816*a**5*b**3*c**6*d**6*(-a*b
**3)**(3/2)/(a*d - b*c)**9 - 18*a**5*b*c*d**5*sqrt(-a*b**3)/(a*d
- b*c)**3 - 1920*a**4*b**4*c**7*d**5*(-a*b**3)**(3/2)/(a*d - b*c)
**9 + 99*a**4*b**2*c**2*d**4*sqrt(-a*b**3)/(a*d - b*c)**3 + 7936*
a**3*b**5*c**8*d**4*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 108*a**3*b*
*3*c**3*d**3*sqrt(-a*b**3)/(a*d - b*c)**3 - 8192*a**2*b**6*c**9*d
**3*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 297*a**2*b**4*c**4*d**2*sqr
t(-a*b**3)/(a*d - b*c)**3 + 3840*a*b**7*c**10*d**2*(-a*b**3)**(3/
2)/(a*d - b*c)**9 - 674*a*b**5*c**5*d*sqrt(-a*b**3)/(a*d - b*c)**
3 - 704*b**8*c**11*d*(-a*b**3)**(3/2)/(a*d - b*c)**9 - 27*b**6*c*
*6*sqrt(-a*b**3)/(a*d - b*c)**3)/(a**3*b**2*d**3 - 15*a**2*b**3*c
*d**2 + 51*a*b**4*c**2*d + 27*b**5*c**3))/(2*(a*d - b*c)**3) - sq
```

```
rt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)*log(x + (-a
**8*c**3*d**9*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**
2*c**2)**3/(8*(a*d - b*c)**9) + 3*a**7*b*c**4*d**8*(-1/(c**3*d))*
*(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9
) - 5*a**6*b**2*c**5*d**7*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b
*c*d - 3*b**2*c**2)**3/(a*d - b*c)**9 - a**6*d**6*sqrt(-1/(c**3*d
))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)**3) + 11*
a**5*b**3*c**6*d**6*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d -
 3*b**2*c**2)**3/(2*(a*d - b*c)**9) + 9*a**5*b*c*d**5*sqrt(-1/(c*
*3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(4*(a*d - b*c)**3) +
 15*a**4*b**4*c**7*d**5*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c
*d - 3*b**2*c**2)**3/(4*(a*d - b*c)**9) - 99*a**4*b**2*c**2*d**4*
sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d -
 b*c)**3) - 31*a**3*b**5*c**8*d**4*(-1/(c**3*d))**(3/2)*(a**2*d**
2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) + 27*a**3*b**3
*c**3*d**3*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2
)/(2*(a*d - b*c)**3) + 16*a**2*b**6*c**9*d**3*(-1/(c**3*d))**(3/2
)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(a*d - b*c)**9 + 297*a
**2*b**4*c**4*d**2*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b
**2*c**2)/(8*(a*d - b*c)**3) - 15*a*b**7*c**10*d**2*(-1/(c**3*d))
**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**
9) + 337*a*b**5*c**5*d*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d -
 3*b**2*c**2)/(4*(a*d - b*c)**3) + 11*b**8*c**11*d*(-1/(c**3*d))*
*(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(8*(a*d - b*c)**9
) + 27*b**6*c**6*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**
2*c**2)/(8*(a*d - b*c)**3))/(a**3*b**2*d**3 - 15*a**2*b**3*c*d**2
 + 51*a*b**4*c**2*d + 27*b**5*c**3))/(16*(a*d - b*c)**3) + sqrt(-
1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)*log(x + (a**8*c
**3*d**9*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**
2)**3/(8*(a*d - b*c)**9) - 3*a**7*b*c**4*d**8*(-1/(c**3*d))**(3/2
)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) + 5
*a**6*b**2*c**5*d**7*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d
- 3*b**2*c**2)**3/(a*d - b*c)**9 + a**6*d**6*sqrt(-1/(c**3*d))*(a
**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)**3) - 11*a**5*
b**3*c**6*d**6*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d - 3*b*
*2*c**2)**3/(2*(a*d - b*c)**9) - 9*a**5*b*c*d**5*sqrt(-1/(c**3*d)
)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(4*(a*d - b*c)**3) - 15*a
**4*b**4*c**7*d**5*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6*a*b*c*d -
3*b**2*c**2)**3/(4*(a*d - b*c)**9) + 99*a**4*b**2*c**2*d**4*sqrt(
-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(8*(a*d - b*c)
**3) + 31*a**3*b**5*c**8*d**4*(-1/(c**3*d))**(3/2)*(a**2*d**2 - 6
*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) - 27*a**3*b**3*c**3
*d**3*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)/(2*
(a*d - b*c)**3) - 16*a**2*b**6*c**9*d**3*(-1/(c**3*d))**(3/2)*(a*
*2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(a*d - b*c)**9 - 297*a**2*b
**4*c**4*d**2*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c
**2)/(8*(a*d - b*c)**3) + 15*a*b**7*c**10*d**2*(-1/(c**3*d))**(3/
2)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(2*(a*d - b*c)**9) -
337*a*b**5*c**5*d*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b*
*2*c**2)/(4*(a*d - b*c)**3) - 11*b**8*c**11*d*(-1/(c**3*d))**(3/2
)*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**2)**3/(8*(a*d - b*c)**9) - 2
7*b**6*c**6*sqrt(-1/(c**3*d))*(a**2*d**2 - 6*a*b*c*d - 3*b**2*c**
2)/(8*(a*d - b*c)**3))/(a**3*b**2*d**3 - 15*a**2*b**3*c*d**2 + 51
*a*b**4*c**2*d + 27*b**5*c**3))/(16*(a*d - b*c)**3) + (x**3*(a*d*
*2 + 3*b*c*d) + x*(-a*c*d + 5*b*c**2))/(8*a**2*c**3*d**2 - 16*a*b
*c**4*d + 8*b**2*c**5 + x**4*(8*a**2*c*d**4 - 16*a*b*c**2*d**3 +
8*b**2*c**3*d**2) + x**2*(16*a**2*c**2*d**3 - 32*a*b*c**3*d**2 +
16*b**2*c**4*d))
```

GIAC/XCAS [A] time = 0.289198, size = 278, normalized size = 1.79

$$-\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{ab}} + \frac{(3\,b^2c^2 + 6\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3)\sqrt{cd}} + \frac{3\,bcdx^3 + ad^2x^3 + 5\,bc^2x - acdx}{8\,(b^2c^3 - 2\,abc^2d + a^2cd^2)(dx^2 + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] -a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^
2)*arctan(d*x/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*
d^2 - a^3*c*d^3)*sqrt(c*d)) + 1/8*(3*b*c*d*x^3 + a*d^2*x^3 + 5*b*
c^2*x - a*c*d*x)/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(d*x^2 + c)
^2)
```

3.255 $\int \frac{x}{(a+bx^2)(c+dx^2)^3} \, dx$

Optimal. Leaf size=98

$$\frac{b^2 \log\left(a+bx^2\right)}{2(bc-ad)^3} - \frac{b^2 \log\left(c+dx^2\right)}{2(bc-ad)^3} + \frac{b}{2\left(c+dx^2\right)(bc-ad)^2} + \frac{1}{4\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] 1/(4*(b*c - a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2))
 + (b^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/
(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.166613, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{b^2 \log\left(a+bx^2\right)}{2(bc-ad)^3} - \frac{b^2 \log\left(c+dx^2\right)}{2(bc-ad)^3} + \frac{b}{2\left(c+dx^2\right)(bc-ad)^2} + \frac{1}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] 1/(4*(b*c - a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2))
 + (b^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/
(2*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 31.7724, size = 80, normalized size = 0.82

$$-\frac{b^2 \log\left(a+bx^2\right)}{2(ad-bc)^3} + \frac{b^2 \log\left(c+dx^2\right)}{2(ad-bc)^3} + \frac{b}{2\left(c+dx^2\right)(ad-bc)^2} - \frac{1}{4\left(c+dx^2\right)^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -b**2*log(a + b*x**2)/(2*(a*d - b*c)**3) + b**2*log(c + d*x**2)/(
2*(a*d - b*c)**3) + b/(2*(c + d*x**2)*(a*d - b*c)**2) - 1/(4*(c +
 d*x**2)**2*(a*d - b*c))
```

Mathematica [A] time = 0.0793567, size = 98, normalized size = 1.

$$\frac{b^2 \log\left(a+bx^2\right)}{2(bc-ad)^3} - \frac{b^2 \log\left(c+dx^2\right)}{2(bc-ad)^3} + \frac{b}{2\left(c+dx^2\right)(bc-ad)^2} - \frac{1}{4\left(c+dx^2\right)^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(4*(-(b*c) + a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x
^2)) + (b^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^
2])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.018, size = 176, normalized size = 1.8

$$\frac{abd}{2\left(ad-bc\right)^{3}\left(dx^{2}+c\right)}-\frac{b^{2}c}{2\left(ad-bc\right)^{3}\left(dx^{2}+c\right)}-\frac{a^{2}d^{2}}{4\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{cabd}{2\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}-\frac{b^{2}c^{2}}{4\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{\ln\left(dx^{2}+c\right)b^{2}}{2\left(ad-bc\right)^{3}}-\frac{b^{2}\ln\left(bx^{2}+a\right)}{2\left(ad-bc\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/2*d/(a*d-b*c)^3*b/(d*x^2+c)*a-1/2/(a*d-b*c)^3*b^2/(d*x^2+c)*c-1
/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*a^2+1/2*d/(a*d-b*c)^3/(d*x^2+c)^2*
c*a*b-1/4/(a*d-b*c)^3/(d*x^2+c)^2*b^2*c^2+1/2/(a*d-b*c)^3*ln(d*x^
2+c)*b^2-1/2*b^2/(a*d-b*c)^3*ln(b*x^2+a)
```

Maxima [A] time = 1.36669, size = 285, normalized size = 2.91

$$\frac{b^{2}\log\left(bx^{2}+a\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{b^{2}\log\left(dx^{2}+c\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{2\,bdx^{2}+3\,bc-ad}{4\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)x^{4}+2\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3) - 1/2*b^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3) + 1/4*(2*b*d*x^2 + 3*b*c - a*d)/(b^2*c^4 -
2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)
```

Fricas [A] time = 0.241356, size = 343, normalized size = 3.5

$$\frac{3\,b^{2}c^{2}-4\,abcd+a^{2}d^{2}+2\left(b^{2}cd-abd^{2}\right)x^{2}+2\left(b^{2}d^{2}x^{4}+2\,b^{2}cdx^{2}+b^{2}c^{2}\right)\log\left(bx^{2}+a\right)-2\left(b^{2}d^{2}x^{4}+2\,b^{2}cdx^{2}+b^{2}c^{2}\right)\log}{4\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}+\left(b^{3}c^{3}d^{2}-3\,ab^{2}c^{2}d^{3}+3\,a^{2}bcd^{4}-a^{3}d^{5}\right)x^{4}+2\left(b^{3}c^{4}d-3\,ab^{2}c^{3}d^{2}+3\,a^{2}bc^{2}d^{3}-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2
+ 2*(b^2*d^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(b*x^2 + a) - 2*(b
^2*d^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(b^3*c^5 -
3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*
a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^4 + 2*(b^3*c^4*d - 3*a
*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)
```

Sympy [A] time = 15.7124, size = 391, normalized size = 3.99

$$\frac{b^2\log\left(x^2+\frac{-\frac{a^4b^2d^4}{(ad-bc)^3}+\frac{4a^3b^3cd^3}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2}{(ad-bc)^3}+\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d-\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{2(ad-bc)^3} - \frac{b^2\log\left(x^2+\frac{\frac{a^4b^2d^4}{(ad-bc)^3}-\frac{4a^3b^3cd^3}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2}{(ad-bc)^3}-\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d+\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{2(ad-bc)^3} + \frac{-ad+3bc+2bdx^2}{4a^2c^2d^2-8abc^3d+4b^2c^4+x^4(4a^2d^4-8abcd^3+4b^2c^2d^2)+x^2(8a^2cd^3-16abc^2d^2+8b^2c^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(x**2 + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d
**3/(a*d - b*c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b
**5*c**3*d/(a*d - b*c)**3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 +
 b**3*c)/(2*b**3*d))/(2*(a*d - b*c)**3) - b**2*log(x**2 + (a**4*b
**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d - b*c)**3 + 6*a
**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b*c)**
3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(2*
(a*d - b*c)**3) + (-a*d + 3*b*c + 2*b*d*x**2)/(4*a**2*c**2*d**2 -
 8*a*b*c**3*d + 4*b**2*c**4 + x**4*(4*a**2*d**4 - 8*a*b*c*d**3 +
4*b**2*c**2*d**2) + x**2*(8*a**2*c*d**3 - 16*a*b*c**2*d**2 + 8*b*
*2*c**3*d))
```

GIAC/XCAS [A] time = 0.259293, size = 235, normalized size = 2.4

$$\frac{b^3\ln\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)} - \frac{b^2d\ln\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)} + \frac{3\,b^2c^2-4\,abcd+a^2d^2+2\left(b^2cd-abd^2\right)x^2}{4\left(dx^2+c\right)^2(bc-ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] 1/2*b^3*ln(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c
*d^2 - a^3*b*d^3) - 1/2*b^2*d*ln(abs(d*x^2 + c))/(b^3*c^3*d - 3*a
*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) + 1/4*(3*b^2*c^2 - 4*a*b*
c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2)/((d*x^2 + c)^2*(b*c -
a*d)^3)
```

3.256 $\int \frac{1}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2(c+dx^2)(bc-ad)^2}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8
*c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*(b*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*
d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d
)^3)
```

Rubi [A] time = 0.459235, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2(c+dx^2)(bc-ad)^2}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8
*c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*(b*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*
d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d
)^3)
```

Rubi in Sympy [A] time = 99.1836, size = 146, normalized size = 0.91

$$\frac{dx}{4c(c+dx^2)^2(ad-bc)}+\frac{dx(3ad-7bc)}{8c^2(c+dx^2)(ad-bc)^2}+\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}(ad-bc)^3}-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] d*x/(4*c*(c + d*x**2)**2*(a*d - b*c)) + d*x*(3*a*d - 7*b*c)/(8*c*
*2*(c + d*x**2)*(a*d - b*c)**2) + sqrt(d)*(3*a**2*d**2 - 10*a*b*c
*d + 15*b**2*c**2)*atan(sqrt(d)*x/sqrt(c))/(8*c**(5/2)*(a*d - b*c
)**3) - b**(5/2)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.507882, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3}+\frac{dx(3ad-7bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{2dx}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/
(c^2*(b*c - a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(Sqrt[a]*(-(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a
*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c -
a*d)^3))/8
```

Maple [B] time = 0.002, size = 310, normalized size = 1.9

$$\frac{3\,d^4x^3a^2}{8\,(ad-bc)^3\,(dx^2+c)^2\,c^2}-\frac{5\,d^3x^3ab}{4\,(ad-bc)^3\,(dx^2+c)^2\,c}+\frac{7\,d^2b^2x^3}{8\,(ad-bc)^3\,(dx^2+c)^2}+\frac{5\,d^3xa^2}{8\,(ad-bc)^3\,(dx^2+c)^2\,c}-\frac{7\,d^2xab}{4\,(ad-bc)^3\,(dx^2+c)^2}+\frac{9\,dxb^2c}{8\,(ad-bc)^3\,(dx^2+c)^2}+\frac{3\,a^2d^3}{8\,(ad-bc)^3\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{5\,ad^2b}{4\,(ad-bc)^3\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{15\,db^2}{8\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3}{(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^3*a^2-5/4*d^3/(a*d-b*c)^3/(
d*x^2+c)^2/c*x^3*a*b+7/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*b^2*x^3+5/8*
d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x*a^2-7/4*d^2/(a*d-b*c)^3/(d*x^2+c)
^2*x*a*b+9/8*d/(a*d-b*c)^3/(d*x^2+c)^2*x*b^2*c+3/8*d^3/(a*d-b*c)^
3/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-5/4*d^2/(a*d-b*c)^3
/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+15/8*d/(a*d-b*c)^3/(c*
d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-b^3/(a*d-b*c)^3/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.01554, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d
+ 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c^3*
d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3
*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3
*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^
2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 1
0*a*b*c*d^3 + 3*a^2*d^4)*x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2
*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(1
5*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(d/c)*arctan
(d*x/(c*sqrt(d/c))) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*
c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
 (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b
^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2
*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*
a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/16*(2*(7*
b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 16*(b^2*c^2*d^2*x^4
 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a)))
 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 -
 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2
 + 3*a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c
)/(d*x^2 + c)) + 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3
*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 +
 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*
x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b
^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(b*x/
(a*sqrt(b/a))) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15
*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d -
10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d
/c))) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7
- 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6
*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.262079, size = 293, normalized size = 1.83

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{ab}} - \frac{\left(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3)\sqrt{cd}} - \frac{7\,bcd^2x^3 - 3\,ad^3x^3 + 9\,bc^2dx - 5\,acd^2x}{8\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)(dx^2 + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a
^2*d^3)*arctan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*sqrt(c*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3
*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c
^2*d^2)*(d*x^2 + c)^2)
```

3.257 $\int \frac{1}{x(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=149

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\log(x)}{ac^3}$$

```
[Out] -d/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(2*c^2*(b*
c - a*d)^2*(c + d*x^2)) + Log[x]/(a*c^3) - (b^3*Log[a + b*x^2])/(
2*a*(b*c - a*d)^3) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c +
 d*x^2])/(2*c^3*(b*c - a*d)^3)
```

Rubi [A] time = 0.348319, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -d/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(2*c^2*(b*
c - a*d)^2*(c + d*x^2)) + Log[x]/(a*c^3) - (b^3*Log[a + b*x^2])/(
2*a*(b*c - a*d)^3) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c +
 d*x^2])/(2*c^3*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 64.1219, size = 133, normalized size = 0.89

$$\frac{d}{4c\left(c+dx^2\right)^2(ad-bc)}+\frac{d(ad-2bc)}{2c^2\left(c+dx^2\right)(ad-bc)^2}-\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(ad-bc)^3}+\frac{b^3\log\left(a+bx^2\right)}{2a(ad-bc)^3}+\frac{\log\left(x^2\right)}{2ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] d/(4*c*(c + d*x**2)**2*(a*d - b*c)) + d*(a*d - 2*b*c)/(2*c**2*(c
+ d*x**2)*(a*d - b*c)**2) - d*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**
2)*log(c + d*x**2)/(2*c**3*(a*d - b*c)**3) + b**3*log(a + b*x**2)
/(2*a*(a*d - b*c)**3) + log(x**2)/(2*a*c**3)
```

Mathematica [A] time = 0.469545, size = 141, normalized size = 0.95

$$\frac{\frac{d\left(\frac{c\left(a^2d^2\left(3c+2dx^2\right)-2abcd\left(4c+3dx^2\right)+b^2c^2\left(5c+4dx^2\right)\right)}{\left(c+dx^2\right)^2}-2\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)\right)}{c^3}+\frac{2b^3\log\left(a+bx^2\right)}{a}}{4(ad-bc)^3}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Log[x]/(a*c^3) + ((2*b^3*Log[a + b*x^2])/a + (d*((c*(a^2*d^2*(3*c
 + 2*d*x^2) - 2*a*b*c*d*(4*c + 3*d*x^2) + b^2*c^2*(5*c + 4*d*x^2)
))/(c + d*x^2)^2 - 2*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*
x^2]))/c^3)/(4*(-(b*c) + a*d)^3)
```

Maple [B] time = 0.026, size = 286, normalized size = 1.9

$$\frac{\ln(x)}{ac^3}+\frac{a^2d^3}{2c^2(ad-bc)^3(dx^2+c)}-\frac{3ad^2b}{2c(ad-bc)^3(dx^2+c)}+\frac{b^2d}{(ad-bc)^3(dx^2+c)}+\frac{a^2d^3}{4c(ad-bc)^3(dx^2+c)^2}-\frac{ad^2b}{2(ad-bc)^3(dx^2+c)^2}+\frac{b^2cd}{4(ad-bc)^3(dx^2+c)^2}-\frac{d^3\ln(dx^2+c)a^2}{2c^3(ad-bc)^3}+\frac{3d^2\ln(dx^2+c)ab}{2c^2(ad-bc)^3}-\frac{3d\ln(dx^2+c)b^2}{2c(ad-bc)^3}+\frac{b^3\ln(bx^2+a)}{2a(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] ln(x)/a/c^3+1/2*d^3/c^2/(a*d-b*c)^3/(d*x^2+c)*a^2-3/2*d^2/c/(a*d-
b*c)^3/(d*x^2+c)*a*b+d/(a*d-b*c)^3/(d*x^2+c)*b^2+1/4*d^3/c/(a*d-b
*c)^3/(d*x^2+c)^2*a^2-1/2*d^2/(a*d-b*c)^3/(d*x^2+c)^2*a*b+1/4*d*c
/(a*d-b*c)^3/(d*x^2+c)^2*b^2-1/2*d^3/c^3/(a*d-b*c)^3*ln(d*x^2+c)*
a^2+3/2*d^2/c^2/(a*d-b*c)^3*ln(d*x^2+c)*a*b-3/2*d/c/(a*d-b*c)^3*l
n(d*x^2+c)*b^2+1/2*b^3/a/(a*d-b*c)^3*ln(b*x^2+a)
```

Maxima [A] time = 1.37478, size = 375, normalized size = 2.52

$$-\frac{b^3\log(bx^2+a)}{2(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3)}+\frac{(3b^2c^2d-3abcd^2+a^2d^3)\log(dx^2+c)}{2(b^3c^6-3ab^2c^5d+3a^2bc^4d^2-a^3c^3d^3)}-\frac{5bc^2d-3acd^2+2(2bcd^2-ad^3)x^2}{4(b^2c^6-2abc^5d+a^2c^4d^2+(b^2c^4d^2-2abc^3d^3+a^2c^2d^4)x^4+2(b^2c^5d-2abc^4d^2+a^2c^3d^3)x^2)}+\frac{\log(x^2)}{2ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2 - a^4*d^3) + 1/2*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*
x^2 + c)/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
) - 1/4*(5*b*c^2*d - 3*a*c*d^2 + 2*(2*b*c*d^2 - a*d^3)*x^2)/(b^2*
c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 +
a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^
2) + 1/2*log(x^2)/(a*c^3)
```

Fricas [A] time = 3.67369, size = 702, normalized size = 4.71

$$-\frac{5ab^2c^4d-8a^2bc^3d^2+3a^3c^2d^3+2(2ab^2c^3d^2-3a^2bc^2d^3+a^3cd^4)x^2+2(b^3c^3d^2x^4+2b^3c^4dx^2+b^3c^5)\log(bx^2+a)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x),x, algorithm="fricas")
```

```
[Out] -1/4*(5*a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 3*a^3*c^2*d^3 + 2*(2*a*b^
2*c^3*d^2 - 3*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2 + 2*(b^3*c^3*d^2*x^4
 + 2*b^3*c^4*d*x^2 + b^3*c^5)*log(b*x^2 + a) - 2*(3*a*b^2*c^4*d -
```

```
 3*a^2*b*c^3*d^2 + a^3*c^2*d^3 + (3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4
 + a^3*d^5)*x^4 + 2*(3*a*b^2*c^3*d^2 - 3*a^2*b*c^2*d^3 + a^3*c*d^
4)*x^2)*log(d*x^2 + c) - 4*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3
*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d
^4 - a^3*d^5)*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*
d^3 - a^3*c*d^4)*x^2)*log(x))/(a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^
3*b*c^6*d^2 - a^4*c^5*d^3 + (a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 +
3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^4 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c
^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.258378, size = 425, normalized size = 2.85

$$-\frac{b^4 \ln\left(\left|bx^2+a\right|\right)}{2\,(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3)}+\frac{\left(3\,b^2c^2d^2-3\,abcd^3+a^2d^4\right)\ln\left(\left|dx^2+c\right|\right)}{2\,(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4)}-\frac{9\,b^2c^2d^3x^4-9\,abcd^4x^4+3\,a^2d^5x^4+22\,b^2c^3d^2x^2-24\,abc^2d^3x^2+8\,a^2cd^4x^2+14\,b^2c^4d-17\,abc^3d^2+6\,a^2c^2d^3}{4\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)(dx^2+c)^2}+\frac{\ln\left(x^2\right)}{2\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x),x, algorithm="giac")
```

```
[Out] -1/2*b^4*ln(abs(b*x^2 + a))/(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*
b^2*c*d^2 - a^4*b*d^3) + 1/2*(3*b^2*c^2*d^2 - 3*a*b*c*d^3 + a^2*d
^4)*ln(abs(d*x^2 + c))/(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4) - 1/4*(9*b^2*c^2*d^3*x^4 - 9*a*b*c*d^4*x^4 +
3*a^2*d^5*x^4 + 22*b^2*c^3*d^2*x^2 - 24*a*b*c^2*d^3*x^2 + 8*a^2*c
*d^4*x^2 + 14*b^2*c^4*d - 17*a*b*c^3*d^2 + 6*a^2*c^2*d^3)/((b^3*c
^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2
) + 1/2*ln(x^2)/(a*c^3)
```

3.258 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=211

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}-\frac{d(9bc-5ad)}{8c^2x\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(8*b^2*c^2 - 27*a*b*c*d + 15*a^2*d^2)/(8*a*c^3*(b*c - a*d)^2*x)
- d/(4*c*(b*c - a*d)*x*(c + d*x^2)^2) - (d*(9*b*c - 5*a*d))/(8*c^
2*(b*c - a*d)^2*x*(c + d*x^2)) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d
 + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c - a*d
)^3)
```

Rubi [A] time = 0.827151, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}-\frac{d(9bc-5ad)}{8c^2x\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(8*b^2*c^2 - 27*a*b*c*d + 15*a^2*d^2)/(8*a*c^3*(b*c - a*d)^2*x)
- d/(4*c*(b*c - a*d)*x*(c + d*x^2)^2) - (d*(9*b*c - 5*a*d))/(8*c^
2*(b*c - a*d)^2*x*(c + d*x^2)) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d
 + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c - a*d
)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.677219, size = 172, normalized size = 0.82

$$\frac{1}{8}\left(\frac{8b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}+\frac{d^2x(11bc-7ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{2d^2x}{c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{8}{ac^3x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a*c^3*x) + (2*d^2*x)/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*
(11*b*c - 7*a*d)*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (8*b^(7/2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^3) + (d^(3/2
)*(35*b^2*c^2 - 42*a*b*c*d + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(c^(7/2)*(b*c - a*d)^3))/8
```

Maple [A] time = 0.025, size = 335, normalized size = 1.6

$$-\frac{1}{ac^3x}-\frac{7\,d^5x^3a^2}{8\,c^3\,(ad-bc)^3\,(dx^2+c)^2}+\frac{9\,d^4x^3ab}{4\,c^2\,(ad-bc)^3\,(dx^2+c)^2}-\frac{11\,d^3x^3b^2}{8\,c\,(ad-bc)^3\,(dx^2+c)^2}-\frac{9\,d^4xa^2}{8\,c^2\,(ad-bc)^3\,(dx^2+c)^2}+\frac{11\,d^3xab}{4\,c\,(ad-bc)^3\,(dx^2+c)^2}-\frac{13\,d^2xb^2}{8\,(ad-bc)^3\,(dx^2+c)^2}-\frac{15\,a^2d^4}{8\,c^3\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{21\,abd^3}{4\,c^2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{35\,d^2b^2}{8\,c\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^4}{a\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/a/c^3/x-7/8*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a^2+9/4*d^4/c^
2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*b-11/8*d^3/c/(a*d-b*c)^3/(d*x^2+c
)^2*x^3*b^2-9/8*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x*a^2+11/4*d^3/c/
(a*d-b*c)^3/(d*x^2+c)^2*x*a*b-13/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x*
b^2-15/8*d^4/c^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*
a^2+21/4*d^3/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*
a*b-35/8*d^2/c/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^
2+1/a*b^4/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36682, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^3*d^2 - 16*a^3*c
^2*d^3 + 2*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 1
5*a^3*d^5)*x^4 + 2*(16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^
2*d^3 - 25*a^3*c*d^4)*x^2 + 8*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3
+ b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5)*x^5 +
2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*
a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*lo
```

```
g((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 -
3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3
*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 +
 (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a^4*c^5*d^3)*x)
, -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b*c^3*d^2 - 8*a^3*c^2
*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 15*a^
3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3
- 25*a^3*c*d^4)*x^2 - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^
3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^
4)*x^3 + (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*
sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + 4*(b^3*c^3*d^2*x^5 + 2*b^3*
c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c
^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 +
3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d
 + 3*a^3*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/16*(16*b^3*c^5 - 48*a*b^
2*c^4*d + 48*a^2*b*c^3*d^2 - 16*a^3*c^2*d^3 + 2*(8*b^3*c^3*d^2 -
35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 15*a^3*d^5)*x^4 + 2*(16*b^3*c
^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 +
16*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(b/a)*arct
an(b*x/(a*sqrt(b/a))) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*
a^3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*
d^4)*x^3 + (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x
)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a
*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)
*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4
*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 -
a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b*c^3*
d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*
b*c*d^4 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70
*a^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 + 8*(b^3*c^3*d^2*x^5 + 2*b^3*c
^4*d*x^3 + b^3*c^5*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - ((35*
a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5)*x^5 + 2*(35*a*b^2*c^
3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*b^2*c^4*d -
42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(d*x/(c*sqr
t(d/c))))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 -
 a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*
c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3
*b*c^6*d^2 - a^4*c^5*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.253936, size = 319, normalized size = 1.51

$$-\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\sqrt{ab}} + \frac{\left(35\,b^2c^2d^2 - 42\,abcd^3 + 15\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3)\sqrt{cd}} + \frac{11\,bcd^3x^3 - 7\,ad^4x^3 + 13\,bc^2d^2x - 9\,acd^3x}{8\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2)(dx^2+c)^2} - \frac{1}{ac^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^2),x, algorithm="giac")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*
b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^
```

```
3 + 15*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d +
 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*sqrt(c*d)) + 1/8*(11*b*c*d^3*x^3
- 7*a*d^4*x^3 + 13*b*c^2*d^2*x - 9*a*c*d^3*x)/((b^2*c^5 - 2*a*b*c
^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 1/(a*c^3*x)
```

3.259 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=178

$$\frac{b^4\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}-\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2}{4c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{1}{2ac^3x^2}$$

```
[Out] -1/(2*a*c^3*x^2) + d^2/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(
3*b*c - 2*a*d))/(2*c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d
)*Log[x])/(a^2*c^4) + (b^4*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3)
- (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4
*(b*c - a*d)^3)
```

Rubi [A] time = 0.467233, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^4\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}-\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2}{4c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{1}{2ac^3x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(2*a*c^3*x^2) + d^2/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(
3*b*c - 2*a*d))/(2*c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d
)*Log[x])/(a^2*c^4) + (b^4*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3)
- (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4
*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 75.2056, size = 167, normalized size = 0.94

$$-\frac{d^2}{4c^2\left(c+dx^2\right)^2(ad-bc)}-\frac{d^2(2ad-3bc)}{2c^3\left(c+dx^2\right)(ad-bc)^2}+\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(ad-bc)^3}-\frac{1}{2ac^3x^2}-\frac{b^4\log\left(a+bx^2\right)}{2a^2(ad-bc)^3}-\frac{(3ad+bc)\log\left(x^2\right)}{2a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -d**2/(4*c**2*(c + d*x**2)**2*(a*d - b*c)) - d**2*(2*a*d - 3*b*c)
/(2*c**3*(c + d*x**2)*(a*d - b*c)**2) + d**2*(3*a**2*d**2 - 8*a*b
*c*d + 6*b**2*c**2)*log(c + d*x**2)/(2*c**4*(a*d - b*c)**3) - 1/(
2*a*c**3*x**2) - b**4*log(a + b*x**2)/(2*a**2*(a*d - b*c)**3) - (
3*a*d + b*c)*log(x**2)/(2*a**2*c**4)
```

Mathematica [A] time = 1.32424, size = 171, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2b^4\log\left(a+bx^2\right)}{a^2(ad-bc)^3}-\frac{2d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{c^4(bc-ad)^3}-\frac{4\log(x)(3ad+bc)}{a^2c^4}+\frac{2d^2(3bc-2ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2}{c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{2}{ac^3x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a*c^3*x^2) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (2*d^2*(3
*b*c - 2*a*d))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (4*(b*c + 3*a*d)
*Log[x])/(a^2*c^4) - (2*b^4*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)^3
) - (2*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(c
^4*(b*c - a*d)^3))/4
```

Maple [A] time = 0.031, size = 322, normalized size = 1.8

$$-\frac{1}{2\,ac^3x^2}-3\,\frac{\ln(x)\,d}{ac^4}-\frac{b\ln(x)}{a^2c^3}-\frac{a^2d^4}{c^3\,(ad-bc)^3\,(dx^2+c)}+\frac{5\,abd^3}{2\,c^2\,(ad-bc)^3\,(dx^2+c)}-\frac{3\,d^2b^2}{2\,c\,(ad-bc)^3\,(dx^2+c)}-\frac{a^2d^4}{4\,c^2\,(ad-bc)^3\,(dx^2+c)^2}+\frac{abd^3}{2\,c\,(ad-bc)^3\,(dx^2+c)^2}-\frac{d^2b^2}{4\,(ad-bc)^3\,(dx^2+c)^2}+\frac{3\,d^4\ln\left(dx^2+c\right)\,a^2}{2\,c^4\,(ad-bc)^3}-4\,\frac{d^3\ln\left(dx^2+c\right)\,ab}{c^3\,(ad-bc)^3}+3\,\frac{d^2\ln\left(dx^2+c\right)\,b^2}{c^2\,(ad-bc)^3}-\frac{b^4\ln\left(bx^2+a\right)}{2\,a^2\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/2/a/c^3/x^2-3/a/c^4*ln(x)*d-1/a^2/c^3*ln(x)*b-d^4/c^3/(a*d-b*c
)^3/(d*x^2+c)*a^2+5/2*d^3/c^2/(a*d-b*c)^3/(d*x^2+c)*a*b-3/2*d^2/c
/(a*d-b*c)^3/(d*x^2+c)*b^2-1/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*a^
2+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*a*b-1/4*d^2/(a*d-b*c)^3/(d*x^
2+c)^2*b^2+3/2*d^4/c^4/(a*d-b*c)^3*ln(d*x^2+c)*a^2-4*d^3/c^3/(a*d
-b*c)^3*ln(d*x^2+c)*a*b+3*d^2/c^2/(a*d-b*c)^3*ln(d*x^2+c)*b^2-1/2
*b^4/a^2/(a*d-b*c)^3*ln(b*x^2+a)
```

Maxima [A] time = 1.38239, size = 491, normalized size = 2.76

$$\frac{b^4\log\left(bx^2+a\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)}-\frac{\left(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4\right)\log\left(dx^2+c\right)}{2\,(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3)}-\frac{2\,b^2c^4-4\,abc^3d+2\,a^2c^2d^2+2\left(b^2c^2d^2-5\,abcd^3+3\,a^2d^4\right)x^4+\left(4\,b^2c^3d-15\,abc^2d^2+9\,a^2cd^3\right)x^2}{4\,((ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4)x^6+2\,(ab^2c^6d-2\,a^2bc^5d^2+a^3c^4d^3)x^4+(ab^2c^7-2\,a^2bc^6d+a^3c^5d^2)x^2)}-\frac{(bc+3\,ad)\log\left(x^2\right)}{2\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c
*d^2 - a^5*d^3) - 1/2*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*l
og(d*x^2 + c)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^
4*d^3) - 1/4*(2*b^2*c^4 - 4*a*b*c^3*d + 2*a^2*c^2*d^2 + 2*(b^2*c^
2*d^2 - 5*a*b*c*d^3 + 3*a^2*d^4)*x^4 + (4*b^2*c^3*d - 15*a*b*c^2*
d^2 + 9*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c
^3*d^4)*x^6 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^4
 + (a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*x^2) - 1/2*(b*c + 3*
a*d)*log(x^2)/(a^2*c^4)
```

Fricas [A] time = 8.59421, size = 864, normalized size = 4.85

$$-\frac{2\,ab^3c^6-6\,a^2b^2c^5d+6\,a^3bc^4d^2-2\,a^4c^3d^3+2\left(ab^3c^4d^2-6\,a^2b^2c^3d^3+8\,a^3bc^2d^4-3\,a^4cd^5\right)x^4+\left(4\,ab^3c^5d-19\,a^2b^2c^4d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b^3*c^6 - 6*a^2*b^2*c^5*d + 6*a^3*b*c^4*d^2 - 2*a^4*c^3
*d^3 + 2*(a*b^3*c^4*d^2 - 6*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 - 3
*a^4*c*d^5)*x^4 + (4*a*b^3*c^5*d - 19*a^2*b^2*c^4*d^2 + 24*a^3*b*
c^3*d^3 - 9*a^4*c^2*d^4)*x^2 - 2*(b^4*c^4*d^2*x^6 + 2*b^4*c^5*d*x
^4 + b^4*c^6*x^2)*log(b*x^2 + a) + 2*((6*a^2*b^2*c^2*d^4 - 8*a^3*
b*c*d^5 + 3*a^4*d^6)*x^6 + 2*(6*a^2*b^2*c^3*d^3 - 8*a^3*b*c^2*d^4
 + 3*a^4*c*d^5)*x^4 + (6*a^2*b^2*c^4*d^2 - 8*a^3*b*c^3*d^3 + 3*a^
4*c^2*d^4)*x^2)*log(d*x^2 + c) + 4*((b^4*c^4*d^2 - 6*a^2*b^2*c^2*
d^4 + 8*a^3*b*c*d^5 - 3*a^4*d^6)*x^6 + 2*(b^4*c^5*d - 6*a^2*b^2*c
^3*d^3 + 8*a^3*b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (b^4*c^6 - 6*a^2*b^
2*c^4*d^2 + 8*a^3*b*c^3*d^3 - 3*a^4*c^2*d^4)*x^2)*log(x))/((a^2*b
^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x
^6 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5
*c^5*d^4)*x^4 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2
- a^5*c^6*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.253661, size = 482, normalized size = 2.71

$$\frac{b^5 \ln\left(\left|bx^2+a\right|\right)}{2\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)}-\frac{\left(6\,b^2c^2d^3-8\,abcd^4+3\,a^2d^5\right)\ln\left(\left|dx^2+c\right|\right)}{2\left(b^3c^7d-3\,ab^2c^6d^2+3\,a^2bc^5d^3-a^3c^4d^4\right)}+\frac{18\,b^2c^2d^4x^4-24\,abcd^5x^4+9\,a^2d^6x^4+42\,b^2c^3d^3x^2-58\,abc^2d^4x^2+22\,a^2cd^5x^2+25\,b^2c^4d^2-36\,abc^3d^3+14\,a^2c^2d^4}{4\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3\right)\left(dx^2+c\right)^2}-\frac{\left(bc+3\,ad\right)\ln\left(x^2\right)}{2\,a^2c^4}+\frac{bcx^2+3\,adx^2-ac}{2\,a^2c^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^3),x, algorithm="giac")
```

```
[Out] 1/2*b^5*ln(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4
*b^2*c*d^2 - a^5*b*d^3) - 1/2*(6*b^2*c^2*d^3 - 8*a*b*c*d^4 + 3*a^
2*d^5)*ln(abs(d*x^2 + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*
c^5*d^3 - a^3*c^4*d^4) + 1/4*(18*b^2*c^2*d^4*x^4 - 24*a*b*c*d^5*x
^4 + 9*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^2 - 58*a*b*c^2*d^4*x^2 + 22
*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 36*a*b*c^3*d^3 + 14*a^2*c^2*d^4
)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x
^2 + c)^2) - 1/2*(b*c + 3*a*d)*ln(x^2)/(a^2*c^4) + 1/2*(b*c*x^2 +
 3*a*d*x^2 - a*c)/(a^2*c^4*x^2)
```

3.260 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=270

$$\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{35a^3d^3-55a^2bcd^2+8ab^2c^2d+8b^3c^3}{8a^2c^4x(bc-ad)^2}-\frac{d(11bc-7ad)}{8c^2x^3\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(8*b^2*c^2 - 55*a*b*c*d + 35*a^2*d^2)/(24*a*c^3*(b*c - a*d)^2*x^
3) + (8*b^3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(8
*a^2*c^4*(b*c - a*d)^2*x) - d/(4*c*(b*c - a*d)*x^3*(c + d*x^2)^2)
 - (d*(11*b*c - 7*a*d))/(8*c^2*(b*c - a*d)^2*x^3*(c + d*x^2)) + (
b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^3) - (d
^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Rubi [A] time = 1.20236, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{35a^3d^3-55a^2bcd^2+8ab^2c^2d+8b^3c^3}{8a^2c^4x(bc-ad)^2}-\frac{d(11bc-7ad)}{8c^2x^3\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```


```
[Out] -(8*b^2*c^2 - 55*a*b*c*d + 35*a^2*d^2)/(24*a*c^3*(b*c - a*d)^2*x^
3) + (8*b^3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(8
*a^2*c^4*(b*c - a*d)^2*x) - d/(4*c*(b*c - a*d)*x^3*(c + d*x^2)^2)
 - (d*(11*b*c - 7*a*d))/(8*c^2*(b*c - a*d)^2*x^3*(c + d*x^2)) + (
b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^3) - (d
^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.81582, size = 196, normalized size = 0.73

$$-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}+\frac{3ad+bc}{a^2c^4x}-\frac{d^3x(15bc-11ad)}{8c^4\left(c+dx^2\right)(bc-ad)^2}-\frac{d^3x}{4c^3\left(c+dx^2\right)^2(bc-ad)}-\frac{1}{3ac^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(3*a*c^3*x^3) + (b*c + 3*a*d)/(a^2*c^4*x) - (d^3*x)/(4*c^3*(b*
c - a*d)*(c + d*x^2)^2) - (d^3*(15*b*c - 11*a*d)*x)/(8*c^4*(b*c -
 a*d)^2*(c + d*x^2)) - (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(
5/2)*(-(b*c) + a*d)^3) - (d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a
^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Maple [A] time = 0.028, size = 362, normalized size = 1.3

$$-\frac{1}{3\,ac^3x^3}+3\,\frac{d}{axc^4}+\frac{b}{a^2c^3x}+\frac{11\,d^6x^3a^2}{8\,c^4\,(ad-bc)^3\,(dx^2+c)^2}-\frac{13\,d^5x^3ab}{4\,c^3\,(ad-bc)^3\,(dx^2+c)^2}+\frac{15\,d^4x^3b^2}{8\,c^2\,(ad-bc)^3\,(dx^2+c)^2}+\frac{13\,d^5xa^2}{8\,c^3\,(ad-bc)^3\,(dx^2+c)^2}-\frac{15\,d^4xab}{4\,c^2\,(ad-bc)^3\,(dx^2+c)^2}+\frac{17\,d^3xb^2}{8\,c\,(ad-bc)^3\,(dx^2+c)^2}+\frac{35\,a^2d^5}{8\,c^4\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{45\,abd^4}{4\,c^3\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{63\,d^3b^2}{8\,c^2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^5}{a^2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/3/a/c^3/x^3+3/x/a/c^4*d+1/x/a^2/c^3*b+11/8*d^6/c^4/(a*d-b*c)^3
/(d*x^2+c)^2*x^3*a^2-13/4*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*b
+15/8*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+13/8*d^5/c^3/(a*d-b
*c)^3/(d*x^2+c)^2*x*a^2-15/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x*a*
b+17/8*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x*b^2+35/8*d^5/c^4/(a*d-b*c)
^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-45/4*d^4/c^3/(a*d-b*c)
^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+63/8*d^3/c^2/(a*d-b*c)
^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/a^2*b^5/(a*d-b*c)^3/
(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.28237, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^4),x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^2 - 16*a
^4*c^3*d^3 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d
^5 - 35*a^4*d^6)*x^6 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^
2*b^2*c^3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^
```

```
4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7
*a^4*c^2*d^4)*x^2 + 24*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c
^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2
*(63*a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63
*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-
d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c
^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 +
 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5
*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^
5*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b
*c^4*d^2 - 8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4
+ 90*a^3*b*c*d^5 - 35*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*
d^2 - 315*a^2*b^2*c^3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^
4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*
c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*
c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^
4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d^3 +
35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + 12*(b^
4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*
x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^3*c^7*d^2 - 3*a
^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*
c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^5 +
(a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x
^3), -1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^2 -
16*a^4*c^3*d^3 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b
*c*d^5 - 35*a^4*d^6)*x^6 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 31
5*a^2*b^2*c^3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 16*(
3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3
 - 7*a^4*c^2*d^4)*x^2 - 48*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b
^4*c^6*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 3*((63*a^2*b^2*
c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^
3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 -
90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2
*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*
c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d -
 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^
3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^3), -1
/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b*c^4*d^2 - 8*a^4*c^
3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 -
35*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c
^3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 -
2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2
*d^4)*x^2 - 24*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*
sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 3*((63*a^2*b^2*c^2*d^4 - 90
*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 - 90*a^3*b
*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3
*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/
((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4
*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^
3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c
^7*d^2 - a^5*c^6*d^3)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239924, size = 346, normalized size = 1.28

$$\frac{b^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)\sqrt{ab}} - \frac{\left(63\,b^2c^2d^3 - 90\,abcd^4 + 35\,a^2d^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3)\sqrt{cd}} - \frac{15\,bcd^4x^3 - 11\,ad^5x^3 + 17\,bc^2d^3x - 13\,acd^4x}{8\,(b^2c^6 - 2\,abc^5d + a^2c^4d^2)(dx^2 + c)^2} + \frac{3\,bcx^2 + 9\,adx^2 - ac}{3\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^4),x, algorithm="giac")
```

```
[Out] b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4
*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d
^4 + 35*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d
+ 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*sqrt(c*d)) - 1/8*(15*b*c*d^4*x^3
 - 11*a*d^5*x^3 + 17*b*c^2*d^3*x - 13*a*c*d^4*x)/((b^2*c^6 - 2*a*
b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 1/3*(3*b*c*x^2 + 9*a*d*x^
2 - a*c)/(a^2*c^4*x^3)
```

3.261 $\int \frac{x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi [A] time = 0.0377833, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x^2)*(4 + x^2)),x]

[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi in Sympy [A] time = 6.70731, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+1)/(x**2+4),x)

[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

Mathematica [A] time = 0.00697595, size = 21, normalized size = 1.

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x^2)*(4 + x^2)),x]

[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] time = 0.009, size = 18, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+1)/(x^2+4),x)

[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A] time = 1.33336, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 1)),x, algorithm="maxima")

[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A] time = 0.222726, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 1)),x, algorithm="fricas")

[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.205388, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+1)/(x**2+4),x)

[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

GIAC/XCAS [A] time = 0.23364, size = 23, normalized size = 1.1

$$-\frac{1}{6}\ln\left(x^2+4\right)+\frac{1}{6}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 1)),x, algorithm="giac")

[Out] -1/6*ln(x^2 + 4) + 1/6*ln(x^2 + 1)
```

3.262 $\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(bc-ad)}{2b^3(a+bx^2)}+\frac{x(bc-2ad)}{b^3}+\frac{dx^3}{3b^2}$$

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + (a*(b*c - a*d)*x)/(2*b^
3*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.179617, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\sqrt{a}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(bc-ad)}{2b^3(a+bx^2)}+\frac{x(bc-2ad)}{b^3}+\frac{dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + (a*(b*c - a*d)*x)/(2*b^
3*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*b^(7/2))
```

Rubi in Sympy [A] time = 42.2409, size = 80, normalized size = 0.92

$$\frac{\sqrt{a}\,(5ad-3bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{7}{2}}}-\frac{ax\,(ad-bc)}{2b^3\,(a+bx^2)}+\frac{dx^3}{3b^2}-\frac{x\,(2ad-bc)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] sqrt(a)*(5*a*d - 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(2*b**(7/2)) - a*
x*(a*d - b*c)/(2*b**3*(a + b*x**2)) + d*x**3/(3*b**2) - x*(2*a*d
- b*c)/b**3
```

Mathematica [A] time = 0.119615, size = 89, normalized size = 1.02

$$\frac{x\left(abc-a^2d\right)}{2b^3(a+bx^2)}+\frac{\sqrt{a}(5ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(bc-2ad)}{b^3}+\frac{dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + ((a*b*c - a^2*d)*x)/(2*
b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.013, size = 105, normalized size = 1.2

$$\frac{dx^3}{3b^2} - 2\frac{adx}{b^3} + \frac{cx}{b^2} - \frac{xa^2d}{2b^3(bx^2+a)} + \frac{acx}{2b^2(bx^2+a)} + \frac{5a^2d}{2b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{3ac}{2b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d*x^3/b^2-2/b^3*a*d*x+1/b^2*x*c-1/2*a^2/b^3*x/(b*x^2+a)*d+1/2
*a/b^2*x/(b*x^2+a)*c+5/2*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*d-3/2*a/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234294, size = 1, normalized size = 0.01

$$\left[\frac{4b^2dx^5 + 4\left(3b^2c - 5abd\right)x^3 - 3\left(3abc - 5a^2d + \left(3b^2c - 5abd\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\left(3abc - 5a^2d\right)x}{12\left(b^4x^2 + ab^3\right)}, \frac{2b^2 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*d*x^5 + 4*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a
^2*d + (3*b^2*c - 5*a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqr
t(-a/b) - a)/(b*x^2 + a)) + 6*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a
*b^3), 1/6*(2*b^2*d*x^5 + 2*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c
- 5*a^2*d + (3*b^2*c - 5*a*b*d)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)
) + 3*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 3.07707, size = 128, normalized size = 1.47

$$-\frac{x\left(a^2d - abc\right)}{2ab^3 + 2b^4x^2} - \frac{\sqrt{-\frac{a}{b^7}}\,(5ad - 3bc)\log\left(-b^3\sqrt{-\frac{a}{b^7}} + x\right)}{4} + \frac{\sqrt{-\frac{a}{b^7}}\,(5ad - 3bc)\log\left(b^3\sqrt{-\frac{a}{b^7}} + x\right)}{4} + \frac{dx^3}{3b^2} - \frac{x\left(2ad - bc\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**2*d - a*b*c)/(2*a*b**3 + 2*b**4*x**2) - sqrt(-a/b**7)*(5*a
*d - 3*b*c)*log(-b**3*sqrt(-a/b**7) + x)/4 + sqrt(-a/b**7)*(5*a*d
 - 3*b*c)*log(b**3*sqrt(-a/b**7) + x)/4 + d*x**3/(3*b**2) - x*(2*
a*d - b*c)/b**3
```

GIAC/XCAS [A] time = 0.235402, size = 119, normalized size = 1.37

$$-\frac{\left(3\,abc - 5\,a^{2}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^{3}} + \frac{abcx - a^{2}dx}{2\,(bx^{2} + a)b^{3}} + \frac{b^{4}dx^{3} + 3\,b^{4}cx - 6\,ab^{3}dx}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c - 5*a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
1/2*(a*b*c*x - a^2*d*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d*x^3 + 3*b^
4*c*x - 6*a*b^3*d*x)/b^6
```

3.263 $\int \frac{x^3(c+dx^2)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=60

$$\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}+\frac{dx^2}{2b^2}$$

```
[Out] (d*x^2)/(2*b^2) + (a*(b*c - a*d))/(2*b^3*(a + b*x^2)) + ((b*c - 2
*a*d)*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] time = 0.150472, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}+\frac{dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x^2)/(2*b^2) + (a*(b*c - a*d))/(2*b^3*(a + b*x^2)) + ((b*c - 2
*a*d)*Log[a + b*x^2])/(2*b^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a(ad-bc)}{2b^3(a+bx^2)}+\frac{\int^{x^2} d\,dx}{2b^2}-\frac{(2ad-bc)\log(a+bx^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -a*(a*d - b*c)/(2*b**3*(a + b*x**2)) + Integral(d, (x, x**2))/(2*
b**2) - (2*a*d - b*c)*log(a + b*x**2)/(2*b**3)
```

Mathematica [A] time = 0.0582615, size = 50, normalized size = 0.83

$$\frac{\frac{a(bc-ad)}{a+bx^2}+(bc-2ad)\log(a+bx^2)+bdx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*d*x^2 + (a*(b*c - a*d))/(a + b*x^2) + (b*c - 2*a*d)*Log[a + b*
x^2])/(2*b^3)
```

Maple [A] time = 0.015, size = 74, normalized size = 1.2

$$\frac{dx^2}{2\,b^2}-\frac{\ln(bx^2+a)\,ad}{b^3}+\frac{c\ln(bx^2+a)}{2\,b^2}-\frac{a^2d}{2\,b^3(bx^2+a)}+\frac{ac}{2\,b^2(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d*x^2/b^2-1/b^3*ln(b*x^2+a)*a*d+1/2*c*ln(b*x^2+a)/b^2-1/2/b^3
*a^2/(b*x^2+a)*d+1/2*a*c/b^2/(b*x^2+a)
```

Maxima [A] time = 1.33961, size = 80, normalized size = 1.33

$$\frac{dx^2}{2\,b^2} + \frac{abc - a^2d}{2\,(b^4x^2 + ab^3)} + \frac{(bc - 2\,ad)\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b^2 + 1/2*(a*b*c - a^2*d)/(b^4*x^2 + a*b^3) + 1/2*(b*c
- 2*a*d)*log(b*x^2 + a)/b^3
```

Fricas [A] time = 0.216687, size = 105, normalized size = 1.75

$$\frac{b^2dx^4 + abdx^2 + abc - a^2d + \left(abc - 2\,a^2d + \left(b^2c - 2\,abd\right)x^2\right)\log\left(bx^2 + a\right)}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^4 + a*b*d*x^2 + a*b*c - a^2*d + (a*b*c - 2*a^2*d + (
b^2*c - 2*a*b*d)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 2.76607, size = 56, normalized size = 0.93

$$-\frac{a^2d - abc}{2ab^3 + 2b^4x^2} + \frac{dx^2}{2b^2} - \frac{(2ad - bc)\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -(a**2*d - a*b*c)/(2*a*b**3 + 2*b**4*x**2) + d*x**2/(2*b**2) - (2
*a*d - b*c)*log(a + b*x**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.23864, size = 122, normalized size = 2.03

$$\frac{\frac{(bx^2+a)\,d}{b^2} - \frac{(bc-2\,ad)\ln\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^2} + \frac{\frac{ab^2c}{bx^2+a} - \frac{a^2bd}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*d/b^2 - (b*c - 2*a*d)*ln(abs(b*x^2 + a)/((b*x^2
+ a)^2*abs(b)))/b^2 + (a*b^2*c/(b*x^2 + a) - a^2*b*d/(b*x^2 + a))
/b^3)/b
```

3.264 $\int \frac{x^2(c+dx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=67

$$\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(bc-ad)}{2b^2(a+bx^2)}+\frac{dx}{b^2}$$

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) + ((b*c - 3*a*d)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.131405, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(bc-ad)}{2b^2(a+bx^2)}+\frac{dx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) + ((b*c - 3*a*d)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rubi in Sympy [A] time = 24.151, size = 60, normalized size = 0.9

$$\frac{dx}{b^2}+\frac{x(ad-bc)}{2b^2(a+bx^2)}-\frac{(3ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] d*x/b**2 + x*(a*d - b*c)/(2*b**2*(a + b*x**2)) - (3*a*d - b*c)*at
an(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b**(5/2))
```

Mathematica [A] time = 0.11306, size = 68, normalized size = 1.01

$$-\frac{(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(bc-ad)}{2b^2(a+bx^2)}+\frac{dx}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) - ((-(b*c) + 3*a*
d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.012, size = 82, normalized size = 1.2

$$\frac{dx}{b^2}+\frac{axd}{2\,b^2\,(bx^2+a)}-\frac{cx}{2\,b\,(bx^2+a)}-\frac{3\,ad}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] d*x/b^2+1/2/b^2*x/(b*x^2+a)*a*d-1/2*c*x/b/(b*x^2+a)-3/2/b^2/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+1/2*c/b/(a*b)^(1/2)*arctan(x*b
/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227732, size = 1, normalized size = 0.01

$$\left[-\frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(2\,bdx^3-\left(bc-3\,ad\right)x\right)\sqrt{-ab}}{4\left(b^3x^2+ab^2\right)\sqrt{-ab}}, \frac{\left(abc-3\,a^2d+\left(b^2c-3\,ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*log(-(2*a*b*x -
(b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(2*b*d*x^3 - (b*c - 3*a*
d)*x)*sqrt(-a*b))/((b^3*x^2 + a*b^2)*sqrt(-a*b)), 1/2*((a*b*c - 3
*a^2*d + (b^2*c - 3*a*b*d)*x^2)*arctan(sqrt(a*b)*x/a) + (2*b*d*x^
3 - (b*c - 3*a*d)*x)*sqrt(a*b))/((b^3*x^2 + a*b^2)*sqrt(a*b))]
```

Sympy [A] time = 2.58635, size = 114, normalized size = 1.7

$$\frac{x\left(ad-bc\right)}{2ab^2+2b^3x^2}+\frac{\sqrt{-\frac{1}{ab^5}}\left(3ad-bc\right)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^5}}\left(3ad-bc\right)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}+\frac{dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(3*a*d
 - b*c)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*
(3*a*d - b*c)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/4 + d*x/b**2
```

GIAC/XCAS [A] time = 0.248684, size = 78, normalized size = 1.16

$$\frac{dx}{b^2}+\frac{\left(bc-3\,ad\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}-\frac{bcx-adx}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
 - 1/2*(b*c*x - a*d*x)/((b*x^2 + a)*b^2)
```

3.265 $\int \frac{x(c+dx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=41

$$\frac{d\log(a+bx^2)}{2b^2} - \frac{bc-ad}{2b^2(a+bx^2)}$$

```
[Out] -(b*c - a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] time = 0.0879048, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{d\log(a+bx^2)}{2b^2} - \frac{bc-ad}{2b^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2))/(a + b*x^2)^2,x]

[Out] -(b*c - a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Rubi in Sympy [A] time = 14.0521, size = 32, normalized size = 0.78

$$\frac{d\log(a+bx^2)}{2b^2} + \frac{ad-bc}{2b^2(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)/(b*x**2+a)**2,x)

[Out] d*log(a + b*x**2)/(2*b**2) + (a*d - b*c)/(2*b**2*(a + b*x**2))
```

Mathematica [A] time = 0.01979, size = 41, normalized size = 1.

$$\frac{ad-bc}{2b^2(a+bx^2)} + \frac{d\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2))/(a + b*x^2)^2,x]

[Out] (-(b*c) + a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.013, size = 47, normalized size = 1.2

$$\frac{d\ln(bx^2+a)}{2\,b^2} + \frac{ad}{(2\,bx^2+2\,a)\,b^2} - \frac{c}{2\,b\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b^2+1/2/(b*x^2+a)/b^2*a*d-1/2*c/b/(b*x^2+a)
```

Maxima [A] time = 1.3458, size = 54, normalized size = 1.32

$$-\frac{bc-ad}{2\,(b^3x^2+ab^2)}+\frac{d\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)/(b^3*x^2 + a*b^2) + 1/2*d*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.22004, size = 61, normalized size = 1.49

$$-\frac{bc-ad-\left(bdx^2+ad\right)\log\left(bx^2+a\right)}{2\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d - (b*d*x^2 + a*d)*log(b*x^2 + a))/(b^3*x^2 + a*b^
2)
```

Sympy [A] time = 2.03878, size = 36, normalized size = 0.88

$$\frac{ad-bc}{2ab^2+2b^3x^2}+\frac{d\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] (a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + d*log(a + b*x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.233476, size = 88, normalized size = 2.15

$$-\frac{d\left(\frac{\ln\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2\,b}-\frac{c}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d*(ln(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 +
 a)*b))/b - 1/2*c/((b*x^2 + a)*b)
```

3.266 $\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\,(a+bx^2)}$$

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0634542, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi in Sympy [A] time = 9.73318, size = 51, normalized size = 0.81

$$-\frac{x\,(ad-bc)}{2ab\,(a+bx^2)}+\frac{(ad+bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(2*a*b*(a + b*x**2)) + (a*d + b*c)*atan(sqrt(b)*x/
sqrt(a))/(2*a**(3/2)*b**(3/2))
```

Mathematica [A] time = 0.0729257, size = 63, normalized size = 1.

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(ad-bc)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -((-(b*c) + a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.001, size = 68, normalized size = 1.1

$$-\frac{(ad-bc)\,x}{2\,ab\,(bx^2+a)}+\frac{d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*d+1/2*c/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.225961, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{-ab}(bc-ad)x+\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)}{4\left(ab^2x^2+a^2b\right)\sqrt{-ab}}, \frac{\sqrt{ab}(bc-ad)x+\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)}{2\left(ab^2x^2+a^2b\right)\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b)*(b*c - a*d)*x + (a*b*c + a^2*d + (b^2*c + a*b*
d)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)))/((a*
b^2*x^2 + a^2*b)*sqrt(-a*b)), 1/2*(sqrt(a*b)*(b*c - a*d)*x + (a*b
*c + a^2*d + (b^2*c + a*b*d)*x^2)*arctan(sqrt(a*b)*x/a))/((a*b^2*
x^2 + a^2*b)*sqrt(a*b))]
```

Sympy [A] time = 2.11197, size = 112, normalized size = 1.78

$$-\frac{x\left(ad-bc\right)}{2a^2b+2ab^2x^2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad+bc\right)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad+bc\right)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*
(a*d + b*c)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a*
*3*b**3))*(a*d + b*c)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

GIAC/XCAS [A] time = 0.252562, size = 77, normalized size = 1.22

$$\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}+\frac{bcx-adx}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*
x - a*d*x)/((b*x^2 + a)*a*b)
```

3.267 $\int \frac{c+dx^2}{x(a+bx^2)^2} \, dx$

Optimal. Leaf size=51

$$-\frac{c\log(a+bx^2)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab(a+bx^2)}$$

```
[Out] (b*c - a*d)/(2*a*b*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x
^2])/(2*a^2)
```

Rubi [A] time = 0.113667, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{c\log(a+bx^2)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)/(2*a*b*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x
^2])/(2*a^2)
```

Rubi in Sympy [A] time = 16.6897, size = 44, normalized size = 0.86

$$-\frac{ad-bc}{2ab(a+bx^2)}+\frac{c\log(x^2)}{2a^2}-\frac{c\log(a+bx^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/x/(b*x**2+a)**2,x)
```

```
[Out] -(a*d - b*c)/(2*a*b*(a + b*x**2)) + c*log(x**2)/(2*a**2) - c*log(
a + b*x**2)/(2*a**2)
```

Mathematica [A] time = 0.0488924, size = 46, normalized size = 0.9

$$\frac{\frac{a(bc-ad)}{b(a+bx^2)}-c\log(a+bx^2)+2c\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(b*(a + b*x^2)) + 2*c*Log[x] - c*Log[a + b*x^2])
/(2*a^2)
```

Maple [A] time = 0.017, size = 53, normalized size = 1.

$$\frac{c\ln(x)}{a^2}-\frac{c\ln(bx^2+a)}{2a^2}-\frac{d}{2b(bx^2+a)}+\frac{c}{2a(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x/(b*x^2+a)^2,x)
```

```
[Out] c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2-1/2/b/(b*x^2+a)*d+1/2*c/a/(b*x^
2+a)
```

Maxima [A] time = 1.34692, size = 69, normalized size = 1.35

$$\frac{bc - ad}{2\left(ab^2x^2 + a^2b\right)} - \frac{c\log\left(bx^2 + a\right)}{2\,a^2} + \frac{c\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)/(a*b^2*x^2 + a^2*b) - 1/2*c*log(b*x^2 + a)/a^2 +
1/2*c*log(x^2)/a^2
```

Fricas [A] time = 0.231858, size = 96, normalized size = 1.88

$$\frac{abc - a^2d - \left(b^2cx^2 + abc\right)\log\left(bx^2 + a\right) + 2\left(b^2cx^2 + abc\right)\log\left(x\right)}{2\left(a^2b^2x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d - (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) + 2*(b^2*
c*x^2 + a*b*c)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

Sympy [A] time = 2.2005, size = 46, normalized size = 0.9

$$-\frac{ad - bc}{2a^2b + 2ab^2x^2} + \frac{c\log\left(x\right)}{a^2} - \frac{c\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x/(b*x**2+a)**2,x)
```

```
[Out] -(a*d - b*c)/(2*a**2*b + 2*a*b**2*x**2) + c*log(x)/a**2 - c*log(a
/b + x**2)/(2*a**2)
```

GIAC/XCAS [A] time = 0.251333, size = 85, normalized size = 1.67

$$\frac{c\ln\left(x^2\right)}{2\,a^2} - \frac{c\ln\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{b^2cx^2 + 2\,abc - a^2d}{2\left(bx^2 + a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*c*ln(x^2)/a^2 - 1/2*c*ln(abs(b*x^2 + a))/a^2 + 1/2*(b^2*c*x^2
 + 2*a*b*c - a^2*d)/((b*x^2 + a)*a^2*b)
```

3.268 $\int \frac{c+dx^2}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(bc-ad)}{2a^2\left(a+bx^2\right)}-\frac{c}{a^2x}$$

```
[Out] -(c/(a^2*x)) - ((b*c - a*d)*x)/(2*a^2*(a + b*x^2)) - ((3*b*c - a*
d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.145078, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(bc-ad)}{2a^2\left(a+bx^2\right)}-\frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) - ((b*c - a*d)*x)/(2*a^2*(a + b*x^2)) - ((3*b*c - a*
d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 20.0822, size = 60, normalized size = 0.85

$$-\frac{c}{a^2x}+\frac{x\,(ad-bc)}{2a^2\left(a+bx^2\right)}+\frac{(ad-3bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -c/(a**2*x) + x*(a*d - b*c)/(2*a**2*(a + b*x**2)) + (a*d - 3*b*c)
*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/2)*sqrt(b))
```

Mathematica [A] time = 0.0553049, size = 70, normalized size = 0.99

$$\frac{(ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{x(ad-bc)}{2a^2\left(a+bx^2\right)}-\frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + ((-(b*c) + a*d)*x)/(2*a^2*(a + b*x^2)) + ((-3*b*c
+ a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Maple [A] time = 0.014, size = 85, normalized size = 1.2

$$-\frac{c}{a^2x}+\frac{dx}{2\,a\,(bx^2+a)}-\frac{bcx}{2\,a^2\,(bx^2+a)}+\frac{d}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,bc}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c/a^2/x+1/2/a*x/(b*x^2+a)*d-1/2*c*b/a^2*x/(b*x^2+a)+1/2/a/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))*d-3/2*c*b/a^2/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241225, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(3\,b^2c-abd\right)x^3+\left(3\,abc-a^2d\right)x\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\,\left((3\,bc-ad)x^2+2\,ac\right)\sqrt{-ab}}{4\,(a^2bx^3+a^3x)\sqrt{-ab}},\ -\frac{\left(\left(3\,b^2c-abd\right)x^3+\left(3\,abc-a^2d\right)x\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left((3\,bc-ad)x^2+2\,ac\right)\sqrt{ab}}{2\,(a^2bx^3+a^3x)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b^2*c - a*b*d)*x^3 + (3*a*b*c - a^2*d)*x)*log((2*a*b*x
 + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*((3*b*c - a*d)*x^2 +
2*a*c)*sqrt(-a*b))/((a^2*b*x^3 + a^3*x)*sqrt(-a*b)), -1/2*(((3*b^
2*c - a*b*d)*x^3 + (3*a*b*c - a^2*d)*x)*arctan(sqrt(a*b)*x/a) + (
(3*b*c - a*d)*x^2 + 2*a*c)*sqrt(a*b))/((a^2*b*x^3 + a^3*x)*sqrt(a
*b))]
```

Sympy [A] time = 2.49772, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2ac+x^2\,(ad-3bc)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - 3*b*c)*log(-a**3*sqrt(-1/(a**5*b)) + x)
/4 + sqrt(-1/(a**5*b))*(a*d - 3*b*c)*log(a**3*sqrt(-1/(a**5*b)) +
 x)/4 + (-2*a*c + x**2*(a*d - 3*b*c))/(2*a**3*x + 2*a**2*b*x**3)
```

GIAC/XCAS [A] time = 0.254733, size = 86, normalized size = 1.21

$$-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{3\,bcx^2-adx^2+2\,ac}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3
*b*c*x^2 - a*d*x^2 + 2*a*c)/((b*x^3 + a*x)*a^2)
```

3.269 $\int \frac{c+dx^2}{x^3(a+bx^2)^2} dx$

Optimal. Leaf size=76

$$\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{bc-ad}{2a^2\left(a+bx^2\right)}-\frac{c}{2a^2x^2}$$

```
[Out] -c/(2*a^2*x^2) - (b*c - a*d)/(2*a^2*(a + b*x^2)) - ((2*b*c - a*d)
*Log[x])/a^3 + ((2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Rubi [A] time = 0.177952, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{bc-ad}{2a^2\left(a+bx^2\right)}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - (b*c - a*d)/(2*a^2*(a + b*x^2)) - ((2*b*c - a*d)
*Log[x])/a^3 + ((2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 22.2363, size = 68, normalized size = 0.89

$$-\frac{c}{2a^2x^2}+\frac{ad-bc}{2a^2\left(a+bx^2\right)}+\frac{(ad-2bc)\log\left(x^2\right)}{2a^3}-\frac{(ad-2bc)\log\left(a+bx^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] -c/(2*a**2*x**2) + (a*d - b*c)/(2*a**2*(a + b*x**2)) + (a*d - 2*b
*c)*log(x**2)/(2*a**3) - (a*d - 2*b*c)*log(a + b*x**2)/(2*a**3)
```

Mathematica [A] time = 0.0833559, size = 64, normalized size = 0.84

$$\frac{\frac{a(ad-bc)}{a+bx^2}+(2bc-ad)\log\left(a+bx^2\right)+2\log(x)(ad-2bc)-\frac{ac}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c)/x^2) + (a*(-(b*c) + a*d))/(a + b*x^2) + 2*(-2*b*c + a*d)
*Log[x] + (2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.02, size = 86, normalized size = 1.1

$$-\frac{c}{2\,a^2x^2}+\frac{\ln(x)\,d}{a^2}-2\,\frac{bc\ln(x)}{a^3}-\frac{\ln\left(bx^2+a\right)d}{2\,a^2}+\frac{bc\ln\left(bx^2+a\right)}{a^3}+\frac{d}{2\,a\,(bx^2+a)}-\frac{bc}{2\,a^2\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c/a^2/x^2+1/a^2*ln(x)*d-2*b*c*ln(x)/a^3-1/2/a^2*ln(b*x^2+a)*
d+b*c*ln(b*x^2+a)/a^3+1/2/a/(b*x^2+a)*d-1/2*b*c/a^2/(b*x^2+a)
```

Maxima [A] time = 1.34356, size = 105, normalized size = 1.38

$$-\frac{(2\,bc-ad)x^2+ac}{2\,(a^2bx^4+a^3x^2)}+\frac{(2\,bc-ad)\log\left(bx^2+a\right)}{2\,a^3}-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*c - a*d)*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + 1/2*(2*b*c
 - a*d)*log(b*x^2 + a)/a^3 - 1/2*(2*b*c - a*d)*log(x^2)/a^3
```

Fricas [A] time = 0.231165, size = 165, normalized size = 2.17

$$-\frac{a^2c+\left(2\,abc-a^2d\right)x^2-\left(\left(2\,b^2c-abd\right)x^4+\left(2\,abc-a^2d\right)x^2\right)\log\left(bx^2+a\right)+2\left(\left(2\,b^2c-abd\right)x^4+\left(2\,abc-a^2d\right)x^2\right)\log(}{2\,(a^3bx^4+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c + (2*a*b*c - a^2*d)*x^2 - ((2*b^2*c - a*b*d)*x^4 + (2
*a*b*c - a^2*d)*x^2)*log(b*x^2 + a) + 2*((2*b^2*c - a*b*d)*x^4 +
(2*a*b*c - a^2*d)*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 3.86831, size = 70, normalized size = 0.92

$$\frac{-ac+x^2\,(ad-2bc)}{2a^3x^2+2a^2bx^4}+\frac{(ad-2bc)\log{(x)}}{a^3}-\frac{(ad-2bc)\log{\left(\frac{a}{b}+x^2\right)}}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*c + x**2*(a*d - 2*b*c))/(2*a**3*x**2 + 2*a**2*b*x**4) + (a*d
- 2*b*c)*log(x)/a**3 - (a*d - 2*b*c)*log(a/b + x**2)/(2*a**3)
```

GIAC/XCAS [A] time = 0.23943, size = 113, normalized size = 1.49

$$-\frac{(2\,bc-ad)\ln\left(x^2\right)}{2\,a^3}-\frac{2\,bcx^2-adx^2+ac}{2\,(bx^4+ax^2)a^2}+\frac{\left(2\,b^2c-abd\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c - a*d)*ln(x^2)/a^3 - 1/2*(2*b*c*x^2 - a*d*x^2 + a*c)/
((b*x^4 + a*x^2)*a^2) + 1/2*(2*b^2*c - a*b*d)*ln(abs(b*x^2 + a))/
(a^3*b)
```

3.270 $\int \frac{c+dx^2}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{b}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(bc-ad)}{2a^3(a+bx^2)}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (b*(b*c - a*d)*x)/(2*a^3
*(a + b*x^2)) + (Sqrt[b]*(5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2))
```

Rubi [A] time = 0.261874, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\sqrt{b}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(bc-ad)}{2a^3(a+bx^2)}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (b*(b*c - a*d)*x)/(2*a^3
*(a + b*x^2)) + (Sqrt[b]*(5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2))
```

Rubi in Sympy [A] time = 40.5816, size = 82, normalized size = 0.91

$$-\frac{c}{3a^2x^3}-\frac{bx(ad-bc)}{2a^3(a+bx^2)}-\frac{ad-2bc}{a^3x}-\frac{\sqrt{b}(3ad-5bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -c/(3*a**2*x**3) - b*x*(a*d - b*c)/(2*a**3*(a + b*x**2)) - (a*d -
 2*b*c)/(a**3*x) - sqrt(b)*(3*a*d - 5*b*c)*atan(sqrt(b)*x/sqrt(a)
)/(2*a**(7/2))
```

Mathematica [A] time = 0.117096, size = 90, normalized size = 1.

$$-\frac{\sqrt{b}(3ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{bx(ad-bc)}{2a^3(a+bx^2)}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) - (b*(-(b*c) + a*d)*x)/(2*
a^3*(a + b*x^2)) - (Sqrt[b]*(-5*b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.016, size = 110, normalized size = 1.2

$$-\frac{c}{3\,a^2x^3}-\frac{d}{a^2x}+2\,\frac{bc}{a^3x}-\frac{bxd}{2\,a^2\,(bx^2+a)}+\frac{xb^2c}{2\,a^3\,(bx^2+a)}-\frac{3\,bd}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^2c}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c/a^2/x^3-1/a^2/x*d+2/a^3/x*b*c-1/2/a^2*b*x/(b*x^2+a)*d+1/2/
a^3*b^2*x/(b*x^2+a)*c-3/2/a^2*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2
))*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.246263, size = 1, normalized size = 0.01

$$\left[\frac{6\left(5\,b^2c-3\,abd\right)x^4-4\,a^2c+4\left(5\,abc-3\,a^2d\right)x^2-3\left(\left(5\,b^2c-3\,abd\right)x^5+\left(5\,abc-3\,a^2d\right)x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{12\left(a^3bx^5+a^4x^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(6*(5*b^2*c - 3*a*b*d)*x^4 - 4*a^2*c + 4*(5*a*b*c - 3*a^2*d
)*x^2 - 3*((5*b^2*c - 3*a*b*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqr
t(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x
^5 + a^4*x^3), 1/6*(3*(5*b^2*c - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*
b*c - 3*a^2*d)*x^2 + 3*((5*b^2*c - 3*a*b*d)*x^5 + (5*a*b*c - 3*a^
2*d)*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^3*b*x^5 + a^4*x
^3)]
```

Sympy [A] time = 3.15965, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}-\frac{2a^2c+x^4\left(9abd-15b^2c\right)+x^2\left(6a^2d-10abc\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(-a**4*sqrt(-b/a**7)*(3*a*d - 5*
b*c)/(3*a*b*d - 5*b**2*c) + x)/4 - sqrt(-b/a**7)*(3*a*d - 5*b*c)*
log(a**4*sqrt(-b/a**7)*(3*a*d - 5*b*c)/(3*a*b*d - 5*b**2*c) + x)/
4 - (2*a**2*c + x**4*(9*a*b*d - 15*b**2*c) + x**2*(6*a**2*d - 10*
a*b*c))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

GIAC/XCAS [A] time = 0.228804, size = 116, normalized size = 1.29

$$\frac{\left(5\, b^{2} c - 3\, a b d\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2\, \sqrt{a b} a^{3}} + \frac{b^{2} c x - a b d x}{2\, {\left(b x^{2} + a\right)} a^{3}} + \frac{6\, b c x^{2} - 3\, a d x^{2} - a c}{3\, a^{3} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1
/2*(b^2*c*x - a*b*d*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c*x^2 - 3*a*d
*x^2 - a*c)/(a^3*x^3)
```

3.271 $\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{a}(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x^5(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^5}{5b^2}$$

```
[Out] ((3*b*c - 7*a*d)*(b*c - a*d)*x)/(2*b^4) - ((3*b*c - 7*a*d)*(b*c -
 a*d)*x^3)/(6*a*b^3) + (d^2*x^5)/(5*b^2) + ((b*c - a*d)^2*x^5)/(2
*a*b^2*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 7*a*d)*(b*c - a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] time = 0.341792, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{\sqrt{a}(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x^5(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*c - 7*a*d)*(b*c - a*d)*x)/(2*b^4) - ((3*b*c - 7*a*d)*(b*c -
 a*d)*x^3)/(6*a*b^3) + (d^2*x^5)/(5*b^2) + ((b*c - a*d)^2*x^5)/(2
*a*b^2*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 7*a*d)*(b*c - a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{a}\,(ad-bc)\,(7ad-3bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{9}{2}}}+\frac{d^2x^5}{5b^2}+\frac{x^5\,(ad-bc)^2}{2ab^2\,(a+bx^2)}-\frac{x^3\,(ad-bc)\,(7ad-3bc)}{6ab^3}+\frac{(ad-bc)\,(7ad-3bc)\int a\,dx}{2ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(a)*(a*d - b*c)*(7*a*d - 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(2*b
**(9/2)) + d**2*x**5/(5*b**2) + x**5*(a*d - b*c)**2/(2*a*b**2*(a
+ b*x**2)) - x**3*(a*d - b*c)*(7*a*d - 3*b*c)/(6*a*b**3) + (a*d -
 b*c)*(7*a*d - 3*b*c)*Integral(a, x)/(2*a*b**4)
```

Mathematica [A] time = 0.145263, size = 138, normalized size = 0.95

$$-\frac{\sqrt{a}\left(7a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x\left(3a^2d^2-4abcd+b^2c^2\right)}{b^4}+\frac{ax(bc-ad)^2}{2b^4(a+bx^2)}+\frac{2dx^3(bc-ad)}{3b^3}+\frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d*(b*c - a*d)*x^3)
/(3*b^3) + (d^2*x^5)/(5*b^2) + (a*(b*c - a*d)^2*x)/(2*b^4*(a + b*
x^2)) - (Sqrt[a]*(3*b^2*c^2 - 10*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.014, size = 196, normalized size = 1.4

$$\frac{d^2x^5}{5\,b^2}-\frac{2\,x^3ad^2}{3\,b^3}+\frac{2\,cx^3d}{3\,b^2}+3\,\frac{a^2d^2x}{b^4}-4\,\frac{acdx}{b^3}+\frac{c^2x}{b^2}+\frac{a^3xd^2}{2\,b^4\,(bx^2+a)}-\frac{xa^2cd}{b^3\,(bx^2+a)}+\frac{axc^2}{2\,b^2\,(bx^2+a)}-\frac{7\,a^3d^2}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+5\,\frac{a^2cd}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\frac{3\,ac^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^2*x^5/b^2-2/3/b^3*x^3*a*d^2+2/3/b^2*x^3*c*d+3/b^4*a^2*d^2*x
-4/b^3*a*c*d*x+1/b^2*c^2*x+1/2*a^3/b^4*x/(b*x^2+a)*d^2-a^2/b^3*x/
(b*x^2+a)*c*d+1/2*a/b^2*x/(b*x^2+a)*c^2-7/2*a^3/b^4/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*d^2+5*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*c*d-3/2*a/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240178, size = 1, normalized size = 0.01

$$\left[\frac{12\,b^3d^2x^7+4\left(10\,b^3cd-7\,ab^2d^2\right)x^5+20\left(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2\right)x^3+15\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2+\left(3\,b^3c^2-10\right.\right.}{60\left(b^5x^2+ab^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*d^2*x^7 + 4*(10*b^3*c*d - 7*a*b^2*d^2)*x^5 + 20*(3*
b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 + 15*(3*a*b^2*c^2 - 10*
a^2*b*c*d + 7*a^3*d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*
x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) +
 30*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*x)/(b^5*x^2 + a*b^4)
, 1/30*(6*b^3*d^2*x^7 + 2*(10*b^3*c*d - 7*a*b^2*d^2)*x^5 + 10*(3*
b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 - 15*(3*a*b^2*c^2 - 10*
a^2*b*c*d + 7*a^3*d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*
x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 15*(3*a*b^2*c^2 - 10*a^2*b*c
*d + 7*a^3*d^2)*x)/(b^5*x^2 + a*b^4)]
```

Sympy [A] time = 4.45831, size = 280, normalized size = 1.93

$$\frac{x\left(a^3d^2-2a^2bcd+ab^2c^2\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)\,(7ad-3bc)\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)\,(7ad-3bc)\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4}+\frac{d^2x^5}{5b^2}-\frac{x^3\left(2ad^2-2bcd\right)}{3b^3}+\frac{x\left(3a^2d^2-4abcd+b^2c^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4 + 2*b**5*x**
2) + sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(-b**4*sqrt(-a/
b**9)*(a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b
**2*c**2) + x)/4 - sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(
b**4*sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*
a*b*c*d + 3*b**2*c**2) + x)/4 + d**2*x**5/(5*b**2) - x**3*(2*a*d*
*2 - 2*b*c*d)/(3*b**3) + x*(3*a**2*d**2 - 4*a*b*c*d + b**2*c**2)/
b**4
```

GIAC/XCAS [A] time = 0.250102, size = 211, normalized size = 1.46

$$-\frac{\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}+\frac{ab^2c^2x-2\,a^2bcdx+a^3d^2x}{2\,(bx^2+a)b^4}+\frac{3\,b^8d^2x^5+10\,b^8cdx^3-10\,ab^7d^2x^3+15\,b^8c^2x-60\,ab^7cdx+45\,a^2b^6d^2x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^4) + 1/2*(a*b^2*c^2*x - 2*a^2*b*c*d*x + a^3*d^2*x
)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*d^2*x^5 + 10*b^8*c*d*x^3 - 10*a
*b^7*d^2*x^3 + 15*b^8*c^2*x - 60*a*b^7*c*d*x + 45*a^2*b^6*d^2*x)/
b^10
```

3.272 $\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$\frac{a(bc-ad)^2}{2b^4(a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{dx^2(bc-ad)}{b^3}+\frac{d^2x^4}{4b^2}$$

```
[Out] (d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(4*b^2) + (a*(b*c - a*d)^2)/(
2*b^4*(a + b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/(
2*b^4)
```

Rubi [A] time = 0.261381, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a(bc-ad)^2}{2b^4(a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{dx^2(bc-ad)}{b^3}+\frac{d^2x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(4*b^2) + (a*(b*c - a*d)^2)/(
2*b^4*(a + b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/(
2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(ad-bc\right)^2}{2b^4\left(a+bx^2\right)}+\frac{d^2\int^{x^2}x\,dx}{2b^2}-\frac{dx^2\left(ad-bc\right)}{b^3}+\frac{\left(ad-bc\right)\left(3ad-bc\right)\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] a*(a*d - b*c)**2/(2*b**4*(a + b*x**2)) + d**2*Integral(x, (x, x**
2))/(2*b**2) - d*x**2*(a*d - b*c)/b**3 + (a*d - b*c)*(3*a*d - b*c
)*log(a + b*x**2)/(2*b**4)
```

Mathematica [A] time = 0.106417, size = 87, normalized size = 0.99

$$\frac{2\left(3a^2d^2-4abcd+b^2c^2\right)\log\left(a+bx^2\right)+4bdx^2(bc-ad)+\frac{2a(bc-ad)^2}{a+bx^2}+b^2d^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (4*b*d*(b*c - a*d)*x^2 + b^2*d^2*x^4 + (2*a*(b*c - a*d)^2)/(a + b
*x^2) + 2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Log[a + b*x^2])/(4*b^
4)
```

Maple [A] time = 0.015, size = 142, normalized size = 1.6

$$\frac{d^2x^4}{4\,b^2}-\frac{a d^2x^2}{b^3}+\frac{d x^2 c}{b^2}+\frac{3\,\ln\left(b x^2+a\right)a^2d^2}{2\,b^4}-2\,\frac{\ln\left(b x^2+a\right)adc}{b^3}+\frac{\ln\left(b x^2+a\right)c^2}{2\,b^2}+\frac{a^3d^2}{2\,b^4\left(b x^2+a\right)}-\frac{a^2cd}{b^3\left(b x^2+a\right)}+\frac{a c^2}{2\,b^2\left(b x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^2*x^4/b^2-d^2/b^3*a*x^2+d/b^2*x^2*c+3/2/b^4*ln(b*x^2+a)*a^2
*d^2-2/b^3*ln(b*x^2+a)*a*d*c+1/2/b^2*ln(b*x^2+a)*c^2+1/2/b^4*a^3/
(b*x^2+a)*d^2-1/b^3*a^2/(b*x^2+a)*d*c+1/2/b^2*a/(b*x^2+a)*c^2
```

Maxima [A] time = 1.33687, size = 144, normalized size = 1.64

$$\frac{ab^2c^2-2\,a^2bcd+a^3d^2}{2\,(b^5x^2+ab^4)}+\frac{bd^2x^4+4\left(bcd-ad^2\right)x^2}{4\,b^3}+\frac{\left(b^2c^2-4\,abcd+3\,a^2d^2\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b^5*x^2 + a*b^4) + 1/4*(
b*d^2*x^4 + 4*(b*c*d - a*d^2)*x^2)/b^3 + 1/2*(b^2*c^2 - 4*a*b*c*d
 + 3*a^2*d^2)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.224757, size = 216, normalized size = 2.45

$$\frac{b^3d^2x^6+2\,ab^2c^2-4\,a^2bcd+2\,a^3d^2+\left(4\,b^3cd-3\,ab^2d^2\right)x^4+4\left(ab^2cd-a^2bd^2\right)x^2+2\left(ab^2c^2-4\,a^2bcd+3\,a^3d^2+\left(b^3c^2-\right.\right.}{4\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^2*x^6 + 2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (4*b^3
*c*d - 3*a*b^2*d^2)*x^4 + 4*(a*b^2*c*d - a^2*b*d^2)*x^2 + 2*(a*b^
2*c^2 - 4*a^2*b*c*d + 3*a^3*d^2 + (b^3*c^2 - 4*a*b^2*c*d + 3*a^2*
b*d^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 4.25568, size = 97, normalized size = 1.1

$$\frac{a^3d^2-2a^2bcd+ab^2c^2}{2ab^4+2b^5x^2}+\frac{d^2x^4}{4b^2}-\frac{x^2\left(ad^2-bcd\right)}{b^3}+\frac{(ad-bc)(3ad-bc)\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**3*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] (a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4 + 2*b**5*x**2)
 + d**2*x**4/(4*b**2) - x**2*(a*d**2 - b*c*d)/b**3 + (a*d - b*c)*
(3*a*d - b*c)*log(a + b*x**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.237655, size = 220, normalized size = 2.5

$$\frac{\frac{(bx^2+a)^2\left(d^2+\frac{2\left(2\,b^2cd-3\,abd^2\right)}{(bx^2+a)b}\right)}{b^3}-\frac{2\left(b^2c^2-4\,abcd+3\,a^2d^2\right)\ln\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^3}+\frac{2\left(\frac{ab^4c^2}{bx^2+a}-\frac{2\,a^2b^3cd}{bx^2+a}+\frac{a^3b^2d^2}{bx^2+a}\right)}{b^5}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)^2*(d^2 + 2*(2*b^2*c*d - 3*a*b*d^2)/((b*x^2 + a)*
b))/b^3 - 2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*ln(abs(b*x^2 + a)/(
(b*x^2 + a)^2*abs(b)))/b^3 + 2*(a*b^4*c^2/(b*x^2 + a) - 2*a^2*b^3
*c*d/(b*x^2 + a) + a^3*b^2*d^2/(b*x^2 + a))/b^5)/b
```

3.273 $\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=116

$$\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{x^3(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^3}{3b^2}$$

```
[Out] -((b*c - 5*a*d)*(b*c - a*d)*x)/(2*a*b^3) + (d^2*x^3)/(3*b^2) + ((
b*c - a*d)^2*x^3)/(2*a*b^2*(a + b*x^2)) + ((b*c - 5*a*d)*(b*c - a
*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.288539, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{x^3(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] -((b*c - 5*a*d)*(b*c - a*d)*x)/(2*a*b^3) + (d^2*x^3)/(3*b^2) + ((
b*c - a*d)^2*x^3)/(2*a*b^2*(a + b*x^2)) + ((b*c - 5*a*d)*(b*c - a
*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Rubi in Sympy [A] time = 45.041, size = 100, normalized size = 0.86

$$\frac{d^2x^3}{3b^2}+\frac{x^3(ad-bc)^2}{2ab^2(a+bx^2)}-\frac{x(ad-bc)(5ad-bc)}{2ab^3}+\frac{(ad-bc)(5ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] d**2*x**3/(3*b**2) + x**3*(a*d - b*c)**2/(2*a*b**2*(a + b*x**2))
- x*(a*d - b*c)*(5*a*d - b*c)/(2*a*b**3) + (a*d - b*c)*(5*a*d - b
*c)*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b**(7/2))
```

Mathematica [A] time = 0.113475, size = 105, normalized size = 0.91

$$\frac{(5a^2d^2-6abcd+b^2c^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}-\frac{x(bc-ad)^2}{2b^3(a+bx^2)}+\frac{2dx(bc-ad)}{b^3}+\frac{d^2x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (2*d*(b*c - a*d)*x)/b^3 + (d^2*x^3)/(3*b^2) - ((b*c - a*d)^2*x)/(
2*b^3*(a + b*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.012, size = 156, normalized size = 1.3

$$\frac{d^2x^3}{3\,b^2}-2\,\frac{ad^2x}{b^3}+2\,\frac{dxc}{b^2}-\frac{a^2d^2x}{2\,b^3\,(bx^2+a)}+\frac{acxd}{b^2\,(bx^2+a)}-\frac{xc^2}{2\,b\,(bx^2+a)}+\frac{5\,a^2d^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-3\,\frac{acd}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{c^2}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^2*x^3/b^2-2*d^2/b^3*a*x+2*d/b^2*x*c-1/2/b^3*x/(b*x^2+a)*a^2
*d^2+1/b^2*x/(b*x^2+a)*c*a*d-1/2/b*x/(b*x^2+a)*c^2+5/2/b^3/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*d^2-3/b^2/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*c*a*d+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248215, size = 1, normalized size = 0.01

$$\left[\frac{3\,\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\,\left(2\,b^2d^2x^5+2\,\left(6\,b^2cd-5\,abd^2\right)x^3\right.}{12\,(b^4x^2+ab^3)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^
2*c*d + 5*a^2*b*d^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/
(b*x^2 + a)) + 2*(2*b^2*d^2*x^5 + 2*(6*b^2*c*d - 5*a*b*d^2)*x^3 -
 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x)*sqrt(-a*b))/((b^4*x^2 + a
*b^3)*sqrt(-a*b)), 1/6*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 +
(b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*arctan(sqrt(a*b)*x/a)
+ (2*b^2*d^2*x^5 + 2*(6*b^2*c*d - 5*a*b*d^2)*x^3 - 3*(b^2*c^2 - 6
*a*b*c*d + 5*a^2*d^2)*x)*sqrt(a*b))/((b^4*x^2 + a*b^3)*sqrt(a*b))
]
```

Sympy [A] time = 4.02083, size = 245, normalized size = 2.11

$$-\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2ab^3+2b^4x^2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)\,(5ad-bc)\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)\,(5ad-bc)\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{d^2x^3}{3b^2}-\frac{x\left(2ad^2-2bcd\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) -
 sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(-a*b**3*sqrt(-1/
(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b*
*2*c**2) + x)/4 + sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log
(a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2
- 6*a*b*c*d + b**2*c**2) + x)/4 + d**2*x**3/(3*b**2) - x*(2*a*d**
2 - 2*b*c*d)/b**3
```

GIAC/XCAS [A] time = 0.23674, size = 154, normalized size = 1.33

$$\frac{\left(b^{2} c^{2} - 6 \, abcd + 5 \, a^{2} d^{2}\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab} b^{3}} - \frac{b^{2} c^{2} x - 2 \, abcdx + a^{2} d^{2} x}{2 \, {\left(bx^{2} + a\right)} b^{3}} + \frac{b^{4} d^{2} x^{3} + 6 \, b^{4} cdx - 6 \, ab^{3} d^{2} x}{3 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^3) - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 +
a)*b^3) + 1/3*(b^4*d^2*x^3 + 6*b^4*c*d*x - 6*a*b^3*d^2*x)/b^6
```

3.274 $\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}+\frac{d^2x^2}{2b^2}$$

```
[Out] (d^2*x^2)/(2*b^2) - (b*c - a*d)^2/(2*b^3*(a + b*x^2)) + (d*(b*c -
 a*d)*Log[a + b*x^2])/b^3
```

Rubi [A] time = 0.147973, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}+\frac{d^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x^2)/(2*b^2) - (b*c - a*d)^2/(2*b^3*(a + b*x^2)) + (d*(b*c -
 a*d)*Log[a + b*x^2])/b^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^2\int^{x^2}\frac{1}{b^2}\,dx}{2}-\frac{d\,(ad-bc)\log\left(a+bx^2\right)}{b^3}-\frac{(ad-bc)^2}{2b^3\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] d**2*Integral(b**(-2), (x, x**2))/2 - d*(a*d - b*c)*log(a + b*x**
2)/b**3 - (a*d - b*c)**2/(2*b**3*(a + b*x**2))
```

Mathematica [A] time = 0.0759352, size = 56, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx^2}+2d(bc-ad)\log\left(a+bx^2\right)+bd^2x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (b*d^2*x^2 - (b*c - a*d)^2/(a + b*x^2) + 2*d*(b*c - a*d)*Log[a +
b*x^2])/(2*b^3)
```

Maple [A] time = 0.014, size = 97, normalized size = 1.6

$$\frac{d^2x^2}{2\,b^2}-\frac{\ln\left(bx^2+a\right)d^2a}{b^3}+\frac{\ln\left(bx^2+a\right)dc}{b^2}-\frac{a^2d^2}{2\,b^3\left(bx^2+a\right)}+\frac{acd}{b^2\left(bx^2+a\right)}-\frac{c^2}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d^2*x^2/b^2-1/b^3*ln(b*x^2+a)*d^2*a+1/b^2*ln(b*x^2+a)*d*c-1/2
/b^3/(b*x^2+a)*a^2*d^2+1/b^2/(b*x^2+a)*a*d*c-1/2/b/(b*x^2+a)*c^2
```

Maxima [A] time = 1.3282, size = 99, normalized size = 1.62

$$\frac{d^2x^2}{2\,b^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\,(b^4x^2 + ab^3)} + \frac{\left(bcd - ad^2\right)\log\left(bx^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x^2 +
a*b^3) + (b*c*d - a*d^2)*log(b*x^2 + a)/b^3
```

Fricas [A] time = 0.219055, size = 136, normalized size = 2.23

$$\frac{b^2d^2x^4 + abd^2x^2 - b^2c^2 + 2\,abcd - a^2d^2 + 2\left(abcd - a^2d^2 + \left(b^2cd - abd^2\right)x^2\right)\log\left(bx^2 + a\right)}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + a*b*d^2*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 +
2*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*log(b*x^2 + a))/(
b^4*x^2 + a*b^3)
```

Sympy [A] time = 3.60889, size = 68, normalized size = 1.11

$$-\frac{a^2d^2 - 2abcd + b^2c^2}{2ab^3 + 2b^4x^2} + \frac{d^2x^2}{2b^2} - \frac{d\,(ad - bc)\log\left(a + bx^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] -(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) + d
**2*x**2/(2*b**2) - d*(a*d - b*c)*log(a + b*x**2)/b**3
```

GIAC/XCAS [A] time = 0.24152, size = 150, normalized size = 2.46

$$\frac{\left(bx^2 + a\right)d^2}{2\,b^3} - \frac{\left(bcd - ad^2\right)\ln\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2\left|b\right|}\right)}{b^3} - \frac{\frac{b^3c^2}{bx^2+a} - \frac{2\,ab^2cd}{bx^2+a} + \frac{a^2bd^2}{bx^2+a}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^2*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)*d^2/b^3 - (b*c*d - a*d^2)*ln(abs(b*x^2 + a)/((b*x
^2 + a)^2*abs(b)))/b^3 - 1/2*(b^3*c^2/(b*x^2 + a) - 2*a*b^2*c*d/(
b*x^2 + a) + a^2*b*d^2/(b*x^2 + a))/b^4
```

3.275 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a
*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2)
)
```

Rubi [A] time = 0.216862, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a
*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2)
)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\int\frac{1}{b^2}\,dx+\frac{x(ad-bc)^2}{2ab^2(a+bx^2)}-\frac{(ad-bc)(3ad+bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] d**2*Integral(b**(-2), x) + x*(a*d - b*c)**2/(2*a*b**2*(a + b*x**
2)) - (a*d - b*c)*(3*a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/
2)*b**(5/2))
```

Mathematica [A] time = 0.101781, size = 88, normalized size = 1.07

$$\frac{(-3a^2d^2+2abcd+b^2c^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2
 + 2*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)
*b^(5/2))
```

Maple [A] time = 0.002, size = 129, normalized size = 1.6

$$\frac{d^2x}{b^2} + \frac{axd^2}{2\,b^2\,(bx^2+a)} - \frac{cxd}{b\,(bx^2+a)} + \frac{xc^2}{2\,a\,(bx^2+a)} - \frac{3\,ad^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{cd}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{c^2}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] d^2*x/b^2+1/2/b^2*x*a/(b*x^2+a)*d^2-1/b*x/(b*x^2+a)*c*d+1/2*x/a/(
b*x^2+a)*c^2-3/2/b^2*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^2+1/
b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d+1/2/a/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234421, size = 1, normalized size = 0.01

$$\Bigg[-\frac{\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\left(2\,abd^2x^3+\left(b^2c^2-2\,abcd+3\right.\right.}{4\,(ab^3x^2+a^2b^2)\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*x^2)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(
b*x^2 + a)) - 2*(2*a*b*d^2*x^3 + (b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2
)*x)*sqrt(-a*b))/((a*b^3*x^2 + a^2*b^2)*sqrt(-a*b)), 1/2*((a*b^2*
c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*x^2)*arctan(sqrt(a*b)*x/a) + (2*a*b*d^2*x^3 + (b^2*c^2 - 2*a
*b*c*d + 3*a^2*d^2)*x)*sqrt(a*b))/((a*b^3*x^2 + a^2*b^2)*sqrt(a*b
))]
```

Sympy [A] time = 3.47001, size = 236, normalized size = 2.88

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2a^2b^2+2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**
2) + sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**
2*sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2
*a*b*c*d - b**2*c**2) + x)/4 - sqrt(-1/(a**3*b**5))*(a*d - b*c)*(
3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*
d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 + d**2*x/b*
*2
```

GIAC/XCAS [A] time = 0.241747, size = 127, normalized size = 1.55

$$\frac{d^2x}{b^2} + \frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(bx^2 + a)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2
*x)/((b*x^2 + a)*a*b^2)
```

3.276 $\int \frac{(c+dx^2)^2}{x(a+bx^2)^2} \, dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}$$

```
[Out] (b*c - a*d)^2/(2*a*b^2*(a + b*x^2)) + (c^2*Log[x])/a^2 - ((c^2/a^
2 - d^2/b^2)*Log[a + b*x^2])/2
```

Rubi [A] time = 0.159148, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*a*b^2*(a + b*x^2)) + (c^2*Log[x])/a^2 - ((c^2/a^
2 - d^2/b^2)*Log[a + b*x^2])/2
```

Rubi in Sympy [A] time = 28.6669, size = 60, normalized size = 0.9

$$-\left(-\frac{d^2}{2b^2}+\frac{c^2}{2a^2}\right)\log\left(a+bx^2\right)+\frac{(ad-bc)^2}{2ab^2\left(a+bx^2\right)}+\frac{c^2\log\left(x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x/(b*x**2+a)**2,x)
```

```
[Out] -(-d**2/(2*b**2) + c**2/(2*a**2))*log(a + b*x**2) + (a*d - b*c)**
2/(2*a*b**2*(a + b*x**2)) + c**2*log(x**2)/(2*a**2)
```

Mathematica [A] time = 0.0711076, size = 70, normalized size = 1.04

$$\frac{\frac{(ad-bc)\left((a+bx^2)(ad+bc)\log(a+bx^2)+a(ad-bc)\right)}{b^2(a+bx^2)}+2c^2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^2*Log[x] + ((-(b*c) + a*d)*(a*(-(b*c) + a*d) + (b*c + a*d)*(
a + b*x^2)*Log[a + b*x^2]))/(b^2*(a + b*x^2)))/(2*a^2)
```

Maple [A] time = 0.017, size = 94, normalized size = 1.4

$$\frac{c^2\ln(x)}{a^2}+\frac{\ln\left(bx^2+a\right)d^2}{2\,b^2}-\frac{\ln\left(bx^2+a\right)c^2}{2\,a^2}+\frac{ad^2}{2\,b^2\left(bx^2+a\right)}-\frac{cd}{b\left(bx^2+a\right)}+\frac{c^2}{2\,a\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x/(b*x^2+a)^2,x)
```

```
[Out] c^2*ln(x)/a^2+1/2/b^2*ln(b*x^2+a)*d^2-1/2/a^2*ln(b*x^2+a)*c^2+1/2
*a/b^2/(b*x^2+a)*d^2-1/b/(b*x^2+a)*d*c+1/2/a/(b*x^2+a)*c^2
```

Maxima [A] time = 1.33789, size = 116, normalized size = 1.73

$$\frac{c^2 \log\left(x^2\right)}{2\,a^2} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\,(ab^3x^2 + a^2b^2)} - \frac{\left(b^2c^2 - a^2d^2\right)\log\left(bx^2 + a\right)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a*b^3
*x^2 + a^2*b^2) - 1/2*(b^2*c^2 - a^2*d^2)*log(b*x^2 + a)/(a^2*b^2
)
```

Fricas [A] time = 0.22979, size = 158, normalized size = 2.36

$$\frac{ab^2c^2 - 2\,a^2bcd + a^3d^2 - \left(ab^2c^2 - a^3d^2 + \left(b^3c^2 - a^2bd^2\right)x^2\right)\log\left(bx^2 + a\right) + 2\left(b^3c^2x^2 + ab^2c^2\right)\log\left(x\right)}{2\,(a^2b^3x^2 + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 - (a*b^2*c^2 - a^3*d^2 + (
b^3*c^2 - a^2*b*d^2)*x^2)*log(b*x^2 + a) + 2*(b^3*c^2*x^2 + a*b^2
*c^2)*log(x))/(a^2*b^3*x^2 + a^3*b^2)
```

Sympy [A] time = 5.35883, size = 80, normalized size = 1.19

$$\frac{a^2d^2 - 2abcd + b^2c^2}{2a^2b^2 + 2ab^3x^2} + \frac{c^2\log\left(x\right)}{a^2} + \frac{(ad - bc)\,(ad + bc)\log\left(\frac{a}{b} + x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x/(b*x**2+a)**2,x)
```

```
[Out] (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2)
 + c**2*log(x)/a**2 + (a*d - b*c)*(a*d + b*c)*log(a/b + x**2)/(2*
a**2*b**2)
```

GIAC/XCAS [A] time = 0.259983, size = 134, normalized size = 2.

$$\frac{c^2\ln\left(x^2\right)}{2\,a^2} - \frac{\left(b^2c^2 - a^2d^2\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,a^2b^2} + \frac{b^2c^2x^2 - a^2d^2x^2 + 2\,abc^2 - 2\,a^2cd}{2\,(bx^2 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*c^2*ln(x^2)/a^2 - 1/2*(b^2*c^2 - a^2*d^2)*ln(abs(b*x^2 + a))/
(a^2*b^2) + 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*
d)/((b*x^2 + a)*a^2*b)
```

3.277 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=103

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2a(a+bx^2)}-\frac{c^2}{ax(a+bx^2)}$$

```
[Out] -(c^2/(a*x*(a + b*x^2))) - (((3*b*c^2)/a - 2*c*d + (a*d^2)/b)*x)/
(2*a*(a + b*x^2)) - ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.184359, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2a(a+bx^2)}-\frac{c^2}{ax(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2/(a*x*(a + b*x^2))) - (((3*b*c^2)/a - 2*c*d + (a*d^2)/b)*x)/
(2*a*(a + b*x^2)) - ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 24.1629, size = 88, normalized size = 0.85

$$-\frac{c^2}{ax(a+bx^2)}-\frac{x\left(a^2d^2-bc(2ad-3bc)\right)}{2a^2b(a+bx^2)}+\frac{(ad-bc)(ad+3bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x**2/(b*x**2+a)**2,x)
```

```
[Out] -c**2/(a*x*(a + b*x**2)) - x*(a**2*d**2 - b*c*(2*a*d - 3*b*c))/(2
*a**2*b*(a + b*x**2)) + (a*d - b*c)*(a*d + 3*b*c)*atan(sqrt(b)*x/
sqrt(a))/(2*a**(5/2)*b**(3/2))
```

Mathematica [A] time = 0.111834, size = 91, normalized size = 0.88

$$-\frac{x(ad-bc)^2}{2a^2b(a+bx^2)}-\frac{c^2}{a^2x}+\frac{\left(a^2d^2+2abcd-3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2/(a^2*x)) - ((-(b*c) + a*d)^2*x)/(2*a^2*b*(a + b*x^2)) + ((-
3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(5/2)*b^(3/2))
```

Maple [A] time = 0.015, size = 131, normalized size = 1.3

$$-\frac{c^2}{a^2x}-\frac{xd^2}{2\,b\,(bx^2+a)}+\frac{cxd}{a\,(bx^2+a)}-\frac{bxc^2}{2\,a^2\,(bx^2+a)}+\frac{d^2}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{cd}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,bc^2}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c^2/a^2/x-1/2/b*x/(b*x^2+a)*d^2+1/a*x/(b*x^2+a)*c*d-1/2/a^2*b*x/
(b*x^2+a)*c^2+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^2+1/a/(
a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d-3/2/a^2*b/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238065, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(3\,b^3c^2-2\,ab^2cd-a^2bd^2\right)x^3+\left(3\,ab^2c^2-2\,a^2bcd-a^3d^2\right)x\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,abc^2+\left(3\,b^2c^2-2\,abcd+a\right.\right.}{4\left(a^2b^2x^3+a^3bx\right)\sqrt{-ab}}\right.$$

$$-\frac{\left(\left(3\,b^3c^2-2\,ab^2cd-a^2bd^2\right)x^3+\left(3\,ab^2c^2-2\,a^2bcd-a^3d^2\right)x\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(2\,abc^2+\left(3\,b^2c^2-2\,abcd+a^2d^2\right)x^2\right)\sqrt{a}}{2\left(a^2b^2x^3+a^3bx\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2
- 2*a^2*b*c*d - a^3*d^2)*x)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b)
)/(b*x^2 + a)) + 2*(2*a*b*c^2 + (3*b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*x^2)*sqrt(-a*b))/((a^2*b^2*x^3 + a^3*b*x)*sqrt(-a*b)), -1/2*(((3
*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2*a^2*b*
c*d - a^3*d^2)*x)*arctan(sqrt(a*b)*x/a) + (2*a*b*c^2 + (3*b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*x^2)*sqrt(a*b))/((a^2*b^2*x^3 + a^3*b*x)*
sqrt(a*b))]
```

Sympy [A] time = 4.00616, size = 238, normalized size = 2.31

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}-\frac{2abc^2+x^2\left(a^2d^2-2abcd+3b^2c^2\right)}{2a^3bx+2a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(-a**3*b*sqrt(
-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d
- 3*b**2*c**2) + x)/4 + sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3
*b*c)*log(a**3*b*sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)/(
a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)/4 - (2*a*b*c**2 + x**2*
(a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2))/(2*a**3*b*x + 2*a**2*b**2*
x**3)
```

GIAC/XCAS [A] time = 0.242098, size = 139, normalized size = 1.35

$$-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b}-\frac{3\,b^2c^2x^2-2\,abcdx^2+a^2d^2x^2+2\,abc^2}{2\,(bx^3+ax)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2*b) - 1/2*(3*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2
+ 2*a*b*c^2)/((b*x^3 + a*x)*a^2*b)
```

3.278 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2} dx$

Optimal. Leaf size=80

$$\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{2a^2x^2}$$

```
[Out] -c^2/(2*a^2*x^2) - (b*c - a*d)^2/(2*a^2*b*(a + b*x^2)) - (2*c*(b*
c - a*d)*Log[x])/a^3 + (c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Rubi [A] time = 0.200642, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c^2/(2*a^2*x^2) - (b*c - a*d)^2/(2*a^2*b*(a + b*x^2)) - (2*c*(b*
c - a*d)*Log[x])/a^3 + (c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Rubi in Sympy [A] time = 27.9634, size = 70, normalized size = 0.88

$$-\frac{c^2}{2a^2x^2}-\frac{(ad-bc)^2}{2a^2b\left(a+bx^2\right)}+\frac{c\,(ad-bc)\log\left(x^2\right)}{a^3}-\frac{c\,(ad-bc)\log\left(a+bx^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**2/x**3/(b*x**2+a)**2,x)
```

```
[Out] -c**2/(2*a**2*x**2) - (a*d - b*c)**2/(2*a**2*b*(a + b*x**2)) + c*
(a*d - b*c)*log(x**2)/a**3 - c*(a*d - b*c)*log(a + b*x**2)/a**3
```

Mathematica [A] time = 0.173657, size = 72, normalized size = 0.9

$$-\frac{\frac{a(bc-ad)^2}{b(a+bx^2)}-2c(bc-ad)\log\left(a+bx^2\right)+4c\log(x)(bc-ad)+\frac{ac^2}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((a*c^2)/x^2 + (a*(b*c - a*d)^2)/(b*(a + b*x^2)) + 4*c*(b*c - a*
d)*Log[x] - 2*c*(b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.02, size = 114, normalized size = 1.4

$$-\frac{c^2}{2\,a^2x^2}+2\,\frac{c\ln(x)\,d}{a^2}-2\,\frac{c^2\ln(x)\,b}{a^3}-\frac{c\ln\left(bx^2+a\right)d}{a^2}+\frac{c^2\ln\left(bx^2+a\right)b}{a^3}-\frac{d^2}{2\,b\,(bx^2+a)}+\frac{cd}{a\,(bx^2+a)}-\frac{bc^2}{2\,a^2\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c^2/a^2/x^2+2*c/a^2*ln(x)*d-2*c^2/a^3*ln(x)*b-1/a^2*c*ln(b*x
^2+a)*d+1/a^3*c^2*ln(b*x^2+a)*b-1/2/b/(b*x^2+a)*d^2+1/a/(b*x^2+a)
*d*c-1/2/a^2/(b*x^2+a)*c^2*b
```

Maxima [A] time = 1.33887, size = 135, normalized size = 1.69

$$-\frac{abc^2 + \left(2\,b^2c^2 - 2\,abcd + a^2d^2\right)x^2}{2\left(a^2b^2x^4 + a^3bx^2\right)} + \frac{\left(bc^2 - acd\right)\log\left(bx^2 + a\right)}{a^3} - \frac{\left(bc^2 - acd\right)\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c^2 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x
^4 + a^3*b*x^2) + (b*c^2 - a*c*d)*log(b*x^2 + a)/a^3 - (b*c^2 - a
*c*d)*log(x^2)/a^3
```

Fricas [A] time = 0.232108, size = 215, normalized size = 2.69

$$-\frac{a^2bc^2 + \left(2\,ab^2c^2 - 2\,a^2bcd + a^3d^2\right)x^2 - 2\left(\left(b^3c^2 - ab^2cd\right)x^4 + \left(ab^2c^2 - a^2bcd\right)x^2\right)\log\left(bx^2 + a\right) + 4\left(\left(b^3c^2 - ab^2cd\right)x^4 + \right.}{2\left(a^3b^2x^4 + a^4bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2 - 2*(
(b^3*c^2 - a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(b*x^
2 + a) + 4*((b^3*c^2 - a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x
^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2)
```

Sympy [A] time = 6.19215, size = 92, normalized size = 1.15

$$-\frac{abc^2 + x^2\left(a^2d^2 - 2abcd + 2b^2c^2\right)}{2a^3bx^2 + 2a^2b^2x^4} + \frac{2c\left(ad - bc\right)\log\left(x\right)}{a^3} - \frac{c\left(ad - bc\right)\log\left(\frac{a}{b} + x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x**3/(b*x**2+a)**2,x)
```

```
[Out] -(a*b*c**2 + x**2*(a**2*d**2 - 2*a*b*c*d + 2*b**2*c**2))/(2*a**3*
b*x**2 + 2*a**2*b**2*x**4) + 2*c*(a*d - b*c)*log(x)/a**3 - c*(a*d
 - b*c)*log(a/b + x**2)/a**3
```

GIAC/XCAS [A] time = 0.237616, size = 147, normalized size = 1.84

$$-\frac{\left(bc^2 - acd\right)\ln\left(x^2\right)}{a^3} + \frac{\left(b^2c^2 - abcd\right)\ln\left(\left|bx^2 + a\right|\right)}{a^3b} - \frac{2\,b^2c^2x^2 - 2\,abcdx^2 + a^2d^2x^2 + abc^2}{2\left(bx^4 + ax^2\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d)*ln(x^2)/a^3 + (b^2*c^2 - a*b*c*d)*ln(abs(b*x^2 +
 a))/(a^3*b) - 1/2*(2*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 +
 a*b*c^2)/((b*x^4 + a*x^2)*a^2*b)
```

3.279 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=127

$$\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}+\frac{c(5bc-6ad)}{3a^3x}+\frac{x\left(3a^2d^2-6abcd+5b^2c^2\right)}{6a^3\left(a+bx^2\right)}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

```
[Out] (c*(5*b*c - 6*a*d))/(3*a^3*x) - c^2/(3*a*x^3*(a + b*x^2)) + ((5*b
^2*c^2 - 6*a*b*c*d + 3*a^2*d^2)*x)/(6*a^3*(a + b*x^2)) + ((b*c -
a*d)*(5*b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b
])
```

Rubi [A] time = 0.363003, antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}+\frac{c(5bc-6ad)}{3a^3x}+\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{6a\left(a+bx^2\right)}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (c*(5*b*c - 6*a*d))/(3*a^3*x) - c^2/(3*a*x^3*(a + b*x^2)) + ((3*d
^2 + (b*c*(5*b*c - 6*a*d))/a^2)*x)/(6*a*(a + b*x^2)) + ((b*c - a*
d)*(5*b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 42.1185, size = 110, normalized size = 0.87

$$-\frac{c^2}{3ax^3\left(a+bx^2\right)}-\frac{c\left(6ad-5bc\right)}{3a^3x}+\frac{x\left(3a^2d^2-bc\left(6ad-5bc\right)\right)}{6a^3\left(a+bx^2\right)}+\frac{\left(ad-5bc\right)\left(ad-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{7}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x**4/(b*x**2+a)**2,x)
```

```
[Out] -c**2/(3*a*x**3*(a + b*x**2)) - c*(6*a*d - 5*b*c)/(3*a**3*x) + x*
(3*a**2*d**2 - b*c*(6*a*d - 5*b*c))/(6*a**3*(a + b*x**2)) + (a*d
- 5*b*c)*(a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(7/2)*sqrt(b))
```

Mathematica [A] time = 0.113971, size = 107, normalized size = 0.84

$$\frac{x(ad-bc)^2}{2a^3\left(a+bx^2\right)}-\frac{2c(ad-bc)}{a^3x}-\frac{c^2}{3a^2x^3}+\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c^2/(3*a^2*x^3) - (2*c*(-(b*c) + a*d))/(a^3*x) + ((-(b*c) + a*d)
^2*x)/(2*a^3*(a + b*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.017, size = 161, normalized size = 1.3

$$-\frac{c^2}{3a^2x^3}-2\frac{cd}{a^2x}+2\frac{bc^2}{a^3x}+\frac{xd^2}{2a(bx^2+a)}-\frac{cxbd}{a^2(bx^2+a)}+\frac{b^2c^2x}{2a^3(bx^2+a)}+\frac{d^2}{2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-3\frac{bdc}{a^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{5b^2c^2}{2a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c^2/a^2/x^3-2*c/a^2/x*d+2*c^2/a^3/x*b+1/2/a*x/(b*x^2+a)*d^2-
1/a^2*x/(b*x^2+a)*c*b*d+1/2/a^3*x/(b*x^2+a)*b^2*c^2+1/2/a/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*d^2-3/a^2/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*c*b*d+5/2/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b^2*c
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245404, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(5b^3c^2-6ab^2cd+a^2bd^2\right)x^5+\left(5ab^2c^2-6a^2bcd+a^3d^2\right)x^3\right)\log\left(\frac{2abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(3\left(5b^2c^2-6abcd+a^2d^2\right)x\right.}{12\left(a^3bx^5+a^4x^3\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^5 + (5*a*b^2*c^
2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*log((2*a*b*x + (b*x^2 - a)*sqrt(-
a*b))/(b*x^2 + a)) + 2*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4 -
 2*a^2*c^2 + 2*(5*a*b*c^2 - 6*a^2*c*d)*x^2)*sqrt(-a*b))/((a^3*b*x
^5 + a^4*x^3)*sqrt(-a*b)), 1/6*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2
*b*d^2)*x^5 + (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*arctan(s
qrt(a*b)*x/a) + (3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4 - 2*a^2*
c^2 + 2*(5*a*b*c^2 - 6*a^2*c*d)*x^2)*sqrt(a*b))/((a^3*b*x^5 + a^4
*x^3)*sqrt(a*b))]
```

Sympy [A] time = 4.74846, size = 248, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)\log\left(-\frac{a^4\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)\log\left(\frac{a^4\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{-2a^2c^2+x^4\left(3a^2d^2-18abcd+15b^2c^2\right)+x^2\left(-12a^2cd+10abc^2\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(-a**4*sqrt(-1/(a
**7*b))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2
*c**2) + x)/4 + sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(a
**4*sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*
b*c*d + 5*b**2*c**2) + x)/4 + (-2*a**2*c**2 + x**4*(3*a**2*d**2 -
 18*a*b*c*d + 15*b**2*c**2) + x**2*(-12*a**2*c*d + 10*a*b*c**2))/
(6*a**4*x**3 + 6*a**3*b*x**5)
```

GIAC/XCAS [A] time = 0.248913, size = 151, normalized size = 1.19

$$\frac{\left(5\,b^2c^2 - 6\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(bx^2 + a)a^3} + \frac{6\,bc^2x^2 - 6\,acdx^2 - ac^2}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^3) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 +
a)*a^3) + 1/3*(6*b*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^3*x^3)
```

3.280 $\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=169

$$\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{2b^4}-\frac{3\sqrt{a}(bc-3ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}+\frac{3d^2x^5(7bc-3ad)}{10b^3}-\frac{x^3(c+dx^2)^3}{2b(a+bx^2)}+\frac{9d^3x^7}{14b^2}$$

```
[Out] (3*(b*c - 3*a*d)*(b*c - a*d)^2*x)/(2*b^5) + (d*(5*b^2*c^2 - 7*a*b
*c*d + 3*a^2*d^2)*x^3)/(2*b^4) + (3*d^2*(7*b*c - 3*a*d)*x^5)/(10*
b^3) + (9*d^3*x^7)/(14*b^2) - (x^3*(c + d*x^2)^3)/(2*b*(a + b*x^2
)) - (3*Sqrt[a]*(b*c - 3*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*b^(11/2))
```

Rubi [A] time = 0.383735, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{2b^4}-\frac{3\sqrt{a}(bc-3ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}+\frac{3d^2x^5(7bc-3ad)}{10b^3}-\frac{x^3(c+dx^2)^3}{2b(a+bx^2)}+\frac{9d^3x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*(b*c - 3*a*d)*(b*c - a*d)^2*x)/(2*b^5) + (d*(5*b^2*c^2 - 7*a*b
*c*d + 3*a^2*d^2)*x^3)/(2*b^4) + (3*d^2*(7*b*c - 3*a*d)*x^5)/(10*
b^3) + (9*d^3*x^7)/(14*b^2) - (x^3*(c + d*x^2)^3)/(2*b*(a + b*x^2
)) - (3*Sqrt[a]*(b*c - 3*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*b^(11/2))
```

Rubi in Sympy [A] time = 64.7086, size = 163, normalized size = 0.96

$$\frac{3\sqrt{a}(ad-bc)^2(3ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{11}{2}}}-\frac{x^3(c+dx^2)^3}{2b(a+bx^2)}+\frac{9d^3x^7}{14b^2}-\frac{3d^2x^5(3ad-7bc)}{10b^3}+\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{2b^4}-\frac{3x(ad-bc)^2(3ad-bc)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(a)*(a*d - b*c)**2*(3*a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/(2
*b**(11/2)) - x**3*(c + d*x**2)**3/(2*b*(a + b*x**2)) + 9*d**3*x*
*7/(14*b**2) - 3*d**2*x**5*(3*a*d - 7*b*c)/(10*b**3) + d*x**3*(3*
a**2*d**2 - 7*a*b*c*d + 5*b**2*c**2)/(2*b**4) - 3*x*(a*d - b*c)**
2*(3*a*d - b*c)/(2*b**5)
```

Mathematica [A] time = 0.133784, size = 151, normalized size = 0.89

$$\frac{3\sqrt{a}(bc-ad)^2(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{ax(bc-ad)^3}{2b^5(a+bx^2)}+\frac{x(bc-4ad)(bc-ad)^2}{b^5}+\frac{dx^3(bc-ad)^2}{b^4}+\frac{d^2x^5(3bc-2ad)}{5b^3}+\frac{d^3x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 4*a*d)*(b*c - a*d)^2*x)/b^5 + (d*(b*c - a*d)^2*x^3)/b^4 +
 (d^2*(3*b*c - 2*a*d)*x^5)/(5*b^3) + (d^3*x^7)/(7*b^2) + (a*(b*c
- a*d)^3*x)/(2*b^5*(a + b*x^2)) + (3*Sqrt[a]*(b*c - a*d)^2*(-(b*c
) + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [B] time = 0.017, size = 302, normalized size = 1.8

$$\frac{d^3x^7}{7\,b^2}-\frac{2\,x^5ad^3}{5\,b^3}+\frac{3\,x^5cd^2}{5\,b^2}+\frac{x^3a^2d^3}{b^4}-2\,\frac{x^3acd^2}{b^3}+\frac{x^3c^2d}{b^2}-4\,\frac{a^3d^3x}{b^5}+9\,\frac{xa^2cd^2}{b^4}-6\,\frac{ac^2dx}{b^3}+\frac{c^3x}{b^2}-\frac{a^4xd^3}{2\,b^5\,(bx^2+a)}+\frac{3\,a^3xcd^2}{2\,b^4\,(bx^2+a)}-\frac{3\,xa^2c^2d}{2\,b^3\,(bx^2+a)}+\frac{axc^3}{2\,b^2\,(bx^2+a)}+\frac{9\,a^4d^3}{2\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{21\,a^3cd^2}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{15\,a^2c^2d}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,ac^3}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/7*d^3*x^7/b^2-2/5/b^3*x^5*a*d^3+3/5/b^2*x^5*c*d^2+1/b^4*x^3*a^2
*d^3-2/b^3*x^3*a*c*d^2+1/b^2*x^3*c^2*d-4/b^5*a^3*d^3*x+9/b^4*a^2*
c*d^2*x-6/b^3*a*c^2*d*x+1/b^2*c^3*x-1/2*a^4/b^5*x/(b*x^2+a)*d^3+3
/2*a^3/b^4*x/(b*x^2+a)*c*d^2-3/2*a^2/b^3*x/(b*x^2+a)*c^2*d+1/2*a/
b^2*x/(b*x^2+a)*c^3+9/2*a^4/b^5/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2
))*d^3-21/2*a^3/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^2+15/
2*a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d-3/2*a/b^2/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242363, size = 1, normalized size = 0.01

$$\left[\frac{20\,b^4d^3x^9+12\left(7\,b^4cd^2-3\,ab^3d^3\right)x^7+28\left(5\,b^4c^2d-7\,ab^3cd^2+3\,a^2b^2d^3\right)x^5+140\left(b^4c^3-5\,ab^3c^2d+7\,a^2b^2cd^2-3\,a^3bd^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*d^3*x^9 + 12*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 28*
(5*b^4*c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 140*(b^4*c^3
- 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3
*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5
*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^2)*sqrt(-a/b)*log
```

```
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(a*b^3*c^3 - 5
*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5),
 1/70*(10*b^4*d^3*x^9 + 6*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 14*(5
*b^4*c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 70*(b^4*c^3 - 5
*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^
3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*
b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^2)*sqrt(a/b)*arctan(
x/sqrt(a/b)) + 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 -
 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [A] time = 6.08175, size = 382, normalized size = 2.26

$$-\frac{x\left(a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3\right)}{2ab^5+2b^6x^2}-\frac{3\sqrt{-\frac{a}{b^{11}}}\,(ad-bc)^2\,(3ad-bc)\log\left(-\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}+\frac{3\sqrt{-\frac{a}{b^{11}}}\,(ad-bc)^2\,(3ad-bc)\log\left(\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}+\frac{d^3x^7}{7b^2}-\frac{x^5\left(2ad^3-3bcd^2\right)}{5b^3}+\frac{x^3\left(a^2d^3-2abcd^2+b^2c^2d\right)}{b^4}-\frac{x\left(4a^3d^3-9a^2bcd^2+6ab^2c^2d-b^3c^3\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**
3)/(2*a*b**5 + 2*b**6*x**2) - 3*sqrt(-a/b**11)*(a*d - b*c)**2*(3*
a*d - b*c)*log(-3*b**5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c
)/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 3*b**3*c**
3) + x)/4 + 3*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)*log(3*b
**5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)/(9*a**3*d**3 - 21
*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 3*b**3*c**3) + x)/4 + d**3*x*
*7/(7*b**2) - x**5*(2*a*d**3 - 3*b*c*d**2)/(5*b**3) + x**3*(a**2*
d**3 - 2*a*b*c*d**2 + b**2*c**2*d)/b**4 - x*(4*a**3*d**3 - 9*a**2
*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)/b**5
```

GIAC/XCAS [A] time = 0.253876, size = 325, normalized size = 1.92

$$-\frac{3\left(ab^3c^3-5\,a^2b^2c^2d+7\,a^3bcd^2-3\,a^4d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}+\frac{ab^3c^3x-3\,a^2b^2c^2dx+3\,a^3bcd^2x-a^4d^3x}{2\,(bx^2+a)b^5}+\frac{5\,b^{12}d^3x^7+21\,b^{12}cd^2x^5-14\,ab^{11}d^3x^5+35\,b^{12}c^2dx^3-70\,ab^{11}cd^2x^3+35\,a^2b^{10}d^3x^3+35\,b^{12}c^3x-210\,ab^{11}c^2dx+315\,a^2b^1}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c^3*x - 3*a^2*b^
2*c^2*d*x + 3*a^3*b*c*d^2*x - a^4*d^3*x)/((b*x^2 + a)*b^5) + 1/35
*(5*b^12*d^3*x^7 + 21*b^12*c*d^2*x^5 - 14*a*b^11*d^3*x^5 + 35*b^1
2*c^2*d*x^3 - 70*a*b^11*c*d^2*x^3 + 35*a^2*b^10*d^3*x^3 + 35*b^12
*c^3*x - 210*a*b^11*c^2*d*x + 315*a^2*b^10*c*d^2*x - 140*a^3*b^9*
d^3*x)/b^14
```

3.281 $\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=117

$$\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{d^3x^6}{6b^2}$$

```
[Out] (3*d*(b*c - a*d)^2*x^2)/(2*b^4) + (d^2*(3*b*c - 2*a*d)*x^4)/(4*b^
3) + (d^3*x^6)/(6*b^2) + (a*(b*c - a*d)^3)/(2*b^5*(a + b*x^2)) +
((b*c - 4*a*d)*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] time = 0.328727, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{d^3x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x^2)/(2*b^4) + (d^2*(3*b*c - 2*a*d)*x^4)/(4*b^
3) + (d^3*x^6)/(6*b^2) + (a*(b*c - a*d)^3)/(2*b^5*(a + b*x^2)) +
((b*c - 4*a*d)*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a(ad-bc)^3}{2b^5(a+bx^2)}+\frac{d^3x^6}{6b^2}-\frac{d^2(2ad-3bc)\int^{x^2} x\,dx}{2b^3}+\frac{3dx^2(ad-bc)^2}{2b^4}-\frac{(ad-bc)^2(4ad-bc)\log(a+bx^2)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -a*(a*d - b*c)**3/(2*b**5*(a + b*x**2)) + d**3*x**6/(6*b**2) - d*
*2*(2*a*d - 3*b*c)*Integral(x, (x, x**2))/(2*b**3) + 3*d*x**2*(a*
d - b*c)**2/(2*b**4) - (a*d - b*c)**2*(4*a*d - b*c)*log(a + b*x**
2)/(2*b**5)
```

Mathematica [A] time = 0.152384, size = 106, normalized size = 0.91

$$\frac{3b^2d^2x^4(3bc-2ad)+18bdx^2(bc-ad)^2-\frac{6a(ad-bc)^3}{a+bx^2}+6(bc-4ad)(bc-ad)^2\log(a+bx^2)+2b^3d^3x^6}{12b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (18*b*d*(b*c - a*d)^2*x^2 + 3*b^2*d^2*(3*b*c - 2*a*d)*x^4 + 2*b^3
*d^3*x^6 - (6*a*(-(b*c) + a*d)^3)/(a + b*x^2) + 6*(b*c - 4*a*d)*(
b*c - a*d)^2*Log[a + b*x^2])/(12*b^5)
```

Maple [B] time = 0.017, size = 229, normalized size = 2.

$$\frac{d^3x^6}{6\,b^2}-\frac{d^3x^4a}{2\,b^3}+\frac{3\,d^2x^4c}{4\,b^2}+\frac{3\,d^3x^2a^2}{2\,b^4}-3\,\frac{d^2x^2ac}{b^3}+\frac{3\,dx^2c^2}{2\,b^2}-2\,\frac{\ln\left(bx^2+a\right)a^3d^3}{b^5}+\frac{9\,\ln\left(bx^2+a\right)a^2d^2c}{2\,b^4}-3\,\frac{\ln\left(bx^2+a\right)adc^2}{b^3}+\frac{\ln\left(bx^2+a\right)c^3}{2\,b^2}-\frac{a^4d^3}{2\,b^5\left(bx^2+a\right)}+\frac{3\,a^3d^2c}{2\,b^4\left(bx^2+a\right)}-\frac{3\,a^2c^2d}{2\,b^3\left(bx^2+a\right)}+\frac{ac^3}{2\,b^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/6*d^3*x^6/b^2-1/2*d^3/b^3*x^4*a+3/4*d^2/b^2*x^4*c+3/2*d^3/b^4*x
^2*a^2-3*d^2/b^3*x^2*a*c+3/2*d/b^2*x^2*c^2-2/b^5*ln(b*x^2+a)*a^3*
d^3+9/2/b^4*ln(b*x^2+a)*a^2*d^2*c-3/b^3*ln(b*x^2+a)*a*d*c^2+1/2/b
^2*ln(b*x^2+a)*c^3-1/2/b^5*a^4/(b*x^2+a)*d^3+3/2/b^4*a^3/(b*x^2+a
)*d^2*c-3/2/b^3*a^2/(b*x^2+a)*d*c^2+1/2/b^2*a/(b*x^2+a)*c^3
```

Maxima [A] time = 1.34705, size = 235, normalized size = 2.01

$$\frac{ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3}{2\left(b^6x^2+ab^5\right)}+\frac{2\,b^2d^3x^6+3\left(3\,b^2cd^2-2\,abd^3\right)x^4+18\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2}{12\,b^4}+\frac{\left(b^3c^3-6\,ab^2c^2d+9\,a^2bcd^2-4\,a^3d^3\right)\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)/(b^6*
x^2 + a*b^5) + 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - 2*a*b*d^3)*
x^4 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)/b^4 + 1/2*(b^3*
c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*log(b*x^2 + a)/b
^5
```

Fricas [A] time = 0.225204, size = 343, normalized size = 2.93

$$\frac{2\,b^4d^3x^8+6\,ab^3c^3-18\,a^2b^2c^2d+18\,a^3bcd^2-6\,a^4d^3+\left(9\,b^4cd^2-4\,ab^3d^3\right)x^6+3\left(6\,b^4c^2d-9\,ab^3cd^2+4\,a^2b^2d^3\right)x^4+18\,(a}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^4*d^3*x^8 + 6*a*b^3*c^3 - 18*a^2*b^2*c^2*d + 18*a^3*b*c
*d^2 - 6*a^4*d^3 + (9*b^4*c*d^2 - 4*a*b^3*d^3)*x^6 + 3*(6*b^4*c^2
*d - 9*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x^4 + 18*(a*b^3*c^2*d - 2*a^2
*b^2*c*d^2 + a^3*b*d^3)*x^2 + 6*(a*b^3*c^3 - 6*a^2*b^2*c^2*d + 9*
a^3*b*c*d^2 - 4*a^4*d^3 + (b^4*c^3 - 6*a*b^3*c^2*d + 9*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x^2)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] time = 6.1864, size = 158, normalized size = 1.35

$$-\frac{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}{2ab^5+2b^6x^2}+\frac{d^3x^6}{6b^2}-\frac{x^4\left(2ad^3-3bcd^2\right)}{4b^3}+\frac{x^2\left(3a^2d^3-6abcd^2+3b^2c^2d\right)}{2b^4}-\frac{(ad-bc)^2\left(4ad-bc\right)\log\left(a+bx^2\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**2*b**2*c**2*d - a*b**3*c**3)
/(2*a*b**5 + 2*b**6*x**2) + d**3*x**6/(6*b**2) - x**4*(2*a*d**3 -
 3*b*c*d**2)/(4*b**3) + x**2*(3*a**2*d**3 - 6*a*b*c*d**2 + 3*b**2
*c**2*d)/(2*b**4) - (a*d - b*c)**2*(4*a*d - b*c)*log(a + b*x**2)/
(2*b**5)
```

GIAC/XCAS [A] time = 0.247111, size = 336, normalized size = 2.87

$$\frac{\frac{\left(2\,d^3+\frac{3\left(3\,b^2cd^2-4\,abd^3\right)}{\left(bx^2+a\right)b}+\frac{18\left(b^4c^2d-3\,ab^3cd^2+2\,a^2b^2d^3\right)}{\left(bx^2+a\right)^2b^2}\right)\left(bx^2+a\right)^3}{b^4}-\frac{6\left(b^3c^3-6\,ab^2c^2d+9\,a^2bcd^2-4\,a^3d^3\right)\ln\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^4}+\frac{6\left(\frac{ab^6c^3}{bx^2+a}-\frac{3\,a^2b^5c^2d}{bx^2+a}+\frac{3\,a^3b^4cd^2}{bx^2+a}\right.}{b^7}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/12*((2*d^3 + 3*(3*b^2*c*d^2 - 4*a*b*d^3)/((b*x^2 + a)*b) + 18*(
b^4*c^2*d - 3*a*b^3*c*d^2 + 2*a^2*b^2*d^3)/((b*x^2 + a)^2*b^2))*(
b*x^2 + a)^3/b^4 - 6*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4
*a^3*d^3)*ln(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b^4 + 6*(a*b^
6*c^3/(b*x^2 + a) - 3*a^2*b^5*c^2*d/(b*x^2 + a) + 3*a^3*b^4*c*d^2
/(b*x^2 + a) - a^4*b^3*d^3/(b*x^2 + a))/b^7)/b
```

3.282 $\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=147

$$\frac{dx\left(105a^2d^2-190abcd+81b^2c^2\right)}{30b^4}+\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}+\frac{dx\left(c+dx^2\right)(33bc-35ad)}{30b^3}-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}$$

```
[Out] (d*(81*b^2*c^2 - 190*a*b*c*d + 105*a^2*d^2)*x)/(30*b^4) + (d*(33*
b*c - 35*a*d)*x*(c + d*x^2))/(30*b^3) + (7*d*x*(c + d*x^2)^2)/(10
*b^2) - (x*(c + d*x^2)^3)/(2*b*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c
 - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rubi [A] time = 0.44753, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{dx\left(105a^2d^2-190abcd+81b^2c^2\right)}{30b^4}+\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}+\frac{dx\left(c+dx^2\right)(33bc-35ad)}{30b^3}-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(81*b^2*c^2 - 190*a*b*c*d + 105*a^2*d^2)*x)/(30*b^4) + (d*(33*
b*c - 35*a*d)*x*(c + d*x^2))/(30*b^3) + (7*d*x*(c + d*x^2)^2)/(10
*b^2) - (x*(c + d*x^2)^3)/(2*b*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c
 - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rubi in Sympy [A] time = 69.5053, size = 150, normalized size = 1.02

$$-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}-\frac{dx\left(c(7ad-5bc)+dx^2(35ad-33bc)\right)}{30b^3}+\frac{dx\left(105a^2d^2-218abcd+109b^2c^2\right)}{30b^4}-\frac{(ad-bc)^2(7ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(c + d*x**2)**3/(2*b*(a + b*x**2)) + 7*d*x*(c + d*x**2)**2/(10
*b**2) - d*x*(c*(7*a*d - 5*b*c) + d*x**2*(35*a*d - 33*b*c))/(30*b
**3) + d*x*(105*a**2*d**2 - 218*a*b*c*d + 109*b**2*c**2)/(30*b**4
) - (a*d - b*c)**2*(7*a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(
a)*b**(9/2))
```

Mathematica [A] time = 0.115066, size = 125, normalized size = 0.85

$$\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}-\frac{x(bc-ad)^3}{2b^4\left(a+bx^2\right)}+\frac{3dx(bc-ad)^2}{b^4}+\frac{d^2x^3(3bc-2ad)}{3b^3}+\frac{d^3x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x)/b^4 + (d^2*(3*b*c - 2*a*d)*x^3)/(3*b^3) + (
d^3*x^5)/(5*b^2) - ((b*c - a*d)^3*x)/(2*b^4*(a + b*x^2)) + ((b*c
- 7*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^
(9/2))
```

Maple [A] time = 0.013, size = 247, normalized size = 1.7

$$\frac{d^3x^5}{5\,b^2}-\frac{2\,d^3x^3a}{3\,b^3}+\frac{d^2x^3c}{b^2}+3\,\frac{a^2d^3x}{b^4}-6\,\frac{acd^2x}{b^3}+3\,\frac{c^2dx}{b^2}+\frac{xa^3d^3}{2\,b^4\,(bx^2+a)} -\frac{3\,xa^2cd^2}{2\,b^3\,(bx^2+a)}+\frac{3\,axc^2d}{2\,b^2\,(bx^2+a)}-\frac{xc^3}{2\,b\,(bx^2+a)}-\frac{7\,a^3d^3}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} +\frac{15\,a^2cd^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{9\,ac^2d}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^3}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^3/b^2*x^5-2/3*d^3/b^3*x^3*a+d^2/b^2*x^3*c+3*d^3/b^4*a^2*x-6
*d^2/b^3*a*c*x+3*d/b^2*c^2*x+1/2/b^4*x/(b*x^2+a)*a^3*d^3-3/2/b^3*
x/(b*x^2+a)*a^2*c*d^2+3/2/b^2*x/(b*x^2+a)*a*c^2*d-1/2/b*x/(b*x^2+
a)*c^3-7/2/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^3*d^3+15/2/b
^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*c*d^2-9/2/b^2/(a*b)^(1
/2)*arctan(x*b/(a*b)^(1/2))*a*c^2*d+1/2/b/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243132, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(ab^3c^3-9\,a^2b^2c^2d+15\,a^3bcd^2-7\,a^4d^3+\left(b^4c^3-9\,ab^3c^2d+15\,a^2b^2cd^2-7\,a^3bd^3\right)x^2\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2}{60\,(b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b^3*c^3 - 9*a^2*b^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*
d^3 + (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^2*b^2*c*d^2 - 7*a^3*b*d^3)*
x^2)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(6*
b^3*d^3*x^7 + 2*(15*b^3*c*d^2 - 7*a*b^2*d^3)*x^5 + 10*(9*b^3*c^2*
d - 15*a*b^2*c*d^2 + 7*a^2*b*d^3)*x^3 - 15*(b^3*c^3 - 9*a*b^2*c^2
*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*x)*sqrt(-a*b))/((b^5*x^2 + a*b^4
```

```
)*sqrt(-a*b)), 1/30*(15*(a*b^3*c^3 - 9*a^2*b^2*c^2*d + 15*a^3*b*c
*d^2 - 7*a^4*d^3 + (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^2*b^2*c*d^2 -
7*a^3*b*d^3)*x^2)*arctan(sqrt(a*b)*x/a) + (6*b^3*d^3*x^7 + 2*(15*
b^3*c*d^2 - 7*a*b^2*d^3)*x^5 + 10*(9*b^3*c^2*d - 15*a*b^2*c*d^2 +
 7*a^2*b*d^3)*x^3 - 15*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2
- 7*a^3*d^3)*x)*sqrt(a*b))/((b^5*x^2 + a*b^4)*sqrt(a*b))]
```

Sympy [A] time = 5.57005, size = 337, normalized size = 2.29

$$\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^2\,(7ad-bc)\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^2(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^2\,(7ad-bc)\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^2(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4}+\frac{d^3x^5}{5b^2}-\frac{x^3\left(2ad^3-3bcd^2\right)}{3b^3}+\frac{x\left(3a^2d^3-6abcd^2+3b^2c^2d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*
a*b**4 + 2*b**5*x**2) + sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d -
 b*c)*log(-a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/
(7*a**3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b**3*c**3) +
x)/4 - sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)*log(a*b**4*
sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/(7*a**3*d**3 - 15*
a**2*b*c*d**2 + 9*a*b**2*c**2*d - b**3*c**3) + x)/4 + d**3*x**5/(
5*b**2) - x**3*(2*a*d**3 - 3*b*c*d**2)/(3*b**3) + x*(3*a**2*d**3
- 6*a*b*c*d**2 + 3*b**2*c**2*d)/b**4
```

GIAC/XCAS [A] time = 0.248581, size = 248, normalized size = 1.69

$$\frac{\left(b^3c^3-9\,ab^2c^2d+15\,a^2bcd^2-7\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}-\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)b^4}+\frac{3\,b^8d^3x^5+15\,b^8cd^2x^3-10\,ab^7d^3x^3+45\,b^8c^2dx-90\,ab^7cd^2x+45\,a^2b^6d^3x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*
x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*
d^3*x^5 + 15*b^8*c*d^2*x^3 - 10*a*b^7*d^3*x^3 + 45*b^8*c^2*d*x -
90*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^10
```

3.283 $\int \frac{x(c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal. Leaf size=88

$$-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) - (b*c - a*
d)^3/(2*b^4*(a + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/(2*
b^4)
```

Rubi [A] time = 0.218004, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) - (b*c - a*
d)^3/(2*b^4*(a + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/(2*
b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^2(2ad-3bc)\int^{x^2}\frac{1}{b^3}\,dx}{2}+\frac{d^3\int^{x^2}x\,dx}{2b^2}+\frac{3d(ad-bc)^2\log(a+bx^2)}{2b^4}+\frac{(ad-bc)^3}{2b^4(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -d**2*(2*a*d - 3*b*c)*Integral(b**(-3), (x, x**2))/2 + d**3*Integ
ral(x, (x, x**2))/(2*b**2) + 3*d*(a*d - b*c)**2*log(a + b*x**2)/(
2*b**4) + (a*d - b*c)**3/(2*b**4*(a + b*x**2))
```

Mathematica [A] time = 0.0741487, size = 127, normalized size = 1.44

$$\frac{3(a^2d^3-2abcd^2+b^2c^2d)\log(a+bx^2)}{2b^4}+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2b^4(a+bx^2)}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) + (-(b^3*c^
3) + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)/(2*b^4*(a + b*x^2))
 + (3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*Log[a + b*x^2])/(2*b^4)
```

Maple [B] time = 0.014, size = 168, normalized size = 1.9

$$\frac{d^3x^4}{4\,b^2}-\frac{ad^3x^2}{b^3}+\frac{3\,d^2x^2c}{2\,b^2}+\frac{3\,\ln\left(bx^2+a\right)d^3a^2}{2\,b^4}-3\,\frac{\ln\left(bx^2+a\right)d^2ca}{b^3}+\frac{3\,\ln\left(bx^2+a\right)dc^2}{2\,b^2}+\frac{a^3d^3}{2\,b^4\left(bx^2+a\right)}-\frac{3\,a^2cd^2}{2\,b^3\left(bx^2+a\right)}+\frac{3\,adc^2}{2\,b^2\left(bx^2+a\right)}-\frac{c^3}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-d^3/b^3*a*x^2+3/2*d^2/b^2*x^2*c+3/2/b^4*ln(b*x^2+
a)*d^3*a^2-3/b^3*ln(b*x^2+a)*d^2*c*a+3/2/b^2*ln(b*x^2+a)*d*c^2+1/
2/b^4/(b*x^2+a)*a^3*d^3-3/2/b^3/(b*x^2+a)*a^2*d^2*c+3/2/b^2/(b*x^
2+a)*a*d*c^2-1/2/b/(b*x^2+a)*c^3
```

Maxima [A] time = 1.35585, size = 167, normalized size = 1.9

$$-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\left(b^5x^2+ab^4\right)}+\frac{bd^3x^4+2\left(3\,bcd^2-2\,ad^3\right)x^2}{4\,b^3}+\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x^2
 + a*b^4) + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - 2*a*d^3)*x^2)/b^3 + 3
/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.216234, size = 244, normalized size = 2.77

$$\frac{b^3d^3x^6-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^4+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x^2+6\left(ab^2c^2d-2\,a^2bcd^2+\right.}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^3*x^6 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*
a^3*d^3 + 3*(2*b^3*c*d^2 - a*b^2*d^3)*x^4 + 2*(3*a*b^2*c*d^2 - 2*
a^2*b*d^3)*x^2 + 6*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*
c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2)*log(b*x^2 + a))/(b^5*x^2
+ a*b^4)
```

Sympy [A] time = 5.27337, size = 112, normalized size = 1.27

$$\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2ab^4+2b^5x^2}+\frac{d^3x^4}{4b^2}-\frac{x^2\left(2ad^3-3bcd^2\right)}{2b^3}+\frac{3d\left(ad-bc\right)^2\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a*
b**4 + 2*b**5*x**2) + d**3*x**4/(4*b**2) - x**2*(2*a*d**3 - 3*b*c
```

```
*d**2)/(2*b**3) + 3*d*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.236639, size = 247, normalized size = 2.81

$$\frac{\left(d^3 + \frac{6\,(b^2cd^2 - abd^3)}{(bx^2+a)b}\right)\left(bx^2 + a\right)^2}{4\,b^4} - \frac{3\,\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\ln\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{2\,b^4} - \frac{\frac{b^5c^3}{bx^2+a} - \frac{3\,ab^4c^2d}{bx^2+a} + \frac{3\,a^2b^3cd^2}{bx^2+a} - \frac{a^3b^2d^3}{bx^2+a}}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x^2 + a)*b))*(b*x^2 + a)^2
/b^4 - 3/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*ln(abs(b*x^2 + a)/
((b*x^2 + a)^2*abs(b)))/b^4 - 1/2*(b^5*c^3/(b*x^2 + a) - 3*a*b^4*
c^2*d/(b*x^2 + a) + 3*a^2*b^3*c*d^2/(b*x^2 + a) - a^3*b^2*d^3/(b*
x^2 + a))/b^6
```

3.284 $\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=106

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*
x)/(2*a*b^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi [A] time = 0.204906, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*
x)/(2*a*b^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^2\left(2ad-3bc\right)\int \frac{1}{b^3}\,dx+\frac{d^3x^3}{3b^2}-\frac{x\left(ad-bc\right)^3}{2ab^3\left(a+bx^2\right)}+\frac{\left(ad-bc\right)^2\left(5ad+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -d**2*(2*a*d - 3*b*c)*Integral(b**(-3), x) + d**3*x**3/(3*b**2) -
 x*(a*d - b*c)**3/(2*a*b**3*(a + b*x**2)) + (a*d - b*c)**2*(5*a*d
 + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*b**(7/2))
```

Mathematica [A] time = 0.100068, size = 106, normalized size = 1.

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*
x)/(2*a*b^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Maple [B] time = 0., size = 205, normalized size = 1.9

$$\frac{d^3x^3}{3\,b^2}-2\,\frac{d^3ax}{b^3}+3\,\frac{d^2xc}{b^2}-\frac{a^2xd^3}{2\,b^3\,(bx^2+a)}+\frac{3\,acxd^2}{2\,b^2\,(bx^2+a)}-\frac{3\,xc^2d}{2\,b\,(bx^2+a)}+\frac{xc^3}{2\,a\,(bx^2+a)}+\frac{5\,a^2d^3}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{9\,acd^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^2d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^3}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^3*x^3/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/2/b^3*x*a^2/(b*x^2+
a)*d^3+3/2/b^2*x*a/(b*x^2+a)*c*d^2-3/2/b*x/(b*x^2+a)*c^2*d+1/2*x/
a/(b*x^2+a)*c^3+5/2/b^3*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d
^3-9/2/b^2*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^2+3/2/b/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d+1/2/a/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239902, size = 1, normalized size = 0.01

$$\left[\frac{3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\,a^3bd^3\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,ab^2d\right.}{12\left(ab^4x^2+a^2b^3\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3
 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)
*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(2*a*b^2
*d^3*x^5 + 2*(9*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 + 3*(b^3*c^3 - 3*a
*b^2*c^2*d + 9*a^2*b*c*d^2 - 5*a^3*d^3)*x)*sqrt(-a*b))/((a*b^4*x^
2 + a^2*b^3)*sqrt(-a*b)), 1/6*(3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9
*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c
*d^2 + 5*a^3*b*d^3)*x^2)*arctan(sqrt(a*b)*x/a) + (2*a*b^2*d^3*x^5
 + 2*(9*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 + 3*(b^3*c^3 - 3*a*b^2*c^2
*d + 9*a^2*b*c*d^2 - 5*a^3*d^3)*x)*sqrt(a*b))/((a*b^4*x^2 + a^2*b
^3)*sqrt(a*b))]
```

Sympy [A] time = 4.90501, size = 313, normalized size = 2.95

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2a^2b^3+2ab^4x^2}-\frac{\sqrt{-\frac{1}{a^3b^7}}\left(ad-bc\right)^2\left(5ad+bc\right)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^7}}\left(ad-bc\right)^2\left(5ad+bc\right)\log\left(\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4}+\frac{d^3x^3}{3b^2}-\frac{x\left(2ad^3-3bcd^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2
*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(a*d - b*c)**2
*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2
*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d +
 b**3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d +
 b*c)*log(a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d +
b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3
) + x)/4 + d**3*x**3/(3*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

GIAC/XCAS [A] time = 0.26075, size = 205, normalized size = 1.93

$$\frac{\left(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)ab^3}+\frac{b^4d^3x^3+9\,b^4cd^2x-6\,ab^3d^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d
*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*
d^3*x^3 + 9*b^4*c*d^2*x - 6*a*b^3*d^3*x)/b^6
```

3.285 $\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

```
[Out] (d^3*x^2)/(2*b^2) + (b*c - a*d)^3/(2*a*b^3*(a + b*x^2)) + (c^3*Lo
g[x])/a^2 - ((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(2*a^2*b
^3)
```

Rubi [A] time = 0.203636, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (d^3*x^2)/(2*b^2) + (b*c - a*d)^3/(2*a*b^3*(a + b*x^2)) + (c^3*Lo
g[x])/a^2 - ((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(2*a^2*b
^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^3\int^{x^2}\frac{1}{b^2}\,dx}{2}-\frac{(ad-bc)^3}{2ab^3\left(a+bx^2\right)}+\frac{c^3\log\left(x^2\right)}{2a^2}-\frac{(ad-bc)^2\,(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x/(b*x**2+a)**2,x)
```

```
[Out] d**3*Integral(b**(-2), (x, x**2))/2 - (a*d - b*c)**3/(2*a*b**3*(a
 + b*x**2)) + c**3*log(x**2)/(2*a**2) - (a*d - b*c)**2*(2*a*d + b
*c)*log(a + b*x**2)/(2*a**2*b**3)
```

Mathematica [A] time = 0.176408, size = 111, normalized size = 1.26

$$\frac{\frac{\frac{a\left(-a^3d^3+a^2bd^2\left(3c+dx^2\right)+ab^2\left(d^3x^4-3c^2d\right)+b^3c^3\right)}{a+bx^2}-(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{b^3}+2c^3\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^3*Log[x] + ((a*(b^3*c^3 - a^3*d^3 + a^2*b*d^2*(3*c + d*x^2)
+ a*b^2*(-3*c^2*d + d^3*x^4)))/(a + b*x^2) - (b*c - a*d)^2*(b*c +
 2*a*d)*Log[a + b*x^2])/b^3)/(2*a^2)
```

Maple [A] time = 0.023, size = 146, normalized size = 1.7

$$\frac{d^3x^2}{2\,b^2}+\frac{c^3\ln(x)}{a^2}-\frac{a\ln\left(bx^2+a\right)d^3}{b^3}+\frac{3\,\ln\left(bx^2+a\right)d^2c}{2\,b^2}-\frac{\ln\left(bx^2+a\right)c^3}{2\,a^2}-\frac{a^2d^3}{2\,b^3\,(bx^2+a)}+\frac{3\,ad^2c}{2\,b^2\,(bx^2+a)}-\frac{3\,dc^2}{2\,b\,(bx^2+a)}+\frac{c^3}{2\,a\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x^2+c)^3/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d^3*x^2/b^2+c^3*ln(x)/a^2-a/b^3*ln(b*x^2+a)*d^3+3/2/b^2*ln(b*
x^2+a)*d^2*c-1/2/a^2*ln(b*x^2+a)*c^3-1/2*a^2/b^3/(b*x^2+a)*d^3+3/
2*a/b^2/(b*x^2+a)*d^2*c-3/2/b/(b*x^2+a)*d*c^2+1/2/a/(b*x^2+a)*c^3
```

Maxima [A] time = 1.35421, size = 165, normalized size = 1.88

$$\frac{d^3x^2}{2\,b^2}+\frac{c^3\log\left(x^2\right)}{2\,a^2}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\,(ab^4x^2+a^2b^3)}-\frac{\left(b^3c^3-3\,a^2bcd^2+2\,a^3d^3\right)\log\left(bx^2+a\right)}{2\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(a*b^4*x^2 + a^2*b^3) - 1/2*(b^3*
c^3 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*log(b*x^2 + a)/(a^2*b^3)
```

Fricas [A] time = 0.239041, size = 240, normalized size = 2.73

$$\frac{a^2b^2d^3x^4+a^3bd^3x^2+ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3-\left(ab^3c^3-3\,a^3bcd^2+2\,a^4d^3+\left(b^4c^3-3\,a^2b^2cd^2+2\,a^3bd^3\right)x^2\right)}{2\,(a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b^2*d^3*x^4 + a^3*b*d^3*x^2 + a*b^3*c^3 - 3*a^2*b^2*c^2*
d + 3*a^3*b*c*d^2 - a^4*d^3 - (a*b^3*c^3 - 3*a^3*b*c*d^2 + 2*a^4*
d^3 + (b^4*c^3 - 3*a^2*b^2*c*d^2 + 2*a^3*b*d^3)*x^2)*log(b*x^2 +
a) + 2*(b^4*c^3*x^2 + a*b^3*c^3)*log(x))/(a^2*b^4*x^2 + a^3*b^3)
```

Sympy [A] time = 10.2533, size = 110, normalized size = 1.25

$$-\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2a^2b^3+2ab^4x^2}+\frac{d^3x^2}{2b^2}+\frac{c^3\log(x)}{a^2}-\frac{(ad-bc)^2\,(2ad+bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x**2+c)**3/x/(b*x**2+a)**2,x)
```

```
[Out] -(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a
**2*b**3 + 2*a*b**4*x**2) + d**3*x**2/(2*b**2) + c**3*log(x)/a**2
 - (a*d - b*c)**2*(2*a*d + b*c)*log(a/b + x**2)/(2*a**2*b**3)
```

GIAC/XCAS [A] time = 0.232261, size = 203, normalized size = 2.31

$$\frac{d^3x^2}{2\,b^2} + \frac{c^3\ln\left(x^2\right)}{2\,a^2} - \frac{\left(b^3c^3 - 3\,a^2bcd^2 + 2\,a^3d^3\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,a^2b^3} + \frac{b^4c^3x^2 - 3\,a^2b^2cd^2x^2 + 2\,a^3bd^3x^2 + 2\,ab^3c^3 - 3\,a^2b^2c^2d + a^4d^3}{2\,(bx^2 + a)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*ln(x^2)/a^2 - 1/2*(b^3*c^3 - 3*a^2*b*c*
d^2 + 2*a^3*d^3)*ln(abs(b*x^2 + a))/(a^2*b^3) + 1/2*(b^4*c^3*x^2
- 3*a^2*b^2*c*d^2*x^2 + 2*a^3*b*d^3*x^2 + 2*a*b^3*c^3 - 3*a^2*b^2
*c^2*d + a^4*d^3)/((b*x^2 + a)*a^2*b^3)
```

3.286 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{3(bc-ad)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2abx(a+bx^2)}$$

```
[Out] -(c^2*(3*b*c - a*d))/(2*a^2*b*x) - (d^2*(b*c - 3*a*d)*x)/(2*a*b^2
) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) - (3*(b*c -
 a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/
2))
```

Rubi [A] time = 0.334847, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{3(bc-ad)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2abx(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(3*b*c - a*d))/(2*a^2*b*x) - (d^2*(b*c - 3*a*d)*x)/(2*a*b^2
) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) - (3*(b*c -
 a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/
2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^2(3ad-bc)\int\frac{1}{b}\,dx}{2ab}-\frac{(c+dx^2)^2(ad-bc)}{2abx(a+bx^2)}+\frac{c^2(ad-3bc)}{2a^2bx}-\frac{3(ad-bc)^2(ad+bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**2/(b*x**2+a)**2,x)
```

```
[Out] d**2*(3*a*d - b*c)*Integral(1/b, x)/(2*a*b) - (c + d*x**2)**2*(a*
d - b*c)/(2*a*b*x*(a + b*x**2)) + c**2*(a*d - 3*b*c)/(2*a**2*b*x)
 - 3*(a*d - b*c)**2*(a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/
2)*b**(5/2))
```

Mathematica [A] time = 0.0973548, size = 94, normalized size = 0.72

$$-\frac{3(ad-bc)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}+\frac{x(ad-bc)^3}{2a^2b^2(a+bx^2)}-\frac{c^3}{a^2x}+\frac{d^3x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^3/(a^2*x)) + (d^3*x)/b^2 + ((-(b*c) + a*d)^3*x)/(2*a^2*b^2*(a
 + b*x^2)) - (3*(-(b*c) + a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Maple [A] time = 0.017, size = 189, normalized size = 1.4

$$\frac{d^3x}{b^2}-\frac{c^3}{a^2x}+\frac{axd^3}{2\,b^2\,(bx^2+a)}-\frac{3\,cxd^2}{2\,b\,(bx^2+a)}+\frac{3\,xc^2d}{2\,a\,(bx^2+a)}-\frac{bxc^3}{2\,a^2\,(bx^2+a)}-\frac{3\,ad^3}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,cd^2}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^2d}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,bc^3}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^2/(b*x^2+a)^2,x)
```

```
[Out] d^3*x/b^2-c^3/x/a^2+1/2*a/b^2*x/(b*x^2+a)*d^3-3/2/b*x/(b*x^2+a)*c
*d^2+3/2/a*x/(b*x^2+a)*c^2*d-1/2/a^2*b*x/(b*x^2+a)*c^3-3/2*a/b^2/
(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^3+3/2/b/(a*b)^(1/2)*arctan(
x*b/(a*b)^(1/2))*c*d^2+3/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*
c^2*d-3/2/a^2*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240846, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(b^4c^3-ab^3c^2d-a^2b^2cd^2+a^3bd^3\right)x^3+\left(ab^3c^3-a^2b^2c^2d-a^3bcd^2+a^4d^3\right)x\right)\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,a^2bd^3x^4\right.}{4\left(a^2b^3x^3+a^3b^2x\right)\sqrt{-ab}}\right.$$

$$-\frac{3\left(\left(b^4c^3-ab^3c^2d-a^2b^2cd^2+a^3bd^3\right)x^3+\left(ab^3c^3-a^2b^2c^2d-a^3bcd^2+a^4d^3\right)x\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(2\,a^2bd^3x^4-2\,ab^2c^3-\right.}{2\left(a^2b^3x^3+a^3b^2x\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 + a^3*b*d^3)*x^3
+ (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*log(-(2*
a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(2*a^2*b*d^3*x^4
 - 2*a*b^2*c^3 - 3*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3
)*x^2)*sqrt(-a*b))/((a^2*b^3*x^3 + a^3*b^2*x)*sqrt(-a*b)), -1/2*(
3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 + a^3*b*d^3)*x^3 + (a*b
^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*arctan(sqrt(a*
b)*x/a) - (2*a^2*b*d^3*x^4 - 2*a*b^2*c^3 - 3*(b^3*c^3 - a*b^2*c^2
*d + a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(a*b))/((a^2*b^3*x^3 + a^3*b
^2*x)*sqrt(a*b))]
```

Sympy [A] time = 7.05431, size = 309, normalized size = 2.36

$$\frac{3\sqrt{-\frac{1}{a^5b^5}}\,(ad-bc)^2\,(ad+bc)\log\left(-\frac{3a^3b^2\sqrt{-\frac{1}{a^5b^5}}(ad-bc)^2(ad+bc)}{3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3}+x\right)}{4} - \frac{3\sqrt{-\frac{1}{a^5b^5}}\,(ad-bc)^2\,(ad+bc)\log\left(\frac{3a^3b^2\sqrt{-\frac{1}{a^5b^5}}(ad-bc)^2(ad+bc)}{3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3}+x\right)}{4} + \frac{-2ab^2c^3+x^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-3b^3c^3\right)}{2a^3b^2x+2a^2b^3x^3}+\frac{d^3x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)*log(-3*a**3*b**
2*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)/(3*a**3*d**3 -
3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + 3*b**3*c**3) + x)/4 - 3*sqrt(
-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)*log(3*a**3*b**2*sqrt(-
1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)/(3*a**3*d**3 - 3*a**2*b
*c*d**2 - 3*a*b**2*c**2*d + 3*b**3*c**3) + x)/4 + (-2*a*b**2*c**3
 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 3*b**3*c
**3))/(2*a**3*b**2*x + 2*a**2*b**3*x**3) + d**3*x/b**2
```

GIAC/XCAS [A] time = 0.247841, size = 193, normalized size = 1.47

$$\frac{d^3x}{b^2} - \frac{3\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2b^2} - \frac{3\,b^3c^3x^2-3\,ab^2c^2dx^2+3\,a^2bcd^2x^2-a^3d^3x^2+2\,ab^2c^3}{2\,(bx^3+ax)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] d^3*x/b^2 - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*a
rctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c^3*x^2 - 3
*a*b^2*c^2*d*x^2 + 3*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)
/((b*x^3 + a*x)*a^2*b^2)
```

3.287 $\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2} \, dx$

Optimal. Leaf size=98

$$\frac{(bc-ad)^2(ad+2bc)\log(a+bx^2)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{(bc-ad)^3}{2a^2b^2(a+bx^2)}-\frac{c^3}{2a^2x^2}$$

```
[Out] -c^3/(2*a^2*x^2) - (b*c - a*d)^3/(2*a^2*b^2*(a + b*x^2)) - (c^2*(
2*b*c - 3*a*d)*Log[x])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a +
 b*x^2])/(2*a^3*b^2)
```

Rubi [A] time = 0.243098, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(bc-ad)^2(ad+2bc)\log(a+bx^2)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{(bc-ad)^3}{2a^2b^2(a+bx^2)}-\frac{c^3}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c^3/(2*a^2*x^2) - (b*c - a*d)^3/(2*a^2*b^2*(a + b*x^2)) - (c^2*(
2*b*c - 3*a*d)*Log[x])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a +
 b*x^2])/(2*a^3*b^2)
```

Rubi in Sympy [A] time = 41.4223, size = 92, normalized size = 0.94

$$-\frac{c^3}{2a^2x^2}+\frac{(ad-bc)^3}{2a^2b^2(a+bx^2)}+\frac{c^2(3ad-2bc)\log(x^2)}{2a^3}+\frac{(ad-bc)^2(ad+2bc)\log(a+bx^2)}{2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**3/(b*x**2+a)**2,x)
```

```
[Out] -c**3/(2*a**2*x**2) + (a*d - b*c)**3/(2*a**2*b**2*(a + b*x**2)) +
 c**2*(3*a*d - 2*b*c)*log(x**2)/(2*a**3) + (a*d - b*c)**2*(a*d +
2*b*c)*log(a + b*x**2)/(2*a**3*b**2)
```

Mathematica [A] time = 0.157241, size = 87, normalized size = 0.89

$$\frac{\frac{a(ad-bc)^3}{b^2(a+bx^2)}+\frac{(bc-ad)^2(ad+2bc)\log(a+bx^2)}{b^2}+2c^2\log(x)(3ad-2bc)-\frac{ac^3}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c^3)/x^2) + (a*(-(b*c) + a*d)^3)/(b^2*(a + b*x^2)) + 2*c^2*
(-2*b*c + 3*a*d)*Log[x] + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*
x^2])/b^2)/(2*a^3)
```

Maple [A] time = 0.022, size = 156, normalized size = 1.6

$$-\frac{c^3}{2\,a^2x^2}+3\,\frac{c^2\ln(x)\,d}{a^2}-2\,\frac{c^3\ln(x)\,b}{a^3}+\frac{\ln\left(bx^2+a\right)d^3}{2\,b^2}-\frac{3\,\ln\left(bx^2+a\right)dc^2}{2\,a^2}+\frac{b\ln\left(bx^2+a\right)c^3}{a^3}+\frac{ad^3}{2\,b^2\left(bx^2+a\right)}-\frac{3\,cd^2}{2\,b\left(bx^2+a\right)}+\frac{3\,c^2d}{2\,a\left(bx^2+a\right)}-\frac{c^3b}{2\,a^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c^3/a^2/x^2+3*c^2/a^2*ln(x)*d-2*c^3/a^3*ln(x)*b+1/2/b^2*ln(b
*x^2+a)*d^3-3/2/a^2*ln(b*x^2+a)*d*c^2+1/a^3*b*ln(b*x^2+a)*c^3+1/2
*a/b^2/(b*x^2+a)*d^3-3/2/b/(b*x^2+a)*d^2*c+3/2/a/(b*x^2+a)*d*c^2-
1/2/a^2*b/(b*x^2+a)*c^3
```

Maxima [A] time = 1.35682, size = 190, normalized size = 1.94

$$-\frac{ab^2c^3+\left(2\,b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^2}{2\left(a^2b^3x^4+a^3b^2x^2\right)}-\frac{\left(2\,bc^3-3\,ac^2d\right)\log\left(x^2\right)}{2\,a^3}+\frac{\left(2\,b^3c^3-3\,ab^2c^2d+a^3d^3\right)\log\left(bx^2+a\right)}{2\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^2*c^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*x^2)/(a^2*b^3*x^4 + a^3*b^2*x^2) - 1/2*(2*b*c^3 - 3*a*c^2*
d)*log(x^2)/a^3 + 1/2*(2*b^3*c^3 - 3*a*b^2*c^2*d + a^3*d^3)*log(b
*x^2 + a)/(a^3*b^2)
```

Fricas [A] time = 0.239245, size = 282, normalized size = 2.88

$$-\frac{a^2b^2c^3+\left(2\,ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)x^2-\left(\left(2\,b^4c^3-3\,ab^3c^2d+a^3bd^3\right)x^4+\left(2\,ab^3c^3-3\,a^2b^2c^2d+a^4d^3\right)x^2\right)}{2\left(a^3b^3x^4+a^4b^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^
2 - a^4*d^3)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 +
 (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + a^4*d^3)*x^2)*log(b*x^2 + a) +
2*((2*b^4*c^3 - 3*a*b^3*c^2*d)*x^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2
*d)*x^2)*log(x))/(a^3*b^3*x^4 + a^4*b^2*x^2)
```

Sympy [A] time = 13.5534, size = 128, normalized size = 1.31

$$\frac{-ab^2c^3+x^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-2b^3c^3\right)}{2a^3b^2x^2+2a^2b^3x^4}+\frac{c^2\left(3ad-2bc\right)\log\left(x\right)}{a^3}+\frac{\left(ad-bc\right)^2\left(ad+2bc\right)\log\left(\frac{a}{b}+x^2\right)}{2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2
*d - 2*b**3*c**3))/(2*a**3*b**2*x**2 + 2*a**2*b**3*x**4) + c**2*(
3*a*d - 2*b*c)*log(x)/a**3 + (a*d - b*c)**2*(a*d + 2*b*c)*log(a/b
 + x**2)/(2*a**3*b**2)
```

GIAC/XCAS [A] time = 0.251165, size = 212, normalized size = 2.16

$$-\frac{\left(2\,bc^3-3\,ac^2d\right)\ln\left(x^2\right)}{2\,a^3}+\frac{\left(2\,b^3c^3-3\,ab^2c^2d+a^3d^3\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^3b^2}-\frac{a^2bd^3x^4+4\,b^3c^3x^2-6\,ab^2c^2dx^2+6\,a^2bcd^2x^2-a^3d^3x^2+2\,ab^2c^3}{4\,(bx^4+ax^2)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c^3 - 3*a*c^2*d)*ln(x^2)/a^3 + 1/2*(2*b^3*c^3 - 3*a*b^2
*c^2*d + a^3*d^3)*ln(abs(b*x^2 + a))/(a^3*b^2) - 1/4*(a^2*b*d^3*x
^4 + 4*b^3*c^3*x^2 - 6*a*b^2*c^2*d*x^2 + 6*a^2*b*c*d^2*x^2 - a^3*
d^3*x^2 + 2*a*b^2*c^3)/((b*x^4 + a*x^2)*a^2*b^2)
```

3.288 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=147

$$\frac{(bc-ad)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

```
[Out] -(c^2*(5*b*c - 3*a*d))/(6*a^2*b*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d
+ 2*a^2*d^2))/(2*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^
3*(a + b*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.351205, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{(bc-ad)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(5*b*c - 3*a*d))/(6*a^2*b*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d
+ 2*a^2*d^2))/(2*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^
3*(a + b*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rubi in Sympy [A] time = 56.6265, size = 131, normalized size = 0.89

$$-\frac{\left(c+dx^2\right)^2(ad-bc)}{2abx^3\left(a+bx^2\right)}+\frac{c^2(3ad-5bc)}{6a^2bx^3}+\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{(ad-bc)^2(ad+5bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{7}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**4/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**2*(a*d - b*c)/(2*a*b*x**3*(a + b*x**2)) + c**2*(3*
a*d - 5*b*c)/(6*a**2*b*x**3) + c*(2*a**2*d**2 - 9*a*b*c*d + 5*b**
2*c**2)/(2*a**3*b*x) + (a*d - b*c)**2*(a*d + 5*b*c)*atan(sqrt(b)*
x/sqrt(a))/(2*a**(7/2)*b**(3/2))
```

Mathematica [A] time = 0.102152, size = 109, normalized size = 0.74

$$\frac{(ad-bc)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(3ad-2bc)}{a^3x}-\frac{x(ad-bc)^3}{2a^3b\left(a+bx^2\right)}-\frac{c^3}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c^3/(3*a^2*x^3) - (c^2*(-2*b*c + 3*a*d))/(a^3*x) - ((-(b*c) + a*
d)^3*x)/(2*a^3*b*(a + b*x^2)) + ((-(b*c) + a*d)^2*(5*b*c + a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Maple [A] time = 0.019, size = 209, normalized size = 1.4

$$-\frac{c^3}{3\,a^2x^3}-3\,\frac{c^2d}{a^2x}+2\,\frac{c^3b}{a^3x}-\frac{xd^3}{2\,b\,(bx^2+a)}+\frac{3\,cxd^2}{2\,a\,(bx^2+a)}-\frac{3\,bxc^2d}{2\,a^2\,(bx^2+a)}+\frac{xb^2c^3}{2\,a^3\,(bx^2+a)}+\frac{d^3}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,cd^2}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{9\,bdc^2}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^2c^3}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3*c^3/a^2/x^3-3*c^2/a^2/x*d+2*c^3/a^3/x*b-1/2/b*x/(b*x^2+a)*d^
3+3/2/a*x/(b*x^2+a)*c*d^2-3/2/a^2*b*x/(b*x^2+a)*c^2*d+1/2/a^3*b^2
*x/(b*x^2+a)*c^3+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^3+3/
2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^2-9/2/a^2*b/(a*b)^(1/
2)*arctan(x*b/(a*b)^(1/2))*c^2*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(x
*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241413, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(5\,b^4c^3-9\,ab^3c^2d+3\,a^2b^2cd^2+a^3bd^3\right)x^5+\left(5\,ab^3c^3-9\,a^2b^2c^2d+3\,a^3bcd^2+a^4d^3\right)x^3\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)-2\,(2}{12\,(a^3b^2x^5+a^4bx^3)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((5*b^4*c^3 - 9*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + a^3*b*d^
3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 + a^4*d^3
)*x^3)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(2
*a^2*b*c^3 - 3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*x^4 - 2*(5*a*b^2*c^3 - 9*a^2*b*c^2*d)*x^2)*sqrt(-a*b))/((a^3*
b^2*x^5 + a^4*b*x^3)*sqrt(-a*b)), 1/6*(3*((5*b^4*c^3 - 9*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b^2
*c^2*d + 3*a^3*b*c*d^2 + a^4*d^3)*x^3)*arctan(sqrt(a*b)*x/a) - (2
*a^2*b*c^3 - 3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*x^4 - 2*(5*a*b^2*c^3 - 9*a^2*b*c^2*d)*x^2)*sqrt(a*b))/((a^3*b
^2*x^5 + a^4*b*x^3)*sqrt(a*b))]
```

Sympy [A] time = 8.50912, size = 321, normalized size = 2.18

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\left(ad-bc\right)^2\left(ad+5bc\right)\log\left(-\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\left(ad-bc\right)^2\left(ad+5bc\right)\log\left(\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}-\frac{2a^2bc^3+x^4\left(3a^3d^3-9a^2bcd^2+27ab^2c^2d-15b^3c^3\right)+x^2\left(18a^2bc^2d-10ab^2c^3\right)}{6a^4bx^3+6a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-a**4*b*sq
rt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a*
*2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(a*
*7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)*log(a**4*b*sqrt(-1/(a**7*b
**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 -
 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 - (2*a**2*b*c**3 + x**4*(3
*a**3*d**3 - 9*a**2*b*c*d**2 + 27*a*b**2*c**2*d - 15*b**3*c**3) +
 x**2*(18*a**2*b*c**2*d - 10*a*b**2*c**3))/(6*a**4*b*x**3 + 6*a**
3*b**2*x**5)
```

GIAC/XCAS [A] time = 0.252069, size = 203, normalized size = 1.38

$$\frac{\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)a^3b}+\frac{6\,bc^3x^2-9\,ac^2dx^2-ac^3}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d
*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^2 + a)*a^3*b) + 1/3*(6*b*
c^3*x^2 - 9*a*c^2*d*x^2 - a*c^3)/(a^3*x^3)
```

3.289 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{a}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}+\frac{ax}{2b\,(a+bx^2)\,(bc-ad)}$$

```
[Out] (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)^2) + (c^(3/2)*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)^2)
```

Rubi [A] time = 0.23461, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\sqrt{a}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}+\frac{ax}{2b\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)^2) + (c^(3/2)*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 36.8098, size = 94, normalized size = 0.86

$$\frac{\sqrt{a}\,(ad-3bc)\,\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}\,(ad-bc)^2}-\frac{ax}{2b\,(a+bx^2)\,(ad-bc)}+\frac{c^{\frac{3}{2}}\,\text{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] sqrt(a)*(a*d - 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(2*b**(3/2)*(a*d -
b*c)**2) - a*x/(2*b*(a + b*x**2)*(a*d - b*c)) + c**(3/2)*atan(sqr
t(d)*x/sqrt(c))/(sqrt(d)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.249216, size = 95, normalized size = 0.87

$$\frac{\frac{\sqrt{a}(ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{ax(bc-ad)}{b(a+bx^2)}+\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] ((a*(b*c - a*d)*x)/(b*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + (2*c^(3/2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/Sqrt[d])/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.016, size = 144, normalized size = 1.3

$$\frac{c^2}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{xa^2d}{2\,(ad-bc)^2\,b\,(bx^2+a)}+\frac{acx}{2\,(ad-bc)^2\,(bx^2+a)}+\frac{a^2d}{2\,(ad-bc)^2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,ac}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*a^2/(a*d-
b*c)^2/b*x/(b*x^2+a)*d+1/2*a/(a*d-b*c)^2*c*x/(b*x^2+a)+1/2*a^2/(a
*d-b*c)^2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-3/2*a/(a*d-b*c)
^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.311494, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\left(b^2cx^2+abc\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\left(abc-a^2d\right)}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}\right.$$

$$-\frac{\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-\left(b^2cx^2+abc\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-\left(abc-a^2d\right)x}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)},\ \frac{4\left(b^2cx^2+\right.}{}$$

$$\left.-\frac{\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-2\left(b^2cx^2+abc\right)\sqrt{\frac{c}{d}}\arctan\left(\frac{x}{\sqrt{\frac{c}{d}}}\right)-\left(abc-a^2d\right)x}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(-a/b)*log((
b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b^2*c*x^2 + a*b*c
)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*
(a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*
c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), -1/2*((3*a*b*c - a^2*d + (
3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - (b^2*c*x^2
+ a*b*c)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c
)) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 +
(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*(4*(b^2*c*x^2 + a
*b*c)*sqrt(c/d)*arctan(x/sqrt(c/d)) - (3*a*b*c - a^2*d + (3*b^2*c
 - a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)) + 2*(a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*
b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), -1/2*((3*a*b*
```

```
c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b))
- 2*(b^2*c*x^2 + a*b*c)*sqrt(c/d)*arctan(x/sqrt(c/d)) - (a*b*c -
a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*x^2)]
```

Sympy [A] time = 42.8901, size = 1850, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a*x/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - s
qrt(-a/b**3)*(a*d - 3*b*c)*log(x + (-a**5*b**3*d**6*(-a/b**3)**(3
/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 9*a**4*b**4*c*d**5*(-a/
b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - a**4*d**4*sqrt
(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - 13*a**3*b**5*c**2*d*
*4*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + 9*a**3*b*c*
d**3*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 17*a**2*b**
6*c**3*d**3*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 27
*a**2*b**2*c**2*d**2*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**
2) - 21*a*b**7*c**4*d**2*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*
d - b*c)**6) + 27*a*b**3*c**3*d*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a
*d - b*c)**2) + 5*b**8*c**5*d*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(
2*(a*d - b*c)**6) + 4*b**4*c**4*sqrt(-a/b**3)*(a*d - 3*b*c)/(a*d
- b*c)**2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**3))/(4*(a*d -
 b*c)**2) + sqrt(-a/b**3)*(a*d - 3*b*c)*log(x + (a**5*b**3*d**6*(
-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 9*a**4*b**4
*c*d**5*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + a*
*4*d**4*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 13*a**3*
b**5*c**2*d**4*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 -
 9*a**3*b*c*d**3*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) -
 17*a**2*b**6*c**3*d**3*(-a/b**3)**(3/2)*(a*d - 3*b*c)**3/(a*d -
b*c)**6 + 27*a**2*b**2*c**2*d**2*sqrt(-a/b**3)*(a*d - 3*b*c)/(2*(
a*d - b*c)**2) + 21*a*b**7*c**4*d**2*(-a/b**3)**(3/2)*(a*d - 3*b*
c)**3/(2*(a*d - b*c)**6) - 27*a*b**3*c**3*d*sqrt(-a/b**3)*(a*d -
3*b*c)/(2*(a*d - b*c)**2) - 5*b**8*c**5*d*(-a/b**3)**(3/2)*(a*d -
 3*b*c)**3/(2*(a*d - b*c)**6) - 4*b**4*c**4*sqrt(-a/b**3)*(a*d -
3*b*c)/(a*d - b*c)**2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**3
))/(4*(a*d - b*c)**2) - sqrt(-c**3/d)*log(x + (-4*a**5*b**3*d**6*
(-c**3/d)**(3/2)/(a*d - b*c)**6 + 36*a**4*b**4*c*d**5*(-c**3/d)**
(3/2)/(a*d - b*c)**6 - a**4*d**4*sqrt(-c**3/d)/(a*d - b*c)**2 - 1
04*a**3*b**5*c**2*d**4*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 9*a**3*b
*c*d**3*sqrt(-c**3/d)/(a*d - b*c)**2 + 136*a**2*b**6*c**3*d**3*(-
c**3/d)**(3/2)/(a*d - b*c)**6 - 27*a**2*b**2*c**2*d**2*sqrt(-c**3
/d)/(a*d - b*c)**2 - 84*a*b**7*c**4*d**2*(-c**3/d)**(3/2)/(a*d -
b*c)**6 + 27*a*b**3*c**3*d*sqrt(-c**3/d)/(a*d - b*c)**2 + 20*b**8
*c**5*d*(-c**3/d)**(3/2)/(a*d - b*c)**6 + 8*b**4*c**4*sqrt(-c**3/
d)/(a*d - b*c)**2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**3))/(
2*(a*d - b*c)**2) + sqrt(-c**3/d)*log(x + (4*a**5*b**3*d**6*(-c**
3/d)**(3/2)/(a*d - b*c)**6 - 36*a**4*b**4*c*d**5*(-c**3/d)**(3/2)
/(a*d - b*c)**6 + a**4*d**4*sqrt(-c**3/d)/(a*d - b*c)**2 + 104*a*
*3*b**5*c**2*d**4*(-c**3/d)**(3/2)/(a*d - b*c)**6 - 9*a**3*b*c*d*
*3*sqrt(-c**3/d)/(a*d - b*c)**2 - 136*a**2*b**6*c**3*d**3*(-c**3/
d)**(3/2)/(a*d - b*c)**6 + 27*a**2*b**2*c**2*d**2*sqrt(-c**3/d)/(
a*d - b*c)**2 + 84*a*b**7*c**4*d**2*(-c**3/d)**(3/2)/(a*d - b*c)*
*6 - 27*a*b**3*c**3*d*sqrt(-c**3/d)/(a*d - b*c)**2 - 20*b**8*c**5
*d*(-c**3/d)**(3/2)/(a*d - b*c)**6 - 8*b**4*c**4*sqrt(-c**3/d)/(a
*d - b*c)**2)/(a**2*c*d**2 - 7*a*b*c**2*d + 12*b**2*c**3))/(2*(a*
d - b*c)**2)
```

GIAC/XCAS [A] time = 0.241564, size = 165, normalized size = 1.51

$$\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}-\frac{\left(3\,abc-a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{ab}}+\frac{ax}{2\,(b^2c-abd)(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) - 1/2*(3*a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*sqrt(a*b)) + 1/2*a*x/((b^2*c - a*b*d)*(b*x
^2 + a))
```

3.290 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=74

$$\frac{a}{2b\,(a+bx^2)\,(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*
d)^2) - (c*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.159262, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a}{2b\,(a+bx^2)\,(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*
d)^2) - (c*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 24.5667, size = 58, normalized size = 0.78

$$-\frac{a}{2b\,(a+bx^2)\,(ad-bc)}+\frac{c\log\left(a+bx^2\right)}{2\,(ad-bc)^2}-\frac{c\log\left(c+dx^2\right)}{2\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a/(2*b*(a + b*x**2)*(a*d - b*c)) + c*log(a + b*x**2)/(2*(a*d - b
*c)**2) - c*log(c + d*x**2)/(2*(a*d - b*c)**2)
```

Mathematica [A] time = 0.055822, size = 74, normalized size = 1.

$$\frac{a}{2b\,(a+bx^2)\,(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*
d)^2) - (c*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.018, size = 95, normalized size = 1.3

$$-\frac{c\ln\left(dx^2+c\right)}{2\,(ad-bc)^2}+\frac{c\ln\left(bx^2+a\right)}{2\,(ad-bc)^2}-\frac{a^2d}{2\,(ad-bc)^2\,b\,(bx^2+a)}+\frac{ac}{2\,(ad-bc)^2\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*c/(a*d-b*c)^2*ln(d*x^2+c)+1/2/(a*d-b*c)^2*c*ln(b*x^2+a)-1/2/
(a*d-b*c)^2*a^2/b/(b*x^2+a)*d+1/2/(a*d-b*c)^2*a/(b*x^2+a)*c
```

Maxima [A] time = 1.35115, size = 142, normalized size = 1.92

$$\frac{c\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{c\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{a}{2\left(ab^2c-a^2bd+(b^3c-ab^2d)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*log(
d*x^2 + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a/(a*b^2*c - a^2
*b*d + (b^3*c - a*b^2*d)*x^2)
```

Fricas [A] time = 0.233411, size = 158, normalized size = 2.14

$$\frac{abc-a^2d+\left(b^2cx^2+abc\right)\log\left(bx^2+a\right)-\left(b^2cx^2+abc\right)\log\left(dx^2+c\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d + (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) - (b^2*c*
x^2 + a*b*c)*log(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2)
```

Sympy [A] time = 8.54609, size = 253, normalized size = 3.42

$$-\frac{a}{2a^2bd-2ab^2c+x^2\left(2ab^2d-2b^3c\right)}-\frac{c\log\left(x^2+\frac{-\frac{a^3cd^3}{(ad-bc)^2}+\frac{3a^2bc^2d^2}{(ad-bc)^2}-\frac{3ab^2c^3d}{(ad-bc)^2}+acd+\frac{b^3c^4}{(ad-bc)^2}+bc^2}{2bcd}\right)}{2\left(ad-bc\right)^2}+\frac{c\log\left(x^2+\frac{\frac{a^3cd^3}{(ad-bc)^2}-\frac{3a^2bc^2d^2}{(ad-bc)^2}+\frac{3ab^2c^3d}{(ad-bc)^2}+acd-\frac{b^3c^4}{(ad-bc)^2}+bc^2}{2bcd}\right)}{2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - c*l
og(x**2 + (-a**3*c*d**3/(a*d - b*c)**2 + 3*a**2*b*c**2*d**2/(a*d
- b*c)**2 - 3*a*b**2*c**3*d/(a*d - b*c)**2 + a*c*d + b**3*c**4/(a
*d - b*c)**2 + b*c**2)/(2*b*c*d))/(2*(a*d - b*c)**2) + c*log(x**2
 + (a**3*c*d**3/(a*d - b*c)**2 - 3*a**2*b*c**2*d**2/(a*d - b*c)**
2 + 3*a*b**2*c**3*d/(a*d - b*c)**2 + a*c*d - b**3*c**4/(a*d - b*c
)**2 + b*c**2)/(2*b*c*d))/(2*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.242386, size = 124, normalized size = 1.68

$$-\frac{\frac{b^2c\ln\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{ab}{(b^2c-abd)(bx^2+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*ln(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2) - a*b/((b^2*c - a*b*d)*(b*x^2 + a))
)/b
```

3.291 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=104

$$-\frac{x}{2(a+bx^2)(bc-ad)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)^2}-\frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

```
[Out] -x/(2*(b*c - a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2) - (Sqrt[c]*Sqrt[d]*Ar
cTan[(Sqrt[d]*x)/Sqrt[c]])/(b*c - a*d)^2
```

Rubi [A] time = 0.175464, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{x}{2(a+bx^2)(bc-ad)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)^2}-\frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -x/(2*(b*c - a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2) - (Sqrt[c]*Sqrt[d]*Ar
cTan[(Sqrt[d]*x)/Sqrt[c]])/(b*c - a*d)^2
```

Rubi in Sympy [A] time = 34.173, size = 88, normalized size = 0.85

$$-\frac{\sqrt{c}\sqrt{d}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(ad-bc)^2}+\frac{x}{2(a+bx^2)(ad-bc)}+\frac{(ad+bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -sqrt(c)*sqrt(d)*atan(sqrt(d)*x/sqrt(c))/(a*d - b*c)**2 + x/(2*(a
 + b*x**2)*(a*d - b*c)) + (a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*
sqrt(a)*sqrt(b)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.245064, size = 104, normalized size = 1.

$$\frac{x}{2(a+bx^2)(ad-bc)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(ad-bc)^2}-\frac{\sqrt{c}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] x/(2*(-(b*c) + a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*(-(b*c) + a*d)^2) - (Sqrt[c]*Sqrt[
d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(b*c - a*d)^2
```

Maple [A] time = 0.014, size = 134, normalized size = 1.3

$$-\frac{cd}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{axd}{2\,(ad-bc)^2\,(bx^2+a)}-\frac{bxc}{2\,(ad-bc)^2\,(bx^2+a)}+\frac{ad}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{bc}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -c*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))+1/2/(a*d-b*c
)^2*x/(b*x^2+a)*a*d-1/2/(a*d-b*c)^2*x/(b*x^2+a)*b*c+1/2/(a*d-b*c)
^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+1/2/(a*d-b*c)^2/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300203, size = 1, normalized size = 0.01

$$\left[\frac{2\left(bx^2+a\right)\sqrt{-ab}\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)-2\sqrt{-ab}(bc-ad)x+\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{4\left(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^2\right)\sqrt{-ab}},\right.$$
$$-\frac{4\left(bx^2+a\right)\sqrt{-ab}\sqrt{cd}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+2\sqrt{-ab}(bc-ad)x-\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{4\left(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^2\right)\sqrt{-ab}},$$
$$\left.-\frac{2\left(bx^2+a\right)\sqrt{ab}\sqrt{cd}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\sqrt{ab}(bc-ad)x-\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^2\right)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*x^2 + a)*sqrt(-a*b)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*
d)*x - c)/(d*x^2 + c)) - 2*sqrt(-a*b)*(b*c - a*d)*x + (a*b*c + a^
2*d + (b^2*c + a*b*d)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))
/(b*x^2 + a)))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(-a*b)), 1/2*((b*x^2 + a)*sqrt(a
*b)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - sq
rt(a*b)*(b*c - a*d)*x + (a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*arc
tan(sqrt(a*b)*x/a))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(a*b)), -1/4*(4*(b*x^2 + a)
*sqrt(-a*b)*sqrt(c*d)*arctan(d*x/sqrt(c*d)) + 2*sqrt(-a*b)*(b*c -
 a*d)*x - (a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*log((2*a*b*x + (b
*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)))/((a*b^2*c^2 - 2*a^2*b*c*d + a
^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(-a*b)), -1
/2*(2*(b*x^2 + a)*sqrt(a*b)*sqrt(c*d)*arctan(d*x/sqrt(c*d)) + sqr
t(a*b)*(b*c - a*d)*x - (a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*arct
an(sqrt(a*b)*x/a))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2
```

```
- 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(a*b))]
```

Sympy [A] time = 23.5807, size = 1530, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] x/(2*a**2*d - 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c)) + sqrt(-1/(a*b
))*(a*d + b*c)*log(x + (-3*a**6*b*d**5*(-1/(a*b))**(3/2)*(a*d + b
*c)**3/(2*(a*d - b*c)**6) + 11*a**5*b**2*c*d**4*(-1/(a*b))**(3/2)
*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 7*a**4*b**3*c**2*d**3*(-1/(a
*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 + 3*a**3*b**4*c**3*d**2
*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)**6 - a**3*d**3*sqrt
(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2) + a**2*b**5*c**4*d*(-1/
(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 11*a**2*b*c*d**
2*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2) - a*b**6*c**5*(-1
/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 3*a*b**2*c**2*
d*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2) - b**3*c**3*sqrt(
-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**2))/(a*d**2 + b*c*d))/(4*(a
*d - b*c)**2) - sqrt(-1/(a*b))*(a*d + b*c)*log(x + (3*a**6*b*d**5
*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) - 11*a**5*b*
*2*c*d**4*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d - b*c)**6) + 7
*a**4*b**3*c**2*d**3*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d - b*c)
**6 - 3*a**3*b**4*c**3*d**2*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(a*d
 - b*c)**6 + a**3*d**3*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)*
*2) - a**2*b**5*c**4*d*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d -
 b*c)**6) + 11*a**2*b*c*d**2*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d -
 b*c)**2) + a*b**6*c**5*(-1/(a*b))**(3/2)*(a*d + b*c)**3/(2*(a*d
- b*c)**6) + 3*a*b**2*c**2*d*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d -
 b*c)**2) + b**3*c**3*sqrt(-1/(a*b))*(a*d + b*c)/(2*(a*d - b*c)**
2))/(a*d**2 + b*c*d))/(4*(a*d - b*c)**2) + sqrt(-c*d)*log(x + (-1
2*a**6*b*d**5*(-c*d)**(3/2)/(a*d - b*c)**6 + 44*a**5*b**2*c*d**4*
(-c*d)**(3/2)/(a*d - b*c)**6 - 56*a**4*b**3*c**2*d**3*(-c*d)**(3/
2)/(a*d - b*c)**6 + 24*a**3*b**4*c**3*d**2*(-c*d)**(3/2)/(a*d - b
*c)**6 - a**3*d**3*sqrt(-c*d)/(a*d - b*c)**2 + 4*a**2*b**5*c**4*d
*(-c*d)**(3/2)/(a*d - b*c)**6 - 11*a**2*b*c*d**2*sqrt(-c*d)/(a*d
- b*c)**2 - 4*a*b**6*c**5*(-c*d)**(3/2)/(a*d - b*c)**6 - 3*a*b**2
*c**2*d*sqrt(-c*d)/(a*d - b*c)**2 - b**3*c**3*sqrt(-c*d)/(a*d - b
*c)**2)/(a*d**2 + b*c*d))/(2*(a*d - b*c)**2) - sqrt(-c*d)*log(x +
 (12*a**6*b*d**5*(-c*d)**(3/2)/(a*d - b*c)**6 - 44*a**5*b**2*c*d*
*4*(-c*d)**(3/2)/(a*d - b*c)**6 + 56*a**4*b**3*c**2*d**3*(-c*d)**
(3/2)/(a*d - b*c)**6 - 24*a**3*b**4*c**3*d**2*(-c*d)**(3/2)/(a*d
- b*c)**6 + a**3*d**3*sqrt(-c*d)/(a*d - b*c)**2 - 4*a**2*b**5*c**
4*d*(-c*d)**(3/2)/(a*d - b*c)**6 + 11*a**2*b*c*d**2*sqrt(-c*d)/(a
*d - b*c)**2 + 4*a*b**6*c**5*(-c*d)**(3/2)/(a*d - b*c)**6 + 3*a*b
**2*c**2*d*sqrt(-c*d)/(a*d - b*c)**2 + b**3*c**3*sqrt(-c*d)/(a*d
- b*c)**2)/(a*d**2 + b*c*d))/(2*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.23621, size = 149, normalized size = 1.43

$$-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{(bc + ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} - \frac{x}{2\,(bx^2 + a)(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
c*d)) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(a*b)) - 1/2*x/((b*x^2 + a)*(b*c - a*d))
```

3.292 $\int \frac{x}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] -1/(2*(b*c - a*d)*(a + b*x^2)) - (d*Log[a + b*x^2])/(2*(b*c - a*d
)^2) + (d*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi [A] time = 0.120471, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x^2)) - (d*Log[a + b*x^2])/(2*(b*c - a*d
)^2) + (d*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 21.8142, size = 56, normalized size = 0.8

$$-\frac{d\log\left(a+bx^2\right)}{2\left(ad-bc\right)^2}+\frac{d\log\left(c+dx^2\right)}{2\left(ad-bc\right)^2}+\frac{1}{2\left(a+bx^2\right)(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -d*log(a + b*x**2)/(2*(a*d - b*c)**2) + d*log(c + d*x**2)/(2*(a*d
 - b*c)**2) + 1/(2*(a + b*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.04828, size = 66, normalized size = 0.94

$$\frac{d\left(a+bx^2\right)\log\left(c+dx^2\right)-d\left(a+bx^2\right)\log\left(a+bx^2\right)+ad-bc}{2\left(a+bx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x^2)*Log[a + b*x^2] + d*(a + b*x^2)*Log[
c + d*x^2])/(2*(b*c - a*d)^2*(a + b*x^2))
```

Maple [A] time = 0.017, size = 90, normalized size = 1.3

$$\frac{d\ln\left(dx^2+c\right)}{2\left(ad-bc\right)^2}-\frac{\ln\left(bx^2+a\right)d}{2\left(ad-bc\right)^2}+\frac{ad}{2\left(ad-bc\right)^2\left(bx^2+a\right)}-\frac{bc}{2\left(ad-bc\right)^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*d/(a*d-b*c)^2*ln(d*x^2+c)-1/2/(a*d-b*c)^2*ln(b*x^2+a)*d+1/2/(
a*d-b*c)^2/(b*x^2+a)*a*d-1/2*b/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.35271, size = 134, normalized size = 1.91

$$-\frac{d\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{d\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{1}{2\left(abc-a^2d+(b^2c-abd)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] -1/2*d*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*d*log
(d*x^2 + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2/(a*b*c - a^2*d
+ (b^2*c - a*b*d)*x^2)
```

Fricas [A] time = 0.231694, size = 139, normalized size = 1.99

$$-\frac{bc-ad+\left(bdx^2+ad\right)\log\left(bx^2+a\right)-\left(bdx^2+ad\right)\log\left(dx^2+c\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d + (b*d*x^2 + a*d)*log(b*x^2 + a) - (b*d*x^2 + a*d
)*log(d*x^2 + c))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*x^2)
```

Sympy [A] time = 8.12281, size = 248, normalized size = 3.54

$$\frac{d\log\left(x^2+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\left(ad-bc\right)^2}-\frac{d\log\left(x^2+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\left(ad-bc\right)^2}+\frac{1}{2a^2d-2abc+x^2\left(2abd-2b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d*log(x**2 + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d -
b*c)**2 - 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*
d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2) - d*log(
x**2 + (a**3*d**4/(a*d - b*c)**2 - 3*a**2*b*c*d**3/(a*d - b*c)**2
 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b**3*c**3*d/(a*d
- b*c)**2 + b*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2) + 1/(2*a**2*d -
 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c))
```

GIAC/XCAS [A] time = 0.265773, size = 115, normalized size = 1.64

$$\frac{bd\ln\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}-\frac{b}{2\left(b^2c-abd\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 1/2*b*d*ln(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2) - 1/2*b/((b^2*c - a*b*d)*(b*x^2 + a))
```

3.293 $\int \frac{1}{(a+bx^2)^2(c+dx^2)} dx$

Optimal. Leaf size=108

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a(a+bx^2)(bc-ad)}$$

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rubi [A] time = 0.195759, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 41.6443, size = 94, normalized size = 0.87

$$\frac{d^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\,(ad-bc)^2}-\frac{bx}{2a\,(a+bx^2)\,(ad-bc)}-\frac{\sqrt{b}\,(3ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d**(3/2)*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*(a*d - b*c)**2) - b*x/(
2*a*(a + b*x**2)*(a*d - b*c)) - sqrt(b)*(3*a*d - b*c)*atan(sqrt(b
)*x/sqrt(a))/(2*a**(3/2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.235244, size = 109, normalized size = 1.01

$$-\frac{\sqrt{b}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(ad-bc)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}-\frac{bx}{2a(a+bx^2)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(b*x)/(2*a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*
d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d
^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A] time = 0., size = 144, normalized size = 1.3

$$\frac{d^2}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{bxd}{2\,(ad-bc)^2\,(bx^2+a)}+\frac{xb^2c}{2\,(ad-bc)^2\,a\,(bx^2+a)}-\frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^2c}{2\,(ad-bc)^2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*b/(a*d-b*
c)^2*x/(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^2*x/a/(b*x^2+a)*c-3/2*b/(a*d
-b*c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+1/2*b^2/(a*d-b*c)^2
/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.346089, size = 1, normalized size = 0.01

$$\left[-\frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(b^2c-abd\right.}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((
b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d
)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*
(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*
c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/4*(4*(a*b*d*x^2 + a^2*d)
*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - (a*b*c - 3*a^2*d + (b^2*c
- 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c
- 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b
/a))) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d
/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b
*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/
2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(b*x
/(a*sqrt(b/a))) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(d*x/(c*s
qrt(d/c))) + (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

Sympy [A] time = 47.2932, size = 2033, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -b*x/(2*a**3*d - 2*a**2*b*c + x**2*(2*a**2*b*d - 2*a*b**2*c)) + s
qrt(-b/a**3)*(3*a*d - b*c)*log(x + (-a**9*c*d**6*(-b/a**3)**(3/2)
*(3*a*d - b*c)**3/(a*d - b*c)**6 + 5*a**8*b*c**2*d**5*(-b/a**3)**
(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + a**7*b**2*c**3*d**4*(
-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 7*a**6*b**3
*c**4*d**3*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 8*a
**5*b**4*c**5*d**2*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)*
*6 - 4*a**5*d**5*sqrt(-b/a**3)*(3*a*d - b*c)/(a*d - b*c)**2 - 7*a
**4*b**5*c**6*d*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)*
*6) + a**3*b**6*c**7*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d -
b*c)**6) - 27*a**3*b**2*c**2*d**3*sqrt(-b/a**3)*(3*a*d - b*c)/(2*
(a*d - b*c)**2) + 27*a**2*b**3*c**3*d**2*sqrt(-b/a**3)*(3*a*d - b
*c)/(2*(a*d - b*c)**2) - 9*a*b**4*c**4*d*sqrt(-b/a**3)*(3*a*d - b
*c)/(2*(a*d - b*c)**2) + b**5*c**5*sqrt(-b/a**3)*(3*a*d - b*c)/(2
*(a*d - b*c)**2))/(12*a**2*b*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d
**2))/(4*(a*d - b*c)**2) - sqrt(-b/a**3)*(3*a*d - b*c)*log(x + (a
**9*c*d**6*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 5*a
**8*b*c**2*d**5*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)*
*6) - a**7*b**2*c**3*d**4*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a
*d - b*c)**6) + 7*a**6*b**3*c**4*d**3*(-b/a**3)**(3/2)*(3*a*d - b
*c)**3/(a*d - b*c)**6 - 8*a**5*b**4*c**5*d**2*(-b/a**3)**(3/2)*(3
*a*d - b*c)**3/(a*d - b*c)**6 + 4*a**5*d**5*sqrt(-b/a**3)*(3*a*d
- b*c)/(a*d - b*c)**2 + 7*a**4*b**5*c**6*d*(-b/a**3)**(3/2)*(3*a*
d - b*c)**3/(2*(a*d - b*c)**6) - a**3*b**6*c**7*(-b/a**3)**(3/2)*
(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt
(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 27*a**2*b**3*c**3*d*
*2*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + 9*a*b**4*c**4
*d*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - b**5*c**5*sqr
t(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2))/(12*a**2*b*d**4 - 7*
a*b**2*c*d**3 + b**3*c**2*d**2))/(4*(a*d - b*c)**2) + sqrt(-d**3/
c)*log(x + (-8*a**9*c*d**6*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 20*a
**8*b*c**2*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 4*a**7*b**2*c**
3*d**4*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 56*a**6*b**3*c**4*d**3*(
-d**3/c)**(3/2)/(a*d - b*c)**6 + 64*a**5*b**4*c**5*d**2*(-d**3/c)
**(3/2)/(a*d - b*c)**6 - 8*a**5*d**5*sqrt(-d**3/c)/(a*d - b*c)**2
 - 28*a**4*b**5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 4*a**3*b
**6*c**7*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 27*a**3*b**2*c**2*d**3
*sqrt(-d**3/c)/(a*d - b*c)**2 + 27*a**2*b**3*c**3*d**2*sqrt(-d**3
/c)/(a*d - b*c)**2 - 9*a*b**4*c**4*d*sqrt(-d**3/c)/(a*d - b*c)**2
 + b**5*c**5*sqrt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b*d**4 - 7*a*
b**2*c*d**3 + b**3*c**2*d**2))/(2*(a*d - b*c)**2) - sqrt(-d**3/c)
*log(x + (8*a**9*c*d**6*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 20*a**8
*b*c**2*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 4*a**7*b**2*c**3*d
**4*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 56*a**6*b**3*c**4*d**3*(-d*
*3/c)**(3/2)/(a*d - b*c)**6 - 64*a**5*b**4*c**5*d**2*(-d**3/c)**(
3/2)/(a*d - b*c)**6 + 8*a**5*d**5*sqrt(-d**3/c)/(a*d - b*c)**2 +
28*a**4*b**5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 4*a**3*b**6
*c**7*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 27*a**3*b**2*c**2*d**3*sq
rt(-d**3/c)/(a*d - b*c)**2 - 27*a**2*b**3*c**3*d**2*sqrt(-d**3/c)
/(a*d - b*c)**2 + 9*a*b**4*c**4*d*sqrt(-d**3/c)/(a*d - b*c)**2 -
b**5*c**5*sqrt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b*d**4 - 7*a*b**
2*c*d**3 + b**3*c**2*d**2))/(2*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.235098, size = 163, normalized size = 1.51

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{(b^2c - 3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc - a^2d)(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 -
```

```
2*a^2*b*c*d + a^3*d^2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x
^2 + a))
```

3.294 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}+\frac{b}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] b/(2*a*(b*c - a*d)*(a + b*x^2)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*
d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(
2*c*(b*c - a*d)^2)
```

Rubi [A] time = 0.249087, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}+\frac{b}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] b/(2*a*(b*c - a*d)*(a + b*x^2)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*
d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(
2*c*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 37.3878, size = 85, normalized size = 0.86

$$-\frac{d^2\log\left(c+dx^2\right)}{2c\left(ad-bc\right)^2}-\frac{b}{2a\left(a+bx^2\right)\left(ad-bc\right)}+\frac{b\left(2ad-bc\right)\log\left(a+bx^2\right)}{2a^2\left(ad-bc\right)^2}+\frac{\log\left(x^2\right)}{2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -d**2*log(c + d*x**2)/(2*c*(a*d - b*c)**2) - b/(2*a*(a + b*x**2)*
(a*d - b*c)) + b*(2*a*d - b*c)*log(a + b*x**2)/(2*a**2*(a*d - b*c
)**2) + log(x**2)/(2*a**2*c)
```

Mathematica [A] time = 0.190741, size = 97, normalized size = 0.98

$$\frac{2\log(x)-\frac{a\left(ad^2\left(a+bx^2\right)\log\left(c+dx^2\right)+bc(ad-bc)\right)+bc\left(a+bx^2\right)(bc-2ad)\log\left(a+bx^2\right)}{\left(a+bx^2\right)(bc-ad)^2}}{2a^2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (2*Log[x] - (b*c*(b*c - 2*a*d)*(a + b*x^2)*Log[a + b*x^2] + a*(b*
c*(-(b*c) + a*d) + a*d^2*(a + b*x^2)*Log[c + d*x^2]))/((b*c - a*d
)^2*(a + b*x^2)))/(2*a^2*c)
```

Maple [A] time = 0.022, size = 139, normalized size = 1.4

$$\frac{\ln(x)}{a^2c}-\frac{d^2\ln\left(dx^2+c\right)}{2c\left(ad-bc\right)^2}+\frac{b\ln\left(bx^2+a\right)d}{a\left(ad-bc\right)^2}-\frac{b^2\ln\left(bx^2+a\right)c}{2a^2\left(ad-bc\right)^2}-\frac{bd}{2\left(ad-bc\right)^2\left(bx^2+a\right)}+\frac{b^2c}{2a\left(ad-bc\right)^2\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] ln(x)/a^2/c-1/2*d^2/c/(a*d-b*c)^2*ln(d*x^2+c)+b/a/(a*d-b*c)^2*ln(
b*x^2+a)*d-1/2*b^2/a^2/(a*d-b*c)^2*ln(b*x^2+a)*c-1/2*b/(a*d-b*c)^
2/(b*x^2+a)*d+1/2*b^2/a/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.35937, size = 185, normalized size = 1.87

$$-\frac{d^2\log\left(dx^2+c\right)}{2\left(b^2c^3-2abc^2d+a^2cd^2\right)}-\frac{\left(b^2c-2abd\right)\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)}+\frac{b}{2\left(a^2bc-a^3d+(ab^2c-a^2bd)x^2\right)}+\frac{\log\left(x^2\right)}{2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*log(d*x^2 + c)/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - 1/2
*(b^2*c - 2*a*b*d)*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^
4*d^2) + 1/2*b/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*
log(x^2)/(a^2*c)
```

Fricas [A] time = 1.02741, size = 294, normalized size = 2.97

$$\frac{ab^2c^2-a^2bcd-\left(ab^2c^2-2a^2bcd+\left(b^3c^2-2ab^2cd\right)x^2\right)\log\left(bx^2+a\right)-\left(a^2bd^2x^2+a^3d^2\right)\log\left(dx^2+c\right)+2\left(ab^2c^2-2a^2b\right.}{2\left(a^3b^2c^3-2a^4bc^2d+a^5cd^2+(a^2b^3c^3-2a^3b^2c^2d+a^4bcd^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - a^2*b*c*d - (a*b^2*c^2 - 2*a^2*b*c*d + (b^3*c^2
- 2*a*b^2*c*d)*x^2)*log(b*x^2 + a) - (a^2*b*d^2*x^2 + a^3*d^2)*lo
g(d*x^2 + c) + 2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)*x^2)*log(x))/(a^3*b^2*c^3 - 2*a^4*b*c^2*
d + a^5*c*d^2 + (a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266992, size = 247, normalized size = 2.49

$$-\frac{d^{3}\ln\left(\left|dx^{2}+c\right|\right)}{2\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)}-\frac{\left(b^{3}c-2\,ab^{2}d\right)\ln\left(\left|bx^{2}+a\right|\right)}{2\left(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2}\right)}+\frac{b^{3}cx^{2}-2\,ab^{2}dx^{2}+2\,ab^{2}c-3\,a^{2}bd}{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\left(bx^{2}+a\right)}+\frac{\ln\left(x^{2}\right)}{2\,a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -1/2*d^3*ln(abs(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^
3) - 1/2*(b^3*c - 2*a*b^2*d)*ln(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*
a^3*b^2*c*d + a^4*b*d^2) + 1/2*(b^3*c*x^2 - 2*a*b^2*d*x^2 + 2*a*b
^2*c - 3*a^2*b*d)/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 +
 a)) + 1/2*ln(x^2)/(a^2*c)
```

3.295 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=144

$$-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\,(a+bx^2)\,(bc-ad)}$$

```
[Out] -(3*b*c - 2*a*d)/(2*a^2*c*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x*(
a + b*x^2)) - (b^(3/2)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(2*a^(5/2)*(b*c - a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(c^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.516638, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(3*b*c - 2*a*d)/(2*a^2*c*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x*(
a + b*x^2)) - (b^(3/2)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(2*a^(5/2)*(b*c - a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(c^(3/2)*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 94.9558, size = 121, normalized size = 0.84

$$-\frac{d^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}\,(ad-bc)^2}-\frac{b}{2ax\,(a+bx^2)\,(ad-bc)}-\frac{2ad-3bc}{2a^2cx\,(ad-bc)}+\frac{b^{\frac{3}{2}}\,(5ad-3bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -d**(5/2)*atan(sqrt(d)*x/sqrt(c))/(c**(3/2)*(a*d - b*c)**2) - b/(
2*a*x*(a + b*x**2)*(a*d - b*c)) - (2*a*d - 3*b*c)/(2*a**2*c*x*(a*
d - b*c)) + b**(3/2)*(5*a*d - 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a
**(5/2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.321635, size = 123, normalized size = 0.85

$$\frac{b^{3/2}(5ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(ad-bc)^2}+\frac{b^2x}{2a^2\,(a+bx^2)\,(ad-bc)}-\frac{1}{a^2cx}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(1/(a^2*c*x)) + (b^2*x)/(2*a^2*(-(b*c) + a*d)*(a + b*x^2)) + (b^
(3/2)*(-3*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(-
(b*c) + a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*
(b*c - a*d)^2)
```

Maple [A] time = 0.019, size = 169, normalized size = 1.2

$$-\frac{1}{a^{2}cx}-\frac{d^{3}}{c\left(ad-bc\right)^{2}}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^{2}xd}{2\,a\left(ad-bc\right)^{2}\left(bx^{2}+a\right)}-\frac{b^{3}xc}{2\,a^{2}\left(ad-bc\right)^{2}\left(bx^{2}+a\right)}+\frac{5\,b^{2}d}{2\,a\left(ad-bc\right)^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,b^{3}c}{2\,a^{2}\left(ad-bc\right)^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/a^2/c/x-1/c*d^3/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)
)+1/2*b^2/a/(a*d-b*c)^2*x/(b*x^2+a)*d-1/2*b^3/a^2/(a*d-b*c)^2*x/(
b*x^2+a)*c+5/2*b^2/a/(a*d-b*c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*d-3/2*b^3/a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.620206, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d^2 + 2*(3*b^3*c^2 - 5*a
*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3
*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-
b/a) - a)/(b*x^2 + a)) - 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)
*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^3 -
 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*
d + a^5*c*d^2)*x), -1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d^2 +
2*(3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + 4*(a^2*b*d^2*x^3
+ a^3*d^2*x)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + ((3*b^3*c^2 -
5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log(
(b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^3*c^3 - 2*a^
3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a
^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (3*b^
3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*
d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(b*x/(a*s
qrt(b/a))) - (a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 -
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2
*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)
*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (3*b^3*c^2 - 5
*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 +
(3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))
) + 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/
c))))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b
^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.234968, size = 221, normalized size = 1.53

$$-\frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{cd}} - \frac{\left(3\,b^3c - 5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2)\sqrt{ab}} - \frac{3\,b^2cx^2 - 2\,abdx^2 + 2\,abc - 2\,a^2d}{2\,(a^2bc^2 - a^3cd)(bx^3 + ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*s
qrt(c*d)) - 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b)) - 1/2*(3*b^2*c*x^2 -
 2*a*b*d*x^2 + 2*a*b*c - 2*a^2*d)/((a^2*b*c^2 - a^3*c*d)*(b*x^3 +
 a*x))
```

3.296 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=126

$$\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{b^2}{2a^2\left(a+bx^2\right)(bc-ad)}-\frac{1}{2a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}$$

```
[Out] -1/(2*a^2*c*x^2) - b^2/(2*a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c
+ a*d)*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(
2*a^3*(b*c - a*d)^2) + (d^3*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rubi [A] time = 0.324945, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{b^2}{2a^2\left(a+bx^2\right)(bc-ad)}-\frac{1}{2a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(2*a^2*c*x^2) - b^2/(2*a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c
+ a*d)*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(
2*a^3*(b*c - a*d)^2) + (d^3*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 48.3724, size = 116, normalized size = 0.92

$$\frac{d^3\log\left(c+dx^2\right)}{2c^2\left(ad-bc\right)^2}+\frac{b^2}{2a^2\left(a+bx^2\right)(ad-bc)}-\frac{1}{2a^2cx^2}-\frac{b^2\left(3ad-2bc\right)\log\left(a+bx^2\right)}{2a^3\left(ad-bc\right)^2}-\frac{(ad+2bc)\log\left(x^2\right)}{2a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d**3*log(c + d*x**2)/(2*c**2*(a*d - b*c)**2) + b**2/(2*a**2*(a +
b*x**2)*(a*d - b*c)) - 1/(2*a**2*c*x**2) - b**2*(3*a*d - 2*b*c)*l
og(a + b*x**2)/(2*a**3*(a*d - b*c)**2) - (a*d + 2*b*c)*log(x**2)/
(2*a**3*c**2)
```

Mathematica [A] time = 0.27241, size = 119, normalized size = 0.94

$$\frac{1}{2}\left(\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^2}-\frac{2\log(x)(ad+2bc)}{a^3c^2}+\frac{b^2}{a^2\left(a+bx^2\right)(ad-bc)}-\frac{1}{a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{c^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^2)) + b^2/(a^2*(-(b*c) + a*d)*(a + b*x^2)) - (2*(2*
b*c + a*d)*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2
])/(a^3*(b*c - a*d)^2) + (d^3*Log[c + d*x^2])/(c^2*(b*c - a*d)^2)
)/2
```

Maple [A] time = 0.027, size = 170, normalized size = 1.4

$$-\frac{1}{2\,a^2cx^2}-\frac{\ln(x)\,d}{a^2c^2}-2\,\frac{b\ln(x)}{a^3c}+\frac{d^3\ln\left(dx^2+c\right)}{2\,c^2\,(ad-bc)^2}-\frac{3\,b^2\ln\left(bx^2+a\right)d}{2\,a^2\,(ad-bc)^2}+\frac{b^3\ln\left(bx^2+a\right)c}{a^3\,(ad-bc)^2}+\frac{b^2d}{2\,a\,(ad-bc)^2\,(bx^2+a)}-\frac{b^3c}{2\,a^2\,(ad-bc)^2\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2/a^2/c/x^2-1/a^2/c^2*ln(x)*d-2/a^3/c*ln(x)*b+1/2*d^3/c^2/(a*d
-b*c)^2*ln(d*x^2+c)-3/2*b^2/a^2/(a*d-b*c)^2*ln(b*x^2+a)*d+b^3/a^3
/(a*d-b*c)^2*ln(b*x^2+a)*c+1/2*b^2/a/(a*d-b*c)^2/(b*x^2+a)*d-1/2*
b^3/a^2/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.36836, size = 255, normalized size = 2.02

$$\frac{d^3\log\left(dx^2+c\right)}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2)}+\frac{\left(2\,b^3c-3\,ab^2d\right)\log\left(bx^2+a\right)}{2\,(a^3b^2c^2-2\,a^4bcd+a^5d^2)}-\frac{abc-a^2d+\left(2\,b^2c-abd\right)x^2}{2\,((a^2b^2c^2-a^3bcd)x^4+(a^3bc^2-a^4cd)x^2)}-\frac{(2\,bc+ad)\log\left(x^2\right)}{2\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + 1/
2*(2*b^3*c - 3*a*b^2*d)*log(b*x^2 + a)/(a^3*b^2*c^2 - 2*a^4*b*c*d
 + a^5*d^2) - 1/2*(a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)/((a^2*b
^2*c^2 - a^3*b*c*d)*x^4 + (a^3*b*c^2 - a^4*c*d)*x^2) - 1/2*(2*b*c
 + a*d)*log(x^2)/(a^3*c^2)
```

Fricas [A] time = 2.46711, size = 409, normalized size = 3.25

$$-\frac{a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+\left(2\,ab^3c^3-3\,a^2b^2c^2d+a^3bcd^2\right)x^2-\left(\left(2\,b^4c^3-3\,ab^3c^2d\right)x^4+\left(2\,ab^3c^3-3\,a^2b^2c^2d\right)x^2\right)\log\left(b\right.}{2\left((a^3b^3c^4-2\,a^4b^2c^3d+a^5bc^2d^2)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^3 - 3*
a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d)*x
^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)*log(b*x^2 + a) - (a^3*b
*d^3*x^4 + a^4*d^3*x^2)*log(d*x^2 + c) + 2*((2*b^4*c^3 - 3*a*b^3*
c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + a^4*d^3
)*x^2)*log(x))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x
^4 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239393, size = 347, normalized size = 2.75

$$\frac{d^4 \ln\left(\left|dx^2 + c\right|\right)}{2\left(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3\right)} + \frac{\left(2\,b^4c - 3\,ab^3d\right)\ln\left(\left|bx^2 + a\right|\right)}{2\left(a^3b^3c^2 - 2\,a^4b^2cd + a^5bd^2\right)} + \frac{a^2bd^3x^4 - 4\,b^3c^3x^2 + 6\,ab^2c^2dx^2 - 2\,a^2bcd^2x^2 + a^3d^3x^2 - 2\,ab^2c^3 + 4\,a^2bc^2d - 2\,a^3cd^2}{4\left(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2\right)\left(bx^4 + ax^2\right)} - \frac{(2\,bc + ad)\ln\left(x^2\right)}{2\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^4*ln(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d
^3) + 1/2*(2*b^4*c - 3*a*b^3*d)*ln(abs(b*x^2 + a))/(a^3*b^3*c^2 -
 2*a^4*b^2*c*d + a^5*b*d^2) + 1/4*(a^2*b*d^3*x^4 - 4*b^3*c^3*x^2
+ 6*a*b^2*c^2*d*x^2 - 2*a^2*b*c*d^2*x^2 + a^3*d^3*x^2 - 2*a*b^2*c
^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d +
 a^4*c^2*d^2)*(b*x^4 + a*x^2)) - 1/2*(2*b*c + a*d)*ln(x^2)/(a^3*c
^2)
```

3.297 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=189

$$\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

```
[Out] -(5*b*c - 2*a*d)/(6*a^2*c*(b*c - a*d)*x^3) + (5*b^2*c^2 - 2*a*b*c
*d - 2*a^2*d^2)/(2*a^3*c^2*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^
3*(a + b*x^2)) + (b^(5/2)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(7/2)*(b*c - a*d)^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(5/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.762349, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(5*b*c - 2*a*d)/(6*a^2*c*(b*c - a*d)*x^3) + (5*b^2*c^2 - 2*a*b*c
*d - 2*a^2*d^2)/(2*a^3*c^2*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^
3*(a + b*x^2)) + (b^(5/2)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(7/2)*(b*c - a*d)^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(5/2)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.608628, size = 142, normalized size = 0.75

$$-\frac{b^{5/2}(7ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(ad-bc)^2}-\frac{b^3x}{2a^3\left(a+bx^2\right)(ad-bc)}+\frac{ad+2bc}{a^3c^2x}-\frac{1}{3a^2cx^3}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(3*a^2*c*x^3) + (2*b*c + a*d)/(a^3*c^2*x) - (b^3*x)/(2*a^3*(-(
b*c) + a*d)*(a + b*x^2)) - (b^(5/2)*(-5*b*c + 7*a*d)*ArcTan[(Sqrt
```

```
[b]*x)/Sqrt[a]])/(2*a^(7/2)*(-(b*c) + a*d)^2) + (d^(7/2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.023, size = 191, normalized size = 1.

$$-\frac{1}{3\,a^2cx^3}+\frac{d}{a^2c^2x}+2\,\frac{b}{xa^3c}+\frac{d^4}{c^2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3xd}{2\,a^2\,(ad-bc)^2\,(bx^2+a)}+\frac{b^4xc}{2\,a^3\,(ad-bc)^2\,(bx^2+a)}-\frac{7\,db^3}{2\,a^2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^4c}{2\,a^3\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/3/a^2/c/x^3+1/x/a^2/c^2*d+2/x/a^3/c*b+1/c^2*d^4/(a*d-b*c)^2/(c
*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*b^3/a^2/(a*d-b*c)^2*x/(b*x^
2+a)*d+1/2*b^4/a^3/(a*d-b*c)^2*x/(b*x^2+a)*c-7/2*b^3/a^2/(a*d-b*c
)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+5/2*b^4/a^3/(a*d-b*c)^2
/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40961, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^2*c^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^
3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2
*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 + 3*((5*b^4*c^3 - 7*a*b^3*c
^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log((
b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6*(a^3*b*d^3*x^5 + a
^4*d^3*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2
+ c)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^
4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3), -1/12*(4*a^2*b^2*c
^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^3*c^2*d +
 2*a^3*b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^
2 + 3*a^4*d^3)*x^2 - 12*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*a
rctan(d*x/(c*sqrt(d/c))) + 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (
5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x
*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d +
a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*
x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*(5*b^
4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2
*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b
^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(b/a)*ar
```

```
ctan(b*x/(a*sqrt(b/a))) - 3*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^3*c^
4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b
*c^3*d + a^6*c^2*d^2)*x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d +
 2*a^4*c*d^2 - 3*(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 -
2*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 -
 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^
2*d)*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 6*(a^3*b*d^3*x^5
+ a^4*d^3*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/((a^3*b^3*c^4
 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*
c^3*d + a^6*c^2*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283637, size = 223, normalized size = 1.18

$$\frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\sqrt{cd}} + \frac{b^3x}{2\,(a^3bc - a^4d)(bx^2 + a)} + \frac{\left(5\,b^4c - 7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{ab}} + \frac{6\,bcx^2 + 3\,adx^2 - ac}{3\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*
sqrt(c*d)) + 1/2*b^3*x/((a^3*b*c - a^4*d)*(b*x^2 + a)) + 1/2*(5*b
^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*c
*d + a^5*d^2)*sqrt(a*b)) + 1/3*(6*b*c*x^2 + 3*a*d*x^2 - a*c)/(a^3
*c^2*x^3)
```

3.298 $\int \frac{1}{x^5(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=160

$$-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}+\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{d^4\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}$$

```
[Out] -1/(4*a^2*c*x^4) + (2*b*c + a*d)/(2*a^3*c^2*x^2) + b^3/(2*a^3*(b*
c - a*d)*(a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x]
)/(a^4*c^3) - (b^3*(3*b*c - 4*a*d)*Log[a + b*x^2])/(2*a^4*(b*c -
a*d)^2) - (d^4*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rubi [A] time = 0.419323, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}+\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{d^4\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(4*a^2*c*x^4) + (2*b*c + a*d)/(2*a^3*c^2*x^2) + b^3/(2*a^3*(b*
c - a*d)*(a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x]
)/(a^4*c^3) - (b^3*(3*b*c - 4*a*d)*Log[a + b*x^2])/(2*a^4*(b*c -
a*d)^2) - (d^4*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 58.4545, size = 151, normalized size = 0.94

$$-\frac{d^4\log\left(c+dx^2\right)}{2c^3\left(ad-bc\right)^2}-\frac{1}{4a^2cx^4}-\frac{b^3}{2a^3\left(a+bx^2\right)\left(ad-bc\right)}+\frac{ad+2bc}{2a^3c^2x^2}+\frac{b^3\left(4ad-3bc\right)\log\left(a+bx^2\right)}{2a^4\left(ad-bc\right)^2}+\frac{\left(a^2d^2+2abcd+3b^2c^2\right)\log\left(x^2\right)}{2a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -d**4*log(c + d*x**2)/(2*c**3*(a*d - b*c)**2) - 1/(4*a**2*c*x**4)
 - b**3/(2*a**3*(a + b*x**2)*(a*d - b*c)) + (a*d + 2*b*c)/(2*a**3
*c**2*x**2) + b**3*(4*a*d - 3*b*c)*log(a + b*x**2)/(2*a**4*(a*d -
 b*c)**2) + (a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(x**2)/(2*a*
*4*c**3)
```

Mathematica [A] time = 0.365272, size = 155, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2b^3(4ad-3bc)\log\left(a+bx^2\right)}{a^4(bc-ad)^2}-\frac{2b^3}{a^3\left(a+bx^2\right)(ad-bc)}+\frac{2ad+4bc}{a^3c^2x^2}-\frac{1}{a^2cx^4}+\frac{4\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{2d^4\log\left(c+dx^2\right)}{c^3(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^4)) + (4*b*c + 2*a*d)/(a^3*c^2*x^2) - (2*b^3)/(a^3*
(-(b*c) + a*d)*(a + b*x^2)) + (4*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2
)*Log[x])/(a^4*c^3) + (2*b^3*(-3*b*c + 4*a*d)*Log[a + b*x^2])/(a^
4*(b*c - a*d)^2) - (2*d^4*Log[c + d*x^2])/(c^3*(b*c - a*d)^2))/4
```

Maple [A] time = 0.029, size = 209, normalized size = 1.3

$$-\frac{1}{4\,a^2cx^4}+\frac{d}{2\,a^2c^2x^2}+\frac{b}{x^2a^3c}+\frac{\ln(x)\,d^2}{a^2c^3}+2\,\frac{b\ln(x)\,d}{a^3c^2}+3\,\frac{\ln(x)\,b^2}{a^4c}-\frac{d^4\ln\left(dx^2+c\right)}{2\,c^3\,(ad-bc)^2}+2\,\frac{b^3\ln\left(bx^2+a\right)d}{a^3\,(ad-bc)^2}-\frac{3\,b^4\ln\left(bx^2+a\right)c}{2\,a^4\,(ad-bc)^2}-\frac{db^3}{2\,a^2\,(ad-bc)^2\,(bx^2+a)}+\frac{b^4c}{2\,a^3\,(ad-bc)^2\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/4/a^2/c/x^4+1/2/x^2/a^2/c^2*d+1/x^2/a^3/c*b+1/a^2/c^3*ln(x)*d^
2+2/a^3/c^2*ln(x)*b*d+3/a^4/c*ln(x)*b^2-1/2*d^4/c^3/(a*d-b*c)^2*l
n(d*x^2+c)+2*b^3/a^3/(a*d-b*c)^2*ln(b*x^2+a)*d-3/2*b^4/a^4/(a*d-b
*c)^2*ln(b*x^2+a)*c-1/2*b^3/a^2/(a*d-b*c)^2/(b*x^2+a)*d+1/2*b^4/a
^3/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.36299, size = 348, normalized size = 2.17

$$-\frac{d^4\log\left(dx^2+c\right)}{2\,(b^2c^5-2\,abc^4d+a^2c^3d^2)}-\frac{\left(3\,b^4c-4\,ab^3d\right)\log\left(bx^2+a\right)}{2\,(a^4b^2c^2-2\,a^5bcd+a^6d^2)}-\frac{a^2bc^2-a^3cd-2\left(3\,b^3c^2-ab^2cd-a^2bd^2\right)x^4-\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^2}{4\,((a^3b^2c^3-a^4bc^2d)x^6+(a^4bc^3-a^5c^2d)x^4)}+\frac{\left(3\,b^2c^2+2\,abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] -1/2*d^4*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1
/2*(3*b^4*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^4*b^2*c^2 - 2*a^5*b*c*
d + a^6*d^2) - 1/4*(a^2*b*c^2 - a^3*c*d - 2*(3*b^3*c^2 - a*b^2*c*
d - a^2*b*d^2)*x^4 - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)/(
(a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^4) +
1/2*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(x^2)/(a^4*c^3)
```

Fricas [A] time = 6.04552, size = 481, normalized size = 3.01

$$-\frac{a^3b^2c^4-2\,a^4bc^3d+a^5c^2d^2-2\left(3\,ab^4c^4-4\,a^2b^3c^3d+a^4bcd^3\right)x^4-\left(3\,a^2b^3c^4-4\,a^3b^2c^3d-a^4bc^2d^2+2\,a^5cd^3\right)x^2+2\left(\left(3\,b\right.\right.}{4\left(\left(a^4b^3c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] -1/4*(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 - 2*(3*a*b^4*c^4
- 4*a^2*b^3*c^3*d + a^4*b*c*d^3)*x^4 - (3*a^2*b^3*c^4 - 4*a^3*b^2
*c^3*d - a^4*b*c^2*d^2 + 2*a^5*c*d^3)*x^2 + 2*((3*b^5*c^4 - 4*a*b
^4*c^3*d)*x^6 + (3*a*b^4*c^4 - 4*a^2*b^3*c^3*d)*x^4)*log(b*x^2 +
```

```
a) + 2*(a^4*b*d^4*x^6 + a^5*d^4*x^4)*log(d*x^2 + c) - 4*((3*b^5*c
^4 - 4*a*b^4*c^3*d + a^4*b*d^4)*x^6 + (3*a*b^4*c^4 - 4*a^2*b^3*c^
3*d + a^5*d^4)*x^4)*log(x))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6
*b*c^3*d^2)*x^6 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.227361, size = 379, normalized size = 2.37

$$-\frac{d^5\ln\left(\left|dx^2+c\right|\right)}{2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)}-\frac{\left(3\,b^5c-4\,ab^4d\right)\ln\left(\left|bx^2+a\right|\right)}{2\left(a^4b^3c^2-2\,a^5b^2cd+a^6bd^2\right)}+\frac{3\,b^5cx^2-4\,ab^4dx^2+4\,ab^4c-5\,a^2b^3d}{2\left(a^4b^2c^2-2\,a^5bcd+a^6d^2\right)\left(bx^2+a\right)}+\frac{\left(3\,b^2c^2+2\,abcd+a^2d^2\right)\ln\left(x^2\right)}{2\,a^4c^3}-\frac{9\,b^2c^2x^4+6\,abcdx^4+3\,a^2d^2x^4-4\,abc^2x^2-2\,a^2cdx^2+a^2c^2}{4\,a^4c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] -1/2*d^5*ln(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*
d^3) - 1/2*(3*b^5*c - 4*a*b^4*d)*ln(abs(b*x^2 + a))/(a^4*b^3*c^2
- 2*a^5*b^2*c*d + a^6*b*d^2) + 1/2*(3*b^5*c*x^2 - 4*a*b^4*d*x^2 +
 4*a*b^4*c - 5*a^2*b^3*d)/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*
(b*x^2 + a)) + 1/2*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ln(x^2)/(a^4
*c^3) - 1/4*(9*b^2*c^2*x^4 + 6*a*b*c*d*x^4 + 3*a^2*d^2*x^4 - 4*a*
b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^4*c^3*x^4)
```

3.299 $\int \frac{1}{x^6(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=250

$$-\frac{b^{7/2}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}+\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}-\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}+\frac{b}{2ax^5(a+bx^2)(bc-ad)}$$

```
[Out] -(7*b*c - 2*a*d)/(10*a^2*c*(b*c - a*d)*x^5) + (7*b^2*c^2 - 2*a*b*
c*d - 2*a^2*d^2)/(6*a^3*c^2*(b*c - a*d)*x^3) - (7*b^3*c^3 - 2*a*b
^2*c^2*d - 2*a^2*b*c*d^2 - 2*a^3*d^3)/(2*a^4*c^3*(b*c - a*d)*x) +
 b/(2*a*(b*c - a*d)*x^5*(a + b*x^2)) - (b^(7/2)*(7*b*c - 9*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(b*c - a*d)^2) - (d^(9/2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

Rubi [A] time = 1.10313, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{b^{7/2}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}+\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}-\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}+\frac{b}{2ax^5(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(7*b*c - 2*a*d)/(10*a^2*c*(b*c - a*d)*x^5) + (7*b^2*c^2 - 2*a*b*
c*d - 2*a^2*d^2)/(6*a^3*c^2*(b*c - a*d)*x^3) - (7*b^3*c^3 - 2*a*b
^2*c^2*d - 2*a^2*b*c*d^2 - 2*a^3*d^3)/(2*a^4*c^3*(b*c - a*d)*x) +
 b/(2*a*(b*c - a*d)*x^5*(a + b*x^2)) - (b^(7/2)*(7*b*c - 9*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(b*c - a*d)^2) - (d^(9/2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.568445, size = 179, normalized size = 0.72

$$\frac{b^{7/2}(9ad-7bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(ad-bc)^2}+\frac{b^4x}{2a^4(a+bx^2)(ad-bc)}+\frac{ad+2bc}{3a^3c^2x^3}-\frac{1}{5a^2cx^5}+\frac{-a^2d^2-2abcd-3b^2c^2}{a^4c^3x}-\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(5*a^2*c*x^5) + (2*b*c + a*d)/(3*a^3*c^2*x^3) + (-3*b^2*c^2 -
2*a*b*c*d - a^2*d^2)/(a^4*c^3*x) + (b^4*x)/(2*a^4*(-(b*c) + a*d)*
(a + b*x^2)) + (b^(7/2)*(-7*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(9/2)*(-(b*c) + a*d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

Maple [A] time = 0.024, size = 234, normalized size = 0.9

$$-\frac{1}{5\,a^2cx^5}+\frac{d}{3\,a^2c^2x^3}+\frac{2\,b}{3\,x^3a^3c}-\frac{d^2}{a^2c^3x}-2\,\frac{bd}{a^3c^2x}-3\,\frac{b^2}{a^4cx}-\frac{d^5}{c^3\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^4xd}{2\,a^3\,(ad-bc)^2\,(bx^2+a)}-\frac{b^5xc}{2\,a^4\,(ad-bc)^2\,(bx^2+a)}+\frac{9\,db^4}{2\,a^3\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7\,b^5c}{2\,a^4\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/5/a^2/c/x^5+1/3/x^3/a^2/c^2*d+2/3/x^3/a^3/c*b-1/a^2/c^3/x*d^2-
2/a^3/c^2/x*b*d-3/a^4/c/x*b^2-1/c^3*d^5/(a*d-b*c)^2/(c*d)^(1/2)*a
rctan(x*d/(c*d)^(1/2))+1/2*b^4/a^3/(a*d-b*c)^2*x/(b*x^2+a)*d-1/2*
b^5/a^4/(a*d-b*c)^2*x/(b*x^2+a)*c+9/2*b^4/a^3/(a*d-b*c)^2/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))*d-7/2*b^5/a^4/(a*d-b*c)^2/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.12234, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^6),x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*
b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*
a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 -
 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^
5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)
*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 30*
(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(
-d/c) - c)/(d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*
c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5),
-1/60*(12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b
^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a
^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 -
9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 60*(a^4*b*
```

```
d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + 15*(
(7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)
*x^5)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))
/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*
c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 1
2*a^4*b*c^3*d + 6*a^5*c^2*d^2 + 15*(7*b^5*c^4 - 9*a*b^4*c^3*d + 2
*a^4*b*d^4)*x^6 + 10*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3
 + 3*a^5*d^4)*x^4 - 2*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*
c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7
+ (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)*arctan(b*x/(a*sq
rt(b/a))) - 15*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2
*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*
c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*c^3*d + 6*a^5*c^2*
d^2 + 15*(7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 10*(7*a*
b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a
^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2
 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3
*c^3*d)*x^5)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 30*(a^4*b*d^4*
x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/((a^4*b^3
*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^
6*b*c^4*d + a^7*c^3*d^2)*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.238905, size = 279, normalized size = 1.12

$$-\frac{d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}}-\frac{b^4x}{2\,(a^4bc-a^5d)(bx^2+a)}-\frac{(7\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^4b^2c^2-2\,a^5bcd+a^6d^2)\sqrt{ab}}-\frac{45\,b^2c^2x^4+30\,abcdx^4+15\,a^2d^2x^4-10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,a^4c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^6),x, algorithm="giac")
```

```
[Out] -d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)
*sqrt(c*d)) - 1/2*b^4*x/((a^4*b*c - a^5*d)*(b*x^2 + a)) - 1/2*(7*
b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b*
c*d + a^6*d^2)*sqrt(a*b)) - 1/15*(45*b^2*c^2*x^4 + 30*a*b*c*d*x^4
 + 15*a^2*d^2*x^4 - 10*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(
a^4*c^3*x^5)
```

3.300 $\int \frac{1}{x^7(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=210

$$\frac{b^4(4bc-5ad)\log\left(a+bx^2\right)}{2a^5(bc-ad)^2}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{4a^3c^2x^4}-\frac{1}{6a^2cx^6}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}+\frac{d^5\log\left(c+dx^2\right)}{2c^4(bc-ad)^2}$$

```
[Out] -1/(6*a^2*c*x^6) + (2*b*c + a*d)/(4*a^3*c^2*x^4) - (3*b^2*c^2 + 2
*a*b*c*d + a^2*d^2)/(2*a^4*c^3*x^2) - b^4/(2*a^4*(b*c - a*d)*(a +
 b*x^2)) - ((4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)
*Log[x])/(a^5*c^4) + (b^4*(4*b*c - 5*a*d)*Log[a + b*x^2])/(2*a^5*
(b*c - a*d)^2) + (d^5*Log[c + d*x^2])/(2*c^4*(b*c - a*d)^2)
```

Rubi [A] time = 0.540241, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^4(4bc-5ad)\log\left(a+bx^2\right)}{2a^5(bc-ad)^2}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{4a^3c^2x^4}-\frac{1}{6a^2cx^6}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}+\frac{d^5\log\left(c+dx^2\right)}{2c^4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(6*a^2*c*x^6) + (2*b*c + a*d)/(4*a^3*c^2*x^4) - (3*b^2*c^2 + 2
*a*b*c*d + a^2*d^2)/(2*a^4*c^3*x^2) - b^4/(2*a^4*(b*c - a*d)*(a +
 b*x^2)) - ((4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)
*Log[x])/(a^5*c^4) + (b^4*(4*b*c - 5*a*d)*Log[a + b*x^2])/(2*a^5*
(b*c - a*d)^2) + (d^5*Log[c + d*x^2])/(2*c^4*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 83.4612, size = 202, normalized size = 0.96

$$\frac{d^5\log\left(c+dx^2\right)}{2c^4\left(ad-bc\right)^2}-\frac{1}{6a^2cx^6}+\frac{ad+2bc}{4a^3c^2x^4}+\frac{b^4}{2a^4\left(a+bx^2\right)\left(ad-bc\right)}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{b^4\left(5ad-4bc\right)\log\left(a+bx^2\right)}{2a^5\left(ad-bc\right)^2}-\frac{\left(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)\log\left(x^2\right)}{2a^5c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d**5*log(c + d*x**2)/(2*c**4*(a*d - b*c)**2) - 1/(6*a**2*c*x**6)
+ (a*d + 2*b*c)/(4*a**3*c**2*x**4) + b**4/(2*a**4*(a + b*x**2)*(a
*d - b*c)) - (a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)/(2*a**4*c**3*x
**2) - b**4*(5*a*d - 4*b*c)*log(a + b*x**2)/(2*a**5*(a*d - b*c)**
2) - (a**3*d**3 + 2*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 4*b**3*c**3
)*log(x**2)/(2*a**5*c**4)
```

Mathematica [A] time = 0.579743, size = 202, normalized size = 0.96

$$\frac{1}{12}\left(\frac{6b^4(4bc-5ad)\log\left(a+bx^2\right)}{a^5(bc-ad)^2}+\frac{6b^4}{a^4\left(a+bx^2\right)(ad-bc)}+\frac{3ad+6bc}{a^3c^2x^4}-\frac{2}{a^2cx^6}-\frac{6\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3x^2}-\frac{12\log(x)\left(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}+\frac{6d^5\log\left(c+dx^2\right)}{c^4(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-2/(a^2*c*x^6) + (6*b*c + 3*a*d)/(a^3*c^2*x^4) - (6*(3*b^2*c^2 +
 2*a*b*c*d + a^2*d^2))/(a^4*c^3*x^2) + (6*b^4)/(a^4*(-(b*c) + a*d
)*(a + b*x^2)) - (12*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 +
 a^3*d^3)*Log[x])/(a^5*c^4) + (6*b^4*(4*b*c - 5*a*d)*Log[a + b*x^
2])/(a^5*(b*c - a*d)^2) + (6*d^5*Log[c + d*x^2])/(c^4*(b*c - a*d)
^2))/12
```

Maple [A] time = 0.03, size = 268, normalized size = 1.3

$$-\frac{1}{6\,a^{2}c x^{6}}+\frac{d}{4\,a^{2}c^{2}x^{4}}+\frac{b}{2\,x^{4}a^{3}c}-\frac{d^{2}}{2\,a^{2}c^{3}x^{2}}-\frac{bd}{a^{3}c^{2}x^{2}}-\frac{3\,b^{2}}{2\,a^{4}cx^{2}}-\frac{\ln(x)\,d^{3}}{a^{2}c^{4}}-2\,\frac{\ln(x)\,d^{2}b}{a^{3}c^{3}}-3\,\frac{\ln(x)\,db^{2}}{a^{4}c^{2}}-4\,\frac{\ln(x)\,b^{3}}{a^{5}c}+\frac{d^{5}\ln\left(dx^{2}+c\right)}{2\,c^{4}\left(ad-bc\right)^{2}}-\frac{5\,b^{4}\ln\left(bx^{2}+a\right)d}{2\,a^{4}\left(ad-bc\right)^{2}}+2\,\frac{b^{5}\ln\left(bx^{2}+a\right)c}{a^{5}\left(ad-bc\right)^{2}}+\frac{db^{4}}{2\,a^{3}\left(ad-bc\right)^{2}\left(bx^{2}+a\right)}-\frac{b^{5}c}{2\,a^{4}\left(ad-bc\right)^{2}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/6/a^2/c/x^6+1/4/x^4/a^2/c^2*d+1/2/x^4/a^3/c*b-1/2/a^2/c^3/x^2*
d^2-1/a^3/c^2/x^2*b*d-3/2/a^4/c/x^2*b^2-1/a^2/c^4*ln(x)*d^3-2/a^3
/c^3*ln(x)*d^2*b-3/a^4/c^2*ln(x)*d*b^2-4/a^5/c*ln(x)*b^3+1/2*d^5/
c^4/(a*d-b*c)^2*ln(d*x^2+c)-5/2*b^4/a^4/(a*d-b*c)^2*ln(b*x^2+a)*d
+2*b^5/a^5/(a*d-b*c)^2*ln(b*x^2+a)*c+1/2*b^4/a^3/(a*d-b*c)^2/(b*x
^2+a)*d-1/2*b^5/a^4/(a*d-b*c)^2/(b*x^2+a)*c
```

Maxima [A] time = 1.3673, size = 458, normalized size = 2.18

$$\frac{d^{5}\log\left(dx^{2}+c\right)}{2\left(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}\right)}+\frac{\left(4\,b^{5}c-5\,ab^{4}d\right)\log\left(bx^{2}+a\right)}{2\left(a^{5}b^{2}c^{2}-2\,a^{6}bcd+a^{7}d^{2}\right)}-\frac{2\,a^{3}bc^{3}-2\,a^{4}c^{2}d+6\left(4\,b^{4}c^{3}-ab^{3}c^{2}d-a^{2}b^{2}cd^{2}-a^{3}bd^{3}\right)x^{6}+3\left(4\,ab^{3}c^{3}-a^{2}b^{2}c^{2}d-a^{3}bcd^{2}-2\,a^{4}d^{3}\right)x^{4}-\left(4\,a^{2}b^{2}c^{3}-a\right.}{12\left(\left(a^{4}b^{2}c^{4}-a^{5}bc^{3}d\right)x^{8}+\left(a^{5}bc^{4}-a^{6}c^{3}d\right)x^{6}\right)}-\frac{\left(4\,b^{3}c^{3}+3\,ab^{2}c^{2}d+2\,a^{2}bcd^{2}+a^{3}d^{3}\right)\log\left(x^{2}\right)}{2\,a^{5}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^7),x, algorithm="maxima")
```

```
[Out] 1/2*d^5*log(d*x^2 + c)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2) + 1/
2*(4*b^5*c - 5*a*b^4*d)*log(b*x^2 + a)/(a^5*b^2*c^2 - 2*a^6*b*c*d
 + a^7*d^2) - 1/12*(2*a^3*b*c^3 - 2*a^4*c^2*d + 6*(4*b^4*c^3 - a*
b^3*c^2*d - a^2*b^2*c*d^2 - a^3*b*d^3)*x^6 + 3*(4*a*b^3*c^3 - a^2
*b^2*c^2*d - a^3*b*c*d^2 - 2*a^4*d^3)*x^4 - (4*a^2*b^2*c^3 - a^3*
b*c^2*d - 3*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*c^3*d)*x^8 + (a
^5*b*c^4 - a^6*c^3*d)*x^6) - 1/2*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a
^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4)
```

Fricas [A] time = 9.30528, size = 554, normalized size = 2.64

$$-\frac{2\,a^{4}b^{2}c^{5}-4\,a^{5}bc^{4}d+2\,a^{6}c^{3}d^{2}+6\left(4\,ab^{5}c^{5}-5\,a^{2}b^{4}c^{4}d+a^{5}bcd^{4}\right)x^{6}+3\left(4\,a^{2}b^{4}c^{5}-5\,a^{3}b^{3}c^{4}d-a^{5}bc^{2}d^{3}+2\,a^{6}cd^{4}\right)x^{4}-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^7),x, algorithm="fricas")
```

```
[Out] -1/12*(2*a^4*b^2*c^5 - 4*a^5*b*c^4*d + 2*a^6*c^3*d^2 + 6*(4*a*b^5
*c^5 - 5*a^2*b^4*c^4*d + a^5*b*c*d^4)*x^6 + 3*(4*a^2*b^4*c^5 - 5*
a^3*b^3*c^4*d - a^5*b*c^2*d^3 + 2*a^6*c*d^4)*x^4 - (4*a^3*b^3*c^5
 - 5*a^4*b^2*c^4*d - 2*a^5*b*c^3*d^2 + 3*a^6*c^2*d^3)*x^2 - 6*((4
*b^6*c^5 - 5*a*b^5*c^4*d)*x^8 + (4*a*b^5*c^5 - 5*a^2*b^4*c^4*d)*x
^6)*log(b*x^2 + a) - 6*(a^5*b*d^5*x^8 + a^6*d^5*x^6)*log(d*x^2 +
c) + 12*((4*b^6*c^5 - 5*a*b^5*c^4*d + a^5*b*d^5)*x^8 + (4*a*b^5*c
^5 - 5*a^2*b^4*c^4*d + a^6*d^5)*x^6)*log(x))/((a^5*b^3*c^6 - 2*a^
6*b^2*c^5*d + a^7*b*c^4*d^2)*x^8 + (a^6*b^2*c^6 - 2*a^7*b*c^5*d +
 a^8*c^4*d^2)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239585, size = 478, normalized size = 2.28

$$\frac{d^6\ln\left(\left|dx^2+c\right|\right)}{2\left(b^2c^6d-2\,abc^5d^2+a^2c^4d^3\right)}+\frac{\left(4\,b^6c-5\,ab^5d\right)\ln\left(\left|bx^2+a\right|\right)}{2\left(a^5b^3c^2-2\,a^6b^2cd+a^7bd^2\right)}-\frac{4\,b^6cx^2-5\,ab^5dx^2+5\,ab^5c-6\,a^2b^4d}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)\left(bx^2+a\right)}-\frac{\left(4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3\right)\ln\left(x^2\right)}{2\,a^5c^4}+\frac{44\,b^3c^3x^6+33\,ab^2c^2dx^6+22\,a^2bcd^2x^6+11\,a^3d^3x^6-18\,ab^2c^3x^4-12\,a^2bc^2dx^4-6\,a^3cd^2x^4+6\,a^2bc^3x^2+3\,a^3c^2dx^2-2\,a^3}{12\,a^5c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*x^7),x, algorithm="giac")
```

```
[Out] 1/2*d^6*ln(abs(d*x^2 + c))/(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d
^3) + 1/2*(4*b^6*c - 5*a*b^5*d)*ln(abs(b*x^2 + a))/(a^5*b^3*c^2 -
 2*a^6*b^2*c*d + a^7*b*d^2) - 1/2*(4*b^6*c*x^2 - 5*a*b^5*d*x^2 +
5*a*b^5*c - 6*a^2*b^4*d)/((a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*(
b*x^2 + a)) - 1/2*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^
3*d^3)*ln(x^2)/(a^5*c^4) + 1/12*(44*b^3*c^3*x^6 + 33*a*b^2*c^2*d*
x^6 + 22*a^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 18*a*b^2*c^3*x^4 - 12
*a^2*b*c^2*d*x^4 - 6*a^3*c*d^2*x^4 + 6*a^2*b*c^3*x^2 + 3*a^3*c^2*
d*x^2 - 2*a^3*c^3)/(a^5*c^4*x^6)
```

3.301 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2} dx$

Optimal. Leaf size=162

$$\frac{x(ad+bc)}{2b(c+dx^2)(bc-ad)^2}+\frac{ax}{2b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\sqrt{a}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)^3}+\frac{\sqrt{c}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)^3}$$

```
[Out] ((b*c + a*d)*x)/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*x)/(2*b*(b*c
 - a*d)*(a + b*x^2)*(c + d*x^2)) - (Sqrt[a]*(3*b*c + a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*(b*c - a*d)^3) + (Sqrt[c]*(b*c +
 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d)^3)
```

Rubi [A] time = 0.411205, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x(ad+bc)}{2b(c+dx^2)(bc-ad)^2}+\frac{ax}{2b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\sqrt{a}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)^3}+\frac{\sqrt{c}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*c + a*d)*x)/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*x)/(2*b*(b*c
 - a*d)*(a + b*x^2)*(c + d*x^2)) - (Sqrt[a]*(3*b*c + a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*(b*c - a*d)^3) + (Sqrt[c]*(b*c +
 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 77.951, size = 138, normalized size = 0.85

$$\frac{\sqrt{a}\,(ad+3bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\,(ad-bc)^3}-\frac{ax}{2b\,(a+bx^2)\,(c+dx^2)\,(ad-bc)}-\frac{\sqrt{c}\,(3ad+bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}\,(ad-bc)^3}+\frac{x\,(ad+bc)}{2b\,(c+dx^2)\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(a)*(a*d + 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(b)*(a*d - b
*c)**3) - a*x/(2*b*(a + b*x**2)*(c + d*x**2)*(a*d - b*c)) - sqrt(
c)*(3*a*d + b*c)*atan(sqrt(d)*x/sqrt(c))/(2*sqrt(d)*(a*d - b*c)**
3) + x*(a*d + b*c)/(2*b*(c + d*x**2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.324419, size = 133, normalized size = 0.82

$$\frac{1}{2}\left(\frac{ax}{(a+bx^2)(bc-ad)^2}+\frac{cx}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt{a}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad-bc)^3}+\frac{\sqrt{c}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*x)/((b*c - a*d)^2*(a + b*x^2)) + (c*x)/((b*c - a*d)^2*(c + d*
x^2)) + (Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt
[b]*(-(b*c) + a*d)^3) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)^3))/2
```

Maple [A] time = 0.02, size = 222, normalized size = 1.4

$$\frac{acxd}{2\,(ad-bc)^3\,(dx^2+c)}-\frac{c^2xb}{2\,(ad-bc)^3\,(dx^2+c)}-\frac{3\,acd}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{bc^2}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{xa^2d}{2\,(ad-bc)^3\,(bx^2+a)}-\frac{xabc}{2\,(ad-bc)^3\,(bx^2+a)}+\frac{a^2d}{2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,abc}{2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^3*x/(d*x^2+c)*a*d-1/2*c^2/(a*d-b*c)^3*x/(d*x^2+c)
*b-3/2*c/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*d-1/2*
c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*a^2/(a*
d-b*c)^3*x/(b*x^2+a)*d-1/2*a/(a*d-b*c)^3*x/(b*x^2+a)*b*c+1/2*a^2/
(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+3/2*a/(a*d-b*c)
^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.399201, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*
a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-
a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - ((b^2*c*d
+ 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3
*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x
^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d
+ 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*
a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3
*b*c*d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - 2*((3*
b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) - ((b^2*c*d +
3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a
^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2
```

```
 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d +
3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^
2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b
*c*d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 + 2*((b^2*
c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d
 + 3*a^2*d^2)*x^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) - ((3*b^2*c*d +
a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*
a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*
b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c
*d^3 - a^4*d^4)*x^2), 1/2*((b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d
+ a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^2)*sqrt(a/b)*arctan(x/sqrt(a/b)) + ((b^2*c*d + 3*a*b*d^
2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*
x^2)*sqrt(c/d)*arctan(x/sqrt(c/d)) + 2*(a*b*c^2 - a^2*c*d)*x)/(a*
b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^
3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c
^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

Sympy [A] time = 83.168, size = 2378, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-a/b)*(a*d + 3*b*c)*log(x + (-4*a**7*b*d**8*(-a/b)**(3/2)*(a
*d + 3*b*c)**3/(a*d - b*c)**9 + 20*a**6*b**2*c*d**7*(-a/b)**(3/2)
*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 36*a**5*b**3*c**2*d**6*(-a/b)*
*(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 20*a**4*b**4*c**3*d**5*(
-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - a**4*d**4*sqrt(-a/
b)*(a*d + 3*b*c)/(a*d - b*c)**3 + 20*a**3*b**5*c**4*d**4*(-a/b)**
(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 36*a**3*b*c*d**3*sqrt(-a/
b)*(a*d + 3*b*c)/(a*d - b*c)**3 - 36*a**2*b**6*c**5*d**3*(-a/b)**
(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 54*a**2*b**2*c**2*d**2*sq
rt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 + 20*a*b**7*c**6*d**2*(-a/b
)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 36*a*b**3*c**3*d*sqrt(
-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 - 4*b**8*c**7*d*(-a/b)**(3/2)*
(a*d + 3*b*c)**3/(a*d - b*c)**9 - b**4*c**4*sqrt(-a/b)*(a*d + 3*b
*c)/(a*d - b*c)**3)/(3*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2))/(4*
(a*d - b*c)**3) - sqrt(-a/b)*(a*d + 3*b*c)*log(x + (4*a**7*b*d**8
*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 20*a**6*b**2*c*d
**7*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 36*a**5*b**3*
c**2*d**6*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 20*a**4
*b**4*c**3*d**5*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + a
**4*d**4*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 - 20*a**3*b**5*c
**4*d**4*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 36*a**3*
b*c*d**3*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 + 36*a**2*b**6*c
**5*d**3*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 54*a**2*
b**2*c**2*d**2*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 - 20*a*b**
7*c**6*d**2*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 36*a*
b**3*c**3*d*sqrt(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3 + 4*b**8*c**7
*d*(-a/b)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + b**4*c**4*sqrt
(-a/b)*(a*d + 3*b*c)/(a*d - b*c)**3)/(3*a**2*d**2 + 10*a*b*c*d +
3*b**2*c**2))/(4*(a*d - b*c)**3) + sqrt(-c/d)*(3*a*d + b*c)*log(x
 + (-4*a**7*b*d**8*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9
+ 20*a**6*b**2*c*d**7*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)*
*9 - 36*a**5*b**3*c**2*d**6*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d -
 b*c)**9 + 20*a**4*b**4*c**3*d**5*(-c/d)**(3/2)*(3*a*d + b*c)**3/
(a*d - b*c)**9 - a**4*d**4*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**
3 + 20*a**3*b**5*c**4*d**4*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d -
b*c)**9 - 36*a**3*b*c*d**3*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**
3 - 36*a**2*b**6*c**5*d**3*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d -
b*c)**9 - 54*a**2*b**2*c**2*d**2*sqrt(-c/d)*(3*a*d + b*c)/(a*d -
b*c)**3 + 20*a*b**7*c**6*d**2*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d
 - b*c)**9 - 36*a*b**3*c**3*d*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c
)**3 - 4*b**8*c**7*d*(-c/d)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**
9 - b**4*c**4*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*c)**3)/(3*a**2*d*
```

```
*2 + 10*a*b*c*d + 3*b**2*c**2))/(4*(a*d - b*c)**3) - sqrt(-c/d)*(
3*a*d + b*c)*log(x + (4*a**7*b*d**8*(-c/d)**(3/2)*(3*a*d + b*c)**
3/(a*d - b*c)**9 - 20*a**6*b**2*c*d**7*(-c/d)**(3/2)*(3*a*d + b*c
)**3/(a*d - b*c)**9 + 36*a**5*b**3*c**2*d**6*(-c/d)**(3/2)*(3*a*d
 + b*c)**3/(a*d - b*c)**9 - 20*a**4*b**4*c**3*d**5*(-c/d)**(3/2)*
(3*a*d + b*c)**3/(a*d - b*c)**9 + a**4*d**4*sqrt(-c/d)*(3*a*d + b
*c)/(a*d - b*c)**3 - 20*a**3*b**5*c**4*d**4*(-c/d)**(3/2)*(3*a*d
+ b*c)**3/(a*d - b*c)**9 + 36*a**3*b*c*d**3*sqrt(-c/d)*(3*a*d + b
*c)/(a*d - b*c)**3 + 36*a**2*b**6*c**5*d**3*(-c/d)**(3/2)*(3*a*d
+ b*c)**3/(a*d - b*c)**9 + 54*a**2*b**2*c**2*d**2*sqrt(-c/d)*(3*a
*d + b*c)/(a*d - b*c)**3 - 20*a*b**7*c**6*d**2*(-c/d)**(3/2)*(3*a
*d + b*c)**3/(a*d - b*c)**9 + 36*a*b**3*c**3*d*sqrt(-c/d)*(3*a*d
+ b*c)/(a*d - b*c)**3 + 4*b**8*c**7*d*(-c/d)**(3/2)*(3*a*d + b*c)
**3/(a*d - b*c)**9 + b**4*c**4*sqrt(-c/d)*(3*a*d + b*c)/(a*d - b*
c)**3)/(3*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2))/(4*(a*d - b*c)**
3) + (2*a*c*x + x**3*(a*d + b*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*
d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**
3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d
 + 2*b**3*c**3))
```

GIAC/XCAS [A] time = 0.340477, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] Done
```

3.302 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=107

$$\frac{a}{2(a+bx^2)(bc-ad)^2}+\frac{c}{2(c+dx^2)(bc-ad)^2}+\frac{(ad+bc)\log(a+bx^2)}{2(bc-ad)^3}-\frac{(ad+bc)\log(c+dx^2)}{2(bc-ad)^3}$$

```
[Out] a/(2*(b*c - a*d)^2*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2))
 + ((b*c + a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^3) - ((b*c + a*d)*
Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rubi [A] time = 0.246564, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a}{2(a+bx^2)(bc-ad)^2}+\frac{c}{2(c+dx^2)(bc-ad)^2}+\frac{(ad+bc)\log(a+bx^2)}{2(bc-ad)^3}-\frac{(ad+bc)\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] a/(2*(b*c - a*d)^2*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2))
 + ((b*c + a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^3) - ((b*c + a*d)*
Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 39.3241, size = 87, normalized size = 0.81

$$\frac{a}{2(a+bx^2)(ad-bc)^2}+\frac{c}{2(c+dx^2)(ad-bc)^2}-\frac{(ad+bc)\log(a+bx^2)}{2(ad-bc)^3}+\frac{(ad+bc)\log(c+dx^2)}{2(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] a/(2*(a + b*x**2)*(a*d - b*c)**2) + c/(2*(c + d*x**2)*(a*d - b*c)
**2) - (a*d + b*c)*log(a + b*x**2)/(2*(a*d - b*c)**3) + (a*d + b*
c)*log(c + d*x**2)/(2*(a*d - b*c)**3)
```

Mathematica [A] time = 0.111439, size = 86, normalized size = 0.8

$$\frac{\frac{a(bc-ad)}{a+bx^2}+\frac{c(bc-ad)}{c+dx^2}+(ad+bc)\log(a+bx^2)-(ad+bc)\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d))/(c + d*x^2) + (b*c
 + a*d)*Log[a + b*x^2] - (b*c + a*d)*Log[c + d*x^2])/(2*(b*c - a*
d)^3)
```

Maple [A] time = 0.024, size = 188, normalized size = 1.8

$$\frac{acd}{2\left(ad-bc\right)^{3}\left(dx^{2}+c\right)}-\frac{bc^{2}}{2\left(ad-bc\right)^{3}\left(dx^{2}+c\right)}+\frac{d\ln\left(dx^{2}+c\right)a}{2\left(ad-bc\right)^{3}}+\frac{\ln\left(dx^{2}+c\right)bc}{2\left(ad-bc\right)^{3}}-\frac{\ln\left(bx^{2}+a\right)ad}{2\left(ad-bc\right)^{3}}-\frac{b\ln\left(bx^{2}+a\right)c}{2\left(ad-bc\right)^{3}}+\frac{a^{2}d}{2\left(ad-bc\right)^{3}\left(bx^{2}+a\right)}-\frac{abc}{2\left(ad-bc\right)^{3}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d/(a*d-b*c)^3*c/(d*x^2+c)*a-1/2/(a*d-b*c)^3*c^2/(d*x^2+c)*b+1
/2*d/(a*d-b*c)^3*ln(d*x^2+c)*a+1/2/(a*d-b*c)^3*ln(d*x^2+c)*b*c-1/
2/(a*d-b*c)^3*ln(b*x^2+a)*a*d-1/2*b/(a*d-b*c)^3*ln(b*x^2+a)*c+1/2
/(a*d-b*c)^3*a^2/(b*x^2+a)*d-1/2*b/(a*d-b*c)^3*a/(b*x^2+a)*c
```

Maxima [A] time = 1.36032, size = 308, normalized size = 2.88

$$\frac{(bc+ad)\log\left(bx^{2}+a\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{(bc+ad)\log\left(dx^{2}+c\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{(bc+ad)x^{2}+2\,ac}{2\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3})x^{4}+(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3})x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c + a*d)*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3) - 1/2*(b*c + a*d)*log(d*x^2 + c)/(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*((b*c + a*d)*x^2 + 2*
a*c)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^
2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
 a^3*d^3)*x^2)
```

Fricas [A] time = 0.245078, size = 400, normalized size = 3.74

$$\frac{2\,abc^{2}-2\,a^{2}cd+\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}+\left(\left(b^{2}cd+abd^{2}\right)x^{4}+abc^{2}+a^{2}cd+\left(b^{2}c^{2}+2\,abcd+a^{2}d^{2}\right)x^{2}\right)\log\left(bx^{2}+a\right)-\left(\left(b^{2}cd+\right.\right.}{2\left(ab^{3}c^{4}-3\,a^{2}b^{2}c^{3}d+3\,a^{3}bc^{2}d^{2}-a^{4}cd^{3}+(b^{4}c^{3}d-3\,ab^{3}c^{2}d^{2}+3\,a^{2}b^{2}cd^{3}-a^{3}bd^{4})x^{4}+(b^{4}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2 - 2*a^2*c*d + (b^2*c^2 - a^2*d^2)*x^2 + ((b^2*c*d
+ a*b*d^2)*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d + a^2*d
^2)*x^2)*log(b*x^2 + a) - ((b^2*c*d + a*b*d^2)*x^4 + a*b*c^2 + a^
2*c*d + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*x^2)*log(d*x^2 + c))/(a*b
^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3
*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^
4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

Sympy [A] time = 16.2687, size = 507, normalized size = 4.74

$$\frac{2ac+x^2(ad+bc)}{2a^3cd^2-4a^2bc^2d+2ab^2c^3+x^4(2a^2bd^3-4ab^2cd^2+2b^3c^2d)+x^2(2a^3d^3-2a^2bcd^2-2ab^2c^2d+2b^3c^3)} + \frac{(ad+bc)\log\left(x^2+\frac{-\frac{a^4d^4(ad+bc)}{(ad-bc)^3}+\frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3}-\frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3}+a^2d^2+\frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3}+2abcd-\frac{b^4c^4(ad+bc)}{(ad-bc)^3}+b^2c^2}{2abd^2+2b^2cd}\right)}{2(ad-bc)^3} - \frac{(ad+bc)\log\left(x^2+\frac{\frac{a^4d^4(ad+bc)}{(ad-bc)^3}-\frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3}+\frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3}+a^2d^2-\frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3}+2abcd+\frac{b^4c^4(ad+bc)}{(ad-bc)^3}+b^2c^2}{2abd^2+2b^2cd}\right)}{2(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] (2*a*c + x**2*(a*d + b*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*d + 2*a
*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**3*c**2*
d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b*
*3*c**3)) + (a*d + b*c)*log(x**2 + (-a**4*d**4*(a*d + b*c)/(a*d -
 b*c)**3 + 4*a**3*b*c*d**3*(a*d + b*c)/(a*d - b*c)**3 - 6*a**2*b*
*2*c**2*d**2*(a*d + b*c)/(a*d - b*c)**3 + a**2*d**2 + 4*a*b**3*c*
*3*d*(a*d + b*c)/(a*d - b*c)**3 + 2*a*b*c*d - b**4*c**4*(a*d + b*
c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 + 2*b**2*c*d))/(2*(a*d
 - b*c)**3) - (a*d + b*c)*log(x**2 + (a**4*d**4*(a*d + b*c)/(a*d
- b*c)**3 - 4*a**3*b*c*d**3*(a*d + b*c)/(a*d - b*c)**3 + 6*a**2*b
**2*c**2*d**2*(a*d + b*c)/(a*d - b*c)**3 + a**2*d**2 - 4*a*b**3*c
**3*d*(a*d + b*c)/(a*d - b*c)**3 + 2*a*b*c*d + b**4*c**4*(a*d + b
*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 + 2*b**2*c*d))/(2*(a*
d - b*c)**3)
```

GIAC/XCAS [A] time = 0.234937, size = 240, normalized size = 2.24

$$\frac{\frac{ab^3}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)(bx^2+a)}-\frac{(b^3c+ab^2d)\ln\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^2cd}{(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b*x^2 + a)) -
(b^3*c + a*b^2*d)*ln(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^2*c*d
/((b*c - a*d)^3*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d)))/b
```

3.303 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{b}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^3}-\frac{\sqrt{d}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)^3}$$

```
[Out] -((d*x)/((b*c - a*d)^2*(c + d*x^2))) - x/(2*(b*c - a*d)*(a + b*x^
2)*(c + d*x^2)) + (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*Sqrt[a]*(b*c - a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(S
qrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d)^3)
```

Rubi [A] time = 0.339085, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{b}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^3}-\frac{\sqrt{d}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*x)/((b*c - a*d)^2*(c + d*x^2))) - x/(2*(b*c - a*d)*(a + b*x^
2)*(c + d*x^2)) + (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*Sqrt[a]*(b*c - a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(S
qrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 68.3915, size = 126, normalized size = 0.86

$$-\frac{dx}{\left(c+dx^2\right)(ad-bc)^2}+\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(ad-bc)}+\frac{\sqrt{d}\,(ad+3bc)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\,(ad-bc)^3}-\frac{\sqrt{b}\,(3ad+bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -d*x/((c + d*x**2)*(a*d - b*c)**2) + x/(2*(a + b*x**2)*(c + d*x**
2)*(a*d - b*c)) + sqrt(d)*(a*d + 3*b*c)*atan(sqrt(d)*x/sqrt(c))/(
2*sqrt(c)*(a*d - b*c)**3) - sqrt(b)*(3*a*d + b*c)*atan(sqrt(b)*x/
sqrt(a))/(2*sqrt(a)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.2994, size = 137, normalized size = 0.93

$$\frac{1}{2}\left(-\frac{bx}{\left(a+bx^2\right)(bc-ad)^2}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}-\frac{\sqrt{b}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3}-\frac{\sqrt{d}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-((b*x)/((b*c - a*d)^2*(a + b*x^2))) - (d*x)/((b*c - a*d)^2*(c +
 d*x^2)) - (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(S
qrt[a]*(-(b*c) + a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3))/2
```

Maple [A] time = 0.02, size = 222, normalized size = 1.5

$$-\frac{d^2xa}{2\,(ad-bc)^3\,(dx^2+c)}+\frac{dxbc}{2\,(ad-bc)^3\,(dx^2+c)}+\frac{ad^2}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,bcd}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{xabd}{2\,(ad-bc)^3\,(bx^2+a)}+\frac{xb^2c}{2\,(ad-bc)^3\,(bx^2+a)}-\frac{3\,abd}{2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^2c}{2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x/(d*x^2+c)*
b*c+1/2*d^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+3/2
*d/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b*c-1/2*b/(a*d
-b*c)^3*x/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3*x/(b*x^2+a)*c-3/2*b/(
a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d-1/2*b^2/(a*d-b
*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.523413, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + ((b^2*c*d + 3*a*b*d^2)*x^4 + a
*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((3*b^2*c
*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*
x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d
 + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3
*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^
3*b*c*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + 2*((
3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + ((b^
2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c
*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
```

```
)/(b*x^2 + a)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*
c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^
2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d +
 2*a^3*b*c*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 -
 2*((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 +
4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) +
 ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c
) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2
*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^
3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2), -1/2*(2*(b^2*c*d - a*b*d^2)*
x^3 - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2
 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))
) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2
 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c)))
+ (b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c
^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d
^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
a^4*d^4)*x^2)]
```

Sympy [A] time = 97.8929, size = 2399, normalized size = 16.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-b/a)*(3*a*d + b*c)*log(x + (-a**9*c*d**8*(-b/a)**(3/2)*(3*a
*d + b*c)**3/(a*d - b*c)**9 + 20*a**7*b**2*c**3*d**6*(-b/a)**(3/2
)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 64*a**6*b**3*c**4*d**5*(-b/a)
**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 90*a**5*b**4*c**5*d**4*
(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - a**5*d**5*sqrt(-b
/a)*(3*a*d + b*c)/(a*d - b*c)**3 - 64*a**4*b**5*c**6*d**3*(-b/a)*
*(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 9*a**4*b*c*d**4*sqrt(-b/
a)*(3*a*d + b*c)/(a*d - b*c)**3 + 20*a**3*b**6*c**7*d**2*(-b/a)**
(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 54*a**3*b**2*c**2*d**3*sq
rt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 - 54*a**2*b**3*c**3*d**2*sq
rt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 - a*b**8*c**9*(-b/a)**(3/2)
*(3*a*d + b*c)**3/(a*d - b*c)**9 - 9*a*b**4*c**4*d*sqrt(-b/a)*(3*
a*d + b*c)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b/a)*(3*a*d + b*c)/(a
*d - b*c)**3)/(3*a**2*b*d**3 + 10*a*b**2*c*d**2 + 3*b**3*c**2*d))
/(4*(a*d - b*c)**3) - sqrt(-b/a)*(3*a*d + b*c)*log(x + (a**9*c*d*
*8*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 20*a**7*b**2*c
**3*d**6*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 64*a**6*
b**3*c**4*d**5*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 - 90
*a**5*b**4*c**5*d**4*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**
9 + a**5*d**5*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + 64*a**4*b
**5*c**6*d**3*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 9*a
**4*b*c*d**4*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 - 20*a**3*b*
*6*c**7*d**2*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 54*a
**3*b**2*c**2*d**3*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + 54*a
**2*b**3*c**3*d**2*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + a*b*
*8*c**9*(-b/a)**(3/2)*(3*a*d + b*c)**3/(a*d - b*c)**9 + 9*a*b**4*
c**4*d*sqrt(-b/a)*(3*a*d + b*c)/(a*d - b*c)**3 + b**5*c**5*sqrt(-
b/a)*(3*a*d + b*c)/(a*d - b*c)**3)/(3*a**2*b*d**3 + 10*a*b**2*c*d
**2 + 3*b**3*c**2*d))/(4*(a*d - b*c)**3) + sqrt(-d/c)*(a*d + 3*b*
c)*log(x + (-a**9*c*d**8*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*
c)**9 + 20*a**7*b**2*c**3*d**6*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*
d - b*c)**9 - 64*a**6*b**3*c**4*d**5*(-d/c)**(3/2)*(a*d + 3*b*c)*
*3/(a*d - b*c)**9 + 90*a**5*b**4*c**5*d**4*(-d/c)**(3/2)*(a*d + 3
*b*c)**3/(a*d - b*c)**9 - a**5*d**5*sqrt(-d/c)*(a*d + 3*b*c)/(a*d
 - b*c)**3 - 64*a**4*b**5*c**6*d**3*(-d/c)**(3/2)*(a*d + 3*b*c)**
3/(a*d - b*c)**9 - 9*a**4*b*c*d**4*sqrt(-d/c)*(a*d + 3*b*c)/(a*d
- b*c)**3 + 20*a**3*b**6*c**7*d**2*(-d/c)**(3/2)*(a*d + 3*b*c)**3
/(a*d - b*c)**9 - 54*a**3*b**2*c**2*d**3*sqrt(-d/c)*(a*d + 3*b*c)
/(a*d - b*c)**3 - 54*a**2*b**3*c**3*d**2*sqrt(-d/c)*(a*d + 3*b*c)
/(a*d - b*c)**3 - a*b**8*c**9*(-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d
 - b*c)**9 - 9*a*b**4*c**4*d*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)
```

```
**3 - b**5*c**5*sqrt(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3)/(3*a**2*
b*d**3 + 10*a*b**2*c*d**2 + 3*b**3*c**2*d))/(4*(a*d - b*c)**3) -
sqrt(-d/c)*(a*d + 3*b*c)*log(x + (a**9*c*d**8*(-d/c)**(3/2)*(a*d
+ 3*b*c)**3/(a*d - b*c)**9 - 20*a**7*b**2*c**3*d**6*(-d/c)**(3/2)
*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 64*a**6*b**3*c**4*d**5*(-d/c)*
*(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 - 90*a**5*b**4*c**5*d**4*(
-d/c)**(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + a**5*d**5*sqrt(-d/
c)*(a*d + 3*b*c)/(a*d - b*c)**3 + 64*a**4*b**5*c**6*d**3*(-d/c)**
(3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 9*a**4*b*c*d**4*sqrt(-d/c
)*(a*d + 3*b*c)/(a*d - b*c)**3 - 20*a**3*b**6*c**7*d**2*(-d/c)**(
3/2)*(a*d + 3*b*c)**3/(a*d - b*c)**9 + 54*a**3*b**2*c**2*d**3*sqr
t(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3 + 54*a**2*b**3*c**3*d**2*sqr
t(-d/c)*(a*d + 3*b*c)/(a*d - b*c)**3 + a*b**8*c**9*(-d/c)**(3/2)*
(a*d + 3*b*c)**3/(a*d - b*c)**9 + 9*a*b**4*c**4*d*sqrt(-d/c)*(a*d
 + 3*b*c)/(a*d - b*c)**3 + b**5*c**5*sqrt(-d/c)*(a*d + 3*b*c)/(a*
d - b*c)**3)/(3*a**2*b*d**3 + 10*a*b**2*c*d**2 + 3*b**3*c**2*d))/
(4*(a*d - b*c)**3) - (2*b*d*x**3 + x*(a*d + b*c))/(2*a**3*c*d**2
- 4*a**2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**
2*c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 -
 2*a*b**2*c**2*d + 2*b**3*c**3))
```

GIAC/XCAS [A] time = 0.307961, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] Done
```

3.304 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{b}{2(a+bx^2)(bc-ad)^2}-\frac{d}{2(c+dx^2)(bc-ad)^2}-\frac{bd\log(a+bx^2)}{(bc-ad)^3}+\frac{bd\log(c+dx^2)}{(bc-ad)^3}$$

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x^2)) - d/(2*(b*c - a*d)^2*(c + d*x^2)
) - (b*d*Log[a + b*x^2])/(b*c - a*d)^3 + (b*d*Log[c + d*x^2])/(b*
c - a*d)^3
```

Rubi [A] time = 0.176817, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{b}{2(a+bx^2)(bc-ad)^2}-\frac{d}{2(c+dx^2)(bc-ad)^2}-\frac{bd\log(a+bx^2)}{(bc-ad)^3}+\frac{bd\log(c+dx^2)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x^2)) - d/(2*(b*c - a*d)^2*(c + d*x^2)
) - (b*d*Log[a + b*x^2])/(b*c - a*d)^3 + (b*d*Log[c + d*x^2])/(b*
c - a*d)^3
```

Rubi in Sympy [A] time = 33.1255, size = 76, normalized size = 0.83

$$\frac{bd\log(a+bx^2)}{(ad-bc)^3}-\frac{bd\log(c+dx^2)}{(ad-bc)^3}-\frac{b}{2(a+bx^2)(ad-bc)^2}-\frac{d}{2(c+dx^2)(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b*d*log(a + b*x**2)/(a*d - b*c)**3 - b*d*log(c + d*x**2)/(a*d - b
*c)**3 - b/(2*(a + b*x**2)*(a*d - b*c)**2) - d/(2*(c + d*x**2)*(a
*d - b*c)**2)
```

Mathematica [A] time = 0.119363, size = 77, normalized size = 0.84

$$\frac{\frac{b(ad-bc)}{a+bx^2}+\frac{d(ad-bc)}{c+dx^2}-2bd\log(a+bx^2)+2bd\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x^2) + (d*(-(b*c) + a*d))/(c + d*x^2)
- 2*b*d*Log[a + b*x^2] + 2*b*d*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.024, size = 143, normalized size = 1.6

$$-\frac{ad^2}{2(ad-bc)^3(dx^2+c)}+\frac{bdc}{2(ad-bc)^3(dx^2+c)}-\frac{bd\ln(dx^2+c)}{(ad-bc)^3}+\frac{b\ln(bx^2+a)d}{(ad-bc)^3}-\frac{abd}{2(ad-bc)^3(bx^2+a)}+\frac{b^2c}{2(ad-bc)^3(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3/(d*x^2+c)*b*c-
d/(a*d-b*c)^3*b*ln(d*x^2+c)+b/(a*d-b*c)^3*ln(b*x^2+a)*d-1/2*b/(a*
d-b*c)^3/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3/(b*x^2+a)*c
```

Maxima [A] time = 1.36526, size = 290, normalized size = 3.15

$$-\frac{bd\log\left(bx^2+a\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{bd\log\left(dx^2+c\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,bdx^2+bc+ad}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] -b*d*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3) + b*d*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3) - 1/2*(2*b*d*x^2 + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b
*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4
+ (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

Fricas [A] time = 0.255273, size = 342, normalized size = 3.72

$$-\frac{b^2c^2-a^2d^2+2\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)\log\left(bx^2+a\right)-2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right.\right.}{2\,(ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x^4+(b^4c^4-2\,ab^3c^3d+2\,a^3bcd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d^2*
x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)*log(b*x^2 + a) - 2*(b^2*
d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)*log(d*x^2 + c))/(a*b
^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3
*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^
4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

Sympy [A] time = 14.4604, size = 408, normalized size = 4.43

$$-\frac{bd\log\left(x^2+\frac{-\frac{a^4bd^5}{(ad-bc)^3}+\frac{4a^3b^2cd^4}{(ad-bc)^3}-\frac{6a^2b^3c^2d^3}{(ad-bc)^3}+\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2-\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}+\frac{bd\log\left(x^2+\frac{\frac{a^4bd^5}{(ad-bc)^3}-\frac{4a^3b^2cd^4}{(ad-bc)^3}+\frac{6a^2b^3c^2d^3}{(ad-bc)^3}-\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2+\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}-\frac{ad+bc+2bdx^2}{2a^3cd^2-4a^2bc^2d+2ab^2c^3+x^4\left(2a^2bd^3-4ab^2cd^2+2b^3c^2d\right)+x^2\left(2a^3d^3-2a^2bcd^2-2ab^2c^2d+2b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -b*d*log(x**2 + (-a**4*b*d**5/(a*d - b*c)**3 + 4*a**3*b**2*c*d**4
/(a*d - b*c)**3 - 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 4*a*b**4
*c**3*d**2/(a*d - b*c)**3 + a*b*d**2 - b**5*c**4*d/(a*d - b*c)**3
 + b**2*c*d)/(2*b**2*d**2))/(a*d - b*c)**3 + b*d*log(x**2 + (a**4
*b*d**5/(a*d - b*c)**3 - 4*a**3*b**2*c*d**4/(a*d - b*c)**3 + 6*a*
*2*b**3*c**2*d**3/(a*d - b*c)**3 - 4*a*b**4*c**3*d**2/(a*d - b*c)
**3 + a*b*d**2 + b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b**2*d
**2))/(a*d - b*c)**3 - (a*d + b*c + 2*b*d*x**2)/(2*a**3*c*d**2 -
4*a**2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*
c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2
*a*b**2*c**2*d + 2*b**3*c**3))
```

GIAC/XCAS [A] time = 0.251424, size = 220, normalized size = 2.39

$$\frac{b^2 d \ln\left(\left|\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^3}{2\,(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)(bx^2+a)} + \frac{bd^2}{2\,(bc-ad)^3\left(\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] b^2*d*ln(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*b^3/((b^4*c^2 -
 2*a*b^3*c*d + a^2*b^2*d^2)*(b*x^2 + a)) + 1/2*b*d^2/((b*c - a*d)
^3*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))
```

3.305 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}+\frac{dx(ad+bc)}{2ac(c+dx^2)(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5
*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3
)
```

Rubi [A] time = 0.456696, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}+\frac{dx(ad+bc)}{2ac(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5
*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3
)
```

Rubi in Sympy [A] time = 99.6727, size = 141, normalized size = 0.84

$$\frac{dx}{2c(a+bx^2)(c+dx^2)(ad-bc)}+\frac{d^{\frac{3}{2}}(ad-5bc)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}(ad-bc)^3}+\frac{bx(ad+bc)}{2ac(a+bx^2)(ad-bc)^2}+\frac{b^{\frac{3}{2}}(5ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*c*(a + b*x**2)*(c + d*x**2)*(a*d - b*c)) + d**(3/2)*(a*d -
 5*b*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(3/2)*(a*d - b*c)**3) + b*x
*(a*d + b*c)/(2*a*c*(a + b*x**2)*(a*d - b*c)**2) + b**(3/2)*(5*a*
d - b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.555684, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{b^{3/2}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}+\frac{x(bc-ad)\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*
(-(b*c) + a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^
2 + c*d*x^2)) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A] time = 0.002, size = 238, normalized size = 1.4

$$\frac{d^3xa}{2\,(ad-bc)^3\,c\,(dx^2+c)}-\frac{d^2xb}{2\,(ad-bc)^3\,(dx^2+c)}+\frac{d^3a}{2\,(ad-bc)^3\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$
$$-\frac{5\,d^2b}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2xd}{2\,(ad-bc)^3\,(bx^2+a)}-\frac{b^3xc}{2\,(ad-bc)^3\,a\,(bx^2+a)}$$
$$+\frac{5\,b^2d}{2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^3c}{2\,(ad-bc)^3\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+
c)*b+1/2*d^3/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-
5/2*d^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*b^2
/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3*x/a/(b*x^2+a)*c+5/
2*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-1/2*b^3/(
a*d-b*c)^3/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.994173, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d
+ (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*
a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b
*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-
d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3
 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b
^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 -
2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*
c^2*d - a^2*b*d^3)*x^3 + 2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*
```

```
c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3
)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + (a*b^2*c^3 - 5*a^2*b
*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2
*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3
*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c
^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a
^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3
- a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + 2*(a*b^2*
c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3
- 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqr
t(b/a))) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^
3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 -
a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*
c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*
c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a
^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d
 - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a
*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*
sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + (5*a^2*b*c^2*d - a^3*c*d^2
+ (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^
2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + (b^3*c^3
- a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^
2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^
3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2
*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.394462, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")

[Out] Done
```

3.306 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^3}+\frac{d^2}{2c\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] b^2/(2*a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c +
 d*x^2)) + Log[x]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/
(2*a^2*(b*c - a*d)^3) - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(2*c^2
*(b*c - a*d)^3)
```

Rubi [A] time = 0.35751, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^3}+\frac{d^2}{2c\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] b^2/(2*a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c +
 d*x^2)) + Log[x]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/
(2*a^2*(b*c - a*d)^3) - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(2*c^2
*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 62.5929, size = 124, normalized size = 0.88

$$\frac{d^2}{2c\left(c+dx^2\right)(ad-bc)^2}-\frac{d^2\left(ad-3bc\right)\log\left(c+dx^2\right)}{2c^2\left(ad-bc\right)^3}+\frac{b^2}{2a\left(a+bx^2\right)(ad-bc)^2}-\frac{b^2\left(3ad-bc\right)\log\left(a+bx^2\right)}{2a^2\left(ad-bc\right)^3}+\frac{\log\left(x^2\right)}{2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] d**2/(2*c*(c + d*x**2)*(a*d - b*c)**2) - d**2*(a*d - 3*b*c)*log(c
 + d*x**2)/(2*c**2*(a*d - b*c)**3) + b**2/(2*a*(a + b*x**2)*(a*d
- b*c)**2) - b**2*(3*a*d - b*c)*log(a + b*x**2)/(2*a**2*(a*d - b*
c)**3) + log(x**2)/(2*a**2*c**2)
```

Mathematica [A] time = 0.409594, size = 133, normalized size = 0.94

$$\frac{1}{2}\left(\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{a^2(ad-bc)^3}+\frac{2\log(x)}{a^2c^2}+\frac{b^2}{a\left(a+bx^2\right)(bc-ad)^2}+\frac{d^2(ad-3bc)\log\left(c+dx^2\right)}{c^2(bc-ad)^3}+\frac{d^2}{c\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (b^2/(a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*
x^2)) + (2*Log[x])/(a^2*c^2) + (b^2*(b*c - 3*a*d)*Log[a + b*x^2])
/(a^2*(-(b*c) + a*d)^3) + (d^2*(-3*b*c + a*d)*Log[c + d*x^2])/(c^
2*(b*c - a*d)^3))/2
```

Maple [A] time = 0.033, size = 225, normalized size = 1.6

$$\frac{\ln(x)}{a^2c^2}+\frac{d^3a}{2c(ad-bc)^3(dx^2+c)}-\frac{d^2b}{2(ad-bc)^3(dx^2+c)}-\frac{d^3\ln(dx^2+c)a}{2c^2(ad-bc)^3}+\frac{3d^2\ln(dx^2+c)b}{2c(ad-bc)^3}-\frac{3b^2\ln(bx^2+a)d}{2a(ad-bc)^3}+\frac{b^3\ln(bx^2+a)c}{2a^2(ad-bc)^3}+\frac{db^2}{2(ad-bc)^3(bx^2+a)}-\frac{b^3c}{2a(ad-bc)^3(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] ln(x)/a^2/c^2+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)
^3/(d*x^2+c)*b-1/2*d^3/c^2/(a*d-b*c)^3*ln(d*x^2+c)*a+3/2*d^2/c/(a
*d-b*c)^3*ln(d*x^2+c)*b-3/2*b^2/a/(a*d-b*c)^3*ln(b*x^2+a)*d+1/2*b
^3/a^2/(a*d-b*c)^3*ln(b*x^2+a)*c+1/2*b^2/(a*d-b*c)^3/(b*x^2+a)*d-
1/2*b^3/a/(a*d-b*c)^3/(b*x^2+a)*c
```

Maxima [A] time = 1.3684, size = 398, normalized size = 2.82

$$-\frac{(b^3c-3ab^2d)\log(bx^2+a)}{2(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3)}-\frac{(3bcd^2-ad^3)\log(dx^2+c)}{2(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3)}+\frac{b^2c^2+a^2d^2+(b^2cd+abd^2)x^2}{2(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^2)}+\frac{\log(x^2)}{2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - 3*a*b^2*d)*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*
c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/2*(3*b*c*d^2 - a*d^3)*log(d*
x^2 + c)/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3
) + 1/2*(b^2*c^2 + a^2*d^2 + (b^2*c*d + a*b*d^2)*x^2)/(a^2*b^2*c^
4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^
2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2
 + a^4*c*d^3)*x^2) + 1/2*log(x^2)/(a^2*c^2)
```

Fricas [A] time = 4.12246, size = 729, normalized size = 5.17

$$ab^3c^4-a^2b^2c^3d+a^3bc^2d^2-a^4cd^3+\left(ab^3c^3d-a^3bcd^3\right)x^2-\left(ab^3c^4-3a^2b^2c^3d+\left(b^4c^3d-3ab^3c^2d^2\right)x^4+\left(b^4c^4-2ab^3c^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^3*c^4 - a^2*b^2*c^3*d + a^3*b*c^2*d^2 - a^4*c*d^3 + (a*b
^3*c^3*d - a^3*b*c*d^3)*x^2 - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + (b^4
*c^3*d - 3*a*b^3*c^2*d^2)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d - 3*a^2*
```

```
b^2*c^2*d^2)*x^2)*log(b*x^2 + a) - (3*a^3*b*c^2*d^2 - a^4*c*d^3 +
 (3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (3*a^2*b^2*c^2*d^2 + 2*a^3*b
*c*d^3 - a^4*d^4)*x^2)*log(d*x^2 + c) + 2*(a*b^3*c^4 - 3*a^2*b^2*
c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^
2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d +
 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(x))/(a^3*b^3*c^6 - 3*a^4*b^2*c
^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3 + (a^2*b^4*c^5*d - 3*a^3*b^3
*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^4 + (a^2*b^4*c^6
- 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.307 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2} \, dx$

Optimal. Leaf size=218

$$-\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{d^{5/2}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{d(ad+bc)}{2acx\,(c+dx^2)\,(bc-ad)^2}$$

```
[Out] -(3*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*x)
+ (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x*(c + d*x^2)) + b/(2*a*(b
*c - a*d)*x*(a + b*x^2)*(c + d*x^2)) - (b^(5/2)*(3*b*c - 7*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*
(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*
d)^3)
```

Rubi [A] time = 0.812753, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{d^{5/2}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{d(ad+bc)}{2acx\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(3*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*x)
+ (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x*(c + d*x^2)) + b/(2*a*(b
*c - a*d)*x*(a + b*x^2)*(c + d*x^2)) - (b^(5/2)*(3*b*c - 7*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*
(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*
d)^3)
```

Rubi in Sympy [A] time = 164.269, size = 189, normalized size = 0.87

$$-\frac{d^{\frac{5}{2}}\left(3ad-7bc\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{5}{2}}\left(ad-bc\right)^3}-\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)\left(ad-bc\right)}+\frac{d\left(ad+bc\right)}{2acx\left(c+dx^2\right)\left(ad-bc\right)^2}-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x\left(ad-bc\right)^2}-\frac{b^{\frac{5}{2}}\left(7ad-3bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -d**(5/2)*(3*a*d - 7*b*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(5/2)*(a*
d - b*c)**3) - b/(2*a*x*(a + b*x**2)*(c + d*x**2)*(a*d - b*c)) +
d*(a*d + b*c)/(2*a*c*x*(c + d*x**2)*(a*d - b*c)**2) - (3*a**2*d**
2 - 4*a*b*c*d + 3*b**2*c**2)/(2*a**2*c**2*x*(a*d - b*c)**2) - b**
(5/2)*(7*a*d - 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(5/2)*(a*d -
b*c)**3)
```

Mathematica [A] time = 0.586848, size = 158, normalized size = 0.72

$$\frac{1}{2}\left(\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3}-\frac{b^3x}{a^2(a+bx^2)(bc-ad)^2}-\frac{2}{a^2c^2x}+\frac{d^{5/2}(3ad-7bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}-\frac{d^3x}{c^2(c+dx^2)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x) - (b^3*x)/(a^2*(b*c - a*d)^2*(a + b*x^2)) - (d^3*
x)/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*(3*b*c - 7*a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) + (d^(5/2)*(
-7*b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)
^3))/2
```

Maple [A] time = 0.026, size = 261, normalized size = 1.2

$$-\frac{1}{a^2c^2x}-\frac{d^4xa}{2\,c^2\,(ad-bc)^3\,(dx^2+c)}+\frac{d^3xb}{2\,c\,(ad-bc)^3\,(dx^2+c)}-\frac{3\,d^4a}{2\,c^2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{7\,d^3b}{2\,c\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3xd}{2\,a\,(ad-bc)^3\,(bx^2+a)}+\frac{b^4xc}{2\,a^2\,(ad-bc)^3\,(bx^2+a)}-\frac{7\,b^3d}{2\,a\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,b^4c}{2\,a^2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/a^2/c^2/x-1/2*d^4/c^2/(a*d-b*c)^3*x/(d*x^2+c)*a+1/2*d^3/c/(a*d
-b*c)^3*x/(d*x^2+c)*b-3/2*d^4/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(
x*d/(c*d)^(1/2))*a+7/2*d^3/c/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(
c*d)^(1/2))*b-1/2*b^3/a/(a*d-b*c)^3*x/(b*x^2+a)*d+1/2*b^4/a^2/(a*
d-b*c)^3*x/(b*x^2+a)*c-7/2*b^3/a/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x
*b/(a*b)^(1/2))*d+3/2*b^4/a^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23345, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*
c*d^3 + 2*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^
3*b*d^4)*x^4 + 2*(3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a
^4*d^4)*x^2 - ((3*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 -
 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*c^4 - 7*a^2*b^
2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) - ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2
 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^
3)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))
/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*
c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 -
 a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*
d^2 - a^6*c^3*d^3)*x), -1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*
a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 +
7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*b^4*c^4 - 5*a*b^3*c^3*d
 + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((7*a^2*b^2*c*d^3 - 3*a^3*b
*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 +
 (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(d/c)*arctan(d*x/(c*sqrt(
d/c))) - ((3*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*
b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3
*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
)/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b
*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3
- a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4
*d^2 - a^6*c^3*d^3)*x), -1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12
*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 +
 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*b^4*c^4 - 5*a*b^3*c^3*
d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((3*b^4*c^3*d - 7*a*b^3*c^
2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3
+ (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(b*x/(a*sqrt
(b/a))) - ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d
^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*
d^3)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)
))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*
b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3
 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^
4*d^2 - a^6*c^3*d^3)*x), -1/2*(2*a*b^3*c^4 - 6*a^2*b^2*c^3*d + 6*
a^3*b*c^2*d^2 - 2*a^4*c*d^3 + (3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*
a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + (3*b^4*c^4 - 5*a*b^3*c^3*d + 5
*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + ((3*b^4*c^3*d - 7*a*b^3*c^2*d^2)*
x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3 + (3*a*
b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a)))
 + ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 + 4*
a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)
*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/((a^2*b^4*c^5*d - 3*a^3*b^3
*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6
- 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3
*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.461368, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^2),x, algorithm="giac")

[Out] Done
```

3.308 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=156

$$\frac{b^3(bc-2ad)\log(a+bx^2)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{b^3}{2a^2(a+bx^2)(bc-ad)^2}-\frac{1}{2a^2c^2x^2}+\frac{d^3(2bc-ad)\log(c+dx^2)}{c^3(bc-ad)^3}-\frac{d^3}{2c^2(c+dx^2)(bc-ad)^2}$$

```
[Out] -1/(2*a^2*c^2*x^2) - b^3/(2*a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/
(2*c^2*(b*c - a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x])/(a^3*c
^3) + (b^3*(b*c - 2*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (d
^3*(2*b*c - a*d)*Log[c + d*x^2])/(c^3*(b*c - a*d)^3)
```

Rubi [A] time = 0.448724, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{b^3(bc-2ad)\log(a+bx^2)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{b^3}{2a^2(a+bx^2)(bc-ad)^2}-\frac{1}{2a^2c^2x^2}+\frac{d^3(2bc-ad)\log(c+dx^2)}{c^3(bc-ad)^3}-\frac{d^3}{2c^2(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/(2*a^2*c^2*x^2) - b^3/(2*a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/
(2*c^2*(b*c - a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x])/(a^3*c
^3) + (b^3*(b*c - 2*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (d
^3*(2*b*c - a*d)*Log[c + d*x^2])/(c^3*(b*c - a*d)^3)
```

Rubi in Sympy [A] time = 71.6083, size = 143, normalized size = 0.92

$$-\frac{d^3}{2c^2(c+dx^2)(ad-bc)^2}+\frac{d^3(ad-2bc)\log(c+dx^2)}{c^3(ad-bc)^3}-\frac{b^3}{2a^2(a+bx^2)(ad-bc)^2}-\frac{1}{2a^2c^2x^2}+\frac{b^3(2ad-bc)\log(a+bx^2)}{a^3(ad-bc)^3}-\frac{(ad+bc)\log(x^2)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -d**3/(2*c**2*(c + d*x**2)*(a*d - b*c)**2) + d**3*(a*d - 2*b*c)*l
og(c + d*x**2)/(c**3*(a*d - b*c)**3) - b**3/(2*a**2*(a + b*x**2)*
(a*d - b*c)**2) - 1/(2*a**2*c**2*x**2) + b**3*(2*a*d - b*c)*log(a
 + b*x**2)/(a**3*(a*d - b*c)**3) - (a*d + b*c)*log(x**2)/(a**3*c*
*3)
```

Mathematica [A] time = 0.368365, size = 157, normalized size = 1.01

$$\frac{1}{2}\left(\frac{2b^3(2ad-bc)\log(a+bx^2)}{a^3(ad-bc)^3}-\frac{4\log(x)(ad+bc)}{a^3c^3}-\frac{b^3}{a^2(a+bx^2)(bc-ad)^2}-\frac{1}{a^2c^2x^2}+\frac{2d^3(2bc-ad)\log(c+dx^2)}{c^3(bc-ad)^3}-\frac{d^3}{c^2(c+dx^2)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-(1/(a^2*c^2*x^2)) - b^3/(a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(
c^2*(b*c - a*d)^2*(c + d*x^2)) - (4*(b*c + a*d)*Log[x])/(a^3*c^3)
 + (2*b^3*(-(b*c) + 2*a*d)*Log[a + b*x^2])/(a^3*(-(b*c) + a*d)^3)
 + (2*d^3*(2*b*c - a*d)*Log[c + d*x^2])/(c^3*(b*c - a*d)^3))/2
```

Maple [A] time = 0.037, size = 254, normalized size = 1.6

$$-\frac{1}{2\,a^2c^2x^2}-2\,\frac{\ln(x)\,d}{a^2c^3}-2\,\frac{b\ln(x)}{a^3c^2}-\frac{d^4a}{2\,c^2\,(ad-bc)^3\,(dx^2+c)}+\frac{d^3b}{2\,c\,(ad-bc)^3\,(dx^2+c)}+\frac{d^4\ln\left(dx^2+c\right)\,a}{c^3\,(ad-bc)^3}-2\,\frac{d^3\ln\left(dx^2+c\right)\,b}{c^2\,(ad-bc)^3}+2\,\frac{b^3\ln\left(bx^2+a\right)\,d}{a^2\,(ad-bc)^3}-\frac{b^4\ln\left(bx^2+a\right)\,c}{a^3\,(ad-bc)^3}-\frac{db^3}{2\,a\,(ad-bc)^3\,(bx^2+a)}+\frac{b^4c}{2\,a^2\,(ad-bc)^3\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2/a^2/c^2/x^2-2/a^2/c^3*ln(x)*d-2/a^3/c^2*ln(x)*b-1/2*d^4/c^2/
(a*d-b*c)^3/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)*b+d^4/c^3
/(a*d-b*c)^3*ln(d*x^2+c)*a-2*d^3/c^2/(a*d-b*c)^3*ln(d*x^2+c)*b+2*
b^3/a^2/(a*d-b*c)^3*ln(b*x^2+a)*d-b^4/a^3/(a*d-b*c)^3*ln(b*x^2+a)
*c-1/2*b^3/a/(a*d-b*c)^3/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3/(b*x
^2+a)*c
```

Maxima [A] time = 1.37145, size = 514, normalized size = 3.29

$$\frac{\left(b^4c-2\,ab^3d\right)\log\left(bx^2+a\right)}{a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3}+\frac{\left(2\,bcd^3-ad^4\right)\log\left(dx^2+c\right)}{b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3}-\frac{ab^2c^3-2\,a^2bc^2d+a^3cd^2+2\left(b^3c^2d-ab^2cd^2+a^2bd^3\right)x^4+\left(2\,b^3c^3-ab^2c^2d-a^2bcd^2+2\,a^3d^3\right)x^2}{2\left((a^2b^3c^4d-2\,a^3b^2c^3d^2+a^4bc^2d^3)x^6+(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3)x^4+(a^3b^2c^5-2\,a^4bc^4d+a^5c^3d^2)x^2\right)}-\frac{(bc+ad)\log\left(x^2\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^3),x, algorithm="maxima")
```

```
[Out] (b^4*c - 2*a*b^3*d)*log(b*x^2 + a)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3) + (2*b*c*d^3 - a*d^4)*log(d*x^2 + c)/
(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/2*(
a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + 2*(b^3*c^2*d - a*b^2*c*d^
2 + a^2*b*d^3)*x^4 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a
^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)
*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3
)*x^4 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x^2) - (b*c +
 a*d)*log(x^2)/(a^3*c^3)
```

Fricas [A] time = 9.23125, size = 900, normalized size = 5.77

$$-\frac{a^2b^3c^5-3\,a^3b^2c^4d+3\,a^4bc^3d^2-a^5c^2d^3+2\left(ab^4c^4d-2\,a^2b^3c^3d^2+2\,a^3b^2c^2d^3-a^4bcd^4\right)x^4+\left(2\,ab^4c^5-3\,a^2b^3c^4d+3\,a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d
^3 + 2*(a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - a^4
*b*c*d^4)*x^4 + (2*a*b^4*c^5 - 3*a^2*b^3*c^4*d + 3*a^4*b*c^2*d^3
- 2*a^5*c*d^4)*x^2 - 2*((b^5*c^4*d - 2*a*b^4*c^3*d^2)*x^6 + (b^5*
c^5 - a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2)*x^4 + (a*b^4*c^5 - 2*a^2*b
^3*c^4*d)*x^2)*log(b*x^2 + a) - 2*((2*a^3*b^2*c*d^4 - a^4*b*d^5)*
x^6 + (2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (2*a^4*b*
c^2*d^3 - a^5*c*d^4)*x^2)*log(d*x^2 + c) + 4*((b^5*c^4*d - 2*a*b^
4*c^3*d^2 + 2*a^3*b^2*c*d^4 - a^4*b*d^5)*x^6 + (b^5*c^5 - a*b^4*c
^4*d - 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*
d^5)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c
*d^4)*x^2)*log(x))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^
2*c^4*d^3 - a^6*b*c^3*d^4)*x^6 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d +
 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^4 + (a^4*b^3*c^7 - 3*a^5*b^2*c^
6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.309 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=271

$$\frac{b^{7/2}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3}-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^3\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] -(5*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^3
) + ((b*c + a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(2*a^3*c^3*
(b*c - a*d)^2*x) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^3*(c +
d*x^2)) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)) + (b^(7
/2)*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*(b*c
- a*d)^3) + (d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(2*c^(7/2)*(b*c - a*d)^3)
```

Rubi [A] time = 1.17042, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{b^{7/2}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3}-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^3\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(5*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^3
) + ((b*c + a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(2*a^3*c^3*
(b*c - a*d)^2*x) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^3*(c +
d*x^2)) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)) + (b^(7
/2)*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*(b*c
- a*d)^3) + (d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(2*c^(7/2)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.696494, size = 178, normalized size = 0.66

$$\frac{1}{6}\left(\frac{3b^{7/2}(9ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(ad-bc)^3}+\frac{3b^4x}{a^3\left(a+bx^2\right)(bc-ad)^2}+\frac{12(ad+bc)}{a^3c^3x}-\frac{2}{a^2c^2x^3}+\frac{3d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}+\frac{3d^4x}{c^3\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x^3) + (12*(b*c + a*d))/(a^3*c^3*x) + (3*b^4*x)/(a^3
*(b*c - a*d)^2*(a + b*x^2)) + (3*d^4*x)/(c^3*(b*c - a*d)^2*(c + d
*x^2)) + (3*b^(7/2)*(-5*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(7/2)*(-(b*c) + a*d)^3) + (3*d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^3))/6
```

Maple [A] time = 0.03, size = 285, normalized size = 1.1

$$-\frac{1}{3\,a^2c^2x^3}+2\,\frac{d}{a^2c^3x}+2\,\frac{b}{a^3c^2x}+\frac{d^5xa}{2\,c^3\,(ad-bc)^3\,(dx^2+c)}-\frac{d^4xb}{2\,c^2\,(ad-bc)^3\,(dx^2+c)}+\frac{5\,d^5a}{2\,c^3\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{9\,d^4b}{2\,c^2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^4xd}{2\,a^2\,(ad-bc)^3\,(bx^2+a)}-\frac{b^5xc}{2\,a^3\,(ad-bc)^3\,(bx^2+a)}+\frac{9\,b^4d}{2\,a^2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{5\,b^5c}{2\,a^3\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/3/a^2/c^2/x^3+2/x/a^2/c^3*d+2/x/a^3/c^2*b+1/2*d^5/c^3/(a*d-b*c
)^3*x/(d*x^2+c)*a-1/2*d^4/c^2/(a*d-b*c)^3*x/(d*x^2+c)*b+5/2*d^5/c
^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-9/2*d^4/c^2/
(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*b^4/a^2/(a*
d-b*c)^3*x/(b*x^2+a)*d-1/2*b^5/a^3/(a*d-b*c)^3*x/(b*x^2+a)*c+9/2*
b^4/a^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-5/2*b^5
/a^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.59037, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d + 12*a^4*b*c^3*d^2 - 4*a
^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3*b^2*c*d^4 -
 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3*c
^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 -
20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^
2 - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4
```

```
*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)
*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
- 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4
*a^4*b*c*d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x
^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(
(a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^
3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a
^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^
2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d +
12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d
^2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^
4*c^4*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^
4 - 15*a^5*d^5)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c
^2*d^3 - a^5*c*d^4)*x^2 - 6*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7
+ (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*
c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) -
 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^
4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^
3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((
a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3
*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^
7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2
 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d + 1
2*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^
2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4
*c^4*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4
 - 15*a^5*d^5)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^
2*d^3 - a^5*c*d^4)*x^2 - 6*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 +
 (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c
^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) -
3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a
^4*b*c*d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3
)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a
^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*
d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7
*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2
- a^7*c^4*d^3)*x^3), -1/6*(2*a^2*b^3*c^5 - 6*a^3*b^2*c^4*d + 6*a^
4*b*c^3*d^2 - 2*a^5*c^2*d^3 - 3*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 +
9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - (15*b^5*c^5 - 17*a*b^4*c^4*d
 - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*
a^5*d^5)*x^4 - 10*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3
- a^5*c*d^4)*x^2 - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^
5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9
*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 3*((9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c
*d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt
(d/c)*arctan(d*x/(c*sqrt(d/c))))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*
d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a
^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7
- 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.458028, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^4),x, algorithm="giac")

[Out] Done
```

3.310 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3} \, dx$

Optimal. Leaf size=207

$$\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}(bc-ad)^4}+\frac{3x(3ad+bc)}{8\left(c+dx^2\right)(bc-ad)^3}+\frac{x(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{3\sqrt{a}\sqrt{b}(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2(bc-ad)^4}$$

```
[Out] ((b*c + 2*a*d)*x)/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*x)/(2*b*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (3*(b*c + 3*a*d)*x)/(8*(
b*c - a*d)^3*(c + d*x^2)) - (3*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*(b*c - a*d)^4) + (3*(b^2*c^2 + 6*a*b*c*
d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*Sqrt[d]*(b*c
 - a*d)^4)
```

Rubi [A] time = 0.629957, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}(bc-ad)^4}+\frac{3x(3ad+bc)}{8\left(c+dx^2\right)(bc-ad)^3}+\frac{x(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{3\sqrt{a}\sqrt{b}(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*x)/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*x)/(2*b*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (3*(b*c + 3*a*d)*x)/(8*(
b*c - a*d)^3*(c + d*x^2)) - (3*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*(b*c - a*d)^4) + (3*(b^2*c^2 + 6*a*b*c*
d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqrt[c]*Sqrt[d]*(b*c
 - a*d)^4)
```

Rubi in Sympy [A] time = 119.91, size = 187, normalized size = 0.9

$$-\frac{3\sqrt{a}\sqrt{b}\left(ad+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\left(ad-bc\right)^4}-\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2\left(ad-bc\right)}-\frac{3x\left(3ad+bc\right)}{8\left(c+dx^2\right)\left(ad-bc\right)^3}+\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}\left(ad-bc\right)^4}+\frac{x\left(2ad+bc\right)}{4b\left(c+dx^2\right)^2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -3*sqrt(a)*sqrt(b)*(a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*(a*d -
b*c)**4) - a*x/(2*b*(a + b*x**2)*(c + d*x**2)**2*(a*d - b*c)) - 3
*x*(3*a*d + b*c)/(8*(c + d*x**2)*(a*d - b*c)**3) + 3*(a**2*d**2 +
 6*a*b*c*d + b**2*c**2)*atan(sqrt(d)*x/sqrt(c))/(8*sqrt(c)*sqrt(d
)*(a*d - b*c)**4) + x*(2*a*d + b*c)/(4*b*(c + d*x**2)**2*(a*d - b
*c)**2)
```

Mathematica [A] time = 0.611585, size = 166, normalized size = 0.8

$$\frac{\frac{3(a^2d^2+6abcd+b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}+\frac{2cx(bc-ad)^2}{(c+dx^2)^2}+\frac{4abx(bc-ad)}{a+bx^2}+\frac{x(5ad+3bc)(bc-ad)}{c+dx^2}-12\sqrt{a}\sqrt{b}(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((4*a*b*(b*c - a*d)*x)/(a + b*x^2) + (2*c*(b*c - a*d)^2*x)/(c + d
*x^2)^2 + ((b*c - a*d)*(3*b*c + 5*a*d)*x)/(c + d*x^2) - 12*Sqrt[a
]*Sqrt[b]*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + (3*(b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[
d]))/(8*(b*c - a*d)^4)
```

Maple [B] time = 0.023, size = 388, normalized size = 1.9

$$-\frac{5x^3a^2d^3}{8(ad-bc)^4(dx^2+c)^2}+\frac{x^3abcd^2}{4(ad-bc)^4(dx^2+c)^2}+\frac{3x^3b^2c^2d}{8(ad-bc)^4(dx^2+c)^2}-\frac{3xa^2cd^2}{8(ad-bc)^4(dx^2+c)^2}-\frac{xabc^2d}{4(ad-bc)^4(dx^2+c)^2}+\frac{5xb^2c^3}{8(ad-bc)^4(dx^2+c)^2}+\frac{3a^2d^2}{8(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{9cabd}{4(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3b^2c^2}{8(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2bxd}{2(ad-bc)^4(bx^2+a)}+\frac{ab^2xc}{2(ad-bc)^4(bx^2+a)}-\frac{3a^2bd}{2(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3ab^2c}{2(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -5/8/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a^2*d^3+1/4/(a*d-b*c)^4/(d*x^2+c
)^2*x^3*a*b*c*d^2+3/8/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2*c^2*d-3/8/(
a*d-b*c)^4/(d*x^2+c)^2*x*a^2*c*d^2-1/4/(a*d-b*c)^4/(d*x^2+c)^2*x*
a*b*c^2*d+5/8/(a*d-b*c)^4/(d*x^2+c)^2*x*b^2*c^3+3/8/(a*d-b*c)^4/(
c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*d^2+9/4/(a*d-b*c)^4/(c*d)^
(1/2)*arctan(x*d/(c*d)^(1/2))*c*a*b*d+3/8/(a*d-b*c)^4/(c*d)^(1/2)
*arctan(x*d/(c*d)^(1/2))*b^2*c^2-1/2*a^2*b/(a*d-b*c)^4*x/(b*x^2+a
)*d+1/2*a*b^2/(a*d-b*c)^4*x/(b*x^2+a)*c-3/2*a^2*b/(a*d-b*c)^4/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-3/2*a*b^2/(a*d-b*c)^4/(a*b)^(1
/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.10676, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [1/16*(12*((b^2*c*d^2 + a*b*d^3)*x^6 + a*b*c^3 + a^2*c^2*d + (2*b
^2*c^2*d + 3*a*b*c*d^2 + a^2*d^3)*x^4 + (b^2*c^3 + 3*a*b*c^2*d +
2*a^2*c*d^2)*x^2)*sqrt(-a*b)*sqrt(-c*d)*log((b*x^2 - 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)) + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2
 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d +
 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a
*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*log((2*c*d*x +
(d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) + 2*(3*(b^3*c^2*d + 2*a*b^2*
c*d^2 - 3*a^2*b*d^3)*x^5 + (5*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c
*d^2 - 5*a^3*d^3)*x^3 + 3*(3*a*b^2*c^3 - 2*a^2*b*c^2*d - a^3*c*d^
2)*x)*sqrt(-c*d))/((a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d
^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3*d
^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (2*b^
5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4
 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^
2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5
)*x^2)*sqrt(-c*d)), 1/8*(6*((b^2*c*d^2 + a*b*d^3)*x^6 + a*b*c^3 +
 a^2*c^2*d + (2*b^2*c^2*d + 3*a*b*c*d^2 + a^2*d^3)*x^4 + (b^2*c^3
 + 3*a*b*c^2*d + 2*a^2*c*d^2)*x^2)*sqrt(-a*b)*sqrt(c*d)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 3*(a*b^2*c^4 + 6*a^2*b*c^3
*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6
+ (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4
+ (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)
*arctan(sqrt(c*d)*x/c) + (3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*
d^3)*x^5 + (5*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c*d^2 - 5*a^3*d^3
)*x^3 + 3*(3*a*b^2*c^3 - 2*a^2*b*c^2*d - a^3*c*d^2)*x)*sqrt(c*d))
/((a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*
d^3 + a^5*c^2*d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^
2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4
*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5
+ a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8
*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*sqrt(c*d))
, -1/16*(24*((b^2*c*d^2 + a*b*d^3)*x^6 + a*b*c^3 + a^2*c^2*d + (2
*b^2*c^2*d + 3*a*b*c*d^2 + a^2*d^3)*x^4 + (b^2*c^3 + 3*a*b*c^2*d
+ 2*a^2*c*d^2)*x^2)*sqrt(a*b)*sqrt(-c*d)*arctan(b*x/sqrt(a*b)) -
3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b
^2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a
^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c
^2*d^2 + 2*a^3*c*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))
/(d*x^2 + c)) - 2*(3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^
5 + (5*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c*d^2 - 5*a^3*d^3)*x^3 +
 3*(3*a*b^2*c^3 - 2*a^2*b*c^2*d - a^3*c*d^2)*x)*sqrt(-c*d))/((a*b
^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 +
a^5*c^2*d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4
- 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d
^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*
d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b
^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*sqrt(-c*d)), -1/
8*(12*((b^2*c*d^2 + a*b*d^3)*x^6 + a*b*c^3 + a^2*c^2*d + (2*b^2*c
^2*d + 3*a*b*c*d^2 + a^2*d^3)*x^4 + (b^2*c^3 + 3*a*b*c^2*d + 2*a^
2*c*d^2)*x^2)*sqrt(a*b)*sqrt(c*d)*arctan(b*x/sqrt(a*b)) - 3*(a*b^
2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^
3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*
d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2
+ 2*a^3*c*d^3)*x^2)*arctan(sqrt(c*d)*x/c) - (3*(b^3*c^2*d + 2*a*b
^2*c*d^2 - 3*a^2*b*d^3)*x^5 + (5*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*
b*c*d^2 - 5*a^3*d^3)*x^3 + 3*(3*a*b^2*c^3 - 2*a^2*b*c^2*d - a^3*c
*d^2)*x)*sqrt(c*d))/((a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4
*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3
*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (2*
b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d
^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*
a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d
^5)*x^2)*sqrt(c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.247166, size = 406, normalized size = 1.96

$$\frac{abx}{2\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)(bx^2 + a)} - \frac{3\,\left(ab^2c + a^2bd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\sqrt{ab}} + \frac{3\,\left(b^2c^2 + 6\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\sqrt{cd}} + \frac{3\,bcdx^3 + 5\,ad^2x^3 + 5\,bc^2x + 3\,acdx}{8\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)(dx^2 + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] 1/2*a*b*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b
*x^2 + a)) - 3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*sqrt(a*b)) + 3/8*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqr
t(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*b*c*d*x^3 + 5*a*d^2*x^3 + 5*b
*c^2*x + 3*a*c*d*x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*(d*x^2 + c)^2)
```

3.311 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=142

$$\frac{ab}{2(a+bx^2)(bc-ad)^3}+\frac{ad+bc}{2(c+dx^2)(bc-ad)^3}+\frac{c}{4(c+dx^2)^2(bc-ad)^2}+\frac{b(2ad+bc)\log(a+bx^2)}{2(bc-ad)^4}-\frac{b(2ad+bc)\log(c+dx^2)}{2(bc-ad)^4}$$

```
[Out] (a*b)/(2*(b*c - a*d)^3*(a + b*x^2)) + c/(4*(b*c - a*d)^2*(c + d*x
^2)^2) + (b*c + a*d)/(2*(b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*
a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^4) - (b*(b*c + 2*a*d)*Log[c +
 d*x^2])/(2*(b*c - a*d)^4)
```

Rubi [A] time = 0.329991, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{ab}{2(a+bx^2)(bc-ad)^3}+\frac{ad+bc}{2(c+dx^2)(bc-ad)^3}+\frac{c}{4(c+dx^2)^2(bc-ad)^2}+\frac{b(2ad+bc)\log(a+bx^2)}{2(bc-ad)^4}-\frac{b(2ad+bc)\log(c+dx^2)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (a*b)/(2*(b*c - a*d)^3*(a + b*x^2)) + c/(4*(b*c - a*d)^2*(c + d*x
^2)^2) + (b*c + a*d)/(2*(b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*
a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^4) - (b*(b*c + 2*a*d)*Log[c +
 d*x^2])/(2*(b*c - a*d)^4)
```

Rubi in Sympy [A] time = 52.1401, size = 121, normalized size = 0.85

$$-\frac{ab}{2(a+bx^2)(ad-bc)^3}+\frac{b(2ad+bc)\log(a+bx^2)}{2(ad-bc)^4}-\frac{b(2ad+bc)\log(c+dx^2)}{2(ad-bc)^4}+\frac{c}{4(c+dx^2)^2(ad-bc)^2}-\frac{ad+bc}{2(c+dx^2)(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -a*b/(2*(a + b*x**2)*(a*d - b*c)**3) + b*(2*a*d + b*c)*log(a + b*
x**2)/(2*(a*d - b*c)**4) - b*(2*a*d + b*c)*log(c + d*x**2)/(2*(a*
d - b*c)**4) + c/(4*(c + d*x**2)**2*(a*d - b*c)**2) - (a*d + b*c)
/(2*(c + d*x**2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.161484, size = 121, normalized size = 0.85

$$\frac{\frac{c(bc-ad)^2}{(c+dx^2)^2}+\frac{2ab(bc-ad)}{a+bx^2}+\frac{2(ad+bc)(bc-ad)}{c+dx^2}+2b(2ad+bc)\log(a+bx^2)-2b(2ad+bc)\log(c+dx^2)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((2*a*b*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d)^2)/(c + d*x^2)^
2 + (2*(b*c - a*d)*(b*c + a*d))/(c + d*x^2) + 2*b*(b*c + 2*a*d)*L
og[a + b*x^2] - 2*b*(b*c + 2*a*d)*Log[c + d*x^2])/(4*(b*c - a*d)^
4)
```

Maple [B] time = 0.026, size = 283, normalized size = 2.

$$-\frac{a^2d^2}{2\left(ad-bc\right)^4\left(dx^2+c\right)}+\frac{b^2c^2}{2\left(ad-bc\right)^4\left(dx^2+c\right)}+\frac{a^2cd^2}{4\left(ad-bc\right)^4\left(dx^2+c\right)^2}-\frac{abc^2d}{2\left(ad-bc\right)^4\left(dx^2+c\right)^2}+\frac{b^2c^3}{4\left(ad-bc\right)^4\left(dx^2+c\right)^2}-\frac{bd\ln\left(dx^2+c\right)a}{\left(ad-bc\right)^4}-\frac{b^2\ln\left(dx^2+c\right)c}{2\left(ad-bc\right)^4}+\frac{b\ln\left(bx^2+a\right)ad}{\left(ad-bc\right)^4}+\frac{b^2\ln\left(bx^2+a\right)c}{2\left(ad-bc\right)^4}-\frac{ba^2d}{2\left(ad-bc\right)^4\left(bx^2+a\right)}+\frac{ab^2c}{2\left(ad-bc\right)^4\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^4/(d*x^2+c)*a^2+1/2/(a*d-b*c)^4/(d*x^2+c)*b^2*
c^2+1/4*d^2/(a*d-b*c)^4*c/(d*x^2+c)^2*a^2-1/2*d/(a*d-b*c)^4*c^2/(
d*x^2+c)^2*a*b+1/4/(a*d-b*c)^4*c^3/(d*x^2+c)^2*b^2-d/(a*d-b*c)^4*
b*ln(d*x^2+c)*a-1/2/(a*d-b*c)^4*b^2*ln(d*x^2+c)*c+b/(a*d-b*c)^4*l
n(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^4*ln(b*x^2+a)*c-1/2*b/(a*d-b*c)^
4*a^2/(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^4*a/(b*x^2+a)*c
```

Maxima [A] time = 1.38297, size = 560, normalized size = 3.94

$$\frac{\left(b^2c+2\,abd\right)\log\left(bx^2+a\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}-\frac{\left(b^2c+2\,abd\right)\log\left(dx^2+c\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}+\frac{2\left(b^2cd+2\,abd^2\right)x^4+5\,abc^2+a^2cd+\left(3\,b^2c^2+7\,abcd+\cdots\right.}{4\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^6+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c + 2*a*b*d)*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(b^2*c + 2*a*b*
d)*log(d*x^2 + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4) + 1/4*(2*(b^2*c*d + 2*a*b*d^2)*x^4 + 5*a
*b*c^2 + a^2*c*d + (3*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^2)/(a*b^
3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b
^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^
4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b
*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

Fricas [A] time = 0.265148, size = 807, normalized size = 5.68

$$\frac{5\,ab^2c^3-4\,a^2bc^2d-a^3cd^2+2\left(b^3c^2d+ab^2cd^2-2\,a^2bd^3\right)x^4+\left(3\,b^3c^3+4\,ab^2c^2d-5\,a^2bcd^2-2\,a^3d^3\right)x^2+2\left(\left(b^3cd^2+2\,ab\cdots\right.\right.}{4\left(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+\left(b^5c^4d^2-4\,a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(5*a*b^2*c^3 - 4*a^2*b*c^2*d - a^3*c*d^2 + 2*(b^3*c^2*d + a*b
^2*c*d^2 - 2*a^2*b*d^3)*x^4 + (3*b^3*c^3 + 4*a*b^2*c^2*d - 5*a^2*
```

```
b*c*d^2 - 2*a^3*d^3)*x^2 + 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b
^2*c^3 + 2*a^2*b*c^2*d + (2*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d
^3)*x^4 + (b^3*c^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)*x^2)*log(b*x^
2 + a) - 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*c^3 + 2*a^2*b*c
^2*d + (2*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^3*c^3
 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c^6
 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^
2*d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^
3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8
*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x
^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3
*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

Sympy [A] time = 57.4828, size = 780, normalized size = 5.49

$$-\frac{b\left(2ad+bc\right)\log\left(x^2+\frac{-\frac{a^5bd^5(2ad+bc)}{(ad-bc)^4}+\frac{5a^4b^2cd^4(2ad+bc)}{(ad-bc)^4}-\frac{10a^3b^3c^2d^3(2ad+bc)}{(ad-bc)^4}+\frac{10a^2b^4c^3d^2(2ad+bc)}{(ad-bc)^4}+2a^2bd^2-\frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4}+3ab^2cd+\frac{b^6c^5(2ad+bc)}{(ad-bc)^4}+b}{4ab^2d^2+2b^3cd}\right.}{2\left(ad-bc\right)^4}$$

$$+\frac{b\left(2ad+bc\right)\log\left(x^2+\frac{\frac{a^5bd^5(2ad+bc)}{(ad-bc)^4}-\frac{5a^4b^2cd^4(2ad+bc)}{(ad-bc)^4}+\frac{10a^3b^3c^2d^3(2ad+bc)}{(ad-bc)^4}-\frac{10a^2b^4c^3d^2(2ad+bc)}{(ad-bc)^4}+2a^2bd^2+\frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4}+3ab^2cd-\frac{b^6c^5(2ad+bc)}{(ad-bc)^4}+b^3}{4ab^2d^2+2b^3cd}\right.}{2\left(ad-bc\right)^4}$$

$$-\frac{a^2cd+5abc^2+x^4\left(4abd^2+2b^2cd\right)+x^2\left(2a^2d^2\right.}{4a^4c^2d^3-12a^3bc^3d^2+12a^2b^2c^4d-4ab^3c^5+x^6\left(4a^3bd^5-12a^2b^2cd^4+12ab^3c^2d^3-4b^4c^3d^2\right)+x^4\left(4a^4d^5-4a^3bcd^4-12a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -b*(2*a*d + b*c)*log(x**2 + (-a**5*b*d**5*(2*a*d + b*c)/(a*d - b*
c)**4 + 5*a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**3
*b**3*c**2*d**3*(2*a*d + b*c)/(a*d - b*c)**4 + 10*a**2*b**4*c**3*
d**2*(2*a*d + b*c)/(a*d - b*c)**4 + 2*a**2*b*d**2 - 5*a*b**5*c**4
*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*a*b**2*c*d + b**6*c**5*(2*a*d
 + b*c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2*d**2 + 2*b**3*c*d))
/(2*(a*d - b*c)**4) + b*(2*a*d + b*c)*log(x**2 + (a**5*b*d**5*(2*
a*d + b*c)/(a*d - b*c)**4 - 5*a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d
 - b*c)**4 + 10*a**3*b**3*c**2*d**3*(2*a*d + b*c)/(a*d - b*c)**4
- 10*a**2*b**4*c**3*d**2*(2*a*d + b*c)/(a*d - b*c)**4 + 2*a**2*b*
d**2 + 5*a*b**5*c**4*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*a*b**2*c*
d - b**6*c**5*(2*a*d + b*c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2
*d**2 + 2*b**3*c*d))/(2*(a*d - b*c)**4) - (a**2*c*d + 5*a*b*c**2
+ x**4*(4*a*b*d**2 + 2*b**2*c*d) + x**2*(2*a**2*d**2 + 7*a*b*c*d
+ 3*b**2*c**2))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a**2
*b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2
*c*d**4 + 12*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*
d**5 - 4*a**3*b*c*d**4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*
d**2 - 8*b**4*c**4*d) + x**2*(8*a**4*c*d**4 - 20*a**3*b*c**2*d**3
 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*c**4*d - 4*b**4*c**5))
```

GIAC/XCAS [A] time = 0.239851, size = 360, normalized size = 2.54

$$\frac{\frac{2\,ab^5}{(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)(bx^2+a)}-\frac{2\left(b^4c+2\,ab^3d\right)\ln\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{3\,b^3cd^2+2\,ab^2d^3+\frac{2\left(2\,b^5c^2d-ab^4cd^2-a^2b^3d^3\right)}{\left(bx^2+a\right)b}}{(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*b^5/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^
3*d^3)*(b*x^2 + a)) - 2*(b^4*c + 2*a*b^3*d)*ln(abs(b*c/(b*x^2 + a
```

```
) - a*d/(b*x^2 + a) + d))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^
2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - (3*b^3*c*d^2 + 2*a*b^2*d^3
 + 2*(2*b^5*c^2*d - a*b^4*c*d^2 - a^2*b^3*d^3)/((b*x^2 + a)*b))/(
(b*c - a*d)^4*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d)^2))/b
```

3.312 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3dx}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] (-3*d*x)/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x/(2*(b*c - a*d)*(a +
b*x^2)*(c + d*x^2)^2) - (d*(11*b*c + a*d)*x)/(8*c*(b*c - a*d)^3*(
c + d*x^2)) + (b^(3/2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*Sqrt[a]*(b*c - a*d)^4) - (Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d -
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.603195, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3dx}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x)/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x/(2*(b*c - a*d)*(a +
b*x^2)*(c + d*x^2)^2) - (d*(11*b*c + a*d)*x)/(8*c*(b*c - a*d)^3*(
c + d*x^2)) + (b^(3/2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*Sqrt[a]*(b*c - a*d)^4) - (Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d -
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*(b*c - a*d)^4)
```

Rubi in Sympy [A] time = 117.255, size = 178, normalized size = 0.89

$$-\frac{3dx}{4\left(c+dx^2\right)^2(ad-bc)^2}+\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(ad-bc)}+\frac{dx\,(ad+11bc)}{8c\left(c+dx^2\right)(ad-bc)^3}+\frac{\sqrt{d}\left(a^2d^2-10abcd-15b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}(ad-bc)^4}+\frac{b^{\frac{3}{2}}(5ad+bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\,(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -3*d*x/(4*(c + d*x**2)**2*(a*d - b*c)**2) + x/(2*(a + b*x**2)*(c
+ d*x**2)**2*(a*d - b*c)) + d*x*(a*d + 11*b*c)/(8*c*(c + d*x**2)*
(a*d - b*c)**3) + sqrt(d)*(a**2*d**2 - 10*a*b*c*d - 15*b**2*c**2)
*atan(sqrt(d)*x/sqrt(c))/(8*c**(3/2)*(a*d - b*c)**4) + b**(3/2)*(
5*a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*(a*d - b*c)**4)
```

Mathematica [A] time = 0.706733, size = 171, normalized size = 0.86

$$\frac{\frac{\sqrt{d}\left(a^2d^2-10abcd-15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{4b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{4b^2x(bc-ad)}{a+bx^2}+\frac{dx(ad-bc)(ad+7bc)}{c\left(c+dx^2\right)}-\frac{2dx(bc-ad)^2}{\left(c+dx^2\right)^2}}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^2*(b*c - a*d)*x)/(a + b*x^2) - (2*d*(b*c - a*d)^2*x)/(c +
d*x^2)^2 + (d*(-(b*c) + a*d)*(7*b*c + a*d)*x)/(c*(c + d*x^2)) + (
4*b^(3/2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (S
qrt[d]*(-15*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/c^(3/2))/(8*(b*c - a*d)^4)
```

Maple [B] time = 0.022, size = 391, normalized size = 2.

$$\frac{d^4x^3a^2}{8\,(ad-bc)^4\,(dx^2+c)^2\,c}+\frac{3\,d^3x^3ab}{4\,(ad-bc)^4\,(dx^2+c)^2}-\frac{7\,d^2x^3b^2c}{8\,(ad-bc)^4\,(dx^2+c)^2}+\frac{5\,abcd^2x}{4\,(ad-bc)^4\,(dx^2+c)^2}-\frac{9\,b^2c^2dx}{8\,(ad-bc)^4\,(dx^2+c)^2}-\frac{a^2d^3x}{8\,(ad-bc)^4\,(dx^2+c)^2}+\frac{a^2d^3}{8\,(ad-bc)^4\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{5\,abd^2}{4\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{15\,b^2cd}{8\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2xad}{2\,(ad-bc)^4\,(bx^2+a)}-\frac{b^3xc}{2\,(ad-bc)^4\,(bx^2+a)}+\frac{5\,ab^2d}{2\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^3c}{2\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/8*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^3*a^2+3/4*d^3/(a*d-b*c)^4/(d*
x^2+c)^2*x^3*a*b-7/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2*c+5/4*d^
2/(a*d-b*c)^4/(d*x^2+c)^2*a*b*c*x-9/8*d/(a*d-b*c)^4/(d*x^2+c)^2*b
^2*c^2*x-1/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a^2*x+1/8*d^3/(a*d-b*c)^
4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-5/4*d^2/(a*d-b*c)^4/(
c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-15/8*d/(a*d-b*c)^4*c/(c*d)
^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2+1/2*b^2/(a*d-b*c)^4*x/(b*x^2+a
)*a*d-1/2*b^3/(a*d-b*c)^4*x/(b*x^2+a)*c+5/2*b^2/(a*d-b*c)^4/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+1/2*b^3/(a*d-b*c)^4/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96483, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(
17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 -
```

```
4*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6
+ (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4
 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*a*b^2*c^4 + 10*a^2*b*c^
3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)
*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4
)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c
*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
 c)) + 2*(4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^
3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*
c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^
3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d -
 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*
b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3
*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x
^2), -1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + (1
7*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 + (
15*a*b^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 1
0*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2
 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d +
19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt
(d/c))) - 2*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c
*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4
+ (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(-b/a)*lo
g((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^3*c^4 + 5*a*
b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b
^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b
^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*
d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b
^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4
 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d
^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/16*(2*(11*b^3*c^2*
d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*b^3*c^3*d - 11*a*b^
2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 8*(a*b^2*c^4 + 5*a^2*b
*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*
b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*
a^2*b*c^2*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + (15*a*b
^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^
2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a
^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2
*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt
(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*
b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^
2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^
4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*
x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*
b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b
^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^
4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 -
 a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^
3 - a^3*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 +
5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*
d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt
(b/a)*arctan(b*x/(a*sqrt(b/a))) + (15*a*b^2*c^4 + 10*a^2*b*c^3*d
- a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6
 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^
4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3
)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + (4*b^3*c^4 + 5*a*b^2
*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*
c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*
c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5
 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*
c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 +
(b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3
- 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.25718, size = 428, normalized size = 2.14

$$-\frac{b^2x}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(bx^2+a)}+\frac{\left(b^3c+5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}}-\frac{\left(15\,b^2c^2d+10\,abcd^2-a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}}-\frac{7\,bcd^2x^3+ad^3x^3+9\,bc^2dx-acd^2x}{8\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] -1/2*b^2*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(
b*x^2 + a)) + 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d + 10*a*b*c*d^2 - a^2*d^3)*arcta
n(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*(7*b*c*d^2*x^3 + a*
d^3*x^3 + 9*b*c^2*d*x - a*c*d^2*x)/((b^3*c^4 - 3*a*b^2*c^3*d + 3*
a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

3.313 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=126

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x^2)) - d/(4*(b*c - a*d)^2*(c + d*x^
2)^2) - (b*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^
2])/(2*(b*c - a*d)^4) + (3*b^2*d*Log[c + d*x^2])/(2*(b*c - a*d)^4
)
```

Rubi [A] time = 0.24839, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x^2)) - d/(4*(b*c - a*d)^2*(c + d*x^
2)^2) - (b*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^
2])/(2*(b*c - a*d)^4) + (3*b^2*d*Log[c + d*x^2])/(2*(b*c - a*d)^4
)
```

Rubi in Sympy [A] time = 47.8401, size = 109, normalized size = 0.87

$$-\frac{3b^2d\log\left(a+bx^2\right)}{2\left(ad-bc\right)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2\left(ad-bc\right)^4}+\frac{b^2}{2\left(a+bx^2\right)\left(ad-bc\right)^3}+\frac{bd}{\left(c+dx^2\right)\left(ad-bc\right)^3}-\frac{d}{4\left(c+dx^2\right)^2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -3*b**2*d*log(a + b*x**2)/(2*(a*d - b*c)**4) + 3*b**2*d*log(c + d
*x**2)/(2*(a*d - b*c)**4) + b**2/(2*(a + b*x**2)*(a*d - b*c)**3)
+ b*d/((c + d*x**2)*(a*d - b*c)**3) - d/(4*(c + d*x**2)**2*(a*d -
 b*c)**2)
```

Mathematica [A] time = 0.226254, size = 107, normalized size = 0.85

$$-\frac{\frac{2b^2(bc-ad)}{a+bx^2}+6b^2d\log\left(a+bx^2\right)+\frac{4bd(bc-ad)}{c+dx^2}+\frac{d(bc-ad)^2}{\left(c+dx^2\right)^2}-6b^2d\log\left(c+dx^2\right)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -((2*b^2*(b*c - a*d))/(a + b*x^2) + (d*(b*c - a*d)^2)/(c + d*x^2)
^2 + (4*b*d*(b*c - a*d))/(c + d*x^2) + 6*b^2*d*Log[a + b*x^2] - 6
*b^2*d*Log[c + d*x^2])/(4*(b*c - a*d)^4)
```

Maple [A] time = 0.024, size = 234, normalized size = 1.9

$$\frac{abd^{2}}{\left(ad-bc\right)^{4}\left(dx^{2}+c\right)}-\frac{db^{2}c}{\left(ad-bc\right)^{4}\left(dx^{2}+c\right)}-\frac{a^{2}d^{3}}{4\left(ad-bc\right)^{4}\left(dx^{2}+c\right)^{2}}+\frac{abcd^{2}}{2\left(ad-bc\right)^{4}\left(dx^{2}+c\right)^{2}}-\frac{b^{2}c^{2}d}{4\left(ad-bc\right)^{4}\left(dx^{2}+c\right)^{2}}+\frac{3d\ln\left(dx^{2}+c\right)b^{2}}{2\left(ad-bc\right)^{4}}-\frac{3b^{2}\ln\left(bx^{2}+a\right)d}{2\left(ad-bc\right)^{4}}+\frac{b^{2}ad}{2\left(ad-bc\right)^{4}\left(bx^{2}+a\right)}-\frac{b^{3}c}{2\left(ad-bc\right)^{4}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] d^2/(a*d-b*c)^4*b/(d*x^2+c)*a-d/(a*d-b*c)^4*b^2/(d*x^2+c)*c-1/4*d
^3/(a*d-b*c)^4/(d*x^2+c)^2*a^2+1/2*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*
a*b-1/4*d/(a*d-b*c)^4/(d*x^2+c)^2*b^2*c^2+3/2*d/(a*d-b*c)^4*ln(d*
x^2+c)*b^2-3/2*b^2/(a*d-b*c)^4*ln(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^4/
(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4/(b*x^2+a)*c
```

Maxima [A] time = 1.36679, size = 532, normalized size = 4.22

$$-\frac{3b^{2}d\log\left(bx^{2}+a\right)}{2\left(b^{4}c^{4}-4ab^{3}c^{3}d+6a^{2}b^{2}c^{2}d^{2}-4a^{3}bcd^{3}+a^{4}d^{4}\right)}+\frac{3b^{2}d\log\left(dx^{2}+c\right)}{2\left(b^{4}c^{4}-4ab^{3}c^{3}d+6a^{2}b^{2}c^{2}d^{2}-4a^{3}bcd^{3}+a^{4}d^{4}\right)}-\frac{6b^{2}d^{2}x^{4}+2b^{2}c^{2}+5abcd-a^{2}d^{2}+3\left(3b^{2}cd+ab\right.}{4\left(ab^{3}c^{5}-3a^{2}b^{2}c^{4}d+3a^{3}bc^{3}d^{2}-a^{4}c^{2}d^{3}+\left(b^{4}c^{3}d^{2}-3ab^{3}c^{2}d^{3}+3a^{2}b^{2}cd^{4}-a^{3}bd^{5}\right)x^{6}+\left(2b^{4}c^{4}d-5ab^{3}c^{3}d^{2}+3a^{2}b^{2}c^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] -3/2*b^2*d*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 3/2*b^2*d*log(d*x^2 + c)/(b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
) - 1/4*(6*b^2*d^2*x^4 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b
^2*c*d + a*b*d^2)*x^2)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3
*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c
*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^
2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d -
 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

Fricas [A] time = 0.236112, size = 684, normalized size = 5.43

$$-\frac{2b^{3}c^{3}+3ab^{2}c^{2}d-6a^{2}bcd^{2}+a^{3}d^{3}+6\left(b^{3}cd^{2}-ab^{2}d^{3}\right)x^{4}+3\left(3b^{3}c^{2}d-2ab^{2}cd^{2}-a^{2}bd^{3}\right)x^{2}+6\left(b^{3}d^{3}x^{6}+ab^{2}c^{2}d+\left(2b\right.\right.}{4\left(ab^{4}c^{6}-4a^{2}b^{3}c^{5}d+6a^{3}b^{2}c^{4}d^{2}-4a^{4}bc^{3}d^{3}+a^{5}c^{2}d^{4}+\left(b^{5}c^{4}d^{2}-4ab^{4}c^{3}d^{3}+6a^{2}b^{3}c^{2}d^{4}-4a^{3}b^{2}cd^{5}+a^{4}bd^{6}\right)x^{6}+\left(2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 - a*b^2*d^3)*x^4 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b
*d^3)*x^2 + 6*(b^3*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d
^3)*x^4 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x^2)*log(b*x^2 + a) - 6*(b^
3*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^4 + (b^3*c^
```

```
2*d + 2*a*b^2*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c^6 - 4*a^2*b^3*
c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*
c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 +
 a^4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*
d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6
 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*
b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

Sympy [A] time = 56.269, size = 643, normalized size = 5.1

$$\frac{3b^2d\log\left(x^2+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{2(ad-bc)^4}-\frac{3b^2d\log\left(x^2+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\frac{30a^2b^5c^3d^3}{(ad-bc)^4}+\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2-\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{2(ad-bc)^4}+\frac{-a^2d^2+5abcd+2b^2c^2+6b^2d^2x^4+x^2\left(3ab\right.}{4a^4c^2d^3-12a^3bc^3d^2+12a^2b^2c^4d-4ab^3c^5+x^6\left(4a^3bd^5-12a^2b^2cd^4+12ab^3c^2d^3-4b^4c^3d^2\right)+x^4\left(4a^4d^5-4a^3bcd^4-12a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] 3*b**2*d*log(x**2 + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b
**3*c*d**5/(a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4
 + 30*a**2*b**5*c**3*d**3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a
*d - b*c)**4 + 3*a*b**2*d**2 + 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b
**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*c)**4) - 3*b**2*d*log(x**2 +
(3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d - b*c
)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*
d**3/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b*
*2*d**2 - 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2
))/(2*(a*d - b*c)**4) + (-a**2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6
*b**2*d**2*x**4 + x**2*(3*a*b*d**2 + 9*b**2*c*d))/(4*a**4*c**2*d*
*3 - 12*a**3*b*c**3*d**2 + 12*a**2*b**2*c**4*d - 4*a*b**3*c**5 +
x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4 + 12*a*b**3*c**2*d**3 -
 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c*d**4 - 12*a**
2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**2*(8
*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a
*b**3*c**4*d - 4*b**4*c**5))
```

GIAC/XCAS [A] time = 0.233265, size = 309, normalized size = 2.45

$$\frac{3\,b^3d\ln\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)}-\frac{b^5}{2\,(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)(bx^2+a)}+\frac{5\,b^2d^3+\frac{6\,(b^4cd^2-ab^3d^3)}{(bx^2+a)b}}{4\,(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] 3/2*b^3*d*ln(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^
4) - 1/2*b^5/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^
3*d^3)*(b*x^2 + a)) + 1/4*(5*b^2*d^3 + 6*(b^4*c*d^2 - a*b^3*d^3)/
((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a
) + d)^2)
```

3.314 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(
2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*
c + 3*a*d)*x)/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c
 - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^4)
+ (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.727715, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(
2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*
c + 3*a*d)*x)/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c
 - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^4)
+ (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.971143, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(\frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4}-\frac{4b^3x}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{d^2x(11bc-3ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{2d^2x}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c
- a*d)^2*(c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*
d)^3*(c + d*x^2)) + (4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*
c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d
)^4))/8
```

Maple [A] time = 0.002, size = 403, normalized size = 1.8

$$\frac{3\,d^5x^3a^2}{8\,(ad-bc)^4\,(dx^2+c)^2\,c^2}-\frac{7\,d^4x^3ab}{4\,(ad-bc)^4\,(dx^2+c)^2\,c}+\frac{11\,d^3b^2x^3}{8\,(ad-bc)^4\,(dx^2+c)^2}+\frac{5\,d^4xa^2}{8\,(ad-bc)^4\,(dx^2+c)^2\,c}-\frac{9\,d^3xab}{4\,(ad-bc)^4\,(dx^2+c)^2}+\frac{13\,d^2xb^2c}{8\,(ad-bc)^4\,(dx^2+c)^2}+\frac{3\,d^4a^2}{8\,(ad-bc)^4\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,abd^3}{4\,(ad-bc)^4\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{35\,d^2b^2}{8\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3xd}{2\,(ad-bc)^4\,(bx^2+a)}+\frac{b^4xc}{2\,(ad-bc)^4\,a\,(bx^2+a)}-\frac{7\,b^3d}{2\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^4c}{2\,(ad-bc)^4\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^3*a^2-7/4*d^4/(a*d-b*c)^4/(
d*x^2+c)^2/c*x^3*a*b+11/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*b^2*x^3+5/8
*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x*a^2-9/4*d^3/(a*d-b*c)^4/(d*x^2+c
)^2*x*a*b+13/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*x*b^2*c+3/8*d^4/(a*d-b
*c)^4/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-7/4*d^3/(a*d-b*
c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+35/8*d^2/(a*d-b*c)
^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/2*b^3/(a*d-b*c)^4*x/
(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4*x/a/(b*x^2+a)*c-7/2*b^3/(a*d-b*c)
^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+1/2*b^4/(a*d-b*c)^4/a/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.32526, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*
a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2
```

*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*
c^4*d + (b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a
*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d
- 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/
a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a
^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*
x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a
^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2
*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c)
- c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^
3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b
^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a
*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2
+ 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*
c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 +
8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*(
(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^
3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b
*d^5)*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4
+ 3*a^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3
*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))
) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3*c^2*d
^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^
4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a
)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 -
4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d
^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5
*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6
*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*
b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4
*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^
4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3
- 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*
c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*
(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^
6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (b
^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b/a)*arcta
n(b*x/(a*sqrt(b/a))) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a
^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*
x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a
^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2
*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c)
- c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^
3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b
^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a
*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2
+ 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*
c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 +
8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*(
(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^
3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^
4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^
2 - 7*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^
2*c^3*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + (35*a^2*b^2
*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 1
4*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^
2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56
*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)
*arctan(d*x/(c*sqrt(d/c))) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*
b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4
*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^
4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^
4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c
^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5
+ a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6
*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239068, size = 448, normalized size = 1.95

$$\frac{b^3x}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)(bx^2+a)}+\frac{\left(b^4c-7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}}+\frac{\left(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}}+\frac{11\,bcd^3x^3-3\,ad^4x^3+13\,bc^2d^2x-5\,acd^3x}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3
)*(b*x^2 + a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((
a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2
*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2
*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/8*(11*b*
c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^3*c^5
 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

3.315 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^3} \, dx$

Optimal. Leaf size=192

$$-\frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4}-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a\left(a+bx^2\right)(bc-ad)^3}+\frac{d^2(3bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2}{4c\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] b^3/(2*a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(4*c*(b*c - a*d)^2*(c +
 d*x^2)^2) + (d^2*(3*b*c - a*d))/(2*c^2*(b*c - a*d)^3*(c + d*x^2)
) + Log[x]/(a^2*c^3) - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(2*a^2*
(b*c - a*d)^4) - (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d
*x^2])/(2*c^3*(b*c - a*d)^4)
```

Rubi [A] time = 0.489682, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4}-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a\left(a+bx^2\right)(bc-ad)^3}+\frac{d^2(3bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2}{4c\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] b^3/(2*a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(4*c*(b*c - a*d)^2*(c +
 d*x^2)^2) + (d^2*(3*b*c - a*d))/(2*c^2*(b*c - a*d)^3*(c + d*x^2)
) + Log[x]/(a^2*c^3) - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(2*a^2*
(b*c - a*d)^4) - (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d
*x^2])/(2*c^3*(b*c - a*d)^4)
```

Rubi in Sympy [A] time = 101.995, size = 173, normalized size = 0.9

$$\frac{d^2}{4c\left(c+dx^2\right)^2(ad-bc)^2}+\frac{d^2(ad-3bc)}{2c^2\left(c+dx^2\right)(ad-bc)^3}-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(ad-bc)^4}-\frac{b^3}{2a\left(a+bx^2\right)(ad-bc)^3}+\frac{b^3(4ad-bc)\log\left(a+bx^2\right)}{2a^2(ad-bc)^4}+\frac{\log\left(x^2\right)}{2a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] d**2/(4*c*(c + d*x**2)**2*(a*d - b*c)**2) + d**2*(a*d - 3*b*c)/(2
*c**2*(c + d*x**2)*(a*d - b*c)**3) - d**2*(a**2*d**2 - 4*a*b*c*d
+ 6*b**2*c**2)*log(c + d*x**2)/(2*c**3*(a*d - b*c)**4) - b**3/(2*
a*(a + b*x**2)*(a*d - b*c)**3) + b**3*(4*a*d - b*c)*log(a + b*x**
2)/(2*a**2*(a*d - b*c)**4) + log(x**2)/(2*a**2*c**3)
```

Mathematica [A] time = 0.545019, size = 187, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2b^3(4ad-bc)\log\left(a+bx^2\right)}{a^2(bc-ad)^4}-\frac{2d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{c^3(bc-ad)^4}+\frac{4\log(x)}{a^2c^3}-\frac{2b^3}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{2d^2(3bc-ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-2*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2
*(c + d*x^2)^2) + (2*d^2*(3*b*c - a*d))/(c^2*(b*c - a*d)^3*(c + d
*x^2)) + (4*Log[x])/(a^2*c^3) + (2*b^3*(-(b*c) + 4*a*d)*Log[a + b
*x^2])/(a^2*(b*c - a*d)^4) - (2*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*
d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d)^4))/4
```

Maple [B] time = 0.037, size = 374, normalized size = 2.

$$\frac{\ln(x)}{a^2c^3}+\frac{a^2d^4}{2c^2(ad-bc)^4(dx^2+c)}-2\frac{abd^3}{c(ad-bc)^4(dx^2+c)}+\frac{3b^2d^2}{2(ad-bc)^4(dx^2+c)}+\frac{a^2d^4}{4c(ad-bc)^4(dx^2+c)^2}-\frac{abd^3}{2(ad-bc)^4(dx^2+c)^2}+\frac{b^2cd^2}{4(ad-bc)^4(dx^2+c)^2}-\frac{d^4\ln(dx^2+c)a^2}{2c^3(ad-bc)^4}+2\frac{d^3\ln(dx^2+c)ab}{c^2(ad-bc)^4}-3\frac{d^2\ln(dx^2+c)b^2}{c(ad-bc)^4}+2\frac{b^3\ln(bx^2+a)d}{a(ad-bc)^4}-\frac{b^4\ln(bx^2+a)c}{2a^2(ad-bc)^4}-\frac{b^3d}{2(ad-bc)^4(bx^2+a)}+\frac{b^4c}{2a(ad-bc)^4(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] ln(x)/a^2/c^3+1/2*d^4/c^2/(a*d-b*c)^4/(d*x^2+c)*a^2-2*d^3/c/(a*d-
b*c)^4/(d*x^2+c)*a*b+3/2*d^2/(a*d-b*c)^4/(d*x^2+c)*b^2+1/4*d^4/c/
(a*d-b*c)^4/(d*x^2+c)^2*a^2-1/2*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a*b+1
/4*d^2*c/(a*d-b*c)^4/(d*x^2+c)^2*b^2-1/2*d^4/c^3/(a*d-b*c)^4*ln(d
*x^2+c)*a^2+2*d^3/c^2/(a*d-b*c)^4*ln(d*x^2+c)*a*b-3*d^2/c/(a*d-b*
c)^4*ln(d*x^2+c)*b^2+2*b^3/a/(a*d-b*c)^4*ln(b*x^2+a)*d-1/2*b^4/a^
2/(a*d-b*c)^4*ln(b*x^2+a)*c-1/2*b^3/(a*d-b*c)^4/(b*x^2+a)*d+1/2*b
^4/a/(a*d-b*c)^4/(b*x^2+a)*c
```

Maxima [A] time = 1.38927, size = 711, normalized size = 3.7

$$-\frac{(b^4c-4ab^3d)\log(bx^2+a)}{2(a^2b^4c^4-4a^3b^3c^3d+6a^4b^2c^2d^2-4a^5bcd^3+a^6d^4)}-\frac{(6b^2c^2d^2-4abcd^3+a^2d^4)\log(dx^2+c)}{2(b^4c^7-4ab^3c^6d+6a^2b^2c^5d^2-4a^3bc^4d^3+a^4c^3d^4)}+\frac{2b^3c^4+7a^2bc^2d^2-3a^3cd^3+2(b^3c^2d^2+3ab^2cd^3-a^2bd^4)x^4+(4b}{4(a^2b^3c^7-3a^3b^2c^6d+3a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3a^2b^3c^4d^3+3a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2ab^4c^6d-5a^2b^3c^5d^2+3}+\frac{\log(x^2)}{2a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x),x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^2*b^4*c^4 - 4*a^3*b^3*
c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/2*(6*b^2
*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x^2 + c)/(b^4*c^7 - 4*a*b
^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) + 1
/4*(2*b^3*c^4 + 7*a^2*b*c^2*d^2 - 3*a^3*c*d^3 + 2*(b^3*c^2*d^2 +
3*a*b^2*c*d^3 - a^2*b*d^4)*x^4 + (4*b^3*c^3*d + 7*a*b^2*c^2*d^2 +
 3*a^2*b*c*d^3 - 2*a^3*d^4)*x^2)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d +
 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d
^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*
a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5
)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*
```

```
c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 1/2*log(x^2)/(a^2*c^3)
```

Fricas [A] time = 14.5606, size = 1428, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^4*c^6 - 2*a^2*b^3*c^5*d + 7*a^3*b^2*c^4*d^2 - 10*a^4*b
*c^3*d^3 + 3*a^5*c^2*d^4 + 2*(a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^3 -
 4*a^3*b^2*c^2*d^4 + a^4*b*c*d^5)*x^4 + (4*a*b^4*c^5*d + 3*a^2*b^
3*c^4*d^2 - 4*a^3*b^2*c^3*d^3 - 5*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^
2 - 2*(a*b^4*c^6 - 4*a^2*b^3*c^5*d + (b^5*c^4*d^2 - 4*a*b^4*c^3*d
^3)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3)*x^4
 + (b^5*c^6 - 2*a*b^4*c^5*d - 8*a^2*b^3*c^4*d^2)*x^2)*log(b*x^2 +
 a) - 2*(6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (6*a
^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (12*a^2*b^3*c
^3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (6*a^
2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5
)*x^2)*log(d*x^2 + c) + 4*(a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^
2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d^2 - 4*a*b^
4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6
+ (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*
c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d
 - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^
5*c*d^5)*x^2)*log(x))/(a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*
c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4 + (a^2*b^5*c^7*d^2 - 4*a^
3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3
*d^6)*x^6 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*
d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^4 + (a
^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*
d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.258157, size = 635, normalized size = 3.31

$$-\frac{\left(b^{5} c - 4 \, a b^{4} d\right) \ln\left({\left| b x^{2} + a \right|}\right)}{2 \,{\left(a^{2} b^{5} c^{4} - 4 \, a^{3} b^{4} c^{3} d + 6 \, a^{4} b^{3} c^{2} d^{2} - 4 \, a^{5} b^{2} c d^{3} + a^{6} b d^{4}\right)}} - \frac{{\left(6 \, b^{2} c^{2} d^{3} - 4 \, a b c d^{4} + a^{2} d^{5}\right)} \ln\left({\left| d x^{2} + c \right|}\right)}{2 \,{\left(b^{4} c^{7} d - 4 \, a b^{3} c^{6} d^{2} + 6 \, a^{2} b^{2} c^{5} d^{3} - 4 \, a^{3} b c^{4} d^{4} + a^{4} c^{3} d^{5}\right)}} + \frac{b^{5} c x^{2} - 4 \, a b^{4} d x^{2} + 2 \, a b^{4} c - 5 \, a^{2} b^{3} d}{2 \,{\left(a^{2} b^{4} c^{4} - 4 \, a^{3} b^{3} c^{3} d + 6 \, a^{4} b^{2} c^{2} d^{2} - 4 \, a^{5} b c d^{3} + a^{6} d^{4}\right)}{\left(b x^{2} + a\right)}} + \frac{18 \, b^{2} c^{2} d^{4} x^{4} - 12 \, a b c d^{5} x^{4} + 3 \, a^{2} d^{6} x^{4} + 42 \, b^{2} c^{3} d^{3} x^{2} - 32 \, a b c^{2} d^{4} x^{2} + 8 \, a^{2} c d^{5} x^{2} + 25 \, b^{2} c^{4} d^{2} - 22 \, a b c^{3} d^{3} + 6 \, a^{2} c^{2} d^{4}}{4 \,{\left(b^{4} c^{7} - 4 \, a b^{3} c^{6} d + 6 \, a^{2} b^{2} c^{5} d^{2} - 4 \, a^{3} b c^{4} d^{3} + a^{4} c^{3} d^{4}\right)}{\left(d x^{2} + c\right)}^{2}} + \frac{\ln\left(x^{2}\right)}{2 \, a^{2} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x),x, algorithm="giac")
```

```
[Out] -1/2*(b^5*c - 4*a*b^4*d)*ln(abs(b*x^2 + a))/(a^2*b^5*c^4 - 4*a^3*
b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4) - 1/
2*(6*b^2*c^2*d^3 - 4*a*b*c*d^4 + a^2*d^5)*ln(abs(d*x^2 + c))/(b^4
*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 +
a^4*c^3*d^5) + 1/2*(b^5*c*x^2 - 4*a*b^4*d*x^2 + 2*a*b^4*c - 5*a^2
*b^3*d)/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a
^5*b*c*d^3 + a^6*d^4)*(b*x^2 + a)) + 1/4*(18*b^2*c^2*d^4*x^4 - 12
*a*b*c*d^5*x^4 + 3*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^2 - 32*a*b*c^2*
d^4*x^2 + 8*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 22*a*b*c^3*d^3 + 6*a
^2*c^2*d^4)/((b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3
*b*c^4*d^3 + a^4*c^3*d^4)*(d*x^2 + c)^2) + 1/2*ln(x^2)/(a^2*c^3)
```

3.316 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{3b^{7/2}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^4}+\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}-\frac{3(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{8a^2c^3x(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] (-3*(2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c*d + 5*a^2*d^2))/(8*a^2*c^3
*(b*c - a*d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + (d
*(4*b^2*c^2 + 13*a*b*c*d - 5*a^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x*(
c + d*x^2)) - (3*b^(7/2)*(b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(2*a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c - a*d
)^4)
```

Rubi [A] time = 1.27174, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{3b^{7/2}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^4}+\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}-\frac{3(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{8a^2c^3x(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*(2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c*d + 5*a^2*d^2))/(8*a^2*c^3
*(b*c - a*d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + (d
*(4*b^2*c^2 + 13*a*b*c*d - 5*a^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x*(
c + d*x^2)) - (3*b^(7/2)*(b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(2*a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c - a*d
)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.06818, size = 210, normalized size = 0.71

$$\frac{1}{8}\left(\frac{12b^{7/2}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4}+\frac{4b^4x}{a^2(a+bx^2)(ad-bc)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^4}-\frac{8}{a^2c^3x}+\frac{d^3x(7ad-15bc)}{c^3(c+dx^2)(bc-ad)^3}-\frac{2d^3x}{c^2(c+dx^2)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x) + (4*b^4*x)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) -
(2*d^3*x)/(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-15*b*c + 7*a
*d)*x)/(c^3*(b*c - a*d)^3*(c + d*x^2)) + (12*b^(7/2)*(-(b*c) + 3*
a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^4) - (3*d^
(5/2)*(21*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(c^(7/2)*(b*c - a*d)^4))/8
```

Maple [A] time = 0.03, size = 428, normalized size = 1.4

$$-\frac{1}{a^2c^3x}-\frac{7\,d^6x^3a^2}{8\,c^3\,(ad-bc)^4\,(dx^2+c)^2}+\frac{11\,d^5x^3ab}{4\,c^2\,(ad-bc)^4\,(dx^2+c)^2}-\frac{15\,d^4x^3b^2}{8\,c\,(ad-bc)^4\,(dx^2+c)^2}-\frac{9\,d^5xa^2}{8\,c^2\,(ad-bc)^4\,(dx^2+c)^2}+\frac{13\,d^4xab}{4\,c\,(ad-bc)^4\,(dx^2+c)^2}-\frac{17\,d^3xb^2}{8\,(ad-bc)^4\,(dx^2+c)^2}-\frac{15\,a^2d^5}{8\,c^3\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{27\,d^4ab}{4\,c^2\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{63\,b^2d^3}{8\,c\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^4xd}{2\,a\,(ad-bc)^4\,(bx^2+a)}-\frac{b^5xc}{2\,a^2\,(ad-bc)^4\,(bx^2+a)}+\frac{9\,b^4d}{2\,a\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,b^5c}{2\,a^2\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/a^2/c^3/x-7/8*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a^2+11/4*d^5
/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a*b-15/8*d^4/c/(a*d-b*c)^4/(d*x^
2+c)^2*x^3*b^2-9/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x*a^2+13/4*d^4
/c/(a*d-b*c)^4/(d*x^2+c)^2*x*a*b-17/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2
*x*b^2-15/8*d^5/c^3/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2
))*a^2+27/4*d^4/c^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2
))*a*b-63/8*d^3/c/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
*b^2+1/2*b^4/a/(a*d-b*c)^4*x/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*
x/(b*x^2+a)*c+9/2*b^4/a/(a*d-b*c)^4/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*d-3/2*b^5/a^2/(a*d-b*c)^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.14802, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 96*a^3*b^2*c^4*d^2 - 64
*a^4*b*c^3*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^2 - 12*a*b^4*c^3
*d^3 + 21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 +
 2*(24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3
*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + 2*(12*b^5*c^6 -
 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^
4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4*c^3*
d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^
5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6
- 3*a^2*b^3*c^5*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) - 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a
^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*
b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d
^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 -
 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^2 - 2*c
*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^
6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^
7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*
a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c
^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*
a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8
*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x), -1/8*
(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b*c
^3*d^3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21
*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*
c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^
4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + (12*b^5*c^6 - 20*a*b^4*c^5
*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 +
 25*a^5*c*d^5)*x^2 + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 +
5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a
^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^
3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^
2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(d/c)*arctan(d*x/(c*
sqrt(d/c))) + 6*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5
*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*
c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)
*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^
2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2
*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*
d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a
^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*
d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 +
(a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*
d^3 + a^7*c^5*d^4)*x), -1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 9
6*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*
c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^
5 + 5*a^4*b*d^6)*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^
2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)
*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a
^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 24*((b^5*
c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 -
 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*
d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)*sqrt(b/a)*arctan(b*x/
(a*sqrt(b/a))) - 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^
4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b
*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^
3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 -
18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*
x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6
*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7
```

```
 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a
^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^
9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a
^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*
d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x), -1/8*(
8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b*c^
3*d^3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*
a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c
^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4
 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + (12*b^5*c^6 - 20*a*b^4*c^5*
d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 +
25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*
b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 -
 a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^
5*d)*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 3*((21*a^2*b^3*c^2*
d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 -
 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^
3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)
*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)
*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/((a^2*b^5*c^7*d^2 - 4*a^3*b
^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^
6)*x^7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3
 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*
b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3
 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^
3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.260334, size = 581, normalized size = 1.96

$$-\frac{3\left(b^{5}c-3\,ab^{4}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^{2}b^{4}c^{4}-4\,a^{3}b^{3}c^{3}d+6\,a^{4}b^{2}c^{2}d^{2}-4\,a^{5}bcd^{3}+a^{6}d^{4})\sqrt{ab}}-\frac{3\left(21\,b^{2}c^{2}d^{3}-18\,abcd^{4}+5\,a^{2}d^{5}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^{4}c^{7}-4\,ab^{3}c^{6}d+6\,a^{2}b^{2}c^{5}d^{2}-4\,a^{3}bc^{4}d^{3}+a^{4}c^{3}d^{4})\sqrt{cd}}-\frac{3\,b^{4}c^{3}x^{2}-6\,ab^{3}c^{2}dx^{2}+6\,a^{2}b^{2}cd^{2}x^{2}-2\,a^{3}bd^{3}x^{2}+2\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d+6\,a^{3}bcd^{2}-2\,a^{4}d^{3}}{2\,(a^{2}b^{3}c^{6}-3\,a^{3}b^{2}c^{5}d+3\,a^{4}bc^{4}d^{2}-a^{5}c^{3}d^{3})(bx^{3}+ax)}-\frac{15\,bcd^{4}x^{3}-7\,ad^{5}x^{3}+17\,bc^{2}d^{3}x-9\,acd^{4}x}{8\,(b^{3}c^{6}-3\,ab^{2}c^{5}d+3\,a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3})(dx^{2}+c)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^2),x, algorithm="giac")
```

```
[Out] -3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*
a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt
(a*b)) - 3/8*(21*b^2*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d
*x/sqrt(c*d))/((b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a
^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d)) - 1/2*(3*b^4*c^3*x^2 - 6*a
*b^3*c^2*d*x^2 + 6*a^2*b^2*c*d^2*x^2 - 2*a^3*b*d^3*x^2 + 2*a*b^3*
c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3)/((a^2*b^3*c^6
- 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(b*x^3 + a*x))
 - 1/8*(15*b*c*d^4*x^3 - 7*a*d^5*x^3 + 17*b*c^2*d^3*x - 9*a*c*d^4
*x)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d
```

```
*x^2 + c)^2)
```

3.317 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^3} dx$

Optimal. Leaf size=215

$$\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^4}-\frac{1}{2a^2c^3x^2}-\frac{d^3(2bc-ad)}{c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3}{4c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] -1/(2*a^2*c^3*x^2) - b^4/(2*a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/
(4*c^2*(b*c - a*d)^2*(c + d*x^2)^2) - (d^3*(2*b*c - a*d))/(c^3*(b
*c - a*d)^3*(c + d*x^2)) - ((2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (
b^4*(2*b*c - 5*a*d)*Log[a + b*x^2])/(2*a^3*(b*c - a*d)^4) + (d^3*
(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4*(b*c
 - a*d)^4)
```

Rubi [A] time = 0.612914, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^4}-\frac{1}{2a^2c^3x^2}-\frac{d^3(2bc-ad)}{c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3}{4c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/(2*a^2*c^3*x^2) - b^4/(2*a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/
(4*c^2*(b*c - a*d)^2*(c + d*x^2)^2) - (d^3*(2*b*c - a*d))/(c^3*(b
*c - a*d)^3*(c + d*x^2)) - ((2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (
b^4*(2*b*c - 5*a*d)*Log[a + b*x^2])/(2*a^3*(b*c - a*d)^4) + (d^3*
(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4*(b*c
 - a*d)^4)
```

Rubi in Sympy [A] time = 124.796, size = 202, normalized size = 0.94

$$-\frac{d^3}{4c^2\left(c+dx^2\right)^2(ad-bc)^2}-\frac{d^3(ad-2bc)}{c^3\left(c+dx^2\right)(ad-bc)^3}+\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(ad-bc)^4}+\frac{b^4}{2a^2\left(a+bx^2\right)(ad-bc)^3}-\frac{1}{2a^2c^3x^2}-\frac{b^4(5ad-2bc)\log\left(a+bx^2\right)}{2a^3(ad-bc)^4}-\frac{(3ad+2bc)\log\left(x^2\right)}{2a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -d**3/(4*c**2*(c + d*x**2)**2*(a*d - b*c)**2) - d**3*(a*d - 2*b*c
)/(c**3*(c + d*x**2)*(a*d - b*c)**3) + d**3*(3*a**2*d**2 - 10*a*b
*c*d + 10*b**2*c**2)*log(c + d*x**2)/(2*c**4*(a*d - b*c)**4) + b*
*4/(2*a**2*(a + b*x**2)*(a*d - b*c)**3) - 1/(2*a**2*c**3*x**2) -
b**4*(5*a*d - 2*b*c)*log(a + b*x**2)/(2*a**3*(a*d - b*c)**4) - (3
*a*d + 2*b*c)*log(x**2)/(2*a**3*c**4)
```

Mathematica [A] time = 0.62441, size = 208, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2b^4(2bc-5ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^4}-\frac{4\log(x)(3ad+2bc)}{a^3c^4}+\frac{2b^4}{a^2\left(a+bx^2\right)(ad-bc)^3}+\frac{2d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{c^4(bc-ad)^4}-\frac{2}{a^2c^3x^2}+\frac{4d^3(ad-2bc)}{c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3}{c^2\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a^2*c^3*x^2) + (2*b^4)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) -
d^3/(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (4*d^3*(-2*b*c + a*d))/(c
^3*(b*c - a*d)^3*(c + d*x^2)) - (4*(2*b*c + 3*a*d)*Log[x])/(a^3*c
^4) + (2*b^4*(2*b*c - 5*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^4)
+ (2*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(c
^4*(b*c - a*d)^4))/4
```

Maple [A] time = 0.04, size = 405, normalized size = 1.9

$$-\frac{1}{2\,a^2c^3x^2}-3\,\frac{\ln(x)\,d}{a^2c^4}-2\,\frac{b\ln(x)}{a^3c^3}-\frac{d^5a^2}{c^3\,(ad-bc)^4\,(dx^2+c)}+3\,\frac{d^4ab}{c^2\,(ad-bc)^4\,(dx^2+c)}-2\,\frac{b^2d^3}{c\,(ad-bc)^4\,(dx^2+c)}-\frac{d^5a^2}{4\,c^2\,(ad-bc)^4\,(dx^2+c)^2}+\frac{d^4ab}{2\,c\,(ad-bc)^4\,(dx^2+c)^2}-\frac{b^2d^3}{4\,(ad-bc)^4\,(dx^2+c)^2}+\frac{3\,d^5\ln\left(dx^2+c\right)a^2}{2\,c^4\,(ad-bc)^4}-5\,\frac{d^4\ln\left(dx^2+c\right)ab}{c^3\,(ad-bc)^4}+5\,\frac{d^3\ln\left(dx^2+c\right)b^2}{c^2\,(ad-bc)^4}-\frac{5\,b^4\ln\left(bx^2+a\right)d}{2\,a^2\,(ad-bc)^4}+\frac{b^5\ln\left(bx^2+a\right)c}{a^3\,(ad-bc)^4}+\frac{b^4d}{2\,a\,(ad-bc)^4\,(bx^2+a)}-\frac{b^5c}{2\,a^2\,(ad-bc)^4\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2/a^2/c^3/x^2-3/a^2/c^4*ln(x)*d-2/a^3/c^3*ln(x)*b-d^5/c^3/(a*d
-b*c)^4/(d*x^2+c)*a^2+3*d^4/c^2/(a*d-b*c)^4/(d*x^2+c)*a*b-2*d^3/c
/(a*d-b*c)^4/(d*x^2+c)*b^2-1/4*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*a^
2+1/2*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*a*b-1/4*d^3/(a*d-b*c)^4/(d*x^
2+c)^2*b^2+3/2*d^5/c^4/(a*d-b*c)^4*ln(d*x^2+c)*a^2-5*d^4/c^3/(a*d
-b*c)^4*ln(d*x^2+c)*a*b+5*d^3/c^2/(a*d-b*c)^4*ln(d*x^2+c)*b^2-5/2
*b^4/a^2/(a*d-b*c)^4*ln(b*x^2+a)*d+b^5/a^3/(a*d-b*c)^4*ln(b*x^2+a
)*c+1/2*b^4/a/(a*d-b*c)^4/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4/(b*
x^2+a)*c
```

Maxima [A] time = 1.40466, size = 879, normalized size = 4.09

$$\frac{\left(2\,b^5c-5\,ab^4d\right)\log\left(bx^2+a\right)}{2\,(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4)}+\frac{\left(10\,b^2c^2d^3-10\,abcd^4+3\,a^2d^5\right)\log\left(dx^2+c\right)}{2\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)}-\frac{2\,ab^3c^5-6\,a^2b^2c^4d+6\,a^3bc^3d^2-2\,a^4c^2d^3+2\left(2\,b^4c^3d^2-3\,ab^3c^2d^3+7\,a^2b^2cd^4-3\,a^3bd^5\right)x^6+\left(8\,b^4c^4d-10\,ab^3c^3d^2\right.}{4\left((a^2b^4c^6d^2-3\,a^3b^3c^5d^3+3\,a^4b^2c^4d^4-a^5bc^3d^5)x^8+(2\,a^2b^4c^7d-5\,a^3b^3c^6d^2+3\,a^4b^2c^5d^3+a^5bc^4d^4-a^6c^3d^5)x^6+(a^2b^4\right.}-\frac{(2\,bc+3\,ad)\log\left(x^2\right)}{2\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^5*c - 5*a*b^4*d)*log(b*x^2 + a)/(a^3*b^4*c^4 - 4*a^4*b^3
*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4) + 1/2*(10*b
^2*c^2*d^3 - 10*a*b*c*d^4 + 3*a^2*d^5)*log(d*x^2 + c)/(b^4*c^8 -
4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4
) - 1/4*(2*a*b^3*c^5 - 6*a^2*b^2*c^4*d + 6*a^3*b*c^3*d^2 - 2*a^4*
c^2*d^3 + 2*(2*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 7*a^2*b^2*c*d^4 -
3*a^3*b*d^5)*x^6 + (8*b^4*c^4*d - 10*a*b^3*c^3*d^2 + 15*a^2*b^2*c
^2*d^3 + 5*a^3*b*c*d^4 - 6*a^4*d^5)*x^4 + (4*b^4*c^5 - 2*a*b^3*c^
4*d - 6*a^2*b^2*c^3*d^2 + 19*a^3*b*c^2*d^3 - 9*a^4*c*d^4)*x^2)/((
a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c
^3*d^5)*x^8 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^
5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^6 + (a^2*b^4*c^8 - a^3*b^3
*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^4
 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3
)*x^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

Fricas [A] time = 29.6754, size = 1656, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b^4*c^7 - 8*a^3*b^3*c^6*d + 12*a^4*b^2*c^5*d^2 - 8*a^
5*b*c^4*d^3 + 2*a^6*c^3*d^4 + 2*(2*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*
d^3 + 10*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^5 + 3*a^5*b*c*d^6)*x^
6 + (8*a*b^5*c^6*d - 18*a^2*b^4*c^5*d^2 + 25*a^3*b^3*c^4*d^3 - 10
*a^4*b^2*c^3*d^4 - 11*a^5*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (4*a*b^5
*c^7 - 6*a^2*b^4*c^6*d - 4*a^3*b^3*c^5*d^2 + 25*a^4*b^2*c^4*d^3 -
 28*a^5*b*c^3*d^4 + 9*a^6*c^2*d^5)*x^2 - 2*((2*b^6*c^5*d^2 - 5*a*
b^5*c^4*d^3)*x^8 + (4*b^6*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4
*d^3)*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*c^5*d^2)*x^4 +
(2*a*b^5*c^7 - 5*a^2*b^4*c^6*d)*x^2)*log(b*x^2 + a) - 2*((10*a^3*
b^3*c^2*d^5 - 10*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (20*a^3*b^3*c
^3*d^4 - 10*a^4*b^2*c^2*d^5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (1
0*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a^5*b*c^2*d^5 + 6*a^6
*c*d^6)*x^4 + (10*a^4*b^2*c^4*d^3 - 10*a^5*b*c^3*d^4 + 3*a^6*c^2*
d^5)*x^2)*log(d*x^2 + c) + 4*((2*b^6*c^5*d^2 - 5*a*b^5*c^4*d^3 +
10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (4*b^6
*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4
 - 10*a^4*b^2*c^2*d^5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (2*b^6*c
^7 - a*b^5*c^6*d - 10*a^2*b^4*c^5*d^2 + 10*a^3*b^3*c^4*d^3 + 10*a
^4*b^2*c^3*d^4 - 17*a^5*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (2*a*b^5*c
^7 - 5*a^2*b^4*c^6*d + 10*a^4*b^2*c^4*d^3 - 10*a^5*b*c^3*d^4 + 3*
a^6*c^2*d^5)*x^2)*log(x))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 +
 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^8 + (2*
a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2
*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^6 + (a^3*b^5*c^10 - 2
*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*
c^6*d^4 + 2*a^8*c^5*d^5)*x^4 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d +
6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275722, size = 861, normalized size = 4.

$$\frac{\left(2\,b^6c-5\,ab^5d\right)\ln\left(\left|bx^2+a\right|\right)}{2\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)}+\frac{\left(10\,b^2c^2d^4-10\,abcd^5+3\,a^2d^6\right)\ln\left(\left|dx^2+c\right|\right)}{2\,(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5)}+\frac{10\,a^2b^3c^2d^3x^4-10\,a^3b^2cd^4x^4+3\,a^4bd^5x^4-4\,b^5c^5x^2+10\,ab^4c^4dx^2-12\,a^2b^3c^3d^2x^2+18\,a^3b^2c^2d^3x^2-12\,a^4bcd^4x^2+3\,a^5}{4\,(a^2b^4c^8-4\,a^3b^3c^7d+6\,a^4b^2c^6d^2-4\,a^5bc^5d^3+a^6c^4d^4)(bx^4+}-\frac{30\,b^2c^2d^5x^4-30\,abcd^6x^4+9\,a^2d^7x^4+68\,b^2c^3d^4x^2-72\,abc^2d^5x^2+22\,a^2cd^6x^2+39\,b^2c^4d^3-44\,abc^3d^4+14\,a^2c^2d^5}{4\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)(dx^2+c)^2}-\frac{(2\,bc+3\,ad)\ln\left(x^2\right)}{2\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^6*c - 5*a*b^5*d)*ln(abs(b*x^2 + a))/(a^3*b^5*c^4 - 4*a^4
*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4) + 1
/2*(10*b^2*c^2*d^4 - 10*a*b*c*d^5 + 3*a^2*d^6)*ln(abs(d*x^2 + c))
/(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d
^4 + a^4*c^4*d^5) + 1/4*(10*a^2*b^3*c^2*d^3*x^4 - 10*a^3*b^2*c*d^
4*x^4 + 3*a^4*b*d^5*x^4 - 4*b^5*c^5*x^2 + 10*a*b^4*c^4*d*x^2 - 12
*a^2*b^3*c^3*d^2*x^2 + 18*a^3*b^2*c^2*d^3*x^2 - 12*a^4*b*c*d^4*x^
2 + 3*a^5*d^5*x^2 - 2*a*b^4*c^5 + 8*a^2*b^3*c^4*d - 12*a^3*b^2*c^
3*d^2 + 8*a^4*b*c^2*d^3 - 2*a^5*c*d^4)/((a^2*b^4*c^8 - 4*a^3*b^3*
c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4)*(b*x^4
 + a*x^2)) - 1/4*(30*b^2*c^2*d^5*x^4 - 30*a*b*c*d^6*x^4 + 9*a^2*d
^7*x^4 + 68*b^2*c^3*d^4*x^2 - 72*a*b*c^2*d^5*x^2 + 22*a^2*c*d^6*x
^2 + 39*b^2*c^4*d^3 - 44*a*b*c^3*d^4 + 14*a^2*c^2*d^5)/((b^4*c^8
- 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d
^4)*(d*x^2 + c)^2) - 1/2*(2*b*c + 3*a*d)*ln(x^2)/(a^3*c^4)
```

3.318 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=377

$$\frac{b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^4}+\frac{d\left(-7a^2d^2+15abcd+4b^2c^2\right)}{8ac^2x^3\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^4}-\frac{-35a^3d^3+75a^2bcd^2-24ab^2c^2d+20b^3c^3}{24a^2c^3x^3(bc-ad)^3}+\frac{-35a^4d^4+75a^3bcd^3-24a^2b^2c^2d^2-24ab^3c^3d+20b^4c^4}{8a^3c^4x(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx^3\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] -(20*b^3*c^3 - 24*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 35*a^3*d^3)/(24*
a^2*c^3*(b*c - a*d)^3*x^3) + (20*b^4*c^4 - 24*a*b^3*c^3*d - 24*a^
2*b^2*c^2*d^2 + 75*a^3*b*c*d^3 - 35*a^4*d^4)/(8*a^3*c^4*(b*c - a*
d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^
2) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^
2*c^2 + 15*a*b*c*d - 7*a^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x^3*(c +
d*x^2)) + (b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(7/2)*(b*c - a*d)^4) + (d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d +
35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^4
)
```

Rubi [A] time = 1.78343, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^4}+\frac{d\left(-7a^2d^2+15abcd+4b^2c^2\right)}{8ac^2x^3\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^4}-\frac{-35a^3d^3+75a^2bcd^2-24ab^2c^2d+20b^3c^3}{24a^2c^3x^3(bc-ad)^3}+\frac{-35a^4d^4+75a^3bcd^3-24a^2b^2c^2d^2-24ab^3c^3d+20b^4c^4}{8a^3c^4x(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx^3\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(20*b^3*c^3 - 24*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 35*a^3*d^3)/(24*
a^2*c^3*(b*c - a*d)^3*x^3) + (20*b^4*c^4 - 24*a*b^3*c^3*d - 24*a^
2*b^2*c^2*d^2 + 75*a^3*b*c*d^3 - 35*a^4*d^4)/(8*a^3*c^4*(b*c - a*
d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^
2) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^
2*c^2 + 15*a*b*c*d - 7*a^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x^3*(c +
d*x^2)) + (b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(7/2)*(b*c - a*d)^4) + (d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d +
35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^4
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.05648, size = 230, normalized size = 0.61

$$\frac{1}{24}\left(\frac{12b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)^4}-\frac{12b^5x}{a^3\left(a+bx^2\right)(ad-bc)^3}+\frac{72ad+48bc}{a^3c^4x}+\frac{3d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{9/2}(bc-ad)^4}-\frac{8}{a^2c^3x^3}+\frac{3d^4x(19bc-11ad)}{c^4\left(c+dx^2\right)(bc-ad)^3}+\frac{6d^4x}{c^3\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x^3) + (48*b*c + 72*a*d)/(a^3*c^4*x) - (12*b^5*x)/(a
^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (6*d^4*x)/(c^3*(b*c - a*d)^2*(
c + d*x^2)^2) + (3*d^4*(19*b*c - 11*a*d)*x)/(c^4*(b*c - a*d)^3*(c
 + d*x^2)) + (12*b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(7/2)*(b*c - a*d)^4) + (3*d^(7/2)*(99*b^2*c^2 - 110*a*b*
c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(9/2)*(b*c - a*
d)^4))/24
```

Maple [A] time = 0.035, size = 455, normalized size = 1.2

$$-\frac{1}{3a^2c^3x^3}+3\frac{d}{a^2xc^4}+2\frac{b}{xa^3c^3}+\frac{11d^7x^3a^2}{8c^4(ad-bc)^4(dx^2+c)^2}-\frac{15d^6x^3ab}{4c^3(ad-bc)^4(dx^2+c)^2}+\frac{19d^5x^3b^2}{8c^2(ad-bc)^4(dx^2+c)^2}+\frac{13d^6xa^2}{8c^3(ad-bc)^4(dx^2+c)^2}-\frac{17d^5xab}{4c^2(ad-bc)^4(dx^2+c)^2}+\frac{21d^4xb^2}{8c(ad-bc)^4(dx^2+c)^2}+\frac{35d^6a^2}{8c^4(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{55d^5ab}{4c^3(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{99d^4b^2}{8c^2(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^5xd}{2a^2(ad-bc)^4(bx^2+a)}+\frac{b^6xc}{2a^3(ad-bc)^4(bx^2+a)}-\frac{11b^5d}{2a^2(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5b^6c}{2a^3(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/3/a^2/c^3/x^3+3/x/a^2/c^4*d+2/x/a^3/c^3*b+11/8*d^7/c^4/(a*d-b*
c)^4/(d*x^2+c)^2*x^3*a^2-15/4*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3
*a*b+19/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2+13/8*d^6/c^3/(a
*d-b*c)^4/(d*x^2+c)^2*x*a^2-17/4*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*
x*a*b+21/8*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x*b^2+35/8*d^6/c^4/(a*d-
b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-55/4*d^5/c^3/(a*d-
b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+99/8*d^4/c^2/(a*d-
b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/2*b^5/a^2/(a*d-b
*c)^4*x/(b*x^2+a)*d+1/2*b^6/a^3/(a*d-b*c)^4*x/(b*x^2+a)*c-11/2*b^
5/a^2/(a*d-b*c)^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+5/2*b^6/a
^3/(a*d-b*c)^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 20.7274, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^4),x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 -
64*a^5*b*c^4*d^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*
c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*
x^8 - 2*(120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 +
 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 10
5*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5
*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*
d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2
*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*
c^2*d^5)*x^2 + 12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b
^6*c^6*d - 17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^
7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2
*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*
b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*
b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*
d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3
 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c)*log((d*x^2
 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^4*
b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d
^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^
3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3
*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d
^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a^5
*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x
^3), -1/24*(8*a^2*b^4*c^7 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*c^5*d^2
 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4 - 3*(20*b^6*c^5*d^2 - 44*a*b^
5*c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7
)*x^8 - (120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 +
 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 10
5*a^6*d^7)*x^6 - (60*b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5*d
^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*d^
5 + 175*a^6*c*d^6)*x^4 - 8*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^
3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2
*d^5)*x^2 - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b
*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b
*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d
^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3
- 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(d/c)*arctan(d*x/(
c*sqrt(d/c))) + 6*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b
^6*c^6*d - 17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^
7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2
*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6
*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d
- 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a
^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d
 - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^
```

```
8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*
d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/48*(16*a^2*b^4*c^7
- 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c^4*d^3 + 16*a
^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^
2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d
- 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 -
253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*
b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*
d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^
4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a
^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 24*((5*b
^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5
*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^
2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b
/a)*arctan(b*x/(a*sqrt(b/a))) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*
b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^
2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^
3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 +
(99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sq
rt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b
^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^
5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2
 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*
c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^
2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a
^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d
^3 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7 - 32*a^3*b^3*c^6*d +
 48*a^4*b^2*c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4 - 3*(20*b^
6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c
*d^6 + 35*a^5*b*d^7)*x^8 - (120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 8
8*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 1
55*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - (60*b^6*c^7 - 52*a*b^5*c^6*d
- 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4
 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 8*(5*a*b^5*c^7 - 13*a
^2*b^4*c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*
c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*
d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)
*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5 + (5*a*
b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a
))) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*
x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6
 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 1
85*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*
a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(d/c)*arctan(d*x/(c*sqrt
(d/c))))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^
4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7
*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*
b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d -
2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c
^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2
 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.247669, size = 495, normalized size = 1.31

$$\frac{b^5 x}{2\,(a^3 b^3 c^3 - 3\,a^4 b^2 c^2 d + 3\,a^5 b c d^2 - a^6 d^3)(b x^2 + a)} + \frac{\left(5\,b^6 c - 11\,a b^5 d\right)\arctan\left(\frac{b x}{\sqrt{ab}}\right)}{2\,(a^3 b^4 c^4 - 4\,a^4 b^3 c^3 d + 6\,a^5 b^2 c^2 d^2 - 4\,a^6 b c d^3 + a^7 d^4)\sqrt{ab}} + \frac{\left(99\,b^2 c^2 d^4 - 110\,a b c d^5 + 35\,a^2 d^6\right)\arctan\left(\frac{d x}{\sqrt{cd}}\right)}{8\,(b^4 c^8 - 4\,a b^3 c^7 d + 6\,a^2 b^2 c^6 d^2 - 4\,a^3 b c^5 d^3 + a^4 c^4 d^4)\sqrt{cd}} + \frac{19\,b c d^5 x^3 - 11\,a d^6 x^3 + 21\,b c^2 d^4 x - 13\,a c d^5 x}{8\,(b^3 c^7 - 3\,a b^2 c^6 d + 3\,a^2 b c^5 d^2 - a^3 c^4 d^3)(d x^2 + c)^2} + \frac{6\,b c x^2 + 9\,a d x^2 - a c}{3\,a^3 c^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^4),x, algorithm="giac")
```

```
[Out] 1/2*b^5*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d
^3)*(b*x^2 + a)) + 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b
))/((a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4)*sqrt(a*b)) + 1/8*(99*b^2*c^2*d^4 - 110*a*b*c*d^5
 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b^4*c^8 - 4*a*b^3*c^7*d +
6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(c*d)) + 1
/8*(19*b*c*d^5*x^3 - 11*a*d^6*x^3 + 21*b*c^2*d^4*x - 13*a*c*d^5*x
)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x
^2 + c)^2) + 1/3*(6*b*c*x^2 + 9*a*d*x^2 - a*c)/(a^3*c^4*x^3)
```

3.319 $\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right)\,dx$

Optimal. Leaf size=96

$$\frac{a^3Ax^{m+1}}{m+1}+\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

```
[Out] (a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m)
 + (3*a*b*(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(
7 + m))/(7 + m) + (b^3*B*x^(9 + m))/(9 + m)
```

Rubi [A] time = 0.157444, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{a^3Ax^{m+1}}{m+1}+\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m)
 + (3*a*b*(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(
7 + m))/(7 + m) + (b^3*B*x^(9 + m))/(9 + m)
```

Rubi in Sympy [A] time = 20.949, size = 87, normalized size = 0.91

$$\frac{Aa^3x^{m+1}}{m+1}+\frac{Bb^3x^{m+9}}{m+9}+\frac{a^2x^{m+3}\left(3Ab+Ba\right)}{m+3}+\frac{3abx^{m+5}\left(Ab+Ba\right)}{m+5}+\frac{b^2x^{m+7}\left(Ab+3Ba\right)}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] A*a**3*x**(m + 1)/(m + 1) + B*b**3*x**(m + 9)/(m + 9) + a**2*x**(
m + 3)*(3*A*b + B*a)/(m + 3) + 3*a*b*x**(m + 5)*(A*b + B*a)/(m +
5) + b**2*x**(m + 7)*(A*b + 3*B*a)/(m + 7)
```

Mathematica [A] time = 0.119591, size = 88, normalized size = 0.92

$$x^m\left(\frac{a^3Ax}{m+1}+\frac{a^2x^3(aB+3Ab)}{m+3}+\frac{b^2x^7(3aB+Ab)}{m+7}+\frac{3abx^5(aB+Ab)}{m+5}+\frac{b^3Bx^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] x^m*((a^3*A*x)/(1 + m) + (a^2*(3*A*b + a*B)*x^3)/(3 + m) + (3*a*b
*(A*b + a*B)*x^5)/(5 + m) + (b^2*(A*b + 3*a*B)*x^7)/(7 + m) + (b^
3*B*x^9)/(9 + m))
```

Maple [B] time = 0.01, size = 474, normalized size = 4.9

$$x^{1+m}\left(Bb^3m^4x^8+16\,Bb^3m^3x^8+Ab^3m^4x^6+3\,Bab^2m^4x^6+86\,Bb^3m^2x^8+18\,Ab^3m^3x^6+54\,Bab^2m^3x^6+176\,Bb^3mx^8+3\,Aab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b^3*m^4*x^8+16*B*b^3*m^3*x^8+A*b^3*m^4*x^6+3*B*a*b^2*m
^4*x^6+86*B*b^3*m^2*x^8+18*A*b^3*m^3*x^6+54*B*a*b^2*m^3*x^6+176*B
*b^3*m*x^8+3*A*a*b^2*m^4*x^4+104*A*b^3*m^2*x^6+3*B*a^2*b*m^4*x^4+
312*B*a*b^2*m^2*x^6+105*B*b^3*x^8+60*A*a*b^2*m^3*x^4+222*A*b^3*m*
x^6+60*B*a^2*b*m^3*x^4+666*B*a*b^2*m*x^6+3*A*a^2*b*m^4*x^2+390*A*
a*b^2*m^2*x^4+135*A*b^3*x^6+B*a^3*m^4*x^2+390*B*a^2*b*m^2*x^4+405
*B*a*b^2*x^6+66*A*a^2*b*m^3*x^2+900*A*a*b^2*m*x^4+22*B*a^3*m^3*x^
2+900*B*a^2*b*m*x^4+A*a^3*m^4+492*A*a^2*b*m^2*x^2+567*A*a*b^2*x^4
+164*B*a^3*m^2*x^2+567*B*a^2*b*x^4+24*A*a^3*m^3+1374*A*a^2*b*m*x^
2+458*B*a^3*m*x^2+206*A*a^3*m^2+945*A*a^2*b*x^2+315*B*a^3*x^2+744
*A*a^3*m+945*A*a^3)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237212, size = 512, normalized size = 5.33

$$\frac{\left(\left(Bb^3m^4 + 16\,Bb^3m^3 + 86\,Bb^3m^2 + 176\,Bb^3m + 105\,Bb^3\right)x^9 + \left(\left(3\,Bab^2 + Ab^3\right)m^4 + 405\,Bab^2 + 135\,Ab^3 + 18\,\left(3\,Bab^2 + Ab^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^m,x, algorithm="fricas")
```

```
[Out] ((B*b^3*m^4 + 16*B*b^3*m^3 + 86*B*b^3*m^2 + 176*B*b^3*m + 105*B*b
^3)*x^9 + ((3*B*a*b^2 + A*b^3)*m^4 + 405*B*a*b^2 + 135*A*b^3 + 18
*(3*B*a*b^2 + A*b^3)*m^3 + 104*(3*B*a*b^2 + A*b^3)*m^2 + 222*(3*B
*a*b^2 + A*b^3)*m)*x^7 + 3*((B*a^2*b + A*a*b^2)*m^4 + 189*B*a^2*b
 + 189*A*a*b^2 + 20*(B*a^2*b + A*a*b^2)*m^3 + 130*(B*a^2*b + A*a*
b^2)*m^2 + 300*(B*a^2*b + A*a*b^2)*m)*x^5 + ((B*a^3 + 3*A*a^2*b)*
m^4 + 315*B*a^3 + 945*A*a^2*b + 22*(B*a^3 + 3*A*a^2*b)*m^3 + 164*
(B*a^3 + 3*A*a^2*b)*m^2 + 458*(B*a^3 + 3*A*a^2*b)*m)*x^3 + (A*a^3
*m^4 + 24*A*a^3*m^3 + 206*A*a^3*m^2 + 744*A*a^3*m + 945*A*a^3)*x)
*x^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 9.57639, size = 2069, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**3/(8*x**8) - A*a**2*b/(2*x**6) - 3*A*a*b**2/(4*x
**4) - A*b**3/(2*x**2) - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) -
3*B*a*b**2/(2*x**2) + B*b**3*log(x), Eq(m, -9)), (-A*a**3/(6*x**6
) - 3*A*a**2*b/(4*x**4) - 3*A*a*b**2/(2*x**2) + A*b**3*log(x) - B
*a**3/(4*x**4) - 3*B*a**2*b/(2*x**2) + 3*B*a*b**2*log(x) + B*b**3
```

```
*x**2/2, Eq(m, -7)), (-A*a**3/(4*x**4) - 3*A*a**2*b/(2*x**2) + 3*
A*a*b**2*log(x) + A*b**3*x**2/2 - B*a**3/(2*x**2) + 3*B*a**2*b*lo
g(x) + 3*B*a*b**2*x**2/2 + B*b**3*x**4/4, Eq(m, -5)), (-A*a**3/(2
*x**2) + 3*A*a**2*b*log(x) + 3*A*a*b**2*x**2/2 + A*b**3*x**4/4 +
B*a**3*log(x) + 3*B*a**2*b*x**2/2 + 3*B*a*b**2*x**4/4 + B*b**3*x*
*6/6, Eq(m, -3)), (A*a**3*log(x) + 3*A*a**2*b*x**2/2 + 3*A*a*b**2
*x**4/4 + A*b**3*x**6/6 + B*a**3*x**2/2 + 3*B*a**2*b*x**4/4 + B*a
*b**2*x**6/2 + B*b**3*x**8/8, Eq(m, -1)), (A*a**3*m**4*x*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24*A*a**3*m*
*3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 206*A*a**3*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*A*a**3*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 945*A*a**3*x*x**m/(m**5 + 25*m**4 + 230
*m**3 + 950*m**2 + 1689*m + 945) + 3*A*a**2*b*m**4*x**3*x**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 66*A*a**2*b*m
**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 492*A*a**2*b*m**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 1374*A*a**2*b*m*x**3*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*A*a**2*b*x**3*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3*A*a*b**2
*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
945) + 60*A*a*b**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 390*A*a*b**2*m**2*x**5*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*A*a*b**2*m*x**5*x
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 567*A
*a*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + A*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 18*A*b**3*m**3*x**7*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + 104*A*b**3*m**2*x**7*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 222*A*b**3
*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 135*A*b**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + B*a**3*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 22*B*a**3*m**3*x**3*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 164*B*a**3*m**2*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 458
*B*a**3*m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*
m + 945) + 315*B*a**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 3*B*a**2*b*m**4*x**5*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 60*B*a**2*b*m**3*x**5*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*B*a
**2*b*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 900*B*a**2*b*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 567*B*a**2*b*x**5*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3*B*a*b**2*m**4*x**7*x*
*m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 54*B*a
*b**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 312*B*a*b**2*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 666*B*a*b**2*m*x**7*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 405*B*a*b**2*x**7*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + B*b*
*3*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 16*B*b**3*m**3*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 86*B*b**3*m**2*x**9*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*B*b**3*m*x**9*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*B*b**3
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945),
 True))
```

GIAC/XCAS [A] time = 0.271332, size = 909, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^m,x, algorithm="giac")
```

```
[Out] (B*b^3*m^4*x^9*e^(m*ln(x)) + 16*B*b^3*m^3*x^9*e^(m*ln(x)) + 3*B*a
*b^2*m^4*x^7*e^(m*ln(x)) + A*b^3*m^4*x^7*e^(m*ln(x)) + 86*B*b^3*m
^2*x^9*e^(m*ln(x)) + 54*B*a*b^2*m^3*x^7*e^(m*ln(x)) + 18*A*b^3*m^
```

3*x^7*e^(m*ln(x)) + 176*B*b^3*m*x^9*e^(m*ln(x)) + 3*B*a^2*b*m^4*x^5*e^(m*ln(x)) + 3*A*a*b^2*m^4*x^5*e^(m*ln(x)) + 312*B*a*b^2*m^2*x^7*e^(m*ln(x)) + 104*A*b^3*m^2*x^7*e^(m*ln(x)) + 105*B*b^3*x^9*e^(m*ln(x)) + 60*B*a^2*b*m^3*x^5*e^(m*ln(x)) + 60*A*a*b^2*m^3*x^5*e^(m*ln(x)) + 666*B*a*b^2*m*x^7*e^(m*ln(x)) + 222*A*b^3*m*x^7*e^(m*ln(x)) + B*a^3*m^4*x^3*e^(m*ln(x)) + 3*A*a^2*b*m^4*x^3*e^(m*ln(x)) + 390*B*a^2*b*m^2*x^5*e^(m*ln(x)) + 390*A*a*b^2*m^2*x^5*e^(m*ln(x)) + 405*B*a*b^2*x^7*e^(m*ln(x)) + 135*A*b^3*x^7*e^(m*ln(x)) + 22*B*a^3*m^3*x^3*e^(m*ln(x)) + 66*A*a^2*b*m^3*x^3*e^(m*ln(x)) + 900*B*a^2*b*m*x^5*e^(m*ln(x)) + 900*A*a*b^2*m*x^5*e^(m*ln(x)) + A*a^3*m^4*x*e^(m*ln(x)) + 164*B*a^3*m^2*x^3*e^(m*ln(x)) + 492*A*a^2*b*m^2*x^3*e^(m*ln(x)) + 567*B*a^2*b*x^5*e^(m*ln(x)) + 567*A*a*b^2*x^5*e^(m*ln(x)) + 24*A*a^3*m^3*x*e^(m*ln(x)) + 458*B*a^3*m*x^3*e^(m*ln(x)) + 1374*A*a^2*b*m*x^3*e^(m*ln(x)) + 206*A*a^3*m^2*x*e^(m*ln(x)) + 315*B*a^3*x^3*e^(m*ln(x)) + 945*A*a^2*b*x^3*e^(m*ln(x)) + 744*A*a^3*m*x*e^(m*ln(x)) + 945*A*a^3*x*e^(m*ln(x)))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)

3.320 $\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=71

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) +
 (b*(A*b + 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

Rubi [A] time = 0.11568, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) +
 (b*(A*b + 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

Rubi in Sympy [A] time = 15.9866, size = 63, normalized size = 0.89

$$\frac{Aa^2x^{m+1}}{m+1}+\frac{Bb^2x^{m+7}}{m+7}+\frac{ax^{m+3}\left(2Ab+Ba\right)}{m+3}+\frac{bx^{m+5}\left(Ab+2Ba\right)}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**(m + 1)/(m + 1) + B*b**2*x**(m + 7)/(m + 7) + a*x**(m +
 3)*(2*A*b + B*a)/(m + 3) + b*x**(m + 5)*(A*b + 2*B*a)/(m + 5)
```

Mathematica [A] time = 0.0755131, size = 65, normalized size = 0.92

$$x^m\left(\frac{a^2Ax}{m+1}+\frac{bx^5(2aB+Ab)}{m+5}+\frac{ax^3(aB+2Ab)}{m+3}+\frac{b^2Bx^7}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] x^m*((a^2*A*x)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(3 + m) + (b*(A*b
+ 2*a*B)*x^5)/(5 + m) + (b^2*B*x^7)/(7 + m))
```

Maple [B] time = 0.009, size = 262, normalized size = 3.7

$$x^{1+m}\left(Bb^2m^3x^6+9\,Bb^2m^2x^6+Ab^2m^3x^4+2\,Babm^3x^4+23\,Bb^2mx^6+11\,Ab^2m^2x^4+22\,Babm^2x^4+15\,b^2Bx^6+2\,Aabm^3x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b^2*m^3*x^6+9*B*b^2*m^2*x^6+A*b^2*m^3*x^4+2*B*a*b*m^3*
x^4+23*B*b^2*m*x^6+11*A*b^2*m^2*x^4+22*B*a*b*m^2*x^4+15*B*b^2*x^6
+2*A*a*b*m^3*x^2+31*A*b^2*m*x^4+B*a^2*m^3*x^2+62*B*a*b*m*x^4+26*A
*a*b*m^2*x^2+21*A*b^2*x^4+13*B*a^2*m^2*x^2+42*B*a*b*x^4+A*a^2*m^3
+94*A*a*b*m*x^2+47*B*a^2*m*x^2+15*A*a^2*m^2+70*A*a*b*x^2+35*B*a^2
*x^2+71*A*a^2*m+105*A*a^2)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23495, size = 290, normalized size = 4.08

$$\frac{\left(\left(Bb^2m^3 + 9\,Bb^2m^2 + 23\,Bb^2m + 15\,Bb^2\right)x^7 + \left(\left(2\,Bab + Ab^2\right)m^3 + 42\,Bab + 21\,Ab^2 + 11\left(2\,Bab + Ab^2\right)m^2 + 31\left(2\,Bab + Ab^2 \right.\right.\right.}{m^4 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^m,x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 9*B*b^2*m^2 + 23*B*b^2*m + 15*B*b^2)*x^7 + ((2*B*a*
b + A*b^2)*m^3 + 42*B*a*b + 21*A*b^2 + 11*(2*B*a*b + A*b^2)*m^2 +
 31*(2*B*a*b + A*b^2)*m)*x^5 + ((B*a^2 + 2*A*a*b)*m^3 + 35*B*a^2
+ 70*A*a*b + 13*(B*a^2 + 2*A*a*b)*m^2 + 47*(B*a^2 + 2*A*a*b)*m)*x
^3 + (A*a^2*m^3 + 15*A*a^2*m^2 + 71*A*a^2*m + 105*A*a^2)*x)*x^m/(
m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [A] time = 5.53462, size = 1044, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2) -
B*a**2/(4*x**4) - B*a*b/x**2 + B*b**2*log(x), Eq(m, -7)), (-A*a**
2/(4*x**4) - A*a*b/x**2 + A*b**2*log(x) - B*a**2/(2*x**2) + 2*B*a
*b*log(x) + B*b**2*x**2/2, Eq(m, -5)), (-A*a**2/(2*x**2) + 2*A*a*
b*log(x) + A*b**2*x**2/2 + B*a**2*log(x) + B*a*b*x**2 + B*b**2*x*
*4/4, Eq(m, -3)), (A*a**2*log(x) + A*a*b*x**2 + A*b**2*x**4/4 + B
*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2*x**6/6, Eq(m, -1)), (A*a**2*
m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*a**2*
m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*A*a**2*
m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*A*a**2*x*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*A*a*b*m**3*x**3
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*A*a*b*m**2*x*
*3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*A*a*b*m*x**
3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*A*a*b*x**3*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*b**2*m**3*x**5*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*b**2*m**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*A*b**2*m*x**
```

```
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*A*b**2*x**5*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + B*a**2*m**3*x**3*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*B*a**2*m**2*x*
*3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*B*a**2*m*x*
*3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*B*a**2*x**3
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*B*a*b*m**3*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 22*B*a*b*m**2*x
**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 62*B*a*b*m*x*
*5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 42*B*a*b*x**5*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + B*b**2*m**3*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*B*b**2*m**2*x**
7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*B*b**2*m*x**
7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*B*b**2*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

GIAC/XCAS [A] time = 0.278326, size = 513, normalized size = 7.23

$$\frac{Bb^2m^3x^7e^{(m\ln(x))} + 9\,Bb^2m^2x^7e^{(m\ln(x))} + 2\,Babm^3x^5e^{(m\ln(x))} + Ab^2m^3x^5e^{(m\ln(x))} + 23\,Bb^2mx^7e^{(m\ln(x))} + 22\,Babm^2x^5e^{(m\ln(x))} \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^m,x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^7*e^(m*ln(x)) + 9*B*b^2*m^2*x^7*e^(m*ln(x)) + 2*B*a*
b*m^3*x^5*e^(m*ln(x)) + A*b^2*m^3*x^5*e^(m*ln(x)) + 23*B*b^2*m*x^
7*e^(m*ln(x)) + 22*B*a*b*m^2*x^5*e^(m*ln(x)) + 11*A*b^2*m^2*x^5*e
^(m*ln(x)) + 15*B*b^2*x^7*e^(m*ln(x)) + B*a^2*m^3*x^3*e^(m*ln(x))
 + 2*A*a*b*m^3*x^3*e^(m*ln(x)) + 62*B*a*b*m*x^5*e^(m*ln(x)) + 31*
A*b^2*m*x^5*e^(m*ln(x)) + 13*B*a^2*m^2*x^3*e^(m*ln(x)) + 26*A*a*b
*m^2*x^3*e^(m*ln(x)) + 42*B*a*b*x^5*e^(m*ln(x)) + 21*A*b^2*x^5*e^
(m*ln(x)) + A*a^2*m^3*x*e^(m*ln(x)) + 47*B*a^2*m*x^3*e^(m*ln(x))
+ 94*A*a*b*m*x^3*e^(m*ln(x)) + 15*A*a^2*m^2*x*e^(m*ln(x)) + 35*B*
a^2*x^3*e^(m*ln(x)) + 70*A*a*b*x^3*e^(m*ln(x)) + 71*A*a^2*m*x*e^(
m*ln(x)) + 105*A*a^2*x*e^(m*ln(x)))/(m^4 + 16*m^3 + 86*m^2 + 176*
m + 105)
```

3.321 $\int x^m (a + bx^2) (A + Bx^2) \, dx$

Optimal. Leaf size=45

$$\frac{x^{m+3}(aB + Ab)}{m + 3} + \frac{aAx^{m+1}}{m + 1} + \frac{bBx^{m+5}}{m + 5}$$

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*
x^(5 + m))/(5 + m)
```

Rubi [A] time = 0.0614579, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{x^{m+3}(aB + Ab)}{m + 3} + \frac{aAx^{m+1}}{m + 1} + \frac{bBx^{m+5}}{m + 5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*
x^(5 + m))/(5 + m)
```

Rubi in Sympy [A] time = 9.42324, size = 37, normalized size = 0.82

$$\frac{Aax^{m+1}}{m + 1} + \frac{Bbx^{m+5}}{m + 5} + \frac{x^{m+3} (Ab + Ba)}{m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**(m + 1)/(m + 1) + B*b*x**(m + 5)/(m + 5) + x**(m + 3)*(A*b
 + B*a)/(m + 3)
```

Mathematica [A] time = 0.04053, size = 41, normalized size = 0.91

$$x^m \left(\frac{x^3(aB + Ab)}{m + 3} + \frac{aAx}{m + 1} + \frac{bBx^5}{m + 5} \right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] x^m*((a*A*x)/(1 + m) + ((A*b + a*B)*x^3)/(3 + m) + (b*B*x^5)/(5 +
 m))
```

Maple [B] time = 0.005, size = 110, normalized size = 2.4

$$\frac{x^{1+m} \left(Bbm^2x^4 + 4\,Bbmx^4 + Abm^2x^2 + Bam^2x^2 + 3\,bBx^4 + 6\,Abmx^2 + 6\,Bamx^2 + Aam^2 + 5\,Ax^2b + 5\,Bx^2a + 8\,Aam + 15\,Aa \right)}{(5 + m)(3 + m)(1 + m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b*m^2*x^4+4*B*b*m*x^4+A*b*m^2*x^2+B*a*m^2*x^2+3*B*b*x^
4+6*A*b*m*x^2+6*B*a*m*x^2+A*a*m^2+5*A*b*x^2+5*B*a*x^2+8*A*a*m+15*
A*a)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234854, size = 124, normalized size = 2.76

$$\frac{\left(\left(Bbm^2 + 4\,Bbm + 3\,Bb\right)x^5 + \left((Ba + Ab)m^2 + 5\,Ba + 5\,Ab + 6\,(Ba + Ab)m\right)x^3 + \left(Aam^2 + 8\,Aam + 15\,Aa\right)x\right)x^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^m,x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 4*B*b*m + 3*B*b)*x^5 + ((B*a + A*b)*m^2 + 5*B*a + 5*A
*b + 6*(B*a + A*b)*m)*x^3 + (A*a*m^2 + 8*A*a*m + 15*A*a)*x)*x^m/(
m^3 + 9*m^2 + 23*m + 15)
```

Sympy [A] time = 2.86559, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{4x^4} - \frac{Ab}{2x^2} - \frac{Ba}{2x^2} + Bb\log(x) \\ -\frac{Aa}{2x^2} + Ab\log(x) + Ba\log(x) + \frac{Bbx^2}{2} \\ Aa\log(x) + \frac{Abx^2}{2} + \frac{Bax^2}{2} + \frac{Bbx^4}{4} \\ \frac{Aam^2xx^m}{m^3+9m^2+23m+15} + \frac{8Aamxx^m}{m^3+9m^2+23m+15} + \frac{15Aaxx^m}{m^3+9m^2+23m+15} + \frac{Abm^2x^3x^m}{m^3+9m^2+23m+15} + \frac{6Abmx^3x^m}{m^3+9m^2+23m+15} + \frac{5Abx^3x^m}{m^3+9m^2+23m+15} + \frac{Bam^2x^3x^m}{m^3+9m^2+23m+15} + \cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a/(4*x**4) - A*b/(2*x**2) - B*a/(2*x**2) + B*b*log(
x), Eq(m, -5)), (-A*a/(2*x**2) + A*b*log(x) + B*a*log(x) + B*b*x*
*2/2, Eq(m, -3)), (A*a*log(x) + A*b*x**2/2 + B*a*x**2/2 + B*b*x**
4/4, Eq(m, -1)), (A*a*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8
*A*a*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*A*a*x*x**m/(m**3 +
 9*m**2 + 23*m + 15) + A*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m +
 15) + 6*A*b*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*A*b*x**3
*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*a*m**2*x**3*x**m/(m**3 + 9*
m**2 + 23*m + 15) + 6*B*a*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15)
 + 5*B*a*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*b*m**2*x**5*x*
*m/(m**3 + 9*m**2 + 23*m + 15) + 4*B*b*m*x**5*x**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*B*b*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15), True
))
```

GIAC/XCAS [A] time = 0.307916, size = 225, normalized size = 5.

$$\frac{Bbm^2x^5e^{(m\ln(x))} + 4\,Bbmx^5e^{(m\ln(x))} + Bam^2x^3e^{(m\ln(x))} + Abm^2x^3e^{(m\ln(x))} + 3\,Bbx^5e^{(m\ln(x))} + 6\,Bamx^3e^{(m\ln(x))} + 6\,Abmx^3e^{(m\ln(x))} \cdots}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^m,x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^5*e^(m*ln(x)) + 4*B*b*m*x^5*e^(m*ln(x)) + B*a*m^2*x^3*
e^(m*ln(x)) + A*b*m^2*x^3*e^(m*ln(x)) + 3*B*b*x^5*e^(m*ln(x)) + 6
*B*a*m*x^3*e^(m*ln(x)) + 6*A*b*m*x^3*e^(m*ln(x)) + A*a*m^2*x*e^(m
*ln(x)) + 5*B*a*x^3*e^(m*ln(x)) + 5*A*b*x^3*e^(m*ln(x)) + 8*A*a*m
*x*e^(m*ln(x)) + 15*A*a*x*e^(m*ln(x)))/(m^3 + 9*m^2 + 23*m + 15)
```

3.322 $\int \frac{x^m (A+Bx^2)}{a+bx^2}\, dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rubi [A] time = 0.0961959, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rubi in Sympy [A] time = 12.6802, size = 49, normalized size = 0.74

$$\frac{Bx^{m+1}}{b\,(m+1)}+\frac{x^{m+1}\,(Ab-Ba)\,{}_2F_1\left(\begin{matrix}1,\ \frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ab\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**(m + 1)/(b*(m + 1)) + x**(m + 1)*(A*b - B*a)*hyper((1, m/2 +
 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*b*(m + 1))
```

Mathematica [A] time = 0.0679551, size = 55, normalized size = 0.83

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aB\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^m\left(Bx^2+A\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bx^2 + A)x^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Sympy [A] time = 15.5752, size = 190, normalized size = 2.88

$$\frac{Amxx^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Axx^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Bmx^3x^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right)\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3Bx^3x^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right)\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*x*x**m*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3
/2)) + B*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*B*x**3*x**m*lerc
hphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*
a*gamma(m/2 + 5/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

3.323 $\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+A(b-bm))\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{2ab\,(a+bx^2)}$$

```
[Out] ((A*b - a*B)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*B*(1 + m) + A*(
b - b*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/(2*a^2*b*(1 + m))
```

Rubi [A] time = 0.121905, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^{m+1}(aB(m+1)+A(b-bm))\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*B*(1 + m) + A*(
b - b*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/(2*a^2*b*(1 + m))
```

Rubi in Sympy [A] time = 14.8714, size = 71, normalized size = 0.76

$$\frac{x^{m+1}\,(Ab-Ba)}{2ab\,(a+bx^2)}+\frac{x^{m+1}\,(Ab\,(-m+1)+Ba\,(m+1))\,_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{2a^2b\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] x**(m + 1)*(A*b - B*a)/(2*a*b*(a + b*x**2)) + x**(m + 1)*(A*b*(-m
 + 1) + B*a*(m + 1))*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/
a)/(2*a**2*b*(m + 1))
```

Mathematica [A] time = 0.0708068, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aB\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2b(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)] + (A*b - a*B)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)]))/(a^2*b*(1 + m))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(B x^{2}+A\right)}{\left(b x^{2}+a\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B x^{2}+A\right) x^{m}}{\left(b x^{2}+a\right)^{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(B x^{2}+A\right) x^{m}}{b^{2} x^{4}+2\, a b x^{2}+a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 137.154, size = 906, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] A*(-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma
(m/2 + 3/2)) + 2*a*m*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 +
3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + a*x*x**m*lerchphi(b*x**2
*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(
m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*x*x**m*gamma(m
/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/
2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*
gamma(m/2 + 3/2)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**
2*b*x**2*gamma(m/2 + 3/2))) + B*(-a*m**2*x**3*x**m*lerchphi(b*x**
2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma
(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x**3*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/
(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*
```

```
m*x**3*x**m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*
x**2*gamma(m/2 + 5/2)) - 3*a*x**3*x**m*lerchphi(b*x**2*exp_polar(
I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2)
+ 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*x**3*x**m*gamma(m/2 + 3/2
)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - b*
m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*g
amma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/
2 + 5/2)) - 4*b*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*
x**2*gamma(m/2 + 5/2)) - 3*b*x**5*x**m*lerchphi(b*x**2*exp_polar(
I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2)
+ 8*a**2*b*x**2*gamma(m/2 + 5/2)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\,x^m}{(bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

3.324 $\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+Ab(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

```
[Out] ((A*b - a*B)*x^(1 + m))/(4*a*b*(a + b*x^2)^2) + ((A*b*(3 - m) + a
*B*(1 + m))*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/(4*a^3*b*(1 + m))
```

Rubi [A] time = 0.119772, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^{m+1}(aB(m+1)+Ab(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(4*a*b*(a + b*x^2)^2) + ((A*b*(3 - m) + a
*B*(1 + m))*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/(4*a^3*b*(1 + m))
```

Rubi in Sympy [A] time = 14.7396, size = 73, normalized size = 0.78

$$\frac{x^{m+1}\left(Ab-Ba\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^{m+1}\left(Ab\left(-m+3\right)+Ba\left(m+1\right)\right){}_2F_1\left(\left.\begin{matrix}2,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\right|-\frac{bx^2}{a}\right)}{4a^3b\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] x**(m + 1)*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) + x**(m + 1)*(A*b*
(-m + 3) + B*a*(m + 1))*hyper((2, m/2 + 1/2), (m/2 + 3/2,), -b*x*
*2/a)/(4*a**3*b*(m + 1))
```

Mathematica [A] time = 0.0801516, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aB\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)] + (A*b - a*B)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)]))/(a^3*b*(1 + m))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(B x^{2}+A \right)}{\left(b \,x^{2}+a \right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)

[Out] int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{(bx^2 + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3,x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bx^2 + A)x^m}{b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3,x, algorithm="fricas")

[Out] integral((B*x^2 + A)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(B*x**2+A)/(b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^m}{(bx^2 + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3,x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

3.325 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=151

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc)}{m+9}+\frac{b^2d^3x^{m+11}}{m+11}$$

```
[Out] (a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(
3 + m) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m)
+ (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^
2*(3*b*c + 2*a*d)*x^(9 + m))/(9 + m) + (b^2*d^3*x^(11 + m))/(11 +
 m)
```

Rubi [A] time = 0.229958, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc)}{m+9}+\frac{b^2d^3x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(
3 + m) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m)
+ (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^
2*(3*b*c + 2*a*d)*x^(9 + m))/(9 + m) + (b^2*d^3*x^(11 + m))/(11 +
 m)
```

Rubi in Sympy [A] time = 38.0051, size = 144, normalized size = 0.95

$$\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}\left(3ad+2bc\right)}{m+3}+\frac{b^2d^3x^{m+11}}{m+11}+\frac{bd^2x^{m+9}\left(2ad+3bc\right)}{m+9}+\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**(m + 1)/(m + 1) + a*c**2*x**(m + 3)*(3*a*d + 2*b*c)/
(m + 3) + b**2*d**3*x**(m + 11)/(m + 11) + b*d**2*x**(m + 9)*(2*a
*d + 3*b*c)/(m + 9) + c*x**(m + 5)*(3*a**2*d**2 + 6*a*b*c*d + b**
2*c**2)/(m + 5) + d*x**(m + 7)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c*
*2)/(m + 7)
```

Mathematica [A] time = 0.159988, size = 141, normalized size = 0.93

$$x^m\left(\frac{dx^7\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{cx^5\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{a^2c^3x}{m+1}+\frac{ac^2x^3(3ad+2bc)}{m+3}+\frac{bd^2x^9(2ad+3bc)}{m+9}+\frac{b^2d^3x^{11}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] x^m*((a^2*c^3*x)/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^3)/(3 + m) +
(c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^5)/(5 + m) + (d*(3*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^7)/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^9
)/(9 + m) + (b^2*d^3*x^11)/(11 + m))
```

Maple [B] time = 0.012, size = 976, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] x^(1+m)*(b^2*d^3*m^5*x^10+25*b^2*d^3*m^4*x^10+2*a*b*d^3*m^5*x^8+3
*b^2*c*d^2*m^5*x^8+230*b^2*d^3*m^3*x^10+54*a*b*d^3*m^4*x^8+81*b^2
*c*d^2*m^4*x^8+950*b^2*d^3*m^2*x^10+a^2*d^3*m^5*x^6+6*a*b*c*d^2*m
^5*x^6+524*a*b*d^3*m^3*x^8+3*b^2*c^2*d*m^5*x^6+786*b^2*c*d^2*m^3*
x^8+1689*b^2*d^3*m*x^10+29*a^2*d^3*m^4*x^6+174*a*b*c*d^2*m^4*x^6+
2244*a*b*d^3*m^2*x^8+87*b^2*c^2*d*m^4*x^6+3366*b^2*c*d^2*m^2*x^8+
945*b^2*d^3*x^10+3*a^2*c*d^2*m^5*x^4+302*a^2*d^3*m^3*x^6+6*a*b*c^
2*d*m^5*x^4+1812*a*b*c*d^2*m^3*x^6+4082*a*b*d^3*m*x^8+b^2*c^3*m^5
*x^4+906*b^2*c^2*d*m^3*x^6+6123*b^2*c*d^2*m*x^8+93*a^2*c*d^2*m^4*
x^4+1366*a^2*d^3*m^2*x^6+186*a*b*c^2*d*m^4*x^4+8196*a*b*c*d^2*m^2
*x^6+2310*a*b*d^3*x^8+31*b^2*c^3*m^4*x^4+4098*b^2*c^2*d*m^2*x^6+3
465*b^2*c*d^2*x^8+3*a^2*c^2*d*m^5*x^2+1050*a^2*c*d^2*m^3*x^4+2577
*a^2*d^3*m*x^6+2*a*b*c^3*m^5*x^2+2100*a*b*c^2*d*m^3*x^4+15462*a*b
*c*d^2*m*x^6+350*b^2*c^3*m^3*x^4+7731*b^2*c^2*d*m*x^6+99*a^2*c^2*
d*m^4*x^2+5190*a^2*c*d^2*m^2*x^4+1485*a^2*d^3*x^6+66*a*b*c^3*m^4*
x^2+10380*a*b*c^2*d*m^2*x^4+8910*a*b*c*d^2*x^6+1730*b^2*c^3*m^2*x
^4+4455*b^2*c^2*d*x^6+a^2*c^3*m^5+1218*a^2*c^2*d*m^3*x^2+10467*a^
2*c*d^2*m*x^4+812*a*b*c^3*m^3*x^2+20934*a*b*c^2*d*m*x^4+3489*b^2*
c^3*m*x^4+35*a^2*c^3*m^4+6786*a^2*c^2*d*m^2*x^2+6237*a^2*c*d^2*x^
4+4524*a*b*c^3*m^2*x^2+12474*a*b*c^2*d*x^4+2079*b^2*c^3*x^4+470*a
^2*c^3*m^3+16059*a^2*c^2*d*m*x^2+10706*a*b*c^3*m*x^2+3010*a^2*c^3
*m^2+10395*a^2*c^2*d*x^2+6930*a*b*c^3*x^2+9129*a^2*c^3*m+10395*a^
2*c^3)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237777, size = 1044, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^m,x, algorithm="fricas")
```

```
[Out] ((b^2*d^3*m^5 + 25*b^2*d^3*m^4 + 230*b^2*d^3*m^3 + 950*b^2*d^3*m^
2 + 1689*b^2*d^3*m + 945*b^2*d^3)*x^11 + ((3*b^2*c*d^2 + 2*a*b*d^
3)*m^5 + 3465*b^2*c*d^2 + 2310*a*b*d^3 + 27*(3*b^2*c*d^2 + 2*a*b*
```

```
d^3)*m^4 + 262*(3*b^2*c*d^2 + 2*a*b*d^3)*m^3 + 1122*(3*b^2*c*d^2
+ 2*a*b*d^3)*m^2 + 2041*(3*b^2*c*d^2 + 2*a*b*d^3)*m)*x^9 + ((3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^5 + 4455*b^2*c^2*d + 8910*a*b*
c*d^2 + 1485*a^2*d^3 + 29*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m
^4 + 302*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^3 + 1366*(3*b^2*
c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^2 + 2577*(3*b^2*c^2*d + 6*a*b*c*
d^2 + a^2*d^3)*m)*x^7 + ((b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^
5 + 2079*b^2*c^3 + 12474*a*b*c^2*d + 6237*a^2*c*d^2 + 31*(b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^4 + 350*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*m^3 + 1730*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^2
 + 3489*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m)*x^5 + ((2*a*b*c^
3 + 3*a^2*c^2*d)*m^5 + 6930*a*b*c^3 + 10395*a^2*c^2*d + 33*(2*a*b
*c^3 + 3*a^2*c^2*d)*m^4 + 406*(2*a*b*c^3 + 3*a^2*c^2*d)*m^3 + 226
2*(2*a*b*c^3 + 3*a^2*c^2*d)*m^2 + 5353*(2*a*b*c^3 + 3*a^2*c^2*d)*
m)*x^3 + (a^2*c^3*m^5 + 35*a^2*c^3*m^4 + 470*a^2*c^3*m^3 + 3010*a
^2*c^3*m^2 + 9129*a^2*c^3*m + 10395*a^2*c^3)*x)*x^m/(m^6 + 36*m^5
 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [A] time = 17.1084, size = 4345, normalized size = 28.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((-a**2*c**3/(10*x**10) - 3*a**2*c**2*d/(8*x**8) - a**2*
c*d**2/(2*x**6) - a**2*d**3/(4*x**4) - a*b*c**3/(4*x**8) - a*b*c*
*2*d/x**6 - 3*a*b*c*d**2/(2*x**4) - a*b*d**3/x**2 - b**2*c**3/(6*
x**6) - 3*b**2*c**2*d/(4*x**4) - 3*b**2*c*d**2/(2*x**2) + b**2*d*
*3*log(x), Eq(m, -11)), (-a**2*c**3/(8*x**8) - a**2*c**2*d/(2*x**
6) - 3*a**2*c*d**2/(4*x**4) - a**2*d**3/(2*x**2) - a*b*c**3/(3*x*
*6) - 3*a*b*c**2*d/(2*x**4) - 3*a*b*c*d**2/x**2 + 2*a*b*d**3*log(
x) - b**2*c**3/(4*x**4) - 3*b**2*c**2*d/(2*x**2) + 3*b**2*c*d**2*
log(x) + b**2*d**3*x**2/2, Eq(m, -9)), (-a**2*c**3/(6*x**6) - 3*a
**2*c**2*d/(4*x**4) - 3*a**2*c*d**2/(2*x**2) + a**2*d**3*log(x) -
 a*b*c**3/(2*x**4) - 3*a*b*c**2*d/x**2 + 6*a*b*c*d**2*log(x) + a*
b*d**3*x**2 - b**2*c**3/(2*x**2) + 3*b**2*c**2*d*log(x) + 3*b**2*
c*d**2*x**2/2 + b**2*d**3*x**4/4, Eq(m, -7)), (-a**2*c**3/(4*x**4
) - 3*a**2*c**2*d/(2*x**2) + 3*a**2*c*d**2*log(x) + a**2*d**3*x**
2/2 - a*b*c**3/x**2 + 6*a*b*c**2*d*log(x) + 3*a*b*c*d**2*x**2 + a
*b*d**3*x**4/2 + b**2*c**3*log(x) + 3*b**2*c**2*d*x**2/2 + 3*b**2
*c*d**2*x**4/4 + b**2*d**3*x**6/6, Eq(m, -5)), (-a**2*c**3/(2*x**
2) + 3*a**2*c**2*d*log(x) + 3*a**2*c*d**2*x**2/2 + a**2*d**3*x**4
/4 + 2*a*b*c**3*log(x) + 3*a*b*c**2*d*x**2 + 3*a*b*c*d**2*x**4/2
+ a*b*d**3*x**6/3 + b**2*c**3*x**2/2 + 3*b**2*c**2*d*x**4/4 + b**
2*c*d**2*x**6/2 + b**2*d**3*x**8/8, Eq(m, -3)), (a**2*c**3*log(x)
 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c*d**2*x**4/4 + a**2*d**3*x**6/6
 + a*b*c**3*x**2 + 3*a*b*c**2*d*x**4/2 + a*b*c*d**2*x**6 + a*b*d*
*3*x**8/4 + b**2*c**3*x**4/4 + b**2*c**2*d*x**6/2 + 3*b**2*c*d**2
*x**8/8 + b**2*d**3*x**10/10, Eq(m, -1)), (a**2*c**3*m**5*x*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 35*a**2*c**3*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*c**3*m**3*x*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 3010*a**2*c**3*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*c**3*m*x*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 10395*a**2*c**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 3*a**2*c**2*d*m**5*x**
3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1952
4*m + 10395) + 99*a**2*c**2*d*m**4*x**3*x**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1218*a**2*c*
*2*d*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 6786*a**2*c**2*d*m**2*x**3*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 16059*a**2*c**2*d*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*c**2*d*x**3*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 3*a**2*c*d**2*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m*
```

*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 93*a**2*c*d**2*m
4*x5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 1050*a**2*c*d**2*m**3*x**5*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 519
0*a**2*c*d**2*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m*
*3 + 12139*m**2 + 19524*m + 10395) + 10467*a**2*c*d**2*m*x**5*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 6237*a**2*c*d**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + a**2*d**3*m**5*x**7*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 29*a**2*d**3*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 302*a**2*d**3*m*
*3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 1366*a**2*d**3*m**2*x**7*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2577*a
2*d3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 1485*a**2*d**3*x**7*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a
*b*c**3*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 66*a*b*c**3*m**4*x**3*x**m/(m**6 +
 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 812*a*b*c**3*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m*
*3 + 12139*m**2 + 19524*m + 10395) + 4524*a*b*c**3*m**2*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 10706*a*b*c**3*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6930*a*b*c**3*x**3*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 6*a*b*c**2*d*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 186*a*b*c**2*d*m**
4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 2100*a*b*c**2*d*m**3*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10380*
a*b*c**2*d*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 20934*a*b*c**2*d*m*x**5*x**m/(m
6 + 36*m5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 12474*a*b*c**2*d*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 6*a*b*c*d**2*m**5*x**7*x
m/(m6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 174*a*b*c*d**2*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + 1812*a*b*c*d**2
*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 8196*a*b*c*d**2*m**2*x**7*x**m/(m**6 + 36
*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 15
462*a*b*c*d**2*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 8910*a*b*c*d**2*x**7*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2*a*b*d**3*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
3 + 12139*m2 + 19524*m + 10395) + 54*a*b*d**3*m**4*x**9*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 524*a*b*d**3*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2244*a*b*d**3*m**2*x*
*9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 4082*a*b*d**3*m*x**9*x**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + 2310*a*b*d**3*x
9*xm/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19
524*m + 10395) + b**2*c**3*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + 31*b**2*c**3*m*
*4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 350*b**2*c**3*m**3*x**5*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1730*b*
*2*c**3*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 3489*b**2*c**3*m*x**5*x**m/(m**6 +
 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 2079*b**2*c**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 3*b**2*c**2*d*m**5*x**7*x**m/(m
6 + 36*m5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 87*b**2*c**2*d*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 906*b**2*c**2*d*m**3*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 4098*b**2*c**2*d*m**2*x**7*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 7731*b*
*2*c**2*d*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 4455*b**2*c**2*d*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +

```
3*b**2*c*d**2*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m*
*3 + 12139*m**2 + 19524*m + 10395) + 81*b**2*c*d**2*m**4*x**9*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 786*b**2*c*d**2*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3366*b**2*c*d**2
*m**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 6123*b**2*c*d**2*m*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465
*b**2*c*d**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + b**2*d**3*m**5*x**11*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
25*b**2*d**3*m**4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m*
*3 + 12139*m**2 + 19524*m + 10395) + 230*b**2*d**3*m**3*x**11*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 950*b**2*d**3*m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b**2*d**3*m*
x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
19524*m + 10395) + 945*b**2*d**3*x**11*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

GIAC/XCAS [A] time = 0.285031, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^m,x, algorithm="giac")
```

```
[Out] Done
```

3.326 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=109

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

```
[Out] (a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 +
m) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d
*(b*c + a*d)*x^(7 + m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

Rubi [A] time = 0.168043, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 +
m) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d
*(b*c + a*d)*x^(7 + m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

Rubi in Sympy [A] time = 28.2352, size = 100, normalized size = 0.92

$$\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}\left(ad+bc\right)}{m+3}+\frac{b^2d^2x^{m+9}}{m+9}+\frac{2bdx^{m+7}\left(ad+bc\right)}{m+7}+\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**(m + 1)/(m + 1) + 2*a*c*x**(m + 3)*(a*d + b*c)/(m +
3) + b**2*d**2*x**(m + 9)/(m + 9) + 2*b*d*x**(m + 7)*(a*d + b*c)/
(m + 7) + x**(m + 5)*(a**2*d**2 + 4*a*b*c*d + b**2*c**2)/(m + 5)
```

Mathematica [A] time = 0.110726, size = 101, normalized size = 0.93

$$x^m\left(\frac{x^5\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x}{m+1}+\frac{2bdx^7(ad+bc)}{m+7}+\frac{2acx^3(ad+bc)}{m+3}+\frac{b^2d^2x^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] x^m*((a^2*c^2*x)/(1 + m) + (2*a*c*(b*c + a*d)*x^3)/(3 + m) + ((b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^5)/(5 + m) + (2*b*d*(b*c + a*d)*x^
7)/(7 + m) + (b^2*d^2*x^9)/(9 + m))
```

Maple [B] time = 0.009, size = 569, normalized size = 5.2

$$x^{1+m}\left(b^2d^2m^4x^8+16\,b^2d^2m^3x^8+2\,abd^2m^4x^6+2\,b^2cdm^4x^6+86\,b^2d^2m^2x^8+36\,abd^2m^3x^6+36\,b^2cdm^3x^6+176\,b^2d^2mx^8+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] x^(1+m)*(b^2*d^2*m^4*x^8+16*b^2*d^2*m^3*x^8+2*a*b*d^2*m^4*x^6+2*b
^2*c*d*m^4*x^6+86*b^2*d^2*m^2*x^8+36*a*b*d^2*m^3*x^6+36*b^2*c*d*m
^3*x^6+176*b^2*d^2*m*x^8+a^2*d^2*m^4*x^4+4*a*b*c*d*m^4*x^4+208*a*
b*d^2*m^2*x^6+b^2*c^2*m^4*x^4+208*b^2*c*d*m^2*x^6+105*b^2*d^2*x^8
+20*a^2*d^2*m^3*x^4+80*a*b*c*d*m^3*x^4+444*a*b*d^2*m*x^6+20*b^2*c
^2*m^3*x^4+444*b^2*c*d*m*x^6+2*a^2*c*d*m^4*x^2+130*a^2*d^2*m^2*x^
4+2*a*b*c^2*m^4*x^2+520*a*b*c*d*m^2*x^4+270*a*b*d^2*x^6+130*b^2*c
^2*m^2*x^4+270*b^2*c*d*x^6+44*a^2*c*d*m^3*x^2+300*a^2*d^2*m*x^4+4
4*a*b*c^2*m^3*x^2+1200*a*b*c*d*m*x^4+300*b^2*c^2*m*x^4+a^2*c^2*m^
4+328*a^2*c*d*m^2*x^2+189*a^2*d^2*x^4+328*a*b*c^2*m^2*x^2+756*a*b
*c*d*x^4+189*b^2*c^2*x^4+24*a^2*c^2*m^3+916*a^2*c*d*m*x^2+916*a*b
*c^2*m*x^2+206*a^2*c^2*m^2+630*a^2*c*d*x^2+630*a*b*c^2*x^2+744*a^
2*c^2*m+945*a^2*c^2)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237938, size = 597, normalized size = 5.48

$$\frac{\left(\left(b^2d^2m^4 + 16\,b^2d^2m^3 + 86\,b^2d^2m^2 + 176\,b^2d^2m + 105\,b^2d^2\right)x^9 + 2\left(\left(b^2cd + abd^2\right)m^4 + 135\,b^2cd + 135\,abd^2 + 18\left(b^2cd + al\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^m,x, algorithm="fricas")
```

```
[Out] ((b^2*d^2*m^4 + 16*b^2*d^2*m^3 + 86*b^2*d^2*m^2 + 176*b^2*d^2*m +
 105*b^2*d^2)*x^9 + 2*((b^2*c*d + a*b*d^2)*m^4 + 135*b^2*c*d + 13
5*a*b*d^2 + 18*(b^2*c*d + a*b*d^2)*m^3 + 104*(b^2*c*d + a*b*d^2)*
m^2 + 222*(b^2*c*d + a*b*d^2)*m)*x^7 + ((b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*m^4 + 189*b^2*c^2 + 756*a*b*c*d + 189*a^2*d^2 + 20*(b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*m^3 + 130*(b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*m^2 + 300*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m)*x^5 + 2*((a*b*c^2
+ a^2*c*d)*m^4 + 315*a*b*c^2 + 315*a^2*c*d + 22*(a*b*c^2 + a^2*c*
d)*m^3 + 164*(a*b*c^2 + a^2*c*d)*m^2 + 458*(a*b*c^2 + a^2*c*d)*m)
*x^3 + (a^2*c^2*m^4 + 24*a^2*c^2*m^3 + 206*a^2*c^2*m^2 + 744*a^2*
c^2*m + 945*a^2*c^2)*x)*x^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1
689*m + 945)
```

Sympy [A] time = 9.85582, size = 2363, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**2*c**2/(8*x**8) - a**2*c*d/(3*x**6) - a**2*d**2/(4
*x**4) - a*b*c**2/(3*x**6) - a*b*c*d/x**4 - a*b*d**2/x**2 - b**2*
```

```
c**2/(4*x**4) - b**2*c*d/x**2 + b**2*d**2*log(x), Eq(m, -9)), (-a
**2*c**2/(6*x**6) - a**2*c*d/(2*x**4) - a**2*d**2/(2*x**2) - a*b*
c**2/(2*x**4) - 2*a*b*c*d/x**2 + 2*a*b*d**2*log(x) - b**2*c**2/(2
*x**2) + 2*b**2*c*d*log(x) + b**2*d**2*x**2/2, Eq(m, -7)), (-a**2
*c**2/(4*x**4) - a**2*c*d/x**2 + a**2*d**2*log(x) - a*b*c**2/x**2
 + 4*a*b*c*d*log(x) + a*b*d**2*x**2 + b**2*c**2*log(x) + b**2*c*d
*x**2 + b**2*d**2*x**4/4, Eq(m, -5)), (-a**2*c**2/(2*x**2) + 2*a*
*2*c*d*log(x) + a**2*d**2*x**2/2 + 2*a*b*c**2*log(x) + 2*a*b*c*d*
x**2 + a*b*d**2*x**4/2 + b**2*c**2*x**2/2 + b**2*c*d*x**4/2 + b**
2*d**2*x**6/6, Eq(m, -3)), (a**2*c**2*log(x) + a**2*c*d*x**2 + a*
*2*d**2*x**4/4 + a*b*c**2*x**2 + a*b*c*d*x**4 + a*b*d**2*x**6/3 +
 b**2*c**2*x**4/4 + b**2*c*d*x**6/3 + b**2*d**2*x**8/8, Eq(m, -1)
), (a**2*c**2*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 24*a**2*c**2*m**3*x*x**m/(m**5 + 25*m**4 + 230*m
**3 + 950*m**2 + 1689*m + 945) + 206*a**2*c**2*m**2*x*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**2*c**2*m
*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 9
45*a**2*c**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*
m + 945) + 2*a**2*c*d*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 44*a**2*c*d*m**3*x**3*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a**2*c*d*m**2*x
**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
916*a**2*c*d*m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 630*a**2*c*d*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + a**2*d**2*m**4*x**5*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 20*a**2*d**2*m**3*x
**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
130*a**2*d**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 300*a**2*d**2*m*x**5*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 189*a**2*d**2*x**5*x**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a*b*c**2*m*
*4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 44*a*b*c**2*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 328*a*b*c**2*m**2*x**3*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*c**2*m*x**3*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*
c**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 4*a*b*c*d*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 80*a*b*c*d*m**3*x**5*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 520*a*b*c*d*m**2*x**5*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1200*a*b
*c*d*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 756*a*b*c*d*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**2*m**4*x**7*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + 36*a*b*d**2*m**3*x**7*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 208*a*b*d
**2*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 444*a*b*d**2*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 270*a*b*d**2*x**7*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*c**2*m**4*x**5*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 20*b**2*c
**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 130*b**2*c**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 300*b**2*c**2*m*x**5*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 189*b**2*c**2*x**5
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*b
**2*c*d*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 16
89*m + 945) + 36*b**2*c*d*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 208*b**2*c*d*m**2*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*b**2*c*d*m
*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 270*b**2*c*d*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + b**2*d**2*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 16*b**2*d**2*m**3*x**9*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**2*d**2*m
**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 176*b**2*d**2*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 105*b**2*d**2*x**9*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945), True))
```

GIAC/XCAS [A] time = 0.228424, size = 1071, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^m,x, algorithm="giac")
```

```
[Out] (b^2*d^2*m^4*x^9*e^(m*ln(x)) + 16*b^2*d^2*m^3*x^9*e^(m*ln(x)) + 2
*b^2*c*d*m^4*x^7*e^(m*ln(x)) + 2*a*b*d^2*m^4*x^7*e^(m*ln(x)) + 86
*b^2*d^2*m^2*x^9*e^(m*ln(x)) + 36*b^2*c*d*m^3*x^7*e^(m*ln(x)) + 3
6*a*b*d^2*m^3*x^7*e^(m*ln(x)) + 176*b^2*d^2*m*x^9*e^(m*ln(x)) + b
^2*c^2*m^4*x^5*e^(m*ln(x)) + 4*a*b*c*d*m^4*x^5*e^(m*ln(x)) + a^2*
d^2*m^4*x^5*e^(m*ln(x)) + 208*b^2*c*d*m^2*x^7*e^(m*ln(x)) + 208*a
*b*d^2*m^2*x^7*e^(m*ln(x)) + 105*b^2*d^2*x^9*e^(m*ln(x)) + 20*b^2
*c^2*m^3*x^5*e^(m*ln(x)) + 80*a*b*c*d*m^3*x^5*e^(m*ln(x)) + 20*a^
2*d^2*m^3*x^5*e^(m*ln(x)) + 444*b^2*c*d*m*x^7*e^(m*ln(x)) + 444*a
*b*d^2*m*x^7*e^(m*ln(x)) + 2*a*b*c^2*m^4*x^3*e^(m*ln(x)) + 2*a^2*
c*d*m^4*x^3*e^(m*ln(x)) + 130*b^2*c^2*m^2*x^5*e^(m*ln(x)) + 520*a
*b*c*d*m^2*x^5*e^(m*ln(x)) + 130*a^2*d^2*m^2*x^5*e^(m*ln(x)) + 27
0*b^2*c*d*x^7*e^(m*ln(x)) + 270*a*b*d^2*x^7*e^(m*ln(x)) + 44*a*b*
c^2*m^3*x^3*e^(m*ln(x)) + 44*a^2*c*d*m^3*x^3*e^(m*ln(x)) + 300*b^
2*c^2*m*x^5*e^(m*ln(x)) + 1200*a*b*c*d*m*x^5*e^(m*ln(x)) + 300*a^
2*d^2*m*x^5*e^(m*ln(x)) + a^2*c^2*m^4*x*e^(m*ln(x)) + 328*a*b*c^2
*m^2*x^3*e^(m*ln(x)) + 328*a^2*c*d*m^2*x^3*e^(m*ln(x)) + 189*b^2*
c^2*x^5*e^(m*ln(x)) + 756*a*b*c*d*x^5*e^(m*ln(x)) + 189*a^2*d^2*x
^5*e^(m*ln(x)) + 24*a^2*c^2*m^3*x*e^(m*ln(x)) + 916*a*b*c^2*m*x^3
*e^(m*ln(x)) + 916*a^2*c*d*m*x^3*e^(m*ln(x)) + 206*a^2*c^2*m^2*x*
e^(m*ln(x)) + 630*a*b*c^2*x^3*e^(m*ln(x)) + 630*a^2*c*d*x^3*e^(m*
ln(x)) + 744*a^2*c^2*m*x*e^(m*ln(x)) + 945*a^2*c^2*x*e^(m*ln(x)))
/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.327 $\int x^m (a+bx^2)^2 (c+dx^2)\, dx$

Optimal. Leaf size=71

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

```
[Out] (a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) +
 (b*(b*c + 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

Rubi [A] time = 0.11054, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) +
 (b*(b*c + 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

Rubi in Sympy [A] time = 15.8642, size = 63, normalized size = 0.89

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}\,(ad+2bc)}{m+3}+\frac{b^2dx^{m+7}}{m+7}+\frac{bx^{m+5}\,(2ad+bc)}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**(m + 1)/(m + 1) + a*x**(m + 3)*(a*d + 2*b*c)/(m + 3) +
b**2*d*x**(m + 7)/(m + 7) + b*x**(m + 5)*(2*a*d + b*c)/(m + 5)
```

Mathematica [A] time = 0.0816094, size = 65, normalized size = 0.92

$$x^m\left(\frac{a^2cx}{m+1}+\frac{bx^5(2ad+bc)}{m+5}+\frac{ax^3(ad+2bc)}{m+3}+\frac{b^2dx^7}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] x^m*((a^2*c*x)/(1 + m) + (a*(2*b*c + a*d)*x^3)/(3 + m) + (b*(b*c
+ 2*a*d)*x^5)/(5 + m) + (b^2*d*x^7)/(7 + m))
```

Maple [B] time = 0.008, size = 262, normalized size = 3.7

$$x^{1+m}\left(b^2dm^3x^6+9\,b^2dm^2x^6+2\,abdm^3x^4+b^2cm^3x^4+23\,b^2dmx^6+22\,abdm^2x^4+11\,b^2cm^2x^4+15\,b^2dx^6+a^2dm^3x^2+2\,ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] x^(1+m)*(b^2*d*m^3*x^6+9*b^2*d*m^2*x^6+2*a*b*d*m^3*x^4+b^2*c*m^3*
x^4+23*b^2*d*m*x^6+22*a*b*d*m^2*x^4+11*b^2*c*m^2*x^4+15*b^2*d*x^6
+a^2*d*m^3*x^2+2*a*b*c*m^3*x^2+62*a*b*d*m*x^4+31*b^2*c*m*x^4+13*a
^2*d*m^2*x^2+26*a*b*c*m^2*x^2+42*a*b*d*x^4+21*b^2*c*x^4+a^2*c*m^3
+47*a^2*d*m*x^2+94*a*b*c*m*x^2+15*a^2*c*m^2+35*a^2*d*x^2+70*a*b*c
*x^2+71*a^2*c*m+105*a^2*c)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240005, size = 290, normalized size = 4.08

$$\frac{\left(\left(b^2dm^3+9\,b^2dm^2+23\,b^2dm+15\,b^2d\right)x^7+\left(\left(b^2c+2\,abd\right)m^3+21\,b^2c+42\,abd+11\,\left(b^2c+2\,abd\right)m^2+31\,\left(b^2c+2\,abd\right)m}{m^4+16\,m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^m,x, algorithm="fricas")
```

```
[Out] ((b^2*d*m^3 + 9*b^2*d*m^2 + 23*b^2*d*m + 15*b^2*d)*x^7 + ((b^2*c
+ 2*a*b*d)*m^3 + 21*b^2*c + 42*a*b*d + 11*(b^2*c + 2*a*b*d)*m^2 +
 31*(b^2*c + 2*a*b*d)*m)*x^5 + ((2*a*b*c + a^2*d)*m^3 + 70*a*b*c
+ 35*a^2*d + 13*(2*a*b*c + a^2*d)*m^2 + 47*(2*a*b*c + a^2*d)*m)*x
^3 + (a^2*c*m^3 + 15*a^2*c*m^2 + 71*a^2*c*m + 105*a^2*c)*x)*x^m/(
m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [A] time = 5.47498, size = 1044, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] Piecewise((-a**2*c/(6*x**6) - a**2*d/(4*x**4) - a*b*c/(2*x**4) -
a*b*d/x**2 - b**2*c/(2*x**2) + b**2*d*log(x), Eq(m, -7)), (-a**2*
c/(4*x**4) - a**2*d/(2*x**2) - a*b*c/x**2 + 2*a*b*d*log(x) + b**2
*c*log(x) + b**2*d*x**2/2, Eq(m, -5)), (-a**2*c/(2*x**2) + a**2*d
*log(x) + 2*a*b*c*log(x) + a*b*d*x**2 + b**2*c*x**2/2 + b**2*d*x*
*4/4, Eq(m, -3)), (a**2*c*log(x) + a**2*d*x**2/2 + a*b*c*x**2 + a
*b*d*x**4/2 + b**2*c*x**4/4 + b**2*d*x**6/6, Eq(m, -1)), (a**2*c*
m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*a**2*c*
m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*a**2*c*
m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**2*c*x*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + a**2*d*m**3*x**3*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a**2*d*m**2*x*
*3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a**2*d*m*x*
*3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a**2*d*x**3
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*a*b*c*m**3*x**
3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*a*b*c*m**2*x
**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*a*b*c*m*x*
```

```
*3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*a*b*c*x**3*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*a*b*d*m**3*x**5
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 22*a*b*d*m**2*x*
*5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 62*a*b*d*m*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 42*a*b*d*x**5*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**2*c*m**3*x**5*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b**2*c*m**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*b**2*c*m*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b**2*c*x**5*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**2*d*m**3*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*b**2*d*m**2*x**
7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*b**2*d*m*x**
7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*b**2*d*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

GIAC/XCAS [A] time = 0.263233, size = 513, normalized size = 7.23

$$\frac{b^2dm^3x^7e^{(m\ln(x))} + 9\,b^2dm^2x^7e^{(m\ln(x))} + b^2cm^3x^5e^{(m\ln(x))} + 2\,abdm^3x^5e^{(m\ln(x))} + 23\,b^2dmx^7e^{(m\ln(x))} + 11\,b^2cm^2x^5e^{(m\ln(x))} + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^m,x, algorithm="giac")
```

```
[Out] (b^2*d*m^3*x^7*e^(m*ln(x)) + 9*b^2*d*m^2*x^7*e^(m*ln(x)) + b^2*c*
m^3*x^5*e^(m*ln(x)) + 2*a*b*d*m^3*x^5*e^(m*ln(x)) + 23*b^2*d*m*x^
7*e^(m*ln(x)) + 11*b^2*c*m^2*x^5*e^(m*ln(x)) + 22*a*b*d*m^2*x^5*e
^(m*ln(x)) + 15*b^2*d*x^7*e^(m*ln(x)) + 2*a*b*c*m^3*x^3*e^(m*ln(x
)) + a^2*d*m^3*x^3*e^(m*ln(x)) + 31*b^2*c*m*x^5*e^(m*ln(x)) + 62*
a*b*d*m*x^5*e^(m*ln(x)) + 26*a*b*c*m^2*x^3*e^(m*ln(x)) + 13*a^2*d
*m^2*x^3*e^(m*ln(x)) + 21*b^2*c*x^5*e^(m*ln(x)) + 42*a*b*d*x^5*e^
(m*ln(x)) + a^2*c*m^3*x*e^(m*ln(x)) + 94*a*b*c*m*x^3*e^(m*ln(x))
+ 47*a^2*d*m*x^3*e^(m*ln(x)) + 15*a^2*c*m^2*x*e^(m*ln(x)) + 70*a*
b*c*x^3*e^(m*ln(x)) + 35*a^2*d*x^3*e^(m*ln(x)) + 71*a^2*c*m*x*e^(
m*ln(x)) + 105*a^2*c*x*e^(m*ln(x)))/(m^4 + 16*m^3 + 86*m^2 + 176*
m + 105)
```

3.328 $\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=94

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{cd^2(m+1)}-\frac{bx^{m+1}(bc-2ad)}{d^2(m+1)}+\frac{b^2x^{m+3}}{d(m+3)}$$

```
[Out] -((b*(b*c - 2*a*d)*x^(1 + m))/(d^2*(1 + m))) + (b^2*x^(3 + m))/(d
*(3 + m)) + ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^2*(1 + m))
```

Rubi [A] time = 0.159371, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{cd^2(m+1)}-\frac{bx^{m+1}(bc-2ad)}{d^2(m+1)}+\frac{b^2x^{m+3}}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x^(1 + m))/(d^2*(1 + m))) + (b^2*x^(3 + m))/(d
*(3 + m)) + ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^2*(1 + m))
```

Rubi in Sympy [A] time = 25.9419, size = 76, normalized size = 0.81

$$\frac{b^2x^{m+3}}{d\,(m+3)}+\frac{bx^{m+1}\,(2ad-bc)}{d^2\,(m+1)}+\frac{x^{m+1}\,(ad-bc)^2\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{c}\right)}{cd^2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**(m + 3)/(d*(m + 3)) + b*x**(m + 1)*(2*a*d - b*c)/(d**2*(m
 + 1)) + x**(m + 1)*(a*d - b*c)**2*hyper((1, m/2 + 1/2), (m/2 + 3
/2,), -d*x**2/c)/(c*d**2*(m + 1))
```

Mathematica [A] time = 0.151446, size = 118, normalized size = 1.26

$$\frac{x^{m+1}\left(\frac{a^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{m+1}+bx^2\left(\frac{2a\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{dx^2}{c}\right)}{m+3}+\frac{bx^2\,{}_2F_1\left(1,\frac{m+5}{2};\frac{m+7}{2};-\frac{dx^2}{c}\right)}{m+5}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(1 + m) + b*x^2*((2*a*Hypergeometric2F1[1, (3 + m)/2, (
5 + m)/2, -((d*x^2)/c)])/(3 + m) + (b*x^2*Hypergeometric2F1[1, (5
 + m)/2, (7 + m)/2, -((d*x^2)/c)])/(5 + m))))/c
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{x^m (bx^2 + a)^2}{dx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2 x^m}{dx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4 + 2\,abx^2 + a^2)\,x^m}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d*x^2 + c), x)
```

Sympy [A] time = 34.561, size = 299, normalized size = 3.18

$$\frac{a^2mxx^m \Phi\left(\frac{dx^2e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4c\,\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{a^2xx^m \Phi\left(\frac{dx^2e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4c\,\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{abmx^3x^m \Phi\left(\frac{dx^2e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{2c\,\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3abx^3x^m \Phi\left(\frac{dx^2e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{2c\,\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{b^2mx^5x^m \Phi\left(\frac{dx^2e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{5}{2}\right) \Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}{4c\,\Gamma\left(\frac{m}{2} + \frac{7}{2}\right)} + \frac{5b^2x^5x^m \Phi\left(\frac{dx^2e^{i\pi}}{c}, 1, \frac{m}{2} + \frac{5}{2}\right) \Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}{4c\,\Gamma\left(\frac{m}{2} + \frac{7}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2*m*x*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*ga
mma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + a**2*x*x**m*lerchphi(d*x*
*2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m
/2 + 3/2)) + a*b*m*x**3*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1
, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*c*gamma(m/2 + 5/2)) + 3*a*b*x**3
```

```
*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2
+ 3/2)/(2*c*gamma(m/2 + 5/2)) + b**2*m*x**5*x**m*lerchphi(d*x**2*
exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2
+ 7/2)) + 5*b**2*x**5*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1,
m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2 x^m}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

3.329 $\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=120

$$-\frac{x^{m+1}(bc-ad)(ad(1-m)+bc(m+3))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^{m+1}}{d^2(m+1)}$$

```
[Out] (b^2*x^(1 + m))/(d^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*c*d^
2*(c + d*x^2)) - ((b*c - a*d)*(a*d*(1 - m) + b*c*(3 + m))*x^(1 +
m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c
^2*d^2*(1 + m))
```

Rubi [A] time = 0.279261, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{x^{m+1}(bc-ad)(ad(1-m)+bc(m+3))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^{m+1}}{d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^(1 + m))/(d^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*c*d^
2*(c + d*x^2)) - ((b*c - a*d)*(a*d*(1 - m) + b*c*(3 + m))*x^(1 +
m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c
^2*d^2*(1 + m))
```

Rubi in Sympy [A] time = 41.9547, size = 104, normalized size = 0.87

$$\frac{b^2x^{m+1}}{d^2(m+1)}+\frac{x^{m+1}(ad-bc)^2}{2cd^2(c+dx^2)}+\frac{x^{m+1}(ad-bc)(-adm+ad+bcm+3bc)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{c}\right)}{2c^2d^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**(m + 1)/(d**2*(m + 1)) + x**(m + 1)*(a*d - b*c)**2/(2*c*d
**2*(c + d*x**2)) + x**(m + 1)*(a*d - b*c)*(-a*d*m + a*d + b*c*m
+ 3*b*c)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -d*x**2/c)/(2*c**2*d
**2*(m + 1))
```

Mathematica [A] time = 0.162157, size = 118, normalized size = 0.98

$$\frac{x^{m+1}\left(\frac{a^2\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{m+1}+bx^2\left(\frac{2a\,{}_2F_1\left(2,\frac{m+3}{2};\frac{m+5}{2};-\frac{dx^2}{c}\right)}{m+3}+\frac{bx^2\,{}_2F_1\left(2,\frac{m+5}{2};\frac{m+7}{2};-\frac{dx^2}{c}\right)}{m+5}\right)\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(1 + m) + b*x^2*((2*a*Hypergeometric2F1[2, (3 + m)/2, (
```

```
5 + m)/2, -((d*x^2)/c)])/(3 + m) + (b*x^2*Hypergeometric2F1[2, (5
 + m)/2, (7 + m)/2, -((d*x^2)/c)])/(5 + m))))/c^2
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{x^m (bx^2 + a)^2}{(dx^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2 x^m}{(dx^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4 + 2\,abx^2 + a^2)x^m}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^2*x^4 + 2*c*d*x^2 + c
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (a + bx^2)^2}{(c + dx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2 x^m}{(dx^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

3.330 $\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=171

$$\frac{x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)+2abcd\left(1-m^2\right)+b^2c^2\left(m^2+4m+3\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3d^2(m+1)}-\frac{x^{m+1}(bc-ad)(ad(3-m)+bc(m+5))}{8c^2d^2\left(c+dx^2\right)}+\frac{x^{m+1}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

```
[Out] ((b*c - a*d)^2*x^(1 + m))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*
(a*d*(3 - m) + b*c*(5 + m))*x^(1 + m))/(8*c^2*d^2*(c + d*x^2)) +
((2*a*b*c*d*(1 - m^2) + a^2*d^2*(3 - 4*m + m^2) + b^2*c^2*(3 + 4*
m + m^2))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(d*x^2)/c)])/(8*c^3*d^2*(1 + m))
```

Rubi [A] time = 0.386817, antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x^{m+1}\left(\frac{(1-m)\left(4a^2d^2-(m+1)(bc-ad)^2\right)}{c^2(m+1)}+4b^2\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8cd^2}-\frac{x^{m+1}(bc-ad)(ad(3-m)+bc(m+5))}{8c^2d^2\left(c+dx^2\right)}+\frac{x^{m+1}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*x^(1 + m))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*
(a*d*(3 - m) + b*c*(5 + m))*x^(1 + m))/(8*c^2*d^2*(c + d*x^2)) +
((4*b^2 + ((1 - m)*(4*a^2*d^2 - (b*c - a*d)^2*(1 + m)))/(c^2*(1 +
 m)))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x
^2)/c)])/(8*c*d^2)
```

Rubi in Sympy [A] time = 41.7323, size = 148, normalized size = 0.87

$$\frac{x^{m+1}(ad-bc)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{x^{m+1}(ad-bc)(-adm+3ad+bcm+5bc)}{8c^2d^2\left(c+dx^2\right)}+\frac{x^{m+1}\left(4b^2c^2(m+1)+(-m+1)\left(4a^2d^2-(m+1)(ad-bc)^2\right)\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{c}\right)}{8c^3d^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] x**(m + 1)*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) + x**(m + 1)
*(a*d - b*c)*(-a*d*m + 3*a*d + b*c*m + 5*b*c)/(8*c**2*d**2*(c + d
*x**2)) + x**(m + 1)*(4*b**2*c**2*(m + 1) + (-m + 1)*(4*a**2*d**2
 - (m + 1)*(a*d - b*c)**2))*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -
d*x**2/c)/(8*c**3*d**2*(m + 1))
```

Mathematica [A] time = 0.164144, size = 118, normalized size = 0.69

$$\frac{x^{m+1}\left(\frac{a^2\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{m+1}+bx^2\left(\frac{2a\,{}_2F_1\left(3,\frac{m+3}{2};\frac{m+5}{2};-\frac{dx^2}{c}\right)}{m+3}+\frac{bx^2\,{}_2F_1\left(3,\frac{m+5}{2};\frac{m+7}{2};-\frac{dx^2}{c}\right)}{m+5}\right)\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(1 + m) + b*x^2*((2*a*Hypergeometric2F1[3, (3 + m)/2, (
5 + m)/2, -((d*x^2)/c)])/(3 + m) + (b*x^2*Hypergeometric2F1[3, (5
 + m)/2, (7 + m)/2, -((d*x^2)/c)])/(5 + m))))/c^3
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(b x^{2}+a\right)^{2}}{\left(d x^{2}+c\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{2} x^{m}}{\left(d x^{2}+c\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2} x^{4}+2\, a b x^{2}+a^{2}\right) x^{m}}{d^{3} x^{6}+3\, c d^{2} x^{4}+3\, c^{2} d x^{2}+c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^3*x^6 + 3*c*d^2*x^4 +
 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 x^m}{\left(dx^2 + c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

3.331 $\int \frac{x^m (c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=133

$$\frac{dx^{m+1}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3(m+1)}+\frac{x^{m+1}(bc-ad)^3\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^3(m+1)}+\frac{d^2x^{m+3}(3bc-ad)}{b^2(m+3)}+\frac{d^3x^{m+5}}{b(m+5)}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(1 + m))/(b^3*(1 + m)) + (
d^2*(3*b*c - a*d)*x^(3 + m))/(b^2*(3 + m)) + (d^3*x^(5 + m))/(b*(
5 + m)) + ((b*c - a*d)^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(a*b^3*(1 + m))
```

Rubi [A] time = 0.209192, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{dx^{m+1}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3(m+1)}+\frac{x^{m+1}(bc-ad)^3\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^3(m+1)}+\frac{d^2x^{m+3}(3bc-ad)}{b^2(m+3)}+\frac{d^3x^{m+5}}{b(m+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```


```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(1 + m))/(b^3*(1 + m)) + (
d^2*(3*b*c - a*d)*x^(3 + m))/(b^2*(3 + m)) + (d^3*x^(5 + m))/(b*(
5 + m)) + ((b*c - a*d)^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(a*b^3*(1 + m))
```

Rubi in Sympy [A] time = 37.481, size = 116, normalized size = 0.87

$$\frac{d^3x^{m+5}}{b\,(m+5)}-\frac{d^2x^{m+3}\,(ad-3bc)}{b^2\,(m+3)}+\frac{dx^{m+1}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3\,(m+1)}-\frac{x^{m+1}\,(ad-bc)^3\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ab^3\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] d**3*x**(m + 5)/(b*(m + 5)) - d**2*x**(m + 3)*(a*d - 3*b*c)/(b**2
*(m + 3)) + d*x**(m + 1)*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)/(b
**3*(m + 1)) - x**(m + 1)*(a*d - b*c)**3*hyper((1, m/2 + 1/2), (m
/2 + 3/2,), -b*x**2/a)/(a*b**3*(m + 1))
```

Mathematica [A] time = 0.266775, size = 159, normalized size = 1.2

$$\frac{x^{m+1}\left(\frac{c^3\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}+dx^2\left(\frac{3c^2\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{m+3}+dx^2\left(\frac{3c\,{}_2F_1\left(1,\frac{m+5}{2};\frac{m+7}{2};-\frac{bx^2}{a}\right)}{m+5}+\frac{dx^2\,{}_2F_1\left(1,\frac{m+7}{2};\frac{m+9}{2};-\frac{bx^2}{a}\right)}{m+7}\right)\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*((c^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(1 + m) + d*x^2*((3*c^2*Hypergeometric2F1[1, (3 + m)/2,
```

```
 (5 + m)/2, -((b*x^2)/a)])/(3 + m) + d*x^2*((3*c*Hypergeometric2F
1[1, (5 + m)/2, (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (d*x^2*Hyperg
eometric2F1[1, (7 + m)/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))))/a
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(d x^{2}+c \right)^{3}}{b \,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(d x^{2} + c\right)}^{3} x^{m}}{b x^{2} + a}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(d^{3} x^{6} + 3 \, c d^{2} x^{4} + 3 \, c^{2} d x^{2} + c^{3}\right)} x^{m}}{b x^{2} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b*x^2 +
 a), x)
```

Sympy [A] time = 64.6448, size = 411, normalized size = 3.09

$$\frac{c^{3} m x x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4 a \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{c^{3} x x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4 a \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{3 c^{2} d m x^{3} x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \left(\frac{m}{2} + \frac{3}{2}\right)}{4 a \left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{9 c^{2} d x^{3} x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \left(\frac{m}{2} + \frac{3}{2}\right)}{4 a \left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3 c d^{2} m x^{5} x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{5}{2}\right) \left(\frac{m}{2} + \frac{5}{2}\right)}{4 a \left(\frac{m}{2} + \frac{7}{2}\right)} + \frac{15 c d^{2} x^{5} x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{5}{2}\right) \left(\frac{m}{2} + \frac{5}{2}\right)}{4 a \left(\frac{m}{2} + \frac{7}{2}\right)} + \frac{d^{3} m x^{7} x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{7}{2}\right) \left(\frac{m}{2} + \frac{7}{2}\right)}{4 a \left(\frac{m}{2} + \frac{9}{2}\right)} + \frac{7 d^{3} x^{7} x^{m} \left(\frac{b x^{2} e^{i \pi}}{a}, 1, \frac{m}{2} + \frac{7}{2}\right) \left(\frac{m}{2} + \frac{7}{2}\right)}{4 a \left(\frac{m}{2} + \frac{9}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] c**3*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*ga
mma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**3*x*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m
/2 + 3/2)) + 3*c**2*d*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 9*c**
2*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*ga
mma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*c*d**2*m*x**5*x**m*lerc
hphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*
a*gamma(m/2 + 7/2)) + 15*c*d**2*x**5*x**m*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2))
 + d**3*m*x**7*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7
/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2)) + 7*d**3*x**7*x**m*le
rchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(
4*a*gamma(m/2 + 9/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^m/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

3.332 $\int \frac{x^m (c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=94

$$\frac{x^{m+1}(bc-ad)^2 {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^2(m+1)}+\frac{dx^{m+1}(2bc-ad)}{b^2(m+1)}+\frac{d^2x^{m+3}}{b(m+3)}$$

```
[Out] (d*(2*b*c - a*d)*x^(1 + m))/(b^2*(1 + m)) + (d^2*x^(3 + m))/(b*(3
 + m)) + ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
 (3 + m)/2, -((b*x^2)/a)])/(a*b^2*(1 + m))
```

Rubi [A] time = 0.151754, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^{m+1}(bc-ad)^2 {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^2(m+1)}+\frac{dx^{m+1}(2bc-ad)}{b^2(m+1)}+\frac{d^2x^{m+3}}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x^(1 + m))/(b^2*(1 + m)) + (d^2*x^(3 + m))/(b*(3
 + m)) + ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
 (3 + m)/2, -((b*x^2)/a)])/(a*b^2*(1 + m))
```

Rubi in Sympy [A] time = 26.7751, size = 76, normalized size = 0.81

$$\frac{d^2x^{m+3}}{b(m+3)}-\frac{dx^{m+1}(ad-2bc)}{b^2(m+1)}+\frac{x^{m+1}(ad-bc)^2 {}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ab^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] d**2*x**(m + 3)/(b*(m + 3)) - d*x**(m + 1)*(a*d - 2*b*c)/(b**2*(m
 + 1)) + x**(m + 1)*(a*d - b*c)**2*hyper((1, m/2 + 1/2), (m/2 + 3
/2,), -b*x**2/a)/(a*b**2*(m + 1))
```

Mathematica [A] time = 0.14129, size = 118, normalized size = 1.26

$$\frac{x^{m+1}\left(\frac{c^2 {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}+dx^2\left(\frac{2c\, {}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{m+3}+\frac{dx^2 {}_2F_1\left(1,\frac{m+5}{2};\frac{m+7}{2};-\frac{bx^2}{a}\right)}{m+5}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*((c^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(1 + m) + d*x^2*((2*c*Hypergeometric2F1[1, (3 + m)/2, (
5 + m)/2, -((b*x^2)/a)])/(3 + m) + (d*x^2*Hypergeometric2F1[1, (5
 + m)/2, (7 + m)/2, -((b*x^2)/a)])/(5 + m))))/a
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{x^m \left(d x^2+c\right)^2}{b x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^2+c\right)^2 x^m}{b x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(d^2 x^4+2\, c d x^2+c^2\right) x^m}{b x^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b*x^2 + a), x)
```

Sympy [A] time = 34.3137, size = 299, normalized size = 3.18

$$\frac{c^2 m x x^m \Phi\left(\frac{b x^2 e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\,\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^2 x x^m \Phi\left(\frac{b x^2 e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\,\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c d m x^3 x^m \Phi\left(\frac{b x^2 e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\,\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3 c d x^3 x^m \Phi\left(\frac{b x^2 e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\,\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{d^2 m x^5 x^m \Phi\left(\frac{b x^2 e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\,\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{5 d^2 x^5 x^m \Phi\left(\frac{b x^2 e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\,\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] c**2*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*ga
mma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**2*x*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m
/2 + 3/2)) + c*d*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1
, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*a*gamma(m/2 + 5/2)) + 3*c*d*x**3
```

```
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
+ 3/2)/(2*a*gamma(m/2 + 5/2)) + d**2*m*x**5*x**m*lerchphi(b*x**2*
exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2
+ 7/2)) + 5*d**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^2 x^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^m/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

3.333 $\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}(bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{dx^{m+1}}{b(m+1)}$$

```
[Out] (d*x^(1 + m))/(b*(1 + m)) + ((b*c - a*d)*x^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rubi [A] time = 0.107302, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^{m+1}(bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{dx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (d*x^(1 + m))/(b*(1 + m)) + ((b*c - a*d)*x^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rubi in Sympy [A] time = 12.9328, size = 49, normalized size = 0.74

$$\frac{dx^{m+1}}{b\,(m+1)}-\frac{x^{m+1}\,(ad-bc)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ab\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] d*x**(m + 1)/(b*(m + 1)) - x**(m + 1)*(a*d - b*c)*hyper((1, m/2 +
 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*b*(m + 1))
```

Mathematica [A] time = 0.06636, size = 55, normalized size = 0.83

$$\frac{x^{m+1}\left((bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ad\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^m\left(dx^2+c\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)\, x^m}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^m/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2 + c)\, x^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^m/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

Sympy [A] time = 15.4561, size = 190, normalized size = 2.88

$$\frac{cmxx^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{cxx^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4a\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{dmx^3 x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \left(\frac{m}{2} + \frac{3}{2}\right)}{4a\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3dx^3 x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \left(\frac{m}{2} + \frac{3}{2}\right)}{4a\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] c*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c*x*x**m*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3
/2)) + d*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*d*x**3*x**m*lerc
hphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*
a*gamma(m/2 + 5/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)\, x^m}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^m/(b*x^2 + a),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

3.334 $\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=102

$$\frac{bx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}$$

```
[Out] (b*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a*(b*c - a*d)*(1 + m)) - (d*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m))
```

Rubi [A] time = 0.130967, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{bx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a*(b*c - a*d)*(1 + m)) - (d*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m))
```

Rubi in Sympy [A] time = 21.2751, size = 76, normalized size = 0.75

$$\frac{dx^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{c}\right)}{c\,(m+1)\,(ad-bc)}-\frac{bx^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{a\,(m+1)\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] d*x**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -d*x**2/c)/(c*(m
 + 1)*(a*d - b*c)) - b*x**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/
2,), -b*x**2/a)/(a*(m + 1)*(a*d - b*c))
```

Mathematica [A] time = 0.0761294, size = 85, normalized size = 0.83

$$\frac{x^{m+1}\left(ad\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)-bc\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(b*c*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)]) + a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)]))/(a*c*(-(b*c) + a*d)*(1 + m))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Sympy [A] time = 93.4697, size = 354, normalized size = 3.47

$$\frac{amx^m\left(\frac{ae^{i\pi}}{bx^2},1,-\frac{m}{2}+\frac{3}{2}\right)^2\left(-\frac{m}{2}+\frac{3}{2}\right)}{x^3\left(4abd\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)-4b^2c\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)\right)}-\frac{3ax^m\left(\frac{ae^{i\pi}}{bx^2},1,-\frac{m}{2}+\frac{3}{2}\right)^2\left(-\frac{m}{2}+\frac{3}{2}\right)}{x^3\left(4abd\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)-4b^2c\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)\right)}+\frac{bmx^m\left(\frac{ce^{i\pi}}{dx^2},1,-\frac{m}{2}+\frac{1}{2}\right)\left(-\frac{m}{2}+\frac{1}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)}{x\left(4abd\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)-4b^2c\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)\right)}-\frac{bx^m\left(\frac{ce^{i\pi}}{dx^2},1,-\frac{m}{2}+\frac{1}{2}\right)\left(-\frac{m}{2}+\frac{1}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)}{x\left(4abd\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)-4b^2c\left(-\frac{m}{2}+\frac{3}{2}\right)\left(-\frac{m}{2}+\frac{5}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*m*x**m*lerchphi(a*exp_polar(I*pi)/(b*x**2), 1, -m/2 + 3/2)*gamm
a(-m/2 + 3/2)**2/(x**3*(4*a*b*d*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/
2) - 4*b**2*c*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2))) - 3*a*x**m*le
rchphi(a*exp_polar(I*pi)/(b*x**2), 1, -m/2 + 3/2)*gamma(-m/2 + 3/
2)**2/(x**3*(4*a*b*d*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2) - 4*b**2
```

```
*c*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2))) + b*m*x**m*lerchphi(c*ex
p_polar(I*pi)/(d*x**2), 1, -m/2 + 1/2)*gamma(-m/2 + 1/2)*gamma(-m
/2 + 5/2)/(x*(4*a*b*d*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2) - 4*b**
2*c*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2))) - b*x**m*lerchphi(c*exp
_polar(I*pi)/(d*x**2), 1, -m/2 + 1/2)*gamma(-m/2 + 1/2)*gamma(-m/
2 + 5/2)/(x*(4*a*b*d*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2) - 4*b**2
*c*gamma(-m/2 + 3/2)*gamma(-m/2 + 5/2)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)(dx^2 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.335 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)} dx$

Optimal. Leaf size=156

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\,(a+bx^2)\,(bc-ad)}$$

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a
*d*(3 - m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/(2*a^2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c -
a*d)^2*(1 + m))
```

Rubi [A] time = 0.500411, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{bx^{m+1}(ad(3-m)-b(c-cm))\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) - (b*(a*d*(3 - m) - b
*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/(2*a^2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c -
a*d)^2*(1 + m))
```

Rubi in Sympy [A] time = 96.1174, size = 128, normalized size = 0.82

$$\frac{d^2x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{c}\right)}{c\,(m+1)\,(ad-bc)^2}-\frac{bx^{m+1}}{2a\,(a+bx^2)\,(ad-bc)}-\frac{bx^{m+1}\,(-adm+3ad+bcm-bc)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{2a^2\,(m+1)\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d**2*x**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -d*x**2/c)/(c
*(m + 1)*(a*d - b*c)**2) - b*x**(m + 1)/(2*a*(a + b*x**2)*(a*d -
b*c)) - b*x**(m + 1)*(-a*d*m + 3*a*d + b*c*m - b*c)*hyper((1, m/2
 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(2*a**2*(m + 1)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.12763, size = 127, normalized size = 0.81

$$\frac{x^{m+1}\left(a^2d^2\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)-abcd\,_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+bc(bc-ad)\,_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2c(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)] + b*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)]))/(a^2*c*(b*c - a*d)^2*(1 + m))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{b^2dx^6+(b^2c+2\,abd)x^4+a^2c+(2\,abc+a^2d)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*
c + a^2*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.336 $\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx$

Optimal. Leaf size=234

$$\frac{bx^{m+1}(bc(3-m)-ad(7-m))}{8a^2(a+bx^2)(bc-ad)^2}+\frac{bx^{m+1}\left(a^2d^2\left(m^2-8m+15\right)-2abcd\left(m^2-6m+5\right)+b^2c^2\left(m^2-4m+3\right)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{8a^3(m+1)(bc-ad)^3}-\frac{d^3x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^3}+\frac{bx^{m+1}}{4a(a+bx^2)^2(bc-ad)}$$

```
[Out] (b*x^(1 + m))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(b*c*(3 - m) -
 a*d*(7 - m))*x^(1 + m))/(8*a^2*(b*c - a*d)^2*(a + b*x^2)) + (b*(
a^2*d^2*(15 - 8*m + m^2) - 2*a*b*c*d*(5 - 6*m + m^2) + b^2*c^2*(3
 - 4*m + m^2))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((b*x^2)/a)])/(8*a^3*(b*c - a*d)^3*(1 + m)) - (d^3*x^(1 + m)*
Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c
 - a*d)^3*(1 + m))
```

Rubi [A] time = 1.00289, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{bx^{m+1}(bc(3-m)-ad(7-m))}{8a^2(a+bx^2)(bc-ad)^2}+\frac{bx^{m+1}\left(a^2d^2\left(m^2-8m+15\right)-2abcd\left(m^2-6m+5\right)+b^2c^2\left(m^2-4m+3\right)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{8a^3(m+1)(bc-ad)^3}-\frac{d^3x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^3}+\frac{bx^{m+1}}{4a(a+bx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(b*c*(3 - m) -
 a*d*(7 - m))*x^(1 + m))/(8*a^2*(b*c - a*d)^2*(a + b*x^2)) + (b*(
a^2*d^2*(15 - 8*m + m^2) - 2*a*b*c*d*(5 - 6*m + m^2) + b^2*c^2*(3
 - 4*m + m^2))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((b*x^2)/a)])/(8*a^3*(b*c - a*d)^3*(1 + m)) - (d^3*x^(1 + m)*
Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c
 - a*d)^3*(1 + m))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.41484, size = 196, normalized size = 0.84

$$\frac{ac(m+3)x^{m+1}F_1\left(\frac{m+1}{2};3,1;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(m+1)(a+bx^2)^3(c+dx^2)\left(ac(m+3)F_1\left(\frac{m+1}{2};3,1;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(adF_1\left(\frac{m+3}{2};3,2;\frac{m+5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{m}{}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (a*c*(3 + m)*x^(1 + m)*AppellF1[(1 + m)/2, 3, 1, (3 + m)/2, -((b*
x^2)/a), -((d*x^2)/c)])/((1 + m)*(a + b*x^2)^3*(c + d*x^2)*(a*c*(
3 + m)*AppellF1[(1 + m)/2, 3, 1, (3 + m)/2, -((b*x^2)/a), -((d*x^
2)/c)] - 2*x^2*(a*d*AppellF1[(3 + m)/2, 3, 2, (5 + m)/2, -((b*x^2
)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[(3 + m)/2, 4, 1, (5 + m)/2,
-((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^3dx^8+(b^3c+3\,ab^2d)x^6+3\,(ab^2c+a^2bd)x^4+a^3c+(3\,a^2bc+a^3d)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*d*x^8 + (b^3*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c +
a^2*b*d)*x^4 + a^3*c + (3*a^2*b*c + a^3*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

3.337 $\int \frac{x^m (c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal. Leaf size=201

$$\frac{x^{m+1}(bc-ad)^2(ad(m+5)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^3(m+1)} - \frac{dx^{m+1}\left(a^2d^2(m+5)-3abcd(m+3)+2b^2c^2(m+1)\right)}{2ab^3(m+1)} - \frac{d^2x^{m+3}(bc(m+3)-ad(m+5))}{2ab^2(m+3)} + \frac{x^{m+1}\left(c+dx^2\right)^2(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] -(d*(2*b^2*c^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m))*x^(
1 + m))/(2*a*b^3*(1 + m)) - (d^2*(b*c*(3 + m) - a*d*(5 + m))*x^(3
 + m))/(2*a*b^2*(3 + m)) + ((b*c - a*d)*x^(1 + m)*(c + d*x^2)^2)/
(2*a*b*(a + b*x^2)) + ((b*c - a*d)^2*(a*d*(5 + m) + b*(c - c*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)
])/(2*a^2*b^3*(1 + m))
```

Rubi [A] time = 0.55184, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x^{m+1}(bc-ad)^2(ad(m+5)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^3(m+1)} - \frac{dx^{m+1}\left(a^2d^2(m+5)-3abcd(m+3)+2b^2c^2(m+1)\right)}{2ab^3(m+1)} - \frac{d^2x^{m+3}(bc(m+3)-ad(m+5))}{2ab^2(m+3)} + \frac{x^{m+1}\left(c+dx^2\right)^2(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -(d*(2*b^2*c^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m))*x^(
1 + m))/(2*a*b^3*(1 + m)) - (d^2*(b*c*(3 + m) - a*d*(5 + m))*x^(3
 + m))/(2*a*b^2*(3 + m)) + ((b*c - a*d)*x^(1 + m)*(c + d*x^2)^2)/
(2*a*b*(a + b*x^2)) + ((b*c - a*d)^2*(a*d*(5 + m) + b*(c - c*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)
])/(2*a^2*b^3*(1 + m))
```

Rubi in Sympy [A] time = 139.365, size = 206, normalized size = 1.02

$$-\frac{x^{m+1}\left(c+dx^2\right)^2(ad-bc)}{2ab\left(a+bx^2\right)} + \frac{d^2x^{m+3}(adm+5ad-bcm-3bc)}{2ab^2(m+3)} - \frac{dx^{m+1}\left(a^2d^2m+5a^2d^2-3abcdm-9abcd+2b^2c^2m+2b^2c^2\right)}{2ab^3(m+1)} + \frac{x^{m+1}(ad-bc)^2(adm+5ad-bcm+bc)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{2a^2b^3(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x**(m + 1)*(c + d*x**2)**2*(a*d - b*c)/(2*a*b*(a + b*x**2)) + d*
*2*x**(m + 3)*(a*d*m + 5*a*d - b*c*m - 3*b*c)/(2*a*b**2*(m + 3))
- d*x**(m + 1)*(a**2*d**2*m + 5*a**2*d**2 - 3*a*b*c*d*m - 9*a*b*c
```

```
*d + 2*b**2*c**2*m + 2*b**2*c**2)/(2*a*b**3*(m + 1)) + x**(m + 1)
*(a*d - b*c)**2*(a*d*m + 5*a*d - b*c*m + b*c)*hyper((1, m/2 + 1/2
), (m/2 + 3/2,), -b*x**2/a)/(2*a**2*b**3*(m + 1))
```

Mathematica [A] time = 0.267561, size = 159, normalized size = 0.79

$$\frac{x^{m+1}\left(\frac{c^3\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}+dx^2\left(\frac{3c^2\,{}_2F_1\left(2,\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{m+3}+dx^2\left(\frac{3c\,{}_2F_1\left(2,\frac{m+5}{2};\frac{m+7}{2};-\frac{bx^2}{a}\right)}{m+5}+\frac{dx^2\,{}_2F_1\left(2,\frac{m+7}{2};\frac{m+9}{2};-\frac{bx^2}{a}\right)}{m+7}\right)\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*((c^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(1 + m) + d*x^2*((3*c^2*Hypergeometric2F1[2, (3 + m)/2,
 (5 + m)/2, -((b*x^2)/a)])/(3 + m) + d*x^2*((3*c*Hypergeometric2F
1[2, (5 + m)/2, (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (d*x^2*Hyperg
eometric2F1[2, (7 + m)/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))))/a
^2
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m\left(dx^2+c\right)^3}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3x^m}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3\right)x^m}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b^2*x^4
 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^3 x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

3.338 $\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=120

$$\frac{x^{m+1}(bc-ad)(ad(m+3)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^{m+1}}{b^2(m+1)}$$

```
[Out] (d^2*x^(1 + m))/(b^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*a*b^
2*(a + b*x^2)) + ((b*c - a*d)*(a*d*(3 + m) + b*(c - c*m))*x^(1 +
m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a
^2*b^2*(1 + m))
```

Rubi [A] time = 0.280486, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x^{m+1}(bc-ad)(ad(m+3)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^{m+1}}{b^2(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x^(1 + m))/(b^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*a*b^
2*(a + b*x^2)) + ((b*c - a*d)*(a*d*(3 + m) + b*(c - c*m))*x^(1 +
m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a
^2*b^2*(1 + m))
```

Rubi in Sympy [A] time = 42.4356, size = 104, normalized size = 0.87

$$\frac{d^2x^{m+1}}{b^2(m+1)}+\frac{x^{m+1}(ad-bc)^2}{2ab^2(a+bx^2)}-\frac{x^{m+1}(ad-bc)(adm+3ad-bcm+bc)\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{2a^2b^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] d**2*x**(m + 1)/(b**2*(m + 1)) + x**(m + 1)*(a*d - b*c)**2/(2*a*b
**2*(a + b*x**2)) - x**(m + 1)*(a*d - b*c)*(a*d*m + 3*a*d - b*c*m
 + b*c)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(2*a**2*b*
*2*(m + 1))
```

Mathematica [A] time = 0.176373, size = 118, normalized size = 0.98

$$\frac{x^{m+1}\left(\frac{c^2\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}+dx^2\left(\frac{2c\,{}_2F_1\left(2,\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{m+3}+\frac{dx^2\,{}_2F_1\left(2,\frac{m+5}{2};\frac{m+7}{2};-\frac{bx^2}{a}\right)}{m+5}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*((c^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(1 + m) + d*x^2*((2*c*Hypergeometric2F1[2, (3 + m)/2, (
```

```
5 + m)/2, -((b*x^2)/a)])/(3 + m) + (d*x^2*Hypergeometric2F1[2, (5
 + m)/2, (7 + m)/2, -((b*x^2)/a)])/(5 + m))))/a^2
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{x^m (dx^2+c)^2}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^2 x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^2x^4+2\,cdx^2+c^2)\,x^m}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b^2*x^4 + 2*a*b*x^2 + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**m*(c + d*x**2)**2/(a + b*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^2 x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

3.339 $\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(ad(m+1)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(bc-ad)}{2ab\,(a+bx^2)}$$

```
[Out] ((b*c - a*d)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*d*(1 + m) + b*(
c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/(2*a^2*b*(1 + m))
```

Rubi [A] time = 0.121625, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^{m+1}(ad(m+1)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*d*(1 + m) + b*(
c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/(2*a^2*b*(1 + m))
```

Rubi in Sympy [A] time = 14.8568, size = 71, normalized size = 0.76

$$-\frac{x^{m+1}\,(ad-bc)}{2ab\,(a+bx^2)}+\frac{x^{m+1}\,(ad\,(m+1)+bc\,(-m+1))\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{2a^2b\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x**(m + 1)*(a*d - b*c)/(2*a*b*(a + b*x**2)) + x**(m + 1)*(a*d*(m
 + 1) + b*c*(-m + 1))*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2
/a)/(2*a**2*b*(m + 1))
```

Mathematica [A] time = 0.0722982, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((bc-ad)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ad\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2b(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)] + (b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)]))/(a^2*b*(1 + m))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^m (dx^2 + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)

[Out] int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c) x^m}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2 + c) x^m}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2,x, algorithm="fricas")

[Out] integral((d*x^2 + c)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 137.2, size = 906, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(d*x**2+c)/(b*x**2+a)**2,x)

[Out] c*(-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma
(m/2 + 3/2)) + 2*a*m*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 +
3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + a*x*x**m*lerchphi(b*x**2
*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(
m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*x*x**m*gamma(m
/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/
2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*
gamma(m/2 + 3/2)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**
2*b*x**2*gamma(m/2 + 3/2))) + d*(-a*m**2*x**3*x**m*lerchphi(b*x**
2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma
(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x**3*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/
(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*
```

```
m*x**3*x**m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*
x**2*gamma(m/2 + 5/2)) - 3*a*x**3*x**m*lerchphi(b*x**2*exp_polar(
I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2)
+ 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*x**3*x**m*gamma(m/2 + 3/2
)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - b*
m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*g
amma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/
2 + 5/2)) - 4*b*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*
x**2*gamma(m/2 + 5/2)) - 3*b*x**5*x**m*lerchphi(b*x**2*exp_polar(
I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2)
+ 8*a**2*b*x**2*gamma(m/2 + 5/2)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)\,x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

3.340 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=156

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a
*d*(3 - m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/(2*a^2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c -
a*d)^2*(1 + m))
```

Rubi [A] time = 0.468687, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{bx^{m+1}(ad(3-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) - (b*(a*d*(3 - m) - b
*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/(2*a^2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c -
a*d)^2*(1 + m))
```

Rubi in Sympy [A] time = 95.4734, size = 128, normalized size = 0.82

$$\frac{d^2x^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{c}\right)}{c\,(m+1)\,(ad-bc)^2}-\frac{bx^{m+1}}{2a\left(a+bx^2\right)(ad-bc)}-\frac{bx^{m+1}\left(-adm+3ad+bcm-bc\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{2a^2\,(m+1)\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d**2*x**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -d*x**2/c)/(c
*(m + 1)*(a*d - b*c)**2) - b*x**(m + 1)/(2*a*(a + b*x**2)*(a*d -
b*c)) - b*x**(m + 1)*(-a*d*m + 3*a*d + b*c*m - b*c)*hyper((1, m/2
 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(2*a**2*(m + 1)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.112833, size = 127, normalized size = 0.81

$$\frac{x^{m+1}\left(a^2d^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)-abcd\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+bc(bc-ad)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^2c(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)] + b*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)]))/(a^2*c*(b*c - a*d)^2*(1 + m))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*
c + a^2*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.341 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{b^2x^{m+1}(ad(5-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^3}-\frac{d^2x^{m+1}(ad(1-m)-bc(5-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2(m+1)(bc-ad)^3}+\frac{dx^{m+1}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

```
[Out] (d*(b*c + a*d)*x^(1 + m))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*
x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^2*(a*d*
(5 - m) - b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
(3 + m)/2, -((b*x^2)/a)])/(2*a^2*(b*c - a*d)^3*(1 + m)) - (d^2*(a
*d*(1 - m) - b*c*(5 - m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*(b*c - a*d)^3*(1 + m))
```

Rubi [A] time = 0.963905, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{b^2x^{m+1}(ad(5-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^3}-\frac{d^2x^{m+1}(ad(1-m)-bc(5-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2(m+1)(bc-ad)^3}+\frac{dx^{m+1}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x^(1 + m))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*
x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^2*(a*d*
(5 - m) - b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
(3 + m)/2, -((b*x^2)/a)])/(2*a^2*(b*c - a*d)^3*(1 + m)) - (d^2*(a
*d*(1 - m) - b*c*(5 - m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*(b*c - a*d)^3*(1 + m))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.480614, size = 195, normalized size = 0.85

$$\frac{ac(m+3)x^{m+1}F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(m+1)\left(a+bx^2\right)^2\left(c+dx^2\right)^2\left(ac(m+3)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-4x^2\left(adF_1\left(\frac{m+3}{2};2,3;\frac{m+5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{m}{\ldots}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (a*c*(3 + m)*x^(1 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, -((b*
x^2)/a), -((d*x^2)/c)])/((1 + m)*(a + b*x^2)^2*(c + d*x^2)^2*(a*c
*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, -((b*x^2)/a), -((d*
x^2)/c)] - 4*x^2*(a*d*AppellF1[(3 + m)/2, 2, 3, (5 + m)/2, -((b*x
^2)/a), -((d*x^2)/c)] + b*c*AppellF1[(3 + m)/2, 3, 2, (5 + m)/2,
-((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2\,(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2d^2x^8+2\,(b^2cd+abd^2)x^6+(b^2c^2+4\,abcd+a^2d^2)x^4+a^2c^2+2\,(abc^2+a^2cd)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d^2*x^8 + 2*(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.342 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=325

$$-\frac{b^3x^{m+1}(ad(7-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^4}+\frac{dx^{m+1}\left(-a^2d^2(3-m)+abcd(11-m)+4b^2c^2\right)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)-2abcd\left(m^2-8m+7\right)+b^2c^2\left(m^2-12m+35\right)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3(m+1)(bc-ad)^4}+\frac{dx^{m+1}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] (d*(2*b*c + a*d)*x^(1 + m))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) +
 (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(
4*b^2*c^2 - a^2*d^2*(3 - m) + a*b*c*d*(11 - m))*x^(1 + m))/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) - (b^3*(a*d*(7 - m) - b*(c - c*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)
])/(2*a^2*(b*c - a*d)^4*(1 + m)) + (d^2*(b^2*c^2*(35 - 12*m + m^2
) - 2*a*b*c*d*(7 - 8*m + m^2) + a^2*d^2*(3 - 4*m + m^2))*x^(1 + m
)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c^
3*(b*c - a*d)^4*(1 + m))
```

Rubi [A] time = 1.60165, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{b^3x^{m+1}(bc(1-m)-ad(7-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^4}+\frac{dx^{m+1}\left(-a^2d^2(3-m)+abcd(11-m)+4b^2c^2\right)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)-2abcd\left(m^2-8m+7\right)+b^2c^2\left(m^2-12m+35\right)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3(m+1)(bc-ad)^4}+\frac{dx^{m+1}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x^(1 + m))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) +
 (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(
4*b^2*c^2 - a^2*d^2*(3 - m) + a*b*c*d*(11 - m))*x^(1 + m))/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^3*(b*c*(1 - m) - a*d*(7 - m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)
])/(2*a^2*(b*c - a*d)^4*(1 + m)) + (d^2*(b^2*c^2*(35 - 12*m + m^2
) - 2*a*b*c*d*(7 - 8*m + m^2) + a^2*d^2*(3 - 4*m + m^2))*x^(1 + m
)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c^
3*(b*c - a*d)^4*(1 + m))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.663452, size = 197, normalized size = 0.61

$$\frac{ac(m+3)x^{m+1}F_1\left(\frac{m+1}{2};2,3;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(m+1)\left(a+bx^2\right)^2\left(c+dx^2\right)^3\left(ac(m+3)F_1\left(\frac{m+1}{2};2,3;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(3adF_1\left(\frac{m+3}{2};2,4;\frac{m+5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2bcF_1\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (a*c*(3 + m)*x^(1 + m)*AppellF1[(1 + m)/2, 2, 3, (3 + m)/2, -((b*
x^2)/a), -((d*x^2)/c)])/((1 + m)*(a + b*x^2)^2*(c + d*x^2)^3*(a*c
*(3 + m)*AppellF1[(1 + m)/2, 2, 3, (3 + m)/2, -((b*x^2)/a), -((d*
x^2)/c)] - 2*x^2*(3*a*d*AppellF1[(3 + m)/2, 2, 4, (5 + m)/2, -((b
*x^2)/a), -((d*x^2)/c)] + 2*b*c*AppellF1[(3 + m)/2, 3, 3, (5 + m)
/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2d^3x^{10}+(3\,b^2cd^2+2\,abd^3)x^8+(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6+a^2c^3+(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^4+(2\,abc^3+3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d^3*x^10 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + (3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3 + (b^2*c^3 + 6*a*
b*c^2*d + 3*a^2*c*d^2)*x^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

3.343 $\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{17}bBx^{17/2}$$

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2)
)/17
```

Rubi [A] time = 0.0516741, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2)
)/17
```

Rubi in Sympy [A] time = 6.97539, size = 41, normalized size = 1.05

$$\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{17}{2}}}{17}+x^{\frac{13}{2}}\left(\frac{2Ab}{13}+\frac{2Ba}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*B*b*x**(17/2)/17 + x**(13/2)*(2*A*b/13 + 2*B
*a/13)
```

Mathematica [A] time = 0.0204658, size = 33, normalized size = 0.85

$$\frac{2x^{9/2}\left(153x^2(aB+Ab)+221aA+117bBx^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(221*a*A + 153*(A*b + a*B)*x^2 + 117*b*B*x^4))/1989
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$\frac{234\,bBx^4+306\,Ax^2b+306\,Bx^2a+442\,Aa}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/1989*x^(9/2)*(117*B*b*x^4+153*A*b*x^2+153*B*a*x^2+221*A*a)
```

Maxima [A] time = 1.34506, size = 36, normalized size = 0.92

$$\frac{2}{17} Bbx^{\frac{17}{2}} + \frac{2}{13} (Ba + Ab)x^{\frac{13}{2}} + \frac{2}{9} Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/9*A*a*x^(9/2)
```

Fricas [A] time = 0.218747, size = 43, normalized size = 1.1

$$\frac{2}{1989} \left(117 Bbx^8 + 153 (Ba + Ab)x^6 + 221 Aax^4\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*B*b*x^8 + 153*(B*a + A*b)*x^6 + 221*A*a*x^4)*sqrt(x)
```

Sympy [A] time = 36.9228, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9} + \frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Bax^{\frac{13}{2}}}{13} + \frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*
b*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.213472, size = 39, normalized size = 1.

$$\frac{2}{17} Bbx^{\frac{17}{2}} + \frac{2}{13} Bax^{\frac{13}{2}} + \frac{2}{13} Abx^{\frac{13}{2}} + \frac{2}{9} Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/9*A
*a*x^(9/2)
```

3.344 $\int x^{5/2} \left(a + bx^2\right) \left(A + Bx^2\right) \, dx$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(aB + Ab) + \frac{2}{7}aAx^{7/2} + \frac{2}{15}bBx^{15/2}$$

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2)
)/15
```

Rubi [A] time = 0.0500869, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{2}{11}x^{11/2}(aB + Ab) + \frac{2}{7}aAx^{7/2} + \frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2)
)/15
```

Rubi in Sympy [A] time = 7.10942, size = 41, normalized size = 1.05

$$\frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{15}{2}}}{15} + x^{\frac{11}{2}}\left(\frac{2Ab}{11} + \frac{2Ba}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*B*b*x**(15/2)/15 + x**(11/2)*(2*A*b/11 + 2*B
*a/11)
```

Mathematica [A] time = 0.0175703, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(105x^2(aB + Ab) + 165aA + 77bBx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(7/2)*(165*a*A + 105*(A*b + a*B)*x^2 + 77*b*B*x^4))/1155
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$\frac{154\,bBx^4 + 210\,Ax^2b + 210\,Bx^2a + 330\,Aa}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/1155*x^(7/2)*(77*B*b*x^4+105*A*b*x^2+105*B*a*x^2+165*A*a)
```

Maxima [A] time = 1.34799, size = 36, normalized size = 0.92

$$\frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{11}\,(Ba + Ab)x^{\frac{11}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/7*A*a*x^(7/2)
```

Fricas [A] time = 0.225491, size = 43, normalized size = 1.1

$$\frac{2}{1155}\left(77\,Bbx^{7} + 105\,(Ba + Ab)x^{5} + 165\,Aax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b*x^7 + 105*(B*a + A*b)*x^5 + 165*A*a*x^3)*sqrt(x)
```

Sympy [A] time = 20.0121, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Bax^{\frac{11}{2}}}{11} + \frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*
b*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.216875, size = 39, normalized size = 1.

$$\frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{11}\,Bax^{\frac{11}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/7*A
*a*x^(7/2)
```

3.345 $\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{13}bBx^{13/2}$$

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/
13
```

Rubi [A] time = 0.0497932, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/
13
```

Rubi in Sympy [A] time = 7.06465, size = 41, normalized size = 1.05

$$\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{13}{2}}}{13}+x^{\frac{9}{2}}\left(\frac{2Ab}{9}+\frac{2Ba}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*B*b*x**(13/2)/13 + x**(9/2)*(2*A*b/9 + 2*B*a
/9)
```

Mathematica [A] time = 0.0169249, size = 33, normalized size = 0.85

$$\frac{2}{585}x^{5/2}\left(65x^2(aB+Ab)+117aA+45bBx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(117*a*A + 65*(A*b + a*B)*x^2 + 45*b*B*x^4))/585
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$\frac{90\,bBx^4+130\,Ax^2b+130\,Bx^2a+234\,Aa}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/585*x^(5/2)*(45*B*b*x^4+65*A*b*x^2+65*B*a*x^2+117*A*a)
```

Maxima [A] time = 1.35172, size = 36, normalized size = 0.92

$$\frac{2}{13}Bbx^{\frac{13}{2}} + \frac{2}{9}(Ba + Ab)x^{\frac{9}{2}} + \frac{2}{5}Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/5*A*a*x^(5/2)
```

Fricas [A] time = 0.21123, size = 43, normalized size = 1.1

$$\frac{2}{585}\left(45\,Bbx^{6} + 65\,(Ba + Ab)x^{4} + 117\,Aax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b*x^6 + 65*(B*a + A*b)*x^4 + 117*A*a*x^2)*sqrt(x)
```

Sympy [A] time = 7.92613, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5} + \frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Bax^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x*
*(13/2)/13
```

GIAC/XCAS [A] time = 0.216553, size = 39, normalized size = 1.

$$\frac{2}{13}Bbx^{\frac{13}{2}} + \frac{2}{9}Bax^{\frac{9}{2}} + \frac{2}{9}Abx^{\frac{9}{2}} + \frac{2}{5}Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/5*A*a*x
^(5/2)
```

3.346 $\int \sqrt{x} \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=39

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/
11
```

Rubi [A] time = 0.0470413, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/
11
```

Rubi in Sympy [A] time = 7.1917, size = 41, normalized size = 1.05

$$\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{11}{2}}}{11}+x^{\frac{7}{2}}\left(\frac{2Ab}{7}+\frac{2Ba}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*B*b*x**(11/2)/11 + x**(7/2)*(2*A*b/7 + 2*B*a
/7)
```

Mathematica [A] time = 0.0169348, size = 33, normalized size = 0.85

$$\frac{2}{231}x^{3/2}\left(33x^2(aB+Ab)+77aA+21bBx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(77*a*A + 33*(A*b + a*B)*x^2 + 21*b*B*x^4))/231
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$\frac{42\,bBx^4+66\,Ax^2b+66\,Bx^2a+154\,Aa}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*B*b*x^4+33*A*b*x^2+33*B*a*x^2+77*A*a)
```

Maxima [A] time = 1.35088, size = 36, normalized size = 0.92

$$\frac{2}{11}Bbx^{\frac{11}{2}} + \frac{2}{7}(Ba + Ab)x^{\frac{7}{2}} + \frac{2}{3}Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/7*(B*a + A*b)*x^(7/2) + 2/3*A*a*x^(3/2)
```

Fricas [A] time = 0.220071, size = 41, normalized size = 1.05

$$\frac{2}{231}\left(21\,Bbx^{5} + 33\,(Ba + Ab)x^{3} + 77\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b*x^5 + 33*(B*a + A*b)*x^3 + 77*A*a*x)*sqrt(x)
```

Sympy [A] time = 2.64564, size = 37, normalized size = 0.95

$$\frac{2Aax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{11}{2}}}{11} + \frac{2x^{\frac{7}{2}}\,(Ab + Ba)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*B*b*x**(11/2)/11 + 2*x**(7/2)*(A*b + B*a)/7
```

GIAC/XCAS [A] time = 0.250886, size = 39, normalized size = 1.

$$\frac{2}{11}Bbx^{\frac{11}{2}} + \frac{2}{7}Bax^{\frac{7}{2}} + \frac{2}{7}Abx^{\frac{7}{2}} + \frac{2}{3}Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2/3*A*a*x
^(3/2)
```

3.347 $\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

Rubi [A] time = 0.0481667, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]

[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

Rubi in Sympy [A] time = 6.97293, size = 39, normalized size = 1.05

$$2Aa\sqrt{x}+\frac{2Bbx^{\frac{9}{2}}}{9}+x^{\frac{5}{2}}\left(\frac{2Ab}{5}+\frac{2Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)/x**(1/2),x)

[Out] 2*A*a*sqrt(x) + 2*B*b*x**(9/2)/9 + x**(5/2)*(2*A*b/5 + 2*B*a/5)
```

Mathematica [A] time = 0.0169575, size = 33, normalized size = 0.89

$$\frac{2}{45}\sqrt{x}\left(9x^2(aB+Ab)+45aA+5bBx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]

[Out] (2*Sqrt[x]*(45*a*A + 9*(A*b + a*B)*x^2 + 5*b*B*x^4))/45
```

Maple [A] time = 0.006, size = 32, normalized size = 0.9

$$\frac{10\,bBx^4+18\,Ax^2b+18\,Bx^2a+90\,Aa}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)/x^(1/2),x)

[Out] 2/45*x^(1/2)*(5*B*b*x^4+9*A*b*x^2+9*B*a*x^2+45*A*a)
```

Maxima [A] time = 1.34509, size = 36, normalized size = 0.97

$$\frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{5}\,(Ba + Ab)x^{\frac{5}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/5*(B*a + A*b)*x^(5/2) + 2*A*a*sqrt(x)
```

Fricas [A] time = 0.222465, size = 39, normalized size = 1.05

$$\frac{2}{45}\,\left(5\,Bbx^4 + 9\,(Ba + Ab)x^2 + 45\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*b*x^4 + 9*(B*a + A*b)*x^2 + 45*A*a)*sqrt(x)
```

Sympy [A] time = 2.22367, size = 44, normalized size = 1.19

$$2Aa\sqrt{x} + \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Bax^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(9
/2)/9
```

GIAC/XCAS [A] time = 0.213575, size = 39, normalized size = 1.05

$$\frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{5}\,Bax^{\frac{5}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) + 2*A*a*sqrt(
x)
```

3.348 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

Rubi [A] time = 0.0496518, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

Rubi in Sympy [A] time = 6.89538, size = 39, normalized size = 1.05

$$-\frac{2Aa}{\sqrt{x}}+\frac{2Bbx^{\frac{7}{2}}}{7}+x^{\frac{3}{2}}\left(\frac{2Ab}{3}+\frac{2Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a/sqrt(x) + 2*B*b*x**(7/2)/7 + x**(3/2)*(2*A*b/3 + 2*B*a/3)
```

Mathematica [A] time = 0.0176442, size = 33, normalized size = 0.89

$$\frac{2\left(7x^2(aB+Ab)-21aA+3bBx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-21*a*A + 7*(A*b + a*B)*x^2 + 3*b*B*x^4))/(21*Sqrt[x])
```

Maple [A] time = 0.005, size = 32, normalized size = 0.9

$$-\frac{-6\,bBx^4-14\,Ax^2b-14\,Bx^2a+42\,Aa}{21}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/21*(-3*B*b*x^4-7*A*b*x^2-7*B*a*x^2+21*A*a)/x^(1/2)
```

Maxima [A] time = 1.34693, size = 36, normalized size = 0.97

$$\frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{3}\,(Ba + Ab)x^{\frac{3}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b*x^(7/2) + 2/3*(B*a + A*b)*x^(3/2) - 2*A*a/sqrt(x)
```

Fricas [A] time = 0.22322, size = 39, normalized size = 1.05

$$\frac{2\,\left(3\,Bbx^4 + 7\,(Ba + Ab)x^2 - 21\,Aa\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*B*b*x^4 + 7*(B*a + A*b)*x^2 - 21*A*a)/sqrt(x)
```

Sympy [A] time = 3.48273, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(
7/2)/7
```

GIAC/XCAS [A] time = 0.210184, size = 39, normalized size = 1.05

$$\frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{3}\,Bax^{\frac{3}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b*x^(7/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2*A*a/sqrt(
x)
```

3.349 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

```
[Out] (-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

Rubi [A] time = 0.0499817, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

Rubi in Sympy [A] time = 6.97056, size = 37, normalized size = 1.

$$-\frac{2Aa}{3x^{\frac{3}{2}}}+\frac{2Bbx^{\frac{5}{2}}}{5}+\sqrt{x}\,(2Ab+2Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*B*b*x**(5/2)/5 + sqrt(x)*(2*A*b + 2*B*a)
```

Mathematica [A] time = 0.0198245, size = 33, normalized size = 0.89

$$\frac{2\left(15x^2(aB+Ab)-5aA+3bBx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-5*a*A + 15*(A*b + a*B)*x^2 + 3*b*B*x^4))/(15*x^(3/2))
```

Maple [A] time = 0.006, size = 32, normalized size = 0.9

$$-\frac{-6\,bBx^4-30\,Ax^2b-30\,Bx^2a+10\,Aa}{15}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/15*(-3*B*b*x^4-15*A*b*x^2-15*B*a*x^2+5*A*a)/x^(3/2)
```

Maxima [A] time = 1.32995, size = 36, normalized size = 0.97

$$\frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,(Ba + Ab)\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*B*b*x^(5/2) + 2*(B*a + A*b)*sqrt(x) - 2/3*A*a/x^(3/2)
```

Fricas [A] time = 0.224072, size = 39, normalized size = 1.05

$$\frac{2\,\left(3\,Bbx^{4} + 15\,(Ba + Ab)x^{2} - 5\,Aa\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*b*x^4 + 15*(B*a + A*b)*x^2 - 5*A*a)/x^(3/2)
```

Sympy [A] time = 4.07331, size = 42, normalized size = 1.14

$$-\frac{2Aa}{3x^{\frac{3}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(5
/2)/5
```

GIAC/XCAS [A] time = 0.213255, size = 39, normalized size = 1.05

$$\frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*B*b*x^(5/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/3*A*a/x^(3/2)
```

3.350 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

```
[Out] (-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/
3
```

Rubi [A] time = 0.0488016, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/
3
```

Rubi in Sympy [A] time = 6.96375, size = 37, normalized size = 1.

$$-\frac{2Aa}{5x^{\frac{5}{2}}}+\frac{2Bbx^{\frac{3}{2}}}{3}-\frac{2Ab+2Ba}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a/(5*x**(5/2)) + 2*B*b*x**(3/2)/3 - (2*A*b + 2*B*a)/sqrt(x)
```

Mathematica [A] time = 0.0235296, size = 33, normalized size = 0.89

$$\frac{2\left(-15x^2(aB+Ab)-3aA+5bBx^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(-3*a*A - 15*(A*b + a*B)*x^2 + 5*b*B*x^4))/(15*x^(5/2))
```

Maple [A] time = 0.004, size = 32, normalized size = 0.9

$$-\frac{-10\,bBx^4+30\,Ax^2b+30\,Bx^2a+6\,Aa}{15}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/15*(-5*B*b*x^4+15*A*b*x^2+15*B*a*x^2+3*A*a)/x^(5/2)
```

Maxima [A] time = 1.33949, size = 39, normalized size = 1.05

$$\frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\,\left(5\,(Ba + Ab)x^{2} + Aa\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*(B*a + A*b)*x^2 + A*a)/x^(5/2)
```

Fricas [A] time = 0.220654, size = 39, normalized size = 1.05

$$\frac{2\,\left(5\,Bbx^{4} - 15\,(Ba + Ab)x^{2} - 3\,Aa\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*B*b*x^4 - 15*(B*a + A*b)*x^2 - 3*A*a)/x^(5/2)
```

Sympy [A] time = 6.1715, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}} - \frac{2Ab}{\sqrt{x}} - \frac{2Ba}{\sqrt{x}} + \frac{2Bbx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a/(5*x**(5/2)) - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + 2*B*b*x**(3
/2)/3
```

GIAC/XCAS [A] time = 0.208502, size = 42, normalized size = 1.14

$$\frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\,\left(5\,Bax^{2} + 5\,Abx^{2} + Aa\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*B*a*x^2 + 5*A*b*x^2 + A*a)/x^(5/2)
```

3.351 $\int x^{7/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b
 + 2*a*B)*x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

Rubi [A] time = 0.092485, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b
 + 2*a*B)*x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

Rubi in Sympy [A] time = 12.6046, size = 63, normalized size = 1.

$$\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{21}{2}}}{21}+\frac{2ax^{\frac{13}{2}}(2Ab+Ba)}{13}+\frac{2bx^{\frac{17}{2}}(Ab+2Ba)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 2*B*b**2*x**(21/2)/21 + 2*a*x**(13/2)*(2*A*
b + B*a)/13 + 2*b*x**(17/2)*(A*b + 2*B*a)/17
```

Mathematica [A] time = 0.0324754, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547a^2A+819bx^4(2aB+Ab)+1071ax^2(aB+2Ab)+663b^2Bx^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(1547*a^2*A + 1071*a*(2*A*b + a*B)*x^2 + 819*b*(A*b +
2*a*B)*x^4 + 663*b^2*B*x^6))/13923
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$\frac{1326\,b^2Bx^6+1638\,Ab^2x^4+3276\,x^4abB+4284\,aAbx^2+2142\,x^2a^2B+3094\,a^2A}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/13923*x^(9/2)*(663*B*b^2*x^6+819*A*b^2*x^4+1638*B*a*b*x^4+2142*
A*a*b*x^2+1071*B*a^2*x^2+1547*A*a^2)
```

Maxima [A] time = 1.34746, size = 69, normalized size = 1.1

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}} + \frac{2}{17}\left(2\,Bab + Ab^2\right)x^{\frac{17}{2}} + \frac{2}{9}\,Aa^2x^{\frac{9}{2}} + \frac{2}{13}\left(Ba^2 + 2\,Aab\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/9*A*a^2
*x^(9/2) + 2/13*(B*a^2 + 2*A*a*b)*x^(13/2)
```

Fricas [A] time = 0.212822, size = 76, normalized size = 1.21

$$\frac{2}{13923}\left(663\,Bb^2x^{10} + 819\left(2\,Bab + Ab^2\right)x^8 + 1547\,Aa^2x^4 + 1071\left(Ba^2 + 2\,Aab\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*b^2*x^10 + 819*(2*B*a*b + A*b^2)*x^8 + 1547*A*a^2*
x^4 + 1071*(B*a^2 + 2*A*a*b)*x^6)*sqrt(x)
```

Sympy [A] time = 70.7962, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9} + \frac{4Aabx^{\frac{13}{2}}}{13} + \frac{2Ab^2x^{\frac{17}{2}}}{17} + \frac{2Ba^2x^{\frac{13}{2}}}{13} + \frac{4Babx^{\frac{17}{2}}}{17} + \frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**(7/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(17/2)/1
7 + 2*B*a**2*x**(13/2)/13 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(2
1/2)/21
```

GIAC/XCAS [A] time = 0.210345, size = 72, normalized size = 1.14

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}} + \frac{4}{17}\,Babx^{\frac{17}{2}} + \frac{2}{17}\,Ab^2x^{\frac{17}{2}} + \frac{2}{13}\,Ba^2x^{\frac{13}{2}} + \frac{4}{13}\,Aabx^{\frac{13}{2}} + \frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) +
 2/13*B*a^2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/9*A*a^2*x^(9/2)
```

3.352 $\int x^{5/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b
 + 2*a*B)*x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Rubi [A] time = 0.0876206, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b
 + 2*a*B)*x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Rubi in Sympy [A] time = 12.6446, size = 63, normalized size = 1.

$$\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{19}{2}}}{19}+\frac{2ax^{\frac{11}{2}}\left(2Ab+Ba\right)}{11}+\frac{2bx^{\frac{15}{2}}\left(Ab+2Ba\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(5/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 2*B*b**2*x**(19/2)/19 + 2*a*x**(11/2)*(2*A*
b + B*a)/11 + 2*b*x**(15/2)*(A*b + 2*B*a)/15
```

Mathematica [A] time = 0.0304358, size = 63, normalized size = 1.

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b
 + 2*a*B)*x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Maple [A] time = 0.009, size = 56, normalized size = 0.9

$$\frac{2310\,b^2Bx^6+2926\,Ab^2x^4+5852\,x^4abB+7980\,aAbx^2+3990\,x^2a^2B+6270\,a^2A}{21945}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*B*b^2*x^6+1463*A*b^2*x^4+2926*B*a*b*x^4+399
0*A*a*b*x^2+1995*B*a^2*x^2+3135*A*a^2)
```

Maxima [A] time = 1.37332, size = 69, normalized size = 1.1

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{2}{15}\left(2\,Bab + Ab^2\right)x^{\frac{15}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}} + \frac{2}{11}\left(Ba^2 + 2\,Aab\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/7*A*a^2
*x^(7/2) + 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

Fricas [A] time = 0.211977, size = 76, normalized size = 1.21

$$\frac{2}{21945}\left(1155\,Bb^2x^9 + 1463\left(2\,Bab + Ab^2\right)x^7 + 3135\,Aa^2x^3 + 1995\left(Ba^2 + 2\,Aab\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^2*x^9 + 1463*(2*B*a*b + A*b^2)*x^7 + 3135*A*a^2
*x^3 + 1995*(B*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

Sympy [A] time = 37.5169, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7} + \frac{4Aabx^{\frac{11}{2}}}{11} + \frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{2Ba^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(15/2)/1
5 + 2*B*a**2*x**(11/2)/11 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(1
9/2)/19
```

GIAC/XCAS [A] time = 0.215108, size = 72, normalized size = 1.14

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{4}{15}\,Babx^{\frac{15}{2}} + \frac{2}{15}\,Ab^2x^{\frac{15}{2}} + \frac{2}{11}\,Ba^2x^{\frac{11}{2}} + \frac{4}{11}\,Aabx^{\frac{11}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) +
 2/11*B*a^2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/7*A*a^2*x^(7/2)
```

3.353 $\int x^{3/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b +
 2*a*B)*x^(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

Rubi [A] time = 0.0898352, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b +
 2*a*B)*x^(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

Rubi in Sympy [A] time = 12.9284, size = 63, normalized size = 1.

$$\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{2Bb^2x^{\frac{17}{2}}}{17}+\frac{2ax^{\frac{9}{2}}\left(2Ab+Ba\right)}{9}+\frac{2bx^{\frac{13}{2}}\left(Ab+2Ba\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 2*B*b**2*x**(17/2)/17 + 2*a*x**(9/2)*(2*A*b
 + B*a)/9 + 2*b*x**(13/2)*(A*b + 2*B*a)/13
```

Mathematica [A] time = 0.0327183, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989a^2A+765bx^4(2aB+Ab)+1105ax^2(aB+2Ab)+585b^2Bx^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(1989*a^2*A + 1105*a*(2*A*b + a*B)*x^2 + 765*b*(A*b +
2*a*B)*x^4 + 585*b^2*B*x^6))/9945
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$\frac{1170\,b^2Bx^6+1530\,Ab^2x^4+3060\,x^4abB+4420\,aAbx^2+2210\,x^2a^2B+3978\,a^2A}{9945}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/9945*x^(5/2)*(585*B*b^2*x^6+765*A*b^2*x^4+1530*B*a*b*x^4+2210*A
*a*b*x^2+1105*B*a^2*x^2+1989*A*a^2)
```

Maxima [A] time = 1.38131, size = 69, normalized size = 1.1

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{2}{13}\left(2\,Bab + Ab^2\right)x^{\frac{13}{2}} + \frac{2}{5}\,Aa^2x^{\frac{5}{2}} + \frac{2}{9}\left(Ba^2 + 2\,Aab\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/5*A*a^2
*x^(5/2) + 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

Fricas [A] time = 0.215985, size = 76, normalized size = 1.21

$$\frac{2}{9945}\left(585\,Bb^2x^8 + 765\left(2\,Bab + Ab^2\right)x^6 + 1989\,Aa^2x^2 + 1105\left(Ba^2 + 2\,Aab\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*b^2*x^8 + 765*(2*B*a*b + A*b^2)*x^6 + 1989*A*a^2*x^
2 + 1105*(B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

Sympy [A] time = 20.4965, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(13/2)/13
+ 2*B*a**2*x**(9/2)/9 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(17/2)
/17
```

GIAC/XCAS [A] time = 0.251165, size = 72, normalized size = 1.14

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{4}{13}\,Babx^{\frac{13}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{9}\,Ba^2x^{\frac{9}{2}} + \frac{4}{9}\,Aabx^{\frac{9}{2}} + \frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) +
 2/9*B*a^2*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/5*A*a^2*x^(5/2)
```

3.354 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b +
 2*a*B)*x^(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

Rubi [A] time = 0.0854342, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b +
 2*a*B)*x^(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

Rubi in Sympy [A] time = 12.9051, size = 63, normalized size = 1.

$$\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{15}{2}}}{15}+\frac{2ax^{\frac{7}{2}}\left(2Ab+Ba\right)}{7}+\frac{2bx^{\frac{11}{2}}\left(Ab+2Ba\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(15/2)/15 + 2*a*x**(7/2)*(2*A*b
 + B*a)/7 + 2*b*x**(11/2)*(A*b + 2*B*a)/11
```

Mathematica [A] time = 0.0332197, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385a^2A+105bx^4(2aB+Ab)+165ax^2(aB+2Ab)+77b^2Bx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*A + 165*a*(2*A*b + a*B)*x^2 + 105*b*(A*b + 2*
a*B)*x^4 + 77*b^2*B*x^6))/1155
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$\frac{154\,b^2Bx^6+210\,Ab^2x^4+420\,x^4abB+660\,aAbx^2+330\,x^2a^2B+770\,a^2A}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*B*b^2*x^6+105*A*b^2*x^4+210*B*a*b*x^4+330*A*a*
b*x^2+165*B*a^2*x^2+385*A*a^2)
```

Maxima [A] time = 1.33495, size = 69, normalized size = 1.1

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{11}\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/3*A*a^2
*x^(3/2) + 2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

Fricas [A] time = 0.216891, size = 73, normalized size = 1.16

$$\frac{2}{1155}\left(77\,Bb^2x^7+105\left(2\,Bab+Ab^2\right)x^5+385\,Aa^2x+165\left(Ba^2+2\,Aab\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^2*x^7 + 105*(2*B*a*b + A*b^2)*x^5 + 385*A*a^2*x +
165*(B*a^2 + 2*A*a*b)*x^3)*sqrt(x)
```

Sympy [A] time = 4.81044, size = 66, normalized size = 1.05

$$\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(Ab^2+2Bab\right)}{11}+\frac{2x^{\frac{7}{2}}\left(2Aab+Ba^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(15/2)/15 + 2*x**(11/2)*(A*b**2
 + 2*B*a*b)/11 + 2*x**(7/2)*(2*A*a*b + B*a**2)/7
```

GIAC/XCAS [A] time = 0.22732, size = 72, normalized size = 1.14

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) +
 2/7*B*a^2*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2/3*A*a^2*x^(3/2)
```

3.355 $\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a
*B)*x^(9/2))/9 + (2*b^2*B*x^(13/2))/13
```

Rubi [A] time = 0.0878789, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a
*B)*x^(9/2))/9 + (2*b^2*B*x^(13/2))/13
```

Rubi in Sympy [A] time = 12.6494, size = 61, normalized size = 1.

$$2Aa^2\sqrt{x}+\frac{2Bb^2x^{\frac{13}{2}}}{13}+\frac{2ax^{\frac{5}{2}}(2Ab+Ba)}{5}+\frac{2bx^{\frac{9}{2}}(Ab+2Ba)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 2*B*b**2*x**(13/2)/13 + 2*a*x**(5/2)*(2*A*b +
B*a)/5 + 2*b*x**(9/2)*(A*b + 2*B*a)/9
```

Mathematica [A] time = 0.0328795, size = 53, normalized size = 0.87

$$\frac{2}{585}\sqrt{x}\left(585a^2A+65bx^4(2aB+Ab)+117ax^2(aB+2Ab)+45b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(585*a^2*A + 117*a*(2*A*b + a*B)*x^2 + 65*b*(A*b + 2*a
*B)*x^4 + 45*b^2*B*x^6))/585
```

Maple [A] time = 0.009, size = 56, normalized size = 0.9

$$\frac{90\,b^2Bx^6+130\,Ab^2x^4+260\,x^4abB+468\,aAbx^2+234\,x^2a^2B+1170\,a^2A}{585}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/585*x^(1/2)*(45*B*b^2*x^6+65*A*b^2*x^4+130*B*a*b*x^4+234*A*a*b*
x^2+117*B*a^2*x^2+585*A*a^2)
```

Maxima [A] time = 1.33669, size = 69, normalized size = 1.13

$$\frac{2}{13}Bb^2x^{\frac{13}{2}} + \frac{2}{9}\left(2Bab + Ab^2\right)x^{\frac{9}{2}} + 2Aa^2\sqrt{x} + \frac{2}{5}\left(Ba^2 + 2Aab\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2*A*a^2*sqr
t(x) + 2/5*(B*a^2 + 2*A*a*b)*x^(5/2)
```

Fricas [A] time = 0.217327, size = 72, normalized size = 1.18

$$\frac{2}{585}\left(45Bb^2x^6 + 65\left(2Bab + Ab^2\right)x^4 + 585Aa^2 + 117\left(Ba^2 + 2Aab\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^2*x^6 + 65*(2*B*a*b + A*b^2)*x^4 + 585*A*a^2 + 117*
(B*a^2 + 2*A*a*b)*x^2)*sqrt(x)
```

Sympy [A] time = 6.71726, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{2Ba^2x^{\frac{5}{2}}}{5} + \frac{4Babx^{\frac{9}{2}}}{9} + \frac{2Bb^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(9/2)/9 + 2*B
*a**2*x**(5/2)/5 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.209922, size = 72, normalized size = 1.18

$$\frac{2}{13}Bb^2x^{\frac{13}{2}} + \frac{4}{9}Babx^{\frac{9}{2}} + \frac{2}{9}Ab^2x^{\frac{9}{2}} + \frac{2}{5}Ba^2x^{\frac{5}{2}} + \frac{4}{5}Aabx^{\frac{5}{2}} + 2Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/5
*B*a^2*x^(5/2) + 4/5*A*a*b*x^(5/2) + 2*A*a^2*sqrt(x)
```

3.356 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b +
2*a*B)*x^(7/2))/7 + (2*b^2*B*x^(11/2))/11
```

Rubi [A] time = 0.0890327, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b +
2*a*B)*x^(7/2))/7 + (2*b^2*B*x^(11/2))/11
```

Rubi in Sympy [A] time = 12.7595, size = 61, normalized size = 1.

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{2Bb^2x^{\frac{11}{2}}}{11}+\frac{2ax^{\frac{3}{2}}\left(2Ab+Ba\right)}{3}+\frac{2bx^{\frac{7}{2}}\left(Ab+2Ba\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 2*B*b**2*x**(11/2)/11 + 2*a*x**(3/2)*(2*A*b +
 B*a)/3 + 2*b*x**(7/2)*(A*b + 2*B*a)/7
```

Mathematica [A] time = 0.0336501, size = 53, normalized size = 0.87

$$\frac{2\left(-231a^2A+33bx^4(2aB+Ab)+77ax^2(aB+2Ab)+21b^2Bx^6\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-231*a^2*A + 77*a*(2*A*b + a*B)*x^2 + 33*b*(A*b + 2*a*B)*x^4
+ 21*b^2*B*x^6))/(231*Sqrt[x])
```

Maple [A] time = 0.007, size = 56, normalized size = 0.9

$$-\frac{-42\,b^2Bx^6-66\,Ab^2x^4-132\,x^4abB-308\,aAbx^2-154\,x^2a^2B+462\,a^2A}{231}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/231*(-21*B*b^2*x^6-33*A*b^2*x^4-66*B*a*b*x^4-154*A*a*b*x^2-77*
B*a^2*x^2+231*A*a^2)/x^(1/2)
```

Maxima [A] time = 1.34332, size = 69, normalized size = 1.13

$$\frac{2}{11} Bb^2x^{\frac{11}{2}} + \frac{2}{7} \left(2 Bab + Ab^2\right) x^{\frac{7}{2}} - \frac{2 Aa^2}{\sqrt{x}} + \frac{2}{3} \left(Ba^2 + 2 Aab\right) x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) - 2*A*a^2/sqr
t(x) + 2/3*(B*a^2 + 2*A*a*b)*x^(3/2)
```

Fricas [A] time = 0.209474, size = 72, normalized size = 1.18

$$\frac{2 \left(21 Bb^2x^6 + 33 \left(2 Bab + Ab^2\right) x^4 - 231 Aa^2 + 77 \left(Ba^2 + 2 Aab\right) x^2\right)}{231 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b^2*x^6 + 33*(2*B*a*b + A*b^2)*x^4 - 231*A*a^2 + 77*(
B*a^2 + 2*A*a*b)*x^2)/sqrt(x)
```

Sympy [A] time = 8.57423, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{7}{2}}}{7} + \frac{2Bb^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(7/2)/7 + 2*
B*a**2*x**(3/2)/3 + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.221667, size = 72, normalized size = 1.18

$$\frac{2}{11} Bb^2x^{\frac{11}{2}} + \frac{4}{7} Babx^{\frac{7}{2}} + \frac{2}{7} Ab^2x^{\frac{7}{2}} + \frac{2}{3} Ba^2x^{\frac{3}{2}} + \frac{4}{3} Aabx^{\frac{3}{2}} - \frac{2 Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/3
*B*a^2*x^(3/2) + 4/3*A*a*b*x^(3/2) - 2*A*a^2/sqrt(x)
```

3.357 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b +
2*a*B)*x^(5/2))/5 + (2*b^2*B*x^(9/2))/9
```

Rubi [A] time = 0.0899875, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b +
2*a*B)*x^(5/2))/5 + (2*b^2*B*x^(9/2))/9
```

Rubi in Sympy [A] time = 12.6919, size = 61, normalized size = 1.

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}}+\frac{2Bb^2x^{\frac{9}{2}}}{9}+2a\sqrt{x}\,(2Ab+Ba)+\frac{2bx^{\frac{5}{2}}\,(Ab+2Ba)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 2*B*b**2*x**(9/2)/9 + 2*a*sqrt(x)*(2*A*b
 + B*a) + 2*b*x**(5/2)*(A*b + 2*B*a)/5
```

Mathematica [A] time = 0.0343367, size = 53, normalized size = 0.87

$$\frac{2\left(-15a^2A+9bx^4(2aB+Ab)+45ax^2(aB+2Ab)+5b^2Bx^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*A + 45*a*(2*A*b + a*B)*x^2 + 9*b*(A*b + 2*a*B)*x^4 +
5*b^2*B*x^6))/(45*x^(3/2))
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$-\frac{-10\,b^2Bx^6-18\,Ab^2x^4-36\,x^4abB-180\,aAbx^2-90\,Ba^2x^2+30\,a^2A}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/45*(-5*B*b^2*x^6-9*A*b^2*x^4-18*B*a*b*x^4-90*A*a*b*x^2-45*B*a^
2*x^2+15*A*a^2)/x^(3/2)
```

Maxima [A] time = 1.32307, size = 69, normalized size = 1.13

$$\frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{2}{5}\left(2\,Bab + Ab^2\right)x^{\frac{5}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}} + 2\left(Ba^2 + 2\,Aab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) - 2/3*A*a^2/x^(
3/2) + 2*(B*a^2 + 2*A*a*b)*sqrt(x)
```

Fricas [A] time = 0.224474, size = 72, normalized size = 1.18

$$\frac{2\left(5\,Bb^2x^6 + 9\left(2\,Bab + Ab^2\right)x^4 - 15\,Aa^2 + 45\left(Ba^2 + 2\,Aab\right)x^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*b^2*x^6 + 9*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 45*(B*a^
2 + 2*A*a*b)*x^2)/x^(3/2)
```

Sympy [A] time = 10.2123, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}} + 4Aab\sqrt{x} + \frac{2Ab^2x^{\frac{5}{2}}}{5} + 2Ba^2\sqrt{x} + \frac{4Babx^{\frac{5}{2}}}{5} + \frac{2Bb^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(5/2)/5 +
2*B*a**2*sqrt(x) + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(9/2)/9
```

GIAC/XCAS [A] time = 0.212402, size = 72, normalized size = 1.18

$$\frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{4}{5}\,Babx^{\frac{5}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + 2\,Ba^2\sqrt{x} + 4\,Aab\sqrt{x} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2*B*a
^2*sqrt(x) + 4*A*a*b*sqrt(x) - 2/3*A*a^2/x^(3/2)
```

3.358 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

```
[Out] (-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b
+ 2*a*B)*x^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

Rubi [A] time = 0.0885723, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b
+ 2*a*B)*x^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

Rubi in Sympy [A] time = 12.7748, size = 61, normalized size = 1.

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}}+\frac{2Bb^2x^{\frac{7}{2}}}{7}-\frac{2a\,(2Ab+Ba)}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}\,(Ab+2Ba)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) + 2*B*b**2*x**(7/2)/7 - 2*a*(2*A*b + B*a)/
sqrt(x) + 2*b*x**(3/2)*(A*b + 2*B*a)/3
```

Mathematica [A] time = 0.0289204, size = 57, normalized size = 0.93

$$\frac{-42a^2\left(A+5Bx^2\right)+140abx^2\left(Bx^2-3A\right)+10b^2x^4\left(7A+3Bx^2\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (140*a*b*x^2*(-3*A + B*x^2) + 10*b^2*x^4*(7*A + 3*B*x^2) - 42*a^2
*(A + 5*B*x^2))/(105*x^(5/2))
```

Maple [A] time = 0.009, size = 56, normalized size = 0.9

$$-\frac{-30\,b^2Bx^6-70\,Ab^2x^4-140\,x^4abB+420\,aAbx^2+210\,Ba^2x^2+42\,a^2A}{105}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/105*(-15*B*b^2*x^6-35*A*b^2*x^4-70*B*a*b*x^4+210*A*a*b*x^2+105
*B*a^2*x^2+21*A*a^2)/x^(5/2)
```

Maxima [A] time = 1.32982, size = 72, normalized size = 1.18

$$\frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{2}{3}\left(2\,Bab + Ab^2\right)x^{\frac{3}{2}} - \frac{2\left(Aa^2 + 5\left(Ba^2 + 2\,Aab\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) - 2/5*(A*a^2 +
5*(B*a^2 + 2*A*a*b)*x^2)/x^(5/2)
```

Fricas [A] time = 0.218293, size = 72, normalized size = 1.18

$$\frac{2\left(15\,Bb^2x^6 + 35\left(2\,Bab + Ab^2\right)x^4 - 21\,Aa^2 - 105\left(Ba^2 + 2\,Aab\right)x^2\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 - 21*A*a^2 - 105*(
B*a^2 + 2*A*a*b)*x^2)/x^(5/2)
```

Sympy [A] time = 15.4564, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}} - \frac{4Aab}{\sqrt{x}} + \frac{2Ab^2x^{\frac{3}{2}}}{3} - \frac{2Ba^2}{\sqrt{x}} + \frac{4Babx^{\frac{3}{2}}}{3} + \frac{2Bb^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*x**(3/2)/3 -
2*B*a**2/sqrt(x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(7/2)/7
```

GIAC/XCAS [A] time = 0.212193, size = 74, normalized size = 1.21

$$\frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{4}{3}\,Babx^{\frac{3}{2}} + \frac{2}{3}\,Ab^2x^{\frac{3}{2}} - \frac{2\left(5\,Ba^2x^2 + 10\,Aabx^2 + Aa^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) - 2/5*(
5*B*a^2*x^2 + 10*A*a*b*x^2 + A*a^2)/x^(5/2)
```

3.359 $\int x^{7/2} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*
(A*b + a*B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2
*b^3*B*x^(25/2))/25
```

Rubi [A] time = 0.118122, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*
(A*b + a*B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2
*b^3*B*x^(25/2))/25
```

Rubi in Sympy [A] time = 16.5911, size = 85, normalized size = 1.

$$\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{2Bb^3x^{\frac{25}{2}}}{25}+\frac{2a^2x^{\frac{13}{2}}\left(3Ab+Ba\right)}{13}+\frac{6abx^{\frac{17}{2}}\left(Ab+Ba\right)}{17}+\frac{2b^2x^{\frac{21}{2}}\left(Ab+3Ba\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 2*B*b**3*x**(25/2)/25 + 2*a**2*x**(13/2)*(3
*A*b + B*a)/13 + 6*a*b*x**(17/2)*(A*b + B*a)/17 + 2*b**2*x**(21/2
)*(A*b + 3*B*a)/21
```

Mathematica [A] time = 0.0423958, size = 85, normalized size = 1.

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*
(A*b + a*B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2
*b^3*B*x^(25/2))/25
```

Maple [A] time = 0.01, size = 80, normalized size = 0.9

$$\frac{27846\,Bb^3x^8+33150\,x^6Ab^3+99450\,x^6Bab^2+122850\,x^4Aab^2+122850\,x^4Ba^2b+160650\,x^2Aa^2b+53550\,x^2Ba^3+77350\,Aa}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/348075*x^(9/2)*(13923*B*b^3*x^8+16575*A*b^3*x^6+49725*B*a*b^2*x
^6+61425*A*a*b^2*x^4+61425*B*a^2*b*x^4+80325*A*a^2*b*x^2+26775*B*
a^3*x^2+38675*A*a^3)
```

Maxima [A] time = 1.33881, size = 99, normalized size = 1.16

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{21}\left(3\,Bab^2 + Ab^3\right)x^{\frac{21}{2}} + \frac{6}{17}\left(Ba^2b + Aab^2\right)x^{\frac{17}{2}} + \frac{2}{9}\,Aa^3x^{\frac{9}{2}} + \frac{2}{13}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 6/17*(B
*a^2*b + A*a*b^2)*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/13*(B*a^3 + 3*
A*a^2*b)*x^(13/2)
```

Fricas [A] time = 0.213486, size = 105, normalized size = 1.24

$$\frac{2}{348075}\left(13923\,Bb^3x^{12} + 16575\left(3\,Bab^2 + Ab^3\right)x^{10} + 61425\left(Ba^2b + Aab^2\right)x^8 + 38675\,Aa^3x^4 + 26775\left(Ba^3 + 3\,Aa^2b\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*b^3*x^12 + 16575*(3*B*a*b^2 + A*b^3)*x^10 + 614
25*(B*a^2*b + A*a*b^2)*x^8 + 38675*A*a^3*x^4 + 26775*(B*a^3 + 3*A
*a^2*b)*x^6)*sqrt(x)
```

Sympy [A] time = 123.375, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9} + \frac{6Aa^2bx^{\frac{13}{2}}}{13} + \frac{6Aab^2x^{\frac{17}{2}}}{17} + \frac{2Ab^3x^{\frac{21}{2}}}{21} + \frac{2Ba^3x^{\frac{13}{2}}}{13} + \frac{6Ba^2bx^{\frac{17}{2}}}{17} + \frac{2Bab^2x^{\frac{21}{2}}}{7} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(17
/2)/17 + 2*A*b**3*x**(21/2)/21 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2
*b*x**(17/2)/17 + 2*B*a*b**2*x**(21/2)/7 + 2*B*b**3*x**(25/2)/25
```

GIAC/XCAS [A] time = 0.212349, size = 104, normalized size = 1.22

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{7}\,Bab^2x^{\frac{21}{2}} + \frac{2}{21}\,Ab^3x^{\frac{21}{2}} + \frac{6}{17}\,Ba^2bx^{\frac{17}{2}} + \frac{6}{17}\,Aab^2x^{\frac{17}{2}} + \frac{2}{13}\,Ba^3x^{\frac{13}{2}} + \frac{6}{13}\,Aa^2bx^{\frac{13}{2}} + \frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2)
+ 6/17*B*a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/13*B*a^3*x^(1
3/2) + 6/13*A*a^2*b*x^(13/2) + 2/9*A*a^3*x^(9/2)
```

3.360 $\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*
(A*b + a*B)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*
b^3*B*x^(23/2))/23
```

Rubi [A] time = 0.116085, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*
(A*b + a*B)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*
b^3*B*x^(23/2))/23
```

Rubi in Sympy [A] time = 16.528, size = 85, normalized size = 1.

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{2Bb^3x^{\frac{23}{2}}}{23}+\frac{2a^2x^{\frac{11}{2}}\left(3Ab+Ba\right)}{11}+\frac{2abx^{\frac{15}{2}}\left(Ab+Ba\right)}{5}+\frac{2b^2x^{\frac{19}{2}}\left(Ab+3Ba\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 2*B*b**3*x**(23/2)/23 + 2*a**2*x**(11/2)*(3
*A*b + B*a)/11 + 2*a*b*x**(15/2)*(A*b + B*a)/5 + 2*b**2*x**(19/2)
*(A*b + 3*B*a)/19
```

Mathematica [A] time = 0.0380789, size = 85, normalized size = 1.

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*
(A*b + a*B)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*
b^3*B*x^(23/2))/23
```

Maple [A] time = 0.009, size = 80, normalized size = 0.9

$$\frac{14630\,Bb^3x^8+17710\,x^6Ab^3+53130\,x^6Bab^2+67298\,x^4Aab^2+67298\,x^4Ba^2b+91770\,x^2Aa^2b+30590\,x^2Ba^3+48070\,Aa^3}{168245}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/168245*x^(7/2)*(7315*B*b^3*x^8+8855*A*b^3*x^6+26565*B*a*b^2*x^6
+33649*A*a*b^2*x^4+33649*B*a^2*b*x^4+45885*A*a^2*b*x^2+15295*B*a^
3*x^2+24035*A*a^3)
```

Maxima [A] time = 1.35314, size = 99, normalized size = 1.16

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{2}{19}\left(3\,Bab^2 + Ab^3\right)x^{\frac{19}{2}} + \frac{2}{5}\left(Ba^2b + Aab^2\right)x^{\frac{15}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}} + \frac{2}{11}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 2/5*(B*
a^2*b + A*a*b^2)*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/11*(B*a^3 + 3*A
*a^2*b)*x^(11/2)
```

Fricas [A] time = 0.213378, size = 105, normalized size = 1.24

$$\frac{2}{168245}\left(7315\,Bb^3x^{11} + 8855\left(3\,Bab^2 + Ab^3\right)x^9 + 33649\left(Ba^2b + Aab^2\right)x^7 + 24035\,Aa^3x^3 + 15295\left(Ba^3 + 3\,Aa^2b\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*b^3*x^11 + 8855*(3*B*a*b^2 + A*b^3)*x^9 + 33649*
(B*a^2*b + A*a*b^2)*x^7 + 24035*A*a^3*x^3 + 15295*(B*a^3 + 3*A*a^
2*b)*x^5)*sqrt(x)
```

Sympy [A] time = 71.5639, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7} + \frac{6Aa^2bx^{\frac{11}{2}}}{11} + \frac{2Aab^2x^{\frac{15}{2}}}{5} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{11}{2}}}{11} + \frac{2Ba^2bx^{\frac{15}{2}}}{5} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(11/2)/11 + 2*A*a*b**2*x**(15
/2)/5 + 2*A*b**3*x**(19/2)/19 + 2*B*a**3*x**(11/2)/11 + 2*B*a**2*
b*x**(15/2)/5 + 6*B*a*b**2*x**(19/2)/19 + 2*B*b**3*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.214408, size = 104, normalized size = 1.22

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{2}{5}\,Ba^2bx^{\frac{15}{2}} + \frac{2}{5}\,Aab^2x^{\frac{15}{2}} + \frac{2}{11}\,Ba^3x^{\frac{11}{2}} + \frac{6}{11}\,Aa^2bx^{\frac{11}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2)
 + 2/5*B*a^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/11*B*a^3*x^(11
/2) + 6/11*A*a^2*b*x^(11/2) + 2/7*A*a^3*x^(7/2)
```

3.361 $\int x^{3/2} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A
*b + a*B)*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b
^3*B*x^(21/2))/21
```

Rubi [A] time = 0.116648, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A
*b + a*B)*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b
^3*B*x^(21/2))/21
```

Rubi in Sympy [A] time = 16.6313, size = 85, normalized size = 1.

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{2Bb^3x^{\frac{21}{2}}}{21}+\frac{2a^2x^{\frac{9}{2}}(3Ab+Ba)}{9}+\frac{6abx^{\frac{13}{2}}(Ab+Ba)}{13}+\frac{2b^2x^{\frac{17}{2}}(Ab+3Ba)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 2*B*b**3*x**(21/2)/21 + 2*a**2*x**(9/2)*(3*
A*b + B*a)/9 + 6*a*b*x**(13/2)*(A*b + B*a)/13 + 2*b**2*x**(17/2)*
(A*b + 3*B*a)/17
```

Mathematica [A] time = 0.0381439, size = 85, normalized size = 1.

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A
*b + a*B)*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b
^3*B*x^(21/2))/21
```

Maple [A] time = 0.008, size = 80, normalized size = 0.9

$$\frac{6630\,Bb^3x^8+8190\,x^6Ab^3+24570\,x^6Bab^2+32130\,x^4Aab^2+32130\,x^4Ba^2b+46410\,x^2Aa^2b+15470\,x^2Ba^3+27846\,Aa^3}{69615}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/69615*x^(5/2)*(3315*B*b^3*x^8+4095*A*b^3*x^6+12285*B*a*b^2*x^6+
16065*A*a*b^2*x^4+16065*B*a^2*b*x^4+23205*A*a^2*b*x^2+7735*B*a^3*
x^2+13923*A*a^3)
```

Maxima [A] time = 1.36807, size = 99, normalized size = 1.16

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{17}\left(3\,Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/13*(B
*a^2*b + A*a*b^2)*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/9*(B*a^3 + 3*A
*a^2*b)*x^(9/2)
```

Fricas [A] time = 0.216124, size = 105, normalized size = 1.24

$$\frac{2}{69615}\left(3315\,Bb^3x^{10}+4095\left(3\,Bab^2+Ab^3\right)x^8+16065\left(Ba^2b+Aab^2\right)x^6+13923\,Aa^3x^2+7735\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*B*b^3*x^10 + 4095*(3*B*a*b^2 + A*b^3)*x^8 + 16065*(
B*a^2*b + A*a*b^2)*x^6 + 13923*A*a^3*x^2 + 7735*(B*a^3 + 3*A*a^2*
b)*x^4)*sqrt(x)
```

Sympy [A] time = 38.1527, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a*b**2*x**(13/2
)/13 + 2*A*b**3*x**(17/2)/17 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x
**(13/2)/13 + 6*B*a*b**2*x**(17/2)/17 + 2*B*b**3*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.21342, size = 104, normalized size = 1.22

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{6}{17}\,Bab^2x^{\frac{17}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{6}{13}\,Ba^2bx^{\frac{13}{2}}+\frac{6}{13}\,Aab^2x^{\frac{13}{2}}+\frac{2}{9}\,Ba^3x^{\frac{9}{2}}+\frac{2}{3}\,Aa^2bx^{\frac{9}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2)
 + 6/13*B*a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/9*B*a^3*x^(9
/2) + 2/3*A*a^2*b*x^(9/2) + 2/5*A*a^3*x^(5/2)
```

3.362 $\int \sqrt{x}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A
*b + a*B)*x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b
^3*B*x^(19/2))/19
```

Rubi [A] time = 0.112036, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A
*b + a*B)*x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b
^3*B*x^(19/2))/19
```

Rubi in Sympy [A] time = 16.9412, size = 85, normalized size = 1.

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Bb^3x^{\frac{19}{2}}}{19}+\frac{2a^2x^{\frac{7}{2}}\left(3Ab+Ba\right)}{7}+\frac{6abx^{\frac{11}{2}}\left(Ab+Ba\right)}{11}+\frac{2b^2x^{\frac{15}{2}}\left(Ab+3Ba\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(19/2)/19 + 2*a**2*x**(7/2)*(3*
A*b + B*a)/7 + 6*a*b*x**(11/2)*(A*b + B*a)/11 + 2*b**2*x**(15/2)*
(A*b + 3*B*a)/15
```

Mathematica [A] time = 0.0411249, size = 71, normalized size = 0.84

$$\frac{2x^{3/2}\left(7315a^3A+3135a^2x^2(aB+3Ab)+1463b^2x^6(3aB+Ab)+5985abx^4(aB+Ab)+1155b^3Bx^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(7315*a^3*A + 3135*a^2*(3*A*b + a*B)*x^2 + 5985*a*b*(A
*b + a*B)*x^4 + 1463*b^2*(A*b + 3*a*B)*x^6 + 1155*b^3*B*x^8))/219
45
```

Maple [A] time = 0.009, size = 80, normalized size = 0.9

$$\frac{2310\,b^3Bx^8+2926\,x^6b^3A+8778\,x^6ab^2B+11970\,x^4ab^2A+11970\,x^4a^2bB+18810\,x^2Aa^2b+6270\,x^2Ba^3+14630\,a^3A}{21945}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/21945*x^(3/2)*(1155*B*b^3*x^8+1463*A*b^3*x^6+4389*B*a*b^2*x^6+5
985*A*a*b^2*x^4+5985*B*a^2*b*x^4+9405*A*a^2*b*x^2+3135*B*a^3*x^2+
7315*A*a^3)
```

Maxima [A] time = 1.32999, size = 99, normalized size = 1.16

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{2}{15}\left(3\,Bab^2 + Ab^3\right)x^{\frac{15}{2}} + \frac{6}{11}\left(Ba^2b + Aab^2\right)x^{\frac{11}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}} + \frac{2}{7}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 6/11*(B
*a^2*b + A*a*b^2)*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/7*(B*a^3 + 3*A
*a^2*b)*x^(7/2)
```

Fricas [A] time = 0.212201, size = 103, normalized size = 1.21

$$\frac{2}{21945}\left(1155\,Bb^3x^9 + 1463\left(3\,Bab^2 + Ab^3\right)x^7 + 5985\left(Ba^2b + Aab^2\right)x^5 + 7315\,Aa^3x + 3135\left(Ba^3 + 3\,Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^3*x^9 + 1463*(3*B*a*b^2 + A*b^3)*x^7 + 5985*(B*
a^2*b + A*a*b^2)*x^5 + 7315*A*a^3*x + 3135*(B*a^3 + 3*A*a^2*b)*x^
3)*sqrt(x)
```

Sympy [A] time = 10.2179, size = 95, normalized size = 1.12

$$\frac{2Aa^3x^{\frac{3}{2}}}{3} + \frac{2Bb^3x^{\frac{19}{2}}}{19} + \frac{2x^{\frac{15}{2}}\left(Ab^3 + 3Bab^2\right)}{15} + \frac{2x^{\frac{11}{2}}\left(3Aab^2 + 3Ba^2b\right)}{11} + \frac{2x^{\frac{7}{2}}\left(3Aa^2b + Ba^3\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(19/2)/19 + 2*x**(15/2)*(A*b**3
 + 3*B*a*b**2)/15 + 2*x**(11/2)*(3*A*a*b**2 + 3*B*a**2*b)/11 + 2*
x**(7/2)*(3*A*a**2*b + B*a**3)/7
```

GIAC/XCAS [A] time = 0.212652, size = 104, normalized size = 1.22

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{6}{11}\,Ba^2bx^{\frac{11}{2}} + \frac{6}{11}\,Aab^2x^{\frac{11}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2)
+ 6/11*B*a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/7*B*a^3*x^(7/
2) + 6/7*A*a^2*b*x^(7/2) + 2/3*A*a^3*x^(3/2)
```

3.363 $\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=83

$$2a^3A\sqrt{x}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b +
 a*B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x
^(17/2))/17
```

Rubi [A] time = 0.111743, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$2a^3A\sqrt{x}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b +
 a*B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x
^(17/2))/17
```

Rubi in Sympy [A] time = 16.635, size = 83, normalized size = 1.

$$2Aa^3\sqrt{x}+\frac{2Bb^3x^{\frac{17}{2}}}{17}+\frac{2a^2x^{\frac{5}{2}}\left(3Ab+Ba\right)}{5}+\frac{2abx^{\frac{9}{2}}\left(Ab+Ba\right)}{3}+\frac{2b^2x^{\frac{13}{2}}\left(Ab+3Ba\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 2*B*b**3*x**(17/2)/17 + 2*a**2*x**(5/2)*(3*A*b
 + B*a)/5 + 2*a*b*x**(9/2)*(A*b + B*a)/3 + 2*b**2*x**(13/2)*(A*b
+ 3*B*a)/13
```

Mathematica [A] time = 0.0420963, size = 71, normalized size = 0.86

$$\frac{2\sqrt{x}\left(3315a^3A+663a^2x^2(aB+3Ab)+255b^2x^6(3aB+Ab)+1105abx^4(aB+Ab)+195b^3Bx^8\right)}{3315}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3315*a^3*A + 663*a^2*(3*A*b + a*B)*x^2 + 1105*a*b*(A*
b + a*B)*x^4 + 255*b^2*(A*b + 3*a*B)*x^6 + 195*b^3*B*x^8))/3315
```

Maple [A] time = 0.009, size = 80, normalized size = 1.

$$\frac{390\,b^3Bx^8+510\,x^6b^3A+1530\,x^6ab^2B+2210\,x^4ab^2A+2210\,x^4a^2bB+3978\,x^2Aa^2b+1326\,x^2Ba^3+6630\,a^3A}{3315}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/3315*x^(1/2)*(195*B*b^3*x^8+255*A*b^3*x^6+765*B*a*b^2*x^6+1105*
A*a*b^2*x^4+1105*B*a^2*b*x^4+1989*A*a^2*b*x^2+663*B*a^3*x^2+3315*
A*a^3)
```

Maxima [A] time = 1.34168, size = 99, normalized size = 1.19

$$\frac{2}{17}Bb^3x^{\frac{17}{2}} + \frac{2}{13}\left(3\,Bab^2 + Ab^3\right)x^{\frac{13}{2}} + \frac{2}{3}\left(Ba^2b + Aab^2\right)x^{\frac{9}{2}} + 2\,Aa^3\sqrt{x} + \frac{2}{5}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^3/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 2/3*(B*
a^2*b + A*a*b^2)*x^(9/2) + 2*A*a^3*sqrt(x) + 2/5*(B*a^3 + 3*A*a^2
*b)*x^(5/2)
```

Fricas [A] time = 0.225598, size = 101, normalized size = 1.22

$$\frac{2}{3315}\left(195\,Bb^3x^8 + 255\left(3\,Bab^2 + Ab^3\right)x^6 + 1105\left(Ba^2b + Aab^2\right)x^4 + 3315\,Aa^3 + 663\left(Ba^3 + 3\,Aa^2b\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^3/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*B*b^3*x^8 + 255*(3*B*a*b^2 + A*b^3)*x^6 + 1105*(B*a^2
*b + A*a*b^2)*x^4 + 3315*A*a^3 + 663*(B*a^3 + 3*A*a^2*b)*x^2)*sqr
t(x)
```

Sympy [A] time = 16.9509, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{5}{2}}}{5} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{13}{2}}}{13} + \frac{2Ba^3x^{\frac{5}{2}}}{5} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{6Bab^2x^{\frac{13}{2}}}{13} + \frac{2Bb^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**3*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 2*A*a*b**2*x**(9/2)/3
+ 2*A*b**3*x**(13/2)/13 + 2*B*a**3*x**(5/2)/5 + 2*B*a**2*b*x**(9/
2)/3 + 6*B*a*b**2*x**(13/2)/13 + 2*B*b**3*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.210854, size = 104, normalized size = 1.25

$$\frac{2}{17}Bb^3x^{\frac{17}{2}} + \frac{6}{13}Bab^2x^{\frac{13}{2}} + \frac{2}{13}Ab^3x^{\frac{13}{2}} + \frac{2}{3}Ba^2bx^{\frac{9}{2}} + \frac{2}{3}Aab^2x^{\frac{9}{2}} + \frac{2}{5}Ba^3x^{\frac{5}{2}} + \frac{6}{5}Aa^2bx^{\frac{5}{2}} + 2\,Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^3/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2)
 + 2/3*B*a^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/5*B*a^3*x^(5/2)
+ 6/5*A*a^2*b*x^(5/2) + 2*A*a^3*sqrt(x)
```

3.364 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}} dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*
b + a*B)*x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*
B*x^(15/2))/15
```

Rubi [A] time = 0.114803, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*
b + a*B)*x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*
B*x^(15/2))/15
```

Rubi in Sympy [A] time = 16.7358, size = 82, normalized size = 0.99

$$-\frac{2Aa^3}{\sqrt{x}}+\frac{2Bb^3x^{\frac{15}{2}}}{15}+2a^2x^{\frac{3}{2}}\left(Ab+\frac{Ba}{3}\right)+\frac{6abx^{\frac{7}{2}}(Ab+Ba)}{7}+\frac{2b^2x^{\frac{11}{2}}(Ab+3Ba)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 2*B*b**3*x**(15/2)/15 + 2*a**2*x**(3/2)*(A*b
+ B*a/3) + 6*a*b*x**(7/2)*(A*b + B*a)/7 + 2*b**2*x**(11/2)*(A*b +
 3*B*a)/11
```

Mathematica [A] time = 0.0448136, size = 71, normalized size = 0.86

$$\frac{2\left(-1155a^3A+385a^2x^2(aB+3Ab)+105b^2x^6(3aB+Ab)+495abx^4(aB+Ab)+77b^3Bx^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-1155*a^3*A + 385*a^2*(3*A*b + a*B)*x^2 + 495*a*b*(A*b + a*B)
*x^4 + 105*b^2*(A*b + 3*a*B)*x^6 + 77*b^3*B*x^8))/(1155*Sqrt[x])
```

Maple [A] time = 0.008, size = 80, normalized size = 1.

$$-\frac{-154\,b^3Bx^8-210\,x^6b^3A-630\,x^6ab^2B-990\,x^4ab^2A-990\,x^4a^2bB-2310\,x^2Aa^2b-770\,x^2Ba^3+2310\,a^3A}{1155}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/1155*(-77*B*b^3*x^8-105*A*b^3*x^6-315*B*a*b^2*x^6-495*A*a*b^2*
x^4-495*B*a^2*b*x^4-1155*A*a^2*b*x^2-385*B*a^3*x^2+1155*A*a^3)/x^
(1/2)
```

Maxima [A] time = 1.34287, size = 99, normalized size = 1.19

$$\frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{11}\left(3\,Bab^2 + Ab^3\right)x^{\frac{11}{2}} + \frac{6}{7}\left(Ba^2b + Aab^2\right)x^{\frac{7}{2}} - \frac{2\,Aa^3}{\sqrt{x}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 2/11*(3*B*a*b^2 + A*b^3)*x^(11/2) + 6/7*(B*
a^2*b + A*a*b^2)*x^(7/2) - 2*A*a^3/sqrt(x) + 2/3*(B*a^3 + 3*A*a^2
*b)*x^(3/2)
```

Fricas [A] time = 0.224956, size = 101, normalized size = 1.22

$$\frac{2\left(77\,Bb^3x^8 + 105\left(3\,Bab^2 + Ab^3\right)x^6 + 495\left(Ba^2b + Aab^2\right)x^4 - 1155\,Aa^3 + 385\left(Ba^3 + 3\,Aa^2b\right)x^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^3*x^8 + 105*(3*B*a*b^2 + A*b^3)*x^6 + 495*(B*a^2*b
 + A*a*b^2)*x^4 - 1155*A*a^3 + 385*(B*a^3 + 3*A*a^2*b)*x^2)/sqrt(
x)
```

Sympy [A] time = 20.2476, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{\sqrt{x}} + 2Aa^2bx^{\frac{3}{2}} + \frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{6Ba^2bx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{11}{2}}}{11} + \frac{2Bb^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a*b**2*x**(7/2)/7 +
 2*A*b**3*x**(11/2)/11 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(7/2
)/7 + 6*B*a*b**2*x**(11/2)/11 + 2*B*b**3*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.211181, size = 104, normalized size = 1.25

$$\frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{6}{11}\,Bab^2x^{\frac{11}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}} + \frac{6}{7}\,Ba^2bx^{\frac{7}{2}} + \frac{6}{7}\,Aab^2x^{\frac{7}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} - \frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2)
 + 6/7*B*a^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2/3*B*a^3*x^(3/2)
+ 2*A*a^2*b*x^(3/2) - 2*A*a^3/sqrt(x)
```

3.365 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{3x^{3/2}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*
b + a*B)*x^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*
x^(13/2))/13
```

Rubi [A] time = 0.112388, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^3A}{3x^{3/2}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*
b + a*B)*x^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*
x^(13/2))/13
```

Rubi in Sympy [A] time = 16.6911, size = 83, normalized size = 1.

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+2a^2\sqrt{x}\,(3Ab+Ba)+\frac{6abx^{\frac{5}{2}}\,(Ab+Ba)}{5}+\frac{2b^2x^{\frac{9}{2}}\,(Ab+3Ba)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 2*B*b**3*x**(13/2)/13 + 2*a**2*sqrt(x)*(
3*A*b + B*a) + 6*a*b*x**(5/2)*(A*b + B*a)/5 + 2*b**2*x**(9/2)*(A*
b + 3*B*a)/9
```

Mathematica [A] time = 0.0444066, size = 71, normalized size = 0.86

$$\frac{2\left(-195a^3A+585a^2x^2(aB+3Ab)+65b^2x^6(3aB+Ab)+351abx^4(aB+Ab)+45b^3Bx^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-195*a^3*A + 585*a^2*(3*A*b + a*B)*x^2 + 351*a*b*(A*b + a*B)*
x^4 + 65*b^2*(A*b + 3*a*B)*x^6 + 45*b^3*B*x^8))/(585*x^(3/2))
```

Maple [A] time = 0.008, size = 80, normalized size = 1.

$$-\frac{-90\,b^3Bx^8-130\,x^6b^3A-390\,x^6ab^2B-702\,x^4ab^2A-702\,x^4a^2bB-3510\,Aa^2bx^2-1170\,Ba^3x^2+390\,a^3A}{585}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/585*(-45*B*b^3*x^8-65*A*b^3*x^6-195*B*a*b^2*x^6-351*A*a*b^2*x^
4-351*B*a^2*b*x^4-1755*A*a^2*b*x^2-585*B*a^3*x^2+195*A*a^3)/x^(3/
2)
```

Maxima [A] time = 1.33559, size = 99, normalized size = 1.19

$$\frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{2}{9}\left(3\,Bab^2 + Ab^3\right)x^{\frac{9}{2}} + \frac{6}{5}\left(Ba^2b + Aab^2\right)x^{\frac{5}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}} + 2\left(Ba^3 + 3\,Aa^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/9*(3*B*a*b^2 + A*b^3)*x^(9/2) + 6/5*(B*a^
2*b + A*a*b^2)*x^(5/2) - 2/3*A*a^3/x^(3/2) + 2*(B*a^3 + 3*A*a^2*b
)*sqrt(x)
```

Fricas [A] time = 0.220424, size = 101, normalized size = 1.22

$$\frac{2\left(45\,Bb^3x^8 + 65\left(3\,Bab^2 + Ab^3\right)x^6 + 351\left(Ba^2b + Aab^2\right)x^4 - 195\,Aa^3 + 585\left(Ba^3 + 3\,Aa^2b\right)x^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^3*x^8 + 65*(3*B*a*b^2 + A*b^3)*x^6 + 351*(B*a^2*b +
 A*a*b^2)*x^4 - 195*A*a^3 + 585*(B*a^3 + 3*A*a^2*b)*x^2)/x^(3/2)
```

Sympy [A] time = 24.298, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}} + 6Aa^2b\sqrt{x} + \frac{6Aab^2x^{\frac{5}{2}}}{5} + \frac{2Ab^3x^{\frac{9}{2}}}{9} + 2Ba^3\sqrt{x} + \frac{6Ba^2bx^{\frac{5}{2}}}{5} + \frac{2Bab^2x^{\frac{9}{2}}}{3} + \frac{2Bb^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(5/2)
/5 + 2*A*b**3*x**(9/2)/9 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(5/2)
/5 + 2*B*a*b**2*x**(9/2)/3 + 2*B*b**3*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.21813, size = 104, normalized size = 1.25

$$\frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{2}{3}\,Bab^2x^{\frac{9}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{6}{5}\,Ba^2bx^{\frac{5}{2}} + \frac{6}{5}\,Aab^2x^{\frac{5}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 6
/5*B*a^2*b*x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 2*B*a^3*sqrt(x) + 6*A*
a^2*b*sqrt(x) - 2/3*A*a^3/x^(3/2)
```

3.366 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}} \, dx$

Optimal. Leaf size=81

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

```
[Out] (-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A
*b + a*B)*x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^
(11/2))/11
```

Rubi [A] time = 0.112557, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A
*b + a*B)*x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^
(11/2))/11
```

Rubi in Sympy [A] time = 16.6324, size = 82, normalized size = 1.01

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}}+\frac{2Bb^3x^{\frac{11}{2}}}{11}-\frac{2a^2\left(3Ab+Ba\right)}{\sqrt{x}}+2abx^{\frac{3}{2}}\left(Ab+Ba\right)+\frac{2b^2x^{\frac{7}{2}}\left(Ab+3Ba\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) + 2*B*b**3*x**(11/2)/11 - 2*a**2*(3*A*b +
B*a)/sqrt(x) + 2*a*b*x**(3/2)*(A*b + B*a) + 2*b**2*x**(7/2)*(A*b
+ 3*B*a)/7
```

Mathematica [A] time = 0.0374316, size = 78, normalized size = 0.96

$$\frac{2\left(-77a^3\left(A+5Bx^2\right)+385a^2bx^2\left(Bx^2-3A\right)+55ab^2x^4\left(7A+3Bx^2\right)+5b^3x^6\left(11A+7Bx^2\right)\right)}{385x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(385*a^2*b*x^2*(-3*A + B*x^2) + 55*a*b^2*x^4*(7*A + 3*B*x^2) -
 77*a^3*(A + 5*B*x^2) + 5*b^3*x^6*(11*A + 7*B*x^2)))/(385*x^(5/2)
)
```

Maple [A] time = 0.008, size = 80, normalized size = 1.

$$-\frac{-70\,b^3Bx^8-110\,x^6b^3A-330\,x^6ab^2B-770\,Aab^2x^4-770\,Ba^2bx^4+2310\,Aa^2bx^2+770\,Ba^3x^2+154\,a^3A}{385}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/385*(-35*B*b^3*x^8-55*A*b^3*x^6-165*B*a*b^2*x^6-385*A*a*b^2*x^
4-385*B*a^2*b*x^4+1155*A*a^2*b*x^2+385*B*a^3*x^2+77*A*a^3)/x^(5/2
)
```

Maxima [A] time = 1.33754, size = 101, normalized size = 1.25

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{2}{7}\left(3\,Bab^2+Ab^3\right)x^{\frac{7}{2}}+2\left(Ba^2b+Aab^2\right)x^{\frac{3}{2}}-\frac{2\left(Aa^3+5\left(Ba^3+3\,Aa^2b\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 2/7*(3*B*a*b^2 + A*b^3)*x^(7/2) + 2*(B*a^2*
b + A*a*b^2)*x^(3/2) - 2/5*(A*a^3 + 5*(B*a^3 + 3*A*a^2*b)*x^2)/x^
(5/2)
```

Fricas [A] time = 0.223549, size = 101, normalized size = 1.25

$$\frac{2\left(35\,Bb^3x^8+55\left(3\,Bab^2+Ab^3\right)x^6+385\left(Ba^2b+Aab^2\right)x^4-77\,Aa^3-385\left(Ba^3+3\,Aa^2b\right)x^2\right)}{385\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*B*b^3*x^8 + 55*(3*B*a*b^2 + A*b^3)*x^6 + 385*(B*a^2*b +
 A*a*b^2)*x^4 - 77*A*a^3 - 385*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

Sympy [A] time = 33.2381, size = 107, normalized size = 1.32

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}}-\frac{6Aa^2b}{\sqrt{x}}+2Aab^2x^{\frac{3}{2}}+\frac{2Ab^3x^{\frac{7}{2}}}{7}-\frac{2Ba^3}{\sqrt{x}}+2Ba^2bx^{\frac{3}{2}}+\frac{6Bab^2x^{\frac{7}{2}}}{7}+\frac{2Bb^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) - 6*A*a**2*b/sqrt(x) + 2*A*a*b**2*x**(3/2)
 + 2*A*b**3*x**(7/2)/7 - 2*B*a**3/sqrt(x) + 2*B*a**2*b*x**(3/2) +
 6*B*a*b**2*x**(7/2)/7 + 2*B*b**3*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.223142, size = 107, normalized size = 1.32

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{6}{7}\,Bab^2x^{\frac{7}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+2\,Ba^2bx^{\frac{3}{2}}+2\,Aab^2x^{\frac{3}{2}}-\frac{2\left(5\,Ba^3x^2+15\,Aa^2bx^2+Aa^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 2
*B*a^2*b*x^(3/2) + 2*A*a*b^2*x^(3/2) - 2/5*(5*B*a^3*x^2 + 15*A*a^
2*b*x^2 + A*a^3)/x^(5/2)
```

3.367 $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=276

$$-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}-\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{13/4}}-\frac{2a\sqrt{x}(Ab-aB)}{b^3}+\frac{2x^{5/2}(Ab-aB)}{5b^2}+\frac{2Bx^{9/2}}{9b}$$

```
[Out] (-2*a*(A*b - a*B)*Sqrt[x])/b^3 + (2*(A*b - a*B)*x^(5/2))/(5*b^2)
+ (2*B*x^(9/2))/(9*b) - (a^(5/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a
*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13
/4)) - (a^(5/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a*B
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*
Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.567414, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}-\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{13/4}}-\frac{2a\sqrt{x}(Ab-aB)}{b^3}+\frac{2x^{5/2}(Ab-aB)}{5b^2}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[x])/b^3 + (2*(A*b - a*B)*x^(5/2))/(5*b^2)
+ (2*B*x^(9/2))/(9*b) - (a^(5/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a
*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13
/4)) - (a^(5/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a*B
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*
Sqrt[2]*b^(13/4))
```

Rubi in Sympy [A] time = 85.8017, size = 258, normalized size = 0.93

$$\frac{2Bx^{\frac{9}{2}}}{9b}-\frac{\sqrt{2}a^{\frac{5}{4}}\left(Ab-Ba\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{13}{4}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\left(Ab-Ba\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{13}{4}}}-\frac{\sqrt{2}a^{\frac{5}{4}}\left(Ab-Ba\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{13}{4}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\left(Ab-Ba\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{13}{4}}}-\frac{2a\sqrt{x}\left(Ab-Ba\right)}{b^3}+\frac{2x^{\frac{5}{2}}\left(Ab-Ba\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] 2*B*x**(9/2)/(9*b) - sqrt(2)*a**(5/4)*(A*b - B*a)*log(-sqrt(2)*a*
*(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(13/4)) + sq
rt(2)*a**(5/4)*(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x)
+ sqrt(a) + sqrt(b)*x)/(4*b**(13/4)) - sqrt(2)*a**(5/4)*(A*b - B*
a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(13/4)) + sq
rt(2)*a**(5/4)*(A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(
1/4))/(2*b**(13/4)) - 2*a*sqrt(x)*(A*b - B*a)/b**3 + 2*x**(5/2)*(
A*b - B*a)/(5*b**2)
```

Mathematica [A] time = 0.307242, size = 264, normalized size = 0.96

$$\frac{45\sqrt{2}a^{5/4}(aB-Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-45\sqrt{2}a^{5/4}(aB-Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+90\sqrt{2}a^{5/4}(aB-}{1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (360*a*b^(1/4)*(-(A*b) + a*B)*Sqrt[x] + 72*b^(5/4)*(A*b - a*B)*x^
(5/2) + 40*b^(9/4)*B*x^(9/2) + 90*Sqrt[2]*a^(5/4)*(-(A*b) + a*B)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 90*Sqrt[2]*a^(5/4
)*(-(A*b) + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] +
45*Sqrt[2]*a^(5/4)*(-(A*b) + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x] - 45*Sqrt[2]*a^(5/4)*(-(A*b) + a*B)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(180*b
^(13/4))
```

Maple [A] time = 0.017, size = 330, normalized size = 1.2

$$\frac{2B}{9b}x^{\frac{9}{2}}+\frac{2A}{5b}x^{\frac{5}{2}}-\frac{2Ba}{5b^2}x^{\frac{5}{2}}-2\frac{aA\sqrt{x}}{b^2}+2\frac{B\sqrt{x}a^2}{b^3}+\frac{a\sqrt{2}A}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{a\sqrt{2}A}{4b^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{a\sqrt{2}A}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{a^2\sqrt{2}B}{2b^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{a^2\sqrt{2}B}{4b^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{a^2\sqrt{2}B}{2b^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/9*B*x^(9/2)/b+2/5/b*A*x^(5/2)-2/5/b^2*B*x^(5/2)*a-2/b^2*A*x^(1/
2)*a+2/b^3*B*x^(1/2)*a^2+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*A*ln(
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a^2/b^3*(a/b)^(1/4)*2^(1/2)*B*ar
```

```
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4*a^2/b^3*(a/b)^(1/4)*2^(1/
2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2*a^2/b^3*(a/b)^(1/4)*2^(1/2)*B
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249058, size = 805, normalized size = 2.92

$$-\frac{180\,b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}}{(Ba^2-Aab)\sqrt{x}-\sqrt{b^6\sqrt{-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}}+(B^2a}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/90*(180*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4
*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*arctan(-b^3*(-(B^4*a^9
- 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b
^4)/b^13)^(1/4)/((B*a^2 - A*a*b)*sqrt(x) - sqrt(b^6*sqrt(-(B^4*a^
9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5
*b^4)/b^13) + (B^2*a^4 - 2*A*B*a^3*b + A^2*a^2*b^2)*x))) - 45*b^3
*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3
 + A^4*a^5*b^4)/b^13)^(1/4)*log(b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b +
6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4) -
(B*a^2 - A*a*b)*sqrt(x)) + 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*
A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(
-b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6
*b^3 + A^4*a^5*b^4)/b^13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) - 4*(5
*B*b^2*x^4 + 45*B*a^2 - 45*A*a*b - 9*(B*a*b - A*b^2)*x^2)*sqrt(x)
)/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.229697, size = 402, normalized size = 1.46

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^{4}}+\frac{2\left(5\,Bb^{8}x^{\frac{9}{2}}-9\,Bab^{7}x^{\frac{5}{2}}+9\,Ab^{8}x^{\frac{5}{2}}+45\,Ba^{2}b^{6}\sqrt{x}-45\,Aab^{7}\sqrt{x}\right)}{45\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1
/2*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/
4*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*ln(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 + 1/4*sqrt(2)*((a*b^3)^(
1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/b^4 + 2/45*(5*B*b^8*x^(9/2) - 9*B*a*b^7*x^(5/2)
 + 9*A*b^8*x^(5/2) + 45*B*a^2*b^6*sqrt(x) - 45*A*a*b^7*sqrt(x))/b
^9
```

3.368 $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=257

$$-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{11/4}}-\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{11/4}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{7/2}}{7b}$$

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(7/2))/(7*b) + (a^(3/4)*
(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(11/4)) - (a^(3/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*b^(11/4)) - (a^(3/4)*(A*b - a*B)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*b^(11/4)) + (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.477297, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}+\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{11/4}}-\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{11/4}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(7/2))/(7*b) + (a^(3/4)*
(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(11/4)) - (a^(3/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*b^(11/4)) - (a^(3/4)*(A*b - a*B)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*b^(11/4)) + (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4))
```

Rubi in Sympy [A] time = 78.2852, size = 240, normalized size = 0.93

$$\frac{2Bx^{\frac{7}{2}}}{7b}-\frac{\sqrt{2}a^{\frac{3}{4}}(Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{11}{4}}}+\frac{\sqrt{2}a^{\frac{3}{4}}(Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{11}{4}}}+\frac{\sqrt{2}a^{\frac{3}{4}}(Ab-Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{11}{4}}}-\frac{\sqrt{2}a^{\frac{3}{4}}(Ab-Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{11}{4}}}+\frac{2x^{\frac{3}{2}}(Ab-Ba)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] 2*B*x**(7/2)/(7*b) - sqrt(2)*a**(3/4)*(A*b - B*a)*log(-sqrt(2)*a*
*(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(11/4)) + sq
rt(2)*a**(3/4)*(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x)
+ sqrt(a) + sqrt(b)*x)/(4*b**(11/4)) + sqrt(2)*a**(3/4)*(A*b - B*
a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(11/4)) - sq
rt(2)*a**(3/4)*(A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(
1/4))/(2*b**(11/4)) + 2*x**(3/2)*(A*b - B*a)/(3*b**2)
```

Mathematica [A] time = 0.234142, size = 243, normalized size = 0.95

$$\frac{21\sqrt{2}a^{3/4}(aB-Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-21\sqrt{2}a^{3/4}(aB-Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-42\sqrt{2}a^{3/4}(aB-}{84b^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (56*b^(3/4)*(A*b - a*B)*x^(3/2) + 24*b^(7/4)*B*x^(7/2) - 42*Sqrt[
2]*a^(3/4)*(-(A*b) + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)] + 42*Sqrt[2]*a^(3/4)*(-(A*b) + a*B)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)] + 21*Sqrt[2]*a^(3/4)*(-(A*b) + a*B)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 21*Sqrt[2
]*a^(3/4)*(-(A*b) + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(84*b^(11/4))
```

Maple [A] time = 0.011, size = 308, normalized size = 1.2

$$\frac{2\,B}{7\,b}x^{\frac{7}{2}}+\frac{2\,A}{3\,b}x^{\frac{3}{2}}-\frac{2\,Ba}{3\,b^2}x^{\frac{3}{2}}-\frac{a\sqrt{2}A}{2\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{a\sqrt{2}A}{2\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{a\sqrt{2}A}{4\,b^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{a^2\sqrt{2}B}{2\,b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{a^2\sqrt{2}B}{2\,b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{a^2\sqrt{2}B}{4\,b^3}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/7*B*x^(7/2)/b+2/3/b*x^(3/2)*A-2/3/b^2*x^(3/2)*B*a-1/2*a/b^2/(a/
b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a/b^
2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4
*a/b^2/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2*a^2/b
^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/
```

```
2*a^2/b^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)+1/4*a^2/b^3/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254527, size = 1054, normalized size = 4.1

$$-\frac{84\,b^2\left(-\frac{B^4a^7-4\,AB^3a^6b+6\,A^2B^2a^5b^2-4\,A^3Ba^4b^3+A^4a^3b^4}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{}{(B^3a^5-3\,AB^2a^4b+3\,A^2Ba^3b^2-A^3a^2b^3)\sqrt{x}-\sqrt{(B^6a^{10}-6\,AB^5a^9b+15\,A^2B^4a^8b^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/42*(84*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*
A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*arctan(-b^8*(-(B^4*a^7 -
 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^
4)/b^11)^(3/4)/((B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*
a^2*b^3)*sqrt(x) - sqrt((B^6*a^10 - 6*A*B^5*a^9*b + 15*A^2*B^4*a^
8*b^2 - 20*A^3*B^3*a^7*b^3 + 15*A^4*B^2*a^6*b^4 - 6*A^5*B*a^5*b^5
 + A^6*a^4*b^6)*x - (B^4*a^7*b^5 - 4*A*B^3*a^6*b^6 + 6*A^2*B^2*a^
5*b^7 - 4*A^3*B*a^4*b^8 + A^4*a^3*b^9)*sqrt(-(B^4*a^7 - 4*A*B^3*a
^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)))
) + 21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3
*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(b^8*(-(B^4*a^7 - 4*A*B^
3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11
)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^
3)*sqrt(x)) - 21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b
^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(-b^8*(-(B^4*a
^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^
3*b^4)/b^11)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 -
 A^3*a^2*b^3)*sqrt(x)) - 4*(3*B*b*x^3 - 7*(B*a - A*b)*x)*sqrt(x))
/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.256861, size = 356, normalized size = 1.39

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{5}}+\frac{2\left(3Bb^{6}x^{\frac{7}{2}}-7Bab^{5}x^{\frac{3}{2}}+7Ab^{6}x^{\frac{3}{2}}\right)}{21b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 + 1/2*sq
rt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/4*sqrt(2)
*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a
*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/b^5 + 2/21*(3*B*b^6*x^(7/2) - 7*B*a*b^5*x^(3/2) + 7*A*b^6*x^(3/2
))/b^7
```

3.369 $\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=255

$$\frac{\sqrt[4]{a}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{a}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{9/4}}+\frac{2\sqrt{x}(Ab-aB)}{b^2}+\frac{2Bx^{5/2}}{5b}$$

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(5/2))/(5*b) + (a^(1/4)*(A*b
 - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b
^(9/4)) - (a^(1/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(1/4)*(A*b - a*B)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/
4)) - (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.443393, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{\sqrt[4]{a}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{a}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{9/4}}+\frac{2\sqrt{x}(Ab-aB)}{b^2}+\frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(5/2))/(5*b) + (a^(1/4)*(A*b
 - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b
^(9/4)) - (a^(1/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(1/4)*(A*b - a*B)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/
4)) - (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi in Sympy [A] time = 78.6494, size = 238, normalized size = 0.93

$$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{\sqrt{2}\sqrt[4]{a}\,(Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{9}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}\,(Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{9}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}\,(Ab-Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{9}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}\,(Ab-Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{9}{4}}}+\frac{2\sqrt{x}\,(Ab-Ba)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] 2*B*x**(5/2)/(5*b) + sqrt(2)*a**(1/4)*(A*b - B*a)*log(-sqrt(2)*a*
*(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(9/4)) - sqr
```

```
t(2)*a**(1/4)*(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) +
 sqrt(a) + sqrt(b)*x)/(4*b**(9/4)) + sqrt(2)*a**(1/4)*(A*b - B*a)
*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(9/4)) - sqrt(
2)*a**(1/4)*(A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4
))/(2*b**(9/4)) + 2*sqrt(x)*(A*b - B*a)/b**2
```

Mathematica [A] time = 0.221948, size = 243, normalized size = 0.95

$$\frac{40\sqrt[4]{b}\sqrt{x}(Ab-aB)-5\sqrt{2}\sqrt[4]{a}(aB-Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}\sqrt[4]{a}(aB-Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{20b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (40*b^(1/4)*(A*b - a*B)*Sqrt[x] + 8*b^(5/4)*B*x^(5/2) - 10*Sqrt[2
]*a^(1/4)*(-(A*b) + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)] + 10*Sqrt[2]*a^(1/4)*(-(A*b) + a*B)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(1/4)*(-(A*b) + a*B)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a
^(1/4)*(-(A*b) + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(20*b^(9/4))
```

Maple [A] time = 0.011, size = 299, normalized size = 1.2

$$\frac{2B}{5b}x^{\frac{5}{2}}+2\frac{A\sqrt{x}}{b}-2\frac{B\sqrt{x}a}{b^2}-\frac{\sqrt{2}A}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}A}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}A}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}Ba}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}Ba}{2b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{\sqrt{2}Ba}{4b^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/5*B*x^(5/2)/b+2/b*A*x^(1/2)-2/b^2*B*x^(1/2)*a-1/2/b*(a/b)^(1/4)
*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/b*(a/b)^(1/4
)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/b*(a/b)^(1/
4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2/b^2*(a/b)^(1/4)*2^(1/
2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a+1/2/b^2*(a/b)^(1/4)*
2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a+1/4/b^2*(a/b)^(
1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25064, size = 737, normalized size = 2.89

$$\frac{20\,b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}}{(Ba-Ab)\sqrt{x}-\sqrt{b^4\sqrt{-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}}+(B^2a^2-2\,AB}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A
^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4)*arctan(-b^2*(-(B^4*a^5 - 4*A
*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9
)^(1/4)/((B*a - A*b)*sqrt(x) - sqrt(b^4*sqrt(-(B^4*a^5 - 4*A*B^3*
a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9) + (
B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x))) - 5*b^2*(-(B^4*a^5 - 4*A*B^3*
a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/
4)*log(b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3
*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) + 5*b^2
*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3
 + A^4*a*b^4)/b^9)^(1/4)*log(-b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*
A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a
- A*b)*sqrt(x)) + 4*(B*b*x^2 - 5*B*a + 5*A*b)*sqrt(x))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.259624, size = 355, normalized size = 1.39

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3}+\frac{2\left(Bb^4x^{\frac{5}{2}}-5\,Bab^3\sqrt{x}+5\,Ab^4\sqrt{x}\right)}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/2*sq
rt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)
*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a
*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/b^3 + 2/5*(B*b^4*x^(5/2) - 5*B*a*b^3*sqrt(x) + 5*A*b^4*sqrt(x))/
b^5
```

3.370 $\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=237

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{2Bx^{3/2}}{3b}$$

```
[Out] (2*B*x^(3/2))/(3*b) - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4))
 + ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(7/4)) - ((A*b - a*B)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/
4)*b^(7/4))
```

Rubi [A] time = 0.405857, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4))
 + ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(7/4)) - ((A*b - a*B)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/
4)*b^(7/4))
```

Rubi in Sympy [A] time = 70.0701, size = 221, normalized size = 0.93

$$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{\sqrt{2}(Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{a}b^{\frac{7}{4}}}-\frac{\sqrt{2}(Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{a}b^{\frac{7}{4}}}-\frac{\sqrt{2}(Ab-Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{\frac{7}{4}}}+\frac{\sqrt{2}(Ab-Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*x**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*B*x**(3/2)/(3*b) + sqrt(2)*(A*b - B*a)*log(-sqrt(2)*a**(1/4)*b*
*(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(1/4)*b**(7/4)) - sqr
```

```
t(2)*(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a)
+ sqrt(b)*x)/(4*a**(1/4)*b**(7/4)) - sqrt(2)*(A*b - B*a)*atan(1 -
 sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(1/4)*b**(7/4)) + sqrt(
2)*(A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**
(1/4)*b**(7/4))
```

Mathematica [A] time = 0.1969, size = 213, normalized size = 0.9

$$\frac{3\sqrt{2}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-3\sqrt{2}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-6\sqrt{2}(Ab-aB)\tan^{-1}\left(1-\right.}{12\sqrt[4]{a}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (8*a^(1/4)*b^(3/4)*B*x^(3/2) - 6*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 6*Sqrt[2]*(A*b - a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 3*Sqrt[2]*(A*b - a*B)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 3*Sqrt
[2]*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(12*a^(1/4)*b^(7/4))
```

Maple [A] time = 0.011, size = 280, normalized size = 1.2

$$\frac{2B}{3b}x^{\frac{3}{2}}+\frac{\sqrt{2}A}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}A}{4b}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}A}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}Ba}{2b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{\sqrt{2}Ba}{4b^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{\sqrt{2}Ba}{2b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*x^(1/2)/(b*x^2+a),x)
```

```
[Out] 2/3*B*x^(3/2)/b+1/2/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)-1)+1/4/b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2)))+1/2/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)-1/2/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)*a-1/4/b^2/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))*a-1/2/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.251612, size = 977, normalized size = 4.12

$$\frac{4\,Bx^{\frac{3}{2}} + 12\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{}{(B^3a^3-3\,AB^2a^2b+3\,A^2Bab^2-A^3b^3)\sqrt{x}-\sqrt{(B^6a^6-6\,AB^5a^5b+15\,A^2B^4a^4b^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) + 12*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^
2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*arctan(-a*b^5*(-(
B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4
*b^4)/(a*b^7))^(3/4)/((B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 -
A^3*b^3)*sqrt(x) - sqrt((B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)*x - (B^4*a^5*b^3 - 4*A*B^3*a^4*b^4 + 6*A^2*B^2*a^3*b^5 -
 4*A^3*B*a^2*b^6 + A^4*a*b^7)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*
A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))))) + 3*b*(-(B
^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*
b^4)/(a*b^7))^(1/4)*log(a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*
B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3
- 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - 3*b*(-(B^4*
a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4
)/(a*b^7))^(1/4)*log(-a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 -
3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.324584, size = 339, normalized size = 1.43

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba - \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{4}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba - \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{4}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba - \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^{4}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba - \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*
A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(a*b^4) - 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*
b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b
)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) - 1/4*s
qrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
```

3.371 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)}\,dx$

Optimal. Leaf size=235

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}b^{5/4}}+\frac{2B\sqrt{x}}{b}$$

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) - (
(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b
^(5/4))
```

Rubi [A] time = 0.374576, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}b^{5/4}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```


```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) - (
(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b
^(5/4))
```

Rubi in Sympy [A] time = 71.4617, size = 219, normalized size = 0.93

$$\frac{2B\sqrt{x}}{b}-\frac{\sqrt{2}(Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}(Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}(Ab-Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}(Ab-Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*B*sqrt(x)/b - sqrt(2)*(A*b - B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4
)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(3/4)*b**(5/4)) + sqrt(2)*
(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqr
```

```
t(b)*x)/(4*a**(3/4)*b**(5/4)) - sqrt(2)*(A*b - B*a)*atan(1 - sqrt
(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(3/4)*b**(5/4)) + sqrt(2)*(A
*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(3/4)
*b**(5/4))
```

Mathematica [A] time = 0.203323, size = 212, normalized size = 0.9

$$\frac{8a^{3/4}\sqrt[4]{b}B\sqrt{x} - \sqrt{2}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + \sqrt{2}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 2\sqrt{2}(Ab-a}{4a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*B*Sqrt[x] - 2*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*Sqrt[2]*(A*b - a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - Sqrt[2]*(A*b - a*B)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*
(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(4*a^(3/4)*b^(5/4))
```

Maple [A] time = 0.011, size = 277, normalized size = 1.2

$$2\,\frac{B\sqrt{x}}{b}+\frac{\sqrt{2}A}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{\sqrt{2}A}{4\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}A}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}B}{2\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}B}{4\,b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}B}{2\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 2*B*x^(1/2)/b+1/2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)+1/4*(a/b)^(1/4)/a*2^(1/2)*A*ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))+1/2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)-1/2/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)-1/4/b*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2
/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25076, size = 709, normalized size = 3.02

$$-\frac{4\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}}{(Ba-Ab)\sqrt{x}-\sqrt{a^2b^2\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}}+(B^2a^2-2\,ABab\cdots}}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*arctan(-a*b*(-(B^4*a^4 - 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5
))^(1/4)/((B*a - A*b)*sqrt(x) - sqrt(a^2*b^2*sqrt(-(B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^
5)) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x))) - b*(-(B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5)
)^(1/4)*log(a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt(x))
+ b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b
 + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4)
- (B*a - A*b)*sqrt(x)) - 4*B*sqrt(x))/b
```

Sympy [A] time = 41.0706, size = 371, normalized size = 1.58

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ -\frac{\sqrt[4]{-1}A\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b^4\left(\frac{1}{b}\right)^{\frac{15}{4}}}+\frac{\sqrt[4]{-1}A\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}b^4\left(\frac{1}{b}\right)^{\frac{15}{4}}}-\frac{\sqrt[4]{-1}A\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{3}{4}}b^4\left(\frac{1}{b}\right)^{\frac{15}{4}}}+\frac{\sqrt[4]{-1}B\sqrt[4]{a}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^5\left(\frac{1}{b}\right)^{\frac{15}{4}}}-\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b
, 0)), ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqr
t(x) + 2*B*x**(5/2)/5)/a, Eq(b, 0)), (-(-1)**(1/4)*A*log(-(-1)**(
1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)*b**4*(1/b)**(15
/4)) + (-1)**(1/4)*A*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt
(x))/(2*a**(3/4)*b**4*(1/b)**(15/4)) - (-1)**(1/4)*A*atan((-1)**(
3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(3/4)*b**4*(1/b)**(15/4
)) + (-1)**(1/4)*B*a**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4
) + sqrt(x))/(2*b**5*(1/b)**(15/4)) - (-1)**(1/4)*B*a**(1/4)*log(
(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**5*(1/b)**(15/4
)) + (-1)**(1/4)*B*a**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1
/b)**(1/4)))/(b**5*(1/b)**(15/4)) + 2*B*sqrt(x)/b, True))
```

GIAC/XCAS [A] time = 0.295454, size = 339, normalized size = 1.44

$$\frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^2} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^2} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*
b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/
4))/(a*b^2) - 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4
))/(a*b^2) - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*
ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) + 1/4*sqr
t(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
```

3.372 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=235

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{3/4}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}b^{3/4}}-\frac{2A}{a\sqrt{x}}$$

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4))
- ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(3/4)) + ((A*b - a*B)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4
)*b^(3/4))
```

Rubi [A] time = 0.398846, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{3/4}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}b^{3/4}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4))
- ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(3/4)) + ((A*b - a*B)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4
)*b^(3/4))
```

Rubi in Sympy [A] time = 71.3506, size = 219, normalized size = 0.93

$$-\frac{2A}{a\sqrt{x}}-\frac{\sqrt{2}(Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{5}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}(Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{5}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}(Ab-Ba)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{5}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}(Ab-Ba)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{5}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(3/2)/(b*x**2+a),x)
```

```
[Out] -2*A/(a*sqrt(x)) - sqrt(2)*(A*b - B*a)*log(-sqrt(2)*a**(1/4)*b**(
1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(5/4)*b**(3/4)) + sqrt(
2)*(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
```

```
sqrt(b)*x)/(4*a**(5/4)*b**(3/4)) + sqrt(2)*(A*b - B*a)*atan(1 - s
qrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(5/4)*b**(3/4)) - sqrt(2)
*(A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(5
/4)*b**(3/4))
```

Mathematica [A] time = 0.301534, size = 221, normalized size = 0.94

$$\frac{\frac{\sqrt{2}(aB-Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{\sqrt{2}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{2\sqrt{2}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{2\sqrt{2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{b^{3/4}}}{4a^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((-8*a^(1/4)*A)/Sqrt[x] + (2*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) - (2*Sqrt[2]*(A*b - a*B)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (Sqrt[2]*
(-(A*b) + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/b^(3/4) + (Sqrt[2]*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4))/(4*a^(5/4))
```

Maple [A] time = 0.014, size = 277, normalized size = 1.2

$$-\frac{\sqrt{2}A}{2\,a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}A}{4\,a}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{\sqrt{2}A}{2\,a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}B}{2\,b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}B}{4\,b}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}B}{2\,b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-2\,\frac{A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
)-1/4/a/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2/a/(a
/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/b/(
a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/4/b/
(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2/b/(a/b)^(1/4
)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-2*A/a/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250424, size = 996, normalized size = 4.24

$$-\frac{4\,a\sqrt{x}\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\cdots}{(B^3a^3-3\,AB^2a^2b+3\,A^2Bab^2-A^3b^3)\sqrt{x}-\sqrt{(B^6a^6-6\,AB^5a^5b+15\,A^2B^4a^4b^2-20\,A\cdots}}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2
- 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*arctan(-a^4*b^2*(-(B^
4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^5*b^3))^(3/4)/((B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 -
A^3*b^3)*sqrt(x) - sqrt((B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)*x - (B^4*a^7*b - 4*A*B^3*a^6*b^2 + 6*A^2*B^2*a^5*b^3 - 4
*A^3*B*a^4*b^4 + A^4*a^3*b^5)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*
A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))))) + a*sqrt
(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/
4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x))
 - a*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A
^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-(B^4*a^4 -
4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5
*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3
)*sqrt(x)) + 4*A)/(a*sqrt(x))
```

Sympy [A] time = 95.4648, size = 374, normalized size = 1.59

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}}{b} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} \\ -\frac{2A}{a\sqrt{x}}+\frac{(-1)^{\frac{3}{4}}A\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}b^{11}\left(\frac{1}{b}\right)^{\frac{45}{4}}}-\frac{(-1)^{\frac{3}{4}}A\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}b^{11}\left(\frac{1}{b}\right)^{\frac{45}{4}}}-\frac{(-1)^{\frac{3}{4}}A\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}b^{11}\left(\frac{1}{b}\right)^{\frac{45}{4}}}-\frac{(-1)^{\frac{3}{4}}B\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}b^{12}\left(\frac{1}{b}\right)^{\frac{45}{4}}}+\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b
, 0)), ((-2*A/(5*x**(5/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sq
rt(x) + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-2*A/(a*sqrt(x)) + (-1)**(
3/4)*A*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5
```

```
/4)*b**11*(1/b)**(45/4)) - (-1)**(3/4)*A*log((-1)**(1/4)*a**(1/4)
*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*b**11*(1/b)**(45/4)) - (-1)*
*(3/4)*A*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(5
/4)*b**11*(1/b)**(45/4)) - (-1)**(3/4)*B*log(-(-1)**(1/4)*a**(1/4
)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b**12*(1/b)**(45/4)) + (-1)
**(3/4)*B*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**
(1/4)*b**12*(1/b)**(45/4)) + (-1)**(3/4)*B*atan((-1)**(3/4)*sqrt(
x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b**12*(1/b)**(45/4)), True)
)
```

GIAC/XCAS [A] time = 0.319834, size = 339, normalized size = 1.44

$$-\frac{2\,A}{a\sqrt{x}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{2}b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{2}b^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{2}b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(3/2)),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(a^2*b^3) + 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)
*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4
)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
+ 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*ln(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

3.373 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=237

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{2A}{3ax^{3/2}}$$

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)
) + ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7
/4)*b^(1/4))
```

Rubi [A] time = 0.387985, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)
) + ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7
/4)*b^(1/4))
```

Rubi in Sympy [A] time = 67.4079, size = 221, normalized size = 0.93

$$-\frac{2A}{3ax^{\frac{3}{2}}}+\frac{\sqrt{2}(Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{7}{4}}\sqrt[4]{b}}-\frac{\sqrt{2}(Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{7}{4}}\sqrt[4]{b}}+\frac{\sqrt{2}(Ab-Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{7}{4}}\sqrt[4]{b}}-\frac{\sqrt{2}(Ab-Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{7}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(5/2)/(b*x**2+a),x)
```

```
[Out] -2*A/(3*a*x**(3/2)) + sqrt(2)*(A*b - B*a)*log(-sqrt(2)*a**(1/4)*b
**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(7/4)*b**(1/4)) - sq
rt(2)*(A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a)
```

```
 + sqrt(b)*x)/(4*a**(7/4)*b**(1/4)) + sqrt(2)*(A*b - B*a)*atan(1
- sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(7/4)*b**(1/4)) - sqrt
(2)*(A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a*
*(7/4)*b**(1/4))
```

Mathematica [A] time = 0.321666, size = 223, normalized size = 0.94

$$\frac{-\frac{8a^{3/4}A}{x^{3/2}}+\frac{3\sqrt{2}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}(aB-Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}(Ab-a}{}}{12a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] ((-8*a^(3/4)*A)/x^(3/2) + (6*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (6*Sqrt[2]*(A*b - a*B)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (3*Sqrt[2
]*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/b^(1/4) + (3*Sqrt[2]*(-(A*b) + a*B)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(12*a^(7/4))
```

Maple [A] time = 0.014, size = 280, normalized size = 1.2

$$-\frac{\sqrt{2}Ab}{2\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}Ab}{2\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}Ab}{4\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}B}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}B}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{\sqrt{2}B}{4\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{2\,A}{3\,a}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/2/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)*b-1/2/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)-1)*b-1/4/a^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
))*b+1/2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)+1/2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)+1/4/a*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-2/3
*A/a/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2406, size = 722, normalized size = 3.05

$$12\,ax^{\frac{3}{2}}\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}}{(Ba-Ab)\sqrt{x}-\sqrt{a^4\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}}+(B^2a^2-2\,ABab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(12*a*x^(3/2)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2
- 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4)*arctan(-a^2*(-(B^4*a^4
- 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a
^7*b))^(1/4)/((B*a - A*b)*sqrt(x) - sqrt(a^4*sqrt(-(B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b)
) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x))) - 3*a*x^(3/2)*(-(B^4*a^4
 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(
a^7*b))^(1/4)*log(a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*
b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(
x)) + 3*a*x^(3/2)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2
- 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4)*log(-a^2*(-(B^4*a^4 - 4
*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*
b))^(1/4) - (B*a - A*b)*sqrt(x)) - 4*A)/(a*x^(3/2))
```

Sympy [A] time = 170.036, size = 379, normalized size = 1.6

$$\begin{cases}\tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{a} \\ -\frac{2A}{3ax^{\frac{3}{2}}}+\frac{\sqrt[4]{-1}Ab^7\left(\frac{1}{b}\right)^{\frac{25}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}Ab^7\left(\frac{1}{b}\right)^{\frac{25}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}}+\frac{\sqrt[4]{-1}Ab^7\left(\frac{1}{b}\right)^{\frac{25}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}Bb}{}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) &
 Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)),
 ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(3*a*x**(
3/2)) + (-1)**(1/4)*A*b**7*(1/b)**(25/4)*log(-(-1)**(1/4)*a**(1/4
)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) - (-1)**(1/4)*A*b**7*(1/b)
**(25/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(
7/4)) + (-1)**(1/4)*A*b**7*(1/b)**(25/4)*atan((-1)**(3/4)*sqrt(x)
/(a**(1/4)*(1/b)**(1/4)))/a**(7/4) - (-1)**(1/4)*B*b**6*(1/b)**(2
5/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4
)) + (-1)**(1/4)*B*b**6*(1/b)**(25/4)*log((-1)**(1/4)*a**(1/4)*(1
/b)**(1/4) + sqrt(x))/(2*a**(3/4)) - (-1)**(1/4)*B*b**6*(1/b)**(2
5/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4),
True))
```

GIAC/XCAS [A] time = 0.264679, size = 339, normalized size = 1.43

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{2}b}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{2}b}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{2}b}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{2}b}-\frac{2A}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(5/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/
2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/4
*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ln(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 1/4*sqrt(2)*((a*b^3)^(1
/4)*B*a - (a*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
+ sqrt(a/b))/(a^2*b) - 2/3*A/(a*x^(3/2))
```

3.374 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

Optimal. Leaf size=255

$$\frac{\sqrt[4]{b}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{\sqrt[4]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}}+\frac{2(Ab-aB)}{a^2\sqrt{x}}-\frac{2A}{5ax^{5/2}}$$

```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(A*b - a*B))/(a^2*Sqrt[x]) - (b^(1/4)*(
A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*a^(9/4)) + (b^(1/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(1/4)*(A*b - a*B)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(9/4)) - (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rubi [A] time = 0.444201, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{\sqrt[4]{b}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{\sqrt[4]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}}+\frac{2(Ab-aB)}{a^2\sqrt{x}}-\frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(A*b - a*B))/(a^2*Sqrt[x]) - (b^(1/4)*(
A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*a^(9/4)) + (b^(1/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(1/4)*(A*b - a*B)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(9/4)) - (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rubi in Sympy [A] time = 74.8395, size = 240, normalized size = 0.94

$$-\frac{2A}{5ax^{\frac{5}{2}}}+\frac{2\left(Ab-Ba\right)}{a^2\sqrt{x}}+\frac{\sqrt{2}\sqrt[4]{b}\left(Ab-Ba\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{9}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\left(Ab-Ba\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{9}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\left(Ab-Ba\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{9}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\left(Ab-Ba\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(7/2)/(b*x**2+a),x)
```

```
[Out] -2*A/(5*a*x**(5/2)) + 2*(A*b - B*a)/(a**2*sqrt(x)) + sqrt(2)*b**(
1/4)*(A*b - B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a)
 + sqrt(b)*x)/(4*a**(9/4)) - sqrt(2)*b**(1/4)*(A*b - B*a)*log(sqr
```

```
t(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(9/4)
) - sqrt(2)*b**(1/4)*(A*b - B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x
)/a**(1/4))/(2*a**(9/4)) + sqrt(2)*b**(1/4)*(A*b - B*a)*atan(1 +
sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(9/4))
```

Mathematica [A] time = 0.391585, size = 243, normalized size = 0.95

$$\frac{-\frac{8a^{5/4}A}{x^{5/2}}-\frac{40\sqrt[4]{a}(aB-Ab)}{\sqrt{x}}+5\sqrt{2}\sqrt[4]{b}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}\sqrt[4]{b}(aB-Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{20a^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] ((-8*a^(5/4)*A)/x^(5/2) - (40*a^(1/4)*(-(A*b) + a*B))/Sqrt[x] - 1
0*Sqrt[2]*b^(1/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] + 10*Sqrt[2]*b^(1/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] + 5*Sqrt[2]*b^(1/4)*(A*b - a*B)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*b^
(1/4)*(-(A*b) + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(20*a^(9/4))
```

Maple [A] time = 0.017, size = 299, normalized size = 1.2

$$-\frac{2A}{5a}x^{-\frac{5}{2}}+2\frac{Ab}{\sqrt{x}a^2}-2\frac{B}{\sqrt{x}a}+\frac{\sqrt{2}Ab}{2a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}Ab}{2a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}Ab}{4a^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{\sqrt{2}B}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}B}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{\sqrt{2}B}{4a}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(7/2)/(b*x^2+a),x)
```

```
[Out] -2/5*A/a/x^(5/2)+2/x^(1/2)/a^2*A*b-2/x^(1/2)/a*B+1/2/a^2/(a/b)^(1
/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b+1/2/a^2/(a/
b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b+1/4/a^
2/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*b-1/2/a/(a/b)^
(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a/(a/b)
^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/a/(a/b
)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.252987, size = 1031, normalized size = 4.04

$$\frac{20\,a^2x^{\frac{5}{2}}\left(-\frac{B^4a^4b-4\,AB^3a^3b^2+6\,A^2B^2a^2b^3-4\,A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{}{(B^3a^3b-3\,AB^2a^2b^2+3\,A^2Bab^3-A^3b^4)\sqrt{x}-\sqrt{(B^6a^6b^2-6\,AB^5a^5b^3+15\,A^2B^4a^4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(20*a^2*x^(5/2)*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*arctan(-a^7*(-(B^4*a
^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*
b^5)/a^9)^(3/4)/((B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A
^3*b^4)*sqrt(x) - sqrt((B^6*a^6*b^2 - 6*A*B^5*a^5*b^3 + 15*A^2*B^
4*a^4*b^4 - 20*A^3*B^3*a^3*b^5 + 15*A^4*B^2*a^2*b^6 - 6*A^5*B*a*b
^7 + A^6*b^8)*x - (B^4*a^9*b - 4*A*B^3*a^8*b^2 + 6*A^2*B^2*a^7*b^
3 - 4*A^3*B*a^6*b^4 + A^4*a^5*b^5)*sqrt(-(B^4*a^4*b - 4*A*B^3*a^3
*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)))) + 5*a
^2*x^(5/2)*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4
*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(a^7*(-(B^4*a^4*b - 4*A*B^3
*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4
) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(
x)) - 5*a^2*x^(5/2)*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^
2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(-a^7*(-(B^4*a^4*b
 - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)
/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*
b^4)*sqrt(x)) - 20*(B*a - A*b)*x^2 - 4*A*a)/(a^2*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.25155, size = 362, normalized size = 1.42

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^3b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^3b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^3b^2}-\frac{2\left(5Bax^2-5Abx^2+Aa\right)}{5a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)*x^(7/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) -
 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2)
+ 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*ln(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 1/4*sqrt(2)*((a*
b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^3*b^2) - 2/5*(5*B*a*x^2 - 5*A*b*x^2 + A*a)
/(a^2*x^(5/2))
```

3.375 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}+\frac{\sqrt[4]{a}(5Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}b^{13/4}}+\frac{\sqrt{x}(5Ab-9aB)}{2b^3}-\frac{x^{5/2}(5Ab-9aB)}{10ab^2}+\frac{x^{9/2}(Ab-aB)}{2ab(a+bx^2)}$$

```
[Out] ((5*A*b - 9*a*B)*Sqrt[x])/(2*b^3) - ((5*A*b - 9*a*B)*x^(5/2))/(10
*a*b^2) + ((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) + (a^(1/4)*(5
*A*b - 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*S
qrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(13/4)) + (a^(1/4)*(5*A*b
- 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4
))
```

Rubi [A] time = 0.518517, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}+\frac{\sqrt[4]{a}(5Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}b^{13/4}}+\frac{\sqrt{x}(5Ab-9aB)}{2b^3}-\frac{x^{5/2}(5Ab-9aB)}{10ab^2}+\frac{x^{9/2}(Ab-aB)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```


```
[Out] ((5*A*b - 9*a*B)*Sqrt[x])/(2*b^3) - ((5*A*b - 9*a*B)*x^(5/2))/(10
*a*b^2) + ((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) + (a^(1/4)*(5
*A*b - 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*S
qrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(13/4)) + (a^(1/4)*(5*A*b
- 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4
))
```

Rubi in Sympy [A] time = 87.2669, size = 289, normalized size = 0.93

$$\frac{\sqrt{2}\sqrt[4]{a}\left(5Ab-9Ba\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16b^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}\left(5Ab-9Ba\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16b^{\frac{13}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}\left(5Ab-9Ba\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8b^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}\left(5Ab-9Ba\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8b^{\frac{13}{4}}}+\frac{\sqrt{x}\left(5Ab-9Ba\right)}{2b^3}+\frac{x^{\frac{9}{2}}\left(Ab-Ba\right)}{2ab\left(a+bx^2\right)}-\frac{x^{\frac{5}{2}}\left(5Ab-9Ba\right)}{10ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] sqrt(2)*a**(1/4)*(5*A*b - 9*B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)*s
qrt(x) + sqrt(a) + sqrt(b)*x)/(16*b**(13/4)) - sqrt(2)*a**(1/4)*(
5*A*b - 9*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
sqrt(b)*x)/(16*b**(13/4)) + sqrt(2)*a**(1/4)*(5*A*b - 9*B*a)*atan
(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*b**(13/4)) - sqrt(2)*a
**(1/4)*(5*A*b - 9*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4
))/(8*b**(13/4)) + sqrt(x)*(5*A*b - 9*B*a)/(2*b**3) + x**(9/2)*(A
*b - B*a)/(2*a*b*(a + b*x**2)) - x**(5/2)*(5*A*b - 9*B*a)/(10*a*b
**2)
```

Mathematica [A]   time = 0.394001, size = 277, normalized size = 0.89

$$\frac{\frac{40a\sqrt[4]{b}\sqrt{x}(Ab-aB)}{a+bx^2}+160\sqrt[4]{b}\sqrt{x}(Ab-2aB)-5\sqrt{2}\sqrt[4]{a}(9aB-5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}\sqrt[4]{a}(9aB-5Ab)\log\left(\sqrt{\right.}{8}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (160*b^(1/4)*(A*b - 2*a*B)*Sqrt[x] + 32*b^(5/4)*B*x^(5/2) + (40*a
*b^(1/4)*(A*b - a*B)*Sqrt[x])/(a + b*x^2) - 10*Sqrt[2]*a^(1/4)*(-
5*A*b + 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10
*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)] - 5*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(1/4)
*(-5*A*b + 9*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(80*b^(13/4))
```

Maple [A]   time = 0.021, size = 339, normalized size = 1.1

$$\frac{2B}{5b^2}x^{\frac{5}{2}}+2\frac{A\sqrt{x}}{b^2}-4\frac{B\sqrt{x}a}{b^3}+\frac{Aa}{2b^2(bx^2+a)}\sqrt{x}$$
$$-\frac{a^2B}{2b^3(bx^2+a)}\sqrt{x}-\frac{5\sqrt{2}A}{8b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{5\sqrt{2}A}{16b^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{5\sqrt{2}A}{8b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{9a\sqrt{2}B}{8b^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{9a\sqrt{2}B}{16b^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{9a\sqrt{2}B}{8b^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2/5/b^2*B*x^(5/2)+2/b^2*A*x^(1/2)-4/b^3*B*x^(1/2)*a+1/2*a/b^2*x^(
1/2)/(b*x^2+a)*A-1/2*a^2/b^3*x^(1/2)/(b*x^2+a)*B-5/8/b^2*(a/b)^(1
/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-5/16/b^2*(a/b
)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-5/8/b^2*(a/b)^(1/4)*
2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*a/b^3*(a/b)^(
1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+9/16*a/b^3*(
a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+9/8*a/b^3*(a/b)^(
1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254727, size = 851, normalized size = 2.75

$$\frac{20\left(b^4x^2+ab^3\right)\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^3\left(-\frac{6561\,B^4a^5-14580\,A\cdots}{}\right.}{(9\,Ba-5\,Ab)\sqrt{x}-\sqrt{b^6\sqrt{-\frac{6561\,B^4a^5-14580\,AB^3a^4\cdots}{}}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(20*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b +
12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)
^(1/4)*arctan(-b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^
2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)/(
(9*B*a - 5*A*b)*sqrt(x) - sqrt(b^6*sqrt(-(6561*B^4*a^5 - 14580*A*
B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*
a*b^4)/b^13) + (81*B^2*a^2 - 90*A*B*a*b + 25*A^2*b^2)*x))) - 5*(b
^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B
^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(
b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 -
 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a - 5*A*b
)*sqrt(x)) + 5*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^
4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)
/b^13)^(1/4)*log(-b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150
*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4
) - (9*B*a - 5*A*b)*sqrt(x)) + 4*(4*B*b^2*x^4 - 45*B*a^2 + 25*A*a
*b - 4*(9*B*a*b - 5*A*b^2)*x^2)*sqrt(x))/(b^4*x^2 + a*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.249426, size = 402, normalized size = 1.3

$$\frac{\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba-5\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba-5\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba-5\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^4}-\frac{\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba-5\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^4}-\frac{Ba^2\sqrt{x}-Aab\sqrt{x}}{2\,(bx^2+a)b^3}+\frac{2\left(Bb^8x^{\frac{5}{2}}-10\,Bab^7\sqrt{x}+5\,Ab^8\sqrt{x}\right)}{5\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/
8*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/1
6*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*ln(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/16*sqrt(2)*(9*(a*b^3
)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/b^4 - 1/2*(B*a^2*sqrt(x) - A*a*b*sqrt(x))/((b*
x^2 + a)*b^3) + 2/5*(B*b^8*x^(5/2) - 10*B*a*b^7*sqrt(x) + 5*A*b^8
*sqrt(x))/b^10
```

3.376 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{(3Ab-7aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} + \frac{(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{x^{3/2}(3Ab-7aB)}{6ab^2} + \frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

```
[Out] -((3*A*b - 7*a*B)*x^(3/2))/(6*a*b^2) + ((A*b - a*B)*x^(7/2))/(2*a
*b*(a + b*x^2)) - ((3*A*b - 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)
*b^(11/4)) + ((3*A*b - 7*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/4)) - ((3*A*b -
 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(1/4)*b^(11/4))
```

Rubi [A] time = 0.469584, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$\frac{(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{(3Ab-7aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} + \frac{(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{x^{3/2}(3Ab-7aB)}{6ab^2} + \frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((3*A*b - 7*a*B)*x^(3/2))/(6*a*b^2) + ((A*b - a*B)*x^(7/2))/(2*a
*b*(a + b*x^2)) - ((3*A*b - 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)
*b^(11/4)) + ((3*A*b - 7*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/4)) - ((3*A*b -
 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(1/4)*b^(11/4))
```

Rubi in Sympy [A] time = 80.0589, size = 269, normalized size = 0.93

$$\frac{x^{\frac{7}{2}}(Ab-Ba)}{2ab\,(a+bx^2)} - \frac{x^{\frac{3}{2}}(3Ab-7Ba)}{6ab^2} + \frac{\sqrt{2}\,(3Ab-7Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16\sqrt[4]{a}b^{\frac{11}{4}}} - \frac{\sqrt{2}\,(3Ab-7Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16\sqrt[4]{a}b^{\frac{11}{4}}} - \frac{\sqrt{2}\,(3Ab-7Ba)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{\frac{11}{4}}} + \frac{\sqrt{2}\,(3Ab-7Ba)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] x**(7/2)*(A*b - B*a)/(2*a*b*(a + b*x**2)) - x**(3/2)*(3*A*b - 7*B
*a)/(6*a*b**2) + sqrt(2)*(3*A*b - 7*B*a)*log(-sqrt(2)*a**(1/4)*b*
*(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(1/4)*b**(11/4)) - s
qrt(2)*(3*A*b - 7*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sq
rt(a) + sqrt(b)*x)/(16*a**(1/4)*b**(11/4)) - sqrt(2)*(3*A*b - 7*B
*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(1/4)*b**(1
1/4)) + sqrt(2)*(3*A*b - 7*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)
/a**(1/4))/(8*a**(1/4)*b**(11/4))
```

Mathematica [A] time = 0.449962, size = 256, normalized size = 0.89

$$\frac{-\frac{24b^{3/4}x^{3/2}(Ab-aB)}{a+bx^2}+\frac{3\sqrt{2}(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}+\frac{3\sqrt{2}(7aB-3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}+\frac{6\sqrt{2}(7aB-3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right.}{\sqrt[4]{a}}\cdots}{48b^{11/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (32*b^(3/4)*B*x^(3/2) - (24*b^(3/4)*(A*b - a*B)*x^(3/2))/(a + b*x
^2) + (6*Sqrt[2]*(-3*A*b + 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/a^(1/4) + (6*Sqrt[2]*(3*A*b - 7*a*B)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (3*Sqrt[2]*(3*A*b -
7*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/a^(1/4) + (3*Sqrt[2]*(-3*A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1/4))/(48*b^(11/4))
```

Maple [A] time = 0.02, size = 317, normalized size = 1.1

$$\frac{2B}{3b^2}x^{\frac{3}{2}}-\frac{A}{2b\left(bx^2+a\right)}x^{\frac{3}{2}}+\frac{Ba}{2b^2\left(bx^2+a\right)}x^{\frac{3}{2}}$$
$$-\frac{7\sqrt{2}Ba}{16b^3}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{7\sqrt{2}Ba}{8b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{7\sqrt{2}Ba}{8b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{3\sqrt{2}A}{16b^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{3\sqrt{2}A}{8b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\sqrt{2}A}{8b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/b^2-1/2/b*x^(3/2)/(b*x^2+a)*A+1/2/b^2*x^(3/2)/(b*x^
2+a)*B*a-7/16/b^3/(a/b)^(1/4)*2^(1/2)*B*a*ln((x-(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
)))-7/8/b^3/(a/b)^(1/4)*2^(1/2)*B*a*arctan(2^(1/2)/(a/b)^(1/4)*x^
```

```
(1/2)+1)-7/8/b^3/(a/b)^(1/4)*2^(1/2)*B*a*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2)))+3/8/b^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+3/8/b^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254998, size = 1095, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2
646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/
4)*arctan(-a*b^8*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^
2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3/4)/((343*B
^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*sqrt(x)
- sqrt((117649*B^6*a^6 - 302526*A*B^5*a^5*b + 324135*A^2*B^4*a^4*
b^2 - 185220*A^3*B^3*a^3*b^3 + 59535*A^4*B^2*a^2*b^4 - 10206*A^5*
B*a*b^5 + 729*A^6*b^6)*x - (2401*B^4*a^5*b^5 - 4116*A*B^3*a^4*b^6
 + 2646*A^2*B^2*a^3*b^7 - 756*A^3*B*a^2*b^8 + 81*A^4*a*b^9)*sqrt(
-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^
3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))))) + 3*(b^3*x^2 + a*b^2)*(-(240
1*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a
*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(a*b^8*(-(2401*B^4*a^4 - 41
16*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*
b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B
*a*b^2 - 27*A^3*b^3)*sqrt(x)) - 3*(b^3*x^2 + a*b^2)*(-(2401*B^4*a
^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 +
81*A^4*b^4)/(a*b^11))^(1/4)*log(-a*b^8*(-(2401*B^4*a^4 - 4116*A*B
^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(
a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2
 - 27*A^3*b^3)*sqrt(x)) + 4*(4*B*b*x^3 + (7*B*a - 3*A*b)*x)*sqrt(
x))/(b^3*x^2 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.254201, size = 382, normalized size = 1.32

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} + \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{2\,(bx^2+a)b^2} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{3}{4}}Ba - 3\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^5} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{3}{4}}Ba - 3\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^5} + \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{3}{4}}Ba - 3\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^5} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{3}{4}}Ba - 3\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 + 1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*
b^2) - 1/8*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(
a*b^5) - 1/8*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a*b^5) + 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A
*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/1
6*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*ln(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
```

3.377 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{(Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{\sqrt{x}(Ab-5aB)}{2ab^2}+\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

```
[Out] -((A*b - 5*a*B)*Sqrt[x])/(2*a*b^2) + ((A*b - a*B)*x^(5/2))/(2*a*b
*(a + b*x^2)) - ((A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4
)) - ((A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]
*a^(3/4)*b^(9/4))
```

Rubi [A] time = 0.469691, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{(Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{\sqrt{x}(Ab-5aB)}{2ab^2}+\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 5*a*B)*Sqrt[x])/(2*a*b^2) + ((A*b - a*B)*x^(5/2))/(2*a*b
*(a + b*x^2)) - ((A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4
)) - ((A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]
*a^(3/4)*b^(9/4))
```

Rubi in Sympy [A] time = 79.0688, size = 260, normalized size = 0.92

$$\frac{x^{\frac{5}{2}}(Ab-Ba)}{2ab\,(a+bx^2)}-\frac{\sqrt{x}\,(Ab-5Ba)}{2ab^2}-\frac{\sqrt{2}\,(Ab-5Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{3}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}\,(Ab-5Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{3}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}\,(Ab-5Ba)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}\,(Ab-5Ba)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] x**(5/2)*(A*b - B*a)/(2*a*b*(a + b*x**2)) - sqrt(x)*(A*b - 5*B*a)
/(2*a*b**2) - sqrt(2)*(A*b - 5*B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4
)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(3/4)*b**(9/4)) + sqrt(2)
*(A*b - 5*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
sqrt(b)*x)/(16*a**(3/4)*b**(9/4)) - sqrt(2)*(A*b - 5*B*a)*atan(1
- sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(3/4)*b**(9/4)) + sqrt
(2)*(A*b - 5*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*
a**(3/4)*b**(9/4))
```

Mathematica [A] time = 0.545892, size = 252, normalized size = 0.89

$$\frac{\frac{\sqrt{2}(5aB-Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{\sqrt{2}(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{2\sqrt{2}(5aB-Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}(Ab-5aB)\tan^{-1}\left(\cdots\right.}{a^{3/4}}}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (32*b^(1/4)*B*Sqrt[x] - (8*b^(1/4)*(A*b - a*B)*Sqrt[x])/(a + b*x^
2) + (2*Sqrt[2]*(-(A*b) + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*(A*b - 5*a*B)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (Sqrt[2]*(-(A*b) + 5*a*
B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^
(3/4) + (Sqrt[2]*(A*b - 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(16*b^(9/4))
```

Maple [A] time = 0.019, size = 323, normalized size = 1.1

$$\begin{aligned}
&2\,\frac{B\sqrt{x}}{b^{2}}-\frac{A}{2\,b\,(bx^{2}+a)}\sqrt{x}+\frac{Ba}{2\,b^{2}\,(bx^{2}+a)}\sqrt{x}\\
&+\frac{\sqrt{2}A}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}A}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\\
&+\frac{\sqrt{2}A}{16\,ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\\
&-\frac{5\,\sqrt{2}B}{8\,b^{2}}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{5\,\sqrt{2}B}{8\,b^{2}}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\\
&-\frac{5\,\sqrt{2}B}{16\,b^{2}}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2*B*x^(1/2)/b^2-1/2/b*x^(1/2)/(b*x^2+a)*A+1/2/b^2*x^(1/2)/(b*x^2+
a)*B*a+1/8/b*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+1/8/b*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)+1/16/b*(a/b)^(1/4)/a*2^(1/2)*A*ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))-5/8/b^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)-5/8/b^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)
```

```
*x^(1/2)-1)-5/16/b^2*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.255432, size = 817, normalized size = 2.88

$$-\frac{4\left(b^3x^2+ab^2\right)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab^2\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2}{a^3b^9}\right.}{(5\,Ba-Ab)\sqrt{x}-\sqrt{a^2b^4\sqrt{-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-}{a^3b^9}}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*
A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*arct
an(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2
- 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)/((5*B*a - A*b)*sqrt(
x) - sqrt(a^2*b^4*sqrt(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*
B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9)) + (25*B^2*a^2
- 10*A*B*a*b + A^2*b^2)*x))) - (b^3*x^2 + a*b^2)*(-(625*B^4*a^4 -
 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4
)/(a^3*b^9))^(1/4)*log(a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 1
50*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) -
 (5*B*a - A*b)*sqrt(x)) + (b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*
A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^
3*b^9))^(1/4)*log(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A
^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*
B*a - A*b)*sqrt(x)) - 4*(4*B*b*x^2 + 5*B*a - A*b)*sqrt(x))/(b^3*x
^2 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.254144, size = 382, normalized size = 1.35

$$\frac{2\,B\sqrt{x}}{b^2}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3}+\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\,(bx^2+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4
)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(a*b^3) - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4
)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(a*b^3) - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1
/4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3)
+ 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ln(-sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/2*(B*a*sqrt(
x) - A*b*sqrt(x))/((b*x^2 + a)*b^2)
```

3.378 $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=261

$$\frac{(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{7/4}}+\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{7/4}}+\frac{x^{3/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

```
[Out] ((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/
4)) + ((A*b + 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((A*b + 3*a*B)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(
7/4)) - ((A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rubi [A] time = 0.398931, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{7/4}}+\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{7/4}}+\frac{x^{3/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/
4)) + ((A*b + 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((A*b + 3*a*B)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(
7/4)) - ((A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rubi in Sympy [A] time = 71.6248, size = 240, normalized size = 0.92

$$\frac{x^{\frac{3}{2}}(Ab-Ba)}{2ab\,(a+bx^2)}+\frac{\sqrt{2}\,(Ab+3Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{5}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\,(Ab+3Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{5}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\,(Ab+3Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{5}{4}}b^{\frac{7}{4}}}+\frac{\sqrt{2}\,(Ab+3Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{5}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] x**(3/2)*(A*b - B*a)/(2*a*b*(a + b*x**2)) + sqrt(2)*(A*b + 3*B*a)
*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(1
6*a**(5/4)*b**(7/4)) - sqrt(2)*(A*b + 3*B*a)*log(sqrt(2)*a**(1/4)
```

```
*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(5/4)*b**(7/4)) -
 sqrt(2)*(A*b + 3*B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4)
)/(8*a**(5/4)*b**(7/4)) + sqrt(2)*(A*b + 3*B*a)*atan(1 + sqrt(2)*
b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(5/4)*b**(7/4))
```

Mathematica [A] time = 0.307772, size = 228, normalized size = 0.87

$$\frac{-\frac{8\sqrt[4]{a}b^{3/4}x^{3/2}(aB-Ab)}{a+bx^2}+\sqrt{2}(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt{2}(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2\sqrt{}}{16a^{5/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((-8*a^(1/4)*b^(3/4)*(-(A*b) + a*B)*x^(3/2))/(a + b*x^2) - 2*Sqrt
[2]*(A*b + 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] +
 2*Sqrt[2]*(A*b + 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)] + Sqrt[2]*(A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*(A*b + 3*a*B)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*a^(5/4)*b^(7/4))
```

Maple [A] time = 0.019, size = 305, normalized size = 1.2

$$\frac{Ab-Ba}{2\,ab\,(bx^2+a)}x^{\frac{3}{2}}+\frac{\sqrt{2}A}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}A}{16\,ab}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{\sqrt{2}A}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\sqrt{2}B}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{3\sqrt{2}B}{16\,b^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{3\sqrt{2}B}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)+1/8/a/b/(a/b)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/16/a/b/(a/b)^(1/4)*2^(1/
2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/8/a/b/(a/b)^(1/4)*2^(1/2)*A*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/b^2/(a/b)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/b^2/(a/b)^(1/4)*2^(1/2
)*B*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+3/8/b^2/(a/b)^(1/4)*2^(1/2)*B*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244753, size = 1072, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*a - A*b)*x^(3/2) - 4*(a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4
 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^
4)/(a^5*b^7))^(1/4)*arctan(a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*
b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/
4)/((27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3)*sqrt(
x) + sqrt((729*B^6*a^6 + 1458*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2
+ 540*A^3*B^3*a^3*b^3 + 135*A^4*B^2*a^2*b^4 + 18*A^5*B*a*b^5 + A^
6*b^6)*x - (81*B^4*a^7*b^3 + 108*A*B^3*a^6*b^4 + 54*A^2*B^2*a^5*b
^5 + 12*A^3*B*a^4*b^6 + A^4*a^3*b^7)*sqrt(-(81*B^4*a^4 + 108*A*B^
3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7
))))) - (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(
a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12
*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2
*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) + (a*b^2*x^2 + a^2*b)*
(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a
*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-(81*B^4*a^4 + 108
*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^
5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^
3*b^3)*sqrt(x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.244316, size = 369, normalized size = 1.41

$$-\frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{2\,(bx^2 + a)ab} + \frac{\sqrt{2}\Big(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4} + \frac{\sqrt{2}\Big(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4} - \frac{\sqrt{2}\Big(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\Big)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^2b^4} + \frac{\sqrt{2}\Big(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\Big)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*a*b) + 1/8*sqrt(2)*
(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 1/8*sqrt(2
)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 1/16*sq
rt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*ln(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 1/16*sqrt(2)*(3*(a*b^3
)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(a^2*b^4)
```

3.379 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^2} \, dx$

Optimal. Leaf size=261

$$-\frac{(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}b^{5/4}}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx^2)}$$

```
[Out] ((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*A*b + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*A*b + a*B)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(
5/4)) + ((3*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.396892, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}b^{5/4}}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```


```
[Out] ((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*A*b + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*A*b + a*B)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(
5/4)) + ((3*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi in Sympy [A] time = 69.8223, size = 240, normalized size = 0.92

$$\frac{\sqrt{x}\,(Ab-Ba)}{2ab\,(a+bx^2)}-\frac{\sqrt{2}\,(3Ab+Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\,(3Ab+Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\,(3Ab+Ba)\,\text{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\,(3Ab+Ba)\,\text{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] sqrt(x)*(A*b - B*a)/(2*a*b*(a + b*x**2)) - sqrt(2)*(3*A*b + B*a)*
log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16
*a**(7/4)*b**(5/4)) + sqrt(2)*(3*A*b + B*a)*log(sqrt(2)*a**(1/4)*
```

```
b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(7/4)*b**(5/4)) -
sqrt(2)*(3*A*b + B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))
/(8*a**(7/4)*b**(5/4)) + sqrt(2)*(3*A*b + B*a)*atan(1 + sqrt(2)*b
**(1/4)*sqrt(x)/a**(1/4))/(8*a**(7/4)*b**(5/4))
```

Mathematica [A] time = 0.299087, size = 228, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}\sqrt{x}(aB-Ab)}{a+bx^2}-\sqrt{2}(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+\sqrt{2}(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2\sqrt{2}\cdots}{16a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(A*b) + a*B)*Sqrt[x])/(a + b*x^2) - 2*Sqrt
[2]*(3*A*b + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] +
 2*Sqrt[2]*(3*A*b + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)] - Sqrt[2]*(3*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*(3*A*b + a*B)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*a^(7/4)*b^(5/4))
```

Maple [A] time = 0.017, size = 305, normalized size = 1.2

$$\frac{Ab-Ba}{2ab(bx^2+a)}\sqrt{x}+\frac{3\sqrt{2}A}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\sqrt{2}A}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{3\sqrt{2}A}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}B}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}B}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{\sqrt{2}B}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 1/2*(A*b-B*a)*x^(1/2)/a/b/(b*x^2+a)+3/8/a^2*(a/b)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/a^2*(a/b)^(1/4)*2^(1/2
)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/a^2*(a/b)^(1/4)*2^
(1/2)*A*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/8/a/b*(a/b)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/a/b*(a/b)^(1/4)*2^(1/2)
*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/16/a/b*(a/b)^(1/4)*2^(
1/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24986, size = 801, normalized size = 3.07

$$-\frac{4\left(ab^2x^2+a^2b\right)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3\cdots}{a^7b^5}\right.}{(Ba+3\,Ab)\sqrt{x}+\sqrt{a^4b^2\sqrt{-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Ba\cdots}{a^7b^5}}}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*
B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*arct
an(a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A
^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)/((B*a + 3*A*b)*sqrt(x)
+ sqrt(a^4*b^2*sqrt(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b
^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5)) + (B^2*a^2 + 6*A*B*
a*b + 9*A^2*b^2)*x))) - (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3
*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*
b^5))^(1/4)*log(a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^
2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3
*A*b)*sqrt(x)) + (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b
+ 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(
1/4)*log(-a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
+ 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*
sqrt(x)) + 4*(B*a - A*b)*sqrt(x))/(a*b^2*x^2 + a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.244352, size = 369, normalized size = 1.41

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^2}-\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*sqrt(x)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2)
+ 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^
2) + 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*ln(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 1/16*sqrt(
2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 1/2*(B*a*sqrt(x) - A*b*s
qrt(x))/((b*x^2 + a)*a*b)
```

3.380 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^2} dx$

Optimal. Leaf size=289

$$-\frac{(5Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{9/4}b^{3/4}}-\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

```
[Out] -(5*A*b - a*B)/(2*a^2*b*Sqrt[x]) + (A*b - a*B)/(2*a*b*Sqrt[x]*(a
+ b*x^2)) + ((5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) -
 ((5*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(3/4)) + ((5*A*b - a*B)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
9/4)*b^(3/4))
```

Rubi [A] time = 0.463966, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{(5Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{9/4}b^{3/4}}-\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -(5*A*b - a*B)/(2*a^2*b*Sqrt[x]) + (A*b - a*B)/(2*a*b*Sqrt[x]*(a
+ b*x^2)) + ((5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) -
 ((5*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(3/4)) + ((5*A*b - a*B)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
9/4)*b^(3/4))
```

Rubi in Sympy [A] time = 80.1584, size = 260, normalized size = 0.9

$$\frac{Ab-Ba}{2ab\sqrt{x}\,(a+bx^2)}-\frac{5Ab-Ba}{2a^2b\sqrt{x}}-\frac{\sqrt{2}\,(5Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{9}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,(5Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{9}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,(5Ab-Ba)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{9}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\,(5Ab-Ba)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{9}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] (A*b - B*a)/(2*a*b*sqrt(x)*(a + b*x**2)) - (5*A*b - B*a)/(2*a**2*
b*sqrt(x)) - sqrt(2)*(5*A*b - B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)
```

```
*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(9/4)*b**(3/4)) + sqrt(2)*
(5*A*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + s
qrt(b)*x)/(16*a**(9/4)*b**(3/4)) + sqrt(2)*(5*A*b - B*a)*atan(1 -
 sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(9/4)*b**(3/4)) - sqrt(
2)*(5*A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a
**(9/4)*b**(3/4))
```

Mathematica [A] time = 0.458713, size = 253, normalized size = 0.88

$$\frac{\frac{\sqrt{2}(aB-5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{\sqrt{2}(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{2\sqrt{2}(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{2\sqrt{2}(5Ab-aB)\tan^{-1}\left(\right.}{b^{3/4}}}{16a^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*a^(1/4)*A)/Sqrt[x] + (8*a^(1/4)*(-(A*b) + a*B)*x^(3/2))/(a
+ b*x^2) + (2*Sqrt[2]*(5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/b^(3/4) - (2*Sqrt[2]*(5*A*b - a*B)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-5*A*b + a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b
^(3/4) + (Sqrt[2]*(5*A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4))/(16*a^(9/4))
```

Maple [A] time = 0.023, size = 323, normalized size = 1.1

$$\begin{gathered}-2\,\frac{A}{a^2\sqrt{x}}-\frac{Ab}{2\,a^2\,(bx^2+a)}x^{\frac{3}{2}}+\frac{B}{2\,a\,(bx^2+a)}x^{\frac{3}{2}}\\ -\frac{5\,\sqrt{2}A}{16\,a^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\ -\frac{5\,\sqrt{2}A}{8\,a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{5\,\sqrt{2}A}{8\,a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\ +\frac{\sqrt{2}B}{16\,ab}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\ +\frac{\sqrt{2}B}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{\sqrt{2}B}{8\,ab}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\end{gathered}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -2*A/a^2/x^(1/2)-1/2/a^2*x^(3/2)/(b*x^2+a)*A*b+1/2/a*x^(3/2)/(b*x
^2+a)*B-5/16/a^2/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
-5/8/a^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)-5/8/a^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+
1/8/a/b/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+
1)+1/8/a/b/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259386, size = 1087, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(B*a - 5*A*b)*x^2 - 4*(a^2*b*x^2 + a^3)*sqrt(x)*(-(B^4*a^4
 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A
^4*b^4)/(a^9*b^3))^(1/4)*arctan(-a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^
3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b
^3))^(3/4)/((B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*
b^3)*sqrt(x) - sqrt((B^6*a^6 - 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b
^2 - 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 - 18750*A^5*B*a*
b^5 + 15625*A^6*b^6)*x - (B^4*a^9*b - 20*A*B^3*a^8*b^2 + 150*A^2*
B^2*a^7*b^3 - 500*A^3*B*a^6*b^4 + 625*A^4*a^5*b^5)*sqrt(-(B^4*a^4
 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A
^4*b^4)/(a^9*b^3))))) - (a^2*b*x^2 + a^3)*sqrt(x)*(-(B^4*a^4 - 20
*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^
4)/(a^9*b^3))^(1/4)*log(a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150
*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4
) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*sqr
t(x)) + (a^2*b*x^2 + a^3)*sqrt(x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 1
50*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1
/4)*log(-a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^
2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 -
15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*sqrt(x)) - 16*A*a)
/((a^2*b*x^2 + a^3)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.255788, size = 375, normalized size = 1.3

$$\frac{Bax^2 - 5\,Abx^2 - 4\,Aa}{2\left(bx^{\frac{5}{2}} + a\sqrt{x}\right)a^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(3/2)),x, algorithm="giac")
```

```
[Out] 1/2*(B*a*x^2 - 5*A*b*x^2 - 4*A*a)/((b*x^(5/2) + a*sqrt(x))*a^2) +
 1/8*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3)
 + 1/8*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b
^3) - 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*ln(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/16*sqrt
(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

3.381 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}(a+bx^2)}$$

```
[Out] -(7*A*b - 3*a*B)/(6*a^2*b*x^(3/2)) + (A*b - a*B)/(2*a*b*x^(3/2)*(
a + b*x^2)) + ((7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a*B)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^
(1/4)) + ((7*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a
*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi [A] time = 0.456884, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -(7*A*b - 3*a*B)/(6*a^2*b*x^(3/2)) + (A*b - a*B)/(2*a*b*x^(3/2)*(
a + b*x^2)) + ((7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a*B)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^
(1/4)) + ((7*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a
*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi in Sympy [A] time = 78.7587, size = 269, normalized size = 0.93

$$\frac{Ab-Ba}{2abx^{\frac{3}{2}}(a+bx^2)}-\frac{7Ab-3Ba}{6a^2bx^{\frac{3}{2}}}+\frac{\sqrt{2}(7Ab-3Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{11}{4}}\sqrt[4]{b}}-\frac{\sqrt{2}(7Ab-3Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{11}{4}}\sqrt[4]{b}}+\frac{\sqrt{2}(7Ab-3Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{11}{4}}\sqrt[4]{b}}-\frac{\sqrt{2}(7Ab-3Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{11}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] (A*b - B*a)/(2*a*b*x**(3/2)*(a + b*x**2)) - (7*A*b - 3*B*a)/(6*a*
*2*b*x**(3/2)) + sqrt(2)*(7*A*b - 3*B*a)*log(-sqrt(2)*a**(1/4)*b*
*(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(11/4)*b**(1/4)) - s
qrt(2)*(7*A*b - 3*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sq
rt(a) + sqrt(b)*x)/(16*a**(11/4)*b**(1/4)) + sqrt(2)*(7*A*b - 3*B
*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(11/4)*b**(
1/4)) - sqrt(2)*(7*A*b - 3*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)
/a**(1/4))/(8*a**(11/4)*b**(1/4))
```

Mathematica [A] time = 0.599642, size = 256, normalized size = 0.89

$$\frac{\frac{24a^{3/4}\sqrt{x}(aB-Ab)}{a+bx^2}-\frac{32a^{3/4}A}{x^{3/2}}+\frac{3\sqrt{2}(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}(3aB-7Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}(7Ab-3aB)\tan^{-1}(\cdots}{\sqrt[4]{b}}}{48a^{11/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*a^(3/4)*A)/x^(3/2) + (24*a^(3/4)*(-(A*b) + a*B)*Sqrt[x])/(a
 + b*x^2) + (6*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/b^(1/4) - (6*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (3*Sqrt[2]*(7*A
*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/b^(1/4) + (3*Sqrt[2]*(-7*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(48*a^(11/4))
```

Maple [A] time = 0.023, size = 317, normalized size = 1.1

$$-\frac{2A}{3a^2}x^{-\frac{3}{2}}-\frac{Ab}{2a^2(bx^2+a)}\sqrt{x}+\frac{B}{2a(bx^2+a)}\sqrt{x}$$
$$-\frac{7\sqrt{2}Ab}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{7\sqrt{2}Ab}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{7\sqrt{2}Ab}{16a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\sqrt{2}B}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\sqrt{2}B}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{3\sqrt{2}B}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/3*A/a^2/x^(3/2)-1/2/a^2*x^(1/2)/(b*x^2+a)*A*b+1/2/a*x^(1/2)/(b
*x^2+a)*B-7/8/a^3*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)*b-7/8/a^3*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)-1)*b-7/16/a^3*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))*b+3/8/a^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)
```

```
^(1/4)*x^(1/2)+1)+3/8/a^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258555, size = 838, normalized size = 2.9

$$\frac{4\,(3\,Ba-7\,Ab)x^2+12\,\left(a^2bx^3+a^3x\right)\sqrt{x}\left(-\frac{81\,B^4a^4-756\,AB^3a^3b+2646\,A^2B^2a^2b^2-4116\,A^3Bab^3+2401\,A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(-\frac{\cdots}{(3\,Ba-7\,Ab)\sqrt{x}-\sqrt{a^6\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*(3*B*a - 7*A*b)*x^2 + 12*(a^2*b*x^3 + a^3*x)*sqrt(x)*(-(8
1*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a
*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*arctan(-a^3*(-(81*B^4*a^4 -
756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*
A^4*b^4)/(a^11*b))^(1/4)/((3*B*a - 7*A*b)*sqrt(x) - sqrt(a^6*sqrt
(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b)) + (9*B^2*a^2 - 42*A*B*a*b + 49
*A^2*b^2)*x))) - 3*(a^2*b*x^3 + a^3*x)*sqrt(x)*(-(81*B^4*a^4 - 75
6*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^
4*b^4)/(a^11*b))^(1/4)*log(a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b +
2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))
^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) + 3*(a^2*b*x^3 + a^3*x)*sqrt(x)
*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^
3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*log(-a^3*(-(81*B^4*a^4
- 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 240
1*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) - 16*A*a)/(
(a^2*b*x^3 + a^3*x)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239638, size = 382, normalized size = 1.32

$$\frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}Ba-7\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b}+\frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}Ba-7\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b}+\frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}Ba-7\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b}-\frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}Ba-7\,\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\,(bx^2+a)a^2}-\frac{2\,A}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(5/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b)
+ 1/8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*
b) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*ln(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) - 1/16*sqrt(
2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/2*(B*a*sqrt(x) - A*b*s
qrt(x))/((b*x^2 + a)*a^2) - 2/3*A/(a^2*x^(3/2))
```

3.382 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(9Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{Ab-aB}{2abx^{5/2}(a+bx^2)}$$

```
[Out] -(9*A*b - 5*a*B)/(10*a^2*b*x^(5/2)) + (9*A*b - 5*a*B)/(2*a^3*Sqrt
[x]) + (A*b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) - (b^(1/4)*(9*A*b
- 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(13/4)) + (b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)) + (b^(1/4)*(9*A*b - 5*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*a^(13/4)) - (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rubi [A] time = 0.522273, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(9Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{Ab-aB}{2abx^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```


```
[Out] -(9*A*b - 5*a*B)/(10*a^2*b*x^(5/2)) + (9*A*b - 5*a*B)/(2*a^3*Sqrt
[x]) + (A*b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) - (b^(1/4)*(9*A*b
- 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(13/4)) + (b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)) + (b^(1/4)*(9*A*b - 5*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*a^(13/4)) - (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rubi in Sympy [A] time = 88.7177, size = 289, normalized size = 0.93

$$\frac{Ab-Ba}{2abx^{\frac{5}{2}}(a+bx^2)}-\frac{9Ab-5Ba}{10a^2bx^{\frac{5}{2}}}+\frac{9Ab-5Ba}{2a^3\sqrt{x}}+\frac{\sqrt{2}\sqrt[4]{b}\,(9Ab-5Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\,(9Ab-5Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\,(9Ab-5Ba)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{13}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\,(9Ab-5Ba)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] (A*b - B*a)/(2*a*b*x**(5/2)*(a + b*x**2)) - (9*A*b - 5*B*a)/(10*a
**2*b*x**(5/2)) + (9*A*b - 5*B*a)/(2*a**3*sqrt(x)) + sqrt(2)*b**(
1/4)*(9*A*b - 5*B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqr
t(a) + sqrt(b)*x)/(16*a**(13/4)) - sqrt(2)*b**(1/4)*(9*A*b - 5*B*
a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(
16*a**(13/4)) - sqrt(2)*b**(1/4)*(9*A*b - 5*B*a)*atan(1 - sqrt(2)
*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(13/4)) + sqrt(2)*b**(1/4)*(9*A
*b - 5*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(13
/4))
```

Mathematica [A] time = 0.463227, size = 277, normalized size = 0.89

$$\frac{-\frac{32a^{5/4}A}{x^{5/2}}-\frac{40\sqrt[4]{a}bx^{3/2}(aB-Ab)}{a+bx^2}-\frac{160\sqrt[4]{a}(aB-2Ab)}{\sqrt{x}}+5\sqrt{2}\sqrt[4]{b}(9Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}\sqrt[4]{b}(5aB-9Ab)}{80a^{13}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*a^(5/4)*A)/x^(5/2) - (160*a^(1/4)*(-2*A*b + a*B))/Sqrt[x] -
 (40*a^(1/4)*b*(-(A*b) + a*B)*x^(3/2))/(a + b*x^2) - 10*Sqrt[2]*b
^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)] + 10*Sqrt[2]*b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)] + 5*Sqrt[2]*b^(1/4)*(9*A*b - 5*a*B)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*b
^(1/4)*(-9*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(80*a^(13/4))
```

Maple [A] time = 0.026, size = 339, normalized size = 1.1

$$-\frac{2A}{5a^2}x^{-\frac{5}{2}}+4\frac{Ab}{\sqrt{x}a^3}-2\frac{B}{\sqrt{x}a^2}+\frac{b^2A}{2a^3(bx^2+a)}x^{\frac{3}{2}}-\frac{Bb}{2a^2(bx^2+a)}x^{\frac{3}{2}}$$
$$+\frac{9b\sqrt{2}A}{16a^3}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{9b\sqrt{2}A}{8a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{9b\sqrt{2}A}{8a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{5\sqrt{2}B}{16a^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{5\sqrt{2}B}{8a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{5\sqrt{2}B}{8a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/5*A/a^2/x^(5/2)+4/x^(1/2)/a^3*A*b-2/x^(1/2)/a^2*B+1/2/a^3*b^2*
x^(3/2)/(b*x^2+a)*A-1/2/a^2*b*x^(3/2)/(b*x^2+a)*B+9/16/a^3*b/(a/b
)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+9/8/a^3*b/(a/b)^(1/4
)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8/a^3*b/(a/b)
```

```
^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-5/16/a^2/(
a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-5/8/a^2/(a/b)^(1/
4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/a^2/(a/b)^
(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26269, size = 1164, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(7/2)),x, algorithm="fricas")
```

```
[Out] -1/40*(20*(5*B*a*b - 9*A*b^2)*x^4 + 16*A*a^2 + 16*(5*B*a^2 - 9*A*
a*b)*x^2 - 20*(a^3*b*x^4 + a^4*x^2)*sqrt(x)*(-(625*B^4*a^4*b - 45
00*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 65
61*A^4*b^5)/a^13)^(1/4)*arctan(-a^10*(-(625*B^4*a^4*b - 4500*A*B^
3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*
b^5)/a^13)^(3/4)/((125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B
*a*b^3 - 729*A^3*b^4)*sqrt(x) - sqrt((15625*B^6*a^6*b^2 - 168750*
A*B^5*a^5*b^3 + 759375*A^2*B^4*a^4*b^4 - 1822500*A^3*B^3*a^3*b^5
+ 2460375*A^4*B^2*a^2*b^6 - 1771470*A^5*B*a*b^7 + 531441*A^6*b^8)
*x - (625*B^4*a^11*b - 4500*A*B^3*a^10*b^2 + 12150*A^2*B^2*a^9*b^
3 - 14580*A^3*B*a^8*b^4 + 6561*A^4*a^7*b^5)*sqrt(-(625*B^4*a^4*b
- 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4
+ 6561*A^4*b^5)/a^13)))) - 5*(a^3*b*x^4 + a^4*x^2)*sqrt(x)*(-(625
*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A
^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(a^10*(-(625*B^4*a^4*b
- 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4
+ 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2
+ 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) + 5*(a^3*b*x^4 + a^4*x
^2)*sqrt(x)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2
*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(-a^1
0*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 -
 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b -
 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)))/((
a^3*b*x^4 + a^4*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270018, size = 409, normalized size = 1.32

$$-\frac{Babx^{\frac{3}{2}} - Ab^2x^{\frac{3}{2}}}{2\,(bx^2+a)a^3} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}} Ba - 9\,\left(ab^3\right)^{\frac{3}{4}} Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}} Ba - 9\,\left(ab^3\right)^{\frac{3}{4}} Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2} + \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}} Ba - 9\,\left(ab^3\right)^{\frac{3}{4}} Ab\right)\ln\left(\sqrt{2}\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^2} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}} Ba - 9\,\left(ab^3\right)^{\frac{3}{4}} Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^2} - \frac{2\,\left(5\,Bax^2 - 10\,Abx^2 + Aa\right)}{5\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^2*x^(7/2)),x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b*x^(3/2) - A*b^2*x^(3/2))/((b*x^2 + a)*a^3) - 1/8*sqrt
(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 1/8*
sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) +
 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*ln(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*sqrt(2)
*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 2/5*(5*B*a*x^2 - 10*A*b*
x^2 + A*a)/(a^3*x^(5/2))
```

3.383 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=316

$$-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}-\frac{5(Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{3/4}b^{13/4}}-\frac{5\sqrt{x}(Ab-9aB)}{16ab^3}+\frac{x^{5/2}(Ab-9aB)}{16ab^2(a+bx^2)}+\frac{x^{9/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

```
[Out] (-5*(A*b - 9*a*B)*Sqrt[x])/(16*a*b^3) + ((A*b - a*B)*x^(9/2))/(4*
a*b*(a + b*x^2)^2) + ((A*b - 9*a*B)*x^(5/2))/(16*a*b^2*(a + b*x^2
)) - (5*(A*b - 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(32*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*B)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(13/4))
- (5*(A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*B)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqr
t[2]*a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.526481, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}-\frac{5(Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{3/4}b^{13/4}}-\frac{5\sqrt{x}(Ab-9aB)}{16ab^3}+\frac{x^{5/2}(Ab-9aB)}{16ab^2(a+bx^2)}+\frac{x^{9/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-5*(A*b - 9*a*B)*Sqrt[x])/(16*a*b^3) + ((A*b - a*B)*x^(9/2))/(4*
a*b*(a + b*x^2)^2) + ((A*b - 9*a*B)*x^(5/2))/(16*a*b^2*(a + b*x^2
)) - (5*(A*b - 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(32*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*B)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(13/4))
- (5*(A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*B)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqr
t[2]*a^(3/4)*b^(13/4))
```

Rubi in Sympy [A] time = 88.5812, size = 298, normalized size = 0.94

$$\frac{x^{\frac{9}{2}}\left(Ab-Ba\right)}{4ab\left(a+bx^{2}\right)^{2}}+\frac{x^{\frac{5}{2}}\left(Ab-9Ba\right)}{16ab^{2}\left(a+bx^{2}\right)}-\frac{5\sqrt{x}\left(Ab-9Ba\right)}{16ab^{3}}-\frac{5\sqrt{2}\left(Ab-9Ba\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{3}{4}}b^{\frac{13}{4}}}+\frac{5\sqrt{2}\left(Ab-9Ba\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{3}{4}}b^{\frac{13}{4}}}-\frac{5\sqrt{2}\left(Ab-9Ba\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{3}{4}}b^{\frac{13}{4}}}+\frac{5\sqrt{2}\left(Ab-9Ba\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{3}{4}}b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] x**(9/2)*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) + x**(5/2)*(A*b - 9*
B*a)/(16*a*b**2*(a + b*x**2)) - 5*sqrt(x)*(A*b - 9*B*a)/(16*a*b**
3) - 5*sqrt(2)*(A*b - 9*B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(
x) + sqrt(a) + sqrt(b)*x)/(128*a**(3/4)*b**(13/4)) + 5*sqrt(2)*(A
*b - 9*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqr
t(b)*x)/(128*a**(3/4)*b**(13/4)) - 5*sqrt(2)*(A*b - 9*B*a)*atan(1
 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(3/4)*b**(13/4)) + 5
*sqrt(2)*(A*b - 9*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4)
)/(64*a**(3/4)*b**(13/4))
```

Mathematica [A] time = 0.503512, size = 285, normalized size = 0.9

$$\frac{\frac{5\sqrt{2}(9aB-Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{5\sqrt{2}(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{10\sqrt{2}(9aB-Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{10\sqrt{2}(Ab-9aB)\tan}{a^{3/}}}{128b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (256*b^(1/4)*B*Sqrt[x] + (32*a*b^(1/4)*(A*b - a*B)*Sqrt[x])/(a +
b*x^2)^2 - (8*b^(1/4)*(9*A*b - 17*a*B)*Sqrt[x])/(a + b*x^2) + (10
*Sqrt[2]*(-(A*b) + 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/a^(3/4) + (10*Sqrt[2]*(A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (5*Sqrt[2]*(-(A*b) + 9*a*B)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4
) + (5*Sqrt[2]*(A*b - 9*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(128*b^(13/4))
```

Maple [A] time = 0.024, size = 363, normalized size = 1.2

$$2\frac{B\sqrt{x}}{b^3}-\frac{9A}{16b(bx^2+a)^2}x^{\frac{5}{2}}+\frac{17Ba}{16b^2(bx^2+a)^2}x^{\frac{5}{2}}-\frac{5Aa}{16b^2(bx^2+a)^2}\sqrt{x}+\frac{13a^2B}{16b^3(bx^2+a)^2}\sqrt{x}$$
$$+\frac{5\sqrt{2}A}{64b^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{5\sqrt{2}A}{64b^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{5\sqrt{2}A}{128b^2a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{45\sqrt{2}B}{64b^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{45\sqrt{2}B}{64b^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{45\sqrt{2}B}{128b^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*B*x^(1/2)/b^3-9/16/b/(b*x^2+a)^2*x^(5/2)*A+17/16/b^2/(b*x^2+a)^
2*x^(5/2)*a*B-5/16/b^2/(b*x^2+a)^2*A*x^(1/2)*a+13/16/b^3/(b*x^2+a
)^2*B*x^(1/2)*a^2+5/64/b^2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)+1)+5/64/b^2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+5/128/b^2*(a/b)^(1/4)/a*2^(1/2)*A*l
n((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2)))-45/64/b^3*(a/b)^(1/4)*2^(1/2)*B*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-45/64/b^3*(a/b)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-45/128/b^3*(a/b)^(1/4)*2^(1/2
)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245301, size = 909, normalized size = 2.88

$$-\frac{20\left(b^5x^4+2ab^4x^2+a^2b^3\right)\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab^3\left(-\frac{6561B^4a^4-291}{}\right.}{(9Ba-Ab)\sqrt{x}-\sqrt{a^2b^6\sqrt{-\frac{6561B^4a^4-2916A}{}}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 291
6*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(
a^3*b^13))^(1/4)*arctan(-a*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b
 + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1
```

```
/4)/((9*B*a - A*b)*sqrt(x) - sqrt(a^2*b^6*sqrt(-(6561*B^4*a^4 - 2
916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)
/(a^3*b^13)) + (81*B^2*a^2 - 18*A*B*a*b + A^2*b^2)*x))) - 5*(b^5*
x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b +
 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4
)*log(5*a*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^
2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a -
A*b)*sqrt(x)) + 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a
^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^
4*b^4)/(a^3*b^13))^(1/4)*log(-5*a*b^3*(-(6561*B^4*a^4 - 2916*A*B^
3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^
13))^(1/4) - 5*(9*B*a - A*b)*sqrt(x)) - 4*(32*B*b^2*x^4 + 45*B*a^
2 - 5*A*a*b + 9*(9*B*a*b - A*b^2)*x^2)*sqrt(x))/(b^5*x^4 + 2*a*b^
4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.260635, size = 410, normalized size = 1.3

$$\frac{2\,B\sqrt{x}}{b^3} - \frac{5\,\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4} - \frac{5\,\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4} - \frac{5\,\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,ab^4} + \frac{5\,\sqrt{2}\left(9\,\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,ab^4} + \frac{17\,Babx^{\frac{5}{2}} - 9\,Ab^2x^{\frac{5}{2}} + 13\,Ba^2\sqrt{x} - 5\,Aab\sqrt{x}}{16\,(bx^2+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^3 - 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/
4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(a*b^4) - 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1
/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(a*b^4) - 5/128*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)
^(1/4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^
4) + 5/128*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ln(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) + 1/16*(17*B
*a*b*x^(5/2) - 9*A*b^2*x^(5/2) + 13*B*a^2*sqrt(x) - 5*A*a*b*sqrt(
x))/((b*x^2 + a)^2*b^3)
```

3.384 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=293

$$\frac{3(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{11/4}}+\frac{3(7aB+Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{5/4}b^{11/4}}-\frac{x^{3/2}(7aB+Ab)}{16ab^2(a+bx^2)}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

```
[Out] ((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) - ((A*b + 7*a*B)*x^(3
/2))/(16*a*b^2*(a + b*x^2)) - (3*(A*b + 7*a*B)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(
A*b + 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/
4)) - (3*(A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi [A] time = 0.471919, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{3(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{11/4}}+\frac{3(7aB+Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{5/4}b^{11/4}}-\frac{x^{3/2}(7aB+Ab)}{16ab^2(a+bx^2)}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) - ((A*b + 7*a*B)*x^(3
/2))/(16*a*b^2*(a + b*x^2)) - (3*(A*b + 7*a*B)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(
A*b + 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/
4)) - (3*(A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi in Sympy [A] time = 80.7179, size = 275, normalized size = 0.94

$$\frac{x^{\frac{7}{2}}(Ab-Ba)}{4ab(a+bx^2)^2}-\frac{x^{\frac{3}{2}}(Ab+7Ba)}{16ab^2(a+bx^2)}+\frac{3\sqrt{2}(Ab+7Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{5}{4}}b^{\frac{11}{4}}}-\frac{3\sqrt{2}(Ab+7Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{5}{4}}b^{\frac{11}{4}}}-\frac{3\sqrt{2}(Ab+7Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{5}{4}}b^{\frac{11}{4}}}+\frac{3\sqrt{2}(Ab+7Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{5}{4}}b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] x**(7/2)*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) - x**(3/2)*(A*b + 7*
B*a)/(16*a*b**2*(a + b*x**2)) + 3*sqrt(2)*(A*b + 7*B*a)*log(-sqrt
(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(5/4
)*b**(11/4)) - 3*sqrt(2)*(A*b + 7*B*a)*log(sqrt(2)*a**(1/4)*b**(1
/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(5/4)*b**(11/4)) - 3*s
qrt(2)*(A*b + 7*B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/
(64*a**(5/4)*b**(11/4)) + 3*sqrt(2)*(A*b + 7*B*a)*atan(1 + sqrt(2
)*b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(5/4)*b**(11/4))
```

Mathematica [A] time = 0.634128, size = 272, normalized size = 0.93

$$\frac{\frac{3\sqrt{2}(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{3\sqrt{2}(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{6\sqrt{2}(7aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{6\sqrt{2}(7aB+Ab)\tan^{-1}\left(\cdots\right)}{a^{5/4}}}{128b^{11/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-32*b^(3/4)*(A*b - a*B)*x^(3/2))/(a + b*x^2)^2 + (8*b^(3/4)*(3*
A*b - 11*a*B)*x^(3/2))/(a*(a + b*x^2)) - (6*Sqrt[2]*(A*b + 7*a*B)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (6*Sqrt
[2]*(A*b + 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
a^(5/4) + (3*Sqrt[2]*(A*b + 7*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (3*Sqrt[2]*(A*b + 7*a*B)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/
4))/(128*b^(11/4))
```

Maple [A] time = 0.023, size = 325, normalized size = 1.1

$$2\frac{1}{(bx^2+a)^2}\left(1/32\frac{(3Ab-11Ba)x^{7/2}}{ab}-1/32\frac{(Ab+7Ba)x^{3/2}}{b^2}\right)$$
$$+\frac{3\sqrt{2}A}{64b^2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\sqrt{2}A}{64b^2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{3\sqrt{2}A}{128b^2a}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{21\sqrt{2}B}{64b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{21\sqrt{2}B}{64b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{21\sqrt{2}B}{128b^3}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(3*A*b-11*B*a)/a/b*x^(7/2)-1/32*(A*b+7*B*a)/b^2*x^(3/2))/
(b*x^2+a)^2+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)
```

```
/(a/b)^(1/4)*x^(1/2)-1)+3/128/b^2/a/(a/b)^(1/4)*2^(1/2)*A*ln((x-(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2)))+21/64/b^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+21/64/b^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+21/128/b^3/(a/b)^(1/4)*2^(1/2)*B*ln
((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259484, size = 1177, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 +
1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4
)/(a^5*b^11))^(1/4)*arctan(a^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a
^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)
)^(3/4)/((343*B^3*a^3 + 147*A*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^
3)*sqrt(x) + sqrt((117649*B^6*a^6 + 100842*A*B^5*a^5*b + 36015*A^
2*B^4*a^4*b^2 + 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 + 42*A
^5*B*a*b^5 + A^6*b^6)*x - (2401*B^4*a^7*b^5 + 1372*A*B^3*a^6*b^6
+ 294*A^2*B^2*a^5*b^7 + 28*A^3*B*a^4*b^8 + A^4*a^3*b^9)*sqrt(-(24
01*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*
b^3 + A^4*b^4)/(a^5*b^11))))) + 3*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^
3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 +
 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(27*a^4*b^8*(-(24
01*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*
b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2*a^
2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) - 3*(a*b^4*x^4 + 2*a^2*b
^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^
2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(-27*a
^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 +
 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 +
147*A*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) - 4*((11*B*a
*b - 3*A*b^2)*x^3 + (7*B*a^2 + A*a*b)*x)*sqrt(x))/(a*b^4*x^4 + 2*
a^2*b^3*x^2 + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.248179, size = 396, normalized size = 1.35

$$-\frac{11\,Babx^{\frac{7}{2}} - 3\,Ab^{2}x^{\frac{7}{2}} + 7\,Ba^{2}x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{16\,(bx^{2}+a)^{2}ab^{2}} + \frac{3\,\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{5}} + \frac{3\,\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{5}} - \frac{3\,\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{5}} + \frac{3\,\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + \left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*a*b*x^(7/2) - 3*A*b^2*x^(7/2) + 7*B*a^2*x^(3/2) + A*a
*b*x^(3/2))/((b*x^2 + a)^2*a*b^2) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)
*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) + 3/64*sqrt(2)*(7*(a*b^3)^(3
/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 3/128*sqrt(2)*(7*(a*b^
3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(a^2*b^5) + 3/128*sqrt(2)*(7*(a*b^3)^(3/4)*B*a +
 (a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(a^2*b^5)
```

3.385 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}-\frac{(5aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{7/4}b^{9/4}}-\frac{\sqrt{x}(5aB+3Ab)}{16ab^2(a+bx^2)}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

```
[Out] ((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*Sq
rt[x])/(16*a*b^2*(a + b*x^2)) - ((3*A*b + 5*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3
*A*b + 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*
Sqrt[2]*a^(7/4)*b^(9/4)) - ((3*A*b + 5*a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4
)) + ((3*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.45225, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}-\frac{(5aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{7/4}b^{9/4}}-\frac{\sqrt{x}(5aB+3Ab)}{16ab^2(a+bx^2)}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```


```
[Out] ((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*Sq
rt[x])/(16*a*b^2*(a + b*x^2)) - ((3*A*b + 5*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3
*A*b + 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*
Sqrt[2]*a^(7/4)*b^(9/4)) - ((3*A*b + 5*a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4
)) + ((3*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi in Sympy [A] time = 78.9186, size = 277, normalized size = 0.93

$$\frac{x^{\frac{5}{2}}(Ab-Ba)}{4ab(a+bx^2)^2}-\frac{\sqrt{x}(3Ab+5Ba)}{16ab^2(a+bx^2)}-\frac{\sqrt{2}(3Ab+5Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}(3Ab+5Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{7}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}(3Ab+5Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}(3Ab+5Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{7}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] x**(5/2)*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) - sqrt(x)*(3*A*b + 5
*B*a)/(16*a*b**2*(a + b*x**2)) - sqrt(2)*(3*A*b + 5*B*a)*log(-sqr
t(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(7/
4)*b**(9/4)) + sqrt(2)*(3*A*b + 5*B*a)*log(sqrt(2)*a**(1/4)*b**(1
/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(7/4)*b**(9/4)) - sqrt
(2)*(3*A*b + 5*B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(
64*a**(7/4)*b**(9/4)) + sqrt(2)*(3*A*b + 5*B*a)*atan(1 + sqrt(2)*
b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(7/4)*b**(9/4))
```

Mathematica [A] time = 0.498726, size = 274, normalized size = 0.92

$$\frac{-\frac{\sqrt{2}(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{\sqrt{2}(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{2\sqrt{2}(5aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt{2}(5aB+3Ab)\tan}{a}}{128b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-32*b^(1/4)*(A*b - a*B)*Sqrt[x])/(a + b*x^2)^2 + (8*b^(1/4)*(A*
b - 9*a*B)*Sqrt[x])/(a*(a + b*x^2)) - (2*Sqrt[2]*(3*A*b + 5*a*B)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (2*Sqrt[
2]*(3*A*b + 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/a^(7/4) - (Sqrt[2]*(3*A*b + 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (Sqrt[2]*(3*A*b + 5*a*B)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7
/4))/(128*b^(9/4))
```

Maple [A] time = 0.023, size = 334, normalized size = 1.1

$$2\frac{1}{(bx^2+a)^2}\left(1/32\frac{(Ab-9Ba)x^{5/2}}{ab}-1/32\frac{(3Ab+5Ba)\sqrt{x}}{b^2}\right)$$
$$+\frac{3\sqrt{2}A}{64ba^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\sqrt{2}A}{64ba^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{3\sqrt{2}A}{128ba^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{5\sqrt{2}B}{64b^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{5\sqrt{2}B}{64b^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{5\sqrt{2}B}{128b^2a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(A*b-9*B*a)/a/b*x^(5/2)-1/32*(3*A*b+5*B*a)/b^2*x^(1/2))/(
b*x^2+a)^2+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)+3/128/b/a^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(
```

```
1/2)+(a/b)^(1/2)))+5/64/b^2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+5/64/b^2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+5/128/b^2/a*(a/b)^(1/4)*2^(1/2)*B*
ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257932, size = 921, normalized size = 3.09

$$-\frac{4\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b^2\left(-\frac{625B^4a^4}{}\right.}{(5Ba+3Ab)\sqrt{x}+\sqrt{a^4b^4\sqrt{-\frac{625B^4a^4+150}{}}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1
500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^9))^(1/4)*arctan(a^2*b^2*(-(625*B^4*a^4 + 1500*A*B^3
*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^
7*b^9))^(1/4)/((5*B*a + 3*A*b)*sqrt(x) + sqrt(a^4*b^4*sqrt(-(625*
B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b
^3 + 81*A^4*b^4)/(a^7*b^9)) + (25*B^2*a^2 + 30*A*B*a*b + 9*A^2*b^
2)*x))) - (a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 +
1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^
4*b^4)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(625*B^4*a^4 + 1500*A*B^3*a
^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*
b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) + (a*b^4*x^4 + 2*a^2*b^3*x
^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^
2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(-a^2*b
^2*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540
*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqr
t(x)) + 4*(5*B*a^2 + 3*A*a*b + (9*B*a*b - A*b^2)*x^2)*sqrt(x))/(a
*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.25181, size = 402, normalized size = 1.35

$$\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{3}}+\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{3}}+\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{3}}-\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{3}}-\frac{9\,Babx^{\frac{5}{2}}-Ab^{2}x^{\frac{5}{2}}+5\,Ba^{2}\sqrt{x}+3\,Aab\sqrt{x}}{16\,\left(bx^{2}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^
3) + 1/64*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(
a^2*b^3) + 1/128*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A
*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) - 1
/128*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*ln(-sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) - 1/16*(9*B*a*
b*x^(5/2) - A*b^2*x^(5/2) + 5*B*a^2*sqrt(x) + 3*A*a*b*sqrt(x))/((
b*x^2 + a)^2*a^2*b^2)
```

3.386 $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=298

$$\frac{(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{7/4}}+\frac{(3aB+5Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{9/4}b^{7/4}}+\frac{x^{3/2}(3aB+5Ab)}{16a^2b\,(a+bx^2)}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

```
[Out] ((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*x^
(3/2))/(16*a^2*b*(a + b*x^2)) - ((5*A*b + 3*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5
*A*b + 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*
Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4
)) - ((5*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi [A] time = 0.46374, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{7/4}}+\frac{(3aB+5Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{9/4}b^{7/4}}+\frac{x^{3/2}(3aB+5Ab)}{16a^2b\,(a+bx^2)}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*x^
(3/2))/(16*a^2*b*(a + b*x^2)) - ((5*A*b + 3*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5
*A*b + 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*
Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4
)) - ((5*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi in Sympy [A] time = 81.3056, size = 277, normalized size = 0.93

$$\frac{x^{\frac{3}{2}}\,(Ab-Ba)}{4ab\,(a+bx^2)^2}+\frac{x^{\frac{3}{2}}\,(5Ab+3Ba)}{16a^2b\,(a+bx^2)}+\frac{\sqrt{2}\,(5Ab+3Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{9}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\,(5Ab+3Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{9}{4}}b^{\frac{7}{4}}}-\frac{\sqrt{2}\,(5Ab+3Ba)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{9}{4}}b^{\frac{7}{4}}}+\frac{\sqrt{2}\,(5Ab+3Ba)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{9}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] x**(3/2)*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) + x**(3/2)*(5*A*b +
3*B*a)/(16*a**2*b*(a + b*x**2)) + sqrt(2)*(5*A*b + 3*B*a)*log(-sq
rt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(9
/4)*b**(7/4)) - sqrt(2)*(5*A*b + 3*B*a)*log(sqrt(2)*a**(1/4)*b**(
1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(9/4)*b**(7/4)) - sqr
t(2)*(5*A*b + 3*B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/
(64*a**(9/4)*b**(7/4)) + sqrt(2)*(5*A*b + 3*B*a)*atan(1 + sqrt(2)
*b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(9/4)*b**(7/4))
```

Mathematica [A] time = 0.370157, size = 267, normalized size = 0.9

$$\frac{-\frac{32a^{5/4}b^{3/4}x^{3/2}(aB-Ab)}{(a+bx^2)^2}+\frac{8\sqrt[4]{a}b^{3/4}x^{3/2}(3aB+5Ab)}{a+bx^2}+\sqrt{2}(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt{2}(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\cdots\right.}{128a^{9/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-32*a^(5/4)*b^(3/4)*(-(A*b) + a*B)*x^(3/2))/(a + b*x^2)^2 + (8*
a^(1/4)*b^(3/4)*(5*A*b + 3*a*B)*x^(3/2))/(a + b*x^2) - 2*Sqrt[2]*
(5*A*b + 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2
*Sqrt[2]*(5*A*b + 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)] + Sqrt[2]*(5*A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*(5*A*b + 3*a*B)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(128*a^(9/4)*b^(7/
4))
```

Maple [A] time = 0.023, size = 335, normalized size = 1.1

$$\begin{aligned}
&2\,\frac{1}{(bx^2+a)^2}\left(1/32\,\frac{(5\,Ab+3\,Ba)\,x^{7/2}}{a^2}+1/32\,\frac{(9\,Ab-Ba)\,x^{3/2}}{ab}\right)\\
&+\frac{5\,\sqrt{2}A}{64\,a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\,\sqrt{2}A}{64\,a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\
&+\frac{5\,\sqrt{2}A}{128\,a^2b}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\
&+\frac{3\,\sqrt{2}B}{64\,ab^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{3\,\sqrt{2}B}{64\,ab^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}\\
&+\frac{3\,\sqrt{2}B}{128\,ab^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(5*A*b+3*B*a)/a^2*x^(7/2)+1/32*(9*A*b-B*a)/a/b*x^(3/2))/(
b*x^2+a)^2+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)+5/128/a^2/b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a
```

```
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2)))+3/64/a/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+3/64/a/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/128/a/b^2/(a/b)^(1/4)*2^(1/2)*B*
ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259989, size = 1189, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540
*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*
b^4)/(a^9*b^7))^(1/4)*arctan(a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^
3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9
*b^7))^(3/4)/((27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 1
25*A^3*b^3)*sqrt(x) + sqrt((729*B^6*a^6 + 7290*A*B^5*a^5*b + 3037
5*A^2*B^4*a^4*b^2 + 67500*A^3*B^3*a^3*b^3 + 84375*A^4*B^2*a^2*b^4
 + 56250*A^5*B*a*b^5 + 15625*A^6*b^6)*x - (81*B^4*a^9*b^3 + 540*A
*B^3*a^8*b^4 + 1350*A^2*B^2*a^7*b^5 + 1500*A^3*B*a^6*b^6 + 625*A^
4*a^5*b^7)*sqrt(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2
*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))))) + (a^2*b^3*x
^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 135
0*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1
/4)*log(a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^
2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4) + (27*B^
3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x))
 - (a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^
3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/
(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1
350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^
(3/4) + (27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 125*A^3
*b^3)*sqrt(x)) + 4*((3*B*a*b + 5*A*b^2)*x^3 - (B*a^2 - 9*A*a*b)*x
)*sqrt(x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.245794, size = 402, normalized size = 1.35

$$\frac{3\,Babx^{\frac{7}{2}} + 5\,Ab^2x^{\frac{7}{2}} - Ba^2x^{\frac{3}{2}} + 9\,Aabx^{\frac{3}{2}}}{16\,(bx^2+a)^2a^2b} + \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^4} + \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^4} - \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^3b^4} + \frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{3}{4}}Ba + 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*a*b*x^(7/2) + 5*A*b^2*x^(7/2) - B*a^2*x^(3/2) + 9*A*a*b
*x^(3/2))/((b*x^2 + a)^2*a^2*b) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B
*a + 5*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) + 1/64*sqrt(2)*(3*(a*b^3)^(3
/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) - 1/128*sqrt(2)*(3*(a*
b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^3*b^4) + 1/128*sqrt(2)*(3*(a*b^3)^(3/4)*B
*a + 5*(a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(a^3*b^4)
```

3.387 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^3} dx$

Optimal. Leaf size=293

$$-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}-\frac{3(aB+7Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{11/4}b^{5/4}}+\frac{\sqrt{x}(aB+7Ab)}{16a^2b(a+bx^2)}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx^2)^2}$$

```
[Out] ((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*Sqrt
[x])/(16*a^2*b*(a + b*x^2)) - (3*(7*A*b + a*B)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(
7*A*b + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(11/4)*b^(5/4)) - (3*(7*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/
4)) + (3*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.454032, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}-\frac{3(aB+7Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{11/4}b^{5/4}}+\frac{\sqrt{x}(aB+7Ab)}{16a^2b(a+bx^2)}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*Sqrt
[x])/(16*a^2*b*(a + b*x^2)) - (3*(7*A*b + a*B)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(
7*A*b + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(11/4)*b^(5/4)) - (3*(7*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/
4)) + (3*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi in Sympy [A] time = 78.2893, size = 275, normalized size = 0.94

$$\frac{\sqrt{x}(Ab-Ba)}{4ab(a+bx^2)^2}+\frac{\sqrt{x}(7Ab+Ba)}{16a^2b(a+bx^2)}-\frac{3\sqrt{2}(7Ab+Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{3\sqrt{2}(7Ab+Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{3\sqrt{2}(7Ab+Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{3\sqrt{2}(7Ab+Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] sqrt(x)*(A*b - B*a)/(4*a*b*(a + b*x**2)**2) + sqrt(x)*(7*A*b + B*
a)/(16*a**2*b*(a + b*x**2)) - 3*sqrt(2)*(7*A*b + B*a)*log(-sqrt(2
)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(11/4)
*b**(5/4)) + 3*sqrt(2)*(7*A*b + B*a)*log(sqrt(2)*a**(1/4)*b**(1/4
)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(11/4)*b**(5/4)) - 3*sqr
t(2)*(7*A*b + B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(6
4*a**(11/4)*b**(5/4)) + 3*sqrt(2)*(7*A*b + B*a)*atan(1 + sqrt(2)*
b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(11/4)*b**(5/4))
```

Mathematica [A] time = 0.36774, size = 263, normalized size = 0.9

$$\frac{-\frac{32a^{7/4}\sqrt[4]{b}\sqrt{x}(aB-Ab)}{(a+bx^2)^2}+\frac{8a^{3/4}\sqrt[4]{b}\sqrt{x}(aB+7Ab)}{a+bx^2}-3\sqrt{2}(aB+7Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+3\sqrt{2}(aB+7Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{128a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((-32*a^(7/4)*b^(1/4)*(-(A*b) + a*B)*Sqrt[x])/(a + b*x^2)^2 + (8*
a^(3/4)*b^(1/4)*(7*A*b + a*B)*Sqrt[x])/(a + b*x^2) - 6*Sqrt[2]*(7
*A*b + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 6*Sqr
t[2]*(7*A*b + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 3*Sqrt[2]*(7*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x] + 3*Sqrt[2]*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(128*a^(11/4)*b^(5/4))
```

Maple [A] time = 0.022, size = 325, normalized size = 1.1

$$2\frac{1}{(bx^2+a)^2}\left(1/32\frac{(7Ab+Ba)x^{5/2}}{a^2}+1/32\frac{(11Ab-3Ba)\sqrt{x}}{ab}\right)$$
$$+\frac{21\sqrt{2}A}{64a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{21\sqrt{2}A}{64a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{21\sqrt{2}A}{128a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\sqrt{2}B}{64a^2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\sqrt{2}B}{64a^2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{3\sqrt{2}B}{128a^2b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2*(1/32*(7*A*b+B*a)/a^2*x^(5/2)+1/32*(11*A*b-3*B*a)/a/b*x^(1/2))/
(b*x^2+a)^2+21/64/a^3*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)+21/64/a^3*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)+21/128/a^3*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))+3/64/a^2/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+3/64/a^2/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^
```

```
(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/128/a^2/b*(a/b)^(1/4)*2^(1/2)*B*ln
((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^3*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25412, size = 903, normalized size = 3.08

$$-\frac{12\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)\left(-\frac{B^4a^4+28\,AB^3a^3b+294\,A^2B^2a^2b^2+1372\,A^3Bab^3+2401\,A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3b\left(-\frac{B^4a^4+28\,AB^3\cdots}{\cdots}\right.}{(Ba+7\,Ab)\sqrt{x}+\sqrt{a^6b^2\sqrt{-\frac{B^4a^4+28\,AB^3a^3b\cdots}{\cdots}}}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^3*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A
*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^
4)/(a^11*b^5))^(1/4)*arctan(a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 2
94*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))
^(1/4)/((B*a + 7*A*b)*sqrt(x) + sqrt(a^6*b^2*sqrt(-(B^4*a^4 + 28*
A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b
^4)/(a^11*b^5)) + (B^2*a^2 + 14*A*B*a*b + 49*A^2*b^2)*x))) - 3*(a
^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b +
 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5
))^(1/4)*log(3*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^
2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*(B
*a + 7*A*b)*sqrt(x)) + 3*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-
(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^
3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*log(-3*a^3*b*(-(B^4*a^4 + 28*
A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b
^4)/(a^11*b^5))^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) + 4*(3*B*a^2 - 1
1*A*a*b - (B*a*b + 7*A*b^2)*x^2)*sqrt(x))/(a^2*b^3*x^4 + 2*a^3*b^
2*x^2 + a^4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.244603, size = 396, normalized size = 1.35

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba + 7\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^2} + \frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba + 7\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^2} + \frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba + 7\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^3b^2} - \frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba + 7\left(ab^3\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^3b^2} + \frac{Babx^{\frac{5}{2}} + 7\,Ab^2x^{\frac{5}{2}} - 3\,Ba^2\sqrt{x} + 11\,Aab\sqrt{x}}{16\left(bx^2+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*sqrt(x)),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2)
 + 3/64*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*
b^2) + 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*ln
(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 3/128*s
qrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*ln(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 1/16*(B*a*b*x^(5/2)
+ 7*A*b^2*x^(5/2) - 3*B*a^2*sqrt(x) + 11*A*a*b*sqrt(x))/((b*x^2 +
 a)^2*a^2*b)
```

3.388 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3} dx$

Optimal. Leaf size=322

$$-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)}{16a^3b\sqrt{x}}+\frac{9Ab-aB}{16a^2b\sqrt{x}(a+bx^2)}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx^2)^2}$$

```
[Out] (-5*(9*A*b - a*B))/(16*a^3*b*Sqrt[x]) + (A*b - a*B)/(4*a*b*Sqrt[x
]*(a + b*x^2)^2) + (9*A*b - a*B)/(16*a^2*b*Sqrt[x]*(a + b*x^2)) +
 (5*(9*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(32*Sqrt[2]*a^(13/4)*b^(3/4)) - (5*(9*A*b - a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(13/4)*b^(3/4)) - (5
*(9*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(13/4)*b^(3/4)) + (5*(9*A*b - a*B)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(13/4)*b^(3/4))
```

Rubi [A] time = 0.521904, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)}{16a^3b\sqrt{x}}+\frac{9Ab-aB}{16a^2b\sqrt{x}(a+bx^2)}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-5*(9*A*b - a*B))/(16*a^3*b*Sqrt[x]) + (A*b - a*B)/(4*a*b*Sqrt[x
]*(a + b*x^2)^2) + (9*A*b - a*B)/(16*a^2*b*Sqrt[x]*(a + b*x^2)) +
 (5*(9*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(32*Sqrt[2]*a^(13/4)*b^(3/4)) - (5*(9*A*b - a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(13/4)*b^(3/4)) - (5
*(9*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(13/4)*b^(3/4)) + (5*(9*A*b - a*B)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(13/4)*b^(3/4))
```

Rubi in Sympy [A] time = 90.2279, size = 298, normalized size = 0.93

$$\frac{Ab-Ba}{4ab\sqrt{x}(a+bx^2)^2}+\frac{9Ab-Ba}{16a^2b\sqrt{x}(a+bx^2)}-\frac{5(9Ab-Ba)}{16a^3b\sqrt{x}}-\frac{5\sqrt{2}(9Ab-Ba)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{13}{4}}b^{\frac{3}{4}}}+\frac{5\sqrt{2}(9Ab-Ba)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{13}{4}}b^{\frac{3}{4}}}+\frac{5\sqrt{2}(9Ab-Ba)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{13}{4}}b^{\frac{3}{4}}}-\frac{5\sqrt{2}(9Ab-Ba)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{13}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] (A*b - B*a)/(4*a*b*sqrt(x)*(a + b*x**2)**2) + (9*A*b - B*a)/(16*a
**2*b*sqrt(x)*(a + b*x**2)) - 5*(9*A*b - B*a)/(16*a**3*b*sqrt(x))
 - 5*sqrt(2)*(9*A*b - B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x)
 + sqrt(a) + sqrt(b)*x)/(128*a**(13/4)*b**(3/4)) + 5*sqrt(2)*(9*A
*b - B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(
b)*x)/(128*a**(13/4)*b**(3/4)) + 5*sqrt(2)*(9*A*b - B*a)*atan(1 -
 sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(13/4)*b**(3/4)) - 5*s
qrt(2)*(9*A*b - B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/
(64*a**(13/4)*b**(3/4))
```

Mathematica [A] time = 0.495145, size = 285, normalized size = 0.89

$$\frac{\frac{32a^{5/4}x^{3/2}(aB-Ab)}{(a+bx^2)^2}+\frac{5\sqrt{2}(aB-9Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{5\sqrt{2}(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{10\sqrt{2}(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}}}{128a^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-256*a^(1/4)*A)/Sqrt[x] + (32*a^(5/4)*(-(A*b) + a*B)*x^(3/2))/(
a + b*x^2)^2 + (8*a^(1/4)*(-13*A*b + 5*a*B)*x^(3/2))/(a + b*x^2)
+ (10*Sqrt[2]*(9*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/b^(3/4) - (10*Sqrt[2]*(9*A*b - a*B)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (5*Sqrt[2]*(-9*A*b + a*B)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4
) + (5*Sqrt[2]*(9*A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/b^(3/4))/(128*a^(13/4))
```

Maple [A] time = 0.026, size = 363, normalized size = 1.1

$$-2\,\frac{A}{a^3\sqrt{x}}-\frac{13\,b^2A}{16\,a^3\,(bx^2+a)^2}x^{\frac{7}{2}}+\frac{5\,Bb}{16\,a^2\,(bx^2+a)^2}x^{\frac{7}{2}}-\frac{17\,Ab}{16\,a^2\,(bx^2+a)^2}x^{\frac{3}{2}}$$
$$+\frac{9\,B}{16\,a\,(bx^2+a)^2}x^{\frac{3}{2}}-\frac{45\,\sqrt{2}A}{128\,a^3}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{45\,\sqrt{2}A}{64\,a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{45\,\sqrt{2}A}{64\,a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{5\,\sqrt{2}B}{128\,a^2b}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{5\,\sqrt{2}B}{64\,a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{5\,\sqrt{2}B}{64\,a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] -2*A/a^3/x^(1/2)-13/16/a^3/(b*x^2+a)^2*x^(7/2)*b^2*A+5/16/a^2/(b*
x^2+a)^2*x^(7/2)*b*B-17/16/a^2/(b*x^2+a)^2*A*x^(3/2)*b+9/16/a/(b*
```

```
x^2+a)^2*B*x^(3/2)-45/128/a^3/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2)))-45/64/a^3/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)-45/64/a^3/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)+5/128/a^2/b/(a/b)^(1/4)*2^(1/2)*B*ln((x-(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2)))+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*B*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258484, size = 1177, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/64*(20*(B*a*b - 9*A*b^2)*x^4 - 128*A*a^2 + 36*(B*a^2 - 9*A*a*b)
*x^2 - 20*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(x)*(-(B^4*a^4 -
36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^
4*b^4)/(a^13*b^3))^(1/4)*arctan(-a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a
^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^
13*b^3))^(3/4)/((B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729
*A^3*b^3)*sqrt(x) - sqrt((B^6*a^6 - 54*A*B^5*a^5*b + 1215*A^2*B^4
*a^4*b^2 - 14580*A^3*B^3*a^3*b^3 + 98415*A^4*B^2*a^2*b^4 - 354294
*A^5*B*a*b^5 + 531441*A^6*b^6)*x - (B^4*a^11*b - 36*A*B^3*a^10*b^
2 + 486*A^2*B^2*a^9*b^3 - 2916*A^3*B*a^8*b^4 + 6561*A^4*a^7*b^5)*
sqrt(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*
B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))))) - 5*(a^3*b^2*x^4 + 2*a^4*b
*x^2 + a^5)*sqrt(x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2
*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*log(125
*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 291
6*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 -
27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) + 5*(a^3
*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(x)*(-(B^4*a^4 - 36*A*B^3*a^3*b
 + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b
^3))^(1/4)*log(-125*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^
2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4
) - 125*(B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3
)*sqrt(x)))/((a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**3,x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.272413, size = 405, normalized size = 1.26

$$-\frac{2\,A}{a^3\sqrt{x}}+\frac{5\,Babx^{\frac{7}{2}}-13\,Ab^2x^{\frac{7}{2}}+9\,Ba^2x^{\frac{3}{2}}-17\,Aabx^{\frac{3}{2}}}{16\,(bx^2+a)^2a^3}+\frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^3}+\frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^3}-\frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^3}+\frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(3/2)),x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/16*(5*B*a*b*x^(7/2) - 13*A*b^2*x^(7/2) + 9
*B*a^2*x^(3/2) - 17*A*a*b*x^(3/2))/((b*x^2 + a)^2*a^3) + 5/64*sqr
t(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3) + 5/64*
sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3) - 5
/128*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*ln(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) + 5/128*sqrt(2)*(
(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*ln(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(a^4*b^3)
```

3.389 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=322

$$\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{7(11Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{11Ab-3aB}{16a^2bx^{3/2}(a+bx^2)}+\frac{Ab-aB}{4abx^{3/2}(a+bx^2)^2}$$

```
[Out] (-7*(11*A*b - 3*a*B))/(48*a^3*b*x^(3/2)) + (A*b - a*B)/(4*a*b*x^(
3/2)*(a + b*x^2)^2) + (11*A*b - 3*a*B)/(16*a^2*b*x^(3/2)*(a + b*x
^2)) + (7*(11*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*(11*A*b - 3*a*B)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(15/4)*b
^(1/4)) + (7*(11*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*(11*A
*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rubi [A] time = 0.537527, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{7(11Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{11Ab-3aB}{16a^2bx^{3/2}(a+bx^2)}+\frac{Ab-aB}{4abx^{3/2}(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-7*(11*A*b - 3*a*B))/(48*a^3*b*x^(3/2)) + (A*b - a*B)/(4*a*b*x^(
3/2)*(a + b*x^2)^2) + (11*A*b - 3*a*B)/(16*a^2*b*x^(3/2)*(a + b*x
^2)) + (7*(11*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*(11*A*b - 3*a*B)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(15/4)*b
^(1/4)) + (7*(11*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*(11*A
*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rubi in Sympy [A] time = 86.9963, size = 306, normalized size = 0.95

$$\frac{Ab-Ba}{4abx^{\frac{3}{2}}\left(a+bx^2\right)^2}+\frac{11Ab-3Ba}{16a^2bx^{\frac{3}{2}}\left(a+bx^2\right)}-\frac{7\left(11Ab-3Ba\right)}{48a^3bx^{\frac{3}{2}}}+\frac{7\sqrt{2}\left(11Ab-3Ba\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{15}{4}}\sqrt[4]{b}}-\frac{7\sqrt{2}\left(11Ab-3Ba\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{15}{4}}\sqrt[4]{b}}+\frac{7\sqrt{2}\left(11Ab-3Ba\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{15}{4}}\sqrt[4]{b}}-\frac{7\sqrt{2}\left(11Ab-3Ba\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{15}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] (A*b - B*a)/(4*a*b*x**(3/2)*(a + b*x**2)**2) + (11*A*b - 3*B*a)/(
16*a**2*b*x**(3/2)*(a + b*x**2)) - 7*(11*A*b - 3*B*a)/(48*a**3*b*
x**(3/2)) + 7*sqrt(2)*(11*A*b - 3*B*a)*log(-sqrt(2)*a**(1/4)*b**(
1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(128*a**(15/4)*b**(1/4)) - 7*
sqrt(2)*(11*A*b - 3*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) +
sqrt(a) + sqrt(b)*x)/(128*a**(15/4)*b**(1/4)) + 7*sqrt(2)*(11*A*b
 - 3*B*a)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(15/
4)*b**(1/4)) - 7*sqrt(2)*(11*A*b - 3*B*a)*atan(1 + sqrt(2)*b**(1/
4)*sqrt(x)/a**(1/4))/(64*a**(15/4)*b**(1/4))
```

Mathematica [A]   time = 0.497686, size = 286, normalized size = 0.89

$$\frac{\frac{96a^{7/4}\sqrt{x}(aB-Ab)}{(a+bx^2)^2}+\frac{24a^{3/4}\sqrt{x}(7aB-15Ab)}{a+bx^2}-\frac{256a^{3/4}A}{x^{3/2}}+\frac{21\sqrt{2}(11Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{21\sqrt{2}(3aB-11Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\cdots\right.}{\sqrt[4]{b}}}{384a^{15/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-256*a^(3/4)*A)/x^(3/2) + (96*a^(7/4)*(-(A*b) + a*B)*Sqrt[x])/(
a + b*x^2)^2 + (24*a^(3/4)*(-15*A*b + 7*a*B)*Sqrt[x])/(a + b*x^2)
 + (42*Sqrt[2]*(11*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/b^(1/4) - (42*Sqrt[2]*(11*A*b - 3*a*B)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (21*Sqrt[2]*(11*A*b
- 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/b^(1/4) + (21*Sqrt[2]*(-11*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(384*a^(15/4))
```

Maple [A]   time = 0.026, size = 357, normalized size = 1.1

$$-\frac{2A}{3a^3}x^{-\frac{3}{2}}-\frac{15\,b^2A}{16\,a^3\,(bx^2+a)^2}x^{\frac{5}{2}}+\frac{7\,Bb}{16\,a^2\,(bx^2+a)^2}x^{\frac{5}{2}}-\frac{19\,Ab}{16\,a^2\,(bx^2+a)^2}\sqrt{x}$$
$$+\frac{11\,B}{16\,a\,(bx^2+a)^2}\sqrt{x}-\frac{77\,\sqrt{2}Ab}{64\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{77\,\sqrt{2}Ab}{128\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{77\,\sqrt{2}Ab}{64\,a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{21\,\sqrt{2}B}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{21\,\sqrt{2}B}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{21\,\sqrt{2}B}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/3*A/a^3/x^(3/2)-15/16/a^3/(b*x^2+a)^2*x^(5/2)*b^2*A+7/16/a^2/(
b*x^2+a)^2*x^(5/2)*b*B-19/16/a^2/(b*x^2+a)^2*A*x^(1/2)*b+11/16/a/
(b*x^2+a)^2*B*x^(1/2)-77/64/a^4*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)*b-77/128/a^4*(a/b)^(1/4)*2^(1/2)*A*ln(
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))*b-77/64/a^4*(a/b)^(1/4)*2^(1/2)*A*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b+21/64/a^3*(a/b)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+21/128/a^3*(a/b)^(1/4)*2^(1
/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+21/64/a^3*(a/b)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256562, size = 929, normalized size = 2.89

$$28\left(3\,Bab-11\,Ab^2\right)x^4-128\,Aa^2+44\left(3\,Ba^2-11\,Aab\right)x^2+84\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\sqrt{x}\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2}{a^{15}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/192*(28*(3*B*a*b - 11*A*b^2)*x^4 - 128*A*a^2 + 44*(3*B*a^2 - 11
*A*a*b)*x^2 + 84*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqrt(x)*(-(8
1*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B
*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4)*arctan(-a^4*(-(81*B^4*a^4
 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 +
14641*A^4*b^4)/(a^15*b))^(1/4)/((3*B*a - 11*A*b)*sqrt(x) - sqrt(a
^8*sqrt(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 -
15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)) + (9*B^2*a^2 - 66*A*
B*a*b + 121*A^2*b^2)*x))) - 21*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x
)*sqrt(x)*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2
 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4)*log(7*a^4*(
-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^
3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b)*s
qrt(x)) + 21*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqrt(x)*(-(81*B^
4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b
^3 + 14641*A^4*b^4)/(a^15*b))^(1/4)*log(-7*a^4*(-(81*B^4*a^4 - 11
88*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641
*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b)*sqrt(x)))/((a^3*b^
2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26433, size = 410, normalized size = 1.27

$$\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b}+\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b}+\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b}-\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b}-\frac{2\,A}{3\,a^{3}x^{\frac{3}{2}}}+\frac{7\,Babx^{\frac{5}{2}}-15\,Ab^{2}x^{\frac{5}{2}}+11\,Ba^{2}\sqrt{x}-19\,Aab\sqrt{x}}{16\,(bx^{2}+a)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(5/2)),x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b
) + 7/64*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(
a^4*b) + 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*
b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 7/12
8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*ln(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 2/3*A/(a^3*x^(3/
2)) + 1/16*(7*B*a*b*x^(5/2) - 15*A*b^2*x^(5/2) + 11*B*a^2*sqrt(x)
 - 19*A*a*b*sqrt(x))/((b*x^2 + a)^2*a^3)
```

3.390 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=343

$$\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{9\sqrt[4]{b}(13Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{17/4}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{13Ab-5aB}{16a^2bx^{5/2}(a+bx^2)}+\frac{Ab-aB}{4abx^{5/2}(a+bx^2)^2}$$

```
[Out] (-9*(13*A*b - 5*a*B))/(80*a^3*b*x^(5/2)) + (9*(13*A*b - 5*a*B))/(
16*a^4*Sqrt[x]) + (A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + (13
*A*b - 5*a*B)/(16*a^2*b*x^(5/2)*(a + b*x^2)) - (9*b^(1/4)*(13*A*b
 - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt
[2]*a^(17/4)) + (9*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (9*b^(1/4)*(13*
A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(64*Sqrt[2]*a^(17/4)) - (9*b^(1/4)*(13*A*b - 5*a*B)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(17/4))
```

Rubi [A] time = 0.579375, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{9\sqrt[4]{b}(13Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{17/4}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{13Ab-5aB}{16a^2bx^{5/2}(a+bx^2)}+\frac{Ab-aB}{4abx^{5/2}(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```


```
[Out] (-9*(13*A*b - 5*a*B))/(80*a^3*b*x^(5/2)) + (9*(13*A*b - 5*a*B))/(
16*a^4*Sqrt[x]) + (A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + (13
*A*b - 5*a*B)/(16*a^2*b*x^(5/2)*(a + b*x^2)) - (9*b^(1/4)*(13*A*b
 - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt
[2]*a^(17/4)) + (9*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (9*b^(1/4)*(13*
A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(64*Sqrt[2]*a^(17/4)) - (9*b^(1/4)*(13*A*b - 5*a*B)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(17/4))
```

Rubi in Sympy [A] time = 95.6378, size = 326, normalized size = 0.95

$$\frac{Ab-Ba}{4abx^{\frac{5}{2}}\left(a+bx^2\right)^2}+\frac{13Ab-5Ba}{16a^2bx^{\frac{5}{2}}\left(a+bx^2\right)}-\frac{9\left(13Ab-5Ba\right)}{80a^3bx^{\frac{5}{2}}}+\frac{9\left(13Ab-5Ba\right)}{16a^4\sqrt{x}}+\frac{9\sqrt{2}\sqrt[4]{b}\left(13Ab-5Ba\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{17}{4}}}-\frac{9\sqrt{2}\sqrt[4]{b}\left(13Ab-5Ba\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{128a^{\frac{17}{4}}}-\frac{9\sqrt{2}\sqrt[4]{b}\left(13Ab-5Ba\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{17}{4}}}+\frac{9\sqrt{2}\sqrt[4]{b}\left(13Ab-5Ba\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] (A*b - B*a)/(4*a*b*x**(5/2)*(a + b*x**2)**2) + (13*A*b - 5*B*a)/(
16*a**2*b*x**(5/2)*(a + b*x**2)) - 9*(13*A*b - 5*B*a)/(80*a**3*b*
x**(5/2)) + 9*(13*A*b - 5*B*a)/(16*a**4*sqrt(x)) + 9*sqrt(2)*b**(
1/4)*(13*A*b - 5*B*a)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sq
rt(a) + sqrt(b)*x)/(128*a**(17/4)) - 9*sqrt(2)*b**(1/4)*(13*A*b -
 5*B*a)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)
*x)/(128*a**(17/4)) - 9*sqrt(2)*b**(1/4)*(13*A*b - 5*B*a)*atan(1
- sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(64*a**(17/4)) + 9*sqrt(2)*b
**(1/4)*(13*A*b - 5*B*a)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/
4))/(64*a**(17/4))
```

Mathematica [A] time = 0.539769, size = 308, normalized size = 0.9

$$-\frac{160a^{5/4}bx^{3/2}(aB-Ab)}{(a+bx^2)^2}-\frac{256a^{5/4}A}{x^{5/2}}-\frac{40\sqrt[4]{a}bx^{3/2}(13aB-21Ab)}{a+bx^2}-\frac{1280\sqrt[4]{a}(aB-3Ab)}{\sqrt{x}}+45\sqrt{2}\sqrt[4]{b}(13Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-256*a^(5/4)*A)/x^(5/2) - (1280*a^(1/4)*(-3*A*b + a*B))/Sqrt[x]
 - (160*a^(5/4)*b*(-(A*b) + a*B)*x^(3/2))/(a + b*x^2)^2 - (40*a^(
1/4)*b*(-21*A*b + 13*a*B)*x^(3/2))/(a + b*x^2) - 90*Sqrt[2]*b^(1/
4)*(13*A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + 90*Sqrt[2]*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)] + 45*Sqrt[2]*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]*
b^(1/4)*(-13*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(640*a^(17/4))
```

Maple [A] time = 0.033, size = 381, normalized size = 1.1

$$-\frac{2A}{5a^3}x^{-\frac{5}{2}}+6\frac{Ab}{\sqrt{x}a^4}-2\frac{B}{\sqrt{x}a^3}+\frac{21b^3A}{16a^4(bx^2+a)^2}x^{\frac{7}{2}}-\frac{13Bb^2}{16a^3(bx^2+a)^2}x^{\frac{7}{2}}+\frac{25b^2A}{16a^3(bx^2+a)^2}x^{\frac{3}{2}}$$
$$-\frac{17Bb}{16a^2(bx^2+a)^2}x^{\frac{3}{2}}+\frac{117b\sqrt{2}A}{128a^4}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{117b\sqrt{2}A}{64a^4}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{117b\sqrt{2}A}{64a^4}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{45\sqrt{2}B}{128a^3}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{45\sqrt{2}B}{64a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{45\sqrt{2}B}{64a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/5*A/a^3/x^(5/2)+6/x^(1/2)/a^4*A*b-2/x^(1/2)/a^3*B+21/16/a^4*b^
3/(b*x^2+a)^2*x^(7/2)*A-13/16/a^3*b^2/(b*x^2+a)^2*x^(7/2)*B+25/16
/a^3*b^2/(b*x^2+a)^2*A*x^(3/2)-17/16/a^2*b/(b*x^2+a)^2*B*x^(3/2)+
117/128/a^4*b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+11
7/64/a^4*b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)+117/64/a^4*b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)-45/128/a^3/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))-45/64/a^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)-45/64/a^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261387, size = 1257, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(7/2)),x, algorithm="fricas")
```

```
[Out] -1/320*(180*(5*B*a*b^2 - 13*A*b^3)*x^6 + 324*(5*B*a^2*b - 13*A*a*
b^2)*x^4 + 128*A*a^3 + 128*(5*B*a^3 - 13*A*a^2*b)*x^2 - 180*(a^4*
b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)*sqrt(x)*(-(625*B^4*a^4*b - 6500*
A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561
```

```
*A^4*b^5)/a^17)^(1/4)*arctan(-a^13*(-(625*B^4*a^4*b - 6500*A*B^3*
a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b
^5)/a^17)^(3/4)/((125*B^3*a^3*b - 975*A*B^2*a^2*b^2 + 2535*A^2*B*
a*b^3 - 2197*A^3*b^4)*sqrt(x) - sqrt((15625*B^6*a^6*b^2 - 243750*
A*B^5*a^5*b^3 + 1584375*A^2*B^4*a^4*b^4 - 5492500*A^3*B^3*a^3*b^5
 + 10710375*A^4*B^2*a^2*b^6 - 11138790*A^5*B*a*b^7 + 4826809*A^6*
b^8)*x - (625*B^4*a^13*b - 6500*A*B^3*a^12*b^2 + 25350*A^2*B^2*a^
11*b^3 - 43940*A^3*B*a^10*b^4 + 28561*A^4*a^9*b^5)*sqrt(-(625*B^4
*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B
*a*b^4 + 28561*A^4*b^5)/a^17)))) - 45*(a^4*b^2*x^6 + 2*a^5*b*x^4
+ a^6*x^2)*sqrt(x)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*
A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*
log(729*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^
2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*
(125*B^3*a^3*b - 975*A*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*
b^4)*sqrt(x)) + 45*(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)*sqrt(x)*
(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 4
3940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*log(-729*a^13*(-(62
5*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*
A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 9
75*A*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)))/((a
^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.262049, size = 440, normalized size = 1.28

$$-\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b^2}-\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b^2}+\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b^2}-\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b^2}-\frac{13\,Bab^2x^{\frac{7}{2}}-21\,Ab^3x^{\frac{7}{2}}+17\,Ba^2bx^{\frac{3}{2}}-25\,Aab^2x^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^4}-\frac{2\left(5\,Bax^2-15\,Abx^2+Aa\right)}{5\,a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^3*x^(7/2)),x, algorithm="giac")

[Out] -9/64*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*
b^2) - 9/64*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^5*b^2) + 9/128*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/
4)*A*b)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2)
```

```
 - 9/128*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*ln(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) - 1/16*(1
3*B*a*b^2*x^(7/2) - 21*A*b^3*x^(7/2) + 17*B*a^2*b*x^(3/2) - 25*A*
a*b^2*x^(3/2))/((b*x^2 + a)^2*a^4) - 2/5*(5*B*a*x^2 - 15*A*b*x^2
+ A*a)/(a^4*x^(5/2))
```

3.391 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

```
[Out] (2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c
 + 2*a*d)*x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

Rubi [A] time = 0.0932629, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c
 + 2*a*d)*x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

Rubi in Sympy [A] time = 12.0578, size = 63, normalized size = 1.

$$\frac{2a^2cx^{\frac{9}{2}}}{9}+\frac{2ax^{\frac{13}{2}}(ad+2bc)}{13}+\frac{2b^2dx^{\frac{21}{2}}}{21}+\frac{2bx^{\frac{17}{2}}(2ad+bc)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(9/2)/9 + 2*a*x**(13/2)*(a*d + 2*b*c)/13 + 2*b**2*d*x
**(21/2)/21 + 2*b*x**(17/2)*(2*a*d + b*c)/17
```

Mathematica [A] time = 0.0347354, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547a^2c+819bx^4(2ad+bc)+1071ax^2(ad+2bc)+663b^2dx^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(9/2)*(1547*a^2*c + 1071*a*(2*b*c + a*d)*x^2 + 819*b*(b*c +
2*a*d)*x^4 + 663*b^2*d*x^6))/13923
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$\frac{1326\,b^2dx^6+3276\,x^4abd+1638\,b^2cx^4+2142\,x^2a^2d+4284\,abcx^2+3094\,a^2c}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/13923*x^(9/2)*(663*b^2*d*x^6+1638*a*b*d*x^4+819*b^2*c*x^4+1071*
a^2*d*x^2+2142*a*b*c*x^2+1547*a^2*c)
```

Maxima [A] time = 1.35182, size = 69, normalized size = 1.1

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}} + \frac{2}{17}\left(b^2c + 2\,abd\right)x^{\frac{17}{2}} + \frac{2}{9}\,a^2cx^{\frac{9}{2}} + \frac{2}{13}\left(2\,abc + a^2d\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*(b^2*c + 2*a*b*d)*x^(17/2) + 2/9*a^2*c
*x^(9/2) + 2/13*(2*a*b*c + a^2*d)*x^(13/2)
```

Fricas [A] time = 0.216111, size = 76, normalized size = 1.21

$$\frac{2}{13923}\left(663\,b^2dx^{10} + 819\left(b^2c + 2\,abd\right)x^8 + 1547\,a^2cx^4 + 1071\left(2\,abc + a^2d\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^2*d*x^10 + 819*(b^2*c + 2*a*b*d)*x^8 + 1547*a^2*c*
x^4 + 1071*(2*a*b*c + a^2*d)*x^6)*sqrt(x)
```

Sympy [A] time = 70.9966, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{9}{2}}}{9} + \frac{2a^2dx^{\frac{13}{2}}}{13} + \frac{4abcx^{\frac{13}{2}}}{13} + \frac{4abdx^{\frac{17}{2}}}{17} + \frac{2b^2cx^{\frac{17}{2}}}{17} + \frac{2b^2dx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(9/2)/9 + 2*a**2*d*x**(13/2)/13 + 4*a*b*c*x**(13/2)/1
3 + 4*a*b*d*x**(17/2)/17 + 2*b**2*c*x**(17/2)/17 + 2*b**2*d*x**(2
1/2)/21
```

GIAC/XCAS [A] time = 0.227682, size = 72, normalized size = 1.14

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}} + \frac{2}{17}\,b^2cx^{\frac{17}{2}} + \frac{4}{17}\,abdx^{\frac{17}{2}} + \frac{4}{13}\,abcx^{\frac{13}{2}} + \frac{2}{13}\,a^2dx^{\frac{13}{2}} + \frac{2}{9}\,a^2cx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*b^2*c*x^(17/2) + 4/17*a*b*d*x^(17/2) +
 4/13*a*b*c*x^(13/2) + 2/13*a^2*d*x^(13/2) + 2/9*a^2*c*x^(9/2)
```

3.392 $\int x^{5/2} \left(a + bx^2\right)^2 \left(c + dx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2cx^{7/2} + \frac{2}{15}bx^{15/2}(2ad + bc) + \frac{2}{11}ax^{11/2}(ad + 2bc) + \frac{2}{19}b^2dx^{19/2}$$

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c
 + 2*a*d)*x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Rubi [A] time = 0.0926184, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2}{7}a^2cx^{7/2} + \frac{2}{15}bx^{15/2}(2ad + bc) + \frac{2}{11}ax^{11/2}(ad + 2bc) + \frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c
 + 2*a*d)*x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Rubi in Sympy [A] time = 12.0796, size = 63, normalized size = 1.

$$\frac{2a^2cx^{\frac{7}{2}}}{7} + \frac{2ax^{\frac{11}{2}}\left(ad + 2bc\right)}{11} + \frac{2b^2dx^{\frac{19}{2}}}{19} + \frac{2bx^{\frac{15}{2}}\left(2ad + bc\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(7/2)/7 + 2*a*x**(11/2)*(a*d + 2*b*c)/11 + 2*b**2*d*x
**(19/2)/19 + 2*b*x**(15/2)*(2*a*d + b*c)/15
```

Mathematica [A] time = 0.0298307, size = 63, normalized size = 1.

$$\frac{2}{7}a^2cx^{7/2} + \frac{2}{15}bx^{15/2}(2ad + bc) + \frac{2}{11}ax^{11/2}(ad + 2bc) + \frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c
 + 2*a*d)*x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$\frac{2310\,b^2dx^6 + 5852\,x^4abd + 2926\,b^2cx^4 + 3990\,x^2a^2d + 7980\,abcx^2 + 6270\,a^2c}{21945}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*b^2*d*x^6+2926*a*b*d*x^4+1463*b^2*c*x^4+199
5*a^2*d*x^2+3990*a*b*c*x^2+3135*a^2*c)
```

Maxima [A] time = 1.35049, size = 69, normalized size = 1.1

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\left(b^2c+2\,abd\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}+\frac{2}{11}\left(2\,abc+a^2d\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*(b^2*c + 2*a*b*d)*x^(15/2) + 2/7*a^2*c
*x^(7/2) + 2/11*(2*a*b*c + a^2*d)*x^(11/2)
```

Fricas [A] time = 0.217673, size = 76, normalized size = 1.21

$$\frac{2}{21945}\left(1155\,b^2dx^9+1463\left(b^2c+2\,abd\right)x^7+3135\,a^2cx^3+1995\left(2\,abc+a^2d\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d*x^9 + 1463*(b^2*c + 2*a*b*d)*x^7 + 3135*a^2*c
*x^3 + 1995*(2*a*b*c + a^2*d)*x^5)*sqrt(x)
```

Sympy [A] time = 37.752, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{11}{2}}}{11}+\frac{4abcx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{15}{2}}}{15}+\frac{2b^2cx^{\frac{15}{2}}}{15}+\frac{2b^2dx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(7/2)/7 + 2*a**2*d*x**(11/2)/11 + 4*a*b*c*x**(11/2)/1
1 + 4*a*b*d*x**(15/2)/15 + 2*b**2*c*x**(15/2)/15 + 2*b**2*d*x**(1
9/2)/19
```

GIAC/XCAS [A] time = 0.234895, size = 72, normalized size = 1.14

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\,b^2cx^{\frac{15}{2}}+\frac{4}{15}\,abdx^{\frac{15}{2}}+\frac{4}{11}\,abcx^{\frac{11}{2}}+\frac{2}{11}\,a^2dx^{\frac{11}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*b^2*c*x^(15/2) + 4/15*a*b*d*x^(15/2) +
 4/11*a*b*c*x^(11/2) + 2/11*a^2*d*x^(11/2) + 2/7*a^2*c*x^(7/2)
```

3.393 $\int x^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

```
[Out] (2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c +
 2*a*d)*x^(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

Rubi [A] time = 0.0877595, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c +
 2*a*d)*x^(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

Rubi in Sympy [A] time = 12.0836, size = 63, normalized size = 1.

$$\frac{2a^2cx^{\frac{5}{2}}}{5}+\frac{2ax^{\frac{9}{2}}\left(ad+2bc\right)}{9}+\frac{2b^2dx^{\frac{17}{2}}}{17}+\frac{2bx^{\frac{13}{2}}\left(2ad+bc\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(5/2)/5 + 2*a*x**(9/2)*(a*d + 2*b*c)/9 + 2*b**2*d*x**
(17/2)/17 + 2*b*x**(13/2)*(2*a*d + b*c)/13
```

Mathematica [A] time = 0.0335752, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989a^2c+765bx^4(2ad+bc)+1105ax^2(ad+2bc)+585b^2dx^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(5/2)*(1989*a^2*c + 1105*a*(2*b*c + a*d)*x^2 + 765*b*(b*c +
2*a*d)*x^4 + 585*b^2*d*x^6))/9945
```

Maple [A] time = 0.009, size = 56, normalized size = 0.9

$$\frac{1170\,b^2dx^6+3060\,x^4abd+1530\,b^2cx^4+2210\,x^2a^2d+4420\,abcx^2+3978\,a^2c}{9945}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/9945*x^(5/2)*(585*b^2*d*x^6+1530*a*b*d*x^4+765*b^2*c*x^4+1105*a
^2*d*x^2+2210*a*b*c*x^2+1989*a^2*c)
```

Maxima [A] time = 1.35406, size = 69, normalized size = 1.1

$$\frac{2}{17}\, b^{2} d x^{\frac{17}{2}} + \frac{2}{13} \left(b^{2} c + 2\, a b d\right) x^{\frac{13}{2}} + \frac{2}{5}\, a^{2} c x^{\frac{5}{2}} + \frac{2}{9} \left(2\, a b c + a^{2} d\right) x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*(b^2*c + 2*a*b*d)*x^(13/2) + 2/5*a^2*c
*x^(5/2) + 2/9*(2*a*b*c + a^2*d)*x^(9/2)
```

Fricas [A] time = 0.20833, size = 76, normalized size = 1.21

$$\frac{2}{9945} \left(585\, b^{2} d x^{8} + 765 \left(b^{2} c + 2\, a b d\right) x^{6} + 1989\, a^{2} c x^{2} + 1105 \left(2\, a b c + a^{2} d\right) x^{4}\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d*x^8 + 765*(b^2*c + 2*a*b*d)*x^6 + 1989*a^2*c*x^
2 + 1105*(2*a*b*c + a^2*d)*x^4)*sqrt(x)
```

Sympy [A] time = 20.4412, size = 80, normalized size = 1.27

$$\frac{2a^{2}cx^{\frac{5}{2}}}{5} + \frac{2a^{2}dx^{\frac{9}{2}}}{9} + \frac{4abcx^{\frac{9}{2}}}{9} + \frac{4abdx^{\frac{13}{2}}}{13} + \frac{2b^{2}cx^{\frac{13}{2}}}{13} + \frac{2b^{2}dx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(5/2)/5 + 2*a**2*d*x**(9/2)/9 + 4*a*b*c*x**(9/2)/9 +
4*a*b*d*x**(13/2)/13 + 2*b**2*c*x**(13/2)/13 + 2*b**2*d*x**(17/2)
/17
```

GIAC/XCAS [A] time = 0.228096, size = 72, normalized size = 1.14

$$\frac{2}{17}\, b^{2} d x^{\frac{17}{2}} + \frac{2}{13}\, b^{2} c x^{\frac{13}{2}} + \frac{4}{13}\, a b d x^{\frac{13}{2}} + \frac{4}{9}\, a b c x^{\frac{9}{2}} + \frac{2}{9}\, a^{2} d x^{\frac{9}{2}} + \frac{2}{5}\, a^{2} c x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*(d*x^2 + c)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*b^2*c*x^(13/2) + 4/13*a*b*d*x^(13/2) +
 4/9*a*b*c*x^(9/2) + 2/9*a^2*d*x^(9/2) + 2/5*a^2*c*x^(5/2)
```

3.394 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

```
[Out] (2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c +
 2*a*d)*x^(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

Rubi [A] time = 0.0850662, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c +
 2*a*d)*x^(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

Rubi in Sympy [A] time = 12.3509, size = 63, normalized size = 1.

$$\frac{2a^2cx^{\frac{3}{2}}}{3}+\frac{2ax^{\frac{7}{2}}\left(ad+2bc\right)}{7}+\frac{2b^2dx^{\frac{15}{2}}}{15}+\frac{2bx^{\frac{11}{2}}\left(2ad+bc\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)*x**(1/2),x)
```

```
[Out] 2*a**2*c*x**(3/2)/3 + 2*a*x**(7/2)*(a*d + 2*b*c)/7 + 2*b**2*d*x**
(15/2)/15 + 2*b*x**(11/2)*(2*a*d + b*c)/11
```

Mathematica [A] time = 0.0318511, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385a^2c+105bx^4(2ad+bc)+165ax^2(ad+2bc)+77b^2dx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*c + 165*a*(2*b*c + a*d)*x^2 + 105*b*(b*c + 2*
a*d)*x^4 + 77*b^2*d*x^6))/1155
```

Maple [A] time = 0.007, size = 56, normalized size = 0.9

$$\frac{154\,b^2dx^6+420\,x^4abd+210\,b^2cx^4+330\,x^2a^2d+660\,abcx^2+770\,a^2c}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*b^2*d*x^6+210*a*b*d*x^4+105*b^2*c*x^4+165*a^2*
d*x^2+330*a*b*c*x^2+385*a^2*c)
```

Maxima [A] time = 1.37178, size = 69, normalized size = 1.1

$$\frac{2}{15}\,b^{2}dx^{\frac{15}{2}} + \frac{2}{11}\left(b^{2}c + 2\,abd\right)x^{\frac{11}{2}} + \frac{2}{3}\,a^{2}cx^{\frac{3}{2}} + \frac{2}{7}\left(2\,abc + a^{2}d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*(b^2*c + 2*a*b*d)*x^(11/2) + 2/3*a^2*c
*x^(3/2) + 2/7*(2*a*b*c + a^2*d)*x^(7/2)
```

Fricas [A] time = 0.225149, size = 73, normalized size = 1.16

$$\frac{2}{1155}\left(77\,b^{2}dx^{7} + 105\left(b^{2}c + 2\,abd\right)x^{5} + 385\,a^{2}cx + 165\left(2\,abc + a^{2}d\right)x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d*x^7 + 105*(b^2*c + 2*a*b*d)*x^5 + 385*a^2*c*x +
165*(2*a*b*c + a^2*d)*x^3)*sqrt(x)
```

Sympy [A] time = 4.85478, size = 66, normalized size = 1.05

$$\frac{2a^{2}cx^{\frac{3}{2}}}{3} + \frac{2b^{2}dx^{\frac{15}{2}}}{15} + \frac{2x^{\frac{11}{2}}\left(2abd + b^{2}c\right)}{11} + \frac{2x^{\frac{7}{2}}\left(a^{2}d + 2abc\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)*x**(1/2),x)
```

```
[Out] 2*a**2*c*x**(3/2)/3 + 2*b**2*d*x**(15/2)/15 + 2*x**(11/2)*(2*a*b*
d + b**2*c)/11 + 2*x**(7/2)*(a**2*d + 2*a*b*c)/7
```

GIAC/XCAS [A] time = 0.225723, size = 72, normalized size = 1.14

$$\frac{2}{15}\,b^{2}dx^{\frac{15}{2}} + \frac{2}{11}\,b^{2}cx^{\frac{11}{2}} + \frac{4}{11}\,abdx^{\frac{11}{2}} + \frac{4}{7}\,abcx^{\frac{7}{2}} + \frac{2}{7}\,a^{2}dx^{\frac{7}{2}} + \frac{2}{3}\,a^{2}cx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*b^2*c*x^(11/2) + 4/11*a*b*d*x^(11/2) +
 4/7*a*b*c*x^(7/2) + 2/7*a^2*d*x^(7/2) + 2/3*a^2*c*x^(3/2)
```

3.395 $\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

```
[Out] 2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a
*d)*x^(9/2))/9 + (2*b^2*d*x^(13/2))/13
```

Rubi [A] time = 0.0844582, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a
*d)*x^(9/2))/9 + (2*b^2*d*x^(13/2))/13
```

Rubi in Sympy [A] time = 12.0753, size = 61, normalized size = 1.

$$2a^2c\sqrt{x}+\frac{2ax^{\frac{5}{2}}(ad+2bc)}{5}+\frac{2b^2dx^{\frac{13}{2}}}{13}+\frac{2bx^{\frac{9}{2}}(2ad+bc)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**(1/2),x)
```

```
[Out] 2*a**2*c*sqrt(x) + 2*a*x**(5/2)*(a*d + 2*b*c)/5 + 2*b**2*d*x**(13
/2)/13 + 2*b*x**(9/2)*(2*a*d + b*c)/9
```

Mathematica [A] time = 0.0313292, size = 53, normalized size = 0.87

$$\frac{2}{585}\sqrt{x}\left(585a^2c+65bx^4(2ad+bc)+117ax^2(ad+2bc)+45b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(585*a^2*c + 117*a*(2*b*c + a*d)*x^2 + 65*b*(b*c + 2*a
*d)*x^4 + 45*b^2*d*x^6))/585
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$\frac{90\,b^2dx^6+260\,x^4abd+130\,b^2cx^4+234\,x^2a^2d+468\,abcx^2+1170\,a^2c}{585}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x)
```

```
[Out] 2/585*x^(1/2)*(45*b^2*d*x^6+130*a*b*d*x^4+65*b^2*c*x^4+117*a^2*d*
x^2+234*a*b*c*x^2+585*a^2*c)
```

Maxima [A] time = 1.35791, size = 69, normalized size = 1.13

$$\frac{2}{13}\,b^{2}dx^{\frac{13}{2}}+\frac{2}{9}\left(b^{2}c+2\,abd\right)x^{\frac{9}{2}}+2\,a^{2}c\sqrt{x}+\frac{2}{5}\left(2\,abc+a^{2}d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*(b^2*c + 2*a*b*d)*x^(9/2) + 2*a^2*c*sqr
t(x) + 2/5*(2*a*b*c + a^2*d)*x^(5/2)
```

Fricas [A] time = 0.220174, size = 72, normalized size = 1.18

$$\frac{2}{585}\left(45\,b^{2}dx^{6}+65\left(b^{2}c+2\,abd\right)x^{4}+585\,a^{2}c+117\left(2\,abc+a^{2}d\right)x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 65*(b^2*c + 2*a*b*d)*x^4 + 585*a^2*c + 117*
(2*a*b*c + a^2*d)*x^2)*sqrt(x)
```

Sympy [A] time = 6.71769, size = 78, normalized size = 1.28

$$2a^{2}c\sqrt{x}+\frac{2a^{2}dx^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{5}{2}}}{5}+\frac{4abdx^{\frac{9}{2}}}{9}+\frac{2b^{2}cx^{\frac{9}{2}}}{9}+\frac{2b^{2}dx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)/x**(1/2),x)
```

```
[Out] 2*a**2*c*sqrt(x) + 2*a**2*d*x**(5/2)/5 + 4*a*b*c*x**(5/2)/5 + 4*a
*b*d*x**(9/2)/9 + 2*b**2*c*x**(9/2)/9 + 2*b**2*d*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.241127, size = 72, normalized size = 1.18

$$\frac{2}{13}\,b^{2}dx^{\frac{13}{2}}+\frac{2}{9}\,b^{2}cx^{\frac{9}{2}}+\frac{4}{9}\,abdx^{\frac{9}{2}}+\frac{4}{5}\,abcx^{\frac{5}{2}}+\frac{2}{5}\,a^{2}dx^{\frac{5}{2}}+2\,a^{2}c\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*b^2*c*x^(9/2) + 4/9*a*b*d*x^(9/2) + 4/5
*a*b*c*x^(5/2) + 2/5*a^2*d*x^(5/2) + 2*a^2*c*sqrt(x)
```

3.396 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

```
[Out] (-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c +
2*a*d)*x^(7/2))/7 + (2*b^2*d*x^(11/2))/11
```

Rubi [A] time = 0.0875, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c +
2*a*d)*x^(7/2))/7 + (2*b^2*d*x^(11/2))/11
```

Rubi in Sympy [A] time = 12.1622, size = 61, normalized size = 1.

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2ax^{\frac{3}{2}}(ad+2bc)}{3}+\frac{2b^2dx^{\frac{11}{2}}}{11}+\frac{2bx^{\frac{7}{2}}(2ad+bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**(3/2),x)
```

```
[Out] -2*a**2*c/sqrt(x) + 2*a*x**(3/2)*(a*d + 2*b*c)/3 + 2*b**2*d*x**(1
1/2)/11 + 2*b*x**(7/2)*(2*a*d + b*c)/7
```

Mathematica [A] time = 0.0357795, size = 53, normalized size = 0.87

$$\frac{2\left(-231a^2c+33bx^4(2ad+bc)+77ax^2(ad+2bc)+21b^2dx^6\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (2*(-231*a^2*c + 77*a*(2*b*c + a*d)*x^2 + 33*b*(b*c + 2*a*d)*x^4
+ 21*b^2*d*x^6))/(231*Sqrt[x])
```

Maple [A] time = 0.01, size = 56, normalized size = 0.9

$$-\frac{-42\,b^2dx^6-132\,x^4abd-66\,b^2cx^4-154\,x^2a^2d-308\,abcx^2+462\,a^2c}{231}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x)
```

```
[Out] -2/231*(-21*b^2*d*x^6-66*a*b*d*x^4-33*b^2*c*x^4-77*a^2*d*x^2-154*
a*b*c*x^2+231*a^2*c)/x^(1/2)
```

Maxima [A] time = 1.37755, size = 69, normalized size = 1.13

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}} + \frac{2}{7}\left(b^2c + 2\,abd\right)x^{\frac{7}{2}} - \frac{2\,a^2c}{\sqrt{x}} + \frac{2}{3}\left(2\,abc + a^2d\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*(b^2*c + 2*a*b*d)*x^(7/2) - 2*a^2*c/sqr
t(x) + 2/3*(2*a*b*c + a^2*d)*x^(3/2)
```

Fricas [A] time = 0.214484, size = 72, normalized size = 1.18

$$\frac{2\left(21\,b^2dx^6 + 33\left(b^2c + 2\,abd\right)x^4 - 231\,a^2c + 77\left(2\,abc + a^2d\right)x^2\right)}{231\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*d*x^6 + 33*(b^2*c + 2*a*b*d)*x^4 - 231*a^2*c + 77*(
2*a*b*c + a^2*d)*x^2)/sqrt(x)
```

Sympy [A] time = 8.5719, size = 78, normalized size = 1.28

$$-\frac{2a^2c}{\sqrt{x}} + \frac{2a^2dx^{\frac{3}{2}}}{3} + \frac{4abcx^{\frac{3}{2}}}{3} + \frac{4abdx^{\frac{7}{2}}}{7} + \frac{2b^2cx^{\frac{7}{2}}}{7} + \frac{2b^2dx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)/x**(3/2),x)
```

```
[Out] -2*a**2*c/sqrt(x) + 2*a**2*d*x**(3/2)/3 + 4*a*b*c*x**(3/2)/3 + 4*
a*b*d*x**(7/2)/7 + 2*b**2*c*x**(7/2)/7 + 2*b**2*d*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.229528, size = 72, normalized size = 1.18

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}} + \frac{2}{7}\,b^2cx^{\frac{7}{2}} + \frac{4}{7}\,abdx^{\frac{7}{2}} + \frac{4}{3}\,abcx^{\frac{3}{2}} + \frac{2}{3}\,a^2dx^{\frac{3}{2}} - \frac{2\,a^2c}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*b^2*c*x^(7/2) + 4/7*a*b*d*x^(7/2) + 4/3
*a*b*c*x^(3/2) + 2/3*a^2*d*x^(3/2) - 2*a^2*c/sqrt(x)
```

3.397 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

```
[Out] (-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c +
2*a*d)*x^(5/2))/5 + (2*b^2*d*x^(9/2))/9
```

Rubi [A] time = 0.0874254, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c +
2*a*d)*x^(5/2))/5 + (2*b^2*d*x^(9/2))/9
```

Rubi in Sympy [A] time = 12.1246, size = 61, normalized size = 1.

$$-\frac{2a^2c}{3x^{\frac{3}{2}}}+2a\sqrt{x}\,(ad+2bc)+\frac{2b^2dx^{\frac{9}{2}}}{9}+\frac{2bx^{\frac{5}{2}}\,(2ad+bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**(5/2),x)
```

```
[Out] -2*a**2*c/(3*x**(3/2)) + 2*a*sqrt(x)*(a*d + 2*b*c) + 2*b**2*d*x**
(9/2)/9 + 2*b*x**(5/2)*(2*a*d + b*c)/5
```

Mathematica [A] time = 0.033021, size = 53, normalized size = 0.87

$$\frac{2\left(-15a^2c+9bx^4(2ad+bc)+45ax^2(ad+2bc)+5b^2dx^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*c + 45*a*(2*b*c + a*d)*x^2 + 9*b*(b*c + 2*a*d)*x^4 +
5*b^2*d*x^6))/(45*x^(3/2))
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$-\frac{-10\,b^2dx^6-36\,x^4abd-18\,b^2cx^4-90\,x^2a^2d-180\,abcx^2+30\,a^2c}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x)
```

```
[Out] -2/45*(-5*b^2*d*x^6-18*a*b*d*x^4-9*b^2*c*x^4-45*a^2*d*x^2-90*a*b*
c*x^2+15*a^2*c)/x^(3/2)
```

Maxima [A] time = 1.34149, size = 69, normalized size = 1.13

$$\frac{2}{9}\,b^2dx^{\frac{9}{2}}+\frac{2}{5}\left(b^2c+2\,abd\right)x^{\frac{5}{2}}-\frac{2\,a^2c}{3\,x^{\frac{3}{2}}}+2\left(2\,abc+a^2d\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*(b^2*c + 2*a*b*d)*x^(5/2) - 2/3*a^2*c/x^(
3/2) + 2*(2*a*b*c + a^2*d)*sqrt(x)
```

Fricas [A] time = 0.227388, size = 72, normalized size = 1.18

$$\frac{2\left(5\,b^2dx^6+9\left(b^2c+2\,abd\right)x^4-15\,a^2c+45\left(2\,abc+a^2d\right)x^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*d*x^6 + 9*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 45*(2*a*
b*c + a^2*d)*x^2)/x^(3/2)
```

Sympy [A] time = 10.2954, size = 76, normalized size = 1.25

$$-\frac{2a^2c}{3x^{\frac{3}{2}}}+2a^2d\sqrt{x}+4abc\sqrt{x}+\frac{4abdx^{\frac{5}{2}}}{5}+\frac{2b^2cx^{\frac{5}{2}}}{5}+\frac{2b^2dx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)/x**(5/2),x)
```

```
[Out] -2*a**2*c/(3*x**(3/2)) + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt(x) + 4*a
*b*d*x**(5/2)/5 + 2*b**2*c*x**(5/2)/5 + 2*b**2*d*x**(9/2)/9
```

GIAC/XCAS [A] time = 0.233384, size = 72, normalized size = 1.18

$$\frac{2}{9}\,b^2dx^{\frac{9}{2}}+\frac{2}{5}\,b^2cx^{\frac{5}{2}}+\frac{4}{5}\,abdx^{\frac{5}{2}}+4\,abc\sqrt{x}+2\,a^2d\sqrt{x}-\frac{2\,a^2c}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*b^2*c*x^(5/2) + 4/5*a*b*d*x^(5/2) + 4*a*b
*c*sqrt(x) + 2*a^2*d*sqrt(x) - 2/3*a^2*c/x^(3/2)
```

3.398 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

```
[Out] (-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c
+ 2*a*d)*x^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

Rubi [A] time = 0.0879057, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c
+ 2*a*d)*x^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

Rubi in Sympy [A] time = 12.1637, size = 61, normalized size = 1.

$$-\frac{2a^2c}{5x^{\frac{5}{2}}}-\frac{2a\,(ad+2bc)}{\sqrt{x}}+\frac{2b^2dx^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{3}{2}}\,(2ad+bc)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)/x**(7/2),x)
```

```
[Out] -2*a**2*c/(5*x**(5/2)) - 2*a*(a*d + 2*b*c)/sqrt(x) + 2*b**2*d*x**
(7/2)/7 + 2*b*x**(3/2)*(2*a*d + b*c)/3
```

Mathematica [A] time = 0.0296077, size = 57, normalized size = 0.93

$$\frac{-42a^2\left(c+5dx^2\right)+140abx^2\left(dx^2-3c\right)+10b^2x^4\left(7c+3dx^2\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (140*a*b*x^2*(-3*c + d*x^2) + 10*b^2*x^4*(7*c + 3*d*x^2) - 42*a^2
*(c + 5*d*x^2))/(105*x^(5/2))
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$-\frac{-30\,b^2dx^6-140\,x^4abd-70\,b^2cx^4+210\,x^2a^2d+420\,abcx^2+42\,a^2c}{105}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x)
```

```
[Out] -2/105*(-15*b^2*d*x^6-70*a*b*d*x^4-35*b^2*c*x^4+105*a^2*d*x^2+210
*a*b*c*x^2+21*a^2*c)/x^(5/2)
```

Maxima [A] time = 1.34331, size = 72, normalized size = 1.18

$$\frac{2}{7}\,b^{2}dx^{\frac{7}{2}}+\frac{2}{3}\,\left(b^{2}c+2\,abd\right)x^{\frac{3}{2}}-\frac{2\,\left(a^{2}c+5\,\left(2\,abc+a^{2}d\right)x^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*(b^2*c + 2*a*b*d)*x^(3/2) - 2/5*(a^2*c +
5*(2*a*b*c + a^2*d)*x^2)/x^(5/2)
```

Fricas [A] time = 0.221629, size = 72, normalized size = 1.18

$$\frac{2\,\left(15\,b^{2}dx^{6}+35\,\left(b^{2}c+2\,abd\right)x^{4}-21\,a^{2}c-105\,\left(2\,abc+a^{2}d\right)x^{2}\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d*x^6 + 35*(b^2*c + 2*a*b*d)*x^4 - 21*a^2*c - 105*(
2*a*b*c + a^2*d)*x^2)/x^(5/2)
```

Sympy [A] time = 15.5271, size = 76, normalized size = 1.25

$$-\frac{2a^{2}c}{5x^{\frac{5}{2}}}-\frac{2a^{2}d}{\sqrt{x}}-\frac{4abc}{\sqrt{x}}+\frac{4abdx^{\frac{3}{2}}}{3}+\frac{2b^{2}cx^{\frac{3}{2}}}{3}+\frac{2b^{2}dx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)/x**(7/2),x)
```

```
[Out] -2*a**2*c/(5*x**(5/2)) - 2*a**2*d/sqrt(x) - 4*a*b*c/sqrt(x) + 4*a
*b*d*x**(3/2)/3 + 2*b**2*c*x**(3/2)/3 + 2*b**2*d*x**(7/2)/7
```

GIAC/XCAS [A] time = 0.229806, size = 74, normalized size = 1.21

$$\frac{2}{7}\,b^{2}dx^{\frac{7}{2}}+\frac{2}{3}\,b^{2}cx^{\frac{3}{2}}+\frac{4}{3}\,abdx^{\frac{3}{2}}-\frac{2\,\left(10\,abcx^{2}+5\,a^{2}dx^{2}+a^{2}c\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*b^2*c*x^(3/2) + 4/3*a*b*d*x^(3/2) - 2/5*(
10*a*b*c*x^2 + 5*a^2*d*x^2 + a^2*c)/x^(5/2)
```

3.399 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(
21/2))/21 + (2*b^2*d^2*x^(25/2))/25
```

Rubi [A] time = 0.14452, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(
21/2))/21 + (2*b^2*d^2*x^(25/2))/25
```

Rubi in Sympy [A] time = 22.1815, size = 102, normalized size = 1.05

$$\frac{2a^2c^2x^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}(ad+bc)}{13}+\frac{2b^2d^2x^{\frac{25}{2}}}{25}+\frac{4bdx^{\frac{21}{2}}(ad+bc)}{21}+x^{\frac{17}{2}}\left(\frac{2a^2d^2}{17}+\frac{8abcd}{17}+\frac{2b^2c^2}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(9/2)/9 + 4*a*c*x**(13/2)*(a*d + b*c)/13 + 2*b**2*
d**2*x**(25/2)/25 + 4*b*d*x**(21/2)*(a*d + b*c)/21 + x**(17/2)*(2
*a**2*d**2/17 + 8*a*b*c*d/17 + 2*b**2*c**2/17)
```

Mathematica [A] time = 0.0536611, size = 97, normalized size = 1.

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(
21/2))/21 + (2*b^2*d^2*x^(25/2))/25
```

Maple [A] time = 0.01, size = 97, normalized size = 1.

$$\frac{27846\,b^2d^2x^8+66300\,x^6abd^2+66300\,x^6b^2cd+40950\,x^4a^2d^2+163800\,x^4abcd+40950\,x^4b^2c^2+107100\,x^2a^2cd+107100\,ac}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/348075*x^(9/2)*(13923*b^2*d^2*x^8+33150*a*b*d^2*x^6+33150*b^2*c
*d*x^6+20475*a^2*d^2*x^4+81900*a*b*c*d*x^4+20475*b^2*c^2*x^4+5355
0*a^2*c*d*x^2+53550*a*b*c^2*x^2+38675*a^2*c^2)
```

Maxima [A] time = 1.35629, size = 115, normalized size = 1.19

$$\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\left(b^2cd+abd^2\right)x^{\frac{21}{2}}+\frac{2}{17}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}+\frac{4}{13}\left(abc^2+a^2cd\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*(b^2*c*d + a*b*d^2)*x^(21/2) + 2/17*
(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(17/2) + 2/9*a^2*c^2*x^(9/2) +
4/13*(a*b*c^2 + a^2*c*d)*x^(13/2)
```

Fricas [A] time = 0.210854, size = 122, normalized size = 1.26

$$\frac{2}{348075}\left(13923\,b^2d^2x^{12}+33150\left(b^2cd+abd^2\right)x^{10}+20475\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+38675\,a^2c^2x^4+53550\left(abc^2+a^2cd\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^2*x^12 + 33150*(b^2*c*d + a*b*d^2)*x^10 + 2
0475*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 38675*a^2*c^2*x^4 + 53
550*(a*b*c^2 + a^2*c*d)*x^6)*sqrt(x)
```

Sympy [A] time = 125.044, size = 136, normalized size = 1.4

$$\frac{2a^2c^2x^{\frac{9}{2}}}{9}+\frac{4a^2cdx^{\frac{13}{2}}}{13}+\frac{2a^2d^2x^{\frac{17}{2}}}{17}+\frac{4abc^2x^{\frac{13}{2}}}{13}+\frac{8abcdx^{\frac{17}{2}}}{17}+\frac{4abd^2x^{\frac{21}{2}}}{21}+\frac{2b^2c^2x^{\frac{17}{2}}}{17}+\frac{4b^2cdx^{\frac{21}{2}}}{21}+\frac{2b^2d^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(9/2)/9 + 4*a**2*c*d*x**(13/2)/13 + 2*a**2*d**2*x*
*(17/2)/17 + 4*a*b*c**2*x**(13/2)/13 + 8*a*b*c*d*x**(17/2)/17 + 4
*a*b*d**2*x**(21/2)/21 + 2*b**2*c**2*x**(17/2)/17 + 4*b**2*c*d*x*
*(21/2)/21 + 2*b**2*d**2*x**(25/2)/25
```

GIAC/XCAS [A] time = 0.230477, size = 127, normalized size = 1.31

$$\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\,b^2cdx^{\frac{21}{2}}+\frac{4}{21}\,abd^2x^{\frac{21}{2}}+\frac{2}{17}\,b^2c^2x^{\frac{17}{2}}+\frac{8}{17}\,abcdx^{\frac{17}{2}}+\frac{2}{17}\,a^2d^2x^{\frac{17}{2}}+\frac{4}{13}\,abc^2x^{\frac{13}{2}}+\frac{4}{13}\,a^2cdx^{\frac{13}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*b^2*c*d*x^(21/2) + 4/21*a*b*d^2*x^(2
1/2) + 2/17*b^2*c^2*x^(17/2) + 8/17*a*b*c*d*x^(17/2) + 2/17*a^2*d
^2*x^(17/2) + 4/13*a*b*c^2*x^(13/2) + 4/13*a^2*c*d*x^(13/2) + 2/9
*a^2*c^2*x^(9/2)
```

3.400 $\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(
19/2))/19 + (2*b^2*d^2*x^(23/2))/23
```

Rubi [A] time = 0.141816, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(
19/2))/19 + (2*b^2*d^2*x^(23/2))/23
```

Rubi in Sympy [A] time = 22.0447, size = 102, normalized size = 1.05

$$\frac{2a^2c^2x^{\frac{7}{2}}}{7}+\frac{4acx^{\frac{11}{2}}(ad+bc)}{11}+\frac{2b^2d^2x^{\frac{23}{2}}}{23}+\frac{4bdx^{\frac{19}{2}}(ad+bc)}{19}+x^{\frac{15}{2}}\left(\frac{2a^2d^2}{15}+\frac{8abcd}{15}+\frac{2b^2c^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(7/2)/7 + 4*a*c*x**(11/2)*(a*d + b*c)/11 + 2*b**2*
d**2*x**(23/2)/23 + 4*b*d*x**(19/2)*(a*d + b*c)/19 + x**(15/2)*(2
*a**2*d**2/15 + 8*a*b*c*d/15 + 2*b**2*c**2/15)
```

Mathematica [A] time = 0.0513131, size = 97, normalized size = 1.

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(
19/2))/19 + (2*b^2*d^2*x^(23/2))/23
```

Maple [A] time = 0.01, size = 97, normalized size = 1.

$$\frac{43890\,b^2d^2x^8+106260\,x^6abd^2+106260\,x^6b^2cd+67298\,x^4a^2d^2+269192\,x^4abcd+67298\,x^4b^2c^2+183540\,x^2a^2cd+183540}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*b^2*d^2*x^8+53130*a*b*d^2*x^6+53130*b^2*c
*d*x^6+33649*a^2*d^2*x^4+134596*a*b*c*d*x^4+33649*b^2*c^2*x^4+917
70*a^2*c*d*x^2+91770*a*b*c^2*x^2+72105*a^2*c^2)
```

Maxima [A] time = 1.33551, size = 115, normalized size = 1.19

$$\frac{2}{23}\,b^2d^2x^{\frac{23}{2}}+\frac{4}{19}\left(b^2cd+abd^2\right)x^{\frac{19}{2}}+\frac{2}{15}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^2c^2x^{\frac{7}{2}}+\frac{4}{11}\left(abc^2+a^2cd\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*(b^2*c*d + a*b*d^2)*x^(19/2) + 2/15*
(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(15/2) + 2/7*a^2*c^2*x^(7/2) +
4/11*(a*b*c^2 + a^2*c*d)*x^(11/2)
```

Fricas [A] time = 0.218577, size = 122, normalized size = 1.26

$$\frac{2}{504735}\left(21945\,b^2d^2x^{11}+53130\left(b^2cd+abd^2\right)x^9+33649\left(b^2c^2+4\,abcd+a^2d^2\right)x^7+72105\,a^2c^2x^3+91770\left(abc^2+a^2cd\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^2*x^11 + 53130*(b^2*c*d + a*b*d^2)*x^9 + 33
649*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7 + 72105*a^2*c^2*x^3 + 917
70*(a*b*c^2 + a^2*c*d)*x^5)*sqrt(x)
```

Sympy [A] time = 72.403, size = 136, normalized size = 1.4

$$\frac{2a^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cdx^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{15}{2}}}{15}+\frac{4abc^2x^{\frac{11}{2}}}{11}+\frac{8abcdx^{\frac{15}{2}}}{15}+\frac{4abd^2x^{\frac{19}{2}}}{19}+\frac{2b^2c^2x^{\frac{15}{2}}}{15}+\frac{4b^2cdx^{\frac{19}{2}}}{19}+\frac{2b^2d^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(7/2)/7 + 4*a**2*c*d*x**(11/2)/11 + 2*a**2*d**2*x*
*(15/2)/15 + 4*a*b*c**2*x**(11/2)/11 + 8*a*b*c*d*x**(15/2)/15 + 4
*a*b*d**2*x**(19/2)/19 + 2*b**2*c**2*x**(15/2)/15 + 4*b**2*c*d*x*
*(19/2)/19 + 2*b**2*d**2*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.23572, size = 127, normalized size = 1.31

$$\frac{2}{23}\,b^2d^2x^{\frac{23}{2}}+\frac{4}{19}\,b^2cdx^{\frac{19}{2}}+\frac{4}{19}\,abd^2x^{\frac{19}{2}}+\frac{2}{15}\,b^2c^2x^{\frac{15}{2}}+\frac{8}{15}\,abcdx^{\frac{15}{2}}+\frac{2}{15}\,a^2d^2x^{\frac{15}{2}}+\frac{4}{11}\,abc^2x^{\frac{11}{2}}+\frac{4}{11}\,a^2cdx^{\frac{11}{2}}+\frac{2}{7}\,a^2c^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*b^2*c*d*x^(19/2) + 4/19*a*b*d^2*x^(1
9/2) + 2/15*b^2*c^2*x^(15/2) + 8/15*a*b*c*d*x^(15/2) + 2/15*a^2*d
^2*x^(15/2) + 4/11*a*b*c^2*x^(11/2) + 4/11*a^2*c*d*x^(11/2) + 2/7
*a^2*c^2*x^(7/2)
```

3.401 $\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17
/2))/17 + (2*b^2*d^2*x^(21/2))/21
```

Rubi [A] time = 0.138356, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17
/2))/17 + (2*b^2*d^2*x^(21/2))/21
```

Rubi in Sympy [A] time = 22.2002, size = 102, normalized size = 1.05

$$\frac{2a^2c^2x^{\frac{5}{2}}}{5}+\frac{4acx^{\frac{9}{2}}(ad+bc)}{9}+\frac{2b^2d^2x^{\frac{21}{2}}}{21}+\frac{4bdx^{\frac{17}{2}}(ad+bc)}{17}+x^{\frac{13}{2}}\left(\frac{2a^2d^2}{13}+\frac{8abcd}{13}+\frac{2b^2c^2}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(5/2)/5 + 4*a*c*x**(9/2)*(a*d + b*c)/9 + 2*b**2*d*
*2*x**(21/2)/21 + 4*b*d*x**(17/2)*(a*d + b*c)/17 + x**(13/2)*(2*a
**2*d**2/13 + 8*a*b*c*d/13 + 2*b**2*c**2/13)
```

Mathematica [A] time = 0.0508341, size = 97, normalized size = 1.

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17
/2))/17 + (2*b^2*d^2*x^(21/2))/21
```

Maple [A] time = 0.009, size = 97, normalized size = 1.

$$\frac{6630\,b^2d^2x^8+16380\,x^6abd^2+16380\,x^6b^2cd+10710\,x^4a^2d^2+42840\,x^4abcd+10710\,x^4b^2c^2+30940\,x^2a^2cd+30940\,ac^2bx^2}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*b^2*d^2*x^8+8190*a*b*d^2*x^6+8190*b^2*c*d*x
^6+5355*a^2*d^2*x^4+21420*a*b*c*d*x^4+5355*b^2*c^2*x^4+15470*a^2*
c*d*x^2+15470*a*b*c^2*x^2+13923*a^2*c^2)
```

Maxima [A] time = 1.33827, size = 115, normalized size = 1.19

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}} + \frac{4}{17}\left(b^2cd + abd^2\right)x^{\frac{17}{2}} + \frac{2}{13}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{13}{2}} + \frac{2}{5}\,a^2c^2x^{\frac{5}{2}} + \frac{4}{9}\left(abc^2 + a^2cd\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*(b^2*c*d + a*b*d^2)*x^(17/2) + 2/13*
(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(13/2) + 2/5*a^2*c^2*x^(5/2) +
4/9*(a*b*c^2 + a^2*c*d)*x^(9/2)
```

Fricas [A] time = 0.222694, size = 122, normalized size = 1.26

$$\frac{2}{69615}\left(3315\,b^2d^2x^{10} + 8190\left(b^2cd + abd^2\right)x^8 + 5355\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^6 + 13923\,a^2c^2x^2 + 15470\left(abc^2 + a^2cd\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^2*x^10 + 8190*(b^2*c*d + a*b*d^2)*x^8 + 5355*
(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^6 + 13923*a^2*c^2*x^2 + 15470*(
a*b*c^2 + a^2*c*d)*x^4)*sqrt(x)
```

Sympy [A] time = 38.5991, size = 136, normalized size = 1.4

$$\frac{2a^2c^2x^{\frac{5}{2}}}{5} + \frac{4a^2cdx^{\frac{9}{2}}}{9} + \frac{2a^2d^2x^{\frac{13}{2}}}{13} + \frac{4abc^2x^{\frac{9}{2}}}{9} + \frac{8abcdx^{\frac{13}{2}}}{13} + \frac{4abd^2x^{\frac{17}{2}}}{17} + \frac{2b^2c^2x^{\frac{13}{2}}}{13} + \frac{4b^2cdx^{\frac{17}{2}}}{17} + \frac{2b^2d^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(5/2)/5 + 4*a**2*c*d*x**(9/2)/9 + 2*a**2*d**2*x**(
13/2)/13 + 4*a*b*c**2*x**(9/2)/9 + 8*a*b*c*d*x**(13/2)/13 + 4*a*b
*d**2*x**(17/2)/17 + 2*b**2*c**2*x**(13/2)/13 + 4*b**2*c*d*x**(17
/2)/17 + 2*b**2*d**2*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.243952, size = 127, normalized size = 1.31

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}} + \frac{4}{17}\,b^2cdx^{\frac{17}{2}} + \frac{4}{17}\,abd^2x^{\frac{17}{2}} + \frac{2}{13}\,b^2c^2x^{\frac{13}{2}} + \frac{8}{13}\,abcdx^{\frac{13}{2}} + \frac{2}{13}\,a^2d^2x^{\frac{13}{2}} + \frac{4}{9}\,abc^2x^{\frac{9}{2}} + \frac{4}{9}\,a^2cdx^{\frac{9}{2}} + \frac{2}{5}\,a^2c^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*b^2*c*d*x^(17/2) + 4/17*a*b*d^2*x^(1
7/2) + 2/13*b^2*c^2*x^(13/2) + 8/13*a*b*c*d*x^(13/2) + 2/13*a^2*d
^2*x^(13/2) + 4/9*a*b*c^2*x^(9/2) + 4/9*a^2*c*d*x^(9/2) + 2/5*a^2
*c^2*x^(5/2)
```

3.402 $\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

```
[Out] (2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15
/2))/15 + (2*b^2*d^2*x^(19/2))/19
```

Rubi [A] time = 0.132648, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15
/2))/15 + (2*b^2*d^2*x^(19/2))/19
```

Rubi in Sympy [A] time = 22.715, size = 102, normalized size = 1.05

$$\frac{2a^2c^2x^{\frac{3}{2}}}{3}+\frac{4acx^{\frac{7}{2}}(ad+bc)}{7}+\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{4bdx^{\frac{15}{2}}(ad+bc)}{15}+x^{\frac{11}{2}}\left(\frac{2a^2d^2}{11}+\frac{8abcd}{11}+\frac{2b^2c^2}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2*x**(1/2),x)
```

```
[Out] 2*a**2*c**2*x**(3/2)/3 + 4*a*c*x**(7/2)*(a*d + b*c)/7 + 2*b**2*d*
*2*x**(19/2)/19 + 4*b*d*x**(15/2)*(a*d + b*c)/15 + x**(11/2)*(2*a
**2*d**2/11 + 8*a*b*c*d/11 + 2*b**2*c**2/11)
```

Mathematica [A] time = 0.0620533, size = 83, normalized size = 0.86

$$\frac{2x^{3/2}\left(1995x^4\left(a^2d^2+4abcd+b^2c^2\right)+7315a^2c^2+2926bdx^6(ad+bc)+6270acx^2(ad+bc)+1155b^2d^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2*c^2 + 6270*a*c*(b*c + a*d)*x^2 + 1995*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^4 + 2926*b*d*(b*c + a*d)*x^6 + 1155*b
^2*d^2*x^8))/21945
```

Maple [A] time = 0.009, size = 97, normalized size = 1.

$$\frac{2310\,b^2d^2x^8+5852\,x^6abd^2+5852\,x^6b^2cd+3990\,x^4a^2d^2+15960\,x^4abcd+3990\,x^4b^2c^2+12540\,x^2a^2cd+12540\,ac^2bx^2+14}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x)
```

```
[Out] 2/21945*x^(3/2)*(1155*b^2*d^2*x^8+2926*a*b*d^2*x^6+2926*b^2*c*d*x
^6+1995*a^2*d^2*x^4+7980*a*b*c*d*x^4+1995*b^2*c^2*x^4+6270*a^2*c*
d*x^2+6270*a*b*c^2*x^2+7315*a^2*c^2)
```

Maxima [A] time = 1.34431, size = 115, normalized size = 1.19

$$\frac{2}{19}\,b^2d^2x^{\frac{19}{2}}+\frac{4}{15}\left(b^2cd+abd^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^2c^2x^{\frac{3}{2}}+\frac{4}{7}\left(abc^2+a^2cd\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*(b^2*c*d + a*b*d^2)*x^(15/2) + 2/11*
(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(11/2) + 2/3*a^2*c^2*x^(3/2) +
4/7*(a*b*c^2 + a^2*c*d)*x^(7/2)
```

Fricas [A] time = 0.222096, size = 119, normalized size = 1.23

$$\frac{2}{21945}\left(1155\,b^2d^2x^9+2926\left(b^2cd+abd^2\right)x^7+1995\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+7315\,a^2c^2x+6270\left(abc^2+a^2cd\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^2*x^9 + 2926*(b^2*c*d + a*b*d^2)*x^7 + 1995*(
b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^5 + 7315*a^2*c^2*x + 6270*(a*b*c
^2 + a^2*c*d)*x^3)*sqrt(x)
```

Sympy [A] time = 10.2143, size = 110, normalized size = 1.13

$$\frac{2a^2c^2x^{\frac{3}{2}}}{3}+\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{15}{2}}\left(2abd^2+2b^2cd\right)}{15}+\frac{2x^{\frac{11}{2}}\left(a^2d^2+4abcd+b^2c^2\right)}{11}+\frac{2x^{\frac{7}{2}}\left(2a^2cd+2abc^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2*x**(1/2),x)
```

```
[Out] 2*a**2*c**2*x**(3/2)/3 + 2*b**2*d**2*x**(19/2)/19 + 2*x**(15/2)*(
2*a*b*d**2 + 2*b**2*c*d)/15 + 2*x**(11/2)*(a**2*d**2 + 4*a*b*c*d
+ b**2*c**2)/11 + 2*x**(7/2)*(2*a**2*c*d + 2*a*b*c**2)/7
```

GIAC/XCAS [A] time = 0.22658, size = 127, normalized size = 1.31

$$\frac{2}{19}\,b^2d^2x^{\frac{19}{2}}+\frac{4}{15}\,b^2cdx^{\frac{15}{2}}+\frac{4}{15}\,abd^2x^{\frac{15}{2}}+\frac{2}{11}\,b^2c^2x^{\frac{11}{2}}+\frac{8}{11}\,abcdx^{\frac{11}{2}}+\frac{2}{11}\,a^2d^2x^{\frac{11}{2}}+\frac{4}{7}\,abc^2x^{\frac{7}{2}}+\frac{4}{7}\,a^2cdx^{\frac{7}{2}}+\frac{2}{3}\,a^2c^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*b^2*c*d*x^(15/2) + 4/15*a*b*d^2*x^(1
5/2) + 2/11*b^2*c^2*x^(11/2) + 8/11*a*b*c*d*x^(11/2) + 2/11*a^2*d
^2*x^(11/2) + 4/7*a*b*c^2*x^(7/2) + 4/7*a^2*c*d*x^(7/2) + 2/3*a^2
*c^2*x^(3/2)
```

3.403 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/1
3 + (2*b^2*d^2*x^(17/2))/17
```

Rubi [A] time = 0.132047, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/1
3 + (2*b^2*d^2*x^(17/2))/17
```

Rubi in Sympy [A] time = 22.323, size = 100, normalized size = 1.05

$$2a^2c^2\sqrt{x}+\frac{4acx^{\frac{5}{2}}(ad+bc)}{5}+\frac{2b^2d^2x^{\frac{17}{2}}}{17}+\frac{4bdx^{\frac{13}{2}}(ad+bc)}{13}+x^{\frac{9}{2}}\left(\frac{2a^2d^2}{9}+\frac{8abcd}{9}+\frac{2b^2c^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] 2*a**2*c**2*sqrt(x) + 4*a*c*x**(5/2)*(a*d + b*c)/5 + 2*b**2*d**2*
x**(17/2)/17 + 4*b*d*x**(13/2)*(a*d + b*c)/13 + x**(9/2)*(2*a**2*
d**2/9 + 8*a*b*c*d/9 + 2*b**2*c**2/9)
```

Mathematica [A] time = 0.050901, size = 95, normalized size = 1.

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/1
3 + (2*b^2*d^2*x^(17/2))/17
```

Maple [A] time = 0.009, size = 97, normalized size = 1.

$$\frac{1170\,b^2d^2x^8+3060\,x^6abd^2+3060\,x^6b^2cd+2210\,x^4a^2d^2+8840\,x^4abcd+2210\,x^4b^2c^2+7956\,x^2a^2cd+7956\,ac^2bx^2+19890}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*b^2*d^2*x^8+1530*a*b*d^2*x^6+1530*b^2*c*d*x^6
+1105*a^2*d^2*x^4+4420*a*b*c*d*x^4+1105*b^2*c^2*x^4+3978*a^2*c*d*
x^2+3978*a*b*c^2*x^2+9945*a^2*c^2)
```

Maxima [A] time = 1.34102, size = 115, normalized size = 1.21

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}}+\frac{4}{13}\left(b^2cd+abd^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{9}{2}}+2\,a^2c^2\sqrt{x}+\frac{4}{5}\left(abc^2+a^2cd\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*(b^2*c*d + a*b*d^2)*x^(13/2) + 2/9*(
b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(9/2) + 2*a^2*c^2*sqrt(x) + 4/5*
(a*b*c^2 + a^2*c*d)*x^(5/2)
```

Fricas [A] time = 0.22173, size = 117, normalized size = 1.23

$$\frac{2}{9945}\left(585\,b^2d^2x^8+1530\left(b^2cd+abd^2\right)x^6+1105\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+9945\,a^2c^2+3978\left(abc^2+a^2cd\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^2*x^8 + 1530*(b^2*c*d + a*b*d^2)*x^6 + 1105*(b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4 + 9945*a^2*c^2 + 3978*(a*b*c^2 +
 a^2*c*d)*x^2)*sqrt(x)
```

Sympy [A] time = 17.235, size = 134, normalized size = 1.41

$$2a^2c^2\sqrt{x}+\frac{4a^2cdx^{\frac{5}{2}}}{5}+\frac{2a^2d^2x^{\frac{9}{2}}}{9}+\frac{4abc^2x^{\frac{5}{2}}}{5}+\frac{8abcdx^{\frac{9}{2}}}{9}+\frac{4abd^2x^{\frac{13}{2}}}{13}+\frac{2b^2c^2x^{\frac{9}{2}}}{9}+\frac{4b^2cdx^{\frac{13}{2}}}{13}+\frac{2b^2d^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] 2*a**2*c**2*sqrt(x) + 4*a**2*c*d*x**(5/2)/5 + 2*a**2*d**2*x**(9/2
)/9 + 4*a*b*c**2*x**(5/2)/5 + 8*a*b*c*d*x**(9/2)/9 + 4*a*b*d**2*x
**(13/2)/13 + 2*b**2*c**2*x**(9/2)/9 + 4*b**2*c*d*x**(13/2)/13 +
2*b**2*d**2*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.23393, size = 127, normalized size = 1.34

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}}+\frac{4}{13}\,b^2cdx^{\frac{13}{2}}+\frac{4}{13}\,abd^2x^{\frac{13}{2}}+\frac{2}{9}\,b^2c^2x^{\frac{9}{2}}+\frac{8}{9}\,abcdx^{\frac{9}{2}}+\frac{2}{9}\,a^2d^2x^{\frac{9}{2}}+\frac{4}{5}\,abc^2x^{\frac{5}{2}}+\frac{4}{5}\,a^2cdx^{\frac{5}{2}}+2\,a^2c^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*b^2*c*d*x^(13/2) + 4/13*a*b*d^2*x^(1
3/2) + 2/9*b^2*c^2*x^(9/2) + 8/9*a*b*c*d*x^(9/2) + 2/9*a^2*d^2*x^
(9/2) + 4/5*a*b*c^2*x^(5/2) + 4/5*a^2*c*d*x^(5/2) + 2*a^2*c^2*sqr
t(x)
```

3.404 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

```
[Out] (-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2)
)/11 + (2*b^2*d^2*x^(15/2))/15
```

Rubi [A] time = 0.136981, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2)
)/11 + (2*b^2*d^2*x^(15/2))/15
```

Rubi in Sympy [A] time = 22.4702, size = 100, normalized size = 1.05

$$-\frac{2a^2c^2}{\sqrt{x}}+\frac{4acx^{\frac{3}{2}}(ad+bc)}{3}+\frac{2b^2d^2x^{\frac{15}{2}}}{15}+\frac{4bdx^{\frac{11}{2}}(ad+bc)}{11}+x^{\frac{7}{2}}\left(\frac{2a^2d^2}{7}+\frac{8abcd}{7}+\frac{2b^2c^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(3/2),x)
```

```
[Out] -2*a**2*c**2/sqrt(x) + 4*a*c*x**(3/2)*(a*d + b*c)/3 + 2*b**2*d**2
*x**(15/2)/15 + 4*b*d*x**(11/2)*(a*d + b*c)/11 + x**(7/2)*(2*a**2
*d**2/7 + 8*a*b*c*d/7 + 2*b**2*c**2/7)
```

Mathematica [A] time = 0.0510197, size = 83, normalized size = 0.87

$$\frac{2\left(165x^4\left(a^2d^2+4abcd+b^2c^2\right)-1155a^2c^2+210bdx^6(ad+bc)+770acx^2(ad+bc)+77b^2d^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2*c^2 + 770*a*c*(b*c + a*d)*x^2 + 165*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 + 210*b*d*(b*c + a*d)*x^6 + 77*b^2*d^2*x^8))
/(1155*Sqrt[x])
```

Maple [A] time = 0.01, size = 97, normalized size = 1.

$$-\frac{-154\,b^2d^2x^8-420\,x^6abd^2-420\,x^6b^2cd-330\,x^4a^2d^2-1320\,x^4abcd-330\,x^4b^2c^2-1540\,x^2a^2cd-1540\,ac^2bx^2+2310\,a^2 \ldots}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*b^2*d^2*x^8-210*a*b*d^2*x^6-210*b^2*c*d*x^6-165*a^2*
d^2*x^4-660*a*b*c*d*x^4-165*b^2*c^2*x^4-770*a^2*c*d*x^2-770*a*b*c
^2*x^2+1155*a^2*c^2)/x^(1/2)
```

Maxima [A] time = 1.33231, size = 115, normalized size = 1.21

$$\frac{2}{15}\, b^2 d^2 x^{\frac{15}{2}} + \frac{4}{11}\left(b^2 cd + abd^2\right) x^{\frac{11}{2}} + \frac{2}{7}\left(b^2 c^2 + 4\, abcd + a^2 d^2\right) x^{\frac{7}{2}} - \frac{2\, a^2 c^2}{\sqrt{x}} + \frac{4}{3}\left(abc^2 + a^2 cd\right) x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*(b^2*c*d + a*b*d^2)*x^(11/2) + 2/7*(
b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(7/2) - 2*a^2*c^2/sqrt(x) + 4/3*
(a*b*c^2 + a^2*c*d)*x^(3/2)
```

Fricas [A] time = 0.214014, size = 117, normalized size = 1.23

$$\frac{2\left(77\, b^2 d^2 x^8 + 210\left(b^2 cd + abd^2\right) x^6 + 165\left(b^2 c^2 + 4\, abcd + a^2 d^2\right) x^4 - 1155\, a^2 c^2 + 770\left(abc^2 + a^2 cd\right) x^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^8 + 210*(b^2*c*d + a*b*d^2)*x^6 + 165*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^4 - 1155*a^2*c^2 + 770*(a*b*c^2 + a^2
*c*d)*x^2)/sqrt(x)
```

Sympy [A] time = 20.5895, size = 134, normalized size = 1.41

$$-\frac{2a^2c^2}{\sqrt{x}} + \frac{4a^2cdx^{\frac{3}{2}}}{3} + \frac{2a^2d^2x^{\frac{7}{2}}}{7} + \frac{4abc^2x^{\frac{3}{2}}}{3} + \frac{8abcdx^{\frac{7}{2}}}{7} + \frac{4abd^2x^{\frac{11}{2}}}{11} + \frac{2b^2c^2x^{\frac{7}{2}}}{7} + \frac{4b^2cdx^{\frac{11}{2}}}{11} + \frac{2b^2d^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(3/2),x)
```

```
[Out] -2*a**2*c**2/sqrt(x) + 4*a**2*c*d*x**(3/2)/3 + 2*a**2*d**2*x**(7/
2)/7 + 4*a*b*c**2*x**(3/2)/3 + 8*a*b*c*d*x**(7/2)/7 + 4*a*b*d**2*
x**(11/2)/11 + 2*b**2*c**2*x**(7/2)/7 + 4*b**2*c*d*x**(11/2)/11 +
 2*b**2*d**2*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.22915, size = 127, normalized size = 1.34

$$\frac{2}{15}\, b^2 d^2 x^{\frac{15}{2}} + \frac{4}{11}\, b^2 cdx^{\frac{11}{2}} + \frac{4}{11}\, abd^2 x^{\frac{11}{2}} + \frac{2}{7}\, b^2 c^2 x^{\frac{7}{2}} + \frac{8}{7}\, abcdx^{\frac{7}{2}} + \frac{2}{7}\, a^2 d^2 x^{\frac{7}{2}} + \frac{4}{3}\, abc^2 x^{\frac{3}{2}} + \frac{4}{3}\, a^2 cdx^{\frac{3}{2}} - \frac{2\, a^2 c^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*b^2*c*d*x^(11/2) + 4/11*a*b*d^2*x^(1
1/2) + 2/7*b^2*c^2*x^(7/2) + 8/7*a*b*c*d*x^(7/2) + 2/7*a^2*d^2*x^
(7/2) + 4/3*a*b*c^2*x^(3/2) + 4/3*a^2*c*d*x^(3/2) - 2*a^2*c^2/sqr
t(x)
```

3.405 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

```
[Out] (-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))
/9 + (2*b^2*d^2*x^(13/2))/13
```

Rubi [A] time = 0.13448, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))
/9 + (2*b^2*d^2*x^(13/2))/13
```

Rubi in Sympy [A] time = 22.2425, size = 100, normalized size = 1.05

$$-\frac{2a^2c^2}{3x^{\frac{3}{2}}}+4ac\sqrt{x}\,(ad+bc)+\frac{2b^2d^2x^{\frac{13}{2}}}{13}+\frac{4bdx^{\frac{9}{2}}\,(ad+bc)}{9}+x^{\frac{5}{2}}\left(\frac{2a^2d^2}{5}+\frac{8abcd}{5}+\frac{2b^2c^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(5/2),x)
```

```
[Out] -2*a**2*c**2/(3*x**(3/2)) + 4*a*c*sqrt(x)*(a*d + b*c) + 2*b**2*d*
*2*x**(13/2)/13 + 4*b*d*x**(9/2)*(a*d + b*c)/9 + x**(5/2)*(2*a**2
*d**2/5 + 8*a*b*c*d/5 + 2*b**2*c**2/5)
```

Mathematica [A] time = 0.0523489, size = 83, normalized size = 0.87

$$\frac{2\left(117x^4\left(a^2d^2+4abcd+b^2c^2\right)-195a^2c^2+130bdx^6(ad+bc)+1170acx^2(ad+bc)+45b^2d^2x^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (2*(-195*a^2*c^2 + 1170*a*c*(b*c + a*d)*x^2 + 117*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 + 130*b*d*(b*c + a*d)*x^6 + 45*b^2*d^2*x^8))
/(585*x^(3/2))
```

Maple [A] time = 0.009, size = 97, normalized size = 1.

$$-\frac{-90\,b^2d^2x^8-260\,x^6abd^2-260\,x^6b^2cd-234\,x^4a^2d^2-936\,x^4abcd-234\,x^4b^2c^2-2340\,x^2a^2cd-2340\,ac^2bx^2+390\,a^2c^2}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*b^2*d^2*x^8-130*a*b*d^2*x^6-130*b^2*c*d*x^6-117*a^2*d
^2*x^4-468*a*b*c*d*x^4-117*b^2*c^2*x^4-1170*a^2*c*d*x^2-1170*a*b*
c^2*x^2+195*a^2*c^2)/x^(3/2)
```

Maxima [A] time = 1.3287, size = 115, normalized size = 1.21

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}}+\frac{4}{9}\left(b^2cd+abd^2\right)x^{\frac{9}{2}}+\frac{2}{5}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{5}{2}}-\frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}+4\left(abc^2+a^2cd\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*(b^2*c*d + a*b*d^2)*x^(9/2) + 2/5*(b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5/2) - 2/3*a^2*c^2/x^(3/2) + 4*(a
*b*c^2 + a^2*c*d)*sqrt(x)
```

Fricas [A] time = 0.218673, size = 117, normalized size = 1.23

$$\frac{2\left(45\,b^2d^2x^8+130\left(b^2cd+abd^2\right)x^6+117\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-195\,a^2c^2+1170\left(abc^2+a^2cd\right)x^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d^2*x^8 + 130*(b^2*c*d + a*b*d^2)*x^6 + 117*(b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*x^4 - 195*a^2*c^2 + 1170*(a*b*c^2 + a^2*
c*d)*x^2)/x^(3/2)
```

Sympy [A] time = 24.6005, size = 133, normalized size = 1.4

$$-\frac{2a^2c^2}{3x^{\frac{3}{2}}}+4a^2cd\sqrt{x}+\frac{2a^2d^2x^{\frac{5}{2}}}{5}+4abc^2\sqrt{x}+\frac{8abcdx^{\frac{5}{2}}}{5}+\frac{4abd^2x^{\frac{9}{2}}}{9}+\frac{2b^2c^2x^{\frac{5}{2}}}{5}+\frac{4b^2cdx^{\frac{9}{2}}}{9}+\frac{2b^2d^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(5/2),x)
```

```
[Out] -2*a**2*c**2/(3*x**(3/2)) + 4*a**2*c*d*sqrt(x) + 2*a**2*d**2*x**(
5/2)/5 + 4*a*b*c**2*sqrt(x) + 8*a*b*c*d*x**(5/2)/5 + 4*a*b*d**2*x
**(9/2)/9 + 2*b**2*c**2*x**(5/2)/5 + 4*b**2*c*d*x**(9/2)/9 + 2*b*
*2*d**2*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.242861, size = 127, normalized size = 1.34

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}}+\frac{4}{9}\,b^2cdx^{\frac{9}{2}}+\frac{4}{9}\,abd^2x^{\frac{9}{2}}+\frac{2}{5}\,b^2c^2x^{\frac{5}{2}}+\frac{8}{5}\,abcdx^{\frac{5}{2}}+\frac{2}{5}\,a^2d^2x^{\frac{5}{2}}+4\,abc^2\sqrt{x}+4\,a^2cd\sqrt{x}-\frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*b^2*c*d*x^(9/2) + 4/9*a*b*d^2*x^(9/2)
 + 2/5*b^2*c^2*x^(5/2) + 8/5*a*b*c*d*x^(5/2) + 2/5*a^2*d^2*x^(5/2
) + 4*a*b*c^2*sqrt(x) + 4*a^2*c*d*sqrt(x) - 2/3*a^2*c^2/x^(3/2)
```

3.406 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

```
[Out] (-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2
))/7 + (2*b^2*d^2*x^(11/2))/11
```

Rubi [A] time = 0.134953, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2
))/7 + (2*b^2*d^2*x^(11/2))/11
```

Rubi in Sympy [A] time = 22.2182, size = 100, normalized size = 1.05

$$-\frac{2a^2c^2}{5x^{\frac{5}{2}}}-\frac{4ac\,(ad+bc)}{\sqrt{x}}+\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{4bdx^{\frac{7}{2}}\,(ad+bc)}{7}+x^{\frac{3}{2}}\left(\frac{2a^2d^2}{3}+\frac{8abcd}{3}+\frac{2b^2c^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(7/2),x)
```

```
[Out] -2*a**2*c**2/(5*x**(5/2)) - 4*a*c*(a*d + b*c)/sqrt(x) + 2*b**2*d*
*2*x**(11/2)/11 + 4*b*d*x**(7/2)*(a*d + b*c)/7 + x**(3/2)*(2*a**2
*d**2/3 + 8*a*b*c*d/3 + 2*b**2*c**2/3)
```

Mathematica [A] time = 0.0616044, size = 83, normalized size = 0.87

$$\frac{2\left(385x^4\left(a^2d^2+4abcd+b^2c^2\right)-231a^2c^2+330bdx^6(ad+bc)-2310acx^2(ad+bc)+105b^2d^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^2 - 2310*a*c*(b*c + a*d)*x^2 + 385*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 + 330*b*d*(b*c + a*d)*x^6 + 105*b^2*d^2*x^8)
)/(1155*x^(5/2))
```

Maple [A] time = 0.01, size = 97, normalized size = 1.

$$-\frac{-210\,b^2d^2x^8-660\,x^6abd^2-660\,x^6b^2cd-770\,x^4a^2d^2-3080\,x^4abcd-770\,x^4b^2c^2+4620\,x^2a^2cd+4620\,ac^2bx^2+462\,a^2c}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*b^2*d^2*x^8-330*a*b*d^2*x^6-330*b^2*c*d*x^6-385*a^2
*d^2*x^4-1540*a*b*c*d*x^4-385*b^2*c^2*x^4+2310*a^2*c*d*x^2+2310*a
*b*c^2*x^2+231*a^2*c^2)/x^(5/2)
```

Maxima [A] time = 1.34768, size = 117, normalized size = 1.23

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}} + \frac{4}{7}\left(b^2cd + abd^2\right)x^{\frac{7}{2}} + \frac{2}{3}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{3}{2}} - \frac{2\left(a^2c^2 + 10\left(abc^2 + a^2cd\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*(b^2*c*d + a*b*d^2)*x^(7/2) + 2/3*(b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(3/2) - 2/5*(a^2*c^2 + 10*(a*b*c^2
 + a^2*c*d)*x^2)/x^(5/2)
```

Fricas [A] time = 0.222071, size = 117, normalized size = 1.23

$$\frac{2\left(105\,b^2d^2x^8 + 330\left(b^2cd + abd^2\right)x^6 + 385\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^4 - 231\,a^2c^2 - 2310\left(abc^2 + a^2cd\right)x^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^2*d^2*x^8 + 330*(b^2*c*d + a*b*d^2)*x^6 + 385*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^4 - 231*a^2*c^2 - 2310*(a*b*c^2 + a^
2*c*d)*x^2)/x^(5/2)
```

Sympy [A] time = 33.7085, size = 133, normalized size = 1.4

$$-\frac{2a^2c^2}{5x^{\frac{5}{2}}} - \frac{4a^2cd}{\sqrt{x}} + \frac{2a^2d^2x^{\frac{3}{2}}}{3} - \frac{4abc^2}{\sqrt{x}} + \frac{8abcdx^{\frac{3}{2}}}{3} + \frac{4abd^2x^{\frac{7}{2}}}{7} + \frac{2b^2c^2x^{\frac{3}{2}}}{3} + \frac{4b^2cdx^{\frac{7}{2}}}{7} + \frac{2b^2d^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(7/2),x)
```

```
[Out] -2*a**2*c**2/(5*x**(5/2)) - 4*a**2*c*d/sqrt(x) + 2*a**2*d**2*x**(
3/2)/3 - 4*a*b*c**2/sqrt(x) + 8*a*b*c*d*x**(3/2)/3 + 4*a*b*d**2*x
**(7/2)/7 + 2*b**2*c**2*x**(3/2)/3 + 4*b**2*c*d*x**(7/2)/7 + 2*b*
*2*d**2*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.249829, size = 130, normalized size = 1.37

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}} + \frac{4}{7}\,b^2cdx^{\frac{7}{2}} + \frac{4}{7}\,abd^2x^{\frac{7}{2}} + \frac{2}{3}\,b^2c^2x^{\frac{3}{2}} + \frac{8}{3}\,abcdx^{\frac{3}{2}} + \frac{2}{3}\,a^2d^2x^{\frac{3}{2}} - \frac{2\left(10\,abc^2x^2 + 10\,a^2cdx^2 + a^2c^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*b^2*c*d*x^(7/2) + 4/7*a*b*d^2*x^(7/2)
 + 2/3*b^2*c^2*x^(3/2) + 8/3*a*b*c*d*x^(3/2) + 2/3*a^2*d^2*x^(3/2
) - 2/5*(10*a*b*c^2*x^2 + 10*a^2*c*d*x^2 + a^2*c^2)/x^(5/2)
```

3.407 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (
2*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d
)*x^(25/2))/25 + (2*b^2*d^3*x^(29/2))/29
```

Rubi [A] time = 0.184369, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (
2*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d
)*x^(25/2))/25 + (2*b^2*d^3*x^(29/2))/29
```

Rubi in Sympy [A] time = 29.7483, size = 144, normalized size = 1.04

$$\frac{2a^2c^3x^{\frac{9}{2}}}{9}+\frac{2ac^2x^{\frac{13}{2}}(3ad+2bc)}{13}+\frac{2b^2d^3x^{\frac{29}{2}}}{29}+\frac{2bd^2x^{\frac{25}{2}}(2ad+3bc)}{25}+\frac{2cx^{\frac{17}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{17}+\frac{2dx^{\frac{21}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(9/2)/9 + 2*a*c**2*x**(13/2)*(3*a*d + 2*b*c)/13 +
2*b**2*d**3*x**(29/2)/29 + 2*b*d**2*x**(25/2)*(2*a*d + 3*b*c)/25
+ 2*c*x**(17/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/17 + 2*d*x*
*(21/2)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/21
```

Mathematica [A] time = 0.0689301, size = 139, normalized size = 1.

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (
2*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d
)*x^(25/2))/25 + (2*b^2*d^3*x^(29/2))/29
```

Maple [A] time = 0.012, size = 138, normalized size = 1.

$$\frac{696150\,b^2d^3x^{10} + 1615068\,x^8abd^3 + 2422602\,x^8b^2cd^2 + 961350\,x^6a^2d^3 + 5768100\,x^6abcd^2 + 2884050\,x^6b^2c^2d + 3562650\,x^4 \cdots}{10094175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/10094175*x^(9/2)*(348075*b^2*d^3*x^10+807534*a*b*d^3*x^8+121130
1*b^2*c*d^2*x^8+480675*a^2*d^3*x^6+2884050*a*b*c*d^2*x^6+1442025*
b^2*c^2*d*x^6+1781325*a^2*c*d^2*x^4+3562650*a*b*c^2*d*x^4+593775*
b^2*c^3*x^4+2329425*a^2*c^2*d*x^2+1552950*a*b*c^3*x^2+1121575*a^2
*c^3)
```

Maxima [A] time = 1.32698, size = 171, normalized size = 1.23

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}} + \frac{2}{25}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{25}{2}} + \frac{2}{21}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{21}{2}} + \frac{2}{9}\,a^2c^3x^{\frac{9}{2}} + \frac{2}{17}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{17}{2}} + \frac{2}{13}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 2/25*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(25/2) +
 2/21*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(21/2) + 2/9*a^2*c^
3*x^(9/2) + 2/17*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(17/2) +
 2/13*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(13/2)
```

Fricas [A] time = 0.219886, size = 178, normalized size = 1.28

$$\frac{2}{10094175}\left(348075\,b^2d^3x^{14} + 403767\left(3\,b^2cd^2 + 2\,abd^3\right)x^{12} + 480675\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{10} + 1121575\,a^2c^3x^4 + 5937 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/10094175*(348075*b^2*d^3*x^14 + 403767*(3*b^2*c*d^2 + 2*a*b*d^3
)*x^12 + 480675*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1121
575*a^2*c^3*x^4 + 593775*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^
8 + 776475*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.236603, size = 182, normalized size = 1.31

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}} + \frac{6}{25}\,b^2cd^2x^{\frac{25}{2}} + \frac{4}{25}\,abd^3x^{\frac{25}{2}} + \frac{2}{7}\,b^2c^2dx^{\frac{21}{2}} + \frac{4}{7}\,abcd^2x^{\frac{21}{2}} + \frac{2}{21}\,a^2d^3x^{\frac{21}{2}} + \frac{2}{17}\,b^2c^3x^{\frac{17}{2}} + \frac{12}{17}\,abc^2dx^{\frac{17}{2}} + \frac{6}{17}\,a^2cd^2x^{\frac{17}{2}} + \frac{4}{13}\,abc^3x^{\frac{13}{2}} + \frac{6}{13}\,a^2c^2dx^{\frac{13}{2}} + \frac{2}{9}\,a^2c^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 6/25*b^2*c*d^2*x^(25/2) + 4/25*a*b*d^3*x^
(25/2) + 2/7*b^2*c^2*d*x^(21/2) + 4/7*a*b*c*d^2*x^(21/2) + 2/21*a
^2*d^3*x^(21/2) + 2/17*b^2*c^3*x^(17/2) + 12/17*a*b*c^2*d*x^(17/2
) + 6/17*a^2*c*d^2*x^(17/2) + 4/13*a*b*c^3*x^(13/2) + 6/13*a^2*c^
2*d*x^(13/2) + 2/9*a^2*c^3*x^(9/2)
```

3.408 $\int x^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (
2*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d
)*x^(23/2))/23 + (2*b^2*d^3*x^(27/2))/27
```

Rubi [A] time = 0.177869, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (
2*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d
)*x^(23/2))/23 + (2*b^2*d^3*x^(27/2))/27
```

Rubi in Sympy [A] time = 29.4869, size = 144, normalized size = 1.04

$$\frac{2a^2c^3x^{\frac{7}{2}}}{7}+\frac{2ac^2x^{\frac{11}{2}}(3ad+2bc)}{11}+\frac{2b^2d^3x^{\frac{27}{2}}}{27}+\frac{2bd^2x^{\frac{23}{2}}(2ad+3bc)}{23}+\frac{2cx^{\frac{15}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{15}+\frac{2dx^{\frac{19}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(7/2)/7 + 2*a*c**2*x**(11/2)*(3*a*d + 2*b*c)/11 +
2*b**2*d**3*x**(27/2)/27 + 2*b*d**2*x**(23/2)*(2*a*d + 3*b*c)/23
+ 2*c*x**(15/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/15 + 2*d*x*
*(19/2)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/19
```

Mathematica [A] time = 0.064363, size = 139, normalized size = 1.

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (
2*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d
)*x^(23/2))/23 + (2*b^2*d^3*x^(27/2))/27
```

Maple [A] time = 0.01, size = 138, normalized size = 1.

$$\frac{336490\, b^2 d^3 x^{10} + 790020\, x^8 abd^3 + 1185030\, x^8 b^2 cd^2 + 478170\, x^6 a^2 d^3 + 2869020\, x^6 abcd^2 + 1434510\, x^6 b^2 c^2 d + 1817046\, x^4 a^2}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/4542615*x^(7/2)*(168245*b^2*d^3*x^10+395010*a*b*d^3*x^8+592515*
b^2*c*d^2*x^8+239085*a^2*d^3*x^6+1434510*a*b*c*d^2*x^6+717255*b^2
*c^2*d*x^6+908523*a^2*c*d^2*x^4+1817046*a*b*c^2*d*x^4+302841*b^2*
c^3*x^4+1238895*a^2*c^2*d*x^2+825930*a*b*c^3*x^2+648945*a^2*c^3)
```

Maxima [A] time = 1.36623, size = 171, normalized size = 1.23

$$\frac{2}{27}\, b^2 d^3 x^{\frac{27}{2}} + \frac{2}{23}\left(3\, b^2 cd^2 + 2\, abd^3\right) x^{\frac{23}{2}} + \frac{2}{19}\left(3\, b^2 c^2 d + 6\, abcd^2 + a^2 d^3\right) x^{\frac{19}{2}} + \frac{2}{7}\, a^2 c^3 x^{\frac{7}{2}} + \frac{2}{15}\left(b^2 c^3 + 6\, abc^2 d + 3\, a^2 cd^2\right) x^{\frac{15}{2}} + \frac{2}{11}\left(2\, abc^3 + 3\, a^2 c^2 d\right) x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 2/23*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(23/2) +
 2/19*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(19/2) + 2/7*a^2*c^
3*x^(7/2) + 2/15*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(15/2) +
 2/11*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(11/2)
```

Fricas [A] time = 0.218149, size = 178, normalized size = 1.28

$$\frac{2}{4542615}\left(168245\, b^2 d^3 x^{13} + 197505\left(3\, b^2 cd^2 + 2\, abd^3\right) x^{11} + 239085\left(3\, b^2 c^2 d + 6\, abcd^2 + a^2 d^3\right) x^9 + 648945\, a^2 c^3 x^3 + 302841\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*b^2*d^3*x^13 + 197505*(3*b^2*c*d^2 + 2*a*b*d^3)
*x^11 + 239085*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 648945
*a^2*c^3*x^3 + 302841*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 +
 412965*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5)*sqrt(x)
```

Sympy [A] time = 125.663, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{7}{2}}}{7} + \frac{6a^2c^2dx^{\frac{11}{2}}}{11} + \frac{2a^2cd^2x^{\frac{15}{2}}}{5} + \frac{2a^2d^3x^{\frac{19}{2}}}{19} + \frac{4abc^3x^{\frac{11}{2}}}{11} + \frac{4abc^2dx^{\frac{15}{2}}}{5} + \frac{12abcd^2x^{\frac{19}{2}}}{19} + \frac{4abd^3x^{\frac{23}{2}}}{23} + \frac{2b^2c^3x^{\frac{15}{2}}}{15} + \frac{6b^2c^2dx^{\frac{19}{2}}}{19} + \frac{6b^2cd^2x^{\frac{23}{2}}}{23} + \frac{2b^2d^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(7/2)/7 + 6*a**2*c**2*d*x**(11/2)/11 + 2*a**2*c*d*
*2*x**(15/2)/5 + 2*a**2*d**3*x**(19/2)/19 + 4*a*b*c**3*x**(11/2)/
11 + 4*a*b*c**2*d*x**(15/2)/5 + 12*a*b*c*d**2*x**(19/2)/19 + 4*a*
b*d**3*x**(23/2)/23 + 2*b**2*c**3*x**(15/2)/15 + 6*b**2*c**2*d*x*
*(19/2)/19 + 6*b**2*c*d**2*x**(23/2)/23 + 2*b**2*d**3*x**(27/2)/2
7
```

GIAC/XCAS [A] time = 0.229959, size = 182, normalized size = 1.31

$$\frac{2}{27}\,b^2d^3x^{\frac{27}{2}}+\frac{6}{23}\,b^2cd^2x^{\frac{23}{2}}+\frac{4}{23}\,abd^3x^{\frac{23}{2}}+\frac{6}{19}\,b^2c^2dx^{\frac{19}{2}}+\frac{12}{19}\,abcd^2x^{\frac{19}{2}}+\frac{2}{19}\,a^2d^3x^{\frac{19}{2}}+\frac{2}{15}\,b^2c^3x^{\frac{15}{2}}+\frac{4}{5}\,abc^2dx^{\frac{15}{2}}+\frac{2}{5}\,a^2cd^2x^{\frac{15}{2}}+\frac{4}{11}\,abc^3x^{\frac{11}{2}}+\frac{6}{11}\,a^2c^2dx^{\frac{11}{2}}+\frac{2}{7}\,a^2c^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 6/23*b^2*c*d^2*x^(23/2) + 4/23*a*b*d^3*x^
(23/2) + 6/19*b^2*c^2*d*x^(19/2) + 12/19*a*b*c*d^2*x^(19/2) + 2/1
9*a^2*d^3*x^(19/2) + 2/15*b^2*c^3*x^(15/2) + 4/5*a*b*c^2*d*x^(15/
2) + 2/5*a^2*c*d^2*x^(15/2) + 4/11*a*b*c^3*x^(11/2) + 6/11*a^2*c^
2*d*x^(11/2) + 2/7*a^2*c^3*x^(7/2)
```

3.409 $\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*
x^(21/2))/21 + (2*b^2*d^3*x^(25/2))/25
```

Rubi [A] time = 0.175668, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*
x^(21/2))/21 + (2*b^2*d^3*x^(25/2))/25
```

Rubi in Sympy [A] time = 29.827, size = 144, normalized size = 1.04

$$\frac{2a^2c^3x^{\frac{5}{2}}}{5}+\frac{2ac^2x^{\frac{9}{2}}\left(3ad+2bc\right)}{9}+\frac{2b^2d^3x^{\frac{25}{2}}}{25}+\frac{2bd^2x^{\frac{21}{2}}\left(2ad+3bc\right)}{21}+\frac{2cx^{\frac{13}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{13}+\frac{2dx^{\frac{17}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(5/2)/5 + 2*a*c**2*x**(9/2)*(3*a*d + 2*b*c)/9 + 2*
b**2*d**3*x**(25/2)/25 + 2*b*d**2*x**(21/2)*(2*a*d + 3*b*c)/21 +
2*c*x**(13/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/13 + 2*d*x**(
17/2)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/17
```

Mathematica [A] time = 0.0609849, size = 139, normalized size = 1.

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*
x^(21/2))/21 + (2*b^2*d^3*x^(25/2))/25
```

Maple [A] time = 0.011, size = 138, normalized size = 1.

$$\frac{27846\,b^2d^3x^{10} + 66300\,x^8abd^3 + 99450\,x^8b^2cd^2 + 40950\,x^6a^2d^3 + 245700\,x^6abcd^2 + 122850\,x^6b^2c^2d + 160650\,x^4a^2cd^2 + 321}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/348075*x^(5/2)*(13923*b^2*d^3*x^10+33150*a*b*d^3*x^8+49725*b^2*
c*d^2*x^8+20475*a^2*d^3*x^6+122850*a*b*c*d^2*x^6+61425*b^2*c^2*d*
x^6+80325*a^2*c*d^2*x^4+160650*a*b*c^2*d*x^4+26775*b^2*c^3*x^4+11
6025*a^2*c^2*d*x^2+77350*a*b*c^3*x^2+69615*a^2*c^3)
```

Maxima [A] time = 1.3291, size = 171, normalized size = 1.23

$$\frac{2}{25}\,b^2d^3x^{\frac{25}{2}} + \frac{2}{21}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{21}{2}} + \frac{2}{17}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{17}{2}} + \frac{2}{5}\,a^2c^3x^{\frac{5}{2}} + \frac{2}{13}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{13}{2}} + \frac{2}{9}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(21/2) +
 2/17*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(17/2) + 2/5*a^2*c^
3*x^(5/2) + 2/13*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(13/2) +
 2/9*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(9/2)
```

Fricas [A] time = 0.221278, size = 178, normalized size = 1.28

$$\frac{2}{348075}\left(13923\,b^2d^3x^{12} + 16575\left(3\,b^2cd^2 + 2\,abd^3\right)x^{10} + 20475\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^8 + 69615\,a^2c^3x^2 + 26775\left(b^2c^3 +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^3*x^12 + 16575*(3*b^2*c*d^2 + 2*a*b*d^3)*x^
10 + 20475*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 69615*a^2*
c^3*x^2 + 26775*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^6 + 38675
*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4)*sqrt(x)
```

Sympy [A] time = 73.2787, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{5}{2}}}{5} + \frac{2a^2c^2dx^{\frac{9}{2}}}{3} + \frac{6a^2cd^2x^{\frac{13}{2}}}{13} + \frac{2a^2d^3x^{\frac{17}{2}}}{17} + \frac{4abc^3x^{\frac{9}{2}}}{9} + \frac{12abc^2dx^{\frac{13}{2}}}{13} + \frac{12abcd^2x^{\frac{17}{2}}}{17} + \frac{4abd^3x^{\frac{21}{2}}}{21} + \frac{2b^2c^3x^{\frac{13}{2}}}{13} + \frac{6b^2c^2dx^{\frac{17}{2}}}{17} + \frac{2b^2cd^2x^{\frac{21}{2}}}{7} + \frac{2b^2d^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(5/2)/5 + 2*a**2*c**2*d*x**(9/2)/3 + 6*a**2*c*d**2
*x**(13/2)/13 + 2*a**2*d**3*x**(17/2)/17 + 4*a*b*c**3*x**(9/2)/9
+ 12*a*b*c**2*d*x**(13/2)/13 + 12*a*b*c*d**2*x**(17/2)/17 + 4*a*b
*d**3*x**(21/2)/21 + 2*b**2*c**3*x**(13/2)/13 + 6*b**2*c**2*d*x**
(17/2)/17 + 2*b**2*c*d**2*x**(21/2)/7 + 2*b**2*d**3*x**(25/2)/25
```

GIAC/XCAS [A] time = 0.229972, size = 182, normalized size = 1.31

$$\frac{2}{25}\,b^2d^3x^{\frac{25}{2}}+\frac{2}{7}\,b^2cd^2x^{\frac{21}{2}}+\frac{4}{21}\,abd^3x^{\frac{21}{2}}+\frac{6}{17}\,b^2c^2dx^{\frac{17}{2}}+\frac{12}{17}\,abcd^2x^{\frac{17}{2}}+\frac{2}{17}\,a^2d^3x^{\frac{17}{2}}+\frac{2}{13}\,b^2c^3x^{\frac{13}{2}}+\frac{12}{13}\,abc^2dx^{\frac{13}{2}}+\frac{6}{13}\,a^2cd^2x^{\frac{13}{2}}+\frac{4}{9}\,abc^3x^{\frac{9}{2}}+\frac{2}{3}\,a^2c^2dx^{\frac{9}{2}}+\frac{2}{5}\,a^2c^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/7*b^2*c*d^2*x^(21/2) + 4/21*a*b*d^3*x^(
21/2) + 6/17*b^2*c^2*d*x^(17/2) + 12/17*a*b*c*d^2*x^(17/2) + 2/17
*a^2*d^3*x^(17/2) + 2/13*b^2*c^3*x^(13/2) + 12/13*a*b*c^2*d*x^(13
/2) + 6/13*a^2*c*d^2*x^(13/2) + 4/9*a*b*c^3*x^(9/2) + 2/3*a^2*c^2
*d*x^(9/2) + 2/5*a^2*c^3*x^(5/2)
```

3.410 $\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*
x^(19/2))/19 + (2*b^2*d^3*x^(23/2))/23
```

Rubi [A] time = 0.171956, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*
x^(19/2))/19 + (2*b^2*d^3*x^(23/2))/23
```

Rubi in Sympy [A] time = 30.5208, size = 144, normalized size = 1.04

$$\frac{2a^2c^3x^{\frac{3}{2}}}{3}+\frac{2ac^2x^{\frac{7}{2}}\left(3ad+2bc\right)}{7}+\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2bd^2x^{\frac{19}{2}}\left(2ad+3bc\right)}{19}+\frac{2cx^{\frac{11}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{11}+\frac{2dx^{\frac{15}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3*x**(1/2),x)
```

```
[Out] 2*a**2*c**3*x**(3/2)/3 + 2*a*c**2*x**(7/2)*(3*a*d + 2*b*c)/7 + 2*
b**2*d**3*x**(23/2)/23 + 2*b*d**2*x**(19/2)*(2*a*d + 3*b*c)/19 +
2*c*x**(11/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/11 + 2*d*x**(
15/2)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/15
```

Mathematica [A] time = 0.0628594, size = 139, normalized size = 1.

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*
x^(19/2))/19 + (2*b^2*d^3*x^(23/2))/23
```

Maple [A] time = 0.01, size = 138, normalized size = 1.

$$\frac{43890\,b^2d^3x^{10} + 106260\,x^8abd^3 + 159390\,x^8b^2cd^2 + 67298\,x^6a^2d^3 + 403788\,x^6abcd^2 + 201894\,x^6b^2c^2d + 275310\,x^4a^2cd^2 + 5}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x)
```

```
[Out] 2/504735*x^(3/2)*(21945*b^2*d^3*x^10+53130*a*b*d^3*x^8+79695*b^2*
c*d^2*x^8+33649*a^2*d^3*x^6+201894*a*b*c*d^2*x^6+100947*b^2*c^2*d
*x^6+137655*a^2*c*d^2*x^4+275310*a*b*c^2*d*x^4+45885*b^2*c^3*x^4+
216315*a^2*c^2*d*x^2+144210*a*b*c^3*x^2+168245*a^2*c^3)
```

Maxima [A] time = 1.34414, size = 171, normalized size = 1.23

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}} + \frac{2}{19}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{19}{2}} + \frac{2}{15}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{15}{2}} + \frac{2}{3}\,a^2c^3x^{\frac{3}{2}} + \frac{2}{11}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{11}{2}} + \frac{2}{7}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 2/19*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(19/2) +
 2/15*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(15/2) + 2/3*a^2*c^
3*x^(3/2) + 2/11*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(11/2) +
 2/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(7/2)
```

Fricas [A] time = 0.214311, size = 176, normalized size = 1.27

$$\frac{2}{504735}\left(21945\,b^2d^3x^{11} + 26565\left(3\,b^2cd^2 + 2\,abd^3\right)x^9 + 33649\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^7 + 168245\,a^2c^3x + 45885\left(b^2c^3 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^3*x^11 + 26565*(3*b^2*c*d^2 + 2*a*b*d^3)*x^
9 + 33649*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + 168245*a^2*
c^3*x + 45885*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 72105*(
2*a*b*c^3 + 3*a^2*c^2*d)*x^3)*sqrt(x)
```

Sympy [A] time = 16.2215, size = 155, normalized size = 1.12

$$\frac{2a^2c^3x^{\frac{3}{2}}}{3} + \frac{2b^2d^3x^{\frac{23}{2}}}{23} + \frac{2x^{\frac{19}{2}}\left(2abd^3 + 3b^2cd^2\right)}{19} + \frac{2x^{\frac{15}{2}}\left(a^2d^3 + 6abcd^2 + 3b^2c^2d\right)}{15} + \frac{2x^{\frac{11}{2}}\left(3a^2cd^2 + 6abc^2d + b^2c^3\right)}{11} + \frac{2x^{\frac{7}{2}}\left(3a^2c^2d + 2abc^3\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3*x**(1/2),x)
```

```
[Out] 2*a**2*c**3*x**(3/2)/3 + 2*b**2*d**3*x**(23/2)/23 + 2*x**(19/2)*(
2*a*b*d**3 + 3*b**2*c*d**2)/19 + 2*x**(15/2)*(a**2*d**3 + 6*a*b*c
*d**2 + 3*b**2*c**2*d)/15 + 2*x**(11/2)*(3*a**2*c*d**2 + 6*a*b*c*
*2*d + b**2*c**3)/11 + 2*x**(7/2)*(3*a**2*c**2*d + 2*a*b*c**3)/7
```

GIAC/XCAS [A] time = 0.233863, size = 182, normalized size = 1.31

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}} + \frac{6}{19}\,b^2cd^2x^{\frac{19}{2}} + \frac{4}{19}\,abd^3x^{\frac{19}{2}} + \frac{2}{5}\,b^2c^2dx^{\frac{15}{2}} + \frac{4}{5}\,abcd^2x^{\frac{15}{2}} + \frac{2}{15}\,a^2d^3x^{\frac{15}{2}} + \frac{2}{11}\,b^2c^3x^{\frac{11}{2}} + \frac{12}{11}\,abc^2dx^{\frac{11}{2}} + \frac{6}{11}\,a^2cd^2x^{\frac{11}{2}} + \frac{4}{7}\,abc^3x^{\frac{7}{2}} + \frac{6}{7}\,a^2c^2dx^{\frac{7}{2}} + \frac{2}{3}\,a^2c^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 6/19*b^2*c*d^2*x^(19/2) + 4/19*a*b*d^3*x^
(19/2) + 2/5*b^2*c^2*d*x^(15/2) + 4/5*a*b*c*d^2*x^(15/2) + 2/15*a
^2*d^3*x^(15/2) + 2/11*b^2*c^3*x^(11/2) + 12/11*a*b*c^2*d*x^(11/2
) + 6/11*a^2*c*d^2*x^(11/2) + 4/7*a*b*c^3*x^(7/2) + 6/7*a^2*c^2*d
*x^(7/2) + 2/3*a^2*c^3*x^(3/2)
```

3.411 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right) + \frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b
^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/
2))/17 + (2*b^2*d^3*x^(21/2))/21
```

Rubi [A] time = 0.170839, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right) + \frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b
^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/
2))/17 + (2*b^2*d^3*x^(21/2))/21
```

Rubi in Sympy [A] time = 29.9065, size = 143, normalized size = 1.04

$$2a^2c^3\sqrt{x} + \frac{2ac^2x^{\frac{5}{2}}(3ad+2bc)}{5} + \frac{2b^2d^3x^{\frac{21}{2}}}{21} + \frac{2bd^2x^{\frac{17}{2}}(2ad+3bc)}{17} + \frac{2cx^{\frac{9}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{9} + \frac{2dx^{\frac{13}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] 2*a**2*c**3*sqrt(x) + 2*a*c**2*x**(5/2)*(3*a*d + 2*b*c)/5 + 2*b**
2*d**3*x**(21/2)/21 + 2*b*d**2*x**(17/2)*(2*a*d + 3*b*c)/17 + 2*c
*x**(9/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/9 + 2*d*x**(13/2)
*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/13
```

Mathematica [A] time = 0.0642897, size = 137, normalized size = 1.

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right) + \frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b
^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/
2))/17 + (2*b^2*d^3*x^(21/2))/21
```

Maple [A] time = 0.01, size = 138, normalized size = 1.

$$\frac{6630\,b^2d^3x^{10} + 16380\,x^8abd^3 + 24570\,x^8b^2cd^2 + 10710\,x^6a^2d^3 + 64260\,x^6abcd^2 + 32130\,x^6b^2c^2d + 46410\,x^4a^2cd^2 + 92820\,x}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 2/69615*x^(1/2)*(3315*b^2*d^3*x^10+8190*a*b*d^3*x^8+12285*b^2*c*d
^2*x^8+5355*a^2*d^3*x^6+32130*a*b*c*d^2*x^6+16065*b^2*c^2*d*x^6+2
3205*a^2*c*d^2*x^4+46410*a*b*c^2*d*x^4+7735*b^2*c^3*x^4+41769*a^2
*c^2*d*x^2+27846*a*b*c^3*x^2+69615*a^2*c^3)
```

Maxima [A] time = 1.34923, size = 171, normalized size = 1.25

$$\frac{2}{21}\,b^2d^3x^{\frac{21}{2}} + \frac{2}{17}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{17}{2}} + \frac{2}{13}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{13}{2}} + 2\,a^2c^3\sqrt{x} + \frac{2}{9}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{9}{2}} + \frac{2}{5}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 2/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(17/2) +
 2/13*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(13/2) + 2*a^2*c^3*
sqrt(x) + 2/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(9/2) + 2/5
*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(5/2)
```

Fricas [A] time = 0.219056, size = 174, normalized size = 1.27

$$\frac{2}{69615}\left(3315\,b^2d^3x^{10} + 4095\left(3\,b^2cd^2 + 2\,abd^3\right)x^8 + 5355\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^6 + 69615\,a^2c^3 + 7735\left(b^2c^3 + 6\,abc^2d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^3*x^10 + 4095*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 +
 5355*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + 69615*a^2*c^3 +
 7735*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 13923*(2*a*b*c^
3 + 3*a^2*c^2*d)*x^2)*sqrt(x)
```

Sympy [A] time = 36.2062, size = 190, normalized size = 1.39

$$2a^2c^3\sqrt{x} + \frac{6a^2c^2dx^{\frac{5}{2}}}{5} + \frac{2a^2cd^2x^{\frac{9}{2}}}{3} + \frac{2a^2d^3x^{\frac{13}{2}}}{13} + \frac{4abc^3x^{\frac{5}{2}}}{5} + \frac{4abc^2dx^{\frac{9}{2}}}{3} + \frac{12abcd^2x^{\frac{13}{2}}}{13} + \frac{4abd^3x^{\frac{17}{2}}}{17} + \frac{2b^2c^3x^{\frac{9}{2}}}{9} + \frac{6b^2c^2dx^{\frac{13}{2}}}{13} + \frac{6b^2cd^2x^{\frac{17}{2}}}{17} + \frac{2b^2d^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] 2*a**2*c**3*sqrt(x) + 6*a**2*c**2*d*x**(5/2)/5 + 2*a**2*c*d**2*x*
*(9/2)/3 + 2*a**2*d**3*x**(13/2)/13 + 4*a*b*c**3*x**(5/2)/5 + 4*a
*b*c**2*d*x**(9/2)/3 + 12*a*b*c*d**2*x**(13/2)/13 + 4*a*b*d**3*x*
*(17/2)/17 + 2*b**2*c**3*x**(9/2)/9 + 6*b**2*c**2*d*x**(13/2)/13
+ 6*b**2*c*d**2*x**(17/2)/17 + 2*b**2*d**3*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.250612, size = 182, normalized size = 1.33

$$\frac{2}{21}\, b^2 d^3 x^{\frac{21}{2}} + \frac{6}{17}\, b^2 c d^2 x^{\frac{17}{2}} + \frac{4}{17}\, a b d^3 x^{\frac{17}{2}} + \frac{6}{13}\, b^2 c^2 d x^{\frac{13}{2}} + \frac{12}{13}\, a b c d^2 x^{\frac{13}{2}} + \frac{2}{13}\, a^2 d^3 x^{\frac{13}{2}} + \frac{2}{9}\, b^2 c^3 x^{\frac{9}{2}} + \frac{4}{3}\, a b c^2 d x^{\frac{9}{2}} + \frac{2}{3}\, a^2 c d^2 x^{\frac{9}{2}} + \frac{4}{5}\, a b c^3 x^{\frac{5}{2}} + \frac{6}{5}\, a^2 c^2 d x^{\frac{5}{2}} + 2\, a^2 c^3 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 6/17*b^2*c*d^2*x^(17/2) + 4/17*a*b*d^3*x^
(17/2) + 6/13*b^2*c^2*d*x^(13/2) + 12/13*a*b*c*d^2*x^(13/2) + 2/1
3*a^2*d^3*x^(13/2) + 2/9*b^2*c^3*x^(9/2) + 4/3*a*b*c^2*d*x^(9/2)
+ 2/3*a^2*c*d^2*x^(9/2) + 4/5*a*b*c^3*x^(5/2) + 6/5*a^2*c^2*d*x^(
5/2) + 2*a^2*c^3*sqrt(x)
```

3.412 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(
15/2))/15 + (2*b^2*d^3*x^(19/2))/19
```

Rubi [A] time = 0.173131, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(
15/2))/15 + (2*b^2*d^3*x^(19/2))/19
```

Rubi in Sympy [A] time = 29.9344, size = 143, normalized size = 1.04

$$-\frac{2a^2c^3}{\sqrt{x}}+\frac{2ac^2x^{\frac{3}{2}}(3ad+2bc)}{3}+\frac{2b^2d^3x^{\frac{19}{2}}}{19}+\frac{2bd^2x^{\frac{15}{2}}(2ad+3bc)}{15}+\frac{2cx^{\frac{7}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{7}+\frac{2dx^{\frac{11}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(3/2),x)
```

```
[Out] -2*a**2*c**3/sqrt(x) + 2*a*c**2*x**(3/2)*(3*a*d + 2*b*c)/3 + 2*b*
*2*d**3*x**(19/2)/19 + 2*b*d**2*x**(15/2)*(2*a*d + 3*b*c)/15 + 2*
c*x**(7/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/7 + 2*d*x**(11/2
)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/11
```

Mathematica [A] time = 0.0882251, size = 137, normalized size = 1.

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(
15/2))/15 + (2*b^2*d^3*x^(19/2))/19
```

Maple [A] time = 0.01, size = 138, normalized size = 1.

$$-\frac{-2310\,b^2d^3x^{10}-5852\,x^8abd^3-8778\,x^8b^2cd^2-3990\,x^6a^2d^3-23940\,x^6abcd^2-11970\,x^6b^2c^2d-18810\,x^4a^2cd^2-37620}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x)
```

```
[Out] -2/21945*(-1155*b^2*d^3*x^10-2926*a*b*d^3*x^8-4389*b^2*c*d^2*x^8-
1995*a^2*d^3*x^6-11970*a*b*c*d^2*x^6-5985*b^2*c^2*d*x^6-9405*a^2*
c*d^2*x^4-18810*a*b*c^2*d*x^4-3135*b^2*c^3*x^4-21945*a^2*c^2*d*x^
2-14630*a*b*c^3*x^2+21945*a^2*c^3)/x^(1/2)
```

Maxima [A] time = 1.36984, size = 171, normalized size = 1.25

$$\frac{2}{19}\,b^2d^3x^{\frac{19}{2}}+\frac{2}{15}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{15}{2}}+\frac{2}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{11}{2}}-\frac{2\,a^2c^3}{\sqrt{x}}+\frac{2}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{7}{2}}+\frac{2}{3}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/15*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(15/2) +
 2/11*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(11/2) - 2*a^2*c^3/
sqrt(x) + 2/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(7/2) + 2/3
*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(3/2)
```

Fricas [A] time = 0.218475, size = 174, normalized size = 1.27

$$\frac{2\left(1155\,b^2d^3x^{10}+1463\left(3\,b^2cd^2+2\,abd^3\right)x^8+1995\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-21945\,a^2c^3+3135\left(b^2c^3+6\,abc^2d+3\,a\right.\right.}{21945\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^3*x^10 + 1463*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 +
 1995*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 21945*a^2*c^3 +
 3135*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 7315*(2*a*b*c^3
 + 3*a^2*c^2*d)*x^2)/sqrt(x)
```

Sympy [A] time = 41.9939, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{\sqrt{x}}+2a^2c^2dx^{\frac{3}{2}}+\frac{6a^2cd^2x^{\frac{7}{2}}}{7}+\frac{2a^2d^3x^{\frac{11}{2}}}{11}+\frac{4abc^3x^{\frac{3}{2}}}{3}+\frac{12abc^2dx^{\frac{7}{2}}}{7}+\frac{12abcd^2x^{\frac{11}{2}}}{11}+\frac{4abd^3x^{\frac{15}{2}}}{15}+\frac{2b^2c^3x^{\frac{7}{2}}}{7}+\frac{6b^2c^2dx^{\frac{11}{2}}}{11}+\frac{2b^2cd^2x^{\frac{15}{2}}}{5}+\frac{2b^2d^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(3/2),x)
```

```
[Out] -2*a**2*c**3/sqrt(x) + 2*a**2*c**2*d*x**(3/2) + 6*a**2*c*d**2*x**
(7/2)/7 + 2*a**2*d**3*x**(11/2)/11 + 4*a*b*c**3*x**(3/2)/3 + 12*a
*b*c**2*d*x**(7/2)/7 + 12*a*b*c*d**2*x**(11/2)/11 + 4*a*b*d**3*x*
*(15/2)/15 + 2*b**2*c**3*x**(7/2)/7 + 6*b**2*c**2*d*x**(11/2)/11
+ 2*b**2*c*d**2*x**(15/2)/5 + 2*b**2*d**3*x**(19/2)/19
```

GIAC/XCAS [A] time = 0.238057, size = 182, normalized size = 1.33

$$\frac{2}{19}\,b^2d^3x^{\frac{19}{2}}+\frac{2}{5}\,b^2cd^2x^{\frac{15}{2}}+\frac{4}{15}\,abd^3x^{\frac{15}{2}}+\frac{6}{11}\,b^2c^2dx^{\frac{11}{2}}+\frac{12}{11}\,abcd^2x^{\frac{11}{2}}+\frac{2}{11}\,a^2d^3x^{\frac{11}{2}}+\frac{2}{7}\,b^2c^3x^{\frac{7}{2}}+\frac{12}{7}\,abc^2dx^{\frac{7}{2}}+\frac{6}{7}\,a^2cd^2x^{\frac{7}{2}}+\frac{4}{3}\,abc^3x^{\frac{3}{2}}+2\,a^2c^2dx^{\frac{3}{2}}-\frac{2\,a^2c^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/5*b^2*c*d^2*x^(15/2) + 4/15*a*b*d^3*x^(
15/2) + 6/11*b^2*c^2*d*x^(11/2) + 12/11*a*b*c*d^2*x^(11/2) + 2/11
*a^2*d^3*x^(11/2) + 2/7*b^2*c^3*x^(7/2) + 12/7*a*b*c^2*d*x^(7/2)
+ 6/7*a^2*c*d^2*x^(7/2) + 4/3*a*b*c^3*x^(3/2) + 2*a^2*c^2*d*x^(3/
2) - 2*a^2*c^3/sqrt(x)
```

3.413 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13
/2))/13 + (2*b^2*d^3*x^(17/2))/17
```

Rubi [A] time = 0.170047, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13
/2))/13 + (2*b^2*d^3*x^(17/2))/17
```

Rubi in Sympy [A] time = 30.2083, size = 143, normalized size = 1.04

$$-\frac{2a^2c^3}{3x^{\frac{3}{2}}}+2ac^2\sqrt{x}\,(3ad+2bc)+\frac{2b^2d^3x^{\frac{17}{2}}}{17}+\frac{2bd^2x^{\frac{13}{2}}\,(2ad+3bc)}{13}+\frac{2cx^{\frac{5}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{5}+\frac{2dx^{\frac{9}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(5/2),x)
```

```
[Out] -2*a**2*c**3/(3*x**(3/2)) + 2*a*c**2*sqrt(x)*(3*a*d + 2*b*c) + 2*
b**2*d**3*x**(17/2)/17 + 2*b*d**2*x**(13/2)*(2*a*d + 3*b*c)/13 +
2*c*x**(5/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/5 + 2*d*x**(9/
2)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/9
```

Mathematica [A] time = 0.094178, size = 137, normalized size = 1.

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13
/2))/13 + (2*b^2*d^3*x^(17/2))/17
```

Maple [A] time = 0.01, size = 138, normalized size = 1.

$$-\frac{-1170\,b^2d^3x^{10}-3060\,x^8abd^3-4590\,x^8b^2cd^2-2210\,x^6a^2d^3-13260\,x^6abcd^2-6630\,x^6b^2c^2d-11934\,x^4a^2cd^2-23868\,x^{}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x)
```

```
[Out] -2/9945*(-585*b^2*d^3*x^10-1530*a*b*d^3*x^8-2295*b^2*c*d^2*x^8-11
05*a^2*d^3*x^6-6630*a*b*c*d^2*x^6-3315*b^2*c^2*d*x^6-5967*a^2*c*d
^2*x^4-11934*a*b*c^2*d*x^4-1989*b^2*c^3*x^4-29835*a^2*c^2*d*x^2-1
9890*a*b*c^3*x^2+3315*a^2*c^3)/x^(3/2)
```

Maxima [A] time = 1.32868, size = 171, normalized size = 1.25

$$\frac{2}{17}\,b^2d^3x^{\frac{17}{2}}+\frac{2}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{13}{2}}+\frac{2}{9}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{9}{2}}-\frac{2\,a^2c^3}{3\,x^{\frac{3}{2}}}+\frac{2}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{5}{2}}+2\left(2\,abc^3+3\,a^2c^2d\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 2/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(13/2) +
 2/9*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(9/2) - 2/3*a^2*c^3/
x^(3/2) + 2/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(5/2) + 2*(
2*a*b*c^3 + 3*a^2*c^2*d)*sqrt(x)
```

Fricas [A] time = 0.217981, size = 174, normalized size = 1.27

$$\frac{2\left(585\,b^2d^3x^{10}+765\left(3\,b^2cd^2+2\,abd^3\right)x^8+1105\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-3315\,a^2c^3+1989\left(b^2c^3+6\,abc^2d+3\,a^2cd^{}\right.\right.}{9945\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^3*x^10 + 765*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 11
05*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 3315*a^2*c^3 + 198
9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 9945*(2*a*b*c^3 + 3
*a^2*c^2*d)*x^2)/x^(3/2)
```

Sympy [A] time = 44.2084, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{3x^{\frac{3}{2}}}+6a^2c^2d\sqrt{x}+\frac{6a^2cd^2x^{\frac{5}{2}}}{5}+\frac{2a^2d^3x^{\frac{9}{2}}}{9}+4abc^3\sqrt{x}+\frac{12abc^2dx^{\frac{5}{2}}}{5}+\frac{4abcd^2x^{\frac{9}{2}}}{3}+\frac{4abd^3x^{\frac{13}{2}}}{13}+\frac{2b^2c^3x^{\frac{5}{2}}}{5}+\frac{2b^2c^2dx^{\frac{9}{2}}}{3}+\frac{6b^2cd^2x^{\frac{13}{2}}}{13}+\frac{2b^2d^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(5/2),x)
```

```
[Out] -2*a**2*c**3/(3*x**(3/2)) + 6*a**2*c**2*d*sqrt(x) + 6*a**2*c*d**2
*x**(5/2)/5 + 2*a**2*d**3*x**(9/2)/9 + 4*a*b*c**3*sqrt(x) + 12*a*
b*c**2*d*x**(5/2)/5 + 4*a*b*c*d**2*x**(9/2)/3 + 4*a*b*d**3*x**(13
/2)/13 + 2*b**2*c**3*x**(5/2)/5 + 2*b**2*c**2*d*x**(9/2)/3 + 6*b*
*2*c*d**2*x**(13/2)/13 + 2*b**2*d**3*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.230354, size = 182, normalized size = 1.33

$$\frac{2}{17}\,b^2d^3x^{\frac{17}{2}}+\frac{6}{13}\,b^2cd^2x^{\frac{13}{2}}+\frac{4}{13}\,abd^3x^{\frac{13}{2}}+\frac{2}{3}\,b^2c^2dx^{\frac{9}{2}}+\frac{4}{3}\,abcd^2x^{\frac{9}{2}}+\frac{2}{9}\,a^2d^3x^{\frac{9}{2}}+\frac{2}{5}\,b^2c^3x^{\frac{5}{2}}+\frac{12}{5}\,abc^2dx^{\frac{5}{2}}+\frac{6}{5}\,a^2cd^2x^{\frac{5}{2}}+4\,abc^3\sqrt{x}+6\,a^2c^2d\sqrt{x}-\frac{2\,a^2c^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 6/13*b^2*c*d^2*x^(13/2) + 4/13*a*b*d^3*x^
(13/2) + 2/3*b^2*c^2*d*x^(9/2) + 4/3*a*b*c*d^2*x^(9/2) + 2/9*a^2*
d^3*x^(9/2) + 2/5*b^2*c^3*x^(5/2) + 12/5*a*b*c^2*d*x^(5/2) + 6/5*
a^2*c*d^2*x^(5/2) + 4*a*b*c^3*sqrt(x) + 6*a^2*c^2*d*sqrt(x) - 2/3
*a^2*c^3/x^(3/2)
```

3.414 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}b^2d^3x^{15/2}$$

```
[Out] (-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2
*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(
11/2))/11 + (2*b^2*d^3*x^(15/2))/15
```

Rubi [A] time = 0.173065, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}b^2d^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2
*c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(
11/2))/11 + (2*b^2*d^3*x^(15/2))/15
```

Rubi in Sympy [A] time = 29.8333, size = 143, normalized size = 1.04

$$-\frac{2a^2c^3}{5x^{\frac{5}{2}}}-\frac{2ac^2\left(3ad+2bc\right)}{\sqrt{x}}+\frac{2b^2d^3x^{\frac{15}{2}}}{15}+\frac{2bd^2x^{\frac{11}{2}}\left(2ad+3bc\right)}{11}+\frac{2cx^{\frac{3}{2}}\left(3a^2d^2+6abcd+b^2c^2\right)}{3}+\frac{2dx^{\frac{7}{2}}\left(a^2d^2+6abcd+3b^2c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(7/2),x)
```

```
[Out] -2*a**2*c**3/(5*x**(5/2)) - 2*a*c**2*(3*a*d + 2*b*c)/sqrt(x) + 2*
b**2*d**3*x**(15/2)/15 + 2*b*d**2*x**(11/2)*(2*a*d + 3*b*c)/11 +
2*c*x**(3/2)*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/3 + 2*d*x**(7/
2)*(a**2*d**2 + 6*a*b*c*d + 3*b**2*c**2)/7
```

Mathematica [A] time = 0.0895888, size = 121, normalized size = 0.88

$$\frac{2\left(165dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+385cx^4\left(3a^2d^2+6abcd+b^2c^2\right)-231a^2c^3-1155ac^2x^2(3ad+2bc)+105bd^2x^8(2ad+3b\right.}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^3 - 1155*a*c^2*(2*b*c + 3*a*d)*x^2 + 385*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^4 + 165*d*(3*b^2*c^2 + 6*a*b*c*d + a
^2*d^2)*x^6 + 105*b*d^2*(3*b*c + 2*a*d)*x^8 + 77*b^2*d^3*x^10))/(
1155*x^(5/2))
```

Maple [A] time = 0.012, size = 138, normalized size = 1.

$$-\frac{-154\,b^2d^3x^{10}-420\,x^8abd^3-630\,x^8b^2cd^2-330\,x^6a^2d^3-1980\,x^6abcd^2-990\,x^6b^2c^2d-2310\,x^4a^2cd^2-4620\,x^4abc^2d-}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x)
```

```
[Out] -2/1155*(-77*b^2*d^3*x^10-210*a*b*d^3*x^8-315*b^2*c*d^2*x^8-165*a
^2*d^3*x^6-990*a*b*c*d^2*x^6-495*b^2*c^2*d*x^6-1155*a^2*c*d^2*x^4
-2310*a*b*c^2*d*x^4-385*b^2*c^3*x^4+3465*a^2*c^2*d*x^2+2310*a*b*c
^3*x^2+231*a^2*c^3)/x^(5/2)
```

Maxima [A] time = 1.34043, size = 174, normalized size = 1.27

$$\frac{2}{15}\,b^2d^3x^{\frac{15}{2}}+\frac{2}{11}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{11}{2}}+\frac{2}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{7}{2}}+\frac{2}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{3}{2}}-\frac{2\left(a^2c^3+5\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 2/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(11/2) +
 2/7*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(7/2) + 2/3*(b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(3/2) - 2/5*(a^2*c^3 + 5*(2*a*b*c
^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

Fricas [A] time = 0.220253, size = 174, normalized size = 1.27

$$\frac{2\left(77\,b^2d^3x^{10}+105\left(3\,b^2cd^2+2\,abd^3\right)x^8+165\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-231\,a^2c^3+385\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^3*x^10 + 105*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 165
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 231*a^2*c^3 + 385*(b
^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 1155*(2*a*b*c^3 + 3*a^2
*c^2*d)*x^2)/x^(5/2)
```

Sympy [A] time = 58.5728, size = 185, normalized size = 1.35

$$-\frac{2a^2c^3}{5x^{\frac{5}{2}}}-\frac{6a^2c^2d}{\sqrt{x}}+2a^2cd^2x^{\frac{3}{2}}+\frac{2a^2d^3x^{\frac{7}{2}}}{7}-\frac{4abc^3}{\sqrt{x}}+4abc^2dx^{\frac{3}{2}}+\frac{12abcd^2x^{\frac{7}{2}}}{7}+\frac{4abd^3x^{\frac{11}{2}}}{11}+\frac{2b^2c^3x^{\frac{3}{2}}}{3}+\frac{6b^2c^2dx^{\frac{7}{2}}}{7}+\frac{6b^2cd^2x^{\frac{11}{2}}}{11}+\frac{2b^2d^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(7/2),x)
```

```
[Out] -2*a**2*c**3/(5*x**(5/2)) - 6*a**2*c**2*d/sqrt(x) + 2*a**2*c*d**2
*x**(3/2) + 2*a**2*d**3*x**(7/2)/7 - 4*a*b*c**3/sqrt(x) + 4*a*b*c
**2*d*x**(3/2) + 12*a*b*c*d**2*x**(7/2)/7 + 4*a*b*d**3*x**(11/2)/
11 + 2*b**2*c**3*x**(3/2)/3 + 6*b**2*c**2*d*x**(7/2)/7 + 6*b**2*c
*d**2*x**(11/2)/11 + 2*b**2*d**3*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.232066, size = 185, normalized size = 1.35

$$\frac{2}{15}\,b^2d^3x^{\frac{15}{2}} + \frac{6}{11}\,b^2cd^2x^{\frac{11}{2}} + \frac{4}{11}\,abd^3x^{\frac{11}{2}} + \frac{6}{7}\,b^2c^2dx^{\frac{7}{2}} + \frac{12}{7}\,abcd^2x^{\frac{7}{2}} + \frac{2}{7}\,a^2d^3x^{\frac{7}{2}} + \frac{2}{3}\,b^2c^3x^{\frac{3}{2}} + 4\,abc^2dx^{\frac{3}{2}} + 2\,a^2cd^2x^{\frac{3}{2}} - \frac{2\left(10\,abc^3x^2 + 15\,a^2c^2dx^2 + a^2c^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 6/11*b^2*c*d^2*x^(11/2) + 4/11*a*b*d^3*x^
(11/2) + 6/7*b^2*c^2*d*x^(7/2) + 12/7*a*b*c*d^2*x^(7/2) + 2/7*a^2
*d^3*x^(7/2) + 2/3*b^2*c^3*x^(3/2) + 4*a*b*c^2*d*x^(3/2) + 2*a^2*
c*d^2*x^(3/2) - 2/5*(10*a*b*c^3*x^2 + 15*a^2*c^2*d*x^2 + a^2*c^3)
/x^(5/2)
```

3.415 $\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=311

$$-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{17/4}}-\frac{2c\sqrt{x}(bc-ad)^2}{d^4}+\frac{2x^{5/2}(bc-ad)^2}{5d^3}-\frac{2bx^{9/2}(bc-2ad)}{9d^2}+\frac{2b^2x^{13/2}}{13d}$$

```
[Out] (-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d
^3) - (2*b*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*
d) - (c^(5/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)) - (c^(5/4)
*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(
17/4))
```

Rubi [A] time = 0.630695, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{17/4}}-\frac{2c\sqrt{x}(bc-ad)^2}{d^4}+\frac{2x^{5/2}(bc-ad)^2}{5d^3}-\frac{2bx^{9/2}(bc-2ad)}{9d^2}+\frac{2b^2x^{13/2}}{13d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```


```
[Out] (-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d
^3) - (2*b*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*
d) - (c^(5/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)) - (c^(5/4)
*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(
17/4))
```

Rubi in Sympy [A] time = 102.279, size = 292, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13d}+\frac{2bx^{\frac{9}{2}}(2ad-bc)}{9d^2}-\frac{\sqrt{2}c^{\frac{5}{4}}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{17}{4}}}+\frac{\sqrt{2}c^{\frac{5}{4}}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{17}{4}}}-\frac{\sqrt{2}c^{\frac{5}{4}}(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{17}{4}}}+\frac{\sqrt{2}c^{\frac{5}{4}}(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{17}{4}}}-\frac{2c\sqrt{x}(ad-bc)^2}{d^4}+\frac{2x^{\frac{5}{2}}(ad-bc)^2}{5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] 2*b**2*x**(13/2)/(13*d) + 2*b*x**(9/2)*(2*a*d - b*c)/(9*d**2) - s
qrt(2)*c**(5/4)*(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqr
t(x) + sqrt(c) + sqrt(d)*x)/(4*d**(17/4)) + sqrt(2)*c**(5/4)*(a*d
 - b*c)**2*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt
(d)*x)/(4*d**(17/4)) - sqrt(2)*c**(5/4)*(a*d - b*c)**2*atan(1 - s
qrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d**(17/4)) + sqrt(2)*c**(5/4
)*(a*d - b*c)**2*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d
**(17/4)) - 2*c*sqrt(x)*(a*d - b*c)**2/d**4 + 2*x**(5/2)*(a*d - b
*c)**2/(5*d**3)
```

Mathematica [A] time = 0.224267, size = 299, normalized size = 0.96

$$-585\sqrt{2}c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+585\sqrt{2}c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-1170\sqrt{2}c^{5/4}\ldots$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-4680*c*d^(1/4)*(b*c - a*d)^2*Sqrt[x] + 936*d^(5/4)*(b*c - a*d)^
2*x^(5/2) - 520*b*d^(9/4)*(b*c - 2*a*d)*x^(9/2) + 360*b^2*d^(13/4
)*x^(13/2) - 1170*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 1170*Sqrt[2]*c^(5/4)*(b*c - a*d)^2
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 585*Sqrt[2]*c^(5
/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x] + 585*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2340*d^(17/4))
```

Maple [B] time = 0.023, size = 545, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/13*b^2*x^(13/2)/d+4/9/d*x^(9/2)*a*b-2/9/d^2*x^(9/2)*b^2*c+2/5/d
*x^(5/2)*a^2-4/5/d^2*x^(5/2)*a*b*c+2/5/d^3*x^(5/2)*b^2*c^2-2/d^2*
x^(1/2)*a^2*c+4/d^3*x^(1/2)*a*b*c^2-2/d^4*x^(1/2)*b^2*c^3+1/2*c/d
^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-
c^2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
*a*b+1/2*c^3/d^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*b^2+1/4*c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))*a^2-1/2*c^2/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))*a*b+1/4*c^3/d^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)*b^2+1/2*c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*a^2-c^2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)*a*b+1/2*c^3/d^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.246855, size = 1485, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] -1/1170*(2340*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*
d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d
^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1
/4)*arctan(d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2
 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5
+ 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)
/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x) + sqrt(d^8*sqrt(-(b
^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*
d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^
6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17) + (b^4*c^6 - 4*a*b^3*c^5
*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*x))) - 58
5*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3
*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*
b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(d^4*
(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c
^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^
7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4) + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) + 585*d^4*(-(b^8*c^13 - 8*a*b^7*
c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c
^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8)/d^17)^(1/4)*log(-d^4*(-(b^8*c^13 - 8*a*b^7*c^12*
d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^
4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a
^8*c^5*d^8)/d^17)^(1/4) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqr
t(x)) - 4*(45*b^2*d^3*x^6 - 585*b^2*c^3 + 1170*a*b*c^2*d - 585*a^
2*c*d^2 - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^4 + 117*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/d^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.255166, size = 589, normalized size = 1.89

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3-2\left(cd^3\right)^{\frac{1}{4}}abc^2d+\left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^5}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3-2\left(cd^3\right)^{\frac{1}{4}}abc^2d+\left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^5}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3-2\left(cd^3\right)^{\frac{1}{4}}abc^2d+\left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^5}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3-2\left(cd^3\right)^{\frac{1}{4}}abc^2d+\left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^5}+\frac{2\left(45\,b^2d^{12}x^{\frac{13}{2}}-65\,b^2cd^{11}x^{\frac{9}{2}}+130\,abd^{12}x^{\frac{9}{2}}+117\,b^2c^2d^{10}x^{\frac{5}{2}}-234\,abcd^{11}x^{\frac{5}{2}}+117\,a^2d^{12}x^{\frac{5}{2}}-585\,b^2c^3d^9\sqrt{x}+1170\,abc^2d\right.}{585\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d +
(c*d^3)^(1/4)*a^2*c*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
+ 2*sqrt(x))/(c/d)^(1/4))/d^5 + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^
3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/d^5 +
1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d +
(c*d^3)^(1/4)*a^2*c*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqr
t(c/d))/d^5 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4
)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^
(1/4) + x + sqrt(c/d))/d^5 + 2/585*(45*b^2*d^12*x^(13/2) - 65*b^2
*c*d^11*x^(9/2) + 130*a*b*d^12*x^(9/2) + 117*b^2*c^2*d^10*x^(5/2)
 - 234*a*b*c*d^11*x^(5/2) + 117*a^2*d^12*x^(5/2) - 585*b^2*c^3*d^
9*sqrt(x) + 1170*a*b*c^2*d^10*sqrt(x) - 585*a^2*c*d^11*sqrt(x))/d
^13
```

3.416 $\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=290

$$-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{15/4}}+\frac{2x^{3/2}(bc-ad)^2}{3d^3}-\frac{2bx^{7/2}(bc-2ad)}{7d^2}+\frac{2b^2x^{11/2}}{11d}$$

```
[Out] (2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(
7*d^2) + (2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(
3/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4)) + (c
^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```

Rubi [A] time = 0.532838, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{15/4}}+\frac{2x^{3/2}(bc-ad)^2}{3d^3}-\frac{2bx^{7/2}(bc-2ad)}{7d^2}+\frac{2b^2x^{11/2}}{11d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(
7*d^2) + (2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(
3/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4)) + (c
^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```

Rubi in Sympy [A] time = 96.3314, size = 272, normalized size = 0.94

$$\frac{2b^2x^{\frac{11}{2}}}{11d}+\frac{2bx^{\frac{7}{2}}(2ad-bc)}{7d^2}-\frac{\sqrt{2}c^{\frac{3}{4}}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{15}{4}}}+\frac{\sqrt{2}c^{\frac{3}{4}}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{15}{4}}}+\frac{\sqrt{2}c^{\frac{3}{4}}(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{15}{4}}}-\frac{\sqrt{2}c^{\frac{3}{4}}(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{15}{4}}}+\frac{2x^{\frac{3}{2}}(ad-bc)^2}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] 2*b**2*x**(11/2)/(11*d) + 2*b*x**(7/2)*(2*a*d - b*c)/(7*d**2) - s
qrt(2)*c**(3/4)*(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqr
t(x) + sqrt(c) + sqrt(d)*x)/(4*d**(15/4)) + sqrt(2)*c**(3/4)*(a*d
 - b*c)**2*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt
(d)*x)/(4*d**(15/4)) + sqrt(2)*c**(3/4)*(a*d - b*c)**2*atan(1 - s
qrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d**(15/4)) - sqrt(2)*c**(3/4
)*(a*d - b*c)**2*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d
**(15/4)) + 2*x**(3/2)*(a*d - b*c)**2/(3*d**3)
```

Mathematica [A] time = 0.17283, size = 276, normalized size = 0.95

$$\frac{-231\sqrt{2}c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+231\sqrt{2}c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+462\sqrt{2}c^{3/4}(}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (616*d^(3/4)*(b*c - a*d)^2*x^(3/2) - 264*b*d^(7/4)*(b*c - 2*a*d)*
x^(7/2) + 168*b^2*d^(11/4)*x^(11/2) + 462*Sqrt[2]*c^(3/4)*(b*c -
a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 462*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)] - 231*Sqrt[2]*c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + 231*Sqrt[2]*c^(3/4)*(b*c - a
*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(924*d^(15/4))
```

Maple [B] time = 0.017, size = 504, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/11*b^2*x^(11/2)/d+4/7/d*x^(7/2)*a*b-2/7/d^2*x^(7/2)*b^2*c+2/3/d
*x^(3/2)*a^2-4/3/d^2*x^(3/2)*c*a*b+2/3/d^3*x^(3/2)*b^2*c^2-1/2*c/
d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2
+c^2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)*a*b-1/2*c^3/d^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)*b^2-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)*a^2+c^2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)*a*b-1/2*c^3/d^4/(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4*c/d^2/(c/d)^(1/4)*2^(1/2)
*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/2*c^2/d^3/(c/d)^(1/4)*2^(1/2)*l
n((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2)))*a*b-1/4*c^3/d^4/(c/d)^(1/4)*2^(1/2)*ln(
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2
)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262635, size = 1980, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] -1/462*(924*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2
 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 +
 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*
arctan(d^11*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 5
6*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*
a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(3/4)/((b^
6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 +
 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x) + sq
rt((b^12*c^16 - 12*a*b^11*c^15*d + 66*a^2*b^10*c^14*d^2 - 220*a^3
*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924
*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 2
20*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a^11*b*c^5*d^11 +
a^12*c^4*d^12)*x - (b^8*c^11*d^7 - 8*a*b^7*c^10*d^8 + 28*a^2*b^6*
c^9*d^9 - 56*a^3*b^5*c^8*d^10 + 70*a^4*b^4*c^7*d^11 - 56*a^5*b^3*
c^6*d^12 + 28*a^6*b^2*c^5*d^13 - 8*a^7*b*c^4*d^14 + a^8*c^3*d^15)
*sqrt(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2
*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)))) + 231*d^3*(-(b
^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^
3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6
- 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(d^11*(-(b^8*c^11
 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*
a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7
*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d
+ 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 -
6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) - 231*d^3*(-(b^8*c^11 - 8
*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*
b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c
^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(-d^11*(-(b^8*c^11 - 8*a*b^7
*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^
7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7
 + a^8*c^3*d^8)/d^15)^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b
^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*sqrt(x)) - 4*(21*b^2*d^2*x^5 - 33*(b^2*c*d -
 2*a*b*d^2)*x^3 + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)*sqrt(x))/
d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.249353, size = 520, normalized size = 1.79

$$-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{6}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{6}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{6}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{6}}+\frac{2\left(21b^{2}d^{10}x^{\frac{11}{2}}-33b^{2}cd^{9}x^{\frac{7}{2}}+66abd^{10}x^{\frac{7}{2}}+77b^{2}c^{2}d^{8}x^{\frac{3}{2}}-154abcd^{9}x^{\frac{3}{2}}+77a^{2}d^{10}x^{\frac{3}{2}}\right)}{231d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (
c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2
*sqrt(x))/(c/d)^(1/4))/d^6 - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 -
 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/d^6 + 1/4*sqr
t(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(
3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^6
 - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
 (c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqr
t(c/d))/d^6 + 2/231*(21*b^2*d^10*x^(11/2) - 33*b^2*c*d^9*x^(7/2)
+ 66*a*b*d^10*x^(7/2) + 77*b^2*c^2*d^8*x^(3/2) - 154*a*b*c*d^9*x^
(3/2) + 77*a^2*d^10*x^(3/2))/d^11
```

3.417 $\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}+\frac{\sqrt[4]{c}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{13/4}}+\frac{2\sqrt{x}(bc-ad)^2}{d^3}-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2b^2x^{9/2}}{9d}$$

```
[Out] (2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^
2) + (2*b^2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(
b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4)) - (c^(1/4)
*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

Rubi [A] time = 0.506879, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}+\frac{\sqrt[4]{c}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{13/4}}+\frac{2\sqrt{x}(bc-ad)^2}{d^3}-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2b^2x^{9/2}}{9d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^
2) + (2*b^2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(
b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4)) - (c^(1/4)
*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

Rubi in Sympy [A] time = 97.4904, size = 270, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9d}+\frac{2bx^{\frac{5}{2}}(2ad-bc)}{5d^2}+\frac{\sqrt{2}\sqrt[4]{c}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{c}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{13}{4}}}+\frac{\sqrt{2}\sqrt[4]{c}(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{c}(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{13}{4}}}+\frac{2\sqrt{x}(ad-bc)^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] 2*b**2*x**(9/2)/(9*d) + 2*b*x**(5/2)*(2*a*d - b*c)/(5*d**2) + sqr
t(2)*c**(1/4)*(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(
x) + sqrt(c) + sqrt(d)*x)/(4*d**(13/4)) - sqrt(2)*c**(1/4)*(a*d -
 b*c)**2*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d
)*x)/(4*d**(13/4)) + sqrt(2)*c**(1/4)*(a*d - b*c)**2*atan(1 - sqr
t(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d**(13/4)) - sqrt(2)*c**(1/4)*
(a*d - b*c)**2*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d**
(13/4)) + 2*sqrt(x)*(a*d - b*c)**2/d**3
```

Mathematica [A] time = 0.163061, size = 276, normalized size = 0.96

$$\frac{-72bd^{5/4}x^{5/2}(bc-2ad)+360\sqrt[4]{d}\sqrt{x}(bc-ad)^2+45\sqrt{2}\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-45\sqrt{2}\sqrt[4]{c}(bc-ad)^2\,\mathrm{lo}}{1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (360*d^(1/4)*(b*c - a*d)^2*Sqrt[x] - 72*b*d^(5/4)*(b*c - 2*a*d)*x
^(5/2) + 40*b^2*d^(9/4)*x^(9/2) + 90*Sqrt[2]*c^(1/4)*(b*c - a*d)^
2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 90*Sqrt[2]*c^(1
/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] +
 45*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x] - 45*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(180*d^
(13/4))
```

Maple [B] time = 0.016, size = 495, normalized size = 1.7

$$\frac{2b^2}{9d}x^{\frac{9}{2}}+\frac{4ab}{5d}x^{\frac{5}{2}}-\frac{2b^2c}{5d^2}x^{\frac{5}{2}}+2\frac{a^2\sqrt{x}}{d}-4\frac{abc\sqrt{x}}{d^2}+2\frac{b^2c^2\sqrt{x}}{d^3}-\frac{\sqrt{2}a^2}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{\sqrt{2}abc}{d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{\sqrt{2}b^2c^2}{2d^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{\sqrt{2}a^2}{4d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}abc}{2d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}b^2c^2}{4d^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}a^2}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}abc}{d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$-\frac{\sqrt{2}b^2c^2}{2d^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/9*b^2*x^(9/2)/d+4/5/d*x^(5/2)*a*b-2/5/d^2*x^(5/2)*b^2*c+2/d*a^2
*x^(1/2)-4/d^2*a*b*c*x^(1/2)+2/d^3*b^2*c^2*x^(1/2)-1/2/d*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+1/d^2*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*c-1/2/d^
3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2*c
^2-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/2/d
^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b*c-1/4/d^3*(
c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2*c^2-1/2/d*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/d^2*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*c-1/2
/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^
2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259936, size = 1401, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/90*(180*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 5
6*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*
a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*arctan
(d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5
*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c
^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(x) + sqrt(d^6*sqrt(-(b^8*c^9 - 8*a*b^7*c^8
*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^
8*c*d^8)/d^13) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*x))) - 45*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d
+ 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 -
56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c
*d^8)/d^13)^(1/4)*log(d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6
*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c
^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^
(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 45*d^3*(-(b^8*
c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 7
0*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a
^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*log(-d^3*(-(b^8*c^9 - 8*a*b
^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c
^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^
7 + a^8*c*d^8)/d^13)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(x)) + 4*(5*b^2*d^2*x^4 + 45*b^2*c^2 - 90*a*b*c*d + 45*a^2*d^2 -
9*(b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(x))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.251715, size = 520, normalized size = 1.81

$$-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{4}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{4}}+\frac{2\left(5b^{2}d^{8}x^{\frac{9}{2}}-9b^{2}cd^{7}x^{\frac{5}{2}}+18abd^{8}x^{\frac{5}{2}}+45b^{2}c^{2}d^{6}\sqrt{x}-90abcd^{7}\sqrt{x}+45a^{2}d^{8}\sqrt{x}\right)}{45d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (
c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2
*sqrt(x))/(c/d)^(1/4))/d^4 - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 -
 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/d^4 - 1/4*sqr
t(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^4
 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
 (c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqr
t(c/d))/d^4 + 2/45*(5*b^2*d^8*x^(9/2) - 9*b^2*c*d^7*x^(5/2) + 18*
a*b*d^8*x^(5/2) + 45*b^2*c^2*d^6*sqrt(x) - 90*a*b*c*d^7*sqrt(x) +
 45*a^2*d^8*sqrt(x))/d^9
```

3.418 $\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=268

$$\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{2b^2x^{7/2}}{7d}$$

```
[Out] (-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((
b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sq
rt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4))
```

Rubi [A] time = 0.459181, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{2b^2x^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((
b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sq
rt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4))
```

Rubi in Sympy [A] time = 88.8397, size = 252, normalized size = 0.94

$$\frac{2b^2x^{\frac{7}{2}}}{7d}+\frac{2bx^{\frac{3}{2}}(2ad-bc)}{3d^2}+\frac{\sqrt{2}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt[4]{c}d^{\frac{11}{4}}}-\frac{\sqrt{2}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt[4]{c}d^{\frac{11}{4}}}-\frac{\sqrt{2}(ad-bc)^2\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{c}d^{\frac{11}{4}}}+\frac{\sqrt{2}(ad-bc)^2\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{c}d^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c),x)
```

```
[Out] 2*b**2*x**(7/2)/(7*d) + 2*b*x**(3/2)*(2*a*d - b*c)/(3*d**2) + sqr
t(2)*(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt
```

```
(c) + sqrt(d)*x)/(4*c**(1/4)*d**(11/4)) - sqrt(2)*(a*d - b*c)**2*
log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c
**(1/4)*d**(11/4)) - sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**(
1/4)*sqrt(x)/c**(1/4))/(2*c**(1/4)*d**(11/4)) + sqrt(2)*(a*d - b*
c)**2*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(1/4)*d**
(11/4))
```

Mathematica [A] time = 0.176175, size = 249, normalized size = 0.93

$$\frac{-56b\sqrt[4]{c}d^{3/4}x^{3/2}(bc-2ad)+21\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-21\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{\cdots}\right.}{84\sqrt[4]{c}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-56*b*c^(1/4)*d^(3/4)*(b*c - 2*a*d)*x^(3/2) + 24*b^2*c^(1/4)*d^(
7/4)*x^(7/2) - 42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)] + 42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 21*Sqrt[2]*(b
*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(84*c^(1/4)*d^(11/4))
```

Maple [B] time = 0.014, size = 461, normalized size = 1.7

$$\begin{aligned}
&\frac{2\,b^2}{7\,d}x^{\frac{7}{2}}+\frac{4\,ab}{3\,d}x^{\frac{3}{2}}-\frac{2\,b^2c}{3\,d^2}x^{\frac{3}{2}}+\frac{\sqrt{2}a^2}{2\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&-\frac{\sqrt{2}abc}{d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}b^2c^2}{2\,d^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{\sqrt{2}a^2}{2\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{\sqrt{2}abc}{d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{\sqrt{2}b^2c^2}{2\,d^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{\sqrt{2}a^2}{4\,d}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&-\frac{\sqrt{2}abc}{2\,d^2}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{\sqrt{2}b^2c^2}{4\,d^3}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x)
```

```
[Out] 2/7*b^2*x^(7/2)/d+4/3*b/d*x^(3/2)*a-2/3*b^2/d^2*x^(3/2)*c+1/2/d/(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/d^
2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*c
+1/2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)*b^2*c^2+1/2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*a^2-1/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)*a*b*c+1/2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*b^2*c^2+1/4/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))*a^2-1/2/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2)))*a*b*c+1/4/d^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))*b^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247925, size = 1894, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/42*(84*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*arctan(
c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4)/((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x) + sqrt((b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)*x - (b^8*c^
9*d^5 - 8*a*b^7*c^8*d^6 + 28*a^2*b^6*c^7*d^7 - 56*a^3*b^5*c^6*d^8
 + 70*a^4*b^4*c^5*d^9 - 56*a^5*b^3*c^4*d^10 + 28*a^6*b^2*c^3*d^11
 - 8*a^7*b*c^2*d^12 + a^8*c*d^13)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c*d^11))))) + 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3
*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1
/4)*log(c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 21*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(-c*d^8*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
```

```
^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 4*(3*b^2*d*x^3 - 7*(b^2*c - 2*a*
b*d)*x)*sqrt(x))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265063, size = 487, normalized size = 1.82

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}}abcd + \left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^5} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}}abcd + \left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^5} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}}abcd + \left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,cd^5} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{3}{4}}abcd + \left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,cd^5} + \frac{2\left(3\,b^2d^6x^{\frac{7}{2}} - 7\,b^2cd^5x^{\frac{3}{2}} + 14\,abd^6x^{\frac{3}{2}}\right)}{21\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c
*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(c*d^5) + 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^
2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^5) -
1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c
*d^3)^(3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(c*d^5) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4
)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(c*d^5) + 2/21*(3*b^2*d^6*x^(7/2) - 7*b^2*c*d^
5*x^(3/2) + 14*a*b*d^6*x^(3/2))/d^7
```

3.419 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)}\,dx$

Optimal. Leaf size=266

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{3/4}d^{9/4}}-\frac{2b\sqrt{x}(bc-2ad)}{d^2}+\frac{2b^2x^{5/2}}{5d}$$

```
[Out] (-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c
- a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*
c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rubi [A] time = 0.448353, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{3/4}d^{9/4}}-\frac{2b\sqrt{x}(bc-2ad)}{d^2}+\frac{2b^2x^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c
- a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*
c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rubi in Sympy [A] time = 84.7122, size = 250, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5d}+\frac{2b\sqrt{x}\,(2ad-bc)}{d^2}-\frac{\sqrt{2}\,(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{3}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{3}{4}}d^{\frac{9}{4}}}-\frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{3}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{3}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)/x**(1/2),x)
```

```
[Out] 2*b**2*x**(5/2)/(5*d) + 2*b*sqrt(x)*(2*a*d - b*c)/d**2 - sqrt(2)*
(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) +
```

```
 sqrt(d)*x)/(4*c**(3/4)*d**(9/4)) + sqrt(2)*(a*d - b*c)**2*log(sq
rt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c**(3/4
)*d**(9/4)) - sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**(1/4)*sq
rt(x)/c**(1/4))/(2*c**(3/4)*d**(9/4)) + sqrt(2)*(a*d - b*c)**2*at
an(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(3/4)*d**(9/4))
```

Mathematica [A] time = 0.180767, size = 249, normalized size = 0.94

$$\frac{-40bc^{3/4}\sqrt[4]{d}\sqrt{x}(bc-2ad)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{20c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*Sqrt[x] + 8*b^2*c^(3/4)*d^(5
/4)*x^(5/2) - 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)] + 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)] - 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + 5*Sqrt[2]*(b*c
- a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(20*c^(3/4)*d^(9/4))
```

Maple [B] time = 0.015, size = 452, normalized size = 1.7

$$\frac{2b^2}{5d}x^{\frac{5}{2}}+4\frac{ab\sqrt{x}}{d}-2\frac{b^2\sqrt{x}c}{d^2}+\frac{\sqrt{2}a^2}{2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$-\frac{\sqrt{2}ab}{d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{c\sqrt{2}b^2}{2d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$+\frac{\sqrt{2}a^2}{2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{\sqrt{2}ab}{d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{c\sqrt{2}b^2}{2d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{\sqrt{2}a^2}{4c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}ab}{2d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}b^2}{4d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x)
```

```
[Out] 2/5*b^2*x^(5/2)/d+4*b/d*a*x^(1/2)-2*b^2/d^2*x^(1/2)*c+1/2*(c/d)^(
1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/d*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2/d^2
*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+
1/2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a
```

```
^2-1/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
a*b+1/2/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)-1)*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a
^2-1/2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+1/4/d
^2*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259488, size = 1365, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/10*(20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arcta
n(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)/((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*sqrt(x) + sqrt(c^2*d^4*sqrt(-(b^8*c^8 - 8*a
*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4
*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^3*d^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x))) - 5*d^2*(-(b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 5
6*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 5*
d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b
^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
+ 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(x)) - 4*(b^2*d*x^2 - 5*b^2*c + 10*a*b*d)*sqrt(x))
/d^2
```

Sympy [A] time = 158.427, size = 612, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**
(5/2)/5), Eq(c, 0) & Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)
/5 + 2*b**2*x**(9/2)/9)/c, Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*
a*b*sqrt(x) + 2*b**2*x**(5/2)/5)/d, Eq(c, 0)), (-(-1)**(1/4)*a**2
*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(3/4)*d*
*20*(1/d)**(79/4)) + (-1)**(1/4)*a**2*log((-1)**(1/4)*c**(1/4)*(1
/d)**(1/4) + sqrt(x))/(2*c**(3/4)*d**20*(1/d)**(79/4)) - (-1)**(1
/4)*a**2*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(c**(3
/4)*d**20*(1/d)**(79/4)) + (-1)**(1/4)*a*b*c**(1/4)*log(-(-1)**(1
/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(d**21*(1/d)**(79/4)) - (-1)
**(1/4)*a*b*c**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt
(x))/(d**21*(1/d)**(79/4)) + 2*(-1)**(1/4)*a*b*c**(1/4)*atan((-1)
**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**21*(1/d)**(79/4)) +
4*a*b*sqrt(x)/d - (-1)**(1/4)*b**2*c**(5/4)*log(-(-1)**(1/4)*c**(
1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**22*(1/d)**(79/4)) + (-1)**(1/4
)*b**2*c**(5/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/
(2*d**22*(1/d)**(79/4)) - (-1)**(1/4)*b**2*c**(5/4)*atan((-1)**(3
/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**22*(1/d)**(79/4)) - 2*b*
*2*c*sqrt(x)/d**2 + 2*b**2*x**(5/2)/(5*d), True))
```

GIAC/XCAS [A] time = 0.239309, size = 486, normalized size = 1.83

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{3}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{3}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^{3}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^{3}}+\frac{2\left(b^{2}d^{4}x^{\frac{5}{2}}-5\,b^{2}cd^{3}\sqrt{x}+10\,abd^{4}\sqrt{x}\right)}{5\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(c*d^3) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^
2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^3) +
1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(c*d^3) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4
)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(c*d^3) + 2/5*(b^2*d^4*x^(5/2) - 5*b^2*c*d^3*s
qrt(x) + 10*a*b*d^4*sqrt(x))/d^5
```

3.420 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{c\sqrt{x}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{5/4}d^{7/4}}+\frac{2b^2x^{3/2}}{3d}$$

```
[Out] (-2*a^2)/(c*Sqrt[x]) + (2*b^2*x^(3/2))/(3*d) + ((b*c - a*d)^2*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7
/4)) - ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(Sqrt[2]*c^(5/4)*d^(7/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7
/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7/4))
```

Rubi [A] time = 0.559327, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{2a^2}{c\sqrt{x}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{5/4}d^{7/4}}+\frac{2b^2x^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```


```
[Out] (-2*a^2)/(c*Sqrt[x]) + (2*b^2*x^(3/2))/(3*d) + ((b*c - a*d)^2*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7
/4)) - ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(Sqrt[2]*c^(5/4)*d^(7/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7
/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7/4))
```

Rubi in Sympy [A] time = 100.607, size = 241, normalized size = 0.93

$$-\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3d}-\frac{\sqrt{2}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{5}{4}}d^{\frac{7}{4}}}+\frac{\sqrt{2}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{5}{4}}d^{\frac{7}{4}}}+\frac{\sqrt{2}(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{5}{4}}d^{\frac{7}{4}}}-\frac{\sqrt{2}(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{5}{4}}d^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c),x)
```

```
[Out] -2*a**2/(c*sqrt(x)) + 2*b**2*x**(3/2)/(3*d) - sqrt(2)*(a*d - b*c)
**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)
```

```
/(4*c**(5/4)*d**(7/4)) + sqrt(2)*(a*d - b*c)**2*log(sqrt(2)*c**(1
/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c**(5/4)*d**(7/4))
 + sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1
/4))/(2*c**(5/4)*d**(7/4)) - sqrt(2)*(a*d - b*c)**2*atan(1 + sqrt
(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(5/4)*d**(7/4))
```

Mathematica [A] time = 0.19949, size = 261, normalized size = 1.

$$\frac{-24a^2\sqrt[4]{c}d^{7/4} - 3\sqrt{2}\sqrt{x}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right) + 3\sqrt{2}\sqrt{x}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right) + 6\sqrt{}}{12c^{5/4}d^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

```
[Out] (-24*a^2*c^(1/4)*d^(7/4) + 8*b^2*c^(5/4)*d^(3/4)*x^2 + 6*Sqrt[2]*
(b*c - a*d)^2*Sqrt[x]*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)] - 6*Sqrt[2]*(b*c - a*d)^2*Sqrt[x]*ArcTan[1 + (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)] - 3*Sqrt[2]*(b*c - a*d)^2*Sqrt[x]*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + 3*Sqrt[2]*(b*c -
a*d)^2*Sqrt[x]*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(12*c^(5/4)*d^(7/4)*Sqrt[x])
```

Maple [B] time = 0.017, size = 439, normalized size = 1.7

$$\frac{2b^2}{3d}x^{\frac{3}{2}} - \frac{\sqrt{2}a^2}{2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} + \frac{\sqrt{2}ab}{d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$- \frac{c\sqrt{2}b^2}{2d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} - \frac{\sqrt{2}a^2}{2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$+ \frac{\sqrt{2}ab}{d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} - \frac{c\sqrt{2}b^2}{2d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$- \frac{\sqrt{2}a^2}{4c}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$+ \frac{\sqrt{2}ab}{2d}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$- \frac{c\sqrt{2}b^2}{4d^2}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}} - 2\frac{a^2}{c\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x)
```

```
[Out] 2/3*b^2*x^(3/2)/d-1/2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)*a^2+1/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)*a*b-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+1/d/(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*ar
```

```
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4/c/(c/d)^(1/4)*2^(1/2)
*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/2/d/(c/d)^(1/4)*2^(1/2)*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))*a*b-1/4*c/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2)))*b^2-2*a^2/c/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.253485, size = 1902, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(4*b^2*c*x^2 - 12*c*d*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28
*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c
^5*d^7))^(1/4)*arctan(c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b
^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d
^7))^(3/4)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^
3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqr
t(x) + sqrt((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)*x - (b^8*c^11*d^3 - 8*a*b^7*c^10*d^4 + 28*a^2*b^6*c
^9*d^5 - 56*a^3*b^5*c^8*d^6 + 70*a^4*b^4*c^7*d^7 - 56*a^5*b^3*c^6
*d^8 + 28*a^6*b^2*c^5*d^9 - 8*a^7*b*c^4*d^10 + a^8*c^3*d^11)*sqrt
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^
6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))))) - 3*c*d*sqrt(x)*(-(b^8
*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*log(c^4*d^5*(-(b^8*c^8 -
8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c
*d^7 + a^8*d^8)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5
*b*c*d^5 + a^6*d^6)*sqrt(x)) + 3*c*d*sqrt(x)*(-(b^8*c^8 - 8*a*b^7
*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^5*d^7))^(1/4)*log(-c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^
8)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^
6*d^6)*sqrt(x)) - 12*a^2*d)/(c*d*sqrt(x))
```

Sympy [A] time = 129.418, size = 597, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**
(3/2)/3), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(5*x**(5/2)) - 4*a*b/sq
rt(x) + 2*b**2*x**(3/2)/3)/d, Eq(c, 0)), ((-2*a**2/sqrt(x) + 4*a*
b*x**(3/2)/3 + 2*b**2*x**(7/2)/7)/c, Eq(d, 0)), (-2*a**2/(c*sqrt(
x)) + (-1)**(3/4)*a**2*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + s
qrt(x))/(2*c**(5/4)*d**13*(1/d)**(53/4)) - (-1)**(3/4)*a**2*log((
-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*d**13*(1/
d)**(53/4)) - (-1)**(3/4)*a**2*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)
*(1/d)**(1/4)))/(c**(5/4)*d**13*(1/d)**(53/4)) - (-1)**(3/4)*a*b*
log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(c**(1/4)*d**14
*(1/d)**(53/4)) + (-1)**(3/4)*a*b*log((-1)**(1/4)*c**(1/4)*(1/d)*
*(1/4) + sqrt(x))/(c**(1/4)*d**14*(1/d)**(53/4)) + 2*(-1)**(3/4)*
a*b*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(c**(1/4)*d
**14*(1/d)**(53/4)) + (-1)**(3/4)*b**2*c**(3/4)*log(-(-1)**(1/4)*
c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**15*(1/d)**(53/4)) - (-1)**
(3/4)*b**2*c**(3/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(
x))/(2*d**15*(1/d)**(53/4)) - (-1)**(3/4)*b**2*c**(3/4)*atan((-1)
**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**15*(1/d)**(53/4)) +
2*b**2*x**(3/2)/(3*d), True))
```

GIAC/XCAS [A] time = 0.252086, size = 464, normalized size = 1.78

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d}-\frac{2\,a^2}{c\sqrt{x}}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^4}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^4}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^2d^4}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(3/2)),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/2*sqrt(2)*((c*d^3)^(3/4
)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arct
an(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^
2*d^4) - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b
*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) + 1/4*sqrt(2)*((c*d^3)^
(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*
ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) - 1/4*s
qrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)
^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/
(c^2*d^4)
```

3.421 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{7/4}d^{5/4}}+\frac{2b^2\sqrt{x}}{d}$$

```
[Out] (-2*a^2)/(3*c*x^(3/2)) + (2*b^2*Sqrt[x])/d + ((b*c - a*d)^2*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4
)) - ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4
)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rubi [A] time = 0.554797, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{7/4}d^{5/4}}+\frac{2b^2\sqrt{x}}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```


```
[Out] (-2*a^2)/(3*c*x^(3/2)) + (2*b^2*Sqrt[x])/d + ((b*c - a*d)^2*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4
)) - ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4
)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rubi in Sympy [A] time = 99.1226, size = 241, normalized size = 0.93

$$-\frac{2a^2}{3cx^{\frac{3}{2}}}+\frac{2b^2\sqrt{x}}{d}+\frac{\sqrt{2}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{7}{4}}d^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{7}{4}}d^{\frac{5}{4}}}+\frac{\sqrt{2}(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{7}{4}}d^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{7}{4}}d^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c),x)
```

```
[Out] -2*a**2/(3*c*x**(3/2)) + 2*b**2*sqrt(x)/d + sqrt(2)*(a*d - b*c)**
2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(
4*c**(7/4)*d**(5/4)) - sqrt(2)*(a*d - b*c)**2*log(sqrt(2)*c**(1/4
```

```
)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c**(7/4)*d**(5/4)) +
 sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4
))/(2*c**(7/4)*d**(5/4)) - sqrt(2)*(a*d - b*c)**2*atan(1 + sqrt(2
)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(7/4)*d**(5/4))
```

Mathematica [A] time = 0.202227, size = 261, normalized size = 1.

$$\frac{-8a^2c^{3/4}d^{5/4}+3\sqrt{2}x^{3/2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-3\sqrt{2}x^{3/2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+6}{12c^{7/4}d^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (-8*a^2*c^(3/4)*d^(5/4) + 24*b^2*c^(7/4)*d^(1/4)*x^2 + 6*Sqrt[2]*
(b*c - a*d)^2*x^(3/2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)] - 6*Sqrt[2]*(b*c - a*d)^2*x^(3/2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)] + 3*Sqrt[2]*(b*c - a*d)^2*x^(3/2)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 3*Sqrt[2]*(b*c -
a*d)^2*x^(3/2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(12*c^(7/4)*d^(5/4)*x^(3/2))
```

Maple [B] time = 0.018, size = 439, normalized size = 1.7

$$2\,\frac{b^2\sqrt{x}}{d}-\frac{d\sqrt{2}a^2}{4\,c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}ab}{2\,c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}b^2}{4\,d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{d\sqrt{2}a^2}{2\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}ab}{c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$-\frac{\sqrt{2}b^2}{2\,d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{d\sqrt{2}a^2}{2\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{\sqrt{2}ab}{c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{\sqrt{2}b^2}{2\,d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{2\,a^2}{3\,c}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x)
```

```
[Out] 2*b^2*x^(1/2)/d-1/4/c^2*d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2)))*a^2+1/2/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*
a*b-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-1/2/
c^2*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a
^2+1/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
a*b-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)*b^2-1/2/c^2*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a^2+1/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
```

```
x^(1/2)-1)*a*b-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)*b^2-2/3*a^2/c/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260379, size = 1368, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(12*b^2*c*x^2 + 12*c*d*x^(3/2)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 2
8*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*
a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(
c^7*d^5))^(1/4)*arctan(c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*
b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^
5))^(1/4)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x) + sqrt(c^4*d^2
*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c
^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5)) + (b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x))) - 3*c*
d*x^(3/2)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^
3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(c^2*d
*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d
^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*sqrt(x)) + 3*c*d*x^(3/2)*(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*
d^8)/(c^7*d^5))^(1/4)*log(-c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*
a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^
5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
7*d^5))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*a^2*
d)/(c*d*x^(3/2))
```

Sympy [A] time = 174.05, size = 597, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**
2*sqrt(x)), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/
(3*x**(3/2)) + 2*b**2*sqrt(x))/d, Eq(c, 0)), ((-2*a**2/(3*x**(3/2
)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5)/c, Eq(d, 0)), (-2*a**2/(3
*c*x**(3/2)) + (-1)**(1/4)*a**2*d**7*(1/d)**(25/4)*log(-(-1)**(1/
```

```
4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(7/4)) - (-1)**(1/4)*a*
*2*d**7*(1/d)**(25/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqr
t(x))/(2*c**(7/4)) + (-1)**(1/4)*a**2*d**7*(1/d)**(25/4)*atan((-1
)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/c**(7/4) - (-1)**(1/4)*
a*b*d**6*(1/d)**(25/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + s
qrt(x))/c**(3/4) + (-1)**(1/4)*a*b*d**6*(1/d)**(25/4)*log((-1)**(
1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/c**(3/4) - 2*(-1)**(1/4)*a*
b*d**6*(1/d)**(25/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1
/4)))/c**(3/4) + (-1)**(1/4)*b**2*c**(1/4)*d**5*(1/d)**(25/4)*log
(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/2 - (-1)**(1/4)*b*
*2*c**(1/4)*d**5*(1/d)**(25/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1
/4) + sqrt(x))/2 + (-1)**(1/4)*b**2*c**(1/4)*d**5*(1/d)**(25/4)*a
tan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4))) + 2*b**2*sqrt(x)
/d, True))
```

GIAC/XCAS [A] time = 0.234427, size = 464, normalized size = 1.78

$$\frac{2\,b^2\sqrt{x}}{d} - \frac{2\,a^2}{3\,cx^{\frac{3}{2}}} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^2d^2} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(5/2)),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/2*sqrt(2)*((c*d^3)^(1/4
)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arct
an(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^
2*d^2) - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b
*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/4*sqrt(2)*((c*d^3)^
(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*
ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2) + 1/4*s
qrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)
^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/
(c^2*d^2)
```

3.422 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{2a^2}{5cx^{5/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{9/4}d^{3/4}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}$$

```
[Out] (-2*a^2)/(5*c*x^(5/2)) - (2*a*(2*b*c - a*d))/(c^2*Sqrt[x]) - ((b*
c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2
]*c^(9/4)*d^(3/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*d^(3/4)) + ((b*c - a*d)^2*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2
]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4))
```

Rubi [A] time = 0.597464, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{2a^2}{5cx^{5/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{9/4}d^{3/4}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(5*c*x^(5/2)) - (2*a*(2*b*c - a*d))/(c^2*Sqrt[x]) - ((b*
c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2
]*c^(9/4)*d^(3/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*d^(3/4)) + ((b*c - a*d)^2*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2
]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4))
```

Rubi in Sympy [A] time = 100.367, size = 250, normalized size = 0.94

$$-\frac{2a^2}{5cx^{\frac{5}{2}}}+\frac{2a\,(ad-2bc)}{c^2\sqrt{x}}+\frac{\sqrt{2}\,(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{9}{4}}d^{\frac{3}{4}}}-\frac{\sqrt{2}\,(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{9}{4}}d^{\frac{3}{4}}}-\frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{9}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{9}{4}}d^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c),x)
```

```
[Out] -2*a**2/(5*c*x**(5/2)) + 2*a*(a*d - 2*b*c)/(c**2*sqrt(x)) + sqrt(
2)*(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c
```

```
) + sqrt(d)*x)/(4*c**(9/4)*d**(3/4)) - sqrt(2)*(a*d - b*c)**2*log
(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c**(
9/4)*d**(3/4)) - sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**(1/4)
*sqrt(x)/c**(1/4))/(2*c**(9/4)*d**(3/4)) + sqrt(2)*(a*d - b*c)**2
*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(9/4)*d**(3/4)
)
```

Mathematica [A] time = 0.196287, size = 254, normalized size = 0.95

$$\frac{-\frac{8a^2c^{5/4}}{x^{5/2}}+\frac{5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{10\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{3/4}}+\frac{10\sqrt{2}(b\cdots}{\cdots}}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a^2*c^(5/4))/x^(5/2) + (40*a*c^(1/4)*(-2*b*c + a*d))/Sqrt[x]
 - (10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/d^(3/4) + (10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) + (5*Sqrt[2]*(b*c - a*d)^2*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4
) - (5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(20*c^(9/4))
```

Maple [B] time = 0.02, size = 452, normalized size = 1.7

$$\begin{aligned}
&\frac{d\sqrt{2}a^2}{2\,c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{\sqrt{2}ab}{c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{\sqrt{2}b^2}{2\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{d\sqrt{2}a^2}{2\,c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&-\frac{\sqrt{2}ab}{c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}b^2}{2\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{d\sqrt{2}a^2}{4\,c^2}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&-\frac{\sqrt{2}ab}{2\,c}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&+\frac{\sqrt{2}b^2}{4\,d}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}\\
&-\frac{2\,a^2}{5\,c}x^{-\frac{5}{2}}+2\,\frac{a^2d}{c^2\sqrt{x}}-4\,\frac{ab}{c\sqrt{x}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x)
```

```
[Out] 1/2/c^2*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)*a^2-1/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
```

```
+1)*a*b+1/2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)*b^2+1/2/c^2*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)*a^2-1/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a*b+1/2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)*b^2+1/4/c^2*d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2)))*a^2-1/2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))*a*b+1/4/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^
2-2/5*a^2/c/x^(5/2)+2*a^2/c^2/x^(1/2)*d-4*a/c/x^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266072, size = 1910, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(20*c^2*x^(5/2)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4
)*arctan(c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(3/4)/((
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x) + sqrt((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
*x - (b^8*c^13*d - 8*a*b^7*c^12*d^2 + 28*a^2*b^6*c^11*d^3 - 56*a^
3*b^5*c^10*d^4 + 70*a^4*b^4*c^9*d^5 - 56*a^5*b^3*c^8*d^6 + 28*a^6
*b^2*c^7*d^7 - 8*a^7*b*c^6*d^8 + a^8*c^5*d^9)*sqrt(-(b^8*c^8 - 8*
a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d
^7 + a^8*d^8)/(c^9*d^3))))) + 5*c^2*x^(5/2)*(-(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^9*d^3))^(1/4)*log(c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 5
6*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*
d^6)*sqrt(x)) - 5*c^2*x^(5/2)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b
^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d
^3))^(1/4)*log(-c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3
*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(
3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x))
 - 4*a^2*c - 20*(2*a*b*c - a^2*d)*x^2)/(c^2*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26043, size = 477, normalized size = 1.79

$$-\frac{2\left(10\,abcx^2-5\,a^2dx^2+a^2c\right)}{5\,c^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^3d^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(7/2)),x, algorithm="giac")
```

```
[Out] -2/5*(10*a*b*c*x^2 - 5*a^2*d*x^2 + a^2*c)/(c^2*x^(5/2)) + 1/2*sqr
t(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(
3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x)
)/(c/d)^(1/4))/(c^3*d^3) + 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2
*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) - 1/4
*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^
3)^(3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(c^3*d^3) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)
*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/(c^3*d^3)
```

3.423 $\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)} dx$

Optimal. Leaf size=269

$$-\frac{2a^2}{7cx^{7/2}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}$$

```
[Out] (-2*a^2)/(7*c*x^(7/2)) - (2*a*(2*b*c - a*d))/(3*c^2*x^(3/2)) - ((
b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(11/4)*d^(1/4)) - ((b*c - a*d)^2*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sq
rt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1/4))
```

Rubi [A] time = 0.587205, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{2a^2}{7cx^{7/2}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(7*c*x^(7/2)) - (2*a*(2*b*c - a*d))/(3*c^2*x^(3/2)) - ((
b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(11/4)*d^(1/4)) - ((b*c - a*d)^2*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sq
rt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1/4))
```

Rubi in Sympy [A] time = 97.1718, size = 252, normalized size = 0.94

$$-\frac{2a^2}{7cx^{\frac{7}{2}}}+\frac{2a(ad-2bc)}{3c^2x^{\frac{3}{2}}}-\frac{\sqrt{2}(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{11}{4}}\sqrt[4]{d}}+\frac{\sqrt{2}(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{11}{4}}\sqrt[4]{d}}-\frac{\sqrt{2}(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{11}{4}}\sqrt[4]{d}}+\frac{\sqrt{2}(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{11}{4}}\sqrt[4]{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2/x**(9/2)/(d*x**2+c),x)
```

```
[Out] -2*a**2/(7*c*x**(7/2)) + 2*a*(a*d - 2*b*c)/(3*c**2*x**(3/2)) - sq
rt(2)*(a*d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqr
```

```
t(c) + sqrt(d)*x)/(4*c**(11/4)*d**(1/4)) + sqrt(2)*(a*d - b*c)**2
*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*
c**(11/4)*d**(1/4)) - sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**
(1/4)*sqrt(x)/c**(1/4))/(2*c**(11/4)*d**(1/4)) + sqrt(2)*(a*d - b
*c)**2*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(11/4)*d
**(1/4))
```

Mathematica [A] time = 0.192612, size = 254, normalized size = 0.94

$$\frac{-\frac{24a^2c^{7/4}}{x^{7/2}}+\frac{56ac^{3/4}(ad-2bc)}{x^{3/2}}-\frac{21\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}}+\frac{21\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}}-\frac{42\sqrt{2}(bc-ad)^2\tan^{-1}\left(1\cdots\right.}{\sqrt[4]{d}}}{84c^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] ((-24*a^2*c^(7/4))/x^(7/2) + (56*a*c^(3/4)*(-2*b*c + a*d))/x^(3/2
) - (42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/d^(1/4) + (42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(1/4) - (21*Sqrt[2]*(b*c - a*d)^2
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(1
/4) + (21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/d^(1/4))/(84*c^(11/4))
```

Maple [B] time = 0.02, size = 461, normalized size = 1.7

$$\frac{\sqrt{2}a^2d^2}{2c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}abd}{c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$+\frac{\sqrt{2}b^2}{2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}a^2d^2}{2c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{\sqrt{2}abd}{c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}b^2}{2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{\sqrt{2}a^2d^2}{4c^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}abd}{2c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}b^2}{4c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{2a^2}{7c}x^{-\frac{7}{2}}+\frac{2a^2d}{3c^2}x^{-\frac{3}{2}}-\frac{4ab}{3c}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x)
```

```
[Out] 1/2/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
*a^2*d^2-1/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)*a*b*d+1/2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)+1)*b^2+1/2/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)*a^2*d^2-1/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
```

```
)/(c/d)^(1/4)*x^(1/2)-1)*a*b*d+1/2/c*(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/4/c^3*(c/d)^(1/4)*2^(1/2)*ln(
(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2
)*2^(1/2)+(c/d)^(1/2)))*a^2*d^2-1/2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2)))*a*b*d+1/4/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2)))*b^2-2/7*a^2/c/x^(7/2)+2/3*a^2/c^2/x^(3/2)*d-4/3*a
/c/x^(3/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(9/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257235, size = 1370, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(9/2)),x, algorithm="fricas")
```

```
[Out] -1/42*(84*c^2*x^(7/2)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4
)*arctan(c^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)/((b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x) + sqrt(c^6*sqrt(-(b^8*c^8 - 8*a
*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4
*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^11*d)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x))) - 21*c^2*x^(7/2)*(-(b^8*c^8
- 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(c^3*(-(b^8*c^8 - 8*a*b^7*c^
7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x))
+ 21*c^2*x^(7/2)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log
(-c^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(x)) + 12*a^2*c + 28*(2*a*b*c - a^2*d)*x^
2)/(c^2*x^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(9/2)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239125, size = 478, normalized size = 1.78

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^3d} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^3d} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4c^3d} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4c^3d} - \frac{2\left(14abcx^2 - 7a^2dx^2 + 3a^2c\right)}{21c^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)*x^(9/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(c^3*d) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^
2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d) +
1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(c^3*d) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4
)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(c^3*d) - 2/21*(14*a*b*c*x^2 - 7*a^2*d*x^2 + 3
*a^2*c)/(c^2*x^(7/2))
```

3.424 $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

Optimal. Leaf size=288

$$-\frac{\sqrt[4]{b}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{13/4}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}+\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2c^2}{9ax^{9/2}}$$

```
[Out] (-2*c^2)/(9*a*x^(9/2)) + (2*c*(b*c - 2*a*d))/(5*a^2*x^(5/2)) - (2
*(b*c - a*d)^2)/(a^3*Sqrt[x]) + (b^(1/4)*(b*c - a*d)^2*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4
)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)) + (b^(1
/4)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4))
```

Rubi [A] time = 0.699275, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{\sqrt[4]{b}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{13/4}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}+\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2c^2}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^2)/(9*a*x^(9/2)) + (2*c*(b*c - 2*a*d))/(5*a^2*x^(5/2)) - (2
*(b*c - a*d)^2)/(a^3*Sqrt[x]) + (b^(1/4)*(b*c - a*d)^2*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4
)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)) + (b^(1
/4)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4))
```

Rubi in Sympy [A] time = 110.706, size = 270, normalized size = 0.94

$$-\frac{2c^2}{9ax^{\frac{9}{2}}}-\frac{2c\left(2ad-bc\right)}{5a^2x^{\frac{5}{2}}}-\frac{2\left(ad-bc\right)^2}{a^3\sqrt{x}}-\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{13}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{13}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{13}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/x**(11/2)/(b*x**2+a),x)
```

```
[Out] -2*c**2/(9*a*x**(9/2)) - 2*c*(2*a*d - b*c)/(5*a**2*x**(5/2)) - 2*
(a*d - b*c)**2/(a**3*sqrt(x)) - sqrt(2)*b**(1/4)*(a*d - b*c)**2*l
og(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a
**(13/4)) + sqrt(2)*b**(1/4)*(a*d - b*c)**2*log(sqrt(2)*a**(1/4)*
b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(13/4)) + sqrt(2)*b
**(1/4)*(a*d - b*c)**2*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4)
)/(2*a**(13/4)) - sqrt(2)*b**(1/4)*(a*d - b*c)**2*atan(1 + sqrt(2
)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(13/4))
```

Mathematica [A] time = 0.229892, size = 277, normalized size = 0.96

$$\frac{-\frac{72a^{5/4}c(2ad-bc)}{x^{5/2}}-\frac{40a^{9/4}c^2}{x^{9/2}}-\frac{360\sqrt[4]{a}(bc-ad)^2}{\sqrt{x}}-45\sqrt{2}\sqrt[4]{b}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+45\sqrt{2}\sqrt[4]{b}(bc-ad)^2\log\left(\right.}{180a^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] ((-40*a^(9/4)*c^2)/x^(9/2) - (72*a^(5/4)*c*(-(b*c) + 2*a*d))/x^(5
/2) - (360*a^(1/4)*(b*c - a*d)^2)/Sqrt[x] + 90*Sqrt[2]*b^(1/4)*(b
*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 90*Sq
rt[2]*b^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)] - 45*Sqrt[2]*b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]*b^(1/4)*(b*c -
 a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(180*a^(13/4))
```

Maple [B] time = 0.021, size = 495, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x)
```

```
[Out] -2/9*c^2/a/x^(9/2)-2/a/x^(1/2)*d^2+4/a^2/x^(1/2)*c*b*d-2/a^3/x^(1
/2)*b^2*c^2-4/5*c/a/x^(5/2)*d+2/5*c^2/a^2/x^(5/2)*b-1/2/a/(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^2+1/a^2/(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b*c*d-1/2/a
^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^2*
c^2-1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1)*d^2+1/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)*b*c*d-1/2/a^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)*b^2*c^2-1/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))*d^2+1/2/a^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
))*b*c*d-1/4/a^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*b
^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^2/((b*x^2 + a)*x^(11/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264203, size = 1958, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^2/((b*x^2 + a)*x^(11/2)),x, algorithm="fricas")
```

```
[Out] -1/90*(180*a^3*x^(9/2)*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^
6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*
d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/
4)*arctan(a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 -
56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28
*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(3/4)/((b^7
*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 +
15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*sqrt(x) + sqrt(
(b^14*c^12 - 12*a*b^13*c^11*d + 66*a^2*b^12*c^10*d^2 - 220*a^3*b^
11*c^9*d^3 + 495*a^4*b^10*c^8*d^4 - 792*a^5*b^9*c^7*d^5 + 924*a^6
*b^8*c^6*d^6 - 792*a^7*b^7*c^5*d^7 + 495*a^8*b^6*c^4*d^8 - 220*a^
9*b^5*c^3*d^9 + 66*a^10*b^4*c^2*d^10 - 12*a^11*b^3*c*d^11 + a^12*
b^2*d^12)*x - (a^7*b^9*c^8 - 8*a^8*b^8*c^7*d + 28*a^9*b^7*c^6*d^2
 - 56*a^10*b^6*c^5*d^3 + 70*a^11*b^5*c^4*d^4 - 56*a^12*b^4*c^3*d^
5 + 28*a^13*b^3*c^2*d^6 - 8*a^14*b^2*c*d^7 + a^15*b*d^8)*sqrt(-(b
^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3
+ 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 -
8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)))) + 45*a^3*x^(9/2)*(-(b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a
^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*
b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*log(a^10*(-(b^9*c^8 - 8*a*b^8*
c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*
d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
 a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c
^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^
5 + a^6*b*d^6)*sqrt(x)) - 45*a^3*x^(9/2)*(-(b^9*c^8 - 8*a*b^8*c^7
*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4
 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^
8*b*d^8)/a^13)^(1/4)*log(-a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^
2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*
b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a
^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a
^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)*sqrt(x)) + 180*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 20*a^2*c^2
 - 36*(a*b*c^2 - 2*a^2*c*d)*x^2)/(a^3*x^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**(11/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.259147, size = 527, normalized size = 1.83

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^2}-\frac{2\left(45b^2c^2x^4-90abcdx^4+45a^2d^2x^4-9abc^2x^2+18a^2cdx^2+5a^2c^2\right)}{45a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^2/((b*x^2 + a)*x^(11/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d + (
a*b^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2
*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^2
*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^
2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d
 + (a*b^3)^(3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sq
rt(a/b))/(a^4*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^
3)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 2/45*(45*b^2*c^2*x^4 - 90*a
*b*c*d*x^4 + 45*a^2*d^2*x^4 - 9*a*b*c^2*x^2 + 18*a^2*c*d*x^2 + 5*
a^2*c^2)/(a^3*x^(9/2))
```

3.425 $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=375

$$\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{17/4}}-\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{17/4}}+\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{17/4}}-\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{17/4}}+\frac{\sqrt{x}(13bc-5ad)(bc-ad)}{2d^4}-\frac{x^{5/2}(13bc-5ad)(bc-ad)}{10cd^3}+\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{2b^2x^{9/2}}{9d^2}$$

```
[Out] ((13*b*c - 5*a*d)*(b*c - a*d)*Sqrt[x])/(2*d^4) - ((13*b*c - 5*a*d
)*(b*c - a*d)*x^(5/2))/(10*c*d^3) + (2*b^2*x^(9/2))/(9*d^2) + ((b
*c - a*d)^2*x^(9/2))/(2*c*d^2*(c + d*x^2)) + (c^(1/4)*(13*b*c - 5
*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(4*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*ArcT
an[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(17/4)) +
 (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4)) - (c^(1/4)
*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4))
```

Rubi [A] time = 0.949134, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{17/4}}-\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{17/4}}+\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{17/4}}-\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{17/4}}+\frac{\sqrt{x}(13bc-5ad)(bc-ad)}{2d^4}-\frac{x^{5/2}(13bc-5ad)(bc-ad)}{10cd^3}+\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{2b^2x^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```


```
[Out] ((13*b*c - 5*a*d)*(b*c - a*d)*Sqrt[x])/(2*d^4) - ((13*b*c - 5*a*d
)*(b*c - a*d)*x^(5/2))/(10*c*d^3) + (2*b^2*x^(9/2))/(9*d^2) + ((b
*c - a*d)^2*x^(9/2))/(2*c*d^2*(c + d*x^2)) + (c^(1/4)*(13*b*c - 5
*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(4*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*ArcT
an[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(17/4)) +
 (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4)) - (c^(1/4)
*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4))
```

Rubi in Sympy [A] time = 114.981, size = 348, normalized size = 0.93

$$\frac{2b^2x^{\frac{9}{2}}}{9d^2}+\frac{\sqrt{2}\sqrt[4]{c}\,(ad-bc)\,(5ad-13bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16d^{\frac{17}{4}}}-\frac{\sqrt{2}\sqrt[4]{c}\,(ad-bc)\,(5ad-13bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16d^{\frac{17}{4}}}+\frac{\sqrt{2}\sqrt[4]{c}\,(ad-bc)\,(5ad-13bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8d^{\frac{17}{4}}}-\frac{\sqrt{2}\sqrt[4]{c}\,(ad-bc)\,(5ad-13bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8d^{\frac{17}{4}}}+\frac{\sqrt{x}\,(ad-bc)\,(5ad-13bc)}{2d^4}+\frac{x^{\frac{9}{2}}\,(ad-bc)^2}{2cd^2\,(c+dx^2)}-\frac{x^{\frac{5}{2}}\,(ad-bc)\,(5ad-13bc)}{10cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] 2*b**2*x**(9/2)/(9*d**2) + sqrt(2)*c**(1/4)*(a*d - b*c)*(5*a*d -
13*b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d
)*x)/(16*d**(17/4)) - sqrt(2)*c**(1/4)*(a*d - b*c)*(5*a*d - 13*b*
c)*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(
16*d**(17/4)) + sqrt(2)*c**(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*ata
n(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*d**(17/4)) - sqrt(2)*
c**(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*atan(1 + sqrt(2)*d**(1/4)*s
qrt(x)/c**(1/4))/(8*d**(17/4)) + sqrt(x)*(a*d - b*c)*(5*a*d - 13*
b*c)/(2*d**4) + x**(9/2)*(a*d - b*c)**2/(2*c*d**2*(c + d*x**2)) -
 x**(5/2)*(a*d - b*c)*(5*a*d - 13*b*c)/(10*c*d**3)
```

Mathematica [A] time = 0.698379, size = 372, normalized size = 0.99

$$\frac{1440\sqrt[4]{d}\sqrt{x}\left(a^2d^2-4abcd+3b^2c^2\right)+45\sqrt{2}\sqrt[4]{c}\left(5a^2d^2-18abcd+13b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-45\sqrt{2}\sqrt[4]{c}\left(5a^2d\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (1440*d^(1/4)*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[x] - 576*b*d
^(5/4)*(b*c - a*d)*x^(5/2) + 160*b^2*d^(9/4)*x^(9/2) + (360*c*d^(
1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2) + 90*Sqrt[2]*c^(1/4)*(13*
b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)] - 90*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*
a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 45*Sqrt[
2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 45*Sqrt[2]*c^(1/4)*(
13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(720*d^(17/4))
```

Maple [A] time = 0.024, size = 563, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/9*b^2*x^(9/2)/d^2+4/5/d^2*x^(5/2)*a*b-4/5/d^3*x^(5/2)*b^2*c+2/d
^2*a^2*x^(1/2)-8/d^3*a*b*c*x^(1/2)+6/d^4*b^2*c^2*x^(1/2)+1/2*c/d^
2*x^(1/2)/(d*x^2+c)*a^2-c^2/d^3*x^(1/2)/(d*x^2+c)*a*b+1/2*c^3/d^4
*x^(1/2)/(d*x^2+c)*b^2-5/8/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a^2+9/4*c/d^3*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-13/8*c^2/d^4*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-5/8/d^2*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+9/4*c/d^3*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-13/8*c^2
/d^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^
2-5/16/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+9/8
*c/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-13/16*c
^2/d^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264339, size = 1538, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*
d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*
b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 90
00*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*arctan(d^4*(-(28561
*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a
^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5
+ 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^1
7)^(1/4)/((13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*sqrt(x) + sqrt(d^
8*sqrt(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*
d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^
5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*
a^8*c*d^8)/d^17) + (169*b^4*c^4 - 468*a*b^3*c^3*d + 454*a^2*b^2*c
^2*d^2 - 180*a^3*b*c*d^3 + 25*a^4*d^4)*x))) - 45*(d^5*x^2 + c*d^4
)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2
- 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^
3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*
c*d^8)/d^17)^(1/4)*log(d^4*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d
+ 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^
4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000
*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*
b*c*d + 5*a^2*d^2)*sqrt(x)) + 45*(d^5*x^2 + c*d^4)*(-(28561*b^8*c
^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5
*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 5510
0*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/
4)*log(-d^4*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^
6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186
840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7
+ 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*sqrt(x)) + 4*(20*b^2*d^3*x^6 + 585*b^2*c^3 - 810*a*b*c^2*d +
225*a^2*c*d^2 - 4*(13*b^2*c*d^2 - 18*a*b*d^3)*x^4 + 36*(13*b^2*c^
```

```
2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*x^2)*sqrt(x))/(d^5*x^2 + c*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.25398, size = 594, normalized size = 1.58

$$-\frac{\sqrt{2}\left(13\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\,\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,d^{5}}-\frac{\sqrt{2}\left(13\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\,\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,d^{5}}-\frac{\sqrt{2}\left(13\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\,\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,d^{5}}+\frac{\sqrt{2}\left(13\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\,\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,d^{5}}+\frac{b^{2}c^{3}\sqrt{x}-2\,abc^{2}d\sqrt{x}+a^{2}cd^{2}\sqrt{x}}{2\,(dx^{2}+c)d^{4}}+\frac{2\left(5\,b^{2}d^{16}x^{\frac{9}{2}}-18\,b^{2}cd^{15}x^{\frac{5}{2}}+18\,abd^{16}x^{\frac{5}{2}}+135\,b^{2}c^{2}d^{14}\sqrt{x}-180\,abcd^{15}\sqrt{x}+45\,a^{2}d^{16}\sqrt{x}\right)}{45\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b*c*d
 + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/d^5 - 1/8*sqrt(2)*(13*(c*d^3)^(1/4)*
b^2*c^2 - 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/d
^5 - 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*
b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/d^5 + 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18
*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqr
t(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^5 + 1/2*(b^2*c^3*sqrt(x) - 2*
a*b*c^2*d*sqrt(x) + a^2*c*d^2*sqrt(x))/((d*x^2 + c)*d^4) + 2/45*(
5*b^2*d^16*x^(9/2) - 18*b^2*c*d^15*x^(5/2) + 18*a*b*d^16*x^(5/2)
+ 135*b^2*c^2*d^14*sqrt(x) - 180*a*b*c*d^15*sqrt(x) + 45*a^2*d^16
*sqrt(x))/d^18
```

3.426 $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=346

$$\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}} - \frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}} - \frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{15/4}} + \frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}d^{15/4}} - \frac{x^{3/2}(11bc-3ad)(bc-ad)}{6cd^3} + \frac{x^{7/2}(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{2b^2x^{7/2}}{7d^2}$$

```
[Out] -((11*b*c - 3*a*d)*(b*c - a*d)*x^(3/2))/(6*c*d^3) + (2*b^2*x^(7/2
))/(7*d^2) + ((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) - ((11
*b*c - 3*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a
*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(1
5/4)) - ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(15/4))
```

Rubi [A] time = 0.697139, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}} - \frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}} - \frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{15/4}} + \frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}d^{15/4}} - \frac{x^{3/2}(11bc-3ad)(bc-ad)}{6cd^3} + \frac{x^{7/2}(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{2b^2x^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```


```
[Out] -((11*b*c - 3*a*d)*(b*c - a*d)*x^(3/2))/(6*c*d^3) + (2*b^2*x^(7/2
))/(7*d^2) + ((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) - ((11
*b*c - 3*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a
*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(1
5/4)) - ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(15/4))
```

Rubi in Sympy [A] time = 111.288, size = 321, normalized size = 0.93

$$\frac{2b^2x^{\frac{7}{2}}}{7d^2}+\frac{x^{\frac{7}{2}}(ad-bc)^2}{2cd^2(c+dx^2)}-\frac{x^{\frac{3}{2}}(ad-bc)(3ad-11bc)}{6cd^3}+\frac{\sqrt{2}(ad-bc)(3ad-11bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16\sqrt[4]{c}d^{\frac{15}{4}}}-\frac{\sqrt{2}(ad-bc)(3ad-11bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16\sqrt[4]{c}d^{\frac{15}{4}}}-\frac{\sqrt{2}(ad-bc)(3ad-11bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8\sqrt[4]{c}d^{\frac{15}{4}}}+\frac{\sqrt{2}(ad-bc)(3ad-11bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8\sqrt[4]{c}d^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] 2*b**2*x**(7/2)/(7*d**2) + x**(7/2)*(a*d - b*c)**2/(2*c*d**2*(c +
 d*x**2)) - x**(3/2)*(a*d - b*c)*(3*a*d - 11*b*c)/(6*c*d**3) + sq
rt(2)*(a*d - b*c)*(3*a*d - 11*b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)
*sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**(1/4)*d**(15/4)) - sqrt(2)
*(a*d - b*c)*(3*a*d - 11*b*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(
x) + sqrt(c) + sqrt(d)*x)/(16*c**(1/4)*d**(15/4)) - sqrt(2)*(a*d
- b*c)*(3*a*d - 11*b*c)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4
))/(8*c**(1/4)*d**(15/4)) + sqrt(2)*(a*d - b*c)*(3*a*d - 11*b*c)*
atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(1/4)*d**(15/4)
)
```

Mathematica [A] time = 0.311673, size = 337, normalized size = 0.97

$$\frac{\frac{21\sqrt{2}(3a^2d^2-14abcd+11b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}(3a^2d^2-14abcd+11b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}(3a^2d^2-14abcd+11b}{\sqrt[4]{}}}{336}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-448*b*d^(3/4)*(b*c - a*d)*x^(3/2) + 96*b^2*d^(7/4)*x^(7/2) - (1
68*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2) - (42*Sqrt[2]*(11*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/c^(1/4) + (42*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(1/4)
+ (21*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(1/4) - (21*Sqrt
[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(1/4))/(336*d^(15/4))
```

Maple [A] time = 0.025, size = 523, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/7*b^2*x^(7/2)/d^2+4/3*b/d^2*x^(3/2)*a-4/3*b^2/d^3*x^(3/2)*c-1/2
/d*x^(3/2)/(d*x^2+c)*a^2+1/d^2*x^(3/2)/(d*x^2+c)*c*a*b-1/2/d^3*x^
```

```
(3/2)/(d*x^2+c)*b^2*c^2-7/8/d^3/(c/d)^(1/4)*2^(1/2)*c*a*b*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))-7/4/d^3/(c/d)^(1/4)*2^(1/2)*c*a*b*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)-7/4/d^3/(c/d)^(1/4)*2^(1/2)*c*a*b*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+11/16/d^4/(c/d)^(1/4)*2^(1/2)*b
^2*c^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+11/8/d^4/(c/d)^(1/4)*2^(1/2)*b^
2*c^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+11/8/d^4/(c/d)^(1/4)*
2^(1/2)*b^2*c^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+3/16/d^2/(c
/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+3/8/d^2/(c/d)^(1
/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3/8/d^2/(c/
d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278114, size = 2029, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d
+ 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^
4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*
a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*arctan(c*d^11*(-(14641*
b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3
*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 1
1772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(
3/4)/((1331*b^6*c^6 - 5082*a*b^5*c^5*d + 7557*a^2*b^4*c^4*d^2 - 5
516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a^5*b*c*d^5 + 27
*a^6*d^6)*sqrt(x) + sqrt((1771561*b^12*c^12 - 13528284*a*b^11*c^1
1*d + 45943458*a^2*b^10*c^10*d^2 - 91492940*a^3*b^9*c^9*d^3 + 118
659255*a^4*b^8*c^8*d^4 - 105323064*a^5*b^7*c^7*d^5 + 65490076*a^6
*b^6*c^6*d^6 - 28724472*a^7*b^5*c^5*d^7 + 8825895*a^8*b^4*c^4*d^8
 - 1855980*a^9*b^3*c^3*d^9 + 254178*a^10*b^2*c^2*d^10 - 20412*a^1
1*b*c*d^11 + 729*a^12*d^12)*x - (14641*b^8*c^9*d^7 - 74536*a*b^7*
c^8*d^8 + 158268*a^2*b^6*c^7*d^9 - 181720*a^3*b^5*c^6*d^10 + 1225
66*a^4*b^4*c^5*d^11 - 49560*a^5*b^3*c^4*d^12 + 11772*a^6*b^2*c^3*
d^13 - 1512*a^7*b*c^2*d^14 + 81*a^8*c*d^15)*sqrt(-(14641*b^8*c^8
- 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5
*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6
*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))))) + 21*(
d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^
2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 -
 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7
 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(c*d^11*(-(14641*b^8*c^8 - 7453
6*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 +
 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c
^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b
^6*c^6 - 5082*a*b^5*c^5*d + 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c
^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a^5*b*c*d^5 + 27*a^6*d^6)*sqr
t(x)) - 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d
 + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b
^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512
```

```
*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(-c*d^11*(-(14641*b
^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*
b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11
772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3
/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d + 7557*a^2*b^4*c^4*d^2 - 5
516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a^5*b*c*d^5 + 27
*a^6*d^6)*sqrt(x)) + 4*(12*b^2*d^2*x^5 - 4*(11*b^2*c*d - 14*a*b*d
^2)*x^3 - 7*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(d^
4*x^2 + c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.246771, size = 558, normalized size = 1.61

$$-\frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)d^3} + \frac{\sqrt{2}\left(11\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 14\,\left(cd^3\right)^{\frac{3}{4}}abcd + 3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^6} + \frac{\sqrt{2}\left(11\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 14\,\left(cd^3\right)^{\frac{3}{4}}abcd + 3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^6} - \frac{\sqrt{2}\left(11\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 14\,\left(cd^3\right)^{\frac{3}{4}}abcd + 3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,cd^6} + \frac{\sqrt{2}\left(11\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 14\,\left(cd^3\right)^{\frac{3}{4}}abcd + 3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,cd^6} + \frac{2\left(3\,b^2d^{12}x^{\frac{7}{2}} - 14\,b^2cd^{11}x^{\frac{3}{2}} + 14\,abd^{12}x^{\frac{3}{2}}\right)}{21\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*
x^2 + c)*d^3) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3
)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sq
rt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c*d^6) + 1/8*sqrt(2)
*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)
^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt
(x))/(c/d)^(1/4))/(c*d^6) - 1/16*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^
2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*ln(sqrt(2
)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 1/16*sqrt(2)*(11
*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/
4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d
^6) + 2/21*(3*b^2*d^12*x^(7/2) - 14*b^2*c*d^11*x^(3/2) + 14*a*b*d
^12*x^(3/2))/d^14
```

3.427 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}-\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{3/4}d^{13/4}}-\frac{\sqrt{x}(bc-ad)(9bc-ad)}{2cd^3}+\frac{x^{5/2}(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{2b^2x^{5/2}}{5d^2}$$

```
[Out] -((b*c - a*d)*(9*b*c - a*d)*Sqrt[x])/(2*c*d^3) + (2*b^2*x^(5/2))/
(5*d^2) + ((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) - ((b*c -
 a*d)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*d^(13/4)) + ((b*c - a*d)*(9*b*c - a*d)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1
3/4)) - ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4)) + ((b*
c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
```

Rubi [A] time = 0.679314, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}-\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{3/4}d^{13/4}}-\frac{\sqrt{x}(bc-ad)(9bc-ad)}{2cd^3}+\frac{x^{5/2}(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{2b^2x^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```


```
[Out] -((b*c - a*d)*(9*b*c - a*d)*Sqrt[x])/(2*c*d^3) + (2*b^2*x^(5/2))/
(5*d^2) + ((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) - ((b*c -
 a*d)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*d^(13/4)) + ((b*c - a*d)*(9*b*c - a*d)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1
3/4)) - ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4)) + ((b*
c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
```

Rubi in Sympy [A] time = 109.982, size = 313, normalized size = 0.9

$$\frac{2b^2x^{\frac{5}{2}}}{5d^2}+\frac{x^{\frac{5}{2}}(ad-bc)^2}{2cd^2(c+dx^2)}-\frac{\sqrt{x}(ad-9bc)(ad-bc)}{2cd^3}-\frac{\sqrt{2}(ad-9bc)(ad-bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{3}{4}}d^{\frac{13}{4}}}+\frac{\sqrt{2}(ad-9bc)(ad-bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{3}{4}}d^{\frac{13}{4}}}-\frac{\sqrt{2}(ad-9bc)(ad-bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{3}{4}}d^{\frac{13}{4}}}+\frac{\sqrt{2}(ad-9bc)(ad-bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{3}{4}}d^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] 2*b**2*x**(5/2)/(5*d**2) + x**(5/2)*(a*d - b*c)**2/(2*c*d**2*(c +
 d*x**2)) - sqrt(x)*(a*d - 9*b*c)*(a*d - b*c)/(2*c*d**3) - sqrt(2
)*(a*d - 9*b*c)*(a*d - b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x
) + sqrt(c) + sqrt(d)*x)/(16*c**(3/4)*d**(13/4)) + sqrt(2)*(a*d -
 9*b*c)*(a*d - b*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(
c) + sqrt(d)*x)/(16*c**(3/4)*d**(13/4)) - sqrt(2)*(a*d - 9*b*c)*(
a*d - b*c)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(3/4
)*d**(13/4)) + sqrt(2)*(a*d - 9*b*c)*(a*d - b*c)*atan(1 + sqrt(2)
*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(3/4)*d**(13/4))
```

Mathematica [A] time = 0.303945, size = 333, normalized size = 0.96

$$\frac{-\frac{5\sqrt{2}(a^2d^2-10abcd+9b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}+\frac{5\sqrt{2}(a^2d^2-10abcd+9b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}-\frac{10\sqrt{2}(a^2d^2-10abcd+9b^2c^2)\tan}{c^{3/4}}}{80d^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-320*b*d^(1/4)*(b*c - a*d)*Sqrt[x] + 32*b^2*d^(5/4)*x^(5/2) - (4
0*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2) - (10*Sqrt[2]*(9*b^2
*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/c^(3/4) + (10*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^
2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4) - (5*Sq
rt[2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(3/4) + (5*Sqrt[2]*(9*b^2*c
^2 - 10*a*b*c*d + a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/c^(3/4))/(80*d^(13/4))
```

Maple [A] time = 0.023, size = 523, normalized size = 1.5

$$\frac{2b^2}{5d^2}x^{\frac{5}{2}} + 4\frac{ab\sqrt{x}}{d^2} - 4\frac{b^2\sqrt{x}c}{d^3} - \frac{a^2}{2d(dx^2+c)}\sqrt{x} + \frac{abc}{d^2(dx^2+c)}\sqrt{x} - \frac{b^2c^2}{2d^3(dx^2+c)}\sqrt{x} + \frac{\sqrt{2}a^2}{8cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) - \frac{5\sqrt{2}ab}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{9c\sqrt{2}b^2}{8d^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{\sqrt{2}a^2}{8cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) - \frac{5\sqrt{2}ab}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{9c\sqrt{2}b^2}{8d^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{\sqrt{2}a^2}{16cd}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) - \frac{5\sqrt{2}ab}{8d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) + \frac{9c\sqrt{2}b^2}{16d^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/5*b^2*x^(5/2)/d^2+4/d^2*b*a*x^(1/2)-4/d^3*b^2*x^(1/2)*c-1/2/d*x
^(1/2)/(d*x^2+c)*a^2+1/d^2*x^(1/2)/(d*x^2+c)*c*a*b-1/2/d^3*x^(1/2
)/(d*x^2+c)*b^2*c^2+1/8/d*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)+1)*a^2-5/4/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+9/8/d^3*(c/d)^(1/4)*c*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/8/d*(c/d)^(1/4)/c*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-5/4/d^2*(c/d)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+9/8/d^3*(c/d)^(1/4
)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/16/d*(c/d
)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-5/8/d^2*(c/d)^(1
/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+9/16/d^3*(c/d)^(1/4)*c
*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269553, size = 1482, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d +
 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^
4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d
^7 + a^8*d^8)/(c^3*d^13))^(1/4)*arctan(c*d^3*(-(6561*b^8*c^8 - 29
160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 +
 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d
^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)/((9*b^2*c^2 - 10
*a*b*c*d + a^2*d^2)*sqrt(x) + sqrt(c^2*d^6*sqrt(-(6561*b^8*c^8 -
29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3
 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2
*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13)) + (81*b^4*c^4 - 180*
a*b^3*c^3*d + 118*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + a^4*d^4)*x))
) - 5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 515
16*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^
4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 +
 a^8*d^8)/(c^3*d^13))^(1/4)*log(c*d^3*(-(6561*b^8*c^8 - 29160*a*b
^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*
a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40
*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 10*a*b*c
*d + a^2*d^2)*sqrt(x)) + 5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29
160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 +
 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d
^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*log(-c*d^3*(-(65
61*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^
3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 63
6*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) +
 (9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(4*b^2*d^2*x^4 -
 45*b^2*c^2 + 50*a*b*c*d - 5*a^2*d^2 - 4*(9*b^2*c*d - 10*a*b*d^2)
*x^2)*sqrt(x))/(d^4*x^2 + c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.24736, size = 551, normalized size = 1.59

$$\frac{\sqrt{2}\left(9\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^4} + \frac{\sqrt{2}\left(9\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^4} + \frac{\sqrt{2}\left(9\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,cd^4} - \frac{\sqrt{2}\left(9\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,cd^4} - \frac{b^2c^2\sqrt{x} - 2\,abcd\sqrt{x} + a^2d^2\sqrt{x}}{2\,(dx^2 + c)d^3} + \frac{2\left(b^2d^8x^{\frac{5}{2}} - 10\,b^2cd^7\sqrt{x} + 10\,abd^8\sqrt{x}\right)}{5\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d +
 (c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
 2*sqrt(x))/(c/d)^(1/4))/(c*d^4) + 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b
^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d
^4) + 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c*d^4) - 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 1
0*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt
(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^4) - 1/2*(b^2*c^2*sqrt(x) -
 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*d^3) + 2/5*(b^
2*d^8*x^(5/2) - 10*b^2*c*d^7*sqrt(x) + 10*a*b*d^8*sqrt(x))/d^10
```

3.428 $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}d^{11/4}}-\frac{(bc-ad)(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}d^{11/4}}+\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{2b^2x^{3/2}}{3d^2}$$

```
[Out] (2*b^2*x^(3/2))/(3*d^2) + ((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d
*x^2)) + ((b*c - a*d)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*
b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*
d^(11/4)) + ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*d^(11/4))
```

Rubi [A] time = 0.615305, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}d^{11/4}}-\frac{(bc-ad)(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}d^{11/4}}+\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{2b^2x^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```



```
[Out] (2*b^2*x^(3/2))/(3*d^2) + ((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d
*x^2)) + ((b*c - a*d)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*
b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*
d^(11/4)) + ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*d^(11/4))
```

Rubi in Sympy [A] time = 103.585, size = 286, normalized size = 0.92

$$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{x^{\frac{3}{2}}\left(ad-bc\right)^2}{2cd^2\left(c+dx^2\right)}+\frac{\sqrt{2}\left(ad-bc\right)\left(ad+7bc\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{5}{4}}d^{\frac{11}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)\left(ad+7bc\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{5}{4}}d^{\frac{11}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)\left(ad+7bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{5}{4}}d^{\frac{11}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)\left(ad+7bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{5}{4}}d^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] 2*b**2*x**(3/2)/(3*d**2) + x**(3/2)*(a*d - b*c)**2/(2*c*d**2*(c +
 d*x**2)) + sqrt(2)*(a*d - b*c)*(a*d + 7*b*c)*log(-sqrt(2)*c**(1/
4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**(5/4)*d**(11/4)
) - sqrt(2)*(a*d - b*c)*(a*d + 7*b*c)*log(sqrt(2)*c**(1/4)*d**(1/
4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**(5/4)*d**(11/4)) - sqrt(
2)*(a*d - b*c)*(a*d + 7*b*c)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c*
*(1/4))/(8*c**(5/4)*d**(11/4)) + sqrt(2)*(a*d - b*c)*(a*d + 7*b*c
)*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(5/4)*d**(11/
4))
```

Mathematica [A] time = 0.301308, size = 319, normalized size = 1.03

$$\frac{-\frac{3\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{3\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{6\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\tan}{c^{5/4}}}{48d^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (32*b^2*d^(3/4)*x^(3/2) + (24*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c*(
c + d*x^2)) + (6*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (6*Sqrt[2]*(7*
b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)])/c^(5/4) - (3*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^
2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^
(5/4) + (3*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(48*d^(1
1/4))
```

Maple [B] time = 0.026, size = 499, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 2/3*b^2*x^(3/2)/d^2+1/2/c*x^(3/2)/(d*x^2+c)*a^2-1/d*x^(3/2)/(d*x^
2+c)*a*b+1/2/d^2*c*x^(3/2)/(d*x^2+c)*b^2+1/8/d/c/(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/4/d^2/(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-7/8/d^3*c/(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/8/d/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/
4/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a
*b-7/8/d^3*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)-1)*b^2+1/16/d/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
*a^2+3/8/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-7
/16/d^3*c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270553, size = 2021, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d
 + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4
*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7
 + a^8*d^8)/(c^5*d^11))^(1/4)*arctan(-c^4*d^8*(-(2401*b^8*c^8 - 8
232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1
434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 +
 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4)/((343*b^6*c^6 - 882*
a*b^5*c^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b
^2*c^2*d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x) - sqrt((117649*b^1
2*c^12 - 605052*a*b^11*c^11*d + 1195698*a^2*b^10*c^10*d^2 - 10495
80*a^3*b^9*c^9*d^3 + 247695*a^4*b^8*c^8*d^4 + 184968*a^5*b^7*c^7*
d^5 - 73604*a^6*b^6*c^6*d^6 - 26424*a^7*b^5*c^5*d^7 + 5055*a^8*b^
4*c^4*d^8 + 3060*a^9*b^3*c^3*d^9 + 498*a^10*b^2*c^2*d^10 + 36*a^1
1*b*c*d^11 + a^12*d^12)*x - (2401*b^8*c^11*d^5 - 8232*a*b^7*c^10*
d^6 + 9212*a^2*b^6*c^9*d^7 - 2520*a^3*b^5*c^8*d^8 - 1434*a^4*b^4*
c^7*d^9 + 360*a^5*b^3*c^6*d^10 + 188*a^6*b^2*c^5*d^11 + 24*a^7*b*
c^4*d^12 + a^8*c^3*d^13)*sqrt(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d +
 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d
^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 +
 a^8*d^8)/(c^5*d^11))))) + 3*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^
8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^
3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*
d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)*log(c^4*d^8*(-(
2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3
*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a
^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (3
43*b^6*c^6 - 882*a*b^5*c^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c
^3*d^3 - 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x))
- 3*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 92
12*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4
+ 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^
8*d^8)/(c^5*d^11))^(1/4)*log(-c^4*d^8*(-(2401*b^8*c^8 - 8232*a*b^
7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*
b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*
b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (343*b^6*c^6 - 882*a*b^5*c
^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b^2*c^2*
d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x)) + 4*(4*b^2*c*d*x^3 + (7*
b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(c*d^3*x^2 + c^2*d^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.2549, size = 524, normalized size = 1.69

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^2}+\frac{b^2c^2x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)cd^2}-\frac{\sqrt{2}\left(7\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-6\,\left(cd^3\right)^{\frac{3}{4}}abcd-\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}-\frac{\sqrt{2}\left(7\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-6\,\left(cd^3\right)^{\frac{3}{4}}abcd-\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}+\frac{\sqrt{2}\left(7\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-6\,\left(cd^3\right)^{\frac{3}{4}}abcd-\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^2d^5}-\frac{\sqrt{2}\left(7\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-6\,\left(cd^3\right)^{\frac{3}{4}}abcd-\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^2 + 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) +
a^2*d^2*x^(3/2))/((d*x^2 + c)*c*d^2) - 1/8*sqrt(2)*(7*(c*d^3)^(3/
4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c
^2*d^5) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*
a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/
d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) + 1/16*sqrt(2)*(7*(c
*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2
*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5) -
 1/16*sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d
- (c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/(c^2*d^5)
```

3.429 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2}\,dx$

Optimal. Leaf size=312

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{7/4}d^{9/4}} + \frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{2b^2\sqrt{x}}{d^2}$$

```
[Out] (2*b^2*Sqrt[x])/d^2 + ((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2
)) + ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[
2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*
d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rubi [A] time = 0.709809, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{7/4}d^{9/4}} + \frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{2b^2\sqrt{x}}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (2*b^2*Sqrt[x])/d^2 + ((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2
)) + ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[
2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*
d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rubi in Sympy [A] time = 96.4479, size = 291, normalized size = 0.93

$$\frac{2b^2\sqrt{x}}{d^2} + \frac{\sqrt{x}(ad-bc)^2}{2cd^2(c+dx^2)} - \frac{\sqrt{2}(ad-bc)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{7}{4}}d^{\frac{9}{4}}} + \frac{\sqrt{2}(ad-bc)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{7}{4}}d^{\frac{9}{4}}} - \frac{\sqrt{2}(ad-bc)(3ad+5bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{7}{4}}d^{\frac{9}{4}}} + \frac{\sqrt{2}(ad-bc)(3ad+5bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{7}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] 2*b**2*sqrt(x)/d**2 + sqrt(x)*(a*d - b*c)**2/(2*c*d**2*(c + d*x**
2)) - sqrt(2)*(a*d - b*c)*(3*a*d + 5*b*c)*log(-sqrt(2)*c**(1/4)*d
**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**(7/4)*d**(9/4)) + s
qrt(2)*(a*d - b*c)*(3*a*d + 5*b*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*
sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**(7/4)*d**(9/4)) - sqrt(2)*(
a*d - b*c)*(3*a*d + 5*b*c)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(
1/4))/(8*c**(7/4)*d**(9/4)) + sqrt(2)*(a*d - b*c)*(3*a*d + 5*b*c)
*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(7/4)*d**(9/4)
)
```

Mathematica [A] time = 0.304361, size = 318, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}-\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}+\frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan}{c^{7/4}}}{16d^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (32*b^2*d^(1/4)*Sqrt[x] + (8*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c*(c
 + d*x^2)) + (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2*Sqrt[2]*(5
*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2
*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/c^(7/4) - (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(16*d
^(9/4))
```

Maple [B] time = 0.023, size = 496, normalized size = 1.6

$$
\begin{aligned}
&2\,\frac{b^2\sqrt{x}}{d^2}+\frac{a^2}{2\,c\,(dx^2+c)}\sqrt{x}-\frac{ab}{d\,(dx^2+c)}\sqrt{x}+\frac{b^2c}{2\,d^2\,(dx^2+c)}\sqrt{x}\\
&+\frac{3\,\sqrt{2}a^2}{8\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}ab}{4\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\\
&-\frac{5\,\sqrt{2}b^2}{8\,d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{3\,\sqrt{2}a^2}{8\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\\
&+\frac{\sqrt{2}ab}{4\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{5\,\sqrt{2}b^2}{8\,d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\\
&+\frac{3\,\sqrt{2}a^2}{16\,c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\\
&+\frac{\sqrt{2}ab}{8\,cd}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\\
&-\frac{5\,\sqrt{2}b^2}{16\,d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 2*b^2*x^(1/2)/d^2+1/2/c*x^(1/2)/(d*x^2+c)*a^2-1/d*x^(1/2)/(d*x^2+
c)*a*b+1/2/d^2*c*x^(1/2)/(d*x^2+c)*b^2+3/8/c^2*(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/4/d/c*(c/d)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-5/8/d^2*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/8/c^2*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+1/4/d/
c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-5
/8/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1
/8/d/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-5/16/d^
2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269569, size = 1497, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d -
 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4
 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 +
81*a^8*d^8)/(c^7*d^9))^(1/4)*arctan(-c^2*d^2*(-(625*b^8*c^8 - 100
0*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*
a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)/((5*b^2*c^2 - 2*a*b*c
*d - 3*a^2*d^2)*sqrt(x) - sqrt(c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*
a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^
4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a
^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*b^4*c^4 - 20*a*b^3*c^3*
d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)*x))) - (c*d^
3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*
c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^
3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(
c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 9
00*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81
*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*
sqrt(x)) + (c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*
d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7
 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 100
0*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*
a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*
c*d - 3*a^2*d^2)*sqrt(x)) - 4*(4*b^2*c*d*x^2 + 5*b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(x))/(c*d^3*x^2 + c^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.238933, size = 524, normalized size = 1.68

$$\frac{2\,b^{2}\sqrt{x}}{d^{2}} - \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd - 3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{2}d^{3}} - \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd - 3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{2}d^{3}} - \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd - 3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^{2}d^{3}} + \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd - 3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^{2}d^{3}} + \frac{b^{2}c^{2}\sqrt{x} - 2\,abcd\sqrt{x} + a^{2}d^{2}\sqrt{x}}{2\,(dx^{2} + c)cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*sqrt(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d^2 - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d
^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(
sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/8*sqr
t(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^
3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sq
rt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2
*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*ln(sqrt
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/16*sqrt(2)
*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(
1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c
^2*d^3) + 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt
(x))/((d*x^2 + c)*c*d^2)
```

3.430 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{x^{3/2}(5a^2d^2-2abcd+b^2c^2)}{2c^2d(c+dx^2)}-\frac{2a^2}{c\sqrt{x}(c+dx^2)}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}d^{7/4}}+\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}d^{7/4}}$$

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*
d^2)*x^(3/2))/(2*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(3*b*c + 5*a*d
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/
4)*d^(7/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) + ((b*c - a*
d)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*d^(7/4)) - ((b*c - a*d)*(3*b*c +
 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(8*Sqrt[2]*c^(9/4)*d^(7/4))
```

Rubi [A] time = 0.713546, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{x^{3/2}(5a^2d^2-2abcd+b^2c^2)}{2c^2d(c+dx^2)}-\frac{2a^2}{c\sqrt{x}(c+dx^2)}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}d^{7/4}}+\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```


```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*
d^2)*x^(3/2))/(2*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(3*b*c + 5*a*d
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/
4)*d^(7/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) + ((b*c - a*
d)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*d^(7/4)) - ((b*c - a*d)*(3*b*c +
 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(8*Sqrt[2]*c^(9/4)*d^(7/4))
```

Rubi in Sympy [A] time = 96.6386, size = 308, normalized size = 0.92

$$-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}-\frac{x^{\frac{3}{2}}\left(ad\,(5ad-2bc)+b^2c^2\right)}{2c^2d\,(c+dx^2)}-\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{9}{4}}d^{\frac{7}{4}}}+\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{9}{4}}d^{\frac{7}{4}}}+\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{9}{4}}d^{\frac{7}{4}}}-\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{9}{4}}d^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] -2*a**2/(c*sqrt(x)*(c + d*x**2)) - x**(3/2)*(a*d*(5*a*d - 2*b*c)
+ b**2*c**2)/(2*c**2*d*(c + d*x**2)) - sqrt(2)*(a*d - b*c)*(5*a*d
 + 3*b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt
(d)*x)/(16*c**(9/4)*d**(7/4)) + sqrt(2)*(a*d - b*c)*(5*a*d + 3*b*
c)*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(
16*c**(9/4)*d**(7/4)) + sqrt(2)*(a*d - b*c)*(5*a*d + 3*b*c)*atan(
1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(9/4)*d**(7/4)) - sq
rt(2)*(a*d - b*c)*(5*a*d + 3*b*c)*atan(1 + sqrt(2)*d**(1/4)*sqrt(
x)/c**(1/4))/(8*c**(9/4)*d**(7/4))
```

Mathematica [A] time = 0.303842, size = 317, normalized size = 0.95

$$\frac{\frac{\sqrt{2}\left(-5a^2d^2+2abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}}+\frac{\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}}+\frac{2\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\tan^{-1}\left(\right.}{d^{7/4}}}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(1/4))/Sqrt[x] - (8*c^(1/4)*(b*c - a*d)^2*x^(3/2))/(d
*(c + d*x^2)) + (2*Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (2*Sqrt[2
]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/d^(7/4) + (Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5
*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/d^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4))/
(16*c^(9/4))
```

Maple [A] time = 0.026, size = 495, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x^(3/2)/(d*x^2+c)*a^2+1/c*x^(3/2)/(d*x^2+c)*a*b-1/2/d*
x^(3/2)/(d*x^2+c)*b^2-5/8/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)*a^2+1/4/c/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/8/d^2/(c/d)^(1/4)*2^(1/2)*arcta
```

```
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-5/16/c^2/(c/d)^(1/4)*2^(1/2)
*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+1/8/c/d/(c/d)^(1/4)*2^(1/2)*ln((x
-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2)))*a*b+3/16/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2)))*b^2-5/8/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)*a^2+1/4/c/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/8/d^2/(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2*a^2/c^2/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268178, size = 2039, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/8*(16*a^2*c*d + 4*(b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2 + 4*(c
^2*d^2*x^2 + c^3*d)*sqrt(x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 16
40*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625
*a^8*d^8)/(c^9*d^7))^(1/4)*arctan(-c^7*d^5*(-(81*b^8*c^8 + 216*a*
b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b
^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^
7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4)/((27*b^6*c^6 + 54*a*b^5
*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c
^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*sqrt(x) - sqrt((729*b^12*
c^12 + 2916*a*b^11*c^11*d - 2430*a^2*b^10*c^10*d^2 - 19980*a^3*b^
9*c^9*d^3 + 135*a^4*b^8*c^8*d^4 + 59976*a^5*b^7*c^7*d^5 + 6364*a^
6*b^6*c^6*d^6 - 99960*a^7*b^5*c^5*d^7 + 375*a^8*b^4*c^4*d^8 + 925
00*a^9*b^3*c^3*d^9 - 18750*a^10*b^2*c^2*d^10 - 37500*a^11*b*c*d^1
1 + 15625*a^12*d^12)*x - (81*b^8*c^13*d^3 + 216*a*b^7*c^12*d^4 -
324*a^2*b^6*c^11*d^5 - 984*a^3*b^5*c^10*d^6 + 646*a^4*b^4*c^9*d^7
 + 1640*a^5*b^3*c^8*d^8 - 900*a^6*b^2*c^7*d^9 - 1000*a^7*b*c^6*d^
10 + 625*a^8*c^5*d^11)*sqrt(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*
a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*
a^8*d^8)/(c^9*d^7))))) + (c^2*d^2*x^2 + c^3*d)*sqrt(x)*(-(81*b^8*
c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3
 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d
^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(c^7*d^5
*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*
b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^
6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4)
- (27*b^6*c^6 + 54*a*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3
*c^3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*s
qrt(x)) - (c^2*d^2*x^2 + c^3*d)*sqrt(x)*(-(81*b^8*c^8 + 216*a*b^7
*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*
c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(-c^7*d^5*(-(81*b^8*c^8
 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 +
646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6
- 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4) - (27*b^6*c^6
```

```
+ 54*a*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165
*a^4*b^2*c^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*sqrt(x)))/((c^2
*d^2*x^2 + c^3*d)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26034, size = 525, normalized size = 1.58

$$-\frac{b^2c^2x^2 - 2\,abcdx^2 + 5\,a^2d^2x^2 + 4\,a^2cd}{2\left(dx^{\frac{5}{2}} + c\sqrt{x}\right)c^2d} + \frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 + 2\,\left(cd^3\right)^{\frac{3}{4}}abcd - 5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4} + \frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 + 2\,\left(cd^3\right)^{\frac{3}{4}}abcd - 5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4} - \frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 + 2\,\left(cd^3\right)^{\frac{3}{4}}abcd - 5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^3d^4} + \frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2 + 2\,\left(cd^3\right)^{\frac{3}{4}}abcd - 5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(3/2)),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 5*a^2*d^2*x^2 + 4*a^2*c*d)/((
d*x^(5/2) + c*sqrt(x))*c^2*d) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*
c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^
4) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c
*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) - 1/16*sqrt(2)*(3*(c*d^
3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*
d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4) +
1/16*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d -
 5*(c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c^3*d^4)
```

3.431 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt{x}\left(7a^2d^2-6abcd+3b^2c^2\right)}{6c^2d\left(c+dx^2\right)}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}-\frac{(bc-ad)(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}d^{5/4}}$$

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) - ((3*b^2*c^2 - 6*a*b*c*d + 7*
a^2*d^2)*Sqrt[x])/(6*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(b*c + 7*a
*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
11/4)*d^(5/4)) + ((b*c - a*d)*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) - ((b*c -
a*d)*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b*c +
7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(8*Sqrt[2]*c^(11/4)*d^(5/4))
```

Rubi [A] time = 0.697849, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{\sqrt{x}\left(7a^2d^2-6abcd+3b^2c^2\right)}{6c^2d\left(c+dx^2\right)}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}-\frac{(bc-ad)(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) - ((3*b^2*c^2 - 6*a*b*c*d + 7*
a^2*d^2)*Sqrt[x])/(6*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(b*c + 7*a
*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
11/4)*d^(5/4)) + ((b*c - a*d)*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) - ((b*c -
a*d)*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b*c +
7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(8*Sqrt[2]*c^(11/4)*d^(5/4))
```

Rubi in Sympy [A] time = 95.7014, size = 304, normalized size = 0.92

$$-\frac{2a^2}{3cx^{\frac{3}{2}}(c+dx^2)}-\frac{\sqrt{x}\left(ad(7ad-6bc)+3b^2c^2\right)}{6c^2d(c+dx^2)}+\frac{\sqrt{2}(ad-bc)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{11}{4}}d^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{11}{4}}d^{\frac{5}{4}}}+\frac{\sqrt{2}(ad-bc)(7ad+bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{11}{4}}d^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)(7ad+bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{11}{4}}d^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] -2*a**2/(3*c*x**(3/2)*(c + d*x**2)) - sqrt(x)*(a*d*(7*a*d - 6*b*c
) + 3*b**2*c**2)/(6*c**2*d*(c + d*x**2)) + sqrt(2)*(a*d - b*c)*(7
*a*d + b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sq
rt(d)*x)/(16*c**(11/4)*d**(5/4)) - sqrt(2)*(a*d - b*c)*(7*a*d + b
*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/
(16*c**(11/4)*d**(5/4)) + sqrt(2)*(a*d - b*c)*(7*a*d + b*c)*atan(
1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c**(11/4)*d**(5/4)) - s
qrt(2)*(a*d - b*c)*(7*a*d + b*c)*atan(1 + sqrt(2)*d**(1/4)*sqrt(x
)/c**(1/4))/(8*c**(11/4)*d**(5/4))
```

Mathematica [A] time = 0.320692, size = 315, normalized size = 0.95

$$\frac{-\frac{3\sqrt{2}(-7a^2d^2+6abcd+b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{3\sqrt{2}(-7a^2d^2+6abcd+b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}-\frac{6\sqrt{2}(-7a^2d^2+6abcd+b^2c^2)\tan}{d^{5/4}}}{48c^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(3/4))/x^(3/2) - (24*c^(3/4)*(b*c - a*d)^2*Sqrt[x])/(
d*(c + d*x^2)) - (6*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (6*Sqrt[2]*
(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/d^(5/4) - (3*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2
*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/d^(5/4) + (3*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4))/(48*c
^(11/4))
```

Maple [A] time = 0.027, size = 498, normalized size = 1.5

$$-\frac{a^2 d}{2 c^2 (d x^2 + c)}\sqrt{x} + \frac{ab}{c (d x^2 + c)}\sqrt{x} - \frac{b^2}{2 d (d x^2 + c)}\sqrt{x} - \frac{7 d\sqrt{2}a^2}{8 c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{3\sqrt{2}ab}{4 c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{\sqrt{2}b^2}{8 cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) - \frac{7 d\sqrt{2}a^2}{8 c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{3\sqrt{2}ab}{4 c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{\sqrt{2}b^2}{8 cd}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) - \frac{7 d\sqrt{2}a^2}{16 c^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) + \frac{3\sqrt{2}ab}{8 c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) + \frac{\sqrt{2}b^2}{16 cd}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) - \frac{2 a^2}{3 c^2}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x^(1/2)/(d*x^2+c)*a^2+1/c*x^(1/2)/(d*x^2+c)*a*b-1/2/d*
x^(1/2)/(d*x^2+c)*b^2-7/8/c^3*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/8/c/d*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-7/8/c^3*d*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/4/c^2*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+1/8/c/d*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-7/16/c^3*
d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+3/8/c^2*(c/d
)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x
-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+1/16/c/d*(c/d)^(1/
4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3*a^2/c^2/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269612, size = 1494, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^2*c*d + 4*(3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2 - 1
2*(c^2*d^2*x^3 + c^3*d*x)*sqrt(x)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 1
88*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 -
 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 +
 2401*a^8*d^8)/(c^11*d^5))^(1/4)*arctan(-c^3*d*(-(b^8*c^8 + 24*a*
b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*
b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*
a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4)/((b^2*c^2 + 6*a*b*c
*d - 7*a^2*d^2)*sqrt(x) - sqrt(c^6*d^2*sqrt(-(b^8*c^8 + 24*a*b^7*
c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*
c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*
b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)) + (b^4*c^4 + 12*a*b^3*c^3*d +
 22*a^2*b^2*c^2*d^2 - 84*a^3*b*c*d^3 + 49*a^4*d^4)*x))) + 3*(c^2*
d^2*x^3 + c^3*d*x)*sqrt(x)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*
b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a
^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a
^8*d^8)/(c^11*d^5))^(1/4)*log(c^3*d*(-(b^8*c^8 + 24*a*b^7*c^7*d +
 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4
 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7
 + 2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2
*d^2)*sqrt(x)) - 3*(c^2*d^2*x^3 + c^3*d*x)*sqrt(x)*(-(b^8*c^8 + 2
4*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*
a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8
232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4)*log(-c^3*d*(-(b
^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d
^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c
^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^
2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)))/((c^2*d^2*x^3 + c^3*d*x)
*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.2577, size = 518, normalized size = 1.56

$$-\frac{2\,a^2}{3\,c^2x^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd-7\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd-7\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd-7\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^3d^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd-7\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^3d^2}-\frac{b^2c^2\sqrt{x}-2\,abcd\sqrt{x}+a^2d^2\sqrt{x}}{2\,(dx^2+c)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(5/2)),x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^2*x^(3/2)) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(
c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2
)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/8*
sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d
^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/16*sqrt(2)*((c*d^3)^(1/4)*b^2*
c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*ln(sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/16*sqrt(2)*
((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4
)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*
d^2) - 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x)
)/((d*x^2 + c)*c^2*d)
```

3.432 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=363

$$-\frac{9a^2d^2-10abcd+5b^2c^2}{10c^2d\sqrt{x}\,(c+dx^2)}-\frac{2a^2}{5cx^{5/2}\,(c+dx^2)}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}}$$

```
[Out] ((b*c - 9*a*d)*(b*c - a*d))/(2*c^3*d*Sqrt[x]) - (2*a^2)/(5*c*x^(5
/2)*(c + d*x^2)) - (5*b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)/(10*c^2*d
*Sqrt[x]*(c + d*x^2)) - ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + (
(b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*(b*c - a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(13/4)*d^(3/4)) - ((b*c - 9*a*d)*(b*c - a*d)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13
/4)*d^(3/4))
```

Rubi [A] time = 0.805869, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{9a^2d^2-10abcd+5b^2c^2}{10c^2d\sqrt{x}\,(c+dx^2)}-\frac{2a^2}{5cx^{5/2}\,(c+dx^2)}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```


```
[Out] ((b*c - 9*a*d)*(b*c - a*d))/(2*c^3*d*Sqrt[x]) - (2*a^2)/(5*c*x^(5
/2)*(c + d*x^2)) - (5*b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)/(10*c^2*d
*Sqrt[x]*(c + d*x^2)) - ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + (
(b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*(b*c - a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(13/4)*d^(3/4)) - ((b*c - 9*a*d)*(b*c - a*d)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13
/4)*d^(3/4))
```

Rubi in Sympy [A] time = 108.071, size = 332, normalized size = 0.91

$$-\frac{2a^2}{5cx^{\frac{5}{2}}(c+dx^2)}-\frac{ad(9ad-10bc)+5b^2c^2}{10c^2d\sqrt{x}(c+dx^2)}+\frac{(ad-bc)(9ad-bc)}{2c^3d\sqrt{x}}+\frac{\sqrt{2}(ad-bc)(9ad-bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{13}{4}}d^{\frac{3}{4}}}-\frac{\sqrt{2}(ad-bc)(9ad-bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{16c^{\frac{13}{4}}d^{\frac{3}{4}}}-\frac{\sqrt{2}(ad-bc)(9ad-bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{13}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}(ad-bc)(9ad-bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{8c^{\frac{13}{4}}d^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**2,x)
```

```
[Out] -2*a**2/(5*c*x**(5/2)*(c + d*x**2)) - (a*d*(9*a*d - 10*b*c) + 5*b
**2*c**2)/(10*c**2*d*sqrt(x)*(c + d*x**2)) + (a*d - b*c)*(9*a*d -
 b*c)/(2*c**3*d*sqrt(x)) + sqrt(2)*(a*d - b*c)*(9*a*d - b*c)*log(
-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**
(13/4)*d**(3/4)) - sqrt(2)*(a*d - b*c)*(9*a*d - b*c)*log(sqrt(2)*
c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(16*c**(13/4)*d*
*(3/4)) - sqrt(2)*(a*d - b*c)*(9*a*d - b*c)*atan(1 - sqrt(2)*d**(
1/4)*sqrt(x)/c**(1/4))/(8*c**(13/4)*d**(3/4)) + sqrt(2)*(a*d - b*
c)*(9*a*d - b*c)*atan(1 + sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(8*c
**(13/4)*d**(3/4))
```

Mathematica [A] time = 0.324893, size = 333, normalized size = 0.92

$$\frac{\frac{5\sqrt{2}(9a^2d^2-10abcd+b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{5\sqrt{2}(9a^2d^2-10abcd+b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{10\sqrt{2}(9a^2d^2-10abcd+b^2c^2)\tan^{-}}{d^{3/4}}}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(5/4))/x^(5/2) + (320*a*c^(1/4)*(-(b*c) + a*d))/Sqrt[
x] + (40*c^(1/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2) - (10*Sqrt[2]
*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/d^(3/4) + (10*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4)
+ (5*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4) - (5*Sqrt[2]*(
b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(80*c^(13/4))
```

Maple [A] time = 0.028, size = 524, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/2/c^3*x^(3/2)/(d*x^2+c)*a^2*d^2-1/c^2*x^(3/2)/(d*x^2+c)*a*b*d+1
/2/c*x^(3/2)/(d*x^2+c)*b^2+9/16/c^3*d/(c/d)^(1/4)*2^(1/2)*a^2*ln(
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2
```

```
)*2^(1/2)+(c/d)^(1/2)))+9/8/c^3*d/(c/d)^(1/4)*2^(1/2)*a^2*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+9/8/c^3*d/(c/d)^(1/4)*2^(1/2)*a^2*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-5/8/c^2/(c/d)^(1/4)*2^(1/2)
*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-5/4/c^2/(c/d)^(1/4)*2^(1/2)*a*b*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-5/4/c^2/(c/d)^(1/4)*2^(1/2)*
a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/16/c/d/(c/d)^(1/4)*2^
(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/8/c/d/(c/d)^(1/4)*2^(1/2)*
b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/8/c/d/(c/d)^(1/4)*2^(
1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5*a^2/c^2/x^(5/2
)+4*a^2/c^3/x^(1/2)*d-4*a/c^2/x^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276757, size = 2043, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/40*(20*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 16*a^2*c^2 - 16
*(10*a*b*c^2 - 9*a^2*c*d)*x^2 + 20*(c^3*d*x^4 + c^4*x^2)*sqrt(x)*
(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*
c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a
^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1
/4)*arctan(c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6
*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b
^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8
*d^8)/(c^13*d^3))^(3/4)/((b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*
c^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 2943*a^4*b^2*c^2*d^4 - 2430*a^5*
b*c*d^5 + 729*a^6*d^6)*sqrt(x) + sqrt((b^12*c^12 - 60*a*b^11*c^11
*d + 1554*a^2*b^10*c^10*d^2 - 22700*a^3*b^9*c^9*d^3 + 205215*a^4*
b^8*c^8*d^4 - 1188600*a^5*b^7*c^7*d^5 + 4443580*a^6*b^6*c^6*d^6 -
 10697400*a^7*b^5*c^5*d^7 + 16622415*a^8*b^4*c^4*d^8 - 16548300*a
^9*b^3*c^3*d^9 + 10195794*a^10*b^2*c^2*d^10 - 3542940*a^11*b*c*d^
11 + 531441*a^12*d^12)*x - (b^8*c^15*d - 40*a*b^7*c^14*d^2 + 636*
a^2*b^6*c^13*d^3 - 5080*a^3*b^5*c^12*d^4 + 21286*a^4*b^4*c^11*d^5
 - 45720*a^5*b^3*c^10*d^6 + 51516*a^6*b^2*c^9*d^7 - 29160*a^7*b*c
^8*d^8 + 6561*a^8*c^7*d^9)*sqrt(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*
a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 -
45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7
 + 6561*a^8*d^8)/(c^13*d^3))))) + 5*(c^3*d*x^4 + c^4*x^2)*sqrt(x)
*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5
*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*
a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(
1/4)*log(c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d
^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3
*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d
^8)/(c^13*d^3))^(3/4) + (b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c
^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 2943*a^4*b^2*c^2*d^4 - 2430*a^5*b
*c*d^5 + 729*a^6*d^6)*sqrt(x)) - 5*(c^3*d*x^4 + c^4*x^2)*sqrt(x)*
(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*
c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a
^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1
```

```
/4)*log(-c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d
^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3
*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d
^8)/(c^13*d^3))^(3/4) + (b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c
^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 2943*a^4*b^2*c^2*d^4 - 2430*a^5*b
*c*d^5 + 729*a^6*d^6)*sqrt(x)))/((c^3*d*x^4 + c^4*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.258771, size = 541, normalized size = 1.49

$$\frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)c^3} - \frac{2\left(10\,abcx^2 - 10\,a^2dx^2 + a^2c\right)}{5\,c^3x^{\frac{5}{2}}} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 10\left(cd^3\right)^{\frac{3}{4}}abcd + 9\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^4d^3} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 10\left(cd^3\right)^{\frac{3}{4}}abcd + 9\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^4d^3} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 10\left(cd^3\right)^{\frac{3}{4}}abcd + 9\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^4d^3} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2 - 10\left(cd^3\right)^{\frac{3}{4}}abcd + 9\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^2*x^(7/2)),x, algorithm="giac")

[Out] 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x
^2 + c)*c^3) - 2/5*(10*a*b*c*x^2 - 10*a^2*d*x^2 + a^2*c)/(c^3*x^(
5/2)) + 1/8*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b
*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^3) + 1/8*sqrt(2)*((c*d^3)
^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d
^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(c^4*d^3) - 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3
)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/
d)^(1/4) + x + sqrt(c/d))/(c^4*d^3) + 1/16*sqrt(2)*((c*d^3)^(3/4)
*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*ln
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^3)
```

3.433 $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=440

$$-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}+\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{17/4}}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

```
[Out] -((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Sqrt[x])/(16*c*d^4) + ((
117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*x^(5/2))/(80*c^2*d^3) + ((b
*c - a*d)^2*x^(9/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(17*b
*c - a*d)*x^(9/2))/(16*c^2*d^2*(c + d*x^2)) - ((117*b^2*c^2 - 90*
a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt
[2]*c^(3/4)*d^(17/4)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sq
rt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(3/4)*d^(17/4))
```

Rubi [A] time = 0.824478, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}+\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{17/4}}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```


```
[Out] -((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Sqrt[x])/(16*c*d^4) + ((
117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*x^(5/2))/(80*c^2*d^3) + ((b
*c - a*d)^2*x^(9/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(17*b
*c - a*d)*x^(9/2))/(16*c^2*d^2*(c + d*x^2)) - ((117*b^2*c^2 - 90*
a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt
[2]*c^(3/4)*d^(17/4)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*L
```

```
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sq
rt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(3/4)*d^(17/4))
```

Rubi in Sympy [A] time = 124.299, size = 425, normalized size = 0.97

$$\frac{x^{\frac{9}{2}}(ad-bc)^2}{4cd^2(c+dx^2)^2}-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}-\frac{x^{\frac{9}{2}}(ad-17bc)(ad-bc)}{16c^2d^2(c+dx^2)}+\frac{x^{\frac{5}{2}}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{3}{4}}d^{\frac{17}{4}}}+\frac{\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{3}{4}}d^{\frac{17}{4}}}-\frac{\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{3}{4}}d^{\frac{17}{4}}}+\frac{\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{3}{4}}d^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] x**(9/2)*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) - sqrt(x)*(5*a
**2*d**2 - 90*a*b*c*d + 117*b**2*c**2)/(16*c*d**4) - x**(9/2)*(a*
d - 17*b*c)*(a*d - b*c)/(16*c**2*d**2*(c + d*x**2)) + x**(5/2)*(5
*a**2*d**2 - 90*a*b*c*d + 117*b**2*c**2)/(80*c**2*d**3) - sqrt(2)
*(5*a**2*d**2 - 90*a*b*c*d + 117*b**2*c**2)*log(-sqrt(2)*c**(1/4)
*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(3/4)*d**(17/4))
 + sqrt(2)*(5*a**2*d**2 - 90*a*b*c*d + 117*b**2*c**2)*log(sqrt(2)
*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(3/4)*d
**(17/4)) - sqrt(2)*(5*a**2*d**2 - 90*a*b*c*d + 117*b**2*c**2)*at
an(1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(3/4)*d**(17/4))
 + sqrt(2)*(5*a**2*d**2 - 90*a*b*c*d + 117*b**2*c**2)*atan(1 + sq
rt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(3/4)*d**(17/4))
```

Mathematica [A] time = 0.891102, size = 383, normalized size = 0.87

$$-\frac{40\sqrt[4]{d}\sqrt{x}\left(9a^2d^2-34abcd+25b^2c^2\right)}{c+dx^2}-\frac{5\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}+\frac{5\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}-\right.}{c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-1280*b*d^(1/4)*(3*b*c - 2*a*d)*Sqrt[x] + 256*b^2*d^(5/4)*x^(5/2
) + (160*c*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 - (40*d^(
1/4)*(25*b^2*c^2 - 34*a*b*c*d + 9*a^2*d^2)*Sqrt[x])/(c + d*x^2) -
 (10*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4) + (10*Sqrt[2]*(117*b^2*
c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/c^(3/4) - (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/c^(3/4) + (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(3/4))
```

```
/(640*d^(17/4))
```

Maple [A] time = 0.028, size = 590, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2/5/d^3*b^2*x^(5/2)+4/d^3*b*a*x^(1/2)-6/d^4*b^2*x^(1/2)*c-9/16/d/
(d*x^2+c)^2*x^(5/2)*a^2+17/8/d^2/(d*x^2+c)^2*x^(5/2)*a*b*c-25/16/
d^3/(d*x^2+c)^2*x^(5/2)*b^2*c^2-5/16/d^2/(d*x^2+c)^2*x^(1/2)*a^2*
c+13/8/d^3/(d*x^2+c)^2*x^(1/2)*a*b*c^2-21/16/d^4/(d*x^2+c)^2*x^(1
/2)*b^2*c^3+5/64/d^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)+1)*a^2-45/32/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)*a*b+117/64/d^4*(c/d)^(1/4)*c*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/64/d^2*(c/d)^(1/4)/c*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-45/32/d^3*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+117/64/d^4*(c
/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/1
28/d^2*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-45/64
/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+117/128/d
^4*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280408, size = 1605, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(20*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8
 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*
a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^
3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8
)/(c^3*d^17))^(1/4)*arctan(c*d^4*(-(187388721*b^8*c^8 - 576580680
*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*
d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273
500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17)
)^(1/4)/((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x) + sqrt(c^
2*d^8*sqrt(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 69731766
0*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4
*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 4
5000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17)) + (13689*b^4*c^4 - 21
060*a*b^3*c^3*d + 9270*a^2*b^2*c^2*d^2 - 900*a^3*b*c*d^3 + 25*a^4
```

```
*d^4)*x))) - 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8
*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092
600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^
3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8
*d^8)/(c^3*d^17))^(1/4)*log(c*d^4*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5
*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 127
3500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17
))^(1/4) + (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) + 5*(d
^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 576580680*
a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d
^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 12735
00*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))
^(1/4)*log(-c*d^4*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d +
697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350
*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2
*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4) + (117*
b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) - 4*(32*b^2*d^3*x^6 -
585*b^2*c^3 + 450*a*b*c^2*d - 25*a^2*c*d^2 - 32*(13*b^2*c*d^2 - 1
0*a*b*d^3)*x^4 - 9*(117*b^2*c^2*d - 90*a*b*c*d^2 + 5*a^2*d^3)*x^2
)*sqrt(x))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.247895, size = 609, normalized size = 1.38

$$\frac{\sqrt{2}\left(117\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-90\,\left(cd^3\right)^{\frac{1}{4}}abcd+5\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^5}+\frac{\sqrt{2}\left(117\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-90\,\left(cd^3\right)^{\frac{1}{4}}abcd+5\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^5}+\frac{\sqrt{2}\left(117\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-90\,\left(cd^3\right)^{\frac{1}{4}}abcd+5\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,cd^5}-\frac{\sqrt{2}\left(117\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-90\,\left(cd^3\right)^{\frac{1}{4}}abcd+5\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,cd^5}-\frac{25\,b^2c^2dx^{\frac{5}{2}}-34\,abcd^2x^{\frac{5}{2}}+9\,a^2d^3x^{\frac{5}{2}}+21\,b^2c^3\sqrt{x}-26\,abc^2d\sqrt{x}+5\,a^2cd^2\sqrt{x}}{16\,\left(dx^2+c\right)^2d^4}+\frac{2\left(b^2d^{12}x^{\frac{5}{2}}-15\,b^2cd^{11}\sqrt{x}+10\,abd^{12}\sqrt{x}\right)}{5\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*
d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1
/4) + 2*sqrt(x))/(c/d)^(1/4))/(c*d^5) + 1/64*sqrt(2)*(117*(c*d^3)
^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d
```

```
^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(c*d^5) + 1/128*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*
d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*
(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/128*sqrt(2)*(117*(c*d^3)
^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d
^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/
16*(25*b^2*c^2*d*x^(5/2) - 34*a*b*c*d^2*x^(5/2) + 9*a^2*d^3*x^(5/
2) + 21*b^2*c^3*sqrt(x) - 26*a*b*c^2*d*sqrt(x) + 5*a^2*c*d^2*sqrt
(x))/((d*x^2 + c)^2*d^4) + 2/5*(b^2*d^12*x^(5/2) - 15*b^2*c*d^11*
sqrt(x) + 10*a*b*d^12*sqrt(x))/d^15
```

3.434 $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=401

$$-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{15/4}}-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}d^{15/4}}-\frac{x^{3/2}\left(\frac{3a^2d}{c}+42ab-\frac{77b^2c}{d}\right)}{48cd^2}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

```
[Out] -((42*a*b - (77*b^2*c)/d + (3*a^2*d)/c)*x^(3/2))/(48*c*d^2) + ((b
*c - a*d)^2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(15*b
*c + a*d)*x^(7/2))/(16*c^2*d^2*(c + d*x^2)) + ((77*b^2*c^2 - 42*a
*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a
^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
2]*c^(5/4)*d^(15/4)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(5/4)*d^(15/4))
```

Rubi [A] time = 0.758133, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{15/4}}-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}d^{15/4}}-\frac{x^{3/2}\left(\frac{3a^2d}{c}+42ab-\frac{77b^2c}{d}\right)}{48cd^2}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```


```
[Out] -((42*a*b - (77*b^2*c)/d + (3*a^2*d)/c)*x^(3/2))/(48*c*d^2) + ((b
*c - a*d)^2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(15*b
*c + a*d)*x^(7/2))/(16*c^2*d^2*(c + d*x^2)) + ((77*b^2*c^2 - 42*a
*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a
^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
```

```
2]*c^(5/4)*d^(15/4)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(5/4)*d^(15/4))
```

Rubi in Sympy [A] time = 115.602, size = 388, normalized size = 0.97

$$\frac{x^{\frac{7}{2}}\left(ad-bc\right)^{2}}{4cd^{2}\left(c+dx^{2}\right)^{2}}+\frac{x^{\frac{7}{2}}\left(ad-bc\right)\left(ad+15bc\right)}{16c^{2}d^{2}\left(c+dx^{2}\right)}-\frac{x^{\frac{3}{2}}\left(3a^{2}d^{2}+42abcd-77b^{2}c^{2}\right)}{48c^{2}d^{3}}+\frac{\sqrt{2}\left(3a^{2}d^{2}+42abcd-77b^{2}c^{2}\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{5}{4}}d^{\frac{15}{4}}}-\frac{\sqrt{2}\left(3a^{2}d^{2}+42abcd-77b^{2}c^{2}\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{5}{4}}d^{\frac{15}{4}}}-\frac{\sqrt{2}\left(3a^{2}d^{2}+42abcd-77b^{2}c^{2}\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{5}{4}}d^{\frac{15}{4}}}+\frac{\sqrt{2}\left(3a^{2}d^{2}+42abcd-77b^{2}c^{2}\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{5}{4}}d^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] x**(7/2)*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) + x**(7/2)*(a*
d - b*c)*(a*d + 15*b*c)/(16*c**2*d**2*(c + d*x**2)) - x**(3/2)*(3
*a**2*d**2 + 42*a*b*c*d - 77*b**2*c**2)/(48*c**2*d**3) + sqrt(2)*
(3*a**2*d**2 + 42*a*b*c*d - 77*b**2*c**2)*log(-sqrt(2)*c**(1/4)*d
**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(5/4)*d**(15/4)) -
 sqrt(2)*(3*a**2*d**2 + 42*a*b*c*d - 77*b**2*c**2)*log(sqrt(2)*c*
*(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(5/4)*d**(
15/4)) - sqrt(2)*(3*a**2*d**2 + 42*a*b*c*d - 77*b**2*c**2)*atan(1
 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(5/4)*d**(15/4)) + s
qrt(2)*(3*a**2*d**2 + 42*a*b*c*d - 77*b**2*c**2)*atan(1 + sqrt(2)
*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(5/4)*d**(15/4))
```

Mathematica [A] time = 0.401856, size = 363, normalized size = 0.91

$$\frac{\frac{24d^{3/4}x^{3/2}\left(3a^2d^2-22abcd+19b^2c^2\right)}{c(c+dx^2)}-\frac{3\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{3\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\cdots\right.}{c^{5/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (256*b^2*d^(3/4)*x^(3/2) - (96*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c
+ d*x^2)^2 + (24*d^(3/4)*(19*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*x^
(3/2))/(c*(c + d*x^2)) + (6*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*
a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) -
 (6*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (3*Sqrt[2]*(77*b^2*c^2
- 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/c^(5/4) + (3*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*
d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/c^(5/4))/(384*d^(15/4))
```

Maple [A] time = 0.029, size = 562, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2/3*x^(3/2)*b^2/d^3+3/16/(d*x^2+c)^2/c*x^(7/2)*a^2-11/8/d/(d*x^2+
c)^2*x^(7/2)*a*b+19/16/d^2/(d*x^2+c)^2*c*x^(7/2)*b^2-1/16/d/(d*x^
2+c)^2*x^(3/2)*a^2-7/8/d^2/(d*x^2+c)^2*x^(3/2)*c*a*b+15/16/d^3/(d
*x^2+c)^2*x^(3/2)*b^2*c^2+3/64/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+21/32/d^3/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-77/64/d^4*c/(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/d^2/c/(c/
d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+21/64/d^3/(c/d)^(
1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-77/128/d^4*c/(c/d)^(1
/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+3/64/d^2/c/(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+21/32/d^3/(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-77/64/d
^4*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274627, size = 2140, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(12*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a
^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5
 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^
15))^(1/4)*arctan(-c^4*d^11*(-(35153041*b^8*c^8 - 76697544*a*b^7*
c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 145
7946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(3/4)/((456533*
b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3
*b^3*c^3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*
d^6)*sqrt(x) - sqrt((208422380089*b^12*c^12 - 682109607564*a*b^11
*c^11*d + 881427350034*a^2*b^10*c^10*d^2 - 543593843100*a^3*b^9*c
^9*d^3 + 136525986135*a^4*b^8*c^8*d^4 + 8334677736*a^5*b^7*c^7*d^
5 - 7849956996*a^6*b^6*c^6*d^6 - 324727704*a^7*b^5*c^5*d^7 + 2072
41335*a^8*b^4*c^4*d^8 + 32148900*a^9*b^3*c^3*d^9 + 2030994*a^10*b
^2*c^2*d^10 + 61236*a^11*b*c*d^11 + 729*a^12*d^12)*x - (35153041*
b^8*c^11*d^7 - 76697544*a*b^7*c^10*d^8 + 57274140*a^2*b^6*c^9*d^9
 - 13854456*a^3*b^5*c^8*d^10 - 1457946*a^4*b^4*c^7*d^11 + 539784*
a^5*b^3*c^6*d^12 + 86940*a^6*b^2*c^5*d^13 + 4536*a^7*b*c^4*d^14 +
```

```
81*a^8*c^3*d^15)*sqrt(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d
+ 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946*a
^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 +
 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))))) + 3*(c*d^5*x^4 + 2
*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*
d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946
*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6
 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(c^4*d^11*
(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6
*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 53978
4*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81
*a^8*d^8)/(c^5*d^15))^(3/4) - (456533*b^6*c^6 - 747054*a*b^5*c^5*
d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^3*d^3 - 13797*a^4*b^
2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) - 3*(c*d^5*x^
4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c^8 - 76697544*a*b^7
*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^
2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(-c^4
*d^11*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b
^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 +
 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^
7 + 81*a^8*d^8)/(c^5*d^15))^(3/4) - (456533*b^6*c^6 - 747054*a*b^
5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^3*d^3 - 13797*
a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) + 4*(32
*b^2*c*d^2*x^5 + (121*b^2*c^2*d - 66*a*b*c*d^2 + 9*a^2*d^3)*x^3 +
 (77*b^2*c^3 - 42*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(x))/(c*d^5*x^4
 + 2*c^2*d^4*x^2 + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.262009, size = 576, normalized size = 1.44

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3}+\frac{19\,b^2c^2dx^{\frac{7}{2}}-22\,abcd^2x^{\frac{7}{2}}+3\,a^2d^3x^{\frac{7}{2}}+15\,b^2c^3x^{\frac{3}{2}}-14\,abc^2dx^{\frac{3}{2}}-a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2cd^3}-\frac{\sqrt{2}\left(77\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-42\,\left(cd^3\right)^{\frac{3}{4}}abcd-3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^6}-\frac{\sqrt{2}\left(77\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-42\,\left(cd^3\right)^{\frac{3}{4}}abcd-3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^6}+\frac{\sqrt{2}\left(77\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-42\,\left(cd^3\right)^{\frac{3}{4}}abcd-3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^2d^6}-\frac{\sqrt{2}\left(77\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2-42\,\left(cd^3\right)^{\frac{3}{4}}abcd-3\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^3 + 1/16*(19*b^2*c^2*d*x^(7/2) - 22*a*b*c*d^2*x
^(7/2) + 3*a^2*d^3*x^(7/2) + 15*b^2*c^3*x^(3/2) - 14*a*b*c^2*d*x^
(3/2) - a^2*c*d^2*x^(3/2))/((d*x^2 + c)^2*c*d^3) - 1/64*sqrt(2)*(
77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(
3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x)
)/(c/d)^(1/4))/(c^2*d^6) - 1/64*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2
 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6
) + 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*
b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(c^2*d^6) - 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c
^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^6)
```

3.435 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=402

$$\frac{(-3a^2d^2-10abcd+45b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}-\frac{(-3a^2d^2-10abcd+45b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}+\frac{(-3a^2d^2-10abcd+45b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}d^{13/4}}-\frac{(-3a^2d^2-10abcd+45b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}d^{13/4}}-\frac{\sqrt{x}\left(\frac{3a^2d}{c}+10ab-\frac{45b^2c}{d}\right)}{16cd^2}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{16c^2d^2(c+dx^2)}+\frac{x^{5/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

```
[Out] -((10*a*b - (45*b^2*c)/d + (3*a^2*d)/c)*Sqrt[x])/(16*c*d^2) + ((b
*c - a*d)^2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(13*b
*c + 3*a*d)*x^(5/2))/(16*c^2*d^2*(c + d*x^2)) + ((45*b^2*c^2 - 10
*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt
[2]*c^(7/4)*d^(13/4)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
t[2]*c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sq
rt[2]*c^(7/4)*d^(13/4))
```

Rubi [A] time = 0.726052, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{(-3a^2d^2-10abcd+45b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}-\frac{(-3a^2d^2-10abcd+45b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}+\frac{(-3a^2d^2-10abcd+45b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}d^{13/4}}-\frac{(-3a^2d^2-10abcd+45b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}d^{13/4}}-\frac{\sqrt{x}\left(\frac{3a^2d}{c}+10ab-\frac{45b^2c}{d}\right)}{16cd^2}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{16c^2d^2(c+dx^2)}+\frac{x^{5/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((10*a*b - (45*b^2*c)/d + (3*a^2*d)/c)*Sqrt[x])/(16*c*d^2) + ((b
*c - a*d)^2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(13*b
*c + 3*a*d)*x^(5/2))/(16*c^2*d^2*(c + d*x^2)) + ((45*b^2*c^2 - 10
*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt
[2]*c^(7/4)*d^(13/4)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
```

```
t[2]*c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sq
rt[2]*c^(7/4)*d^(13/4))
```

Rubi in Sympy [A] time = 115.505, size = 389, normalized size = 0.97

$$\frac{x^{\frac{5}{2}}(ad-bc)^2}{4cd^2(c+dx^2)^2}+\frac{x^{\frac{5}{2}}(ad-bc)(3ad+13bc)}{16c^2d^2(c+dx^2)}-\frac{\sqrt{x}\left(3a^2d^2+10abcd-45b^2c^2\right)}{16c^2d^3}-\frac{\sqrt{2}\left(3a^2d^2+10abcd-45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{7}{4}}d^{\frac{13}{4}}}+\frac{\sqrt{2}\left(3a^2d^2+10abcd-45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{7}{4}}d^{\frac{13}{4}}}-\frac{\sqrt{2}\left(3a^2d^2+10abcd-45b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{7}{4}}d^{\frac{13}{4}}}+\frac{\sqrt{2}\left(3a^2d^2+10abcd-45b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{7}{4}}d^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] x**(5/2)*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) + x**(5/2)*(a*
d - b*c)*(3*a*d + 13*b*c)/(16*c**2*d**2*(c + d*x**2)) - sqrt(x)*(
3*a**2*d**2 + 10*a*b*c*d - 45*b**2*c**2)/(16*c**2*d**3) - sqrt(2)
*(3*a**2*d**2 + 10*a*b*c*d - 45*b**2*c**2)*log(-sqrt(2)*c**(1/4)*
d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(7/4)*d**(13/4))
+ sqrt(2)*(3*a**2*d**2 + 10*a*b*c*d - 45*b**2*c**2)*log(sqrt(2)*c
**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(7/4)*d**
(13/4)) - sqrt(2)*(3*a**2*d**2 + 10*a*b*c*d - 45*b**2*c**2)*atan(
1 - sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(7/4)*d**(13/4)) +
sqrt(2)*(3*a**2*d**2 + 10*a*b*c*d - 45*b**2*c**2)*atan(1 + sqrt(2
)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(7/4)*d**(13/4))
```

Mathematica [A] time = 0.792802, size = 361, normalized size = 0.9

$$\frac{\frac{8\sqrt[4]{d}\sqrt{x}\left(a^2d^2-18abcd+17b^2c^2\right)}{c(c+dx^2)}+\frac{\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}-\frac{\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{}\right.}{c^{7/4}}}{128}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (256*b^2*d^(1/4)*Sqrt[x] - (32*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c
+ d*x^2)^2 + (8*d^(1/4)*(17*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*Sqrt[
x])/(c*(c + d*x^2)) + (2*Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2
*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2
*Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) + (Sqrt[2]*(45*b^2*c^2 - 10*
a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/c^(7/4) - (Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a
^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/c^(7/4))/(128*d^(13/4))
```

Maple [A] time = 0.027, size = 568, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2*x^(1/2)*b^2/d^3+1/16/(d*x^2+c)^2/c*x^(5/2)*a^2-9/8/d/(d*x^2+c)^
2*x^(5/2)*a*b+17/16/d^2/(d*x^2+c)^2*c*x^(5/2)*b^2-3/16/d/(d*x^2+c
)^2*x^(1/2)*a^2-5/8/d^2/(d*x^2+c)^2*x^(1/2)*c*a*b+13/16/d^3/(d*x^
2+c)^2*x^(1/2)*b^2*c^2+3/64/d/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+5/32/d^2/c*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-45/64/d^3*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64/d/c^2*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+5/32/d^2/c*
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-45/
64/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
b^2+3/128/d/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^
2+5/64/d^2/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-4
5/128/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270326, size = 1604, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8
 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*
c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6
*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*a
rctan(-c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*
a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4
- 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7
+ 81*a^8*d^8)/(c^7*d^13))^(1/4)/((45*b^2*c^2 - 10*a*b*c*d - 3*a^2
*d^2)*sqrt(x) - sqrt(c^4*d^6*sqrt(-(4100625*b^8*c^8 - 3645000*a*b
^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 4265
0*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 +
 1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13)) + (2025*b^4*c^4 - 900
*a*b^3*c^3*d - 170*a^2*b^2*c^2*d^2 + 60*a^3*b*c*d^3 + 9*a^4*d^4)*
x))) - (c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 -
 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^
5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b
^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*log
(c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^
6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 3660
0*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a
```

```
^8*d^8)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)
*sqrt(x)) + (c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*
c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b
^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*
a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4
)*log(-c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*
a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4
- 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7
+ 81*a^8*d^8)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^
2*d^2)*sqrt(x)) - 4*(32*b^2*c*d^2*x^4 + 45*b^2*c^3 - 10*a*b*c^2*d
 - 3*a^2*c*d^2 + (81*b^2*c^2*d - 18*a*b*c*d^2 + a^2*d^3)*x^2)*sqr
t(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.257104, size = 575, normalized size = 1.43

$$\frac{2\,b^2\sqrt{x}}{d^3} - \frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd - 3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4} - \frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd - 3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4} - \frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd - 3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^2d^4} + \frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 10\,\left(cd^3\right)^{\frac{1}{4}}abcd - 3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^2d^4} + \frac{17\,b^2c^2dx^{\frac{5}{2}} - 18\,abcd^2x^{\frac{5}{2}} + a^2d^3x^{\frac{5}{2}} + 13\,b^2c^3\sqrt{x} - 10\,abc^2d\sqrt{x} - 3\,a^2cd^2\sqrt{x}}{16\,\left(dx^2 + c\right)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^3,x, algorithm="giac")

[Out] 2*b^2*sqrt(x)/d^3 - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(
c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2
)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/64
*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3
*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
- 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/128*sqrt(2)*(45*(c*d^3)^(
1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2
)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1/1
28*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d -
 3*(c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c^2*d^4) + 1/16*(17*b^2*c^2*d*x^(5/2) - 18*a*b*c*d^2*x
^(5/2) + a^2*d^3*x^(5/2) + 13*b^2*c^3*sqrt(x) - 10*a*b*c^2*d*sqrt
(x) - 3*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c*d^3)
```

3.436 $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=364

$$\frac{(5a^2d^2+6abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}} - \frac{(5a^2d^2+6abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}} - \frac{(5a^2d^2+6abcd+21b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}d^{11/4}} + \frac{(5a^2d^2+6abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}d^{11/4}} - \frac{x^{3/2}(5ad+11bc)(bc-ad)}{16c^2d^2(c+dx^2)} + \frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

```
[Out] ((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(1
1*b*c + 5*a*d)*x^(3/2))/(16*c^2*d^2*(c + d*x^2)) - ((21*b^2*c^2 +
 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d +
5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqr
t[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
t[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
t[2]*c^(9/4)*d^(11/4))
```

Rubi [A] time = 0.644137, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{(5a^2d^2+6abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}} - \frac{(5a^2d^2+6abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}} - \frac{(5a^2d^2+6abcd+21b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}d^{11/4}} + \frac{(5a^2d^2+6abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}d^{11/4}} - \frac{x^{3/2}(5ad+11bc)(bc-ad)}{16c^2d^2(c+dx^2)} + \frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```


```
[Out] ((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(1
1*b*c + 5*a*d)*x^(3/2))/(16*c^2*d^2*(c + d*x^2)) - ((21*b^2*c^2 +
 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d +
5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqr
t[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
t[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
t[2]*c^(9/4)*d^(11/4))
```

Rubi in Sympy [A] time = 107.693, size = 350, normalized size = 0.96

$$\frac{x^{\frac{3}{2}}(ad-bc)^2}{4cd^2(c+dx^2)^2}+\frac{x^{\frac{3}{2}}(ad-bc)(5ad+11bc)}{16c^2d^2(c+dx^2)}+\frac{\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{9}{4}}d^{\frac{11}{4}}}-\frac{\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{9}{4}}d^{\frac{11}{4}}}-\frac{\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{9}{4}}d^{\frac{11}{4}}}+\frac{\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{9}{4}}d^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] x**(3/2)*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) + x**(3/2)*(a*
d - b*c)*(5*a*d + 11*b*c)/(16*c**2*d**2*(c + d*x**2)) + sqrt(2)*(
5*a**2*d**2 + 6*a*b*c*d + 21*b**2*c**2)*log(-sqrt(2)*c**(1/4)*d**
(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(9/4)*d**(11/4)) - s
qrt(2)*(5*a**2*d**2 + 6*a*b*c*d + 21*b**2*c**2)*log(sqrt(2)*c**(1
/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(9/4)*d**(11/
4)) - sqrt(2)*(5*a**2*d**2 + 6*a*b*c*d + 21*b**2*c**2)*atan(1 - s
qrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(9/4)*d**(11/4)) + sqrt(
2)*(5*a**2*d**2 + 6*a*b*c*d + 21*b**2*c**2)*atan(1 + sqrt(2)*d**(
1/4)*sqrt(x)/c**(1/4))/(64*c**(9/4)*d**(11/4))
```

Mathematica [A] time = 0.335137, size = 339, normalized size = 0.93

$$\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((32*c^(5/4)*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 - (8*c^
(1/4)*d^(3/4)*(11*b^2*c^2 - 6*a*b*c*d - 5*a^2*d^2)*x^(3/2))/(c +
d*x^2) - 2*Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1
- (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(21*b^2*c^2 + 6*
a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
] + Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*(21*b^2*c^2
+ 6*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(128*c^(9/4)*d^(11/4))
```

Maple [A] time = 0.026, size = 514, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] 2*(1/32*(5*a^2*d^2+6*a*b*c*d-11*b^2*c^2)/c^2/d*x^(7/2)+1/32*(9*a^
2*d^2-2*a*b*c*d-7*b^2*c^2)/c/d^2*x^(3/2))/(d*x^2+c)^2+5/64/d/c^2/
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/3
2/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
*a*b+21/64/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)*b^2+5/64/d/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)*a^2+3/32/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)*a*b+21/64/d^3/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/d/c^2/(c/d)^(1/4)*2^(1/2
)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+3/64/d^2/c/(c/d)^(1/4)*2^(1/2)*l
n((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2)))*a*b+21/128/d^3/(c/d)^(1/4)*2^(1/2)*ln((
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2)))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276313, size = 2126, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8
 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c
^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a
^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4
)*arctan(c^7*d^8*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*
a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d
^7 + 625*a^8*d^8)/(c^9*d^11))^(3/4)/((9261*b^6*c^6 + 7938*a*b^5*c
^5*d + 8883*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 2115*a^4*b^2
*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*d^6)*sqrt(x) + sqrt((8576612
1*b^12*c^12 + 147027636*a*b^11*c^11*d + 227542770*a^2*b^10*c^10*d
^2 + 215040420*a^3*b^9*c^9*d^3 + 181522215*a^4*b^8*c^8*d^4 + 1129
05576*a^5*b^7*c^7*d^5 + 63002556*a^6*b^6*c^6*d^6 + 26882280*a^7*b
^5*c^5*d^7 + 10290375*a^8*b^4*c^4*d^8 + 2902500*a^9*b^3*c^3*d^9 +
 731250*a^10*b^2*c^2*d^10 + 112500*a^11*b*c*d^11 + 15625*a^12*d^1
2)*x - (194481*b^8*c^13*d^5 + 222264*a*b^7*c^12*d^6 + 280476*a^2*
b^6*c^11*d^7 + 176904*a^3*b^5*c^10*d^8 + 112806*a^4*b^4*c^9*d^9 +
 42120*a^5*b^3*c^8*d^10 + 15900*a^6*b^2*c^7*d^11 + 3000*a^7*b*c^6
*d^12 + 625*a^8*c^5*d^13)*sqrt(-(194481*b^8*c^8 + 222264*a*b^7*c^
7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^
4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3
000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))))) + (c^2*d^4*x^4 + 2*
c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 2
80476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c
^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7
*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*log(c^7*d^8*(-(194481*b
^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3
*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 1
5900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11)
)^(3/4) + (9261*b^6*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2
 + 3996*a^3*b^3*c^3*d^3 + 2115*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5
```

```
+ 125*a^6*d^6)*sqrt(x)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)
*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2
+ 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3
*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8
)/(c^9*d^11))^(1/4)*log(-c^7*d^8*(-(194481*b^8*c^8 + 222264*a*b^7
*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806
*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6
+ 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(3/4) + (9261*b^6*c
^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d
^3 + 2115*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*d^6)*sqrt(x
)) - 4*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^3 + (7*b^2*c^3
 + 2*a*b*c^2*d - 9*a^2*c*d^2)*x)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^
3*x^2 + c^4*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283365, size = 562, normalized size = 1.54

$$-\frac{11\,b^2c^2dx^{\frac{7}{2}} - 6\,abcd^2x^{\frac{7}{2}} - 5\,a^2d^3x^{\frac{7}{2}} + 7\,b^2c^3x^{\frac{3}{2}} + 2\,abc^2dx^{\frac{3}{2}} - 9\,a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2c^2d^2} + \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}abcd + 5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^5} + \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}abcd + 5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^5} - \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}abcd + 5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^3d^5} + \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}abcd + 5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*b^2*c^2*d*x^(7/2) - 6*a*b*c*d^2*x^(7/2) - 5*a^2*d^3*x^(
7/2) + 7*b^2*c^3*x^(3/2) + 2*a*b*c^2*d*x^(3/2) - 9*a^2*c*d^2*x^(3
/2))/((d*x^2 + c)^2*c^2*d^2) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2
*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(
1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d
^5) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*
b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/
d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^5) - 1/128*sqrt(2)*(21*
(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)
*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^
5) + 1/128*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*
b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4)
+ x + sqrt(c/d))/(c^3*d^5)
```

3.437 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3} dx$

Optimal. Leaf size=364

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}-\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}d^{9/4}}-\frac{\sqrt{x}(7ad+9bc)(bc-ad)}{16c^2d^2(c+dx^2)}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

```
[Out] ((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9
*b*c + 7*a*d)*Sqrt[x])/(16*c^2*d^2*(c + d*x^2)) - ((5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt
[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(11/4)*d^(9/4))
```

Rubi [A] time = 0.662263, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}-\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}d^{9/4}}-\frac{\sqrt{x}(7ad+9bc)(bc-ad)}{16c^2d^2(c+dx^2)}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9
*b*c + 7*a*d)*Sqrt[x])/(16*c^2*d^2*(c + d*x^2)) - ((5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt
[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(11/4)*d^(9/4))
```

Rubi in Sympy [A] time = 104.197, size = 350, normalized size = 0.96

$$\frac{\sqrt{x}\,(ad-bc)^2}{4cd^2\,(c+dx^2)^2}+\frac{\sqrt{x}\,(ad-bc)\,(7ad+9bc)}{16c^2d^2\,(c+dx^2)}-\frac{\sqrt{2}\,(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{11}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{11}{4}}d^{\frac{9}{4}}}-\frac{\sqrt{2}\,(21a^2d^2+6abcd+5b^2c^2)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{11}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(21a^2d^2+6abcd+5b^2c^2)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{11}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] sqrt(x)*(a*d - b*c)**2/(4*c*d**2*(c + d*x**2)**2) + sqrt(x)*(a*d
- b*c)*(7*a*d + 9*b*c)/(16*c**2*d**2*(c + d*x**2)) - sqrt(2)*(21*
a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2)*log(-sqrt(2)*c**(1/4)*d**(1/
4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(11/4)*d**(9/4)) + sqrt
(2)*(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2)*log(sqrt(2)*c**(1/4)
*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(11/4)*d**(9/4))
 - sqrt(2)*(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2)*atan(1 - sqrt
(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(11/4)*d**(9/4)) + sqrt(2)*
(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2)*atan(1 + sqrt(2)*d**(1/4
)*sqrt(x)/c**(1/4))/(64*c**(11/4)*d**(9/4))
```

Mathematica [A] time = 0.33101, size = 339, normalized size = 0.93

$$\frac{-\sqrt{2}\,(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}\,(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 - (8*c^
(3/4)*d^(1/4)*(9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*Sqrt[x])/(c + d
*x^2) - 2*Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(5*b^2*c^2 + 6*a*
b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
 - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(5*b^2*c^2 +
6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(128*c^(11/4)*d^(9/4))
```

Maple [A] time = 0.026, size = 514, normalized size = 1.4

$$2\frac{1}{\left(dx^2+c\right)^2}\left(1/32\,\frac{\left(7\,a^2d^2+2\,cabd-9\,b^2c^2\right)x^{5/2}}{c^2d}+1/32\,\frac{\left(11\,a^2d^2-6\,cabd-5\,b^2c^2\right)\sqrt{x}}{d^2c}\right)+\frac{21\,\sqrt{2}a^2}{128\,c^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{3\,\sqrt{2}ab}{64\,c^2d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{5\,\sqrt{2}b^2}{128\,d^2c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{21\,\sqrt{2}a^2}{64\,c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{3\,\sqrt{2}ab}{32\,c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{5\,\sqrt{2}b^2}{64\,d^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{21\,\sqrt{2}a^2}{64\,c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{3\,\sqrt{2}ab}{32\,c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{5\,\sqrt{2}b^2}{64\,d^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 2*(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^(5/2)+1/32*(11*a^
2*d^2-6*a*b*c*d-5*b^2*c^2)/d^2/c*x^(1/2))/(d*x^2+c)^2+21/128/c^3*
(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
)/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+3/64/c^2/d*(c/
d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+5/128/c/d^2*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+21/64/c^3*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/32/c^2/d*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+5/64
/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
b^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)-1)*a^2+3/32/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a*b+5/64/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^3*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274786, size = 1593, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^3*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 +
 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3
 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b
^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/
4)*arctan(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b
^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176
904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)/((5*b^2*c^2 + 6*a*b*c*d + 21
*a^2*d^2)*sqrt(x) + sqrt(c^6*d^4*sqrt(-(625*b^8*c^8 + 3000*a*b^7*
c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^
4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 +
 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 +
 60*a*b^3*c^3*d + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4
*d^4)*x))) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c
^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5
*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a
^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))
^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*
b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 17
6904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^
7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d +
21*a^2*d^2)*sqrt(x)) + (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-
(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a
^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5
+ 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(
c^11*d^9))^(1/4)*log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d +
 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c
^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264
*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*sqrt(x)) + 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11
*a^2*c*d^2 + (9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^2)*sqrt(x)
)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.263868, size = 562, normalized size = 1.54

$$\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3}+\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3}+\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^3d^3}-\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^3d^3}-\frac{9\,b^2c^2dx^{\frac{5}{2}}-2\,abcd^2x^{\frac{5}{2}}-7\,a^2d^3x^{\frac{5}{2}}+5\,b^2c^3\sqrt{x}+6\,abc^2d\sqrt{x}-11\,a^2cd^2\sqrt{x}}{16\left(dx^2+c\right)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*sqrt(x)),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d +
 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(
1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3
)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c
/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(
1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2
)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/
16*(9*b^2*c^2*d*x^(5/2) - 2*a*b*c*d^2*x^(5/2) - 7*a^2*d^3*x^(5/2)
 + 5*b^2*c^3*sqrt(x) + 6*a*b*c^2*d*sqrt(x) - 11*a^2*c*d^2*sqrt(x)
)/((d*x^2 + c)^2*c^2*d^2)
```

3.438 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=399

$$-\frac{x^{3/2}\left(9a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}-\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}-\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}d^{7/4}}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}d^{7/4}}+\frac{x^{3/2}\left(5ad(2bc-9ad)+3b^2c^2\right)}{16c^3d\left(c+dx^2\right)}$$

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) - ((b^2*c^2 - 2*a*b*c*d + 9*a^
2*d^2)*x^(3/2))/(4*c^2*d*(c + d*x^2)^2) + ((3*b^2*c^2 + 5*a*d*(2*
b*c - 9*a*d))*x^(3/2))/(16*c^3*d*(c + d*x^2)) - ((3*b^2*c^2 + 5*a
*d*(2*b*c - 9*a*d))*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*
a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*
c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt
[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c
^(13/4)*d^(7/4)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(13/4)*d^(7/4))
```

Rubi [A] time = 0.830139, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{x^{3/2}\left(9a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{x^{3/2}\left(\frac{5a(2bc-9ad)}{c^2}+\frac{3b^2}{d}\right)}{16c\left(c+dx^2\right)}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}-\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}-\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}d^{7/4}}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) - ((b^2*c^2 - 2*a*b*c*d + 9*a^
2*d^2)*x^(3/2))/(4*c^2*d*(c + d*x^2)^2) + (((3*b^2)/d + (5*a*(2*b
*c - 9*a*d))/c^2)*x^(3/2))/(16*c*(c + d*x^2)) - ((3*b^2*c^2 + 5*a
*d*(2*b*c - 9*a*d))*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*
a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*
c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt
[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c
^(13/4)*d^(7/4)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(13/4)*d^(7/4))
```

Rubi in Sympy [A] time = 110.094, size = 381, normalized size = 0.95

$$-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}-\frac{x^{\frac{3}{2}}\left(ad\,(9ad-2bc)+b^2c^2\right)}{4c^2d\,(c+dx^2)^2}+\frac{x^{\frac{3}{2}}\left(-5ad\,(9ad-2bc)+3b^2c^2\right)}{16c^3d\,(c+dx^2)}$$
$$+\frac{\sqrt{2}\left(-5ad\,(9ad-2bc)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{13}{4}}d^{\frac{7}{4}}}$$
$$-\frac{\sqrt{2}\left(-5ad\,(9ad-2bc)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{13}{4}}d^{\frac{7}{4}}}$$
$$-\frac{\sqrt{2}\left(-5ad\,(9ad-2bc)+3b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{13}{4}}d^{\frac{7}{4}}}$$
$$+\frac{\sqrt{2}\left(-5ad\,(9ad-2bc)+3b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{13}{4}}d^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] -2*a**2/(c*sqrt(x)*(c + d*x**2)**2) - x**(3/2)*(a*d*(9*a*d - 2*b*
c) + b**2*c**2)/(4*c**2*d*(c + d*x**2)**2) + x**(3/2)*(-5*a*d*(9*
a*d - 2*b*c) + 3*b**2*c**2)/(16*c**3*d*(c + d*x**2)) + sqrt(2)*(-
5*a*d*(9*a*d - 2*b*c) + 3*b**2*c**2)*log(-sqrt(2)*c**(1/4)*d**(1/
4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(13/4)*d**(7/4)) - sqrt
(2)*(-5*a*d*(9*a*d - 2*b*c) + 3*b**2*c**2)*log(sqrt(2)*c**(1/4)*d
**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(13/4)*d**(7/4)) -
 sqrt(2)*(-5*a*d*(9*a*d - 2*b*c) + 3*b**2*c**2)*atan(1 - sqrt(2)*
d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(13/4)*d**(7/4)) + sqrt(2)*(-5*
a*d*(9*a*d - 2*b*c) + 3*b**2*c**2)*atan(1 + sqrt(2)*d**(1/4)*sqrt
(x)/c**(1/4))/(64*c**(13/4)*d**(7/4))
```

Mathematica [A] time = 0.744102, size = 364, normalized size = 0.91

$$\frac{\frac{8\sqrt[4]{c}x^{3/2}\left(-13a^2d^2+10abcd+3b^2c^2\right)}{d(c+dx^2)}+\frac{\sqrt{2}\left(-45a^2d^2+10abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}}+\frac{\sqrt{2}\left(45a^2d^2-10abcd-3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\cdots\right.}{d^{7/4}}}{128c^{1\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(1/4))/Sqrt[x] - (32*c^(5/4)*(b*c - a*d)^2*x^(3/2))/
(d*(c + d*x^2)^2) + (8*c^(1/4)*(3*b^2*c^2 + 10*a*b*c*d - 13*a^2*d
^2)*x^(3/2))/(d*(c + d*x^2)) + (2*Sqrt[2]*(-3*b^2*c^2 - 10*a*b*c*
d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^
(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (Sqrt[2]*(3*b^2*
c^2 + 10*a*b*c*d - 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 10*a*b
*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/d^(7/4))/(128*c^(13/4))
```

Maple [A] time = 0.031, size = 568, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] -13/16/c^3/(d*x^2+c)^2*x^(7/2)*a^2*d^2+5/8/c^2/(d*x^2+c)^2*x^(7/2
)*a*b*d+3/16/c/(d*x^2+c)^2*x^(7/2)*b^2-17/16/c^2/(d*x^2+c)^2*d*x^
(3/2)*a^2+9/8/c/(d*x^2+c)^2*x^(3/2)*a*b-1/16/(d*x^2+c)^2/d*x^(3/2
)*b^2-45/64/c^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*a^2+5/32/c^2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*a*b+3/64/c/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)*b^2-45/128/c^3/(c/d)^(1/4)*2^(1/2)*ln((x
-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2)))*a^2+5/64/c^2/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2)))*a*b+3/128/c/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2)))*b^2-45/64/c^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a^2+5/32/c^2/d/(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/64/c/d^2/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2*a^2/c^3/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279397, size = 2147, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/64*(128*a^2*c^2*d - 4*(3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3
)*x^4 + 4*(b^2*c^3 - 18*a*b*c^2*d + 81*a^2*c*d^2)*x^2 + 4*(c^3*d^
3*x^4 + 2*c^4*d^2*x^2 + c^5*d)*sqrt(x)*(-(81*b^8*c^8 + 1080*a*b^7
*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*
b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3
645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*arctan(-c
^10*d^5*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 -
36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^
3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^
8*d^8)/(c^13*d^7))^(3/4)/((27*b^6*c^6 + 270*a*b^5*c^5*d - 315*a^2
*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3 + 4725*a^4*b^2*c^2*d^4 + 6075
0*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x) - sqrt((729*b^12*c^12 + 14
580*a*b^11*c^11*d + 55890*a^2*b^10*c^10*d^2 - 553500*a^3*b^9*c^9*
d^3 - 3479625*a^4*b^8*c^8*d^4 + 10305000*a^5*b^7*c^7*d^5 + 753175
00*a^6*b^6*c^6*d^6 - 154575000*a^7*b^5*c^5*d^7 - 782915625*a^8*b^
4*c^4*d^8 + 1868062500*a^9*b^3*c^3*d^9 + 2829431250*a^10*b^2*c^2*
d^10 - 11071687500*a^11*b*c*d^11 + 8303765625*a^12*d^12)*x - (81*
b^8*c^15*d^3 + 1080*a*b^7*c^14*d^4 + 540*a^2*b^6*c^13*d^5 - 36600
*a^3*b^5*c^12*d^6 - 42650*a^4*b^4*c^11*d^7 + 549000*a^5*b^3*c^10*
d^8 + 121500*a^6*b^2*c^9*d^9 - 3645000*a^7*b*c^8*d^10 + 4100625*a
^8*c^7*d^11)*sqrt(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c
^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a
^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4
100625*a^8*d^8)/(c^13*d^7))))) + (c^3*d^3*x^4 + 2*c^4*d^2*x^2 + c
^5*d)*sqrt(x)*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*
d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*
b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100
625*a^8*d^8)/(c^13*d^7))^(1/4)*log(c^10*d^5*(-(81*b^8*c^8 + 1080*
```

```
a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650
*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^
6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) - (2
7*b^6*c^6 + 270*a*b^5*c^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*
c^3*d^3 + 4725*a^4*b^2*c^2*d^4 + 60750*a^5*b*c*d^5 - 91125*a^6*d^
6)*sqrt(x)) - (c^3*d^3*x^4 + 2*c^4*d^2*x^2 + c^5*d)*sqrt(x)*(-(81
*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5
*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 12150
0*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*
d^7))^(1/4)*log(-c^10*d^5*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*
a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 +
 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*
c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) - (27*b^6*c^6 + 270*a*
b^5*c^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3 + 4725*a^4
*b^2*c^2*d^4 + 60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)))/((c^3
*d^3*x^4 + 2*c^4*d^2*x^2 + c^5*d)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.336035, size = 576, normalized size = 1.44

$$-\frac{2\,a^2}{c^3\sqrt{x}}+\frac{3\,b^2c^2dx^{\frac{7}{2}}+10\,abcd^2x^{\frac{7}{2}}-13\,a^2d^3x^{\frac{7}{2}}-b^2c^3x^{\frac{3}{2}}+18\,abc^2dx^{\frac{3}{2}}-17\,a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2c^3d}$$
$$+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+10\,\left(cd^3\right)^{\frac{3}{4}}abcd-45\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^4d^4}$$
$$+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+10\,\left(cd^3\right)^{\frac{3}{4}}abcd-45\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^4d^4}$$
$$-\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+10\,\left(cd^3\right)^{\frac{3}{4}}abcd-45\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^4d^4}$$
$$+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+10\,\left(cd^3\right)^{\frac{3}{4}}abcd-45\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(3/2)),x, algorithm="giac")
```

```
[Out] -2*a^2/(c^3*sqrt(x)) + 1/16*(3*b^2*c^2*d*x^(7/2) + 10*a*b*c*d^2*x
^(7/2) - 13*a^2*d^3*x^(7/2) - b^2*c^3*x^(3/2) + 18*a*b*c^2*d*x^(3
/2) - 17*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^2*c^3*d) + 1/64*sqrt(2)*
(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^
(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x
))/(c/d)^(1/4))/(c^4*d^4) + 1/64*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2
 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^
4) - 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*
b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4)
+ x + sqrt(c/d))/(c^4*d^4) + 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c
^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*ln(-sqr
```

```
t(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^4)
```

3.439 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\sqrt{x}\left(11a^2d^2-6abcd+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}d^{5/4}}+\frac{\sqrt{x}\left(7ad(6bc-11ad)+3b^2c^2\right)}{48c^3d\left(c+dx^2\right)}$$

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) - ((3*b^2*c^2 - 6*a*b*c*d +
11*a^2*d^2)*Sqrt[x])/(12*c^2*d*(c + d*x^2)^2) + ((3*b^2*c^2 + 7*a
*d*(6*b*c - 11*a*d))*Sqrt[x])/(48*c^3*d*(c + d*x^2)) - ((3*b^2*c^
2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*
b*c - 11*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(15/4)*d^(5/4)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d)
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d)
)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(15/4)*d^(5/4))
```

Rubi [A] time = 0.845902, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{\sqrt{x}\left(11a^2d^2-6abcd+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}+\frac{\sqrt{x}\left(\frac{7a(6bc-11ad)}{c^2}+\frac{3b^2}{d}\right)}{48c\left(c+dx^2\right)}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) - ((3*b^2*c^2 - 6*a*b*c*d +
11*a^2*d^2)*Sqrt[x])/(12*c^2*d*(c + d*x^2)^2) + (((3*b^2)/d + (7*
a*(6*b*c - 11*a*d))/c^2)*Sqrt[x])/(48*c*(c + d*x^2)) - ((3*b^2*c^
2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*
b*c - 11*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(15/4)*d^(5/4)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d)
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
```

```
*Sqrt[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d)
)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(15/4)*d^(5/4))
```

Rubi in Sympy [A] time = 106.729, size = 384, normalized size = 0.96

$$-\frac{2a^2}{3cx^{\frac{3}{2}}\left(c+dx^2\right)^2}-\frac{\sqrt{x}\left(ad\left(11ad-6bc\right)+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}+\frac{\sqrt{x}\left(-7ad\left(11ad-6bc\right)+3b^2c^2\right)}{48c^3d\left(c+dx^2\right)}-\frac{\sqrt{2}\left(-7ad\left(11ad-6bc\right)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{15}{4}}d^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-7ad\left(11ad-6bc\right)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{15}{4}}d^{\frac{5}{4}}}-\frac{\sqrt{2}\left(-7ad\left(11ad-6bc\right)+3b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{15}{4}}d^{\frac{5}{4}}}+\frac{\sqrt{2}\left(-7ad\left(11ad-6bc\right)+3b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{15}{4}}d^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] -2*a**2/(3*c*x**(3/2)*(c + d*x**2)**2) - sqrt(x)*(a*d*(11*a*d - 6
*b*c) + 3*b**2*c**2)/(12*c**2*d*(c + d*x**2)**2) + sqrt(x)*(-7*a*
d*(11*a*d - 6*b*c) + 3*b**2*c**2)/(48*c**3*d*(c + d*x**2)) - sqrt
(2)*(-7*a*d*(11*a*d - 6*b*c) + 3*b**2*c**2)*log(-sqrt(2)*c**(1/4)
*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(15/4)*d**(5/4))
 + sqrt(2)*(-7*a*d*(11*a*d - 6*b*c) + 3*b**2*c**2)*log(sqrt(2)*c*
*(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(15/4)*d**
(5/4)) - sqrt(2)*(-7*a*d*(11*a*d - 6*b*c) + 3*b**2*c**2)*atan(1 -
 sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(15/4)*d**(5/4)) + sqr
t(2)*(-7*a*d*(11*a*d - 6*b*c) + 3*b**2*c**2)*atan(1 + sqrt(2)*d**
(1/4)*sqrt(x)/c**(1/4))/(64*c**(15/4)*d**(5/4))
```

Mathematica [A] time = 0.447418, size = 365, normalized size = 0.91

$$\frac{\frac{3\sqrt{2}\left(77a^2d^2-42abcd-3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{3\sqrt{2}\left(-77a^2d^2+42abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{6\sqrt{2}\left(77a^2d^2-42abcd-3b^2c\right.}{d^{5/4}}}{384c^1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(3/4))/x^(3/2) - (96*c^(7/4)*(b*c - a*d)^2*Sqrt[x])/
(d*(c + d*x^2)^2) + (24*c^(3/4)*(b^2*c^2 + 14*a*b*c*d - 15*a^2*d^
2)*Sqrt[x])/(d*(c + d*x^2)) + (6*Sqrt[2]*(-3*b^2*c^2 - 42*a*b*c*d
 + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(
5/4) + (6*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*ArcTan[1
+ (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (3*Sqrt[2]*(-3*b^
2*c^2 - 42*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4) + (3*Sqrt[2]*(3*b^2*c^2 + 42*
a*b*c*d - 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/d^(5/4))/(384*c^(15/4))
```

Maple [A] time = 0.03, size = 562, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] -15/16/c^3/(d*x^2+c)^2*x^(5/2)*a^2*d^2+7/8/c^2/(d*x^2+c)^2*x^(5/2
)*a*b*d+1/16/c/(d*x^2+c)^2*x^(5/2)*b^2-19/16/c^2/(d*x^2+c)^2*d*x^
(1/2)*a^2+11/8/c/(d*x^2+c)^2*x^(1/2)*a*b-3/16/(d*x^2+c)^2/d*x^(1/
2)*b^2-77/64/c^4*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)*a^2+21/32/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)*a*b+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-77/64/c^4*d*(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+21/32/c^3*(c/d)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/64/c^2/d*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/128/c^4
*d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+21/64/c^3*(
c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+3/128/c^2/d*(c/
d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-2/3*a^2/c^3/x^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272909, size = 1617, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/192*(128*a^2*c^2*d - 4*(3*b^2*c^2*d + 42*a*b*c*d^2 - 77*a^2*d^
3)*x^4 + 4*(9*b^2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2 - 12*(c
^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*sqrt(x)*(-(81*b^8*c^8 + 453
6*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 -
1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6
*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5
))^(1/4)*arctan(-c^4*d*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a
^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4
 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544
*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)/((3*b^2*c^2 +
42*a*b*c*d - 77*a^2*d^2)*sqrt(x) - sqrt(c^8*d^2*sqrt(-(81*b^8*c^8
 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*
d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 572741
40*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^
15*d^5)) + (9*b^4*c^4 + 252*a*b^3*c^3*d + 1302*a^2*b^2*c^2*d^2 -
6468*a^3*b*c*d^3 + 5929*a^4*d^4)*x))) + 3*(c^3*d^3*x^5 + 2*c^4*d^
2*x^3 + c^5*d*x)*sqrt(x)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940
*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d
^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 766975
```

```
44*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*log(c^4*d*(-
(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a
^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d
^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a
^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)
*sqrt(x)) - 3*(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*sqrt(x)*(-(
81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^
3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^
5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^
8*d^8)/(c^15*d^5))^(1/4)*log(-c^4*d*(-(81*b^8*c^8 + 4536*a*b^7*c^
7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^
4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d
^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4) -
 (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*sqrt(x)))/((c^3*d^3*x^5 +
2*c^4*d^2*x^3 + c^5*d*x)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.302784, size = 575, normalized size = 1.43

$$-\frac{2\,a^2}{3\,c^3x^{\frac{3}{2}}}+\frac{\sqrt{2}\Big(3\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\,\left(cd^3\right)^{\frac{1}{4}}abcd-77\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^4d^2}+\frac{\sqrt{2}\Big(3\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\,\left(cd^3\right)^{\frac{1}{4}}abcd-77\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^4d^2}+\frac{\sqrt{2}\Big(3\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\,\left(cd^3\right)^{\frac{1}{4}}abcd-77\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\Big)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^4d^2}-\frac{\sqrt{2}\Big(3\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\,\left(cd^3\right)^{\frac{1}{4}}abcd-77\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\Big)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^4d^2}+\frac{b^2c^2dx^{\frac{5}{2}}+14\,abcd^2x^{\frac{5}{2}}-15\,a^2d^3x^{\frac{5}{2}}-3\,b^2c^3\sqrt{x}+22\,abc^2d\sqrt{x}-19\,a^2cd^2\sqrt{x}}{16\left(dx^2+c\right)^2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(5/2)),x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^3*x^(3/2)) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 +
42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) +
 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d
 - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/128*sqrt(2)*(3*(c*d^
3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^
2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2)
- 1/128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c
*d - 77*(c*d^3)^(1/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) +
x + sqrt(c/d))/(c^4*d^2) + 1/16*(b^2*c^2*d*x^(5/2) + 14*a*b*c*d^2
*x^(5/2) - 15*a^2*d^3*x^(5/2) - 3*b^2*c^3*sqrt(x) + 22*a*b*c^2*d*
sqrt(x) - 19*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c^3*d)
```

3.440 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=439

$$-\frac{13a^2d^2-10abcd+5b^2c^2}{20c^2d\sqrt{x}\,(c+dx^2)^2}-\frac{2a^2}{5cx^{5/2}\,(c+dx^2)^2}+\frac{(5b^2c^2-9ad(10bc-13ad))\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{(5b^2c^2-9ad(10bc-13ad))\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{(5b^2c^2-9ad(10bc-13ad))\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}d^{3/4}}+\frac{(5b^2c^2-9ad(10bc-13ad))\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}d^{3/4}}+\frac{5b^2c^2-9ad(10bc-13ad)}{16c^4d\sqrt{x}}-\frac{5b^2c^2-9ad(10bc-13ad)}{80c^3d\sqrt{x}\,(c+dx^2)}$$

```
[Out] (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(16*c^4*d*Sqrt[x]) - (2*a^2
)/(5*c*x^(5/2)*(c + d*x^2)^2) - (5*b^2*c^2 - 10*a*b*c*d + 13*a^2*
d^2)/(20*c^2*d*Sqrt[x]*(c + d*x^2)^2) - (5*b^2*c^2 - 9*a*d*(10*b*
c - 13*a*d))/(80*c^3*d*Sqrt[x]*(c + d*x^2)) - ((5*b^2*c^2 - 9*a*d
*(10*b*c - 13*a*d))*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 1
3*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
2]*c^(17/4)*d^(3/4)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(17/4)*d^(3/4))
```

Rubi [A] time = 0.979314, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$-\frac{13a^2d^2-10abcd+5b^2c^2}{20c^2d\sqrt{x}\,(c+dx^2)^2}-\frac{2a^2}{5cx^{5/2}\,(c+dx^2)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\,(c+dx^2)}+\frac{(5b^2c^2-9ad(10bc-13ad))\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{(5b^2c^2-9ad(10bc-13ad))\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{(5b^2c^2-9ad(10bc-13ad))\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}d^{3/4}}+\frac{(5b^2c^2-9ad(10bc-13ad))\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}d^{3/4}}+\frac{5b^2c^2-9ad(10bc-13ad)}{16c^4d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(16*c^4*d*Sqrt[x]) - (2*a^2
)/(5*c*x^(5/2)*(c + d*x^2)^2) - (5*b^2*c^2 - 10*a*b*c*d + 13*a^2*
d^2)/(20*c^2*d*Sqrt[x]*(c + d*x^2)^2) - ((5*b^2)/d - (9*a*(10*b*c
 - 13*a*d))/c^2)/(80*c*Sqrt[x]*(c + d*x^2)) - ((5*b^2*c^2 - 9*a*d
*(10*b*c - 13*a*d))*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
```

```
)/(32*Sqrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 1
3*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
2]*c^(17/4)*d^(3/4)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(17/4)*d^(3/4))
```

Rubi in Sympy [A] time = 117.404, size = 420, normalized size = 0.96

$$-\frac{2a^2}{5cx^{\frac{5}{2}}(c+dx^2)^2}-\frac{ad(13ad-10bc)+5b^2c^2}{20c^2d\sqrt{x}(c+dx^2)^2}-\frac{9ad(13ad-10bc)+5b^2c^2}{80c^3d\sqrt{x}(c+dx^2)}+\frac{9ad(13ad-10bc)+5b^2c^2}{16c^4d\sqrt{x}}+\frac{\sqrt{2}\left(9ad(13ad-10bc)+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{17}{4}}d^{\frac{3}{4}}}-\frac{\sqrt{2}\left(9ad(13ad-10bc)+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{128c^{\frac{17}{4}}d^{\frac{3}{4}}}-\frac{\sqrt{2}\left(9ad(13ad-10bc)+5b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{17}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\left(9ad(13ad-10bc)+5b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{64c^{\frac{17}{4}}d^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**3,x)
```

```
[Out] -2*a**2/(5*c*x**(5/2)*(c + d*x**2)**2) - (a*d*(13*a*d - 10*b*c) +
 5*b**2*c**2)/(20*c**2*d*sqrt(x)*(c + d*x**2)**2) - (9*a*d*(13*a*
d - 10*b*c) + 5*b**2*c**2)/(80*c**3*d*sqrt(x)*(c + d*x**2)) + (9*
a*d*(13*a*d - 10*b*c) + 5*b**2*c**2)/(16*c**4*d*sqrt(x)) + sqrt(2
)*(9*a*d*(13*a*d - 10*b*c) + 5*b**2*c**2)*log(-sqrt(2)*c**(1/4)*d
**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(17/4)*d**(3/4)) -
 sqrt(2)*(9*a*d*(13*a*d - 10*b*c) + 5*b**2*c**2)*log(sqrt(2)*c**(
1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(128*c**(17/4)*d**(3
/4)) - sqrt(2)*(9*a*d*(13*a*d - 10*b*c) + 5*b**2*c**2)*atan(1 - s
qrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(64*c**(17/4)*d**(3/4)) + sqrt(
2)*(9*a*d*(13*a*d - 10*b*c) + 5*b**2*c**2)*atan(1 + sqrt(2)*d**(1
/4)*sqrt(x)/c**(1/4))/(64*c**(17/4)*d**(3/4))
```

Mathematica [A] time = 0.767862, size = 382, normalized size = 0.87

$$\frac{\frac{40\sqrt[4]{c}x^{3/2}(21a^2d^2-26abcd+5b^2c^2)}{c+dx^2}+\frac{5\sqrt{2}(117a^2d^2-90abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{5\sqrt{2}(117a^2d^2-90abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{d^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(5/4))/x^(5/2) + (1280*a*c^(1/4)*(-2*b*c + 3*a*d))/S
qrt[x] + (160*c^(5/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 + (40*
c^(1/4)*(5*b^2*c^2 - 26*a*b*c*d + 21*a^2*d^2)*x^(3/2))/(c + d*x^2
) - (10*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) + (10*Sqrt[2]*(5*b^2
*c^2 - 90*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/d^(3/4) + (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117
*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/d^(3/4) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*
Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/
```

```
4))/(640*c^(17/4))
```

Maple [A] time = 0.034, size = 590, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x)
```

```
[Out] 21/16/c^4/(d*x^2+c)^2*x^(7/2)*a^2*d^3-13/8/c^3/(d*x^2+c)^2*x^(7/2
)*a*b*d^2+5/16/c^2/(d*x^2+c)^2*x^(7/2)*b^2*d+25/16/c^3/(d*x^2+c)^
2*x^(3/2)*a^2*d^2-17/8/c^2/(d*x^2+c)^2*x^(3/2)*a*b*d+9/16/c/(d*x^
2+c)^2*x^(3/2)*b^2+117/128/c^4*d/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2)))+117/64/c^4*d/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)+117/64/c^4*d/(c/d)^(1/4)*2^(1/2)*a^2
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-45/64/c^3/(c/d)^(1/4)*2^(1
/2)*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-45/32/c^3/(c/d)^(1/4)*2^(1/2)*
a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-45/32/c^3/(c/d)^(1/4)*2
^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+5/128/c^2/d/(c/d
)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
)/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+5/64/c^2/d/(c/d)^(
1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+5/64/c^2/d
/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/
5*a^2/c^3/x^(5/2)+6*a^2/c^4/x^(1/2)*d-4*a/c^3/x^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280623, size = 2168, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/320*(20*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 128*a^
2*c^3 + 36*(5*b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 128*(
10*a*b*c^3 - 13*a^2*c^2*d)*x^2 + 20*(c^4*d^2*x^6 + 2*c^5*d*x^4 +
c^6*x^2)*sqrt(x)*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2
*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d
^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 5765
80680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*arctan(c
^13*d^2*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*
d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 4150
92600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7
*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4)/((125*b^6*c^6 - 6
750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^
3 + 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d
^6)*sqrt(x) + sqrt((15625*b^12*c^12 - 1687500*a*b^11*c^11*d + 781
```

```
31250*a^2*b^10*c^10*d^2 - 2019937500*a^3*b^9*c^9*d^3 + 3183983437
5*a^4*b^8*c^8*d^4 - 314326575000*a^5*b^7*c^7*d^5 + 1936382557500*
a^6*b^6*c^6*d^6 - 7355241855000*a^7*b^5*c^5*d^7 + 17434219710375*
a^8*b^4*c^4*d^8 - 25881265273500*a^9*b^3*c^3*d^9 + 23425464012210
*a^10*b^2*c^2*d^10 - 11839219392780*a^11*b*c*d^11 + 2565164201769
*a^12*d^12)*x - (625*b^8*c^17*d - 45000*a*b^7*c^16*d^2 + 1273500*
a^2*b^6*c^15*d^3 - 17739000*a^3*b^5*c^14*d^4 + 124525350*a^4*b^4*
c^13*d^5 - 415092600*a^5*b^3*c^12*d^6 + 697317660*a^6*b^2*c^11*d^
7 - 576580680*a^7*b*c^10*d^8 + 187388721*a^8*c^9*d^9)*sqrt(-(625*
b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*
a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c
^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 1873
88721*a^8*d^8)/(c^17*d^3))))) + 5*(c^4*d^2*x^6 + 2*c^5*d*x^4 + c^
6*x^2)*sqrt(x)*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b
^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4
 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580
680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(c^13*d
^2*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 -
 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600
*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*
d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4) + (125*b^6*c^6 - 6750*
a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^3 +
3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d^6)*
sqrt(x)) - 5*(c^4*d^2*x^6 + 2*c^5*d*x^4 + c^6*x^2)*sqrt(x)*(-(625
*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000
*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*
c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187
388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(-c^13*d^2*(-(625*b^8*c^8 -
45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^
5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 6
97317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*
d^8)/(c^17*d^3))^(3/4) + (125*b^6*c^6 - 6750*a*b^5*c^5*d + 130275
*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^3 + 3048435*a^4*b^2*c^2*
d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d^6)*sqrt(x)))/((c^4*d^2*
x^6 + 2*c^5*d*x^4 + c^6*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.31651, size = 599, normalized size = 1.36

$$\frac{5\,b^2c^2dx^{\frac{7}{2}} - 26\,abcd^2x^{\frac{7}{2}} + 21\,a^2d^3x^{\frac{7}{2}} + 9\,b^2c^3x^{\frac{3}{2}} - 34\,abc^2dx^{\frac{3}{2}} + 25\,a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2c^4} - \frac{2\,(10\,abcx^2 - 15\,a^2dx^2 + a^2c)}{5\,c^4x^{\frac{5}{2}}} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}abcd + 117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^5d^3} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}abcd + 117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^5d^3} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}abcd + 117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^5d^3} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}abcd + 117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^3*x^(7/2)),x, algorithm="giac")
```

```
[Out] 1/16*(5*b^2*c^2*d*x^(7/2) - 26*a*b*c*d^2*x^(7/2) + 21*a^2*d^3*x^(
7/2) + 9*b^2*c^3*x^(3/2) - 34*a*b*c^2*d*x^(3/2) + 25*a^2*c*d^2*x^
(3/2))/((d*x^2 + c)^2*c^4) - 2/5*(10*a*b*c*x^2 - 15*a^2*d*x^2 + a
^2*c)/(c^4*x^(5/2)) + 1/64*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*
(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqr
t(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^5*d^3) + 1
/64*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d +
 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1
/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^5*d^3) - 1/128*sqrt(2)*(5*(c*d^3
)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^
2*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3)
+ 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c
*d + 117*(c*d^3)^(3/4)*a^2*d^2)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(c^5*d^3)
```

3.441 $\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=328

$$-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{19/4}}-\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{19/4}}+\frac{2dx^{7/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{7b^3}+\frac{2x^{3/2}(bc-ad)^3}{3b^4}+\frac{2d^2x^{11/2}(3bc-ad)}{11b^2}+\frac{2d^3x^{15/2}}{15b}$$

```
[Out] (2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d +
 a^2*d^2)*x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b
^2) + (2*d^3*x^(15/2))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4
)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4)) + (a^(3
/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4))
```

Rubi [A] time = 0.638363, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{19/4}}-\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{19/4}}+\frac{2dx^{7/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{7b^3}+\frac{2x^{3/2}(bc-ad)^3}{3b^4}+\frac{2d^2x^{11/2}(3bc-ad)}{11b^2}+\frac{2d^3x^{15/2}}{15b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d +
 a^2*d^2)*x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b
^2) + (2*d^3*x^(15/2))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4
)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4)) + (a^(3
/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4))
```

Rubi in Sympy [A] time = 106.645, size = 311, normalized size = 0.95

$$\frac{\sqrt{2}a^{\frac{3}{4}}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{19}{4}}}-\frac{\sqrt{2}a^{\frac{3}{4}}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{19}{4}}}-\frac{\sqrt{2}a^{\frac{3}{4}}(ad-bc)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{19}{4}}}+\frac{\sqrt{2}a^{\frac{3}{4}}(ad-bc)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{19}{4}}}+\frac{2d^3x^{\frac{15}{2}}}{15b}-\frac{2d^2x^{\frac{11}{2}}(ad-3bc)}{11b^2}+\frac{2dx^{\frac{7}{2}}\left(a^2d^2-3abcd+3b^2c^2\right)}{7b^3}-\frac{2x^{\frac{3}{2}}(ad-bc)^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(2)*a**(3/4)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*sq
rt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(19/4)) - sqrt(2)*a**(3/4)*(a*
d - b*c)**3*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqr
t(b)*x)/(4*b**(19/4)) - sqrt(2)*a**(3/4)*(a*d - b*c)**3*atan(1 -
sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(19/4)) + sqrt(2)*a**(3/
4)*(a*d - b*c)**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*
b**(19/4)) + 2*d**3*x**(15/2)/(15*b) - 2*d**2*x**(11/2)*(a*d - 3*
b*c)/(11*b**2) + 2*d*x**(7/2)*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**
2)/(7*b**3) - 2*x**(3/2)*(a*d - b*c)**3/(3*b**4)
```

Mathematica [A] time = 0.259935, size = 314, normalized size = 0.96

$$\frac{1155\sqrt{2}a^{3/4}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1155\sqrt{2}a^{3/4}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2310\sqrt{2}a}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (3080*b^(3/4)*(b*c - a*d)^3*x^(3/2) + 1320*b^(7/4)*d*(3*b^2*c^2 -
 3*a*b*c*d + a^2*d^2)*x^(7/2) + 840*b^(11/4)*d^2*(3*b*c - a*d)*x^
(11/2) + 616*b^(15/4)*d^3*x^(15/2) - 2310*Sqrt[2]*a^(3/4)*(-(b*c)
 + a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2310*Sq
rt[2]*a^(3/4)*(-(b*c) + a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)] + 1155*Sqrt[2]*a^(3/4)*(-(b*c) + a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1155*Sqrt[2]*a^(3
/4)*(-(b*c) + a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(4620*b^(19/4))
```

Maple [B] time = 0.015, size = 721, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 2/15*d^3*x^(15/2)/b-2/11/b^2*x^(11/2)*a*d^3+6/11/b*x^(11/2)*c*d^2
+2/7/b^3*x^(7/2)*a^2*d^3-6/7/b^2*x^(7/2)*a*c*d^2+6/7/b*x^(7/2)*c^
2*d-2/3/b^4*x^(3/2)*a^3*d^3+2/b^3*x^(3/2)*a^2*c*d^2-2/b^2*x^(3/2)
*a*c^2*d+2/3/b*x^(3/2)*c^3+1/2*a^4/b^5/(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-3/2*a^3/b^4/(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+3/2*a^2/b^3/(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d-1/2*a/b
^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+
1/2*a^4/b^5/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)*d^3-3/2*a^3/b^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)*c*d^2+3/2*a^2/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/4*a^4/b^5/(a/b)^(1/4)*2^(
1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^3-3/4*a^3/b^4/(a/b)^(1/4)*2^(1/
2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c*d^2+3/4*a^2/b^3/(a/b)^(1/4)*2^(1/
2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2*d-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)
*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(5/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278634, size = 2951, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(5/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2310*(4620*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b
^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^
8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*
a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 -
12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*arctan(-b^14*(-(a^3*b^1
2*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*
c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6
*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12
*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^1
2)/b^19)^(3/4)/((a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d
^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d
^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a^10*b*c*d^8 - a
^11*d^9)*sqrt(x) - sqrt((a^4*b^18*c^18 - 18*a^5*b^17*c^17*d + 153
*a^6*b^16*c^16*d^2 - 816*a^7*b^15*c^15*d^3 + 3060*a^8*b^14*c^14*d
^4 - 8568*a^9*b^13*c^13*d^5 + 18564*a^10*b^12*c^12*d^6 - 31824*a^
11*b^11*c^11*d^7 + 43758*a^12*b^10*c^10*d^8 - 48620*a^13*b^9*c^9*
d^9 + 43758*a^14*b^8*c^8*d^10 - 31824*a^15*b^7*c^7*d^11 + 18564*a
^16*b^6*c^6*d^12 - 8568*a^17*b^5*c^5*d^13 + 3060*a^18*b^4*c^4*d^1
4 - 816*a^19*b^3*c^3*d^15 + 153*a^20*b^2*c^2*d^16 - 18*a^21*b*c*d
^17 + a^22*d^18)*x - (a^3*b^21*c^12 - 12*a^4*b^20*c^11*d + 66*a^5
*b^19*c^10*d^2 - 220*a^6*b^18*c^9*d^3 + 495*a^7*b^17*c^8*d^4 - 79
2*a^8*b^16*c^7*d^5 + 924*a^9*b^15*c^6*d^6 - 792*a^10*b^14*c^5*d^7
 + 495*a^11*b^13*c^4*d^8 - 220*a^12*b^12*c^3*d^9 + 66*a^13*b^11*c
^2*d^10 - 12*a^14*b^10*c*d^11 + a^15*b^9*d^12)*sqrt(-(a^3*b^12*c^
12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*
d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6
*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3
*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b
^19)))) + 1155*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5
*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*
a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 49
5*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10
- 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*log(b^14*(-(a^3*b^12*
c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^
9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c
^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b
^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)
/b^19)^(3/4) - (a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^
2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^
5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a^10*b*c*d^8 - a^
11*d^9)*sqrt(x)) - 1155*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d
 + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d
^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5
*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*
c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*log(-b^14*(-
(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*
a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924
*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 -
```

```
220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 +
a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*
b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*
b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a^10*b*
c*d^8 - a^11*d^9)*sqrt(x)) + 4*(77*b^3*d^3*x^7 + 105*(3*b^3*c*d^2
 - a*b^2*d^3)*x^5 + 165*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)
*x^3 + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)
*sqrt(x))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.293454, size = 717, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(5/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^7 - 1/
2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d +
3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^7 + 1/4*
sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*
(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(sqrt(2)*sqr
t(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^7 - 1/4*sqrt(2)*((a*b^3)^(3/4
)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c
*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/b^7 + 2/1155*(77*b^14*d^3*x^(15/2) + 315*b^14*c*d^2
*x^(11/2) - 105*a*b^13*d^3*x^(11/2) + 495*b^14*c^2*d*x^(7/2) - 49
5*a*b^13*c*d^2*x^(7/2) + 165*a^2*b^12*d^3*x^(7/2) + 385*b^14*c^3*
x^(3/2) - 1155*a*b^13*c^2*d*x^(3/2) + 1155*a^2*b^12*c*d^2*x^(3/2)
 - 385*a^3*b^11*d^3*x^(3/2))/b^15
```

3.442 $\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=326

$$\frac{2dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{\sqrt[4]{a}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}+\frac{\sqrt[4]{a}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{17/4}}+\frac{2\sqrt{x}(bc-ad)^3}{b^4}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2d^3x^{13/2}}{13b}$$

```
[Out] (2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2
*d^2)*x^(5/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) +
(2*d^3*x^(13/2))/(13*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c
 - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]
*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) - (a^(1/4)*(b
*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.575506, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{2dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{\sqrt[4]{a}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}+\frac{\sqrt[4]{a}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{17/4}}+\frac{2\sqrt{x}(bc-ad)^3}{b^4}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2d^3x^{13/2}}{13b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```


```
[Out] (2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2
*d^2)*x^(5/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) +
(2*d^3*x^(13/2))/(13*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c
 - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]
*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) - (a^(1/4)*(b
*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*b^(17/4))
```

Rubi in Sympy [A] time = 105.181, size = 309, normalized size = 0.95

$$-\frac{\sqrt{2}\sqrt[4]{a}\,(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{17}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}\,(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{17}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}\,(ad-bc)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{17}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}\,(ad-bc)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{17}{4}}}+\frac{2d^3x^{\frac{13}{2}}}{13b}-\frac{2d^2x^{\frac{9}{2}}\,(ad-3bc)}{9b^2}+\frac{2dx^{\frac{5}{2}}\,(a^2d^2-3abcd+3b^2c^2)}{5b^3}-\frac{2\sqrt{x}\,(ad-bc)^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -sqrt(2)*a**(1/4)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*s
qrt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(17/4)) + sqrt(2)*a**(1/4)*(a
*d - b*c)**3*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sq
rt(b)*x)/(4*b**(17/4)) - sqrt(2)*a**(1/4)*(a*d - b*c)**3*atan(1 -
 sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(17/4)) + sqrt(2)*a**(1
/4)*(a*d - b*c)**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2
*b**(17/4)) + 2*d**3*x**(13/2)/(13*b) - 2*d**2*x**(9/2)*(a*d - 3*
b*c)/(9*b**2) + 2*d*x**(5/2)*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2
)/(5*b**3) - 2*sqrt(x)*(a*d - b*c)**3/b**4
```

Mathematica [A] time = 0.219743, size = 314, normalized size = 0.96

$$\frac{936b^{5/4}dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)+520b^{9/4}d^2x^{9/2}(3bc-ad)+4680\sqrt[4]{b}\sqrt{x}(bc-ad)^3-585\sqrt{2}\sqrt[4]{a}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (4680*b^(1/4)*(b*c - a*d)^3*Sqrt[x] + 936*b^(5/4)*d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^(5/2) + 520*b^(9/4)*d^2*(3*b*c - a*d)*x^(9
/2) + 360*b^(13/4)*d^3*x^(13/2) - 1170*Sqrt[2]*a^(1/4)*(-(b*c) +
a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1170*Sqrt[
2]*a^(1/4)*(-(b*c) + a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)] - 585*Sqrt[2]*a^(1/4)*(-(b*c) + a*d)^3*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(1/4)*(
-(b*c) + a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2340*b^(17/4))
```

Maple [B] time = 0.015, size = 712, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 2/13*d^3*x^(13/2)/b-2/9/b^2*x^(9/2)*a*d^3+2/3/b*x^(9/2)*c*d^2+2/5
/b^3*x^(5/2)*a^2*d^3-6/5/b^2*x^(5/2)*a*c*d^2+6/5/b*x^(5/2)*c^2*d-
2/b^4*a^3*d^3*x^(1/2)+6/b^3*a^2*c*d^2*x^(1/2)-6/b^2*a*c^2*d*x^(1/
2)+2/b*c^3*x^(1/2)+1/2/b^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)*a^3*d^3-3/2/b^3*(a/b)^(1/4)*2^(1/2)*arctan(2^
```

```
(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^2*c*d^2+3/2/b^2*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a*c^2*d-1/2/b*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+1/2/b^4*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a^3*d^3-3/2/b
^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a^2*
c*d^2+3/2/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)*a*c^2*d-1/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)*c^3+1/4/b^4*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2)))*a^3*d^3-3/4/b^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))*a^2*c*d^2+3/4/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))*a*c^2*d-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(3/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264048, size = 2106, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(3/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/1170*(2340*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^
10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6
*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^
9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*
a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*arctan(-b^4*(-(a*b^12*c^12
 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d
^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3
*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)
^(1/4)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(
x) - sqrt(b^8*sqrt(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^
10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6
*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^
9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*
a^12*b*c*d^11 + a^13*d^12)/b^17) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5
*b*c*d^5 + a^6*d^6)*x))) - 585*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c
^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*
c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5
*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b
^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*log(b^4*(
-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a
^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*
a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220
*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^1
3*d^12)/b^17)^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*sqrt(x)) + 585*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d +
 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4
 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^
7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*
```

```
d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*log(-b^4*(-(a*b^
12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9
*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^
6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*
b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12
)/b^17)^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*sqrt(x)) - 4*(45*b^3*d^3*x^6 + 585*b^3*c^3 - 1755*a*b^2*c^2*d
+ 1755*a^2*b*c*d^2 - 585*a^3*d^3 + 65*(3*b^3*c*d^2 - a*b^2*d^3)*x
^4 + 117*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^2)*sqrt(x))/
b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.317018, size = 717, normalized size = 2.2

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac14}b^3c^3 - 3\left(ab^3\right)^{\frac14}ab^2c^2d + 3\left(ab^3\right)^{\frac14}a^2bcd^2 - \left(ab^3\right)^{\frac14}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2b^5}$$
$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac14}b^3c^3 - 3\left(ab^3\right)^{\frac14}ab^2c^2d + 3\left(ab^3\right)^{\frac14}a^2bcd^2 - \left(ab^3\right)^{\frac14}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2b^5}$$
$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac14}b^3c^3 - 3\left(ab^3\right)^{\frac14}ab^2c^2d + 3\left(ab^3\right)^{\frac14}a^2bcd^2 - \left(ab^3\right)^{\frac14}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4b^5}$$
$$+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac14}b^3c^3 - 3\left(ab^3\right)^{\frac14}ab^2c^2d + 3\left(ab^3\right)^{\frac14}a^2bcd^2 - \left(ab^3\right)^{\frac14}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4b^5}$$
$$+\frac{2\left(45\,b^{12}d^3x^{\frac{13}{2}} + 195\,b^{12}cd^2x^{\frac92} - 65\,ab^{11}d^3x^{\frac92} + 351\,b^{12}c^2dx^{\frac52} - 351\,ab^{11}cd^2x^{\frac52} + 117\,a^2b^{10}d^3x^{\frac52} + 585\,b^{12}c^3\sqrt{x} - 1755\,ab^{11}\right)}{585\,b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(3/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/
2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d +
3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/4*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*
(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(sqrt(2)*sqr
t(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3)^(1/4
)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c
*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/b^5 + 2/585*(45*b^12*d^3*x^(13/2) + 195*b^12*c*d^2*
x^(9/2) - 65*a*b^11*d^3*x^(9/2) + 351*b^12*c^2*d*x^(5/2) - 351*a*
b^11*c*d^2*x^(5/2) + 117*a^2*b^10*d^3*x^(5/2) + 585*b^12*c^3*sqrt
(x) - 1755*a*b^11*c^2*d*sqrt(x) + 1755*a^2*b^10*c*d^2*sqrt(x) - 5
85*a^3*b^9*d^3*sqrt(x))/b^13
```

3.443 $\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=306

$$\frac{2dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{2d^3x^{11/2}}{11b}$$

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*
(3*b*c - a*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*
a^(1/4)*b^(15/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - a*d)^3*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2
]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rubi [A] time = 0.524272, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{2dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{2d^3x^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*
(3*b*c - a*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*
a^(1/4)*b^(15/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - a*d)^3*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2
]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rubi in Sympy [A] time = 98.2093, size = 291, normalized size = 0.95

$$\frac{2d^3x^{\frac{11}{2}}}{11b}-\frac{2d^2x^{\frac{7}{2}}\left(ad-3bc\right)}{7b^2}+\frac{2dx^{\frac{3}{2}}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}-\frac{\sqrt{2}\left(ad-bc\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{a}b^{\frac{15}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{a}b^{\frac{15}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{\frac{15}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*d**3*x**(11/2)/(11*b) - 2*d**2*x**(7/2)*(a*d - 3*b*c)/(7*b**2)
+ 2*d*x**(3/2)*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)/(3*b**3) - s
qrt(2)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sq
rt(a) + sqrt(b)*x)/(4*a**(1/4)*b**(15/4)) + sqrt(2)*(a*d - b*c)**
3*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4
*a**(1/4)*b**(15/4)) + sqrt(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b*
*(1/4)*sqrt(x)/a**(1/4))/(2*a**(1/4)*b**(15/4)) - sqrt(2)*(a*d -
b*c)**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(1/4)*b
**(15/4))
```

Mathematica [A] time = 0.247934, size = 291, normalized size = 0.95

$$616\sqrt[4]{a}b^{3/4}dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)-264\sqrt[4]{a}b^{7/4}d^2x^{7/2}(ad-3bc)+168\sqrt[4]{a}b^{11/4}d^3x^{11/2}+231\sqrt{2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (616*a^(1/4)*b^(3/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2)
- 264*a^(1/4)*b^(7/4)*d^2*(-3*b*c + a*d)*x^(7/2) + 168*a^(1/4)*b^
(11/4)*d^3*x^(11/2) - 462*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 462*Sqrt[2]*(b*c - a*d)^3*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 231*Sqrt[2]*(b*c - a*d)^3
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 231
*Sqrt[2]*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(924*a^(1/4)*b^(15/4))
```

Maple [B] time = 0.015, size = 659, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x)
```

```
[Out] 2/11*d^3*x^(11/2)/b-2/7*d^3/b^2*x^(7/2)*a+6/7*d^2/b*x^(7/2)*c+2/3
*d^3/b^3*x^(3/2)*a^2-2*d^2/b^2*x^(3/2)*c*a+2*d/b*x^(3/2)*c^2-1/4/
b^4/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*a^3*d^3+3/4/b^
3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*a^2*c*d^2-3/4/b^
2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*a*c^2*d+1/4/b/(a
/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3-1/2/b^4/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^3*d^3+3/2/b^3
/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^2*c*
d^2-3/2/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)*a*c^2*d+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)*c^3-1/2/b^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)-1)*a^3*d^3+3/2/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1)*a^2*c*d^2-3/2/b^2/(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a*c^2*d+1/2/b/(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*sqrt(x)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277267, size = 2844, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*sqrt(x)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/462*(924*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*
c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*arctan(-a*b^11*(-(b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^
(3/4)/((b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6
*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3
*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)
- sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 81
6*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13
*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*
a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^
10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^1
3*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 +
 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)*x - (a*b^1
9*c^12 - 12*a^2*b^18*c^11*d + 66*a^3*b^17*c^10*d^2 - 220*a^4*b^16
*c^9*d^3 + 495*a^5*b^15*c^8*d^4 - 792*a^6*b^14*c^7*d^5 + 924*a^7*
b^13*c^6*d^6 - 792*a^8*b^12*c^5*d^7 + 495*a^9*b^11*c^4*d^8 - 220*
a^10*b^10*c^3*d^9 + 66*a^11*b^9*c^2*d^10 - 12*a^12*b^8*c*d^11 + a
^13*b^7*d^12)*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*
b*c*d^11 + a^12*d^12)/(a*b^15))))) + 231*b^3*(-(b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*
log(a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)/(a*b^15))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^
2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^
5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b
*c*d^8 - a^9*d^9)*sqrt(x)) - 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^
8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*log(-a*b
^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924
*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a*b^15))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c
^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c
```

```
^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8
- a^9*d^9)*sqrt(x)) - 4*(21*b^2*d^3*x^5 + 33*(3*b^2*c*d^2 - a*b*d
^3)*x^3 + 77*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)*sqrt(x))/b^
3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.30328, size = 662, normalized size = 2.16

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^{6}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^{6}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^{6}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^{6}}+\frac{2\left(21b^{10}d^{3}x^{\frac{11}{2}}+99b^{10}cd^{2}x^{\frac{7}{2}}-33ab^{9}d^{3}x^{\frac{7}{2}}+231b^{10}c^{2}dx^{\frac{3}{2}}-231ab^{9}cd^{2}x^{\frac{3}{2}}+77a^{2}b^{8}d^{3}x^{\frac{3}{2}}\right)}{231b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*sqrt(x)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
+ 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^6) +
 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^6)
 - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2
*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6) + 1/4*sqrt(2)*((
a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3
/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(a*b^6) + 2/231*(21*b^10*d^3*x^(11/2) +
99*b^10*c*d^2*x^(7/2) - 33*a*b^9*d^3*x^(7/2) + 231*b^10*c^2*d*x^(
3/2) - 231*a*b^9*c*d^2*x^(3/2) + 77*a^2*b^8*d^3*x^(3/2))/b^11
```

3.444 $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)}\,dx$

Optimal. Leaf size=304

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}b^{13/4}}+\frac{2d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{2d^2x^{5/2}(3bc-ad)}{5b^2}+\frac{2d^3x^{9/2}}{9b}$$

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b
*c - a*d)*x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)
^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4
)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3
/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.506925, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}b^{13/4}}+\frac{2d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{2d^2x^{5/2}(3bc-ad)}{5b^2}+\frac{2d^3x^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```


```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b
*c - a*d)*x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)
^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4
)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3
/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rubi in Sympy [A] time = 96.7473, size = 289, normalized size = 0.95

$$\frac{2d^3x^{\frac{9}{2}}}{9b}-\frac{2d^2x^{\frac{5}{2}}(ad-3bc)}{5b^2}+\frac{2d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{\sqrt{2}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{3}{4}}b^{\frac{13}{4}}}-\frac{\sqrt{2}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{3}{4}}b^{\frac{13}{4}}}+\frac{\sqrt{2}(ad-bc)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{13}{4}}}-\frac{\sqrt{2}(ad-bc)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*d**3*x**(9/2)/(9*b) - 2*d**2*x**(5/2)*(a*d - 3*b*c)/(5*b**2) +
2*d*sqrt(x)*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)/b**3 + sqrt(2)*
(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
 sqrt(b)*x)/(4*a**(3/4)*b**(13/4)) - sqrt(2)*(a*d - b*c)**3*log(s
qrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(3/
4)*b**(13/4)) + sqrt(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b**(1/4)*
sqrt(x)/a**(1/4))/(2*a**(3/4)*b**(13/4)) - sqrt(2)*(a*d - b*c)**3
*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(3/4)*b**(13/4
))
```

Mathematica [A] time = 0.260934, size = 291, normalized size = 0.96

$$-72a^{3/4}b^{5/4}d^2x^{5/2}(ad-3bc)+40a^{3/4}b^{9/4}d^3x^{9/2}+360a^{3/4}\sqrt[4]{b}d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)-45\sqrt{2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x]
- 72*a^(3/4)*b^(5/4)*d^2*(-3*b*c + a*d)*x^(5/2) + 40*a^(3/4)*b^(9
/4)*d^3*x^(9/2) - 90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]
*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(180*a^(3/4)*b^(13/4))
```

Maple [B] time = 0.014, size = 650, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/9*d^3*x^(9/2)/b-2/5*d^3/b^2*x^(5/2)*a+6/5*d^2/b*x^(5/2)*c+2*d^3
/b^3*a^2*x^(1/2)-6*d^2/b^2*a*c*x^(1/2)+6*d/b*c^2*x^(1/2)-1/2/b^3*
(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3
+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1)*c*d^2-3/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)-1)*c^2*d+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)*c^3-1/4/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2)))*d^3+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2)))*c*d^2-3/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3-
1/2/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)*d^3+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)*c*d^2-3/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)*c^2*d+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254648, size = 2052, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*
d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arctan(-a*b^3*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))
^(1/4)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(
x) - sqrt(a^2*b^6*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^
10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) + (b^6*c^6 - 6*a*b^5*c^5*d
+ 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
6*a^5*b*c*d^5 + a^6*d^6)*x))) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(
a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*sqrt(x)) + 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(-a*b
^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*
a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220
*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*sqrt(x)) + 4*(5*b^2*d^3*x^4 + 135*b^2*c^2*d - 135*a
*b*c*d^2 + 45*a^2*d^3 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^2)*sqrt(x))/b
^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.284875, size = 662, normalized size = 2.18

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^4} + \frac{2\left(5\,b^8d^3x^{\frac{9}{2}} + 27\,b^8cd^2x^{\frac{5}{2}} - 9\,ab^7d^3x^{\frac{5}{2}} + 135\,b^8c^2d\sqrt{x} - 135\,ab^7cd^2\sqrt{x} + 45\,a^2b^6d^3\sqrt{x}\right)}{45\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
+ 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) +
 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4)
 + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2
*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) - 1/4*sqrt(2)*((
a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(a*b^4) + 2/45*(5*b^8*d^3*x^(9/2) + 27*b
^8*c*d^2*x^(5/2) - 9*a*b^7*d^3*x^(5/2) + 135*b^8*c^2*d*sqrt(x) -
135*a*b^7*c*d^2*sqrt(x) + 45*a^2*b^6*d^3*sqrt(x))/b^9
```

3.445 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=284

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}b^{11/4}}+\frac{2d^2x^{3/2}(3bc-ad)}{3b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{2d^3x^{7/2}}{7b}$$

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2
*d^3*x^(7/2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(
11/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) + ((b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(5/4)*b^(11/4))
```

Rubi [A] time = 0.5904, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}b^{11/4}}+\frac{2d^2x^{3/2}(3bc-ad)}{3b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{2d^3x^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2
*d^3*x^(7/2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(
11/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) + ((b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(5/4)*b^(11/4))
```

Rubi in Sympy [A] time = 110.388, size = 265, normalized size = 0.93

$$\frac{2d^3x^{\frac{7}{2}}}{7b}-\frac{2d^2x^{\frac{3}{2}}(ad-3bc)}{3b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{\sqrt{2}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{5}{4}}b^{\frac{11}{4}}}-\frac{\sqrt{2}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{5}{4}}b^{\frac{11}{4}}}-\frac{\sqrt{2}(ad-bc)^3\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{5}{4}}b^{\frac{11}{4}}}+\frac{\sqrt{2}(ad-bc)^3\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{5}{4}}b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a),x)
```

```
[Out] 2*d**3*x**(7/2)/(7*b) - 2*d**2*x**(3/2)*(a*d - 3*b*c)/(3*b**2) -
2*c**3/(a*sqrt(x)) + sqrt(2)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)
*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(5/4)*b**(11/4)) -
 sqrt(2)*(a*d - b*c)**3*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + s
qrt(a) + sqrt(b)*x)/(4*a**(5/4)*b**(11/4)) - sqrt(2)*(a*d - b*c)*
*3*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(5/4)*b**(11
/4)) + sqrt(2)*(a*d - b*c)**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a
**(1/4))/(2*a**(5/4)*b**(11/4))
```

Mathematica [A] time = 0.186454, size = 283, normalized size = 1.

$$\frac{(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}+\frac{(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}b^{11/4}}+\frac{2d^2x^{3/2}(3bc-ad)}{3b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{2d^3x^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2
*d^3*x^(7/2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) + ((-(b*c) + a*d)^3
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*
b^(11/4)) + ((-(b*c) + a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) + ((b*c - a
*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])
/(2*Sqrt[2]*a^(5/4)*b^(11/4))
```

Maple [B] time = 0.018, size = 622, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x)
```

```
[Out] 2/7*d^3*x^(7/2)/b-2/3*d^3/b^2*x^(3/2)*a+2*d^2/b*x^(3/2)*c+1/2*a^2
/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^
3-3/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)*c*d^2+3/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)*c^2*d-1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)*c^3+1/4*a^2/b^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))*d^3-3/4*a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))*c*d^2+3/4/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
))*c^2*d-1/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
+1/2*a^2/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)*d^3-3/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)*c*d^2+3/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
```

```
^(1/4)*x^(1/2)+1)*c^2*d-1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)*c^3-2*c^3/a/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280197, size = 2843, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/42*(12*a*b*d^3*x^4 - 84*b^2*c^3 + 84*a*b^2*sqrt(x)*(-(b^12*c^12
 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
+ 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^1
1))^(1/4)*arctan(-a^4*b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 49
5*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4)/((b^9*c^9 - 9*a*b
^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*
c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c
^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x) - sqrt((b^18*c^18 - 18*
a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3
060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c
^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 486
20*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7
*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a
^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16
- 18*a^17*b*c*d^17 + a^18*d^18)*x - (a^3*b^17*c^12 - 12*a^4*b^16*
c^11*d + 66*a^5*b^15*c^10*d^2 - 220*a^6*b^14*c^9*d^3 + 495*a^7*b^
13*c^8*d^4 - 792*a^8*b^12*c^7*d^5 + 924*a^9*b^11*c^6*d^6 - 792*a^
10*b^10*c^5*d^7 + 495*a^11*b^9*c^4*d^8 - 220*a^12*b^8*c^3*d^9 + 6
6*a^13*b^7*c^2*d^10 - 12*a^14*b^6*c*d^11 + a^15*b^5*d^12)*sqrt(-(
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(a^5*b^11))))) + 21*a*b^2*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*log(a^4*
b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924
*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7
*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4
*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^
8 - a^9*d^9)*sqrt(x)) - 21*a*b^2*sqrt(x)*(-(b^12*c^12 - 12*a*b^11
*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
```

```
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*lo
g(-a^4*b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*
a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*
a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8
*b*c*d^8 - a^9*d^9)*sqrt(x)) + 28*(3*a*b*c*d^2 - a^2*d^3)*x^2)/(a
*b^2*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.286885, size = 624, normalized size = 2.2

$$-\frac{2\,c^3}{a\sqrt{x}}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^5}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^5}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^5}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^5}+\frac{2\left(3\,b^6d^3x^{\frac{7}{2}}+21\,b^6cd^2x^{\frac{3}{2}}-7\,ab^5d^3x^{\frac{3}{2}}\right)}{21\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(3/2)),x, algorithm="giac")
```

```
[Out] -2*c^3/(a*sqrt(x)) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^
3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4
)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(
a/b)^(1/4))/(a^2*b^5) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a
*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(
3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x
))/(a/b)^(1/4))/(a^2*b^5) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 -
3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^
3)^(3/4)*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^2*b^5) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)
*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^
3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5) + 2
/21*(3*b^6*d^3*x^(7/2) + 21*b^6*c*d^2*x^(3/2) - 7*a*b^5*d^3*x^(3/
2))/b^7
```

3.446 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=284

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}b^{9/4}}+\frac{2d^2\sqrt{x}(3bc-ad)}{b^2}-\frac{2c^3}{3ax^{3/2}}+\frac{2d^3x^{5/2}}{5b}$$

```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d
^3*x^(5/2))/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4
)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.541341, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}b^{9/4}}+\frac{2d^2\sqrt{x}(3bc-ad)}{b^2}-\frac{2c^3}{3ax^{3/2}}+\frac{2d^3x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d
^3*x^(5/2))/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4
)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(7/4)*b^(9/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2d^2(ad-3bc)\int^{\sqrt{x}}\frac{1}{b^2}\,dx+\frac{2d^3x^{\frac{5}{2}}}{5b}-\frac{2c^3}{3ax^{\frac{3}{2}}}-\frac{\sqrt{2}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{7}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}(ad-bc)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}(ad-bc)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{7}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a),x)
```

```
[Out] -2*d**2*(a*d - 3*b*c)*Integral(b**(-2), (x, sqrt(x))) + 2*d**3*x*
*(5/2)/(5*b) - 2*c**3/(3*a*x**(3/2)) - sqrt(2)*(a*d - b*c)**3*log
(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**
(7/4)*b**(9/4)) + sqrt(2)*(a*d - b*c)**3*log(sqrt(2)*a**(1/4)*b**
(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(7/4)*b**(9/4)) - sqrt
(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2
*a**(7/4)*b**(9/4)) + sqrt(2)*(a*d - b*c)**3*atan(1 + sqrt(2)*b**
(1/4)*sqrt(x)/a**(1/4))/(2*a**(7/4)*b**(9/4))
```

Mathematica [A] time = 0.165033, size = 283, normalized size = 1.

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}b^{9/4}}+\frac{(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}b^{9/4}}+\frac{2d^2\sqrt{x}(3bc-ad)}{b^2}-\frac{2c^3}{3ax^{3/2}}+\frac{2d^3x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d
^3*x^(5/2))/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) + ((-(b*c) + a*d)^3*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(
9/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) + ((-(b*c) + a*d)^3
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*S
qrt[2]*a^(7/4)*b^(9/4))
```

Maple [B] time = 0.018, size = 616, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x)
```

```
[Out] 2/5*d^3*x^(5/2)/b-2*d^3/b^2*a*x^(1/2)+6*d^2/b*x^(1/2)*c+1/2*a/b^2
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-3/
2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d
^2+3/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)*c^2*d-1/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)+1)*c^3+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)-1)*d^3-3/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)*c*d^2+3/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-1/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/4*a/b^2*(a/b)^(1/4)*2^(1
/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^3-3/4/b*(a/b)^(1/4)*2^(1/2)*ln((
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))*c*d^2+3/4/a*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))*c^2*d-1/4/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2)))*c^3-2/3*c^3/a/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268429, size = 2052, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/30*(12*a*b*d^3*x^4 - 60*a*b^2*x^(3/2)*(-(b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b
^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*arct
an(-a^2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d
^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^7*b^9))^(1/4)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*sqrt(x) - sqrt(a^4*b^4*sqrt(-(b^12*c^12 - 12
*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9)) +
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x))) + 15*a*b^2
*x^(3/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
+ 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4) -
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 15
*a*b^2*x^(3/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10
*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(
1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x
)) - 20*b^2*c^3 + 60*(3*a*b*c*d^2 - a^2*d^3)*x^2)/(a*b^2*x^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275127, size = 622, normalized size = 2.19

$$-\frac{2\,c^3}{3\,ax^{\frac{3}{2}}} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^2b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^2b^3} + \frac{2\left(b^4d^3x^{\frac{5}{2}} + 15\,b^4cd^2\sqrt{x} - 5\,ab^3d^3\sqrt{x}\right)}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(5/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^3/(a*x^(3/2)) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*
b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1
/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))
/(a/b)^(1/4))/(a^2*b^3) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*
(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)
^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*
b^3)^(1/4)*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/
4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*
d^3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) +
 2/5*(b^4*d^3*x^(5/2) + 15*b^4*c*d^2*sqrt(x) - 5*a*b^3*d^3*sqrt(x
))/b^5
```

3.447 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)} dx$

Optimal. Leaf size=283

$$\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}} - \frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}} - \frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}b^{7/4}} + \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}b^{7/4}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} - \frac{2c^3}{5ax^{5/2}} + \frac{2d^3x^{3/2}}{3b}$$

```
[Out] (-2*c^3)/(5*a*x^(5/2)) + (2*c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (2
*d^3*x^(3/2))/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7
/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) - ((b*c - a*d)^3*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(9/4)*b^(7/4))
```

Rubi [A] time = 0.58631, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}} - \frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}} - \frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}b^{7/4}} + \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}b^{7/4}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} - \frac{2c^3}{5ax^{5/2}} + \frac{2d^3x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(5*a*x^(5/2)) + (2*c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (2
*d^3*x^(3/2))/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7
/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) - ((b*c - a*d)^3*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(9/4)*b^(7/4))
```

Rubi in Sympy [A] time = 112.6, size = 265, normalized size = 0.94

$$\frac{2d^3x^{\frac{3}{2}}}{3b} - \frac{2c^3}{5ax^{\frac{5}{2}}} - \frac{2c^2(3ad-bc)}{a^2\sqrt{x}} - \frac{\sqrt{2}(ad-bc)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{9}{4}}b^{\frac{7}{4}}} + \frac{\sqrt{2}(ad-bc)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{9}{4}}b^{\frac{7}{4}}} + \frac{\sqrt{2}(ad-bc)^3 \operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{9}{4}}b^{\frac{7}{4}}} - \frac{\sqrt{2}(ad-bc)^3 \operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{9}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a),x)
```

```
[Out] 2*d**3*x**(3/2)/(3*b) - 2*c**3/(5*a*x**(5/2)) - 2*c**2*(3*a*d - b
*c)/(a**2*sqrt(x)) - sqrt(2)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)
*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(9/4)*b**(7/4)) +
sqrt(2)*(a*d - b*c)**3*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sq
rt(a) + sqrt(b)*x)/(4*a**(9/4)*b**(7/4)) + sqrt(2)*(a*d - b*c)**3
*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(9/4)*b**(7/4)
) - sqrt(2)*(a*d - b*c)**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(
1/4))/(2*a**(9/4)*b**(7/4))
```

Mathematica [A] time = 0.214999, size = 287, normalized size = 1.01

$$\frac{-24a^{5/4}b^{7/4}c^3 + 40a^{9/4}b^{3/4}d^3x^4 + 120\sqrt[4]{a}b^{7/4}c^2x^2(bc-3ad) + 15\sqrt{2}x^{5/2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 15\sqrt{2}}{60}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-24*a^(5/4)*b^(7/4)*c^3 + 120*a^(1/4)*b^(7/4)*c^2*(b*c - 3*a*d)*
x^2 + 40*a^(9/4)*b^(3/4)*d^3*x^4 - 30*Sqrt[2]*(b*c - a*d)^3*x^(5/
2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 30*Sqrt[2]*(b*
c - a*d)^3*x^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 15*Sqrt[2]*(b*c - a*d)^3*x^(5/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 15*Sqrt[2]*(b*c - a*d)^3*x^(5/2)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(60*a^
(9/4)*b^(7/4)*x^(5/2))
```

Maple [B] time = 0.021, size = 616, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x)
```

```
[Out] 2/3*d^3*x^(3/2)/b-2/5*c^3/a/x^(5/2)-6*c^2/x^(1/2)/a*d+2*c^3/x^(1/
2)/a^2*b-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)*d^3+3/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)+1)*c*d^2-3/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)*c^2*d+1/2/a^2*b/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/2/b/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-3/2/a/(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+1/2/a^2*b/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3-1/4*a/b^
2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^3+3/4/b/(a/b)^
(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c*d^2-3/4/a/(a/b)^(1/4)*
2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2*d+1/4/a^2*b/(a/b)^(1/4)*2^
(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282851, size = 2842, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/30*(20*a^2*d^3*x^4 - 60*a^2*b*x^(5/2)*(-(b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b
^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*arct
an(-a^7*b^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d
^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^9*b^7))^(3/4)/((b^9*c^9 - 9*a*b^8*c^8*d + 36*a
^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*
b*c*d^8 - a^9*d^9)*sqrt(x) - sqrt((b^18*c^18 - 18*a*b^17*c^17*d +
 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^
14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824
*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*
d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a
^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^1
4 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d
^17 + a^18*d^18)*x - (a^5*b^15*c^12 - 12*a^6*b^14*c^11*d + 66*a^7
*b^13*c^10*d^2 - 220*a^8*b^12*c^9*d^3 + 495*a^9*b^11*c^8*d^4 - 79
2*a^10*b^10*c^7*d^5 + 924*a^11*b^9*c^6*d^6 - 792*a^12*b^8*c^5*d^7
 + 495*a^13*b^7*c^4*d^8 - 220*a^14*b^6*c^3*d^9 + 66*a^15*b^5*c^2*
d^10 - 12*a^16*b^4*c*d^11 + a^17*b^3*d^12)*sqrt(-(b^12*c^12 - 12*
a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66
*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7)))))
 - 15*a^2*b*x^(5/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*
c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(a^7*b^5*(-(b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7)
)^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*
b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*
b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(
x)) + 15*a^2*b*x^(5/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b
^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*
a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6
*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9
*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84
*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84
*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*
sqrt(x)) - 12*a*b*c^3 + 60*(b^2*c^3 - 3*a*b*c^2*d)*x^2)/(a^2*b*x^
(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280867, size = 614, normalized size = 2.17

$$\frac{2\,d^3x^{\frac{3}{2}}}{3\,b}+\frac{2\,\left(5\,bc^3x^2-15\,ac^2dx^2-ac^3\right)}{5\,a^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(7/2)),x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b + 2/5*(5*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c^3)/(a
^2*x^(5/2)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4
)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d
^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(a^3*b^4) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(
3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^
3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(a^3*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^
3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4
)*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b
^4) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*
c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4)
```

3.448 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx$

Optimal. Leaf size=283

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{11/4}b^{5/4}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}-\frac{2c^3}{7ax^{7/2}}+\frac{2d^3\sqrt{x}}{b}$$

```
[Out] (-2*c^3)/(7*a*x^(7/2)) + (2*c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) +
(2*d^3*Sqrt[x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5
/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt
[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.558217, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{11/4}b^{5/4}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}-\frac{2c^3}{7ax^{7/2}}+\frac{2d^3\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(7*a*x^(7/2)) + (2*c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) +
(2*d^3*Sqrt[x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5
/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt
[2]*a^(11/4)*b^(5/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2d^3\int^{\sqrt{x}}\frac{1}{b}\,dx-\frac{2c^3}{7ax^{\frac{7}{2}}}-\frac{2c^2(3ad-bc)}{3a^2x^{\frac{3}{2}}}+\frac{\sqrt{2}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}(ad-bc)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a),x)
```

```
[Out] 2*d**3*Integral(1/b, (x, sqrt(x))) - 2*c**3/(7*a*x**(7/2)) - 2*c*
*2*(3*a*d - b*c)/(3*a**2*x**(3/2)) + sqrt(2)*(a*d - b*c)**3*log(-
sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(1
1/4)*b**(5/4)) - sqrt(2)*(a*d - b*c)**3*log(sqrt(2)*a**(1/4)*b**(
1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(11/4)*b**(5/4)) + sqrt
(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2
*a**(11/4)*b**(5/4)) - sqrt(2)*(a*d - b*c)**3*atan(1 + sqrt(2)*b*
*(1/4)*sqrt(x)/a**(1/4))/(2*a**(11/4)*b**(5/4))
```

Mathematica [A] time = 0.191663, size = 287, normalized size = 1.01

$$\frac{-24a^{7/4}b^{5/4}c^3+56a^{3/4}b^{5/4}c^2x^2(bc-3ad)+168a^{11/4}\sqrt[4]{b}d^3x^4-21\sqrt{2}x^{7/2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+21\sqrt{\ldots}}{84\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-24*a^(7/4)*b^(5/4)*c^3 + 56*a^(3/4)*b^(5/4)*c^2*(b*c - 3*a*d)*x
^2 + 168*a^(11/4)*b^(1/4)*d^3*x^4 - 42*Sqrt[2]*(b*c - a*d)^3*x^(7
/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 42*Sqrt[2]*(b
*c - a*d)^3*x^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 - 21*Sqrt[2]*(b*c - a*d)^3*x^(7/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 21*Sqrt[2]*(b*c - a*d)^3*x^(7/2)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(84*a
^(11/4)*b^(5/4)*x^(7/2))
```

Maple [B] time = 0.022, size = 622, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x)
```

```
[Out] 2*d^3*x^(1/2)/b-2/7*c^3/a/x^(7/2)-2*c^2/a/x^(3/2)*d+2/3*c^3/a^2/x
^(3/2)*b-1/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)*d^3+3/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)*c*d^2-3/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)*c^2*d+1/2/a^3*b^2*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))*d^3+3/4/a*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2)))*c*d^2-3/4/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))*c^2*d+1/4/a^3*b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))*c^3-1/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)*d^3+3/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)*c*d^2-3/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/2/a^3*b^2*(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(9/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271045, size = 2043, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(9/2)),x, algorithm="fricas")
```

```
[Out] 1/42*(84*a^2*d^3*x^4 + 84*a^2*b*x^(7/2)*(-(b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b
^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*arc
tan(-a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)/(a^11*b^5))^(1/4)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*sqrt(x) - sqrt(a^6*b^2*sqrt(-(b^12*c^12 - 12
*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5)) +
 (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x))) - 21*a^2*
b*x^(7/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(a^11*b^5))^(1/4)*log(a^3*b*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 2
1*a^2*b*x^(7/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*
c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(-a^3*b*(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^
(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(
x)) - 12*a*b*c^3 + 28*(b^2*c^3 - 3*a*b*c^2*d)*x^2)/(a^2*b*x^(7/2)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283352, size = 614, normalized size = 2.17

$$\frac{2\,d^3\sqrt{x}}{b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2}+\frac{2\left(7\,bc^3x^2-21\,ac^2dx^2-3\,ac^3\right)}{21\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(9/2)),x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^
(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a
^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(a^3*b^2) + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^
3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4
)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/
(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(
a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^
(1/4)*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a
^3*b^2) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*
b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*
ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 2/21
*(7*b*c^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(a^2*x^(7/2))
```

3.449 $\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx$

Optimal. Leaf size=303

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{13/4}b^{3/4}}+\frac{2c^2(bc-3ad)}{5a^2x^{5/2}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{a^3\sqrt{x}}-\frac{2c^3}{9ax^{9/2}}$$

```
[Out] (-2*c^3)/(9*a*x^(9/2)) + (2*c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) -
(2*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a
*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(
13/4)*b^(3/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rubi [A] time = 0.623232, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{13/4}b^{3/4}}+\frac{2c^2(bc-3ad)}{5a^2x^{5/2}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{a^3\sqrt{x}}-\frac{2c^3}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^3)/(9*a*x^(9/2)) + (2*c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) -
(2*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a
*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(
13/4)*b^(3/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rubi in Sympy [A] time = 119.585, size = 289, normalized size = 0.95

$$-\frac{2c^3}{9ax^{\frac{9}{2}}}-\frac{2c^2\left(3ad-bc\right)}{5a^2x^{\frac{5}{2}}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{a^3\sqrt{x}}+\frac{\sqrt{2}\left(ad-bc\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{13}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{13}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{13}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{13}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(11/2)/(b*x**2+a),x)
```

```
[Out] -2*c**3/(9*a*x**(9/2)) - 2*c**2*(3*a*d - b*c)/(5*a**2*x**(5/2)) -
 2*c*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)/(a**3*sqrt(x)) + sqrt(
2)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a
) + sqrt(b)*x)/(4*a**(13/4)*b**(3/4)) - sqrt(2)*(a*d - b*c)**3*lo
g(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**
(13/4)*b**(3/4)) - sqrt(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b**(1/
4)*sqrt(x)/a**(1/4))/(2*a**(13/4)*b**(3/4)) + sqrt(2)*(a*d - b*c)
**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(13/4)*b**(
3/4))
```

Mathematica [A] time = 0.253552, size = 292, normalized size = 0.96

$$\frac{-\frac{72a^{5/4}c^2(3ad-bc)}{x^{5/2}}-\frac{40a^{9/4}c^3}{x^{9/2}}-\frac{360\sqrt[4]{a}c\left(3a^2d^2-3abcd+b^2c^2\right)}{\sqrt{x}}+\frac{45\sqrt{2}(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}+\frac{45\sqrt{2}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{b^{3/4}}}{180a^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] ((-40*a^(9/4)*c^3)/x^(9/2) - (72*a^(5/4)*c^2*(-(b*c) + 3*a*d))/x^
(5/2) - (360*a^(1/4)*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/Sqrt[x]
 + (90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/b^(3/4) - (90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (45*Sqrt[2]*(-(b*c) + a*d)
^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^
(3/4) + (45*Sqrt[2]*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4))/(180*a^(13/4))
```

Maple [B] time = 0.022, size = 650, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x)
```

```
[Out] -2/9*c^3/a/x^(9/2)-6*c/a/x^(1/2)*d^2+6*c^2/a^2/x^(1/2)*b*d-2*c^3/
a^3/x^(1/2)*b^2-6/5*c^2/a/x^(5/2)*d+2/5*c^3/a^2/x^(5/2)*b+1/2/b/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-3/2/
a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2
+3/2/a^2*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1)*c^2*d-1/2/a^3*b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)*c^3+1/4/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2)))*d^3-3/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c
*d^2+3/4/a^2*b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2
*d-1/4/a^3*b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*
d^3-3/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+
1)*c*d^2+3/2/a^2*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)*c^2*d-1/2/a^3*b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(11/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27291, size = 2858, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(11/2)),x, algorithm="fricas")
```

```
[Out] 1/90*(180*a^3*x^(9/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^
10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)*arctan(-a^10*b^2*(-(b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^
3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(a^13*b^3))^(3/4)/((b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2
- 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5
+ 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d
^9)*sqrt(x) - sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c
^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a
^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*
d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^1
0*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^1
2 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^
3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18
)*x - (a^7*b^13*c^12 - 12*a^8*b^12*c^11*d + 66*a^9*b^11*c^10*d^2
- 220*a^10*b^10*c^9*d^3 + 495*a^11*b^9*c^8*d^4 - 792*a^12*b^8*c^7
*d^5 + 924*a^13*b^7*c^6*d^6 - 792*a^14*b^6*c^5*d^7 + 495*a^15*b^5
*c^4*d^8 - 220*a^16*b^4*c^3*d^9 + 66*a^17*b^3*c^2*d^10 - 12*a^18*
b^2*c*d^11 + a^19*b*d^12)*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))))) + 45*a^3*x^(9/2)
*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3
*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(a^13*b^3))^(1/4)*log(a^10*b^2*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4) - (b^9*
c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 1
26*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 3
6*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) - 45*a^3*x^
(9/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^13*b^3))^(1/4)*log(-a^10*b^2*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4) -
 (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d
^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d
^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) - 20*
a^2*c^3 - 180*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 36*(a*b
*c^3 - 3*a^2*c^2*d)*x^2)/(a^3*x^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(11/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281132, size = 652, normalized size = 2.15

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^3}-\frac{2\left(45b^2c^3x^4-135abc^2dx^4+135a^2cd^2x^4-9abc^3x^2+27a^2c^2dx^2+5a^2c^3\right)}{45a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(11/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3
) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^
2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4
*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^
2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln
(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) - 1/4*sqr
t(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*
b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) - 2/45*(45*b^2*c^3*x^4
- 135*a*b*c^2*d*x^4 + 135*a^2*c*d^2*x^4 - 9*a*b*c^3*x^2 + 27*a^2*
c^2*d*x^2 + 5*a^2*c^3)/(a^3*x^(9/2))
```

3.450 $\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx$

Optimal. Leaf size=305

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{3a^3x^{3/2}}-\frac{2c^3}{11ax^{11/2}}$$

```
[Out] (-2*c^3)/(11*a*x^(11/2)) + (2*c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2))
- (2*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c
 - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]
*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1/4)) + ((b*c - a*d)^3*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rubi [A] time = 0.590183, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{3a^3x^{3/2}}-\frac{2c^3}{11ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^3)/(11*a*x^(11/2)) + (2*c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2))
- (2*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c
 - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]
*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1/4)) + ((b*c - a*d)^3*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rubi in Sympy [A] time = 129.571, size = 291, normalized size = 0.95

$$-\frac{2c^3}{11ax^{\frac{11}{2}}}-\frac{2c^2\left(3ad-bc\right)}{7a^2x^{\frac{7}{2}}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{3a^3x^{\frac{3}{2}}}-\frac{\sqrt{2}\left(ad-bc\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{15}{4}}\sqrt[4]{b}}+\frac{\sqrt{2}\left(ad-bc\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{15}{4}}\sqrt[4]{b}}-\frac{\sqrt{2}\left(ad-bc\right)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{15}{4}}\sqrt[4]{b}}+\frac{\sqrt{2}\left(ad-bc\right)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{15}{4}}\sqrt[4]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(13/2)/(b*x**2+a),x)
```

```
[Out] -2*c**3/(11*a*x**(11/2)) - 2*c**2*(3*a*d - b*c)/(7*a**2*x**(7/2))
 - 2*c*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)/(3*a**3*x**(3/2)) -
sqrt(2)*(a*d - b*c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + s
qrt(a) + sqrt(b)*x)/(4*a**(15/4)*b**(1/4)) + sqrt(2)*(a*d - b*c)*
*3*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(
4*a**(15/4)*b**(1/4)) - sqrt(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b
**(1/4)*sqrt(x)/a**(1/4))/(2*a**(15/4)*b**(1/4)) + sqrt(2)*(a*d -
 b*c)**3*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(15/4)
*b**(1/4))
```

Mathematica [A] time = 0.243859, size = 292, normalized size = 0.96

$$\frac{-\frac{264a^{7/4}c^2(3ad-bc)}{x^{7/2}}-\frac{168a^{11/4}c^3}{x^{11/2}}-\frac{616a^{3/4}c\left(3a^2d^2-3abcd+b^2c^2\right)}{x^{3/2}}+\frac{231\sqrt{2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{\sqrt[4]{b}}}{924a^{15/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] ((-168*a^(11/4)*c^3)/x^(11/2) - (264*a^(7/4)*c^2*(-(b*c) + 3*a*d)
)/x^(7/2) - (616*a^(3/4)*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/x^(
3/2) + (462*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/b^(1/4) - (462*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (231*Sqrt[2]*(b*c -
a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/b^(1/4) + (231*Sqrt[2]*(-(b*c) + a*d)^3*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(924*a^(15/4))
```

Maple [B] time = 0.021, size = 659, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x)
```

```
[Out] -2/11*c^3/a/x^(11/2)-2*c/a/x^(3/2)*d^2+2*c^2/a^2/x^(3/2)*b*d-2/3*
c^3/a^3/x^(3/2)*b^2-6/7*c^2/a/x^(7/2)*d+2/7*c^3/a^2/x^(7/2)*b+1/2
/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-
3/2/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
*b*c*d^2+3/2/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)*b^2*c^2*d-1/2/a^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)*b^3*c^3+1/4/a*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))*d^3-3/4/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/
b)^(1/2)))*b*c*d^2+3/4/a^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))*b^2*c^2*d-1/4/a^4*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))*b^3*c^3+1/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)*d^3-3/2/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)*b*c*d^2+3/2/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^2*c^2*d-1/2/a^4*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^3*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(13/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272568, size = 2053, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(13/2)),x, algorithm="fricas")
```

```
[Out] -1/462*(924*a^3*x^(11/2)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*arctan(-a^4*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6
*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^1
5*b))^(1/4)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(x) - sqrt(a^8*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b
^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*
a^11*b*c*d^11 + a^12*d^12)/(a^15*b)) + (b^6*c^6 - 6*a*b^5*c^5*d +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a^5*b*c*d^5 + a^6*d^6)*x))) - 231*a^3*x^(11/2)*(-(b^12*c^12 - 12
*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/
4)*log(a^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)/(a^15*b))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*sqrt(x)) + 231*a^3*x^(11/2)*(-(b^12*c^12 - 12
*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/
4)*log(-a^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d
^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^15*b))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 84*a^2*c^3 + 308*(b^2*c^3 - 3*a*b
*c^2*d + 3*a^2*c*d^2)*x^4 - 132*(a*b*c^3 - 3*a^2*c^2*d)*x^2)/(a^3
*x^(11/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(13/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301801, size = 652, normalized size = 2.14

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b}-\frac{2\left(77b^2c^3x^4-231abc^2dx^4+231a^2cd^2x^4-33abc^3x^2+99a^2c^2dx^2+21a^2c^3\right)}{231a^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(13/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b)
- 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*
d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b
) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^
2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 1/4*sqrt(2)*(
(a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(
1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 2/231*(77*b^2*c^3*x^4 - 231*a
*b*c^2*d*x^4 + 231*a^2*c*d^2*x^4 - 33*a*b*c^3*x^2 + 99*a^2*c^2*d*
x^2 + 21*a^2*c^3)/(a^3*x^(11/2))
```

3.451 $\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{b}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}-\frac{\sqrt[4]{b}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}-\frac{\sqrt[4]{b}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{17/4}}+\frac{\sqrt[4]{b}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{17/4}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{5a^3x^{5/2}}-\frac{2c^3}{13ax^{13/2}}$$

```
[Out] (-2*c^3)/(13*a*x^(13/2)) + (2*c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2))
- (2*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (2*(b
*c - a*d)^3)/(a^4*Sqrt[x]) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(
b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt
[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(17/4)) - (b^(1/4)
*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(2*Sqrt[2]*a^(17/4))
```

Rubi [A] time = 0.655789, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt[4]{b}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}-\frac{\sqrt[4]{b}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}-\frac{\sqrt[4]{b}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{17/4}}+\frac{\sqrt[4]{b}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{17/4}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{5a^3x^{5/2}}-\frac{2c^3}{13ax^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```


```
[Out] (-2*c^3)/(13*a*x^(13/2)) + (2*c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2))
- (2*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (2*(b
*c - a*d)^3)/(a^4*Sqrt[x]) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(
b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt
[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(17/4)) - (b^(1/4)
*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(2*Sqrt[2]*a^(17/4))
```

Rubi in Sympy [A] time = 170.11, size = 309, normalized size = 0.95

$$-\frac{2c^3}{13ax^{\frac{13}{2}}}-\frac{2c^2\left(3ad-bc\right)}{9a^2x^{\frac{9}{2}}}-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{5a^3x^{\frac{5}{2}}}-\frac{2\left(ad-bc\right)^3}{a^4\sqrt{x}}-\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{17}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{17}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{17}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\left(ad-bc\right)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(15/2)/(b*x**2+a),x)
```

```
[Out] -2*c**3/(13*a*x**(13/2)) - 2*c**2*(3*a*d - b*c)/(9*a**2*x**(9/2))
 - 2*c*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)/(5*a**3*x**(5/2)) -
2*(a*d - b*c)**3/(a**4*sqrt(x)) - sqrt(2)*b**(1/4)*(a*d - b*c)**3
*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4
*a**(17/4)) + sqrt(2)*b**(1/4)*(a*d - b*c)**3*log(sqrt(2)*a**(1/4
)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(17/4)) + sqrt(2)
*b**(1/4)*(a*d - b*c)**3*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/
4))/(2*a**(17/4)) - sqrt(2)*b**(1/4)*(a*d - b*c)**3*atan(1 + sqrt
(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(17/4))
```

Mathematica [A] time = 0.285348, size = 314, normalized size = 0.97

$$-\frac{520a^{9/4}c^2(3ad-bc)}{x^{9/2}}-\frac{360a^{13/4}c^3}{x^{13/2}}-\frac{936a^{5/4}c\left(3a^2d^2-3abcd+b^2c^2\right)}{x^{5/2}}-\frac{4680\sqrt[4]{a}(ad-bc)^3}{\sqrt{x}}+585\sqrt{2}\sqrt[4]{b}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] ((-360*a^(13/4)*c^3)/x^(13/2) - (520*a^(9/4)*c^2*(-(b*c) + 3*a*d)
)/x^(9/2) - (936*a^(5/4)*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/x^(
5/2) - (4680*a^(1/4)*(-(b*c) + a*d)^3)/Sqrt[x] - 1170*Sqrt[2]*b^(
1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 1170*Sqrt[2]*b^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)] + 585*Sqrt[2]*b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*b^(
1/4)*(-(b*c) + a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2340*a^(17/4))
```

Maple [B] time = 0.024, size = 712, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x)
```

```
[Out] -2/13*c^3/a/x^(13/2)-2/a/x^(1/2)*d^3+6/a^2/x^(1/2)*c*d^2*b-6/a^3/
x^(1/2)*c^2*d*b^2+2/a^4/x^(1/2)*c^3*b^3-6/5*c/a/x^(5/2)*d^2+6/5*c
^2/a^2/x^(5/2)*b*d-2/5*c^3/a^3/x^(5/2)*b^2-2/3*c^2/a/x^(9/2)*d+2/
9*c^3/a^2/x^(9/2)*b-1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)-1)*d^3+3/2/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)*b*c*d^2-3/2/a^3/(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^2*c^2*d+1/2/a^4/(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^3*c^3-1/4/a/(a/b)^(
1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^3+3/4/a^2/(a/b)^(1/4)*2
^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*b*c*d^2-3/4/a^3/(a/b)^(1/4)*2^(
1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*b^2*c^2*d+1/4/a^4/(a/b)^(1/4)*2^(
1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*b^3*c^3-1/2/a/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/2/a^2/(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b*c*d^2-3/2/a^3/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^2*c^2*d+1/
2/a^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b
^3*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(15/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267904, size = 2927, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(15/2)),x, algorithm="fricas")
```

```
[Out] -1/1170*(2340*a^4*x^(13/2)*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a
^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 7
92*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 +
495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10
- 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*arctan(-a^13*(-(b
^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10
*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^
7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b
^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d
^12)/a^17)^(3/4)/((b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2
- 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5
+ 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9
*b*d^9)*sqrt(x) - sqrt((b^20*c^18 - 18*a*b^19*c^17*d + 153*a^2*b^
18*c^16*d^2 - 816*a^3*b^17*c^15*d^3 + 3060*a^4*b^16*c^14*d^4 - 85
68*a^5*b^15*c^13*d^5 + 18564*a^6*b^14*c^12*d^6 - 31824*a^7*b^13*c
^11*d^7 + 43758*a^8*b^12*c^10*d^8 - 48620*a^9*b^11*c^9*d^9 + 4375
8*a^10*b^10*c^8*d^10 - 31824*a^11*b^9*c^7*d^11 + 18564*a^12*b^8*c
^6*d^12 - 8568*a^13*b^7*c^5*d^13 + 3060*a^14*b^6*c^4*d^14 - 816*a
^15*b^5*c^3*d^15 + 153*a^16*b^4*c^2*d^16 - 18*a^17*b^3*c*d^17 + a
^18*b^2*d^18)*x - (a^9*b^13*c^12 - 12*a^10*b^12*c^11*d + 66*a^11*
b^11*c^10*d^2 - 220*a^12*b^10*c^9*d^3 + 495*a^13*b^9*c^8*d^4 - 79
2*a^14*b^8*c^7*d^5 + 924*a^15*b^7*c^6*d^6 - 792*a^16*b^6*c^5*d^7
+ 495*a^17*b^5*c^4*d^8 - 220*a^18*b^4*c^3*d^9 + 66*a^19*b^3*c^2*d
^10 - 12*a^20*b^2*c*d^11 + a^21*b*d^12)*sqrt(-(b^13*c^12 - 12*a*b
^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^
4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a
^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a
^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)))) + 5
85*a^4*x^(13/2)*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^1
0*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*
c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5
*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(a^13*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*
a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792
*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66
*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(3/4
) - (b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c
^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c
^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt(x
)) - 585*a^4*x^(13/2)*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^
11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^
5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a
^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*
a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(-a^13*(-(b^13*c^12
 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3
 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^
6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d
```

```
^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^1
7)^(3/4) - (b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^
3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^
6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)
*sqrt(x)) - 2340*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*x^6 + 180*a^3*c^3 + 468*(a*b^2*c^3 - 3*a^2*b*c^2*d + 3*a^3*c*
d^2)*x^4 - 260*(a^2*b*c^3 - 3*a^3*c^2*d)*x^2)/(a^4*x^(13/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(15/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.321783, size = 724, normalized size = 2.23

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3}c^{3} - 3\left(ab^{3}\right)^{\frac{3}{4}} ab^{2}c^{2}d + 3\left(ab^{3}\right)^{\frac{3}{4}} a^{2}bcd^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{5}b^{2}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3}c^{3} - 3\left(ab^{3}\right)^{\frac{3}{4}} ab^{2}c^{2}d + 3\left(ab^{3}\right)^{\frac{3}{4}} a^{2}bcd^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{5}b^{2}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3}c^{3} - 3\left(ab^{3}\right)^{\frac{3}{4}} ab^{2}c^{2}d + 3\left(ab^{3}\right)^{\frac{3}{4}} a^{2}bcd^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3}d^{3}\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^{5}b^{2}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3}c^{3} - 3\left(ab^{3}\right)^{\frac{3}{4}} ab^{2}c^{2}d + 3\left(ab^{3}\right)^{\frac{3}{4}} a^{2}bcd^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3}d^{3}\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^{5}b^{2}} + \frac{2\left(585\,b^{3}c^{3}x^{6} - 1755\,ab^{2}c^{2}dx^{6} + 1755\,a^{2}bcd^{2}x^{6} - 585\,a^{3}d^{3}x^{6} - 117\,ab^{2}c^{3}x^{4} + 351\,a^{2}bc^{2}dx^{4} - 351\,a^{3}cd^{2}x^{4} + 65\,a^{2}bc^{3}x^{2} - \right.}{585\,a^{4}x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)*x^(15/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
+ 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2)
 + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2
*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*
b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2
*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) + 1/4*sqrt
(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b
^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) + 2/585*(585*b^3*c^3*x^6
 - 1755*a*b^2*c^2*d*x^6 + 1755*a^2*b*c*d^2*x^6 - 585*a^3*d^3*x^6
- 117*a*b^2*c^3*x^4 + 351*a^2*b*c^2*d*x^4 - 351*a^3*c*d^2*x^4 + 6
5*a^2*b*c^3*x^2 - 195*a^3*c^2*d*x^2 - 45*a^3*c^3)/(a^4*x^(13/2))
```

3.452 $\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=409

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{21/4}}-\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{21/4}}+\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{21/4}}-\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}b^{21/4}}+\frac{\sqrt{x}(5bc-17ad)(bc-ad)^2}{2b^5}+\frac{d^2x^{9/2}(39bc-17ad)}{18b^3}-\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{17d^3x^{13/2}}{26b^2}$$

```
[Out] ((5*b*c - 17*a*d)*(b*c - a*d)^2*Sqrt[x])/(2*b^5) + (d*(27*b^2*c^2
 - 39*a*b*c*d + 17*a^2*d^2)*x^(5/2))/(10*b^4) + (d^2*(39*b*c - 17
*a*d)*x^(9/2))/(18*b^3) + (17*d^3*x^(13/2))/(26*b^2) - (x^(5/2)*(
c + d*x^2)^3)/(2*b*(a + b*x^2)) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c
- a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) + (a^(1/
4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b
*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4))
```

Rubi [A] time = 0.986217, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{21/4}}-\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{21/4}}+\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{21/4}}-\frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}b^{21/4}}+\frac{\sqrt{x}(5bc-17ad)(bc-ad)^2}{2b^5}+\frac{d^2x^{9/2}(39bc-17ad)}{18b^3}-\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{17d^3x^{13/2}}{26b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] ((5*b*c - 17*a*d)*(b*c - a*d)^2*Sqrt[x])/(2*b^5) + (d*(27*b^2*c^2
 - 39*a*b*c*d + 17*a^2*d^2)*x^(5/2))/(10*b^4) + (d^2*(39*b*c - 17
*a*d)*x^(9/2))/(18*b^3) + (17*d^3*x^(13/2))/(26*b^2) - (x^(5/2)*(
c + d*x^2)^3)/(2*b*(a + b*x^2)) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c
- a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) + (a^(1/
4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b
```

```
*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{2}\sqrt[4]{a}(ad-bc)^2(17ad-5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16b^{\frac{21}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}(ad-bc)^2(17ad-5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16b^{\frac{21}{4}}}-\frac{\sqrt{2}\sqrt[4]{a}(ad-bc)^2(17ad-5bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8b^{\frac{21}{4}}}+\frac{\sqrt{2}\sqrt[4]{a}(ad-bc)^2(17ad-5bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8b^{\frac{21}{4}}}-\frac{x^{\frac{5}{2}}\left(c+dx^2\right)^3}{2b(a+bx^2)}-\frac{(ad-bc)^2(17ad-5bc)\int^{\sqrt{x}}\frac{1}{b^4}\,dx}{2b}+\frac{17d^3x^{\frac{13}{2}}}{26b^2}-\frac{d^2x^{\frac{9}{2}}(17ad-39bc)}{18b^3}+\frac{dx^{\frac{5}{2}}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(2)*a**(1/4)*(a*d - b*c)**2*(17*a*d - 5*b*c)*log(-sqrt(2)*a*
*(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*b**(21/4)) + s
qrt(2)*a**(1/4)*(a*d - b*c)**2*(17*a*d - 5*b*c)*log(sqrt(2)*a**(1
/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*b**(21/4)) - sqrt
(2)*a**(1/4)*(a*d - b*c)**2*(17*a*d - 5*b*c)*atan(1 - sqrt(2)*b**
(1/4)*sqrt(x)/a**(1/4))/(8*b**(21/4)) + sqrt(2)*a**(1/4)*(a*d - b
*c)**2*(17*a*d - 5*b*c)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4
))/(8*b**(21/4)) - x**(5/2)*(c + d*x**2)**3/(2*b*(a + b*x**2)) -
(a*d - b*c)**2*(17*a*d - 5*b*c)*Integral(b**(-4), (x, sqrt(x)))/(
2*b) + 17*d**3*x**(13/2)/(26*b**2) - d**2*x**(9/2)*(17*a*d - 39*b
*c)/(18*b**3) + d*x**(5/2)*(17*a**2*d**2 - 39*a*b*c*d + 27*b**2*c
**2)/(10*b**4)
```

Mathematica [A] time = 0.395073, size = 378, normalized size = 0.92

$$\frac{2080b^{9/4}d^2x^{9/2}(3bc-2ad)+11232b^{5/4}dx^{5/2}(bc-ad)^2+\frac{4680a\sqrt[4]{b}\sqrt{x}(bc-ad)^3}{a+bx^2}+18720\sqrt[4]{b}\sqrt{x}(bc-4ad)(bc-ad)^2-585\sqrt{2}\sqrt[4]{a}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (18720*b^(1/4)*(b*c - 4*a*d)*(b*c - a*d)^2*Sqrt[x] + 11232*b^(5/4
)*d*(b*c - a*d)^2*x^(5/2) + 2080*b^(9/4)*d^2*(3*b*c - 2*a*d)*x^(9
/2) + 1440*b^(13/4)*d^3*x^(13/2) + (4680*a*b^(1/4)*(b*c - a*d)^3*
Sqrt[x])/(a + b*x^2) - 1170*Sqrt[2]*a^(1/4)*(b*c - a*d)^2*(-5*b*c
 + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1170*S
qrt[2]*a^(1/4)*(b*c - a*d)^2*(-5*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*a^(1/4)*(b*c - a*d)^2*(
-5*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x] + 585*Sqrt[2]*a^(1/4)*(b*c - a*d)^2*(-5*b*c + 17*a*d)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(9360
*b^(21/4))
```

Maple [B] time = 0.027, size = 804, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 3/2*a^3/b^4*x^(1/2)/(b*x^2+a)*c*d^2-3/2*a^2/b^3*x^(1/2)/(b*x^2+a)
*c^2*d+17/16*a^3/b^5*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
))*d^3+17/8*a^3/b^5*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)*d^3+17/8*a^3/b^5*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)*d^3+18/b^4*a^2*c*d^2*x^(1/2)-12/b^3*a*c^2*
d*x^(1/2)-1/2*a^4/b^5*x^(1/2)/(b*x^2+a)*d^3+1/2*a/b^2*x^(1/2)/(b*
x^2+a)*c^3-5/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)*c^3-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)*c^3+2/13*d^3*x^(13/2)/b^2-39/16*a^2/b^4*(a/b)^(1/4)*2^(1/2
)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))*c*d^2+27/16*a/b^3*(a/b)^(1/4)*2^(1/2
)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2*d-39/8*a^2/b^4*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+27/8*a/b^3*(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d-5/8/b^2*
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3-12/
5/b^3*x^(5/2)*a*c*d^2-4/9/b^3*x^(9/2)*a*d^3+2/3/b^2*x^(9/2)*c*d^2
+6/5/b^4*x^(5/2)*a^2*d^3+6/5/b^2*x^(5/2)*c^2*d-8/b^5*a^3*d^3*x^(1
/2)-39/8*a^2/b^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)*c*d^2+27/8*a/b^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)*c^2*d+2/b^2*c^3*x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(7/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275052, size = 2257, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(7/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4680*(2340*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^
11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2
603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b
^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8
- 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a
^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*arctan(-b^5*(-(625*a*
b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 71
8060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*
c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 1
2631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11
*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/
```

```
4)/((5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*sq
rt(x) - sqrt(b^10*sqrt(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d
+ 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5
*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6
 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*
a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^
11 + 83521*a^13*d^12)/b^21) + (25*b^6*c^6 - 270*a*b^5*c^5*d + 111
9*a^2*b^4*c^4*d^2 - 2276*a^3*b^3*c^3*d^3 + 2439*a^4*b^2*c^2*d^4 -
 1326*a^5*b*c*d^5 + 289*a^6*d^6)*x))) - 585*(b^6*x^2 + a*b^5)*(-(
625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^
2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^
6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d
^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 316801
8*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^2
1)^(1/4)*log(b^5*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128
850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*
c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14
225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*
b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 +
83521*a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a
^2*b*c*d^2 - 17*a^3*d^3)*sqrt(x)) + 585*(b^6*x^2 + a*b^5)*(-(625*
a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 -
718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^
7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 +
 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^
11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(
1/4)*log(-b^5*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850
*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8
*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225
976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3
*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 835
21*a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*
b*c*d^2 - 17*a^3*d^3)*sqrt(x)) - 4*(180*b^4*d^3*x^8 + 2925*a*b^3*
c^3 - 15795*a^2*b^2*c^2*d + 22815*a^3*b*c*d^2 - 9945*a^4*d^3 + 20
*(39*b^4*c*d^2 - 17*a*b^3*d^3)*x^6 + 52*(27*b^4*c^2*d - 39*a*b^3*
c*d^2 + 17*a^2*b^2*d^3)*x^4 + 468*(5*b^4*c^3 - 27*a*b^3*c^2*d + 3
9*a^2*b^2*c*d^2 - 17*a^3*b*d^3)*x^2)*sqrt(x))/(b^6*x^2 + a*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27865, size = 810, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(7/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^
2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b
^6 - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^
2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/b^6 - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4
)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a
^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^6 + 1/1
```

```
6*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*ln(-s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^6 + 1/2*(a*b^3*c^3*
sqrt(x) - 3*a^2*b^2*c^2*d*sqrt(x) + 3*a^3*b*c*d^2*sqrt(x) - a^4*d
^3*sqrt(x))/((b*x^2 + a)*b^5) + 2/585*(45*b^24*d^3*x^(13/2) + 195
*b^24*c*d^2*x^(9/2) - 130*a*b^23*d^3*x^(9/2) + 351*b^24*c^2*d*x^(
5/2) - 702*a*b^23*c*d^2*x^(5/2) + 351*a^2*b^22*d^3*x^(5/2) + 585*
b^24*c^3*sqrt(x) - 3510*a*b^23*c^2*d*sqrt(x) + 5265*a^2*b^22*c*d^
2*sqrt(x) - 2340*a^3*b^21*d^3*sqrt(x))/b^26
```

3.453 $\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=374

$$\frac{dx^{3/2}\left(5a^2d^2-11abcd+7b^2c^2\right)}{2b^4}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{19/4}}+\frac{3(bc-5ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{19/4}}+\frac{3d^2x^{7/2}(11bc-5ad)}{14b^3}-\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{15d^3x^{11/2}}{22b^2}$$

```
[Out] (d*(7*b^2*c^2 - 11*a*b*c*d + 5*a^2*d^2)*x^(3/2))/(2*b^4) + (3*d^2
*(11*b*c - 5*a*d)*x^(7/2))/(14*b^3) + (15*d^3*x^(11/2))/(22*b^2)
- (x^(3/2)*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - (3*(b*c - 5*a*d)*(b
*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(19/4)
) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4)) - (3*(
b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4))
```

Rubi [A] time = 0.9084, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{dx^{3/2}\left(5a^2d^2-11abcd+7b^2c^2\right)}{2b^4}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{19/4}}+\frac{3(bc-5ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{19/4}}+\frac{3d^2x^{7/2}(11bc-5ad)}{14b^3}-\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{15d^3x^{11/2}}{22b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] (d*(7*b^2*c^2 - 11*a*b*c*d + 5*a^2*d^2)*x^(3/2))/(2*b^4) + (3*d^2
*(11*b*c - 5*a*d)*x^(7/2))/(14*b^3) + (15*d^3*x^(11/2))/(22*b^2)
- (x^(3/2)*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - (3*(b*c - 5*a*d)*(b
*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(19/4)
) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4)) - (3*(
b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4))
```

Rubi in Sympy [A] time = 153.945, size = 359, normalized size = 0.96

$$-\frac{x^{\frac{3}{2}}\left(c+dx^{2}\right)^{3}}{2b\left(a+bx^{2}\right)}+\frac{15d^{3}x^{\frac{11}{2}}}{22b^{2}}-\frac{3d^{2}x^{\frac{7}{2}}\left(5ad-11bc\right)}{14b^{3}}+\frac{dx^{\frac{3}{2}}\left(5a^{2}d^{2}-11abcd+7b^{2}c^{2}\right)}{2b^{4}}-\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(5ad-bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16\sqrt[4]{a}b^{\frac{19}{4}}}+\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(5ad-bc\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16\sqrt[4]{a}b^{\frac{19}{4}}}+\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(5ad-bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{\frac{19}{4}}}-\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(5ad-bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{\frac{19}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x**(3/2)*(c + d*x**2)**3/(2*b*(a + b*x**2)) + 15*d**3*x**(11/2)/
(22*b**2) - 3*d**2*x**(7/2)*(5*a*d - 11*b*c)/(14*b**3) + d*x**(3/
2)*(5*a**2*d**2 - 11*a*b*c*d + 7*b**2*c**2)/(2*b**4) - 3*sqrt(2)*
(a*d - b*c)**2*(5*a*d - b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(
x) + sqrt(a) + sqrt(b)*x)/(16*a**(1/4)*b**(19/4)) + 3*sqrt(2)*(a*
d - b*c)**2*(5*a*d - b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) +
 sqrt(a) + sqrt(b)*x)/(16*a**(1/4)*b**(19/4)) + 3*sqrt(2)*(a*d -
b*c)**2*(5*a*d - b*c)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))
/(8*a**(1/4)*b**(19/4)) - 3*sqrt(2)*(a*d - b*c)**2*(5*a*d - b*c)*
atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(1/4)*b**(19/4)
)
```

Mathematica [A] time = 0.409179, size = 345, normalized size = 0.92

$$352b^{7/4}d^2x^{7/2}(3bc-2ad)+2464b^{3/4}dx^{3/2}(bc-ad)^2-\frac{616b^{3/4}x^{3/2}(bc-ad)^3}{a+bx^2}+\frac{231\sqrt{2}(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}+$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (2464*b^(3/4)*d*(b*c - a*d)^2*x^(3/2) + 352*b^(7/4)*d^2*(3*b*c -
2*a*d)*x^(7/2) + 224*b^(11/4)*d^3*x^(11/2) - (616*b^(3/4)*(b*c -
a*d)^3*x^(3/2))/(a + b*x^2) + (462*Sqrt[2]*(b*c - a*d)^2*(-(b*c)
+ 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) +
 (462*Sqrt[2]*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (231*Sqrt[2]*(b*c - 5*a*d)*(b*c
 - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/a^(1/4) + (231*Sqrt[2]*(b*c - a*d)^2*(-(b*c) + 5*a*d)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1/4))/(1
232*b^(19/4))
```

Maple [B] time = 0.026, size = 748, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 2/11*d^3*x^(11/2)/b^2-4/7*d^3/b^3*x^(7/2)*a+6/7*d^2/b^2*x^(7/2)*c
+2*d^3/b^4*x^(3/2)*a^2-4*d^2/b^3*x^(3/2)*a*c+2*d/b^2*x^(3/2)*c^2+
1/2/b^4*x^(3/2)/(b*x^2+a)*a^3*d^3-3/2/b^3*x^(3/2)/(b*x^2+a)*a^2*c
*d^2+3/2/b^2*x^(3/2)/(b*x^2+a)*a*c^2*d-1/2/b*x^(3/2)/(b*x^2+a)*c^
3-15/16/b^5/(a/b)^(1/4)*2^(1/2)*a^3*d^3*ln((x-(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)-15/8/b^5/(a/b)^(1/4)*2^(1/2)*a^3*d^3*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)-15/8/b^5/(a/b)^(1/4)*2^(1/2)*a^3*d^3*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)+33/16/b^4/(a/b)^(1/4)*2^(1/2)*a^2*c*d^2*ln(
(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))+33/8/b^4/(a/b)^(1/4)*2^(1/2)*a^2*c*d^2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+33/8/b^4/(a/b)^(1/4)*2^(1/2)*
a^2*c*d^2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-21/16/b^3/(a/b)^(
1/4)*2^(1/2)*a*c^2*d*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-21/8/b^3/(a/b)^(1
/4)*2^(1/2)*a*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-21/8/b^
3/(a/b)^(1/4)*2^(1/2)*a*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*c^3*ln((x-(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+3/
8/b^2/(a/b)^(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+
1)+3/8/b^2/(a/b)^(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3*x^(5/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286594, size = 2978, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3*x^(5/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/616*(924*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 3
38*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d
^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^
5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650
*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19)
)^(1/4)*arctan(-a*b^14*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*
b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28
856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d
^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b
^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3/4)
/((b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c
^6*d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3716*a^6*b
^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c*d^8 - 125*a^9*d^9
)*sqrt(x) - sqrt((b^18*c^18 - 42*a*b^17*c^17*d + 801*a^2*b^16*c^1
6*d^2 - 9200*a^3*b^15*c^15*d^3 + 71220*a^4*b^14*c^14*d^4 - 394392
*a^5*b^13*c^13*d^5 + 1619684*a^6*b^12*c^12*d^6 - 5050512*a^7*b^11
*c^11*d^7 + 12147630*a^8*b^10*c^10*d^8 - 22765820*a^9*b^9*c^9*d^9
 + 33419166*a^10*b^8*c^8*d^10 - 38446992*a^11*b^7*c^7*d^11 + 3450
3236*a^12*b^6*c^6*d^12 - 23888280*a^13*b^5*c^5*d^13 + 12508500*a^
14*b^4*c^4*d^14 - 4790000*a^15*b^3*c^3*d^15 + 1265625*a^16*b^2*c^
2*d^16 - 206250*a^17*b*c*d^17 + 15625*a^18*d^18)*x - (a*b^21*c^12
 - 28*a^2*b^20*c^11*d + 338*a^3*b^19*c^10*d^2 - 2316*a^4*b^18*c^9
*d^3 + 10015*a^5*b^17*c^8*d^4 - 28856*a^6*b^16*c^7*d^5 + 57148*a^
7*b^15*c^6*d^6 - 78968*a^8*b^14*c^5*d^7 + 76111*a^9*b^13*c^4*d^8
```

```
- 50220*a^10*b^12*c^3*d^9 + 21650*a^11*b^11*c^2*d^10 - 5500*a^12*
b^10*c*d^11 + 625*a^13*b^9*d^12)*sqrt(-(b^12*c^12 - 28*a*b^11*c^1
1*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^
8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968
*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9
+ 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(
a*b^19))))) + 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11
*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8
*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*
a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 +
 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a
*b^19))^(1/4)*log(27*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338
*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4
 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*
c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a
^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^
(3/4) - 27*(b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*
a^3*b^6*c^6*d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3
716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c*d^8 - 12
5*a^9*d^9)*sqrt(x)) - 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b
^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015
*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6
- 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c
^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*
d^12)/(a*b^19))^(1/4)*log(-27*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^1
1*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^
8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968
*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9
+ 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(
a*b^19))^(3/4) - 27*(b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d
^2 - 820*a^3*b^6*c^6*d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^
4*d^5 + 3716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c
*d^8 - 125*a^9*d^9)*sqrt(x)) - 4*(28*b^3*d^3*x^7 + 12*(11*b^3*c*d
^2 - 5*a*b^2*d^3)*x^5 + 44*(7*b^3*c^2*d - 11*a*b^2*c*d^2 + 5*a^2*
b*d^3)*x^3 - 77*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5*a^3
*d^3)*x)*sqrt(x))/(b^5*x^2 + a*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.317961, size = 745, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(5/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(
3/2) - a^3*d^3*x^(3/2))/((b*x^2 + a)*b^4) + 3/8*sqrt(2)*((a*b^3)^
(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^
2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^7) + 3/8*sqrt(2)*((a*b
^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4
)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^7) - 3/16*sqrt(2)
*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3
)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*ln(sqrt(2)*sqrt(x)
```

```
*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^7) + 3/16*sqrt(2)*((a*b^3)^(3/
4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b
*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(a*b^7) + 2/77*(7*b^20*d^3*x^(11/2) + 33*b^20*c
*d^2*x^(7/2) - 22*a*b^19*d^3*x^(7/2) + 77*b^20*c^2*d*x^(3/2) - 15
4*a*b^19*c*d^2*x^(3/2) + 77*a^2*b^18*d^3*x^(3/2))/b^22
```

3.454 $\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=386

$$-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-13ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{90b^4}+\frac{d\sqrt{x}\left(c+dx^2\right)(113bc-117ad)}{90b^3}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2}$$

```
[Out] (d*(497*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/(90*b^4) +
 (d*(113*b*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(90*b^3) + (13*d*Sqr
t[x]*(c + d*x^2)^2)/(18*b^2) - (Sqrt[x]*(c + d*x^2)^3)/(2*b*(a +
b*x^2)) - ((b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - 13*a
*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - 13*a*d)*(b*c - a*d)^2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2
]*a^(3/4)*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)
*b^(17/4))
```

Rubi [A] time = 1.07515, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-13ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{90b^4}+\frac{d\sqrt{x}\left(c+dx^2\right)(113bc-117ad)}{90b^3}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] (d*(497*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/(90*b^4) +
 (d*(113*b*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(90*b^3) + (13*d*Sqr
t[x]*(c + d*x^2)^2)/(18*b^2) - (Sqrt[x]*(c + d*x^2)^3)/(2*b*(a +
b*x^2)) - ((b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - 13*a
*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - 13*a*d)*(b*c - a*d)^2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2
]*a^(3/4)*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt[a] +
```

```
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)
*b^(17/4))
```

Rubi in Sympy [A] time = 172.812, size = 372, normalized size = 0.96

$$-\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2}-\frac{d\sqrt{x}\left(c\left(13ad-9bc\right)+dx^2\left(117ad-113bc\right)\right)}{90b^3}+\frac{d\sqrt{x}\left(585a^2d^2-1202abcd+601b^2c^2\right)}{90b^4}+\frac{\sqrt{2}\left(ad-bc\right)^2\left(13ad-bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{3}{4}}b^{\frac{17}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)^2\left(13ad-bc\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{3}{4}}b^{\frac{17}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)^2\left(13ad-bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}b^{\frac{17}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)^2\left(13ad-bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}b^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(x)*(c + d*x**2)**3/(2*b*(a + b*x**2)) + 13*d*sqrt(x)*(c + d
*x**2)**2/(18*b**2) - d*sqrt(x)*(c*(13*a*d - 9*b*c) + d*x**2*(117
*a*d - 113*b*c))/(90*b**3) + d*sqrt(x)*(585*a**2*d**2 - 1202*a*b*
c*d + 601*b**2*c**2)/(90*b**4) + sqrt(2)*(a*d - b*c)**2*(13*a*d -
 b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*
x)/(16*a**(3/4)*b**(17/4)) - sqrt(2)*(a*d - b*c)**2*(13*a*d - b*c
)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(1
6*a**(3/4)*b**(17/4)) + sqrt(2)*(a*d - b*c)**2*(13*a*d - b*c)*ata
n(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(3/4)*b**(17/4)) -
 sqrt(2)*(a*d - b*c)**2*(13*a*d - b*c)*atan(1 + sqrt(2)*b**(1/4)*
sqrt(x)/a**(1/4))/(8*a**(3/4)*b**(17/4))
```

Mathematica [A] time = 0.441102, size = 345, normalized size = 0.89

$$\frac{\frac{45\sqrt{2}(bc-ad)^2(13ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{45\sqrt{2}(bc-13ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{90\sqrt{2}(bc-ad)^2(13ad-bc)\tan^{-1}\left(1-\sqrt{2}\ldots\right.}{a^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (4320*b^(1/4)*d*(b*c - a*d)^2*Sqrt[x] + 288*b^(5/4)*d^2*(3*b*c -
2*a*d)*x^(5/2) + 160*b^(9/4)*d^3*x^(9/2) - (360*b^(1/4)*(b*c - a*
d)^3*Sqrt[x])/(a + b*x^2) + (90*Sqrt[2]*(b*c - a*d)^2*(-(b*c) + 1
3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (
90*Sqrt[2]*(b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (45*Sqrt[2]*(b*c - a*d)^2*(-(b*c)
 + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/a^(3/4) + (45*Sqrt[2]*(b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(720
*b^(17/4))
```

Maple [B] time = 0.025, size = 748, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 2/9*d^3/b^2*x^(9/2)-4/5*d^3/b^3*x^(5/2)*a+6/5*d^2/b^2*x^(5/2)*c+6
*d^3/b^4*a^2*x^(1/2)-12*d^2/b^3*a*c*x^(1/2)+6*d/b^2*c^2*x^(1/2)+1
/2/b^4*x^(1/2)/(b*x^2+a)*a^3*d^3-3/2/b^3*x^(1/2)/(b*x^2+a)*a^2*c*
d^2+3/2/b^2*x^(1/2)/(b*x^2+a)*a*c^2*d-1/2/b*x^(1/2)/(b*x^2+a)*c^3
-13/8/b^4*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)*d^3+27/8/b^3*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)+1)*c*d^2-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/8/b*(a/b)^(1/4)/a*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-13/8/b^4*(a/b)^(1/4)*a^2*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+27/8/b^3*(a/b)^(1/4)*
a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-15/8/b^2*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+1/8
/b*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^
3-13/16/b^4*(a/b)^(1/4)*a^2*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^
3+27/16/b^3*(a/b)^(1/4)*a*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c*d^
2-15/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2*d+
1/16/b*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(3/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271988, size = 2183, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(3/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 14
58*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8
*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824
*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*
d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^1
2*d^12)/(a^3*b^17))^(1/4)*arctan(-a*b^4*(-(b^12*c^12 - 60*a*b^11*
c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*
a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^
6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a
^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11
+ 28561*a^12*d^12)/(a^3*b^17))^(1/4)/((b^3*c^3 - 15*a*b^2*c^2*d +
 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x) - sqrt(a^2*b^8*sqrt(-(b^12*
c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*
c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 13657
56*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^
4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 2372
76*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17)) + (b^6*c^6 - 30*a
*b^5*c^5*d + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4
*b^2*c^2*d^4 - 702*a^5*b*c*d^5 + 169*a^6*d^6)*x))) - 45*(b^5*x^2
+ a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2
```

```
 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^
7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2
520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^
2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^
(1/4)*log(a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c
^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032
*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*
d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*
a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*
b^17))^(1/4) - (b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^
3*d^3)*sqrt(x)) + 45*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c
^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a
^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6
 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^
9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 +
 28561*a^12*d^12)/(a^3*b^17))^(1/4)*log(-a*b^4*(-(b^12*c^12 - 60*
a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 +
130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6
*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 18
53644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*
c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^3 - 15*a*b^2
*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) + 4*(20*b^3*d^3*x^
6 - 45*b^3*c^3 + 675*a*b^2*c^2*d - 1215*a^2*b*c*d^2 + 585*a^3*d^3
 + 4*(27*b^3*c*d^2 - 13*a*b^2*d^3)*x^4 + 36*(15*b^3*c^2*d - 27*a*
b^2*c*d^2 + 13*a^2*b*d^3)*x^2)*sqrt(x))/(b^5*x^2 + a*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.292189, size = 745, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*x^(3/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b
^5) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2
*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4
))/(a*b^5) + 1/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/
4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*
a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
- 1/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^
2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*ln
(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/2*(b^3
*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^
3*d^3*sqrt(x))/((b*x^2 + a)*b^4) + 2/45*(5*b^16*d^3*x^(9/2) + 27*
b^16*c*d^2*x^(5/2) - 18*a*b^15*d^3*x^(5/2) + 135*b^16*c^2*d*sqrt(
x) - 270*a*b^15*c*d^2*sqrt(x) + 135*a^2*b^14*d^3*sqrt(x))/b^18
```

3.455 $\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=376

$$\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}} - \frac{(bc-ad)^2(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}} - \frac{(bc-ad)^2(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{15/4}} + \frac{(bc-ad)^2(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{15/4}} - \frac{dx^{3/2}\left(11a^2d^2-21abcd+6b^2c^2\right)}{6ab^3} - \frac{d^2x^{7/2}(7bc-11ad)}{14ab^2} + \frac{x^{3/2}\left(c+dx^2\right)^2(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] -(d*(6*b^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/(6*a*b^3) - (d
^2*(7*b*c - 11*a*d)*x^(7/2))/(14*a*b^2) + ((b*c - a*d)*x^(3/2)*(c
 + d*x^2)^2)/(2*a*b*(a + b*x^2)) - ((b*c - a*d)^2*(b*c + 11*a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)
*b^(15/4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) + ((b*c - a
*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4)) - ((b*c - a*d)^2*(b*
c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4))
```

Rubi [A] time = 0.910527, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}} - \frac{(bc-ad)^2(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}} - \frac{(bc-ad)^2(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{15/4}} + \frac{(bc-ad)^2(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{15/4}} - \frac{dx^{3/2}\left(11a^2d^2-21abcd+6b^2c^2\right)}{6ab^3} - \frac{d^2x^{7/2}(7bc-11ad)}{14ab^2} + \frac{x^{3/2}\left(c+dx^2\right)^2(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```


```
[Out] -(d*(6*b^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/(6*a*b^3) - (d
^2*(7*b*c - 11*a*d)*x^(7/2))/(14*a*b^2) + ((b*c - a*d)*x^(3/2)*(c
 + d*x^2)^2)/(2*a*b*(a + b*x^2)) - ((b*c - a*d)^2*(b*c + 11*a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)
*b^(15/4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) + ((b*c - a
*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4)) - ((b*c - a*d)^2*(b*
c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4))
```

Rubi in Sympy [A] time = 152.817, size = 347, normalized size = 0.92

$$-\frac{x^{\frac{3}{2}}\left(c+dx^{2}\right)^{2}(ad-bc)}{2ab\left(a+bx^{2}\right)}+\frac{d^{2}x^{\frac{7}{2}}(11ad-7bc)}{14ab^{2}}-\frac{dx^{\frac{3}{2}}\left(11a^{2}d^{2}-21abcd+6b^{2}c^{2}\right)}{6ab^{3}}+\frac{\sqrt{2}(ad-bc)^{2}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{5}{4}}b^{\frac{15}{4}}}-\frac{\sqrt{2}(ad-bc)^{2}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{5}{4}}b^{\frac{15}{4}}}-\frac{\sqrt{2}(ad-bc)^{2}(11ad+bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{5}{4}}b^{\frac{15}{4}}}+\frac{\sqrt{2}(ad-bc)^{2}(11ad+bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{5}{4}}b^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] -x**(3/2)*(c + d*x**2)**2*(a*d - b*c)/(2*a*b*(a + b*x**2)) + d**2
*x**(7/2)*(11*a*d - 7*b*c)/(14*a*b**2) - d*x**(3/2)*(11*a**2*d**2
 - 21*a*b*c*d + 6*b**2*c**2)/(6*a*b**3) + sqrt(2)*(a*d - b*c)**2*
(11*a*d + b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
 sqrt(b)*x)/(16*a**(5/4)*b**(15/4)) - sqrt(2)*(a*d - b*c)**2*(11*
a*d + b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt
(b)*x)/(16*a**(5/4)*b**(15/4)) - sqrt(2)*(a*d - b*c)**2*(11*a*d +
 b*c)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(5/4)*b**
(15/4)) + sqrt(2)*(a*d - b*c)**2*(11*a*d + b*c)*atan(1 + sqrt(2)*
b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(5/4)*b**(15/4))
```

Mathematica [A] time = 0.410647, size = 323, normalized size = 0.86

$$\frac{\frac{21\sqrt{2}(bc-ad)^{2}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{21\sqrt{2}(bc-ad)^{2}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{42\sqrt{2}(bc-ad)^{2}(11ad+bc)\tan^{-1}\left(1-\sqrt{2}\right.}{a^{5/4}}}{336b^{15/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (224*b^(3/4)*d^2*(3*b*c - 2*a*d)*x^(3/2) + 96*b^(7/4)*d^3*x^(7/2)
 + (168*b^(3/4)*(b*c - a*d)^3*x^(3/2))/(a*(a + b*x^2)) - (42*Sqrt
[2]*(b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/a^(5/4) + (42*Sqrt[2]*(b*c - a*d)^2*(b*c + 11*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (21*Sqr
t[2]*(b*c - a*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (21*Sqrt[2]*(b*c - a*d)^2*
(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/a^(5/4))/(336*b^(15/4))
```

Maple [B] time = 0.024, size = 706, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 2/7*d^3/b^2*x^(7/2)-4/3*d^3/b^3*x^(3/2)*a+2*d^2/b^2*x^(3/2)*c-1/2
/b^3*a^2*x^(3/2)/(b*x^2+a)*d^3+3/2/b^2*a*x^(3/2)/(b*x^2+a)*c*d^2-
3/2/b*x^(3/2)/(b*x^2+a)*c^2*d+1/2/a*x^(3/2)/(b*x^2+a)*c^3+11/8/b^
```

```
4*a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d
^3-21/8/b^3*a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)*c*d^2+9/8/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)*c^2*d+1/8/b/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)*c^3+11/16/b^4*a^2/(a/b)^(1/4)*2^(1/2)*ln((x-
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))*d^3-21/16/b^3*a/(a/b)^(1/4)*2^(1/2)*ln((x-(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2)))*c*d^2+9/16/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)
^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2)))*c^2*d+1/16/b/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))*c^3+11/8/b^4*a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)*d^3-21/8/b^3*a/(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+9/8/b^2/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/8/b/a/(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3*sqrt(x)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284317, size = 2955, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3*sqrt(x)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d +
 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*
d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*
b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 3
68082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)
/(a^5*b^15))^(1/4)*arctan(a^4*b^11*(-(b^12*c^12 + 36*a*b^11*c^11*
d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c
^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a
^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9
+ 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^
12)/(a^5*b^15))^(3/4)/((b^9*c^9 + 27*a*b^8*c^8*d + 180*a^2*b^7*c^
7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13194*a^5*b^
4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 - 7623*
a^8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x) + sqrt((b^18*c^18 + 54*a*b^17
*c^17*d + 1089*a^2*b^16*c^16*d^2 + 8976*a^3*b^15*c^15*d^3 + 5940*
a^4*b^14*c^14*d^4 - 279576*a^5*b^13*c^13*d^5 - 338844*a^6*b^12*c^
12*d^6 + 6001776*a^7*b^11*c^11*d^7 - 6412626*a^8*b^10*c^10*d^8 -
62165180*a^9*b^9*c^9*d^9 + 294333534*a^10*b^8*c^8*d^10 - 67136270
4*a^11*b^7*c^7*d^11 + 974580036*a^12*b^6*c^6*d^12 - 971334936*a^1
3*b^5*c^5*d^13 + 678512340*a^14*b^4*c^4*d^14 - 328575984*a^15*b^3
*c^3*d^15 + 105546969*a^16*b^2*c^2*d^16 - 20292426*a^17*b*c*d^17
+ 1771561*a^18*d^18)*x - (a^3*b^19*c^12 + 36*a^4*b^18*c^11*d + 40
2*a^5*b^17*c^10*d^2 + 692*a^6*b^16*c^9*d^3 - 10017*a^7*b^15*c^8*d
^4 - 5688*a^8*b^14*c^7*d^5 + 160188*a^9*b^13*c^6*d^6 - 486648*a^1
0*b^12*c^5*d^7 + 746703*a^11*b^11*c^4*d^8 - 676588*a^12*b^10*c^3*
d^9 + 368082*a^13*b^9*c^2*d^10 - 111804*a^14*b^8*c*d^11 + 14641*a
^15*b^7*d^12)*sqrt(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*
c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5
*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 +
```

```
746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2
*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))))
) + 21*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 40
2*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4
 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5
*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 3680
82*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a
^5*b^15))^(1/4)*log(a^4*b^11*(-(b^12*c^12 + 36*a*b^11*c^11*d + 40
2*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4
 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5
*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 3680
82*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a
^5*b^15))^(3/4) + (b^9*c^9 + 27*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2
 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13194*a^5*b^4*c^4
*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 - 7623*a^8*b
*c*d^8 + 1331*a^9*d^9)*sqrt(x)) - 21*(a*b^4*x^2 + a^2*b^3)*(-(b^1
2*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c
^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 -
 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*
b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*log(-a^4*b^11*(-(b^
12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*
c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a
^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8
- 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11
*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) + (b^9*c^9 + 27*a*
b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*
b^5*c^5*d^4 + 13194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 178
20*a^7*b^2*c^2*d^7 - 7623*a^8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) +
4*(12*a*b^2*d^3*x^5 + 4*(21*a*b^2*c*d^2 - 11*a^2*b*d^3)*x^3 + 7*(
3*b^3*c^3 - 9*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 11*a^3*d^3)*x)*sqrt(
x))/(a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.308015, size = 697, normalized size = 1.85

$$\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3 + 9\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 21\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 11\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^6} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3 + 9\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 21\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 11\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^6} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3 + 9\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 21\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 11\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^2b^6} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3 + 9\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 21\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 11\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^2b^6} + \frac{2\left(3\,b^{12}d^3x^{\frac{7}{2}} + 21\,b^{12}cd^2x^{\frac{3}{2}} - 14\,ab^{11}d^3x^{\frac{3}{2}}\right)}{21\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*sqrt(x)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3
/2) - a^3*d^3*x^(3/2))/((b*x^2 + a)*a*b^3) + 1/8*sqrt(2)*((a*b^3)
^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a
^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^6) + 1/8*sqrt(2)*(
(a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^
(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^6) - 1/16*
sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21
*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*ln(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^6) + 1/16*sqrt(2)*((
a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(
3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^6) + 2/21*(3*b^12*d^3*x^(7/2)
 + 21*b^12*c*d^2*x^(3/2) - 14*a*b^11*d^3*x^(3/2))/b^14
```

3.456 $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)^2} \, dx$

Optimal. Leaf size=340

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+\frac{2d^2\sqrt{x}(3bc-2ad)}{b^3}+\frac{\sqrt{x}(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{2d^3x^{5/2}}{5b^2}$$

```
[Out] (2*d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (2*d^3*x^(5/2))/(5*b^2) + (
(b*c - a*d)^3*Sqrt[x])/(2*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(
b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sq
rt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(13/4
)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*
(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
```

Rubi [A] time = 0.768115, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+\frac{2d^2\sqrt{x}(3bc-2ad)}{b^3}+\frac{\sqrt{x}(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{2d^3x^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```


```
[Out] (2*d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (2*d^3*x^(5/2))/(5*b^2) + (
(b*c - a*d)^3*Sqrt[x])/(2*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(
b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sq
rt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(13/4
)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*
(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2d^2(2ad-3bc)\int^{\sqrt{x}}\frac{1}{b^3}\,dx+\frac{2d^3x^{\frac{5}{2}}}{5b^2}-\frac{\sqrt{x}\,(ad-bc)^3}{2ab^3(a+bx^2)}-\frac{3\sqrt{2}\,(ad-bc)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{7}{4}}b^{\frac{13}{4}}}+\frac{3\sqrt{2}\,(ad-bc)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{7}{4}}b^{\frac{13}{4}}}-\frac{3\sqrt{2}\,(ad-bc)^2(3ad+bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{13}{4}}}+\frac{3\sqrt{2}\,(ad-bc)^2(3ad+bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{7}{4}}b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] -2*d**2*(2*a*d - 3*b*c)*Integral(b**(-3), (x, sqrt(x))) + 2*d**3*
x**(5/2)/(5*b**2) - sqrt(x)*(a*d - b*c)**3/(2*a*b**3*(a + b*x**2)
) - 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*log(-sqrt(2)*a**(1/4)*
b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(7/4)*b**(13/4)) +
 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*log(sqrt(2)*a**(1/4)*b**(
1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(7/4)*b**(13/4)) - 3*s
qrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*atan(1 - sqrt(2)*b**(1/4)*sqr
t(x)/a**(1/4))/(8*a**(7/4)*b**(13/4)) + 3*sqrt(2)*(a*d - b*c)**2*
(3*a*d + b*c)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(
7/4)*b**(13/4))
```

Mathematica [A] time = 0.388552, size = 323, normalized size = 0.95

$$\frac{-\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{30\sqrt{2}(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\cdots}{\cdots}\right)}{a^{7/4}}\cdots}{80b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*Sqrt[x] + 32*b^(5/4)*d^3*x^(5/2)
 + (40*b^(1/4)*(b*c - a*d)^3*Sqrt[x])/(a*(a + b*x^2)) - (30*Sqrt[
2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) - (15*Sqrt[2
]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c
 + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/a^(7/4))/(80*b^(13/4))
```

Maple [B] time = 0.023, size = 697, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/5*d^3*x^(5/2)/b^2-4*d^3/b^3*a*x^(1/2)+6*d^2/b^2*x^(1/2)*c-1/2/b
^3*a^2*x^(1/2)/(b*x^2+a)*d^3+3/2/b^2*a*x^(1/2)/(b*x^2+a)*c*d^2-3/
2/b*x^(1/2)/(b*x^2+a)*c^2*d+1/2/a*x^(1/2)/(b*x^2+a)*c^3+9/8/b^3*a
```

```
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-15
/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*
c*d^2+3/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)*c^2*d+3/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)*c^3+9/16/b^3*a*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/
b)^(1/2)))*d^3-15/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2)))*c*d^2+3/16/b/a*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
)))*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
*c^3+9/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)*d^3-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)*c*d^2+3/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)*c^2*d+3/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266077, size = 2157, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/40*(60*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d -
14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 +
 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7
+ 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2
*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*arcta
n(a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 -
 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 -
 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8
 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d
^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)/((b^3*c^3 + a*b^2*c^2*d - 5
*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x) + sqrt(a^4*b^6*sqrt(-(b^12*c^12
 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 +
127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 +
 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^
9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a
^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3*
b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x)
)) - 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14
*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 1
36*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 +
1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d
^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*a
^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44
*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 64
4*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 -
1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11
 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5
*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) + 15*(a*b^4*x^2 + a^2*b^3)*(-(
b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c
```

```
^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*
c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b
^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12
*d^12)/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c
^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c
^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*
c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10
*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/
4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x
)) - 4*(4*a*b^2*d^3*x^4 + 5*b^3*c^3 - 15*a*b^2*c^2*d + 75*a^2*b*c
*d^2 - 45*a^3*d^3 + 12*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^2)*sqrt(x)
)/(a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**2/x**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.29263, size = 690, normalized size = 2.03

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^4}+\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^4}+\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b^4}-\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b^4}+\frac{b^3c^3\sqrt{x}-3ab^2c^2d\sqrt{x}+3a^2bcd^2\sqrt{x}-a^3d^3\sqrt{x}}{2(bx^2+a)ab^3}+\frac{2\left(b^8d^3x^{\frac{5}{2}}+15b^8cd^2\sqrt{x}-10ab^7d^3\sqrt{x}\right)}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*sqrt(x)),x, algorithm="giac")

[Out] 3/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d -
5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4)
 + 3/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d
 - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*
b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*
c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*ln
(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) - 3/16*sq
rt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b
^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*ln(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 1/2*(b^3*c^3*sqrt(x)
 - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x
))/((b*x^2 + a)*a*b^3) + 2/5*(b^8*d^3*x^(5/2) + 15*b^8*c*d^2*sqrt
(x) - 10*a*b^7*d^3*sqrt(x))/b^10
```

3.457 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{11/4}}-\frac{(bc-ad)^2(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{9/4}b^{11/4}}-\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}}-\frac{d^2x^{3/2}(3bc-7ad)}{6ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2ab\sqrt{x}(a+bx^2)}$$

```
[Out] -(c^2*(5*b*c - a*d))/(2*a^2*b*Sqrt[x]) - (d^2*(3*b*c - 7*a*d)*x^(
3/2))/(6*a*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*Sqrt[x]*(a +
 b*x^2)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*
d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(9/4)*b^(11/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^
(9/4)*b^(11/4))
```

Rubi [A] time = 0.91854, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{11/4}}-\frac{(bc-ad)^2(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{9/4}b^{11/4}}-\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}}-\frac{d^2x^{3/2}(3bc-7ad)}{6ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2ab\sqrt{x}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```


```
[Out] -(c^2*(5*b*c - a*d))/(2*a^2*b*Sqrt[x]) - (d^2*(3*b*c - 7*a*d)*x^(
3/2))/(6*a*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*Sqrt[x]*(a +
 b*x^2)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*
d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(9/4)*b^(11/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^
(9/4)*b^(11/4))
```

Rubi in Sympy [A] time = 153.543, size = 338, normalized size = 0.92

$$-\frac{(c+dx^2)^2(ad-bc)}{2ab\sqrt{x}(a+bx^2)}+\frac{d^2x^{\frac{3}{2}}(7ad-3bc)}{6ab^2}+\frac{c^2(ad-5bc)}{2a^2b\sqrt{x}}-\frac{\sqrt{2}(ad-bc)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{9}{4}}b^{\frac{11}{4}}}+\frac{\sqrt{2}(ad-bc)^2(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{9}{4}}b^{\frac{11}{4}}}+\frac{\sqrt{2}(ad-bc)^2(7ad+5bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{9}{4}}b^{\frac{11}{4}}}-\frac{\sqrt{2}(ad-bc)^2(7ad+5bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{9}{4}}b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**2*(a*d - b*c)/(2*a*b*sqrt(x)*(a + b*x**2)) + d**2*
x**(3/2)*(7*a*d - 3*b*c)/(6*a*b**2) + c**2*(a*d - 5*b*c)/(2*a**2*
b*sqrt(x)) - sqrt(2)*(a*d - b*c)**2*(7*a*d + 5*b*c)*log(-sqrt(2)*
a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(9/4)*b**
(11/4)) + sqrt(2)*(a*d - b*c)**2*(7*a*d + 5*b*c)*log(sqrt(2)*a**(
1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a**(9/4)*b**(11/
4)) + sqrt(2)*(a*d - b*c)**2*(7*a*d + 5*b*c)*atan(1 - sqrt(2)*b**
(1/4)*sqrt(x)/a**(1/4))/(8*a**(9/4)*b**(11/4)) - sqrt(2)*(a*d - b
*c)**2*(7*a*d + 5*b*c)*atan(1 + sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4)
)/(8*a**(9/4)*b**(11/4))
```

Mathematica [A] time = 0.413562, size = 327, normalized size = 0.89

$$\frac{1}{48}\left(-\frac{3\sqrt{2}(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}b^{11/4}}+\frac{3\sqrt{2}(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}b^{11/4}}+\frac{6\sqrt{2}(bc-ad)^2(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}b^{11/4}}-\frac{6\sqrt{2}(bc-ad)^2(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}b^{11/4}}+\frac{24x^{3/2}(ad-bc)^3}{a^2b^2(a+bx^2)}-\frac{96c^3}{a^2\sqrt{x}}+\frac{32d^3x^{3/2}}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-96*c^3)/(a^2*Sqrt[x]) + (32*d^3*x^(3/2))/b^2 + (24*(-(b*c) + a
*d)^3*x^(3/2))/(a^2*b^2*(a + b*x^2)) + (6*Sqrt[2]*(b*c - a*d)^2*(
5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^
(9/4)*b^(11/4)) - (6*Sqrt[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*b^(11/4)) - (3*
Sqrt[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*b^(11/4)) + (3*Sqrt[2]*
(b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*b^(11/4)))/48
```

Maple [B] time = 0.03, size = 682, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 2/3*d^3*x^(3/2)/b^2-2*c^3/x^(1/2)/a^2+1/2*a/b^2*x^(3/2)/(b*x^2+a)
*d^3-3/2/b*x^(3/2)/(b*x^2+a)*c*d^2+3/2/a*x^(3/2)/(b*x^2+a)*c^2*d-
1/2/a^2*b*x^(3/2)/(b*x^2+a)*c^3-7/16*a/b^3/(a/b)^(1/4)*2^(1/2)*d^
3*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))-7/8*a/b^3/(a/b)^(1/4)*2^(1/2)*d^3*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-7/8*a/b^3/(a/b)^(1/4)*2^(1/2)
*d^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/a/b/(a/b)^(1/4)*2
^(1/2)*c^2*d*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+3/8/a/b/(a/b)^(1/4)*2^(1/
2)*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/a/b/(a/b)^(1/4
)*2^(1/2)*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-5/16/a^2/(a
/b)^(1/4)*2^(1/2)*c^3*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-5/8/a^2/(a/b)^(1
/4)*2^(1/2)*c^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/a^2/(a/
b)^(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+9/16/b
^2/(a/b)^(1/4)*2^(1/2)*c*d^2*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+9/8/b^2/(
a/b)^(1/4)*2^(1/2)*c*d^2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/
8/b^2/(a/b)^(1/4)*2^(1/2)*c*d^2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28554, size = 2971, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/24*(16*a^2*b*d^3*x^4 - 48*a*b^2*c^3 - 4*(15*b^3*c^3 - 9*a*b^2*c
^2*d + 9*a^2*b*c*d^2 - 7*a^3*d^3)*x^2 - 12*(a^2*b^3*x^2 + a^3*b^2
)*sqrt(x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c
^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 3
7665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*
d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*ar
ctan(a^7*b^8*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^1
0*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728
*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7
- 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c
^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(3/4)
/((125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3
*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*
a^6*b^3*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a
^9*d^9)*sqrt(x) + sqrt((15625*b^18*c^18 - 56250*a*b^17*c^17*d - 8
```

```
4375*a^2*b^16*c^16*d^2 + 570000*a^3*b^15*c^15*d^3 - 211500*a^4*b^
14*c^14*d^4 - 2174040*a^5*b^13*c^13*d^5 + 2720004*a^6*b^12*c^12*d
^6 + 3321072*a^7*b^11*c^11*d^7 - 8368866*a^8*b^10*c^10*d^8 + 6404
20*a^9*b^9*c^9*d^9 + 11255310*a^10*b^8*c^8*d^10 - 8509968*a^11*b^
7*c^7*d^11 - 4831644*a^12*b^6*c^6*d^12 + 9537192*a^13*b^5*c^5*d^1
3 - 3095820*a^14*b^4*c^4*d^14 - 2551920*a^15*b^3*c^3*d^15 + 26146
89*a^16*b^2*c^2*d^16 - 907578*a^17*b*c*d^17 + 117649*a^18*d^18)*x
 - (625*a^5*b^17*c^12 - 1500*a^6*b^16*c^11*d - 3150*a^7*b^15*c^10
*d^2 + 11060*a^8*b^14*c^9*d^3 + 1071*a^9*b^13*c^8*d^4 - 28728*a^1
0*b^12*c^7*d^5 + 19068*a^11*b^11*c^6*d^6 + 27144*a^12*b^10*c^5*d^
7 - 37665*a^13*b^9*c^4*d^8 + 2324*a^14*b^8*c^3*d^9 + 19698*a^15*b
^7*c^2*d^10 - 12348*a^16*b^6*c*d^11 + 2401*a^17*b^5*d^12)*sqrt(-(
625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 110
60*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5
 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*
c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*
a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))))) - 3*(a^2*b^3*x^2 +
 a^3*b^2)*sqrt(x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^
2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 -
28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5
*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*
b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^
(1/4)*log(a^7*b^8*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^
2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 -
28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5
*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*
b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^
(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 13
08*a^3*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 +
 1140*a^6*b^3*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 +
 343*a^9*d^9)*sqrt(x)) + 3*(a^2*b^3*x^2 + a^3*b^2)*sqrt(x)*(-(625
*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*
a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 +
19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4
*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^1
1*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(-a^7*b^8*(-(62
5*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060
*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 +
 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^
4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^
11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(3/4) + (125*b^9*c^9 -
225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3*b^6*c^6*d^3 + 34
2*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3*c^3*d^6 +
 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x)))
/((a^2*b^3*x^2 + a^3*b^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265592, size = 680, normalized size = 1.85

$$\frac{2\,d^3x^{\frac{3}{2}}}{3\,b^2} - \frac{5\,b^3c^3x^2 - 3\,ab^2c^2dx^2 + 3\,a^2bcd^2x^2 - a^3d^3x^2 + 4\,ab^2c^3}{2\left(bx^{\frac{5}{2}} + a\sqrt{x}\right)a^2b^2} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 9\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 7\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^5} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 9\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 7\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^5} + \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 9\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 7\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^5} - \frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 3\,\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d - 9\,\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + 7\,\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(3/2)),x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b^2 - 1/2*(5*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3*
a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 4*a*b^2*c^3)/((b*x^(5/2) + a*sqrt
(x))*a^2*b^2) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^
(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a
/b)^(1/4))/(a^3*b^5) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(
a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3
)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b^5) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*
c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 +
 7*(a*b^3)^(3/4)*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sq
rt(a/b))/(a^3*b^5) - 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a
*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)
^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(a^3*b^5)
```

3.458 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=367

$$\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}+\frac{(bc-ad)^2(5ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}b^{9/4}}-\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}}-\frac{d^2\sqrt{x}(bc-5ad)}{2ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2abx^{3/2}(a+bx^2)}$$

```
[Out] -(c^2*(7*b*c - 3*a*d))/(6*a^2*b*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqr
t[x])/(2*a*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(3/2)*(a +
 b*x^2)) + ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*
d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) + ((b*c - a*d)^2*(7*b*c + 5*a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^
(11/4)*b^(9/4))
```

Rubi [A] time = 0.911418, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}+\frac{(bc-ad)^2(5ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}b^{9/4}}-\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}}-\frac{d^2\sqrt{x}(bc-5ad)}{2ab^2}+\frac{(c+dx^2)^2(bc-ad)}{2abx^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(7*b*c - 3*a*d))/(6*a^2*b*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqr
t[x])/(2*a*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(3/2)*(a +
 b*x^2)) + ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*
d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) + ((b*c - a*d)^2*(7*b*c + 5*a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^
(11/4)*b^(9/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^2(5ad-bc)\int^{\sqrt{x}}\frac{1}{b}\,dx}{2ab}-\frac{(c+dx^2)^2(ad-bc)}{2abx^{\frac{3}{2}}(a+bx^2)}+\frac{c^2(3ad-7bc)}{6a^2bx^{\frac{3}{2}}}+\frac{\sqrt{2}(ad-bc)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{11}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}(ad-bc)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{11}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}(ad-bc)^2(5ad+7bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{11}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}(ad-bc)^2(5ad+7bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{11}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] d**2*(5*a*d - b*c)*Integral(1/b, (x, sqrt(x)))/(2*a*b) - (c + d*x
**2)**2*(a*d - b*c)/(2*a*b*x**(3/2)*(a + b*x**2)) + c**2*(3*a*d -
 7*b*c)/(6*a**2*b*x**(3/2)) + sqrt(2)*(a*d - b*c)**2*(5*a*d + 7*b
*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)
/(16*a**(11/4)*b**(9/4)) - sqrt(2)*(a*d - b*c)**2*(5*a*d + 7*b*c)
*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16
*a**(11/4)*b**(9/4)) + sqrt(2)*(a*d - b*c)**2*(5*a*d + 7*b*c)*ata
n(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(11/4)*b**(9/4)) -
 sqrt(2)*(a*d - b*c)**2*(5*a*d + 7*b*c)*atan(1 + sqrt(2)*b**(1/4)
*sqrt(x)/a**(1/4))/(8*a**(11/4)*b**(9/4))
```

Mathematica [A] time = 0.388902, size = 327, normalized size = 0.89

$$\frac{1}{48}\left(\frac{3\sqrt{2}(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}b^{9/4}}-\frac{3\sqrt{2}(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}b^{9/4}}+\frac{6\sqrt{2}(bc-ad)^2(5ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}b^{9/4}}-\frac{6\sqrt{2}(bc-ad)^2(5ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}b^{9/4}}+\frac{24\sqrt{x}(ad-bc)^3}{a^2b^2(a+bx^2)}-\frac{32c^3}{a^2x^{3/2}}+\frac{96d^3\sqrt{x}}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*c^3)/(a^2*x^(3/2)) + (96*d^3*Sqrt[x])/b^2 + (24*(-(b*c) + a
*d)^3*Sqrt[x])/(a^2*b^2*(a + b*x^2)) + (6*Sqrt[2]*(b*c - a*d)^2*(
7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^
(11/4)*b^(9/4)) - (6*Sqrt[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*b^(9/4)) + (3*
Sqrt[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*b^(9/4)) - (3*Sqrt[2]*
(b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*b^(9/4)))/48
```

Maple [B] time = 0.029, size = 682, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] 2*d^3*x^(1/2)/b^2-2/3*c^3/a^2/x^(3/2)+1/2*a/b^2*x^(1/2)/(b*x^2+a)
*d^3-3/2/b*x^(1/2)/(b*x^2+a)*c*d^2+3/2/a*x^(1/2)/(b*x^2+a)*c^2*d-
1/2/a^2*b*x^(1/2)/(b*x^2+a)*c^3-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/8/a/b*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+9/8/a^2*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d-7/8/a^3*b*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-5/8/b^2
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/
8/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c
*d^2+9/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)*c^2*d-7/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)*c^3-5/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b
)^(1/2)))*d^3+3/16/a/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))*c*d^2+9/16/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)*c^2*d-7/16/a^3*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264506, size = 2183, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/24*(48*a^2*b*d^3*x^4 - 16*a*b^2*c^3 - 4*(7*b^3*c^3 - 9*a*b^2*c^
2*d + 9*a^2*b*c*d^2 - 15*a^3*d^3)*x^2 + 12*(a^2*b^3*x^3 + a^3*b^2
*x)*sqrt(x)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b
^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 271
44*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^
7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*
c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*
arctan(a^3*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^
2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 +
27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5
*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b
^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/
4)/((7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(
x) + sqrt(a^6*b^4*sqrt(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 1
9698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8
*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*
```

```
b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150
*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^
9)) + (49*b^6*c^6 - 126*a*b^5*c^5*d + 39*a^2*b^4*c^4*d^2 + 124*a^
3*b^3*c^3*d^3 - 81*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 25*a^6*d^6)
*x))) - 3*(a^2*b^3*x^3 + a^3*b^2*x)*sqrt(x)*(-(2401*b^12*c^12 - 1
2348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d
^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^
6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*
a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 6
25*a^12*d^12)/(a^11*b^9))^(1/4)*log(a^3*b^2*(-(2401*b^12*c^12 - 1
2348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d
^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^
6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*
a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 6
25*a^12*d^12)/(a^11*b^9))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*
a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(x)) + 3*(a^2*b^3*x^3 + a^3*b^2*x)*s
qrt(x)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c
^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1
071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d
^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(-
a^3*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*
c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a
^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 +
1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*
d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4) + (7
*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(x)))/(
(a^2*b^3*x^3 + a^3*b^2*x)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.288542, size = 676, normalized size = 1.84

$$\frac{2\,d^3\sqrt{x}}{b^2} - \frac{2\,c^3}{3\,a^2x^{\frac{3}{2}}} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 9\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 5\,\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 9\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 5\,\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} - \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 9\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 5\,\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3} + \frac{\sqrt{2}\left(7\,\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 9\,\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 3\,\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 5\,\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3} - \frac{b^3c^3\sqrt{x} - 3\,ab^2c^2d\sqrt{x} + 3\,a^2bcd^2\sqrt{x} - a^3d^3\sqrt{x}}{2\,(bx^2 + a)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(5/2)),x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b^2 - 2/3*c^3/(a^2*x^(3/2)) - 1/8*sqrt(2)*(7*(a*b^3
)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a
^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 1/8*sqrt(2)*(7
*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^
(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 1/16*s
qrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3
*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*ln(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/16*sqrt(2)*(7*
(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(
1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(
a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) - 1/2*(b^3*c^3*sqrt(x) - 3*
a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((
b*x^2 + a)*a^2*b^2)
```

3.459 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=376

$$\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}} - \frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}} - \frac{3(bc-ad)^2(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}b^{7/4}} + \frac{3(bc-ad)^2(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}b^{7/4}} - \frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(2a^2d^2-15abcd+9b^2c^2\right)}{2a^3b\sqrt{x}} + \frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{5/2}\left(a+bx^2\right)}$$

```
[Out] -(c^2*(9*b*c - 5*a*d))/(10*a^2*b*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*
b*c*d + 2*a^2*d^2))/(2*a^3*b*Sqrt[x]) + ((b*c - a*d)*(c + d*x^2)^
2)/(2*a*b*x^(5/2)*(a + b*x^2)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)
*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c -
 a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(b*c - a*d)^2*
(3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4))
```

Rubi [A] time = 0.940374, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}} - \frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}} - \frac{3(bc-ad)^2(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}b^{7/4}} + \frac{3(bc-ad)^2(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}b^{7/4}} - \frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(2a^2d^2-15abcd+9b^2c^2\right)}{2a^3b\sqrt{x}} + \frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(9*b*c - 5*a*d))/(10*a^2*b*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*
b*c*d + 2*a^2*d^2))/(2*a^3*b*Sqrt[x]) + ((b*c - a*d)*(c + d*x^2)^
2)/(2*a*b*x^(5/2)*(a + b*x^2)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)
*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c -
 a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(b*c - a*d)^2*
(3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4))
```

Rubi in Sympy [A] time = 157.362, size = 354, normalized size = 0.94

$$-\frac{\left(c+dx^{2}\right)^{2}\left(ad-bc\right)}{2abx^{\frac{5}{2}}\left(a+bx^{2}\right)}+\frac{c^{2}\left(5ad-9bc\right)}{10a^{2}bx^{\frac{5}{2}}}+\frac{c\left(2a^{2}d^{2}-15abcd+9b^{2}c^{2}\right)}{2a^{3}b\sqrt{x}}+\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(ad+3bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{13}{4}}b^{\frac{7}{4}}}-\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(ad+3bc\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{13}{4}}b^{\frac{7}{4}}}-\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(ad+3bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{13}{4}}b^{\frac{7}{4}}}+\frac{3\sqrt{2}\left(ad-bc\right)^{2}\left(ad+3bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{13}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**2*(a*d - b*c)/(2*a*b*x**(5/2)*(a + b*x**2)) + c**2
*(5*a*d - 9*b*c)/(10*a**2*b*x**(5/2)) + c*(2*a**2*d**2 - 15*a*b*c
*d + 9*b**2*c**2)/(2*a**3*b*sqrt(x)) + 3*sqrt(2)*(a*d - b*c)**2*(
a*d + 3*b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + s
qrt(b)*x)/(16*a**(13/4)*b**(7/4)) - 3*sqrt(2)*(a*d - b*c)**2*(a*d
 + 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(
b)*x)/(16*a**(13/4)*b**(7/4)) - 3*sqrt(2)*(a*d - b*c)**2*(a*d + 3
*b*c)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(13/4)*b*
*(7/4)) + 3*sqrt(2)*(a*d - b*c)**2*(a*d + 3*b*c)*atan(1 + sqrt(2)
*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(13/4)*b**(7/4))
```

Mathematica [A] time = 0.416625, size = 323, normalized size = 0.86

$$\frac{-\frac{32a^{5/4}c^{3}}{x^{5/2}}+\frac{15\sqrt{2}(bc-ad)^{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{7/4}}-\frac{15\sqrt{2}(bc-ad)^{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{7/4}}-\frac{30\sqrt{2}(bc-ad)^{2}(ad+3bc)}{b^{7/4}}}{80a^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*a^(5/4)*c^3)/x^(5/2) - (160*a^(1/4)*c^2*(-2*b*c + 3*a*d))/S
qrt[x] - (40*a^(1/4)*(-(b*c) + a*d)^3*x^(3/2))/(b*(a + b*x^2)) -
(30*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(3*b*c
+ a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (
15*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) - (15*Sqrt[2]*(b*c - a*
d)^2*(3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/b^(7/4))/(80*a^(13/4))
```

Maple [B] time = 0.03, size = 697, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/5*c^3/a^2/x^(5/2)-6*c^2/a^2/x^(1/2)*d+4*c^3/a^3/x^(1/2)*b-1/2/
b*x^(3/2)/(b*x^2+a)*d^3+3/2/a*x^(3/2)/(b*x^2+a)*c*d^2-3/2/a^2*b*x
^(3/2)/(b*x^2+a)*c^2*d+1/2/a^3*b^2*x^(3/2)/(b*x^2+a)*c^3+3/8/b^2/
```

```
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/8
/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*
d^2-15/8/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)*c^2*d+9/8/a^3*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)+1)*c^3+3/8/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)*d^3+3/8/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-15/8/a^2/(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+9/8/a^3*b/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+3/16/b^2/(a/b)^(1/4
)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)
^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^3+3/16/a/b/(a/b)^(1/4)*2^(
1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c*d^2-15/16/a^2/(a/b)^(1/4)*2^(1/
2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2*d+9/16/a^3*b/(a/b)^(1/4)*2^(1/2
)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286593, size = 2982, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(7/2)),x, algorithm="fricas")
```

```
[Out] -1/40*(16*a^2*b*c^3 - 20*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*x^4 - 48*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2 - 60*(a
^3*b^2*x^4 + a^4*b*x^2)*sqrt(x)*(-(81*b^12*c^12 - 540*a*b^11*c^11
*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8
*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^
5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^
2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*arcta
n(a^10*b^5*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^
10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^
7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b
^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b
*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4)/((27*b^9*c^9 - 135*a*b^8*c
^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*
d^4 + 78*a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^
7 + 3*a^8*b*c*d^8 + a^9*d^9)*sqrt(x) + sqrt((729*b^18*c^18 - 7290
*a*b^17*c^17*d + 31833*a^2*b^16*c^16*d^2 - 78192*a^3*b^15*c^15*d^
3 + 113940*a^4*b^14*c^14*d^4 - 88920*a^5*b^13*c^13*d^5 + 10180*a^
6*b^12*c^12*d^6 + 46320*a^7*b^11*c^11*d^7 - 35970*a^8*b^10*c^10*d
^8 - 220*a^9*b^9*c^9*d^9 + 12078*a^10*b^8*c^8*d^10 - 3600*a^11*b^
7*c^7*d^11 - 1884*a^12*b^6*c^6*d^12 + 936*a^13*b^5*c^5*d^13 + 180
*a^14*b^4*c^4*d^14 - 112*a^15*b^3*c^3*d^15 - 15*a^16*b^2*c^2*d^16
 + 6*a^17*b*c*d^17 + a^18*d^18)*x - (81*a^7*b^15*c^12 - 540*a^8*b
^14*c^11*d + 1458*a^9*b^13*c^10*d^2 - 1932*a^10*b^12*c^9*d^3 + 10
39*a^11*b^11*c^8*d^4 + 328*a^12*b^10*c^7*d^5 - 644*a^13*b^9*c^6*d
^6 + 136*a^14*b^8*c^5*d^7 + 127*a^15*b^7*c^4*d^8 - 44*a^16*b^6*c^
3*d^9 - 14*a^17*b^5*c^2*d^10 + 4*a^18*b^4*c*d^11 + a^19*b^3*d^12)
*sqrt(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2
 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*
d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4
```

```
*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^1
1 + a^12*d^12)/(a^13*b^7))))) - 15*(a^3*b^2*x^4 + a^4*b*x^2)*sqrt
(x)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2
- 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d
^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*
d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11
 + a^12*d^12)/(a^13*b^7))^(1/4)*log(27*a^10*b^5*(-(81*b^12*c^12 -
 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^
3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*
d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*
d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b
^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b^8*c^8*d + 252*a^2*b^7*c^7*d
^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*a^5*b^4*c^4*d^5
 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 + a^9*
d^9)*sqrt(x)) + 15*(a^3*b^2*x^4 + a^4*b*x^2)*sqrt(x)*(-(81*b^12*c
^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c
^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6
*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3
*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a
^13*b^7))^(1/4)*log(-27*a^10*b^5*(-(81*b^12*c^12 - 540*a*b^11*c^1
1*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^
8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b
^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b
^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27
*(27*b^9*c^9 - 135*a*b^8*c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^
6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*a^5*b^4*c^4*d^5 - 20*a^6*b^3*c
^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 + a^9*d^9)*sqrt(x)))/
((a^3*b^2*x^4 + a^4*b*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.305752, size = 682, normalized size = 1.81

$$\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)a^3b} + \frac{2\left(10\,bc^3x^2 - 15\,ac^2dx^2 - ac^3\right)}{5\,a^3x^{\frac{5}{2}}} + \frac{3\sqrt{2}\left(3\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 5\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d + \left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + \left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^4} + \frac{3\sqrt{2}\left(3\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 5\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d + \left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + \left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^4} - \frac{3\sqrt{2}\left(3\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 5\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d + \left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + \left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^4} + \frac{3\sqrt{2}\left(3\left(ab^3\right)^{\frac{3}{4}}b^3c^3 - 5\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d + \left(ab^3\right)^{\frac{3}{4}}a^2bcd^2 + \left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(7/2)),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3
/2) - a^3*d^3*x^(3/2))/((b*x^2 + a)*a^3*b) + 2/5*(10*b*c^3*x^2 -
15*a*c^2*d*x^2 - a*c^3)/(a^3*x^(5/2)) + 3/8*sqrt(2)*(3*(a*b^3)^(3
/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c
*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) + 3/8*sqrt(2)*(3*(a*b^3)
^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*
b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/
b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) - 3/16*sqrt(2)*(3*(a
*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)
*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^4*b^4) + 3/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^
3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 +
 (a*b^3)^(3/4)*a^3*d^3)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a^4*b^4)
```

3.460 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2} dx$

Optimal. Leaf size=376

$$-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}-\frac{(bc-ad)^2(ad+11bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{15/4}b^{5/4}}-\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}}+\frac{c\left(6a^2d^2-21abcd+11b^2c^2\right)}{6a^3bx^{3/2}}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{7/2}\left(a+bx^2\right)}$$

```
[Out] -(c^2*(11*b*c - 7*a*d))/(14*a^2*b*x^(7/2)) + (c*(11*b^2*c^2 - 21*
a*b*c*d + 6*a^2*d^2))/(6*a^3*b*x^(3/2)) + ((b*c - a*d)*(c + d*x^2
)^2)/(2*a*b*x^(7/2)*(a + b*x^2)) - ((b*c - a*d)^2*(11*b*c + a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4
)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) - ((b*c - a
*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11
*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rubi [A] time = 0.904467, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}-\frac{(bc-ad)^2(ad+11bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{15/4}b^{5/4}}-\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}}+\frac{c\left(6a^2d^2-21abcd+11b^2c^2\right)}{6a^3bx^{3/2}}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```


```
[Out] -(c^2*(11*b*c - 7*a*d))/(14*a^2*b*x^(7/2)) + (c*(11*b^2*c^2 - 21*
a*b*c*d + 6*a^2*d^2))/(6*a^3*b*x^(3/2)) + ((b*c - a*d)*(c + d*x^2
)^2)/(2*a*b*x^(7/2)*(a + b*x^2)) - ((b*c - a*d)^2*(11*b*c + a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4
)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) - ((b*c - a
*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11
*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rubi in Sympy [A] time = 153.168, size = 347, normalized size = 0.92

$$-\frac{\left(c+dx^2\right)^2(ad-bc)}{2abx^{\frac{7}{2}}(a+bx^2)}+\frac{c^2(7ad-11bc)}{14a^2bx^{\frac{7}{2}}}+\frac{c\left(6a^2d^2-21abcd+11b^2c^2\right)}{6a^3bx^{\frac{3}{2}}}-\frac{\sqrt{2}(ad-bc)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{15}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}(ad-bc)^2(ad+11bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{16a^{\frac{15}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}(ad-bc)^2(ad+11bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{15}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}(ad-bc)^2(ad+11bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{15}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**2*(a*d - b*c)/(2*a*b*x**(7/2)*(a + b*x**2)) + c**2
*(7*a*d - 11*b*c)/(14*a**2*b*x**(7/2)) + c*(6*a**2*d**2 - 21*a*b*
c*d + 11*b**2*c**2)/(6*a**3*b*x**(3/2)) - sqrt(2)*(a*d - b*c)**2*
(a*d + 11*b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
 sqrt(b)*x)/(16*a**(15/4)*b**(5/4)) + sqrt(2)*(a*d - b*c)**2*(a*d
 + 11*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt
(b)*x)/(16*a**(15/4)*b**(5/4)) - sqrt(2)*(a*d - b*c)**2*(a*d + 11
*b*c)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(15/4)*b*
*(5/4)) + sqrt(2)*(a*d - b*c)**2*(a*d + 11*b*c)*atan(1 + sqrt(2)*
b**(1/4)*sqrt(x)/a**(1/4))/(8*a**(15/4)*b**(5/4))
```

Mathematica [A] time = 0.413445, size = 323, normalized size = 0.86

$$\frac{-\frac{224a^{3/4}c^2(3ad-2bc)}{x^{3/2}}-\frac{168a^{3/4}\sqrt{x}(ad-bc)^3}{b(a+bx^2)}-\frac{96a^{7/4}c^3}{x^{7/2}}-\frac{21\sqrt{2}(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{5/4}}+\frac{21\sqrt{2}(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\right.}{b^{5/4}}}{336a^{15/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-96*a^(7/4)*c^3)/x^(7/2) - (224*a^(3/4)*c^2*(-2*b*c + 3*a*d))/x
^(3/2) - (168*a^(3/4)*(-(b*c) + a*d)^3*Sqrt[x])/(b*(a + b*x^2)) -
 (42*Sqrt[2]*(b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (42*Sqrt[2]*(b*c - a*d)^2*(11*b
*c + a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4)
- (21*Sqrt[2]*(b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (21*Sqrt[2]*(b*c
- a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/b^(5/4))/(336*a^(15/4))
```

Maple [B] time = 0.03, size = 706, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/7*c^3/a^2/x^(7/2)-2*c^2/a^2/x^(3/2)*d+4/3*c^3/a^3/x^(3/2)*b-1/
2/b*x^(1/2)/(b*x^2+a)*d^3+3/2/a*x^(1/2)/(b*x^2+a)*c*d^2-3/2/a^2*b
*x^(1/2)/(b*x^2+a)*c^2*d+1/2/a^3*b^2*x^(1/2)/(b*x^2+a)*c^3+1/8/a/
```

```
b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+9
/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*
c*d^2-21/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)*c^2*d+11/8/a^4*b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)*c^3+1/8/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+9/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-21/8/a^3*b*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+11/8/a^4*b^2*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/16/a/
b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d^3+9/16/a^2*(a/
b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c*d^2-21/16/a^3*b*(a/
b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^2*d+11/16/a^4*b^2*(
a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(9/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278786, size = 2178, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(9/2)),x, algorithm="fricas")
```

```
[Out] -1/168*(48*a^2*b*c^3 - 28*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*
c*d^2 - 3*a^3*d^3)*x^4 - 16*(11*a*b^2*c^3 - 21*a^2*b*c^2*d)*x^2 +
 84*(a^3*b^2*x^5 + a^4*b*x^3)*sqrt(x)*(-(14641*b^12*c^12 - 111804
*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^
3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*
b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*
a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12
*d^12)/(a^15*b^5))^(1/4)*arctan(a^4*b*(-(14641*b^12*c^12 - 111804
*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^
3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*
b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*
a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12
*d^12)/(a^15*b^5))^(1/4)/((11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*sqrt(x) + sqrt(a^8*b^2*sqrt(-(14641*b^12*c^12 -
111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*
c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 16018
8*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8
+ 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11
+ a^12*d^12)/(a^15*b^5)) + (121*b^6*c^6 - 462*a*b^5*c^5*d + 639*a
^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 39*a^4*b^2*c^2*d^4 + 18*a^
5*b*c*d^5 + a^6*d^6)*x))) - 21*(a^3*b^2*x^5 + a^4*b*x^3)*sqrt(x)*
(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*
d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^
5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 1
0017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^1
0 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(a^4*b*(-(
14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2
 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b
^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 1001
```

```
7*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 +
 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4) + (11*b^3*c^3 -
21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) + 21*(a^3*b^2*
x^5 + a^4*b*x^3)*sqrt(x)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*
d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^
4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 -
 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d
^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*
b^5))^(1/4)*log(-a^4*b*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d
+ 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*
b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5
688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9
 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^
5))^(1/4) + (11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^
3)*sqrt(x)))/((a^3*b^2*x^5 + a^4*b*x^3)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270616, size = 687, normalized size = 1.83

$$\frac{\sqrt{2}\left(11\,(ab^3)^{\frac14}b^3c^3-21\,(ab^3)^{\frac14}ab^2c^2d+9\,(ab^3)^{\frac14}a^2bcd^2+(ab^3)^{\frac14}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8\,a^4b^2}$$
$$+\frac{\sqrt{2}\left(11\,(ab^3)^{\frac14}b^3c^3-21\,(ab^3)^{\frac14}ab^2c^2d+9\,(ab^3)^{\frac14}a^2bcd^2+(ab^3)^{\frac14}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8\,a^4b^2}$$
$$+\frac{\sqrt{2}\left(11\,(ab^3)^{\frac14}b^3c^3-21\,(ab^3)^{\frac14}ab^2c^2d+9\,(ab^3)^{\frac14}a^2bcd^2+(ab^3)^{\frac14}a^3d^3\right)\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b^2}$$
$$-\frac{\sqrt{2}\left(11\,(ab^3)^{\frac14}b^3c^3-21\,(ab^3)^{\frac14}ab^2c^2d+9\,(ab^3)^{\frac14}a^2bcd^2+(ab^3)^{\frac14}a^3d^3\right)\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b^2}$$
$$+\frac{b^3c^3\sqrt{x}-3\,ab^2c^2d\sqrt{x}+3\,a^2bcd^2\sqrt{x}-a^3d^3\sqrt{x}}{2\,(bx^2+a)a^3b}+\frac{2\left(14\,bc^3x^2-21\,ac^2dx^2-3\,ac^3\right)}{21\,a^3x^{\frac72}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/((b*x^2 + a)^2*x^(9/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^
2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*
b^2) + 1/8*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a
*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4
))/(a^4*b^2) + 1/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3
)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)
*a^3*d^3)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^
2) - 1/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*
b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*
ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) + 1/2*
(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x)
```

```
- a^3*d^3*sqrt(x))/((b*x^2 + a)*a^3*b) + 2/21*(14*b*c^3*x^2 - 21*
a*c^2*d*x^2 - 3*a*c^3)/(a^3*x^(7/2))
```

3.461 $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=478

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)}+\frac{a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{7/4}(bc-ad)}-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}+\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{7/4}(bc-ad)}-\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{7/4}(bc-ad)}+\frac{2x^{3/2}}{3bd}$$

```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (a^(7/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d))
 + (c^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[
2]*d^(7/4)*(b*c - a*d)) - (c^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) + (a^(7/4)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b
^(7/4)*(b*c - a*d)) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7
/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
2*Sqrt[2]*d^(7/4)*(b*c - a*d)) + (c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(7/4)*(b*c - a*
d))
```

Rubi [A] time = 1.19064, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)}+\frac{a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{7/4}(bc-ad)}-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}+\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{7/4}(bc-ad)}-\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{7/4}(bc-ad)}+\frac{2x^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (a^(7/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d))
 + (c^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[
2]*d^(7/4)*(b*c - a*d)) - (c^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) + (a^(7/4)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b
^(7/4)*(b*c - a*d)) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7
/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
2*Sqrt[2]*d^(7/4)*(b*c - a*d)) + (c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(7/4)*(b*c - a*
```

```
d))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.503218, size = 411, normalized size = 0.86

$$\frac{\frac{3\sqrt{2}a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{7/4}}-\frac{3\sqrt{2}a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{7/4}}-\frac{6\sqrt{2}a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{7/4}}+\frac{6\sqrt{2}a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{7/4}}-\frac{8ax^3}{b}}{12b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a*x^(3/2))/b + (8*c*x^(3/2))/d - (6*Sqrt[2]*a^(7/4)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (6*Sqrt[2]*a^(7/
4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (6*Sq
rt[2]*c^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7
/4) - (6*Sqrt[2]*c^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/d^(7/4) + (3*Sqrt[2]*a^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) - (3*Sqrt[2]*a^(7/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) -
(3*Sqrt[2]*c^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/d^(7/4) + (3*Sqrt[2]*c^(7/4)*Log[Sqrt[c] + Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4))/(12*b*c - 12*a*d)
```

Maple [A] time = 0.019, size = 351, normalized size = 0.7

$$\frac{2}{3\,bd}x^{\frac{3}{2}}+\frac{c^2\sqrt{2}}{(4\,ad-4\,bc)\,d^2}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$+\frac{c^2\sqrt{2}}{(2\,ad-2\,bc)\,d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{c^2\sqrt{2}}{(2\,ad-2\,bc)\,d^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$-\frac{a^2\sqrt{2}}{(4\,ad-4\,bc)\,b^2}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{a^2\sqrt{2}}{(2\,ad-2\,bc)\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{a^2\sqrt{2}}{(2\,ad-2\,bc)\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 2/3*x^(3/2)/b/d+1/4*c^2/(a*d-b*c)/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))+1/2*c^2/(a*d-b*c)/d^2/(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2*c^2/(a*d-b*c)/d^2/(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/4*a^2/(a*d-b*
c)/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b
)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2*a^2/(a*
d-b*c)/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)-1/2*a^2/(a*d-b*c)/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35166, size = 1777, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -1/6*(12*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4
*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*arctan(-(b^8*c^3 - 3*a*b
^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^11*c^4 - 4*a*b
^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(
3/4)/(a^5*sqrt(x) + sqrt(a^10*x - (a^7*b^5*c^2 - 2*a^8*b^4*c*d +
a^9*b^3*d^2)*sqrt(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2
*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))))) - 12*(-c^7/(b^4*c^4*d^7
 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^1
1))^(1/4)*b*d*arctan(-(b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*
d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c
^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)/(c^5*sqrt(x) + sqrt(c^
10*x - (b^2*c^9*d^3 - 2*a*b*c^8*d^4 + a^2*c^7*d^5)*sqrt(-c^7/(b^4
*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 +
 a^4*d^11))))) - 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c
^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x
) + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a
^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d
^3 + a^4*b^7*d^4))^(3/4)) + 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d +
6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log
(a^5*sqrt(x) - (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b
^5*d^3)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*
a^3*b^8*c*d^3 + a^4*b^7*d^4))^(3/4)) + 3*(-c^7/(b^4*c^4*d^7 - 4*a
*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1
/4)*b*d*log(c^5*sqrt(x) + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*
b*c*d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b
^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)) - 3*(-c^7/(b^4*c^
4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^
4*d^11))^(1/4)*b*d*log(c^5*sqrt(x) - (b^3*c^3*d^5 - 3*a*b^2*c^2*d
^6 + 3*a^2*b*c*d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^
8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)) - 4*x^
(3/2))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{(bx^2+a)(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.462 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=476

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{5/4}(bc-ad)}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{5/4}(bc-ad)}-\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{5/4}(bc-ad)}+\frac{2\sqrt{x}}{bd}$$

```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) +
 (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]
*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(
5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*
Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)
)
```

Rubi [A] time = 1.0304, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{5/4}(bc-ad)}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{5/4}(bc-ad)}-\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{5/4}(bc-ad)}+\frac{2\sqrt{x}}{bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) +
 (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]
*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(
5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*
Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)
```

)

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.403226, size = 409, normalized size = 0.86

$$\frac{-\frac{\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{5/4}}+\frac{\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{5/4}}-\frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{5/4}}-\frac{8a\sqrt{x}}{b}}{4bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a*Sqrt[x])/b + (8*c*Sqrt[x])/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/
4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (2*Sq
rt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5
/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/d^(5/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (Sqr
t[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4))/(4*b*c - 4*a*d)
```

Maple [A] time = 0.019, size = 339, normalized size = 0.7

$$2\,\frac{\sqrt{x}}{bd}+\frac{c\sqrt{2}}{4\,(ad-bc)\,d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}}{2\,(ad-bc)\,d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{c\sqrt{2}}{2\,(ad-bc)\,d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{a\sqrt{2}}{4\,(ad-bc)\,b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{a\sqrt{2}}{2\,(ad-bc)\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{a\sqrt{2}}{2\,(ad-bc)\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 2*x^(1/2)/b/d+1/4/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+
```

```
(c/d)^(1/2)))+1/2/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)+1/2/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)
*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2/b*a/(a*d-b*c)*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/b*a/(a*d-b*c)*
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.503231, size = 1499, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3
*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(-(-a^5/(b^9*c^4 - 4*a
*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^
(1/4)*(b^2*c - a*b*d)/(a*sqrt(x) + sqrt(a^2*x + (b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))))) - 4*(-c^5/(b^4*c^
4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4
*d^9))^(1/4)*b*d*arctan(-(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6
*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2
)/(c*sqrt(x) + sqrt(c^2*x + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*sqrt(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4
*a^3*b*c*d^8 + a^4*d^9))))) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*
a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a
*sqrt(x) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4
*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - (-a^5/(b^
9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4
*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3
*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b
^2*c - a*b*d)) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2
*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) + (-
c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*
c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) + (-c^5/(b^4*c^4*d^5 - 4
*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1
/4)*b*d*log(c*sqrt(x) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*
a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)
) + 4*sqrt(x))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{(bx^2 + a)(dx^2 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")

[Out] integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.463 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{3/4}(bc-ad)}$$

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*
b^(3/4)*(b*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)
) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) +
(a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c
 - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rubi [A] time = 0.778738, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*
b^(3/4)*(b*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)
) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) +
(a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c
 - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rubi in Sympy [A] time = 149.778, size = 420, normalized size = 0.91

$$\frac{\sqrt{2}a^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}a^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4b^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}a^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}a^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4d^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2d^{\frac{3}{4}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(2)*a**(3/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a)
 + sqrt(b)*x)/(4*b**(3/4)*(a*d - b*c)) - sqrt(2)*a**(3/4)*log(sqr
t(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(3/4)
*(a*d - b*c)) - sqrt(2)*a**(3/4)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x
)/a**(1/4))/(2*b**(3/4)*(a*d - b*c)) + sqrt(2)*a**(3/4)*atan(1 +
sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(3/4)*(a*d - b*c)) - sqr
t(2)*c**(3/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) +
sqrt(d)*x)/(4*d**(3/4)*(a*d - b*c)) + sqrt(2)*c**(3/4)*log(sqrt(2
)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*d**(3/4)*(a
*d - b*c)) + sqrt(2)*c**(3/4)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/c
**(1/4))/(2*d**(3/4)*(a*d - b*c)) - sqrt(2)*c**(3/4)*atan(1 + sqr
t(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d**(3/4)*(a*d - b*c))
```

Mathematica [A] time = 0.205574, size = 364, normalized size = 0.79

$$-a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2a$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 - 2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - b^(3/4)*c^(3/4)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*b^
(3/4)*d^(3/4)*(b*c - a*d))
```

Maple [A] time = 0.018, size = 328, normalized size = 0.7

$$-\frac{c\sqrt{2}}{(4\,ad-4\,bc)\,d}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{c\sqrt{2}}{(2\,ad-2\,bc)\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{c\sqrt{2}}{(2\,ad-2\,bc)\,d}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{a\sqrt{2}}{(4\,ad-4\,bc)\,b}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{a\sqrt{2}}{(2\,ad-2\,bc)\,b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{a\sqrt{2}}{(2\,ad-2\,bc)\,b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(5/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
-1/2*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)-1/2*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*ln((
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))+1/2*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a/(a*d-b*c)/b/(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.342606, size = 1737, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*
c*d^3 + a^4*b^3*d^4))^(1/4)*arctan(-(b^5*c^3 - 3*a*b^4*c^2*d + 3*
a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a
^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)/(a^2*sqrt(
x) + sqrt(a^4*x - (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*sqrt(
-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d
^3 + a^4*b^3*d^4))))) - 2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 +
6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(-(b^3*
c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c
^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^
```

```
4*d^7))^(3/4)/(c^2*sqrt(x) + sqrt(c^4*x - (b^2*c^5*d - 2*a*b*c^4*
d^2 + a^2*c^3*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a
^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))))) - 1/2*(-a^3/(b^7*c^
4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3
*d^4))^(1/4)*log(a^2*sqrt(x) + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b
^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^
5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-a^3/(b
^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^
4*b^3*d^4))^(1/4)*log(a^2*sqrt(x) - (b^5*c^3 - 3*a*b^4*c^2*d + 3*
a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a
^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-c
^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c
*d^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x) + (b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^
3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/
2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^
3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x) - (b^3*c^3*d^2 - 3*a*
b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b
^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{(bx^2 + a)(dx^2 + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")

[Out] integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.464 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*
b^(1/4)*(b*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)
) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)) -
(a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c
 - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rubi [A] time = 0.766528, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*
b^(1/4)*(b*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)
) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)) -
(a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c
 - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rubi in Sympy [A] time = 147.127, size = 420, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{b}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{b}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{a}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{b}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{a}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{b}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt[4]{d}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt[4]{d}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{c}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{d}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{c}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{d}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(2)*a**(1/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a
) + sqrt(b)*x)/(4*b**(1/4)*(a*d - b*c)) + sqrt(2)*a**(1/4)*log(sq
rt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*b**(1/4
)*(a*d - b*c)) - sqrt(2)*a**(1/4)*atan(1 - sqrt(2)*b**(1/4)*sqrt(
x)/a**(1/4))/(2*b**(1/4)*(a*d - b*c)) + sqrt(2)*a**(1/4)*atan(1 +
 sqrt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*b**(1/4)*(a*d - b*c)) + sq
rt(2)*c**(1/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) +
 sqrt(d)*x)/(4*d**(1/4)*(a*d - b*c)) - sqrt(2)*c**(1/4)*log(sqrt(
2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*d**(1/4)*(
a*d - b*c)) + sqrt(2)*c**(1/4)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x)/
c**(1/4))/(2*d**(1/4)*(a*d - b*c)) - sqrt(2)*c**(1/4)*atan(1 + sq
rt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*d**(1/4)*(a*d - b*c))
```

Mathematica [A] time = 0.195736, size = 364, normalized size = 0.79

$$\sqrt[4]{a}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 - 2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + b^(1/4)*c^(1/4)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*b^
(1/4)*d^(1/4)*(b*c - a*d))
```

Maple [A] time = 0.017, size = 304, normalized size = 0.7

$$-\frac{\sqrt{2}}{4\,ad-4\,bc}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}}{2\,ad-2\,bc}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}}{2\,ad-2\,bc}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{\sqrt{2}}{4\,ad-4\,bc}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}}{2\,ad-2\,bc}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}}{2\,ad-2\,bc}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^(3/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-1/2
/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)-1/2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)+1/4/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2)))+1/2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+1/2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271234, size = 1327, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c
*d^3 + a^4*b*d^4))^(1/4)*arctan(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4
)/(sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)) + x)
+ sqrt(x))) + 2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*
d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*arctan(-(b*c - a*d)*(-c/(b^
4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a
^4*d^5))^(1/4)/(sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4
*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^
4*d^5)) + x) + sqrt(x))) - 1/2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log((b*c - a
```

```
*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*
c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-a/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*
log(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-c/(b^4*
c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4
*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6
*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + sqrt(x)) - 1
/2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b
*c*d^4 + a^4*d^5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^
3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) +
 sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(bx^2 + a)(dx^2 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.465 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*a^(1/4)*(b*c - a*d))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a
*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(S
qrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d))
 - (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(
b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rubi [A] time = 0.776791, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*a^(1/4)*(b*c - a*d))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a
*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(S
qrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d))
 - (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(
b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rubi in Sympy [A] time = 148.044, size = 420, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt[4]{c}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt[4]{c}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{d}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{c}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{d}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{c}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{a}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt[4]{a}\,(ad-bc)}+\frac{\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,(ad-bc)}-\frac{\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(2)*d**(1/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c)
 + sqrt(d)*x)/(4*c**(1/4)*(a*d - b*c)) - sqrt(2)*d**(1/4)*log(sqr
t(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c**(1/4)
*(a*d - b*c)) - sqrt(2)*d**(1/4)*atan(1 - sqrt(2)*d**(1/4)*sqrt(x
)/c**(1/4))/(2*c**(1/4)*(a*d - b*c)) + sqrt(2)*d**(1/4)*atan(1 +
sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(1/4)*(a*d - b*c)) - sqr
t(2)*b**(1/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
sqrt(b)*x)/(4*a**(1/4)*(a*d - b*c)) + sqrt(2)*b**(1/4)*log(sqrt(2
)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(1/4)*(a
*d - b*c)) + sqrt(2)*b**(1/4)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/a
**(1/4))/(2*a**(1/4)*(a*d - b*c)) - sqrt(2)*b**(1/4)*atan(1 + sqr
t(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(1/4)*(a*d - b*c))
```

Mathematica [A] time = 0.272425, size = 364, normalized size = 0.79

$$\sqrt[4]{b}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\tan$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
] + 2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + a^(1/4)*d^(1/4)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*a
^(1/4)*c^(1/4)*(b*c - a*d))
```

Maple [A] time = 0.016, size = 304, normalized size = 0.7

$$\frac{\sqrt{2}}{4ad-4bc}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}}{2ad-2bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}}{2ad-2bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{\sqrt{2}}{4ad-4bc}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{2ad-2bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{2ad-2bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/2/
(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)+1/2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)-1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))-1/2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)-1/2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277904, size = 1624, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b
*c*d^3 + a^5*d^4))^(1/4)*arctan(-(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3
*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*
b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)/(b*sqrt(x) + sqrt(b
^2*x - (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(-b/(a*b^4*c^4
 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)
)))) + 2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3
*b*c^2*d^3 + a^4*c*d^4))^(1/4)*arctan(-(b^3*c^4 - 3*a*b^2*c^3*d +
 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^
2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)/(d*sqrt(x) +
```

```
sqrt(d^2*x - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*sqrt(-d/(b^4
*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*
c*d^4))))) + 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2
*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log((a*b^3*c^3 - 3*a^2*b^2
*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*
d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(
x)) - 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 -
4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(-(a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) -
1/2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^
2*d^3 + a^4*c*d^4))^(1/4)*log((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*
c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3
*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4) + d*sqrt(x)) + 1/2*(-d
/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 +
 a^4*c*d^4))^(1/4)*log(-(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^
2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4) + d*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(bx^2 + a)(dx^2 + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")

[Out] integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.466 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{3/4}(bc-ad)}$$

```
[Out] -((b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*a^(3/4)*(b*c - a*d))) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a
*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(S
qrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d))
 + (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(
b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rubi [A] time = 0.716622, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] -((b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*a^(3/4)*(b*c - a*d))) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a
*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(S
qrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d))
 + (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(
b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rubi in Sympy [A] time = 145.862, size = 420, normalized size = 0.91

$$-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4c^{\frac{3}{4}}(ad-bc)} - \frac{\sqrt{2}d^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}d^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2c^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{3}{4}}(ad-bc)} - \frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4a^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{\frac{3}{4}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)/x**(1/2),x)
```

```
[Out] -sqrt(2)*d**(3/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c
) + sqrt(d)*x)/(4*c**(3/4)*(a*d - b*c)) + sqrt(2)*d**(3/4)*log(sq
rt(2)*c**(1/4)*d**(1/4)*sqrt(x) + sqrt(c) + sqrt(d)*x)/(4*c**(3/4
)*(a*d - b*c)) - sqrt(2)*d**(3/4)*atan(1 - sqrt(2)*d**(1/4)*sqrt(
x)/c**(1/4))/(2*c**(3/4)*(a*d - b*c)) + sqrt(2)*d**(3/4)*atan(1 +
 sqrt(2)*d**(1/4)*sqrt(x)/c**(1/4))/(2*c**(3/4)*(a*d - b*c)) + sq
rt(2)*b**(3/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) +
 sqrt(b)*x)/(4*a**(3/4)*(a*d - b*c)) - sqrt(2)*b**(3/4)*log(sqrt(
2)*a**(1/4)*b**(1/4)*sqrt(x) + sqrt(a) + sqrt(b)*x)/(4*a**(3/4)*(
a*d - b*c)) + sqrt(2)*b**(3/4)*atan(1 - sqrt(2)*b**(1/4)*sqrt(x)/
a**(1/4))/(2*a**(3/4)*(a*d - b*c)) - sqrt(2)*b**(3/4)*atan(1 + sq
rt(2)*b**(1/4)*sqrt(x)/a**(1/4))/(2*a**(3/4)*(a*d - b*c))
```

Mathematica [A] time = 0.254887, size = 364, normalized size = 0.79

$$a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-2a^{3/}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
] + 2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - a^(3/4)*d^(3/4)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*a
^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A] time = 0.016, size = 328, normalized size = 0.7

$$\frac{d\sqrt{2}}{(4\,ad-4\,bc)\,c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{d\sqrt{2}}{(2\,ad-2\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{d\sqrt{2}}{(2\,ad-2\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{b\sqrt{2}}{(4\,ad-4\,bc)\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{b\sqrt{2}}{(2\,ad-2\,bc)\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{b\sqrt{2}}{(2\,ad-2\,bc)\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x)
```

```
[Out] 1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+
1/2*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)+1/2*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*ln((x
+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2)))-1/2*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)*(a/b)^(1/4)/a*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.417589, size = 1467, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^
6*b*c*d^3 + a^7*d^4))^(1/4)*arctan(-(a*b*c - a^2*d)*(-b^3/(a^3*b^
4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7
*d^4))^(1/4)/(b*sqrt(x) + sqrt(b^2*x + (a^2*b^2*c^2 - 2*a^3*b*c*d
 + a^4*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*
c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))))) - 2*(-d^3/(b^4*c^7 - 4*a*b
^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1
/4)*arctan(-(b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^
2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)/(d*sqrt(x)
+ sqrt(d^2*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b
^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^
4*c^3*d^4))))) + 1/2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5
```

```
*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) + (a
*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^
2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/2*(-b^3/(a^3*b^4*c^4
 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*sqrt(x) - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^
4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)
) - 1/2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*sqrt(x) + (b*c^2 - a*c*d)
*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4
*d^3 + a^4*c^3*d^4))^(1/4)) + 1/2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d
*sqrt(x) - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2
*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(x)), x)
```

3.467 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=476

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}(bc-ad)}-\frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}(bc-ad)}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{5/4}(bc-ad)}-\frac{2}{ac\sqrt{x}}$$

```
[Out] -2/(a*c*Sqrt[x]) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) -
(d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5
/4)*(b*c - a*d)) + (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)) + (d^(5/4)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*S
qrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rubi [A] time = 1.15675, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}(bc-ad)}-\frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}(bc-ad)}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{5/4}(bc-ad)}-\frac{2}{ac\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] -2/(a*c*Sqrt[x]) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) -
(d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5
/4)*(b*c - a*d)) + (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)) + (d^(5/4)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*S
qrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.496649, size = 409, normalized size = 0.86

$$\frac{\frac{\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{2\sqrt{2}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{2\sqrt{2}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}}+\frac{8b}{a\sqrt{x}}-\cdots}{4ad-\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*Sqrt[x]) - (8*d)/(c*Sqrt[x]) - (2*Sqrt[2]*b^(5/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*b^
(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2
*Sqrt[2]*d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c
^(5/4) - (2*Sqrt[2]*d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/c^(5/4) + (Sqrt[2]*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (Sqrt[2]*b^(5/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (
Sqrt[2]*d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/c^(5/4) + (Sqrt[2]*d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(-4*b*c + 4*a*d)
```

Maple [A] time = 0.02, size = 339, normalized size = 0.7

$$-\frac{d\sqrt{2}}{4\,c\,(ad-bc)}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$-\frac{d\sqrt{2}}{2\,c\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{d\sqrt{2}}{2\,c\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$+\frac{b\sqrt{2}}{4\,a\,(ad-bc)}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{b\sqrt{2}}{2\,a\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$+\frac{b\sqrt{2}}{2\,a\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-2\,\frac{1}{ac\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
-1/2*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)-1/2*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))+1/2*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b/a/(a*d-b*c)/(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2/a/c/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.649198, size = 1797, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*sqrt(x)*arctan(-(a^4*b^3*c^3
 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4
- 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))
^(3/4)/(b^4*sqrt(x) + sqrt(b^8*x - (a^3*b^7*c^2 - 2*a^4*b^6*c*d +
 a^5*b^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^
2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))))) - 4*(-d^5/(b^4*c^9 - 4*a
*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^
(1/4)*a*c*sqrt(x)*arctan(-(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*
d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7
*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)/(d^4*sqrt(x) + sqrt(
d^8*x - (b^2*c^5*d^5 - 2*a*b*c^4*d^6 + a^2*c^3*d^7)*sqrt(-d^5/(b^
4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4
*c^5*d^4))))) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*
c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*sqrt(x)*log(b^4*sqr
t(x) + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*
(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*
b*c*d^3 + a^9*d^4))^(3/4)) + (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d
 + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*sqrt(x
)*log(b^4*sqrt(x) - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^
2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2
*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4*a*b^
3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/
4)*a*c*sqrt(x)*log(d^4*sqrt(x) + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*
b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) - (-d^5/(b^4
*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*
c^5*d^4))^(1/4)*a*c*sqrt(x)*log(d^4*sqrt(x) - (b^3*c^7 - 3*a*b^2*
c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c
^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4))
 - 4)/(a*c*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(3/2)), x)
```

3.468 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=478

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{7/4}(bc-ad)}-\frac{2}{3acx^{3/2}}$$

```
[Out] -2/(3*a*c*x^(3/2)) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d))
- (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2
]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(7/4)*(b*c - a*d)) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/
4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2
*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d
))
```

Rubi [A] time = 1.02252, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{7/4}(bc-ad)}-\frac{2}{3acx^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```


```
[Out] -2/(3*a*c*x^(3/2)) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d))
- (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2
]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(7/4)*(b*c - a*d)) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/
4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2
*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d
```

```
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.518259, size = 411, normalized size = 0.86

$$\frac{-\frac{3\sqrt{2}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{3\sqrt{2}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{6\sqrt{2}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8\ldots}{ax\ldots}}{12a\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*x^(3/2)) - (8*d)/(c*x^(3/2)) - (6*Sqrt[2]*b^(7/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^
(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6
*Sqrt[2]*d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c
^(7/4) - (6*Sqrt[2]*d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4)
 + (3*Sqrt[2]*d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/c^(7/4) - (3*Sqrt[2]*d^(7/4)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(-12*b*c + 12*a
*d)
```

Maple [A] time = 0.02, size = 351, normalized size = 0.7

$$-\frac{d^2\sqrt{2}}{4\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{d^2\sqrt{2}}{2\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{d^2\sqrt{2}}{2\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{b^2\sqrt{2}}{4\,a^2\,(ad-bc)}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{b^2\sqrt{2}}{2\,a^2\,(ad-bc)}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$+\frac{b^2\sqrt{2}}{2\,a^2\,(ad-bc)}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)-\frac{2}{3\,ac}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))-1/2/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)-1/2/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1
/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2/a^2*b^2/(a*d-b*c)*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/a^2*b^2
/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1)-2/3/a/c/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.24354, size = 1557, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/6*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2
 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^(3/2)*arctan(-(-b^7/(a
^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3
 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)/(b^2*sqrt(x) + sqrt(b^4*x +
 (a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4
*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4)))
)) - 12*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*
a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^(3/2)*arctan(-(-d^7/(b^
4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a
^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)/(d^2*sqrt(x) + sqrt(d^4*x +
(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*sqrt(-d^7/(b^4*c^11 - 4*a*b
^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4)))
) + 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 -
4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^(3/2)*log(b^2*sqrt(x) + (
-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) - 3*(-b^7/(a^7*b^4*
c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11
*d^4))^(1/4)*a*c*x^(3/2)*log(b^2*sqrt(x) - (-b^7/(a^7*b^4*c^4 - 4
*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^
(1/4)*(a^2*b*c - a^3*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6
*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^(3
/2)*log(d^2*sqrt(x) + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^
2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*
d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*
a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^(3/2)*log(d^2*sqrt(x) -
 (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c
^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 4)/(a*c*x^(3/2)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)x^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(5/2)), x)
```

3.469 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=498

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{2(ad+bc)}{a^2c^2\sqrt{x}}-\frac{d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}(bc-ad)}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{9/4}(bc-ad)}-\frac{2}{5acx^{5/2}}$$

```
[Out] -2/(5*a*c*x^(5/2)) + (2*(b*c + a*d))/(a^2*c^2*Sqrt[x]) - (b^(9/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*
(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(
9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9
/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*S
qrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
 + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Rubi [A] time = 1.48478, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{2(ad+bc)}{a^2c^2\sqrt{x}}-\frac{d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}(bc-ad)}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{9/4}(bc-ad)}-\frac{2}{5acx^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```



```
[Out] -2/(5*a*c*x^(5/2)) + (2*(b*c + a*d))/(a^2*c^2*Sqrt[x]) - (b^(9/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*
(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(
9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9
/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*S
qrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
 + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
```

```
d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.638943, size = 437, normalized size = 0.88

$$\frac{-\frac{5\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}+\frac{5\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}+\frac{10\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}}-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*x^(5/2)) - (8*d)/(c*x^(5/2)) - (40*b^2)/(a^2*Sqrt[x]) +
 (40*d^2)/(c^2*Sqrt[x]) + (10*Sqrt[2]*b^(9/4)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(
9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(9/4) + (10
*Sqrt[2]*d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c
^(9/4) - (5*Sqrt[2]*b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/a^(9/4) + (5*Sqrt[2]*b^(9/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4) + (5*Sqrt[
2]*d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(9/4))/(-20*b*c + 20*a*d)
```

Maple [A] time = 0.023, size = 375, normalized size = 0.8

$$\frac{d^2\sqrt{2}}{4\,c^2\,(ad-bc)}\ln\left(1\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$+\frac{d^2\sqrt{2}}{2\,c^2\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}$$
$$+\frac{d^2\sqrt{2}}{2\,c^2\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{2}{5\,ac}x^{-\frac{5}{2}}+2\,\frac{d}{\sqrt{x}ac^2}+2\,\frac{b}{a^2c\sqrt{x}}$$
$$-\frac{b^2\sqrt{2}}{4\,a^2\,(ad-bc)}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$
$$-\frac{b^2\sqrt{2}}{2\,a^2\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{b^2\sqrt{2}}{2\,a^2\,(ad-bc)}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))+1/2*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)+1/2*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c/x^(5/2)+2/x^(1/2)/a/c^2
*d+2/x^(1/2)/a^2/c*b-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln
((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2)))-1/2*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^2/a^2/(a*d-b*c)/(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.37911, size = 1852, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] -1/10*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*
d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^(5/2)*arctan(-(
a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(
a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*
d^3 + a^13*d^4))^(3/4)/(b^7*sqrt(x) + sqrt(b^14*x - (a^5*b^11*c^2
 - 2*a^6*b^10*c*d + a^7*b^9*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*
b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))))) -
 20*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3
*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^(5/2)*arctan(-(b^3*c^
10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^
13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4
*c^9*d^4))^(3/4)/(d^7*sqrt(x) + sqrt(d^14*x - (b^2*c^7*d^9 - 2*a*
b*c^6*d^10 + a^2*c^5*d^11)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d +
 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))))) - 5*(-b
^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*
b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^(5/2)*log(b^7*sqrt(x) + (a^7
*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9
*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3
 + a^13*d^4))^(3/4)) + 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d +
6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^
(5/2)*log(b^7*sqrt(x) - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*
c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*
b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) + 5*(-d^9/(b^4*c
^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^
4*c^9*d^4))^(1/4)*a^2*c^2*x^(5/2)*log(d^7*sqrt(x) + (b^3*c^10 - 3
*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4
*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d
^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11
*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^(5/2)*log
(d^7*sqrt(x) - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*
c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 -
```

```
4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) - 20*(b*c + a*d)*x^2 + 4*
a*c)/(a^2*c^2*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*x^(7/2)), x)
```

3.470 $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=570

$$\frac{a^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{5/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{9/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{9/4}(bc-ad)^2}+\frac{\sqrt{x}(5bc-4ad)}{2bd^2(bc-ad)}-\frac{cx^{5/2}}{2d\,(c+dx^2)\,(bc-ad)}$$

```
[Out] ((5*b*c - 4*a*d)*Sqrt[x])/(2*b*d^2*(b*c - a*d)) - (c*x^(5/2))/(2*
d*(b*c - a*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9/4)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*
c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(5/4)
*(5*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) + (a^(9/4)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c -
a*d)^2) - (a^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c
 - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(5/4)*(5*b*c - 9*a*d)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*d^(9/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.7406, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$\frac{a^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{5/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{9/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{9/4}(bc-ad)^2}+\frac{\sqrt{x}(5bc-4ad)}{2bd^2(bc-ad)}-\frac{cx^{5/2}}{2d\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((5*b*c - 4*a*d)*Sqrt[x])/(2*b*d^2*(b*c - a*d)) - (c*x^(5/2))/(2*
d*(b*c - a*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
```

```
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9/4)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*
c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(5/4)
*(5*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) + (a^(9/4)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c -
a*d)^2) - (a^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c
 - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(5/4)*(5*b*c - 9*a*d)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*d^(9/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.619396, size = 563, normalized size = 0.99

$$\frac{4\sqrt{2}a^{9/4}d^{9/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}a^{9/4}d^{9/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+8\sqrt{2}a^{9/4}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*b^(5/4)*c^2*d^(1/4)*(b*c - a*d)*Sqrt[x] + 32*b^(1/4)*d^(1/4)*(
b*c - a*d)^2*Sqrt[x]*(c + d*x^2) + 8*Sqrt[2]*a^(9/4)*d^(9/4)*(c +
 d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[2]
*a^(9/4)*d^(9/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] + 2*Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2*Sqrt[2]*b^(5/4
)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt[2]*
a^(9/4)*d^(9/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(
c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x] - Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(16*b^(5/4)
*d^(9/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.026, size = 582, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 2/b/d^2*x^(1/2)-1/2*c^2/d/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a+1/2*c^3
/d^2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*b+9/8*c/d/(a*d-b*c)^2*(c/d)^(1
```

```
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-5/8*c^2/d^2/(
a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1)*b+9/16*c/d/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2)))*a-5/16*c^2/d^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2)))*b+9/8*c/d/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-5/8*c^2/d^2/(a*d-b*c)^2*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-1/4/b*a^2/(
a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2/b*a
^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)-1/2/b*a^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 50.7359, size = 3723, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(16*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 -
56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 2
8*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^
2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*arctan((-a^9/(b^13
*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3
 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 -
 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)/(a^2*sqrt(x) + sqrt(a^4*x + (b^6*c^4 - 4*a*b^5*c^3*d +
6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*sqrt(-a^9/(b^1
3*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^
3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6
- 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))))) - 4*(b^2*c^2*d^2 - a*b*c*d^3
 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d +
 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/
(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^
5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b
^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)*arctan(-(b^2*c^2*
d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d +
12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(
b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5
*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^
2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)/((5*b*c^2 - 9*a*c*
d)*sqrt(x) - sqrt((25*b^2*c^4 - 90*a*b*c^3*d + 81*a^2*c^2*d^2)*x
+ (b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*
d^7 + a^4*d^8)*sqrt(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*
b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^
9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12
+ 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15
 - 8*a^7*b*c*d^16 + a^8*d^17))))) - 4*(-a^9/(b^13*c^8 - 8*a*b^12*
c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^
4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7
 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*
```

```
b*d^4)*x^2)*log(a^2*sqrt(x) + (-a^9/(b^13*c^8 - 8*a*b^12*c^7*d +
28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 -
56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b
^5*d^8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) + 4*(-a^9/(b^
13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d
^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6
 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3
 + (b^2*c*d^3 - a*b*d^4)*x^2)*log(a^2*sqrt(x) - (-a^9/(b^13*c^8 -
 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*
a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7
*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)) + (b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(6
25*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3
*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 +
28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 -
 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8
*d^17))^(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a
^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8
*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^
12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d
^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)) - (b^2*c^2*d^2 - a*b*c*d
^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d
 + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4
)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*
b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6
*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)*log(-(5*b*c^2 -
 9*a*c*d)*sqrt(x) - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(625*
b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*
c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*
a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56
*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^
17))^(1/4)) + 4*(5*b*c^2 - 4*a*c*d + 4*(b*c*d - a*d^2)*x^2)*sqrt(
x))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.365905, size = 969, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt
(2)*a^2*b^2*d^2) - (a*b^3)^(1/4)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(
2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/2*(a*b^3)^(1/4)*a^2*ln(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2*s
qrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*(a*b^3)^(1/4)*a^2*l
n(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2
- 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/4*(5*(c*d^3)^(1/
4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/
d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(
2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) - 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*
```

```
(c*d^3)^(1/4)*a*c*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2
*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4
 + sqrt(2)*a^2*d^5) - 1/8*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4
)*a*c*d)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)
*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/8*(5*(c
*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*ln(-sqrt(2)*sqrt(x)*(c
/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c
*d^4 + sqrt(2)*a^2*d^5) + 1/2*c^2*sqrt(x)/((b*c*d^2 - a*d^3)*(d*x
^2 + c)) + 2*sqrt(x)/(b*d^2)
```

3.471 $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=536

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{3/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{7/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{7/4}(bc-ad)^2}-\frac{cx^{3/2}}{2d\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(c*x^(3/2))/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^
2) + (a^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
t[2]*b^(3/4)*(b*c - a*d)^2) - (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*
d)^2) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (a^(7/4)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*b^(3/4)*(b*c - a*d)^2) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)^2) +
 (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) - (c^(3/4)
*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.2961, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{3/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{7/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{7/4}(bc-ad)^2}-\frac{cx^{3/2}}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*x^(3/2))/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^
2) + (a^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
```

```
t[2]*b^(3/4)*(b*c - a*d)^2) - (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*
d)^2) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (a^(7/4)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*b^(3/4)*(b*c - a*d)^2) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)^2) +
 (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) - (c^(3/4)
*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.588122, size = 527, normalized size = 0.98

$$\frac{4\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{2}a^{7/4}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-8*b^(3/4)*c*d^(3/4)*(b*c - a*d)*x^(3/2) - 8*Sqrt[2]*a^(7/4)*d^(
7/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] +
8*Sqrt[2]*a^(7/4)*d^(7/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c
 + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[
2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(c + d*x^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4
*Sqrt[2]*a^(7/4)*d^(7/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c -
 7*a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^
2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(1
6*b^(3/4)*d^(7/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.025, size = 566, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x^(3/
2)/(d*x^2+c)*b-7/8*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+3/8*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-7/8*c/(a*d-b*c)^
2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+3
```

```
/8*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)*b-7/16*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*ln((x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2)))*a+3/16*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/
2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/4*a^2/(a*d-b*c)^2/b/(a/b)^(1/4)
*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2*a^2/(a*d-b*c)^2/b/(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a^2/(a*d-b
*c)^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.2834, size = 4362, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*c*x^(3/2) - 16*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*
b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^
6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^
(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*arctan((b^8*c^6
 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a
^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 -
 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^
4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b
^4*c*d^7 + a^8*b^3*d^8))^(3/4)/(a^5*sqrt(x) + sqrt(a^10*x - (a^7*
b^5*c^4 - 4*a^8*b^4*c^3*d + 6*a^9*b^3*c^2*d^2 - 4*a^10*b^2*c*d^3
+ a^11*b*d^4)*sqrt(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c
^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3
*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))))) +
4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 - 756
*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a
^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 -
 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12
+ 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*arctan(
-(b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*b^4*c^4*d^7 - 20*a^3*b^3
*c^3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^6*d^11)*(-(81*
b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4
*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*
b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*
b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^
(3/4)/((27*b^3*c^5 - 189*a*b^2*c^4*d + 441*a^2*b*c^3*d^2 - 343*a^
3*c^2*d^3)*sqrt(x) - sqrt((729*b^6*c^10 - 10206*a*b^5*c^9*d + 595
35*a^2*b^4*c^8*d^2 - 185220*a^3*b^3*c^7*d^3 + 324135*a^4*b^2*c^6*
d^4 - 302526*a^5*b*c^5*d^5 + 117649*a^6*c^4*d^6)*x - (81*b^8*c^11
*d^3 - 1080*a*b^7*c^10*d^4 + 6156*a^2*b^6*c^9*d^5 - 19560*a^3*b^5
*c^8*d^6 + 37846*a^4*b^4*c^7*d^7 - 45640*a^5*b^3*c^6*d^8 + 33516*
a^6*b^2*c^5*d^9 - 13720*a^7*b*c^4*d^10 + 2401*a^8*c^3*d^11)*sqrt(
-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*
b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28
*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56
*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^
15))))) - 4*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2
```

```
 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 +
 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*(b*c^
2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) + (b^8*c^6
 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a
^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 -
 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^
4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b
^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + 4*(-a^7/(b^11*c^8 - 8*a*b^10*c^
7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^
4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a
^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*lo
g(a^5*sqrt(x) - (b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 2
0*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^
2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56
*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a
^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + (b*c^2*
d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 - 756*a*b^3*c^
6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^
4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b
^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*
b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*log((b^6*c^6*d^5
 - 6*a*b^5*c^5*d^6 + 15*a^2*b^4*c^4*d^7 - 20*a^3*b^3*c^3*d^8 + 15
*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^6*d^11)*(-(81*b^4*c^7 - 756
*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a
^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 -
 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12
+ 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(3/4) - (27*b
^3*c^5 - 189*a*b^2*c^4*d + 441*a^2*b*c^3*d^2 - 343*a^3*c^2*d^3)*s
qrt(x)) - (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c
^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3
+ 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c
^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c
^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)
*log(-(b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*b^4*c^4*d^7 - 20*a^
3*b^3*c^3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^6*d^11)*(
-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*
b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28
*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56
*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^
15))^(3/4) - (27*b^3*c^5 - 189*a*b^2*c^4*d + 441*a^2*b*c^3*d^2 -
343*a^3*c^2*d^3)*sqrt(x)))/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)
*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.336445, size = 919, normalized size = 1.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)
*a^2*b^3*d^2) + (a*b^3)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*
```

```
b^4*c*d + sqrt(2)*a^2*b^3*d^2) - 1/2*(a*b^3)^(3/4)*a*ln(sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*
a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/2*(a*b^3)^(3/4)*a*ln(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c^2 - 2*sqrt(
2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/4*(3*(c*d^3)^(3/4)*b*c -
7*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5
+ sqrt(2)*a^2*d^6) + 1/4*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^
6) - 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*ln(sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt
(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) + 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(
c*d^3)^(3/4)*a*d)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) -
 1/2*c*x^(3/2)/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.472 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=532

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}-\frac{c\sqrt{x}}{2d\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(c*Sqrt[x])/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^
2) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
t[2]*b^(1/4)*(b*c - a*d)^2) - (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)
^2) + (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) - (a^(5/4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(
1/4)*(b*c - a*d)^2) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)^2) - (c^
(1/4)*(b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) + (c^(1/4)*(b*c
- 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.13101, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}-\frac{c\sqrt{x}}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*Sqrt[x])/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^
2) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
```

```
t[2]*b^(1/4)*(b*c - a*d)^2) - (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)
^2) + (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) - (a^(5/4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(
1/4)*(b*c - a*d)^2) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)^2) - (c^
(1/4)*(b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) + (c^(1/4)*(b*c
- 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.515788, size = 523, normalized size = 0.98

$$\frac{-4\sqrt{2}a^{5/4}d^{5/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\sqrt{2}a^{5/4}d^{5/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{2}a^{5/}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-8*b^(1/4)*c*d^(1/4)*(b*c - a*d)*Sqrt[x] - 8*Sqrt[2]*a^(5/4)*d^(
5/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] +
8*Sqrt[2]*a^(5/4)*d^(5/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c +
 d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]
*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*a^(5/4)*d^(5/4)*(c + d*x^2)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4*Sqr
t[2]*a^(5/4)*d^(5/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d
)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x] + Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(16*b^(1/4
)*d^(5/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.024, size = 533, normalized size = 1.

$$\frac{ac}{2\left(ad-bc\right)^{2}\left(dx^{2}+c\right)}\sqrt{x}-\frac{bc^{2}}{2\left(ad-bc\right)^{2}d\left(dx^{2}+c\right)}\sqrt{x}$$
$$-\frac{5\sqrt{2}a}{8\left(ad-bc\right)^{2}}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{c\sqrt{2}b}{8\left(ad-bc\right)^{2}d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$-\frac{5\sqrt{2}a}{8\left(ad-bc\right)^{2}}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{c\sqrt{2}b}{8\left(ad-bc\right)^{2}d}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{5\sqrt{2}a}{16\left(ad-bc\right)^{2}}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{c\sqrt{2}b}{16\left(ad-bc\right)^{2}d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}a}{4\left(ad-bc\right)^{2}}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}a}{2\left(ad-bc\right)^{2}}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}a}{2\left(ad-bc\right)^{2}}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x^(1/
2)/(d*x^2+c)*b-5/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a+1/8*c/(a*d-b*c)^2/d*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-5/8/(a*d-b*c)^2*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+1/8*c/(a*d-b*
c)^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
b-5/16/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
*a+1/16*c/(a*d-b*c)^2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))*b+1/4*a/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2)))+1/2*a/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+1/2*a/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.54025, size = 3505, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(16*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56
*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a
^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b*c^2*d - a
*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*arctan((-a^5/(b^9*c^8 - 8*a*b^8*c
^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d
^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
a^8*b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a*sqrt(x) + sq
rt(a^2*x + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*sqrt(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7
*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c
^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))))) -
4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b
^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^
4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b
^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^
2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)*arctan(-(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b
^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*
a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*
b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b
*c*d^12 + a^8*d^13))^(1/4)/((b*c - 5*a*d)*sqrt(x) - sqrt((b^2*c^2
 - 10*a*b*c*d + 25*a^2*d^2)*x + (b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 +
6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5 + a^4*d^6)*sqrt(-(b^4*c^5 - 20*
a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c
*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^
3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6
*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))))) + (b*c^2*d - a*c*d
^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2
*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 -
8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^
4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7
*b*c*d^12 + a^8*d^13))^(1/4)*log(-(b*c - 5*a*d)*sqrt(x) + (b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^
2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 -
 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a
^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^
7*b*c*d^12 + a^8*d^13))^(1/4)) - (b*c^2*d - a*c*d^2 + (b*c*d^2 -
a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 5
00*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6
+ 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 -
56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*
d^13))^(1/4)*log(-(b*c - 5*a*d)*sqrt(x) - (b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 -
500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6
 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 -
 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8
*d^13))^(1/4)) - 4*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^
6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*
d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b
*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) + (-a^5/(
b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3
 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 -
 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)) + 4*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56
*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a
^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b*c^2*d - a
*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) - (-a^5/(b^9*c^8 -
8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*
b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2
*c*d^7 + a^8*b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*c
*sqrt(x))/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.345843, size = 903, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)
*a^2*b*d^2) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^
2*c*d + sqrt(2)*a^2*b*d^2) + 1/2*(a*b^3)^(1/4)*a*ln(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^
2*c*d + sqrt(2)*a^2*b*d^2) - 1/2*(a*b^3)^(1/4)*a*ln(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b
^2*c*d + sqrt(2)*a^2*b*d^2) + 1/4*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^
(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(
c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*
a^2*d^4) + 1/4*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) + 1/8*((c
*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*ln(sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3
+ sqrt(2)*a^2*d^4) - 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d
)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c
^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) - 1/2*c*sqrt(x)/(
(b*c*d - a*d^2)*(d*x^2 + c))
```

3.473 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=528

$$-\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^2}+\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}+\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}+\frac{x^{3/2}}{2\left(c+dx^2\right)(bc-ad)}$$

```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a
^(3/4)*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*(b*c - a*d)^2) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) + (
(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b
*c - a*d)^2) + (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) + ((b*c + 3*
a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - ((b*c + 3*a*d)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*
c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.23022, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^2}+\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}+\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}+\frac{x^{3/2}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a
^(3/4)*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*(b*c - a*d)^2) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) + (
(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b
*c - a*d)^2) + (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) + ((b*c + 3*
a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
```

```
(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - ((b*c + 3*a*d)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*
c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.50265, size = 522, normalized size = 0.99

$$-4\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*c^(1/4)*d^(3/4)*(b*c - a*d)*x^(3/2) + 8*Sqrt[2]*a^(3/4)*b^(1/4
)*c^(1/4)*d^(3/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 8*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*(c + d*x^2
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(b*c
+ 3*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
] + 2*Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*
(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x] + 4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*(c + d*x^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]
*(b*c + 3*a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(16*c^(1/4)*d
^(3/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.024, size = 528, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x^(3/2)/(d
*x^2+c)*b*c+3/16/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2)))+3/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)+3/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/16/(a*d-b*c)^2/d/(c/d)^(1/4)
*2^(1/2)*b*c*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+1/8/(a*d-b*c)^2/d/(c/d)^(
1/4)*2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/8/(a*d-b
*c)^2/d/(c/d)^(1/4)*2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)-1/4*a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))-1/2*a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
```

```
1/4)*x^(1/2)+1)-1/2*a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.35201, size = 4201, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(16*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d
 + (b*c*d - a*d^2)*x^2)*arctan((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*
b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c
*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)/(a^2*b*
sqrt(x) + sqrt(a^4*b^2*x - (a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5
*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*sqrt(-a^3*b/(b^8*c^8
- 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8))))) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(
-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3
 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^
5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8
+ 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*arct
an((b^6*c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b
^3*c^4*d^5 + 15*a^4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-
(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3
+ 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5
 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 +
 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4)/((b^3
*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x) + sqr
t((b^6*c^6 + 18*a*b^5*c^5*d + 135*a^2*b^4*c^4*d^2 + 540*a^3*b^3*c
^3*d^3 + 1215*a^4*b^2*c^2*d^4 + 1458*a^5*b*c*d^5 + 729*a^6*d^6)*x
 - (b^8*c^9*d + 8*a*b^7*c^8*d^2 + 12*a^2*b^6*c^7*d^3 - 40*a^3*b^5
*c^6*d^4 - 74*a^4*b^4*c^5*d^5 + 120*a^5*b^3*c^4*d^6 + 108*a^6*b^2
*c^3*d^7 - 216*a^7*b*c^2*d^8 + 81*a^8*c*d^9)*sqrt(-(b^4*c^4 + 12*
a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/
(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*
c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^
3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))))) + 4*(-a^3*b/(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log
(a^2*b*sqrt(x) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6
)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - 4*(-a^3*b/(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*l
```

```
og(a^2*b*sqrt(x) - (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d
^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^
3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - (b*c^2 - a*c*d +
 (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^
2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*
d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^
7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 +
a^8*c*d^11))^(1/4)*log((b^6*c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^
4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4*b^2*c^3*d^6 - 6*a^5*b*c^2
*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^
2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4
+ 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 -
56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*
c*d^11))^(3/4) + (b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a
^3*d^3)*sqrt(x)) + (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c
^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a
^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*
a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^
6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log(-(b^6*c
^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^
5 + 15*a^4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4
+ 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*
d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3
*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b
^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4) + (b^3*c^3 + 9
*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x)) - 4*x^(3/2))
/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.344711, size = 922, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d
^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) + 1/4*((c*d^3)^(3
/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)
*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) - 1/8*((c*d^3)^(3/4)*b*c + 3*(c
*d^3)^(3/4)*a*d)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/
(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5)
 + 1/8*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*ln(-sqrt(2)*sqrt
(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)
*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) - (a*b^3)^(3/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c
^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - (a*b^3)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) +
1/2*(a*b^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) -
```

```
1/2*(a*b^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) +
 1/2*x^(3/2)/((d*x^2 + c)*(b*c - a*d))
```

3.474 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=528

$$\frac{\sqrt[4]{a}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}-\frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-\frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^2}-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{\sqrt{x}}{2\left(c+dx^2\right)(bc-ad)}$$

```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a
^(1/4)*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*(b*c - a*d)^2) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (
(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (a^(1/4)*b^(3/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b
*c - a*d)^2) - (a^(1/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) - ((3*b*c +
a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + ((3*b*c + a*d)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*
c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.987408, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{\sqrt[4]{a}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}-\frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}+\frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-\frac{\sqrt[4]{a}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^2}-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{\sqrt{x}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```


```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a
^(1/4)*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*(b*c - a*d)^2) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (
(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (a^(1/4)*b^(3/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b
*c - a*d)^2) - (a^(1/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) - ((3*b*c +
a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
```

```
(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + ((3*b*c + a*d)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*
c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.494756, size = 522, normalized size = 0.99

$$4\sqrt{2}\sqrt[4]{a}b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}\sqrt[4]{a}b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*c^(3/4)*d^(1/4)*(b*c - a*d)*Sqrt[x] + 8*Sqrt[2]*a^(1/4)*b^(3/4
)*c^(3/4)*d^(1/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 8*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*(c + d*x^2
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(3*b*
c + a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
] + 2*Sqrt[2]*(3*b*c + a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*
(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x] - 4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*(c + d*x^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]
*(3*b*c + a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(3*b*c + a*d)*(c + d*x^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(16*c^(3/4)*d
^(1/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.023, size = 528, normalized size = 1.

$$-\frac{ad}{2\left(ad-bc\right)^{2}\left(dx^{2}+c\right)}\sqrt{x}+\frac{bc}{2\left(ad-bc\right)^{2}\left(dx^{2}+c\right)}\sqrt{x}+\frac{\sqrt{2}ad}{8\left(ad-bc\right)^{2}c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{3\sqrt{2}b}{8\left(ad-bc\right)^{2}}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}ad}{8\left(ad-bc\right)^{2}c}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{3\sqrt{2}b}{8\left(ad-bc\right)^{2}}\sqrt[4]{\frac{c}{d}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}ad}{16\left(ad-bc\right)^{2}c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{3\sqrt{2}b}{16\left(ad-bc\right)^{2}}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x+\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x-\sqrt[4]{\frac{c}{d}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)-\frac{\sqrt{2}b}{4\left(ad-bc\right)^{2}}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{\sqrt{2}b}{2\left(ad-bc\right)^{2}}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}b}{2\left(ad-bc\right)^{2}}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x^(1/2)/(d
*x^2+c)*b*c+1/8/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)*a*d+3/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+1/8/(a*d-b*c)^2*(c/d)^(1/4)
/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*d+3/8/(a*d-b*c
)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b+1
/16/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*
a*d+3/16/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2
)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
))*b-1/4*b/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))-1/2*b/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.97694, size = 3430, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(16*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d
+ (b*c*d - a*d^2)*x^2)*arctan((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-a
*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)/(b*sqrt(x) + sqrt(b^2*x +
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))))) - 4*(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a
^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*
c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c
^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^
8 + a^8*c^3*d^9))^(1/4)*arctan((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2
)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b
*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9
*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d
^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)/(
(3*b*c + a*d)*sqrt(x) + sqrt((9*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
+ (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3
+ a^4*c^2*d^4)*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2
 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 -
 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*
c^3*d^9))))) + (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^
4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d
^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*
b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^
2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)*log((3*b*c + a*
d)*sqrt(x) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(-(81*b^4*c^4 +
108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/
(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*
c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^
5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - (b*c^2 - a*c*d +
 (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^
2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*
d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^
5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a
^8*c^3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) - (b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^
2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2
+ 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 -
56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c
^3*d^9))^(1/4)) - 4*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*
c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) + (b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3
*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)) + 4*
(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d
 - a*d^2)*x^2)*log(b*sqrt(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-
a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)) + 4*sqrt(x))/(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.335396, size = 884, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d
 - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/4*(3*(c*d^3)^(1
/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b
*c^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/8*(3*(c*d^3)^(1/4)*b*c + (c*d^3
)^(1/4)*a*d)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqr
t(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) - 1/8
*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*ln(-sqrt(2)*sqrt(x)*(c
/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2
*d^2 + sqrt(2)*a^2*c*d^3) - (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^2*c^2 - 2*s
qrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - (a*b^3)^(1/4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^2*
c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/2*(a*b^3)^(1/4)*ln
(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 -
2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/2*(a*b^3)^(1/4)*ln(-sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqr
t(2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/2*sqrt(x)/((d*x^2 + c)*(b*c -
 a*d))
```

3.475 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=536

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{\sqrt[4]{d}(5bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{d}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2}-\frac{dx^{3/2}}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*x^(3/2))/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^
2) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
t[2]*a^(1/4)*(b*c - a*d)^2) + (d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)
^2) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (b^(5/4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
1/4)*(b*c - a*d)^2) - (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^2) - (d^
(1/4)*(5*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (d^(1/4)*(5*b*
c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.26398, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{\sqrt[4]{d}(5bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{d}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2}-\frac{dx^{3/2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x^(3/2))/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^
2) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
```

```
t[2]*a^(1/4)*(b*c - a*d)^2) + (d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)
^2) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (b^(5/4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
1/4)*(b*c - a*d)^2) - (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^2) - (d^
(1/4)*(5*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (d^(1/4)*(5*b*
c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.534663, size = 523, normalized size = 0.98

$$4\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{2}b^{5/4}c$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*a^(1/4)*c^(1/4)*d*(-(b*c) + a*d)*x^(3/2) - 8*Sqrt[2]*b^(5/4)*c
^(5/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 8*Sqrt[2]*b^(5/4)*c^(5/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(
c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt
[2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*b^(5/4)*c^(5/4)*(c + d*x^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4
*Sqrt[2]*b^(5/4)*c^(5/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*a^(1/4)*d^(1/4)*(-5*b*c
+ a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x] + Sqrt[2]*a^(1/4)*d^(1/4)*(5*b*c - a*d)*(c + d*x^2)*
Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(16*a
^(1/4)*c^(5/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.023, size = 533, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x^(3/2)/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x^(3/
2)/(d*x^2+c)*b+1/8*d/(a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-5/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+1/8*d/(a*d-b*c)^2/c/(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-5/8/(a*d-
```

```
b*c)^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
b+1/16*d/(a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))*a-5/16/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))*b+1/4*b/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2)))+1/2*b/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+1/2*b/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.90806, size = 4293, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*d*x^(3/2) - 16*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^
3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*
b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4
)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*arctan((a*b^6*c^6 -
 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a
^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^8 - 8*a^
2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^
4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d
^7 + a^9*d^8))^(3/4)/(b^4*sqrt(x) + sqrt(b^8*x - (a*b^9*c^4 - 4*a
^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)
*sqrt(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56
*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a
^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))))) - 4*(b*c^3 - a*c^2*
d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2
 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*
a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4
*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*
c^6*d^7 + a^8*c^5*d^8))^(1/4)*arctan(-(b^6*c^10 - 6*a*b^5*c^9*d +
 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6
*a^5*b*c^5*d^5 + a^6*c^4*d^6)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^
2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8
*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^
4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
*c^6*d^7 + a^8*c^5*d^8))^(3/4)/((125*b^3*c^3*d - 75*a*b^2*c^2*d^2
 + 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x) - sqrt((15625*b^6*c^6*d^2 -
18750*a*b^5*c^5*d^3 + 9375*a^2*b^4*c^4*d^4 - 2500*a^3*b^3*c^3*d^5
 + 375*a^4*b^2*c^2*d^6 - 30*a^5*b*c*d^7 + a^6*d^8)*x - (625*b^8*c
^11*d - 3000*a*b^7*c^10*d^2 + 5900*a^2*b^6*c^9*d^3 - 6120*a^3*b^5
*c^8*d^4 + 3606*a^4*b^4*c^7*d^5 - 1224*a^5*b^3*c^6*d^6 + 236*a^6*
b^2*c^5*d^7 - 24*a^7*b*c^4*d^8 + a^8*c^3*d^9)*sqrt(-(625*b^4*c^4*
d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^
4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*
b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b
^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))))) - 4*(-b^5/(a*b^8*
c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 +
 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8
```

```
*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*
d^2)*x^2)*log(b^4*sqrt(x) + (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3
*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*
c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*
c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^
3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(3/4)) + 4
*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4
*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b
^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (
b*c^2*d - a*c*d^2)*x^2)*log(b^4*sqrt(x) - (a*b^6*c^6 - 6*a^2*b^5*
c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*
d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d
 + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 -
 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^
8))^(3/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b
^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d
^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 -
56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 2
8*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(1/4)*log((b^
6*c^10 - 6*a*b^5*c^9*d + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3
+ 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5*d^5 + a^6*c^4*d^6)*(-(625*b^4*
c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4
+ a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*
a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a
^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3
*c^3*d - 75*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)) +
(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^4*c^4*d - 50
0*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)
/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^
10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7
*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(1/4)*log(-(b^6*c^10 - 6*a
*b^5*c^9*d + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2
*c^6*d^4 - 6*a^5*b*c^5*d^5 + a^6*c^4*d^6)*(-(625*b^4*c^4*d - 500*
a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(
b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10
*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3*c^3*d - 75*
a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)))/(b*c^3 - a*c^
2*d + (b*c^2*d - a*c*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.361784, size = 946, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")

[Out] -1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)
*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4*
d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) - 1/4*(5*(c*d^
3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c
/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt
(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) + 1/8*(5*(c*d^3)^(3/4)*b*c
 - (c*d^3)^(3/4)*a*d)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
```

```
/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c
^2*d^4) - 1/8*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*ln(-sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*
sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) + (a*b^3)^(3/4)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) + (a*b
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(
a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*
a^3*b*d^2) - 1/2*(a*b^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2
)*a^3*b*d^2) + 1/2*(a*b^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqr
t(2)*a^3*b*d^2) - 1/2*d*x^(3/2)/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.476 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=536

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d\sqrt{x}}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*Sqrt[x])/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^
2) + (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
t[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*
d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) +
 (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)
*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.11823, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d\sqrt{x}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```



```
[Out] -(d*Sqrt[x])/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^
2) + (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
```

```
t[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*
d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) +
 (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)
*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.550348, size = 526, normalized size = 0.98

$$8a^{3/4}c^{3/4}d\sqrt{x}(ad-bc)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(3ad-7$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*Sqrt[x] - 8*Sqrt[2]*b^(7/4)*c
^(7/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)
*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sq
rt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
 + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*a^(3/4)*d^(3/4)*(7*b
*c - 3*a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c +
 d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(16*a^(3/4)*c^(7/4)*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A] time = 0.023, size = 566, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x^(1/2)/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x^(1/
2)/(d*x^2+c)*b+3/8*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-7/8*d/(a*d-b*c)^2/c*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+3/8*d^2/(a*d-b*c
)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

```
*a-7/8*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*ln
((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2)))*a-7/16*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/
2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a
*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2*b^2/(a*d-b*c)^2*(a/b)^(1/
4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^2/(a*d-b
*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 23.5627, size = 3611, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^
2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5
+ 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 -
 a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*arctan((-b^7/(a^3*b^8*c^8 - 8
*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7
*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b
*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b^
2*sqrt(x) + sqrt(b^4*x + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b
^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*
a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*
b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*
c*d^7 + a^11*d^8))))) + 4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*
x^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*
d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d +
28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8
*c^7*d^8))^(1/4)*arctan(-(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-
(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 7
56*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*
b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5
*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^
8))^(1/4)/((7*b*c*d - 3*a*d^2)*sqrt(x) - sqrt((49*b^2*c^2*d^2 - 4
2*a*b*c*d^3 + 9*a^2*d^4)*x + (b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2
*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4*c^4*d^3
 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 +
81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56
*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 2
8*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))))) - 4*(-b^7/
(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*
c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^
2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c
^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) + (-b^7/(a^3*b^8*c^8 - 8*a^4
*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4
*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d
^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) + 4*(-
b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*
```

```
b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^
2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d +
(b*c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) - (-b^7/(a^3*b^8*c^8 - 8
*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7
*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b
*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) -
(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^4*c^4*d^3 -
 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a
^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*
a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b
*c*d - 3*a*d^2)*sqrt(x) + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(
-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 -
756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2
*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^
5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d
^8))^(1/4)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401
*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^
3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c
^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*
c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(
1/4)*log(-(7*b*c*d - 3*a*d^2)*sqrt(x) - (b^2*c^4 - 2*a*b*c^3*d +
a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*
b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c
^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^
11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d
^7 + a^8*c^7*d^8))^(1/4)) + 4*d*sqrt(x))/(b*c^3 - a*c^2*d + (b*c^
2*d - a*c*d^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.334679, size = 909, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*sqrt(x)),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(
2)*a^3*d^2) + (a*b^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^
2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b^3)^(1/4)*b*ln(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^
2*b*c*d + sqrt(2)*a^3*d^2) - 1/2*(a*b^3)^(1/4)*b*ln(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a
^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3
)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^
2*c^2*d^2) - 1/4*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8
*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*ln(sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*
d + sqrt(2)*a^2*c^2*d^2) + 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(
1/4)*a*d)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(
```

```
2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/2*d*s
qrt(x)/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.477 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=570

$$-\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}(bc-ad)^2}-\frac{4bc-5ad}{2ac^2\sqrt{x}(bc-ad)}-\frac{d}{2c\sqrt{x}\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(4*b*c - 5*a*d)/(2*a*c^2*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*
d)*Sqrt[x]*(c + d*x^2)) + (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (b^(9/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c -
 a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) + (d^(5/4)*(9
*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(9/4)*(b*c - a*d)^2) - (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d
)^2) + (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^2) + (d^(5/4)*(9*b*c -
5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt
[2]*c^(9/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.69739, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}(bc-ad)^2}-\frac{4bc-5ad}{2ac^2\sqrt{x}(bc-ad)}-\frac{d}{2c\sqrt{x}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 5*a*d)/(2*a*c^2*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*
d)*Sqrt[x]*(c + d*x^2)) + (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
```

```
rt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (b^(9/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c -
 a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) + (d^(5/4)*(9
*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(9/4)*(b*c - a*d)^2) - (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d
)^2) + (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^2) + (d^(5/4)*(9*b*c -
5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt
[2]*c^(9/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.68955, size = 540, normalized size = 0.95

$$\frac{1}{16}\left(-\frac{4\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^2}+\frac{4\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^2}+\frac{8\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}(bc-ad)^2}-\frac{8\sqrt{2}b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}(bc-ad)^2}+\frac{\sqrt{2}d^{5/4}(9bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^2}+\frac{\sqrt{2}d^{5/4}(5ad-9bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^2}+\frac{2\sqrt{2}d^{5/4}(5ad-9bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}(bc-ad)^2}+\frac{2\sqrt{2}d^{5/4}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}(bc-ad)^2}+\frac{8d^2x^{3/2}}{c^2\left(c+dx^2\right)(bc-ad)}-\frac{32}{ac^2\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a*c^2*Sqrt[x]) + (8*d^2*x^(3/2))/(c^2*(b*c - a*d)*(c + d*x^
2)) + (8*Sqrt[2]*b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(a^(5/4)*(b*c - a*d)^2) - (8*Sqrt[2]*b^(9/4)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(b*c - a*d)^2) + (2*Sq
rt[2]*d^(5/4)*(-9*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^2) + (2*Sqrt[2]*d^(5/4)*(9*b*c
- 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*
(b*c - a*d)^2) - (4*Sqrt[2]*b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^2) + (4*Sqrt[
2]*b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(a^(5/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5/4)*(9*b*c - 5*a*d)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9
/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5/4)*(-9*b*c + 5*a*d)*Log[Sqrt[c
```

```
] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c -
 a*d)^2))/16
```

Maple [A] time = 0.029, size = 582, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^3/c^2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^
2*x^(3/2)/(d*x^2+c)*b-5/16*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2
)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-5/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4
)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-5/8*d^2/c^2/(a*
d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1)+9/16*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2)))+9/8*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)+9/8*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a/c^2/x^(1/2)-1/4*b^2/a/
(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2*b^
2/a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)-1/2*b^2/a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.8001, size = 4498, normalized size = 7.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/8*(16*b*c^2 - 16*a*c*d + 4*(4*b*c*d - 5*a*d^2)*x^2 + 16*(-b^9/
(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*
c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*
c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(a*b*c^4 - a^2*c^3*d
+ (a*b*c^3*d - a^2*c^2*d^2)*x^2)*sqrt(x)*arctan((a^4*b^6*c^6 - 6*
a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*
b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8 - 8*a^
6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^
4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*
c*d^7 + a^13*d^8))^(3/4)/(b^7*sqrt(x) + sqrt(b^14*x - (a^3*b^13*c
^4 - 4*a^4*b^12*c^3*d + 6*a^5*b^11*c^2*d^2 - 4*a^6*b^10*c*d^3 + a
^7*b^9*d^4)*sqrt(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6
*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*
c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))))) +
```

```
4*(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)*sqrt(x)*(
-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7
- 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28
*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 5
6*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8
*c^9*d^8))^(1/4)*arctan(-(b^6*c^13 - 6*a*b^5*c^12*d + 15*a^2*b^4*
c^11*d^2 - 20*a^3*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^4 - 6*a^5*b*c^8
*d^5 + a^6*c^7*d^6)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 1
2150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17
- 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70
*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8
*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4)/((729*b^3*c^3*d^4 - 1215*a*
b^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt(x) - sqrt((5314
41*b^6*c^6*d^8 - 1771470*a*b^5*c^5*d^9 + 2460375*a^2*b^4*c^4*d^10
 - 1822500*a^3*b^3*c^3*d^11 + 759375*a^4*b^2*c^2*d^12 - 168750*a^
5*b*c*d^13 + 15625*a^6*d^14)*x - (6561*b^8*c^13*d^5 - 40824*a*b^7
*c^12*d^6 + 109836*a^2*b^6*c^11*d^7 - 166824*a^3*b^5*c^10*d^8 + 1
56406*a^4*b^4*c^9*d^9 - 92680*a^5*b^3*c^8*d^10 + 33900*a^6*b^2*c^
7*d^11 - 7000*a^7*b*c^6*d^12 + 625*a^8*c^5*d^13)*sqrt(-(6561*b^4*
c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*
b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^
15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c
^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8)))
)) + 4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2
- 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 +
 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(a*b*c^4
 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)*sqrt(x)*log(b^7*sqr
t(x) + (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a
^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(
-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8
*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11
*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(3/4)) - 4*(-b^9/(a^5*
b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d
^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d
^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(a*b*c^4 - a^2*c^3*d + (a*
b*c^3*d - a^2*c^2*d^2)*x^2)*sqrt(x)*log(b^7*sqrt(x) - (a^4*b^6*c^
6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8
- 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*
a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a
^12*b*c*d^7 + a^13*d^8))^(3/4)) + (a*b*c^4 - a^2*c^3*d + (a*b*c^3
*d - a^2*c^2*d^2)*x^2)*sqrt(x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*
c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)
/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^
14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c
^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4)*log((b^6*c^13 -
6*a*b^5*c^12*d + 15*a^2*b^4*c^11*d^2 - 20*a^3*b^3*c^10*d^3 + 15*a
^4*b^2*c^9*d^4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6561*b^4*c^4*d
^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d
^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^
2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d
^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4)
 - (729*b^3*c^3*d^4 - 1215*a*b^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*
a^3*d^7)*sqrt(x)) - (a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d
^2)*x^2)*sqrt(x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 1215
0*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8
*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^
4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^
7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4)*log(-(b^6*c^13 - 6*a*b^5*c^12*
d + 15*a^2*b^4*c^11*d^2 - 20*a^3*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^
4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6561*b^4*c^4*d^5 - 14580*a*
b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*
d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^
5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b
^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4) - (729*b^3*c
^3*d^4 - 1215*a*b^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt
(x)))/((a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)*sqrt
(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A]   time = 0.357802, size = 979, normalized size = 1.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(3/2)),x, algorithm="giac")
```

```
[Out] 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2
)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5
*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) + 1/4*(9*(c*d^3
)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5*d - 2*sqrt(
2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) - 1/8*(9*(c*d^3)^(3/4)*b*c
- 5*(c*d^3)^(3/4)*a*d)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^
3*d^3) + 1/8*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*ln(-sqrt
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*s
qrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) - (a*b^3)^(3/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(
2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - (a*b^3)
^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4
*d^2) + 1/2*(a*b^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sq
rt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4
*d^2) - 1/2*(a*b^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^
4*d^2) - 1/2*(4*b*c*d*x^2 - 5*a*d^2*x^2 + 4*b*c^2 - 4*a*c*d)/((a*
b*c^3 - a^2*c^2*d)*(d*x^(5/2) + c*sqrt(x)))
```

3.478 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=570

$$\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}(bc-ad)^2}-\frac{4bc-7ad}{6ac^2x^{3/2}(bc-ad)}-\frac{d}{2cx^{3/2}\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(4*b*c - 7*a*d)/(6*a*c^2*(b*c - a*d)*x^(3/2)) - d/(2*c*(b*c - a*
d)*x^(3/2)*(c + d*x^2)) + (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c
 - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (d^(7/4
)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*
c - a*d)^2) - (b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(
11*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (d^(7/4)*(11*b*c
- 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

Rubi [A] time = 1.72427, antiderivative size = 570, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}(bc-ad)^2}-\frac{4bc-7ad}{6ac^2x^{3/2}(bc-ad)}-\frac{d}{2cx^{3/2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 7*a*d)/(6*a*c^2*(b*c - a*d)*x^(3/2)) - d/(2*c*(b*c - a*
d)*x^(3/2)*(c + d*x^2)) + (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
```

```
qrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c
 - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (d^(7/4
)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*
c - a*d)^2) - (b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(
11*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (d^(7/4)*(11*b*c
- 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.63879, size = 542, normalized size = 0.95

$$\frac{1}{48}\left(\frac{12\sqrt{2}b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^2}-\frac{12\sqrt{2}b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^2}+\frac{24\sqrt{2}b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^2}-\frac{24\sqrt{2}b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}(bc-ad)^2}+\frac{3\sqrt{2}d^{7/4}(7ad-11bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^2}+\frac{3\sqrt{2}d^{7/4}(11bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^2}+\frac{6\sqrt{2}d^{7/4}(7ad-11bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{11/4}(bc-ad)^2}+\frac{6\sqrt{2}d^{7/4}(11bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{11/4}(bc-ad)^2}+\frac{24d^2\sqrt{x}}{c^2\left(c+dx^2\right)(bc-ad)}-\frac{32}{ac^2x^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a*c^2*x^(3/2)) + (24*d^2*Sqrt[x])/(c^2*(b*c - a*d)*(c + d*x
^2)) + (24*Sqrt[2]*b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(a^(7/4)*(b*c - a*d)^2) - (24*Sqrt[2]*b^(11/4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(b*c - a*d)^2) +
(6*Sqrt[2]*d^(7/4)*(-11*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^2) + (6*Sqrt[2]*d^(7/4)*
(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
c^(11/4)*(b*c - a*d)^2) + (12*Sqrt[2]*b^(11/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^2)
 - (12*Sqrt[2]*b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^2) + (3*Sqrt[2]*d^(7/4)*(
-11*b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^2) + (3*Sqrt[2]*d^(7/4)*(11*b*c
 - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(c^(11/4)*(b*c - a*d)^2))/48
```

Maple [A] time = 0.028, size = 588, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^3/c^2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^
2*x^(1/2)/(d*x^2+c)*b-7/8*d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+11/8*d^2/c^2/(a*d-b*c)^2
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-7/8*
d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)-1)*a+11/8*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*d^3/c^3/(a*d-b*c)^2*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a+11/16*d^2/c^2/(a*d-b*
c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b-2/3/a/c^2/x
^(3/2)-1/4/a^2*b^3/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))-1/2/a^2*b^3/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a^2*b^3/(a*d-b*c)^2*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 120.048, size = 3791, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*b*c^2 - 16*a*c*d + 4*(4*b*c*d - 7*a*d^2)*x^2 - 48*(-b^1
1/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b
^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*
b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*((a*b*c^3*d - a^2
*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*sqrt(x)*arctan((-b^11/(a
^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c
^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*
c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*(a^2*b^2*c^2 - 2*a^3*
b*c*d + a^4*d^2)/(b^3*sqrt(x) + sqrt(b^6*x + (a^4*b^4*c^4 - 4*a^5
*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)*sqrt(-b
^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10
```

```
*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^1
3*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))))) - 12*((a*b*c^3*d -
 a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*sqrt(x)*(-(14641*b^4
*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^
3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b
^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*
b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*
d^8))^(1/4)*arctan(-(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(-(1464
1*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 150
92*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*
a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56
*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*
c^11*d^8))^(1/4)/((11*b*c*d^2 - 7*a*d^3)*sqrt(x) - sqrt((121*b^2*
c^2*d^4 - 154*a*b*c*d^5 + 49*a^2*d^6)*x + (b^4*c^10 - 4*a*b^3*c^9
*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b*c^7*d^3 + a^4*c^6*d^4)*sqrt(-(14
641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 1
5092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 2
8*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 -
56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^
8*c^11*d^8))))) + 12*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a
^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a
^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8
))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x
)*sqrt(x)*log(b^3*sqrt(x) + (-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d
 + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4
 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a
^15*d^8))^(1/4)*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)) - 12*(-b^1
1/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b
^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*
b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*((a*b*c^3*d - a^2
*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*sqrt(x)*log(b^3*sqrt(x)
- (-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56
*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 2
8*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*(a^2*b^2*c
^2 - 2*a^3*b*c*d + a^4*d^2)) + 3*((a*b*c^3*d - a^2*c^2*d^2)*x^3 +
 (a*b*c^4 - a^2*c^3*d)*x)*sqrt(x)*(-(14641*b^4*c^4*d^7 - 37268*a*
b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a
^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^
3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a
^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(1/4)*log(-(1
1*b*c*d^2 - 7*a*d^3)*sqrt(x) + (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d
^2)*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^
2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c
^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^
15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12
*d^7 + a^8*c^11*d^8))^(1/4)) - 3*((a*b*c^3*d - a^2*c^2*d^2)*x^3 +
 (a*b*c^4 - a^2*c^3*d)*x)*sqrt(x)*(-(14641*b^4*c^4*d^7 - 37268*a*
b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a
^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^
3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a
^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(1/4)*log(-(1
1*b*c*d^2 - 7*a*d^3)*sqrt(x) - (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d
^2)*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^
2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c
^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^
15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12
*d^7 + a^8*c^11*d^8))^(1/4)))/(((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (
a*b*c^4 - a^2*c^3*d)*x)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.341204, size = 969, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(5/2)),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d +
sqrt(2)*a^4*d^2) - (a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*s
qrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(a*b^3)^(1/4)*b^2*ln(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 -
 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/2*(a*b^3)^(1/4)*b^2*l
n(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*
c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/4*(11*(c*d^3)^(1
/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c
/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*
a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/4*(11*(c*d^3)^(1/4)*b*c*d -
7*(c*d^3)^(1/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d +
 sqrt(2)*a^2*c^3*d^2) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(
1/4)*a*d^2)*ln(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt
(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) - 1/8*(1
1*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*ln(-sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c
^4*d + sqrt(2)*a^2*c^3*d^2) + 1/2*d^2*sqrt(x)/((b*c^3 - a*c^2*d)*
(d*x^2 + c)) - 2/3/(a*c^2*x^(3/2))
```

3.479 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=618

$$\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^2}+\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}(bc-ad)^2}+\frac{-9a^2d^2+4abcd+4b^2c^2}{2a^2c^3\sqrt{x}(bc-ad)}-\frac{d^{9/4}(13bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^2}-\frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}(bc-ad)^2}-\frac{4bc-9ad}{10ac^2x^{5/2}(bc-ad)}-\frac{d}{2cx^{5/2}\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(4*b*c - 9*a*d)/(10*a*c^2*(b*c - a*d)*x^(5/2)) + (4*b^2*c^2 + 4*
a*b*c*d - 9*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*
c - a*d)*x^(5/2)*(c + d*x^2)) - (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (b^(13/
4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4
)*(b*c - a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) - (
d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (b^(13/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/
4)*(b*c - a*d)^2) - (b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (d^(
9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (d^(9/4)*(1
3*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2)
```

Rubi [A] time = 2.14629, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^2}+\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}(bc-ad)^2}+\frac{-9a^2d^2+4abcd+4b^2c^2}{2a^2c^3\sqrt{x}(bc-ad)}-\frac{d^{9/4}(13bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^2}-\frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}(bc-ad)^2}-\frac{4bc-9ad}{10ac^2x^{5/2}(bc-ad)}-\frac{d}{2cx^{5/2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 9*a*d)/(10*a*c^2*(b*c - a*d)*x^(5/2)) + (4*b^2*c^2 + 4*
a*b*c*d - 9*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*
```

```
c - a*d)*x^(5/2)*(c + d*x^2)) - (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (b^(13/
4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4
)*(b*c - a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) - (
d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (b^(13/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/
4)*(b*c - a*d)^2) - (b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (d^(
9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (d^(9/4)*(1
3*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)

[Out] Timed out
```

Mathematica [A] time = 1.51029, size = 563, normalized size = 0.91

$$\frac{1}{80}\left(\frac{20\sqrt{2}b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^2}-\frac{20\sqrt{2}b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^2}-\frac{40\sqrt{2}b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}(bc-ad)^2}+\frac{40\sqrt{2}b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}(bc-ad)^2}+\frac{160(2ad+bc)}{a^2c^3\sqrt{x}}+\frac{5\sqrt{2}d^{9/4}(9ad-13bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^2}+\frac{5\sqrt{2}d^{9/4}(13bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^2}+\frac{10\sqrt{2}d^{9/4}(13bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{13/4}(bc-ad)^2}+\frac{10\sqrt{2}d^{9/4}(9ad-13bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{13/4}(bc-ad)^2}-\frac{40d^3x^{3/2}}{c^3\left(c+dx^2\right)(bc-ad)}-\frac{32}{ac^2x^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]

[Out] (-32/(a*c^2*x^(5/2)) + (160*(b*c + 2*a*d))/(a^2*c^3*Sqrt[x]) - (4
0*d^3*x^(3/2))/(c^3*(b*c - a*d)*(c + d*x^2)) - (40*Sqrt[2]*b^(13/
4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c -
 a*d)^2) + (40*Sqrt[2]*b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(a^(9/4)*(b*c - a*d)^2) + (10*Sqrt[2]*d^(9/4)*(13*b
*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13
/4)*(b*c - a*d)^2) + (10*Sqrt[2]*d^(9/4)*(-13*b*c + 9*a*d)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)*(b*c - a*d)^2)
 + (20*Sqrt[2]*b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[x] + Sqrt[b]*x])/(a^(9/4)*(b*c - a*d)^2) - (20*Sqrt[2]*b^(13/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(
9/4)*(b*c - a*d)^2) + (5*Sqrt[2]*d^(9/4)*(-13*b*c + 9*a*d)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(
b*c - a*d)^2) + (5*Sqrt[2]*d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(b*c - a
*d)^2))/80
```

Maple [A] time = 0.033, size = 612, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^4/c^3/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a-1/2*d^3/c^2/(a*d-b*c)
^2*x^(3/2)/(d*x^2+c)*b+9/16*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/
2)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+9/8*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/
4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+9/8*d^3/c^3/(a
*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)-13/16*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))-13/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-13/8*d^2/c^2/(a*d-b*c)^2/(c/d)
^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c^2/
x^(5/2)+4/x^(1/2)/a/c^3*d+2/x^(1/2)/a^2/c^2*b+1/4*b^3/a^2/(a*d-b*
c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2*b^3/a^2/(
a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)+1/2*b^3/a^2/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 88.2567, size = 4652, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(7/2)),x, algorithm="fricas")
```

```
[Out] -1/40*(16*a*b*c^3 - 16*a^2*c^2*d - 20*(4*b^2*c^2*d + 4*a*b*c*d^2
- 9*a^2*d^3)*x^4 - 16*(5*b^2*c^3 + 4*a*b*c^2*d - 9*a^2*c*d^2)*x^2
 - 80*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^
2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d
^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^
2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(
x)*arctan((a^7*b^6*c^6 - 6*a^8*b^5*c^5*d + 15*a^9*b^4*c^4*d^2 - 2
0*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^2*d^4 - 6*a^12*b*c*d^5 + a^13*
```

d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)/(b^10*sqrt(x) + sqrt(b^20*x - (a^5*b^17*c^4 - 4*a^6*b^16*c^3*d + 6*a^7*b^15*c^2*d^2 - 4*a^8*b^14*c*d^3 + a^9*b^13*d^4)*sqrt(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))))) - 20*((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(x)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(1/4)*arctan(-(b^6*c^16 - 6*a*b^5*c^15*d + 15*a^2*b^4*c^14*d^2 - 20*a^3*b^3*c^13*d^3 + 15*a^4*b^2*c^12*d^4 - 6*a^5*b*c^11*d^5 + a^6*c^10*d^6)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(3/4)/((2197*b^3*c^3*d^7 - 4563*a*b^2*c^2*d^8 + 3159*a^2*b*c*d^9 - 729*a^3*d^10)*sqrt(x) - sqrt((4826809*b^6*c^6*d^14 - 20049822*a*b^5*c^5*d^15 + 34701615*a^2*b^4*c^4*d^16 - 32032260*a^3*b^3*c^3*d^17 + 16632135*a^4*b^2*c^2*d^18 - 4605822*a^5*b*c*d^19 + 531441*a^6*d^20)*x - (28561*b^8*c^15*d^9 - 193336*a*b^7*c^14*d^10 + 569868*a^2*b^6*c^13*d^11 - 955240*a^3*b^5*c^12*d^12 + 995926*a^4*b^4*c^11*d^13 - 661320*a^5*b^3*c^10*d^14 + 273132*a^6*b^2*c^9*d^15 - 64152*a^7*b*c^8*d^16 + 6561*a^8*c^7*d^17)*sqrt(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))))) - 20*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(x)*log(b^10*sqrt(x) + (a^7*b^6*c^6 - 6*a^8*b^5*c^5*d + 15*a^9*b^4*c^4*d^2 - 20*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^2*d^4 - 6*a^12*b*c*d^5 + a^13*d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)) + 20*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(x)*log(b^10*sqrt(x) - (a^7*b^6*c^6 - 6*a^8*b^5*c^5*d + 15*a^9*b^4*c^4*d^2 - 20*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^2*d^4 - 6*a^12*b*c*d^5 + a^13*d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)) - 5*((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(x)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(1/4)*log((b^6*c^16 - 6*a*b^5*c^15*d + 15*a^2*b^4*c^14*d^2 - 20*a^3*b^3*c^13*d^3 + 15*a^4*b^2*c^12*d^4 - 6*a^5*b*c^11*d^5 + a^6*c^10*d^6)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(3/4) - (2197*b^3*c^3*d^7 - 4563*a*b^2*c^2*d^8 + 3159*a^2*b*c*d^9 - 729*a^3*d^10)*sqrt(x)) + 5*((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(x)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(1/4)*log(-(b^6*c^16 - 6*a*b^5*c^15*d + 15*a^2*b^4*c^14*d^2 - 20*a^3*b^3*c^13*d^3 + 15*a^4*b^2*c^12*d^4 - 6*a^5*b*c^11*d^5 + a^6*c^10*d^6)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5

```
*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13
*d^8))^(3/4) - (2197*b^3*c^3*d^7 - 4563*a*b^2*c^2*d^8 + 3159*a^2*
b*c*d^9 - 729*a^3*d^10)*sqrt(x)))/(((a^2*b*c^4*d - a^3*c^3*d^2)*x
^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.368401, size = 965, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2*x^(7/2)),x, algorithm="giac")
```

```
[Out] -1/2*d^3*x^(3/2)/((b*c^4 - a*c^3*d)*(d*x^2 + c)) + (a*b^3)^(3/4)*
b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4
))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2)
+ (a*b^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sq
rt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d +
sqrt(2)*a^5*d^2) - 1/2*(a*b^3)^(3/4)*b*ln(sqrt(2)*sqrt(x)*(a/b)^(
1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d
+ sqrt(2)*a^5*d^2) + 1/2*(a*b^3)^(3/4)*b*ln(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c
*d + sqrt(2)*a^5*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4
*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) + 1/8*(13
*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*ln(sqrt(2)*sqrt(x)*(c/d
)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d +
 sqrt(2)*a^2*c^4*d^2) - 1/8*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/
4)*a*d)*ln(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)
*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) + 2/5*(5*b*
c*x^2 + 10*a*d*x^2 - a*c)/(a^2*c^3*x^(5/2))
```

3.480 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=631

$$-\frac{a^{5/4}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}-\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}-\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\sqrt{x}(bc-9ad)}{16d\left(c+dx^2\right)(bc-ad)^2}-\frac{c\sqrt{x}}{4d\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(c*Sqrt[x])/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 9*a*d)*Sqr
t[x])/(16*d*(b*c - a*d)^2*(c + d*x^2)) - (a^(5/4)*b^(3/4)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) +
 (a^(5/4)*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)
*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)
*d^(5/4)*(b*c - a*d)^3) - (a^(5/4)*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) +
 (a^(5/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 - 30*a*b*c*d
 - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2
 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.67614, antiderivative size = 631, normalized size of antiderivative = 1., number of

steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{a^{5/4}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}-\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}-\frac{(-5a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{(-5a^2d^2-30abcd+3b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\sqrt{x}(bc-9ad)}{16d(c+dx^2)(bc-ad)^2}-\frac{c\sqrt{x}}{4d(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(c*Sqrt[x])/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 9*a*d)*Sqr
t[x])/(16*d*(b*c - a*d)^2*(c + d*x^2)) - (a^(5/4)*b^(3/4)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) +
 (a^(5/4)*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)
*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)
*d^(5/4)*(b*c - a*d)^3) - (a^(5/4)*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) +
 (a^(5/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 - 30*a*b*c*d
 - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2
 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.05653, size = 640, normalized size = 1.01

$$\frac{-32\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+32\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*Sqrt[x] + 8*c^(3/4)*d^(1/4)*(b
*c - 9*a*d)*(b*c - a*d)*Sqrt[x]*(c + d*x^2) - 64*Sqrt[2]*a^(5/4)*
b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] + 64*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(
c + d*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sq
rt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(3*b^2*c^2 - 30
*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)] - 32*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(c
+ d*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x] + 32*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[
2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(3*b^2*
c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(128*c^(3/4)*d^(5/4)*(b*c
- a*d)^3*(c + d*x^2)^2)
```

Maple [A] time = 0.029, size = 839, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -9/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a^2*d^2+5/8/(a*d-b*c)^3/(d*
x^2+c)^2*x^(5/2)*c*a*b*d-1/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2
*c^2-5/16/(a*d-b*c)^3/(d*x^2+c)^2*c*d*x^(1/2)*a^2+1/8/(a*d-b*c)^3
/(d*x^2+c)^2*c^2*x^(1/2)*a*b+3/16/(a*d-b*c)^3/(d*x^2+c)^2*c^3/d*x
^(1/2)*b^2+5/64/(a*d-b*c)^3*d*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)*a^2+15/32/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-3/64/(a*d-b*c)^3/d*(c
/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/6
4/(a*d-b*c)^3*d*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)*a^2+15/32/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-3/64/(a*d-b*c)^3/d*(c/d)^(1/4)*c*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/(a*d-b*c)^3
*d*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+15/64/(a*
d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-3/128
/(a*d-b*c)^3/d*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b
^2-1/4*a*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))-1/2*a*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)-1/2*a*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 101.111, size = 6067, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] 1/64*(128*(-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*
b*c*d^11 + a^12*d^12))^(1/4)*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2
*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2
 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)*arctan(-(-a^5*b^3/(b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(a*b*sqrt(x) + s
qrt(a^2*b^2*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)
*sqrt(-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d
^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*
d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12))))) - 4*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3
 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2
*a*b*c^2*d^3 + a^2*c*d^4)*x^2)*(-(81*b^8*c^8 - 3240*a*b^7*c^7*d +
 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*
c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 + 15000
*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11*c^14*d^6 +
 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4*b^8*c^11*d
^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 792*a^7*b^5*c
^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a^10*b
^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))^(1/4)*arctan((
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*(-(81*
b^8*c^8 - 3240*a*b^7*c^7*d + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b
^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 13
3500*a^6*b^2*c^2*d^6 + 15000*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^1
5*d^5 - 12*a*b^11*c^14*d^6 + 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c
^12*d^8 + 495*a^4*b^8*c^11*d^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*
b^6*c^9*d^11 - 792*a^7*b^5*c^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*
a^9*b^3*c^6*d^14 + 66*a^10*b^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^
12*c^3*d^17))^(1/4)/((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*sqrt(x)
 - sqrt((9*b^4*c^4 - 180*a*b^3*c^3*d + 870*a^2*b^2*c^2*d^2 + 300*
a^3*b*c*d^3 + 25*a^4*d^4)*x + (b^6*c^8*d^2 - 6*a*b^5*c^7*d^3 + 15
*a^2*b^4*c^6*d^4 - 20*a^3*b^3*c^5*d^5 + 15*a^4*b^2*c^4*d^6 - 6*a^
5*b*c^3*d^7 + a^6*c^2*d^8)*sqrt(-(81*b^8*c^8 - 3240*a*b^7*c^7*d +
 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*
c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 + 15000
*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11*c^14*d^6 +
 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4*b^8*c^11*d
^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 792*a^7*b^5*c
^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a^10*b
^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))))) + 32*(-a^5*
b^3/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^
3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*
a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12))^(1/4)*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*
d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2)*log(a*b*sqrt(x) + (-a^5*b^3/(b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^3*c^3
- 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)) - 32*(-a^5*b^3/(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6
*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/
4)*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*
```

```
b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d
^4)*x^2)*log(a*b*sqrt(x) - (-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)) - (b^2*c^4*d - 2*a*b*c^3*d^2 +
a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*
c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)*(-(81*b^8*c^8 - 3240*a*
b^7*c^7*d + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 6493
50*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*
d^6 + 15000*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11
*c^14*d^6 + 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4
*b^8*c^11*d^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 79
2*a^7*b^5*c^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14
+ 66*a^10*b^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))^(1/
4)*log(-(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*sqrt(x) + (b^3*c^4*d
 - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*(-(81*b^8*c^8 -
 3240*a*b^7*c^7*d + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^
3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*
b^2*c^2*d^6 + 15000*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 1
2*a*b^11*c^14*d^6 + 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 +
 495*a^4*b^8*c^11*d^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d
^11 - 792*a^7*b^5*c^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c
^6*d^14 + 66*a^10*b^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^
17))^(1/4)) + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2
*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^
3 + a^2*c*d^4)*x^2)*(-(81*b^8*c^8 - 3240*a*b^7*c^7*d + 48060*a^2*
b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 5
13000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 + 15000*a^7*b*c*d^
7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11*c^14*d^6 + 66*a^2*b^1
0*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4*b^8*c^11*d^9 - 792*a^
5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 792*a^7*b^5*c^8*d^12 + 4
95*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a^10*b^2*c^5*d^15
 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))^(1/4)*log(-(3*b^2*c^2 - 3
0*a*b*c*d - 5*a^2*d^2)*sqrt(x) - (b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3
*a^2*b*c^2*d^3 - a^3*c*d^4)*(-(81*b^8*c^8 - 3240*a*b^7*c^7*d + 48
060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4
*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 + 15000*a^
7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11*c^14*d^6 + 66
*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4*b^8*c^11*d^9
- 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 792*a^7*b^5*c^8*
d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a^10*b^2*
c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))^(1/4)) - 4*(3*b*c
^2 + 5*a*c*d - (b*c*d - 9*a*d^2)*x^2)*sqrt(x))/(b^2*c^4*d - 2*a*b
*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^
4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.394611, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.481 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=628

$$-\frac{a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}$$
$$+\frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}$$
$$+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}$$
$$-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}$$
$$-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}$$
$$+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}$$
$$+\frac{x^{3/2}(3ad+5bc)}{16c\,(c+dx^2)\,(bc-ad)^2}+\frac{x^{3/2}}{4\,(c+dx^2)^2\,(bc-ad)}$$

```
[Out] x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)^2) + ((5*b*c + 3*a*d)*x^(3/2))
/(16*c*(b*c - a*d)^2*(c + d*x^2)) + (a^(3/4)*b^(5/4)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(
3/4)*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt
[2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3
/4)*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3
/4)*(b*c - a*d)^3) - (a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(
3/4)*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*
a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) - ((5*b^2*c^2 + 30
*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.65786, antiderivative size = 628, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}+\frac{x^{3/2}(3ad+5bc)}{16c\left(c+dx^2\right)(bc-ad)^2}+\frac{x^{3/2}}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)^2) + ((5*b*c + 3*a*d)*x^(3/2))
/(16*c*(b*c - a*d)^2*(c + d*x^2)) + (a^(3/4)*b^(5/4)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(
3/4)*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt
[2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3
/4)*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3
/4)*(b*c - a*d)^3) - (a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(
3/4)*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*
a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) - ((5*b^2*c^2 + 30
*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.66167, size = 544, normalized size = 0.87

$$-32\sqrt{2}a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+32\sqrt{2}a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+64\sqrt{2}a^{3/4}b^{5/4}\tan^{-1}\left(1-\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((32*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 + (8*(b*c - a*d)*(5*b*c
 + 3*a*d)*x^(3/2))/(c*(c + d*x^2)) + 64*Sqrt[2]*a^(3/4)*b^(5/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 64*Sqrt[2]*a^(3/4)
*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[
2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*d^(3/4)) + (2*Sqrt[2]*(5*b^2*c^2 +
 30*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(c^(5/4)*d^(3/4)) - 32*Sqrt[2]*a^(3/4)*b^(5/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 32*Sqrt[2]*a^(3
/4)*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x] + (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(5/4)*d^(3/4)
) - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(5/4)*d^(3/4)))/(
128*(b*c - a*d)^3)
```

Maple [A] time = 0.028, size = 839, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/(a*d-b*c)^3/(d*x^2+c)^2*d^3/c*x^(7/2)*a^2+1/8/(a*d-b*c)^3/(d
*x^2+c)^2*d^2*x^(7/2)*a*b-5/16/(a*d-b*c)^3/(d*x^2+c)^2*d*c*x^(7/2
)*b^2-1/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a^2*d^2+5/8/(a*d-b*c)^
3/(d*x^2+c)^2*x^(3/2)*c*a*b*d-9/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2
)*b^2*c^2+3/64/(a*d-b*c)^3/c*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a^2-15/32/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-5/64/(a*d-b*c)^3*c/d/(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64
/(a*d-b*c)^3/c*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a^2-15/32/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)*a*b-5/64/(a*d-b*c)^3*c/d/(c/d)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/(a*d-b*c)^3/
c*d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-15/64/(a*d
-b*c)^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-5/128/
(a*d-b*c)^3*c/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^
2+1/4*a*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
)))+1/2*a*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)+1/2*a*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 64.5506, size = 7329, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] 1/64*(128*(-a^3*b^5/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*
b*c*d^11 + a^12*d^12))^(1/4)*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2
 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d -
 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*arctan(-(b^9*c^9 - 9*a*b^8*c^8
*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^
4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7
 + 9*a^8*b*c*d^8 - a^9*d^9)*(-a^3*b^5/(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(3/4)/(a^2*b^4*sqrt(x) +
 sqrt(a^4*b^8*x - (a^3*b^11*c^6 - 6*a^4*b^10*c^5*d + 15*a^5*b^9*c
^4*d^2 - 20*a^6*b^8*c^3*d^3 + 15*a^7*b^7*c^2*d^4 - 6*a^8*b^6*c*d^
5 + a^9*b^5*d^6)*sqrt(-a^3*b^5/(b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12))))) + 4*(b^2*c^5 - 2*a*b*c^4*d +
 a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*
(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*(-(625*b^8*c^8 + 1
5000*a*b^7*c^7*d + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^
3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b
^2*c^2*d^6 - 3240*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^17*d^3 - 12*a
*b^11*c^16*d^4 + 66*a^2*b^10*c^15*d^5 - 220*a^3*b^9*c^14*d^6 + 49
5*a^4*b^8*c^13*d^7 - 792*a^5*b^7*c^12*d^8 + 924*a^6*b^6*c^11*d^9
- 792*a^7*b^5*c^10*d^10 + 495*a^8*b^4*c^9*d^11 - 220*a^9*b^3*c^8*
d^12 + 66*a^10*b^2*c^7*d^13 - 12*a^11*b*c^6*d^14 + a^12*c^5*d^15)
)^(1/4)*arctan((b^9*c^13*d^2 - 9*a*b^8*c^12*d^3 + 36*a^2*b^7*c^11
*d^4 - 84*a^3*b^6*c^10*d^5 + 126*a^4*b^5*c^9*d^6 - 126*a^5*b^4*c^
8*d^7 + 84*a^6*b^3*c^7*d^8 - 36*a^7*b^2*c^6*d^9 + 9*a^8*b*c^5*d^1
0 - a^9*c^4*d^11)*(-(625*b^8*c^8 + 15000*a*b^7*c^7*d + 133500*a^2
*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 -
307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 - 3240*a^7*b*c*d^7
 + 81*a^8*d^8)/(b^12*c^17*d^3 - 12*a*b^11*c^16*d^4 + 66*a^2*b^10*
c^15*d^5 - 220*a^3*b^9*c^14*d^6 + 495*a^4*b^8*c^13*d^7 - 792*a^5*
b^7*c^12*d^8 + 924*a^6*b^6*c^11*d^9 - 792*a^7*b^5*c^10*d^10 + 495
*a^8*b^4*c^9*d^11 - 220*a^9*b^3*c^8*d^12 + 66*a^10*b^2*c^7*d^13 -
 12*a^11*b*c^6*d^14 + a^12*c^5*d^15))^(3/4)/((125*b^6*c^6 + 2250*
a*b^5*c^5*d + 13275*a^2*b^4*c^4*d^2 + 24300*a^3*b^3*c^3*d^3 - 796
5*a^4*b^2*c^2*d^4 + 810*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x) - sqrt(
(15625*b^12*c^12 + 562500*a*b^11*c^11*d + 8381250*a^2*b^10*c^10*d
^2 + 65812500*a^3*b^9*c^9*d^3 + 283584375*a^4*b^8*c^8*d^4 + 60952
5000*a^5*b^7*c^7*d^5 + 382657500*a^6*b^6*c^6*d^6 - 365715000*a^7*
b^5*c^5*d^7 + 102090375*a^8*b^4*c^4*d^8 - 14215500*a^9*b^3*c^3*d^
9 + 1086210*a^10*b^2*c^2*d^10 - 43740*a^11*b*c*d^11 + 729*a^12*d^
12)*x - (625*b^14*c^17*d + 11250*a*b^13*c^16*d^2 + 52875*a^2*b^12
*c^15*d^3 - 75500*a^3*b^11*c^14*d^4 - 716775*a^4*b^10*c^13*d^5 +
1042350*a^5*b^9*c^12*d^6 + 3288235*a^6*b^8*c^11*d^7 - 10986600*a^
7*b^7*c^10*d^8 + 13692171*a^8*b^6*c^9*d^9 - 9010386*a^9*b^5*c^8*d
^10 + 3283065*a^10*b^4*c^7*d^11 - 646380*a^11*b^3*c^6*d^12 + 6871
5*a^12*b^2*c^5*d^13 - 3726*a^13*b*c^4*d^14 + 81*a^14*c^3*d^15)*sq
rt(-(625*b^8*c^8 + 15000*a*b^7*c^7*d + 133500*a^2*b^6*c^6*d^2 + 5
13000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c
^3*d^5 + 48060*a^6*b^2*c^2*d^6 - 3240*a^7*b*c*d^7 + 81*a^8*d^8)/(
b^12*c^17*d^3 - 12*a*b^11*c^16*d^4 + 66*a^2*b^10*c^15*d^5 - 220*a
^3*b^9*c^14*d^6 + 495*a^4*b^8*c^13*d^7 - 792*a^5*b^7*c^12*d^8 + 9
24*a^6*b^6*c^11*d^9 - 792*a^7*b^5*c^10*d^10 + 495*a^8*b^4*c^9*d^1
1 - 220*a^9*b^3*c^8*d^12 + 66*a^10*b^2*c^7*d^13 - 12*a^11*b*c^6*d
^14 + a^12*c^5*d^15))))) - 32*(-a^3*b^5/(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c
^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*
c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
```

```
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^2*c^5 - 2*a*b
*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*
x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*log(a^2*b^
4*sqrt(x) + (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^
6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*(-a
^3*b^5/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 92
4*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12))^(3/4)) + 32*(-a^3*b^5/(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^2*c^5 - 2*a*b*c^4*d +
 a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*
(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*log(a^2*b^4*sqrt(x
) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^
6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^
3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*(-a^3*b^5/(
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
)^(3/4)) - (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 -
2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a
^2*c^2*d^3)*x^2)*(-(625*b^8*c^8 + 15000*a*b^7*c^7*d + 133500*a^2*
b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 3
07800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 - 3240*a^7*b*c*d^7
+ 81*a^8*d^8)/(b^12*c^17*d^3 - 12*a*b^11*c^16*d^4 + 66*a^2*b^10*c
^15*d^5 - 220*a^3*b^9*c^14*d^6 + 495*a^4*b^8*c^13*d^7 - 792*a^5*b
^7*c^12*d^8 + 924*a^6*b^6*c^11*d^9 - 792*a^7*b^5*c^10*d^10 + 495*
a^8*b^4*c^9*d^11 - 220*a^9*b^3*c^8*d^12 + 66*a^10*b^2*c^7*d^13 -
12*a^11*b*c^6*d^14 + a^12*c^5*d^15))^(1/4)*log((b^9*c^13*d^2 - 9*
a*b^8*c^12*d^3 + 36*a^2*b^7*c^11*d^4 - 84*a^3*b^6*c^10*d^5 + 126*
a^4*b^5*c^9*d^6 - 126*a^5*b^4*c^8*d^7 + 84*a^6*b^3*c^7*d^8 - 36*a
^7*b^2*c^6*d^9 + 9*a^8*b*c^5*d^10 - a^9*c^4*d^11)*(-(625*b^8*c^8
+ 15000*a*b^7*c^7*d + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5
*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^
6*b^2*c^2*d^6 - 3240*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^17*d^3 - 1
2*a*b^11*c^16*d^4 + 66*a^2*b^10*c^15*d^5 - 220*a^3*b^9*c^14*d^6 +
 495*a^4*b^8*c^13*d^7 - 792*a^5*b^7*c^12*d^8 + 924*a^6*b^6*c^11*d
^9 - 792*a^7*b^5*c^10*d^10 + 495*a^8*b^4*c^9*d^11 - 220*a^9*b^3*c
^8*d^12 + 66*a^10*b^2*c^7*d^13 - 12*a^11*b*c^6*d^14 + a^12*c^5*d^
15))^(3/4) - (125*b^6*c^6 + 2250*a*b^5*c^5*d + 13275*a^2*b^4*c^4*
d^2 + 24300*a^3*b^3*c^3*d^3 - 7965*a^4*b^2*c^2*d^4 + 810*a^5*b*c*
d^5 - 27*a^6*d^6)*sqrt(x)) + (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2
 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d -
 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*(-(625*b^8*c^8 + 15000*a*b^7*c
^7*d + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a
^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 -
 3240*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^17*d^3 - 12*a*b^11*c^16*d
^4 + 66*a^2*b^10*c^15*d^5 - 220*a^3*b^9*c^14*d^6 + 495*a^4*b^8*c^
13*d^7 - 792*a^5*b^7*c^12*d^8 + 924*a^6*b^6*c^11*d^9 - 792*a^7*b^
5*c^10*d^10 + 495*a^8*b^4*c^9*d^11 - 220*a^9*b^3*c^8*d^12 + 66*a^
10*b^2*c^7*d^13 - 12*a^11*b*c^6*d^14 + a^12*c^5*d^15))^(1/4)*log(
-(b^9*c^13*d^2 - 9*a*b^8*c^12*d^3 + 36*a^2*b^7*c^11*d^4 - 84*a^3*
b^6*c^10*d^5 + 126*a^4*b^5*c^9*d^6 - 126*a^5*b^4*c^8*d^7 + 84*a^6
*b^3*c^7*d^8 - 36*a^7*b^2*c^6*d^9 + 9*a^8*b*c^5*d^10 - a^9*c^4*d^
11)*(-(625*b^8*c^8 + 15000*a*b^7*c^7*d + 133500*a^2*b^6*c^6*d^2 +
 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3
*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 - 3240*a^7*b*c*d^7 + 81*a^8*d^8)
/(b^12*c^17*d^3 - 12*a*b^11*c^16*d^4 + 66*a^2*b^10*c^15*d^5 - 220
*a^3*b^9*c^14*d^6 + 495*a^4*b^8*c^13*d^7 - 792*a^5*b^7*c^12*d^8 +
 924*a^6*b^6*c^11*d^9 - 792*a^7*b^5*c^10*d^10 + 495*a^8*b^4*c^9*d
^11 - 220*a^9*b^3*c^8*d^12 + 66*a^10*b^2*c^7*d^13 - 12*a^11*b*c^6
*d^14 + a^12*c^5*d^15))^(3/4) - (125*b^6*c^6 + 2250*a*b^5*c^5*d +
 13275*a^2*b^4*c^4*d^2 + 24300*a^3*b^3*c^3*d^3 - 7965*a^4*b^2*c^2
*d^4 + 810*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) + 4*((5*b*c*d + 3*a
*d^2)*x^3 + (9*b*c^2 - a*c*d)*x)*sqrt(x))/(b^2*c^5 - 2*a*b*c^4*d
+ a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2
*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.443891, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.482 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=627

$$-\frac{(-3a^2d^2+14abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{(-3a^2d^2+14abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}-\frac{(-3a^2d^2+14abcd+21b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{(-3a^2d^2+14abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{a}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{\sqrt[4]{a}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{\sqrt[4]{a}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{\sqrt[4]{a}b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}+\frac{\sqrt{x}(ad+7bc)}{16c(c+dx^2)(bc-ad)^2}+\frac{\sqrt{x}}{4(c+dx^2)^2(bc-ad)}$$

```
[Out] Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)^2) + ((7*b*c + a*d)*Sqrt[x])/(
16*c*(b*c - a*d)^2*(c + d*x^2)) + (a^(1/4)*b^(7/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(1/
4)*b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1/
4)*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1
/4)*(b*c - a*d)^3) + (a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - (a^(
1/4)*b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b*c*d - 3
*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*b^2*c^2 +
14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.49367, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{(-3a^2d^2+14abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{(-3a^2d^2+14abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}-\frac{(-3a^2d^2+14abcd+21b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{(-3a^2d^2+14abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{a}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{\sqrt[4]{a}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{\sqrt[4]{a}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{\sqrt[4]{a}b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}+\frac{\sqrt{x}(ad+7bc)}{16c(c+dx^2)(bc-ad)^2}+\frac{\sqrt{x}}{4(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)^2) + ((7*b*c + a*d)*Sqrt[x])/(
16*c*(b*c - a*d)^2*(c + d*x^2)) + (a^(1/4)*b^(7/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(1/
4)*b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2
]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1/
4)*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1
/4)*(b*c - a*d)^3) + (a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - (a^(
1/4)*b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b*c*d - 3
*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*b^2*c^2 +
14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.66039, size = 543, normalized size = 0.87

$$-\frac{\frac{\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}\sqrt[4]{d}}+\frac{\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}\sqrt[4]{d}}-\frac{2\sqrt{2}\left(-3a^2d^2+14abcd+21b\right.}{c^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((32*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 + (8*(b*c - a*d)*(7*b*c
 + a*d)*Sqrt[x])/(c*(c + d*x^2)) + 64*Sqrt[2]*a^(1/4)*b^(7/4)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 64*Sqrt[2]*a^(1/4)*b
^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]
*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(c^(7/4)*d^(1/4)) + (2*Sqrt[2]*(21*b^2*c^2 +
 14*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(c^(7/4)*d^(1/4)) + 32*Sqrt[2]*a^(1/4)*b^(7/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 32*Sqrt[2]*a^(1
/4)*b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x] - (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c
] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)*d^(1/4
)) + (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)*d^(1/4)))
/(128*(b*c - a*d)^3)
```

Maple [A] time = 0.027, size = 848, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/16/(a*d-b*c)^3/(d*x^2+c)^2*d^3/c*x^(5/2)*a^2+3/8/(a*d-b*c)^3/(d
*x^2+c)^2*d^2*x^(5/2)*a*b-7/16/(a*d-b*c)^3/(d*x^2+c)^2*d*c*x^(5/2
)*b^2+7/8/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*c*a*b*d-11/16/(a*d-b*c)
^3/(d*x^2+c)^2*x^(1/2)*b^2*c^2-3/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/
2)*a^2*d^2+3/64/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2-7/32/(a*d-b*c)^3/c*(c/d)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d-21/64/(a*d-b*c
)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
+3/64/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a^2*d^2-7/32/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*d-21/64/(a*d-b*c)^3*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/(a*
d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2*d
^2-7/64/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))*a*b*d-21/128/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4
)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2)))*b^2+1/4*b^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))+1/2*b^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.398723, size = 1, normalized size = 0.

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.483 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{dx^{3/2}(13bc-5ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(13*b*c - 5*a*d
)*x^(3/2))/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(9/4)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*
d)^3) + (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c
*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(3
2*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c -
a*d)^3) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^
2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) + (
d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c
 - a*d)^3)
```

Rubi [A] time = 1.77318, antiderivative size = 633, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{dx^{3/2}(13bc-5ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(13*b*c - 5*a*d
)*x^(3/2))/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(9/4)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*
d)^3) + (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c
*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(3
2*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c -
a*d)^3) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^
2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) + (
d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c
 - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.20567, size = 620, normalized size = 0.98

$$\frac{1}{128}\left(\frac{\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(ad-bc)^3}+\frac{\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^3}+\frac{2\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}(bc-ad)^3}-\frac{2\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}(bc-ad)^3}+\frac{32\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}(bc-ad)^3}+\frac{32\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}(ad-bc)^3}+\frac{64\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}(ad-bc)^3}-\frac{64\sqrt{2}b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}(ad-bc)^3}+\frac{8dx^{3/2}(5ad-13bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{32dx^{3/2}}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-32*d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + (8*d*(-13*b*c +
5*a*d)*x^(3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (64*Sqrt[2]*b^(
9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b
*c) + a*d)^3) - (64*Sqrt[2]*b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*d^(1/4)
*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(1/4)
*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(9/4
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^
(1/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(9/4)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(-(b*c) + a*d)^3)
+ (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt
[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(-(b
*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2
*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(c^(9/4)*(b*c - a*d)^3))/128
```

Maple [A] time = 0.027, size = 855, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 5/16*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^(7/2)*a^2-9/8*d^3/(a*d-b*c
)^3/(d*x^2+c)^2/c*x^(7/2)*a*b+13/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x
^(7/2)*b^2+9/16*d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x^(3/2)*a^2-13/8*d^
2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a*b+17/16*d/(a*d-b*c)^3/(d*x^2+
c)^2*c*x^(3/2)*b^2+5/64*d^2/(a*d-b*c)^3/c^2/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-9/32*d/(a*d-b*c)^3/c/(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+45/64/
(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*b^2+5/128*d^2/(a*d-b*c)^3/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2)))*a^2-9/64*d/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*ln(
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2
)*2^(1/2)+(c/d)^(1/2)))*a*b+45/128/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2
)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+5/64*d^2/(a*d-b*c)^3/c^2/(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-9/32*d/(a*
d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)*a*b+45/64/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)*b^2-1/4*b^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln
((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2)))-1/2*b^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^2/(a*d-b*c)^3/(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 83.491, size = 7324, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] -1/64*(128*(-b^9/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^
7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b
^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^1
2*b*c*d^11 + a^13*d^12))^(1/4)*(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d
^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5
*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)*arctan(-(a*b^9*c^9 - 9*a^2
*b^8*c^8*d + 36*a^3*b^7*c^7*d^2 - 84*a^4*b^6*c^6*d^3 + 126*a^5*b^
5*c^5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a^7*b^3*c^3*d^6 - 36*a^8*b^2
*c^2*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-b^9/(a*b^12*c^12 - 12*a^2*
b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^
5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a
^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*
a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(3/4)/(b^7*sqr
t(x) + sqrt(b^14*x - (a*b^15*c^6 - 6*a^2*b^14*c^5*d + 15*a^3*b^13
*c^4*d^2 - 20*a^4*b^12*c^3*d^3 + 15*a^5*b^11*c^2*d^4 - 6*a^6*b^10
*c*d^5 + a^7*b^9*d^6)*sqrt(-b^9/(a*b^12*c^12 - 12*a^2*b^11*c^11*d
 + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d
^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*
d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^
2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))))) - 4*(b^2*c^6 - 2*a*b*c
^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*
x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)*(-(4100625
*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 32
36760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*
c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8 + 625*a^8*d^9)
/(b^12*c^21 - 12*a*b^11*c^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b
^9*c^18*d^3 + 495*a^4*b^8*c^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a
^6*b^6*c^15*d^6 - 792*a^7*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 - 2
20*a^9*b^3*c^12*d^9 + 66*a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11
 + a^12*c^9*d^12))^(1/4)*arctan(-(b^9*c^16 - 9*a*b^8*c^15*d + 36*
a^2*b^7*c^14*d^2 - 84*a^3*b^6*c^13*d^3 + 126*a^4*b^5*c^12*d^4 - 1
26*a^5*b^4*c^11*d^5 + 84*a^6*b^3*c^10*d^6 - 36*a^7*b^2*c^9*d^7 +
9*a^8*b*c^8*d^8 - a^9*c^7*d^9)*(-(4100625*b^8*c^8*d - 6561000*a*b
^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 +
1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*
c^2*d^7 - 9000*a^7*b*c*d^8 + 625*a^8*d^9)/(b^12*c^21 - 12*a*b^11*
c^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495*a^4*b^
8*c^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^
7*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d^9 + 66
*a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11 + a^12*c^9*d^12))^(3/4)
/((91125*b^6*c^6*d - 109350*a*b^5*c^5*d^2 + 74115*a^2*b^4*c^4*d^3
 - 30132*a^3*b^3*c^3*d^4 + 8235*a^4*b^2*c^2*d^5 - 1350*a^5*b*c*d^
6 + 125*a^6*d^7)*sqrt(x) + sqrt((8303765625*b^12*c^12*d^2 - 19929
037500*a*b^11*c^11*d^3 + 25464881250*a^2*b^10*c^10*d^4 - 21700507
500*a^3*b^9*c^9*d^5 + 13583730375*a^4*b^8*c^8*d^6 - 6513498360*a^
5*b^7*c^7*d^7 + 2446637724*a^6*b^6*c^6*d^8 - 723722040*a^7*b^5*c^
5*d^9 + 167700375*a^8*b^4*c^4*d^10 - 29767500*a^9*b^3*c^3*d^11 +
3881250*a^10*b^2*c^2*d^12 - 337500*a^11*b*c*d^13 + 15625*a^12*d^1
4)*x - (4100625*b^14*c^19*d - 31164750*a*b^13*c^18*d^2 + 10663447
5*a^2*b^12*c^17*d^3 - 218218860*a^3*b^11*c^16*d^4 + 299819961*a^4
*b^10*c^15*d^5 - 294812946*a^5*b^9*c^14*d^6 + 216070155*a^6*b^8*c
^13*d^7 - 121167720*a^7*b^7*c^12*d^8 + 52746475*a^8*b^6*c^11*d^9
- 17893266*a^9*b^5*c^10*d^10 + 4697241*a^10*b^4*c^9*d^11 - 933740
*a^11*b^3*c^8*d^12 + 134475*a^12*b^2*c^7*d^13 - 12750*a^13*b*c^6*
d^14 + 625*a^14*c^5*d^15)*sqrt(-(4100625*b^8*c^8*d - 6561000*a*b^
7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1
283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c
^2*d^7 - 9000*a^7*b*c*d^8 + 625*a^8*d^9)/(b^12*c^21 - 12*a*b^11*c
^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495*a^4*b^8
*c^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^7
*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d^9 + 66*
a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11 + a^12*c^9*d^12))))) - 3
2*(-b^9/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2
- 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^
8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^1
1 + a^13*d^12))^(1/4)*(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2
*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*
```

```
b*c^4*d^2 + a^2*c^3*d^3)*x^2)*log(b^7*sqrt(x) + (a*b^9*c^9 - 9*a^
2*b^8*c^8*d + 36*a^3*b^7*c^7*d^2 - 84*a^4*b^6*c^6*d^3 + 126*a^5*b
^5*c^5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a^7*b^3*c^3*d^6 - 36*a^8*b^
2*c^2*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-b^9/(a*b^12*c^12 - 12*a^2
*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a
^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*
a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66
*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(3/4)) + 32*(
-b^9/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 2
20*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 +
924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 -
 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 +
 a^13*d^12))^(1/4)*(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^
4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c
^4*d^2 + a^2*c^3*d^3)*x^2)*log(b^7*sqrt(x) - (a*b^9*c^9 - 9*a^2*b
^8*c^8*d + 36*a^3*b^7*c^7*d^2 - 84*a^4*b^6*c^6*d^3 + 126*a^5*b^5*
c^5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a^7*b^3*c^3*d^6 - 36*a^8*b^2*c
^2*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-b^9/(a*b^12*c^12 - 12*a^2*b^
11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*
b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8
*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^
11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(3/4)) + (b^2*c^
6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^
2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
*(-(4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c
^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 3596
40*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8 + 6
25*a^8*d^9)/(b^12*c^21 - 12*a*b^11*c^20*d + 66*a^2*b^10*c^19*d^2
- 220*a^3*b^9*c^18*d^3 + 495*a^4*b^8*c^17*d^4 - 792*a^5*b^7*c^16*
d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^7*b^5*c^14*d^7 + 495*a^8*b^4*c
^13*d^8 - 220*a^9*b^3*c^12*d^9 + 66*a^10*b^2*c^11*d^10 - 12*a^11*
b*c^10*d^11 + a^12*c^9*d^12))^(1/4)*log((b^9*c^16 - 9*a*b^8*c^15*
d + 36*a^2*b^7*c^14*d^2 - 84*a^3*b^6*c^13*d^3 + 126*a^4*b^5*c^12*
d^4 - 126*a^5*b^4*c^11*d^5 + 84*a^6*b^3*c^10*d^6 - 36*a^7*b^2*c^9
*d^7 + 9*a^8*b*c^8*d^8 - a^9*c^7*d^9)*(-(4100625*b^8*c^8*d - 6561
000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5
*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a
^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8 + 625*a^8*d^9)/(b^12*c^21 - 12*
a*b^11*c^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495
*a^4*b^8*c^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15*d^6 -
 792*a^7*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d
^9 + 66*a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11 + a^12*c^9*d^12)
)^(3/4) + (91125*b^6*c^6*d - 109350*a*b^5*c^5*d^2 + 74115*a^2*b^4
*c^4*d^3 - 30132*a^3*b^3*c^3*d^4 + 8235*a^4*b^2*c^2*d^5 - 1350*a^
5*b*c*d^6 + 125*a^6*d^7)*sqrt(x)) - (b^2*c^6 - 2*a*b*c^5*d + a^2*
c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^
2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)*(-(4100625*b^8*c^8*d
- 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b
^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 7
1100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8 + 625*a^8*d^9)/(b^12*c^21
 - 12*a*b^11*c^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3
 + 495*a^4*b^8*c^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15
*d^6 - 792*a^7*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*
c^12*d^9 + 66*a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11 + a^12*c^9
*d^12))^(1/4)*log(-(b^9*c^16 - 9*a*b^8*c^15*d + 36*a^2*b^7*c^14*d
^2 - 84*a^3*b^6*c^13*d^3 + 126*a^4*b^5*c^12*d^4 - 126*a^5*b^4*c^1
1*d^5 + 84*a^6*b^3*c^10*d^6 - 36*a^7*b^2*c^9*d^7 + 9*a^8*b*c^8*d^
8 - a^9*c^7*d^9)*(-(4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5
759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^
4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000
*a^7*b*c*d^8 + 625*a^8*d^9)/(b^12*c^21 - 12*a*b^11*c^20*d + 66*a^
2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495*a^4*b^8*c^17*d^4 - 7
92*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^7*b^5*c^14*d^7
 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d^9 + 66*a^10*b^2*c^11
*d^10 - 12*a^11*b*c^10*d^11 + a^12*c^9*d^12))^(3/4) + (91125*b^6*
c^6*d - 109350*a*b^5*c^5*d^2 + 74115*a^2*b^4*c^4*d^3 - 30132*a^3*
b^3*c^3*d^4 + 8235*a^4*b^2*c^2*d^5 - 1350*a^5*b*c*d^6 + 125*a^6*d
^7)*sqrt(x)) + 4*((13*b*c*d^2 - 5*a*d^3)*x^3 + (17*b*c^2*d - 9*a*
c*d^2)*x)*sqrt(x))/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^
4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c
^4*d^2 + a^2*c^3*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.426414, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.484 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^3}-\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}+\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d\sqrt{x}(15bc-7ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d\sqrt{x}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*Sqrt[x])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(15*b*c - 7*a*d
)*Sqrt[x])/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(11/4)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a
*d)^3) + (b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(77*b^2*c^2 - 66*a*b*
c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a
*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (b^(11/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*
(b*c - a*d)^3) + (b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4
)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b*c - a
*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c
^(11/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.73775, antiderivative size = 633, normalized size of antiderivative = 1., number of

steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^3}-\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}+\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d\sqrt{x}(15bc-7ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d\sqrt{x}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*Sqrt[x])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(15*b*c - 7*a*d
)*Sqrt[x])/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(11/4)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a
*d)^3) + (b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(77*b^2*c^2 - 66*a*b*
c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a
*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (b^(11/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*
(b*c - a*d)^3) + (b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4
)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b*c - a
*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c
^(11/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.68668, size = 620, normalized size = 0.98

$$\frac{1}{128}\left(\frac{32\sqrt{2}b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}(ad-bc)^3}+\frac{32\sqrt{2}b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}(bc-ad)^3}+\frac{64\sqrt{2}b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}(ad-bc)^3}-\frac{64\sqrt{2}b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}(ad-bc)^3}+\frac{\sqrt{2}d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^3}+\frac{\sqrt{2}d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(ad-bc)^3}+\frac{2\sqrt{2}d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{11/4}(bc-ad)^3}-\frac{2\sqrt{2}d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{11/4}(bc-ad)^3}+\frac{8d\sqrt{x}(7ad-15bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{32d\sqrt{x}}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-32*d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)^2) + (8*d*(-15*b*c +
7*a*d)*Sqrt[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (64*Sqrt[2]*b^(
11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(
b*c) + a*d)^3) - (64*Sqrt[2]*b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*d^(3/
4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(
3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) + (32*Sqrt[2]*
b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(a^(3/4)*(-(b*c) + a*d)^3) + (32*Sqrt[2]*b^(11/4)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*(b*c -
a*d)^3) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(
11/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d
+ 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(c^(11/4)*(-(b*c) + a*d)^3))/128
```

Maple [A] time = 0.027, size = 882, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 7/16*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^(5/2)*a^2-11/8*d^3/(a*d-b*
c)^3/(d*x^2+c)^2/c*x^(5/2)*a*b+15/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*
x^(5/2)*b^2+11/16*d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x^(1/2)*a^2-15/8*
d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*a*b+19/16*d/(a*d-b*c)^3/(d*x^
2+c)^2*c*x^(1/2)*b^2+21/64*d^3/(a*d-b*c)^3/c^3*(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-33/32*d^2/(a*d-b*c)^3
/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*
```

```
b+77/64*d/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)*b^2+21/64*d^3/(a*d-b*c)^3/c^3*(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-33/32*d^2/(a*d-b*c)^3
/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*
b+77/64*d/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)*b^2+21/128*d^3/(a*d-b*c)^3/c^3*(c/d)^(1/4)*2^(1/
2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-33/64*d^2/(a*d-b*c)^3/c^2*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+77/128*d/(a*d-b*c)^
3/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2-1/4*b^3/(a
*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2*b^
3/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)-1/2*b^3/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*sqrt(x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.394472, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*sqrt(x)),x, algorithm="giac")
```

```
[Out] Done
```

3.485 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=681

$$-\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}$$
$$+\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}$$
$$+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}$$
$$-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}$$
$$-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}$$
$$+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{45a^2d^2-85abcd+32b^2c^2}{16ac^3\sqrt{x}(bc-ad)^2}$$
$$-\frac{d(17bc-9ad)}{16c^2\sqrt{x}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\sqrt{x}\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(32*b^2*c^2 - 85*a*b*c*d + 45*a^2*d^2)/(16*a*c^3*(b*c - a*d)^2*S
qrt[x]) - d/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) - (d*(17*b*c
- 9*a*d))/(16*c^2*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(
b*c - a*d)^3) - (b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 -
 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*
c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) + (b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3
) + (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
3/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^
2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^3)
```

Rubi [A] time = 2.27686, antiderivative size = 681, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$-\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{45a^2d^2-85abcd+32b^2c^2}{16ac^3\sqrt{x}(bc-ad)^2}-\frac{d(17bc-9ad)}{16c^2\sqrt{x}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\sqrt{x}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 85*a*b*c*d + 45*a^2*d^2)/(16*a*c^3*(b*c - a*d)^2*S
qrt[x]) - d/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) - (d*(17*b*c
- 9*a*d))/(16*c^2*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(
b*c - a*d)^3) - (b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 -
 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*
c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) + (b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3
) + (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
3/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^
2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.81875, size = 637, normalized size = 0.94

$$\frac{1}{128}\left(\frac{32\sqrt{2}b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(ad-bc)^3}+\frac{32\sqrt{2}b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^3}\right.$$
$$-\frac{64\sqrt{2}b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}(ad-bc)^3}+\frac{64\sqrt{2}b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}(ad-bc)^3}$$
$$+\frac{\sqrt{2}d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^3}$$
$$+\frac{\sqrt{2}d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(ad-bc)^3}$$
$$-\frac{2\sqrt{2}d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{13/4}(bc-ad)^3}$$
$$+\frac{2\sqrt{2}d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{13/4}(bc-ad)^3}$$
$$\left.+\frac{8d^2x^{3/2}(21bc-13ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{32d^2x^{3/2}}{c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{256}{ac^3\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a*c^3*Sqrt[x]) + (32*d^2*x^(3/2))/(c^2*(b*c - a*d)*(c + d*
x^2)^2) + (8*d^2*(21*b*c - 13*a*d)*x^(3/2))/(c^3*(b*c - a*d)^2*(c
 + d*x^2)) - (64*Sqrt[2]*b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(a^(5/4)*(-(b*c) + a*d)^3) + (64*Sqrt[2]*b^(13/4)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-(b*c)
+ a*d)^3) - (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^
2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)*(
b*c - a*d)^3) + (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 4
5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/
4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(-(b*c) + a*d)^3) +
(32*Sqrt[2]*b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(5/4)*(117*b
^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(b*c - a*d)^3) + (Sqrt[2]
*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(-(b*c) +
a*d)^3))/128
```

Maple [A] time = 0.033, size = 900, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -13/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a^2+17/8*d^4/c^2/(
a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a*b-21/16*d^3/c/(a*d-b*c)^3/(d*x^2
+c)^2*x^(7/2)*b^2-17/16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a
^2+21/8*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a*b-25/16*d^2/(a*d-
b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2-45/128*d^3/c^3/(a*d-b*c)^3/(c/d)^(
1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
```

```
x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-45/64*d^3/c^3/(a*d-b*
c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)-45/64*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)+65/64*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^
(1/2)*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+65/32*d^2/c^2/(a*d-b*c)^3/(c
/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+65/32
*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)-117/128*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2
*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))-117/64*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^
(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-117/64*d/c/(a*d-b
*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)-2/a/c^3/x^(1/2)+1/4*b^3/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))+1/2*b^3/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^3/a/(a*d-b*c)^3/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.420483, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(3/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.486 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=681

$$\frac{b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{15/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3}-\frac{77a^2d^2-133abcd+32b^2c^2}{48ac^3x^{3/2}(bc-ad)^2}-\frac{d(19bc-11ad)}{16c^2x^{3/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(32*b^2*c^2 - 133*a*b*c*d + 77*a^2*d^2)/(48*a*c^3*(b*c - a*d)^2*
x^(3/2)) - d/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) - (d*(19*b*c
 - 11*a*d))/(16*c^2*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + (b^(15/4
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)
*(b*c - a*d)^3) - (b^(15/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2
 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^
2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)
^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*
a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^3)
```

Rubi [A] time = 2.01855, antiderivative size = 681, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458

$$\frac{b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{15/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3}-\frac{77a^2d^2-133abcd+32b^2c^2}{48ac^3x^{3/2}(bc-ad)^2}-\frac{d(19bc-11ad)}{16c^2x^{3/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 133*a*b*c*d + 77*a^2*d^2)/(48*a*c^3*(b*c - a*d)^2*
x^(3/2)) - d/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) - (d*(19*b*c
 - 11*a*d))/(16*c^2*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + (b^(15/4
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)
*(b*c - a*d)^3) - (b^(15/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2
 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^
2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)
^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*
a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.61724, size = 639, normalized size = 0.94

$$\frac{1}{384}\left(\frac{96\sqrt{2}b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^3}+\frac{96\sqrt{2}b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(ad-bc)^3}-\frac{192\sqrt{2}b^{15/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}(ad-bc)^3}+\frac{192\sqrt{2}b^{15/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}(ad-bc)^3}+\frac{3\sqrt{2}d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{15/4}(ad-bc)^3}+\frac{3\sqrt{2}d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{15/4}(bc-ad)^3}-\frac{6\sqrt{2}d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{15/4}(bc-ad)^3}+\frac{6\sqrt{2}d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{15/4}(bc-ad)^3}+\frac{24d^2\sqrt{x}(23bc-15ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{96d^2\sqrt{x}}{c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{256}{ac^3x^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a*c^3*x^(3/2)) + (96*d^2*Sqrt[x])/(c^2*(b*c - a*d)*(c + d*
x^2)^2) + (24*d^2*(23*b*c - 15*a*d)*Sqrt[x])/(c^3*(b*c - a*d)^2*(
c + d*x^2)) - (192*Sqrt[2]*b^(15/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) + (192*Sqrt[2]*b^(15
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(-(b*
c) + a*d)^3) - (6*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77
*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(15/4
)*(b*c - a*d)^3) + (6*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d
+ 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(
15/4)*(b*c - a*d)^3) + (96*Sqrt[2]*b^(15/4)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^3) +
(96*Sqrt[2]*b^(15/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(a^(7/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]*d^(7/4)*(
165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(15/4)*(-(b*c) + a*d)^3) +
(3*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(15/4)*
(b*c - a*d)^3))/384
```

Maple [A] time = 0.035, size = 906, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -15/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a^2+19/8*d^4/c^2/(
a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a*b-23/16*d^3/c/(a*d-b*c)^3/(d*x^2
+c)^2*x^(5/2)*b^2-19/16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*a
^2+23/8*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*a*b-27/16*d^2/(a*d-
b*c)^3/(d*x^2+c)^2*x^(1/2)*b^2-77/64*d^4/c^4/(a*d-b*c)^3*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+105/32*d^3/
```

```
c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*a*b-165/64*d^2/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/128*d^4/c^4/(a*d-b*c)^3*(
c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+105/64*d^3/c^3/
(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-16
5/128*d^2/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2)))*b^2-77/64*d^4/c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+105/32*d^3/c^3/(a*d-b*c)^3*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-165/64
*d^2/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)*b^2-2/3/a/c^3/x^(3/2)+1/4/a^2*b^4/(a*d-b*c)^3*(a/b)
^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+1/2/a^2*b^4/(a*d-b*c)^3
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/a^
2*b^4/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(5/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.402415, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(5/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.487 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=743

$$\frac{b^{17/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{17/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3}-\frac{117a^2d^2-189abcd+32b^2c^2}{80ac^3x^{5/2}(bc-ad)^2}+\frac{117a^3d^3-189a^2bcd^2+32ab^2c^2d+32b^3c^3}{16a^2c^4\sqrt{x}(bc-ad)^2}-\frac{d(21bc-13ad)}{16c^2x^{5/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(32*b^2*c^2 - 189*a*b*c*d + 117*a^2*d^2)/(80*a*c^3*(b*c - a*d)^2
*x^(5/2)) + (32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2 + 117*
a^3*d^3)/(16*a^2*c^4*(b*c - a*d)^2*Sqrt[x]) - d/(4*c*(b*c - a*d)*
x^(5/2)*(c + d*x^2)^2) - (d*(21*b*c - 13*a*d))/(16*c^2*(b*c - a*d
)^2*x^(5/2)*(c + d*x^2)) - (b^(17/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(17/4)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/
4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2
*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*
c^(17/4)*(b*c - a*d)^3) + (b^(17/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3)
- (b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2
- 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (d^
(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(
b*c - a*d)^3)
```

Rubi [A] time = 2.75816, antiderivative size = 743, normalized size of antiderivative = 1., number of

steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$\frac{b^{17/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{17/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3}-\frac{117a^2d^2-189abcd+32b^2c^2}{80ac^3x^{5/2}(bc-ad)^2}+\frac{117a^3d^3-189a^2bcd^2+32ab^2c^2d+32b^3c^3}{16a^2c^4\sqrt{x}(bc-ad)^2}-\frac{d(21bc-13ad)}{16c^2x^{5/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 189*a*b*c*d + 117*a^2*d^2)/(80*a*c^3*(b*c - a*d)^2
*x^(5/2)) + (32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2 + 117*
a^3*d^3)/(16*a^2*c^4*(b*c - a*d)^2*Sqrt[x]) - d/(4*c*(b*c - a*d)*
x^(5/2)*(c + d*x^2)^2) - (d*(21*b*c - 13*a*d))/(16*c^2*(b*c - a*d
)^2*x^(5/2)*(c + d*x^2)) - (b^(17/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(17/4)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/
4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2
*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*
c^(17/4)*(b*c - a*d)^3) + (b^(17/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3)
- (b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2
- 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (d^
(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(
b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.25835, size = 660, normalized size = 0.89

$$\frac{1}{640}\left(\frac{160\sqrt{2}b^{17/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^3}+\frac{160\sqrt{2}b^{17/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(ad-bc)^3}+\frac{320\sqrt{2}b^{17/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}(ad-bc)^3}-\frac{320\sqrt{2}b^{17/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}(ad-bc)^3}+\frac{5\sqrt{2}d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{17/4}(ad-bc)^3}+\frac{5\sqrt{2}d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{17/4}(bc-ad)^3}+\frac{10\sqrt{2}d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{17/4}(bc-ad)^3}-\frac{10\sqrt{2}d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{17/4}(bc-ad)^3}+\frac{1280(3ad+bc)}{a^2c^4\sqrt{x}}+\frac{40d^3x^{3/2}(21ad-29bc)}{c^4\left(c+dx^2\right)(bc-ad)^2}-\frac{160d^3x^{3/2}}{c^3\left(c+dx^2\right)^2(bc-ad)}-\frac{256}{ac^3x^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a*c^3*x^(5/2)) + (1280*(b*c + 3*a*d))/(a^2*c^4*Sqrt[x]) -
(160*d^3*x^(3/2))/(c^3*(b*c - a*d)*(c + d*x^2)^2) + (40*d^3*(-29*
b*c + 21*a*d)*x^(3/2))/(c^4*(b*c - a*d)^2*(c + d*x^2)) + (320*Sqr
t[2]*b^(17/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(
9/4)*(-(b*c) + a*d)^3) - (320*Sqrt[2]*b^(17/4)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(-(b*c) + a*d)^3) + (10*Sqr
t[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(17/4)*(b*c - a*d)^3) - (
10*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcT
an[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(17/4)*(b*c - a*d)^
3) + (160*Sqrt[2]*b^(17/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(a^(9/4)*(b*c - a*d)^3) + (160*Sqrt[2]*b^(1
7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(a^(9/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 30
6*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(c^(17/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9
/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(17/4)*(b*c - a*d)^3)
)/640
```

Maple [A] time = 0.038, size = 933, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 21/16*d^6/c^4/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a^2-25/8*d^5/c^3/(a
*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a*b+29/16*d^4/c^2/(a*d-b*c)^3/(d*x^
```

```
2+c)^2*x^(7/2)*b^2+25/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*
a^2-29/8*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a*b+33/16*d^3/c/
(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2+117/128*d^4/c^4/(a*d-b*c)^3/(
c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+117/64*d^4/c^4/
(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)+117/64*d^4/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-153/64*d^3/c^3/(a*d-b*c)^3/(c/d)
^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-153/32*d^3/c^3/(a*d
-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)-153/32*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)+221/128*d^2/c^2/(a*d-b*c)^3/(c/d)^(1
/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+221/64*d^2/c^2/(a*d-b*
c)^3/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)+221/64*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c^3/x^(5/2)+6/x^(1/2)/a/c^4*d+2/x
^(1/2)/a^2/c^3*b-1/4*b^4/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))-1/2*b^4/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^4/a^2/(a*d-b*c)^3/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(7/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(7/2)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.456854, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3*x^(7/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.488 $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=624

$$\frac{a\sqrt{x}}{2b\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{\sqrt{x}(ad+bc)}{2b\,(c+dx^2)\,(bc-ad)^2}+\frac{\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{a}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{c}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{c}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)^3}$$

```
[Out] ((b*c + a*d)*Sqrt[x])/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*Sqrt[x
])/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (a^(1/4)*(5*b*c +
3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*
b^(1/4)*(b*c - a*d)^3) - (a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3)
 - (c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^(1/4)*(3*b*c + 5
*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d
^(1/4)*(b*c - a*d)^3) + (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b
*c - a*d)^3) - (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b*c - a*d)
^3) - (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^
(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.60135, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{a\sqrt{x}}{2b\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{\sqrt{x}(ad+bc)}{2b\,(c+dx^2)\,(bc-ad)^2}+\frac{\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{a}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{c}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{c}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)^3}+\frac{\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*c + a*d)*Sqrt[x])/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*Sqrt[x
])/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (a^(1/4)*(5*b*c +
3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*
```

```
b^(1/4)*(b*c - a*d)^3) - (a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3)
 - (c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^(1/4)*(3*b*c + 5
*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d
^(1/4)*(b*c - a*d)^3) + (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b
*c - a*d)^3) - (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b*c - a*d)
^3) - (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^
(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.32611, size = 585, normalized size = 0.94

$$\frac{1}{16}\left(\frac{8a\sqrt{x}}{(a+bx^2)(bc-ad)^2}+\frac{8c\sqrt{x}}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt{2}\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt{2}\sqrt[4]{a}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}(bc-ad)^3}+\frac{\sqrt{2}\sqrt[4]{c}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}(ad-bc)^3}-\frac{\sqrt{2}\sqrt[4]{c}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}(ad-bc)^3}+\frac{2\sqrt{2}\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}(bc-ad)^3}-\frac{2\sqrt{2}\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}(bc-ad)^3}+\frac{2\sqrt{2}\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt[4]{d}(ad-bc)^3}+\frac{2\sqrt{2}\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt[4]{d}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*a*Sqrt[x])/((b*c - a*d)^2*(a + b*x^2)) + (8*c*Sqrt[x])/((b*c
- a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b^(1/4)*(b*c - a*d)^3)
- (2*Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(b^(1/4)*(b*c - a*d)^3) + (2*Sqrt[2]*c^(1/4)*(
3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(d^
(1/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*ArcT
```

```
an[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(d^(1/4)*(b*c - a*d)^3
) + (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(b^(1/4)*(b*c - a*d)^3) - (Sqrt[2
]*a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(b^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*c^(1/4)*(
3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(d^(1/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*c^(1/4)*(3*b*c +
5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(d^(1/4)*(-(b*c) + a*d)^3))/16
```

Maple [A] time = 0.03, size = 740, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*c/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a*d-1/2*c^2/(a*d-b*c)^3*x^(1/
2)/(d*x^2+c)*b-5/8/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)*a*d-3/8*c/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-5/16/(a*d-b*c)^3*(c/d)^(
1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*d-3/16*c/(a*d-b*c)^3*(c
/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b-5/8/(a*d-b*c)^3*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*d-3/8*c
/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)+1)*b+1/2*a^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*d-1/2*a/(a*d-b*c)^3
*x^(1/2)/(b*x^2+a)*b*c+3/8*a/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d+5/8/(a*d-b*c)^3*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b*c+3/16*a/(a*d-b*c)
^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d+5/16/(a*d-b*c
)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*b*c+3/8*a/(a*d
-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
*d+5/8/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 27.9399, size = 5917, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a
*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^
```

```
2 + a^3*d^3)*x^2)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^
3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b
^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^
4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a
^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a
^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*arcta
n(-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(625*a*b
^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*
d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^1
0*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*
c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5
*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12))^(1/4)/((5*b*c + 3*a*d)*sqrt(x) + sqrt((
25*b^2*c^2 + 30*a*b*c*d + 9*a^2*d^2)*x + (b^6*c^6 - 6*a*b^5*c^5*d
 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3
*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c
^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*
d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6
*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^
3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))
))) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a
*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^
2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*
c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a
*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 79
2*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 +
66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(1/4)*arcta
n(-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(81*b^4*
c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3
 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10
*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*
b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^
11*b*c*d^12 + a^12*d^13))^(1/4)/((3*b*c + 5*a*d)*sqrt(x) + sqrt((
9*b^2*c^2 + 30*a*b*c*d + 25*a^2*d^2)*x + (b^6*c^6 - 6*a*b^5*c^5*d
 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1
350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c
^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c
^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*
c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3
*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))
))) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b
^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2
+ a^3*d^3)*x^2)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*
b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^1
2*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*
b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7
*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^1
0*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*log((5*
b*c + 3*a*d)*sqrt(x) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^
2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11
*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c
^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*
c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)) - (a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b
*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*
(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 54
0*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^
2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 79
2*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 -
 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*log((5*b*c + 3*a*d)*sqr
t(x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(625
*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*
b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11
*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*
b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8
*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^
11*b^2*c*d^11 + a^12*b*d^12))^(1/4)) - (a*b^2*c^3 - 2*a^2*b*c^2*d
```

```
 + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3
*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 +
540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625
*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*
d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7
*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^
4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c
*d^12 + a^12*d^13))^(1/4)*log((3*b*c + 5*a*d)*sqrt(x) + (b^3*c^3
- 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(81*b^4*c^5 + 540*a*
b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c
*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 -
220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 +
 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
- 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12
+ a^12*d^13))^(1/4)) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (
b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2
*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d
+ 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^1
2*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^
9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b
^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*
b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^1
3))^(1/4)*log((3*b*c + 5*a*d)*sqrt(x) - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350
*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12
*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*
d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6
*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^
3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(1
/4)) - 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(x))/(a*b^2*c^3 - 2*a^2*b*
c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{(bx^2 + a)^2 (dx^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.489 $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=609

$$-\frac{x^{3/2}}{2(a+bx^2)(c+dx^2)(bc-ad)}-\frac{dx^{3/2}}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt[4]{b}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{\sqrt[4]{b}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{\sqrt[4]{d}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}(bc-ad)^3}+\frac{\sqrt[4]{d}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}(bc-ad)^3}-\frac{\sqrt[4]{b}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{\sqrt[4]{b}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{\sqrt[4]{d}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)^3}-\frac{\sqrt[4]{d}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)^3}$$

```
[Out] -((d*x^(3/2))/((b*c - a*d)^2*(c + d*x^2))) - x^(3/2)/(2*(b*c - a*
d)*(a + b*x^2)*(c + d*x^2)) - (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*
d)^3) + (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(5*b*
c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*c^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d
)^3) + (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b
^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5
*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(5*b*c + 3*
a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.65718, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{x^{3/2}}{2(a+bx^2)(c+dx^2)(bc-ad)}-\frac{dx^{3/2}}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt[4]{b}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{\sqrt[4]{b}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{\sqrt[4]{d}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}(bc-ad)^3}+\frac{\sqrt[4]{d}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}(bc-ad)^3}-\frac{\sqrt[4]{b}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{\sqrt[4]{b}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{\sqrt[4]{d}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)^3}-\frac{\sqrt[4]{d}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] -((d*x^(3/2))/((b*c - a*d)^2*(c + d*x^2))) - x^(3/2)/(2*(b*c - a*
d)*(a + b*x^2)*(c + d*x^2)) - (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*
d)^3) + (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
```

```
[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(5*b*
c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*c^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d
)^3) + (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b
^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5
*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(5*b*c + 3*
a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.8252, size = 583, normalized size = 0.96

$$\frac{1}{16}\left(-\frac{8bx^{3/2}}{(a+bx^2)(bc-ad)^2}-\frac{8dx^{3/2}}{(c+dx^2)(bc-ad)^2}+\frac{\sqrt{2}\sqrt[4]{b}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}(bc-ad)^3}+\frac{\sqrt{2}\sqrt[4]{b}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}(ad-bc)^3}+\frac{\sqrt{2}\sqrt[4]{d}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}(ad-bc)^3}+\frac{\sqrt{2}\sqrt[4]{d}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}(bc-ad)^3}+\frac{2\sqrt{2}\sqrt[4]{b}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}(ad-bc)^3}-\frac{2\sqrt{2}\sqrt[4]{b}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}(ad-bc)^3}+\frac{2\sqrt{2}\sqrt[4]{d}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}(bc-ad)^3}-\frac{2\sqrt{2}\sqrt[4]{d}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt[4]{c}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-8*b*x^(3/2))/((b*c - a*d)^2*(a + b*x^2)) - (8*d*x^(3/2))/((b*c
 - a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)
^3) - (2*Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*d^
(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(c^(1/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*
ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(1/4)*(b*c - a*
```

```
d)^3) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)^3) + (Sq
rt[2]*b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^
(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(c^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(5
*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(c^(1/4)*(b*c - a*d)^3))/16
```

Maple [A] time = 0.03, size = 740, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x^(3/2
)/(d*x^2+c)*b*c+5/16/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*c*ln((x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2)))+5/8/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*c*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+5/8/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/
2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+3/16*d/(a*d-b*c)^3/(
c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+3/8*d/(a*d-b*c)^3
/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3/8*
d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)-1/2*b/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c
)^3*x^(3/2)/(b*x^2+a)*c-5/16/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*a*d*
ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2)))-5/8/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*a*
d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/(a*d-b*c)^3/(a/b)^(1/
4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-3/16*b/(a*d-
b*c)^3/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-3/8*b/(a*
d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)-3/8*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 24.3847, size = 7214, normalized size = 11.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a
*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^
2 + a^3*d^3)*x^2)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*
```

```
c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a
^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495
*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 79
2*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 +
66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(1/4)*arcta
n(-(a*b^9*c^9 - 9*a^2*b^8*c^8*d + 36*a^3*b^7*c^7*d^2 - 84*a^4*b^6
*c^6*d^3 + 126*a^5*b^5*c^5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a^7*b^3
*c^3*d^6 - 36*a^8*b^2*c^2*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-(81*b
^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*
d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b
^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^
6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a
^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12
*a^12*b*c*d^11 + a^13*d^12))^(3/4)/((27*b^4*c^3 + 135*a*b^3*c^2*d
 + 225*a^2*b^2*c*d^2 + 125*a^3*b*d^3)*sqrt(x) + sqrt((729*b^8*c^6
 + 7290*a*b^7*c^5*d + 30375*a^2*b^6*c^4*d^2 + 67500*a^3*b^5*c^3*d
^3 + 84375*a^4*b^4*c^2*d^4 + 56250*a^5*b^3*c*d^5 + 15625*a^6*b^2*
d^6)*x - (81*a*b^11*c^10 + 54*a^2*b^10*c^9*d - 675*a^3*b^9*c^8*d^
2 - 120*a^4*b^8*c^7*d^3 + 2290*a^5*b^7*c^6*d^4 - 636*a^6*b^6*c^5*
d^5 - 3534*a^7*b^5*c^4*d^6 + 2440*a^8*b^4*c^3*d^7 + 1725*a^9*b^3*
c^2*d^8 - 2250*a^10*b^2*c*d^9 + 625*a^11*b*d^10)*sqrt(-(81*b^5*c^
4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 +
 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c
^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7
*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^
4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12
*b*c*d^11 + a^13*d^12))))) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c
*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a
*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*b^4*c^4*d + 1500*
a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d
^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^
3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*
a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220
*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^
12*c*d^12))^(1/4)*arctan(-(b^9*c^10 - 9*a*b^8*c^9*d + 36*a^2*b^7*
c^8*d^2 - 84*a^3*b^6*c^7*d^3 + 126*a^4*b^5*c^6*d^4 - 126*a^5*b^4*
c^5*d^5 + 84*a^6*b^3*c^4*d^6 - 36*a^7*b^2*c^3*d^7 + 9*a^8*b*c^2*d
^8 - a^9*c*d^9)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*
b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^1
1*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*
b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7
*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^1
0*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(3/4)/((125*b
^3*c^3*d + 225*a*b^2*c^2*d^2 + 135*a^2*b*c*d^3 + 27*a^3*d^4)*sqrt
(x) + sqrt((15625*b^6*c^6*d^2 + 56250*a*b^5*c^5*d^3 + 84375*a^2*b
^4*c^4*d^4 + 67500*a^3*b^3*c^3*d^5 + 30375*a^4*b^2*c^2*d^6 + 7290
*a^5*b*c*d^7 + 729*a^6*d^8)*x - (625*b^10*c^11*d - 2250*a*b^9*c^1
0*d^2 + 1725*a^2*b^8*c^9*d^3 + 2440*a^3*b^7*c^8*d^4 - 3534*a^4*b^
6*c^7*d^5 - 636*a^5*b^5*c^6*d^6 + 2290*a^6*b^4*c^5*d^7 - 120*a^7*
b^3*c^4*d^8 - 675*a^8*b^2*c^3*d^9 + 54*a^9*b*c^2*d^10 + 81*a^10*c
*d^11)*sqrt(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c
^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^1
2*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c
^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*
c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2
*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))))) - (a*b^2*c^3 -
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(
81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^
2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a
^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 79
2*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 4
95*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10
- 12*a^12*b*c*d^11 + a^13*d^12))^(1/4)*log((a*b^9*c^9 - 9*a^2*b^8
*c^8*d + 36*a^3*b^7*c^7*d^2 - 84*a^4*b^6*c^6*d^3 + 126*a^5*b^5*c^
5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a^7*b^3*c^3*d^6 - 36*a^8*b^2*c^2
*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d
+ 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b
^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^
9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b
^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10
*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^1
2))^(3/4) + (27*b^4*c^3 + 135*a*b^3*c^2*d + 225*a^2*b^2*c*d^2 + 1
25*a^3*b*d^3)*sqrt(x)) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 +
```

```
(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c
^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^5*c^4 + 540*a*b^4*c^3*
d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a
*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*
b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7
*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^
10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d
^12))^(1/4)*log(-(a*b^9*c^9 - 9*a^2*b^8*c^8*d + 36*a^3*b^7*c^7*d^
2 - 84*a^4*b^6*c^6*d^3 + 126*a^5*b^5*c^5*d^4 - 126*a^6*b^4*c^4*d^
5 + 84*a^7*b^3*c^3*d^6 - 36*a^8*b^2*c^2*d^7 + 9*a^9*b*c*d^8 - a^1
0*d^9)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1
500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^1
1*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^
8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c
^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2
*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(3/4) + (27*b^4*c^3 +
135*a*b^3*c^2*d + 225*a^2*b^2*c*d^2 + 125*a^3*b*d^3)*sqrt(x)) + (
a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^
2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^2)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2
*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*
d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9
*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^
6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c
^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1/4)*log((b^9*c^10
- 9*a*b^8*c^9*d + 36*a^2*b^7*c^8*d^2 - 84*a^3*b^6*c^7*d^3 + 126*a
^4*b^5*c^6*d^4 - 126*a^5*b^4*c^5*d^5 + 84*a^6*b^3*c^4*d^6 - 36*a^
7*b^2*c^3*d^7 + 9*a^8*b*c^2*d^8 - a^9*c*d^9)*(-(625*b^4*c^4*d + 1
500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a
^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 22
0*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 +
924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 -
 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11
+ a^12*c*d^12))^(3/4) + (125*b^3*c^3*d + 225*a*b^2*c^2*d^2 + 135*
a^2*b*c*d^3 + 27*a^3*d^4)*sqrt(x)) - (a*b^2*c^3 - 2*a^2*b*c^2*d +
 a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c
^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*b^4*c^4*d +
 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81
*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 -
220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5
+ 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8
 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^1
1 + a^12*c*d^12))^(1/4)*log(-(b^9*c^10 - 9*a*b^8*c^9*d + 36*a^2*b
^7*c^8*d^2 - 84*a^3*b^6*c^7*d^3 + 126*a^4*b^5*c^6*d^4 - 126*a^5*b
^4*c^5*d^5 + 84*a^6*b^3*c^4*d^6 - 36*a^7*b^2*c^3*d^7 + 9*a^8*b*c^
2*d^8 - a^9*c*d^9)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a
^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*
b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a
^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*
a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*
a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(3/4) + (1
25*b^3*c^3*d + 225*a*b^2*c^2*d^2 + 135*a^2*b*c*d^3 + 27*a^3*d^4)*
sqrt(x)) + 4*(2*b*d*x^3 + (b*c + a*d)*x)*sqrt(x))/(a*b^2*c^3 - 2*
a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)
*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{(bx^2 + a)^2 (dx^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.490 $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2} dx$

Optimal. Leaf size=601

$$-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}-\frac{b^{3/4}(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{3/4}(bc-ad)^3}-\frac{d\sqrt{x}}{(c+dx^2)(bc-ad)^2}-\frac{\sqrt{x}}{2(a+bx^2)(c+dx^2)(bc-ad)}$$

```
[Out] -((d*Sqrt[x])/((b*c - a*d)^2*(c + d*x^2))) - Sqrt[x]/(2*(b*c - a*
d)*(a + b*x^2)*(c + d*x^2)) - (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)
^3) + (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(7*b*c +
a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^
(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (
b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*
c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(7*b*c + a*d)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sq
rt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/
4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.41324, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}-\frac{b^{3/4}(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{3/4}(bc-ad)^3}-\frac{d\sqrt{x}}{(c+dx^2)(bc-ad)^2}-\frac{\sqrt{x}}{2(a+bx^2)(c+dx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*Sqrt[x])/((b*c - a*d)^2*(c + d*x^2))) - Sqrt[x]/(2*(b*c - a*
d)*(a + b*x^2)*(c + d*x^2)) - (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)
^3) + (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(7*b*c +
a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^
(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (
b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*
c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(7*b*c + a*d)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sq
rt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/
4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.75995, size = 575, normalized size = 0.96

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}(ad-bc)^3}+\frac{\sqrt{2}b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}(bc-ad)^3}+\frac{2\sqrt{2}b^{3/4}(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}(ad-bc)^3}-\frac{2\sqrt{2}b^{3/4}(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}(ad-bc)^3}+\frac{\sqrt{2}d^{3/4}(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}(bc-ad)^3}+\frac{\sqrt{2}d^{3/4}(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}(ad-bc)^3}+\frac{2\sqrt{2}d^{3/4}(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{3/4}(bc-ad)^3}-\frac{2\sqrt{2}d^{3/4}(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{3/4}(bc-ad)^3}-\frac{8b\sqrt{x}}{(a+bx^2)(bc-ad)^2}-\frac{8d\sqrt{x}}{(c+dx^2)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-8*b*Sqrt[x])/((b*c - a*d)^2*(a + b*x^2)) - (8*d*Sqrt[x])/((b*c
 - a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3
) - (2*Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*d^(3/4
)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c
^(3/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(3/4)*(7*b*c + a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(3/4)*(b*c - a*d)^3) +
(Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*b
```

```
^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(a^(3/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(7*b*c
 + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(c^(3/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(7*b*c + a*d)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*(
-(b*c) + a*d)^3))/16
```

Maple [A] time = 0.031, size = 770, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x^(1/2
)/(d*x^2+c)*b*c+1/8*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+7/8*d/(a*d-b*c)^3*(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+1/8*d^2/(a*d-b*c)^3
*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+7/
8*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*b+1/16*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2
)+(c/d)^(1/2)))*a+7/16*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2)))*b-1/2*b/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*a*d+1/2*
b^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*c-7/8*b/(a*d-b*c)^3*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d-1/8*b^2/(a*d-b*c
)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c
-7/8*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)*d-1/8*b^2/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c-7/16*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(
1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d-1/16*b^2/(a*d-b*c)^3*(a/b)^(1/4
)/a*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 75.9416, size = 6036, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2
 + a^3*d^3)*x^2)*(-(b^7*c^4 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^
2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^
4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*
```

a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792
*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 +
66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*arct
an(-(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-(b^
7*c^4 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3
+ 2401*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5
*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*
a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 49
5*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10
- 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)/((b^2*c + 7*a*b*d)*sqrt(x)
+ sqrt((b^4*c^2 + 14*a*b^3*c*d + 49*a^2*b^2*d^2)*x + (a^2*b^6*c^
6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 1
5*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*sqrt(-(b^7*c^4 + 28*
a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*
d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7
*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*
c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b
*c*d^11 + a^15*d^12))))) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d
^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b
^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c^4*d^3 + 1372
*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/
(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^
9*c^12*d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^
6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220*a
^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^4*d^11 + a^12
*c^3*d^12))^(1/4)*arctan(-(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*
d^2 - a^3*c*d^3)*(-(2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a
^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*a*b^11
*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b
^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^
7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^
10*b^2*c^5*d^10 - 12*a^11*b*c^4*d^11 + a^12*c^3*d^12))^(1/4)/((7*
b*c*d + a*d^2)*sqrt(x) + sqrt((49*b^2*c^2*d^2 + 14*a*b*c*d^3 + a^
2*d^4)*x + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3
*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6
)*sqrt(-(2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*
d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*a*b^11*c^14*d + 6
6*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^4
- 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d
^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*
d^10 - 12*a^11*b*c^4*d^11 + a^12*c^3*d^12))))) + (a*b^2*c^3 - 2*a
^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(b^7
*c^4 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3
+ 2401*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*
b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a
^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495
*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 -
12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*log((b^2*c + 7*a*b*d)*sqrt(
x) + (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-(b
^7*c^4 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^
3 + 2401*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^
5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792
*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 4
95*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10
- 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)) - (a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 +
(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(b^7*c^4 +
28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401
*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c
^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7
*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*
b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^
14*b*c*d^11 + a^15*d^12))^(1/4)*log((b^2*c + 7*a*b*d)*sqrt(x) - (
a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-(b^7*c^4
+ 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 24
01*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10
*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b
^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^1
1*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*
a^14*b*c*d^11 + a^15*d^12))^(1/4)) - (a*b^2*c^3 - 2*a^2*b*c^2*d +
a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c
^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c^4*d^

```
3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a
^4*d^7)/(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 22
0*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5
+ 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8
 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^4*d^1
1 + a^12*c^3*d^12))^(1/4)*log((7*b*c*d + a*d^2)*sqrt(x) + (b^3*c^
4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-(2401*b^4*c^4*
d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 +
 a^4*d^7)/(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 -
220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^
5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d
^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^4*d
^11 + a^12*c^3*d^12))^(1/4)) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c
*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a
*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c^4*d^3 + 13
72*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7
)/(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*
b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*
a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220
*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^4*d^11 + a^
12*c^3*d^12))^(1/4)*log((7*b*c*d + a*d^2)*sqrt(x) - (b^3*c^4 - 3*
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-(2401*b^4*c^4*d^3 +
1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d
^7)/(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^
3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 92
4*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 2
20*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^4*d^11 +
a^12*c^3*d^12))^(1/4)) - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(x))/(a*b^
2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 +
a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(bx^2+a)^2(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.491 $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=624

$$\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}(bc-ad)^3}+\frac{dx^{3/2}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

```
[Out] (d*(b*c + a*d)*x^(3/2))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^
(3/2))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(5/4)*(b*c
- 9*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(5/4)*(b*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^3)
 - (d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c - a*d
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/
4)*(b*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a
*d)^3) - (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (d
^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(9*b
*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.83762, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}(bc-ad)^3}+\frac{dx^{3/2}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x^(3/2))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^
(3/2))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(5/4)*(b*c
- 9*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(5/4)*(b*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^3)
 - (d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c - a*d
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/
4)*(b*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a
*d)^3) - (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (d
^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(9*b
*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.30798, size = 589, normalized size = 0.94

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{5/4}(9ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(ad-bc)^3}+\frac{\sqrt{2}b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(ad-bc)^3}+\frac{2\sqrt{2}b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}(ad-bc)^3}+\frac{2\sqrt{2}b^{5/4}(9ad-bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}(ad-bc)^3}+\frac{8b^2x^{3/2}}{a\left(a+bx^2\right)(bc-ad)^2}+\frac{\sqrt{2}d^{5/4}(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}(bc-ad)^3}+\frac{\sqrt{2}d^{5/4}(ad-9bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{5/4}(ad-9bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{5/4}(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}(bc-ad)^3}+\frac{8d^2x^{3/2}}{c\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*b^2*x^(3/2))/(a*(b*c - a*d)^2*(a + b*x^2)) + (8*d^2*x^(3/2))/
(c*(b*c - a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(5/4)*(b*c - 9*a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-(b*c) +
 a*d)^3) + (2*Sqrt[2]*b^(5/4)*(-(b*c) + 9*a*d)*ArcTan[1 + (Sqrt[2
```

```
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-(b*c) + a*d)^3) + (2*Sqrt
[2]*d^(5/4)*(-9*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(c^(5/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(5/4)*(9*b*c - a*
d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*(b*c -
 a*d)^3) + (Sqrt[2]*b^(5/4)*(-(b*c) + 9*a*d)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(-(b*c) + a*d)^3
) + (Sqrt[2]*b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(-(b*c) + a*d)^3) + (Sqrt[
2]*d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(c^(5/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(5/4)*(-
9*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(c^(5/4)*(b*c - a*d)^3))/16
```

Maple [A] time = 0.03, size = 778, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x^(3/2)/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x^(
3/2)/(d*x^2+c)*b+1/8*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-9/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b+1/16*d^2/(a*d-b*c)
^3/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a-9/16*d/(a*d
-b*c)^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/8*d^2/
(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)*a-9/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)*b+1/2*b^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d-1/2
*b^3/(a*d-b*c)^3/a*x^(3/2)/(b*x^2+a)*c+9/8*b/(a*d-b*c)^3/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d-1/8*b^2/(a*d-
b*c)^3/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
)*c+9/16*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))*d-1/16*b^2/(a*d-b*c)^3/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2)))*c+9/8*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)*d-1/8*b^2/(a*d-b*c)^3/a/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 73.8499, size = 7488, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d
- 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3
*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*(-(b^9*c^4 - 36*a*b^8*c^3*d
+ 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a
^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^
8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*
a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d^8 -
220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 +
a^17*d^12))^(1/4)*arctan((a^4*b^9*c^9 - 9*a^5*b^8*c^8*d + 36*a^6*
b^7*c^7*d^2 - 84*a^7*b^6*c^6*d^3 + 126*a^8*b^5*c^5*d^4 - 126*a^9*
b^4*c^4*d^5 + 84*a^10*b^3*c^3*d^6 - 36*a^11*b^2*c^2*d^7 + 9*a^12*
b*c*d^8 - a^13*d^9)*(-(b^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2
*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 - 12
*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 4
95*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*a^11*b^6*c^6*d^6
- 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d^8 - 220*a^14*b^3*c^3*
d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 + a^17*d^12))^(3/4)
/((b^7*c^3 - 27*a*b^6*c^2*d + 243*a^2*b^5*c*d^2 - 729*a^3*b^4*d^3
)*sqrt(x) - sqrt((b^14*c^6 - 54*a*b^13*c^5*d + 1215*a^2*b^12*c^4*
d^2 - 14580*a^3*b^11*c^3*d^3 + 98415*a^4*b^10*c^2*d^4 - 354294*a^
5*b^9*c*d^5 + 531441*a^6*b^8*d^6)*x - (a^3*b^15*c^10 - 42*a^4*b^1
4*c^9*d + 717*a^5*b^13*c^8*d^2 - 6392*a^6*b^12*c^7*d^3 + 32082*a^
7*b^11*c^6*d^4 - 93372*a^8*b^10*c^5*d^5 + 164242*a^9*b^9*c^4*d^6
- 177912*a^10*b^8*c^3*d^7 + 116397*a^11*b^7*c^2*d^8 - 42282*a^12*
b^6*c*d^9 + 6561*a^13*b^5*d^10)*sqrt(-(b^9*c^4 - 36*a*b^8*c^3*d +
 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^
5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^8
*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*a
^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d^8 - 2
20*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 + a
^17*d^12))))) + 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a
*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 -
a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*(-(6561*b^4*c^4*d
^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 +
a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 2
20*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5
 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*
d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*
d^11 + a^12*c^5*d^12))^(1/4)*arctan((b^9*c^13 - 9*a*b^8*c^12*d +
36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^4*b^5*c^9*d^4 -
 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d^7 +
9*a^8*b*c^5*d^8 - a^9*c^4*d^9)*(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c
^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^
17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d
^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^
11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3
*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d^11 + a^12*c^5*d
^12))^(3/4)/((729*b^3*c^3*d^4 - 243*a*b^2*c^2*d^5 + 27*a^2*b*c*d^
6 - a^3*d^7)*sqrt(x) - sqrt((531441*b^6*c^6*d^8 - 354294*a*b^5*c^
5*d^9 + 98415*a^2*b^4*c^4*d^10 - 14580*a^3*b^3*c^3*d^11 + 1215*a^
4*b^2*c^2*d^12 - 54*a^5*b*c*d^13 + a^6*d^14)*x - (6561*b^10*c^13*
d^5 - 42282*a*b^9*c^12*d^6 + 116397*a^2*b^8*c^11*d^7 - 177912*a^3
*b^7*c^10*d^8 + 164242*a^4*b^6*c^9*d^9 - 93372*a^5*b^5*c^8*d^10 +
 32082*a^6*b^4*c^7*d^11 - 6392*a^7*b^3*c^6*d^12 + 717*a^8*b^2*c^5
*d^13 - 42*a^9*b*c^4*d^14 + a^10*c^3*d^15)*sqrt(-(6561*b^4*c^4*d^
5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a
^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 22
0*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5
+ 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d
^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d
^11 + a^12*c^5*d^12))))) - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2
*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b
^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*(-(b^9*c
^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 +
6561*a^4*b^5*d^4)/(a^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^7*b^
10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^1
0*b^7*c^7*d^5 + 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495
*a^13*b^4*c^4*d^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 -
 12*a^16*b*c*d^11 + a^17*d^12))^(1/4)*log((a^4*b^9*c^9 - 9*a^5*b^
8*c^8*d + 36*a^6*b^7*c^7*d^2 - 84*a^7*b^6*c^6*d^3 + 126*a^8*b^5*c
^5*d^4 - 126*a^9*b^4*c^4*d^5 + 84*a^10*b^3*c^3*d^6 - 36*a^11*b^2*
c^2*d^7 + 9*a^12*b*c*d^8 - a^13*d^9)*(-(b^9*c^4 - 36*a*b^8*c^3*d
+ 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a
^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^
```

```
8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*
a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d^8 -
220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 +
a^17*d^12))^(3/4) - (b^7*c^3 - 27*a*b^6*c^2*d + 243*a^2*b^5*c*d^2
 - 729*a^3*b^4*d^3)*sqrt(x)) + (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4
*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 +
(a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*(-(b
^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^
3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^
7*b^10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792
*a^10*b^7*c^7*d^5 + 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 +
 495*a^13*b^4*c^4*d^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^
10 - 12*a^16*b*c*d^11 + a^17*d^12))^(1/4)*log(-(a^4*b^9*c^9 - 9*a
^5*b^8*c^8*d + 36*a^6*b^7*c^7*d^2 - 84*a^7*b^6*c^6*d^3 + 126*a^8*
b^5*c^5*d^4 - 126*a^9*b^4*c^4*d^5 + 84*a^10*b^3*c^3*d^6 - 36*a^11
*b^2*c^2*d^7 + 9*a^12*b*c*d^8 - a^13*d^9)*(-(b^9*c^4 - 36*a*b^8*c
^3*d + 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^
4)/(a^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 2
20*a^8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 +
 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d
^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^
11 + a^17*d^12))^(3/4) - (b^7*c^3 - 27*a*b^6*c^2*d + 243*a^2*b^5*
c*d^2 - 729*a^3*b^4*d^3)*sqrt(x)) - (a^2*b^2*c^4 - 2*a^3*b*c^3*d
+ a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x
^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
*(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 -
 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2
*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 79
2*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7
+ 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^1
0 - 12*a^11*b*c^6*d^11 + a^12*c^5*d^12))^(1/4)*log((b^9*c^13 - 9*
a*b^8*c^12*d + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7
*b^2*c^6*d^7 + 9*a^8*b*c^5*d^8 - a^9*c^4*d^9)*(-(6561*b^4*c^4*d^5
 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^
4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 220
*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5 +
 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d^
8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d^
11 + a^12*c^5*d^12))^(3/4) - (729*b^3*c^3*d^4 - 243*a*b^2*c^2*d^5
 + 27*a^2*b*c*d^6 - a^3*d^7)*sqrt(x)) + (a^2*b^2*c^4 - 2*a^3*b*c^
3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^
3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*
x^2)*(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d
^7 - 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66
*a^2*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4
- 792*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*
d^7 + 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7
*d^10 - 12*a^11*b*c^6*d^11 + a^12*c^5*d^12))^(1/4)*log(-(b^9*c^13
 - 9*a*b^8*c^12*d + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 1
26*a^4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 3
6*a^7*b^2*c^6*d^7 + 9*a^8*b*c^5*d^8 - a^9*c^4*d^9)*(-(6561*b^4*c^
4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8
 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2
- 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*
d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c
^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c
^6*d^11 + a^12*c^5*d^12))^(3/4) - (729*b^3*c^3*d^4 - 243*a*b^2*c^
2*d^5 + 27*a^2*b*c*d^6 - a^3*d^7)*sqrt(x)) + 4*((b^2*c*d + a*b*d^
2)*x^3 + (b^2*c^2 + a^2*d^2)*x)*sqrt(x))/(a^2*b^2*c^4 - 2*a^3*b*c
^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d
^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)
*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(bx^2 + a)^2 (dx^2 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.492 $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=628

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{b\sqrt{x}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d\sqrt{x}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*Sqrt[x])/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*Sq
rt[x])/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(7/4)*(3*b*
c - 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a
*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11
*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a
^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(
b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*
d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```

Rubi [A] time = 1.74051, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{b\sqrt{x}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d\sqrt{x}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*Sqrt[x])/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*Sq
rt[x])/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(7/4)*(3*b*
c - 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a
*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11
*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a
^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(
b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*
d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.36155, size = 593, normalized size = 0.94

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^3}+\frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(ad-bc)^3}+\frac{2\sqrt{2}b^{7/4}(11ad-3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^3}+\frac{2\sqrt{2}b^{7/4}(11ad-3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}(ad-bc)^3}+\frac{8b^2\sqrt{x}}{a\left(a+bx^2\right)(bc-ad)^2}+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}(ad-bc)^3}+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{7/4}(3ad-11bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{7/4}(bc-ad)^3}+\frac{8d^2\sqrt{x}}{c\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*b^2*Sqrt[x])/(a*(b*c - a*d)^2*(a + b*x^2)) + (8*d^2*Sqrt[x])/
(c*(b*c - a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a
```

```
*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(b*c
- a*d)^3) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) + (2*Sq
rt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(7/4)*(11*b*
c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(7/4
)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c -
a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(-(b*c) + a*d)^3
) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)*(-(b*c) + a*d)^3) + (Sq
rt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)*(b*c - a*d)^3))/16
```

Maple [A] time = 0.029, size = 808, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x^(1/2)/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x^(
1/2)/(d*x^2+c)*b+3/8*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-11/8*d^2/(a*d-b*c)^3/c*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+3/8*d^3/(a*
d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)-1)*a-11/8*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^
(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^
(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b+1/2*b^2/(a*d-b*c)^3*x^
(1/2)/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3/a*x^(1/2)/(b*x^2+a)*c+11/8*
b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)*d-3/8*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/
4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d-3/16*b^3/(a*d-b*c)^3/a^2*
(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
)/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c+11/8*b^2/(a*d-b*
c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*
d-3/8*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^2 (dx^2 + c)^2 \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*sqrt(x)), x)
```

3.493 $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=676

$$-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{9/4}(5bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{5a^2d^2-8abcd+5b^2c^2}{2a^2c^2\sqrt{x}(bc-ad)^2}-\frac{d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{b}{2a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2ac\sqrt{x}\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] -(5*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*Sqr
t[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2))
 + b/(2*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (b^(9/4)
*(5*b*c - 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b
*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(9/
4)*(13*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(9/4)*(b*c - a*d)^3) + (b^(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
9/4)*(b*c - a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)
^3)
```

Rubi [A] time = 2.31032, antiderivative size = 676, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{9/4}(5bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{5a^2d^2-8abcd+5b^2c^2}{2a^2c^2\sqrt{x}(bc-ad)^2}-\frac{d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{b}{2a\sqrt{x}\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{d(ad+bc)}{2ac\sqrt{x}\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(5*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*Sqr
t[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2))
 + b/(2*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (b^(9/4)
*(5*b*c - 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b
*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(9/
4)*(13*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(9/4)*(b*c - a*d)^3) + (b^(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
9/4)*(b*c - a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)
^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.80456, size = 606, normalized size = 0.9

$$\frac{1}{16}\left(\frac{\sqrt{2}b^{9/4}(13ad-5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^3}+\frac{\sqrt{2}b^{9/4}(13ad-5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(ad-bc)^3}+\frac{2\sqrt{2}b^{9/4}(13ad-5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}(ad-bc)^3}+\frac{2\sqrt{2}b^{9/4}(13ad-5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}(bc-ad)^3}-\frac{8b^3x^{3/2}}{a^2\left(a+bx^2\right)(bc-ad)^2}-\frac{32}{a^2c^2\sqrt{x}}+\frac{\sqrt{2}d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(ad-bc)^3}+\frac{\sqrt{2}d^{9/4}(13bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{9/4}(13bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{9/4}(5ad-13bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}(bc-ad)^3}-\frac{8d^3x^{3/2}}{c^2\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a^2*c^2*Sqrt[x]) - (8*b^3*x^(3/2))/(a^2*(b*c - a*d)^2*(a +
b*x^2)) - (8*d^3*x^(3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (2*Sq
rt[2]*b^(9/4)*(-5*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(a^(9/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*b^(9/4)*(-5
*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^
(9/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(9/4)*(13*b*c - 5*a*d)*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3)
+ (2*Sqrt[2]*d^(9/4)*(-13*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(9/4)*(
-5*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(a^(9/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(9/4)*(-5*b*c +
13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(a^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9/4)*(13*b*c - 5*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(
9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9/4)*(13*b*c - 5*a*d)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b
*c - a*d)^3))/16
```

Maple [A] time = 0.036, size = 825, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^4/c^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^
3*x^(3/2)/(d*x^2+c)*b-5/16*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2
)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-5/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4
)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-5/8*d^3/c^2/(a*
d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1)+13/16*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
```

```
/d)^(1/2)))+13/8*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+13/8*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*
2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a^2/c^2/x^(1/2)
-1/2*b^3/a/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^
3*x^(3/2)/(b*x^2+a)*c-13/16*b^2/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)
*d*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))-13/8*b^2/a/(a*d-b*c)^3/(a/b)^(1/4)*
2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-13/8*b^2/a/(a*d-b
*c)^3/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
+5/16*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b
)^(1/2)))+5/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+5/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*
2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^2x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(3/2)), x)
```

3.494 $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=676

$$\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3} + \frac{b^{11/4}(7bc-15ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{b^{11/4}(7bc-15ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{7a^2d^2-8abcd+7b^2c^2}{6a^2c^2x^{3/2}(bc-ad)^2} + \frac{d^{11/4}(15bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3} - \frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3} + \frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3} - \frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3} + \frac{b}{2ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)} + \frac{d(ad+bc)}{2acx^{3/2}\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] -(7*b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^(
3/2)) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(3/2)*(c + d*x^2))
 + b/(2*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (b^(11/4
)*(7*b*c - 15*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b^(11/4)*(7*b*c - 15*a*d)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4
)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) -
(d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(7*b*c -
15*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b^(11/4)*(7*b*c - 15*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2
]*c^(11/4)*(b*c - a*d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1
1/4)*(b*c - a*d)^3)
```

Rubi [A] time = 2.177, antiderivative size = 676, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3} + \frac{b^{11/4}(7bc-15ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{b^{11/4}(7bc-15ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{7a^2d^2-8abcd+7b^2c^2}{6a^2c^2x^{3/2}(bc-ad)^2} + \frac{d^{11/4}(15bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3} - \frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3} + \frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3} - \frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3} + \frac{b}{2ax^{3/2}(a+bx^2)(c+dx^2)(bc-ad)} + \frac{d(ad+bc)}{2acx^{3/2}(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(7*b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^(
3/2)) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(3/2)*(c + d*x^2))
 + b/(2*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (b^(11/4
)*(7*b*c - 15*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b^(11/4)*(7*b*c - 15*a*d)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4
)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) -
(d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(7*b*c -
15*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b^(11/4)*(7*b*c - 15*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2
]*c^(11/4)*(b*c - a*d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1
1/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.75799, size = 610, normalized size = 0.9

$$\frac{1}{48}\left(\frac{3\sqrt{2}b^{11/4}(15ad-7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}(ad-bc)^3}+\frac{3\sqrt{2}b^{11/4}(15ad-7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}(bc-ad)^3}+\frac{6\sqrt{2}b^{11/4}(15ad-7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}(ad-bc)^3}+\frac{6\sqrt{2}b^{11/4}(15ad-7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}(bc-ad)^3}-\frac{24b^3\sqrt{x}}{a^2(a+bx^2)(bc-ad)^2}-\frac{32}{a^2c^2x^{3/2}}+\frac{3\sqrt{2}d^{11/4}(15bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^3}+\frac{3\sqrt{2}d^{11/4}(15bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(ad-bc)^3}+\frac{6\sqrt{2}d^{11/4}(15bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{11/4}(bc-ad)^3}+\frac{6\sqrt{2}d^{11/4}(7ad-15bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{11/4}(bc-ad)^3}-\frac{24d^3\sqrt{x}}{c^2(c+dx^2)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a^2*c^2*x^(3/2)) - (24*b^3*Sqrt[x])/(a^2*(b*c - a*d)^2*(a +
 b*x^2)) - (24*d^3*Sqrt[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (6*
Sqrt[2]*b^(11/4)*(-7*b*c + 15*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(a^(11/4)*(-(b*c) + a*d)^3) + (6*Sqrt[2]*b^(11/4
)*(-7*b*c + 15*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(a^(11/4)*(b*c - a*d)^3) + (6*Sqrt[2]*d^(11/4)*(15*b*c - 7*a*d)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c -
a*d)^3) + (6*Sqrt[2]*d^(11/4)*(-15*b*c + 7*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) + (3*Sqrt[
2]*b^(11/4)*(-7*b*c + 15*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]
*b^(11/4)*(-7*b*c + 15*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*(b*c - a*d)^3) + (3*Sqrt[2]*d^(1
1/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^3) + (3*Sqrt[2]*d^(11/4)*(
15*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(c^(11/4)*(-(b*c) + a*d)^3))/48
```

Maple [A] time = 0.034, size = 825, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*d^4/c^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^
3*x^(1/2)/(d*x^2+c)*b-7/8*d^4/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+15/8*d^3/c^2/(a*d-b*c)^3
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-7/8*
d^4/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)-1)*a+15/8*d^3/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*d^4/c^3/(a*d-b*c)^3*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-
```

```
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a+15/16*d^3/c^2/(a*d-b*
c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b-2/3/a^2/c^2
/x^(3/2)-1/2*b^3/a/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*d+1/2*b^4/a^2/(a
*d-b*c)^3*x^(1/2)/(b*x^2+a)*c-15/8*b^3/a^2/(a*d-b*c)^3*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d+7/8*b^4/a^3/(a*
d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)*c-15/8*b^3/a^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)*d+7/8*b^4/a^3/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c-15/16*b^3/a^2/(a*d-b*c
)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d+7/16*b^4/a^3
/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^2 (dx^2 + c)^2 x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(5/2)), x)
```

3.495 $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=731

$$\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3} - \frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3} - \frac{b^{13/4}(9bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3} + \frac{b^{13/4}(9bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3} - \frac{9a^2d^2-8abcd+9b^2c^2}{10a^2c^2x^{5/2}(bc-ad)^2} + \frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{2a^3c^3\sqrt{x}(bc-ad)^2} + \frac{d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3} - \frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3} - \frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3} + \frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3} + \frac{b}{2ax^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)} + \frac{d(ad+bc)}{2acx^{5/2}\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] -(9*b^2*c^2 - 8*a*b*c*d + 9*a^2*d^2)/(10*a^2*c^2*(b*c - a*d)^2*x^
(5/2)) + ((b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(2*a^
3*c^3*(b*c - a*d)^2*Sqrt[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)
^2*x^(5/2)*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*
(c + d*x^2)) - (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (b^(
13/4)*(9*b*c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a
*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
13/4)*(b*c - a*d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3
) + (b^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (b
^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (d^(13/4
)*(17*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*
b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3)
```

Rubi [A] time = 2.84229, antiderivative size = 731, normalized size of antiderivative = 1., number of

steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3} - \frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3} - \frac{b^{13/4}(9bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3} + \frac{b^{13/4}(9bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3} - \frac{9a^2d^2-8abcd+9b^2c^2}{10a^2c^2x^{5/2}(bc-ad)^2} + \frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{2a^3c^3\sqrt{x}(bc-ad)^2} + \frac{d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3} - \frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3} - \frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3} + \frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3} + \frac{b}{2ax^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)} + \frac{d(ad+bc)}{2acx^{5/2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(9*b^2*c^2 - 8*a*b*c*d + 9*a^2*d^2)/(10*a^2*c^2*(b*c - a*d)^2*x^
(5/2)) + ((b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(2*a^
3*c^3*(b*c - a*d)^2*Sqrt[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)
^2*x^(5/2)*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*
(c + d*x^2)) - (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (b^(
13/4)*(9*b*c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a
*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
13/4)*(b*c - a*d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3
) + (b^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (b
^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (d^(13/4
)*(17*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*
b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.7427, size = 630, normalized size = 0.86

$$\frac{1}{80}\left(\frac{5\sqrt{2}b^{13/4}(17ad-9bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}(ad-bc)^3}+\frac{5\sqrt{2}b^{13/4}(17ad-9bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}(bc-ad)^3}+\frac{10\sqrt{2}b^{13/4}(17ad-9bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{13/4}(bc-ad)^3}+\frac{10\sqrt{2}b^{13/4}(17ad-9bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{13/4}(ad-bc)^3}+\frac{40b^4x^{3/2}}{a^3(a+bx^2)(bc-ad)^2}+\frac{320(ad+bc)}{a^3c^3\sqrt{x}}-\frac{32}{a^2c^2x^{5/2}}+\frac{5\sqrt{2}d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^3}+\frac{5\sqrt{2}d^{13/4}(17bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(ad-bc)^3}+\frac{10\sqrt{2}d^{13/4}(9ad-17bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{13/4}(bc-ad)^3}+\frac{10\sqrt{2}d^{13/4}(17bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{13/4}(bc-ad)^3}+\frac{40d^4x^{3/2}}{c^3(c+dx^2)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a^2*c^2*x^(5/2)) + (320*(b*c + a*d))/(a^3*c^3*Sqrt[x]) + (4
0*b^4*x^(3/2))/(a^3*(b*c - a*d)^2*(a + b*x^2)) + (40*d^4*x^(3/2))
/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (10*Sqrt[2]*b^(13/4)*(-9*b*c +
 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(13/4)
*(b*c - a*d)^3) + (10*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(13/4)*(-(b*c) + a*d)^3
) + (10*Sqrt[2]*d^(13/4)*(-17*b*c + 9*a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(c^(13/4)*(b*c - a*d)^3) + (10*Sqrt[2]*d
^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(c^(13/4)*(b*c - a*d)^3) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 1
7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(a^(13/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 17*
a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(a^(13/4)*(b*c - a*d)^3) + (5*Sqrt[2]*d^(13/4)*(17*b*c - 9*a*d)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13
/4)*(b*c - a*d)^3) + (5*Sqrt[2]*d^(13/4)*(17*b*c - 9*a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(-
(b*c) + a*d)^3))/80
```

Maple [A] time = 0.041, size = 849, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^5/c^3/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a-1/2*d^4/c^2/(a*d-b*c)
^3*x^(3/2)/(d*x^2+c)*b+9/16*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/
2)*a*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4
```

```
)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+9/8*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/
4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+9/8*d^4/c^3/(a
*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)-17/16*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))-17/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-17/8*d^3/c^2/(a*d-b*c)^3/(c/d)
^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a^2/c^
2/x^(5/2)+4/x^(1/2)/a^2/c^3*d+4/x^(1/2)/a^3/c^2*b+1/2*b^4/a^2/(a*
d-b*c)^3*x^(3/2)/(b*x^2+a)*d-1/2*b^5/a^3/(a*d-b*c)^3*x^(3/2)/(b*x
^2+a)*c+17/16*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*ln((x-(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2)))+17/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+17/8*b^3/a^2/(a*d-b*c)^3/(a
/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-9/16*b^
4/a^3/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)-9/8*b^4/a^3/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)-9/8*b^4/a^3/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*
c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^2x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2*x^(7/2)), x)
```

3.496 $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=718

$$-\frac{(5a^2d^2+70abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{(5a^2d^2+70abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}-\frac{(5a^2d^2+70abcd+21b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{(5a^2d^2+70abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}(bc-ad)^4}-\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}(bc-ad)^4}+\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}(bc-ad)^4}-\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}(bc-ad)^4}+\frac{a\sqrt{x}}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}+\frac{\sqrt{x}(17ad+7bc)}{16(c+dx^2)(bc-ad)^3}+\frac{\sqrt{x}(2ad+bc)}{4b(c+dx^2)^2(bc-ad)^2}$$

```
[Out] ((b*c + 2*a*d)*Sqrt[x])/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*Sq
rt[x])/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((7*b*c + 17
*a*d)*Sqrt[x])/(16*(b*c - a*d)^3*(c + d*x^2)) + (a^(1/4)*b^(3/4)*
(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*(b*c - a*d)^4
) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^
4) + ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)
^4) + (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - (a^
(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - ((21*b^2*c^2
 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4)
+ ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(
b*c - a*d)^4)
```

Rubi [A] time = 2.1069, antiderivative size = 718, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{(5a^2d^2+70abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{(5a^2d^2+70abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}-\frac{(5a^2d^2+70abcd+21b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{(5a^2d^2+70abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}+\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}(bc-ad)^4}-\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}(bc-ad)^4}+\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}(bc-ad)^4}-\frac{\sqrt[4]{a}b^{3/4}(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}(bc-ad)^4}+\frac{a\sqrt{x}}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}+\frac{\sqrt{x}(17ad+7bc)}{16(c+dx^2)(bc-ad)^3}+\frac{\sqrt{x}(2ad+bc)}{4b(c+dx^2)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*Sqrt[x])/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*Sq
rt[x])/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((7*b*c + 17
*a*d)*Sqrt[x])/(16*(b*c - a*d)^3*(c + d*x^2)) + (a^(1/4)*b^(3/4)*
(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*(b*c - a*d)^4
) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^
4) + ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)
^4) + (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - (a^
(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - ((21*b^2*c^2
 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4)
+ ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(
b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.50188, size = 604, normalized size = 0.84

$$-\frac{\sqrt{2}(5a^2d^2+70abcd+21b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}\sqrt[4]{d}}+\frac{\sqrt{2}(5a^2d^2+70abcd+21b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}\sqrt[4]{d}}-\frac{2\sqrt{2}(5a^2d^2+70abcd+21b^2c^2)}{c^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((64*a*b*(b*c - a*d)*Sqrt[x])/(a + b*x^2) + (32*c*(b*c - a*d)^2*S
qrt[x])/(c + d*x^2)^2 + (8*(b*c - a*d)*(7*b*c + 9*a*d)*Sqrt[x])/(
c + d*x^2) + 16*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 16*Sqrt[2]*a^(1/4)*b^(3/4)
*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] -
(2*Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(3/4)*d^(1/4)) + (2*Sqrt[2]*(21
*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(c^(3/4)*d^(1/4)) + 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5
*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] - 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (Sqrt[2]*(21*b^2*
c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*d^(1/4)) + (Sqrt[2]*(21*b^2*c^2
 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*d^(1/4)))/(128*(b*c - a*d)^4)
```

Maple [A] time = 0.033, size = 1066, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -9/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a^2*d^3+1/8/(a*d-b*c)^4/(d*
x^2+c)^2*x^(5/2)*a*b*c*d^2+7/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*b
^2*c^2*d-5/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a^2*c*d^2-3/8/(a*d-
b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b*c^2*d+11/16/(a*d-b*c)^4/(d*x^2+c)^
2*x^(1/2)*b^2*c^3+5/64/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2+35/32/(a*d-b*c)^4*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d+21/64/(a*d
-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)*b^2+5/64/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*a^2*d^2+35/32/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*d+21/64/(a*d-b*c)^4*(c/
d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/12
8/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^
2*d^2+35/64/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))*a*b*d+21/128/(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))*b^2-1/2*a^2*b/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d+1/2*
a*b^2/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*c-7/8*a*b/(a*d-b*c)^4*(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d-5/8*b^2/(a*d
-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
*c-7/8*a*b/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)-1)*d-5/8*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c-7/16*a*b/(a*d-b*c)^4*(a/b)^(1/4)
*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d-5/16*b^2/(a*d-b*c)^4*(a/b)^
(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.508318, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.497 $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4}-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3dx^{3/2}(ad+7bc)}{16c\left(c+dx^2\right)(bc-ad)^3}-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{3dx^{3/2}}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] (-3*d*x^(3/2))/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x^(3/2)/(2*(b*c
- a*d)*(a + b*x^2)*(c + d*x^2)^2) - (3*d*(7*b*c + a*d)*x^(3/2))/(
16*c*(b*c - a*d)^3*(c + d*x^2)) - (3*b^(5/4)*(b*c + 3*a*d)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c
- a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) + (3*d^(1/4)
*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*d^(1/4
)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*b^(5/
4)*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*b^(5/4)*(b*c +
 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15*b^2*c^2 + 1
8*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*d^(1/4)*(
15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4)
```

Rubi [A] time = 2.19898, antiderivative size = 703, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4}-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3dx^{3/2}(ad+7bc)}{16c\left(c+dx^2\right)(bc-ad)^3}-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{3dx^{3/2}}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x^(3/2))/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x^(3/2)/(2*(b*c
- a*d)*(a + b*x^2)*(c + d*x^2)^2) - (3*d*(7*b*c + a*d)*x^(3/2))/(
16*c*(b*c - a*d)^3*(c + d*x^2)) - (3*b^(5/4)*(b*c + 3*a*d)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c
- a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) + (3*d^(1/4)
*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*d^(1/4
)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*b^(5/
4)*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*b^(5/4)*(b*c +
 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15*b^2*c^2 + 1
8*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*d^(1/4)*(
15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.22736, size = 604, normalized size = 0.86

$$\frac{\frac{3\sqrt{2}\sqrt[4]{d}(a^2d^2-18abcd-15b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{3\sqrt{2}\sqrt[4]{d}(-a^2d^2+18abcd+15b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{6\sqrt{2}\sqrt[4]{d}(-a^2d^2+18ab\ldots}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^2*(b*c - a*d)*x^(3/2))/(a + b*x^2) - (32*d*(b*c - a*d)^2*
x^(3/2))/(c + d*x^2)^2 + (8*d*(-(b*c) + a*d)*(13*b*c + 3*a*d)*x^(
3/2))/(c*(c + d*x^2)) - (48*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (48*Sqrt[2]*b^(
5/4)*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/a^(1/4) + (6*Sqrt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (6*Sqrt
[2]*d^(1/4)*(-15*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*ArcTan[1 + (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (24*Sqrt[2]*b^(5/4)*(b*c
 + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/a^(1/4) - (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1/4) + (3*Sqrt[2]*
d^(1/4)*(-15*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4) + (3*Sqrt[2]*d^(1
/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(128*(b*c - a*d)^4)
```

Maple [A] time = 0.035, size = 1067, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(7/2)*a^2+5/8*d^3/(a*d-b*c)^
4/(d*x^2+c)^2*x^(7/2)*a*b-13/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(
7/2)*b^2-1/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+9/8*d^2/(a*
d-b*c)^4/(d*x^2+c)^2*x^(3/2)*c*a*b-17/16*d/(a*d-b*c)^4/(d*x^2+c)^
2*x^(3/2)*b^2*c^2+3/64*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-27/32*d/(a*d-b*c)^4/(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-45/64/(a*d
-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)*b^2+3/64*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)*a^2-27/32*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-45/64/(a*d-b*c)^4*c/(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/12
8*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))
)*a^2-27/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2)))*a*b-45/128/(a*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2)))*b^2+1/2*b^2/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*a*d-1/2*
b^3/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*c+9/16*b/(a*d-b*c)^4/(a/b)^(1/4
)*2^(1/2)*a*d*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+9/8*b/(a*d-b*c)^4/(a/b)^
(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*b/(a*
d-b*c)^4/(a/b)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)+3/16*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/
b)^(1/2)))+3/8*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+3/8*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)
*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.529946, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.498 $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}-\frac{b^{7/4}(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}+\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d\sqrt{x}(ad+23bc)}{16c\,(c+dx^2)\,(bc-ad)^3}-\frac{3d\sqrt{x}}{4\,(c+dx^2)^2\,(bc-ad)^2}-\frac{\sqrt{x}}{2\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)}$$

```
[Out] (-3*d*Sqrt[x])/(4*(b*c - a*d)^2*(c + d*x^2)^2) - Sqrt[x]/(2*(b*c
- a*d)*(a + b*x^2)*(c + d*x^2)^2) - (d*(23*b*c + a*d)*Sqrt[x])/(1
6*c*(b*c - a*d)^3*(c + d*x^2)) - (b^(7/4)*(b*c + 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c -
a*d)^4) + (b^(7/4)*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*
b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77
*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (b^(7/4)*(b
*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (b^(7/4)*(b*c + 11*a*
d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8
*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*b^2*c^2 + 22*a*b*c
*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*
c^2 + 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4)
```

Rubi [A] time = 2.08094, antiderivative size = 703, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}-\frac{b^{7/4}(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}+\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d\sqrt{x}(ad+23bc)}{16c\left(c+dx^2\right)(bc-ad)^3}-\frac{3d\sqrt{x}}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{\sqrt{x}}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*Sqrt[x])/(4*(b*c - a*d)^2*(c + d*x^2)^2) - Sqrt[x]/(2*(b*c
- a*d)*(a + b*x^2)*(c + d*x^2)^2) - (d*(23*b*c + a*d)*Sqrt[x])/(1
6*c*(b*c - a*d)^3*(c + d*x^2)) - (b^(7/4)*(b*c + 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c -
a*d)^4) + (b^(7/4)*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*
b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77
*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (b^(7/4)*(b
*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (b^(7/4)*(b*c + 11*a*
d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8
*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*b^2*c^2 + 22*a*b*c
*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*
c^2 + 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.44656, size = 603, normalized size = 0.86

$$-\frac{8\sqrt{2}b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{8\sqrt{2}b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{16\sqrt{2}b^{7/4}(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{16}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^2*(b*c - a*d)*Sqrt[x])/(a + b*x^2) - (32*d*(b*c - a*d)^2*
Sqrt[x])/(c + d*x^2)^2 + (8*d*(-(b*c) + a*d)*(15*b*c + a*d)*Sqrt[
x])/(c*(c + d*x^2)) - (16*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (16*Sqrt[2]*b^(7
/4)*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/a^(3/4) + (2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^
2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2*Sq
rt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (8*Sqrt[2]*b^(7/4)*(b
*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/a^(3/4) + (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]
*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4) + (Sqrt[2]*d^(3
/4)*(-77*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(128*(b*c - a*d)^4
)
```

Maple [A] time = 0.034, size = 1094, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(5/2)*a^2+7/8*d^3/(a*d-b*c)^
4/(d*x^2+c)^2*x^(5/2)*a*b-15/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(
5/2)*b^2+11/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*c*a*b-19/16*d/(
a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*b^2*c^2-3/16*d^3/(a*d-b*c)^4/(d*x^
2+c)^2*x^(1/2)*a^2+3/64*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-11/32*d^2/(a*d-b*c)^4/c*
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-77/
64*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)+1)*b^2+3/64*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-11/32*d^2/(a*d-b*c)^4/c*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-77/64*d/
(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*b^2+3/128*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2)))*a^2-11/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*
ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))*a*b-77/128*d/(a*d-b*c)^4*(c/d)^(1/4)*2
^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+1/2*b^2/(a*d-b*c)^4*x^(1/2)
/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*c+11/16*b^2/
(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d+1/16
*b^3/(a*d-b*c)^4*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
*c+11/8*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)*d+1/8*b^3/(a*d-b*c)^4*(a/b)^(1/4)/a*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c+11/8*b^2/(a*d-b*c)^4*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d+1/8*b^3/(a*d-
b*c)^4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.561003, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.499 $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=739

$$\frac{b^{9/4}(bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4} - \frac{b^{9/4}(bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4} - \frac{b^{9/4}(bc-13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4} + \frac{b^{9/4}(bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4} + \frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3} + \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4} - \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4} - \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4} + \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4} + \frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)} + \frac{dx^{3/2}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] (d*(2*b*c + a*d)*x^(3/2))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*x^(3/2))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^
2*c^2 + 21*a*b*c*d - 5*a^2*d^2)*x^(3/2))/(16*a*c^2*(b*c - a*d)^3*
(c + d*x^2)) - (b^(9/4)*(b*c - 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) + (b^(9/4)
*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*
c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(9/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*
b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^4) - (b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)
*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(9/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c
*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^4)
```

Rubi [A] time = 2.45956, antiderivative size = 739, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$\frac{b^{9/4}(bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4} - \frac{b^{9/4}(bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4} - \frac{b^{9/4}(bc-13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4} + \frac{b^{9/4}(bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4} + \frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3} + \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4} - \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4} - \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4} + \frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4} + \frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)} + \frac{dx^{3/2}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x^(3/2))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*x^(3/2))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^
2*c^2 + 21*a*b*c*d - 5*a^2*d^2)*x^(3/2))/(16*a*c^2*(b*c - a*d)^3*
(c + d*x^2)) - (b^(9/4)*(b*c - 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) + (b^(9/4)
*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*
c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(9/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*
b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^4) - (b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)
*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(9/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c
*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.42775, size = 691, normalized size = 0.94

$$\frac{1}{128}\left(\frac{8\sqrt{2}b^{9/4}(bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^4}+\frac{8\sqrt{2}b^{9/4}(13ad-bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^4}+\frac{16\sqrt{2}b^{9/4}(13ad-bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}(bc-ad)^4}+\frac{16\sqrt{2}b^{9/4}(bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}(bc-ad)^4}+\frac{\sqrt{2}d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^4}-\frac{\sqrt{2}d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^4}-\frac{2\sqrt{2}d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}(bc-ad)^4}+\frac{2\sqrt{2}d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}(bc-ad)^4}-\frac{64b^3x^{3/2}}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{8d^2x^{3/2}(21bc-5ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{32d^2x^{3/2}}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^3*x^(3/2))/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (32*d^2*x^(
3/2))/(c*(b*c - a*d)^2*(c + d*x^2)^2) + (8*d^2*(21*b*c - 5*a*d)*x
^(3/2))/(c^2*(b*c - a*d)^3*(c + d*x^2)) + (16*Sqrt[2]*b^(9/4)*(-(
b*c) + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^
(5/4)*(b*c - a*d)^4) + (16*Sqrt[2]*b^(9/4)*(b*c - 13*a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(b*c - a*d)^4) -
 (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^4)
+ (2*Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^4)
 + (8*Sqrt[2]*b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^4) + (8*Sqrt
[2]*b^(9/4)*(-(b*c) + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^4) + (Sqrt[2]*d^(5/
4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c - a*d)^4) - (S
qrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c -
a*d)^4))/128
```

Maple [A] time = 0.034, size = 1100, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 5/16*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^(7/2)*a^2-13/8*d^4/(a*d-b*
c)^4/(d*x^2+c)^2/c*x^(7/2)*a*b+21/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*
```

```
x^(7/2)*b^2+9/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(3/2)*a^2-17/8*d
^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a*b+25/16*d^2/(a*d-b*c)^4/(d*x
^2+c)^2*c*x^(3/2)*b^2+5/64*d^3/(a*d-b*c)^4/c^2/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-13/32*d^2/(a*d-b*c)^4
/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+
117/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)*b^2+5/128*d^3/(a*d-b*c)^4/c^2/(c/d)^(1/4)*2^(1/2)*l
n((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2)))*a^2-13/64*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)
*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+117/128*d/(a*d-b*c)^4/(c/
d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+5/64*d^3/(a*d-b*c
)^4/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
*a^2-13/32*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)*a*b+117/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/2*b^3/(a*d-b*c)^4*x
^(3/2)/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x^(3/2)/(b*x^2+a)*c-13/8
*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)*d+1/8*b^3/(a*d-b*c)^4/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)*c-13/16*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^
(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d+1/16*b^3/(a*d-b*c)^4/a/(a/b)^(
1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c-13/8*b^2/(a*d-b*c)^4/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d+1/8*b^3
/(a*d-b*c)^4/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.566899, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.500 $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=739

$$-\frac{3b^{11/4}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}-\frac{3b^{11/4}(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{b\sqrt{x}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] (d*(2*b*c + a*d)*Sqrt[x])/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*Sqrt[x])/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^
2*c^2 + 23*a*b*c*d - 7*a^2*d^2)*Sqrt[x])/(16*a*c^2*(b*c - a*d)^3*
(c + d*x^2)) - (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(
11/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30
*a*b*c*d + 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2
- 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) - (3*b^(11/4)*(b*c -
5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(11/4)*(b*c - 5*a*d)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
 + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2
*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

Rubi [A] time = 2.05292, antiderivative size = 739, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{3b^{11/4}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}-\frac{3b^{11/4}(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{b\sqrt{x}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*Sqrt[x])/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*Sqrt[x])/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^
2*c^2 + 23*a*b*c*d - 7*a^2*d^2)*Sqrt[x])/(16*a*c^2*(b*c - a*d)^3*
(c + d*x^2)) - (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(
11/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30
*a*b*c*d + 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2
- 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) - (3*b^(11/4)*(b*c -
5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(11/4)*(b*c - 5*a*d)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
 + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2
*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.70825, size = 692, normalized size = 0.94

$$\frac{1}{128}\left(\frac{24\sqrt{2}b^{11/4}(5ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^4}\right.$$
$$+\frac{24\sqrt{2}b^{11/4}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}(bc-ad)^4}$$
$$+\frac{48\sqrt{2}b^{11/4}(5ad-bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^4}+\frac{48\sqrt{2}b^{11/4}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}(bc-ad)^4}$$
$$-\frac{3\sqrt{2}d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^4}$$
$$+\frac{3\sqrt{2}d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{11/4}(bc-ad)^4}$$
$$-\frac{6\sqrt{2}d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{11/4}(bc-ad)^4}$$
$$+\frac{6\sqrt{2}d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{11/4}(bc-ad)^4}$$
$$\left.-\frac{64b^3\sqrt{x}}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{8d^2\sqrt{x}(23bc-7ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{32d^2\sqrt{x}}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^3*Sqrt[x])/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (32*d^2*Sqr
t[x])/(c*(b*c - a*d)^2*(c + d*x^2)^2) + (8*d^2*(23*b*c - 7*a*d)*S
qrt[x])/(c^2*(b*c - a*d)^3*(c + d*x^2)) + (48*Sqrt[2]*b^(11/4)*(-
(b*c) + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^
(7/4)*(b*c - a*d)^4) + (48*Sqrt[2]*b^(11/4)*(b*c - 5*a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*(b*c - a*d)^4) -
 (6*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[
1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^4)
+ (6*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^4)
 + (24*Sqrt[2]*b^(11/4)*(-(b*c) + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^4) + (24
*Sqrt[2]*b^(11/4)*(b*c - 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*(b*c - a*d)^4) - (3*Sqrt[2]*d
^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(11/4)*(b*c - a*d)^4)
+ (3*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(11/4)*(
b*c - a*d)^4))/128
```

Maple [A] time = 0.033, size = 1124, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 7/16*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^(5/2)*a^2-15/8*d^4/(a*d-b*
c)^4/(d*x^2+c)^2/c*x^(5/2)*a*b+23/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*
```

```
x^(5/2)*b^2+11/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(1/2)*a^2-19/8*
d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b+27/16*d^2/(a*d-b*c)^4/(d*
x^2+c)^2*c*x^(1/2)*b^2+21/64*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-45/32*d^3/(a*d-b*c)
^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
a*b+165/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)*b^2+21/64*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-45/32*d^3/(a*d-b
*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)*a*b+165/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)*b^2+21/128*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/
4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2-45/64*d^3/(a*d-b*c)^4/c
^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+165/128*d^2
/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2
-1/2*b^3/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x^
(1/2)/(b*x^2+a)*c-15/8*b^3/(a*d-b*c)^4/a*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d+3/8*b^4/(a*d-b*c)^4/a^2*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c-15/8*b^3/(
a*d-b*c)^4/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)*d+3/8*b^4/(a*d-b*c)^4/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)*c-15/16*b^3/(a*d-b*c)^4/a*(a/b)^(1/4)*2^
(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*d+3/16*b^4/(a*d-b*c)^4/a^2*(a/b)
^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*sqrt(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*sqrt(x)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.498943, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*sqrt(x)),x, algorithm="giac")
```

```
[Out] Done
```

3.501 $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=805

$$\frac{(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}$$
$$-\frac{(5bc-17ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}$$
$$+\frac{(5bc-17ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)\sqrt{x}\,(bx^2+a)\,(dx^2+c)^2}$$
$$+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$-\frac{40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}}$$
$$+\frac{d\left(8b^2c^2+25abdc-9a^2d^2\right)}{16ac^2(bc-ad)^3\sqrt{x}\,(dx^2+c)}+\frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,(dx^2+c)^2}$$

```
[Out] -(40*b^3*c^3 - 96*a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(16
*a^2*c^3*(b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c -
 a*d)^2*Sqrt[x]*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*Sqrt[x]*(a +
b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 25*a*b*c*d - 9*a^2*d^2))/
(16*a*c^2*(b*c - a*d)^3*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*(5*b*c -
 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(9/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d
)^4) + (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c
- a*d)^4) - (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*
(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c -
a*d)^4) + (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4)
 - (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13
/4)*(b*c - a*d)^4) + (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2
*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

Rubi [A] time = 3.05594, antiderivative size = 805, normalized size of antiderivative = 1., number of

steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$\frac{(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{(5bc-17ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)\sqrt{x}\,(bx^2+a)\,(dx^2+c)^2}+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}}+\frac{d\left(8b^2c^2+25abdc-9a^2d^2\right)}{16ac^2(bc-ad)^3\sqrt{x}\,(dx^2+c)}+\frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,(dx^2+c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(40*b^3*c^3 - 96*a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(16
*a^2*c^3*(b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c -
 a*d)^2*Sqrt[x]*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*Sqrt[x]*(a +
b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 25*a*b*c*d - 9*a^2*d^2))/
(16*a*c^2*(b*c - a*d)^3*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*(5*b*c -
 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(9/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d
)^4) + (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c
- a*d)^4) - (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*
(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c -
a*d)^4) + (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4)
 - (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13
/4)*(b*c - a*d)^4) + (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2
*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.14473, size = 706, normalized size = 0.88

$$\frac{1}{128}\left(\frac{8\sqrt{2}b^{13/4}(17ad-5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^4}+\frac{8\sqrt{2}b^{13/4}(5bc-17ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}(bc-ad)^4}+\frac{16\sqrt{2}b^{13/4}(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}(bc-ad)^4}+\frac{16\sqrt{2}b^{13/4}(17ad-5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}(bc-ad)^4}+\frac{64b^4x^{3/2}}{a^2(a+bx^2)(ad-bc)^3}-\frac{\sqrt{2}d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^4}+\frac{\sqrt{2}d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{13/4}(bc-ad)^4}+\frac{2\sqrt{2}d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{13/4}(bc-ad)^4}-\frac{2\sqrt{2}d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{13/4}(bc-ad)^4}-\frac{256}{a^2c^3\sqrt{x}}+\frac{8d^3x^{3/2}(13ad-29bc)}{c^3(c+dx^2)(bc-ad)^3}-\frac{32d^3x^{3/2}}{c^2(c+dx^2)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a^2*c^3*Sqrt[x]) + (64*b^4*x^(3/2))/(a^2*(-(b*c) + a*d)^3*
(a + b*x^2)) - (32*d^3*x^(3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)^2)
 + (8*d^3*(-29*b*c + 13*a*d)*x^(3/2))/(c^3*(b*c - a*d)^3*(c + d*x
^2)) + (16*Sqrt[2]*b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(9/4)*(b*c - a*d)^4) + (16*Sqrt[2]*
b^(13/4)*(-5*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(a^(9/4)*(b*c - a*d)^4) + (2*Sqrt[2]*d^(9/4)*(221*b^2*c^
2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(13/4)*(b*c - a*d)^4) - (2*Sqrt[2]*d^(9/4)*(221*b^
2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(c^(13/4)*(b*c - a*d)^4) + (8*Sqrt[2]*b^(13/4)*(-
5*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(a^(9/4)*(b*c - a*d)^4) + (8*Sqrt[2]*b^(13/4)*(5*b*c -
 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(a^(9/4)*(b*c - a*d)^4) - (Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 170
*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(c^(13/4)*(b*c - a*d)^4) + (Sqrt[2]*d^(9/4)*(22
1*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(13/4)*(b*c - a*d)^4))/128
```

Maple [A] time = 0.043, size = 1143, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -13/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a^2+21/8*d^5/c^2/(
a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a*b-29/16*d^4/c/(a*d-b*c)^4/(d*x^2
+c)^2*x^(7/2)*b^2-17/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a
^2+25/8*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a*b-33/16*d^3/(a*d-
b*c)^4/(d*x^2+c)^2*x^(3/2)*b^2-45/128*d^4/c^3/(a*d-b*c)^4/(c/d)^(
1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))-45/64*d^4/c^3/(a*d-b*
c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)-45/64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)+85/64*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^
(1/2)*a*b*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+85/32*d^3/c^2/(a*d-b*c)^4/(c
/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+85/32
*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)-221/128*d^2/c/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b
^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2)))-221/64*d^2/c/(a*d-b*c)^4/(c/d)^(1/4
)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-221/64*d^2/c/
(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)-2/a^2/c^3/x^(1/2)+1/2*b^4/a/(a*d-b*c)^4*x^(3/2)/(b*x^2+a
)*d-1/2*b^5/a^2/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*c+17/16*b^3/a/(a*d-
b*c)^4/(a/b)^(1/4)*2^(1/2)*d*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+17/8*b^3/
a/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)+17/8*b^3/a/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)-5/16*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2
^(1/2)*c*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-5/8*b^4/a^2/(a*d-b*c)^4/(a/b)
^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8*b^4/a^
2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)-1)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.664914, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(3/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.502 $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=805

$$\frac{(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}$$
$$+\frac{(7bc-19ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}a^{11/4}(bc-ad)^4}$$
$$-\frac{(7bc-19ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{3/2}(bx^2+a)(dx^2+c)^2}$$
$$+\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}$$
$$-\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}$$
$$+\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}$$
$$-\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}$$
$$+\frac{d\left(8b^2c^2+27abdc-11a^2d^2\right)}{16ac^2(bc-ad)^3x^{3/2}(dx^2+c)}$$
$$-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}(dx^2+c)^2}$$

```
[Out] -(56*b^3*c^3 - 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(48
*a^2*c^3*(b*c - a*d)^3*x^(3/2)) + (d*(2*b*c + a*d))/(4*a*c*(b*c -
 a*d)^2*x^(3/2)*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(3/2)*(a +
b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 27*a*b*c*d - 11*a^2*d^2))
/(16*a*c^2*(b*c - a*d)^3*x^(3/2)*(c + d*x^2)) + (b^(15/4)*(7*b*c
- 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c -
a*d)^4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcT
an[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(
b*c - a*d)^4) - (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(1
5/4)*(b*c - a*d)^4) + (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(
b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c -
a*d)^4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
2]*c^(15/4)*(b*c - a*d)^4) - (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d
 + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

Rubi [A] time = 2.92887, antiderivative size = 805, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$\frac{(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{(7bc-19ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{3/2}(bx^2+a)(dx^2+c)^2}+\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}-\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}+\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}-\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}+\frac{d(8b^2c^2+27abdc-11a^2d^2)}{16ac^2(bc-ad)^3x^{3/2}(dx^2+c)}-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}(dx^2+c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(56*b^3*c^3 - 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(48
*a^2*c^3*(b*c - a*d)^3*x^(3/2)) + (d*(2*b*c + a*d))/(4*a*c*(b*c -
 a*d)^2*x^(3/2)*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(3/2)*(a +
b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 27*a*b*c*d - 11*a^2*d^2))
/(16*a*c^2*(b*c - a*d)^3*x^(3/2)*(c + d*x^2)) + (b^(15/4)*(7*b*c
- 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c -
a*d)^4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcT
an[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(
b*c - a*d)^4) - (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(1
5/4)*(b*c - a*d)^4) + (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(
b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c -
a*d)^4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
2]*c^(15/4)*(b*c - a*d)^4) - (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d
 + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.37201, size = 707, normalized size = 0.88

$$\frac{1}{384}\left(\frac{24\sqrt{2}b^{15/4}(7bc-19ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}(bc-ad)^4}+\frac{24\sqrt{2}b^{15/4}(19ad-7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}(bc-ad)^4}+\frac{48\sqrt{2}b^{15/4}(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}(bc-ad)^4}+\frac{48\sqrt{2}b^{15/4}(19ad-7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}(bc-ad)^4}+\frac{192b^4\sqrt{x}}{a^2(a+bx^2)(ad-bc)^3}+\frac{3\sqrt{2}d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{15/4}(bc-ad)^4}-\frac{3\sqrt{2}d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{15/4}(bc-ad)^4}+\frac{6\sqrt{2}d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{15/4}(bc-ad)^4}-\frac{6\sqrt{2}d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{15/4}(bc-ad)^4}-\frac{256}{a^2c^3x^{3/2}}+\frac{24d^3\sqrt{x}(15ad-31bc)}{c^3(c+dx^2)(bc-ad)^3}-\frac{96d^3\sqrt{x}}{c^2(c+dx^2)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a^2*c^3*x^(3/2)) + (192*b^4*Sqrt[x])/(a^2*(-(b*c) + a*d)^3
*(a + b*x^2)) - (96*d^3*Sqrt[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)^2
) + (24*d^3*(-31*b*c + 15*a*d)*Sqrt[x])/(c^3*(b*c - a*d)^3*(c + d
*x^2)) + (48*Sqrt[2]*b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*(b*c - a*d)^4) + (48*Sqrt[
2]*b^(15/4)*(-7*b*c + 19*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(a^(11/4)*(b*c - a*d)^4) + (6*Sqrt[2]*d^(11/4)*(285*b
^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(c^(15/4)*(b*c - a*d)^4) - (6*Sqrt[2]*d^(11/4)*(
285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)])/(c^(15/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(1
5/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(a^(11/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(15/4)*
(-7*b*c + 19*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(a^(11/4)*(b*c - a*d)^4) + (3*Sqrt[2]*d^(11/4)*(285*
b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(15/4)*(b*c - a*d)^4) - (3*Sqrt
[2]*d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(15/4)*(b*c -
 a*d)^4))/384
```

Maple [A] time = 0.049, size = 1143, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -15/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a^2+23/8*d^5/c^2/(
a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a*b-31/16*d^4/c/(a*d-b*c)^4/(d*x^2
+c)^2*x^(5/2)*b^2-19/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a
^2+27/8*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b-35/16*d^3/(a*d-
b*c)^4/(d*x^2+c)^2*x^(1/2)*b^2-77/64*d^5/c^4/(a*d-b*c)^4*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+133/32*d^4/
c^3/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*a*b-285/64*d^3/c^2/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-77/64*d^5/c^4/(a*d-b*c)^4*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+133/3
2*d^4/c^3/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)*a*b-285/64*d^3/c^2/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/128*d^5/c^4/(a*d-b*
c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a^2+133/64*d^
4/c^3/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*
a*b-285/128*d^3/c^2/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2)))*b^2-2/3/a^2/c^3/x^(3/2)+1/2*b^4/a/(a*d-b*c)^4*x^(1/
2)/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*c+19/8*b
^4/a^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)*d-7/8*b^5/a^3/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c+19/8*b^4/a^2/(a*d-b*c)^4*(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d-7/8*b^5/a^3/
(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)*c+19/16*b^4/a^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2)))*d-7/16*b^5/a^3/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*ln(
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(5/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.534704, size = 1, normalized size = 0.

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(5/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.503 $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=881

$$-\frac{3(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}$$
$$+\frac{3(3bc-7ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}a^{13/4}(bc-ad)^4}$$
$$-\frac{3(3bc-7ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{5/2}(bx^2+a)(dx^2+c)^2}$$
$$-\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$-\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$+\frac{3\left(24b^4c^4-32ab^3dc^3-32a^2b^2d^2c^2+87a^3bd^3c-39a^4d^4\right)}{16a^3c^4(bc-ad)^3\sqrt{x}}+\frac{d\left(8b^2c^2+29abdc-13a^2d^2\right)}{16ac^2(bc-ad)^3x^{5/2}(dx^2+c)}$$
$$+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}(dx^2+c)^2}-\frac{3\left(24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}}$$

```
[Out] (-3*(24*b^3*c^3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3))/
(80*a^2*c^3*(b*c - a*d)^3*x^(5/2)) + (3*(24*b^4*c^4 - 32*a*b^3*c^
3*d - 32*a^2*b^2*c^2*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4))/(16*a^3*
c^4*(b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)
^2*x^(5/2)*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(5/2)*(a + b*x^2
)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 29*a*b*c*d - 13*a^2*d^2))/(16*
a*c^2*(b*c - a*d)^3*x^(5/2)*(c + d*x^2)) - (3*b^(17/4)*(3*b*c - 7
*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a
^(13/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d
)^4) - (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b
*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^
2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(
17/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)
*(b*c - a*d)^4) - (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c
 - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(17/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119*b^2*c^2 - 126*
a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

Rubi [A] time = 3.69103, antiderivative size = 881, normalized size of antiderivative = 1., number of

steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$

$$-\frac{3(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}a^{13/4}(bc-ad)^4}-\frac{3(3bc-7ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{5/2}(bx^2+a)(dx^2+c)^2}-\frac{3d^{13/4}(119b^2c^2-126abdc+39a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}+\frac{3d^{13/4}(119b^2c^2-126abdc+39a^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}+\frac{3d^{13/4}(119b^2c^2-126abdc+39a^2d^2)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{17/4}(bc-ad)^4}-\frac{3d^{13/4}(119b^2c^2-126abdc+39a^2d^2)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{17/4}(bc-ad)^4}+\frac{3(24b^4c^4-32ab^3dc^3-32a^2b^2d^2c^2+87a^3bd^3c-39a^4d^4)}{16a^3c^4(bc-ad)^3\sqrt{x}}+\frac{d(8b^2c^2+29abdc-13a^2d^2)}{16ac^2(bc-ad)^3x^{5/2}(dx^2+c)}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}(dx^2+c)^2}-\frac{3(24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*(24*b^3*c^3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3))/
(80*a^2*c^3*(b*c - a*d)^3*x^(5/2)) + (3*(24*b^4*c^4 - 32*a*b^3*c^
3*d - 32*a^2*b^2*c^2*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4))/(16*a^3*
c^4*(b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)
^2*x^(5/2)*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(5/2)*(a + b*x^2
)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 29*a*b*c*d - 13*a^2*d^2))/(16*
a*c^2*(b*c - a*d)^3*x^(5/2)*(c + d*x^2)) - (3*b^(17/4)*(3*b*c - 7
*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a
^(13/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d
)^4) - (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b
*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^
2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(
17/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)
*(b*c - a*d)^4) - (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c
 - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(17/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119*b^2*c^2 - 126*
a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

[Out] Timed out

Mathematica [A] time = 4.51477, size = 729, normalized size = 0.83

$$\frac{1}{640}\left(\frac{120\sqrt{2}b^{17/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}(bc-ad)^4}+\frac{120\sqrt{2}b^{17/4}(7ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}(bc-ad)^4}+\frac{240\sqrt{2}b^{17/4}(7ad-3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{13/4}(bc-ad)^4}+\frac{240\sqrt{2}b^{17/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{13/4}(bc-ad)^4}-\frac{320b^5x^{3/2}}{a^3\left(a+bx^2\right)(ad-bc)^3}+\frac{1280(3ad+2bc)}{a^3c^4\sqrt{x}}+\frac{15\sqrt{2}d^{13/4}\left(39a^2d^2-126abcd+119b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{17/4}(bc-ad)^4}-\frac{15\sqrt{2}d^{13/4}\left(39a^2d^2-126abcd+119b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{17/4}(bc-ad)^4}-\frac{30\sqrt{2}d^{13/4}\left(39a^2d^2-126abcd+119b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{17/4}(bc-ad)^4}+\frac{30\sqrt{2}d^{13/4}\left(39a^2d^2-126abcd+119b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{17/4}(bc-ad)^4}-\frac{256}{a^2c^3x^{5/2}}+\frac{40d^4x^{3/2}(37bc-21ad)}{c^4\left(c+dx^2\right)(bc-ad)^3}+\frac{160d^4x^{3/2}}{c^3\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-256/(a^2*c^3*x^(5/2)) + (1280*(2*b*c + 3*a*d))/(a^3*c^4*Sqrt[x]
) - (320*b^5*x^(3/2))/(a^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (160*d
^4*x^(3/2))/(c^3*(b*c - a*d)^2*(c + d*x^2)^2) + (40*d^4*(37*b*c -
 21*a*d)*x^(3/2))/(c^4*(b*c - a*d)^3*(c + d*x^2)) + (240*Sqrt[2]*
b^(17/4)*(-3*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(a^(13/4)*(b*c - a*d)^4) + (240*Sqrt[2]*b^(17/4)*(3*b*c -
 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(13/4)*
(b*c - a*d)^4) - (30*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d
+ 39*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(
17/4)*(b*c - a*d)^4) + (30*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*
b*c*d + 39*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(c^(17/4)*(b*c - a*d)^4) + (120*Sqrt[2]*b^(17/4)*(3*b*c - 7*a*d
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^
(13/4)*(b*c - a*d)^4) + (120*Sqrt[2]*b^(17/4)*(-3*b*c + 7*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(13/
4)*(b*c - a*d)^4) + (15*Sqrt[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c
*d + 39*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(c^(17/4)*(b*c - a*d)^4) - (15*Sqrt[2]*d^(13/4)*(119*
b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(17/4)*(b*c - a*d)^4))/640
```

Maple [A] time = 0.046, size = 1170, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -189/32*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)-189/32*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2
^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+6/x^(1/2)/a^2/c^
4*d+4/x^(1/2)/a^3/c^3*b+21/16*d^7/c^4/(a*d-b*c)^4/(d*x^2+c)^2*x^(
7/2)*a^2+37/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*b^2+25/16*
d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+41/16*d^4/c/(a*d-b*c)
^4/(d*x^2+c)^2*x^(3/2)*b^2-1/2*b^5/a^2/(a*d-b*c)^4*x^(3/2)/(b*x^2
+a)*d-189/64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*ln((x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2)))-2/5/a^2/c^3/x^(5/2)+117/128*d^5/c^4/(a*d-b*c)^
4/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+117/64*d^5/c
^4/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)+117/64*d^5/c^4/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+357/128*d^3/c^2/(a*d-b*c)^4/(
c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+357/64*d^3/c^2/
(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)+357/64*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-21/16*b^4/a^2/(a*d-b*c)^4/(a/b)^
(1/4)*2^(1/2)*d*ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x
+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-21/8*b^4/a^2/(a*d-b*c)
^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-21
/8*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)-1)+9/16*b^5/a^3/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*
ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2)))+9/8*b^5/a^3/(a*d-b*c)^4/(a/b)^(1/4)*2^
(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*b^5/a^3/(a*d-b*
c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+
1/2*b^6/a^3/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*c-29/8*d^6/c^3/(a*d-b*c
)^4/(d*x^2+c)^2*x^(7/2)*a*b-33/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*
x^(3/2)*a*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(7/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.5731, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3*x^(7/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.504 $\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=103

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (a*(2*A*b - 3*a*B)*
(a + b*x^2)^(5/2))/(5*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7
*b^4) + (B*(a + b*x^2)^(9/2))/(9*b^4)
```

Rubi [A] time = 0.232375, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (a*(2*A*b - 3*a*B)*
(a + b*x^2)^(5/2))/(5*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7
*b^4) + (B*(a + b*x^2)^(9/2))/(9*b^4)
```

Rubi in Sympy [A] time = 25.5864, size = 92, normalized size = 0.89

$$\frac{B\left(a+bx^2\right)^{\frac{9}{2}}}{9b^4}+\frac{a^2\left(a+bx^2\right)^{\frac{3}{2}}(Ab-Ba)}{3b^4}-\frac{a\left(a+bx^2\right)^{\frac{5}{2}}(2Ab-3Ba)}{5b^4}+\frac{\left(a+bx^2\right)^{\frac{7}{2}}(Ab-3Ba)}{7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] B*(a + b*x**2)**(9/2)/(9*b**4) + a**2*(a + b*x**2)**(3/2)*(A*b -
B*a)/(3*b**4) - a*(a + b*x**2)**(5/2)*(2*A*b - 3*B*a)/(5*b**4) +
(a + b*x**2)**(7/2)*(A*b - 3*B*a)/(7*b**4)
```

Mathematica [A] time = 0.0829751, size = 75, normalized size = 0.73

$$\frac{\left(a+bx^2\right)^{3/2}\left(-16a^3B+24a^2b\left(A+Bx^2\right)-6ab^2x^2\left(6A+5Bx^2\right)+5b^3x^4\left(9A+7Bx^2\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^2) - 6*a*b^2*x^
2*(6*A + 5*B*x^2) + 5*b^3*x^4*(9*A + 7*B*x^2)))/(315*b^4)
```

Maple [A] time = 0.009, size = 77, normalized size = 0.8

$$\frac{35\,Bx^6b^3+45\,Ab^3x^4-30\,Bab^2x^4-36\,Aab^2x^2+24\,Ba^2bx^2+24\,Aa^2b-16\,Ba^3}{315\,b^4}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(3/2)*(35*B*b^3*x^6+45*A*b^3*x^4-30*B*a*b^2*x^4-3
6*A*a*b^2*x^2+24*B*a^2*b*x^2+24*A*a^2*b-16*B*a^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234769, size = 134, normalized size = 1.3

$$\frac{\left(35\,Bb^4x^8 + 5\,\left(Bab^3 + 9\,Ab^4\right)x^6 - 16\,Ba^4 + 24\,Aa^3b - 3\,\left(2\,Ba^2b^2 - 3\,Aab^3\right)x^4 + 4\,\left(2\,Ba^3b - 3\,Aa^2b^2\right)x^2\right)\sqrt{bx^2 + a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^5,x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(B*a*b^3 + 9*A*b^4)*x^6 - 16*B*a^4 + 24*A
*a^3*b - 3*(2*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 4*(2*B*a^3*b - 3*A*a^2
*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 4.5192, size = 212, normalized size = 2.06

$$\begin{cases} \frac{8Aa^3\sqrt{a+bx^2}}{105b^3} - \frac{4Aa^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{Aax^4\sqrt{a+bx^2}}{35b} + \frac{Ax^6\sqrt{a+bx^2}}{7} - \frac{16Ba^4\sqrt{a+bx^2}}{315b^4} + \frac{8Ba^3x^2\sqrt{a+bx^2}}{315b^3} - \frac{2Ba^2x^4\sqrt{a+bx^2}}{105b^2} + \frac{Bax^6\sqrt{a+bx^2}}{63b} + \frac{Bx^8\sqrt{a+bx^2}}{9} \\ \sqrt{a}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*A*a**2*x**2*s
qrt(a + b*x**2)/(105*b**2) + A*a*x**4*sqrt(a + b*x**2)/(35*b) + A
*x**6*sqrt(a + b*x**2)/7 - 16*B*a**4*sqrt(a + b*x**2)/(315*b**4)
+ 8*B*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*B*a**2*x**4*sqrt(
a + b*x**2)/(105*b**2) + B*a*x**6*sqrt(a + b*x**2)/(63*b) + B*x**
8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*(A*x**6/6 + B*x**8/8),
True))
```

GIAC/XCAS [A] time = 0.239926, size = 144, normalized size = 1.4

$$\frac{\frac{3\left(15\,(bx^2+a)^{\frac{7}{2}} - 42\,(bx^2+a)^{\frac{5}{2}}a + 35\,(bx^2+a)^{\frac{3}{2}}a^2\right)A}{b^2} + \frac{\left(35\,(bx^2+a)^{\frac{9}{2}} - 135\,(bx^2+a)^{\frac{7}{2}}a + 189\,(bx^2+a)^{\frac{5}{2}}a^2 - 105\,(bx^2+a)^{\frac{3}{2}}a^3\right)B}{b^3}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^5,x, algorithm="giac")
```

```
[Out] 1/315*(3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x
^2 + a)^(3/2)*a^2)*A/b^2 + (35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a
)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3
)*B/b^3)/b
```

3.505 $\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=155

$$\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}-\frac{a^2x\sqrt{a+bx^2}(8Ab-5aB)}{128b^3}+\frac{ax^3\sqrt{a+bx^2}(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}(8Ab-5aB)}{48b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

```
[Out] -(a^2*(8*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(128*b^3) + (a*(8*A*b -
5*a*B)*x^3*Sqrt[a + b*x^2])/(192*b^2) + ((8*A*b - 5*a*B)*x^5*Sqrt
[a + b*x^2])/(48*b) + (B*x^5*(a + b*x^2)^(3/2))/(8*b) + (a^3*(8*A
*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rubi [A] time = 0.223511, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}-\frac{a^2x\sqrt{a+bx^2}(8Ab-5aB)}{128b^3}+\frac{ax^3\sqrt{a+bx^2}(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}(8Ab-5aB)}{48b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -(a^2*(8*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(128*b^3) + (a*(8*A*b -
5*a*B)*x^3*Sqrt[a + b*x^2])/(192*b^2) + ((8*A*b - 5*a*B)*x^5*Sqrt
[a + b*x^2])/(48*b) + (B*x^5*(a + b*x^2)^(3/2))/(8*b) + (a^3*(8*A
*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rubi in Sympy [A] time = 22.8884, size = 144, normalized size = 0.93

$$\frac{Bx^5\left(a+bx^2\right)^{\frac{3}{2}}}{8b}+\frac{a^3\left(8Ab-5Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{7}{2}}}-\frac{a^2x\sqrt{a+bx^2}\left(8Ab-5Ba\right)}{128b^3}+\frac{ax^3\sqrt{a+bx^2}\left(8Ab-5Ba\right)}{192b^2}+\frac{x^5\sqrt{a+bx^2}\left(8Ab-5Ba\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] B*x**5*(a + b*x**2)**(3/2)/(8*b) + a**3*(8*A*b - 5*B*a)*atanh(sqr
t(b)*x/sqrt(a + b*x**2))/(128*b**(7/2)) - a**2*x*sqrt(a + b*x**2)
*(8*A*b - 5*B*a)/(128*b**3) + a*x**3*sqrt(a + b*x**2)*(8*A*b - 5*
B*a)/(192*b**2) + x**5*sqrt(a + b*x**2)*(8*A*b - 5*B*a)/(48*b)
```

Mathematica [A] time = 0.131558, size = 123, normalized size = 0.79

$$\sqrt{a+bx^2}\left(\frac{a^2x(5aB-8Ab)}{128b^3}-\frac{ax^3(5aB-8Ab)}{192b^2}+\frac{x^5(aB+8Ab)}{48b}+\frac{Bx^7}{8}\right)-\frac{a^3(5aB-8Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{128b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((a^2*(-8*A*b + 5*a*B)*x)/(128*b^3) - (a*(-8*A*b
+ 5*a*B)*x^3)/(192*b^2) + ((8*A*b + a*B)*x^5)/(48*b) + (B*x^7)/8)
 - (a^3*(-8*A*b + 5*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(128
*b^(7/2))
```

Maple [A] time = 0.011, size = 181, normalized size = 1.2

$$\frac{Ax^3}{6b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{aAx}{8b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{a^2Ax}{16b^2}\sqrt{bx^2+a}+\frac{Aa^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{Bx^5}{8b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5Bax^3}{48b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5Bxa^2}{64b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5Ba^3x}{128b^3}\sqrt{bx^2+a}-\frac{5Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*A*x^3*(b*x^2+a)^(3/2)/b-1/8*A*a/b^2*x*(b*x^2+a)^(3/2)+1/16*A*
a^2/b^2*x*(b*x^2+a)^(1/2)+1/16*A*a^3/b^(5/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))+1/8*B*x^5*(b*x^2+a)^(3/2)/b-5/48*B*a/b^2*x^3*(b*x^2+a)^
(3/2)+5/64*B*a^2/b^3*x*(b*x^2+a)^(3/2)-5/128*B*a^3/b^3*x*(b*x^2+a
)^(1/2)-5/128*B*a^4/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305985, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48Bb^3x^7+8\left(Bab^2+8Ab^3\right)x^5-2\left(5Ba^2b-8Aab^2\right)x^3+3\left(5Ba^3-8Aa^2b\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(5Ba^4-8Aa^3b\right)\log\left(-\right.}{768b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^4,x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*B*b^3*x^7 + 8*(B*a*b^2 + 8*A*b^3)*x^5 - 2*(5*B*a^2*
b - 8*A*a*b^2)*x^3 + 3*(5*B*a^3 - 8*A*a^2*b)*x)*sqrt(b*x^2 + a)*s
qrt(b) - 3*(5*B*a^4 - 8*A*a^3*b)*log(-2*sqrt(b*x^2 + a)*b*x - (2*
b*x^2 + a)*sqrt(b)))/b^(7/2), 1/384*((48*B*b^3*x^7 + 8*(B*a*b^2 +
 8*A*b^3)*x^5 - 2*(5*B*a^2*b - 8*A*a*b^2)*x^3 + 3*(5*B*a^3 - 8*A*
a^2*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) - 3*(5*B*a^4 - 8*A*a^3*b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [A] time = 46.6973, size = 286, normalized size = 1.85

$$-\frac{Aa^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5A\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Abx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x^3}{384b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^5}{192b\sqrt{1+\frac{bx^2}{a}}}+\frac{7B\sqrt{a}x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{Bbx^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] -A*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - A*a**(3/2)*x**3/(48*
b*sqrt(1 + b*x**2/a)) + 5*A*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a))
+ A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) + A*b*x**7/(6*sqr
t(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(7/2)*x/(128*b**3*sqrt(1 + b*x*
*2/a)) + 5*B*a**(5/2)*x**3/(384*b**2*sqrt(1 + b*x**2/a)) - B*a**(
3/2)*x**5/(192*b*sqrt(1 + b*x**2/a)) + 7*B*sqrt(a)*x**7/(48*sqrt(
1 + b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(7/2))
 + B*b*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.236525, size = 178, normalized size = 1.15

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2+\frac{Bab^5+8\,Ab^6}{b^6}\right)x^2-\frac{5\,Ba^2b^4-8\,Aab^5}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^3-8\,Aa^2b^4\right)}{b^6}\right)\sqrt{bx^2+a}x+\frac{\left(5\,Ba^4-8\,Aa^3b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^4,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2 + (B*a*b^5 + 8*A*b^6)/b^6)*x^2 - (5*B*a^2*b^
4 - 8*A*a*b^5)/b^6)*x^2 + 3*(5*B*a^3*b^3 - 8*A*a^2*b^4)/b^6)*sqrt
(b*x^2 + a)*x + 1/128*(5*B*a^4 - 8*A*a^3*b)*ln(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(7/2)
```

3.506 $\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^3) + ((A*b - 2*a*B)*(a +
b*x^2)^(5/2))/(5*b^3) + (B*(a + b*x^2)^(7/2))/(7*b^3)
```

Rubi [A] time = 0.165699, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^3) + ((A*b - 2*a*B)*(a +
b*x^2)^(5/2))/(5*b^3) + (B*(a + b*x^2)^(7/2))/(7*b^3)
```

Rubi in Sympy [A] time = 19.0767, size = 63, normalized size = 0.86

$$\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b^3}-\frac{a\left(a+bx^2\right)^{\frac{3}{2}}(Ab-Ba)}{3b^3}+\frac{\left(a+bx^2\right)^{\frac{5}{2}}(Ab-2Ba)}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] B*(a + b*x**2)**(7/2)/(7*b**3) - a*(a + b*x**2)**(3/2)*(A*b - B*a
)/(3*b**3) + (a + b*x**2)**(5/2)*(A*b - 2*B*a)/(5*b**3)
```

Mathematica [A] time = 0.0571598, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{3/2}\left(8a^2B-2ab\left(7A+6Bx^2\right)+3b^2x^2\left(7A+5Bx^2\right)\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(8*a^2*B + 3*b^2*x^2*(7*A + 5*B*x^2) - 2*a*b*(
7*A + 6*B*x^2)))/(105*b^3)
```

Maple [A] time = 0.008, size = 53, normalized size = 0.7

$$-\frac{-15\,b^2Bx^4-21\,Ax^2b^2+12\,Bx^2ab+14\,abA-8\,a^2B}{105\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(-15*B*b^2*x^4-21*A*b^2*x^2+12*B*a*b*x^2+1
4*A*a*b-8*B*a^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227005, size = 101, normalized size = 1.38

$$\frac{\left(15\,Bb^{3}x^{6}+3\left(Bab^{2}+7\,Ab^{3}\right)x^{4}+8\,Ba^{3}-14\,Aa^{2}b-\left(4\,Ba^{2}b-7\,Aab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^3,x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 + 3*(B*a*b^2 + 7*A*b^3)*x^4 + 8*B*a^3 - 14*A*
a^2*b - (4*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 2.15881, size = 162, normalized size = 2.22

$$\begin{cases} -\frac{2Aa^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{Aax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{Ax^{4}\sqrt{a+bx^{2}}}{5}+\frac{8Ba^{3}\sqrt{a+bx^{2}}}{105b^{3}}-\frac{4Ba^{2}x^{2}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{Bax^{4}\sqrt{a+bx^{2}}}{35b}+\frac{Bx^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\: b \neq 0 \\ \sqrt{a}\left(\frac{Ax^{4}}{4}+\frac{Bx^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a**2*sqrt(a + b*x**2)/(15*b**2) + A*a*x**2*sqrt(a
 + b*x**2)/(15*b) + A*x**4*sqrt(a + b*x**2)/5 + 8*B*a**3*sqrt(a +
 b*x**2)/(105*b**3) - 4*B*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) +
 B*a*x**4*sqrt(a + b*x**2)/(35*b) + B*x**6*sqrt(a + b*x**2)/7, Ne
(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**6/6), True))
```

GIAC/XCAS [A] time = 0.228996, size = 107, normalized size = 1.47

$$\frac{\frac{7\left(3\left(bx^{2}+a\right)^{\frac{5}{2}}-5\left(bx^{2}+a\right)^{\frac{3}{2}}a\right)A}{b}+\frac{\left(15\left(bx^{2}+a\right)^{\frac{7}{2}}-42\left(bx^{2}+a\right)^{\frac{5}{2}}a+35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}\right)B}{b^{2}}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^3,x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*A/b + (15*
(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)
*a^2)*B/b^2)/b
```

3.507 $\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=122

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}(2Ab-aB)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

```
[Out] (a*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b^2) + ((2*A*b - a*B)*x^3
*Sqrt[a + b*x^2])/(8*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a^2*
(2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] time = 0.182169, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}(2Ab-aB)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b^2) + ((2*A*b - a*B)*x^3
*Sqrt[a + b*x^2])/(8*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a^2*
(2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi in Sympy [A] time = 19.4402, size = 107, normalized size = 0.88

$$\frac{Bx^3\left(a+bx^2\right)^{\frac{3}{2}}}{6b}-\frac{a^2\left(2Ab-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{5}{2}}}+\frac{ax\sqrt{a+bx^2}\left(2Ab-Ba\right)}{16b^2}+\frac{x^3\sqrt{a+bx^2}\left(2Ab-Ba\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] B*x**3*(a + b*x**2)**(3/2)/(6*b) - a**2*(2*A*b - B*a)*atanh(sqrt(
b)*x/sqrt(a + b*x**2))/(16*b**(5/2)) + a*x*sqrt(a + b*x**2)*(2*A*
b - B*a)/(16*b**2) + x**3*sqrt(a + b*x**2)*(2*A*b - B*a)/(8*b)
```

Mathematica [A] time = 0.0965216, size = 99, normalized size = 0.81

$$\frac{a^2(aB-2Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{5/2}}+\sqrt{a+bx^2}\left(-\frac{ax(aB-2Ab)}{16b^2}+\frac{x^3(aB+6Ab)}{24b}+\frac{Bx^5}{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*(-(a*(-2*A*b + a*B)*x)/(16*b^2) + ((6*A*b + a*B)*
x^3)/(24*b) + (B*x^5)/6) + (a^2*(-2*A*b + a*B)*Log[b*x + Sqrt[b]*
Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Maple [A] time = 0.01, size = 139, normalized size = 1.1

$$\frac{Ax}{4\,b}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{aAx}{8\,b}\sqrt{bx^{2}+a}-\frac{Aa^{2}}{8}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{3}{2}}+\frac{Bx^{3}}{6\,b}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{Bxa}{8\,b^{2}}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{Bxa^{2}}{16\,b^{2}}\sqrt{bx^{2}+a}+\frac{Ba^{3}}{16}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)/b-1/8*A*a/b*x*(b*x^2+a)^(1/2)-1/8*A*a^2/b
^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*B*x^3*(b*x^2+a)^(3/2)/b-
1/8*B*a/b^2*x*(b*x^2+a)^(3/2)+1/16*B*a^2/b^2*x*(b*x^2+a)^(1/2)+1/
16*B*a^3/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257115, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,Bb^{2}x^{5}+2\left(Bab+6\,Ab^{2}\right)x^{3}-3\left(Ba^{2}-2\,Aab\right)x\right)\sqrt{bx^{2}+a}\sqrt{b}-3\left(Ba^{3}-2\,Aa^{2}b\right)\log\left(2\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)}{96\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*B*b^2*x^5 + 2*(B*a*b + 6*A*b^2)*x^3 - 3*(B*a^2 - 2*A*
a*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(B*a^3 - 2*A*a^2*b)*log(2*sqr
t(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(5/2), 1/48*((8*B*b^
2*x^5 + 2*(B*a*b + 6*A*b^2)*x^3 - 3*(B*a^2 - 2*A*a*b)*x)*sqrt(b*x
^2 + a)*sqrt(-b) + 3*(B*a^3 - 2*A*a^2*b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 29.6152, size = 226, normalized size = 1.85

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3A\sqrt{a}x^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{2}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{5}{2}}x}{16b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{3}{2}}x^{3}}{48b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5B\sqrt{a}x^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}+\frac{Ba^{3}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Bbx^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(
1 + b*x**2/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A
*b*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(5/2)*x/(16*b**2*sq
rt(1 + b*x**2/a)) - B*a**(3/2)*x**3/(48*b*sqrt(1 + b*x**2/a)) + 5
*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + B*a**3*asinh(sqrt(b)*x/
sqrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.252508, size = 135, normalized size = 1.11

$$\frac{1}{48}\left(2\left(4\,Bx^2+\frac{Bab^3+6\,Ab^4}{b^4}\right)x^2-\frac{3\left(Ba^2b^2-2\,Aab^3\right)}{b^4}\right)\sqrt{bx^2+a}x-\frac{\left(Ba^3-2\,Aa^2b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2 + (B*a*b^3 + 6*A*b^4)/b^4)*x^2 - 3*(B*a^2*b^2 -
2*A*a*b^3)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(B*a^3 - 2*A*a^2*b)*ln(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.508 $\int x\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (B*(a + b*x^2)^(5/2))/(
5*b^2)
```

Rubi [A] time = 0.100122, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (B*(a + b*x^2)^(5/2))/(
5*b^2)
```

Rubi in Sympy [A] time = 13.1255, size = 37, normalized size = 0.8

$$\frac{B\left(a+bx^2\right)^{\frac{5}{2}}}{5b^2}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}(Ab-Ba)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] B*(a + b*x**2)**(5/2)/(5*b**2) + (a + b*x**2)**(3/2)*(A*b - B*a)/
(3*b**2)
```

Mathematica [A] time = 0.0392104, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(-2aB+5Ab+3bBx^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^2))/(15*b^2)
```

Maple [A] time = 0.006, size = 31, normalized size = 0.7

$$\frac{3\,bBx^2+5\,Ab-2\,Ba}{15\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(3/2)*(3*B*b*x^2+5*A*b-2*B*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212523, size = 68, normalized size = 1.48

$$\frac{\left(3\,Bb^2x^4 - 2\,Ba^2 + 5\,Aab + \left(Bab + 5\,Ab^2\right)x^2\right)\sqrt{bx^2 + a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 - 2*B*a^2 + 5*A*a*b + (B*a*b + 5*A*b^2)*x^2)*sq
rt(b*x^2 + a)/b^2
```

Sympy [A] time = 0.982783, size = 110, normalized size = 2.39

$$\begin{cases} \frac{Aa\sqrt{a+bx^2}}{3b} + \frac{Ax^2\sqrt{a+bx^2}}{3} - \frac{2Ba^2\sqrt{a+bx^2}}{15b^2} + \frac{Bax^2\sqrt{a+bx^2}}{15b} + \frac{Bx^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^2}{2} + \frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*a*sqrt(a + b*x**2)/(3*b) + A*x**2*sqrt(a + b*x**2)/3
 - 2*B*a**2*sqrt(a + b*x**2)/(15*b**2) + B*a*x**2*sqrt(a + b*x**2
)/(15*b) + B*x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*(A*x**2
/2 + B*x**4/4), True))
```

GIAC/XCAS [A] time = 0.240444, size = 63, normalized size = 1.37

$$\frac{5\left(bx^2 + a\right)^{\frac{3}{2}}A + \frac{\left(3\left(bx^2+a\right)^{\frac{5}{2}} - 5\left(bx^2+a\right)^{\frac{3}{2}}a\right)B}{b}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x,x, algorithm="giac")
```

```
[Out] 1/15*(5*(b*x^2 + a)^(3/2)*A + (3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a
)^(3/2)*a)*B/b)/b
```

3.509 $\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=87

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4Ab-aB)}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

```
[Out] ((4*A*b - a*B)*x*Sqrt[a + b*x^2])/(8*b) + (B*x*(a + b*x^2)^(3/2))
/(4*b) + (a*(4*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
8*b^(3/2))
```

Rubi [A] time = 0.0817528, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4Ab-aB)}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((4*A*b - a*B)*x*Sqrt[a + b*x^2])/(8*b) + (B*x*(a + b*x^2)^(3/2))
/(4*b) + (a*(4*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
8*b^(3/2))
```

Rubi in Sympy [A] time = 9.79674, size = 75, normalized size = 0.86

$$\frac{Bx\left(a+bx^2\right)^{\frac{3}{2}}}{4b}+\frac{a\left(4Ab-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{3}{2}}}+\frac{x\sqrt{a+bx^2}\left(4Ab-Ba\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] B*x*(a + b*x**2)**(3/2)/(4*b) + a*(4*A*b - B*a)*atanh(sqrt(b)*x/s
qrt(a + b*x**2))/(8*b**(3/2)) + x*sqrt(a + b*x**2)*(4*A*b - B*a)/
(8*b)
```

Mathematica [A] time = 0.0695272, size = 78, normalized size = 0.9

$$\sqrt{a+bx^2}\left(\frac{x(aB+4Ab)}{8b}+\frac{Bx^3}{4}\right)-\frac{a(aB-4Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*(((4*A*b + a*B)*x)/(8*b) + (B*x^3)/4) - (a*(-4*A*
b + a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A] time = 0.008, size = 96, normalized size = 1.1

$$\frac{Ax}{2}\sqrt{bx^2+a}+\frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{Bx}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bxa}{8\,b}\sqrt{bx^2+a}-\frac{a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))+1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*B*a/b*x*(b*x^2+a)^(1/2)-1/8*B*
a^2/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.225792, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,Bbx^3+(Ba+4\,Ab)x\right)\sqrt{bx^2+a}\sqrt{b}-\left(Ba^2-4\,Aab\right)\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{16\,b^{\frac{3}{2}}},\frac{\left(2\,Bbx^3+(Ba+4\,Ab)x\right)\sqrt{\cdot}}{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*B*b*x^3 + (B*a + 4*A*b)*x)*sqrt(b*x^2 + a)*sqrt(b) -
(B*a^2 - 4*A*a*b)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt
(b)))/b^(3/2), 1/8*((2*B*b*x^3 + (B*a + 4*A*b)*x)*sqrt(b*x^2 + a)
*sqrt(-b) - (B*a^2 - 4*A*a*b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))
/(sqrt(-b)*b)]
```

Sympy [A] time = 17.382, size = 144, normalized size = 1.66

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(
2*sqrt(b)) + B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*
x**3/(8*sqrt(1 + b*x**2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*
b**(3/2)) + B*b*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.235181, size = 93, normalized size = 1.07

$$\frac{1}{8}\left(2\,Bx^2+\frac{Bab+4\,Ab^2}{b^2}\right)\sqrt{bx^2+a}x+\frac{\left(Ba^2-4\,Aab\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2 + (B*a*b + 4*A*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/8*(B*
a^2 - 4*A*a*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.510 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=59

$$A\sqrt{a+bx^2}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

```
[Out] A*Sqrt[a + b*x^2] + (B*(a + b*x^2)^(3/2))/(3*b) - Sqrt[a]*A*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.133804, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$A\sqrt{a+bx^2}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] A*Sqrt[a + b*x^2] + (B*(a + b*x^2)^(3/2))/(3*b) - Sqrt[a]*A*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 13.6356, size = 49, normalized size = 0.83

$$-A\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+A\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*atanh(sqrt(a + b*x**2)/sqrt(a)) + A*sqrt(a + b*x**2) +
 B*(a + b*x**2)**(3/2)/(3*b)
```

Mathematica [A] time = 0.101877, size = 70, normalized size = 1.19

$$\frac{\sqrt{a+bx^2}\left(aB+3Ab+bBx^2\right)}{3b}-\sqrt{a}A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\sqrt{a}A\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*A*b + a*B + b*B*x^2))/(3*b) + Sqrt[a]*A*Log[x
] - Sqrt[a]*A*Log[a + Sqrt[a]*Sqrt[a + b*x^2]]
```

Maple [A] time = 0.01, size = 57, normalized size = 1.

$$-A\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+A\sqrt{bx^2+a}+\frac{B}{3\,b}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] -A*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+A*(b*x^2+a)^(1/2
)+1/3*B*(b*x^2+a)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.223702, size = 1, normalized size = 0.02

$$\left[\frac{3\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\left(Bbx^2+Ba+3\,Ab\right)\sqrt{bx^2+a}}{6\,b},\; -\frac{3\,A\sqrt{-a}b\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-\left(Bbx^2+Ba+3\,Ab\right)\sqrt{bx^2+a}}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b, -1/3*(3*A*
sqrt(-a)*b*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) - (B*b*x^2 + B*a
+ 3*A*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 12.4458, size = 117, normalized size = 1.98

$$-Aa\left(\begin{cases}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a>0\\ \frac{\operatorname{acoth}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ -a<0\wedge a<a+bx^2\\ \frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ a>a+bx^2\wedge -a<0\end{cases}\right)+A\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*a*Piecewise((-atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a), -a > 0
), (acoth(sqrt(a + b*x**2)/sqrt(a))/sqrt(a), (-a < 0) & (a < a +
b*x**2)), (atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a), (-a < 0) & (a
 > a + b*x**2))) + A*sqrt(a + b*x**2) + B*(a + b*x**2)**(3/2)/(3*
b)
```

GIAC/XCAS [A] time = 0.225121, size = 81, normalized size = 1.37

$$\frac{Aa\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bb^2+3\sqrt{bx^2+a}Ab^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x,x, algorithm="giac")
```

```
[Out] A*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*((b*x^2 + a)^
(3/2)*B*b^2 + 3*sqrt(b*x^2 + a)*A*b^3)/b^3
```

3.511 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{x\sqrt{a+bx^2}(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

```
[Out] ((2*A*b + a*B)*x*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(
a*x) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sq
rt[b])
```

Rubi [A] time = 0.10249, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{x\sqrt{a+bx^2}(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] ((2*A*b + a*B)*x*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(
a*x) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sq
rt[b])
```

Rubi in Sympy [A] time = 10.8135, size = 71, normalized size = 0.85

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{ax}+\frac{(2Ab+Ba)\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{x\sqrt{a+bx^2}\,(2Ab+Ba)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(a*x) + (2*A*b + B*a)*atanh(sqrt(b)*x/sqrt
(a + b*x**2))/(2*sqrt(b)) + x*sqrt(a + b*x**2)*(2*A*b + B*a)/(2*a
)
```

Mathematica [A] time = 0.0637051, size = 65, normalized size = 0.77

$$\sqrt{a+bx^2}\left(\frac{Bx}{2}-\frac{A}{x}\right)+\frac{(aB+2Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] (-(A/x) + (B*x)/2)*Sqrt[a + b*x^2] + ((2*A*b + a*B)*Log[b*x + Sqr
t[b]*Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A] time = 0.013, size = 93, normalized size = 1.1

$$\frac{Bx}{2}\sqrt{bx^2+a}+\frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-\frac{A}{ax}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Axb}{a}\sqrt{bx^2+a}+A\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] 1/2*x*B*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))-A*(b*x^2+a)^(3/2)/a/x+A*b/a*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221886, size = 1, normalized size = 0.01

$$\left[\frac{(Ba+2\,Ab)x\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(Bx^2-2\,A\right)\sqrt{bx^2+a}\sqrt{b}}{4\,\sqrt{b}x},\ \frac{(Ba+2\,Ab)x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(Bx^2-\right.}{2\,\sqrt{-b}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((B*a + 2*A*b)*x*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*
sqrt(b)) + 2*(B*x^2 - 2*A)*sqrt(b*x^2 + a)*sqrt(b))/(sqrt(b)*x),
1/2*((B*a + 2*A*b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*x^2
- 2*A)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*x)]
```

Sympy [A] time = 10.9012, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqr
t(a)) - A*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + B*sqrt(a)*x*sqrt(1 +
 b*x**2/a)/2 + B*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

GIAC/XCAS [A] time = 0.241461, size = 113, normalized size = 1.35

$$\frac{1}{2}\sqrt{bx^{2}+a}Bx+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a}-\frac{\left(Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a) - 1/4*(B*a*sqrt(b) + 2*A*b^(3/2))*ln((sqrt(b)*x - sq
rt(b*x^2 + a))^2)/b
```

3.512 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^2}(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(2*
a*x^2) - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt
[a])
```

Rubi [A] time = 0.180363, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{\sqrt{a+bx^2}(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(2*
a*x^2) - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt
[a])
```

Rubi in Sympy [A] time = 16.0302, size = 68, normalized size = 0.81

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{2ax^2}+\frac{\sqrt{a+bx^2}\left(\frac{Ab}{2}+Ba\right)}{a}-\frac{\left(\frac{Ab}{2}+Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(2*a*x**2) + sqrt(a + b*x**2)*(A*b/2 + B*a
)/a - (A*b/2 + B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a)
```

Mathematica [A] time = 0.116005, size = 81, normalized size = 0.96

$$\sqrt{a+bx^2}\left(B-\frac{A}{2x^2}\right)+\frac{(-2aB-Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{2\sqrt{a}}-\frac{\log(x)(-2aB-Ab)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] (B - A/(2*x^2))*Sqrt[a + b*x^2] - ((-(A*b) - 2*a*B)*Log[x])/(2*Sq
rt[a]) + ((-(A*b) - 2*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(2*S
qrt[a])
```

Maple [A] time = 0.012, size = 106, normalized size = 1.3

$$-\frac{A}{2\,a x^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{A b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{A b}{2\,a}\sqrt{b x^{2}+a}-B\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)+B\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(3/2)/a/x^2-1/2*A*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)+1/2*A*b/a*(b*x^2+a)^(1/2)-B*a^(1/2)*ln((2*a+2*a
^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23009, size = 1, normalized size = 0.01

$$\left[\frac{(2\,Ba+Ab)x^{2}\log\left(-\frac{(bx^{2}+2\,a)\,\sqrt{a}-2\,\sqrt{bx^{2}+a}a}{x^{2}}\right)+2\left(2\,Bx^{2}-A\right)\sqrt{bx^{2}+a}\sqrt{a}}{4\,\sqrt{a}x^{2}},\ -\frac{(2\,Ba+Ab)x^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(2\,Bx^{2}-A\right)\sqrt{bx^{2}+a}\sqrt{-a}}{2\,\sqrt{-a}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*a + A*b)*x^2*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^
2 + a)*a)/x^2) + 2*(2*B*x^2 - A)*sqrt(b*x^2 + a)*sqrt(a))/(sqrt(a
)*x^2), -1/2*((2*B*a + A*b)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a))
- (2*B*x^2 - A)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*x^2)]
```

Sympy [A] time = 31.7916, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}-B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b
)*x))/(2*sqrt(a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a/(s
qrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

GIAC/XCAS [A] time = 0.242215, size = 92, normalized size = 1.1

$$\frac{2\sqrt{bx^2+a}Bb + \frac{(2\,Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^2+a}Ab}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(b*x^2 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^2
+ a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)*A*b/x^2)/b
```

3.513 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=66

$$-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -((B*Sqrt[a + b*x^2])/x) - (A*(a + b*x^2)^(3/2))/(3*a*x^3) + Sqrt
[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.0851174, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] -((B*Sqrt[a + b*x^2])/x) - (A*(a + b*x^2)^(3/2))/(3*a*x^3) + Sqrt
[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi in Sympy [A] time = 10.9052, size = 56, normalized size = 0.85

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{3ax^3}+B\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{B\sqrt{a+bx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(3*a*x**3) + B*sqrt(b)*atanh(sqrt(b)*x/sqr
t(a + b*x**2)) - B*sqrt(a + b*x**2)/x
```

Mathematica [A] time = 0.0697585, size = 70, normalized size = 1.06

$$\sqrt{a+bx^2}\left(\frac{-3aB-Ab}{3ax}-\frac{A}{3x^3}\right)+\sqrt{b}B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] (-A/(3*x^3) + (-(A*b) - 3*a*B)/(3*a*x))*Sqrt[a + b*x^2] + Sqrt[b]
*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]
```

Maple [A] time = 0.013, size = 75, normalized size = 1.1

$$-\frac{A}{3\,ax^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{B}{ax}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{bBx}{a}\sqrt{bx^2+a}+B\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*A*(b*x^2+a)^(3/2)/a/x^3-B/a/x*(b*x^2+a)^(3/2)+B*b/a*x*(b*x^2
+a)^(1/2)+B*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.224419, size = 1, normalized size = 0.02

$$\left[\frac{3\,Ba\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left((3\,Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}}{6\,ax^3},\ \frac{3\,Ba\sqrt{-b}x^3\arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right)-\left((3\,B\right.}{3\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a))/(a*x^3), 1/
3*(3*B*a*sqrt(-b)*x^3*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - ((
3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a))/(a*x^3)]
```

Sympy [A] time = 7.74585, size = 107, normalized size = 1.62

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(3*a) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*as
inh(sqrt(b)*x/sqrt(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.248308, size = 204, normalized size = 3.09

$$-\frac{1}{2}B\sqrt{b}\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+3\,Ba^3\sqrt{b}+Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(b)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b) + 3*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqr
t(b) + 3*B*a^3*sqrt(b) + A*a^2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3
```

3.514 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}(Ab-4aB)}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a*x^2) - (A*(a + b*x^2)^(3/2))
/(4*a*x^4) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
8*a^(3/2))
```

Rubi [A] time = 0.195805, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}(Ab-4aB)}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a*x^2) - (A*(a + b*x^2)^(3/2))
/(4*a*x^4) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
8*a^(3/2))
```

Rubi in Sympy [A] time = 16.3015, size = 76, normalized size = 0.86

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{4ax^4}+\frac{\sqrt{a+bx^2}\,(Ab-4Ba)}{8ax^2}+\frac{b\,(Ab-4Ba)\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(4*a*x**4) + sqrt(a + b*x**2)*(A*b - 4*B*a
)/(8*a*x**2) + b*(A*b - 4*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(8
*a**(3/2))
```

Mathematica [A] time = 0.110759, size = 99, normalized size = 1.12

$$\frac{b(Ab-4aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{8a^{3/2}}-\frac{b\log(x)(Ab-4aB)}{8a^{3/2}}+\sqrt{a+bx^2}\left(\frac{-4aB-Ab}{8ax^2}-\frac{A}{4x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] (-A/(4*x^4) + (-(A*b) - 4*a*B)/(8*a*x^2))*Sqrt[a + b*x^2] - (b*(A
*b - 4*a*B)*Log[x])/(8*a^(3/2)) + (b*(A*b - 4*a*B)*Log[a + Sqrt[a
]*Sqrt[a + b*x^2]])/(8*a^(3/2))
```

Maple [B] time = 0.013, size = 153, normalized size = 1.7

$$-\frac{A}{4\,a x^4}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{A b}{8\,a^2 x^2}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{b^2 A}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{b^2 A}{8\,a^2}\sqrt{b x^2+a}-\frac{B}{2\,a x^2}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{B b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{B b}{2\,a}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(3/2)/a/x^4+1/8*A*b/a^2/x^2*(b*x^2+a)^(3/2)+1/8*
A*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/8*A*b^2/a^2
*(b*x^2+a)^(1/2)-1/2*B/a/x^2*(b*x^2+a)^(3/2)-1/2*B*b/a^(1/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+1/2*B*b/a*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227569, size = 1, normalized size = 0.01

$$\left[-\frac{\left(4\,B a b-A b^2\right)x^4\log\left(-\frac{\left(b x^2+2\,a\right)\sqrt{a}+2\,\sqrt{b x^2+a}a}{x^2}\right)+2\left(\left(4\,B a+A b\right)x^2+2\,A a\right)\sqrt{b x^2+a}\sqrt{a}}{16\,a^{\frac{3}{2}}x^4},\right.$$
$$\left.-\frac{\left(4\,B a b-A b^2\right)x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2+a}}\right)+\left(\left(4\,B a+A b\right)x^2+2\,A a\right)\sqrt{b x^2+a}\sqrt{-a}}{8\,\sqrt{-a}a x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - A*b^2)*x^4*log(-((b*x^2 + 2*a)*sqrt(a) + 2*sqr
t(b*x^2 + a)*a)/x^2) + 2*((4*B*a + A*b)*x^2 + 2*A*a)*sqrt(b*x^2 +
 a)*sqrt(a))/(a^(3/2)*x^4), -1/8*((4*B*a*b - A*b^2)*x^4*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) + ((4*B*a + A*b)*x^2 + 2*A*a)*sqrt(b*x^2
+ a)*sqrt(-a))/(sqrt(-a)*a*x^4)]
```

Sympy [A] time = 70.4917, size = 144, normalized size = 1.64

$$-\frac{A a}{4\sqrt{b}x^5\sqrt{\frac{a}{b x^2}+1}}-\frac{3A\sqrt{b}}{8x^3\sqrt{\frac{a}{b x^2}+1}}-\frac{A b^{\frac{3}{2}}}{8a x\sqrt{\frac{a}{b x^2}+1}}+\frac{A b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{b x^2}+1}}{2x}-\frac{B b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -A*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)/(8*x**3*
sqrt(a/(b*x**2) + 1)) - A*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) +
 A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*sqrt(
a/(b*x**2) + 1)/(2*x) - B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a)
)
```

GIAC/XCAS [A] time = 0.231908, size = 162, normalized size = 1.84

$$\frac{\frac{\left(4\,Bab^2-Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{4\,\left(bx^2+a\right)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^2+a}Ba^2b^2+\left(bx^2+a\right)^{\frac{3}{2}}Ab^3+\sqrt{bx^2+a}Aab^3}{ab^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^
2 + (b*x^2 + a)^(3/2)*A*b^3 + sqrt(b*x^2 + a)*A*a*b^3)/(a*b^2*x^4
))/b
```

3.515 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

```
[Out] -(A*(a + b*x^2)^(3/2))/(5*a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(
3/2))/(15*a^2*x^3)
```

Rubi [A] time = 0.0819841, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(a+bx^2)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(5*a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(
3/2))/(15*a^2*x^3)
```

Rubi in Sympy [A] time = 9.2663, size = 46, normalized size = 0.87

$$-\frac{A(a+bx^2)^{\frac{3}{2}}}{5ax^5}+\frac{(a+bx^2)^{\frac{3}{2}}(2Ab-5Ba)}{15a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(5*a*x**5) + (a + b*x**2)**(3/2)*(2*A*b -
5*B*a)/(15*a**2*x**3)
```

Mathematica [A] time = 0.0498783, size = 40, normalized size = 0.75

$$-\frac{(a+bx^2)^{3/2}(3aA+5aBx^2-2Abx^2)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -((a + b*x^2)^(3/2)*(3*a*A - 2*A*b*x^2 + 5*a*B*x^2))/(15*a^2*x^5)
```

Maple [A] time = 0.008, size = 37, normalized size = 0.7

$$-\frac{-2\,Abx^2+5\,Bax^2+3\,Aa}{15\,x^5a^2}(bx^2+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-2*A*b*x^2+5*B*a*x^2+3*A*a)/x^5/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220597, size = 74, normalized size = 1.4

$$-\frac{\left(\left(5\,Bab - 2\,Ab^2\right)x^4 + 3\,Aa^2 + \left(5\,Ba^2 + Aab\right)x^2\right)\sqrt{bx^2 + a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*a*b - 2*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + A*a*b)*x^2)
*sqrt(b*x^2 + a)/(a^2*x^5)
```

Sympy [A] time = 5.89713, size = 119, normalized size = 2.25

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**
2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqrt(a/
(b*x**2) + 1)/(3*a)
```

GIAC/XCAS [A] time = 0.231074, size = 313, normalized size = 5.91

$$\frac{2\left(15\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Bb^{\frac{3}{2}} - 30\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 Bab^{\frac{3}{2}} + 30\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 Ab^{\frac{5}{2}} + 20\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 Ba^2b^{\frac{3}{2}} + \cdots\right.}{15\left(\left(\sqrt{b}x - \sqrt{bx^2 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(3/2) - 30*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*B*a*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 +
```

```
 a))^6*A*b^(5/2) + 20*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/
2) + 10*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(5/2) - 10*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2) + 10*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a^2*b^(5/2) + 5*B*a^4*b^(3/2) - 2*A*a^3*b^(5/2))/((sq
rt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.516 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=120

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}(Ab-2aB)}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

```
[Out] ((A*b - 2*a*B)*Sqrt[a + b*x^2])/(8*a*x^4) + (b*(A*b - 2*a*B)*Sqrt
[a + b*x^2])/(16*a^2*x^2) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) - (b^
2*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rubi [A] time = 0.253423, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}(Ab-2aB)}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] ((A*b - 2*a*B)*Sqrt[a + b*x^2])/(8*a*x^4) + (b*(A*b - 2*a*B)*Sqrt
[a + b*x^2])/(16*a^2*x^2) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) - (b^
2*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rubi in Sympy [A] time = 21.0993, size = 107, normalized size = 0.89

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{6ax^6}+\frac{\sqrt{a+bx^2}\left(\frac{Ab}{2}-Ba\right)}{4ax^4}+\frac{b\sqrt{a+bx^2}\,(Ab-2Ba)}{16a^2x^2}-\frac{b^2\left(\frac{Ab}{2}-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(6*a*x**6) + sqrt(a + b*x**2)*(A*b/2 - B*a
)/(4*a*x**4) + b*sqrt(a + b*x**2)*(A*b - 2*B*a)/(16*a**2*x**2) -
b**2*(A*b/2 - B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(8*a**(5/2))
```

Mathematica [A] time = 0.165432, size = 123, normalized size = 1.02

$$-\frac{b^2(Ab-2aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{16a^{5/2}}+\frac{b^2\log(x)(Ab-2aB)}{16a^{5/2}}+\sqrt{a+bx^2}\left(-\frac{b(2aB-Ab)}{16a^2x^2}+\frac{-6aB-Ab}{24ax^4}-\frac{A}{6x^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] (-A/(6*x^6) + (-(A*b) - 6*a*B)/(24*a*x^4) - (b*(-(A*b) + 2*a*B))/
(16*a^2*x^2))*Sqrt[a + b*x^2] + (b^2*(A*b - 2*a*B)*Log[x])/(16*a^
(5/2)) - (b^2*(A*b - 2*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(16
*a^(5/2))
```

Maple [A] time = 0.013, size = 197, normalized size = 1.6

$$-\frac{A}{6\,a x^6}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{A b}{8\,a^2 x^4}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{b^2 A}{16\,a^3 x^2}\left(b x^2+a\right)^{\frac{3}{2}} -\frac{A b^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{A b^3}{16\,a^3}\sqrt{b x^2+a}-\frac{B}{4\,a x^4}\left(b x^2+a\right)^{\frac{3}{2}} +\frac{B b}{8\,a^2 x^2}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{B b^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{B b^2}{8\,a^2}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(3/2)/a/x^6+1/8*A*b/a^2/x^4*(b*x^2+a)^(3/2)-1/16
*A*b^2/a^3/x^2*(b*x^2+a)^(3/2)-1/16*A*b^3/a^(5/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)+1/16*A*b^3/a^3*(b*x^2+a)^(1/2)-1/4*B/a/x^4
*(b*x^2+a)^(3/2)+1/8*B*b/a^2/x^2*(b*x^2+a)^(3/2)+1/8*B*b^2/a^(3/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/8*B*b^2/a^2*(b*x^2+a)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247209, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(2\,Bab^2-Ab^3\right)x^6\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)+2\left(3\left(2\,Bab-Ab^2\right)x^4+8\,Aa^2+2\left(6\,Ba^2+Aab\right)x^2\right)\sqrt{bx^2+a}\sqrt{a}}{96\,a^{\frac{5}{2}}x^6},\ \frac{3(2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*B*a*b^2 - A*b^3)*x^6*log(-((b*x^2 + 2*a)*sqrt(a) - 2
*sqrt(b*x^2 + a)*a)/x^2) + 2*(3*(2*B*a*b - A*b^2)*x^4 + 8*A*a^2 +
 2*(6*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(5/2)*x^6),
 1/48*(3*(2*B*a*b^2 - A*b^3)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 - (3*(2*B*a*b - A*b^2)*x^4 + 8*A*a^2 + 2*(6*B*a^2 + A*a*b)*x^2)*
sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^2*x^6)]
```

Sympy [A] time = 101.49, size = 226, normalized size = 1.88

$$-\frac{Aa}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{5A\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} -\frac{Ba}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3B\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Bb^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)/(24*x**5
*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) +
1)) + A*b**(5/2)/(16*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*asinh(
sqrt(a)/(sqrt(b)*x))/(16*a**(5/2)) - B*a/(4*sqrt(b)*x**5*sqrt(a/(
b*x**2) + 1)) - 3*B*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*b**
(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + B*b**2*asinh(sqrt(a)/(sqrt(b
)*x))/(8*a**(3/2))
```

GIAC/XCAS [A] time = 0.229216, size = 189, normalized size = 1.58

$$-\frac{\frac{3\left(2 Bab^3-Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{6\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-6\sqrt{bx^2+a}Ba^3b^3-3\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+8\left(bx^2+a\right)^{\frac{3}{2}}Aab^4+3\sqrt{bx^2+a}Aa^2b^4}{a^2b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(2*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sq
rt(-a)*a^2) + (6*(b*x^2 + a)^(5/2)*B*a*b^3 - 6*sqrt(b*x^2 + a)*B*
a^3*b^3 - 3*(b*x^2 + a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b^4
 + 3*sqrt(b*x^2 + a)*A*a^2*b^4)/(a^2*b^3*x^6))/b
```

3.517 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{105a^3x^3}+\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

```
[Out] -(A*(a + b*x^2)^(3/2))/(7*a*x^7) + ((4*A*b - 7*a*B)*(a + b*x^2)^(
3/2))/(35*a^2*x^5) - (2*b*(4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(105
*a^3*x^3)
```

Rubi [A] time = 0.118273, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{2b\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{105a^3x^3}+\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(7*a*x^7) + ((4*A*b - 7*a*B)*(a + b*x^2)^(
3/2))/(35*a^2*x^5) - (2*b*(4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(105
*a^3*x^3)
```

Rubi in Sympy [A] time = 12.4272, size = 78, normalized size = 0.93

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{7ax^7}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}(4Ab-7Ba)}{35a^2x^5}-\frac{2b\left(a+bx^2\right)^{\frac{3}{2}}(4Ab-7Ba)}{105a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(7*a*x**7) + (a + b*x**2)**(3/2)*(4*A*b -
7*B*a)/(35*a**2*x**5) - 2*b*(a + b*x**2)**(3/2)*(4*A*b - 7*B*a)/(
105*a**3*x**3)
```

Mathematica [A] time = 0.0691115, size = 63, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{3/2}\left(-3a^2\left(5A+7Bx^2\right)+2abx^2\left(6A+7Bx^2\right)-8Ab^2x^4\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-8*A*b^2*x^4 - 3*a^2*(5*A + 7*B*x^2) + 2*a*b*
x^2*(6*A + 7*B*x^2)))/(105*a^3*x^7)
```

Maple [A] time = 0.008, size = 59, normalized size = 0.7

$$-\frac{8\,Ab^2x^4-14\,Babx^4-12\,aAbx^2+21\,Ba^2x^2+15\,Aa^2}{105\,x^7a^3}\left(bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(8*A*b^2*x^4-14*B*a*b*x^4-12*A*a*b*x^2+21*
B*a^2*x^2+15*A*a^2)/x^7/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249552, size = 109, normalized size = 1.3

$$\frac{\left(2\left(7\,Bab^{2}-4\,Ab^{3}\right)x^{6}-\left(7\,Ba^{2}b-4\,Aab^{2}\right)x^{4}-15\,Aa^{3}-3\left(7\,Ba^{3}+Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\,a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*a*b^2 - 4*A*b^3)*x^6 - (7*B*a^2*b - 4*A*a*b^2)*x^4
- 15*A*a^3 - 3*(7*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^3*x^7)
```

Sympy [A] time = 8.58804, size = 442, normalized size = 5.26

$$-\frac{15Aa^{5}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{33Aa^{4}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{17Aa^{3}b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{3Aa^{2}b^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{12Aab^{\frac{17}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{8Ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{4}}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15ax^{2}}+\frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**
2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 +
 105*a**3*b**6*x**10) - 17*A*a**3*b**(13/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4
*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a*b**(17
/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2
) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x
**10) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt
```

```
(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/
(15*a**2)
```

GIAC/XCAS [A] time = 0.241779, size = 389, normalized size = 4.63

$$\frac{4\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Bab^{\frac{5}{2}}+280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^8,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*b^(5/2) - 175*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*B*a*b^(5/2) + 280*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*A*b^(7/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*
b^(5/2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(7/2) - 42*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(5/2) + 84*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*A*a^2*b^(7/2) + 49*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*B*a^4*b^(5/2) - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(7/2
) - 7*B*a^5*b^(5/2) + 4*A*a^4*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 +
 a))^2 - a)^7
```

3.518 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=156

$$\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{a+bx^2}(5Ab-8aB)}{128a^3x^2}+\frac{b\sqrt{a+bx^2}(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}(5Ab-8aB)}{48ax^6}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

```
[Out] ((5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(48*a*x^6) + (b*(5*A*b - 8*a*B)
*Sqrt[a + b*x^2])/(192*a^2*x^4) - (b^2*(5*A*b - 8*a*B)*Sqrt[a + b
*x^2])/(128*a^3*x^2) - (A*(a + b*x^2)^(3/2))/(8*a*x^8) + (b^3*(5*
A*b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Rubi [A] time = 0.314677, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{a+bx^2}(5Ab-8aB)}{128a^3x^2}+\frac{b\sqrt{a+bx^2}(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}(5Ab-8aB)}{48ax^6}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] ((5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(48*a*x^6) + (b*(5*A*b - 8*a*B)
*Sqrt[a + b*x^2])/(192*a^2*x^4) - (b^2*(5*A*b - 8*a*B)*Sqrt[a + b
*x^2])/(128*a^3*x^2) - (A*(a + b*x^2)^(3/2))/(8*a*x^8) + (b^3*(5*
A*b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Rubi in Sympy [A] time = 25.992, size = 144, normalized size = 0.92

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{8ax^8}+\frac{\sqrt{a+bx^2}\,(5Ab-8Ba)}{48ax^6}+\frac{b\sqrt{a+bx^2}\,(5Ab-8Ba)}{192a^2x^4}-\frac{b^2\sqrt{a+bx^2}\,(5Ab-8Ba)}{128a^3x^2}+\frac{b^3\,(5Ab-8Ba)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(8*a*x**8) + sqrt(a + b*x**2)*(5*A*b - 8*B
*a)/(48*a*x**6) + b*sqrt(a + b*x**2)*(5*A*b - 8*B*a)/(192*a**2*x*
*4) - b**2*sqrt(a + b*x**2)*(5*A*b - 8*B*a)/(128*a**3*x**2) + b**
3*(5*A*b - 8*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(128*a**(7/2))
```

Mathematica [A] time = 0.192892, size = 147, normalized size = 0.94

$$\frac{b^3(5Ab-8aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{128a^{7/2}}-\frac{b^3\log(x)(5Ab-8aB)}{128a^{7/2}}+\sqrt{a+bx^2}\left(\frac{b^2(8aB-5Ab)}{128a^3x^2}-\frac{b(8aB-5Ab)}{192a^2x^4}+\frac{-8aB-Ab}{48ax^6}-\frac{A}{8x^8}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] (-A/(8*x^8) + (-(A*b) - 8*a*B)/(48*a*x^6) - (b*(-5*A*b + 8*a*B))/
(192*a^2*x^4) + (b^2*(-5*A*b + 8*a*B))/(128*a^3*x^2))*Sqrt[a + b*
x^2] - (b^3*(5*A*b - 8*a*B)*Log[x])/(128*a^(7/2)) + (b^3*(5*A*b -
 8*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(128*a^(7/2))
```

Maple [A] time = 0.024, size = 239, normalized size = 1.5

$$-\frac{A}{8\,a x^8}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,A b}{48\,a^2 x^6}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{5\,b^2 A}{64\,a^3 x^4}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,A b^3}{128\,a^4 x^2}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,A b^4}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{5\,A b^4}{128\,a^4}\sqrt{b x^2+a}-\frac{B}{6\,a x^6}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{B b}{8\,a^2 x^4}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{B b^2}{16\,a^3 x^2}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{B b^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{B b^3}{16\,a^3}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*A*(b*x^2+a)^(3/2)/a/x^8+5/48*A*b/a^2/x^6*(b*x^2+a)^(3/2)-5/6
4*A*b^2/a^3/x^4*(b*x^2+a)^(3/2)+5/128*A*b^3/a^4/x^2*(b*x^2+a)^(3/
2)+5/128*A*b^4/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5/12
8*A*b^4/a^4*(b*x^2+a)^(1/2)-1/6*B/a/x^6*(b*x^2+a)^(3/2)+1/8*B*b/a
^2/x^4*(b*x^2+a)^(3/2)-1/16*B*b^2/a^3/x^2*(b*x^2+a)^(3/2)-1/16*B*
b^3/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+1/16*B*b^3/a^3*
(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289363, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(8\,Bab^3-5\,Ab^4\right)x^8\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}+2\,\sqrt{bx^2+a}a}{x^2}\right)-2\left(3\left(8\,Bab^2-5\,Ab^3\right)x^6-2\left(8\,Ba^2b-5\,Aab^2\right)x^4-48\,Aa^3-8\left(8\,B\right.\right.}{768\,a^{\frac{7}{2}}x^8}\right.$$

$$-\frac{3\left(8\,Bab^3-5\,Ab^4\right)x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(3\left(8\,Bab^2-5\,Ab^3\right)x^6-2\left(8\,Ba^2b-5\,Aab^2\right)x^4-48\,Aa^3-8\left(8\,Ba^3+Aa^2b\right)x^2\right)\sqrt{}}{384\,\sqrt{-a}a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 5*A*b^4)*x^8*log(-((b*x^2 + 2*a)*sqrt(a)
+ 2*sqrt(b*x^2 + a)*a)/x^2) - 2*(3*(8*B*a*b^2 - 5*A*b^3)*x^6 - 2*
```

```
(8*B*a^2*b - 5*A*a*b^2)*x^4 - 48*A*a^3 - 8*(8*B*a^3 + A*a^2*b)*x^
2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(7/2)*x^8), -1/384*(3*(8*B*a*b^3 -
 5*A*b^4)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*(8*B*a*b^2 -
5*A*b^3)*x^6 - 2*(8*B*a^2*b - 5*A*a*b^2)*x^4 - 48*A*a^3 - 8*(8*B*
a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^8)]
```

Sympy [A] time = 178.175, size = 286, normalized size = 1.83

$$-\frac{Aa}{8\sqrt{b}x^{9}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{7A\sqrt{b}}{48x^{7}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{3}{2}}}{192ax^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{5}{2}}}{384a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{7}{2}}}{128a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5Ab^{4}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{7}{2}}}-\frac{Ba}{6\sqrt{b}x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5B\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Bb^{\frac{3}{2}}}{48ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Bb^{\frac{5}{2}}}{16a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Bb^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] -A*a/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 7*A*sqrt(b)/(48*x**7
*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)/(192*a*x**5*sqrt(a/(b*x**2) +
 1)) - 5*A*b**(5/2)/(384*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b*
*(7/2)/(128*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**4*asinh(sqrt(a)
/(sqrt(b)*x))/(128*a**(7/2)) - B*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2
) + 1)) - 5*B*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)
/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)/(16*a**2*x*sqrt(a/
(b*x**2) + 1)) - B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

GIAC/XCAS [A] time = 0.235266, size = 262, normalized size = 1.68

$$\frac{\frac{3\left(8\,Bab^{4}-5\,Ab^{5}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{3}}+\frac{24\left(bx^{2}+a\right)^{\frac{7}{2}}Bab^{4}-88\left(bx^{2}+a\right)^{\frac{5}{2}}Ba^{2}b^{4}+40\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{3}b^{4}+24\sqrt{bx^{2}+a}Ba^{4}b^{4}-15\left(bx^{2}+a\right)^{\frac{7}{2}}Ab^{5}+55\left(bx^{2}+a\right)^{\frac{5}{2}}Aab^{5}-}{a^{3}b^{4}x^{8}}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(3*(8*B*a*b^4 - 5*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(
sqrt(-a)*a^3) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 - 88*(b*x^2 + a)^(5
/2)*B*a^2*b^4 + 40*(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 +
a)*B*a^4*b^4 - 15*(b*x^2 + a)^(7/2)*A*b^5 + 55*(b*x^2 + a)^(5/2)*
A*a*b^5 - 73*(b*x^2 + a)^(3/2)*A*a^2*b^5 - 15*sqrt(b*x^2 + a)*A*a
^3*b^5)/(a^3*b^4*x^8))/b
```

3.519 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=117

$$\frac{8b^2(a+bx^2)^{3/2}(2Ab-3aB)}{315a^4x^3}-\frac{4b(a+bx^2)^{3/2}(2Ab-3aB)}{105a^3x^5}+\frac{(a+bx^2)^{3/2}(2Ab-3aB)}{21a^2x^7}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

```
[Out] -(A*(a + b*x^2)^(3/2))/(9*a*x^9) + ((2*A*b - 3*a*B)*(a + b*x^2)^(
3/2))/(21*a^2*x^7) - (4*b*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(105
*a^3*x^5) + (8*b^2*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(315*a^4*x^
3)
```

Rubi [A] time = 0.166954, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{8b^2(a+bx^2)^{3/2}(2Ab-3aB)}{315a^4x^3}-\frac{4b(a+bx^2)^{3/2}(2Ab-3aB)}{105a^3x^5}+\frac{(a+bx^2)^{3/2}(2Ab-3aB)}{21a^2x^7}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(9*a*x^9) + ((2*A*b - 3*a*B)*(a + b*x^2)^(
3/2))/(21*a^2*x^7) - (4*b*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(105
*a^3*x^5) + (8*b^2*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(315*a^4*x^
3)
```

Rubi in Sympy [A] time = 16.5738, size = 112, normalized size = 0.96

$$-\frac{A(a+bx^2)^{\frac{3}{2}}}{9ax^9}+\frac{(a+bx^2)^{\frac{3}{2}}(2Ab-3Ba)}{21a^2x^7}-\frac{4b(a+bx^2)^{\frac{3}{2}}(2Ab-3Ba)}{105a^3x^5}+\frac{8b^2(a+bx^2)^{\frac{3}{2}}(2Ab-3Ba)}{315a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(9*a*x**9) + (a + b*x**2)**(3/2)*(2*A*b -
3*B*a)/(21*a**2*x**7) - 4*b*(a + b*x**2)**(3/2)*(2*A*b - 3*B*a)/(
105*a**3*x**5) + 8*b**2*(a + b*x**2)**(3/2)*(2*A*b - 3*B*a)/(315*
a**4*x**3)
```

Mathematica [A] time = 0.0859574, size = 81, normalized size = 0.69

$$\frac{(a+bx^2)^{3/2}(-5a^3(7A+9Bx^2)+6a^2bx^2(5A+6Bx^2)-24ab^2x^4(A+Bx^2)+16Ab^3x^6)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(16*A*b^3*x^6 - 24*a*b^2*x^4*(A + B*x^2) + 6*a
^2*b*x^2*(5*A + 6*B*x^2) - 5*a^3*(7*A + 9*B*x^2)))/(315*a^4*x^9)
```

Maple [A] time = 0.01, size = 83, normalized size = 0.7

$$-\frac{-16\,Ab^{3}x^{6}+24\,Bab^{2}x^{6}+24\,Aab^{2}x^{4}-36\,Ba^{2}bx^{4}-30\,Aa^{2}bx^{2}+45\,Ba^{3}x^{2}+35\,Aa^{3}}{315\,x^{9}a^{4}}\left(bx^{2}+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-16*A*b^3*x^6+24*B*a*b^2*x^6+24*A*a*b^2*x
^4-36*B*a^2*b*x^4-30*A*a^2*b*x^2+45*B*a^3*x^2+35*A*a^3)/x^9/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.321932, size = 142, normalized size = 1.21

$$-\frac{\left(8\left(3\,Bab^{3}-2\,Ab^{4}\right)x^{8}-4\left(3\,Ba^{2}b^{2}-2\,Aab^{3}\right)x^{6}+35\,Aa^{4}+3\left(3\,Ba^{3}b-2\,Aa^{2}b^{2}\right)x^{4}+5\left(9\,Ba^{4}+Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{315\,a^{4}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*a*b^3 - 2*A*b^4)*x^8 - 4*(3*B*a^2*b^2 - 2*A*a*b^3)
*x^6 + 35*A*a^4 + 3*(3*B*a^3*b - 2*A*a^2*b^2)*x^4 + 5*(9*B*a^4 +
A*a^3*b)*x^2)*sqrt(b*x^2 + a)/(a^4*x^9)
```

Sympy [A] time = 13.5257, size = 957, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*A*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 9
45*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14
) - 110*A*a**6*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9
*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b*
*12*x**14) - 114*A*a**5*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 31
5*a**4*b**12*x**14) - 40*A*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2) +
1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14) + 5*A*a**3*b**(27/2)*x**8*sqrt(a/(b*x
**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) + 30*A*a**2*b**(29/2)*x**10*sq
rt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 40*A*a*b**(31/2)*x*
*12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**
10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*A*b**(33/2
```

```
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10
*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*B*a**5
*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**4*b**(11/2)*x**2*sqrt(a/(
b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*
b**6*x**10) - 17*B*a**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*
a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*
a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a*b**(17/2)*x**8*s
qrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - 8*B*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(10
5*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
```

GIAC/XCAS [A] time = 0.23492, size = 464, normalized size = 3.97

$$16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{7}{2}}+630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{9}{2}}+63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(7/2) - 315*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*B*a*b^(7/2) + 630*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*A*b^(9/2) + 63*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a
^2*b^(7/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(9/2) - 42
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(7/2) + 168*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*A*a^2*b^(9/2) + 108*(sqrt(b)*x - sqrt(b*x^2 +
 a))^4*B*a^4*b^(7/2) - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b
^(9/2) - 27*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(7/2) + 18*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(9/2) + 3*B*a^6*b^(7/2) - 2
*A*a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.520 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b^3\sqrt{a+bx^2}(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-10aB)}{384a^3x^4}+\frac{b\sqrt{a+bx^2}(7Ab-10aB)}{480a^2x^6}+\frac{\sqrt{a+bx^2}(7Ab-10aB)}{80ax^8}-\frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}}$$

```
[Out] ((7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(80*a*x^8) + (b*(7*A*b - 10*a*
B)*Sqrt[a + b*x^2])/(480*a^2*x^6) - (b^2*(7*A*b - 10*a*B)*Sqrt[a
+ b*x^2])/(384*a^3*x^4) + (b^3*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/
(256*a^4*x^2) - (A*(a + b*x^2)^(3/2))/(10*a*x^10) - (b^4*(7*A*b -
 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(9/2))
```

Rubi [A] time = 0.371806, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b^3\sqrt{a+bx^2}(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-10aB)}{384a^3x^4}+\frac{b\sqrt{a+bx^2}(7Ab-10aB)}{480a^2x^6}+\frac{\sqrt{a+bx^2}(7Ab-10aB)}{80ax^8}-\frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] ((7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(80*a*x^8) + (b*(7*A*b - 10*a*
B)*Sqrt[a + b*x^2])/(480*a^2*x^6) - (b^2*(7*A*b - 10*a*B)*Sqrt[a
+ b*x^2])/(384*a^3*x^4) + (b^3*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/
(256*a^4*x^2) - (A*(a + b*x^2)^(3/2))/(10*a*x^10) - (b^4*(7*A*b -
 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(9/2))
```

Rubi in Sympy [A] time = 31.8797, size = 177, normalized size = 0.94

$$-\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{10ax^{10}}+\frac{\sqrt{a+bx^2}\,(7Ab-10Ba)}{80ax^8}+\frac{b\sqrt{a+bx^2}\,(7Ab-10Ba)}{480a^2x^6}-\frac{b^2\sqrt{a+bx^2}\,(7Ab-10Ba)}{384a^3x^4}+\frac{b^3\sqrt{a+bx^2}\,(7Ab-10Ba)}{256a^4x^2}-\frac{b^4\,(7Ab-10Ba)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] -A*(a + b*x**2)**(3/2)/(10*a*x**10) + sqrt(a + b*x**2)*(7*A*b - 1
0*B*a)/(80*a*x**8) + b*sqrt(a + b*x**2)*(7*A*b - 10*B*a)/(480*a**
2*x**6) - b**2*sqrt(a + b*x**2)*(7*A*b - 10*B*a)/(384*a**3*x**4)
+ b**3*sqrt(a + b*x**2)*(7*A*b - 10*B*a)/(256*a**4*x**2) - b**4*(
7*A*b - 10*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(256*a**(9/2))
```

Mathematica [A] time = 0.251981, size = 169, normalized size = 0.89

$$-\frac{b^4(7Ab-10aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{256a^{9/2}}+\frac{b^4\log(x)(7Ab-10aB)}{256a^{9/2}}+\sqrt{a+bx^2}\left(-\frac{b^3(10aB-7Ab)}{256a^4x^2}+\frac{b^2(10aB-7Ab)}{384a^3x^4}-\frac{b(10aB-7Ab)}{480a^2x^6}+\frac{-10aB-Ab}{80ax^8}-\frac{A}{10x^{10}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] (-A/(10*x^10) + (-(A*b) - 10*a*B)/(80*a*x^8) - (b*(-7*A*b + 10*a*
B))/(480*a^2*x^6) + (b^2*(-7*A*b + 10*a*B))/(384*a^3*x^4) - (b^3*
(-7*A*b + 10*a*B))/(256*a^4*x^2))*Sqrt[a + b*x^2] + (b^4*(7*A*b -
 10*a*B)*Log[x])/(256*a^(9/2)) - (b^4*(7*A*b - 10*a*B)*Log[a + Sq
rt[a]*Sqrt[a + b*x^2]])/(256*a^(9/2))
```

Maple [A] time = 0.038, size = 281, normalized size = 1.5

$$-\frac{A}{10\,a x^{10}}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{7\,Ab}{80\,a^2 x^8}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{7\,b^2 A}{96\,a^3 x^6}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{7\,Ab^3}{128\,a^4 x^4}\left(b x^2+a\right)^{\frac{3}{2}}$$
$$-\frac{7\,Ab^4}{256\,a^5 x^2}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{7\,Ab^5}{256}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{9}{2}}$$
$$+\frac{7\,Ab^5}{256\,a^5}\sqrt{b x^2+a}-\frac{B}{8\,a x^8}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,Bb}{48\,a^2 x^6}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{5\,Bb^2}{64\,a^3 x^4}\left(b x^2+a\right)^{\frac{3}{2}}$$
$$+\frac{5\,Bb^3}{128\,a^4 x^2}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,Bb^4}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{5\,Bb^4}{128\,a^4}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(3/2)/a/x^10+7/80*A*b/a^2/x^8*(b*x^2+a)^(3/2)-7
/96*A*b^2/a^3/x^6*(b*x^2+a)^(3/2)+7/128*A*b^3/a^4/x^4*(b*x^2+a)^(
3/2)-7/256*A*b^4/a^5/x^2*(b*x^2+a)^(3/2)-7/256*A*b^5/a^(9/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+7/256*A*b^5/a^5*(b*x^2+a)^(1/2)
-1/8*B/a/x^8*(b*x^2+a)^(3/2)+5/48*B*b/a^2/x^6*(b*x^2+a)^(3/2)-5/6
4*B*b^2/a^3/x^4*(b*x^2+a)^(3/2)+5/128*B*b^3/a^4/x^2*(b*x^2+a)^(3/
2)+5/128*B*b^4/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5/12
8*B*b^4/a^4*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.410732, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(10\,Bab^4-7\,Ab^5\right)x^{10}\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)+2\left(15\left(10\,Bab^3-7\,Ab^4\right)x^8-10\left(10\,Ba^2b^2-7\,Aab^3\right)x^6+384\,Aa\right.}{7680\,a^{\frac{9}{2}}x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 7*A*b^5)*x^10*log(-((b*x^2 + 2*a)*sqrt
(a) - 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(15*(10*B*a*b^3 - 7*A*b^4)*x^
```

```
8 - 10*(10*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 384*A*a^4 + 8*(10*B*a^3*b
 - 7*A*a^2*b^2)*x^4 + 48*(10*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a
)*sqrt(a))/(a^(9/2)*x^10), 1/3840*(15*(10*B*a*b^4 - 7*A*b^5)*x^10
*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (15*(10*B*a*b^3 - 7*A*b^4)*x^
8 - 10*(10*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 384*A*a^4 + 8*(10*B*a^3*b
 - 7*A*a^2*b^2)*x^4 + 48*(10*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a
)*sqrt(-a))/(sqrt(-a)*a^4*x^10)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239782, size = 311, normalized size = 1.65

$$-\frac{\frac{15\left(10\,Bab^5-7\,Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{150\left(bx^2+a\right)^{\frac{9}{2}}Bab^5-700\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5+1280\left(bx^2+a\right)^{\frac{5}{2}}Ba^3b^5-580\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-150\sqrt{bx^2+a}Ba^5b^5-105\,(b}{a^4b^5x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 7*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a
))/(sqrt(-a)*a^4) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 - 700*(b*x^2 +
 a)^(7/2)*B*a^2*b^5 + 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 - 580*(b*x
^2 + a)^(3/2)*B*a^4*b^5 - 150*sqrt(b*x^2 + a)*B*a^5*b^5 - 105*(b*
x^2 + a)^(9/2)*A*b^6 + 490*(b*x^2 + a)^(7/2)*A*a*b^6 - 896*(b*x^2
 + a)^(5/2)*A*a^2*b^6 + 790*(b*x^2 + a)^(3/2)*A*a^3*b^6 + 105*sqr
t(b*x^2 + a)*A*a^4*b^6)/(a^4*b^5*x^10))/b
```

3.521 $\int x^5 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=103

$$\frac{a^2 \left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2} (Ab-3aB)}{9b^4}-\frac{a \left(a+bx^2\right)^{7/2} (2Ab-3aB)}{7b^4}+\frac{B \left(a+bx^2\right)^{11/2}}{11b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (a*(2*A*b - 3*a*B)*
(a + b*x^2)^(7/2))/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9
*b^4) + (B*(a + b*x^2)^(11/2))/(11*b^4)
```

Rubi [A] time = 0.227871, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^2 \left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2} (Ab-3aB)}{9b^4}-\frac{a \left(a+bx^2\right)^{7/2} (2Ab-3aB)}{7b^4}+\frac{B \left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (a*(2*A*b - 3*a*B)*
(a + b*x^2)^(7/2))/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9
*b^4) + (B*(a + b*x^2)^(11/2))/(11*b^4)
```

Rubi in Sympy [A] time = 25.747, size = 92, normalized size = 0.89

$$\frac{B \left(a+bx^2\right)^{\frac{11}{2}}}{11b^4}+\frac{a^2 \left(a+bx^2\right)^{\frac{5}{2}} (Ab-Ba)}{5b^4}-\frac{a \left(a+bx^2\right)^{\frac{7}{2}} (2Ab-3Ba)}{7b^4}+\frac{\left(a+bx^2\right)^{\frac{9}{2}} (Ab-3Ba)}{9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**(11/2)/(11*b**4) + a**2*(a + b*x**2)**(5/2)*(A*b
- B*a)/(5*b**4) - a*(a + b*x**2)**(7/2)*(2*A*b - 3*B*a)/(7*b**4)
+ (a + b*x**2)**(9/2)*(A*b - 3*B*a)/(9*b**4)
```

Mathematica [A] time = 0.0964467, size = 78, normalized size = 0.76

$$\frac{\left(a+bx^2\right)^{5/2} \left(-48a^3B+8a^2b \left(11A+15Bx^2\right)-10ab^2x^2 \left(22A+21Bx^2\right)+35b^3x^4 \left(11A+9Bx^2\right)\right)}{3465b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-48*a^3*B + 35*b^3*x^4*(11*A + 9*B*x^2) + 8*a
^2*b*(11*A + 15*B*x^2) - 10*a*b^2*x^2*(22*A + 21*B*x^2)))/(3465*b
^4)
```

Maple [A] time = 0.008, size = 77, normalized size = 0.8

$$\frac{315\,Bx^6b^3+385\,Ab^3x^4-210\,Bab^2x^4-220\,Aab^2x^2+120\,Ba^2bx^2+88\,Aa^2b-48\,Ba^3}{3465\,b^4} \left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/3465*(b*x^2+a)^(5/2)*(315*B*b^3*x^6+385*A*b^3*x^4-210*B*a*b^2*x
^4-220*A*a*b^2*x^2+120*B*a^2*b*x^2+88*A*a^2*b-48*B*a^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215693, size = 167, normalized size = 1.62

$$\frac{\left(315\,Bb^5x^{10} + 35\,\left(12\,Bab^4 + 11\,Ab^5\right)x^8 + 5\,\left(3\,Ba^2b^3 + 110\,Aab^4\right)x^6 - 48\,Ba^5 + 88\,Aa^4b - 3\,\left(6\,Ba^3b^2 - 11\,Aa^2b^3\right)x^4 + 4\,\left(6\,B\right.\right.}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*b^5*x^10 + 35*(12*B*a*b^4 + 11*A*b^5)*x^8 + 5*(3*B*
a^2*b^3 + 110*A*a*b^4)*x^6 - 48*B*a^5 + 88*A*a^4*b - 3*(6*B*a^3*b
^2 - 11*A*a^2*b^3)*x^4 + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b
*x^2 + a)/b^4
```

Sympy [A] time = 12.8816, size = 260, normalized size = 2.52

$$\begin{cases} \frac{8Aa^4\sqrt{a+bx^2}}{315b^3} - \frac{4Aa^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{Aa^2x^4\sqrt{a+bx^2}}{105b} + \frac{10Aax^6\sqrt{a+bx^2}}{63} + \frac{Abx^8\sqrt{a+bx^2}}{9} - \frac{16Ba^5\sqrt{a+bx^2}}{1155b^4} + \frac{8Ba^4x^2\sqrt{a+bx^2}}{1155b^3} - \frac{2Ba^3x^4\sqrt{a+bx^2}}{385b^2} + B \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*A*a**3*x**2*s
qrt(a + b*x**2)/(315*b**2) + A*a**2*x**4*sqrt(a + b*x**2)/(105*b)
 + 10*A*a*x**6*sqrt(a + b*x**2)/63 + A*b*x**8*sqrt(a + b*x**2)/9
- 16*B*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*B*a**4*x**2*sqrt(a +
 b*x**2)/(1155*b**3) - 2*B*a**3*x**4*sqrt(a + b*x**2)/(385*b**2)
+ B*a**2*x**6*sqrt(a + b*x**2)/(231*b) + 4*B*a*x**8*sqrt(a + b*x*
*2)/33 + B*b*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(3/2)*(A*x
**6/6 + B*x**8/8), True))
```

GIAC/XCAS [A] time = 0.248812, size = 323, normalized size = 3.14

$$\frac{\frac{33\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Aa}{b^2} + \frac{11\left(35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)Ba}{b^3} + \frac{11\left(35\left(bx^2+a\right)^{\frac{9}{2}}-13\right.}{}}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b
*x^2 + a)^(3/2)*a^2)*A*a/b^2 + 11*(35*(b*x^2 + a)^(9/2) - 135*(b*
x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3
/2)*a^3)*B*a/b^3 + 11*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/
2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)*A/b
^2 + (315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9/2)*a + 2970*(b
*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3 + 1155*(b*x^2 +
a)^(3/2)*a^4)*B/b^3)/b
```

3.522 $\int x^4\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=188

$$\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}-\frac{3a^3x\sqrt{a+bx^2}(2Ab-aB)}{256b^3}+\frac{a^2x^3\sqrt{a+bx^2}(2Ab-aB)}{128b^2}+\frac{x^5\left(a+bx^2\right)^{3/2}(2Ab-aB)}{16b}+\frac{ax^5\sqrt{a+bx^2}(2Ab-aB)}{32b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}$$

```
[Out] (-3*a^3*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(256*b^3) + (a^2*(2*A*b
- a*B)*x^3*Sqrt[a + b*x^2])/(128*b^2) + (a*(2*A*b - a*B)*x^5*Sqrt
[a + b*x^2])/(32*b) + ((2*A*b - a*B)*x^5*(a + b*x^2)^(3/2))/(16*b
) + (B*x^5*(a + b*x^2)^(5/2))/(10*b) + (3*a^4*(2*A*b - a*B)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(7/2))
```

Rubi [A] time = 0.28575, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}-\frac{3a^3x\sqrt{a+bx^2}(2Ab-aB)}{256b^3}+\frac{a^2x^3\sqrt{a+bx^2}(2Ab-aB)}{128b^2}+\frac{x^5\left(a+bx^2\right)^{3/2}(2Ab-aB)}{16b}+\frac{ax^5\sqrt{a+bx^2}(2Ab-aB)}{32b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (-3*a^3*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(256*b^3) + (a^2*(2*A*b
- a*B)*x^3*Sqrt[a + b*x^2])/(128*b^2) + (a*(2*A*b - a*B)*x^5*Sqrt
[a + b*x^2])/(32*b) + ((2*A*b - a*B)*x^5*(a + b*x^2)^(3/2))/(16*b
) + (B*x^5*(a + b*x^2)^(5/2))/(10*b) + (3*a^4*(2*A*b - a*B)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(7/2))
```

Rubi in Sympy [A] time = 27.8905, size = 170, normalized size = 0.9

$$\frac{Bx^5\left(a+bx^2\right)^{\frac{5}{2}}}{10b}+\frac{3a^4\left(2Ab-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{\frac{7}{2}}}-\frac{3a^3x\sqrt{a+bx^2}\left(2Ab-Ba\right)}{256b^3}+\frac{a^2x^3\sqrt{a+bx^2}\left(2Ab-Ba\right)}{128b^2}+\frac{ax^5\sqrt{a+bx^2}\left(2Ab-Ba\right)}{32b}+\frac{x^5\left(a+bx^2\right)^{\frac{3}{2}}\left(2Ab-Ba\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] B*x**5*(a + b*x**2)**(5/2)/(10*b) + 3*a**4*(2*A*b - B*a)*atanh(sq
rt(b)*x/sqrt(a + b*x**2))/(256*b**(7/2)) - 3*a**3*x*sqrt(a + b*x*
*2)*(2*A*b - B*a)/(256*b**3) + a**2*x**3*sqrt(a + b*x**2)*(2*A*b
- B*a)/(128*b**2) + a*x**5*sqrt(a + b*x**2)*(2*A*b - B*a)/(32*b)
+ x**5*(a + b*x**2)**(3/2)*(2*A*b - B*a)/(16*b)
```

Mathematica [A] time = 0.157687, size = 140, normalized size = 0.74

$$\sqrt{a+bx^2}\left(\frac{3a^3x(aB-2Ab)}{256b^3}-\frac{a^2x^3(aB-2Ab)}{128b^2}+\frac{1}{80}x^7(11aB+10Ab)+\frac{ax^5(aB+30Ab)}{160b}+\frac{1}{10}bBx^9\right)-\frac{3a^4(aB-2Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{256b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((3*a^3*(-2*A*b + a*B)*x)/(256*b^3) - (a^2*(-2*A*
b + a*B)*x^3)/(128*b^2) + (a*(30*A*b + a*B)*x^5)/(160*b) + ((10*A
*b + 11*a*B)*x^7)/80 + (b*B*x^9)/10) - (3*a^4*(-2*A*b + a*B)*Log[
b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(256*b^(7/2))
```

Maple [A] time = 0.012, size = 219, normalized size = 1.2

$$\frac{Ax^3}{8b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{aAx}{16b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a^2Ax}{64b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3Aa^3x}{128b^2}\sqrt{bx^2+a}+\frac{3Aa^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{Bx^5}{10b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bax^3}{16b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Bxa^2}{32b^3}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Ba^3x}{128b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3Ba^4x}{256b^3}\sqrt{bx^2+a}-\frac{3Ba^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/8*A*x^3*(b*x^2+a)^(5/2)/b-1/16*A*a/b^2*x*(b*x^2+a)^(5/2)+1/64*A
*a^2/b^2*x*(b*x^2+a)^(3/2)+3/128*A*a^3/b^2*x*(b*x^2+a)^(1/2)+3/12
8*A*a^4/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/10*B*x^5*(b*x^2+a
)^(5/2)/b-1/16*B*a/b^2*x^3*(b*x^2+a)^(5/2)+1/32*B*a^2/b^3*x*(b*x^
2+a)^(5/2)-1/128*B*a^3/b^3*x*(b*x^2+a)^(3/2)-3/256*B*a^4/b^3*x*(b
*x^2+a)^(1/2)-3/256*B*a^5/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.367371, size = 1, normalized size = 0.01

$$\left[\frac{2\left(128\,Bb^4x^9+16\left(11\,Bab^3+10\,Ab^4\right)x^7+8\left(Ba^2b^2+30\,Aab^3\right)x^5-10\left(Ba^3b-2\,Aa^2b^2\right)x^3+15\left(Ba^4-2\,Aa^3b\right)x\right)\sqrt{bx^2+}}{2560\,b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/2560*(2*(128*B*b^4*x^9 + 16*(11*B*a*b^3 + 10*A*b^4)*x^7 + 8*(B
*a^2*b^2 + 30*A*a*b^3)*x^5 - 10*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 15*
(B*a^4 - 2*A*a^3*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 15*(B*a^5 - 2*A*
a^4*b)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(7/
2), 1/1280*((128*B*b^4*x^9 + 16*(11*B*a*b^3 + 10*A*b^4)*x^7 + 8*(
B*a^2*b^2 + 30*A*a*b^3)*x^5 - 10*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 15
*(B*a^4 - 2*A*a^3*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) - 15*(B*a^5 - 2*
A*a^4*b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [A] time = 112.338, size = 345, normalized size = 1.84

$$-\frac{3Aa^{\frac{7}{2}}x}{128b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{\frac{5}{2}}x^{3}}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{13Aa^{\frac{3}{2}}x^{5}}{64\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5A\sqrt{a}bx^{7}}{16\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Aa^{4}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Ab^{2}x^{9}}{8\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba^{\frac{9}{2}}x}{256b^{3}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{Ba^{\frac{7}{2}}x^{3}}{256b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{5}{2}}x^{5}}{640b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{23Ba^{\frac{3}{2}}x^{7}}{160\sqrt{1+\frac{bx^{2}}{a}}}+\frac{19B\sqrt{a}bx^{9}}{80\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3Ba^{5}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{7}{2}}}+\frac{Bb^{2}x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] -3*A*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - A*a**(5/2)*x**3/(
128*b*sqrt(1 + b*x**2/a)) + 13*A*a**(3/2)*x**5/(64*sqrt(1 + b*x**
2/a)) + 5*A*sqrt(a)*b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*A*a**4*asi
nh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + A*b**2*x**9/(8*sqrt(a)*sqr
t(1 + b*x**2/a)) + 3*B*a**(9/2)*x/(256*b**3*sqrt(1 + b*x**2/a)) +
 B*a**(7/2)*x**3/(256*b**2*sqrt(1 + b*x**2/a)) - B*a**(5/2)*x**5/
(640*b*sqrt(1 + b*x**2/a)) + 23*B*a**(3/2)*x**7/(160*sqrt(1 + b*x
**2/a)) + 19*B*sqrt(a)*b*x**9/(80*sqrt(1 + b*x**2/a)) - 3*B*a**5*
asinh(sqrt(b)*x/sqrt(a))/(256*b**(7/2)) + B*b**2*x**11/(10*sqrt(a
)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.233907, size = 215, normalized size = 1.14

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bbx^{2}+\frac{11\,Bab^{8}+10\,Ab^{9}}{b^{8}}\right)x^{2}+\frac{Ba^{2}b^{7}+30\,Aab^{8}}{b^{8}}\right)x^{2}-\frac{5\left(Ba^{3}b^{6}-2\,Aa^{2}b^{7}\right)}{b^{8}}\right)x^{2}+\frac{15\left(Ba^{4}b^{5}-2\,Aa^{3}b^{6}\right)}{b^{8}}\right)$$
$$+\frac{3\left(Ba^{5}-2\,Aa^{4}b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*b*x^2 + (11*B*a*b^8 + 10*A*b^9)/b^8)*x^2 + (
B*a^2*b^7 + 30*A*a*b^8)/b^8)*x^2 - 5*(B*a^3*b^6 - 2*A*a^2*b^7)/b^
8)*x^2 + 15*(B*a^4*b^5 - 2*A*a^3*b^6)/b^8)*sqrt(b*x^2 + a)*x + 3/
256*(B*a^5 - 2*A*a^4*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
7/2)
```

3.523 $\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{a\left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^3}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^3) + ((A*b - 2*a*B)*(a +
b*x^2)^(7/2))/(7*b^3) + (B*(a + b*x^2)^(9/2))/(9*b^3)
```

Rubi [A] time = 0.167402, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\left(a+bx^2\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{a\left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^3}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^3) + ((A*b - 2*a*B)*(a +
b*x^2)^(7/2))/(7*b^3) + (B*(a + b*x^2)^(9/2))/(9*b^3)
```

Rubi in Sympy [A] time = 19.6681, size = 63, normalized size = 0.86

$$\frac{B\left(a+bx^2\right)^{\frac{9}{2}}}{9b^3}-\frac{a\left(a+bx^2\right)^{\frac{5}{2}} (Ab-Ba)}{5b^3}+\frac{\left(a+bx^2\right)^{\frac{7}{2}} (Ab-2Ba)}{7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**(9/2)/(9*b**3) - a*(a + b*x**2)**(5/2)*(A*b - B*a
)/(5*b**3) + (a + b*x**2)**(7/2)*(A*b - 2*B*a)/(7*b**3)
```

Mathematica [A] time = 0.0652535, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{5/2} \left(8a^2B-2ab\left(9A+10Bx^2\right)+5b^2x^2\left(9A+7Bx^2\right)\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(8*a^2*B + 5*b^2*x^2*(9*A + 7*B*x^2) - 2*a*b*(
9*A + 10*B*x^2)))/(315*b^3)
```

Maple [A] time = 0.007, size = 53, normalized size = 0.7

$$-\frac{-35\,b^2Bx^4-45\,Ab^2x^2+20\,Babx^2+18\,abA-8\,a^2B}{315\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(-35*B*b^2*x^4-45*A*b^2*x^2+20*B*a*b*x^2+1
8*A*a*b-8*B*a^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215094, size = 134, normalized size = 1.84

$$\frac{\left(35\,Bb^{4}x^{8}+5\,\left(10\,Bab^{3}+9\,Ab^{4}\right)x^{6}+8\,Ba^{4}-18\,Aa^{3}b+3\,\left(Ba^{2}b^{2}+24\,Aab^{3}\right)x^{4}-\left(4\,Ba^{3}b-9\,Aa^{2}b^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{315\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(10*B*a*b^3 + 9*A*b^4)*x^6 + 8*B*a^4 - 18
*A*a^3*b + 3*(B*a^2*b^2 + 24*A*a*b^3)*x^4 - (4*B*a^3*b - 9*A*a^2*
b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 7.30306, size = 209, normalized size = 2.86

$$\begin{cases} -\frac{2Aa^{3}\sqrt{a+bx^{2}}}{35b^{2}}+\frac{Aa^{2}x^{2}\sqrt{a+bx^{2}}}{35b}+\frac{8Aax^{4}\sqrt{a+bx^{2}}}{35}+\frac{Abx^{6}\sqrt{a+bx^{2}}}{7}+\frac{8Ba^{4}\sqrt{a+bx^{2}}}{315b^{3}}-\frac{4Ba^{3}x^{2}\sqrt{a+bx^{2}}}{315b^{2}}+\frac{Ba^{2}x^{4}\sqrt{a+bx^{2}}}{105b}+\frac{10Bax^{6}\sqrt{a+bx^{2}}}{63}+ Bb \\ a^{\frac{3}{2}}\left(\frac{Ax^{4}}{4}+\frac{Bx^{6}}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**3*sqrt(a + b*x**2)/(35*b**2) + A*a**2*x**2*sqr
t(a + b*x**2)/(35*b) + 8*A*a*x**4*sqrt(a + b*x**2)/35 + A*b*x**6*
sqrt(a + b*x**2)/7 + 8*B*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*B*a
**3*x**2*sqrt(a + b*x**2)/(315*b**2) + B*a**2*x**4*sqrt(a + b*x**
2)/(105*b) + 10*B*a*x**6*sqrt(a + b*x**2)/63 + B*b*x**8*sqrt(a +
b*x**2)/9, Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**6/6), True))
```

GIAC/XCAS [A] time = 0.22422, size = 247, normalized size = 3.38

$$\frac{\frac{21\left(3\left(bx^{2}+a\right)^{\frac{5}{2}}-5\left(bx^{2}+a\right)^{\frac{3}{2}}a\right)Aa}{b}+\frac{3\left(15\left(bx^{2}+a\right)^{\frac{7}{2}}-42\left(bx^{2}+a\right)^{\frac{5}{2}}a+35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}\right)Ba}{b^{2}}+\frac{3\left(15\left(bx^{2}+a\right)^{\frac{7}{2}}-42\left(bx^{2}+a\right)^{\frac{5}{2}}a+35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}\right)A}{b}+ (35 (b}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*A*a/b + 3
*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^
(3/2)*a^2)*B*a/b^2 + 3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/
2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*A/b + (35*(b*x^2 + a)^(9/2) - 13
5*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 +
a)^(3/2)*a^3)*B/b^2)/b
```

3.524 $\int x^2 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=155

$$-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x\sqrt{a+bx^2}(8Ab-3aB)}{128b^2}+\frac{ax^3\sqrt{a+bx^2}(8Ab-3aB)}{64b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8Ab-3aB)}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

```
[Out] (a^2*(8*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(128*b^2) + (a*(8*A*b - 3
*a*B)*x^3*Sqrt[a + b*x^2])/(64*b) + ((8*A*b - 3*a*B)*x^3*(a + b*x
^2)^(3/2))/(48*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a^3*(8*A*b
 - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.210596, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x\sqrt{a+bx^2}(8Ab-3aB)}{128b^2}+\frac{ax^3\sqrt{a+bx^2}(8Ab-3aB)}{64b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8Ab-3aB)}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(128*b^2) + (a*(8*A*b - 3
*a*B)*x^3*Sqrt[a + b*x^2])/(64*b) + ((8*A*b - 3*a*B)*x^3*(a + b*x
^2)^(3/2))/(48*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a^3*(8*A*b
 - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi in Sympy [A] time = 24.4759, size = 143, normalized size = 0.92

$$\frac{Bx^3\left(a+bx^2\right)^{\frac{5}{2}}}{8b}-\frac{a^3\left(8Ab-3Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{5}{2}}}+\frac{a^2x\sqrt{a+bx^2}\left(8Ab-3Ba\right)}{128b^2}+\frac{ax^3\sqrt{a+bx^2}\left(8Ab-3Ba\right)}{64b}+\frac{x^3\left(a+bx^2\right)^{\frac{3}{2}}\left(8Ab-3Ba\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] B*x**3*(a + b*x**2)**(5/2)/(8*b) - a**3*(8*A*b - 3*B*a)*atanh(sqr
t(b)*x/sqrt(a + b*x**2))/(128*b**(5/2)) + a**2*x*sqrt(a + b*x**2)
*(8*A*b - 3*B*a)/(128*b**2) + a*x**3*sqrt(a + b*x**2)*(8*A*b - 3*
B*a)/(64*b) + x**3*(a + b*x**2)**(3/2)*(8*A*b - 3*B*a)/(48*b)
```

Mathematica [A] time = 0.125664, size = 122, normalized size = 0.79

$$\frac{a^3(3aB-8Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{128b^{5/2}}+\sqrt{a+bx^2}\left(-\frac{a^2x(3aB-8Ab)}{128b^2}+\frac{1}{48}x^5(9aB+8Ab)+\frac{ax^3(3aB+56Ab)}{192b}+\frac{1}{8}bBx^7\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*(-(a^2*(-8*A*b + 3*a*B)*x)/(128*b^2) + (a*(56*A*b
 + 3*a*B)*x^3)/(192*b) + ((8*A*b + 9*a*B)*x^5)/48 + (b*B*x^7)/8)
+ (a^3*(-8*A*b + 3*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(128*
b^(5/2))
```

Maple [A] time = 0.01, size = 177, normalized size = 1.1

$$\frac{Ax}{6\,b}\left(bx^{2}+a\right)^{\frac{5}{2}}-\frac{aAx}{24\,b}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{a^{2}Ax}{16\,b}\sqrt{bx^{2}+a}-\frac{Aa^{3}}{16}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{3}{2}}+\frac{Bx^{3}}{8\,b}\left(bx^{2}+a\right)^{\frac{5}{2}}-\frac{Bxa}{16\,b^{2}}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{Bxa^{2}}{64\,b^{2}}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{3\,Ba^{3}x}{128\,b^{2}}\sqrt{bx^{2}+a}+\frac{3\,Ba^{4}}{128}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/6*A*x*(b*x^2+a)^(5/2)/b-1/24*A*a/b*x*(b*x^2+a)^(3/2)-1/16*A*a^2
/b*x*(b*x^2+a)^(1/2)-1/16*A*a^3/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))+1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/16*B*a/b^2*x*(b*x^2+a)^(5/2)+1
/64*B*a^2/b^2*x*(b*x^2+a)^(3/2)+3/128*B*a^3/b^2*x*(b*x^2+a)^(1/2)
+3/128*B*a^4/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279023, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,Bb^{3}x^{7}+8\left(9\,Bab^{2}+8\,Ab^{3}\right)x^{5}+2\left(3\,Ba^{2}b+56\,Aab^{2}\right)x^{3}-3\left(3\,Ba^{3}-8\,Aa^{2}b\right)x\right)\sqrt{bx^{2}+a}\sqrt{b}-3\left(3\,Ba^{4}-8\,Aa^{3}b\right)\log}{768\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*B*b^3*x^7 + 8*(9*B*a*b^2 + 8*A*b^3)*x^5 + 2*(3*B*a^
2*b + 56*A*a*b^2)*x^3 - 3*(3*B*a^3 - 8*A*a^2*b)*x)*sqrt(b*x^2 + a
)*sqrt(b) - 3*(3*B*a^4 - 8*A*a^3*b)*log(2*sqrt(b*x^2 + a)*b*x - (
2*b*x^2 + a)*sqrt(b)))/b^(5/2), 1/384*((48*B*b^3*x^7 + 8*(9*B*a*b
^2 + 8*A*b^3)*x^5 + 2*(3*B*a^2*b + 56*A*a*b^2)*x^3 - 3*(3*B*a^3 -
 8*A*a^2*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 3*(3*B*a^4 - 8*A*a^3*b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 73.7097, size = 287, normalized size = 1.85

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Aa^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11A\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Ba^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Bb^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*s
qrt(1 + b*x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a))
- A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*
sqrt(a)*sqrt(1 + b*x**2/a)) - 3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b
*x**2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(1 + b*x**2/a)) + 13*B*a**
(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b*x**7/(16*sqrt(
1 + b*x**2/a)) + 3*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2))
 + B*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.23891, size = 180, normalized size = 1.16

$$\frac{1}{384}\left(2\left(4\left(6\,Bbx^2+\frac{9\,Bab^6+8\,Ab^7}{b^6}\right)x^2+\frac{3\,Ba^2b^5+56\,Aab^6}{b^6}\right)x^2-\frac{3\left(3\,Ba^3b^4-8\,Aa^2b^5\right)}{b^6}\right)\sqrt{bx^2+a}x-\frac{\left(3\,Ba^4-8\,Aa^3b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b*x^2 + (9*B*a*b^6 + 8*A*b^7)/b^6)*x^2 + (3*B*a^
2*b^5 + 56*A*a*b^6)/b^6)*x^2 - 3*(3*B*a^3*b^4 - 8*A*a^2*b^5)/b^6)
*sqrt(b*x^2 + a)*x - 1/128*(3*B*a^4 - 8*A*a^3*b)*ln(abs(-sqrt(b)*
x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.525 $\int x \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (B*(a + b*x^2)^(7/2))/(
7*b^2)
```

Rubi [A] time = 0.102413, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (B*(a + b*x^2)^(7/2))/(
7*b^2)
```

Rubi in Sympy [A] time = 13.0507, size = 37, normalized size = 0.8

$$\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b^2}+\frac{\left(a+bx^2\right)^{\frac{5}{2}}(Ab-Ba)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**(7/2)/(7*b**2) + (a + b*x**2)**(5/2)*(A*b - B*a)/
(5*b**2)
```

Mathematica [A] time = 0.0502828, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(-2aB+7Ab+5bBx^2\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^2))/(35*b^2)
```

Maple [A] time = 0.006, size = 31, normalized size = 0.7

$$\frac{5\,bBx^2+7\,Ab-2\,Ba}{35\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/35*(b*x^2+a)^(5/2)*(5*B*b*x^2+7*A*b-2*B*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212391, size = 99, normalized size = 2.15

$$\frac{\left(5\,Bb^{3}x^{6} + \left(8\,Bab^{2} + 7\,Ab^{3}\right)x^{4} - 2\,Ba^{3} + 7\,Aa^{2}b + \left(Ba^{2}b + 14\,Aab^{2}\right)x^{2}\right)\sqrt{bx^{2} + a}}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*b^3*x^6 + (8*B*a*b^2 + 7*A*b^3)*x^4 - 2*B*a^3 + 7*A*a^2
*b + (B*a^2*b + 14*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 3.69978, size = 158, normalized size = 3.43

$$\begin{cases} \frac{Aa^{2}\sqrt{a+bx^{2}}}{5b} + \frac{2Aax^{2}\sqrt{a+bx^{2}}}{5} + \frac{Abx^{4}\sqrt{a+bx^{2}}}{5} - \frac{2Ba^{3}\sqrt{a+bx^{2}}}{35b^{2}} + \frac{Ba^{2}x^{2}\sqrt{a+bx^{2}}}{35b} + \frac{8Bax^{4}\sqrt{a+bx^{2}}}{35} + \frac{Bbx^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\ b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^{2}}{2} + \frac{Bx^{4}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**2*sqrt(a + b*x**2)/(5*b) + 2*A*a*x**2*sqrt(a + b*
x**2)/5 + A*b*x**4*sqrt(a + b*x**2)/5 - 2*B*a**3*sqrt(a + b*x**2)
/(35*b**2) + B*a**2*x**2*sqrt(a + b*x**2)/(35*b) + 8*B*a*x**4*sqr
t(a + b*x**2)/35 + B*b*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (a**(3
/2)*(A*x**2/2 + B*x**4/4), True))
```

GIAC/XCAS [A] time = 0.245832, size = 162, normalized size = 3.52

$$\frac{35\left(bx^{2} + a\right)^{\frac{3}{2}}Aa + 7\left(3\left(bx^{2} + a\right)^{\frac{5}{2}} - 5\left(bx^{2} + a\right)^{\frac{3}{2}}a\right)A + \frac{7\left(3\left(bx^{2}+a\right)^{\frac{5}{2}} - 5\left(bx^{2}+a\right)^{\frac{3}{2}}a\right)Ba}{b} + \frac{\left(15\left(bx^{2}+a\right)^{\frac{7}{2}} - 42\left(bx^{2}+a\right)^{\frac{5}{2}}a + 35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}{b}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/105*(35*(b*x^2 + a)^(3/2)*A*a + 7*(3*(b*x^2 + a)^(5/2) - 5*(b*x
^2 + a)^(3/2)*a)*A + 7*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)
*a)*B*a/b + (15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(
b*x^2 + a)^(3/2)*a^2)*B/b)/b
```

3.526 $\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=118

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}(6Ab-aB)}{16b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

```
[Out] (a*(6*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b) + ((6*A*b - a*B)*x*(a
+ b*x^2)^(3/2))/(24*b) + (B*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*
A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.110344, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}(6Ab-aB)}{16b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a*(6*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b) + ((6*A*b - a*B)*x*(a
+ b*x^2)^(3/2))/(24*b) + (B*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*
A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi in Sympy [A] time = 12.0059, size = 102, normalized size = 0.86

$$\frac{Bx\left(a+bx^2\right)^{\frac{5}{2}}}{6b}+\frac{a^2\left(6Ab-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{3}{2}}}+\frac{ax\sqrt{a+bx^2}\left(6Ab-Ba\right)}{16b}+\frac{x\left(a+bx^2\right)^{\frac{3}{2}}\left(6Ab-Ba\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] B*x*(a + b*x**2)**(5/2)/(6*b) + a**2*(6*A*b - B*a)*atanh(sqrt(b)*
x/sqrt(a + b*x**2))/(16*b**(3/2)) + a*x*sqrt(a + b*x**2)*(6*A*b -
 B*a)/(16*b) + x*(a + b*x**2)**(3/2)*(6*A*b - B*a)/(24*b)
```

Mathematica [A] time = 0.124259, size = 98, normalized size = 0.83

$$\sqrt{a+bx^2}\left(\frac{1}{24}x^3(7aB+6Ab)+\frac{ax(aB+10Ab)}{16b}+\frac{1}{6}bBx^5\right)-\frac{a^2(aB-6Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((a*(10*A*b + a*B)*x)/(16*b) + ((6*A*b + 7*a*B)*x
^3)/24 + (b*B*x^5)/6) - (a^2*(-6*A*b + a*B)*Log[b*x + Sqrt[b]*Sqr
t[a + b*x^2]])/(16*b^(3/2))
```

Maple [A] time = 0.008, size = 131, normalized size = 1.1

$$\frac{Ax}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,aAx}{8}\sqrt{bx^2+a}+\frac{3\,Aa^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{Bx}{6\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bxa}{24\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{Bxa^2}{16\,b}\sqrt{bx^2+a}-\frac{Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)+3/8*A*a*x*(b*x^2+a)^(1/2)+3/8*A*a^2/b^(1/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*B
*a/b*x*(b*x^2+a)^(3/2)-1/16*B*a^2/b*x*(b*x^2+a)^(1/2)-1/16*B*a^3/
b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238467, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,Bb^2x^5+2\left(7\,Bab+6\,Ab^2\right)x^3+3\left(Ba^2+10\,Aab\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(Ba^3-6\,Aa^2b\right)\log\left(-2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\right.}{96\,b^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*B*b^2*x^5 + 2*(7*B*a*b + 6*A*b^2)*x^3 + 3*(B*a^2 + 10
*A*a*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(B*a^3 - 6*A*a^2*b)*log(-2
*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/48*((8*
B*b^2*x^5 + 2*(7*B*a*b + 6*A*b^2)*x^3 + 3*(B*a^2 + 10*A*a*b)*x)*s
qrt(b*x^2 + a)*sqrt(-b) - 3*(B*a^3 - 6*A*a^2*b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)))/(sqrt(-b)*b)]
```

Sympy [A] time = 43.7418, size = 253, normalized size = 2.14

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11B\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Bb^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x*
*2/a)) + 3*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asi
nh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + B*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*B*a**
(3/2)*x**3/(48*sqrt(1 + b*x**2/a)) + 11*B*sqrt(a)*b*x**5/(24*sqrt
(1 + b*x**2/a)) - B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) +
 B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.248828, size = 138, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,Bbx^2+\frac{7\,Bab^4+6\,Ab^5}{b^4}\right)x^2+\frac{3\left(Ba^2b^3+10\,Aab^4\right)}{b^4}\right)\sqrt{bx^2+a}x+\frac{\left(Ba^3-6\,Aa^2b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*b*x^2 + (7*B*a*b^4 + 6*A*b^5)/b^4)*x^2 + 3*(B*a^2*b^
3 + 10*A*a*b^4)/b^4)*sqrt(b*x^2 + a)*x + 1/16*(B*a^3 - 6*A*a^2*b)
*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.527 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=76

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] a*A*Sqrt[a + b*x^2] + (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5
/2))/(5*b) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.164721, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] a*A*Sqrt[a + b*x^2] + (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5
/2))/(5*b) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 16.0131, size = 65, normalized size = 0.86

$$-Aa^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+Aa\sqrt{a+bx^2}+\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{3}+\frac{B\left(a+bx^2\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x,x)
```

```
[Out] -A*a**(3/2)*atanh(sqrt(a + b*x**2)/sqrt(a)) + A*a*sqrt(a + b*x**2
) + A*(a + b*x**2)**(3/2)/3 + B*(a + b*x**2)**(5/2)/(5*b)
```

Mathematica [A] time = 0.151229, size = 93, normalized size = 1.22

$$-a^{3/2}A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+a^{3/2}A\log(x)+\sqrt{a+bx^2}\left(\frac{1}{15}x^2(6aB+5Ab)+\frac{a(3aB+20Ab)}{15b}+\frac{1}{5}bBx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] Sqrt[a + b*x^2]*((a*(20*A*b + 3*a*B))/(15*b) + ((5*A*b + 6*a*B)*x
^2)/15 + (b*B*x^4)/5) + a^(3/2)*A*Log[x] - a^(3/2)*A*Log[a + Sqrt
[a]*Sqrt[a + b*x^2]]
```

Maple [A] time = 0.01, size = 70, normalized size = 0.9

$$\frac{A}{3}\left(bx^2+a\right)^{\frac{3}{2}}-Aa^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+aA\sqrt{bx^2+a}+\frac{B}{5\,b}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x,x)
```

```
[Out] 1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2)
)/x)+a*A*(b*x^2+a)^(1/2)+1/5*B*(b*x^2+a)^(5/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231901, size = 1, normalized size = 0.01

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\left(3\,Bb^2x^4+3\,Ba^2+20\,Aab+\left(6\,Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{30\,b},\; -\frac{15\,A\sqrt{-a}ab\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-\left(3\,Bb^2x^4+3\,Ba^2+20\,Aab+\left(6\,Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(3*B*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b
^2)*x^2)*sqrt(b*x^2 + a))/b, -1/15*(15*A*sqrt(-a)*a*b*arctan(a/(s
qrt(b*x^2 + a)*sqrt(-a))) - (3*B*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (
6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 20.0003, size = 134, normalized size = 1.76

$$-Aa^2\left(\begin{cases} -\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } -a>0 \\ \frac{\operatorname{acoth}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for } -a<0\wedge a<a+bx^2 \\ \frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for } a>a+bx^2\wedge -a<0 \end{cases}\right)+Aa\sqrt{a+bx^2}+\frac{A\left(a+bx^2\right)^{\frac{3}{2}}}{3}+\frac{B\left(a+bx^2\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x,x)
```

```
[Out] -A*a**2*Piecewise((-atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a), -a
> 0), (acoth(sqrt(a + b*x**2)/sqrt(a))/sqrt(a), (-a < 0) & (a < a
 + b*x**2)), (atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a), (-a < 0) &
 (a > a + b*x**2))) + A*a*sqrt(a + b*x**2) + A*(a + b*x**2)**(3/2
)/3 + B*(a + b*x**2)**(5/2)/(5*b)
```

GIAC/XCAS [A] time = 0.247243, size = 107, normalized size = 1.41

$$\frac{Aa^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{3\left(bx^2+a\right)^{\frac{5}{2}}Bb^4 + 5\left(bx^2+a\right)^{\frac{3}{2}}Ab^5 + 15\sqrt{bx^2+a}Aab^5}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] A*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(3*(b*x^2
+ a)^(5/2)*B*b^4 + 5*(b*x^2 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^2 + a)
*A*a*b^5)/b^5
```

3.528 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=109

$$\frac{x\left(a+bx^2\right)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

```
[Out] (3*(4*A*b + a*B)*x*Sqrt[a + b*x^2])/8 + ((4*A*b + a*B)*x*(a + b*x
^2)^(3/2))/(4*a) - (A*(a + b*x^2)^(5/2))/(a*x) + (3*a*(4*A*b + a*
B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.125961, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x\left(a+bx^2\right)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (3*(4*A*b + a*B)*x*Sqrt[a + b*x^2])/8 + ((4*A*b + a*B)*x*(a + b*x
^2)^(3/2))/(4*a) - (A*(a + b*x^2)^(5/2))/(a*x) + (3*a*(4*A*b + a*
B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi in Sympy [A] time = 12.6289, size = 100, normalized size = 0.92

$$-\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{ax}+\frac{3a\,(4Ab+Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+x\sqrt{a+bx^2}\left(\frac{3Ab}{2}+\frac{3Ba}{8}\right)+\frac{x\left(a+bx^2\right)^{\frac{3}{2}}(4Ab+Ba)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(a*x) + 3*a*(4*A*b + B*a)*atanh(sqrt(b)*x/
sqrt(a + b*x**2))/(8*sqrt(b)) + x*sqrt(a + b*x**2)*(3*A*b/2 + 3*B
*a/8) + x*(a + b*x**2)**(3/2)*(4*A*b + B*a)/(4*a)
```

Mathematica [A] time = 0.125061, size = 84, normalized size = 0.77

$$\frac{3a(aB+4Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8\sqrt{b}}+\sqrt{a+bx^2}\left(\frac{1}{8}x(5aB+4Ab)-\frac{aA}{x}+\frac{1}{4}bBx^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] Sqrt[a + b*x^2]*(-((a*A)/x) + ((4*A*b + 5*a*B)*x)/8 + (b*B*x^3)/4
) + (3*a*(4*A*b + a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*Sqr
t[b])
```

Maple [A] time = 0.012, size = 125, normalized size = 1.2

$$\frac{Bx}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{3\,Bxa}{8}\sqrt{bx^{2}+a}+\frac{3\,a^{2}B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)\frac{1}{\sqrt{b}}-\frac{A}{ax}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{Axb}{a}\left(bx^{2}+a\right)^{\frac{3}{2}}+\frac{3\,Axb}{2}\sqrt{bx^{2}+a}+\frac{3\,Aa}{2}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/4*x*B*(b*x^2+a)^(3/2)+3/8*B*a*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(1/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-A*(b*x^2+a)^(5/2)/a/x+A*b/a*x*(b
*x^2+a)^(3/2)+3/2*A*b*x*(b*x^2+a)^(1/2)+3/2*A*b^(1/2)*a*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229091, size = 1, normalized size = 0.01

$$\left[\frac{3\left(Ba^{2}+4\,Aab\right)x\log\left(-2\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)+2\left(2\,Bbx^{4}+\left(5\,Ba+4\,Ab\right)x^{2}-8\,Aa\right)\sqrt{bx^{2}+a}\sqrt{b}}{16\sqrt{b}x},\ \frac{3\left(Ba^{2}+4\,Aa\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*a^2 + 4*A*a*b)*x*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^
2 + a)*sqrt(b)) + 2*(2*B*b*x^4 + (5*B*a + 4*A*b)*x^2 - 8*A*a)*sqr
t(b*x^2 + a)*sqrt(b))/(sqrt(b)*x), 1/8*(3*(B*a^2 + 4*A*a*b)*x*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^4 + (5*B*a + 4*A*b)*x^
2 - 8*A*a)*sqrt(b*x^2 + a)*sqrt(-b))/(sqrt(-b)*x)]
```

Sympy [A] time = 26.9625, size = 216, normalized size = 1.98

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^{2}}{a}}}+\frac{A\sqrt{a}bx\sqrt{1+\frac{bx^{2}}{a}}}{2}-\frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3B\sqrt{a}bx^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba^{2}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Bb^{2}x^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**
2/a)/2 - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(s
qrt(b)*x/sqrt(a))/2 + B*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + B*a**(3
/2)*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b*x**3/(8*sqrt(1 + b*x
**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + B*b**2*
x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.246829, size = 154, normalized size = 1.41

$$\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{8}\left(2\,Bbx^2+\frac{5\,Bab^2+4\,Ab^3}{b^2}\right)\sqrt{bx^2+a}x-\frac{3\left(Ba^2\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*B*
b*x^2 + (5*B*a*b^2 + 4*A*b^3)/b^2)*sqrt(b*x^2 + a)*x - 3/16*(B*a^
2*sqrt(b) + 4*A*a*b^(3/2))*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

3.529 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{(a+bx^2)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Ab)-\frac{1}{2}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((3*A*b + 2*a*B)*(a + b*x^2
)^(3/2))/(6*a) - (A*(a + b*x^2)^(5/2))/(2*a*x^2) - (Sqrt[a]*(3*A*
b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.21969, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{(a+bx^2)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Ab)-\frac{1}{2}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((3*A*b + 2*a*B)*(a + b*x^2
)^(3/2))/(6*a) - (A*(a + b*x^2)^(5/2))/(2*a*x^2) - (Sqrt[a]*(3*A*
b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi in Sympy [A] time = 18.9377, size = 94, normalized size = 0.85

$$-\frac{A(a+bx^2)^{\frac{5}{2}}}{2ax^2}-\sqrt{a}\left(\frac{3Ab}{2}+Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\sqrt{a+bx^2}\left(\frac{3Ab}{2}+Ba\right)+\frac{(a+bx^2)^{\frac{3}{2}}\left(\frac{3Ab}{2}+Ba\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(2*a*x**2) - sqrt(a)*(3*A*b/2 + B*a)*atanh
(sqrt(a + b*x**2)/sqrt(a)) + sqrt(a + b*x**2)*(3*A*b/2 + B*a) + (
a + b*x**2)**(3/2)*(3*A*b/2 + B*a)/(3*a)
```

Mathematica [A] time = 0.189435, size = 100, normalized size = 0.91

$$\frac{1}{6}\left(-3\sqrt{a}(2aB+3Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\frac{\sqrt{a+bx^2}\left(-3aA+8aBx^2+6Abx^2+2bBx^4\right)}{x^2}+3\sqrt{a}\log(x)(2aB+3Ab)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-3*a*A + 6*A*b*x^2 + 8*a*B*x^2 + 2*b*B*x^4))/x
^2 + 3*Sqrt[a]*(3*A*b + 2*a*B)*Log[x] - 3*Sqrt[a]*(3*A*b + 2*a*B)
*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/6
```

Maple [A] time = 0.013, size = 132, normalized size = 1.2

$$-\frac{A}{2\,a x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b}{2\,a}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{3\,A b}{2}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)+\frac{3\,A b}{2}\sqrt{b x^{2}+a}+\frac{B}{3}\left(b x^{2}+a\right)^{\frac{3}{2}}-B a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)+B\sqrt{b x^{2}+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(5/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(3/2)-3/2*A*b*a^
(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/
2)+1/3*B*(b*x^2+a)^(3/2)-B*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x)+B*(b*x^2+a)^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230479, size = 1, normalized size = 0.01

$$\left[\frac{3\,(2\,B a+3\,A b)\sqrt{a}x^{2}\log\left(-\frac{b x^{2}-2\,\sqrt{b x^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(2\,B b x^{4}+2\,(4\,B a+3\,A b)x^{2}-3\,A a\right)\sqrt{b x^{2}+a}}{12\,x^{2}},\; -\frac{3\,(2\,B a+3\,A b)\sqrt{-a}x^{2}\arctan\left(\frac{a}{\sqrt{b x^{2}+a}\sqrt{-a}}\right)-\left(2\,B b x^{4}+2\,(4\,B a+3\,A b)x^{2}-3\,A a\right)\sqrt{b x^{2}+a}}{6\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*a + 3*A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 -
3*A*a)*sqrt(b*x^2 + a))/x^2, -1/6*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^2
*arctan(a/(sqrt(b*x^2 + a)*sqrt(-a))) - (2*B*b*x^4 + 2*(4*B*a + 3
*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] time = 42.7624, size = 184, normalized size = 1.67

$$-\frac{3A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}-Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^{2}}{\sqrt{b}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^{2}}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}x^{2}}{2} & \text{for}\: b=0\\\frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/
(b*x**2) + 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b*
*(3/2)*x/sqrt(a/(b*x**2) + 1) - B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)
*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a*sqrt(b)*x/sq
rt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((
a + b*x**2)**(3/2)/(3*b), True))
```

GIAC/XCAS [A] time = 0.233839, size = 139, normalized size = 1.26

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb+6\sqrt{bx^2+a}Bab+6\sqrt{bx^2+a}Ab^2-\frac{3\sqrt{bx^2+a}Aab}{x^2}+\frac{3\left(2Ba^2b+3Aab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b*x^2 + a)^(3/2)*B*b + 6*sqrt(b*x^2 + a)*B*a*b + 6*sqrt(b
*x^2 + a)*A*b^2 - 3*sqrt(b*x^2 + a)*A*a*b/x^2 + 3*(2*B*a^2*b + 3*
A*a*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

3.530 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{(a+bx^2)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

```
[Out] (b*(2*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/(2*a) - ((2*A*b + 3*a*B)*(a
 + b*x^2)^(3/2))/(3*a*x) - (A*(a + b*x^2)^(5/2))/(3*a*x^3) + (Sqr
t[b]*(2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi [A] time = 0.141663, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{(a+bx^2)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (b*(2*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/(2*a) - ((2*A*b + 3*a*B)*(a
 + b*x^2)^(3/2))/(3*a*x) - (A*(a + b*x^2)^(5/2))/(3*a*x^3) + (Sqr
t[b]*(2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi in Sympy [A] time = 13.9124, size = 105, normalized size = 0.88

$$-\frac{A(a+bx^2)^{\frac{5}{2}}}{3ax^3}+\frac{\sqrt{b}(2Ab+3Ba)\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2}+\frac{bx\sqrt{a+bx^2}(2Ab+3Ba)}{2a}-\frac{(a+bx^2)^{\frac{3}{2}}(2Ab+3Ba)}{3ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(3*a*x**3) + sqrt(b)*(2*A*b + 3*B*a)*atanh
(sqrt(b)*x/sqrt(a + b*x**2))/2 + b*x*sqrt(a + b*x**2)*(2*A*b + 3*
B*a)/(2*a) - (a + b*x**2)**(3/2)*(2*A*b + 3*B*a)/(3*a*x)
```

Mathematica [A] time = 0.100132, size = 86, normalized size = 0.72

$$\frac{1}{2}\sqrt{b}(3aB+2Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{a+bx^2}\left(\frac{-3aB-4Ab}{3x}-\frac{aA}{3x^3}+\frac{bBx}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (-(a*A)/(3*x^3) + (-4*A*b - 3*a*B)/(3*x) + (b*B*x)/2)*Sqrt[a + b*
x^2] + (Sqrt[b]*(2*A*b + 3*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]
])/2
```

Maple [A] time = 0.013, size = 168, normalized size = 1.4

$$-\frac{A}{3\,a\,x^{3}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{2\,A b}{3\,a^{2} x}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{2\,A x \,b^{2}}{3\,a^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{A x \,b^{2}}{a}\sqrt{b x^{2}+a}+A\,b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right)-\frac{B}{a x}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{b B x}{a}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{3\,b B x}{2}\sqrt{b x^{2}+a}+\frac{3\,B a}{2}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x)
```

```
[Out] -1/3*A*(b*x^2+a)^(5/2)/a/x^3-2/3*A*b/a^2/x*(b*x^2+a)^(5/2)+2/3*A*
b^2/a^2*x*(b*x^2+a)^(3/2)+A*b^2/a*x*(b*x^2+a)^(1/2)+A*b^(3/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))-B/a/x*(b*x^2+a)^(5/2)+B*b/a*x*(b*x^2+a
)^(3/2)+3/2*B*b*x*(b*x^2+a)^(1/2)+3/2*B*b^(1/2)*a*ln(x*b^(1/2)+(b
*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228589, size = 1, normalized size = 0.01

$$\left[\frac{3\,(3\,Ba+2\,Ab)\sqrt{b}x^{3}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\left(3\,Bbx^{4}-2\,(3\,Ba+4\,Ab)x^{2}-2\,Aa\right)\sqrt{bx^{2}+a}}{12\,x^{3}},\ \frac{3\,(3\,Ba+2\,Ab)\sqrt{}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*a + 2*A*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) + 2*(3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*
a)*sqrt(b*x^2 + a))/x^3, 1/6*(3*(3*B*a + 2*A*b)*sqrt(-b)*x^3*arct
an(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) + (3*B*b*x^4 - 2*(3*B*a + 4*A*
b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] time = 17.0085, size = 202, normalized size = 1.7

$$-\frac{A\sqrt{a}b}{x\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3}+Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Ab^{2}x}{\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^{2}}{a}}}+\frac{B\sqrt{a}bx\sqrt{1+\frac{bx^{2}}{a}}}{2}-\frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2)
 + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*a
sinh(sqrt(b)*x/sqrt(a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) -
 B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x**
2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*B*a*sqrt(b)*asinh(s
qrt(b)*x/sqrt(a))/2
```

<u>GIAC/XCAS</u> [A] time = 0.242217, size = 279, normalized size = 2.34

$$\frac{1}{2}\sqrt{bx^2+a}Bbx - \frac{1}{4}\left(3\,Ba\sqrt{b} + 2\,Ab^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right) + \frac{2\left(3\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^4 Ba^2\sqrt{b} + 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^4 Aab^{\frac{3}{2}} - 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 Ba^3\sqrt{b} - 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 Aa^2b \cdots\right.}{3\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 - a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b*x - 1/4*(3*B*a*sqrt(b) + 2*A*b^(3/2))*ln(
(sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*B*a^2*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^
(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) - 6*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2) + 3*B*a^4*sqrt(b) + 4*A*
a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.531 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=115

$$-\frac{(a+bx^2)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

```
[Out] (3*b*(A*b + 4*a*B)*Sqrt[a + b*x^2])/(8*a) - ((A*b + 4*a*B)*(a + b
*x^2)^(3/2))/(8*a*x^2) - (A*(a + b*x^2)^(5/2))/(4*a*x^4) - (3*b*(
A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] time = 0.23321, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{(a+bx^2)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (3*b*(A*b + 4*a*B)*Sqrt[a + b*x^2])/(8*a) - ((A*b + 4*a*B)*(a + b
*x^2)^(3/2))/(8*a*x^2) - (A*(a + b*x^2)^(5/2))/(4*a*x^4) - (3*b*(
A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi in Sympy [A] time = 19.2761, size = 104, normalized size = 0.9

$$-\frac{A(a+bx^2)^{\frac{5}{2}}}{4ax^4}+\frac{3b\sqrt{a+bx^2}(Ab+4Ba)}{8a}-\frac{(a+bx^2)^{\frac{3}{2}}(Ab+4Ba)}{8ax^2}-\frac{3b(Ab+4Ba)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(4*a*x**4) + 3*b*sqrt(a + b*x**2)*(A*b + 4
*B*a)/(8*a) - (a + b*x**2)**(3/2)*(A*b + 4*B*a)/(8*a*x**2) - 3*b*
(A*b + 4*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(8*sqrt(a))
```

Mathematica [A] time = 0.222913, size = 100, normalized size = 0.87

$$-\frac{3b(4aB+Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{8\sqrt{a}}+\sqrt{a+bx^2}\left(\frac{-4aB-5Ab}{8x^2}-\frac{aA}{4x^4}+bB\right)+\frac{3b\log(x)(4aB+Ab)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (b*B - (a*A)/(4*x^4) + (-5*A*b - 4*a*B)/(8*x^2))*Sqrt[a + b*x^2]
+ (3*b*(A*b + 4*a*B)*Log[x])/(8*Sqrt[a]) - (3*b*(A*b + 4*a*B)*Log
[a + Sqrt[a]*Sqrt[a + b*x^2]])/(8*Sqrt[a])
```

Maple [A] time = 0.014, size = 184, normalized size = 1.6

$$-\frac{A}{4\,a x^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{A b}{8\,a^{2} x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{b^{2} A}{8\,a^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}} -\frac{3\,b^{2} A}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{3\,b^{2} A}{8\,a}\sqrt{b x^{2}+a}-\frac{B}{2\,a x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}} +\frac{B b}{2\,a}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{3\,B b}{2}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)+\frac{3\,B b}{2}\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(5/2)/a/x^4-1/8*A*b/a^2/x^2*(b*x^2+a)^(5/2)+1/8*
A*b^2/a^2*(b*x^2+a)^(3/2)-3/8*A*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)+3/8*A*b^2/a*(b*x^2+a)^(1/2)-1/2*B/a/x^2*(b*x^2+a
)^(5/2)+1/2*B*b/a*(b*x^2+a)^(3/2)-3/2*B*b*a^(1/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)+3/2*B*b*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.22874, size = 1, normalized size = 0.01

$$\left[\frac{3\left(4\,Bab+Ab^{2}\right)x^{4}\log\left(-\frac{\left(bx^{2}+2\,a\right)\sqrt{a}-2\,\sqrt{bx^{2}+a}a}{x^{2}}\right)+2\left(8\,Bbx^{4}-\left(4\,Ba+5\,Ab\right)x^{2}-2\,Aa\right)\sqrt{bx^{2}+a}\sqrt{a}}{16\,\sqrt{a}x^{4}}, -\frac{3\left(4\,Bab+Ab^{2}\right)x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(8\,Bbx^{4}-\left(4\,Ba+5\,Ab\right)x^{2}-2\,Aa\right)\sqrt{bx^{2}+a}\sqrt{-a}}{8\,\sqrt{-a}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*a*b + A*b^2)*x^4*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sq
rt(b*x^2 + a)*a)/x^2) + 2*(8*B*b*x^4 - (4*B*a + 5*A*b)*x^2 - 2*A*
a)*sqrt(b*x^2 + a)*sqrt(a))/(sqrt(a)*x^4), -1/8*(3*(4*B*a*b + A*b
^2)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (8*B*b*x^4 - (4*B*a +
5*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*x^4)]
```

Sympy [A] time = 89.0397, size = 216, normalized size = 1.88

$$-\frac{Aa^{2}}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Aa\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{Ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} -\frac{3B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2x}+\frac{Ba\sqrt{b}}{x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Bb^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -A*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a*sqrt(b)/(8*
x**3*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x
) - A*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(sqrt(a
)/(sqrt(b)*x))/(8*sqrt(a)) - 3*B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)
*x))/2 - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + B*a*sqrt(b)/(x*
sqrt(a/(b*x**2) + 1)) + B*b**(3/2)*x/sqrt(a/(b*x**2) + 1)
```

GIAC/XCAS [A] time = 0.230297, size = 177, normalized size = 1.54

$$\frac{8\sqrt{bx^2+a}Bb^2+\frac{3\left(4Bab^2+Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab^2-4\sqrt{bx^2+a}Ba^2b^2+5\left(bx^2+a\right)^{\frac{3}{2}}Ab^3-3\sqrt{bx^2+a}Aab^3}{b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/sqrt(-a) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*
sqrt(b*x^2 + a)*B*a^2*b^2 + 5*(b*x^2 + a)^(3/2)*A*b^3 - 3*sqrt(b*
x^2 + a)*A*a*b^3)/(b^2*x^4))/b
```

3.532 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{A(a+bx^2)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{bB\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{3/2}}{3x^3}$$

```
[Out] -((b*B*Sqrt[a + b*x^2])/x) - (B*(a + b*x^2)^(3/2))/(3*x^3) - (A*(
a + b*x^2)^(5/2))/(5*a*x^5) + b^(3/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]]
```

Rubi [A] time = 0.110976, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{A(a+bx^2)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{bB\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -((b*B*Sqrt[a + b*x^2])/x) - (B*(a + b*x^2)^(3/2))/(3*x^3) - (A*(
a + b*x^2)^(5/2))/(5*a*x^5) + b^(3/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]]
```

Rubi in Sympy [A] time = 13.8667, size = 75, normalized size = 0.87

$$-\frac{A(a+bx^2)^{\frac{5}{2}}}{5ax^5}+Bb^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{Bb\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{\frac{3}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(5*a*x**5) + B*b**(3/2)*atanh(sqrt(b)*x/sq
rt(a + b*x**2)) - B*b*sqrt(a + b*x**2)/x - B*(a + b*x**2)**(3/2)/
(3*x**3)
```

Mathematica [A] time = 0.10902, size = 88, normalized size = 1.02

$$\sqrt{a+bx^2}\left(\frac{-5aB-6Ab}{15x^3}-\frac{b(20aB+3Ab)}{15ax}-\frac{aA}{5x^5}\right)+b^{3/2}B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (-(a*A)/(5*x^5) + (-6*A*b - 5*a*B)/(15*x^3) - (b*(3*A*b + 20*a*B)
)/(15*a*x))*Sqrt[a + b*x^2] + b^(3/2)*B*Log[b*x + Sqrt[b]*Sqrt[a
+ b*x^2]]
```

Maple [A] time = 0.015, size = 115, normalized size = 1.3

$$-\frac{A}{5\,a x^{5}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{B}{3\,a x^{3}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{2\,Bb}{3\,a^{2}x}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{2\,b^{2}Bx}{3\,a^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{b^{2}Bx}{a}\sqrt{b x^{2}+a}+Bb^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*A*(b*x^2+a)^(5/2)/a/x^5-1/3*B/a/x^3*(b*x^2+a)^(5/2)-2/3*B*b/
a^2/x*(b*x^2+a)^(5/2)+2/3*B*b^2/a^2*x*(b*x^2+a)^(3/2)+B*b^2/a*x*(
b*x^2+a)^(1/2)+B*b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231343, size = 1, normalized size = 0.01

$$\left[\frac{15\,Bab^{\frac{3}{2}}x^{5}\log\left(-2\,b x^{2}-2\,\sqrt{b x^{2}+a}\sqrt{b}x-a\right)-2\left(\left(20\,Bab+3\,Ab^{2}\right)x^{4}+3\,Aa^{2}+\left(5\,Ba^{2}+6\,Aab\right)x^{2}\right)\sqrt{b x^{2}+a}}{30\,a x^{5}},\ \frac{15\,Ba\sqrt{-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*a*b^(3/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A
*a*b)*x^2)*sqrt(b*x^2 + a))/(a*x^5), 1/15*(15*B*a*sqrt(-b)*b*x^5*
arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - ((20*B*a*b + 3*A*b^2)*x^
4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a*x^5)]
```

Sympy [A] time = 12.7144, size = 184, normalized size = 2.14

$$-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{4}}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{2}}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5a}-\frac{B\sqrt{ab}}{x\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3}+Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bb^{2}x}{\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/
(b*x**2) + 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) -
```

```
B*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2)
+ 1)/(3*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + B*b**(3/2)*as
inh(sqrt(b)*x/sqrt(a)) - B*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.250725, size = 319, normalized size = 3.71

$$-\frac{1}{2}Bb^{\frac{3}{2}}\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+110\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\cdots\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(3/2)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(30*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*A*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2
*b^(3/2) + 110*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 30
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 70*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) + 20*B*a^5*b^(3/2) + 3*A*a^4*b^(
5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.533 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=120

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{b\sqrt{a+bx^2}(Ab-6aB)}{16ax^2}+\frac{(a+bx^2)^{3/2}(Ab-6aB)}{24ax^4}-\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

```
[Out] (b*(A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a*x^2) + ((A*b - 6*a*B)*(a
+ b*x^2)^(3/2))/(24*a*x^4) - (A*(a + b*x^2)^(5/2))/(6*a*x^6) + (b
^2*(A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rubi [A] time = 0.25046, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{b\sqrt{a+bx^2}(Ab-6aB)}{16ax^2}+\frac{(a+bx^2)^{3/2}(Ab-6aB)}{24ax^4}-\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (b*(A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a*x^2) + ((A*b - 6*a*B)*(a
+ b*x^2)^(3/2))/(24*a*x^4) - (A*(a + b*x^2)^(5/2))/(6*a*x^6) + (b
^2*(A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rubi in Sympy [A] time = 20.5432, size = 105, normalized size = 0.88

$$-\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{6ax^6}+\frac{b\sqrt{a+bx^2}\,(Ab-6Ba)}{16ax^2}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}(Ab-6Ba)}{24ax^4}+\frac{b^2\,(Ab-6Ba)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(6*a*x**6) + b*sqrt(a + b*x**2)*(A*b - 6*B
*a)/(16*a*x**2) + (a + b*x**2)**(3/2)*(A*b - 6*B*a)/(24*a*x**4) +
 b**2*(A*b - 6*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(16*a**(3/2))
```

Mathematica [A] time = 0.157332, size = 120, normalized size = 1.

$$\frac{b^2(Ab-6aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{16a^{3/2}}-\frac{b^2\log(x)(Ab-6aB)}{16a^{3/2}}+\sqrt{a+bx^2}\left(\frac{-6aB-7Ab}{24x^4}-\frac{b(10aB+Ab)}{16ax^2}-\frac{aA}{6x^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (-(a*A)/(6*x^6) + (-7*A*b - 6*a*B)/(24*x^4) - (b*(A*b + 10*a*B))/
(16*a*x^2))*Sqrt[a + b*x^2] - (b^2*(A*b - 6*a*B)*Log[x])/(16*a^(3
/2)) + (b^2*(A*b - 6*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(16*a
^(3/2))
```

Maple [B] time = 0.013, size = 233, normalized size = 1.9

$$-\frac{A}{6\,a x^{6}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b}{24\,a^{2} x^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{b^{2} A}{48\,a^{3} x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{A b^{3}}{48\,a^{3}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{A b^{3}}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{3}{2}}-\frac{A b^{3}}{16\,a^{2}}\sqrt{b x^{2}+a}-\frac{B}{4\,a x^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{B b}{8\,a^{2} x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{B b^{2}}{8\,a^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{3\,B b^{2}}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{3\,B b^{2}}{8\,a}\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(5/2)/a/x^6+1/24*A*b/a^2/x^4*(b*x^2+a)^(5/2)+1/4
8*A*b^2/a^3/x^2*(b*x^2+a)^(5/2)-1/48*A*b^3/a^3*(b*x^2+a)^(3/2)+1/
16*A*b^3/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/16*A*b^3
/a^2*(b*x^2+a)^(1/2)-1/4*B/a/x^4*(b*x^2+a)^(5/2)-1/8*B*b/a^2/x^2*
(b*x^2+a)^(5/2)+1/8*B*b^2/a^2*(b*x^2+a)^(3/2)-3/8*B*b^2/a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/8*B*b^2/a*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258684, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(6\,B a b^{2}-A b^{3}\right)x^{6}\log\left(-\frac{\left(b x^{2}+2\,a\right)\sqrt{a}+2\,\sqrt{b x^{2}+a}a}{x^{2}}\right)+2\left(3\left(10\,B a b+A b^{2}\right)x^{4}+8\,A a^{2}+2\left(6\,B a^{2}+7\,A a b\right)x^{2}\right)\sqrt{b x^{2}+a}\sqrt{a}}{96\,a^{\frac{3}{2}}x^{6}},\ -\frac{3\left(6\,B a b^{2}-A b^{3}\right)x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2}+a}}\right)+\left(3\left(10\,B a b+A b^{2}\right)x^{4}+8\,A a^{2}+2\left(6\,B a^{2}+7\,A a b\right)x^{2}\right)\sqrt{b x^{2}+a}\sqrt{-a}}{48\,\sqrt{-a}a x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - A*b^3)*x^6*log(-((b*x^2 + 2*a)*sqrt(a) + 2
*sqrt(b*x^2 + a)*a)/x^2) + 2*(3*(10*B*a*b + A*b^2)*x^4 + 8*A*a^2
+ 2*(6*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(3/2)*x^
6), -1/48*(3*(6*B*a*b^2 - A*b^3)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 +
 a)) + (3*(10*B*a*b + A*b^2)*x^4 + 8*A*a^2 + 2*(6*B*a^2 + 7*A*a*b
)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a*x^6)]
```

Sympy [A] time = 139.909, size = 253, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{11Aa\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{17Ab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}}-\frac{Ba^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ba\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Bb^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}-\frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)/(2
4*x**5*sqrt(a/(b*x**2) + 1)) - 17*A*b**(3/2)/(48*x**3*sqrt(a/(b*x
**2) + 1)) - A*b**(5/2)/(16*a*x*sqrt(a/(b*x**2) + 1)) + A*b**3*as
inh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2)) - B*a**2/(4*sqrt(b)*x**5*s
qrt(a/(b*x**2) + 1)) - 3*B*a*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)
) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b**(3/2)/(8*x*sqrt(
a/(b*x**2) + 1)) - 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)
)
```

GIAC/XCAS [A] time = 0.241533, size = 215, normalized size = 1.79

$$\frac{\frac{3\left(6Bab^3-Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{30\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-48\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+18\sqrt{bx^2+a}Ba^3b^3+3\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+8\left(bx^2+a\right)^{\frac{3}{2}}Aab^4-3\sqrt{bx^2+a}Aa^2b^4}{ab^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqr
t(-a)*a) - (30*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B
*a^2*b^3 + 18*sqrt(b*x^2 + a)*B*a^3*b^3 + 3*(b*x^2 + a)^(5/2)*A*b
^4 + 8*(b*x^2 + a)^(3/2)*A*a*b^4 - 3*sqrt(b*x^2 + a)*A*a^2*b^4)/(
a*b^3*x^6))/b
```

3.534 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8} dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

```
[Out] -(A*(a + b*x^2)^(5/2))/(7*a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(
5/2))/(35*a^2*x^5)
```

Rubi [A] time = 0.0838279, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(a+bx^2)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(7*a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(
5/2))/(35*a^2*x^5)
```

Rubi in Sympy [A] time = 9.25107, size = 46, normalized size = 0.87

$$-\frac{A(a+bx^2)^{\frac{5}{2}}}{7ax^7}+\frac{(a+bx^2)^{\frac{5}{2}}(2Ab-7Ba)}{35a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(7*a*x**7) + (a + b*x**2)**(5/2)*(2*A*b -
7*B*a)/(35*a**2*x**5)
```

Mathematica [A] time = 0.0708727, size = 40, normalized size = 0.75

$$-\frac{(a+bx^2)^{5/2}(5aA+7aBx^2-2Abx^2)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -((a + b*x^2)^(5/2)*(5*a*A - 2*A*b*x^2 + 7*a*B*x^2))/(35*a^2*x^7)
```

Maple [A] time = 0.009, size = 37, normalized size = 0.7

$$-\frac{-2\,Abx^2+7\,Bax^2+5\,Aa}{35\,x^7a^2}(bx^2+a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-2*A*b*x^2+7*B*a*x^2+5*A*a)/x^7/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249138, size = 105, normalized size = 1.98

$$-\frac{\left(\left(7\,Bab^{2}-2\,Ab^{3}\right)x^{6}+\left(14\,Ba^{2}b+Aab^{2}\right)x^{4}+5\,Aa^{3}+\left(7\,Ba^{3}+8\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{35\,a^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*((7*B*a*b^2 - 2*A*b^3)*x^6 + (14*B*a^2*b + A*a*b^2)*x^4 + 5
*A*a^3 + (7*B*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^2*x^7)
```

Sympy [A] time = 13.9259, size = 518, normalized size = 9.77

$$-\frac{15Aa^{6}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{33Aa^{5}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{17Aa^{4}b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{3Aa^{3}b^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{12Aa^{2}b^{\frac{17}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{8Aab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{105a^{5}b^{4}x^{6}+210a^{4}b^{5}x^{8}+105a^{3}b^{6}x^{10}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{4}}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15ax^{2}}+\frac{2Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{2}}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{4}}-\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{2}}-\frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**
2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 +
 105*a**3*b**6*x**10) - 17*A*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4
*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**2*b**
(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b
**5*x**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b
*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b
**6*x**10) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2
)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*A*b**(7/2)*sqrt(a/(b*x**2)
 + 1)/(15*a**2) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*B
*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(5/2)*sqrt(a/(b*x*
```

```
*2) + 1)/(5*a)
```

GIAC/XCAS [A] time = 0.251773, size = 464, normalized size = 8.75

$$\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{5}{2}}-70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{5}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{7}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(5/2) - 70*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*B*a*b^(5/2) + 70*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*A*b^(7/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b
^(5/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(7/2) - 140*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/2) + 140*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a^2*b^(7/2) + 77*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*B*a^4*b^(5/2) + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(7/2
) - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5/2) + 14*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(7/2) + 7*B*a^6*b^(5/2) - 2*A*a^
5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.535 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=156

$$-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b^2\sqrt{a+bx^2}(3Ab-8aB)}{128a^2x^2}+\frac{(a+bx^2)^{3/2}(3Ab-8aB)}{48ax^6}+\frac{b\sqrt{a+bx^2}(3Ab-8aB)}{64ax^4}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

```
[Out] (b*(3*A*b - 8*a*B)*Sqrt[a + b*x^2])/(64*a*x^4) + (b^2*(3*A*b - 8*
a*B)*Sqrt[a + b*x^2])/(128*a^2*x^2) + ((3*A*b - 8*a*B)*(a + b*x^2
)^(3/2))/(48*a*x^6) - (A*(a + b*x^2)^(5/2))/(8*a*x^8) - (b^3*(3*A
*b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rubi [A] time = 0.309678, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b^2\sqrt{a+bx^2}(3Ab-8aB)}{128a^2x^2}+\frac{(a+bx^2)^{3/2}(3Ab-8aB)}{48ax^6}+\frac{b\sqrt{a+bx^2}(3Ab-8aB)}{64ax^4}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (b*(3*A*b - 8*a*B)*Sqrt[a + b*x^2])/(64*a*x^4) + (b^2*(3*A*b - 8*
a*B)*Sqrt[a + b*x^2])/(128*a^2*x^2) + ((3*A*b - 8*a*B)*(a + b*x^2
)^(3/2))/(48*a*x^6) - (A*(a + b*x^2)^(5/2))/(8*a*x^8) - (b^3*(3*A
*b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rubi in Sympy [A] time = 25.6058, size = 143, normalized size = 0.92

$$-\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{8ax^8}+\frac{b\sqrt{a+bx^2}\,(3Ab-8Ba)}{64ax^4}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}(3Ab-8Ba)}{48ax^6}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-8Ba)}{128a^2x^2}-\frac{b^3\,(3Ab-8Ba)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**9,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(8*a*x**8) + b*sqrt(a + b*x**2)*(3*A*b - 8
*B*a)/(64*a*x**4) + (a + b*x**2)**(3/2)*(3*A*b - 8*B*a)/(48*a*x**
6) + b**2*sqrt(a + b*x**2)*(3*A*b - 8*B*a)/(128*a**2*x**2) - b**3
*(3*A*b - 8*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(128*a**(5/2))
```

Mathematica [A] time = 0.200736, size = 145, normalized size = 0.93

$$-\frac{b^3(3Ab-8aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{128a^{5/2}}+\frac{b^3\log(x)(3Ab-8aB)}{128a^{5/2}}+\sqrt{a+bx^2}\left(-\frac{b^2(8aB-3Ab)}{128a^2x^2}+\frac{-8aB-9Ab}{48x^6}-\frac{b(56aB+3Ab)}{192ax^4}-\frac{aA}{8x^8}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (-(a*A)/(8*x^8) + (-9*A*b - 8*a*B)/(48*x^6) - (b*(3*A*b + 56*a*B)
)/(192*a*x^4) - (b^2*(-3*A*b + 8*a*B))/(128*a^2*x^2))*Sqrt[a + b*
x^2] + (b^3*(3*A*b - 8*a*B)*Log[x])/(128*a^(5/2)) - (b^3*(3*A*b -
 8*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(128*a^(5/2))
```

Maple [B] time = 0.014, size = 275, normalized size = 1.8

$$-\frac{A}{8\,a x^8}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ab}{16\,a^2x^6}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{b^2A}{64\,a^3x^4}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Ab^3}{128\,a^4x^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ab^4}{128\,a^4}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,Ab^4}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{3\,Ab^4}{128\,a^3}\sqrt{bx^2+a}-\frac{B}{6\,a x^6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Bb}{24\,a^2x^4}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Bb^2}{48\,a^3x^2}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{Bb^3}{48\,a^3}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{Bb^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{Bb^3}{16\,a^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x)
```

```
[Out] -1/8*A*(b*x^2+a)^(5/2)/a/x^8+1/16*A*b/a^2/x^6*(b*x^2+a)^(5/2)-1/6
4*A*b^2/a^3/x^4*(b*x^2+a)^(5/2)-1/128*A*b^3/a^4/x^2*(b*x^2+a)^(5/
2)+1/128*A*b^4/a^4*(b*x^2+a)^(3/2)-3/128*A*b^4/a^(5/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x)+3/128*A*b^4/a^3*(b*x^2+a)^(1/2)-1/6*B
/a/x^6*(b*x^2+a)^(5/2)+1/24*B*b/a^2/x^4*(b*x^2+a)^(5/2)+1/48*B*b^
2/a^3/x^2*(b*x^2+a)^(5/2)-1/48*B*b^3/a^3*(b*x^2+a)^(3/2)+1/16*B*b
^3/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/16*B*b^3/a^2*(
b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296432, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(8\,Bab^3-3\,Ab^4\right)x^8\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)+2\left(3\left(8\,Bab^2-3\,Ab^3\right)x^6+2\left(56\,Ba^2b+3\,Aab^2\right)x^4+48\,Aa^3+8\left(8\,B\right.\right.}{768\,a^{\frac{5}{2}}x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 3*A*b^4)*x^8*log(-((b*x^2 + 2*a)*sqrt(a)
- 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(3*(8*B*a*b^2 - 3*A*b^3)*x^6 + 2*
(56*B*a^2*b + 3*A*a*b^2)*x^4 + 48*A*a^3 + 8*(8*B*a^3 + 9*A*a^2*b)
```

```
*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(5/2)*x^8), 1/384*(3*(8*B*a*b^3
 - 3*A*b^4)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*(8*B*a*b^2
- 3*A*b^3)*x^6 + 2*(56*B*a^2*b + 3*A*a*b^2)*x^4 + 48*A*a^3 + 8*(8
*B*a^3 + 9*A*a^2*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^2*
x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**9,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.252521, size = 262, normalized size = 1.68

$$-\frac{\frac{3\left(8\,Bab^{4}-3\,Ab^{5}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{24\left(bx^{2}+a\right)^{\frac{7}{2}}Bab^{4}+40\left(bx^{2}+a\right)^{\frac{5}{2}}Ba^{2}b^{4}-88\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{3}b^{4}+24\sqrt{bx^{2}+a}Ba^{4}b^{4}-9\left(bx^{2}+a\right)^{\frac{7}{2}}Ab^{5}+33\left(bx^{2}+a\right)^{\frac{5}{2}}Aab^{5}}{a^{2}b^{4}x^{8}}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*a*b^4 - 3*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/
(sqrt(-a)*a^2) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 + 40*(b*x^2 + a)^(
5/2)*B*a^2*b^4 - 88*(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 +
 a)*B*a^4*b^4 - 9*(b*x^2 + a)^(7/2)*A*b^5 + 33*(b*x^2 + a)^(5/2)*
A*a*b^5 + 33*(b*x^2 + a)^(3/2)*A*a^2*b^5 - 9*sqrt(b*x^2 + a)*A*a^
3*b^5)/(a^2*b^4*x^8))/b
```

3.536 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

```
[Out] -(A*(a + b*x^2)^(5/2))/(9*a*x^9) + ((4*A*b - 9*a*B)*(a + b*x^2)^(
5/2))/(63*a^2*x^7) - (2*b*(4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(315
*a^3*x^5)
```

Rubi [A] time = 0.118607, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{2b\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(9*a*x^9) + ((4*A*b - 9*a*B)*(a + b*x^2)^(
5/2))/(63*a^2*x^7) - (2*b*(4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(315
*a^3*x^5)
```

Rubi in Sympy [A] time = 12.4541, size = 78, normalized size = 0.93

$$-\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{9ax^9}+\frac{\left(a+bx^2\right)^{\frac{5}{2}}(4Ab-9Ba)}{63a^2x^7}-\frac{2b\left(a+bx^2\right)^{\frac{5}{2}}(4Ab-9Ba)}{315a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(9*a*x**9) + (a + b*x**2)**(5/2)*(4*A*b -
9*B*a)/(63*a**2*x**7) - 2*b*(a + b*x**2)**(5/2)*(4*A*b - 9*B*a)/(
315*a**3*x**5)
```

Mathematica [A] time = 0.0888618, size = 63, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{5/2}\left(-5a^2\left(7A+9Bx^2\right)+2abx^2\left(10A+9Bx^2\right)-8Ab^2x^4\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-8*A*b^2*x^4 - 5*a^2*(7*A + 9*B*x^2) + 2*a*b*
x^2*(10*A + 9*B*x^2)))/(315*a^3*x^9)
```

Maple [A] time = 0.008, size = 59, normalized size = 0.7

$$-\frac{8\,Ab^2x^4-18\,Babx^4-20\,aAbx^2+45\,Ba^2x^2+35\,Aa^2}{315\,x^9a^3}\left(bx^2+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(8*A*b^2*x^4-18*B*a*b*x^4-20*A*a*b*x^2+45*
B*a^2*x^2+35*A*a^2)/x^9/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.332746, size = 142, normalized size = 1.69

$$\frac{\left(2\left(9 Bab^3-4 Ab^4\right)x^8-\left(9 Ba^2b^2-4 Aab^3\right)x^6-35 Aa^4-3\left(24 Ba^3b+Aa^2b^2\right)x^4-5\left(9 Ba^4+10 Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{315 a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*a*b^3 - 4*A*b^4)*x^8 - (9*B*a^2*b^2 - 4*A*a*b^3)*x^
6 - 35*A*a^4 - 3*(24*B*a^3*b + A*a^2*b^2)*x^4 - 5*(9*B*a^4 + 10*A
*a^3*b)*x^2)*sqrt(b*x^2 + a)/(a^3*x^9)
```

Sympy [A] time = 21.2061, size = 1408, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 9
45*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14
) - 110*A*a**7*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9
*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b*
*12*x**14) - 114*A*a**6*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 31
5*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/(b*x**2) +
1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x*
*10) + 5*A*a**4*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**
9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b
**12*x**14) - 33*A*a**4*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*
a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 30*A
*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**1
4) - 17*A*a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4
*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**
(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*
b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 3*A*
a**2*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
```

```
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a*b**(33/2)*x**14*
sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 12*A*a*b**(19/2)*
x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 8*A*b**(21/2)*x**10*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10
) - 15*B*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 +
 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**5*b**(11/2)*
x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 17*B*a**4*b**(13/2)*x**4*sqrt(a/(b*x**
2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*
x**10) - 3*B*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b
**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a**2*
b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**
4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a*b**(19/2)*x**10*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**
3*b**6*x**10) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(
5/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*b**(7/2)*sqrt(a/(b*x*
*2) + 1)/(15*a**2)
```

GIAC/XCAS [A] time = 0.256708, size = 540, normalized size = 6.43

$$\frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{7}{2}}+840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*b^(7/2) - 315*(sqrt
(b)*x - sqrt(b*x^2 + a))^12*B*a*b^(7/2) + 840*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*A*b^(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*
a^2*b^(7/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(9/2) -
 819*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(7/2) + 1764*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(9/2) + 441*(sqrt(b)*x - sqrt(b*
x^2 + a))^6*B*a^4*b^(7/2) + 504*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a^3*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(7/2) +
144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(9/2) + 81*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*B*a^6*b^(7/2) - 36*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*A*a^5*b^(9/2) - 9*B*a^7*b^(7/2) + 4*A*a^6*b^(9/2))/((sqrt
(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.537 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}} dx$

Optimal. Leaf size=184

$$\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}} - \frac{3b^3\sqrt{a+bx^2}(Ab-2aB)}{256a^3x^2} + \frac{b^2\sqrt{a+bx^2}(Ab-2aB)}{128a^2x^4} + \frac{(a+bx^2)^{3/2}(Ab-2aB)}{16ax^8} + \frac{b\sqrt{a+bx^2}(Ab-2aB)}{32ax^6} - \frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

```
[Out] (b*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(32*a*x^6) + (b^2*(A*b - 2*a*B)
*Sqrt[a + b*x^2])/(128*a^2*x^4) - (3*b^3*(A*b - 2*a*B)*Sqrt[a + b
*x^2])/(256*a^3*x^2) + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/(16*a*x^
8) - (A*(a + b*x^2)^(5/2))/(10*a*x^10) + (3*b^4*(A*b - 2*a*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(7/2))
```

Rubi [A] time = 0.367655, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}} - \frac{3b^3\sqrt{a+bx^2}(Ab-2aB)}{256a^3x^2} + \frac{b^2\sqrt{a+bx^2}(Ab-2aB)}{128a^2x^4} + \frac{(a+bx^2)^{3/2}(Ab-2aB)}{16ax^8} + \frac{b\sqrt{a+bx^2}(Ab-2aB)}{32ax^6} - \frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(32*a*x^6) + (b^2*(A*b - 2*a*B)
*Sqrt[a + b*x^2])/(128*a^2*x^4) - (3*b^3*(A*b - 2*a*B)*Sqrt[a + b
*x^2])/(256*a^3*x^2) + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/(16*a*x^
8) - (A*(a + b*x^2)^(5/2))/(10*a*x^10) + (3*b^4*(A*b - 2*a*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(7/2))
```

Rubi in Sympy [A] time = 31.9427, size = 170, normalized size = 0.92

$$-\frac{A(a+bx^2)^{\frac{5}{2}}}{10ax^{10}} + \frac{b\sqrt{a+bx^2}(Ab-2Ba)}{32ax^6} + \frac{(a+bx^2)^{\frac{3}{2}}\left(\frac{Ab}{2}-Ba\right)}{8ax^8} + \frac{b^2\sqrt{a+bx^2}\left(\frac{Ab}{2}-Ba\right)}{64a^2x^4} - \frac{3b^3\sqrt{a+bx^2}\left(\frac{Ab}{2}-Ba\right)}{128a^3x^2} + \frac{3b^4\left(\frac{Ab}{2}-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**11,x)
```

```
[Out] -A*(a + b*x**2)**(5/2)/(10*a*x**10) + b*sqrt(a + b*x**2)*(A*b - 2
*B*a)/(32*a*x**6) + (a + b*x**2)**(3/2)*(A*b/2 - B*a)/(8*a*x**8)
+ b**2*sqrt(a + b*x**2)*(A*b/2 - B*a)/(64*a**2*x**4) - 3*b**3*sqr
t(a + b*x**2)*(A*b/2 - B*a)/(128*a**3*x**2) + 3*b**4*(A*b/2 - B*a
)*atanh(sqrt(a + b*x**2)/sqrt(a))/(128*a**(7/2))
```

Mathematica [A] time = 0.290653, size = 164, normalized size = 0.89

$$\frac{3b^4(Ab-2aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{256a^{7/2}} - \frac{3b^4\log(x)(Ab-2aB)}{256a^{7/2}} + \sqrt{a+bx^2}\left(\frac{3b^3(2aB-Ab)}{256a^3x^2} - \frac{b^2(2aB-Ab)}{128a^2x^4} + \frac{-10aB-11Ab}{80x^8} - \frac{b(30aB+Ab)}{160ax^6} - \frac{aA}{10x^{10}}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (-(a*A)/(10*x^10) + (-11*A*b - 10*a*B)/(80*x^8) - (b*(A*b + 30*a*
B))/(160*a*x^6) - (b^2*(-(A*b) + 2*a*B))/(128*a^2*x^4) + (3*b^3*(
-(A*b) + 2*a*B))/(256*a^3*x^2))*Sqrt[a + b*x^2] - (3*b^4*(A*b - 2
*a*B)*Log[x])/(256*a^(7/2)) + (3*b^4*(A*b - 2*a*B)*Log[a + Sqrt[a
]*Sqrt[a + b*x^2]])/(256*a^(7/2))
```

Maple [B] time = 0.046, size = 317, normalized size = 1.7

$$-\frac{A}{10\,a x^{10}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b}{16\,a^{2} x^{8}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{b^{2} A}{32\,a^{3} x^{6}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b^{3}}{128\,a^{4} x^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b^{4}}{256\,a^{5} x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{A b^{5}}{256\,a^{5}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{3\,A b^{5}}{256}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{7}{2}}-\frac{3\,A b^{5}}{256\,a^{4}}\sqrt{b x^{2}+a}-\frac{B}{8\,a x^{8}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{B b}{16\,a^{2} x^{6}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{B b^{2}}{64\,a^{3} x^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{B b^{3}}{128\,a^{4} x^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{B b^{4}}{128\,a^{4}}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{3\,B b^{4}}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{5}{2}}+\frac{3\,B b^{4}}{128\,a^{3}}\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(5/2)/a/x^10+1/16*A*b/a^2/x^8*(b*x^2+a)^(5/2)-1
/32*A*b^2/a^3/x^6*(b*x^2+a)^(5/2)+1/128*A*b^3/a^4/x^4*(b*x^2+a)^(
5/2)+1/256*A*b^4/a^5/x^2*(b*x^2+a)^(5/2)-1/256*A*b^5/a^5*(b*x^2+a
)^(3/2)+3/256*A*b^5/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
-3/256*A*b^5/a^4*(b*x^2+a)^(1/2)-1/8*B/a/x^8*(b*x^2+a)^(5/2)+1/16
*B*b/a^2/x^6*(b*x^2+a)^(5/2)-1/64*B*b^2/a^3/x^4*(b*x^2+a)^(5/2)-1
/128*B*b^3/a^4/x^2*(b*x^2+a)^(5/2)+1/128*B*b^4/a^4*(b*x^2+a)^(3/2
)-3/128*B*b^4/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/128
*B*b^4/a^3*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.420074, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(2\,Bab^{4}-Ab^{5}\right)x^{10}\log\left(-\frac{\left(bx^{2}+2\,a\right)\sqrt{a}+2\,\sqrt{bx^{2}+a}a}{x^{2}}\right)-2\left(15\left(2\,Bab^{3}-Ab^{4}\right)x^{8}-10\left(2\,Ba^{2}b^{2}-Aab^{3}\right)x^{6}-128\,Aa^{4}-8\left(30\right.\right.}{2560\,a^{\frac{7}{2}}x^{10}}\right.$$

$$-\frac{15\left(2\,Bab^{4}-Ab^{5}\right)x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(15\left(2\,Bab^{3}-Ab^{4}\right)x^{8}-10\left(2\,Ba^{2}b^{2}-Aab^{3}\right)x^{6}-128\,Aa^{4}-8\left(30\,Ba^{3}b+Aa^{2}b^{2}\right.\right.}{1280\,\sqrt{-a}a^{3}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*a*b^4 - A*b^5)*x^10*log(-((b*x^2 + 2*a)*sqrt(a)
 + 2*sqrt(b*x^2 + a)*a)/x^2) - 2*(15*(2*B*a*b^3 - A*b^4)*x^8 - 10
*(2*B*a^2*b^2 - A*a*b^3)*x^6 - 128*A*a^4 - 8*(30*B*a^3*b + A*a^2*
b^2)*x^4 - 16*(10*B*a^4 + 11*A*a^3*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a
))/(a^(7/2)*x^10), -1/1280*(15*(2*B*a*b^4 - A*b^5)*x^10*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) - (15*(2*B*a*b^3 - A*b^4)*x^8 - 10*(2*B*a
^2*b^2 - A*a*b^3)*x^6 - 128*A*a^4 - 8*(30*B*a^3*b + A*a^2*b^2)*x^
4 - 16*(10*B*a^4 + 11*A*a^3*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sq
rt(-a)*a^3*x^10)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.252197, size = 286, normalized size = 1.55

$$\frac{\frac{15\left(2\,Bab^5-Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{30\left(bx^2+a\right)^{\frac{9}{2}}Bab^5-140\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5+140\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-30\sqrt{bx^2+a}Ba^5b^5-15\left(bx^2+a\right)^{\frac{9}{2}}Ab^6+70\left(bx^2+a\right)^{\frac{7}{2}}Aab}{a^3b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*a*b^5 - A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(
sqrt(-a)*a^3) + (30*(b*x^2 + a)^(9/2)*B*a*b^5 - 140*(b*x^2 + a)^(
7/2)*B*a^2*b^5 + 140*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 30*sqrt(b*x^2
+ a)*B*a^5*b^5 - 15*(b*x^2 + a)^(9/2)*A*b^6 + 70*(b*x^2 + a)^(7/2
)*A*a*b^6 - 128*(b*x^2 + a)^(5/2)*A*a^2*b^6 - 70*(b*x^2 + a)^(3/2
)*A*a^3*b^6 + 15*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^3*b^5*x^10))/b
```

3.538 $\int x^5 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=103

$$\frac{a^2\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^4}+\frac{\left(a+bx^2\right)^{11/2}(Ab-3aB)}{11b^4}-\frac{a\left(a+bx^2\right)^{9/2}(2Ab-3aB)}{9b^4}+\frac{B\left(a+bx^2\right)^{13/2}}{13b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (a*(2*A*b - 3*a*B)*
(a + b*x^2)^(9/2))/(9*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(11/2))/(
11*b^4) + (B*(a + b*x^2)^(13/2))/(13*b^4)
```

Rubi [A] time = 0.225091, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^2\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^4}+\frac{\left(a+bx^2\right)^{11/2}(Ab-3aB)}{11b^4}-\frac{a\left(a+bx^2\right)^{9/2}(2Ab-3aB)}{9b^4}+\frac{B\left(a+bx^2\right)^{13/2}}{13b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (a*(2*A*b - 3*a*B)*
(a + b*x^2)^(9/2))/(9*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(11/2))/(
11*b^4) + (B*(a + b*x^2)^(13/2))/(13*b^4)
```

Rubi in Sympy [A] time = 26.0401, size = 92, normalized size = 0.89

$$\frac{B\left(a+bx^2\right)^{\frac{13}{2}}}{13b^4}+\frac{a^2\left(a+bx^2\right)^{\frac{7}{2}}(Ab-Ba)}{7b^4}-\frac{a\left(a+bx^2\right)^{\frac{9}{2}}(2Ab-3Ba)}{9b^4}+\frac{\left(a+bx^2\right)^{\frac{11}{2}}(Ab-3Ba)}{11b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**(13/2)/(13*b**4) + a**2*(a + b*x**2)**(7/2)*(A*b
- B*a)/(7*b**4) - a*(a + b*x**2)**(9/2)*(2*A*b - 3*B*a)/(9*b**4)
+ (a + b*x**2)**(11/2)*(A*b - 3*B*a)/(11*b**4)
```

Mathematica [A] time = 0.10486, size = 78, normalized size = 0.76

$$\frac{\left(a+bx^2\right)^{7/2}\left(-48a^3B+8a^2b\left(13A+21Bx^2\right)-14ab^2x^2\left(26A+27Bx^2\right)+63b^3x^4\left(13A+11Bx^2\right)\right)}{9009b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-48*a^3*B + 63*b^3*x^4*(13*A + 11*B*x^2) + 8*
a^2*b*(13*A + 21*B*x^2) - 14*a*b^2*x^2*(26*A + 27*B*x^2)))/(9009*
b^4)
```

Maple [A] time = 0.008, size = 77, normalized size = 0.8

$$\frac{693\,Bx^6b^3+819\,Ab^3x^4-378\,Bab^2x^4-364\,Aab^2x^2+168\,Ba^2bx^2+104\,Aa^2b-48\,Ba^3}{9009\,b^4}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/9009*(b*x^2+a)^(7/2)*(693*B*b^3*x^6+819*A*b^3*x^4-378*B*a*b^2*x
^4-364*A*a*b^2*x^2+168*B*a^2*b*x^2+104*A*a^2*b-48*B*a^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221847, size = 198, normalized size = 1.92

$$\frac{\left(693 \, B b^{6} x^{12} + 63 \, \left(27 \, B a b^{5} + 13 \, A b^{6}\right) x^{10} + 7 \, \left(159 \, B a^{2} b^{4} + 299 \, A a b^{5}\right) x^{8} - 48 \, B a^{6} + 104 \, A a^{5} b + \left(15 \, B a^{3} b^{3} + 1469 \, A a^{2} b^{4}\right) x^{6} - \cdots}{9009 \, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/9009*(693*B*b^6*x^12 + 63*(27*B*a*b^5 + 13*A*b^6)*x^10 + 7*(159
*B*a^2*b^4 + 299*A*a*b^5)*x^8 - 48*B*a^6 + 104*A*a^5*b + (15*B*a^
3*b^3 + 1469*A*a^2*b^4)*x^6 - 3*(6*B*a^4*b^2 - 13*A*a^3*b^3)*x^4
+ 4*(6*B*a^5*b - 13*A*a^4*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 30.8489, size = 313, normalized size = 3.04

$$\begin{cases} \frac{8Aa^5\sqrt{a+bx^2}}{693b^3} - \frac{4Aa^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{Aa^3x^4\sqrt{a+bx^2}}{231b} + \frac{113Aa^2x^6\sqrt{a+bx^2}}{693} + \frac{23Aabx^8\sqrt{a+bx^2}}{99} + \frac{Ab^2x^{10}\sqrt{a+bx^2}}{11} - \frac{16Ba^6\sqrt{a+bx^2}}{3003b^4} + \frac{8Ba^5x^2\sqrt{a+bx^2}}{3003b^3} \cdots \\ a^{\frac{5}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*A*a**4*x**2*s
qrt(a + b*x**2)/(693*b**2) + A*a**3*x**4*sqrt(a + b*x**2)/(231*b)
 + 113*A*a**2*x**6*sqrt(a + b*x**2)/693 + 23*A*a*b*x**8*sqrt(a +
b*x**2)/99 + A*b**2*x**10*sqrt(a + b*x**2)/11 - 16*B*a**6*sqrt(a
+ b*x**2)/(3003*b**4) + 8*B*a**5*x**2*sqrt(a + b*x**2)/(3003*b**3
) - 2*B*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*B*a**3*x**6*sq
rt(a + b*x**2)/(3003*b) + 53*B*a**2*x**8*sqrt(a + b*x**2)/429 + 2
7*B*a*b*x**10*sqrt(a + b*x**2)/143 + B*b**2*x**12*sqrt(a + b*x**2
)/13, Ne(b, 0)), (a**(5/2)*(A*x**6/6 + B*x**8/8), True))
```

GIAC/XCAS [A] time = 0.234473, size = 545, normalized size = 5.29

$$\frac{\frac{429\left(15\left(bx^2+a\right)^{\frac{7}{2}} - 42\left(bx^2+a\right)^{\frac{5}{2}}a + 35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Aa^2}{b^2} + \frac{143\left(35\left(bx^2+a\right)^{\frac{9}{2}} - 135\left(bx^2+a\right)^{\frac{7}{2}}a + 189\left(bx^2+a\right)^{\frac{5}{2}}a^2 - 105\left(bx^2+a\right)^{\frac{3}{2}}a^3\right)Ba^2}{b^3} + \frac{286\left(35\left(bx^2+a\right)\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/45045*(429*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*
(b*x^2 + a)^(3/2)*a^2)*A*a^2/b^2 + 143*(35*(b*x^2 + a)^(9/2) - 13
5*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 +
a)^(3/2)*a^3)*B*a^2/b^3 + 286*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2
+ a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 + a)^(3/2)*
a^3)*A*a/b^2 + 26*(315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(9/2
)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3 + 1
155*(b*x^2 + a)^(3/2)*a^4)*B*a/b^3 + 13*(315*(b*x^2 + a)^(11/2) -
 1540*(b*x^2 + a)^(9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*
x^2 + a)^(5/2)*a^3 + 1155*(b*x^2 + a)^(3/2)*a^4)*A/b^2 + 5*(693*(
b*x^2 + a)^(13/2) - 4095*(b*x^2 + a)^(11/2)*a + 10010*(b*x^2 + a)
^(9/2)*a^2 - 12870*(b*x^2 + a)^(7/2)*a^3 + 9009*(b*x^2 + a)^(5/2)
*a^4 - 3003*(b*x^2 + a)^(3/2)*a^5)*B/b^3)/b
```

3.539 $\int x^4 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=221

$$\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}-\frac{a^4x\sqrt{a+bx^2}(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}(12Ab-5aB)}{1536b^2}+\frac{a^2x^5\sqrt{a+bx^2}(12Ab-5aB)}{384b}+\frac{ax^5\left(a+bx^2\right)^{3/2}(12Ab-5aB)}{192b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12Ab-5aB)}{120b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

```
[Out] -(a^4*(12*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(1024*b^3) + (a^3*(12*A
*b - 5*a*B)*x^3*Sqrt[a + b*x^2])/(1536*b^2) + (a^2*(12*A*b - 5*a*
B)*x^5*Sqrt[a + b*x^2])/(384*b) + (a*(12*A*b - 5*a*B)*x^5*(a + b*
x^2)^(3/2))/(192*b) + ((12*A*b - 5*a*B)*x^5*(a + b*x^2)^(5/2))/(1
20*b) + (B*x^5*(a + b*x^2)^(7/2))/(12*b) + (a^5*(12*A*b - 5*a*B)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rubi [A] time = 0.314075, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}-\frac{a^4x\sqrt{a+bx^2}(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}(12Ab-5aB)}{1536b^2}+\frac{a^2x^5\sqrt{a+bx^2}(12Ab-5aB)}{384b}+\frac{ax^5\left(a+bx^2\right)^{3/2}(12Ab-5aB)}{192b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12Ab-5aB)}{120b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -(a^4*(12*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(1024*b^3) + (a^3*(12*A
*b - 5*a*B)*x^3*Sqrt[a + b*x^2])/(1536*b^2) + (a^2*(12*A*b - 5*a*
B)*x^5*Sqrt[a + b*x^2])/(384*b) + (a*(12*A*b - 5*a*B)*x^5*(a + b*
x^2)^(3/2))/(192*b) + ((12*A*b - 5*a*B)*x^5*(a + b*x^2)^(5/2))/(1
20*b) + (B*x^5*(a + b*x^2)^(7/2))/(12*b) + (a^5*(12*A*b - 5*a*B)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rubi in Sympy [A] time = 33.5531, size = 206, normalized size = 0.93

$$\frac{Bx^5\left(a+bx^2\right)^{\frac{7}{2}}}{12b}+\frac{a^5\left(12Ab-5Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{\frac{7}{2}}}-\frac{a^4x\sqrt{a+bx^2}\left(12Ab-5Ba\right)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}\left(12Ab-5Ba\right)}{1536b^2}+\frac{a^2x^5\sqrt{a+bx^2}\left(12Ab-5Ba\right)}{384b}+\frac{ax^5\left(a+bx^2\right)^{\frac{3}{2}}\left(12Ab-5Ba\right)}{192b}+\frac{x^5\left(a+bx^2\right)^{\frac{5}{2}}\left(12Ab-5Ba\right)}{120b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] B*x**5*(a + b*x**2)**(7/2)/(12*b) + a**5*(12*A*b - 5*B*a)*atanh(s
qrt(b)*x/sqrt(a + b*x**2))/(1024*b**(7/2)) - a**4*x*sqrt(a + b*x*
*2)*(12*A*b - 5*B*a)/(1024*b**3) + a**3*x**3*sqrt(a + b*x**2)*(12
*A*b - 5*B*a)/(1536*b**2) + a**2*x**5*sqrt(a + b*x**2)*(12*A*b -
5*B*a)/(384*b) + a*x**5*(a + b*x**2)**(3/2)*(12*A*b - 5*B*a)/(192
```

```
*b) + x**5*(a + b*x**2)**(5/2)*(12*A*b - 5*B*a)/(120*b)
```

Mathematica [A] time = 0.187, size = 166, normalized size = 0.75

$$\sqrt{a+bx^2}\left(\frac{a^4x(5aB-12Ab)}{1024b^3}-\frac{a^3x^3(5aB-12Ab)}{1536b^2}+\frac{a^2x^5(5aB+372Ab)}{1920b}+\frac{1}{120}bx^9(25aB+12Ab)+\frac{3}{320}ax^7(15aB+28Ab)+\frac{1}{12}b^2Bx^{11}\right)-\frac{a^5(5aB-12Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{1024b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((a^4*(-12*A*b + 5*a*B)*x)/(1024*b^3) - (a^3*(-12
*A*b + 5*a*B)*x^3)/(1536*b^2) + (a^2*(372*A*b + 5*a*B)*x^5)/(1920
*b) + (3*a*(28*A*b + 15*a*B)*x^7)/320 + (b*(12*A*b + 25*a*B)*x^9)
/120 + (b^2*B*x^11)/12) - (a^5*(-12*A*b + 5*a*B)*Log[b*x + Sqrt[b
]*Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Maple [A] time = 0.012, size = 257, normalized size = 1.2

$$\frac{Ax^3}{10\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3\,aAx}{80\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{a^2Ax}{160\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Aa^3x}{128\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,Aa^4x}{256\,b^2}\sqrt{bx^2+a}+\frac{3\,Aa^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{Bx^5}{12\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Bax^3}{24\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Bxa^2}{64\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Ba^3x}{384\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,Ba^4x}{1536\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,Bxa^5}{1024\,b^3}\sqrt{bx^2+a}-\frac{5\,Ba^6}{1024}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/10*A*x^3*(b*x^2+a)^(7/2)/b-3/80*A*a/b^2*x*(b*x^2+a)^(7/2)+1/160
*A*a^2/b^2*x*(b*x^2+a)^(5/2)+1/128*A*a^3/b^2*x*(b*x^2+a)^(3/2)+3/
256*A*a^4/b^2*x*(b*x^2+a)^(1/2)+3/256*A*a^5/b^(5/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))+1/12*B*x^5*(b*x^2+a)^(7/2)/b-1/24*B*a/b^2*x^3*(b
*x^2+a)^(7/2)+1/64*B*a^2/b^3*x*(b*x^2+a)^(7/2)-1/384*B*a^3/b^3*x*
(b*x^2+a)^(5/2)-5/1536*B*a^4/b^3*x*(b*x^2+a)^(3/2)-5/1024*B*a^5/b
^3*x*(b*x^2+a)^(1/2)-5/1024*B*a^6/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.552754, size = 1, normalized size = 0.

$$\left[\frac{2\left(1280\,Bb^5x^{11}+128\left(25\,Bab^4+12\,Ab^5\right)x^9+144\left(15\,Ba^2b^3+28\,Aab^4\right)x^7+8\left(5\,Ba^3b^2+372\,Aa^2b^3\right)x^5-10\left(5\,Ba^4b-12\,A\right.}{30720\,b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30720*(2*(1280*B*b^5*x^11 + 128*(25*B*a*b^4 + 12*A*b^5)*x^9 +
144*(15*B*a^2*b^3 + 28*A*a*b^4)*x^7 + 8*(5*B*a^3*b^2 + 372*A*a^2*
b^3)*x^5 - 10*(5*B*a^4*b - 12*A*a^3*b^2)*x^3 + 15*(5*B*a^5 - 12*A
*a^4*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 15*(5*B*a^6 - 12*A*a^5*b)*lo
g(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(7/2), 1/153
60*((1280*B*b^5*x^11 + 128*(25*B*a*b^4 + 12*A*b^5)*x^9 + 144*(15*
B*a^2*b^3 + 28*A*a*b^4)*x^7 + 8*(5*B*a^3*b^2 + 372*A*a^2*b^3)*x^5
 - 10*(5*B*a^4*b - 12*A*a^3*b^2)*x^3 + 15*(5*B*a^5 - 12*A*a^4*b)*
x)*sqrt(b*x^2 + a)*sqrt(-b) - 15*(5*B*a^6 - 12*A*a^5*b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.247971, size = 263, normalized size = 1.19

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bb^2x^2+\frac{25\,Bab^{11}+12\,Ab^{12}}{b^{10}}\right)x^2+\frac{9\left(15\,Ba^2b^{10}+28\,Aab^{11}\right)}{b^{10}}\right)x^2+\frac{5\,Ba^3b^9+372\,Aa^2b^{10}}{b^{10}}\right)x^2-\frac{5\,(5\,Ba\cdots}{}\right.\right.$$
$$+\frac{\left(5\,Ba^6-12\,Aa^5b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*b^2*x^2 + (25*B*a*b^11 + 12*A*b^12)/b^1
0)*x^2 + 9*(15*B*a^2*b^10 + 28*A*a*b^11)/b^10)*x^2 + (5*B*a^3*b^9
 + 372*A*a^2*b^10)/b^10)*x^2 - 5*(5*B*a^4*b^8 - 12*A*a^3*b^9)/b^1
0)*x^2 + 15*(5*B*a^5*b^7 - 12*A*a^4*b^8)/b^10)*sqrt(b*x^2 + a)*x
+ 1/1024*(5*B*a^6 - 12*A*a^5*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 +
a)))/b^(7/2)
```

3.540 $\int x^3 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{9/2} (Ab-2aB)}{9b^3}-\frac{a \left(a+bx^2\right)^{7/2} (Ab-aB)}{7b^3}+\frac{B \left(a+bx^2\right)^{11/2}}{11b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^3) + ((A*b - 2*a*B)*(a +
b*x^2)^(9/2))/(9*b^3) + (B*(a + b*x^2)^(11/2))/(11*b^3)
```

Rubi [A] time = 0.165741, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(a+bx^2\right)^{9/2} (Ab-2aB)}{9b^3}-\frac{a \left(a+bx^2\right)^{7/2} (Ab-aB)}{7b^3}+\frac{B \left(a+bx^2\right)^{11/2}}{11b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^3) + ((A*b - 2*a*B)*(a +
b*x^2)^(9/2))/(9*b^3) + (B*(a + b*x^2)^(11/2))/(11*b^3)
```

Rubi in Sympy [A] time = 19.7899, size = 63, normalized size = 0.86

$$\frac{B \left(a+bx^2\right)^{\frac{11}{2}}}{11b^3}-\frac{a \left(a+bx^2\right)^{\frac{7}{2}} (Ab-Ba)}{7b^3}+\frac{\left(a+bx^2\right)^{\frac{9}{2}} (Ab-2Ba)}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**(11/2)/(11*b**3) - a*(a + b*x**2)**(7/2)*(A*b - B
*a)/(7*b**3) + (a + b*x**2)**(9/2)*(A*b - 2*B*a)/(9*b**3)
```

Mathematica [A] time = 0.0784186, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{7/2} \left(8a^2B-2ab\left(11A+14Bx^2\right)+7b^2x^2\left(11A+9Bx^2\right)\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(8*a^2*B + 7*b^2*x^2*(11*A + 9*B*x^2) - 2*a*b*
(11*A + 14*B*x^2)))/(693*b^3)
```

Maple [A] time = 0.009, size = 53, normalized size = 0.7

$$-\frac{-63\,b^2Bx^4-77\,Ab^2x^2+28\,Babx^2+22\,abA-8\,a^2B}{693\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] -1/693*(b*x^2+a)^(7/2)*(-63*B*b^2*x^4-77*A*b^2*x^2+28*B*a*b*x^2+2
2*A*a*b-8*B*a^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218933, size = 165, normalized size = 2.26

$$\frac{\left(63\,Bb^5x^{10} + 7\,\left(23\,Bab^4 + 11\,Ab^5\right)x^8 + \left(113\,Ba^2b^3 + 209\,Aab^4\right)x^6 + 8\,Ba^5 - 22\,Aa^4b + 3\,\left(Ba^3b^2 + 55\,Aa^2b^3\right)x^4 - \left(4\,Ba^4b - \right.}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^10 + 7*(23*B*a*b^4 + 11*A*b^5)*x^8 + (113*B*a^2
*b^3 + 209*A*a*b^4)*x^6 + 8*B*a^5 - 22*A*a^4*b + 3*(B*a^3*b^2 + 5
5*A*a^2*b^3)*x^4 - (4*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a
)/b^3
```

Sympy [A] time = 19.4106, size = 260, normalized size = 3.56

$$\begin{cases} -\frac{2Aa^4\sqrt{a+bx^2}}{63b^2} + \frac{Aa^3x^2\sqrt{a+bx^2}}{63b} + \frac{5Aa^2x^4\sqrt{a+bx^2}}{21} + \frac{19Aabx^6\sqrt{a+bx^2}}{63} + \frac{Ab^2x^8\sqrt{a+bx^2}}{9} + \frac{8Ba^5\sqrt{a+bx^2}}{693b^3} - \frac{4Ba^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{Ba^3x^4\sqrt{a+bx^2}}{231b} + \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4} + \frac{Bx^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**4*sqrt(a + b*x**2)/(63*b**2) + A*a**3*x**2*sqr
t(a + b*x**2)/(63*b) + 5*A*a**2*x**4*sqrt(a + b*x**2)/21 + 19*A*a
*b*x**6*sqrt(a + b*x**2)/63 + A*b**2*x**8*sqrt(a + b*x**2)/9 + 8*
B*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*B*a**4*x**2*sqrt(a + b*x**
2)/(693*b**2) + B*a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*B*a**2
*x**6*sqrt(a + b*x**2)/693 + 23*B*a*b*x**8*sqrt(a + b*x**2)/99 +
B*b**2*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(5/2)*(A*x**4/4
+ B*x**6/6), True))
```

GIAC/XCAS [A] time = 0.25041, size = 431, normalized size = 5.9

$$\frac{\frac{231\left(3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Aa^2}{b} + \frac{33\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Ba^2}{b^2} + \frac{66\left(15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2\right)Aa}{b} + }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/3465*(231*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*A*a^2/b
 + 33*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2
+ a)^(3/2)*a^2)*B*a^2/b^2 + 66*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2
+ a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*A*a/b + 22*(35*(b*x^2 +
a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 -
105*(b*x^2 + a)^(3/2)*a^3)*B*a/b^2 + 11*(35*(b*x^2 + a)^(9/2) - 1
35*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a^2 - 105*(b*x^2 +
 a)^(3/2)*a^3)*A/b + (315*(b*x^2 + a)^(11/2) - 1540*(b*x^2 + a)^(
9/2)*a + 2970*(b*x^2 + a)^(7/2)*a^2 - 2772*(b*x^2 + a)^(5/2)*a^3
+ 1155*(b*x^2 + a)^(3/2)*a^4)*B/b^2)/b
```

3.541 $\int x^2\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=188

$$-\frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x\sqrt{a+bx^2}(10Ab-3aB)}{256b^2}+\frac{a^2x^3\sqrt{a+bx^2}(10Ab-3aB)}{128b}+\frac{ax^3\left(a+bx^2\right)^{3/2}(10Ab-3aB)}{96b}+\frac{x^3\left(a+bx^2\right)^{5/2}(10Ab-3aB)}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

```
[Out] (a^3*(10*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(256*b^2) + (a^2*(10*A*b
 - 3*a*B)*x^3*Sqrt[a + b*x^2])/(128*b) + (a*(10*A*b - 3*a*B)*x^3*
(a + b*x^2)^(3/2))/(96*b) + ((10*A*b - 3*a*B)*x^3*(a + b*x^2)^(5/
2))/(80*b) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a^4*(10*A*b - 3*
a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rubi [A] time = 0.251955, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x\sqrt{a+bx^2}(10Ab-3aB)}{256b^2}+\frac{a^2x^3\sqrt{a+bx^2}(10Ab-3aB)}{128b}+\frac{ax^3\left(a+bx^2\right)^{3/2}(10Ab-3aB)}{96b}+\frac{x^3\left(a+bx^2\right)^{5/2}(10Ab-3aB)}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(256*b^2) + (a^2*(10*A*b
 - 3*a*B)*x^3*Sqrt[a + b*x^2])/(128*b) + (a*(10*A*b - 3*a*B)*x^3*
(a + b*x^2)^(3/2))/(96*b) + ((10*A*b - 3*a*B)*x^3*(a + b*x^2)^(5/
2))/(80*b) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a^4*(10*A*b - 3*
a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rubi in Sympy [A] time = 29.735, size = 173, normalized size = 0.92

$$\frac{Bx^3\left(a+bx^2\right)^{\frac{7}{2}}}{10b}-\frac{a^4\left(10Ab-3Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{\frac{5}{2}}}+\frac{a^3x\sqrt{a+bx^2}\left(10Ab-3Ba\right)}{256b^2}+\frac{a^2x^3\sqrt{a+bx^2}\left(10Ab-3Ba\right)}{128b}+\frac{ax^3\left(a+bx^2\right)^{\frac{3}{2}}\left(10Ab-3Ba\right)}{96b}+\frac{x^3\left(a+bx^2\right)^{\frac{5}{2}}\left(10Ab-3Ba\right)}{80b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] B*x**3*(a + b*x**2)**(7/2)/(10*b) - a**4*(10*A*b - 3*B*a)*atanh(s
qrt(b)*x/sqrt(a + b*x**2))/(256*b**(5/2)) + a**3*x*sqrt(a + b*x**
2)*(10*A*b - 3*B*a)/(256*b**2) + a**2*x**3*sqrt(a + b*x**2)*(10*A
*b - 3*B*a)/(128*b) + a*x**3*(a + b*x**2)**(3/2)*(10*A*b - 3*B*a)
/(96*b) + x**3*(a + b*x**2)**(5/2)*(10*A*b - 3*B*a)/(80*b)
```

Mathematica [A] time = 0.167115, size = 144, normalized size = 0.77

$$\frac{a^4(3aB-10Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{256b^{5/2}}+\sqrt{a+bx^2}\left(-\frac{a^3x(3aB-10Ab)}{256b^2}+\frac{a^2x^3(3aB+118Ab)}{384b}+\frac{1}{80}bx^7(21aB+10Ab)+\frac{1}{480}ax^5(93aB+170Ab)+\frac{1}{10}b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*(-(a^3*(-10*A*b + 3*a*B)*x)/(256*b^2) + (a^2*(118
*A*b + 3*a*B)*x^3)/(384*b) + (a*(170*A*b + 93*a*B)*x^5)/480 + (b*
(10*A*b + 21*a*B)*x^7)/80 + (b^2*B*x^9)/10) + (a^4*(-10*A*b + 3*a
*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Maple [A] time = 0.01, size = 215, normalized size = 1.1

$$\frac{Ax}{8b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{aAx}{48b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5a^2Ax}{192b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5Aa^3x}{128b}\sqrt{bx^2+a}-\frac{5Aa^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{Bx^3}{10b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3Bxa}{80b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Bxa^2}{160b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{Ba^3x}{128b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3Ba^4x}{256b^2}\sqrt{bx^2+a}+\frac{3Ba^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/8*A*x*(b*x^2+a)^(7/2)/b-1/48*A*a/b*x*(b*x^2+a)^(5/2)-5/192*A*a^
2/b*x*(b*x^2+a)^(3/2)-5/128*A*a^3/b*x*(b*x^2+a)^(1/2)-5/128*A*a^4
/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/10*B*x^3*(b*x^2+a)^(7/2)
/b-3/80*B*a/b^2*x*(b*x^2+a)^(7/2)+1/160*B*a^2/b^2*x*(b*x^2+a)^(5/
2)+1/128*B*a^3/b^2*x*(b*x^2+a)^(3/2)+3/256*B*a^4/b^2*x*(b*x^2+a)^
(1/2)+3/256*B*a^5/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.365279, size = 1, normalized size = 0.01

$$\left[\frac{2\left(384Bb^4x^9+48\left(21Bab^3+10Ab^4\right)x^7+8\left(93Ba^2b^2+170Aab^3\right)x^5+10\left(3Ba^3b+118Aa^2b^2\right)x^3-15\left(3Ba^4-10Aa^3b\right)\right.}{7680b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(2*(384*B*b^4*x^9 + 48*(21*B*a*b^3 + 10*A*b^4)*x^7 + 8*(9
3*B*a^2*b^2 + 170*A*a*b^3)*x^5 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*x
^3 - 15*(3*B*a^4 - 10*A*a^3*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 15*(3
*B*a^5 - 10*A*a^4*b)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sq
rt(b)))/b^(5/2), 1/3840*((384*B*b^4*x^9 + 48*(21*B*a*b^3 + 10*A*b
^4)*x^7 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*x^5 + 10*(3*B*a^3*b + 11
8*A*a^2*b^2)*x^3 - 15*(3*B*a^4 - 10*A*a^3*b)*x)*sqrt(b*x^2 + a)*s
qrt(-b) + 15*(3*B*a^5 - 10*A*a^4*b)*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 138.628, size = 348, normalized size = 1.85

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129Ba^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73Ba^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29B\sqrt{a}b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{Bb^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(
384*sqrt(1 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x
**2/a)) + 23*A*sqrt(a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a*
*4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + A*b**3*x**9/(8*sqrt(
a)*sqrt(1 + b*x**2/a)) - 3*B*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2
/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a)) + 129*B*a**(5/2
)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*sqrt(
1 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) +
 3*B*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/
(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.250301, size = 223, normalized size = 1.19

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bb^2x^2+\frac{21\,Bab^9+10\,Ab^{10}}{b^8}\right)x^2+\frac{93\,Ba^2b^8+170\,Aab^9}{b^8}\right)x^2+\frac{5\left(3\,Ba^3b^7+118\,Aa^2b^8\right)}{b^8}\right)x^2-\frac{15\left(3\,Ba^4b^6-\right.}{b^8}\right.$$
$$-\frac{\left(3\,Ba^5-10\,Aa^4b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*b^2*x^2 + (21*B*a*b^9 + 10*A*b^10)/b^8)*x^2
+ (93*B*a^2*b^8 + 170*A*a*b^9)/b^8)*x^2 + 5*(3*B*a^3*b^7 + 118*A*
a^2*b^8)/b^8)*x^2 - 15*(3*B*a^4*b^6 - 10*A*a^3*b^7)/b^8)*sqrt(b*x
^2 + a)*x - 1/256*(3*B*a^5 - 10*A*a^4*b)*ln(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(5/2)
```

3.542 $\int x\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (B*(a + b*x^2)^(9/2))/(
9*b^2)
```

Rubi [A] time = 0.100662, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (B*(a + b*x^2)^(9/2))/(
9*b^2)
```

Rubi in Sympy [A] time = 13.2136, size = 37, normalized size = 0.8

$$\frac{B\left(a+bx^2\right)^{\frac{9}{2}}}{9b^2}+\frac{\left(a+bx^2\right)^{\frac{7}{2}}(Ab-Ba)}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] B*(a + b*x**2)**(9/2)/(9*b**2) + (a + b*x**2)**(7/2)*(A*b - B*a)/
(7*b**2)
```

Mathematica [A] time = 0.0579492, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(-2aB+9Ab+7bBx^2\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x^2))/(63*b^2)
```

Maple [A] time = 0.006, size = 31, normalized size = 0.7

$$\frac{7\,bBx^2+9\,Ab-2\,Ba}{63\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/63*(b*x^2+a)^(7/2)*(7*B*b*x^2+9*A*b-2*B*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.214506, size = 131, normalized size = 2.85

$$\frac{\left(7\,Bb^4x^8+\left(19\,Bab^3+9\,Ab^4\right)x^6-2\,Ba^4+9\,Aa^3b+3\left(5\,Ba^2b^2+9\,Aab^3\right)x^4+\left(Ba^3b+27\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^4*x^8 + (19*B*a*b^3 + 9*A*b^4)*x^6 - 2*B*a^4 + 9*A*a^
3*b + 3*(5*B*a^2*b^2 + 9*A*a*b^3)*x^4 + (B*a^3*b + 27*A*a^2*b^2)*
x^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 11.8294, size = 209, normalized size = 4.54

$$\begin{cases} \frac{Aa^3\sqrt{a+bx^2}}{7b} + \frac{3Aa^2x^2\sqrt{a+bx^2}}{7} + \frac{3Aabx^4\sqrt{a+bx^2}}{7} + \frac{Ab^2x^6\sqrt{a+bx^2}}{7} - \frac{2Ba^4\sqrt{a+bx^2}}{63b^2} + \frac{Ba^3x^2\sqrt{a+bx^2}}{63b} + \frac{5Ba^2x^4\sqrt{a+bx^2}}{21} + \frac{19Babx^6\sqrt{a+bx^2}}{63} + B \\ a^{\frac{5}{2}}\left(\frac{Ax^2}{2} + \frac{Bx^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**3*sqrt(a + b*x**2)/(7*b) + 3*A*a**2*x**2*sqrt(a +
 b*x**2)/7 + 3*A*a*b*x**4*sqrt(a + b*x**2)/7 + A*b**2*x**6*sqrt(a
 + b*x**2)/7 - 2*B*a**4*sqrt(a + b*x**2)/(63*b**2) + B*a**3*x**2*
sqrt(a + b*x**2)/(63*b) + 5*B*a**2*x**4*sqrt(a + b*x**2)/21 + 19*
B*a*b*x**6*sqrt(a + b*x**2)/63 + B*b**2*x**8*sqrt(a + b*x**2)/9,
Ne(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**4/4), True))
```

GIAC/XCAS [A] time = 0.236887, size = 304, normalized size = 6.61

$$105\left(bx^2+a\right)^{\frac{3}{2}}Aa^2 + 42\left(3\left(bx^2+a\right)^{\frac{5}{2}} - 5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Aa + \frac{21\left(3\left(bx^2+a\right)^{\frac{5}{2}} - 5\left(bx^2+a\right)^{\frac{3}{2}}a\right)Ba^2}{b} + 3\left(15\left(bx^2+a\right)^{\frac{7}{2}} - 42\left(bx^2+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)*x,x, algorithm="giac")
```

```
[Out] 1/315*(105*(b*x^2 + a)^(3/2)*A*a^2 + 42*(3*(b*x^2 + a)^(5/2) - 5*
(b*x^2 + a)^(3/2)*a)*A*a + 21*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a
)^(3/2)*a)*B*a^2/b + 3*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/
```

```
2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*A + 6*(15*(b*x^2 + a)^(7/2) - 42
*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)*a^2)*B*a/b + (35*(b*x
^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/2)*a
^2 - 105*(b*x^2 + a)^(3/2)*a^3)*B/b)/b
```

3.543 $\int (a+bx^2)^{5/2} (A+Bx^2)\, dx$

Optimal. Leaf size=149

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8Ab-aB)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8Ab-aB)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8Ab-aB)}{192b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

```
[Out] (5*a^2*(8*A*b - a*B)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*A*b - a
*B)*x*(a + b*x^2)^(3/2))/(192*b) + ((8*A*b - a*B)*x*(a + b*x^2)^(
5/2))/(48*b) + (B*x*(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*A*b - a*
B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.137666, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8Ab-aB)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8Ab-aB)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8Ab-aB)}{192b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*A*b - a
*B)*x*(a + b*x^2)^(3/2))/(192*b) + ((8*A*b - a*B)*x*(a + b*x^2)^(
5/2))/(48*b) + (B*x*(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*A*b - a*
B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi in Sympy [A] time = 14.5118, size = 134, normalized size = 0.9

$$\frac{Bx\left(a+bx^2\right)^{\frac{7}{2}}}{8b}+\frac{5a^3\left(8Ab-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{3}{2}}}+\frac{5a^2x\sqrt{a+bx^2}\left(8Ab-Ba\right)}{128b}+\frac{5ax\left(a+bx^2\right)^{\frac{3}{2}}\left(8Ab-Ba\right)}{192b}+\frac{x\left(a+bx^2\right)^{\frac{5}{2}}\left(8Ab-Ba\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] B*x*(a + b*x**2)**(7/2)/(8*b) + 5*a**3*(8*A*b - B*a)*atanh(sqrt(b
)*x/sqrt(a + b*x**2))/(128*b**(3/2)) + 5*a**2*x*sqrt(a + b*x**2)*
(8*A*b - B*a)/(128*b) + 5*a*x*(a + b*x**2)**(3/2)*(8*A*b - B*a)/(
192*b) + x*(a + b*x**2)**(5/2)*(8*A*b - B*a)/(48*b)
```

Mathematica [A] time = 0.121125, size = 121, normalized size = 0.81

$$\sqrt{a+bx^2}\left(\frac{a^2x(5aB+88Ab)}{128b}+\frac{1}{48}bx^5(17aB+8Ab)+\frac{1}{192}ax^3(59aB+104Ab)+\frac{1}{8}b^2Bx^7\right)-\frac{5a^3(aB-8Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{128b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((a^2*(88*A*b + 5*a*B)*x)/(128*b) + (a*(104*A*b +
 59*a*B)*x^3)/192 + (b*(8*A*b + 17*a*B)*x^5)/48 + (b^2*B*x^7)/8)
- (5*a^3*(-8*A*b + a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(128*
b^(3/2))
```

Maple [A] time = 0.008, size = 166, normalized size = 1.1

$$\frac{Ax}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,aAx}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2Ax}{16}\sqrt{bx^2+a}+\frac{5\,Aa^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{Bx}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{Bxa}{48\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,Bxa^2}{192\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,Ba^3x}{128\,b}\sqrt{bx^2+a}-\frac{5\,Ba^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/6*A*x*(b*x^2+a)^(5/2)+5/24*A*a*x*(b*x^2+a)^(3/2)+5/16*A*a^2*x*(
b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1
/8*B*x*(b*x^2+a)^(7/2)/b-1/48*B*a/b*x*(b*x^2+a)^(5/2)-5/192*B*a^2
/b*x*(b*x^2+a)^(3/2)-5/128*B*a^3/b*x*(b*x^2+a)^(1/2)-5/128*B*a^4/
b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.308742, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,Bb^3x^7+8\left(17\,Bab^2+8\,Ab^3\right)x^5+2\left(59\,Ba^2b+104\,Aab^2\right)x^3+3\left(5\,Ba^3+88\,Aa^2b\right)x\right)\sqrt{bx^2+a}\sqrt{b}-15\left(Ba^4-8\,Aa^3b\right)}{768\,b^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*B*b^3*x^7 + 8*(17*B*a*b^2 + 8*A*b^3)*x^5 + 2*(59*B*
a^2*b + 104*A*a*b^2)*x^3 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^2
 + a)*sqrt(b) - 15*(B*a^4 - 8*A*a^3*b)*log(-2*sqrt(b*x^2 + a)*b*x
 - (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/384*((48*B*b^3*x^7 + 8*(17*
B*a*b^2 + 8*A*b^3)*x^5 + 2*(59*B*a^2*b + 104*A*a*b^2)*x^3 + 3*(5*
B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) - 15*(B*a^4 - 8*A
*a^3*b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b)]
```

Sympy [A] time = 87.4629, size = 316, normalized size = 2.12

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Ba^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Ba^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23B\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Bb^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b
*x**2/a)) + 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*s
qrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)
*x/sqrt(a))/(16*sqrt(b)) + A*b**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2
/a)) + 5*B*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*B*a**(5/2)
*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**(3/2)*b*x**5/(192*sqrt(
1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) -
 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**9/(
8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.249302, size = 181, normalized size = 1.21

$$\frac{1}{384}\left(2\left(4\left(6\,Bb^2x^2+\frac{17\,Bab^7+8\,Ab^8}{b^6}\right)x^2+\frac{59\,Ba^2b^6+104\,Aab^7}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^5+88\,Aa^2b^6\right)}{b^6}\right)\sqrt{bx^2+a}x+\frac{5\left(Ba^4-8\,Aa^3b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b^2*x^2 + (17*B*a*b^7 + 8*A*b^8)/b^6)*x^2 + (59*
B*a^2*b^6 + 104*A*a*b^7)/b^6)*x^2 + 3*(5*B*a^3*b^5 + 88*A*a^2*b^6
)/b^6)*sqrt(b*x^2 + a)*x + 5/128*(B*a^4 - 8*A*a^3*b)*ln(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.544 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=95

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+a^2A\sqrt{a+bx^2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

```
[Out] a^2*A*Sqrt[a + b*x^2] + (a*A*(a + b*x^2)^(3/2))/3 + (A*(a + b*x^2
)^(5/2))/5 + (B*(a + b*x^2)^(7/2))/(7*b) - a^(5/2)*A*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]]
```

Rubi [A] time = 0.190377, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+a^2A\sqrt{a+bx^2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] a^2*A*Sqrt[a + b*x^2] + (a*A*(a + b*x^2)^(3/2))/3 + (A*(a + b*x^2
)^(5/2))/5 + (B*(a + b*x^2)^(7/2))/(7*b) - a^(5/2)*A*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]]
```

Rubi in Sympy [A] time = 19.0256, size = 82, normalized size = 0.86

$$-Aa^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+Aa^2\sqrt{a+bx^2}+\frac{Aa\left(a+bx^2\right)^{\frac{3}{2}}}{3}+\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{5}+\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x,x)
```

```
[Out] -A*a**(5/2)*atanh(sqrt(a + b*x**2)/sqrt(a)) + A*a**2*sqrt(a + b*x
**2) + A*a*(a + b*x**2)**(3/2)/3 + A*(a + b*x**2)**(5/2)/5 + B*(a
 + b*x**2)**(7/2)/(7*b)
```

Mathematica [A] time = 0.231555, size = 115, normalized size = 1.21

$$-a^{5/2}A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+a^{5/2}A\log(x)+\sqrt{a+bx^2}\left(\frac{a^2(15aB+161Ab)}{105b}+\frac{1}{35}bx^4(15aB+7Ab)+\frac{1}{105}ax^2(45aB+77Ab)+\frac{1}{7}b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] Sqrt[a + b*x^2]*((a^2*(161*A*b + 15*a*B))/(105*b) + (a*(77*A*b +
45*a*B)*x^2)/105 + (b*(7*A*b + 15*a*B)*x^4)/35 + (b^2*B*x^6)/7) +
 a^(5/2)*A*Log[x] - a^(5/2)*A*Log[a + Sqrt[a]*Sqrt[a + b*x^2]]
```

Maple [A] time = 0.011, size = 85, normalized size = 0.9

$$\frac{A}{5}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{Aa}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}-Aa^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^{2}+a}\right)\right)+a^{2}A\sqrt{bx^{2}+a}+\frac{B}{7\,b}\left(bx^{2}+a\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x,x)
```

```
[Out] 1/5*A*(b*x^2+a)^(5/2)+1/3*a*A*(b*x^2+a)^(3/2)-A*a^(5/2)*ln((2*a+2
*a^(1/2)*(b*x^2+a)^(1/2))/x)+a^2*A*(b*x^2+a)^(1/2)+1/7*B*(b*x^2+a
)^(7/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239964, size = 1, normalized size = 0.01

$$\left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(15\,Bb^{3}x^{6}+3\,\left(15\,Bab^{2}+7\,Ab^{3}\right)x^{4}+15\,Ba^{3}+161\,Aa^{2}b+\left(45\,Ba^{2}b+77\,Aab^{2}\right)x^{2}\right)}{210\,b}\right.$$

$$-\frac{105\,A\sqrt{-a}a^{2}b\arctan\left(\frac{a}{\sqrt{bx^{2}+a}\sqrt{-a}}\right)-\left(15\,Bb^{3}x^{6}+3\,\left(15\,Bab^{2}+7\,Ab^{3}\right)x^{4}+15\,Ba^{3}+161\,Aa^{2}b+\left(45\,Ba^{2}b+77\,Aab^{2}\right)x^{2}\right)\sqrt{b}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*(15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*A*b^3)*x^4 + 15*
B*a^3 + 161*A*a^2*b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b*x^2 +
 a))/b, -1/105*(105*A*sqrt(-a)*a^2*b*arctan(a/(sqrt(b*x^2 + a)*sq
rt(-a))) - (15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*A*b^3)*x^4 + 15*B*a^
3 + 161*A*a^2*b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b*x^2 + a))
/b]
```

Sympy [A] time = 33.3996, size = 151, normalized size = 1.59

$$-Aa^{3}\left(\begin{cases}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\: -a>0\\ \frac{\operatorname{acoth}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\: -a<0\wedge a<a+bx^{2}\\ \frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\: a>a+bx^{2}\wedge -a<0\end{cases}\right)+Aa^{2}\sqrt{a+bx^{2}}+\frac{Aa\left(a+bx^{2}\right)^{\frac{3}{2}}}{3}+\frac{A\left(a+bx^{2}\right)^{\frac{5}{2}}}{5}+\frac{B\left(a+bx^{2}\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x,x)
```

```
[Out] -A*a**3*Piecewise((-atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a), -a
> 0), (acoth(sqrt(a + b*x**2)/sqrt(a))/sqrt(a), (-a < 0) & (a < a
 + b*x**2)), (atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a), (-a < 0) &
 (a > a + b*x**2))) + A*a**2*sqrt(a + b*x**2) + A*a*(a + b*x**2)*
*(3/2)/3 + A*(a + b*x**2)**(5/2)/5 + B*(a + b*x**2)**(7/2)/(7*b)
```

GIAC/XCAS [A] time = 0.224855, size = 131, normalized size = 1.38

$$\frac{Aa^3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{15\left(bx^2+a\right)^{\frac{7}{2}}Bb^6 + 21\left(bx^2+a\right)^{\frac{5}{2}}Ab^7 + 35\left(bx^2+a\right)^{\frac{3}{2}}Aab^7 + 105\sqrt{bx^2+a}Aa^2b^7}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] A*a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/105*(15*(b*x^
2 + a)^(7/2)*B*b^6 + 21*(b*x^2 + a)^(5/2)*A*b^7 + 35*(b*x^2 + a)^
(3/2)*A*a*b^7 + 105*sqrt(b*x^2 + a)*A*a^2*b^7)/b^7
```

3.545 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=136

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}(aB+6Ab)-\frac{A\left(a+bx^2\right)^{7/2}}{ax}$$

```
[Out] (5*a*(6*A*b + a*B)*x*Sqrt[a + b*x^2])/16 + (5*(6*A*b + a*B)*x*(a
+ b*x^2)^(3/2))/24 + ((6*A*b + a*B)*x*(a + b*x^2)^(5/2))/(6*a) -
(A*(a + b*x^2)^(7/2))/(a*x) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] time = 0.154543, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}(aB+6Ab)-\frac{A\left(a+bx^2\right)^{7/2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (5*a*(6*A*b + a*B)*x*Sqrt[a + b*x^2])/16 + (5*(6*A*b + a*B)*x*(a
+ b*x^2)^(3/2))/24 + ((6*A*b + a*B)*x*(a + b*x^2)^(5/2))/(6*a) -
(A*(a + b*x^2)^(7/2))/(a*x) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi in Sympy [A] time = 14.9891, size = 128, normalized size = 0.94

$$-\frac{A\left(a+bx^2\right)^{\frac{7}{2}}}{ax}+\frac{5a^2\left(6Ab+Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5ax\sqrt{a+bx^2}\left(6Ab+Ba\right)}{16}+x\left(a+bx^2\right)^{\frac{3}{2}}\left(\frac{5Ab}{4}+\frac{5Ba}{24}\right)+\frac{x\left(a+bx^2\right)^{\frac{5}{2}}\left(6Ab+Ba\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(a*x) + 5*a**2*(6*A*b + B*a)*atanh(sqrt(b)
*x/sqrt(a + b*x**2))/(16*sqrt(b)) + 5*a*x*sqrt(a + b*x**2)*(6*A*b
 + B*a)/16 + x*(a + b*x**2)**(3/2)*(5*A*b/4 + 5*B*a/24) + x*(a +
b*x**2)**(5/2)*(6*A*b + B*a)/(6*a)
```

Mathematica [A] time = 0.177886, size = 108, normalized size = 0.79

$$\sqrt{a+bx^2}\left(-\frac{a^2A}{x}+\frac{1}{24}bx^3(13aB+6Ab)+\frac{1}{16}ax(11aB+18Ab)+\frac{1}{6}b^2Bx^5\right)+\frac{5a^2(aB+6Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] Sqrt[a + b*x^2]*(-((a^2*A)/x) + (a*(18*A*b + 11*a*B)*x)/16 + (b*(
6*A*b + 13*a*B)*x^3)/24 + (b^2*B*x^5)/6) + (5*a^2*(6*A*b + a*B)*L
og[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Maple [A] time = 0.012, size = 158, normalized size = 1.2

$$\frac{Bx}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,Bxa}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,Bxa^2}{16}\sqrt{bx^2+a}+\frac{5\,Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-\frac{A}{ax}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Axb}{a}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,Axb}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{15\,Axab}{8}\sqrt{bx^2+a}+\frac{15\,Aa^2}{8}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/6*x*B*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(
b*x^2+a)^(1/2)+5/16*B*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-A
*(b*x^2+a)^(7/2)/a/x+A*b/a*x*(b*x^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a)^
(3/2)+15/8*A*b*a*x*(b*x^2+a)^(1/2)+15/8*A*b^(1/2)*a^2*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264099, size = 1, normalized size = 0.01

$$\left[\frac{15\left(Ba^3+6\,Aa^2b\right)x\log\left(-2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(8\,Bb^2x^6+2\left(13\,Bab+6\,Ab^2\right)x^4-48\,Aa^2+3\left(11\,Ba^2+18\,\ldots\right.\right.}{96\sqrt{b}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(B*a^3 + 6*A*a^2*b)*x*log(-2*sqrt(b*x^2 + a)*b*x - (2*b
*x^2 + a)*sqrt(b)) + 2*(8*B*b^2*x^6 + 2*(13*B*a*b + 6*A*b^2)*x^4
- 48*A*a^2 + 3*(11*B*a^2 + 18*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(b)
)/(sqrt(b)*x), 1/48*(15*(B*a^3 + 6*A*a^2*b)*x*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (8*B*b^2*x^6 + 2*(13*B*a*b + 6*A*b^2)*x^4 - 48*
A*a^2 + 3*(11*B*a^2 + 18*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-b))/(s
qrt(-b)*x)]
```

Sympy [A] time = 57.1551, size = 306, normalized size = 2.25

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{Ab^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Ba^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17B\sqrt{a}b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Bb^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x*
*2/a) - 7*A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**
2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x
/sqrt(a))/8 + A*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**(
5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*B*a**(5/2)*x/(16*sqrt(1 + b*x**2/
a)) + 35*B*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*B*sqrt(a)
*b**2*x**5/(24*sqrt(1 + b*x**2/a)) + 5*B*a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.250509, size = 197, normalized size = 1.45

$$\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{48}\left(2\left(4\,Bb^2x^2+\frac{13\,Bab^5+6\,Ab^6}{b^4}\right)x^2+\frac{3\left(11\,Ba^2b^4+18\,Aab^5\right)}{b^4}\right)\sqrt{bx^2+a}x-\frac{5\left(Ba^3\sqrt{b}+6\,Aa^2b^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^3*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/48*(2*(
4*B*b^2*x^2 + (13*B*a*b^5 + 6*A*b^6)/b^4)*x^2 + 3*(11*B*a^2*b^4 +
 18*A*a*b^5)/b^4)*sqrt(b*x^2 + a)*x - 5/32*(B*a^3*sqrt(b) + 6*A*a
^2*b^(3/2))*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

3.546 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}(2aB+5Ab)-\frac{A(a+bx^2)^{7/2}}{2ax^2}$$

```
[Out] (a*(5*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((5*A*b + 2*a*B)*(a + b*x
^2)^(3/2))/6 + ((5*A*b + 2*a*B)*(a + b*x^2)^(5/2))/(10*a) - (A*(a
 + b*x^2)^(7/2))/(2*a*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] time = 0.267193, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}(2aB+5Ab)-\frac{A(a+bx^2)^{7/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (a*(5*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((5*A*b + 2*a*B)*(a + b*x
^2)^(3/2))/6 + ((5*A*b + 2*a*B)*(a + b*x^2)^(5/2))/(10*a) - (A*(a
 + b*x^2)^(7/2))/(2*a*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/2
```

Rubi in Sympy [A] time = 22.277, size = 119, normalized size = 0.88

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{2ax^2}-a^{\frac{3}{2}}\left(\frac{5Ab}{2}+Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{a\sqrt{a+bx^2}(5Ab+2Ba)}{2}+(a+bx^2)^{\frac{3}{2}}\left(\frac{5Ab}{6}+\frac{Ba}{3}\right)+\frac{(a+bx^2)^{\frac{5}{2}}\left(\frac{5Ab}{2}+Ba\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(2*a*x**2) - a**(3/2)*(5*A*b/2 + B*a)*atan
h(sqrt(a + b*x**2)/sqrt(a)) + a*sqrt(a + b*x**2)*(5*A*b + 2*B*a)/
2 + (a + b*x**2)**(3/2)*(5*A*b/6 + B*a/3) + (a + b*x**2)**(5/2)*(
5*A*b/2 + B*a)/(5*a)
```

Mathematica [A] time = 0.272987, size = 117, normalized size = 0.87

$$\frac{1}{30}\left(-15a^{3/2}(2aB+5Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+15a^{3/2}\log(x)(2aB+5Ab)+\sqrt{a+bx^2}\left(-\frac{15a^2A}{x^2}+2bx^2(11aB+5Ab)+2a(23aB+35Ab)+6b^2Bx^4\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*a*(35*A*b + 23*a*B) - (15*a^2*A)/x^2 + 2*b*(5
*A*b + 11*a*B)*x^2 + 6*b^2*B*x^4) + 15*a^(3/2)*(5*A*b + 2*a*B)*Lo
g[x] - 15*a^(3/2)*(5*A*b + 2*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]
])/30
```

Maple [A] time = 0.012, size = 161, normalized size = 1.2

$$-\frac{A}{2\,a x^2}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{A b}{2\,a}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{5\,A b}{6}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{5\,A b}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)+\frac{5\,abA}{2}\sqrt{b x^2+a}+\frac{B}{5}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{Ba}{3}\left(b x^2+a\right)^{\frac{3}{2}}-Ba^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)+B\sqrt{b x^2+a}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(7/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(5/2)+5/6*A*b*(b
*x^2+a)^(3/2)-5/2*A*b*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)+5/2*A*b*a*(b*x^2+a)^(1/2)+1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2
+a)^(3/2)-B*a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x^
2+a)^(1/2)*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257479, size = 1, normalized size = 0.01

$$\left[\frac{15\left(2\,Ba^2+5\,Aab\right)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^6+2\left(11\,Bab+5\,Ab^2\right)x^4-15\,Aa^2+2\left(23\,Ba^2+35\,Aab\right)x^2\right)\ldots}{60\,x^2}, -\frac{15\left(2\,Ba^2+5\,Aab\right)\sqrt{-a}x^2\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-\left(6\,Bb^2x^6+2\left(11\,Bab+5\,Ab^2\right)x^4-15\,Aa^2+2\left(23\,Ba^2+35\,Aab\right)x^2\right)\sqrt{bx\ldots}}{30\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(2*B*a^2 + 5*A*a*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B*b^2*x^6 + 2*(11*B*a*b + 5*A
*b^2)*x^4 - 15*A*a^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2 +
a))/x^2, -1/30*(15*(2*B*a^2 + 5*A*a*b)*sqrt(-a)*x^2*arctan(a/(sqr
t(b*x^2 + a)*sqrt(-a))) - (6*B*b^2*x^6 + 2*(11*B*a*b + 5*A*b^2)*x
^4 - 15*A*a^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^2
]
```

Sympy [A] time = 57.2278, size = 296, normalized size = 2.19

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+2Bab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqr
t(a/(b*x**2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1
)) + 2*A*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sq
rt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) - B*a
**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3/(sqrt(b)*x*sqrt(a/(b*
x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*B*a*b*Pie
cewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), Tr
ue)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x
**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)),
 (sqrt(a)*x**4/4, True))
```

GIAC/XCAS [A] time = 0.243336, size = 188, normalized size = 1.39

$$\frac{6\left(bx^2+a\right)^{\frac{5}{2}}Bb+10\left(bx^2+a\right)^{\frac{3}{2}}Bab+30\sqrt{bx^2+a}Ba^2b+10\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+60\sqrt{bx^2+a}Aab^2-\frac{15\sqrt{bx^2+a}Aa^2b}{x^2}+\frac{15\left(2Ba^3b+\right.}{}}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(b*x^2 + a)^(5/2)*B*b + 10*(b*x^2 + a)^(3/2)*B*a*b + 30*s
qrt(b*x^2 + a)*B*a^2*b + 10*(b*x^2 + a)^(3/2)*A*b^2 + 60*sqrt(b*x
^2 + a)*A*a*b^2 - 15*sqrt(b*x^2 + a)*A*a^2*b/x^2 + 15*(2*B*a^3*b
+ 5*A*a^2*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

3.547 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=146

$$-\frac{(a+bx^2)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx(a+bx^2)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}(3aB+4Ab)+\frac{5}{8}a\sqrt{b}(3aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{7/2}}{3ax^3}$$

```
[Out] (5*b*(4*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/8 + (5*b*(4*A*b + 3*a*B)*
x*(a + b*x^2)^(3/2))/(12*a) - ((4*A*b + 3*a*B)*(a + b*x^2)^(5/2))
/(3*a*x) - (A*(a + b*x^2)^(7/2))/(3*a*x^3) + (5*a*Sqrt[b]*(4*A*b
+ 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rubi [A] time = 0.168715, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{(a+bx^2)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx(a+bx^2)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}(3aB+4Ab)+\frac{5}{8}a\sqrt{b}(3aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{7/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (5*b*(4*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/8 + (5*b*(4*A*b + 3*a*B)*
x*(a + b*x^2)^(3/2))/(12*a) - ((4*A*b + 3*a*B)*(a + b*x^2)^(5/2))
/(3*a*x) - (A*(a + b*x^2)^(7/2))/(3*a*x^3) + (5*a*Sqrt[b]*(4*A*b
+ 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rubi in Sympy [A] time = 16.1999, size = 138, normalized size = 0.95

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{3ax^3}+\frac{5a\sqrt{b}(4Ab+3Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8}+\frac{5bx\sqrt{a+bx^2}(4Ab+3Ba)}{8}+\frac{5bx(a+bx^2)^{\frac{3}{2}}(4Ab+3Ba)}{12a}-\frac{(a+bx^2)^{\frac{5}{2}}(4Ab+3Ba)}{3ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(3*a*x**3) + 5*a*sqrt(b)*(4*A*b + 3*B*a)*a
tanh(sqrt(b)*x/sqrt(a + b*x**2))/8 + 5*b*x*sqrt(a + b*x**2)*(4*A*
b + 3*B*a)/8 + 5*b*x*(a + b*x**2)**(3/2)*(4*A*b + 3*B*a)/(12*a) -
 (a + b*x**2)**(5/2)*(4*A*b + 3*B*a)/(3*a*x)
```

Mathematica [A] time = 0.176564, size = 106, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(-8a^2A+3bx^4(9aB+4Ab)-8ax^2(3aB+7Ab)+6b^2Bx^6\right)}{24x^3}+\frac{5}{8}a\sqrt{b}(3aB+4Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 8*a*(7*A*b + 3*a*B)*x^2 + 3*b*(4*A*b
 + 9*a*B)*x^4 + 6*b^2*B*x^6))/(24*x^3) + (5*a*Sqrt[b]*(4*A*b + 3*
a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/8
```

Maple [A] time = 0.013, size = 204, normalized size = 1.4

$$-\frac{A}{3\,a x^3}\left(b x^2+a\right)^{\frac{7}{2}}-\frac{4\,A b}{3\,a^2 x}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{4\,A x b^2}{3\,a^2}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{5\,A x b^2}{3\,a}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,A x b^2}{2}\sqrt{b x^2+a}+\frac{5\,A a}{2}b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)-\frac{B}{a x}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{b B x}{a}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{5\,b B x}{4}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{15\,B x a b}{8}\sqrt{b x^2+a}+\frac{15\,a^2 B}{8}\sqrt{b}\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x)
```

```
[Out] -1/3*A*(b*x^2+a)^(7/2)/a/x^3-4/3*A*b/a^2/x*(b*x^2+a)^(7/2)+4/3*A*
b^2/a^2*x*(b*x^2+a)^(5/2)+5/3*A*b^2/a*x*(b*x^2+a)^(3/2)+5/2*A*b^2
*x*(b*x^2+a)^(1/2)+5/2*A*b^(3/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-
B/a/x*(b*x^2+a)^(7/2)+B*b/a*x*(b*x^2+a)^(5/2)+5/4*B*b*x*(b*x^2+a)
^(3/2)+15/8*B*b*a*x*(b*x^2+a)^(1/2)+15/8*B*b^(1/2)*a^2*ln(x*b^(1/
2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250702, size = 1, normalized size = 0.01

$$\left[\frac{15\left(3\,B a^2+4\,A a b\right)\sqrt{b}x^3\log\left(-2\,b x^2-2\,\sqrt{b x^2+a}\sqrt{b}x-a\right)+2\left(6\,B b^2 x^6+3\left(9\,B a b+4\,A b^2\right)x^4-8\,A a^2-8\left(3\,B a^2+7\,A a b\right.\right.}{48\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(3*B*a^2 + 4*A*a*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^6 + 3*(9*B*a*b + 4*A*b^2)
*x^4 - 8*A*a^2 - 8*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^3,
 1/24*(15*(3*B*a^2 + 4*A*a*b)*sqrt(-b)*x^3*arctan(b*x/(sqrt(b*x^2
 + a)*sqrt(-b))) + (6*B*b^2*x^6 + 3*(9*B*a*b + 4*A*b^2)*x^4 - 8*A
*a^2 - 8*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] time = 34.9128, size = 299, normalized size = 2.05

$$-\frac{2Aa^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}b^2x\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{2A\sqrt{a}b^2x}{\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+\frac{5Aab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Ba^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Ba^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Ba^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{Bb^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -2*A*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b**2*x*sqrt(1
+ b*x**2/a)/2 - 2*A*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - A*a**2*sq
rt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*a*b**(3/2)*sqrt(a/(b*x**2
) + 1)/3 + 5*A*a*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/2 - B*a**(5/2)
/(x*sqrt(1 + b*x**2/a)) + B*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*B
*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b**2*x**3/(8*s
qrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8
 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.249823, size = 321, normalized size = 2.2

$$\frac{1}{8}\left(2\,Bb^2x^2+\frac{9\,Bab^3+4\,Ab^4}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{5}{16}\left(3\,Ba^2\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^3\sqrt{b}+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^4\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^3}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*b^2*x^2 + (9*B*a*b^3 + 4*A*b^4)/b^2)*sqrt(b*x^2 + a)*x -
 5/16*(3*B*a^2*sqrt(b) + 4*A*a*b^(3/2))*ln((sqrt(b)*x - sqrt(b*x^
2 + a))^2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b)
 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/2) - 6*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*B*a^4*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*A*a^3*b^(3/2) + 3*B*a^5*sqrt(b) + 7*A*a^4*b^(3/2))/((sqrt
(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.548 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=143

$$-\frac{(a+bx^2)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b(a+bx^2)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}(4aB+3Ab)-\frac{5}{8}\sqrt{a}b(4aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A(a+bx^2)^{7/2}}{4ax^4}$$

```
[Out] (5*b*(3*A*b + 4*a*B)*Sqrt[a + b*x^2])/8 + (5*b*(3*A*b + 4*a*B)*(a
 + b*x^2)^(3/2))/(24*a) - ((3*A*b + 4*a*B)*(a + b*x^2)^(5/2))/(8*
a*x^2) - (A*(a + b*x^2)^(7/2))/(4*a*x^4) - (5*Sqrt[a]*b*(3*A*b +
4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rubi [A] time = 0.276589, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{(a+bx^2)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b(a+bx^2)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}(4aB+3Ab)-\frac{5}{8}\sqrt{a}b(4aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A(a+bx^2)^{7/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (5*b*(3*A*b + 4*a*B)*Sqrt[a + b*x^2])/8 + (5*b*(3*A*b + 4*a*B)*(a
 + b*x^2)^(3/2))/(24*a) - ((3*A*b + 4*a*B)*(a + b*x^2)^(5/2))/(8*
a*x^2) - (A*(a + b*x^2)^(7/2))/(4*a*x^4) - (5*Sqrt[a]*b*(3*A*b +
4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rubi in Sympy [A] time = 22.6851, size = 134, normalized size = 0.94

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{4ax^4}-\frac{5\sqrt{a}b(3Ab+4Ba)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8}+\frac{5b\sqrt{a+bx^2}(3Ab+4Ba)}{8}+\frac{5b(a+bx^2)^{\frac{3}{2}}(3Ab+4Ba)}{24a}-\frac{(a+bx^2)^{\frac{5}{2}}(3Ab+4Ba)}{8ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(4*a*x**4) - 5*sqrt(a)*b*(3*A*b + 4*B*a)*a
tanh(sqrt(a + b*x**2)/sqrt(a))/8 + 5*b*sqrt(a + b*x**2)*(3*A*b +
4*B*a)/8 + 5*b*(a + b*x**2)**(3/2)*(3*A*b + 4*B*a)/(24*a) - (a +
b*x**2)**(5/2)*(3*A*b + 4*B*a)/(8*a*x**2)
```

Mathematica [A] time = 0.326401, size = 119, normalized size = 0.83

$$\frac{1}{24}\left(\sqrt{a+bx^2}\left(-\frac{6a^2A}{x^4}-\frac{3a(4aB+9Ab)}{x^2}+8b(7aB+3Ab)+8b^2Bx^2\right)-15\sqrt{a}b(4aB+3Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+15\sqrt{a}b\log(x)(4aB+3Ab)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b*(3*A*b + 7*a*B) - (6*a^2*A)/x^4 - (3*a*(9*A
*b + 4*a*B))/x^2 + 8*b^2*B*x^2) + 15*Sqrt[a]*b*(3*A*b + 4*a*B)*Lo
g[x] - 15*Sqrt[a]*b*(3*A*b + 4*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^
2]])/24
```

Maple [A] time = 0.012, size = 213, normalized size = 1.5

$$-\frac{A}{4\,ax^4}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3\,Ab}{8\,a^2x^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{3\,b^2A}{8\,a^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,b^2A}{8\,a}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{15\,b^2A}{8}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+\frac{15\,b^2A}{8}\sqrt{bx^2+a}-\frac{B}{2\,ax^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{Bb}{2\,a}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,Bb}{6}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,Bb}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)+\frac{5\,abB}{2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(7/2)/a/x^4-3/8*A*b/a^2/x^2*(b*x^2+a)^(7/2)+3/8*
A*b^2/a^2*(b*x^2+a)^(5/2)+5/8*A*b^2/a*(b*x^2+a)^(3/2)-15/8*A*b^2*
a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+15/8*A*b^2*(b*x^2+a
)^(1/2)-1/2*B/a/x^2*(b*x^2+a)^(7/2)+1/2*B*b/a*(b*x^2+a)^(5/2)+5/6
*B*b*(b*x^2+a)^(3/2)-5/2*B*b*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)+5/2*B*b*a*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24765, size = 1, normalized size = 0.01

$$\left[\frac{15\left(4\,Bab+3\,Ab^2\right)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(8\,Bb^2x^6+8\left(7\,Bab+3\,Ab^2\right)x^4-6\,Aa^2-3\left(4\,Ba^2+9\,Aab\right)x^2\right)\sqrt{bx}}{48\,x^4}\right.$$

$$-\frac{15\left(4\,Bab+3\,Ab^2\right)\sqrt{-a}x^4\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right)-\left(8\,Bb^2x^6+8\left(7\,Bab+3\,Ab^2\right)x^4-6\,Aa^2-3\left(4\,Ba^2+9\,Aab\right)x^2\right)\sqrt{bx^2+a}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(4*B*a*b + 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*B*b^2*x^6 + 8*(7*B*a*b + 3*A*
b^2)*x^4 - 6*A*a^2 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/
x^4, -1/24*(15*(4*B*a*b + 3*A*b^2)*sqrt(-a)*x^4*arctan(a/(sqrt(b*
x^2 + a)*sqrt(-a))) - (8*B*b^2*x^6 + 8*(7*B*a*b + 3*A*b^2)*x^4 -
6*A*a^2 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A] time = 105.41, size = 279, normalized size = 1.95

$$-\frac{15A\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8}-\frac{Aa^3}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3Aa^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{x}+\frac{7Aab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ba^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Ba^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Bab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Bb^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -15*A*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - A*a**3/(4*sqrt(
b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a**2*sqrt(b)/(8*x**3*sqrt(a/(
b*x**2) + 1)) - A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/x + 7*A*a*b**(3
/2)/(8*x*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)*x/sqrt(a/(b*x**2) + 1
) - 5*B*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - B*a**2*sqrt(b)*
sqrt(a/(b*x**2) + 1)/(2*x) + 2*B*a**2*sqrt(b)/(x*sqrt(a/(b*x**2)
+ 1)) + 2*B*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + B*b**2*Piecewise(
(sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

GIAC/XCAS [A] time = 0.238546, size = 231, normalized size = 1.62

$$\frac{8\left(bx^2+a\right)^{\frac{3}{2}}Bb^2+48\sqrt{bx^2+a}Bab^2+24\sqrt{bx^2+a}Ab^3+\frac{15\left(4Ba^2b^2+3Aab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\left(4\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^2-4\sqrt{bx^2+a}Ba^3b^2+9\left(\right.\right.}{b^2x^4}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(b*x^2 + a)^(3/2)*B*b^2 + 48*sqrt(b*x^2 + a)*B*a*b^2 + 24
*sqrt(b*x^2 + a)*A*b^3 + 15*(4*B*a^2*b^2 + 3*A*a*b^3)*arctan(sqrt
(b*x^2 + a)/sqrt(-a))/sqrt(-a) - 3*(4*(b*x^2 + a)^(3/2)*B*a^2*b^2
 - 4*sqrt(b*x^2 + a)*B*a^3*b^2 + 9*(b*x^2 + a)^(3/2)*A*a*b^3 - 7*
sqrt(b*x^2 + a)*A*a^2*b^3)/(b^2*x^4))/b
```

3.549 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=152

$$\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{b^2x\sqrt{a+bx^2}(5aB+2Ab)}{2a}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2Ab)}{3ax}-\frac{\left(a+bx^2\right)^{5/2}(5aB+2Ab)}{15ax^3}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}$$

```
[Out] (b^2*(2*A*b + 5*a*B)*x*Sqrt[a + b*x^2])/(2*a) - (b*(2*A*b + 5*a*B
)*(a + b*x^2)^(3/2))/(3*a*x) - ((2*A*b + 5*a*B)*(a + b*x^2)^(5/2)
)/(15*a*x^3) - (A*(a + b*x^2)^(7/2))/(5*a*x^5) + (b^(3/2)*(2*A*b
+ 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi [A] time = 0.180825, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{b^2x\sqrt{a+bx^2}(5aB+2Ab)}{2a}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2Ab)}{3ax}-\frac{\left(a+bx^2\right)^{5/2}(5aB+2Ab)}{15ax^3}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (b^2*(2*A*b + 5*a*B)*x*Sqrt[a + b*x^2])/(2*a) - (b*(2*A*b + 5*a*B
)*(a + b*x^2)^(3/2))/(3*a*x) - ((2*A*b + 5*a*B)*(a + b*x^2)^(5/2)
)/(15*a*x^3) - (A*(a + b*x^2)^(7/2))/(5*a*x^5) + (b^(3/2)*(2*A*b
+ 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rubi in Sympy [A] time = 18.2327, size = 136, normalized size = 0.89

$$-\frac{A\left(a+bx^2\right)^{\frac{7}{2}}}{5ax^5}+\frac{b^{\frac{3}{2}}(2Ab+5Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2}+\frac{b^2x\sqrt{a+bx^2}\,(2Ab+5Ba)}{2a}-\frac{b\left(a+bx^2\right)^{\frac{3}{2}}(2Ab+5Ba)}{3ax}-\frac{\left(a+bx^2\right)^{\frac{5}{2}}(2Ab+5Ba)}{15ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(5*a*x**5) + b**(3/2)*(2*A*b + 5*B*a)*atan
h(sqrt(b)*x/sqrt(a + b*x**2))/2 + b**2*x*sqrt(a + b*x**2)*(2*A*b
+ 5*B*a)/(2*a) - b*(a + b*x**2)**(3/2)*(2*A*b + 5*B*a)/(3*a*x) -
(a + b*x**2)**(5/2)*(2*A*b + 5*B*a)/(15*a*x**3)
```

Mathematica [A] time = 0.169395, size = 105, normalized size = 0.69

$$\frac{1}{2}b^{3/2}(5aB+2Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)-\frac{\sqrt{a+bx^2}\left(6a^2A+2bx^4(35aB+23Ab)+2ax^2(5aB+11Ab)-15b^2Bx^6\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(Sqrt[a + b*x^2]*(6*a^2*A + 2*a*(11*A*b + 5*a*B)*x^2 + 2*b*(23*A
*b + 35*a*B)*x^4 - 15*b^2*B*x^6))/(30*x^5) + (b^(3/2)*(2*A*b + 5*
a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/2
```

Maple [A] time = 0.013, size = 251, normalized size = 1.7

$$-\frac{A}{5\,a x^{5}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{2\,A b}{15\,a^{2} x^{3}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{8\,b^{2} A}{15\,a^{3} x}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{8\,A b^{3} x}{15\,a^{3}}\left(b x^{2}+a\right)^{\frac{5}{2}} +\frac{2\,A b^{3} x}{3\,a^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{A b^{3} x}{a}\sqrt{b x^{2}+a}+A b^{\frac{5}{2}}\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right) -\frac{B}{3\,a x^{3}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{4\,B b}{3\,a^{2} x}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{4\,b^{2} B x}{3\,a^{2}}\left(b x^{2}+a\right)^{\frac{5}{2}} +\frac{5\,b^{2} B x}{3\,a}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{5\,b^{2} B x}{2}\sqrt{b x^{2}+a}+\frac{5\,B a}{2} b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*A*(b*x^2+a)^(7/2)/a/x^5-2/15*A*b/a^2/x^3*(b*x^2+a)^(7/2)-8/1
5*A*b^2/a^3/x*(b*x^2+a)^(7/2)+8/15*A*b^3/a^3*x*(b*x^2+a)^(5/2)+2/
3*A*b^3/a^2*x*(b*x^2+a)^(3/2)+A*b^3/a*x*(b*x^2+a)^(1/2)+A*b^(5/2)
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-1/3*B/a/x^3*(b*x^2+a)^(7/2)-4/3*B*
b/a^2/x*(b*x^2+a)^(7/2)+4/3*B*b^2/a^2*x*(b*x^2+a)^(5/2)+5/3*B*b^2
/a*x*(b*x^2+a)^(3/2)+5/2*B*b^2*x*(b*x^2+a)^(1/2)+5/2*B*b^(3/2)*a*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249151, size = 1, normalized size = 0.01

$$\left[\frac{15\left(5\,B a b+2\,A b^{2}\right)\sqrt{b} x^{5}\log\left(-2\,b x^{2}-2\,\sqrt{b x^{2}+a}\sqrt{b} x-a\right)+2\left(15\,B b^{2} x^{6}-2\left(35\,B a b+23\,A b^{2}\right)x^{4}-6\,A a^{2}-2\left(5\,B a^{2}+1\right.\right.}{60\,x^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(5*B*a*b + 2*A*b^2)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 2*(15*B*b^2*x^6 - 2*(35*B*a*b + 23*A*b
^2)*x^4 - 6*A*a^2 - 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2 + a))/
x^5, 1/30*(15*(5*B*a*b + 2*A*b^2)*sqrt(-b)*x^5*arctan(b*x/(sqrt(b
*x^2 + a)*sqrt(-b))) + (15*B*b^2*x^6 - 2*(35*B*a*b + 23*A*b^2)*x^
4 - 6*A*a^2 - 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A] time = 23.3963, size = 292, normalized size = 1.92

$$-\frac{A\sqrt{a}b^2}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{8Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}+Ab^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Ab^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{2Ba^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}b^2x\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{2B\sqrt{a}b^2x}{\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+\frac{5Bab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*sqrt(a)*b**2/(x*sqrt(1 + b*x**2/a)) - A*a**2*sqrt(b)*sqrt(a/(b
*x**2) + 1)/(5*x**4) - 11*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x
**2) - 8*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 + A*b**(5/2)*asinh(sq
rt(b)*x/sqrt(a)) - A*b**3*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - 2*B*a*
*(3/2)*b/(x*sqrt(1 + b*x**2/a)) + B*sqrt(a)*b**2*x*sqrt(1 + b*x**
2/a)/2 - 2*B*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - B*a**2*sqrt(b)*s
qrt(a/(b*x**2) + 1)/(3*x**2) - B*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/
3 + 5*B*a*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/2
```

GIAC/XCAS [A] time = 0.259681, size = 433, normalized size = 2.85

$$\frac{1}{2}\sqrt{bx^2+a}Bb^2x-\frac{1}{4}\left(5\,Bab^{\frac{3}{2}}+2\,Ab^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ba^2b^{\frac{3}{2}}+45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Aab^{\frac{5}{2}}-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^3b^{\frac{3}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b^2*x - 1/4*(5*B*a*b^(3/2) + 2*A*b^(5/2))*l
n((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*B*a^2*b^(3/2) + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A
*a*b^(5/2) - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(3/2) -
90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(5/2) + 200*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^4*b^(3/2) + 140*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*A*a^3*b^(5/2) - 130*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^
5*b^(3/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(5/2) + 35
*B*a^6*b^(3/2) + 23*A*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))
^2 - a)^5
```

3.550 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=149

$$\frac{5b^2\sqrt{a+bx^2}(6aB+Ab)}{16a}-\frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b\left(a+bx^2\right)^{3/2}(6aB+Ab)}{48ax^2}-\frac{\left(a+bx^2\right)^{5/2}(6aB+Ab)}{24ax^4}-\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6}$$

```
[Out] (5*b^2*(A*b + 6*a*B)*Sqrt[a + b*x^2])/(16*a) - (5*b*(A*b + 6*a*B)
*(a + b*x^2)^(3/2))/(48*a*x^2) - ((A*b + 6*a*B)*(a + b*x^2)^(5/2)
)/(24*a*x^4) - (A*(a + b*x^2)^(7/2))/(6*a*x^6) - (5*b^2*(A*b + 6*
a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

Rubi [A] time = 0.28421, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{5b^2\sqrt{a+bx^2}(6aB+Ab)}{16a}-\frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b\left(a+bx^2\right)^{3/2}(6aB+Ab)}{48ax^2}-\frac{\left(a+bx^2\right)^{5/2}(6aB+Ab)}{24ax^4}-\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (5*b^2*(A*b + 6*a*B)*Sqrt[a + b*x^2])/(16*a) - (5*b*(A*b + 6*a*B)
*(a + b*x^2)^(3/2))/(48*a*x^2) - ((A*b + 6*a*B)*(a + b*x^2)^(5/2)
)/(24*a*x^4) - (A*(a + b*x^2)^(7/2))/(6*a*x^6) - (5*b^2*(A*b + 6*
a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

Rubi in Sympy [A] time = 24.3423, size = 136, normalized size = 0.91

$$-\frac{A\left(a+bx^2\right)^{\frac{7}{2}}}{6ax^6}+\frac{5b^2\sqrt{a+bx^2}\,(Ab+6Ba)}{16a}-\frac{5b\left(a+bx^2\right)^{\frac{3}{2}}(Ab+6Ba)}{48ax^2}-\frac{\left(a+bx^2\right)^{\frac{5}{2}}(Ab+6Ba)}{24ax^4}-\frac{5b^2\,(Ab+6Ba)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(6*a*x**6) + 5*b**2*sqrt(a + b*x**2)*(A*b
+ 6*B*a)/(16*a) - 5*b*(a + b*x**2)**(3/2)*(A*b + 6*B*a)/(48*a*x**
2) - (a + b*x**2)**(5/2)*(A*b + 6*B*a)/(24*a*x**4) - 5*b**2*(A*b
+ 6*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(16*sqrt(a))
```

Mathematica [A] time = 0.296847, size = 121, normalized size = 0.81

$$\frac{1}{48}\left(\sqrt{a+bx^2}\left(-\frac{8a^2A}{x^6}-\frac{2a(6aB+13Ab)}{x^4}-\frac{3b(18aB+11Ab)}{x^2}+48b^2B\right)-\frac{15b^2(6aB+Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}+\frac{15b^2\log(x)(6aB+Ab)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] ((48*b^2*B - (8*a^2*A)/x^6 - (2*a*(13*A*b + 6*a*B))/x^4 - (3*b*(1
1*A*b + 18*a*B))/x^2)*Sqrt[a + b*x^2] + (15*b^2*(A*b + 6*a*B)*Log
[x])/Sqrt[a] - (15*b^2*(A*b + 6*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x
^2]])/Sqrt[a])/48
```

Maple [B] time = 0.013, size = 266, normalized size = 1.8

$$-\frac{A}{6\,ax^{6}}\left(bx^{2}+a\right)^{\frac{7}{2}}-\frac{Ab}{24\,a^{2}x^{4}}\left(bx^{2}+a\right)^{\frac{7}{2}}-\frac{b^{2}A}{16\,a^{3}x^{2}}\left(bx^{2}+a\right)^{\frac{7}{2}}+\frac{Ab^{3}}{16\,a^{3}}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{5\,Ab^{3}}{48\,a^{2}}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{5\,Ab^{3}}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^{2}+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{5\,Ab^{3}}{16\,a}\sqrt{bx^{2}+a}-\frac{B}{4\,ax^{4}}\left(bx^{2}+a\right)^{\frac{7}{2}}-\frac{3\,Bb}{8\,a^{2}x^{2}}\left(bx^{2}+a\right)^{\frac{7}{2}}+\frac{3\,Bb^{2}}{8\,a^{2}}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{5\,Bb^{2}}{8\,a}\left(bx^{2}+a\right)^{\frac{3}{2}}-\frac{15\,Bb^{2}}{8}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^{2}+a}\right)\right)+\frac{15\,Bb^{2}}{8}\sqrt{bx^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(7/2)/a/x^6-1/24*A*b/a^2/x^4*(b*x^2+a)^(7/2)-1/1
6*A*b^2/a^3/x^2*(b*x^2+a)^(7/2)+1/16*A*b^3/a^3*(b*x^2+a)^(5/2)+5/
48*A*b^3/a^2*(b*x^2+a)^(3/2)-5/16*A*b^3/a^(1/2)*ln((2*a+2*a^(1/2)
*(b*x^2+a)^(1/2))/x)+5/16*A*b^3/a*(b*x^2+a)^(1/2)-1/4*B/a/x^4*(b*
x^2+a)^(7/2)-3/8*B*b/a^2/x^2*(b*x^2+a)^(7/2)+3/8*B*b^2/a^2*(b*x^2
+a)^(5/2)+5/8*B*b^2/a*(b*x^2+a)^(3/2)-15/8*B*b^2*a^(1/2)*ln((2*a+
2*a^(1/2)*(b*x^2+a)^(1/2))/x)+15/8*B*b^2*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233035, size = 1, normalized size = 0.01

$$\left[\frac{15\left(6\,Bab^{2}+Ab^{3}\right)x^{6}\log\left(-\frac{\left(bx^{2}+2\,a\right)\sqrt{a}-2\,\sqrt{bx^{2}+a}a}{x^{2}}\right)+2\left(48\,Bb^{2}x^{6}-3\left(18\,Bab+11\,Ab^{2}\right)x^{4}-8\,Aa^{2}-2\left(6\,Ba^{2}+13\,Aab\right)x^{2}\right)}{96\,\sqrt{a}x^{6}}\right.$$

$$-\frac{15\left(6\,Bab^{2}+Ab^{3}\right)x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(48\,Bb^{2}x^{6}-3\left(18\,Bab+11\,Ab^{2}\right)x^{4}-8\,Aa^{2}-2\left(6\,Ba^{2}+13\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{-a}}{48\,\sqrt{-a}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(6*B*a*b^2 + A*b^3)*x^6*log(-((b*x^2 + 2*a)*sqrt(a) - 2
*sqrt(b*x^2 + a)*a)/x^2) + 2*(48*B*b^2*x^6 - 3*(18*B*a*b + 11*A*b
^2)*x^4 - 8*A*a^2 - 2*(6*B*a^2 + 13*A*a*b)*x^2)*sqrt(b*x^2 + a)*s
qrt(a))/(sqrt(a)*x^6), -1/48*(15*(6*B*a*b^2 + A*b^3)*x^6*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) - (48*B*b^2*x^6 - 3*(18*B*a*b + 11*A*b^2
)*x^4 - 8*A*a^2 - 2*(6*B*a^2 + 13*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqr
t(-a))/(sqrt(-a)*x^6)]
```

Sympy [A] time = 163.34, size = 306, normalized size = 2.05

$$-\frac{Aa^3}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{17Aa^2\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35Aab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{3Ab^{\frac{5}{2}}}{16x\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16\sqrt{a}}-\frac{15B\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8}-\frac{Ba^3}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ba^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{x}+\frac{7Bab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*A*a**2*sqrt(b)
/(24*x**5*sqrt(a/(b*x**2) + 1)) - 35*A*a*b**(3/2)/(48*x**3*sqrt(a
/(b*x**2) + 1)) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*A*b**
(5/2)/(16*x*sqrt(a/(b*x**2) + 1)) - 5*A*b**3*asinh(sqrt(a)/(sqrt(
b)*x))/(16*sqrt(a)) - 15*B*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x)
)/8 - B*a**3/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*a**2*sqr
t(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/x + 7*B*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)*
x/sqrt(a/(b*x**2) + 1)
```

GIAC/XCAS [A] time = 0.253609, size = 225, normalized size = 1.51

$$\frac{48\sqrt{bx^2+a}Bb^3+\frac{15\left(6Bab^3+Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{54\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-96\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+42\sqrt{bx^2+a}Ba^3b^3+33\left(bx^2+a\right)^{\frac{5}{2}}Ab^4-40\left(bx^2+a\right)^{\frac{3}{2}}Aab}{b^3x^6}}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(b*x^2 + a)*B*b^3 + 15*(6*B*a*b^3 + A*b^4)*arctan(sq
rt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (54*(b*x^2 + a)^(5/2)*B*a*b^3
- 96*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 42*sqrt(b*x^2 + a)*B*a^3*b^3 +
 33*(b*x^2 + a)^(5/2)*A*b^4 - 40*(b*x^2 + a)^(3/2)*A*a*b^4 + 15*s
qrt(b*x^2 + a)*A*a^2*b^4)/(b^3*x^6))/b
```

3.551 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=108

$$-\frac{A(a+bx^2)^{7/2}}{7ax^7}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{bB(a+bx^2)^{3/2}}{3x^3}$$

```
[Out] -((b^2*B*Sqrt[a + b*x^2])/x) - (b*B*(a + b*x^2)^(3/2))/(3*x^3) -
(B*(a + b*x^2)^(5/2))/(5*x^5) - (A*(a + b*x^2)^(7/2))/(7*a*x^7) +
 b^(5/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi [A] time = 0.138362, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{A(a+bx^2)^{7/2}}{7ax^7}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{bB(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -((b^2*B*Sqrt[a + b*x^2])/x) - (b*B*(a + b*x^2)^(3/2))/(3*x^3) -
(B*(a + b*x^2)^(5/2))/(5*x^5) - (A*(a + b*x^2)^(7/2))/(7*a*x^7) +
 b^(5/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rubi in Sympy [A] time = 17.5634, size = 95, normalized size = 0.88

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{7ax^7}+Bb^{\frac{5}{2}}\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{Bb^2\sqrt{a+bx^2}}{x}-\frac{Bb(a+bx^2)^{\frac{3}{2}}}{3x^3}-\frac{B(a+bx^2)^{\frac{5}{2}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(7*a*x**7) + B*b**(5/2)*atanh(sqrt(b)*x/sq
rt(a + b*x**2)) - B*b**2*sqrt(a + b*x**2)/x - B*b*(a + b*x**2)**(
3/2)/(3*x**3) - B*(a + b*x**2)**(5/2)/(5*x**5)
```

Mathematica [A] time = 0.168297, size = 106, normalized size = 0.98

$$b^{5/2}B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)-\frac{\sqrt{a+bx^2}\left(15a^3A+3a^2x^2(7aB+15Ab)+b^2x^6(161aB+15Ab)+abx^4(77aB+45Ab)\right)}{105ax^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -(Sqrt[a + b*x^2]*(15*a^3*A + 3*a^2*(15*A*b + 7*a*B)*x^2 + a*b*(4
5*A*b + 77*a*B)*x^4 + b^2*(15*A*b + 161*a*B)*x^6))/(105*a*x^7) +
b^(5/2)*B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]
```

Maple [A] time = 0.023, size = 155, normalized size = 1.4

$$-\frac{A}{7\,a x^{7}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{B}{5\,a x^{5}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{2\,B b}{15\,a^{2} x^{3}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{8\,B b^{2}}{15\,a^{3} x}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{8\,B b^{3} x}{15\,a^{3}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{2\,B b^{3} x}{3\,a^{2}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{B b^{3} x}{a}\sqrt{b x^{2}+a}+B b^{\frac{5}{2}}\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x)
```

```
[Out] -1/7*A*(b*x^2+a)^(7/2)/a/x^7-1/5*B/a/x^5*(b*x^2+a)^(7/2)-2/15*B*b
/a^2/x^3*(b*x^2+a)^(7/2)-8/15*B*b^2/a^3/x*(b*x^2+a)^(7/2)+8/15*B*
b^3/a^3*x*(b*x^2+a)^(5/2)+2/3*B*b^3/a^2*x*(b*x^2+a)^(3/2)+B*b^3/a
*x*(b*x^2+a)^(1/2)+B*b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273926, size = 1, normalized size = 0.01

$$\left[\frac{105\,B a b^{\frac{5}{2}} x^{7}\log\left(-2\,b x^{2}-2\,\sqrt{b x^{2}+a}\sqrt{b}x-a\right)-2\left(\left(161\,B a b^{2}+15\,A b^{3}\right)x^{6}+\left(77\,B a^{2} b+45\,A a b^{2}\right)x^{4}+15\,A a^{3}+3\left(7\,B a^{3}+\right.\right.}{210\,a x^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/210*(105*B*a*b^(5/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*((161*B*a*b^2 + 15*A*b^3)*x^6 + (77*B*a^2*b + 45*A
*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + 15*A*a^2*b)*x^2)*sqrt(b*x^2
 + a))/(a*x^7), 1/105*(105*B*a*sqrt(-b)*b^2*x^7*arctan(b*x/(sqrt(
b*x^2 + a)*sqrt(-b))) - ((161*B*a*b^2 + 15*A*b^3)*x^6 + (77*B*a^2
*b + 45*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + 15*A*a^2*b)*x^2)*s
qrt(b*x^2 + a))/(a*x^7)]
```

Sympy [A] time = 22.199, size = 592, normalized size = 5.48

$$-\frac{15Aa^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^6b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^5b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^4b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aa^3b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Aa^2b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{2Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{7Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a}-\frac{B\sqrt{a}b^2}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{8Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}+Bb^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bb^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**6*b**(11/2)*x**
2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 +
 105*a**3*b**6*x**10) - 17*A*a**5*b**(13/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 3*A*a**4*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4
*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**3*b**
(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b
**5*x**8 + 105*a**3*b**6*x**10) - 8*A*a**2*b**(19/2)*x**10*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**
3*b**6*x**10) - 2*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*
A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - A*b**(7/2)*sqrt(a/(b*
x**2) + 1)/(15*a) - B*sqrt(a)*b**2/(x*sqrt(1 + b*x**2/a)) - B*a**
2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*B*a*b**(3/2)*sqrt(a/
(b*x**2) + 1)/(15*x**2) - 8*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 +
B*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**3*x/(sqrt(a)*sqrt(1 +
b*x**2/a))
```

GIAC/XCAS [A] time = 0.245414, size = 432, normalized size = 4.

$$-\frac{1}{2}Bb^{\frac{5}{2}}\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{5}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{7}{2}}-1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{5}{2}}+2555\left(\sqrt{b}x-\sqrt{bx^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(5/2)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/105*(315*(
sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a*b^(5/2) + 105*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*A*b^(7/2) - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^
10*B*a^2*b^(5/2) + 2555*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(
5/2) + 525*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 3080*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 2121*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) + 315*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*A*a^4*b^(7/2) - 812*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^6*b
^(5/2) + 161*B*a^7*b^(5/2) + 15*A*a^6*b^(7/2))/((sqrt(b)*x - sqrt
(b*x^2 + a))^2 - a)^7
```

3.552 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9} dx$

Optimal. Leaf size=152

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}(Ab-8aB)}{128ax^2}+\frac{(a+bx^2)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b(a+bx^2)^{3/2}(Ab-8aB)}{192ax^4}-\frac{A(a+bx^2)^{7/2}}{8ax^8}$$

```
[Out] (5*b^2*(A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a*x^2) + (5*b*(A*b - 8
*a*B)*(a + b*x^2)^(3/2))/(192*a*x^4) + ((A*b - 8*a*B)*(a + b*x^2)
^(5/2))/(48*a*x^6) - (A*(a + b*x^2)^(7/2))/(8*a*x^8) + (5*b^3*(A*
b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rubi [A] time = 0.302208, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}(Ab-8aB)}{128ax^2}+\frac{(a+bx^2)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b(a+bx^2)^{3/2}(Ab-8aB)}{192ax^4}-\frac{A(a+bx^2)^{7/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (5*b^2*(A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a*x^2) + (5*b*(A*b - 8
*a*B)*(a + b*x^2)^(3/2))/(192*a*x^4) + ((A*b - 8*a*B)*(a + b*x^2)
^(5/2))/(48*a*x^6) - (A*(a + b*x^2)^(7/2))/(8*a*x^8) + (5*b^3*(A*
b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rubi in Sympy [A] time = 25.2564, size = 139, normalized size = 0.91

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{8ax^8}+\frac{5b^2\sqrt{a+bx^2}(Ab-8Ba)}{128ax^2}+\frac{5b(a+bx^2)^{\frac{3}{2}}(Ab-8Ba)}{192ax^4}+\frac{(a+bx^2)^{\frac{5}{2}}(Ab-8Ba)}{48ax^6}+\frac{5b^3(Ab-8Ba)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**9,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(8*a*x**8) + 5*b**2*sqrt(a + b*x**2)*(A*b
- 8*B*a)/(128*a*x**2) + 5*b*(a + b*x**2)**(3/2)*(A*b - 8*B*a)/(19
2*a*x**4) + (a + b*x**2)**(5/2)*(A*b - 8*B*a)/(48*a*x**6) + 5*b**
3*(A*b - 8*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(128*a**(3/2))
```

Mathematica [A] time = 0.272862, size = 143, normalized size = 0.94

$$\frac{5b^3(Ab-8aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{128a^{3/2}}-\frac{5b^3\log(x)(Ab-8aB)}{128a^{3/2}}+\sqrt{a+bx^2}\left(-\frac{a^2A}{8x^8}-\frac{b^2(88aB+5Ab)}{128ax^2}-\frac{a(8aB+17Ab)}{48x^6}-\frac{b(104aB+59Ab)}{192x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (-(a^2*A)/(8*x^8) - (a*(17*A*b + 8*a*B))/(48*x^6) - (b*(59*A*b +
104*a*B))/(192*x^4) - (b^2*(5*A*b + 88*a*B))/(128*a*x^2))*Sqrt[a
+ b*x^2] - (5*b^3*(A*b - 8*a*B)*Log[x])/(128*a^(3/2)) + (5*b^3*(A
*b - 8*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(128*a^(3/2))
```

Maple [B] time = 0.015, size = 311, normalized size = 2.1

$$-\frac{A}{8\,a x^8}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{A b}{48\,a^2 x^6}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{b^2 A}{192\,a^3 x^4}\left(b x^2+a\right)^{\frac{7}{2}}+\frac{A b^3}{128\,a^4 x^2}\left(b x^2+a\right)^{\frac{7}{2}} -\frac{A b^4}{128\,a^4}\left(b x^2+a\right)^{\frac{5}{2}}-\frac{5\,A b^4}{384\,a^3}\left(b x^2+a\right)^{\frac{3}{2}}+\frac{5\,A b^4}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}} -\frac{5\,A b^4}{128\,a^2}\sqrt{b x^2+a}-\frac{B}{6\,a x^6}\left(b x^2+a\right)^{\frac{7}{2}}-\frac{B b}{24\,a^2 x^4}\left(b x^2+a\right)^{\frac{7}{2}}-\frac{B b^2}{16\,a^3 x^2}\left(b x^2+a\right)^{\frac{7}{2}} +\frac{B b^3}{16\,a^3}\left(b x^2+a\right)^{\frac{5}{2}}+\frac{5\,B b^3}{48\,a^2}\left(b x^2+a\right)^{\frac{3}{2}}-\frac{5\,B b^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{5\,B b^3}{16\,a}\sqrt{b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x)
```

```
[Out] -1/8*A*(b*x^2+a)^(7/2)/a/x^8+1/48*A*b/a^2/x^6*(b*x^2+a)^(7/2)+1/1
92*A*b^2/a^3/x^4*(b*x^2+a)^(7/2)+1/128*A*b^3/a^4/x^2*(b*x^2+a)^(7
/2)-1/128*A*b^4/a^4*(b*x^2+a)^(5/2)-5/384*A*b^4/a^3*(b*x^2+a)^(3/
2)+5/128*A*b^4/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5/12
8*A*b^4/a^2*(b*x^2+a)^(1/2)-1/6*B/a/x^6*(b*x^2+a)^(7/2)-1/24*B*b/
a^2/x^4*(b*x^2+a)^(7/2)-1/16*B*b^2/a^3/x^2*(b*x^2+a)^(7/2)+1/16*B
*b^3/a^3*(b*x^2+a)^(5/2)+5/48*B*b^3/a^2*(b*x^2+a)^(3/2)-5/16*B*b^
3/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+5/16*B*b^3/a*(b*x
^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296411, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(8\,B a b^3-A b^4\right)x^8\log\left(-\frac{\left(b x^2+2\,a\right)\sqrt{a}+2\,\sqrt{b x^2+a}a}{x^2}\right)+2\left(3\left(88\,B a b^2+5\,A b^3\right)x^6+2\left(104\,B a^2 b+59\,A a b^2\right)x^4+48\,A a^3+8\,(8\right.}{768\,a^{\frac{3}{2}}x^8}\right.$$

$$-\frac{15\left(8\,B a b^3-A b^4\right)x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2+a}}\right)+\left(3\left(88\,B a b^2+5\,A b^3\right)x^6+2\left(104\,B a^2 b+59\,A a b^2\right)x^4+48\,A a^3+8\left(8\,B a^3+17\,A a^2 b\right.}{384\,\sqrt{-a}a x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*B*a*b^3 - A*b^4)*x^8*log(-((b*x^2 + 2*a)*sqrt(a) +
 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(3*(88*B*a*b^2 + 5*A*b^3)*x^6 + 2*
(104*B*a^2*b + 59*A*a*b^2)*x^4 + 48*A*a^3 + 8*(8*B*a^3 + 17*A*a^2
*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(3/2)*x^8), -1/384*(15*(8*B*
a*b^3 - A*b^4)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(88*B*a*
b^2 + 5*A*b^3)*x^6 + 2*(104*B*a^2*b + 59*A*a*b^2)*x^4 + 48*A*a^3
+ 8*(8*B*a^3 + 17*A*a^2*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-
a)*a*x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**9,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.235339, size = 263, normalized size = 1.73

$$\frac{\frac{15\left(8\,Bab^4-Ab^5\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{264\left(bx^2+a\right)^{\frac{7}{2}}Bab^4-584\left(bx^2+a\right)^{\frac{5}{2}}Ba^2b^4+440\left(bx^2+a\right)^{\frac{3}{2}}Ba^3b^4-120\sqrt{bx^2+a}Ba^4b^4+15\left(bx^2+a\right)^{\frac{7}{2}}Ab^5+73\left(bx^2+a\right)^{\frac{5}{2}}A}{ab^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(15*(8*B*a*b^4 - A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(s
qrt(-a)*a) - (264*(b*x^2 + a)^(7/2)*B*a*b^4 - 584*(b*x^2 + a)^(5/
2)*B*a^2*b^4 + 440*(b*x^2 + a)^(3/2)*B*a^3*b^4 - 120*sqrt(b*x^2 +
 a)*B*a^4*b^4 + 15*(b*x^2 + a)^(7/2)*A*b^5 + 73*(b*x^2 + a)^(5/2)
*A*a*b^5 - 55*(b*x^2 + a)^(3/2)*A*a^2*b^5 + 15*sqrt(b*x^2 + a)*A*
a^3*b^5)/(a*b^4*x^8))/b
```

3.553 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

```
[Out] -(A*(a + b*x^2)^(7/2))/(9*a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(
7/2))/(63*a^2*x^7)
```

Rubi [A] time = 0.083881, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a+bx^2)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(7/2))/(9*a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(
7/2))/(63*a^2*x^7)
```

Rubi in Sympy [A] time = 9.31011, size = 46, normalized size = 0.87

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{9ax^9}+\frac{(a+bx^2)^{\frac{7}{2}}(2Ab-9Ba)}{63a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(9*a*x**9) + (a + b*x**2)**(7/2)*(2*A*b -
9*B*a)/(63*a**2*x**7)
```

Mathematica [A] time = 0.0888106, size = 40, normalized size = 0.75

$$-\frac{(a+bx^2)^{7/2}(7aA+9aBx^2-2Abx^2)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -((a + b*x^2)^(7/2)*(7*a*A - 2*A*b*x^2 + 9*a*B*x^2))/(63*a^2*x^9)
```

Maple [A] time = 0.007, size = 37, normalized size = 0.7

$$-\frac{-2\,Abx^2+9\,Bax^2+7\,Aa}{63\,x^9a^2}(bx^2+a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-2*A*b*x^2+9*B*a*x^2+7*A*a)/x^9/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.322045, size = 138, normalized size = 2.6

$$-\frac{\left(\left(9\,Bab^{3}-2\,Ab^{4}\right)x^{8}+\left(27\,Ba^{2}b^{2}+Aab^{3}\right)x^{6}+7\,Aa^{4}+3\left(9\,Ba^{3}b+5\,Aa^{2}b^{2}\right)x^{4}+\left(9\,Ba^{4}+19\,Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{63\,a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/63*((9*B*a*b^3 - 2*A*b^4)*x^8 + (27*B*a^2*b^2 + A*a*b^3)*x^6 +
 7*A*a^4 + 3*(9*B*a^3*b + 5*A*a^2*b^2)*x^4 + (9*B*a^4 + 19*A*a^3*
b)*x^2)*sqrt(b*x^2 + a)/(a^2*x^9)
```

Sympy [A] time = 28.2878, size = 1489, normalized size = 28.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 9
45*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14
) - 110*A*a**8*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9
*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b*
*12*x**14) - 114*A*a**7*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 31
5*a**4*b**12*x**14) - 40*A*a**6*b**(25/2)*x**6*sqrt(a/(b*x**2) +
1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14) - 30*A*a**6*b**(11/2)*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x*
*10) + 5*A*a**5*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**
9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b
**12*x**14) - 66*A*a**5*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*
a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 30*A
*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**1
4) - 34*A*a**4*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4
*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**3*b**
(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*
b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 6*A*
a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a**2*b**(33/2)*x**
14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 24*A*a**2*b**(
```

```
19/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 16*A*a*b**(21/2)*x**10*sqrt(a/(b
*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b
**6*x**10) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(7/2
)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*A*b**(9/2)*sqrt(a/(b*x**2)
 + 1)/(15*a**2) - 15*B*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a
**6*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210
*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**5*b**(13/2)*x**4
*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 +
105*a**3*b**6*x**10) - 3*B*a**4*b**(15/2)*x**6*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10
) - 12*B*a**3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*
x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a**2*b**(1
9/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 2*B*a*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(5*x**4) - 7*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - B*b*
*(7/2)*sqrt(a/(b*x**2) + 1)/(15*a)
```

GIAC/XCAS [A] time = 0.269149, size = 616, normalized size = 11.62

$$\frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}Bb^{\frac{7}{2}}-126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bab^{\frac{7}{2}}+126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Ab^{\frac{9}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*b^(7/2) - 126*(sqrt(b
)*x - sqrt(b*x^2 + a))^14*B*a*b^(7/2) + 126*(sqrt(b)*x - sqrt(b*x
^2 + a))^14*A*b^(9/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^
2*b^(7/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a*b^(9/2) - 63
0*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^3*b^(7/2) + 630*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*A*a^2*b^(9/2) + 504*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*B*a^4*b^(7/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a
^3*b^(9/2) - 378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2) +
378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^4*b^(9/2) + 198*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*B*a^6*b^(7/2) + 54*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*A*a^5*b^(9/2) - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^7
*b^(7/2) + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^6*b^(9/2) + 9*B
*a^8*b^(7/2) - 2*A*a^7*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^9
```

3.554 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^3\sqrt{a+bx^2}(3Ab-10aB)}{256a^2x^2}+\frac{b^2\sqrt{a+bx^2}(3Ab-10aB)}{128ax^4}+\frac{(a+bx^2)^{5/2}(3Ab-10aB)}{80ax^8}+\frac{b(a+bx^2)^{3/2}(3Ab-10aB)}{96ax^6}-\frac{A(a+bx^2)^{7/2}}{10ax^{10}}$$

```
[Out] (b^2*(3*A*b - 10*a*B)*Sqrt[a + b*x^2])/(128*a*x^4) + (b^3*(3*A*b
- 10*a*B)*Sqrt[a + b*x^2])/(256*a^2*x^2) + (b*(3*A*b - 10*a*B)*(a
 + b*x^2)^(3/2))/(96*a*x^6) + ((3*A*b - 10*a*B)*(a + b*x^2)^(5/2)
)/(80*a*x^8) - (A*(a + b*x^2)^(7/2))/(10*a*x^10) - (b^4*(3*A*b -
10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2))
```

Rubi [A] time = 0.367694, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^3\sqrt{a+bx^2}(3Ab-10aB)}{256a^2x^2}+\frac{b^2\sqrt{a+bx^2}(3Ab-10aB)}{128ax^4}+\frac{(a+bx^2)^{5/2}(3Ab-10aB)}{80ax^8}+\frac{b(a+bx^2)^{3/2}(3Ab-10aB)}{96ax^6}-\frac{A(a+bx^2)^{7/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b^2*(3*A*b - 10*a*B)*Sqrt[a + b*x^2])/(128*a*x^4) + (b^3*(3*A*b
- 10*a*B)*Sqrt[a + b*x^2])/(256*a^2*x^2) + (b*(3*A*b - 10*a*B)*(a
 + b*x^2)^(3/2))/(96*a*x^6) + ((3*A*b - 10*a*B)*(a + b*x^2)^(5/2)
)/(80*a*x^8) - (A*(a + b*x^2)^(7/2))/(10*a*x^10) - (b^4*(3*A*b -
10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2))
```

Rubi in Sympy [A] time = 30.8619, size = 173, normalized size = 0.92

$$-\frac{A(a+bx^2)^{\frac{7}{2}}}{10ax^{10}}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-10Ba)}{128ax^4}+\frac{b(a+bx^2)^{\frac{3}{2}}(3Ab-10Ba)}{96ax^6}+\frac{(a+bx^2)^{\frac{5}{2}}(3Ab-10Ba)}{80ax^8}+\frac{b^3\sqrt{a+bx^2}\,(3Ab-10Ba)}{256a^2x^2}-\frac{b^4(3Ab-10Ba)\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**11,x)
```

```
[Out] -A*(a + b*x**2)**(7/2)/(10*a*x**10) + b**2*sqrt(a + b*x**2)*(3*A*
b - 10*B*a)/(128*a*x**4) + b*(a + b*x**2)**(3/2)*(3*A*b - 10*B*a)
/(96*a*x**6) + (a + b*x**2)**(5/2)*(3*A*b - 10*B*a)/(80*a*x**8) +
 b**3*sqrt(a + b*x**2)*(3*A*b - 10*B*a)/(256*a**2*x**2) - b**4*(3
*A*b - 10*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(256*a**(5/2))
```

Mathematica [A] time = 0.32626, size = 167, normalized size = 0.88

$$-\frac{b^4(3Ab-10aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{256a^{5/2}}+\frac{b^4\log(x)(3Ab-10aB)}{256a^{5/2}}+\sqrt{a+bx^2}\left(-\frac{b^3(10aB-3Ab)}{256a^2x^2}-\frac{a^2A}{10x^{10}}-\frac{b^2(118aB+3Ab)}{384ax^4}-\frac{a(10aB+21Ab)}{80x^8}-\frac{b(170aB+93Ab)}{480x^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (-(a^2*A)/(10*x^10) - (a*(21*A*b + 10*a*B))/(80*x^8) - (b*(93*A*b
 + 170*a*B))/(480*x^6) - (b^2*(3*A*b + 118*a*B))/(384*a*x^4) - (b
^3*(-3*A*b + 10*a*B))/(256*a^2*x^2))*Sqrt[a + b*x^2] + (b^4*(3*A*
b - 10*a*B)*Log[x])/(256*a^(5/2)) - (b^4*(3*A*b - 10*a*B)*Log[a +
 Sqrt[a]*Sqrt[a + b*x^2]])/(256*a^(5/2))
```

Maple [B] time = 0.013, size = 353, normalized size = 1.9

$$-\frac{A}{10\,a x^{10}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{3\,A b}{80\,a^{2} x^{8}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{b^{2} A}{160\,a^{3} x^{6}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{A b^{3}}{640\,a^{4} x^{4}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{3\,A b^{4}}{1280\,a^{5} x^{2}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{3\,A b^{5}}{1280\,a^{5}}\left(b x^{2}+a\right)^{\frac{5}{2}}+\frac{A b^{5}}{256\,a^{4}}\left(b x^{2}+a\right)^{\frac{3}{2}}-\frac{3\,A b^{5}}{256}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{5}{2}}+\frac{3\,A b^{5}}{256\,a^{3}}\sqrt{b x^{2}+a}-\frac{B}{8\,a x^{8}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{B b}{48\,a^{2} x^{6}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{B b^{2}}{192\,a^{3} x^{4}}\left(b x^{2}+a\right)^{\frac{7}{2}}+\frac{B b^{3}}{128\,a^{4} x^{2}}\left(b x^{2}+a\right)^{\frac{7}{2}}-\frac{B b^{4}}{128\,a^{4}}\left(b x^{2}+a\right)^{\frac{5}{2}}-\frac{5\,B b^{4}}{384\,a^{3}}\left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{5\,B b^{4}}{128}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{3}{2}}-\frac{5\,B b^{4}}{128\,a^{2}}\sqrt{b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(7/2)/a/x^10+3/80*A*b/a^2/x^8*(b*x^2+a)^(7/2)-1
/160*A*b^2/a^3/x^6*(b*x^2+a)^(7/2)-1/640*A*b^3/a^4/x^4*(b*x^2+a)^
(7/2)-3/1280*A*b^4/a^5/x^2*(b*x^2+a)^(7/2)+3/1280*A*b^5/a^5*(b*x^
2+a)^(5/2)+1/256*A*b^5/a^4*(b*x^2+a)^(3/2)-3/256*A*b^5/a^(5/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+3/256*A*b^5/a^3*(b*x^2+a)^(1/
2)-1/8*B/a/x^8*(b*x^2+a)^(7/2)+1/48*B*b/a^2/x^6*(b*x^2+a)^(7/2)+1
/192*B*b^2/a^3/x^4*(b*x^2+a)^(7/2)+1/128*B*b^3/a^4/x^2*(b*x^2+a)^
(7/2)-1/128*B*b^4/a^4*(b*x^2+a)^(5/2)-5/384*B*b^4/a^3*(b*x^2+a)^(
3/2)+5/128*B*b^4/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-5/
128*B*b^4/a^2*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.419708, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(10\,B a b^{4}-3\,A b^{5}\right)x^{10}\log\left(-\frac{\left(b x^{2}+2\,a\right)\sqrt{a}-2\,\sqrt{b x^{2}+a}\,a}{x^{2}}\right)+2\left(15\left(10\,B a b^{3}-3\,A b^{4}\right)x^{8}+10\left(118\,B a^{2} b^{2}+3\,A a b^{3}\right)x^{6}+384\,A a\right.}{7680\,a^{\frac{5}{2}}x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 3*A*b^5)*x^10*log(-((b*x^2 + 2*a)*sqrt
(a) - 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(15*(10*B*a*b^3 - 3*A*b^4)*x^
8 + 10*(118*B*a^2*b^2 + 3*A*a*b^3)*x^6 + 384*A*a^4 + 8*(170*B*a^3
*b + 93*A*a^2*b^2)*x^4 + 48*(10*B*a^4 + 21*A*a^3*b)*x^2)*sqrt(b*x
^2 + a)*sqrt(a))/(a^(5/2)*x^10), 1/3840*(15*(10*B*a*b^4 - 3*A*b^5
)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (15*(10*B*a*b^3 - 3*A*b
^4)*x^8 + 10*(118*B*a^2*b^2 + 3*A*a*b^3)*x^6 + 384*A*a^4 + 8*(170
*B*a^3*b + 93*A*a^2*b^2)*x^4 + 48*(10*B*a^4 + 21*A*a^3*b)*x^2)*sq
rt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^2*x^10)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.246964, size = 311, normalized size = 1.65

$$-\frac{\frac{15\,(10\,Bab^5-3\,Ab^6)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{150\,(bx^2+a)^{\frac{9}{2}}Bab^5+580\,(bx^2+a)^{\frac{7}{2}}Ba^2b^5-1280\,(bx^2+a)^{\frac{5}{2}}Ba^3b^5+700\,(bx^2+a)^{\frac{3}{2}}Ba^4b^5-150\sqrt{bx^2+a}Ba^5b^5-45\,(bx}{a^2b^5x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 3*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a
))/(sqrt(-a)*a^2) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 + 580*(b*x^2 +
 a)^(7/2)*B*a^2*b^5 - 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 + 700*(b*x
^2 + a)^(3/2)*B*a^4*b^5 - 150*sqrt(b*x^2 + a)*B*a^5*b^5 - 45*(b*x
^2 + a)^(9/2)*A*b^6 + 210*(b*x^2 + a)^(7/2)*A*a*b^6 + 384*(b*x^2
+ a)^(5/2)*A*a^2*b^6 - 210*(b*x^2 + a)^(3/2)*A*a^3*b^6 + 45*sqrt(
b*x^2 + a)*A*a^4*b^6)/(a^2*b^5*x^10))/b
```

3.555 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{a^2\sqrt{a+bx^2}(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] (a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (a*(2*A*b - 3*a*B)*(a + b
*x^2)^(3/2))/(3*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4)
+ (B*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.227582, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2\sqrt{a+bx^2}(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (a*(2*A*b - 3*a*B)*(a + b
*x^2)^(3/2))/(3*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4)
+ (B*(a + b*x^2)^(7/2))/(7*b^4)
```

Rubi in Sympy [A] time = 25.0849, size = 90, normalized size = 0.9

$$\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b^4}+\frac{a^2\sqrt{a+bx^2}\,(Ab-Ba)}{b^4}-\frac{a\left(a+bx^2\right)^{\frac{3}{2}}(2Ab-3Ba)}{3b^4}+\frac{\left(a+bx^2\right)^{\frac{5}{2}}(Ab-3Ba)}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] B*(a + b*x**2)**(7/2)/(7*b**4) + a**2*sqrt(a + b*x**2)*(A*b - B*a
)/b**4 - a*(a + b*x**2)**(3/2)*(2*A*b - 3*B*a)/(3*b**4) + (a + b*
x**2)**(5/2)*(A*b - 3*B*a)/(5*b**4)
```

Mathematica [A] time = 0.0716602, size = 78, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(-48a^3B+8a^2b\left(7A+3Bx^2\right)-2ab^2x^2\left(14A+9Bx^2\right)+3b^3x^4\left(7A+5Bx^2\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^2) + 3*b^3*x^4
*(7*A + 5*B*x^2) - 2*a*b^2*x^2*(14*A + 9*B*x^2)))/(105*b^4)
```

Maple [A] time = 0.009, size = 77, normalized size = 0.8

$$\frac{15\,x^6Bb^3+21\,Ab^3x^4-18\,Bab^2x^4-28\,Aab^2x^2+24\,Ba^2bx^2+56\,Aa^2b-48\,Ba^3}{105\,b^4}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(1/2)*(15*B*b^3*x^6+21*A*b^3*x^4-18*B*a*b^2*x^4-2
8*A*a*b^2*x^2+24*B*a^2*b*x^2+56*A*a^2*b-48*B*a^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228859, size = 103, normalized size = 1.03

$$\frac{\left(15\,Bb^{3}x^{6} - 3\,\left(6\,Bab^{2} - 7\,Ab^{3}\right)x^{4} - 48\,Ba^{3} + 56\,Aa^{2}b + 4\,\left(6\,Ba^{2}b - 7\,Aab^{2}\right)x^{2}\right)\sqrt{bx^{2} + a}}{105\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 - 3*(6*B*a*b^2 - 7*A*b^3)*x^4 - 48*B*a^3 + 56
*A*a^2*b + 4*(6*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] time = 4.77603, size = 172, normalized size = 1.72

$$\begin{cases} \frac{8Aa^{2}\sqrt{a+bx^{2}}}{15b^{3}} - \frac{4Aax^{2}\sqrt{a+bx^{2}}}{15b^{2}} + \frac{Ax^{4}\sqrt{a+bx^{2}}}{5b} - \frac{16Ba^{3}\sqrt{a+bx^{2}}}{35b^{4}} + \frac{8Ba^{2}x^{2}\sqrt{a+bx^{2}}}{35b^{3}} - \frac{6Bax^{4}\sqrt{a+bx^{2}}}{35b^{2}} + \frac{Bx^{6}\sqrt{a+bx^{2}}}{7b} & \text{for}\: b \neq 0 \\ \frac{\frac{Ax^{6}}{6} + \frac{Bx^{8}}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(
a + b*x**2)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3
*sqrt(a + b*x**2)/(35*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*
b**3) - 6*B*a*x**4*sqrt(a + b*x**2)/(35*b**2) + B*x**6*sqrt(a + b
*x**2)/(7*b), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.232575, size = 140, normalized size = 1.4

$$\frac{15\,\left(bx^{2} + a\right)^{\frac{7}{2}}B - 63\,\left(bx^{2} + a\right)^{\frac{5}{2}}Ba + 105\,\left(bx^{2} + a\right)^{\frac{3}{2}}Ba^{2} - 105\,\sqrt{bx^{2} + a}Ba^{3} + 21\,\left(bx^{2} + a\right)^{\frac{5}{2}}Ab - 70\,\left(bx^{2} + a\right)^{\frac{3}{2}}Aab + 105\,\sqrt{}}{105\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2)*B - 63*(b*x^2 + a)^(5/2)*B*a + 105*(b
*x^2 + a)^(3/2)*B*a^2 - 105*sqrt(b*x^2 + a)*B*a^3 + 21*(b*x^2 + a
)^(5/2)*A*b - 70*(b*x^2 + a)^(3/2)*A*a*b + 105*sqrt(b*x^2 + a)*A*
a^2*b)/b^4
```

3.556 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}-\frac{ax\sqrt{a+bx^2}(6Ab-5aB)}{16b^3}+\frac{x^3\sqrt{a+bx^2}(6Ab-5aB)}{24b^2}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

```
[Out] -(a*(6*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*A*b - 5*a*B
)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (B*x^5*Sqrt[a + b*x^2])/(6*b) +
 (a^2*(6*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b
^(7/2))
```

Rubi [A] time = 0.171011, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}-\frac{ax\sqrt{a+bx^2}(6Ab-5aB)}{16b^3}+\frac{x^3\sqrt{a+bx^2}(6Ab-5aB)}{24b^2}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(6*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*A*b - 5*a*B
)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (B*x^5*Sqrt[a + b*x^2])/(6*b) +
 (a^2*(6*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b
^(7/2))
```

Rubi in Sympy [A] time = 18.1425, size = 114, normalized size = 0.93

$$\frac{Bx^5\sqrt{a+bx^2}}{6b}+\frac{a^2\left(6Ab-5Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{7}{2}}}-\frac{ax\sqrt{a+bx^2}\left(6Ab-5Ba\right)}{16b^3}+\frac{x^3\sqrt{a+bx^2}\left(6Ab-5Ba\right)}{24b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] B*x**5*sqrt(a + b*x**2)/(6*b) + a**2*(6*A*b - 5*B*a)*atanh(sqrt(b
)*x/sqrt(a + b*x**2))/(16*b**(7/2)) - a*x*sqrt(a + b*x**2)*(6*A*b
 - 5*B*a)/(16*b**3) + x**3*sqrt(a + b*x**2)*(6*A*b - 5*B*a)/(24*b
**2)
```

Mathematica [A] time = 0.114659, size = 105, normalized size = 0.86

$$\sqrt{a+bx^2}\left(\frac{ax(5aB-6Ab)}{16b^3}+\frac{x^3(6Ab-5aB)}{24b^2}+\frac{Bx^5}{6b}\right)-\frac{a^2(5aB-6Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*((a*(-6*A*b + 5*a*B)*x)/(16*b^3) + ((6*A*b - 5*a*
B)*x^3)/(24*b^2) + (B*x^5)/(6*b)) - (a^2*(-6*A*b + 5*a*B)*Log[b*x
 + Sqrt[b]*Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Maple [A] time = 0.011, size = 143, normalized size = 1.2

$$\frac{Ax^3}{4b}\sqrt{bx^2+a}-\frac{3aAx}{8b^2}\sqrt{bx^2+a}+\frac{3Aa^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{x^5B}{6b}\sqrt{bx^2+a}-\frac{5Bax^3}{24b^2}\sqrt{bx^2+a}+\frac{5Bxa^2}{16b^3}\sqrt{bx^2+a}-\frac{5Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*A*x^3/b*(b*x^2+a)^(1/2)-3/8*A*a/b^2*x*(b*x^2+a)^(1/2)+3/8*A*a
^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*B*x^5*(b*x^2+a)^(1/2
)/b-5/24*B*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*B*a^2/b^3*x*(b*x^2+a)^(
1/2)-5/16*B*a^3/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258052, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8Bb^2x^5-2\left(5Bab-6Ab^2\right)x^3+3\left(5Ba^2-6Aab\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(5Ba^3-6Aa^2b\right)\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+\right.\right.}{96b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*B*b^2*x^5 - 2*(5*B*a*b - 6*A*b^2)*x^3 + 3*(5*B*a^2 -
6*A*a*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(5*B*a^3 - 6*A*a^2*b)*log
(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(7/2), 1/48*(
(8*B*b^2*x^5 - 2*(5*B*a*b - 6*A*b^2)*x^3 + 3*(5*B*a^2 - 6*A*a*b)*
x)*sqrt(b*x^2 + a)*sqrt(-b) - 3*(5*B*a^3 - 6*A*a^2*b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [A] time = 33.1091, size = 235, normalized size = 1.93

$$-\frac{3Aa^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{A\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Ax^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x}{16b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{3}{2}}x^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}x^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{Bx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - A*sqrt(a)*x**3/(8*b
*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5
/2)) + A*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x/(16
*b**3*sqrt(1 + b*x**2/a)) + 5*B*a**(3/2)*x**3/(48*b**2*sqrt(1 + b
*x**2/a)) - B*sqrt(a)*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*B*a**3*a
sinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + B*x**7/(6*sqrt(a)*sqrt(1
+ b*x**2/a))
```

GIAC/XCAS [A] time = 0.234347, size = 144, normalized size = 1.18

$$\frac{1}{48}\left(2\left(\frac{4\,Bx^2}{b}-\frac{5\,Bab^3-6\,Ab^4}{b^5}\right)x^2+\frac{3\left(5\,Ba^2b^2-6\,Aab^3\right)}{b^5}\right)\sqrt{bx^2+a}\,x+\frac{\left(5\,Ba^3-6\,Aa^2b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2/b - (5*B*a*b^3 - 6*A*b^4)/b^5)*x^2 + 3*(5*B*a^2*
b^2 - 6*A*a*b^3)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*B*a^3 - 6*A*a^2
*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.557 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}(Ab-aB)}{b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}$$

```
[Out] -((a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3) + ((A*b - 2*a*B)*(a + b*x^
2)^(3/2))/(3*b^3) + (B*(a + b*x^2)^(5/2))/(5*b^3)
```

Rubi [A] time = 0.167749, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}(Ab-aB)}{b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3) + ((A*b - 2*a*B)*(a + b*x^
2)^(3/2))/(3*b^3) + (B*(a + b*x^2)^(5/2))/(5*b^3)
```

Rubi in Sympy [A] time = 19.0719, size = 61, normalized size = 0.86

$$\frac{B\left(a+bx^2\right)^{\frac{5}{2}}}{5b^3}-\frac{a\sqrt{a+bx^2}\,(Ab-Ba)}{b^3}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}(Ab-2Ba)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] B*(a + b*x**2)**(5/2)/(5*b**3) - a*sqrt(a + b*x**2)*(A*b - B*a)/b
**3 + (a + b*x**2)**(3/2)*(A*b - 2*B*a)/(3*b**3)
```

Mathematica [A] time = 0.0567112, size = 56, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(8a^2B-2ab\left(5A+2Bx^2\right)+b^2x^2\left(5A+3Bx^2\right)\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^2) + b^2*x^2*(5*A
+ 3*B*x^2)))/(15*b^3)
```

Maple [A] time = 0.007, size = 53, normalized size = 0.8

$$-\frac{-3\,b^2Bx^4-5\,Ab^2x^2+4\,Babx^2+10\,abA-8\,a^2B}{15\,b^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(-3*B*b^2*x^4-5*A*b^2*x^2+4*B*a*b*x^2+10*A*
a*b-8*B*a^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233551, size = 70, normalized size = 0.99

$$\frac{\left(3\,Bb^2x^4 + 8\,Ba^2 - 10\,Aab - \left(4\,Bab - 5\,Ab^2\right)x^2\right)\sqrt{bx^2 + a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 + 8*B*a^2 - 10*A*a*b - (4*B*a*b - 5*A*b^2)*x^2)
*sqrt(b*x^2 + a)/b^3
```

Sympy [A] time = 2.94968, size = 121, normalized size = 1.7

$$\begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2} + \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{8Ba^2\sqrt{a+bx^2}}{15b^3} - \frac{4Bax^2\sqrt{a+bx^2}}{15b^2} + \frac{Bx^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x
**2)/(3*b) + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqr
t(a + b*x**2)/(15*b**2) + B*x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)
), ((A*x**4/4 + B*x**6/6)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.239829, size = 99, normalized size = 1.39

$$\frac{3\left(bx^2 + a\right)^{\frac{5}{2}}B - 10\left(bx^2 + a\right)^{\frac{3}{2}}Ba + 15\sqrt{bx^2 + a}Ba^2 + 5\left(bx^2 + a\right)^{\frac{3}{2}}Ab - 15\sqrt{bx^2 + a}Aab}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 10*(b*x^2 + a)^(3/2)*B*a + 15*sqrt(
b*x^2 + a)*B*a^2 + 5*(b*x^2 + a)^(3/2)*A*b - 15*sqrt(b*x^2 + a)*A
*a*b)/b^3
```

3.558 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4Ab-3aB)}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] ((4*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(8*b^2) + (B*x^3*Sqrt[a + b*x
^2])/(4*b) - (a*(4*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(8*b^(5/2))
```

Rubi [A] time = 0.127204, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4Ab-3aB)}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] ((4*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(8*b^2) + (B*x^3*Sqrt[a + b*x
^2])/(4*b) - (a*(4*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(8*b^(5/2))
```

Rubi in Sympy [A] time = 15.1568, size = 82, normalized size = 0.92

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a\,(4Ab-3Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{5}{2}}}+\frac{x\sqrt{a+bx^2}\,(4Ab-3Ba)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] B*x**3*sqrt(a + b*x**2)/(4*b) - a*(4*A*b - 3*B*a)*atanh(sqrt(b)*x
/sqrt(a + b*x**2))/(8*b**(5/2)) + x*sqrt(a + b*x**2)*(4*A*b - 3*B
*a)/(8*b**2)
```

Mathematica [A] time = 0.0728713, size = 77, normalized size = 0.87

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(-3aB+4Ab+2bBx^2\right)+a(3aB-4Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(4*A*b - 3*a*B + 2*b*B*x^2) + a*(-4*A*
b + 3*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] time = 0.01, size = 101, normalized size = 1.1

$$\frac{Ax}{2b}\sqrt{bx^2+a}-\frac{Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{x^3B}{4b}\sqrt{bx^2+a}-\frac{3\,Bxa}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*A*x/b*(b*x^2+a)^(1/2)-1/2*A*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))+1/4*B*x^3*(b*x^2+a)^(1/2)/b-3/8*B*a/b^2*x*(b*x^2+a)^(1/2)+
3/8*B*a^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243589, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,Bbx^3-(3\,Ba-4\,Ab)x\right)\sqrt{bx^2+a}\sqrt{b}-\left(3\,Ba^2-4\,Aab\right)\log\left(2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{16\,b^{\frac{5}{2}}},\ \frac{\left(2\,Bbx^3-(3\,Ba-4\,Ab)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*B*b*x^3 - (3*B*a - 4*A*b)*x)*sqrt(b*x^2 + a)*sqrt(b)
- (3*B*a^2 - 4*A*a*b)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*s
qrt(b)))/b^(5/2), 1/8*((2*B*b*x^3 - (3*B*a - 4*A*b)*x)*sqrt(b*x^2
 + a)*sqrt(-b) + (3*B*a^2 - 4*A*a*b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 19.907, size = 150, normalized size = 1.69

$$\frac{A\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a
))/(2*b**(3/2)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - B*
sqrt(a)*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/
sqrt(a))/(8*b**(5/2)) + B*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.246939, size = 101, normalized size = 1.13

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,Bx^2}{b}-\frac{3\,Bab-4\,Ab^2}{b^3}\right)x-\frac{\left(3\,Ba^2-4\,Aab\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*B*x^2/b - (3*B*a*b - 4*A*b^2)/b^3)*x - 1/8
*(3*B*a^2 - 4*A*a*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2
)
```

3.559 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{a+bx^2}(Ab-aB)}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}$$

```
[Out] ((A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (B*(a + b*x^2)^(3/2))/(3*b^2)
```

Rubi [A] time = 0.100898, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\sqrt{a+bx^2}(Ab-aB)}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] ((A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (B*(a + b*x^2)^(3/2))/(3*b^2)
```

Rubi in Sympy [A] time = 13.1456, size = 36, normalized size = 0.84

$$\frac{B\left(a+bx^2\right)^{\frac{3}{2}}}{3b^2}+\frac{\sqrt{a+bx^2}\,(Ab-Ba)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] B*(a + b*x**2)**(3/2)/(3*b**2) + sqrt(a + b*x**2)*(A*b - B*a)/b**
2
```

Mathematica [A] time = 0.0250252, size = 33, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-2aB+3Ab+bBx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*A*b - 2*a*B + b*B*x^2))/(3*b^2)
```

Maple [A] time = 0.005, size = 30, normalized size = 0.7

$$\frac{bBx^2+3\,Ab-2\,Ba}{3\,b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(B*b*x^2+3*A*b-2*B*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226943, size = 39, normalized size = 0.91

$$\frac{\left(Bbx^2 - 2\,Ba + 3\,Ab\right)\sqrt{bx^2 + a}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^2 - 2*B*a + 3*A*b)*sqrt(b*x^2 + a)/b^2
```

Sympy [A] time = 1.99033, size = 70, normalized size = 1.63

$$\begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2)
 + B*x**2*sqrt(a + b*x**2)/(3*b), Ne(b, 0)), ((A*x**2/2 + B*x**4/
4)/sqrt(a), True))
```

GIAC/XCAS [A] time = 0.234658, size = 58, normalized size = 1.35

$$\frac{\left(bx^2 + a\right)^{\frac{3}{2}}B - 3\,\sqrt{bx^2 + a}Ba + 3\,\sqrt{bx^2 + a}Ab}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2)*B - 3*sqrt(b*x^2 + a)*B*a + 3*sqrt(b*x^2 +
 a)*A*b)/b^2
```

3.560 $\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.055447, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 8.05785, size = 49, normalized size = 0.84

$$\frac{Bx\sqrt{a+bx^2}}{2b}+\frac{(2Ab-Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] B*x*sqrt(a + b*x**2)/(2*b) + (2*A*b - B*a)*atanh(sqrt(b)*x/sqrt(a
 + b*x**2))/(2*b**(3/2))
```

Mathematica [A] time = 0.0404801, size = 61, normalized size = 1.05

$$\frac{(2Ab-aB)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*Log[b*x + Sqrt[b]*Sq
rt[a + b*x^2]])/(2*b^(3/2))
```

Maple [A] time = 0.007, size = 62, normalized size = 1.1

$$A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{Bx}{2\,b}\sqrt{bx^2+a}-\frac{Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+1/2*B*x*(b*x^2+a)^(1/2)/b
-1/2*B*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235402, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{bx^2+a}B\sqrt{b}x-(Ba-2Ab)\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{4b^{\frac{3}{2}}},\ \frac{\sqrt{bx^2+a}B\sqrt{-b}x-(Ba-2Ab)\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*B*sqrt(b)*x - (B*a - 2*A*b)*log(-2*sqrt(b
*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/2*(sqrt(b*x^2
+ a)*B*sqrt(-b)*x - (B*a - 2*A*b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)))/(sqrt(-b)*b)]
```

Sympy [A] time = 8.33778, size = 126, normalized size = 2.17

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+\frac{B\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b
< 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)),
 (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) +
B*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a
))/(2*b**(3/2))
```

GIAC/XCAS [A] time = 0.237252, size = 65, normalized size = 1.12

$$\frac{\sqrt{bx^2+a}Bx}{2b}+\frac{(Ba-2Ab)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b + 1/2*(B*a - 2*A*b)*ln(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(3/2)
```

3.561 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt
[a]
```

Rubi [A] time = 0.111687, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt
[a]
```

Rubi in Sympy [A] time = 11.814, size = 36, normalized size = 0.84

$$-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{B\sqrt{a+bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a) + B*sqrt(a + b*x**2)/b
```

Mathematica [A] time = 0.0635797, size = 54, normalized size = 1.26

$$-\frac{A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}+\frac{A\log(x)}{\sqrt{a}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*Log[x])/Sqrt[a] - (A*Log[a + Sqrt[a]*S
qrt[a + b*x^2]])/Sqrt[a]
```

Maple [A] time = 0.01, size = 45, normalized size = 1.1

$$-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{B}{b}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] -A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*(b*x^2+a)^(1/2
)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237013, size = 1, normalized size = 0.02

$$\left[\frac{Ab\log\left(-\frac{(bx^2+2a)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)+2\sqrt{bx^2+a}B\sqrt{a}}{2\sqrt{a}b}, -\frac{Ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}B\sqrt{-a}}{\sqrt{-a}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(A*b*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2)
 + 2*sqrt(b*x^2 + a)*B*sqrt(a))/(sqrt(a)*b), -(A*b*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*B*sqrt(-a))/(sqrt(-a)*b)]
```

Sympy [A] time = 8.06921, size = 136, normalized size = 3.16

$$A\left(\begin{cases}\frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}} & \text{for } -\frac{1}{a}>0\\ -\frac{\operatorname{acoth}\left(\frac{1}{\sqrt{a+bx^2}\sqrt{\frac{1}{a}}}\right)}{a\sqrt{\frac{1}{a}}} & \text{for } -\frac{1}{a}<0\wedge\frac{1}{a}<\frac{1}{a+bx^2}\\ -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{a+bx^2}\sqrt{\frac{1}{a}}}\right)}{a\sqrt{\frac{1}{a}}} & \text{for } \frac{1}{a}>\frac{1}{a+bx^2}\wedge-\frac{1}{a}<0\end{cases}\right)+\frac{B\sqrt{a+bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((atan(1/(sqrt(-1/a)*sqrt(a + b*x**2)))/(a*sqrt(-1/a))
, -1/a > 0), (-acoth(1/(sqrt(a + b*x**2)*sqrt(1/a)))/(a*sqrt(1/a)
), (-1/a < 0) & (1/a < 1/(a + b*x**2))), (-atanh(1/(sqrt(a + b*x*
*2)*sqrt(1/a)))/(a*sqrt(1/a)), (-1/a < 0) & (1/a > 1/(a + b*x**2)
))) + B*sqrt(a + b*x**2)/b
```

GIAC/XCAS [A] time = 0.228632, size = 51, normalized size = 1.19

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{bx^2+a}B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)*B/b
```

3.562 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0639019, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 8.69821, size = 39, normalized size = 0.83

$$-\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(a*x) + B*atanh(sqrt(b)*x/sqrt(a + b*x**2))/s
qrt(b)
```

Mathematica [A] time = 0.0418477, size = 50, normalized size = 1.06

$$\frac{B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^
2]])/Sqrt[b]
```

Maple [A] time = 0.012, size = 41, normalized size = 0.9

$$B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}-\frac{A}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232797, size = 1, normalized size = 0.02

$$\left[\frac{Bax\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)-2\sqrt{bx^2+a}A\sqrt{b}}{2a\sqrt{b}x}, \frac{Bax\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}A\sqrt{-b}}{a\sqrt{-b}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*x*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)) -
 2*sqrt(b*x^2 + a)*A*sqrt(b))/(a*sqrt(b)*x), (B*a*x*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*A*sqrt(-b))/(a*sqrt(-b)*x
)]
```

Sympy [A] time = 3.03454, size = 99, normalized size = 2.11

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + B*Piecewise((sqrt(-a/b)*asin(
x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqr
t(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/
a))/sqrt(-a), (b > 0) & (a < 0)))
```

GIAC/XCAS [A] time = 0.245287, size = 78, normalized size = 1.66

$$-\frac{B\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*B*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.563 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] time = 0.142435, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rubi in Sympy [A] time = 13.108, size = 48, normalized size = 0.83

$$-\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{\left(\frac{Ab}{2}-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(2*a*x**2) + (A*b/2 - B*a)*atanh(sqrt(a + b*x
**2)/sqrt(a))/a**(3/2)
```

Mathematica [A] time = 0.0721462, size = 81, normalized size = 1.4

$$-\frac{(2aB-Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{2a^{3/2}}+\frac{\log(x)(2aB-Ab)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) + ((-(A*b) + 2*a*B)*Log[x])/(2*a^(
3/2)) - ((-(A*b) + 2*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(2*a^
(3/2))
```

Maple [A] time = 0.013, size = 79, normalized size = 1.4

$$-\frac{A}{2\,a{x}^{2}}\sqrt{b{x}^{2}+a}+\frac{Ab}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b{x}^{2}+a}\right)\right)a^{-\frac{3}{2}}-B\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b{x}^{2}+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/2*A*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)-B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23573, size = 1, normalized size = 0.02

$$\left[-\frac{(2\,Ba-Ab)x^2\log\left(-\frac{(bx^2+2\,a)\sqrt{a}+2\,\sqrt{bx^2+a}a}{x^2}\right)+2\,\sqrt{bx^2+a}A\sqrt{a}}{4\,a^{\frac{3}{2}}x^2},\ -\frac{(2\,Ba-Ab)x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}A\sqrt{-a}}{2\,\sqrt{-a}ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*x^2*log(-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x
^2 + a)*a)/x^2) + 2*sqrt(b*x^2 + a)*A*sqrt(a))/(a^(3/2)*x^2), -1/
2*((2*B*a - A*b)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^
2 + a)*A*sqrt(-a))/(sqrt(-a)*a*x^2)]
```

Sympy [A] time = 23.598, size = 66, normalized size = 1.14

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt
(b)*x))/(2*a**(3/2)) - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

GIAC/XCAS [A] time = 0.244565, size = 84, normalized size = 1.45

$$\frac{\frac{(2\,Bab-Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{\sqrt{bx^2+a}Ab}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)
*a) - sqrt(b*x^2 + a)*A*b/(a*x^2))/b
```

3.564 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{a+bx^2}(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2]
)/(3*a^2*x)
```

Rubi [A] time = 0.0850896, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt{a+bx^2}(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2]
)/(3*a^2*x)
```

Rubi in Sympy [A] time = 9.52643, size = 44, normalized size = 0.83

$$-\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{\sqrt{a+bx^2}\,(2Ab-3Ba)}{3a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(3*a*x**3) + sqrt(a + b*x**2)*(2*A*b - 3*B*a)
/(3*a**2*x)
```

Mathematica [A] time = 0.042519, size = 39, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}\left(a\left(A+3Bx^2\right)-2Abx^2\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/(3*a^2*x^3)
```

Maple [A] time = 0.006, size = 36, normalized size = 0.7

$$-\frac{-2\,Abx^2+3\,Bax^2+Aa}{3\,x^3a^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*A*b*x^2+3*B*a*x^2+A*a)/x^3/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^4),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.225003, size = 46, normalized size = 0.87

$$-\frac{\left((3\,Ba-2\,Ab)x^2+Aa\right)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^4),x, algorithm="fricas")

[Out] -1/3*((3*B*a - 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] time = 4.64634, size = 70, normalized size = 1.32

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)

[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/
(b*x**2) + 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

GIAC/XCAS [A] time = 0.246316, size = 162, normalized size = 3.06

$$\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4B\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ab^{\frac{3}{2}}+3\,Ba^2\sqrt{b}-2\,Aab^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^4),x, algorithm="giac")

[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*sqrt(b) - 6*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*B*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*A*b^(3/2) + 3*B*a^2*sqrt(b) - 2*A*a*b^(3/2))/((sqrt(b)*x - sqr
t(b*x^2 + a))^2 - a)^3
```

3.565 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}(3Ab-4aB)}{8a^2x^2}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2]
)/(8*a^2*x^2) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/(8*a^(5/2))
```

Rubi [A] time = 0.194912, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}(3Ab-4aB)}{8a^2x^2}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2]
)/(8*a^2*x^2) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/(8*a^(5/2))
```

Rubi in Sympy [A] time = 16.538, size = 82, normalized size = 0.91

$$-\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{\sqrt{a+bx^2}\,(3Ab-4Ba)}{8a^2x^2}-\frac{b\,(3Ab-4Ba)\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(4*a*x**4) + sqrt(a + b*x**2)*(3*A*b - 4*B*a)
/(8*a**2*x**2) - b*(3*A*b - 4*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a)
)/(8*a**(5/2))
```

Mathematica [A] time = 0.123471, size = 99, normalized size = 1.1

$$\frac{bx^4\log(x)(3Ab-4aB)+\sqrt{a}\sqrt{a+bx^2}\left(3Abx^2-2a\left(A+2Bx^2\right)\right)+bx^4(4aB-3Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{8a^{5/2}x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b*x^2]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)) + b*(3*A
*b - 4*a*B)*x^4*Log[x] + b*(-3*A*b + 4*a*B)*x^4*Log[a + Sqrt[a]*S
qrt[a + b*x^2]])/(8*a^(5/2)*x^4)
```

Maple [A] time = 0.013, size = 119, normalized size = 1.3

$$-\frac{A}{4\,a x^4}\sqrt{b x^2+a}+\frac{3\,Ab}{8\,a^2 x^2}\sqrt{b x^2+a}-\frac{3\,b^2 A}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{B}{2\,a x^2}\sqrt{b x^2+a}+\frac{Bb}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*A*(b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*
A*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*B/a/x^2*(
b*x^2+a)^(1/2)+1/2*B*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244537, size = 1, normalized size = 0.01

$$\left[-\frac{\left(4\,Bab-3\,Ab^2\right)x^4\log\left(-\frac{\left(bx^2+2\,a\right)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)+2\left(\left(4\,Ba-3\,Ab\right)x^2+2\,Aa\right)\sqrt{bx^2+a}\sqrt{a}}{16\,a^{\frac{5}{2}}x^4},\frac{\left(4\,Bab-3\,Ab^2\right)x^4\arctan\left(\frac{\sqrt{}}{\sqrt{b}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - 3*A*b^2)*x^4*log(-((b*x^2 + 2*a)*sqrt(a) - 2*s
qrt(b*x^2 + a)*a)/x^2) + 2*((4*B*a - 3*A*b)*x^2 + 2*A*a)*sqrt(b*x
^2 + a)*sqrt(a))/(a^(5/2)*x^4), 1/8*((4*B*a*b - 3*A*b^2)*x^4*arct
an(sqrt(-a)/sqrt(b*x^2 + a)) - ((4*B*a - 3*A*b)*x^2 + 2*A*a)*sqrt
(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^2*x^4)]
```

Sympy [A] time = 49.724, size = 150, normalized size = 1.67

$$-\frac{A}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3Ab^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sq
rt(a/(b*x**2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)
) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*
```

```
sqrt(a/(b*x**2) + 1)/(2*a*x) + B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*
a**(3/2))
```

GIAC/XCAS [A] time = 0.241999, size = 163, normalized size = 1.81

$$-\frac{\frac{\left(4\,Bab^{2}-3\,Ab^{3}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{4\,\left(bx^{2}+a\right)^{\frac{3}{2}}Bab^{2}-4\,\sqrt{bx^{2}+a}Ba^{2}b^{2}-3\,\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{3}+5\,\sqrt{bx^{2}+a}Aab^{3}}{a^{2}b^{2}x^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] -1/8*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqr
t(-a)*a^2) + (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a
^2*b^2 - 3*(b*x^2 + a)^(3/2)*A*b^3 + 5*sqrt(b*x^2 + a)*A*a*b^3)/(
a^2*b^2*x^4))/b
```

3.566 $\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}} dx$

Optimal. Leaf size=84

$$-\frac{2b\sqrt{a+bx^2}(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*A*b - 5*a*B)*Sqrt[a + b*x^2]
)/(15*a^2*x^3) - (2*b*(4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^3*x)
```

Rubi [A] time = 0.117464, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{2b\sqrt{a+bx^2}(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*A*b - 5*a*B)*Sqrt[a + b*x^2]
)/(15*a^2*x^3) - (2*b*(4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^3*x)
```

Rubi in Sympy [A] time = 12.8754, size = 76, normalized size = 0.9

$$-\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{\sqrt{a+bx^2}\,(4Ab-5Ba)}{15a^2x^3}-\frac{2b\sqrt{a+bx^2}\,(4Ab-5Ba)}{15a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(5*a*x**5) + sqrt(a + b*x**2)*(4*A*b - 5*B*a)
/(15*a**2*x**3) - 2*b*sqrt(a + b*x**2)*(4*A*b - 5*B*a)/(15*a**3*x
)
```

Mathematica [A] time = 0.0558745, size = 63, normalized size = 0.75

$$\sqrt{a+bx^2}\left(\frac{2b(5aB-4Ab)}{15a^3x}+\frac{4Ab-5aB}{15a^2x^3}-\frac{A}{5ax^5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] (-A/(5*a*x^5) + (4*A*b - 5*a*B)/(15*a^2*x^3) + (2*b*(-4*A*b + 5*a
*B))/(15*a^3*x))*Sqrt[a + b*x^2]
```

Maple [A] time = 0.008, size = 59, normalized size = 0.7

$$-\frac{8\,Ab^2x^4-10\,Babx^4-4\,aAbx^2+5\,Ba^2x^2+3\,Aa^2}{15\,x^5a^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(8*A*b^2*x^4-10*B*a*b*x^4-4*A*a*b*x^2+5*B*a
^2*x^2+3*A*a^2)/x^5/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23782, size = 78, normalized size = 0.93

$$\frac{\left(2\,\left(5\,Bab-4\,Ab^{2}\right)x^{4}-3\,Aa^{2}-\left(5\,Ba^{2}-4\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\,a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^6),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 - (5*B*a^2 - 4*A*a*b)*x
^2)*sqrt(b*x^2 + a)/(a^3*x^5)
```

Sympy [A] time = 7.16687, size = 355, normalized size = 4.23

$$-\frac{3Aa^{4}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{2Aa^{3}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{3Aa^{2}b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{12Aab^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{8Ab^{\frac{17}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}+\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt
(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3
*b**6*x**8) - 3*A*a**2*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a*
*5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*A*a*b*
*(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b*
*5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt(a/(b*x**2)
 + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*B*b**(3/2)*sqrt(
a/(b*x**2) + 1)/(3*a**2)
```

GIAC/XCAS [A] time = 0.243855, size = 238, normalized size = 2.83

$$\frac{4\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bb^{\frac{3}{2}}-35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Bab^{\frac{3}{2}}+40\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{5}{2}}+25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2b^{\frac{3}{2}}-\right.}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*b^(3/2) - 35*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*B*a*b^(3/2) + 40*(sqrt(b)*x - sqrt(b*x^2 +
 a))^4*A*b^(5/2) + 25*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^(3/
2) - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(5/2) - 5*B*a^3*b^(
3/2) + 4*A*a^2*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.567 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=123

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2]
)/(24*a^2*x^4) - (b*(5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^3*x^2)
 + (b^2*(5*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(
7/2))
```

Rubi [A] time = 0.251277, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2]
)/(24*a^2*x^4) - (b*(5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^3*x^2)
 + (b^2*(5*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(
7/2))
```

Rubi in Sympy [A] time = 20.9943, size = 114, normalized size = 0.93

$$-\frac{A\sqrt{a+bx^2}}{6ax^6}+\frac{\sqrt{a+bx^2}\,(5Ab-6Ba)}{24a^2x^4}-\frac{b\sqrt{a+bx^2}\,(5Ab-6Ba)}{16a^3x^2}+\frac{b^2\,(5Ab-6Ba)\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(6*a*x**6) + sqrt(a + b*x**2)*(5*A*b - 6*B*a)
/(24*a**2*x**4) - b*sqrt(a + b*x**2)*(5*A*b - 6*B*a)/(16*a**3*x**
2) + b**2*(5*A*b - 6*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(16*a**
(7/2))
```

Mathematica [A] time = 0.166233, size = 128, normalized size = 1.04

$$\frac{b^2(5Ab-6aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{16a^{7/2}}-\frac{b^2\log(x)(5Ab-6aB)}{16a^{7/2}}+\sqrt{a+bx^2}\left(\frac{b(6aB-5Ab)}{16a^3x^2}+\frac{5Ab-6aB}{24a^2x^4}-\frac{A}{6ax^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (-A/(6*a*x^6) + (5*A*b - 6*a*B)/(24*a^2*x^4) + (b*(-5*A*b + 6*a*B
))/(16*a^3*x^2))*Sqrt[a + b*x^2] - (b^2*(5*A*b - 6*a*B)*Log[x])/(
```

```
16*a^(7/2)) + (b^2*(5*A*b - 6*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2
]])/(16*a^(7/2))
```

Maple [A] time = 0.016, size = 161, normalized size = 1.3

$$-\frac{A}{6\,a x^6}\sqrt{bx^2+a}+\frac{5\,Ab}{24\,a^2x^4}\sqrt{bx^2+a}-\frac{5\,b^2A}{16\,a^3x^2}\sqrt{bx^2+a}+\frac{5\,Ab^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{B}{4\,a x^4}\sqrt{bx^2+a}+\frac{3\,Bb}{8\,a^2x^2}\sqrt{bx^2+a}-\frac{3\,Bb^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*A*(b*x^2+a)^(1/2)/a/x^6+5/24*A*b/a^2/x^4*(b*x^2+a)^(1/2)-5/1
6*A*b^2/a^3/x^2*(b*x^2+a)^(1/2)+5/16*A*b^3/a^(7/2)*ln((2*a+2*a^(1
/2)*(b*x^2+a)^(1/2))/x)-1/4*B/a/x^4*(b*x^2+a)^(1/2)+3/8*B*b/a^2/x
^2*(b*x^2+a)^(1/2)-3/8*B*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262967, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(6\,Bab^2-5\,Ab^3\right)x^6\log\left(-\frac{(bx^2+2\,a)\sqrt{a}+2\,\sqrt{bx^2+a}a}{x^2}\right)-2\left(3\left(6\,Bab-5\,Ab^2\right)x^4-8\,Aa^2-2\left(6\,Ba^2-5\,Aab\right)x^2\right)\sqrt{bx^2+a}\sqrt{a}}{96\,a^{\frac{7}{2}}x^6},\ -\frac{3\left(6\,Bab^2-5\,Ab^3\right)x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(3\left(6\,Bab-5\,Ab^2\right)x^4-8\,Aa^2-2\left(6\,Ba^2-5\,Aab\right)x^2\right)\sqrt{bx^2+a}\sqrt{-a}}{48\,\sqrt{-a}a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^7),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - 5*A*b^3)*x^6*log(-((b*x^2 + 2*a)*sqrt(a) +
 2*sqrt(b*x^2 + a)*a)/x^2) - 2*(3*(6*B*a*b - 5*A*b^2)*x^4 - 8*A*a
^2 - 2*(6*B*a^2 - 5*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a))/(a^(7/2)
*x^6), -1/48*(3*(6*B*a*b^2 - 5*A*b^3)*x^6*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) - (3*(6*B*a*b - 5*A*b^2)*x^4 - 8*A*a^2 - 2*(6*B*a^2 - 5
*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^6)]
```

Sympy [A] time = 86.287, size = 235, normalized size = 1.91

$$-\frac{A}{6\sqrt{b}x^{7}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{A\sqrt{b}}{24ax^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{3}{2}}}{48a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{5}{2}}}{16a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5Ab^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}}-\frac{B}{4\sqrt{b}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{B\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3Bb^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Bb^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(24*a*x**5*s
qrt(a/(b*x**2) + 1)) - 5*A*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2)
 + 1)) - 5*A*b**(5/2)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3
*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(7/2)) - B/(4*sqrt(b)*x**5*sqr
t(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*x**3*sqrt(a/(b*x**2) + 1)) +
3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**2*asinh(sqr
t(a)/(sqrt(b)*x))/(8*a**(5/2))
```

GIAC/XCAS [A] time = 0.238615, size = 213, normalized size = 1.73

$$\frac{\frac{3\left(6Bab^{3}-5Ab^{4}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{3}}+\frac{18\left(bx^{2}+a\right)^{\frac{5}{2}}Bab^{3}-48\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}b^{3}+30\sqrt{bx^{2}+a}Ba^{3}b^{3}-15\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{4}+40\left(bx^{2}+a\right)^{\frac{3}{2}}Aab^{4}-33\sqrt{bx^{2}+a}Aa^{2}b^{4}}{a^{3}b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^7),x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - 5*A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(s
qrt(-a)*a^3) + (18*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/
2)*B*a^2*b^3 + 30*sqrt(b*x^2 + a)*B*a^3*b^3 - 15*(b*x^2 + a)^(5/2
)*A*b^4 + 40*(b*x^2 + a)^(3/2)*A*a*b^4 - 33*sqrt(b*x^2 + a)*A*a^2
*b^4)/(a^3*b^3*x^6))/b
```

3.568 $\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{8b^2\sqrt{a+bx^2}(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

```
[Out] -(A*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*A*b - 7*a*B)*Sqrt[a + b*x^2]
)/(35*a^2*x^5) - (4*b*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^3*x
^3) + (8*b^2*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rubi [A] time = 0.158491, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{8b^2\sqrt{a+bx^2}(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*A*b - 7*a*B)*Sqrt[a + b*x^2]
)/(35*a^2*x^5) - (4*b*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^3*x
^3) + (8*b^2*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rubi in Sympy [A] time = 16.9291, size = 110, normalized size = 0.94

$$-\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{\sqrt{a+bx^2}\,(6Ab-7Ba)}{35a^2x^5}-\frac{4b\sqrt{a+bx^2}\,(6Ab-7Ba)}{105a^3x^3}+\frac{8b^2\sqrt{a+bx^2}\,(6Ab-7Ba)}{105a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2)/(7*a*x**7) + sqrt(a + b*x**2)*(6*A*b - 7*B*a)
/(35*a**2*x**5) - 4*b*sqrt(a + b*x**2)*(6*A*b - 7*B*a)/(105*a**3*
x**3) + 8*b**2*sqrt(a + b*x**2)*(6*A*b - 7*B*a)/(105*a**4*x)
```

Mathematica [A] time = 0.0768327, size = 85, normalized size = 0.73

$$\sqrt{a+bx^2}\left(-\frac{8b^2(7aB-6Ab)}{105a^4x}+\frac{4b(7aB-6Ab)}{105a^3x^3}+\frac{6Ab-7aB}{35a^2x^5}-\frac{A}{7ax^7}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (-A/(7*a*x^7) + (6*A*b - 7*a*B)/(35*a^2*x^5) + (4*b*(-6*A*b + 7*a
*B))/(105*a^3*x^3) - (8*b^2*(-6*A*b + 7*a*B))/(105*a^4*x))*Sqrt[a
 + b*x^2]
```

Maple [A] time = 0.008, size = 83, normalized size = 0.7

$$-\frac{-48\,Ab^3x^6+56\,Bab^2x^6+24\,Aab^2x^4-28\,Ba^2bx^4-18\,Aa^2bx^2+21\,Ba^3x^2+15\,Aa^3}{105\,x^7a^4}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-48*A*b^3*x^6+56*B*a*b^2*x^6+24*A*a*b^2*x
^4-28*B*a^2*b*x^4-18*A*a^2*b*x^2+21*B*a^3*x^2+15*A*a^3)/x^7/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268216, size = 111, normalized size = 0.95

$$-\frac{\left(8\left(7\,Bab^2-6\,Ab^3\right)x^6-4\left(7\,Ba^2b-6\,Aab^2\right)x^4+15\,Aa^3+3\left(7\,Ba^3-6\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^8),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*a*b^2 - 6*A*b^3)*x^6 - 4*(7*B*a^2*b - 6*A*a*b^2)*x
^4 + 15*A*a^3 + 3*(7*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4
*x^7)
```

Sympy [A] time = 10.6679, size = 819, normalized size = 7.

$$-\frac{5Aa^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{9Aa^5b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{5Aa^4b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{5Aa^3b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{30Aa^2b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{40Aab^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{16Ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{3Ba^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2Ba^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3Ba^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12Bab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{8Bb^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*A*a**6*b**(19/2)*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
9*A*a**5*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 +
 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12
) - 5*A*a**4*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x*
*6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x
**12) + 5*A*a**3*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**
9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**
12*x**12) + 30*A*a**2*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**
7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**
4*b**12*x**12) + 40*A*a*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*
a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*
a**4*b**12*x**12) + 16*A*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(35
*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35
*a**4*b**12*x**12) - 3*B*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a
**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*a**3
*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4
*b**5*x**6 + 15*a**3*b**6*x**8) - 3*B*a**2*b**(13/2)*x**4*sqrt(a/
(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b*
*6*x**8) - 12*B*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b*
*4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*B*b**(17/2)*
x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6
+ 15*a**3*b**6*x**8)
```

GIAC/XCAS [A] time = 0.247798, size = 313, normalized size = 2.68

$$\frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{7}{2}}+147\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\cdots}{105\left(\left(\sqrt{b}x-\sqrt{b\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^8),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(5/2) - 175*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*B*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*A*b^(7/2) + 147*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*
b^(5/2) - 126*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(7/2) - 49*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(5/2) + 42*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*A*a^2*b^(7/2) + 7*B*a^4*b^(5/2) - 6*A*a^3*b^(7/2)
)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.569 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{5ax\sqrt{a+bx^2}(6Ab-7aB)}{16b^4}+\frac{5x^3\sqrt{a+bx^2}(6Ab-7aB)}{24b^3}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

```
[Out] -((6*A*b - 7*a*B)*x^5)/(6*b^2*Sqrt[a + b*x^2]) + (B*x^7)/(6*b*Sqr
t[a + b*x^2]) - (5*a*(6*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(16*b^4)
+ (5*(6*A*b - 7*a*B)*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*a^2*(6*A*
b - 7*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rubi [A] time = 0.217056, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{5ax\sqrt{a+bx^2}(6Ab-7aB)}{16b^4}+\frac{5x^3\sqrt{a+bx^2}(6Ab-7aB)}{24b^3}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((6*A*b - 7*a*B)*x^5)/(6*b^2*Sqrt[a + b*x^2]) + (B*x^7)/(6*b*Sqr
t[a + b*x^2]) - (5*a*(6*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(16*b^4)
+ (5*(6*A*b - 7*a*B)*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*a^2*(6*A*
b - 7*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rubi in Sympy [A] time = 23.2889, size = 148, normalized size = 0.97

$$\frac{Bx^7}{6b\sqrt{a+bx^2}}+\frac{5a^2\left(6Ab-7Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{9}{2}}}-\frac{5ax\sqrt{a+bx^2}\left(6Ab-7Ba\right)}{16b^4}-\frac{x^5\left(6Ab-7Ba\right)}{6b^2\sqrt{a+bx^2}}+\frac{5x^3\sqrt{a+bx^2}\left(6Ab-7Ba\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*x**7/(6*b*sqrt(a + b*x**2)) + 5*a**2*(6*A*b - 7*B*a)*atanh(sqrt
(b)*x/sqrt(a + b*x**2))/(16*b**(9/2)) - 5*a*x*sqrt(a + b*x**2)*(6
*A*b - 7*B*a)/(16*b**4) - x**5*(6*A*b - 7*B*a)/(6*b**2*sqrt(a + b
*x**2)) + 5*x**3*sqrt(a + b*x**2)*(6*A*b - 7*B*a)/(24*b**3)
```

Mathematica [A] time = 0.216914, size = 123, normalized size = 0.81

$$\frac{5a^2(6Ab-7aB)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{9/2}}+\frac{x\left(105a^3B+a^2\left(35bBx^2-90Ab\right)-2ab^2x^2\left(15A+7Bx^2\right)+4b^3x^4\left(3A+2Bx^2\right)\right)}{48b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(105*a^3*B + 4*b^3*x^4*(3*A + 2*B*x^2) - 2*a*b^2*x^2*(15*A + 7
*B*x^2) + a^2*(-90*A*b + 35*b*B*x^2)))/(48*b^4*Sqrt[a + b*x^2]) +
 (5*a^2*(6*A*b - 7*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(16*b
^(9/2))
```

Maple [A] time = 0.011, size = 185, normalized size = 1.2

$$\frac{Ax^5}{4b}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,aAx^3}{8\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,a^2Ax}{8\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,Aa^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{x^7B}{6\,b}\frac{1}{\sqrt{bx^2+a}}-\frac{7\,Bax^5}{24\,b^2}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,a^2Bx^3}{48\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,Ba^3x}{16\,b^4}\frac{1}{\sqrt{bx^2+a}}-\frac{35\,Ba^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*A*x^5/b/(b*x^2+a)^(1/2)-5/8*A*a/b^2*x^3/(b*x^2+a)^(1/2)-15/8*
A*a^2/b^3*x/(b*x^2+a)^(1/2)+15/8*A*a^2/b^(7/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))+1/6*B*x^7/b/(b*x^2+a)^(1/2)-7/24*B*a/b^2*x^5/(b*x^2+a
)^(1/2)+35/48*B*a^2/b^3*x^3/(b*x^2+a)^(1/2)+35/16*B*a^3/b^4*x/(b*
x^2+a)^(1/2)-35/16*B*a^3/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277213, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,Bb^3x^7-2\left(7\,Bab^2-6\,Ab^3\right)x^5+5\left(7\,Ba^2b-6\,Aab^2\right)x^3+15\left(7\,Ba^3-6\,Aa^2b\right)x\right)\sqrt{bx^2+a}\sqrt{b}-15\left(7\,Ba^4-6\,Aa^3b+(\right.}{96\left(b^5x^2+ab^4\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*B*b^3*x^7 - 2*(7*B*a*b^2 - 6*A*b^3)*x^5 + 5*(7*B*a^2*
b - 6*A*a*b^2)*x^3 + 15*(7*B*a^3 - 6*A*a^2*b)*x)*sqrt(b*x^2 + a)*
sqrt(b) - 15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2
)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^5*x^2
+ a*b^4)*sqrt(b)), 1/48*((8*B*b^3*x^7 - 2*(7*B*a*b^2 - 6*A*b^3)*x
^5 + 5*(7*B*a^2*b - 6*A*a*b^2)*x^3 + 15*(7*B*a^3 - 6*A*a^2*b)*x)*
sqrt(b*x^2 + a)*sqrt(-b) - 15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b -
 6*A*a^2*b^2)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^5*x^2
+ a*b^4)*sqrt(-b))]
```

Sympy [A] time = 61.5541, size = 233, normalized size = 1.53

$$A\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\frac{35a^{\frac{5}{2}}x}{16b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}x^3}{48b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{7\sqrt{a}x^5}{24b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}}+\frac{x^7}{6\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**6*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8
*b**2*sqrt(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b
**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(35*a**(5/2
)*x/(16*b**4*sqrt(1 + b*x**2/a)) + 35*a**(3/2)*x**3/(48*b**3*sqrt
(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24*b**2*sqrt(1 + b*x**2/a)) - 3
5*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) + x**7/(6*sqrt(a)*b
*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.238776, size = 184, normalized size = 1.21

$$\frac{\left(\left(2\left(\frac{4\,Bx^2}{b}-\frac{7\,Bab^5-6\,Ab^6}{b^7}\right)x^2+\frac{5\left(7\,Ba^2b^4-6\,Aab^5\right)}{b^7}\right)x^2+\frac{15\left(7\,Ba^3b^3-6\,Aa^2b^4\right)}{b^7}\right)x}{48\sqrt{bx^2+a}}+\frac{5\left(7\,Ba^3-6\,Aa^2b\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^6/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*B*x^2/b - (7*B*a*b^5 - 6*A*b^6)/b^7)*x^2 + 5*(7*B*a^2
*b^4 - 6*A*a*b^5)/b^7)*x^2 + 15*(7*B*a^3*b^3 - 6*A*a^2*b^4)/b^7)*
x/sqrt(b*x^2 + a) + 5/16*(7*B*a^3 - 6*A*a^2*b)*ln(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(9/2)
```

3.570 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{(a+bx^2)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}(2Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{5/2}}{5b^4}$$

```
[Out] -((a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2])) - (a*(2*A*b - 3*a*B)*S
qrt[a + b*x^2])/b^4 + ((A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) +
 (B*(a + b*x^2)^(5/2))/(5*b^4)
```

Rubi [A] time = 0.226423, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{(a+bx^2)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}(2Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2])) - (a*(2*A*b - 3*a*B)*S
qrt[a + b*x^2])/b^4 + ((A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) +
 (B*(a + b*x^2)^(5/2))/(5*b^4)
```

Rubi in Sympy [A] time = 25.0018, size = 88, normalized size = 0.89

$$\frac{B(a+bx^2)^{\frac{5}{2}}}{5b^4}-\frac{a^2(Ab-Ba)}{b^4\sqrt{a+bx^2}}-\frac{a\sqrt{a+bx^2}(2Ab-3Ba)}{b^4}+\frac{(a+bx^2)^{\frac{3}{2}}(Ab-3Ba)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*(a + b*x**2)**(5/2)/(5*b**4) - a**2*(A*b - B*a)/(b**4*sqrt(a +
b*x**2)) - a*sqrt(a + b*x**2)*(2*A*b - 3*B*a)/b**4 + (a + b*x**2)
**(3/2)*(A*b - 3*B*a)/(3*b**4)
```

Mathematica [A] time = 0.0747605, size = 77, normalized size = 0.78

$$\frac{48a^3B-8a^2b(5A-3Bx^2)-2ab^2x^2(10A+3Bx^2)+b^3x^4(5A+3Bx^2)}{15b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (48*a^3*B - 8*a^2*b*(5*A - 3*B*x^2) + b^3*x^4*(5*A + 3*B*x^2) - 2
*a*b^2*x^2*(10*A + 3*B*x^2))/(15*b^4*Sqrt[a + b*x^2])
```

Maple [A] time = 0.009, size = 77, normalized size = 0.8

$$-\frac{-3x^6Bb^3-5Ab^3x^4+6Bab^2x^4+20Aab^2x^2-24Ba^2bx^2+40Aa^2b-48Ba^3}{15b^4}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/15*(-3*B*b^3*x^6-5*A*b^3*x^4+6*B*a*b^2*x^4+20*A*a*b^2*x^2-24*B
*a^2*b*x^2+40*A*a^2*b-48*B*a^3)/(b*x^2+a)^(1/2)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215972, size = 119, normalized size = 1.2

$$\frac{\left(3\,Bb^{3}x^{6} - \left(6\,Bab^{2} - 5\,Ab^{3}\right)x^{4} + 48\,Ba^{3} - 40\,Aa^{2}b + 4\left(6\,Ba^{2}b - 5\,Aab^{2}\right)x^{2}\right)\sqrt{bx^{2} + a}}{15\left(b^{5}x^{2} + ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^3*x^6 - (6*B*a*b^2 - 5*A*b^3)*x^4 + 48*B*a^3 - 40*A*a
^2*b + 4*(6*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^5*x^2 +
a*b^4)
```

Sympy [A] time = 5.34624, size = 172, normalized size = 1.74

$$\begin{cases} -\frac{8Aa^{2}}{3b^{3}\sqrt{a+bx^{2}}} - \frac{4Aax^{2}}{3b^{2}\sqrt{a+bx^{2}}} + \frac{Ax^{4}}{3b\sqrt{a+bx^{2}}} + \frac{16Ba^{3}}{5b^{4}\sqrt{a+bx^{2}}} + \frac{8Ba^{2}x^{2}}{5b^{3}\sqrt{a+bx^{2}}} - \frac{2Bax^{4}}{5b^{2}\sqrt{a+bx^{2}}} + \frac{Bx^{6}}{5b\sqrt{a+bx^{2}}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^{6}}{6} + \frac{Bx^{8}}{8}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*A*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*A*a*x**2/(3*b*
*2*sqrt(a + b*x**2)) + A*x**4/(3*b*sqrt(a + b*x**2)) + 16*B*a**3/
(5*b**4*sqrt(a + b*x**2)) + 8*B*a**2*x**2/(5*b**3*sqrt(a + b*x**2
)) - 2*B*a*x**4/(5*b**2*sqrt(a + b*x**2)) + B*x**6/(5*b*sqrt(a +
b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/a**(3/2), True))
```

GIAC/XCAS [A] time = 0.240234, size = 131, normalized size = 1.32

$$\frac{3\left(bx^{2} + a\right)^{\frac{5}{2}}B - 15\left(bx^{2} + a\right)^{\frac{3}{2}}Ba + 45\sqrt{bx^{2} + a}Ba^{2} + 5\left(bx^{2} + a\right)^{\frac{3}{2}}Ab - 30\sqrt{bx^{2} + a}Aab + \frac{15\left(Ba^{3} - Aa^{2}b\right)}{\sqrt{bx^{2}+a}}}{15\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 15*(b*x^2 + a)^(3/2)*B*a + 45*sqrt(
b*x^2 + a)*B*a^2 + 5*(b*x^2 + a)^(3/2)*A*b - 30*sqrt(b*x^2 + a)*A
*a*b + 15*(B*a^3 - A*a^2*b)/sqrt(b*x^2 + a))/b^4
```

3.571 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{3x\sqrt{a+bx^2}(4Ab-5aB)}{8b^3}-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

```
[Out] -((4*A*b - 5*a*B)*x^3)/(4*b^2*Sqrt[a + b*x^2]) + (B*x^5)/(4*b*Sqr
t[a + b*x^2]) + (3*(4*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(8*b^3) - (
3*a*(4*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7
/2))
```

Rubi [A] time = 0.165322, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{3x\sqrt{a+bx^2}(4Ab-5aB)}{8b^3}-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((4*A*b - 5*a*B)*x^3)/(4*b^2*Sqrt[a + b*x^2]) + (B*x^5)/(4*b*Sqr
t[a + b*x^2]) + (3*(4*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(8*b^3) - (
3*a*(4*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7
/2))
```

Rubi in Sympy [A] time = 18.0281, size = 114, normalized size = 0.96

$$\frac{Bx^5}{4b\sqrt{a+bx^2}}-\frac{3a\left(4Ab-5Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{7}{2}}}-\frac{x^3\left(4Ab-5Ba\right)}{4b^2\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}\left(4Ab-5Ba\right)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*x**5/(4*b*sqrt(a + b*x**2)) - 3*a*(4*A*b - 5*B*a)*atanh(sqrt(b)
*x/sqrt(a + b*x**2))/(8*b**(7/2)) - x**3*(4*A*b - 5*B*a)/(4*b**2*
sqrt(a + b*x**2)) + 3*x*sqrt(a + b*x**2)*(4*A*b - 5*B*a)/(8*b**3)
```

Mathematica [A] time = 0.16007, size = 99, normalized size = 0.83

$$\frac{-15a^2Bx+abx\left(12A-5Bx^2\right)+2b^2x^3\left(2A+Bx^2\right)}{8b^3\sqrt{a+bx^2}}+\frac{3a(5aB-4Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-15*a^2*B*x + a*b*x*(12*A - 5*B*x^2) + 2*b^2*x^3*(2*A + B*x^2))/
(8*b^3*Sqrt[a + b*x^2]) + (3*a*(-4*A*b + 5*a*B)*Log[b*x + Sqrt[b]
*Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Maple [A] time = 0.01, size = 141, normalized size = 1.2

$$\frac{Ax^3}{2\,b}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,aAx}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{x^5B}{4\,b}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,Bax^3}{8\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,Bxa^2}{8\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*A*x^3/b/(b*x^2+a)^(1/2)+3/2*A*a/b^2*x/(b*x^2+a)^(1/2)-3/2*A*a
/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*B*x^5/b/(b*x^2+a)^(1/2
)-5/8*B*a/b^2*x^3/(b*x^2+a)^(1/2)-15/8*B*a^2/b^3*x/(b*x^2+a)^(1/2
)+15/8*B*a^2/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^4/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232635, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,Bb^2x^5-\left(5\,Bab-4\,Ab^2\right)x^3-3\left(5\,Ba^2-4\,Aab\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(5\,Ba^3-4\,Aa^2b+\left(5\,Ba^2b-4\,Aab^2\right)x^2\right)\log\left(2\,\sqrt{b}\right.}{16\left(b^4x^2+ab^3\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^4/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*B*b^2*x^5 - (5*B*a*b - 4*A*b^2)*x^3 - 3*(5*B*a^2 - 4*
A*a*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(5*B*a^3 - 4*A*a^2*b + (5*B
*a^2*b - 4*A*a*b^2)*x^2)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a
)*sqrt(b)))/((b^4*x^2 + a*b^3)*sqrt(b)), 1/8*((2*B*b^2*x^5 - (5*B
*a*b - 4*A*b^2)*x^3 - 3*(5*B*a^2 - 4*A*a*b)*x)*sqrt(b*x^2 + a)*sq
rt(-b) + 3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^4*x^2 + a*b^3)*sqrt(-b))]
```

Sympy [A] time = 36.9472, size = 177, normalized size = 1.49

$$A\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/
sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8
*b**2*sqrt(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b
**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.230698, size = 140, normalized size = 1.18

$$\frac{\left(\left(\frac{2\,Bx^2}{b} - \frac{5\,Bab^3 - 4\,Ab^4}{b^5}\right)x^2 - \frac{3\left(5\,Ba^2b^2 - 4\,Aab^3\right)}{b^5}\right)x}{8\,\sqrt{bx^2 + a}} - \frac{3\left(5\,Ba^2 - 4\,Aab\right)\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*B*x^2/b - (5*B*a*b^3 - 4*A*b^4)/b^5)*x^2 - 3*(5*B*a^2*b^2
 - 4*A*a*b^3)/b^5)*x/sqrt(b*x^2 + a) - 3/8*(5*B*a^2 - 4*A*a*b)*ln
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.572 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{a+bx^2}(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}$$

```
[Out] (a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + ((A*b - 2*a*B)*Sqrt[a + b
*x^2])/b^3 + (B*(a + b*x^2)^(3/2))/(3*b^3)
```

Rubi [A] time = 0.16596, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt{a+bx^2}(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + ((A*b - 2*a*B)*Sqrt[a + b
*x^2])/b^3 + (B*(a + b*x^2)^(3/2))/(3*b^3)
```

Rubi in Sympy [A] time = 18.9901, size = 60, normalized size = 0.9

$$\frac{B\left(a+bx^2\right)^{\frac{3}{2}}}{3b^3}+\frac{a\left(Ab-Ba\right)}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\left(Ab-2Ba\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*(a + b*x**2)**(3/2)/(3*b**3) + a*(A*b - B*a)/(b**3*sqrt(a + b*x
**2)) + sqrt(a + b*x**2)*(A*b - 2*B*a)/b**3
```

Mathematica [A] time = 0.060288, size = 55, normalized size = 0.82

$$\frac{-8a^2B+a\left(6Ab-4bBx^2\right)+b^2x^2\left(3A+Bx^2\right)}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^2*B + b^2*x^2*(3*A + B*x^2) + a*(6*A*b - 4*b*B*x^2))/(3*b^3
*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 52, normalized size = 0.8

$$\frac{b^2Bx^4+3\,Ab^2x^2-4\,Babx^2+6\,abA-8\,a^2B}{3\,b^3}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/3*(B*b^2*x^4+3*A*b^2*x^2-4*B*a*b*x^2+6*A*a*b-8*B*a^2)/(b*x^2+a)
^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.211825, size = 85, normalized size = 1.27

$$\frac{\left(Bb^2x^4 - 8\,Ba^2 + 6\,Aab - \left(4\,Bab - 3\,Ab^2\right)x^2\right)\sqrt{bx^2 + a}}{3\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^4 - 8*B*a^2 + 6*A*a*b - (4*B*a*b - 3*A*b^2)*x^2)*sqr
t(b*x^2 + a)/(b^4*x^2 + a*b^3)
```

Sympy [A] time = 3.43039, size = 117, normalized size = 1.75

$$\begin{cases} \frac{2Aa}{b^2\sqrt{a+bx^2}} + \frac{Ax^2}{b\sqrt{a+bx^2}} - \frac{8Ba^2}{3b^3\sqrt{a+bx^2}} - \frac{4Bax^2}{3b^2\sqrt{a+bx^2}} + \frac{Bx^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*A*a/(b**2*sqrt(a + b*x**2)) + A*x**2/(b*sqrt(a + b*x
**2)) - 8*B*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*B*a*x**2/(3*b**2*s
qrt(a + b*x**2)) + B*x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*
x**4/4 + B*x**6/6)/a**(3/2), True))
```

GIAC/XCAS [A] time = 0.228403, size = 88, normalized size = 1.31

$$\frac{\left(bx^2 + a\right)^{\frac{3}{2}}B - 6\,\sqrt{bx^2 + a}Ba + 3\,\sqrt{bx^2 + a}Ab - \frac{3\,(Ba^2 - Aab)}{\sqrt{bx^2 + a}}}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2)*B - 6*sqrt(b*x^2 + a)*B*a + 3*sqrt(b*x^2 +
 a)*A*b - 3*(B*a^2 - A*a*b)/sqrt(b*x^2 + a))/b^3
```

3.573 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

```
[Out] -(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + (B*x*Sqrt[a + b*x^2])/
(2*b^2) + ((2*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*b^(5/2))
```

Rubi [A] time = 0.164013, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + (B*x*Sqrt[a + b*x^2])/
(2*b^2) + ((2*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*b^(5/2))
```

Rubi in Sympy [A] time = 22.5442, size = 75, normalized size = 0.9

$$\frac{Bx\sqrt{a+bx^2}}{2b^2}-\frac{x\,(Ab-Ba)}{b^2\sqrt{a+bx^2}}+\frac{(2Ab-3Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*x*sqrt(a + b*x**2)/(2*b**2) - x*(A*b - B*a)/(b**2*sqrt(a + b*x*
*2)) + (2*A*b - 3*B*a)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(5
/2))
```

Mathematica [A] time = 0.103335, size = 75, normalized size = 0.9

$$\frac{\frac{\sqrt{b}x(3aB-2Ab+bBx^2)}{\sqrt{a+bx^2}}+(2Ab-3aB)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[b]*x*(-2*A*b + 3*a*B + b*B*x^2))/Sqrt[a + b*x^2] + (2*A*b
- 3*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A] time = 0.01, size = 97, normalized size = 1.2

$$-\frac{Ax}{b}\frac{1}{\sqrt{bx^2+a}}+A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{x^3B}{2b}\frac{1}{\sqrt{bx^2+a}}+\frac{3Bxa}{2b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -A*x/b/(b*x^2+a)^(1/2)+A/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/
2*B*x^3/b/(b*x^2+a)^(1/2)+3/2*B*a/b^2*x/(b*x^2+a)^(1/2)-3/2*B*a/b
^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^2/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23767, size = 1, normalized size = 0.01

$$\left[\frac{2\left(Bbx^3+(3Ba-2Ab)x\right)\sqrt{bx^2+a}\sqrt{b}-\left(3Ba^2-2Aab+\left(3Bab-2Ab^2\right)x^2\right)\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{4\left(b^3x^2+ab^2\right)\sqrt{b}}, \frac{(Bb\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((B*x^2 + A)*x^2/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b*x^3 + (3*B*a - 2*A*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - (
3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x^2)*log(-2*sqrt(b*x^2 +
a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^3*x^2 + a*b^2)*sqrt(b)), 1/2
*((B*b*x^3 + (3*B*a - 2*A*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) - (3*B*a
^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x^2)*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)))/((b^3*x^2 + a*b^2)*sqrt(-b))]
```

Sympy [A] time = 20.5681, size = 114, normalized size = 1.37

$$A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x*
*2/a))) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(
sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x*
```

```
*2/a)))
```

GIAC/XCAS [A] time = 0.259891, size = 95, normalized size = 1.14

$$\frac{\left(\frac{Bx^2}{b} + \frac{3\,Bab-2\,Ab^2}{b^3}\right)x}{2\,\sqrt{bx^2+a}} + \frac{(3\,Ba-2\,Ab)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(B*x^2/b + (3*B*a*b - 2*A*b^2)/b^3)*x/sqrt(b*x^2 + a) + 1/2*(
3*B*a - 2*A*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.574 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

```
[Out] -((A*b - a*B)/(b^2*Sqrt[a + b*x^2])) + (B*Sqrt[a + b*x^2])/b^2
```

Rubi [A] time = 0.0977305, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]

[Out] -((A*b - a*B)/(b^2*Sqrt[a + b*x^2])) + (B*Sqrt[a + b*x^2])/b^2
```

Rubi in Sympy [A] time = 12.9567, size = 34, normalized size = 0.83

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-Ba}{b^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(b*x**2+a)**(3/2),x)

[Out] B*sqrt(a + b*x**2)/b**2 - (A*b - B*a)/(b**2*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0242189, size = 30, normalized size = 0.73

$$\frac{2aB-Ab+bBx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]

[Out] (-(A*b) + 2*a*B + b*B*x^2)/(b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.006, size = 30, normalized size = 0.7

$$-\frac{-bBx^2+Ab-2\,Ba}{b^2}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -(-B*b*x^2+A*b-2*B*a)/(b*x^2+a)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229803, size = 54, normalized size = 1.32

$$\frac{\left(Bbx^2 + 2\,Ba - Ab\right)\sqrt{bx^2 + a}}{b^3x^2 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] (B*b*x^2 + 2*B*a - A*b)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 2.2509, size = 66, normalized size = 1.61

$$\begin{cases} -\frac{A}{b\sqrt{a+bx^2}} + \frac{2Ba}{b^2\sqrt{a+bx^2}} + \frac{Bx^2}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-A/(b*sqrt(a + b*x**2)) + 2*B*a/(b**2*sqrt(a + b*x**2)
) + B*x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**2/2 + B*x**4/4
)/a**(3/2), True))
```

GIAC/XCAS [A] time = 0.227732, size = 46, normalized size = 1.12

$$\frac{\sqrt{bx^2 + a}B + \frac{Ba - Ab}{\sqrt{bx^2 + a}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(b*x^2 + a)*B + (B*a - A*b)/sqrt(b*x^2 + a))/b^2
```

3.575 $\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] ((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/b^(3/2)
```

Rubi [A] time = 0.052163, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/b^(3/2)
```

Rubi in Sympy [A] time = 8.16072, size = 46, normalized size = 0.85

$$\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}}+\frac{x\,(Ab-Ba)}{ab\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] B*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(3/2) + x*(A*b - B*a)/(a*b
*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.062452, size = 58, normalized size = 1.07

$$\frac{B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{3/2}}-\frac{x(aB-Ab)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((-(A*b) + a*B)*x)/(a*b*Sqrt[a + b*x^2])) + (B*Log[b*x + Sqrt[b
]*Sqrt[a + b*x^2]])/b^(3/2)
```

Maple [A] time = 0.007, size = 54, normalized size = 1.

$$\frac{Ax}{a}\frac{1}{\sqrt{bx^2+a}}-\frac{Bx}{b}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] A*x/a/(b*x^2+a)^(1/2)-B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237452, size = 1, normalized size = 0.02

$$\left[-\frac{2\sqrt{bx^2+a}(Ba-Ab)\sqrt{b}x-\left(Babx^2+Ba^2\right)\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{2\left(ab^2x^2+a^2b\right)\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{bx^2+a}(Ba-Ab)\sqrt{-b}x-\left(Babx^2+Ba^2\right)\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{\left(ab^2x^2+a^2b\right)\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b*x^2 + a)*(B*a - A*b)*sqrt(b)*x - (B*a*b*x^2 + B*a
^2)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a*b^2*
x^2 + a^2*b)*sqrt(b)), -(sqrt(b*x^2 + a)*(B*a - A*b)*sqrt(-b)*x -
 (B*a*b*x^2 + B*a^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a*b^2*
x^2 + a^2*b)*sqrt(-b))]
```

Sympy [A] time = 11.0108, size = 60, normalized size = 1.11

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b
**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.243235, size = 69, normalized size = 1.28

$$-\frac{B\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{(Ba-Ab)x}{\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -B*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (B*a - A*b)*x/
(sqrt(b*x^2 + a)*a*b)
```

3.576 $\int \frac{A+Bx^2}{x(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=53

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sq
rt[a]])/a^(3/2)
```

Rubi [A] time = 0.128484, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sq
rt[a]])/a^(3/2)
```

Rubi in Sympy [A] time = 13.1833, size = 42, normalized size = 0.79

$$-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{Ab-Ba}{ab\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] -A*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(3/2) + (A*b - B*a)/(a*b*sq
rt(a + b*x**2))
```

Mathematica [A] time = 0.118918, size = 64, normalized size = 1.21

$$-\frac{A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{3/2}}+\frac{A\log(x)}{a^{3/2}}+\frac{Ab-aB}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) + (A*Log[x])/a^(3/2) - (A*Log[a
 + Sqrt[a]*Sqrt[a + b*x^2]])/a^(3/2)
```

Maple [A] time = 0.011, size = 60, normalized size = 1.1

$$\frac{A}{a}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{B}{b}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)-B/b/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239646, size = 1, normalized size = 0.02

$$\left[-\frac{2\sqrt{bx^2+a}(Ba-Ab)\sqrt{a}-\left(Ab^2x^2+Aab\right)\log\left(-\frac{(bx^2+2a)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)}{2\left(ab^2x^2+a^2b\right)\sqrt{a}},\right.$$
$$\left.-\frac{\sqrt{bx^2+a}(Ba-Ab)\sqrt{-a}+\left(Ab^2x^2+Aab\right)\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{(ab^2x^2+a^2b)\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b*x^2 + a)*(B*a - A*b)*sqrt(a) - (A*b^2*x^2 + A*a*b
)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a*b^
2*x^2 + a^2*b)*sqrt(a)), -(sqrt(b*x^2 + a)*(B*a - A*b)*sqrt(-a) +
 (A*b^2*x^2 + A*a*b)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/((a*b^2*x^
2 + a^2*b)*sqrt(-a))]
```

Sympy [A] time = 15.0147, size = 212, normalized size = 4.

$$A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right.$$
$$\left.-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+B\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a
**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(s
qrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*
```

```
x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x*
*2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
+ B*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3
/2)), True))
```

GIAC/XCAS [A] time = 0.226369, size = 70, normalized size = 1.32

$$\frac{A \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{Ba - Ab}{\sqrt{bx^2 + a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (B*a - A*b)/(sq
rt(b*x^2 + a)*a*b)
```

3.577 $\int \frac{A+Bx^2}{x^2(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=47

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

```
[Out] -(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^
2])
```

Rubi [A] time = 0.0707354, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^
2])
```

Rubi in Sympy [A] time = 8.23623, size = 39, normalized size = 0.83

$$-\frac{A}{ax\sqrt{a+bx^2}}-\frac{x\,(2Ab-Ba)}{a^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(a*x*sqrt(a + b*x**2)) - x*(2*A*b - B*a)/(a**2*sqrt(a + b*x**2
))
```

Mathematica [A] time = 0.0397755, size = 36, normalized size = 0.77

$$\frac{-aA+aBx^2-2Abx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 2*A*b*x^2 + a*B*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$-\frac{2\,Abx^2-Bax^2+Aa}{xa^2}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -(2*A*b*x^2-B*a*x^2+A*a)/(b*x^2+a)^(1/2)/x/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.224098, size = 58, normalized size = 1.23

$$\frac{\left((Ba - 2\,Ab)x^2 - Aa\right)\sqrt{bx^2 + a}}{a^2bx^3 + a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] ((B*a - 2*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

Sympy [A] time = 14.1452, size = 68, normalized size = 1.45

$$A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2} + 1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2} + 1}}\right) + \frac{Bx}{a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqr
t(a/(b*x**2) + 1))) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.235366, size = 77, normalized size = 1.64

$$\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 - a\right)a} + \frac{(Ba - Ab)x}{\sqrt{bx^2 + a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + (B*a - A*
b)*x/(sqrt(b*x^2 + a)*a^2)
```

3.578 $\int \frac{A+Bx^2}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

```
[Out] -(3*A*b - 2*a*B)/(2*a^2*Sqrt[a + b*x^2]) - A/(2*a*x^2*Sqrt[a + b*
x^2]) + ((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(
5/2))
```

Rubi [A] time = 0.190206, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(3*A*b - 2*a*B)/(2*a^2*Sqrt[a + b*x^2]) - A/(2*a*x^2*Sqrt[a + b*
x^2]) + ((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(
5/2))
```

Rubi in Sympy [A] time = 16.8769, size = 73, normalized size = 0.85

$$-\frac{A}{2ax^2\sqrt{a+bx^2}}-\frac{\frac{3Ab}{2}-Ba}{a^2\sqrt{a+bx^2}}+\frac{\left(\frac{3Ab}{2}-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(2*a*x**2*sqrt(a + b*x**2)) - (3*A*b/2 - B*a)/(a**2*sqrt(a + b
*x**2)) + (3*A*b/2 - B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(5/2
)
```

Mathematica [A] time = 0.227495, size = 91, normalized size = 1.06

$$\frac{\frac{\sqrt{a}(-aA+2aBx^2-3Abx^2)}{x^2\sqrt{a+bx^2}}+(3Ab-2aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\log(x)(2aB-3Ab)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[a]*(-(a*A) - 3*A*b*x^2 + 2*a*B*x^2))/(x^2*Sqrt[a + b*x^2])
 + (-3*A*b + 2*a*B)*Log[x] + (3*A*b - 2*a*B)*Log[a + Sqrt[a]*Sqrt
[a + b*x^2]])/(2*a^(5/2))
```

Maple [A] time = 0.013, size = 109, normalized size = 1.3

$$-\frac{A}{2\,a x^2}\frac{1}{\sqrt{b x^2+a}}-\frac{3\,Ab}{2\,a^2}\frac{1}{\sqrt{b x^2+a}}+\frac{3\,Ab}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{B}{a}\frac{1}{\sqrt{b x^2+a}}-B\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(1/2)-3/2*A*b/a^2/(b*x^2+a)^(1/2)+3/2*A*b/
a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B/a/(b*x^2+a)^(1/2)
-B/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.246394, size = 1, normalized size = 0.01

$$\left[\frac{2\left((2\,Ba-3\,Ab)x^2-Aa\right)\sqrt{bx^2+a}\sqrt{a}-\left(\left(2\,Bab-3\,Ab^2\right)x^4+\left(2\,Ba^2-3\,Aab\right)x^2\right)\log\left(-\frac{(bx^2+2\,a)\sqrt{a}+2\sqrt{bx^2+a}a}{x^2}\right)}{4\,(a^2bx^4+a^3x^2)\sqrt{a}}, \frac{((2\,Ba-3\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(2*((2*B*a - 3*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)*sqrt(a) - ((2
*B*a*b - 3*A*b^2)*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*log(-((b*x^2 + 2
*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^2))/((a^2*b*x^4 + a^3*x^2)*s
qrt(a)), 1/2*(((2*B*a - 3*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)*sqrt(-a
) - ((2*B*a*b - 3*A*b^2)*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)))/((a^2*b*x^4 + a^3*x^2)*sqrt(-a))]
```

Sympy [A] time = 29.4305, size = 262, normalized size = 3.05

$$A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2
*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(
5/2))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*
x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a
**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)
+ a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*
a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*
b*x**2))
```

GIAC/XCAS [A] time = 0.2282, size = 134, normalized size = 1.56

$$\frac{(2\,Ba - 3\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^2} + \frac{2\,\left(bx^2+a\right)Ba - 2\,Ba^2 - 3\,\left(bx^2+a\right)Ab + 2\,Aab}{2\,\left(\left(bx^2+a\right)^{\frac{3}{2}} - \sqrt{bx^2+a}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 3*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^
2) + 1/2*(2*(b*x^2 + a)*B*a - 2*B*a^2 - 3*(b*x^2 + a)*A*b + 2*A*a
*b)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2)
```

3.579 $\int \frac{A+Bx^2}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

```
[Out] -A/(3*a*x^3*Sqrt[a + b*x^2]) + (4*A*b - 3*a*B)/(3*a^2*x*Sqrt[a +
b*x^2]) + (2*b*(4*A*b - 3*a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.10774, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(3*a*x^3*Sqrt[a + b*x^2]) + (4*A*b - 3*a*B)/(3*a^2*x*Sqrt[a +
b*x^2]) + (2*b*(4*A*b - 3*a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 11.4035, size = 75, normalized size = 0.91

$$-\frac{A}{3ax^3\sqrt{a+bx^2}}+\frac{4Ab-3Ba}{3a^2x\sqrt{a+bx^2}}+\frac{2bx\,(4Ab-3Ba)}{3a^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(3*a*x**3*sqrt(a + b*x**2)) + (4*A*b - 3*B*a)/(3*a**2*x*sqrt(a
 + b*x**2)) + 2*b*x*(4*A*b - 3*B*a)/(3*a**3*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0626492, size = 61, normalized size = 0.74

$$\frac{-a^2\left(A+3Bx^2\right)+a\left(4Abx^2-6bBx^4\right)+8Ab^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] (8*A*b^2*x^4 - a^2*(A + 3*B*x^2) + a*(4*A*b*x^2 - 6*b*B*x^4))/(3*
a^3*x^3*Sqrt[a + b*x^2])
```

Maple [A] time = 0.007, size = 58, normalized size = 0.7

$$-\frac{-8\,Ab^2x^4+6\,Babx^4-4\,aAbx^2+3\,Ba^2x^2+Aa^2}{3\,x^3a^3}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-8*A*b^2*x^4+6*B*a*b*x^4-4*A*a*b*x^2+3*B*a^2*x^2+A*a^2)/(b*
x^2+a)^(1/2)/x^3/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231173, size = 92, normalized size = 1.12

$$-\frac{\left(2\left(3\,Bab - 4\,Ab^2\right)x^4 + Aa^2 + \left(3\,Ba^2 - 4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3bx^5 + a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*a*b - 4*A*b^2)*x^4 + A*a^2 + (3*B*a^2 - 4*A*a*b)*x^2
)*sqrt(b*x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [A] time = 25.6826, size = 284, normalized size = 3.46

$$A\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4
*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*
x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**
6) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 +
 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x
**6)) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(
a**2*sqrt(a/(b*x**2) + 1)))
```

GIAC/XCAS [A] time = 0.240637, size = 244, normalized size = 2.98

$$-\frac{\left(Bab-Ab^{2}\right)x}{\sqrt{bx^{2}+a}a^{3}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba\sqrt{b}-3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{2}\sqrt{b}+12\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aab^{\frac{3}{2}}+\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*x/(sqrt(b*x^2 + a)*a^3) + 2/3*(3*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*B*a*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
A*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 12*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 5
*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

3.580 $\int \frac{A+Bx^2}{x^5(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{3\sqrt{a+bx^2}(5Ab-4aB)}{8a^3x^2}-\frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

```
[Out] -A/(4*a*x^4*Sqrt[a + b*x^2]) - (5*A*b - 4*a*B)/(4*a^2*x^2*Sqrt[a
+ b*x^2]) + (3*(5*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^3*x^2) - (3*
b*(5*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] time = 0.242107, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{3\sqrt{a+bx^2}(5Ab-4aB)}{8a^3x^2}-\frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(4*a*x^4*Sqrt[a + b*x^2]) - (5*A*b - 4*a*B)/(4*a^2*x^2*Sqrt[a
+ b*x^2]) + (3*(5*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^3*x^2) - (3*
b*(5*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

Rubi in Sympy [A] time = 20.4755, size = 114, normalized size = 0.95

$$-\frac{A}{4ax^4\sqrt{a+bx^2}}-\frac{5Ab-4Ba}{4a^2x^2\sqrt{a+bx^2}}+\frac{3\sqrt{a+bx^2}\,(5Ab-4Ba)}{8a^3x^2}-\frac{3b\,(5Ab-4Ba)\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(4*a*x**4*sqrt(a + b*x**2)) - (5*A*b - 4*B*a)/(4*a**2*x**2*sqr
t(a + b*x**2)) + 3*sqrt(a + b*x**2)*(5*A*b - 4*B*a)/(8*a**3*x**2)
 - 3*b*(5*A*b - 4*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(8*a**(7/2
))
```

Mathematica [A] time = 0.275984, size = 115, normalized size = 0.96

$$\frac{\frac{\sqrt{a}\left(-2a^2\left(A+2Bx^2\right)+abx^2\left(5A-12Bx^2\right)+15Ab^2x^4\right)}{x^4\sqrt{a+bx^2}}+3b(4aB-5Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+3b\log(x)(5Ab-4aB)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[a]*(15*A*b^2*x^4 + a*b*x^2*(5*A - 12*B*x^2) - 2*a^2*(A + 2
*B*x^2)))/(x^4*Sqrt[a + b*x^2]) + 3*b*(5*A*b - 4*a*B)*Log[x] + 3*
b*(-5*A*b + 4*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(8*a^(7/2))
```

Maple [A] time = 0.015, size = 153, normalized size = 1.3

$$-\frac{A}{4\,a x^4}\frac{1}{\sqrt{b x^2+a}}+\frac{5\,Ab}{8\,a^2x^2}\frac{1}{\sqrt{b x^2+a}}+\frac{15\,b^2A}{8\,a^3}\frac{1}{\sqrt{b x^2+a}}-\frac{15\,b^2A}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{B}{2\,a x^2}\frac{1}{\sqrt{b x^2+a}}-\frac{3\,Bb}{2\,a^2}\frac{1}{\sqrt{b x^2+a}}+\frac{3\,Bb}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/4*A/a/x^4/(b*x^2+a)^(1/2)+5/8*A*b/a^2/x^2/(b*x^2+a)^(1/2)+15/8
*A*b^2/a^3/(b*x^2+a)^(1/2)-15/8*A*b^2/a^(7/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)-1/2*B/a/x^2/(b*x^2+a)^(1/2)-3/2*B*b/a^2/(b*x^2
+a)^(1/2)+3/2*B*b/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245352, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(3\left(4\,Bab-5\,Ab^2\right)x^4+2\,Aa^2+\left(4\,Ba^2-5\,Aab\right)x^2\right)\sqrt{b x^2+a}\sqrt{a}+3\left(\left(4\,Bab^2-5\,Ab^3\right)x^6+\left(4\,Ba^2b-5\,Aab^2\right)x^4\right)\log\left(-\right.}{16\left(a^3bx^6+a^4x^4\right)\sqrt{a}},\ -\frac{\left(3\left(4\,Bab-5\,Ab^2\right)x^4+2\,Aa^2+\left(4\,Ba^2-5\,Aab\right)x^2\right)\sqrt{b x^2+a}\sqrt{-a}-3\left(\left(4\,Bab^2-5\,Ab^3\right)x^6+\left(4\,Ba^2b-5\,Aab^2\right)x^4\right)\arctan}{8\left(a^3bx^6+a^4x^4\right)\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(3*(4*B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + (4*B*a^2 - 5*A*a
*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a) + 3*((4*B*a*b^2 - 5*A*b^3)*x^6 +
 (4*B*a^2*b - 5*A*a*b^2)*x^4)*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqr
t(b*x^2 + a)*a)/x^2))/((a^3*b*x^6 + a^4*x^4)*sqrt(a)), -1/8*((3*(
4*B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + (4*B*a^2 - 5*A*a*b)*x^2)*sqrt(
b*x^2 + a)*sqrt(-a) - 3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b -
 5*A*a*b^2)*x^4)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/((a^3*b*x^6 +
a^4*x^4)*sqrt(-a))]
```

Sympy [A] time = 56.0351, size = 180, normalized size = 1.5

$$A\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2
*x**3*sqrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**
2) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(
-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*
sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2
)))
```

GIAC/XCAS [A] time = 0.233742, size = 185, normalized size = 1.54

$$-\frac{3\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3}-\frac{Bab-Ab^2}{\sqrt{bx^2+a}a^3}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^2+a}Ba^2b-7\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^2+a}Aab^2}{8\,a^3b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^5),x, algorithm="giac")
```

```
[Out] -3/8*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^3) - (B*a*b - A*b^2)/(sqrt(b*x^2 + a)*a^3) - 1/8*(4*(b*x^2 +
 a)^(3/2)*B*a*b - 4*sqrt(b*x^2 + a)*B*a^2*b - 7*(b*x^2 + a)^(3/2)
*A*b^2 + 9*sqrt(b*x^2 + a)*A*a*b^2)/(a^3*b^2*x^4)
```

3.581 $\int \frac{A+Bx^2}{x^6(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^2]) + (6*A*b - 5*a*B)/(15*a^2*x^3*Sqrt[a
 + b*x^2]) - (4*b*(6*A*b - 5*a*B))/(15*a^3*x*Sqrt[a + b*x^2]) - (
8*b^2*(6*A*b - 5*a*B)*x)/(15*a^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.145938, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^2]) + (6*A*b - 5*a*B)/(15*a^2*x^3*Sqrt[a
 + b*x^2]) - (4*b*(6*A*b - 5*a*B))/(15*a^3*x*Sqrt[a + b*x^2]) - (
8*b^2*(6*A*b - 5*a*B)*x)/(15*a^4*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 15.4474, size = 109, normalized size = 0.95

$$-\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5Ba}{15a^2x^3\sqrt{a+bx^2}}-\frac{4b\left(6Ab-5Ba\right)}{15a^3x\sqrt{a+bx^2}}-\frac{8b^2x\left(6Ab-5Ba\right)}{15a^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(5*a*x**5*sqrt(a + b*x**2)) + (6*A*b - 5*B*a)/(15*a**2*x**3*sq
rt(a + b*x**2)) - 4*b*(6*A*b - 5*B*a)/(15*a**3*x*sqrt(a + b*x**2)
) - 8*b**2*x*(6*A*b - 5*B*a)/(15*a**4*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0866188, size = 84, normalized size = 0.73

$$\frac{-a^3\left(3A+5Bx^2\right)+a^2\left(6Abx^2+20bBx^4\right)+8ab^2x^4\left(5Bx^2-3A\right)-48Ab^3x^6}{15a^4x^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-48*A*b^3*x^6 + 8*a*b^2*x^4*(-3*A + 5*B*x^2) - a^3*(3*A + 5*B*x^
2) + a^2*(6*A*b*x^2 + 20*b*B*x^4))/(15*a^4*x^5*Sqrt[a + b*x^2])
```

Maple [A] time = 0.008, size = 83, normalized size = 0.7

$$-\frac{48\,Ab^3x^6-40\,Bab^2x^6+24\,Aab^2x^4-20\,Ba^2bx^4-6\,Aa^2bx^2+5\,Ba^3x^2+3\,Aa^3}{15\,x^5a^4}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/15*(48*A*b^3*x^6-40*B*a*b^2*x^6+24*A*a*b^2*x^4-20*B*a^2*b*x^4-
6*A*a^2*b*x^2+5*B*a^3*x^2+3*A*a^3)/(b*x^2+a)^(1/2)/x^5/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233874, size = 127, normalized size = 1.1

$$\frac{\left(8\left(5\,Bab^{2}-6\,Ab^{3}\right)x^{6}+4\left(5\,Ba^{2}b-6\,Aab^{2}\right)x^{4}-3\,Aa^{3}-\left(5\,Ba^{3}-6\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\left(a^{4}bx^{7}+a^{5}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^6),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*a*b^2 - 6*A*b^3)*x^6 + 4*(5*B*a^2*b - 6*A*a*b^2)*x^4
 - 3*A*a^3 - (5*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b*x^
7 + a^5*x^5)
```

Sympy [A] time = 39.9199, size = 593, normalized size = 5.16

$$A\left(-\frac{a^{5}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{5a^{3}b^{\frac{23}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{30a^{2}b^{\frac{25}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{40ab^{\frac{27}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}\right)+B\left(-\frac{a^{3}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}+\frac{3a^{2}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}+\frac{12ab^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}+\frac{8b^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a*
*6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3
*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*
```

```
b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 30*a**2*b
**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b*
*10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 40*a*b**(27
/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x
**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**
10*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 +
15*a**5*b**11*x**8 + 5*a**4*b**12*x**10)) + B*(-a**3*b**(9/2)*sqr
t(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b
**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b*
*4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x
**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3
*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*
b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6))
```

GIAC/XCAS [A] time = 0.24083, size = 397, normalized size = 3.45

$$\frac{\left(Bab^2 - Ab^3\right)x}{\sqrt{bx^2 + a}a^4} - \frac{2\left(15\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Bab^{\frac{3}{2}} - 15\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Ab^{\frac{5}{2}} - 90\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 Ba^2b^{\frac{3}{2}} + 90\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 Aab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^6),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^4) - 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*B*a*b^(3/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*A*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)
+ 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 160*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 240*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*A*a^2*b^(5/2) - 110*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^
4*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) + 2
5*B*a^5*b^(3/2) - 33*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a
))^2 - a)^5*a^3)
```

3.582 $\int \frac{A+Bx^2}{x^7(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b\sqrt{a+bx^2}(7Ab-6aB)}{16a^4x^2}+\frac{5\sqrt{a+bx^2}(7Ab-6aB)}{24a^3x^4}-\frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^2]) - (7*A*b - 6*a*B)/(6*a^2*x^4*Sqrt[a
+ b*x^2]) + (5*(7*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^3*x^4) - (5
*b*(7*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^4*x^2) + (5*b^2*(7*A*b
- 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(9/2))
```

Rubi [A] time = 0.306121, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b\sqrt{a+bx^2}(7Ab-6aB)}{16a^4x^2}+\frac{5\sqrt{a+bx^2}(7Ab-6aB)}{24a^3x^4}-\frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^2]) - (7*A*b - 6*a*B)/(6*a^2*x^4*Sqrt[a
+ b*x^2]) + (5*(7*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^3*x^4) - (5
*b*(7*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^4*x^2) + (5*b^2*(7*A*b
- 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(9/2))
```

Rubi in Sympy [A] time = 25.6741, size = 148, normalized size = 0.97

$$-\frac{A}{6ax^6\sqrt{a+bx^2}}-\frac{7Ab-6Ba}{6a^2x^4\sqrt{a+bx^2}}+\frac{5\sqrt{a+bx^2}\,(7Ab-6Ba)}{24a^3x^4}-\frac{5b\sqrt{a+bx^2}\,(7Ab-6Ba)}{16a^4x^2}+\frac{5b^2\,(7Ab-6Ba)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(6*a*x**6*sqrt(a + b*x**2)) - (7*A*b - 6*B*a)/(6*a**2*x**4*sqr
t(a + b*x**2)) + 5*sqrt(a + b*x**2)*(7*A*b - 6*B*a)/(24*a**3*x**4
) - 5*b*sqrt(a + b*x**2)*(7*A*b - 6*B*a)/(16*a**4*x**2) + 5*b**2*
(7*A*b - 6*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(16*a**(9/2))
```

Mathematica [A] time = 0.336931, size = 143, normalized size = 0.93

$$\frac{\frac{\sqrt{a}\left(-4a^3(2A+3Bx^2)+2a^2bx^2(7A+15Bx^2)+5ab^2x^4(18Bx^2-7A)-105Ab^3x^6\right)}{x^6\sqrt{a+bx^2}}+15b^2(7Ab-6aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+15b^2\log(x)(6aB-}{48a^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[a]*(-105*A*b^3*x^6 - 4*a^3*(2*A + 3*B*x^2) + 2*a^2*b*x^2*(
7*A + 15*B*x^2) + 5*a*b^2*x^4*(-7*A + 18*B*x^2)))/(x^6*Sqrt[a + b
*x^2]) + 15*b^2*(-7*A*b + 6*a*B)*Log[x] + 15*b^2*(7*A*b - 6*a*B)*
Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(48*a^(9/2))
```

Maple [A] time = 0.018, size = 197, normalized size = 1.3

$$-\frac{A}{6\,ax^6}\frac{1}{\sqrt{bx^2+a}}+\frac{7\,Ab}{24\,a^2x^4}\frac{1}{\sqrt{bx^2+a}}-\frac{35\,b^2A}{48\,a^3x^2}\frac{1}{\sqrt{bx^2+a}}-\frac{35\,Ab^3}{16\,a^4}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,Ab^3}{16}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{9}{2}}-\frac{B}{4\,ax^4}\frac{1}{\sqrt{bx^2+a}}+\frac{5\,Bb}{8\,a^2x^2}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,Bb^2}{8\,a^3}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,Bb^2}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6*A/a/x^6/(b*x^2+a)^(1/2)+7/24*A*b/a^2/x^4/(b*x^2+a)^(1/2)-35/
48*A*b^2/a^3/x^2/(b*x^2+a)^(1/2)-35/16*A*b^3/a^4/(b*x^2+a)^(1/2)+
35/16*A*b^3/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/4*B/a
/x^4/(b*x^2+a)^(1/2)+5/8*B*b/a^2/x^2/(b*x^2+a)^(1/2)+15/8*B*b^2/a
^3/(b*x^2+a)^(1/2)-15/8*B*b^2/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234418, size = 1, normalized size = 0.01

$$\left[\frac{2\left(15\left(6\,Bab^2-7\,Ab^3\right)x^6+5\left(6\,Ba^2b-7\,Aab^2\right)x^4-8\,Aa^3-2\left(6\,Ba^3-7\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}\sqrt{a}-15\left(\left(6\,Bab^3-7\,Ab^4\right)x^8\right.}{96\left(a^4bx^8+a^5x^6\right)\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^7),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(15*(6*B*a*b^2 - 7*A*b^3)*x^6 + 5*(6*B*a^2*b - 7*A*a*b^2
)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a)*sq
rt(a) - 15*((6*B*a*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)
*x^6)*log(-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^2))/((
a^4*b*x^8 + a^5*x^6)*sqrt(a)), 1/48*((15*(6*B*a*b^2 - 7*A*b^3)*x^
6 + 5*(6*B*a^2*b - 7*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 7*A*a^
2*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-a) - 15*((6*B*a*b^3 - 7*A*b^4)*x^
8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*arctan(sqrt(-a)/sqrt(b*x^2 + a
)))/((a^4*b*x^8 + a^5*x^6)*sqrt(-a))]
```

Sympy [A] time = 91.6751, size = 236, normalized size = 1.54

$$A\left(-\frac{1}{6a\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{7\sqrt{b}}{24a^2x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{3}{2}}}{48a^3x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{5}{2}}}{16a^4x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{9}{2}}}\right)$$
$$+B\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + 7*sqrt(b)/(24*a**
2*x**5*sqrt(a/(b*x**2) + 1)) - 35*b**(3/2)/(48*a**3*x**3*sqrt(a/(
b*x**2) + 1)) - 35*b**(5/2)/(16*a**4*x*sqrt(a/(b*x**2) + 1)) + 35
*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(9/2))) + B*(-1/(4*a*sqrt
(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqrt(a/(b
*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b
**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2)))
```

GIAC/XCAS [A] time = 0.231239, size = 243, normalized size = 1.59

$$\frac{5\left(6\,Bab^2-7\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{16\sqrt{-a}a^4}+\frac{Bab^2-Ab^3}{\sqrt{bx^2+a}a^4}$$
$$+\frac{42\left(bx^2+a\right)^{\frac{5}{2}}Bab^2-96\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^2+54\sqrt{bx^2+a}Ba^3b^2-57\left(bx^2+a\right)^{\frac{5}{2}}Ab^3+136\left(bx^2+a\right)^{\frac{3}{2}}Aab^3-87\sqrt{bx^2+a}Aa}{48\,a^4b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^7),x, algorithm="giac")
```

```
[Out] 5/16*(6*B*a*b^2 - 7*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt
(-a)*a^4) + (B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*a^4) + 1/48*(42*(b
*x^2 + a)^(5/2)*B*a*b^2 - 96*(b*x^2 + a)^(3/2)*B*a^2*b^2 + 54*sqr
t(b*x^2 + a)*B*a^3*b^2 - 57*(b*x^2 + a)^(5/2)*A*b^3 + 136*(b*x^2
+ a)^(3/2)*A*a*b^3 - 87*sqrt(b*x^2 + a)*A*a^2*b^3)/(a^4*b^3*x^6)
```

3.583 $\int \frac{A+Bx^2}{x^8(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^2]) + (8*A*b - 7*a*B)/(35*a^2*x^5*Sqrt[a
 + b*x^2]) - (2*b*(8*A*b - 7*a*B))/(35*a^3*x^3*Sqrt[a + b*x^2]) +
 (8*b^2*(8*A*b - 7*a*B))/(35*a^4*x*Sqrt[a + b*x^2]) + (16*b^3*(8*
A*b - 7*a*B)*x)/(35*a^5*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.184789, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^2]) + (8*A*b - 7*a*B)/(35*a^2*x^5*Sqrt[a
 + b*x^2]) - (2*b*(8*A*b - 7*a*B))/(35*a^3*x^3*Sqrt[a + b*x^2]) +
 (8*b^2*(8*A*b - 7*a*B))/(35*a^4*x*Sqrt[a + b*x^2]) + (16*b^3*(8*
A*b - 7*a*B)*x)/(35*a^5*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 20.1098, size = 143, normalized size = 0.97

$$-\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7Ba}{35a^2x^5\sqrt{a+bx^2}}-\frac{2b\,(8Ab-7Ba)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8b^2\,(8Ab-7Ba)}{35a^4x\sqrt{a+bx^2}}+\frac{16b^3x\,(8Ab-7Ba)}{35a^5\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

```
[Out] -A/(7*a*x**7*sqrt(a + b*x**2)) + (8*A*b - 7*B*a)/(35*a**2*x**5*sq
rt(a + b*x**2)) - 2*b*(8*A*b - 7*B*a)/(35*a**3*x**3*sqrt(a + b*x*
*2)) + 8*b**2*(8*A*b - 7*B*a)/(35*a**4*x*sqrt(a + b*x**2)) + 16*b
**3*x*(8*A*b - 7*B*a)/(35*a**5*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.108209, size = 105, normalized size = 0.71

$$\frac{-a^4\left(5A+7Bx^2\right)+2a^3bx^2\left(4A+7Bx^2\right)-8a^2b^2x^4\left(2A+7Bx^2\right)+16ab^3x^6\left(4A-7Bx^2\right)+128Ab^4x^8}{35a^5x^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] (128*A*b^4*x^8 + 16*a*b^3*x^6*(4*A - 7*B*x^2) - 8*a^2*b^2*x^4*(2*
A + 7*B*x^2) + 2*a^3*b*x^2*(4*A + 7*B*x^2) - a^4*(5*A + 7*B*x^2))
/(35*a^5*x^7*Sqrt[a + b*x^2])
```

Maple [A] time = 0.01, size = 107, normalized size = 0.7

$$-\frac{-128\,A b^{4} x^{8}+112\,B a b^{3} x^{8}-64\,A a b^{3} x^{6}+56\,B a^{2} b^{2} x^{6}+16\,A a^{2} b^{2} x^{4}-14\,B a^{3} b x^{4}-8\,A a^{3} b x^{2}+7\,B a^{4} x^{2}+5\,A a^{4}}{35\,x^{7} a^{5}}\frac{1}{\sqrt{b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*(-128*A*b^4*x^8+112*B*a*b^3*x^8-64*A*a*b^3*x^6+56*B*a^2*b^2
*x^6+16*A*a^2*b^2*x^4-14*B*a^3*b*x^4-8*A*a^3*b*x^2+7*B*a^4*x^2+5*
A*a^4)/(b*x^2+a)^(1/2)/x^7/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289299, size = 158, normalized size = 1.07

$$-\frac{\left(16\,\left(7\,B a b^{3}-8\,A b^{4}\right) x^{8}+8\,\left(7\,B a^{2} b^{2}-8\,A a b^{3}\right) x^{6}+5\,A a^{4}-2\,\left(7\,B a^{3} b-8\,A a^{2} b^{2}\right) x^{4}+\left(7\,B a^{4}-8\,A a^{3} b\right) x^{2}\right) \sqrt{b x^{2}+a}}{35\,\left(a^{5} b x^{9}+a^{6} x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^8),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*a*b^3 - 8*A*b^4)*x^8 + 8*(7*B*a^2*b^2 - 8*A*a*b^3)
*x^6 + 5*A*a^4 - 2*(7*B*a^3*b - 8*A*a^2*b^2)*x^4 + (7*B*a^4 - 8*A
*a^3*b)*x^2)*sqrt(b*x^2 + a)/(a^5*b*x^9 + a^6*x^7)
```

Sympy [A] time = 83.8316, size = 1030, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-5*a**7*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 1
40*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12
+ 35*a**5*b**20*x**14) - 7*a**6*b**(35/2)*x**2*sqrt(a/(b*x**2) +
1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**
10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) - 7*a**5*b**(37/
2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17
*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**
20*x**14) + 35*a**4*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*
b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**
6*b**19*x**12 + 35*a**5*b**20*x**14) + 280*a**3*b**(41/2)*x**8*sq
rt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 21
0*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14)
+ 560*a**2*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x*
```

```
*6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*
x**12 + 35*a**5*b**20*x**14) + 448*a*b**(45/2)*x**12*sqrt(a/(b*x*
*2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**
18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 128*b**(
47/2)*x**14*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b
**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5
*b**20*x**14)) + B*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*
b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**1
2*x**10) - 5*a**3*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(5*a**7*b**
9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x
**10) - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*
x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**
10) - 40*a*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4
+ 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) -
 16*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a
**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10))
```

GIAC/XCAS [A] time = 0.250755, size = 549, normalized size = 3.71

$$-\frac{\left(Bab^3 - Ab^4\right)x}{\sqrt{bx^2+a}a^5} + \frac{2\left(35\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{12}Bab^{\frac{5}{2}} - 35\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{12}Ab^{\frac{7}{2}} - 280\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{5}{2}} + 280\left(\sqrt{b}x - \sqrt{bx^2+a}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^8),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*x/(sqrt(b*x^2 + a)*a^5) + 2/35*(35*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*B*a*b^(5/2) - 35*(sqrt(b)*x - sqrt(b*x^2 +
a))^12*A*b^(7/2) - 280*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(
5/2) + 280*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1015*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 1015*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 1680*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*B*a^4*b^(5/2) + 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*
b^(7/2) + 1337*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 16
73*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 504*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*B*a^6*b^(5/2) + 616*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*A*a^5*b^(7/2) + 77*B*a^7*b^(5/2) - 93*A*a^6*b^(7/2))/(((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^4)
```

3.584 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}(Ab-2aB)}{b^5}+\frac{(a+bx^2)^{3/2}(Ab-4aB)}{3b^5}+\frac{B(a+bx^2)^{5/2}}{5b^5}$$

```
[Out] (a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (a^2*(3*A*b - 4*a*B
))/(b^5*Sqrt[a + b*x^2]) - (3*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^
5 + ((A*b - 4*a*B)*(a + b*x^2)^(3/2))/(3*b^5) + (B*(a + b*x^2)^(5
/2))/(5*b^5)
```

Rubi [A] time = 0.275654, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}(Ab-2aB)}{b^5}+\frac{(a+bx^2)^{3/2}(Ab-4aB)}{3b^5}+\frac{B(a+bx^2)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (a^2*(3*A*b - 4*a*B
))/(b^5*Sqrt[a + b*x^2]) - (3*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^
5 + ((A*b - 4*a*B)*(a + b*x^2)^(3/2))/(3*b^5) + (B*(a + b*x^2)^(5
/2))/(5*b^5)
```

Rubi in Sympy [A] time = 31.7032, size = 117, normalized size = 0.91

$$\frac{B(a+bx^2)^{\frac{5}{2}}}{5b^5}+\frac{a^3(Ab-Ba)}{3b^5(a+bx^2)^{\frac{3}{2}}}-\frac{a^2(3Ab-4Ba)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}(Ab-2Ba)}{b^5}+\frac{(a+bx^2)^{\frac{3}{2}}(Ab-4Ba)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*(a + b*x**2)**(5/2)/(5*b**5) + a**3*(A*b - B*a)/(3*b**5*(a + b*
x**2)**(3/2)) - a**2*(3*A*b - 4*B*a)/(b**5*sqrt(a + b*x**2)) - 3*
a*sqrt(a + b*x**2)*(A*b - 2*B*a)/b**5 + (a + b*x**2)**(3/2)*(A*b
- 4*B*a)/(3*b**5)
```

Mathematica [A] time = 0.100143, size = 98, normalized size = 0.77

$$\frac{128a^4B+a^3(192bBx^2-80Ab)+24a^2b^2x^2(2Bx^2-5A)-2ab^3x^4(15A+4Bx^2)+b^4x^6(5A+3Bx^2)}{15b^5(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (128*a^4*B + 24*a^2*b^2*x^2*(-5*A + 2*B*x^2) + b^4*x^6*(5*A + 3*B
*x^2) - 2*a*b^3*x^4*(15*A + 4*B*x^2) + a^3*(-80*A*b + 192*b*B*x^2
))/(15*b^5*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 101, normalized size = 0.8

$$-\frac{-3 x^8 B b^4 - 5 A b^4 x^6 + 8 B a b^3 x^6 + 30 A a b^3 x^4 - 48 B a^2 b^2 x^4 + 120 A a^2 b^2 x^2 - 192 B a^3 b x^2 + 80 A a^3 b - 128 B a^4}{15 b^5} (b x^2 + a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/15*(-3*B*b^4*x^8-5*A*b^4*x^6+8*B*a*b^3*x^6+30*A*a*b^3*x^4-48*B
*a^2*b^2*x^4+120*A*a^2*b^2*x^2-192*B*a^3*b*x^2+80*A*a^3*b-128*B*a
^4)/(b*x^2+a)^(3/2)/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233815, size = 166, normalized size = 1.3

$$\frac{\left(3 B b^4 x^8 - \left(8 B a b^3 - 5 A b^4\right) x^6 + 128 B a^4 - 80 A a^3 b + 6 \left(8 B a^2 b^2 - 5 A a b^3\right) x^4 + 24 \left(8 B a^3 b - 5 A a^2 b^2\right) x^2\right) \sqrt{b x^2 + a}}{15 \left(b^7 x^4 + 2 a b^6 x^2 + a^2 b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^4*x^8 - (8*B*a*b^3 - 5*A*b^4)*x^6 + 128*B*a^4 - 80*A*
a^3*b + 6*(8*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 24*(8*B*a^3*b - 5*A*a^2
*b^2)*x^2)*sqrt(b*x^2 + a)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

Sympy [A] time = 10.0174, size = 437, normalized size = 3.41

$$\begin{cases} -\frac{80 A a^3 b}{15 a b^5 \sqrt{a+bx^2} + 15 b^6 x^2 \sqrt{a+bx^2}} - \frac{120 A a^2 b^2 x^2}{15 a b^5 \sqrt{a+bx^2} + 15 b^6 x^2 \sqrt{a+bx^2}} - \frac{30 A a b^3 x^4}{15 a b^5 \sqrt{a+bx^2} + 15 b^6 x^2 \sqrt{a+bx^2}} + \frac{5 A b^4 x^6}{15 a b^5 \sqrt{a+bx^2} + 15 b^6 x^2 \sqrt{a+bx^2}} + \frac{1}{15 a b^5 \sqrt{a+bx}} \\ \frac{\frac{A x^8}{8} + \frac{B x^{10}}{10}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-80*A*a**3*b/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**
2*sqrt(a + b*x**2)) - 120*A*a**2*b**2*x**2/(15*a*b**5*sqrt(a + b*
x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) - 30*A*a*b**3*x**4/(15*a*b
**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 5*A*b**4*
x**6/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2))
 + 128*B*a**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a +
 b*x**2)) + 192*B*a**3*b*x**2/(15*a*b**5*sqrt(a + b*x**2) + 15*b*
*6*x**2*sqrt(a + b*x**2)) + 48*B*a**2*b**2*x**4/(15*a*b**5*sqrt(a
 + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) - 8*B*a*b**3*x**6/(15
```

```
*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 3*B*b
**4*x**8/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x*
*2)), Ne(b, 0)), ((A*x**8/8 + B*x**10/10)/a**(5/2), True))
```

GIAC/XCAS [A] time = 0.230089, size = 167, normalized size = 1.3

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}B-20\left(bx^2+a\right)^{\frac{3}{2}}Ba+90\sqrt{bx^2+a}Ba^2+5\left(bx^2+a\right)^{\frac{3}{2}}Ab-45\sqrt{bx^2+a}Aab+\frac{5\left(12\left(bx^2+a\right)Ba^3-Ba^4-9\left(bx^2+a\right)Aa^2b\right.}{\left(bx^2+a\right)^{\frac{3}{2}}}}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 20*(b*x^2 + a)^(3/2)*B*a + 90*sqrt(
b*x^2 + a)*B*a^2 + 5*(b*x^2 + a)^(3/2)*A*b - 45*sqrt(b*x^2 + a)*A
*a*b + 5*(12*(b*x^2 + a)*B*a^3 - B*a^4 - 9*(b*x^2 + a)*A*a^2*b +
A*a^3*b)/(b*x^2 + a)^(3/2))/b^5
```

3.585 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{5x\sqrt{a+bx^2}(4Ab-7aB)}{8b^4}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}-\frac{x^5(4Ab-7aB)}{12b^2(a+bx^2)^{3/2}}+\frac{Bx^7}{4b(a+bx^2)^{3/2}}$$

```
[Out] -((4*A*b - 7*a*B)*x^5)/(12*b^2*(a + b*x^2)^(3/2)) + (B*x^7)/(4*b*
(a + b*x^2)^(3/2)) - (5*(4*A*b - 7*a*B)*x^3)/(12*b^3*Sqrt[a + b*x
^2]) + (5*(4*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(8*b^4) - (5*a*(4*A*
b - 7*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] time = 0.205493, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{5x\sqrt{a+bx^2}(4Ab-7aB)}{8b^4}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}-\frac{x^5(4Ab-7aB)}{12b^2(a+bx^2)^{3/2}}+\frac{Bx^7}{4b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -((4*A*b - 7*a*B)*x^5)/(12*b^2*(a + b*x^2)^(3/2)) + (B*x^7)/(4*b*
(a + b*x^2)^(3/2)) - (5*(4*A*b - 7*a*B)*x^3)/(12*b^3*Sqrt[a + b*x
^2]) + (5*(4*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(8*b^4) - (5*a*(4*A*
b - 7*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi in Sympy [A] time = 23.2873, size = 144, normalized size = 0.97

$$\frac{Bx^7}{4b\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{5a\left(4Ab-7Ba\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{9}{2}}}-\frac{x^5\left(4Ab-7Ba\right)}{12b^2\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{5x^3\left(4Ab-7Ba\right)}{12b^3\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}\left(4Ab-7Ba\right)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*x**7/(4*b*(a + b*x**2)**(3/2)) - 5*a*(4*A*b - 7*B*a)*atanh(sqrt
(b)*x/sqrt(a + b*x**2))/(8*b**(9/2)) - x**5*(4*A*b - 7*B*a)/(12*b
**2*(a + b*x**2)**(3/2)) - 5*x**3*(4*A*b - 7*B*a)/(12*b**3*sqrt(a
 + b*x**2)) + 5*x*sqrt(a + b*x**2)*(4*A*b - 7*B*a)/(8*b**4)
```

Mathematica [A] time = 0.175981, size = 120, normalized size = 0.81

$$\frac{-105a^3Bx+20a^2bx\left(3A-7Bx^2\right)+ab^2x^3\left(80A-21Bx^2\right)+6b^3x^5\left(2A+Bx^2\right)}{24b^4\left(a+bx^2\right)^{3/2}}+\frac{5a(7aB-4Ab)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-105*a^3*B*x + a*b^2*x^3*(80*A - 21*B*x^2) + 20*a^2*b*x*(3*A - 7
*B*x^2) + 6*b^3*x^5*(2*A + B*x^2))/(24*b^4*(a + b*x^2)^(3/2)) + (
5*a*(-4*A*b + 7*a*B)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(9/
2))
```

Maple [A] time = 0.011, size = 181, normalized size = 1.2

$$\frac{Ax^5}{2b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5\,aAx^3}{6\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5\,aAx}{2\,b^3}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,Aa}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{x^7B}{4\,b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{7\,Bax^5}{8\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{35\,a^2Bx^3}{24\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{35\,Bxa^2}{8\,b^4}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,a^2B}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*A*x^5/b/(b*x^2+a)^(3/2)+5/6*A*a/b^2*x^3/(b*x^2+a)^(3/2)+5/2*A
*a/b^3*x/(b*x^2+a)^(1/2)-5/2*A*a/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))+1/4*B*x^7/b/(b*x^2+a)^(3/2)-7/8*B*a/b^2*x^5/(b*x^2+a)^(3/2)
-35/24*B*a^2/b^3*x^3/(b*x^2+a)^(3/2)-35/8*B*a^2/b^4*x/(b*x^2+a)^(
1/2)+35/8*B*a^2/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29066, size = 1, normalized size = 0.01

$$\left[\frac{2\left(6\,Bb^3x^7-3\left(7\,Bab^2-4\,Ab^3\right)x^5-20\left(7\,Ba^2b-4\,Aab^2\right)x^3-15\left(7\,Ba^3-4\,Aa^2b\right)x\right)\sqrt{bx^2+a}\sqrt{b}-15\left(7\,Ba^4-4\,Aa^3b+\right.}{48\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*B*b^3*x^7 - 3*(7*B*a*b^2 - 4*A*b^3)*x^5 - 20*(7*B*a^2
*b - 4*A*a*b^2)*x^3 - 15*(7*B*a^3 - 4*A*a^2*b)*x)*sqrt(b*x^2 + a)
*sqrt(b) - 15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*x^
4 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x^2)*log(2*sqrt(b*x^2 + a)*b*x -
(2*b*x^2 + a)*sqrt(b)))/((b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*sqrt(b
)), 1/24*((6*B*b^3*x^7 - 3*(7*B*a*b^2 - 4*A*b^3)*x^5 - 20*(7*B*a^
2*b - 4*A*a*b^2)*x^3 - 15*(7*B*a^3 - 4*A*a^2*b)*x)*sqrt(b*x^2 + a
)*sqrt(-b) + 15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*
x^4 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x
```

```
^2 + a)))/((b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*sqrt(-b))]
```

Sympy [A] time = 108.445, size = 804, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a)
)/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/
2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*
x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 +
b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a
**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a*
*(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)*x
**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(5
3/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/
2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt
(1 + b*x**2/a))) + B*(105*a**(157/2)*b**41*sqrt(1 + b*x**2/a)*asi
nh(sqrt(b)*x/sqrt(a))/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a)
 + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) + 105*a**(155
/2)*b**42*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(24*a*
*(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x
**2*sqrt(1 + b*x**2/a)) - 105*a**78*b**(83/2)*x/(24*a**(153/2)*b*
*(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1
+ b*x**2/a)) - 140*a**77*b**(85/2)*x**3/(24*a**(153/2)*b**(91/2)*
sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2
/a)) - 21*a**76*b**(87/2)*x**5/(24*a**(153/2)*b**(91/2)*sqrt(1 +
b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) + 6*
a**75*b**(89/2)*x**7/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a)
+ 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.232782, size = 200, normalized size = 1.34

$$\frac{\left(\left(3\left(\frac{2\,Bx^2}{b}-\frac{7\,Ba^2b^5-4\,Aab^6}{ab^7}\right)x^2-\frac{20\left(7\,Ba^3b^4-4\,Aa^2b^5\right)}{ab^7}\right)x^2-\frac{15\left(7\,Ba^4b^3-4\,Aa^3b^4\right)}{ab^7}\right)x}{24\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\left(7\,Ba^2-4\,Aab\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*B*x^2/b - (7*B*a^2*b^5 - 4*A*a*b^6)/(a*b^7))*x^2 - 20
*(7*B*a^3*b^4 - 4*A*a^2*b^5)/(a*b^7))*x^2 - 15*(7*B*a^4*b^3 - 4*A
*a^3*b^4)/(a*b^7))*x/(b*x^2 + a)^(3/2) - 5/8*(7*B*a^2 - 4*A*a*b)*
ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.586 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}(Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

```
[Out] -(a^2*(A*b - a*B))/(3*b^4*(a + b*x^2)^(3/2)) + (a*(2*A*b - 3*a*B)
)/(b^4*Sqrt[a + b*x^2]) + ((A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (
B*(a + b*x^2)^(3/2))/(3*b^4)
```

Rubi [A] time = 0.223697, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}(Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a^2*(A*b - a*B))/(3*b^4*(a + b*x^2)^(3/2)) + (a*(2*A*b - 3*a*B)
)/(b^4*Sqrt[a + b*x^2]) + ((A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (
B*(a + b*x^2)^(3/2))/(3*b^4)
```

Rubi in Sympy [A] time = 25.2659, size = 88, normalized size = 0.91

$$\frac{B(a+bx^2)^{\frac{3}{2}}}{3b^4}-\frac{a^2(Ab-Ba)}{3b^4(a+bx^2)^{\frac{3}{2}}}+\frac{a(2Ab-3Ba)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}(Ab-3Ba)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*(a + b*x**2)**(3/2)/(3*b**4) - a**2*(A*b - B*a)/(3*b**4*(a + b*
x**2)**(3/2)) + a*(2*A*b - 3*B*a)/(b**4*sqrt(a + b*x**2)) + sqrt(
a + b*x**2)*(A*b - 3*B*a)/b**4
```

Mathematica [A] time = 0.0813512, size = 73, normalized size = 0.75

$$\frac{-16a^3B+8a^2b(A-3Bx^2)-6ab^2x^2(Bx^2-2A)+b^3x^4(3A+Bx^2)}{3b^4(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-16*a^3*B + 8*a^2*b*(A - 3*B*x^2) - 6*a*b^2*x^2*(-2*A + B*x^2) +
 b^3*x^4*(3*A + B*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.008, size = 76, normalized size = 0.8

$$\frac{x^6Bb^3+3\,Ab^3x^4-6\,Bab^2x^4+12\,Aab^2x^2-24\,Ba^2bx^2+8\,Aa^2b-16\,Ba^3}{3\,b^4}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(B*b^3*x^6+3*A*b^3*x^4-6*B*a*b^2*x^4+12*A*a*b^2*x^2-24*B*a^2*
b*x^2+8*A*a^2*b-16*B*a^3)/(b*x^2+a)^(3/2)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23518, size = 132, normalized size = 1.36

$$\frac{\left(Bb^3x^6 - 3\left(2\,Bab^2 - Ab^3\right)x^4 - 16\,Ba^3 + 8\,Aa^2b - 12\left(2\,Ba^2b - Aab^2\right)x^2\right)\sqrt{bx^2 + a}}{3\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^3*x^6 - 3*(2*B*a*b^2 - A*b^3)*x^4 - 16*B*a^3 + 8*A*a^2*b
 - 12*(2*B*a^2*b - A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4)
```

Sympy [A] time = 6.3404, size = 337, normalized size = 3.47

$$\begin{cases} \frac{8Aa^2b}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{12Aab^2x^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{3Ab^3x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{16Ba^3}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{24Ba^2bx^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*A*a**2*b/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sq
rt(a + b*x**2)) + 12*A*a*b**2*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3
*b**5*x**2*sqrt(a + b*x**2)) + 3*A*b**3*x**4/(3*a*b**4*sqrt(a + b
*x**2) + 3*b**5*x**2*sqrt(a + b*x**2)) - 16*B*a**3/(3*a*b**4*sqrt
(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x**2)) - 24*B*a**2*b*x**2/(
3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x**2)) - 6*B*a
*b**2*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x*
*2)) + B*b**3*x**6/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(
a + b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/a**(5/2), True))
```

GIAC/XCAS [A] time = 0.231641, size = 124, normalized size = 1.28

$$\frac{\left(bx^2 + a\right)^{\frac{3}{2}}B - 9\sqrt{bx^2 + a}Ba + 3\sqrt{bx^2 + a}Ab - \frac{9\left(bx^2+a\right)Ba^2 - Ba^3 - 6\left(bx^2+a\right)Aab + Aa^2b}{\left(bx^2+a\right)^{\frac{3}{2}}}}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2)*B - 9*sqrt(b*x^2 + a)*B*a + 3*sqrt(b*x^2 +
 a)*A*b - (9*(b*x^2 + a)*B*a^2 - B*a^3 - 6*(b*x^2 + a)*A*a*b + A*
a^2*b)/(b*x^2 + a)^(3/2))/b^4
```

3.587 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

```
[Out] (a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - ((4*A*b - 7*a*B)*x)
/(3*b^3*Sqrt[a + b*x^2]) + (B*x*Sqrt[a + b*x^2])/(2*b^3) + ((2*A*
b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] time = 0.232127, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - ((4*A*b - 7*a*B)*x)
/(3*b^3*Sqrt[a + b*x^2]) + (B*x*Sqrt[a + b*x^2])/(2*b^3) + ((2*A*
b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi in Sympy [A] time = 43.9397, size = 105, normalized size = 0.92

$$\frac{Bx\sqrt{a+bx^2}}{2b^3}+\frac{ax\left(Ab-Ba\right)}{3b^3\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x\left(4Ab-7Ba\right)}{3b^3\sqrt{a+bx^2}}+\frac{(2Ab-5Ba)\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*x*sqrt(a + b*x**2)/(2*b**3) + a*x*(A*b - B*a)/(3*b**3*(a + b*x*
*2)**(3/2)) - x*(4*A*b - 7*B*a)/(3*b**3*sqrt(a + b*x**2)) + (2*A*
b - 5*B*a)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(7/2))
```

Mathematica [A] time = 0.130437, size = 98, normalized size = 0.86

$$\frac{x\left(15a^2B+a\left(20bBx^2-6Ab\right)+b^2x^2\left(3Bx^2-8A\right)\right)}{6b^3\left(a+bx^2\right)^{3/2}}+\frac{(2Ab-5aB)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(15*a^2*B + b^2*x^2*(-8*A + 3*B*x^2) + a*(-6*A*b + 20*b*B*x^2)
))/(6*b^3*(a + b*x^2)^(3/2)) + ((2*A*b - 5*a*B)*Log[b*x + Sqrt[b]
*Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Maple [A] time = 0.012, size = 134, normalized size = 1.2

$$-\frac{Ax^3}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Ax}{b^2}\frac{1}{\sqrt{bx^2+a}}+A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{x^5B}{2b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5\,Bax^3}{6\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5\,Bxa}{2\,b^3}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,Ba}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*A*x^3/b/(b*x^2+a)^(3/2)-A/b^2*x/(b*x^2+a)^(1/2)+A/b^(5/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*B*x^5/b/(b*x^2+a)^(3/2)+5/6*B*a/b
^2*x^3/(b*x^2+a)^(3/2)+5/2*B*a/b^3*x/(b*x^2+a)^(1/2)-5/2*B*a/b^(7
/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2575, size = 1, normalized size = 0.01

$$\left[\frac{2\left(3\,Bb^2x^5+4\left(5\,Bab-2\,Ab^2\right)x^3+3\left(5\,Ba^2-2\,Aab\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(\left(5\,Bab^2-2\,Ab^3\right)x^4+5\,Ba^3-2\,Aa^2b+2\left(5\,Ba^2b\right.\right.}{12\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(3*B*b^2*x^5 + 4*(5*B*a*b - 2*A*b^2)*x^3 + 3*(5*B*a^2 -
2*A*a*b)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*((5*B*a*b^2 - 2*A*b^3)*x^
4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^2)*log(-2*s
qrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^5*x^4 + 2*a*b^4*
x^2 + a^2*b^3)*sqrt(b)), 1/6*((3*B*b^2*x^5 + 4*(5*B*a*b - 2*A*b^2
)*x^3 + 3*(5*B*a^2 - 2*A*a*b)*x)*sqrt(b*x^2 + a)*sqrt(-b) - 3*((5
*B*a*b^2 - 2*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*
A*a*b^2)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^5*x^4 + 2*a
*b^4*x^2 + a^2*b^3)*sqrt(-b))]
```

Sympy [A] time = 65.9155, size = 675, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)
```

```
*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**
2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x
**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19
*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37
/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(
3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*
x**2*sqrt(1 + b*x**2/a))) + B*(-15*a**(81/2)*b**22*sqrt(1 + b*x**
2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x
**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(
79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a
**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**
2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/
2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**
2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b
*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**
38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a
**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.235055, size = 151, normalized size = 1.32

$$\frac{\left(\left(\frac{3\,Bx^2}{b}+\frac{4\,\left(5\,Ba^2b^3-2\,Aab^4\right)}{ab^5}\right)x^2+\frac{3\,\left(5\,Ba^3b^2-2\,Aa^2b^3\right)}{ab^5}\right)x}{6\,\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{(5\,Ba-2\,Ab)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*B*x^2/b + 4*(5*B*a^2*b^3 - 2*A*a*b^4)/(a*b^5))*x^2 + 3*(5
*B*a^3*b^2 - 2*A*a^2*b^3)/(a*b^5))*x/(b*x^2 + a)^(3/2) + 1/2*(5*B
*a - 2*A*b)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.588 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

```
[Out] (a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (A*b - 2*a*B)/(b^3*Sq
rt[a + b*x^2]) + (B*Sqrt[a + b*x^2])/b^3
```

Rubi [A] time = 0.171386, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (A*b - 2*a*B)/(b^3*Sq
rt[a + b*x^2]) + (B*Sqrt[a + b*x^2])/b^3
```

Rubi in Sympy [A] time = 19.6958, size = 60, normalized size = 0.88

$$\frac{B\sqrt{a+bx^2}}{b^3}+\frac{a(Ab-Ba)}{3b^3(a+bx^2)^{\frac{3}{2}}}-\frac{Ab-2Ba}{b^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] B*sqrt(a + b*x**2)/b**3 + a*(A*b - B*a)/(3*b**3*(a + b*x**2)**(3/
2)) - (A*b - 2*B*a)/(b**3*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0640779, size = 54, normalized size = 0.79

$$\frac{8a^2B-2ab\left(A-6Bx^2\right)+3b^2x^2\left(Bx^2-A\right)}{3b^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (8*a^2*B - 2*a*b*(A - 6*B*x^2) + 3*b^2*x^2*(-A + B*x^2))/(3*b^3*(
a + b*x^2)^(3/2))
```

Maple [A] time = 0.008, size = 53, normalized size = 0.8

$$-\frac{-3\,b^2Bx^4+3\,Ab^2x^2-12\,Babx^2+2\,abA-8\,a^2B}{3\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-3*B*b^2*x^4+3*A*b^2*x^2-12*B*a*b*x^2+2*A*a*b-8*B*a^2)/(b*x
^2+a)^(3/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226029, size = 101, normalized size = 1.49

$$\frac{\left(3\,Bb^{2}x^{4}+8\,Ba^{2}-2\,Aab+3\,\left(4\,Bab-Ab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{3\,\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b^2*x^4 + 8*B*a^2 - 2*A*a*b + 3*(4*B*a*b - A*b^2)*x^2)*s
qrt(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [A] time = 4.12173, size = 240, normalized size = 3.53

$$\begin{cases} -\frac{2Aab}{3ab^{3}\sqrt{a+bx^{2}}+3b^{4}x^{2}\sqrt{a+bx^{2}}}-\frac{3Ab^{2}x^{2}}{3ab^{3}\sqrt{a+bx^{2}}+3b^{4}x^{2}\sqrt{a+bx^{2}}}+\frac{8Ba^{2}}{3ab^{3}\sqrt{a+bx^{2}}+3b^{4}x^{2}\sqrt{a+bx^{2}}}+\frac{12Babx^{2}}{3ab^{3}\sqrt{a+bx^{2}}+3b^{4}x^{2}\sqrt{a+bx^{2}}}+\frac{3Bb^{2}x^{4}}{3ab^{3}\sqrt{a+bx^{2}}+3b^{4}x^{2}\sqrt{} \\ \frac{\frac{Ax^{4}}{4}+\frac{Bx^{6}}{6}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*a*b/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt
(a + b*x**2)) - 3*A*b**2*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4
*x**2*sqrt(a + b*x**2)) + 8*B*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3
*b**4*x**2*sqrt(a + b*x**2)) + 12*B*a*b*x**2/(3*a*b**3*sqrt(a + b
*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) + 3*B*b**2*x**4/(3*a*b**3*
sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)), Ne(b, 0)), ((A*
x**4/4 + B*x**6/6)/a**(5/2), True))
```

GIAC/XCAS [A] time = 0.231109, size = 82, normalized size = 1.21

$$\frac{3\,\sqrt{bx^{2}+a}B+\frac{6\,\left(bx^{2}+a\right)Ba-Ba^{2}-3\,\left(bx^{2}+a\right)Ab+Aab}{\left(bx^{2}+a\right)^{\frac{3}{2}}}}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(b*x^2 + a)*B + (6*(b*x^2 + a)*B*a - B*a^2 - 3*(b*x^2
+ a)*A*b + A*a*b)/(b*x^2 + a)^(3/2))/b^3
```

3.589 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{x^3(Ab-aB)}{3ab\,(a+bx^2)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}$$

```
[Out] ((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) - (B*x)/(b^2*Sqrt[a +
 b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi [A] time = 0.107159, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x^3(Ab-aB)}{3ab\,(a+bx^2)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) - (B*x)/(b^2*Sqrt[a +
 b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi in Sympy [A] time = 15.7865, size = 66, normalized size = 0.86

$$-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{5}{2}}}+\frac{x^3\,(Ab-Ba)}{3ab\,(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] -B*x/(b**2*sqrt(a + b*x**2)) + B*atanh(sqrt(b)*x/sqrt(a + b*x**2)
)/b**(5/2) + x**3*(A*b - B*a)/(3*a*b*(a + b*x**2)**(3/2))
```

Mathematica [A] time = 0.147081, size = 75, normalized size = 0.97

$$\frac{-3a^2Bx-4abBx^3+Ab^2x^3}{3ab^2\,(a+bx^2)^{3/2}}+\frac{B\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^2*B*x + A*b^2*x^3 - 4*a*b*B*x^3)/(3*a*b^2*(a + b*x^2)^(3/2)
) + (B*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/b^(5/2)
```

Maple [A] time = 0.009, size = 92, normalized size = 1.2

$$-\frac{Ax}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{Ax}{3\,ab}\frac{1}{\sqrt{bx^2+a}}-\frac{x^3B}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{Bx}{b^2}\frac{1}{\sqrt{bx^2+a}}+B\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*A/b*x/(b*x^2+a)^(3/2)+1/3*A/a/b*x/(b*x^2+a)^(1/2)-1/3*B*x^3/
b/(b*x^2+a)^(3/2)-B*x/b^2/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238117, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(3\,Ba^2x + \left(4\,Bab - Ab^2\right)x^3\right)\sqrt{bx^2 + a}\sqrt{b} - 3\left(Bab^2x^4 + 2\,Ba^2bx^2 + Ba^3\right)\log\left(-2\,\sqrt{bx^2 + a}bx - \left(2\,bx^2 + a\right)\sqrt{b}\right)}{6\left(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2\right)\sqrt{b}},\right.$$
$$\left.-\frac{\left(3\,Ba^2x + \left(4\,Bab - Ab^2\right)x^3\right)\sqrt{bx^2 + a}\sqrt{-b} - 3\left(Bab^2x^4 + 2\,Ba^2bx^2 + Ba^3\right)\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{3\left(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2\right)\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(3*B*a^2*x + (4*B*a*b - A*b^2)*x^3)*sqrt(b*x^2 + a)*sqrt
(b) - 3*(B*a*b^2*x^4 + 2*B*a^2*b*x^2 + B*a^3)*log(-2*sqrt(b*x^2 +
 a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a
^3*b^2)*sqrt(b)), -1/3*((3*B*a^2*x + (4*B*a*b - A*b^2)*x^3)*sqrt(
b*x^2 + a)*sqrt(-b) - 3*(B*a*b^2*x^4 + 2*B*a^2*b*x^2 + B*a^3)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^
3*b^2)*sqrt(-b))]
```

Sympy [A] time = 44.1447, size = 352, normalized size = 4.57

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}} + B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1 + \frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1 + \frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1
+ b*x**2/a)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt
(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(
37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2
*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/
2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**
2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b
**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(3
7/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.236512, size = 93, normalized size = 1.21

$$-\frac{x\left(\frac{3\,Ba}{b^2}+\frac{\left(4\,Bab^2-Ab^3\right)x^2}{ab^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{B\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(3*B*a/b^2 + (4*B*a*b^2 - A*b^3)*x^2/(a*b^3))/(b*x^2 + a)^
(3/2) - B*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.590 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=44

$$-\frac{Ab-aB}{3b^2(a+bx^2)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^2)^(3/2)) - B/(b^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.101192, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{Ab-aB}{3b^2(a+bx^2)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^2)^(3/2)) - B/(b^2*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 13.1774, size = 37, normalized size = 0.84

$$-\frac{B}{b^2\sqrt{a+bx^2}}-\frac{Ab-Ba}{3b^2(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] -B/(b**2*sqrt(a + b*x**2)) - (A*b - B*a)/(3*b**2*(a + b*x**2)**(3
/2))
```

Mathematica [A] time = 0.0264469, size = 34, normalized size = 0.77

$$\frac{-2aB-Ab-3bBx^2}{3b^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(A*b) - 2*a*B - 3*b*B*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.006, size = 30, normalized size = 0.7

$$-\frac{3\,bBx^2+Ab+2\,Ba}{3\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(3*B*b*x^2+A*b+2*B*a)/(b*x^2+a)^(3/2)/b^2
```

Maxima [A] time = 1.35378, size = 68, normalized size = 1.55

$$-\frac{Bx^2}{(bx^2+a)^{\frac{3}{2}}b}-\frac{2\,Ba}{3\,(bx^2+a)^{\frac{3}{2}}b^2}-\frac{A}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2) -
1/3*A/((b*x^2 + a)^(3/2)*b)
```

Fricas [A] time = 0.213887, size = 70, normalized size = 1.59

$$-\frac{\left(3\,Bbx^2+2\,Ba+Ab\right)\sqrt{bx^2+a}}{3\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*B*b*x^2 + 2*B*a + A*b)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3
*x^2 + a^2*b^2)
```

Sympy [A] time = 3.85138, size = 143, normalized size = 3.25

$$\begin{cases} -\frac{Ab}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{2Ba}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3Bbx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{a^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-A*b/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a +
 b*x**2)) - 2*B*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a
 + b*x**2)) - 3*B*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2
*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/a**(5/2), T
rue))
```

GIAC/XCAS [A] time = 0.256997, size = 43, normalized size = 0.98

$$-\frac{3\,\left(bx^2+a\right)B-Ba+Ab}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(b*x^2 + a)*B - B*a + A*b)/((b*x^2 + a)^(3/2)*b^2)
```

3.591 $\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2
)^(3/2))
```

Rubi [A] time = 0.0372268, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2
)^(3/2))
```

Rubi in Sympy [A] time = 6.34653, size = 41, normalized size = 0.87

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] 2*A*x/(3*a**2*sqrt(a + b*x**2)) + x*(A + B*x**2)/(3*a*(a + b*x**2
)**(3/2))
```

Mathematica [A] time = 0.0391493, size = 37, normalized size = 0.79

$$\frac{x\left(3aA+aBx^2+2Abx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*A + 2*A*b*x^2 + a*B*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 34, normalized size = 0.7

$$\frac{x\left(2\,Abx^2+Bax^2+3\,aA\right)}{3\,a^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*A*b*x^2+B*a*x^2+3*A*a)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 1.35526, size = 92, normalized size = 1.96

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2}+\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Bx}{3\,\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1
/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [A] time = 0.210902, size = 73, normalized size = 1.55

$$\frac{\left((Ba+2\,Ab)x^3+3\,Aax\right)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((B*a + 2*A*b)*x^3 + 3*A*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 +
2*a^3*b*x^2 + a^4)
```

Sympy [A] time = 36.0942, size = 144, normalized size = 3.06

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(
1 + b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(
5/2)*b*x**2*sqrt(1 + b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*
x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.233579, size = 54, normalized size = 1.15

$$\frac{x\left(\frac{3\,A}{a}+\frac{(Bab+2\,Ab^2)x^2}{a^2b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*A/a + (B*a*b + 2*A*b^2)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

3.592 $\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{Ab-aB}{3ab\,(a+bx^2)^{3/2}}$$

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^2)^(3/2)) + A/(a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.155064, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{Ab-aB}{3ab\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^2)^(3/2)) + A/(a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rubi in Sympy [A] time = 16.2788, size = 60, normalized size = 0.83

$$\frac{A}{a^2\sqrt{a+bx^2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{Ab-Ba}{3ab\,(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A/(a**2*sqrt(a + b*x**2)) - A*atanh(sqrt(a + b*x**2)/sqrt(a))/a**
(5/2) + (A*b - B*a)/(3*a*b*(a + b*x**2)**(3/2))
```

Mathematica [A] time = 0.260301, size = 79, normalized size = 1.1

$$\frac{\frac{\sqrt{a}(a^2(-B)+4aAb+3Ab^2x^2)}{b(a+bx^2)^{3/2}}-3A\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+3A\log(x)}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[a]*(4*a*A*b - a^2*B + 3*A*b^2*x^2))/(b*(a + b*x^2)^(3/2))
+ 3*A*Log[x] - 3*A*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(3*a^(5/2))
```

Maple [A] time = 0.012, size = 75, normalized size = 1.

$$\frac{A}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{A}{a^2}\frac{1}{\sqrt{bx^2+a}}-A\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}}-\frac{B}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*A/a/(b*x^2+a)^(3/2)+A/a^2/(b*x^2+a)^(1/2)-A/a^(5/2)*ln((2*a+2
*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/3*B/b/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2275, size = 1, normalized size = 0.01

$$\left[\frac{2\left(3\,Ab^2x^2 - Ba^2 + 4\,Aab\right)\sqrt{bx^2+a}\sqrt{a} + 3\left(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b\right)\log\left(-\frac{(bx^2+2\,a)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)}{6\left(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b\right)\sqrt{a}}, \frac{(3\,Ab^2x^2 - Ba^2 + 4\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(3*A*b^2*x^2 - B*a^2 + 4*A*a*b)*sqrt(b*x^2 + a)*sqrt(a) +
 3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log(-((b*x^2 + 2*a)*sqrt
(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 +
a^4*b)*sqrt(a)), 1/3*((3*A*b^2*x^2 - B*a^2 + 4*A*a*b)*sqrt(b*x^2
+ a)*sqrt(-a) - 3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)))/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*s
qrt(-a))]
```

Sympy [A] time = 62.4018, size = 790, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x
**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(
6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a*
*(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/
2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b
**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*
a**6*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a
**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x*
*2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*b**
2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
```

```
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*lo
g(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**
2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b
*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*Piecewise((-1/(3*a*b*sqrt(
a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*
a**(5/2)), True))
```

GIAC/XCAS [A] time = 0.236707, size = 89, normalized size = 1.24

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{Ba^2 - 3\left(bx^2+a\right)Ab - Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(B*a^2 -
3*(b*x^2 + a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a^2*b)
```

3.593 $\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2(a+bx^2)^{3/2}}-\frac{A}{ax(a+bx^2)^{3/2}}$$

```
[Out] -(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*x)/(3*a^2*(a + b*x^
2)^(3/2)) - (2*(4*A*b - a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0933518, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2(a+bx^2)^{3/2}}-\frac{A}{ax(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*x)/(3*a^2*(a + b*x^
2)^(3/2)) - (2*(4*A*b - a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 10.5702, size = 68, normalized size = 0.88

$$-\frac{A}{ax\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x\left(4Ab-Ba\right)}{3a^2\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{2x\left(4Ab-Ba\right)}{3a^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] -A/(a*x*(a + b*x**2)**(3/2)) - x*(4*A*b - B*a)/(3*a**2*(a + b*x**
2)**(3/2)) - 2*x*(4*A*b - B*a)/(3*a**3*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0629823, size = 60, normalized size = 0.78

$$\frac{-3a^2\left(A-Bx^2\right)+2abx^2\left(Bx^2-6A\right)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 - 3*a^2*(A - B*x^2) + 2*a*b*x^2*(-6*A + B*x^2))/(3*
a^3*x*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.007, size = 59, normalized size = 0.8

$$-\frac{8\,Ab^2x^4-2\,Babx^4+12\,aAbx^2-3\,Ba^2x^2+3\,Aa^2}{3\,xa^3}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(8*A*b^2*x^4-2*B*a*b*x^4+12*A*a*b*x^2-3*B*a^2*x^2+3*A*a^2)/(
b*x^2+a)^(3/2)/x/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234176, size = 104, normalized size = 1.35

$$\frac{\left(2\left(Bab-4\,Ab^2\right)x^4-3\,Aa^2+3\left(Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 + 3*(B*a^2 - 4*A*a*b)*x^2)
*sqrt(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

Sympy [A] time = 77.4416, size = 265, normalized size = 3.44

$$A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2)
 + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**
(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2
+ 3*a**3*b**6*x**4)) + B*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) +
3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt
(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

GIAC/XCAS [A] time = 0.231271, size = 136, normalized size = 1.77

$$\frac{x\left(\frac{\left(2\,Ba^3b^2-5\,Aa^2b^3\right)x^2}{a^5b}+\frac{3\left(Ba^4b-2\,Aa^3b^2\right)}{a^5b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] 1/3*x*((2*B*a^3*b^2 - 5*A*a^2*b^3)*x^2/(a^5*b) + 3*(B*a^4*b - 2*A
*a^3*b^2)/(a^5*b))/(b*x^2 + a)^(3/2) + 2*A*sqrt(b)/(((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)*a^2)
```

3.594 $\int \frac{A+Bx^2}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}-\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^
2)^(3/2)) - (5*A*b - 2*a*B)/(2*a^3*Sqrt[a + b*x^2]) + ((5*A*b - 2
*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.240657, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}-\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^
2)^(3/2)) - (5*A*b - 2*a*B)/(2*a^3*Sqrt[a + b*x^2]) + ((5*A*b - 2
*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rubi in Sympy [A] time = 21.3942, size = 99, normalized size = 0.88

$$-\frac{A}{2ax^2\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{\frac{5Ab}{2}-Ba}{3a^2\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{\frac{5Ab}{2}-Ba}{a^3\sqrt{a+bx^2}}+\frac{\left(\frac{5Ab}{2}-Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] -A/(2*a*x**2*(a + b*x**2)**(3/2)) - (5*A*b/2 - B*a)/(3*a**2*(a +
b*x**2)**(3/2)) - (5*A*b/2 - B*a)/(a**3*sqrt(a + b*x**2)) + (5*A*
b/2 - B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(7/2)
```

Mathematica [A] time = 0.318537, size = 114, normalized size = 1.01

$$\frac{\frac{\sqrt{a}\left(-3a^2A+8a^2Bx^2-20aAbx^2+6abBx^4-15Ab^2x^4\right)}{x^2\left(a+bx^2\right)^{3/2}}+3(5Ab-2aB)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\log(x)(6aB-15Ab)}{6a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[a]*(-3*a^2*A - 20*a*A*b*x^2 + 8*a^2*B*x^2 - 15*A*b^2*x^4 +
 6*a*b*B*x^4))/(x^2*(a + b*x^2)^(3/2)) + (-15*A*b + 6*a*B)*Log[x]
 + 3*(5*A*b - 2*a*B)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(6*a^(7/2)
)
```

Maple [A] time = 0.013, size = 140, normalized size = 1.2

$$-\frac{A}{2\,a{x}^{2}} \left(b{x}^{2}+a \right) ^{-\frac{3}{2}}-\frac{5\,Ab}{6\,{a}^{2}} \left(b{x}^{2}+a \right) ^{-\frac{3}{2}}-\frac{5\,Ab}{2\,{a}^{3}}\frac{1}{\sqrt{b{x}^{2}+a}}+\frac{5\,Ab}{2}\ln \left(\frac{1}{x} \left(2\,a+2\,\sqrt{a}\sqrt{b{x}^{2}+a} \right) \right) {a}^{-\frac{7}{2}}+\frac{B}{3\,a} \left(b{x}^{2}+a \right) ^{-\frac{3}{2}}+\frac{B}{{a}^{2}}\frac{1}{\sqrt{b{x}^{2}+a}}-B\ln \left(\frac{1}{x} \left(2\,a+2\,\sqrt{a}\sqrt{b{x}^{2}+a} \right) \right) {a}^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A*b/a^2/(b*x^2+a)^(3/2)-5/2*A*b/
a^3/(b*x^2+a)^(1/2)+5/2*A*b/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(
1/2))/x)+1/3*B/a/(b*x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B/a^(5/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.251177, size = 1, normalized size = 0.01

$$\left[\frac{2\left(3\left(2\,Bab-5\,Ab^{2}\right)x^{4}-3\,Aa^{2}+4\left(2\,Ba^{2}-5\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{a}-3\left(\left(2\,Bab^{2}-5\,Ab^{3}\right)x^{6}+2\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{4}+\left(2\right.\right.}{12\left(a^{3}b^{2}x^{6}+2\,a^{4}bx^{4}+a^{5}x^{2}\right)\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^3),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(3*(2*B*a*b - 5*A*b^2)*x^4 - 3*A*a^2 + 4*(2*B*a^2 - 5*A*
a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(a) - 3*((2*B*a*b^2 - 5*A*b^3)*x^6
+ 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B*a^3 - 5*A*a^2*b)*x^2)*log(
-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^2))/((a^3*b^2*x^
6 + 2*a^4*b*x^4 + a^5*x^2)*sqrt(a)), 1/6*((3*(2*B*a*b - 5*A*b^2)*
x^4 - 3*A*a^2 + 4*(2*B*a^2 - 5*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(-
a) - 3*((2*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4
 + (2*B*a^3 - 5*A*a^2*b)*x^2)*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(
(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*sqrt(-a))]
```

Sympy [A] time = 119.557, size = 1608, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*
b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a*
```

```
*16*b*x**2*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b
*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**
16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4
+ 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**16*b*x
**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)
*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a
**15*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/
2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**15*b**2*x**4*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*
b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a*
*15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36
*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*
x**8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**
8) + 90*a**14*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a
**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12
*a**(33/2)*b**3*x**8)) + B*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b*
*3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**
2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt
(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/
(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a
**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/
(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a
**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(
19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2
)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a
**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
 - 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x*
*6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*
b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4
*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

GIAC/XCAS [A] time = 0.235652, size = 136, normalized size = 1.2

$$\frac{(2\,Ba - 5\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^3} + \frac{3\,\left(bx^2 + a\right)Ba + Ba^2 - 6\,\left(bx^2 + a\right)Ab - Aab}{3\,\left(bx^2 + a\right)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^2 + a}A}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 5*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^
3) + 1/3*(3*(b*x^2 + a)*B*a + B*a^2 - 6*(b*x^2 + a)*A*b - A*a*b)/
((b*x^2 + a)^(3/2)*a^3) - 1/2*sqrt(b*x^2 + a)*A/(a^3*x^2)
```

3.595 $\int \frac{A+Bx^2}{x^4(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*A*b - a*B)/(a^2*x*(a + b*x^2)
^(3/2)) + (4*b*(2*A*b - a*B)*x)/(3*a^3*(a + b*x^2)^(3/2)) + (8*b*
(2*A*b - a*B)*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.145678, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*A*b - a*B)/(a^2*x*(a + b*x^2)
^(3/2)) + (4*b*(2*A*b - a*B)*x)/(3*a^3*(a + b*x^2)^(3/2)) + (8*b*
(2*A*b - a*B)*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 13.9613, size = 99, normalized size = 0.92

$$-\frac{A}{3ax^3\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{2Ab-Ba}{a^2x\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{4bx\left(2Ab-Ba\right)}{3a^3\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{8bx\left(2Ab-Ba\right)}{3a^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] -A/(3*a*x**3*(a + b*x**2)**(3/2)) + (2*A*b - B*a)/(a**2*x*(a + b*
x**2)**(3/2)) + 4*b*x*(2*A*b - B*a)/(3*a**3*(a + b*x**2)**(3/2))
+ 8*b*x*(2*A*b - B*a)/(3*a**4*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0841386, size = 79, normalized size = 0.73

$$\frac{-a^3\left(A+3Bx^2\right)+6a^2bx^2\left(A-2Bx^2\right)-8ab^2x^4\left(Bx^2-3A\right)+16Ab^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (16*A*b^3*x^6 + 6*a^2*b*x^2*(A - 2*B*x^2) - 8*a*b^2*x^4*(-3*A + B
*x^2) - a^3*(A + 3*B*x^2))/(3*a^4*x^3*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 82, normalized size = 0.8

$$-\frac{-16\,Ab^3x^6+8\,Bab^2x^6-24\,Aab^2x^4+12\,Ba^2bx^4-6\,Aa^2bx^2+3\,Ba^3x^2+Aa^3}{3\,x^3a^4}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-16*A*b^3*x^6+8*B*a*b^2*x^6-24*A*a*b^2*x^4+12*B*a^2*b*x^4-6
*A*a^2*b*x^2+3*B*a^3*x^2+A*a^3)/(b*x^2+a)^(3/2)/x^3/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254945, size = 136, normalized size = 1.26

$$-\frac{\left(8\left(Bab^2 - 2Ab^3\right)x^6 + 12\left(Ba^2b - 2Aab^2\right)x^4 + Aa^3 + 3\left(Ba^3 - 2Aa^2b\right)x^2\right)\sqrt{bx^2 + a}}{3\left(a^4b^2x^7 + 2a^5bx^5 + a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^4),x, algorithm="fricas")
```

```
[Out] -1/3*(8*(B*a*b^2 - 2*A*b^3)*x^6 + 12*(B*a^2*b - 2*A*a*b^2)*x^4 +
A*a^3 + 3*(B*a^3 - 2*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b^2*x^7 +
 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [A] time = 141.831, size = 524, normalized size = 4.85

$$A\left(-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\right) + B\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4} - \frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4} - \frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**
6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b*
*(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**1
0*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 30*a**2*b**(23/
2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**
4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*b**(25/2)*x**6*
sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a*
```

```
*5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*
x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x
**6 + 3*a**4*b**12*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) +
 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b*
*(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2
 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a
**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

GIAC/XCAS [A] time = 0.244826, size = 302, normalized size = 2.8

$$-\frac{x\left(\frac{(5\,Ba^4b^3-8\,Aa^3b^4)\,x^2}{a^7b}+\frac{3\,(2\,Ba^5b^2-3\,Aa^4b^3)}{a^7b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aab^{\frac{3}{2}}+\cdots\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^4),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*B*a^4*b^3 - 8*A*a^3*b^4)*x^2/(a^7*b) + 3*(2*B*a^5*b^2
- 3*A*a^4*b^3)/(a^7*b))/(b*x^2 + a)^(3/2) + 2/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*A*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 18
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) -
8*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^3)
```

3.596 $\int \frac{A+Bx^2}{x^5(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{5\sqrt{a+bx^2}(7Ab-4aB)}{8a^4x^2}-\frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}}-\frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}$$

```
[Out] -A/(4*a*x^4*(a + b*x^2)^(3/2)) - (7*A*b - 4*a*B)/(12*a^2*x^2*(a +
 b*x^2)^(3/2)) - (5*(7*A*b - 4*a*B))/(12*a^3*x^2*Sqrt[a + b*x^2])
 + (5*(7*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^4*x^2) - (5*b*(7*A*b
- 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] time = 0.295204, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{5\sqrt{a+bx^2}(7Ab-4aB)}{8a^4x^2}-\frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}}-\frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(4*a*x^4*(a + b*x^2)^(3/2)) - (7*A*b - 4*a*B)/(12*a^2*x^2*(a +
 b*x^2)^(3/2)) - (5*(7*A*b - 4*a*B))/(12*a^3*x^2*Sqrt[a + b*x^2])
 + (5*(7*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^4*x^2) - (5*b*(7*A*b
- 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(9/2))
```

Rubi in Sympy [A] time = 25.3753, size = 143, normalized size = 0.95

$$-\frac{A}{4ax^4\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{7Ab-4Ba}{12a^2x^2\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{5\left(7Ab-4Ba\right)}{12a^3x^2\sqrt{a+bx^2}}+\frac{5\sqrt{a+bx^2}\left(7Ab-4Ba\right)}{8a^4x^2}-\frac{5b\left(7Ab-4Ba\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] -A/(4*a*x**4*(a + b*x**2)**(3/2)) - (7*A*b - 4*B*a)/(12*a**2*x**2
*(a + b*x**2)**(3/2)) - 5*(7*A*b - 4*B*a)/(12*a**3*x**2*sqrt(a +
b*x**2)) + 5*sqrt(a + b*x**2)*(7*A*b - 4*B*a)/(8*a**4*x**2) - 5*b
*(7*A*b - 4*B*a)*atanh(sqrt(a + b*x**2)/sqrt(a))/(8*a**(9/2))
```

Mathematica [A] time = 0.345666, size = 136, normalized size = 0.91

$$\frac{\frac{\sqrt{a}\left(-6a^3\left(A+2Bx^2\right)+a^2bx^2\left(21A-80Bx^2\right)+20ab^2x^4\left(7A-3Bx^2\right)+105Ab^3x^6\right)}{x^4\left(a+bx^2\right)^{3/2}}+15b(4aB-7Ab)\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+15b\log(x)(7Ab-4a}{24a^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[a]*(105*A*b^3*x^6 + a^2*b*x^2*(21*A - 80*B*x^2) + 20*a*b^2
*x^4*(7*A - 3*B*x^2) - 6*a^3*(A + 2*B*x^2)))/(x^4*(a + b*x^2)^(3/
2)) + 15*b*(7*A*b - 4*a*B)*Log[x] + 15*b*(-7*A*b + 4*a*B)*Log[a +
 Sqrt[a]*Sqrt[a + b*x^2]])/(24*a^(9/2))
```

Maple [A] time = 0.018, size = 187, normalized size = 1.3

$$-\frac{A}{4\,a x^4}\left(b x^2+a\right)^{-\frac{3}{2}}+\frac{7\,Ab}{8\,a^2x^2}\left(b x^2+a\right)^{-\frac{3}{2}}+\frac{35\,b^2A}{24\,a^3}\left(b x^2+a\right)^{-\frac{3}{2}}+\frac{35\,b^2A}{8\,a^4}\frac{1}{\sqrt{b x^2+a}}-\frac{35\,b^2A}{8}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{9}{2}}-\frac{B}{2\,a x^2}\left(b x^2+a\right)^{-\frac{3}{2}}-\frac{5\,Bb}{6\,a^2}\left(b x^2+a\right)^{-\frac{3}{2}}-\frac{5\,Bb}{2\,a^3}\frac{1}{\sqrt{b x^2+a}}+\frac{5\,Bb}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/4*A/a/x^4/(b*x^2+a)^(3/2)+7/8*A*b/a^2/x^2/(b*x^2+a)^(3/2)+35/2
4*A*b^2/a^3/(b*x^2+a)^(3/2)+35/8*A*b^2/a^4/(b*x^2+a)^(1/2)-35/8*A
*b^2/a^(9/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*B/a/x^2/(b
*x^2+a)^(3/2)-5/6*B*b/a^2/(b*x^2+a)^(3/2)-5/2*B*b/a^3/(b*x^2+a)^(
1/2)+5/2*B*b/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259379, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(15\left(4\,Bab^2-7\,Ab^3\right)x^6+20\left(4\,Ba^2b-7\,Aab^2\right)x^4+6\,Aa^3+3\left(4\,Ba^3-7\,Aa^2b\right)x^2\right)\sqrt{b x^2+a}\sqrt{a}+15\left(\left(4\,Bab^3-7\,Ab^4\right)x\right.}{48\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)\sqrt{a}}\right.$$

$$-\frac{\left(15\left(4\,Bab^2-7\,Ab^3\right)x^6+20\left(4\,Ba^2b-7\,Aab^2\right)x^4+6\,Aa^3+3\left(4\,Ba^3-7\,Aa^2b\right)x^2\right)\sqrt{b x^2+a}\sqrt{-a}-15\left(\left(4\,Bab^3-7\,Ab^4\right)x\right.}{24\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*(15*(4*B*a*b^2 - 7*A*b^3)*x^6 + 20*(4*B*a^2*b - 7*A*a*b
^2)*x^4 + 6*A*a^3 + 3*(4*B*a^3 - 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a)*
sqrt(a) + 15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2*(4*B*a^2*b^2 - 7*A*a*
b^3)*x^6 + (4*B*a^3*b - 7*A*a^2*b^2)*x^4)*log(-((b*x^2 + 2*a)*sqr
t(a) - 2*sqrt(b*x^2 + a)*a)/x^2))/((a^4*b^2*x^8 + 2*a^5*b*x^6 + a
^6*x^4)*sqrt(a)), -1/24*((15*(4*B*a*b^2 - 7*A*b^3)*x^6 + 20*(4*B*
```

```
a^2*b - 7*A*a*b^2)*x^4 + 6*A*a^3 + 3*(4*B*a^3 - 7*A*a^2*b)*x^2)*s
qrt(b*x^2 + a)*sqrt(-a) - 15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2*(4*B*
a^2*b^2 - 7*A*a*b^3)*x^6 + (4*B*a^3*b - 7*A*a^2*b^2)*x^4)*arctan(
sqrt(-a)/sqrt(b*x^2 + a)))/((a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4)
*sqrt(-a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.253074, size = 223, normalized size = 1.49

$$-\frac{5\left(4\,Bab-7\,Ab^{2}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^{4}}-\frac{6\left(bx^{2}+a\right)Bab+Ba^{2}b-9\left(bx^{2}+a\right)Ab^{2}-Aab^{2}}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{4}}-\frac{4\left(bx^{2}+a\right)^{\frac{3}{2}}Bab-4\sqrt{bx^{2}+a}Ba^{2}b-11\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{2}+13\sqrt{bx^{2}+a}Aab^{2}}{8\,a^{4}b^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^5),x, algorithm="giac")
```

```
[Out] -5/8*(4*B*a*b - 7*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^4) - 1/3*(6*(b*x^2 + a)*B*a*b + B*a^2*b - 9*(b*x^2 + a)*A*b^
2 - A*a*b^2)/((b*x^2 + a)^(3/2)*a^4) - 1/8*(4*(b*x^2 + a)^(3/2)*B
*a*b - 4*sqrt(b*x^2 + a)*B*a^2*b - 11*(b*x^2 + a)^(3/2)*A*b^2 + 1
3*sqrt(b*x^2 + a)*A*a*b^2)/(a^4*b^2*x^4)
```

3.597 $\int \frac{A+Bx^2}{x^6(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4(a+bx^2)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x(a+bx^2)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3(a+bx^2)^{3/2}}-\frac{A}{5ax^5(a+bx^2)^{3/2}}$$

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(3/2)) + (8*A*b - 5*a*B)/(15*a^2*x^3*(a +
 b*x^2)^(3/2)) - (2*b*(8*A*b - 5*a*B))/(5*a^3*x*(a + b*x^2)^(3/2)
) - (8*b^2*(8*A*b - 5*a*B)*x)/(15*a^4*(a + b*x^2)^(3/2)) - (16*b^
2*(8*A*b - 5*a*B)*x)/(15*a^5*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.176362, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4(a+bx^2)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x(a+bx^2)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3(a+bx^2)^{3/2}}-\frac{A}{5ax^5(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(3/2)) + (8*A*b - 5*a*B)/(15*a^2*x^3*(a +
 b*x^2)^(3/2)) - (2*b*(8*A*b - 5*a*B))/(5*a^3*x*(a + b*x^2)^(3/2)
) - (8*b^2*(8*A*b - 5*a*B)*x)/(15*a^4*(a + b*x^2)^(3/2)) - (16*b^
2*(8*A*b - 5*a*B)*x)/(15*a^5*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 18.9269, size = 141, normalized size = 0.97

$$-\frac{A}{5ax^5\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{8Ab-5Ba}{15a^2x^3\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{2b\left(8Ab-5Ba\right)}{5a^3x\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{8b^2x\left(8Ab-5Ba\right)}{15a^4\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{16b^2x\left(8Ab-5Ba\right)}{15a^5\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] -A/(5*a*x**5*(a + b*x**2)**(3/2)) + (8*A*b - 5*B*a)/(15*a**2*x**3
*(a + b*x**2)**(3/2)) - 2*b*(8*A*b - 5*B*a)/(5*a**3*x*(a + b*x**2
)**(3/2)) - 8*b**2*x*(8*A*b - 5*B*a)/(15*a**4*(a + b*x**2)**(3/2)
) - 16*b**2*x*(8*A*b - 5*B*a)/(15*a**5*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.113889, size = 105, normalized size = 0.72

$$\frac{-a^4\left(3A+5Bx^2\right)+a^3\left(8Abx^2+30bBx^4\right)+24a^2b^2x^4\left(5Bx^2-2A\right)+16ab^3x^6\left(5Bx^2-12A\right)-128Ab^4x^8}{15a^5x^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-128*A*b^4*x^8 + 16*a*b^3*x^6*(-12*A + 5*B*x^2) + 24*a^2*b^2*x^4
*(-2*A + 5*B*x^2) - a^4*(3*A + 5*B*x^2) + a^3*(8*A*b*x^2 + 30*b*B
*x^4))/(15*a^5*x^5*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.01, size = 107, normalized size = 0.7

$$-\frac{128 Ab^4x^8 - 80 Bab^3x^8 + 192 Aab^3x^6 - 120 Ba^2b^2x^6 + 48 Aa^2b^2x^4 - 30 Ba^3bx^4 - 8 Aa^3bx^2 + 5 Ba^4x^2 + 3 Aa^4}{15 x^5a^5} \left(bx^2 + a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/15*(128*A*b^4*x^8-80*B*a*b^3*x^8+192*A*a*b^3*x^6-120*B*a^2*b^2
*x^6+48*A*a^2*b^2*x^4-30*B*a^3*b*x^4-8*A*a^3*b*x^2+5*B*a^4*x^2+3*
A*a^4)/(b*x^2+a)^(3/2)/x^5/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.317007, size = 174, normalized size = 1.19

$$\frac{\left(16 \left(5 Bab^3 - 8 Ab^4\right) x^8 + 24 \left(5 Ba^2b^2 - 8 Aab^3\right) x^6 - 3 Aa^4 + 6 \left(5 Ba^3b - 8 Aa^2b^2\right) x^4 - \left(5 Ba^4 - 8 Aa^3b\right) x^2\right) \sqrt{bx^2 + a}}{15 \left(a^5b^2x^9 + 2 a^6bx^7 + a^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^6),x, algorithm="fricas")
```

```
[Out] 1/15*(16*(5*B*a*b^3 - 8*A*b^4)*x^8 + 24*(5*B*a^2*b^2 - 8*A*a*b^3)
*x^6 - 3*A*a^4 + 6*(5*B*a^3*b - 8*A*a^2*b^2)*x^4 - (5*B*a^4 - 8*A
*a^3*b)*x^2)*sqrt(b*x^2 + a)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.246527, size = 454, normalized size = 3.11

$$\frac{x\left(\frac{\left(8 Ba^5b^4-11 Aa^4b^5\right) x^2}{a^9b} + \frac{3 \left(3 Ba^6b^3-4 Aa^5b^4\right)}{a^9b}\right)}{3 \left(bx^2 + a\right)^{\frac{3}{2}}}$$

$$-\frac{2 \left(30 \left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Bab^{\frac{3}{2}} - 45 \left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Ab^{\frac{5}{2}} - 150 \left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 Ba^2b^{\frac{3}{2}} + 240 \left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^6 A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^6),x, algorithm="giac")
```

```
[Out] 1/3*x*((8*B*a^5*b^4 - 11*A*a^4*b^5)*x^2/(a^9*b) + 3*(3*B*a^6*b^3
- 4*A*a^5*b^4)/(a^9*b))/(b*x^2 + a)^(3/2) - 2/15*(30*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*B*a*b^(3/2) - 45*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*A*b^(5/2) - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)
 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 250*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 490*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*A*a^2*b^(5/2) - 170*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*
a^4*b^(3/2) + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) +
 40*B*a^5*b^(3/2) - 73*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 +
 a))^2 - a)^5*a^4)
```

3.598 $\int x^5 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=157

$$\frac{\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-ad)(2bc-ad)}{5d^5}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^5}$$

```
[Out] (c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (2*c*(b*c - a*d)*
(2*b*c - a*d)*(c + d*x^2)^(5/2))/(5*d^5) + ((6*b^2*c^2 - 6*a*b*c*
d + a^2*d^2)*(c + d*x^2)^(7/2))/(7*d^5) - (2*b*(2*b*c - a*d)*(c +
 d*x^2)^(9/2))/(9*d^5) + (b^2*(c + d*x^2)^(11/2))/(11*d^5)
```

Rubi [A] time = 0.338917, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-ad)(2bc-ad)}{5d^5}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (2*c*(b*c - a*d)*
(2*b*c - a*d)*(c + d*x^2)^(5/2))/(5*d^5) + ((6*b^2*c^2 - 6*a*b*c*
d + a^2*d^2)*(c + d*x^2)^(7/2))/(7*d^5) - (2*b*(2*b*c - a*d)*(c +
 d*x^2)^(9/2))/(9*d^5) + (b^2*(c + d*x^2)^(11/2))/(11*d^5)
```

Rubi in Sympy [A] time = 44.6652, size = 144, normalized size = 0.92

$$\frac{b^2\left(c+dx^2\right)^{\frac{11}{2}}}{11d^5}+\frac{2b\left(c+dx^2\right)^{\frac{9}{2}}(ad-2bc)}{9d^5}+\frac{c^2\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)^2}{3d^5}-\frac{2c\left(c+dx^2\right)^{\frac{5}{2}}(ad-2bc)(ad-bc)}{5d^5}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*(c + d*x**2)**(11/2)/(11*d**5) + 2*b*(c + d*x**2)**(9/2)*(a*
d - 2*b*c)/(9*d**5) + c**2*(c + d*x**2)**(3/2)*(a*d - b*c)**2/(3*
d**5) - 2*c*(c + d*x**2)**(5/2)*(a*d - 2*b*c)*(a*d - b*c)/(5*d**5
) + (c + d*x**2)**(7/2)*(a**2*d**2 - 6*a*b*c*d + 6*b**2*c**2)/(7*
d**5)
```

Mathematica [A] time = 0.108793, size = 132, normalized size = 0.84

$$\frac{\left(c+dx^2\right)^{3/2}\left(33a^2d^2\left(8c^2-12cdx^2+15d^2x^4\right)+22abd\left(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6\right)+b^2\left(128c^4-192c^3dx^2+24\right.\right.}{3465d^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(33*a^2*d^2*(8*c^2 - 12*c*d*x^2 + 15*d^2*x^4)
+ 22*a*b*d*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6) +
 b^2*(128*c^4 - 192*c^3*d*x^2 + 240*c^2*d^2*x^4 - 280*c*d^3*x^6 +
 315*d^4*x^8)))/(3465*d^5)
```

Maple [A] time = 0.011, size = 149, normalized size = 1.

$$\frac{315 b^2 x^8 d^4 + 770 abd^4 x^6 - 280 b^2 cd^3 x^6 + 495 a^2 d^4 x^4 - 660 abcd^3 x^4 + 240 b^2 c^2 d^2 x^4 - 396 a^2 cd^3 x^2 + 528 abc^2 d^2 x^2 - 192 b^2 c}{3465 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3465*(d*x^2+c)^(3/2)*(315*b^2*d^4*x^8+770*a*b*d^4*x^6-280*b^2*c
*d^3*x^6+495*a^2*d^4*x^4-660*a*b*c*d^3*x^4+240*b^2*c^2*d^2*x^4-39
6*a^2*c*d^3*x^2+528*a*b*c^2*d^2*x^2-192*b^2*c^3*d*x^2+264*a^2*c^2
*d^2-352*a*b*c^3*d+128*b^2*c^4)/d^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220806, size = 242, normalized size = 1.54

$$\frac{\left(315 b^2 d^5 x^{10} + 35 \left(b^2 cd^4 + 22 abd^5\right) x^8 + 128 b^2 c^5 - 352 abc^4 d + 264 a^2 c^3 d^2 - 5 \left(8 b^2 c^2 d^3 - 22 abcd^4 - 99 a^2 d^5\right) x^6 + 3 \left(16 b\right.}{3465 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^5,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 35*(b^2*c*d^4 + 22*a*b*d^5)*x^8 + 128*
b^2*c^5 - 352*a*b*c^4*d + 264*a^2*c^3*d^2 - 5*(8*b^2*c^2*d^3 - 22
*a*b*c*d^4 - 99*a^2*d^5)*x^6 + 3*(16*b^2*c^3*d^2 - 44*a*b*c^2*d^3
 + 33*a^2*c*d^4)*x^4 - 4*(16*b^2*c^4*d - 44*a*b*c^3*d^2 + 33*a^2*
c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^5
```

Sympy [A] time = 8.71261, size = 389, normalized size = 2.48

$$\begin{cases} \frac{8a^2c^3\sqrt{c+dx^2}}{105d^3} - \frac{4a^2c^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{a^2cx^4\sqrt{c+dx^2}}{35d} + \frac{a^2x^6\sqrt{c+dx^2}}{7} - \frac{32abc^4\sqrt{c+dx^2}}{315d^4} + \frac{16abc^3x^2\sqrt{c+dx^2}}{315d^3} - \frac{4abc^2x^4\sqrt{c+dx^2}}{105d^2} + \frac{2abcx^6\sqrt{c+dx^2}}{63d} + \\ \sqrt{c}\left(\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*a**2*c**2*
x**2*sqrt(c + d*x**2)/(105*d**2) + a**2*c*x**4*sqrt(c + d*x**2)/(
```

```
35*d) + a**2*x**6*sqrt(c + d*x**2)/7 - 32*a*b*c**4*sqrt(c + d*x**
2)/(315*d**4) + 16*a*b*c**3*x**2*sqrt(c + d*x**2)/(315*d**3) - 4*
a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d**2) + 2*a*b*c*x**6*sqrt(c +
 d*x**2)/(63*d) + 2*a*b*x**8*sqrt(c + d*x**2)/9 + 128*b**2*c**5*s
qrt(c + d*x**2)/(3465*d**5) - 64*b**2*c**4*x**2*sqrt(c + d*x**2)/
(3465*d**4) + 16*b**2*c**3*x**4*sqrt(c + d*x**2)/(1155*d**3) - 8*
b**2*c**2*x**6*sqrt(c + d*x**2)/(693*d**2) + b**2*c*x**8*sqrt(c +
 d*x**2)/(99*d) + b**2*x**10*sqrt(c + d*x**2)/11, Ne(d, 0)), (sqr
t(c)*(a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10), True))
```

GIAC/XCAS [A] time = 0.228502, size = 248, normalized size = 1.58

$$\frac{\frac{33\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)a^2}{d^2}+\frac{22\left(35\left(dx^2+c\right)^{\frac{9}{2}}-135\left(dx^2+c\right)^{\frac{7}{2}}c+189\left(dx^2+c\right)^{\frac{5}{2}}c^2-105\left(dx^2+c\right)^{\frac{3}{2}}c^3\right)ab}{d^3}+\frac{\left(315\left(dx^2+c\right)^{\frac{11}{2}}-1540\left(\right.\right.}{}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^5,x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d
*x^2 + c)^(3/2)*c^2)*a^2/d^2 + 22*(35*(d*x^2 + c)^(9/2) - 135*(d*
x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3
/2)*c^3)*a*b/d^3 + (315*(d*x^2 + c)^(11/2) - 1540*(d*x^2 + c)^(9/
2)*c + 2970*(d*x^2 + c)^(7/2)*c^2 - 2772*(d*x^2 + c)^(5/2)*c^3 +
1155*(d*x^2 + c)^(3/2)*c^4)*b^2/d^4)/d
```

3.599 $\int x^3 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^4) + ((b*c - a*d)*(3*b*
c - a*d)*(c + d*x^2)^(5/2))/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x
^2)^(7/2))/(7*d^4) + (b^2*(c + d*x^2)^(9/2))/(9*d^4)
```

Rubi [A] time = 0.266011, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^4) + ((b*c - a*d)*(3*b*
c - a*d)*(c + d*x^2)^(5/2))/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x
^2)^(7/2))/(7*d^4) + (b^2*(c + d*x^2)^(9/2))/(9*d^4)
```

Rubi in Sympy [A] time = 31.965, size = 100, normalized size = 0.88

$$\frac{b^2\left(c+dx^2\right)^{\frac{9}{2}}}{9d^4}+\frac{b\left(c+dx^2\right)^{\frac{7}{2}}(2ad-3bc)}{7d^4}-\frac{c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)^2}{3d^4}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}(ad-3bc)(ad-bc)}{5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*(c + d*x**2)**(9/2)/(9*d**4) + b*(c + d*x**2)**(7/2)*(2*a*d
- 3*b*c)/(7*d**4) - c*(c + d*x**2)**(3/2)*(a*d - b*c)**2/(3*d**4)
 + (c + d*x**2)**(5/2)*(a*d - 3*b*c)*(a*d - b*c)/(5*d**4)
```

Mathematica [A] time = 0.101741, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{3/2}\left(21a^2d^2\left(3dx^2-2c\right)+6abd\left(8c^2-12cdx^2+15d^2x^4\right)+b^2\left(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6\right)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(21*a^2*d^2*(-2*c + 3*d*x^2) + 6*a*b*d*(8*c^2
- 12*c*d*x^2 + 15*d^2*x^4) + b^2*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d
^2*x^4 + 35*d^3*x^6)))/(315*d^4)
```

Maple [A] time = 0.011, size = 108, normalized size = 1.

$$-\frac{-35\,b^2x^6d^3-90\,abd^3x^4+30\,b^2cd^2x^4-63\,a^2d^3x^2+72\,abcd^2x^2-24\,b^2c^2dx^2+42\,a^2cd^2-48\,abc^2d+16\,b^2c^3}{315\,d^4}\left(dx^2+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] -1/315*(d*x^2+c)^(3/2)*(-35*b^2*d^3*x^6-90*a*b*d^3*x^4+30*b^2*c*d
^2*x^4-63*a^2*d^3*x^2+72*a*b*c*d^2*x^2-24*b^2*c^2*d*x^2+42*a^2*c*
d^2-48*a*b*c^2*d+16*b^2*c^3)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215846, size = 189, normalized size = 1.66

$$\frac{\left(35\, b^2 d^4 x^8 + 5\, \left(b^2 c d^3 + 18\, a b d^4\right) x^6 - 16\, b^2 c^4 + 48\, a b c^3 d - 42\, a^2 c^2 d^2 - 3\, \left(2\, b^2 c^2 d^2 - 6\, a b c d^3 - 21\, a^2 d^4\right) x^4 + \left(8\, b^2 c^3 d - 24\, a \cdots\right.}{315\, d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 5*(b^2*c*d^3 + 18*a*b*d^4)*x^6 - 16*b^2*c
^4 + 48*a*b*c^3*d - 42*a^2*c^2*d^2 - 3*(2*b^2*c^2*d^2 - 6*a*b*c*d
^3 - 21*a^2*d^4)*x^4 + (8*b^2*c^3*d - 24*a*b*c^2*d^2 + 21*a^2*c*d
^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 4.91189, size = 308, normalized size = 2.7

$$\begin{cases} -\frac{2a^2c^2\sqrt{c+dx^2}}{15d^2} + \frac{a^2cx^2\sqrt{c+dx^2}}{15d} + \frac{a^2x^4\sqrt{c+dx^2}}{5} + \frac{16abc^3\sqrt{c+dx^2}}{105d^3} - \frac{8abc^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{2abcx^4\sqrt{c+dx^2}}{35d} + \frac{2abx^6\sqrt{c+dx^2}}{7} - \frac{16b^2c^4\sqrt{c+dx^2}}{315d^4} + 8b \cdots \\ \sqrt{c}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c**2*sqrt(c + d*x**2)/(15*d**2) + a**2*c*x**2*
sqrt(c + d*x**2)/(15*d) + a**2*x**4*sqrt(c + d*x**2)/5 + 16*a*b*c
**3*sqrt(c + d*x**2)/(105*d**3) - 8*a*b*c**2*x**2*sqrt(c + d*x**2
)/(105*d**2) + 2*a*b*c*x**4*sqrt(c + d*x**2)/(35*d) + 2*a*b*x**6*
sqrt(c + d*x**2)/7 - 16*b**2*c**4*sqrt(c + d*x**2)/(315*d**4) + 8
*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**3) - 2*b**2*c**2*x**4*sq
rt(c + d*x**2)/(105*d**2) + b**2*c*x**6*sqrt(c + d*x**2)/(63*d) +
 b**2*x**8*sqrt(c + d*x**2)/9, Ne(d, 0)), (sqrt(c)*(a**2*x**4/4 +
 a*b*x**6/3 + b**2*x**8/8), True))
```

GIAC/XCAS [A] time = 0.237007, size = 192, normalized size = 1.68

$$\frac{\frac{21\left(3\left(dx^2+c\right)^{\frac{5}{2}} - 5\left(dx^2+c\right)^{\frac{3}{2}} c\right) a^2}{d} + \frac{6\left(15\left(dx^2+c\right)^{\frac{7}{2}} - 42\left(dx^2+c\right)^{\frac{5}{2}} c + 35\left(dx^2+c\right)^{\frac{3}{2}} c^2\right) ab}{d^2} + \frac{\left(35\left(dx^2+c\right)^{\frac{9}{2}} - 135\left(dx^2+c\right)^{\frac{7}{2}} c + 189\left(dx^2+c\right)^{\frac{5}{2}} c^2 - 105\left(dx^2+c\right)^{\frac{3}{2}} \cdots\right.}{d^3}}{315\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3,x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a^2/d + 6
*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^
(3/2)*c^2)*a*b/d^2 + (35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2
)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*b^2/
d^3)/d
```

3.600 $\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (2*b*(b*c - a*d)*(c +
 d*x^2)^(5/2))/(5*d^3) + (b^2*(c + d*x^2)^(7/2))/(7*d^3)
```

Rubi [A] time = 0.158102, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (2*b*(b*c - a*d)*(c +
 d*x^2)^(5/2))/(5*d^3) + (b^2*(c + d*x^2)^(7/2))/(7*d^3)
```

Rubi in Sympy [A] time = 23.0662, size = 66, normalized size = 0.86

$$\frac{b^2\left(c+dx^2\right)^{\frac{7}{2}}}{7d^3}+\frac{2b\left(c+dx^2\right)^{\frac{5}{2}}(ad-bc)}{5d^3}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)^2}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*(c + d*x**2)**(7/2)/(7*d**3) + 2*b*(c + d*x**2)**(5/2)*(a*d
- b*c)/(5*d**3) + (c + d*x**2)**(3/2)*(a*d - b*c)**2/(3*d**3)
```

Mathematica [A] time = 0.0652225, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{3/2}\left(35a^2d^2+14abd\left(3dx^2-2c\right)+b^2\left(8c^2-12cdx^2+15d^2x^4\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x^2) + b^2*
(8*c^2 - 12*c*d*x^2 + 15*d^2*x^4)))/(105*d^3)
```

Maple [A] time = 0.009, size = 69, normalized size = 0.9

$$\frac{15\,b^2d^2x^4+42\,abd^2x^2-12\,b^2cdx^2+35\,a^2d^2-28\,cabd+8\,b^2c^2}{105\,d^3}\left(dx^2+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(3/2)*(15*b^2*d^2*x^4+42*a*b*d^2*x^2-12*b^2*c*d*x
^2+35*a^2*d^2-28*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215168, size = 139, normalized size = 1.81

$$\frac{\left(15\,b^2d^3x^6 + 8\,b^2c^3 - 28\,abc^2d + 35\,a^2cd^2 + 3\,\left(b^2cd^2 + 14\,abd^3\right)x^4 - \left(4\,b^2c^2d - 14\,abcd^2 - 35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 +
 3*(b^2*c*d^2 + 14*a*b*d^3)*x^4 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 3
5*a^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 2.35255, size = 226, normalized size = 2.94

$$\begin{cases} \frac{a^2c\sqrt{c+dx^2}}{3d} + \frac{a^2x^2\sqrt{c+dx^2}}{3} - \frac{4abc^2\sqrt{c+dx^2}}{15d^2} + \frac{2abcx^2\sqrt{c+dx^2}}{15d} + \frac{2abx^4\sqrt{c+dx^2}}{5} + \frac{8b^2c^3\sqrt{c+dx^2}}{105d^3} - \frac{4b^2c^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{b^2cx^4\sqrt{c+dx^2}}{35d} + \frac{b^2x^6\sqrt{c+}}{7} \\ \sqrt{c}\left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*c*sqrt(c + d*x**2)/(3*d) + a**2*x**2*sqrt(c + d*x
**2)/3 - 4*a*b*c**2*sqrt(c + d*x**2)/(15*d**2) + 2*a*b*c*x**2*sqr
t(c + d*x**2)/(15*d) + 2*a*b*x**4*sqrt(c + d*x**2)/5 + 8*b**2*c**
3*sqrt(c + d*x**2)/(105*d**3) - 4*b**2*c**2*x**2*sqrt(c + d*x**2)
/(105*d**2) + b**2*c*x**4*sqrt(c + d*x**2)/(35*d) + b**2*x**6*sqr
t(c + d*x**2)/7, Ne(d, 0)), (sqrt(c)*(a**2*x**2/2 + a*b*x**4/2 +
b**2*x**6/6), True))
```

GIAC/XCAS [A] time = 0.227028, size = 130, normalized size = 1.69

$$\frac{35\,\left(dx^2+c\right)^{\frac{3}{2}}a^2 + \frac{14\left(3\,\left(dx^2+c\right)^{\frac{5}{2}} - 5\,\left(dx^2+c\right)^{\frac{3}{2}}c\right)ab}{d} + \frac{\left(15\,\left(dx^2+c\right)^{\frac{7}{2}} - 42\,\left(dx^2+c\right)^{\frac{5}{2}}c + 35\,\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)b^2}{d^2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x,x, algorithm="giac")
```

```
[Out] 1/105*(35*(d*x^2 + c)^(3/2)*a^2 + 14*(3*(d*x^2 + c)^(5/2) - 5*(d*
x^2 + c)^(3/2)*c)*a*b/d + (15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^
(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*b^2/d^2)/d
```

3.601 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx$

Optimal. Leaf size=92

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

```
[Out] a^2*Sqrt[c + d*x^2] - (b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2)
 + (b^2*(c + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.219607, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] a^2*Sqrt[c + d*x^2] - (b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2)
 + (b^2*(c + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]]
```

Rubi in Sympy [A] time = 24.5989, size = 82, normalized size = 0.89

$$-a^2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2\sqrt{c+dx^2}+\frac{b^2\left(c+dx^2\right)^{\frac{5}{2}}}{5d^2}+\frac{b\left(c+dx^2\right)^{\frac{3}{2}}(2ad-bc)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] -a**2*sqrt(c)*atanh(sqrt(c + d*x**2)/sqrt(c)) + a**2*sqrt(c + d*x
**2) + b**2*(c + d*x**2)**(5/2)/(5*d**2) + b*(c + d*x**2)**(3/2)*
(2*a*d - b*c)/(3*d**2)
```

Mathematica [A] time = 0.233078, size = 105, normalized size = 1.14

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2+10abd\left(c+dx^2\right)+b^2\left(-2c^2+cdx^2+3d^2x^4\right)\right)}{15d^2} - a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+a^2\sqrt{c}\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(c + d*x^2) + b^2*(-2*c^2
 + c*d*x^2 + 3*d^2*x^4)))/(15*d^2) + a^2*Sqrt[c]*Log[x] - a^2*Sqr
t[c]*Log[c + Sqrt[c]*Sqrt[c + d*x^2]]
```

Maple [A] time = 0.014, size = 100, normalized size = 1.1

$$-\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)\sqrt{c}a^2+a^2\sqrt{dx^2+c}+\frac{b^2x^2}{5d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{2b^2c}{15d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{2ab}{3d}\left(dx^2+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] -ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*c^(1/2)*a^2+a^2*(d*x^2+c)^
(1/2)+1/5*b^2*x^2*(d*x^2+c)^(3/2)/d-2/15*b^2*c/d^2*(d*x^2+c)^(3/2
)+2/3*a*b*(d*x^2+c)^(3/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239042, size = 1, normalized size = 0.01

$$\left[\frac{15\,a^2\sqrt{c}d^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\left(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+\left(b^2cd+10\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{30\,d^2},\right.$$
$$\left.-\frac{15\,a^2\sqrt{-c}d^2\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+\left(b^2cd+10\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) + 2*(3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2
*d^2 + (b^2*c*d + 10*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2, -1/15*(1
5*a^2*sqrt(-c)*d^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (3*b^2*
d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2 + (b^2*c*d + 10*a*b
*d^2)*x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A] time = 15.1976, size = 153, normalized size = 1.66

$$-a^2c\left(\begin{cases}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for } -c>0\\ \frac{\operatorname{acoth}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for } -c<0\wedge c<c+dx^2\\ \frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for } c>c+dx^2\wedge -c<0\end{cases}\right)+a^2\sqrt{c+dx^2}+\frac{b^2\left(c+dx^2\right)^{\frac{5}{2}}}{5d^2}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(4abd-2b^2c\right)}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] -a**2*c*Piecewise((-atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c), -c
> 0), (acoth(sqrt(c + d*x**2)/sqrt(c))/sqrt(c), (-c < 0) & (c < c
 + d*x**2)), (atanh(sqrt(c + d*x**2)/sqrt(c))/sqrt(c), (-c < 0) &
 (c > c + d*x**2))) + a**2*sqrt(c + d*x**2) + b**2*(c + d*x**2)**
(5/2)/(5*d**2) + (c + d*x**2)**(3/2)*(4*a*b*d - 2*b**2*c)/(6*d**2
)
```

GIAC/XCAS [A] time = 0.235753, size = 136, normalized size = 1.48

$$\frac{a^2c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^2d^8-5\left(dx^2+c\right)^{\frac{3}{2}}b^2cd^8+10\left(dx^2+c\right)^{\frac{3}{2}}abd^9+15\sqrt{dx^2+c}a^2d^{10}}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x,x, algorithm="giac")
```

```
[Out] a^2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/15*(3*(d*x^2
+ c)^(5/2)*b^2*d^8 - 5*(d*x^2 + c)^(3/2)*b^2*c*d^8 + 10*(d*x^2 +
c)^(3/2)*a*b*d^9 + 15*sqrt(d*x^2 + c)*a^2*d^10)/d^10
```

3.602 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d}$$

```
[Out] (a*(4*b*c + a*d)*Sqrt[c + d*x^2])/(2*c) + (b^2*(c + d*x^2)^(3/2))
/(3*d) - (a^2*(c + d*x^2)^(3/2))/(2*c*x^2) - (a*(4*b*c + a*d)*Arc
Tanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rubi [A] time = 0.264589, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (a*(4*b*c + a*d)*Sqrt[c + d*x^2])/(2*c) + (b^2*(c + d*x^2)^(3/2))
/(3*d) - (a^2*(c + d*x^2)^(3/2))/(2*c*x^2) - (a*(4*b*c + a*d)*Arc
Tanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rubi in Sympy [A] time = 24.7308, size = 94, normalized size = 0.86

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{2cx^2}+\frac{a\sqrt{c+dx^2}\,(ad+4bc)}{2c}-\frac{a\,(ad+4bc)\,\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2\left(c+dx^2\right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(2*c*x**2) + a*sqrt(c + d*x**2)*(a*d +
4*b*c)/(2*c) - a*(a*d + 4*b*c)*atanh(sqrt(c + d*x**2)/sqrt(c))/(2
*sqrt(c)) + b**2*(c + d*x**2)**(3/2)/(3*d)
```

Mathematica [A] time = 0.223724, size = 103, normalized size = 0.94

$$\sqrt{c+dx^2}\left(-\frac{a^2}{2x^2}+2ab+\frac{b^2\left(c+dx^2\right)}{3d}\right)-\frac{a(ad+4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{2\sqrt{c}}+\frac{a\log(x)(ad+4bc)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] Sqrt[c + d*x^2]*(2*a*b - a^2/(2*x^2) + (b^2*(c + d*x^2))/(3*d)) +
 (a*(4*b*c + a*d)*Log[x])/(2*Sqrt[c]) - (a*(4*b*c + a*d)*Log[c +
Sqrt[c]*Sqrt[c + d*x^2]])/(2*Sqrt[c])
```

Maple [A] time = 0.016, size = 132, normalized size = 1.2

$$\frac{b^2}{3d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2}{2\,cx^2}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{a^2d}{2\,c}\sqrt{dx^2+c}-2\,\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)\sqrt{c}ab+2\,\sqrt{dx^2+c}ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d-1/2*a^2*(d*x^2+c)^(3/2)/c/x^2-1/2*a^2*d
/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2*a^2*d/c*(d*x^2
+c)^(1/2)-2*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*c^(1/2)*a*b+2*(
d*x^2+c)^(1/2)*a*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240473, size = 1, normalized size = 0.01

$$\left[\frac{3\,\left(4\,abcd+a^2d^2\right)x^2\log\left(-\frac{\left(dx^2+2\,c\right)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)+2\,\left(2\,b^2dx^4-3\,a^2d+2\,\left(b^2c+6\,abd\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}}{12\,\sqrt{c}dx^2},\ -\frac{3\,\left(4\,abcd+a^2d^2\right)x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(2\,b^2dx^4-3\,a^2d+2\,\left(b^2c+6\,abd\right)x^2\right)\sqrt{dx^2+c}\sqrt{-c}}{6\,\sqrt{-c}dx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(4*a*b*c*d + a^2*d^2)*x^2*log(-((d*x^2 + 2*c)*sqrt(c) -
2*sqrt(d*x^2 + c)*c)/x^2) + 2*(2*b^2*d*x^4 - 3*a^2*d + 2*(b^2*c +
 6*a*b*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c))/(sqrt(c)*d*x^2), -1/6*(3*
(4*a*b*c*d + a^2*d^2)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*b
^2*d*x^4 - 3*a^2*d + 2*(b^2*c + 6*a*b*d)*x^2)*sqrt(d*x^2 + c)*sqr
t(-c))/(sqrt(-c)*d*x^2)]
```

Sympy [A] time = 31.6659, size = 148, normalized size = 1.36

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\sqrt{c}}-2ab\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{2abc}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{2ab\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}+b^2\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for } d=0\\ \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*asinh(sqrt(c)/(
sqrt(d)*x))/(2*sqrt(c)) - 2*a*b*sqrt(c)*asinh(sqrt(c)/(sqrt(d)*x)
) + 2*a*b*c/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*sqrt(d)*x/sq
rt(c/(d*x**2) + 1) + b**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), (
(c + d*x**2)**(3/2)/(3*d), True))
```

GIAC/XCAS [A] time = 0.229893, size = 120, normalized size = 1.1

$$\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2+12\sqrt{dx^2+c}abd-\frac{3\sqrt{dx^2+c}a^2d}{x^2}+\frac{3\left(4\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x^2 + c)^(3/2)*b^2 + 12*sqrt(d*x^2 + c)*a*b*d - 3*sqrt(
d*x^2 + c)*a^2*d/x^2 + 3*(4*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2
+ c)/sqrt(-c))/sqrt(-c))/d
```

3.603 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{\sqrt{c+dx^2}\left(ad(8bc-ad)+8b^2c^2\right)}{8c^2}-\frac{\left(ad(8bc-ad)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a\left(c+dx^2\right)^{3/2}(8bc-ad)}{8c^2x^2}$$

```
[Out] ((8*b^2*c^2 + a*d*(8*b*c - a*d))*Sqrt[c + d*x^2])/(8*c^2) - (a^2*
(c + d*x^2)^(3/2))/(4*c*x^4) - (a*(8*b*c - a*d)*(c + d*x^2)^(3/2)
)/(8*c^2*x^2) - ((8*b^2*c^2 + a*d*(8*b*c - a*d))*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rubi [A] time = 0.406551, antiderivative size = 140, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{1}{8}\sqrt{c+dx^2}\left(\frac{ad(8bc-ad)}{c^2}+8b^2\right)-\frac{\left(ad(8bc-ad)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a\left(c+dx^2\right)^{3/2}(8bc-ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] ((8*b^2 + (a*d*(8*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/8 - (a^2*(c +
 d*x^2)^(3/2))/(4*c*x^4) - (a*(8*b*c - a*d)*(c + d*x^2)^(3/2))/(8
*c^2*x^2) - ((8*b^2*c^2 + a*d*(8*b*c - a*d))*ArcTanh[Sqrt[c + d*x
^2]/Sqrt[c]])/(8*c^(3/2))
```

Rubi in Sympy [A] time = 28.889, size = 128, normalized size = 0.9

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{4cx^4}+\frac{a\left(c+dx^2\right)^{\frac{3}{2}}(ad-8bc)}{8c^2x^2}+\frac{\sqrt{c+dx^2}\left(-ad\,(ad-8bc)+8b^2c^2\right)}{8c^2}-\frac{\left(-ad\,(ad-8bc)+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**5,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(4*c*x**4) + a*(c + d*x**2)**(3/2)*(a*d
 - 8*b*c)/(8*c**2*x**2) + sqrt(c + d*x**2)*(-a*d*(a*d - 8*b*c) +
8*b**2*c**2)/(8*c**2) - (-a*d*(a*d - 8*b*c) + 8*b**2*c**2)*atanh(
sqrt(c + d*x**2)/sqrt(c))/(8*c**(3/2))
```

Mathematica [A] time = 0.297864, size = 130, normalized size = 0.91

$$\frac{\left(a^2d^2-8abcd-8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{8c^{3/2}}-\frac{\log(x)\left(a^2d^2-8abcd-8b^2c^2\right)}{8c^{3/2}}+\sqrt{c+dx^2}\left(-\frac{a^2}{4x^4}-\frac{a(ad+8bc)}{8cx^2}+b^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] (b^2 - a^2/(4*x^4) - (a*(8*b*c + a*d))/(8*c*x^2))*Sqrt[c + d*x^2]
 - ((-8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*Log[x])/(8*c^(3/2)) + ((-8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])
/(8*c^(3/2))
```

Maple [A] time = 0.016, size = 207, normalized size = 1.5

$$-\frac{a^2}{4\,cx^4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d}{8\,c^2x^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d^2}{8}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}\\-\frac{a^2d^2}{8\,c^2}\sqrt{dx^2+c}-\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\sqrt{c}b^2+\sqrt{dx^2+c}b^2\\-\frac{ab}{cx^2}\left(dx^2+c\right)^{\frac{3}{2}}-abd\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{abd}{c}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(3/2)/c/x^4+1/8*a^2*d/c^2/x^2*(d*x^2+c)^(3/2)+
1/8*a^2*d^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/8*a^2
*d^2/c^2*(d*x^2+c)^(1/2)-ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*c^
(1/2)*b^2+(d*x^2+c)^(1/2)*b^2-a*b/c/x^2*(d*x^2+c)^(3/2)-a*b*d/c^(
1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+a*b*d/c*(d*x^2+c)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233486, size = 1, normalized size = 0.01

$$\left[-\frac{\left(8\,b^2c^2+8\,abcd-a^2d^2\right)x^4\log\left(-\frac{\left(dx^2+2\,c\right)\sqrt{c}+2\,\sqrt{dx^2+c}c}{x^2}\right)-2\left(8\,b^2cx^4-2\,a^2c-\left(8\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}}{16\,c^{\frac{3}{2}}x^4},\right.\\\left.-\frac{\left(8\,b^2c^2+8\,abcd-a^2d^2\right)x^4\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(8\,b^2cx^4-2\,a^2c-\left(8\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{-c}}{8\,\sqrt{-c}cx^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*x^4*log(-((d*x^2 + 2*c)
*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2) - 2*(8*b^2*c*x^4 - 2*a^2*c -
 (8*a*b*c + a^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c))/(c^(3/2)*x^4), -
```

```
1/8*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*x^4*arctan(sqrt(-c)/sqrt(d
*x^2 + c)) - (8*b^2*c*x^4 - 2*a^2*c - (8*a*b*c + a^2*d)*x^2)*sqrt
(d*x^2 + c)*sqrt(-c))/(sqrt(-c)*c*x^4)]
```

Sympy [A] time = 66.9338, size = 219, normalized size = 1.53

$$-\frac{a^2c}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{3}{2}}}{8cx\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8c^{\frac{3}{2}}}-\frac{ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x}$$
$$-\frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}}-b^2\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{b^2c}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**5,x)
```

```
[Out] -a**2*c/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*sqrt(d)/(8
*x**3*sqrt(c/(d*x**2) + 1)) - a**2*d**(3/2)/(8*c*x*sqrt(c/(d*x**2
) + 1)) + a**2*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(3/2)) - a*b
*sqrt(d)*sqrt(c/(d*x**2) + 1)/x - a*b*d*asinh(sqrt(c)/(sqrt(d)*x)
)/sqrt(c) - b**2*sqrt(c)*asinh(sqrt(c)/(sqrt(d)*x)) + b**2*c/(sqr
t(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*sqrt(d)*x/sqrt(c/(d*x**2) + 1
)
```

GIAC/XCAS [A] time = 0.239792, size = 207, normalized size = 1.45

$$\frac{8\sqrt{dx^2+c}b^2d+\frac{\left(8b^2c^2d+8abcd^2-a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{8\left(dx^2+c\right)^{\frac{3}{2}}abcd^2-8\sqrt{dx^2+c}abc^2d^2+\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3+\sqrt{dx^2+c}a^2cd^3}{cd^2x^4}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(d*x^2 + c)*b^2*d + (8*b^2*c^2*d + 8*a*b*c*d^2 - a^2*d
^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (8*(d*x^2 + c
)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(
3/2)*a^2*d^3 + sqrt(d*x^2 + c)*a^2*c*d^3)/(c*d^2*x^4))/d
```

3.604 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{c+dx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6}-\frac{a\left(c+dx^2\right)^{3/2}(4bc-ad)}{8c^2x^4}$$

```
[Out] -((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2])/(16*c^2*x^2)
 - (a^2*(c + d*x^2)^(3/2))/(6*c*x^6) - (a*(4*b*c - a*d)*(c + d*x^
2)^(3/2))/(8*c^2*x^4) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rubi [A] time = 0.382887, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sqrt{c+dx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6}-\frac{a\left(c+dx^2\right)^{3/2}(4bc-ad)}{8c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] -((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2])/(16*c^2*x^2)
 - (a^2*(c + d*x^2)^(3/2))/(6*c*x^6) - (a*(4*b*c - a*d)*(c + d*x^
2)^(3/2))/(8*c^2*x^4) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rubi in Sympy [A] time = 29.117, size = 133, normalized size = 0.89

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{6cx^6}+\frac{a\left(c+dx^2\right)^{\frac{3}{2}}(ad-4bc)}{8c^2x^4}-\frac{\sqrt{c+dx^2}\left(ad(ad-4bc)+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(ad(ad-4bc)+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**7,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(6*c*x**6) + a*(c + d*x**2)**(3/2)*(a*d
 - 4*b*c)/(8*c**2*x**4) - sqrt(c + d*x**2)*(a*d*(a*d - 4*b*c) + 8
*b**2*c**2)/(16*c**2*x**2) - d*(a*d*(a*d - 4*b*c) + 8*b**2*c**2)*
atanh(sqrt(c + d*x**2)/sqrt(c))/(16*c**(5/2))
```

Mathematica [A] time = 0.206289, size = 161, normalized size = 1.08

$$\sqrt{c+dx^2}\left(\frac{a^2d^2-4abcd-8b^2c^2}{16c^2x^2}-\frac{a^2}{6x^6}-\frac{a(ad+12bc)}{24cx^4}\right)-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{16c^{5/2}}+\frac{d\log(x)\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] (-a^2/(6*x^6) - (a*(12*b*c + a*d))/(24*c*x^4) + (-8*b^2*c^2 - 4*a
*b*c*d + a^2*d^2)/(16*c^2*x^2))*Sqrt[c + d*x^2] + (d*(8*b^2*c^2 -
 4*a*b*c*d + a^2*d^2)*Log[x])/(16*c^(5/2)) - (d*(8*b^2*c^2 - 4*a*
b*c*d + a^2*d^2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/(16*c^(5/2))
```

Maple [B] time = 0.018, size = 281, normalized size = 1.9

$$
\begin{aligned}
&-\frac{a^2}{6\,cx^6}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d}{8\,c^2x^4}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2d^2}{16\,c^3x^2}\left(dx^2+c\right)^{\frac{3}{2}}\\
&-\frac{a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}+\frac{a^2d^3}{16\,c^3}\sqrt{dx^2+c}-\frac{b^2}{2\,cx^2}\left(dx^2+c\right)^{\frac{3}{2}}\\
&-\frac{b^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{b^2d}{2\,c}\sqrt{dx^2+c}-\frac{ab}{2\,cx^4}\left(dx^2+c\right)^{\frac{3}{2}}\\
&+\frac{abd}{4\,c^2x^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{abd^2}{4}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}-\frac{abd^2}{4\,c^2}\sqrt{dx^2+c}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(3/2)/c/x^6+1/8*a^2*d/c^2/x^4*(d*x^2+c)^(3/2)-
1/16*a^2*d^2/c^3/x^2*(d*x^2+c)^(3/2)-1/16*a^2*d^3/c^(5/2)*ln((2*c
+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/16*a^2*d^3/c^3*(d*x^2+c)^(1/2)-1
/2*b^2/c/x^2*(d*x^2+c)^(3/2)-1/2*b^2*d/c^(1/2)*ln((2*c+2*c^(1/2)*
(d*x^2+c)^(1/2))/x)+1/2*b^2*d/c*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*
x^2+c)^(3/2)+1/4*a*b*d/c^2/x^2*(d*x^2+c)^(3/2)+1/4*a*b*d^2/c^(3/2
)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/4*a*b*d^2/c^2*(d*x^2+c)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28499, size = 1, normalized size = 0.01

$$
\left[\frac{3\left(8\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^6\log\left(-\frac{\left(dx^2+2\,c\right)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)-2\left(3\left(8\,b^2c^2+4\,abcd-a^2d^2\right)x^4+8\,a^2c^2+2\left(12\,abc^2+a^2cd\right.\right.}{96\,c^{\frac{5}{2}}x^6}\right.
$$

$$
-\frac{3\left(8\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(3\left(8\,b^2c^2+4\,abcd-a^2d^2\right)x^4+8\,a^2c^2+2\left(12\,abc^2+a^2cd\right)x^2\right)\sqrt{dx^2+c}}{48\,\sqrt{-c}c^2x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^6*log(-((d*x^2 +
 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2) - 2*(3*(8*b^2*c^2 + 4*a
*b*c*d - a^2*d^2)*x^4 + 8*a^2*c^2 + 2*(12*a*b*c^2 + a^2*c*d)*x^2)
*sqrt(d*x^2 + c)*sqrt(c))/(c^(5/2)*x^6), -1/48*(3*(8*b^2*c^2*d -
4*a*b*c*d^2 + a^2*d^3)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (3*
(8*b^2*c^2 + 4*a*b*c*d - a^2*d^2)*x^4 + 8*a^2*c^2 + 2*(12*a*b*c^2
 + a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c))/(sqrt(-c)*c^2*x^6)]
```

Sympy [A] time = 96.0702, size = 291, normalized size = 1.95

$$-\frac{a^2c}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}-\frac{5a^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{3}{2}}}{48cx^3\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{5}{2}}}{16c^2x\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{5}{2}}}-\frac{abc}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3ab\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{abd^{\frac{3}{2}}}{4cx\sqrt{\frac{c}{dx^2}+1}}+\frac{abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4c^{\frac{3}{2}}}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**7,x)
```

```
[Out] -a**2*c/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 5*a**2*sqrt(d)/(2
4*x**5*sqrt(c/(d*x**2) + 1)) + a**2*d**(3/2)/(48*c*x**3*sqrt(c/(d
*x**2) + 1)) + a**2*d**(5/2)/(16*c**2*x*sqrt(c/(d*x**2) + 1)) - a
**2*d**3*asinh(sqrt(c)/(sqrt(d)*x))/(16*c**(5/2)) - a*b*c/(2*sqrt
(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a*b*sqrt(d)/(4*x**3*sqrt(c/(d*
x**2) + 1)) - a*b*d**(3/2)/(4*c*x*sqrt(c/(d*x**2) + 1)) + a*b*d**
2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(3/2)) - b**2*sqrt(d)*sqrt(c/(
d*x**2) + 1)/(2*x) - b**2*d*asinh(sqrt(c)/(sqrt(d)*x))/(2*sqrt(c)
)
```

GIAC/XCAS [A] time = 0.24728, size = 300, normalized size = 2.01

$$\frac{\frac{3\left(8b^2c^2d^2-4abcd^3+a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2+12\left(dx^2+c\right)^{\frac{5}{2}}abcd^3-12\sqrt{dx^2+c}abc^3d^3-3\left(dx\right.}{c^2d^3x^6}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(8*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*arctan(sqrt(d*x^2
 + c)/sqrt(-c))/(sqrt(-c)*c^2) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^
2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4
*d^2 + 12*(d*x^2 + c)^(5/2)*a*b*c*d^3 - 12*sqrt(d*x^2 + c)*a*b*c^
3*d^3 - 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*(d*x^2 + c)^(3/2)*a^2*c*d
^4 + 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c^2*d^3*x^6))/d
```

3.605 $\int x^2 (a+bx^2)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=191

$$-\frac{c^2(16a^2d^2+bc(5bc-16ad))\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{x^3\sqrt{c+dx^2}(16a^2d^2+bc(5bc-16ad))}{64d^2}+\frac{cx\sqrt{c+dx^2}(16a^2d^2+bc(5bc-16ad))}{128d^3}-\frac{bx^3(c+dx^2)^{3/2}(5bc-16ad)}{48d^2}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}$$

```
[Out] (c*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^
3) + ((16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x^3*Sqrt[c + d*x^2])/(6
4*d^2) - (b*(5*b*c - 16*a*d)*x^3*(c + d*x^2)^(3/2))/(48*d^2) + (b
^2*x^5*(c + d*x^2)^(3/2))/(8*d) - (c^2*(16*a^2*d^2 + b*c*(5*b*c -
 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(7/2))
```

Rubi [A] time = 0.481757, antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{c^2(16a^2d^2+bc(5bc-16ad))\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{1}{64}x^3\sqrt{c+dx^2}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)+\frac{cx\sqrt{c+dx^2}(16a^2d^2+bc(5bc-16ad))}{128d^3}-\frac{bx^3(c+dx^2)^{3/2}(5bc-16ad)}{48d^2}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^
3) + ((16*a^2 + (b*c*(5*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/
64 - (b*(5*b*c - 16*a*d)*x^3*(c + d*x^2)^(3/2))/(48*d^2) + (b^2*x
^5*(c + d*x^2)^(3/2))/(8*d) - (c^2*(16*a^2*d^2 + b*c*(5*b*c - 16*
a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(7/2))
```

Rubi in Sympy [A] time = 40.2692, size = 182, normalized size = 0.95

$$\frac{b^2x^5(c+dx^2)^{\frac{3}{2}}}{8d}+\frac{bx^3(c+dx^2)^{\frac{3}{2}}(16ad-5bc)}{48d^2}-\frac{c^2(16a^2d^2-bc(16ad-5bc))\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{\frac{7}{2}}}+\frac{cx\sqrt{c+dx^2}(16a^2d^2-bc(16ad-5bc))}{128d^3}+\frac{x^3\sqrt{c+dx^2}(16a^2d^2-bc(16ad-5bc))}{64d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*x**5*(c + d*x**2)**(3/2)/(8*d) + b*x**3*(c + d*x**2)**(3/2)*
(16*a*d - 5*b*c)/(48*d**2) - c**2*(16*a**2*d**2 - b*c*(16*a*d - 5
*b*c))*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(128*d**(7/2)) + c*x*sqr
t(c + d*x**2)*(16*a**2*d**2 - b*c*(16*a*d - 5*b*c))/(128*d**3) +
x**3*sqrt(c + d*x**2)*(16*a**2*d**2 - b*c*(16*a*d - 5*b*c))/(64*d
**2)
```

Mathematica [A] time = 0.160128, size = 157, normalized size = 0.82

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2(c+2dx^2)+16abd(-3c^2+2cdx^2+8d^2x^4)+b^2(15c^3-10c^2dx^2+8cd^2x^4+48d^3x^6)\right)-3c^2(16a^2d^2}{384d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(c + 2*d*x^2) + 16*a*b*d*(
-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4) + b^2*(15*c^3 - 10*c^2*d*x^2 + 8*
c*d^2*x^4 + 48*d^3*x^6)) - 3*c^2*(5*b^2*c^2 - 16*a*b*c*d + 16*a^2
*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(384*d^(7/2))
```

Maple [A] time = 0.02, size = 259, normalized size = 1.4

$$\frac{a^2x}{4d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2cx}{8d}\sqrt{dx^2+c}-\frac{a^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+\frac{b^2x^5}{8d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,b^2cx^3}{48\,d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{5\,b^2c^2x}{64\,d^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,xb^2c^3}{128\,d^3}\sqrt{dx^2+c}-\frac{5\,b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}+\frac{abx^3}{3\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{abcx}{4\,d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{abc^2x}{8\,d^2}\sqrt{dx^2+c}+\frac{abc^3}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*a^2*x*(d*x^2+c)^(3/2)/d-1/8*a^2*c/d*x*(d*x^2+c)^(1/2)-1/8*a^2
*c^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/8*b^2*x^5*(d*x^2+c)^
(3/2)/d-5/48*b^2*c/d^2*x^3*(d*x^2+c)^(3/2)+5/64*b^2*c^2/d^3*x*(d*
x^2+c)^(3/2)-5/128*b^2*c^3/d^3*x*(d*x^2+c)^(1/2)-5/128*b^2*c^4/d^
(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x^3*(d*x^2+c)^(3/2)/d
-1/4*a*b*c/d^2*x*(d*x^2+c)^(3/2)+1/8*a*b*c^2/d^2*x*(d*x^2+c)^(1/2
)+1/8*a*b*c^3/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.32494, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,b^2d^3x^7+8\left(b^2cd^2+16\,abd^3\right)x^5-2\left(5\,b^2c^2d-16\,abcd^2-48\,a^2d^3\right)x^3+3\left(5\,b^2c^3-16\,abc^2d+16\,a^2cd^2\right)x\right)\sqrt{dx^2+c}\cdots}{768\,d^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2,x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^2*d^3*x^7 + 8*(b^2*c*d^2 + 16*a*b*d^3)*x^5 - 2*(5
*b^2*c^2*d - 16*a*b*c*d^2 - 48*a^2*d^3)*x^3 + 3*(5*b^2*c^3 - 16*a
*b*c^2*d + 16*a^2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(5*b^2*c^
4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*log(2*sqrt(d*x^2 + c)*d*x - (2
*d*x^2 + c)*sqrt(d)))/d^(7/2), 1/384*((48*b^2*d^3*x^7 + 8*(b^2*c*
d^2 + 16*a*b*d^3)*x^5 - 2*(5*b^2*c^2*d - 16*a*b*c*d^2 - 48*a^2*d^
3)*x^3 + 3*(5*b^2*c^3 - 16*a*b*c^2*d + 16*a^2*c*d^2)*x)*sqrt(d*x^
2 + c)*sqrt(-d) - 3*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*a
```

```
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^3)]
```

Sympy [A] time = 57.1382, size = 411, normalized size = 2.15

$$\frac{a^2c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}x^3}{8\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{a^2dx^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{3}{2}}x^3}{24d\sqrt{1+\frac{dx^2}{c}}}+\frac{5ab\sqrt{c}x^5}{12\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}}+\frac{abdx^7}{3\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{7}{2}}x}{128d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{5}{2}}x^3}{384d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}x^5}{192d\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2\sqrt{c}x^7}{48\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{7}{2}}}+\frac{b^2dx^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*x**3/(8
*sqrt(1 + d*x**2/c)) - a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(
3/2)) + a**2*d*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)
*x/(8*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(3/2)*x**3/(24*d*sqrt(1 +
 d*x**2/c)) + 5*a*b*sqrt(c)*x**5/(12*sqrt(1 + d*x**2/c)) + a*b*c*
*3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a*b*d*x**7/(3*sqrt(c)*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d**3*sqrt(1 + d*x**2
/c)) + 5*b**2*c**(5/2)*x**3/(384*d**2*sqrt(1 + d*x**2/c)) - b**2*
c**(3/2)*x**5/(192*d*sqrt(1 + d*x**2/c)) + 7*b**2*sqrt(c)*x**7/(4
8*sqrt(1 + d*x**2/c)) - 5*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128
*d**(7/2)) + b**2*d*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.247241, size = 235, normalized size = 1.23

$$\frac{1}{384}\left(2\left(4\left(6\,b^2x^2+\frac{b^2cd^5+16\,abd^6}{d^6}\right)x^2-\frac{5\,b^2c^2d^4-16\,abcd^5-48\,a^2d^6}{d^6}\right)x^2+\frac{3\left(5\,b^2c^3d^3-16\,abc^2d^4+16\,a^2cd^5\right)}{d^6}\right)\sqrt{dx^2+c}\,x+\frac{\left(5\,b^2c^4-16\,abc^3d+16\,a^2c^2d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2 + (b^2*c*d^5 + 16*a*b*d^6)/d^6)*x^2 - (5*b
^2*c^2*d^4 - 16*a*b*c*d^5 - 48*a^2*d^6)/d^6)*x^2 + 3*(5*b^2*c^3*d
^3 - 16*a*b*c^2*d^4 + 16*a^2*c*d^5)/d^6)*sqrt(d*x^2 + c)*x + 1/12
8*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*ln(abs(-sqrt(d)*x +
 sqrt(d*x^2 + c)))/d^(7/2)
```

3.606 $\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}$$

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^2) -
(b*(3*b*c - 8*a*d)*x*(c + d*x^2)^(3/2))/(24*d^2) + (b*x*(a + b*x^
2)*(c + d*x^2)^(3/2))/(6*d) + (c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2
)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2))
```

Rubi [A] time = 0.210795, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^2) -
(b*(3*b*c - 8*a*d)*x*(c + d*x^2)^(3/2))/(24*d^2) + (b*x*(a + b*x^
2)*(c + d*x^2)^(3/2))/(6*d) + (c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2
)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2))
```

Rubi in Sympy [A] time = 23.3195, size = 143, normalized size = 0.96

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}{6d}+\frac{bx\left(c+dx^2\right)^{\frac{3}{2}}(8ad-3bc)}{24d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{\frac{5}{2}}}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] b*x*(a + b*x**2)*(c + d*x**2)**(3/2)/(6*d) + b*x*(c + d*x**2)**(3
/2)*(8*a*d - 3*b*c)/(24*d**2) + c*(8*a**2*d**2 - 4*a*b*c*d + b**2
*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(16*d**(5/2)) + x*sqrt(c
 + d*x**2)*(8*a**2*d**2 - 4*a*b*c*d + b**2*c**2)/(16*d**2)
```

Mathematica [A] time = 0.111786, size = 122, normalized size = 0.82

$$\frac{3c\left(8a^2d^2-4abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}x\sqrt{c+dx^2}\left(24a^2d^2+12abd\left(c+2dx^2\right)+b^2\left(-3c^2+2cdx^2+8d^2x^4\right.\right.}{48d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(c + 2*d*x^2) +
 b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c*(b^2*c^2 - 4*a*b*c*d
 + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*d^(5/2))
```

Maple [A] time = 0.012, size = 190, normalized size = 1.3

$$\frac{a^2x}{2}\sqrt{dx^2+c}+\frac{a^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}+\frac{b^2x^3}{6d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{b^2cx}{8d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{b^2c^2x}{16d^2}\sqrt{dx^2+c}+\frac{b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{abx}{2d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{abcx}{4d}\sqrt{dx^2+c}-\frac{abc^2}{4}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*a^2*x*(d*x^2+c)^(1/2)+1/2*a^2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))+1/6*b^2*x^3*(d*x^2+c)^(3/2)/d-1/8*b^2*c/d^2*x*(d*x^2+c)^
(3/2)+1/16*b^2*c^2/d^2*x*(d*x^2+c)^(1/2)+1/16*b^2*c^3/d^(5/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a*b*x*(d*x^2+c)^(3/2)/d-1/4*a*b*c/
d*x*(d*x^2+c)^(1/2)-1/4*a*b*c^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257659, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,b^2d^2x^5+2\left(b^2cd+12\,abd^2\right)x^3-3\left(b^2c^2-4\,abcd-8\,a^2d^2\right)x\right)\sqrt{dx^2+c}\sqrt{d}+3\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\log\left(-2\sqrt{dx\right.}}{96\,d^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*d^2*x^5 + 2*(b^2*c*d + 12*a*b*d^2)*x^3 - 3*(b^2*c
^2 - 4*a*b*c*d - 8*a^2*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(b^2*c
^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d
*x^2 + c)*sqrt(d)))/d^(5/2), 1/48*((8*b^2*d^2*x^5 + 2*(b^2*c*d +
12*a*b*d^2)*x^3 - 3*(b^2*c^2 - 4*a*b*c*d - 8*a^2*d^2)*x)*sqrt(d*x
^2 + c)*sqrt(-d) + 3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*arctan
(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^2)]
```

Sympy [A] time = 34.6228, size = 291, normalized size = 1.95

$$\frac{a^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}}+\frac{abc^{\frac{3}{2}}x}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}x^3}{4\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{4d^{\frac{3}{2}}}+\frac{abdx^5}{2\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x}{16d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}x^3}{48d\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2\sqrt{c}x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16d^{\frac{5}{2}}}+\frac{b^2dx^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/2 + a**2*c*asinh(sqrt(d)*x/sqrt
(c))/(2*sqrt(d)) + a*b*c**(3/2)*x/(4*d*sqrt(1 + d*x**2/c)) + 3*a*
b*sqrt(c)*x**3/(4*sqrt(1 + d*x**2/c)) - a*b*c**2*asinh(sqrt(d)*x/
sqrt(c))/(4*d**(3/2)) + a*b*d*x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c))
 - b**2*c**(5/2)*x/(16*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)*x
**3/(48*d*sqrt(1 + d*x**2/c)) + 5*b**2*sqrt(c)*x**5/(24*sqrt(1 +
d*x**2/c)) + b**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(5/2)) + b
**2*d*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.246155, size = 173, normalized size = 1.16

$$\frac{1}{48}\left(2\left(4\,b^2x^2+\frac{b^2cd^3+12\,abd^4}{d^4}\right)x^2-\frac{3\left(b^2c^2d^2-4\,abcd^3-8\,a^2d^4\right)}{d^4}\right)\sqrt{dx^2+c}x-\frac{\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2 + (b^2*c*d^3 + 12*a*b*d^4)/d^4)*x^2 - 3*(b^2*c
^2*d^2 - 4*a*b*c*d^3 - 8*a^2*d^4)/d^4)*sqrt(d*x^2 + c)*x - 1/16*(
b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*ln(abs(-sqrt(d)*x + sqrt(d*x
^2 + c)))/d^(5/2)
```

3.607 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a^2(c+dx^2)^{3/2}}{cx}-\frac{(b^2c^2-8ad(ad+bc))\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}-\frac{x\sqrt{c+dx^2}\,(b^2c^2-8ad(ad+bc))}{8cd}+\frac{b^2x(c+dx^2)^{3/2}}{4d}$$

```
[Out] -((b^2*c^2 - 8*a*d*(b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c*d) - (a^2
*(c + d*x^2)^(3/2))/(c*x) + (b^2*x*(c + d*x^2)^(3/2))/(4*d) - ((b
^2*c^2 - 8*a*d*(b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])
/(8*d^(3/2))
```

Rubi [A] time = 0.234853, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{1}{8}x\sqrt{c+dx^2}\left(\frac{8a^2d}{c}+8ab-\frac{b^2c}{d}\right)-\frac{a^2(c+dx^2)^{3/2}}{cx}-\frac{(b^2c^2-8ad(ad+bc))\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}+\frac{b^2x(c+dx^2)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] ((8*a*b - (b^2*c)/d + (8*a^2*d)/c)*x*Sqrt[c + d*x^2])/8 - (a^2*(c
 + d*x^2)^(3/2))/(c*x) + (b^2*x*(c + d*x^2)^(3/2))/(4*d) - ((b^2*
c^2 - 8*a*d*(b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8
*d^(3/2))
```

Rubi in Sympy [A] time = 24.2129, size = 114, normalized size = 0.86

$$-\frac{a^2(c+dx^2)^{\frac{3}{2}}}{cx}+\frac{b^2x(c+dx^2)^{\frac{3}{2}}}{4d}-\frac{(-8ad(ad+bc)+b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{\frac{3}{2}}}-\frac{x\sqrt{c+dx^2}\,(-8ad(ad+bc)+b^2c^2)}{8cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(c*x) + b**2*x*(c + d*x**2)**(3/2)/(4*d
) - (-8*a*d*(a*d + b*c) + b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x
**2))/(8*d**(3/2)) - x*sqrt(c + d*x**2)*(-8*a*d*(a*d + b*c) + b**
2*c**2)/(8*c*d)
```

Mathematica [A] time = 0.183502, size = 99, normalized size = 0.74

$$\frac{(8a^2d^2+8abcd-b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{3/2}}+\sqrt{c+dx^2}\left(-\frac{a^2}{x}+abx+\frac{b^2x(c+2dx^2)}{8d}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-(a^2/x) + a*b*x + (b^2*x*(c + 2*d*x^2))/(8*d))
+ ((-(b^2*c^2) + 8*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c
+ d*x^2]])/(8*d^(3/2))
```

Maple [A] time = 0.016, size = 163, normalized size = 1.2

$$\frac{b^2x}{4d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{b^2cx}{8d}\sqrt{dx^2+c}-\frac{b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}-\frac{a^2}{cx}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2dx}{c}\sqrt{dx^2+c}+a^2\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)+abx\sqrt{dx^2+c}+abc\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] 1/4*b^2*x*(d*x^2+c)^(3/2)/d-1/8*b^2*c/d*x*(d*x^2+c)^(1/2)-1/8*b^2
*c^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-a^2*(d*x^2+c)^(3/2)/c/
x+a^2*d/c*x*(d*x^2+c)^(1/2)+a^2*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1
/2))+a*b*x*(d*x^2+c)^(1/2)+a*b*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236915, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2c^2-8\,abcd-8\,a^2d^2\right)x\log\left(-2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)-2\left(2\,b^2dx^4-8\,a^2d+\left(b^2c+8\,abd\right)x^2\right)\sqrt{dx^2+c}\sqrt{d}}{16\,d^{\frac{3}{2}}x},\ -\frac{\left(b^2c^2-8\,abcd-8\,a^2d^2\right)x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(2\,b^2dx^4-8\,a^2d+\left(b^2c+8\,abd\right)x^2\right)\sqrt{dx^2+c}\sqrt{-d}}{8\,\sqrt{-d}dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*x*log(-2*sqrt(d*x^2 + c
)*d*x - (2*d*x^2 + c)*sqrt(d)) - 2*(2*b^2*d*x^4 - 8*a^2*d + (b^2*
c + 8*a*b*d)*x^2)*sqrt(d*x^2 + c)*sqrt(d))/(d^(3/2)*x), -1/8*((b^
2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c
)) - (2*b^2*d*x^4 - 8*a^2*d + (b^2*c + 8*a*b*d)*x^2)*sqrt(d*x^2 +
 c)*sqrt(-d))/(sqrt(-d)*d*x)]
```

Sympy [A] time = 21.7864, size = 219, normalized size = 1.65

$$-\frac{a^{2}\sqrt{c}}{x\sqrt{1+\frac{dx^{2}}{c}}}+a^{2}\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{a^{2}dx}{\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+ab\sqrt{c}x\sqrt{1+\frac{dx^{2}}{c}}+\frac{abc\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}+\frac{b^{2}c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3b^{2}\sqrt{c}x^{3}}{8\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}c^{2}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{b^{2}dx^{5}}{4\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] -a**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(d)*asinh(sqrt(d)
*x/sqrt(c)) - a**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)
*x*sqrt(1 + d*x**2/c) + a*b*c*asinh(sqrt(d)*x/sqrt(c))/sqrt(d) +
b**2*c**(3/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 3*b**2*sqrt(c)*x**3/(8
*sqrt(1 + d*x**2/c)) - b**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(
3/2)) + b**2*d*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.24256, size = 170, normalized size = 1.28

$$\frac{2\,a^{2}c\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c}+\frac{1}{8}\left(2\,b^{2}x^{2}+\frac{b^{2}cd+8\,abd^{2}}{d^{2}}\right)\sqrt{dx^{2}+c}x+\frac{\left(b^{2}c^{2}\sqrt{d}-8\,abcd^{\frac{3}{2}}-8\,a^{2}d^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}\right)}{16\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/8*(2*b^
2*x^2 + (b^2*c*d + 8*a*b*d^2)/d^2)*sqrt(d*x^2 + c)*x + 1/16*(b^2*
c^2*sqrt(d) - 8*a*b*c*d^(3/2) - 8*a^2*d^(5/2))*ln((sqrt(d)*x - sq
rt(d*x^2 + c))^2)/d^2
```

3.608 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx$

Optimal. Leaf size=111

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

```
[Out] (b*(b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (a^2*(c + d*x^2)^(3/2
))/(3*c*x^3) - (2*a*b*(c + d*x^2)^(3/2))/(c*x) + (b*(b*c + 4*a*d)
*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

Rubi [A] time = 0.19783, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (b*(b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (a^2*(c + d*x^2)^(3/2
))/(3*c*x^3) - (2*a*b*(c + d*x^2)^(3/2))/(c*x) + (b*(b*c + 4*a*d)
*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

Rubi in Sympy [A] time = 23.6834, size = 99, normalized size = 0.89

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{\frac{3}{2}}}{cx}+\frac{b\,(4ad+bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{bx\sqrt{c+dx^2}\,(4ad+bc)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(3*c*x**3) - 2*a*b*(c + d*x**2)**(3/2)/
(c*x) + b*(4*a*d + b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*sqrt
(d)) + b*x*sqrt(c + d*x**2)*(4*a*d + b*c)/(2*c)
```

Mathematica [A] time = 0.143353, size = 91, normalized size = 0.82

$$\sqrt{c+dx^2}\left(-\frac{a^2}{3x^3}-\frac{a(ad+6bc)}{3cx}+\frac{b^2x}{2}\right)+\frac{b(4ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (-a^2/(3*x^3) - (a*(6*b*c + a*d))/(3*c*x) + (b^2*x)/2)*Sqrt[c + d
*x^2] + (b*(b*c + 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*S
qrt[d])
```

Maple [A] time = 0.016, size = 122, normalized size = 1.1

$$\frac{xb^2}{2}\sqrt{dx^2+c}+\frac{b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}-\frac{a^2}{3\,cx^3}\left(dx^2+c\right)^{\frac{3}{2}}-2\,\frac{ab\left(dx^2+c\right)^{3/2}}{cx}+2\,\frac{dabx\sqrt{dx^2+c}}{c}+2\,ab\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x)
```

```
[Out] 1/2*x*b^2*(d*x^2+c)^(1/2)+1/2*b^2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))-1/3*a^2*(d*x^2+c)^(3/2)/c/x^3-2*a*b*(d*x^2+c)^(3/2)/c/x+
2*a*b*d/c*x*(d*x^2+c)^(1/2)+2*a*b*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233574, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^2c^2+4\,abcd\right)x^3\log\left(-2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+2\left(3\,b^2cx^4-2\,a^2c-2\left(6\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{d}}{12\,c\sqrt{d}x^3},\ \frac{3\left(b^2c^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^2 + 4*a*b*c*d)*x^3*log(-2*sqrt(d*x^2 + c)*d*x - (
2*d*x^2 + c)*sqrt(d)) + 2*(3*b^2*c*x^4 - 2*a^2*c - 2*(6*a*b*c + a
^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(d))/(c*sqrt(d)*x^3), 1/6*(3*(b^2*
c^2 + 4*a*b*c*d)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*b^2*
c*x^4 - 2*a^2*c - 2*(6*a*b*c + a^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-
d))/(c*sqrt(-d)*x^3)]
```

Sympy [A] time = 12.9028, size = 170, normalized size = 1.53

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}-\frac{2ab\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+2ab\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{2abdx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*d**(3/2)*sqrt(
c/(d*x**2) + 1)/(3*c) - 2*a*b*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + 2*
a*b*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) - 2*a*b*d*x/(sqrt(c)*sqrt(1
+ d*x**2/c)) + b**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/2 + b**2*c*asinh
(sqrt(d)*x/sqrt(c))/(2*sqrt(d))
```

GIAC/XCAS [A] time = 0.249634, size = 254, normalized size = 2.29

$$\frac{1}{2}\sqrt{dx^2+c}b^2x - \frac{\left(b^2c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,d} + \frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+6\,abc^3\sqrt{d}+a^2c^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x - 1/4*(b^2*c*sqrt(d) + 4*a*b*d^(3/2))*l
n((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d
*x^2 + c))^4*a*b*c*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^
2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) +
6*a*b*c^3*sqrt(d) + a^2*c^2*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c
))^2 - c)^3
```

3.609 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(c+dx^2)^{3/2}(5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

```
[Out] -((b^2*Sqrt[c + d*x^2])/x) - (a^2*(c + d*x^2)^(3/2))/(5*c*x^5) -
(2*a*(5*b*c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^3) + b^2*Sqrt[d]*
ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
```

Rubi [A] time = 0.187503, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(c+dx^2)^{3/2}(5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -((b^2*Sqrt[c + d*x^2])/x) - (a^2*(c + d*x^2)^(3/2))/(5*c*x^5) -
(2*a*(5*b*c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^3) + b^2*Sqrt[d]*
ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
```

Rubi in Sympy [A] time = 24.6507, size = 92, normalized size = 0.89

$$-\frac{a^2(c+dx^2)^{\frac{3}{2}}}{5cx^5}+\frac{2a(c+dx^2)^{\frac{3}{2}}(ad-5bc)}{15c^2x^3}+b^2\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{b^2\sqrt{c+dx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**6,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(5*c*x**5) + 2*a*(c + d*x**2)**(3/2)*(a
*d - 5*b*c)/(15*c**2*x**3) + b**2*sqrt(d)*atanh(sqrt(d)*x/sqrt(c
+ d*x**2)) - b**2*sqrt(c + d*x**2)/x
```

Mathematica [A] time = 0.131193, size = 104, normalized size = 1.01

$$b^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)-\frac{\sqrt{c+dx^2}\left(a^2\left(3c^2+cdx^2-2d^2x^4\right)+10abcx^2\left(c+dx^2\right)+15b^2c^2x^4\right)}{15c^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c + d*x^2) + a^
2*(3*c^2 + c*d*x^2 - 2*d^2*x^4)))/(15*c^2*x^5) + b^2*Sqrt[d]*Log[
d*x + Sqrt[d]*Sqrt[c + d*x^2]]
```

Maple [A] time = 0.017, size = 123, normalized size = 1.2

$$-\frac{a^{2}}{5\,c x^{5}}\left(d x^{2}+c\right)^{\frac{3}{2}}+\frac{2\,a^{2} d}{15\,c^{2} x^{3}}\left(d x^{2}+c\right)^{\frac{3}{2}}-\frac{b^{2}}{c x}\left(d x^{2}+c\right)^{\frac{3}{2}}+\frac{b^{2} d x}{c}\sqrt{d x^{2}+c}+b^{2}\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{d x^{2}+c}\right)-\frac{2\,a b}{3\,c x^{3}}\left(d x^{2}+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(3/2)/c/x^5+2/15*a^2*d/c^2/x^3*(d*x^2+c)^(3/2)
-b^2/c/x*(d*x^2+c)^(3/2)+b^2*d/c*x*(d*x^2+c)^(1/2)+b^2*d^(1/2)*ln
(x*d^(1/2)+(d*x^2+c)^(1/2))-2/3*a*b/c/x^3*(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.251361, size = 1, normalized size = 0.01

$$\left[\frac{15\,b^{2} c^{2}\sqrt{d} x^{5}\log\left(-2\,d x^{2}-2\,\sqrt{d x^{2}+c}\sqrt{d} x-c\right)-2\,\left(\left(15\,b^{2} c^{2}+10\,a b c d-2\,a^{2} d^{2}\right) x^{4}+3\,a^{2} c^{2}+\left(10\,a b c^{2}+a^{2} c d\right) x^{2}\right)\sqrt{d x^{2}+\cdots}}{30\,c^{2} x^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*c^2*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) - 2*((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2)*x^4 + 3*a
^2*c^2 + (10*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5),
1/15*(15*b^2*c^2*sqrt(-d)*x^5*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d
))) - ((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2)*x^4 + 3*a^2*c^2 + (1
0*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5)]
```

Sympy [A] time = 10.4667, size = 199, normalized size = 1.93

$$-\frac{a^{2}\sqrt{d}\sqrt{\frac{c}{d x^{2}}+1}}{5x^{4}}-\frac{a^{2} d^{\frac{3}{2}}\sqrt{\frac{c}{d x^{2}}+1}}{15c x^{2}}+\frac{2a^{2} d^{\frac{5}{2}}\sqrt{\frac{c}{d x^{2}}+1}}{15c^{2}}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{d x^{2}}+1}}{3x^{2}}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{d x^{2}}+1}}{3c}-\frac{b^{2}\sqrt{c}}{x\sqrt{1+\frac{d x^{2}}{c}}}+b^{2}\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{b^{2} d x}{\sqrt{c}\sqrt{1+\frac{d x^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**6,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - a**2*d**(3/2)*sqrt(
c/(d*x**2) + 1)/(15*c*x**2) + 2*a**2*d**(5/2)*sqrt(c/(d*x**2) + 1
```

```
)/(15*c**2) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b
*d**(3/2)*sqrt(c/(d*x**2) + 1)/(3*c) - b**2*sqrt(c)/(x*sqrt(1 + d
*x**2/c)) + b**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) - b**2*d*x/(sqr
t(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.260052, size = 544, normalized size = 5.28

$$-\frac{1}{2}b^2\sqrt{d}\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)$$
$$+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c\sqrt{d}+30\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abd^{\frac{3}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^2*sqrt(d)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(
sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c*sqrt(d) + 30*(sqrt(d)*x - sq
rt(d*x^2 + c))^8*a*b*d^(3/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*b^2*c^2*sqrt(d) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c*d^(3/
2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*d^(5/2) + 90*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*b^2*c^3*sqrt(d) + 40*(sqrt(d)*x - sqrt(d*
x^2 + c))^4*a*b*c^2*d^(3/2) + 10*(sqrt(d)*x - sqrt(d*x^2 + c))^4*
a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^4*sqrt(d
) - 20*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^3*d^(3/2) + 10*(sqrt
(d)*x - sqrt(d*x^2 + c))^2*a^2*c^2*d^(5/2) + 15*b^2*c^5*sqrt(d) +
 10*a*b*c^4*d^(3/2) - 2*a^2*c^3*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2
 + c))^2 - c)^5
```

3.610 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{\left(c+dx^2\right)^{3/2}\left(35b^2c^2-4ad(7bc-2ad)\right)}{105c^3x^3}-\frac{2a\left(c+dx^2\right)^{3/2}(7bc-2ad)}{35c^2x^5}$$

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(7*c*x^7) - (2*a*(7*b*c - 2*a*d)*(c + d*
x^2)^(3/2))/(35*c^2*x^5) - ((35*b^2*c^2 - 4*a*d*(7*b*c - 2*a*d))*
(c + d*x^2)^(3/2))/(105*c^3*x^3)
```

Rubi [A] time = 0.214255, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\left(c+dx^2\right)^{3/2}\left(8a^2d^2-28abcd+35b^2c^2\right)}{105c^3x^3}-\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{2a\left(c+dx^2\right)^{3/2}(7bc-2ad)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(7*c*x^7) - (2*a*(7*b*c - 2*a*d)*(c + d*
x^2)^(3/2))/(35*c^2*x^5) - ((35*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)
*(c + d*x^2)^(3/2))/(105*c^3*x^3)
```

Rubi in Sympy [A] time = 22.0912, size = 94, normalized size = 0.95

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{7cx^7}+\frac{2a\left(c+dx^2\right)^{\frac{3}{2}}(2ad-7bc)}{35c^2x^5}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(4ad\left(2ad-7bc\right)+35b^2c^2\right)}{105c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**8,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(7*c*x**7) + 2*a*(c + d*x**2)**(3/2)*(2
*a*d - 7*b*c)/(35*c**2*x**5) - (c + d*x**2)**(3/2)*(4*a*d*(2*a*d
- 7*b*c) + 35*b**2*c**2)/(105*c**3*x**3)
```

Mathematica [A] time = 0.0832567, size = 76, normalized size = 0.77

$$-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(15c^2-12cdx^2+8d^2x^4\right)+14abcx^2\left(3c-2dx^2\right)+35b^2c^2x^4\right)}{105c^3x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -((c + d*x^2)^(3/2)*(35*b^2*c^2*x^4 + 14*a*b*c*x^2*(3*c - 2*d*x^2
) + a^2*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4)))/(105*c^3*x^7)
```

Maple [A] time = 0.01, size = 78, normalized size = 0.8

$$-\frac{8\,x^4a^2d^2-28\,x^4abcd+35\,x^4b^2c^2-12\,x^2a^2cd+42\,ac^2bx^2+15\,a^2c^2}{105\,x^7c^3}\left(dx^2+c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x)
```

```
[Out] -1/105*(d*x^2+c)^(3/2)*(8*a^2*d^2*x^4-28*a*b*c*d*x^4+35*b^2*c^2*x
^4-12*a^2*c*d*x^2+42*a*b*c^2*x^2+15*a^2*c^2)/x^7/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.307874, size = 144, normalized size = 1.45

$$-\frac{\left(\left(35\, b^{2} c^{2} d - 28\, a b c d^{2} + 8\, a^{2} d^{3}\right) x^{6} + 15\, a^{2} c^{3} + \left(35\, b^{2} c^{3} + 14\, a b c^{2} d - 4\, a^{2} c d^{2}\right) x^{4} + 3\, \left(14\, a b c^{3} + a^{2} c^{2} d\right) x^{2}\right) \sqrt{d x^{2} + c}}{105\, c^{3} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*((35*b^2*c^2*d - 28*a*b*c*d^2 + 8*a^2*d^3)*x^6 + 15*a^2*c^
3 + (35*b^2*c^3 + 14*a*b*c^2*d - 4*a^2*c*d^2)*x^4 + 3*(14*a*b*c^3
 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c)/(c^3*x^7)
```

Sympy [A] time = 10.6966, size = 510, normalized size = 5.15

$$-\frac{15a^{2}c^{5}d^{\frac{9}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{33a^{2}c^{4}d^{\frac{11}{2}}x^{2}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{17a^{2}c^{3}d^{\frac{13}{2}}x^{4}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{3a^{2}c^{2}d^{\frac{15}{2}}x^{6}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{12a^{2}cd^{\frac{17}{2}}x^{8}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{8a^{2}d^{\frac{19}{2}}x^{10}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{5x^{4}}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{15cx^{2}}+\frac{4abd^{\frac{5}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{15c^{2}}-\frac{b^{2}\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{3x^{2}}-\frac{b^{2}d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**8,x)
```

```
[Out] -15*a**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 +
 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 33*a**2*c**4*d**(11/
2)*x**2*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*
x**8 + 105*c**3*d**6*x**10) - 17*a**2*c**3*d**(13/2)*x**4*sqrt(c/
(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3
*d**6*x**10) - 3*a**2*c**2*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)/(1
05*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 1
2*a**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
+ 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*a**2*d**(19/2)*x*
*10*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8
```

```
 + 105*c**3*d**6*x**10) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x
**4) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*c*x**2) + 4*a*b*d*
*(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - b**2*sqrt(d)*sqrt(c/(d*x*
*2) + 1)/(3*x**2) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(3*c)
```

GIAC/XCAS [A] time = 0.253373, size = 662, normalized size = 6.69

$$\frac{2\left(105\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2d^{\frac{3}{2}}-420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2cd^{\frac{3}{2}}+420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abd^{\frac{5}{2}}+665\left(\sqrt{d}x-\sqrt{dx^2+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^8,x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*d^(3/2) - 420*(sq
rt(d)*x - sqrt(d*x^2 + c))^10*b^2*c*d^(3/2) + 420*(sqrt(d)*x - sq
rt(d*x^2 + c))^10*a*b*d^(5/2) + 665*(sqrt(d)*x - sqrt(d*x^2 + c))
^8*b^2*c^2*d^(3/2) - 700*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^
(5/2) + 560*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(7/2) - 560*(sq
rt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*d^(3/2) + 280*(sqrt(d)*x - s
qrt(d*x^2 + c))^6*a*b*c^2*d^(5/2) + 280*(sqrt(d)*x - sqrt(d*x^2 +
 c))^6*a^2*c*d^(7/2) + 315*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^
4*d^(3/2) - 168*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(5/2) +
 168*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^(7/2) - 140*(sqrt(
d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*d^(3/2) + 196*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c^4*d^(5/2) - 56*(sqrt(d)*x - sqrt(d*x^2 + c))
^2*a^2*c^3*d^(7/2) + 35*b^2*c^6*d^(3/2) - 28*a*b*c^5*d^(5/2) + 8*
a^2*c^4*d^(7/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^7
```

3.611 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{2d\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{315c^4x^3}-\frac{\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{105c^3x^5}-\frac{2a\left(c+dx^2\right)^{3/2}(3bc-ad)}{21c^2x^7}$$

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(9*c*x^9) - (2*a*(3*b*c - a*d)*(c + d*x^
2)^(3/2))/(21*c^2*x^7) - ((21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(c +
 d*x^2)^(3/2))/(105*c^3*x^5) + (2*d*(21*b^2*c^2 - 8*a*d*(3*b*c -
a*d))*(c + d*x^2)^(3/2))/(315*c^4*x^3)
```

Rubi [A] time = 0.337251, antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\left(c+dx^2\right)^{3/2}\left(8a^2d^2-24abcd+21b^2c^2\right)}{105c^3x^5}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{2d\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{315c^4x^3}-\frac{2a\left(c+dx^2\right)^{3/2}(3bc-ad)}{21c^2x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(9*c*x^9) - (2*a*(3*b*c - a*d)*(c + d*x^
2)^(3/2))/(21*c^2*x^7) - ((21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*(
c + d*x^2)^(3/2))/(105*c^3*x^5) + (2*d*(21*b^2*c^2 - 8*a*d*(3*b*c
 - a*d))*(c + d*x^2)^(3/2))/(315*c^4*x^3)
```

Rubi in Sympy [A] time = 26.9728, size = 134, normalized size = 0.94

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{9cx^9}+\frac{2a\left(c+dx^2\right)^{\frac{3}{2}}(ad-3bc)}{21c^2x^7}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(8ad\,(ad-3bc)+21b^2c^2\right)}{105c^3x^5}+\frac{2d\left(c+dx^2\right)^{\frac{3}{2}}\left(8ad\,(ad-3bc)+21b^2c^2\right)}{315c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**10,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(9*c*x**9) + 2*a*(c + d*x**2)**(3/2)*(a
*d - 3*b*c)/(21*c**2*x**7) - (c + d*x**2)**(3/2)*(8*a*d*(a*d - 3*
b*c) + 21*b**2*c**2)/(105*c**3*x**5) + 2*d*(c + d*x**2)**(3/2)*(8
*a*d*(a*d - 3*b*c) + 21*b**2*c**2)/(315*c**4*x**3)
```

Mathematica [A] time = 0.1055, size = 108, normalized size = 0.76

$$-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6\right)+6abcx^2\left(15c^2-12cdx^2+8d^2x^4\right)+21b^2c^2x^4\left(3c-2dx^2\right)\right)}{315c^4x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -((c + d*x^2)^(3/2)*(21*b^2*c^2*x^4*(3*c - 2*d*x^2) + 6*a*b*c*x^2
*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4) + a^2*(35*c^3 - 30*c^2*d*x^2 +
 24*c*d^2*x^4 - 16*d^3*x^6)))/(315*c^4*x^9)
```

Maple [A] time = 0.011, size = 117, normalized size = 0.8

$$-\frac{-16\,x^6a^2d^3+48\,x^6abcd^2-42\,x^6b^2c^2d+24\,x^4a^2cd^2-72\,x^4abc^2d+63\,x^4b^2c^3-30\,x^2a^2c^2d+90\,x^2abc^3+35\,a^2c^3}{315\,x^9c^4}\left(dx^2+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x)
```

```
[Out] -1/315*(d*x^2+c)^(3/2)*(-16*a^2*d^3*x^6+48*a*b*c*d^2*x^6-42*b^2*c
^2*d*x^6+24*a^2*c*d^2*x^4-72*a*b*c^2*d*x^4+63*b^2*c^3*x^4-30*a^2*
c^2*d*x^2+90*a*b*c^3*x^2+35*a^2*c^3)/x^9/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.476799, size = 198, normalized size = 1.38

$$\frac{\left(2\left(21\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4\right)x^8-\left(21\,b^2c^3d-24\,abc^2d^2+8\,a^2cd^3\right)x^6-35\,a^2c^4-3\left(21\,b^2c^4+6\,abc^3d-2\,a^2c^2d^2\right)x^4-}{315\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(21*b^2*c^2*d^2 - 24*a*b*c*d^3 + 8*a^2*d^4)*x^8 - (21*b^
2*c^3*d - 24*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^6 - 35*a^2*c^4 - 3*(21*
b^2*c^4 + 6*a*b*c^3*d - 2*a^2*c^2*d^2)*x^4 - 5*(18*a*b*c^4 + a^2*
c^3*d)*x^2)*sqrt(d*x^2 + c)/(c^4*x^9)
```

Sympy [A] time = 15.8475, size = 1061, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**10,x)
```

```
[Out] -35*a**2*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8
+ 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x*
*14) - 110*a**2*c**6*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**
7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c
**4*d**12*x**14) - 114*a**2*c**5*d**(23/2)*x**4*sqrt(c/(d*x**2) +
 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x
```

```
**12 + 315*c**4*d**12*x**14) - 40*a**2*c**4*d**(25/2)*x**6*sqrt(c
/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c
**5*d**11*x**12 + 315*c**4*d**12*x**14) + 5*a**2*c**3*d**(27/2)*x
**8*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**
10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 30*a**2*c**2*
d**(29/2)*x**10*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c*
*6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 4
0*a**2*c*d**(31/2)*x**12*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8
 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x
**14) + 16*a**2*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*d*
*9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*
d**12*x**14) - 30*a*b*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**
5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 66*a*b*
c**4*d**(11/2)*x**2*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 21
0*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 34*a*b*c**3*d**(13/2)*x
**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8
 + 105*c**3*d**6*x**10) - 6*a*b*c**2*d**(15/2)*x**6*sqrt(c/(d*x**
2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*
x**10) - 24*a*b*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d
**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 16*a*b*d**
(19/2)*x**10*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*
d**5*x**8 + 105*c**3*d**6*x**10) - b**2*sqrt(d)*sqrt(c/(d*x**2) +
 1)/(5*x**4) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*c*x**2) + 2
*b**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2)
```

GIAC/XCAS [A] time = 0.268721, size = 782, normalized size = 5.47

$$\frac{4\left(315\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2d^{\frac{5}{2}}-1155\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2cd^{\frac{5}{2}}+1680\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}abd^{\frac{7}{2}}+1575\left(\sqrt{d}x-\sqrt{dx^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(d)*x - sqrt(d*x^2 + c))^14*b^2*d^(5/2) - 1155*(s
qrt(d)*x - sqrt(d*x^2 + c))^12*b^2*c*d^(5/2) + 1680*(sqrt(d)*x -
sqrt(d*x^2 + c))^12*a*b*d^(7/2) + 1575*(sqrt(d)*x - sqrt(d*x^2 +
c))^10*b^2*c^2*d^(5/2) - 2520*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*
b*c*d^(7/2) + 2520*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a^2*d^(9/2) -
 1071*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*d^(5/2) + 504*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(7/2) + 1512*(sqrt(d)*x - sq
rt(d*x^2 + c))^8*a^2*c*d^(9/2) + 609*(sqrt(d)*x - sqrt(d*x^2 + c)
)^6*b^2*c^4*d^(5/2) - 336*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3
*d^(7/2) + 672*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(9/2) -
441*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^5*d^(5/2) + 864*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^(7/2) - 288*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a^2*c^3*d^(9/2) + 189*(sqrt(d)*x - sqrt(d*x^2 + c))
^2*b^2*c^6*d^(5/2) - 216*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^5*
d^(7/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(9/2) - 21
*b^2*c^7*d^(5/2) + 24*a*b*c^6*d^(7/2) - 8*a^2*c^5*d^(9/2))/((sqrt
(d)*x - sqrt(d*x^2 + c))^2 - c)^9
```

3.612 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{8d^2\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{3465c^5x^3}+\frac{4d\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{1155c^4x^5}-\frac{\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{231c^3x^7}-\frac{2a\left(c+dx^2\right)^{3/2}(11bc-4ad)}{99c^2x^9}$$

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(11*c*x^11) - (2*a*(11*b*c - 4*a*d)*(c +
 d*x^2)^(3/2))/(99*c^2*x^9) - ((33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*
d))*(c + d*x^2)^(3/2))/(231*c^3*x^7) + (4*d*(33*b^2*c^2 - 4*a*d*(
11*b*c - 4*a*d))*(c + d*x^2)^(3/2))/(1155*c^4*x^5) - (8*d^2*(33*b
^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2)^(3/2))/(3465*c^5*x^3
)
```

Rubi [A] time = 0.421208, antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\left(c+dx^2\right)^{3/2}\left(16a^2d^2-44abcd+33b^2c^2\right)}{231c^3x^7}-\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{8d^2\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{3465c^5x^3}+\frac{4d\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{1155c^4x^5}-\frac{2a\left(c+dx^2\right)^{3/2}(11bc-4ad)}{99c^2x^9}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(11*c*x^11) - (2*a*(11*b*c - 4*a*d)*(c +
 d*x^2)^(3/2))/(99*c^2*x^9) - ((33*b^2*c^2 - 44*a*b*c*d + 16*a^2*
d^2)*(c + d*x^2)^(3/2))/(231*c^3*x^7) + (4*d*(33*b^2*c^2 - 4*a*d*
(11*b*c - 4*a*d))*(c + d*x^2)^(3/2))/(1155*c^4*x^5) - (8*d^2*(33*
b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2)^(3/2))/(3465*c^5*x^
3)
```

Rubi in Sympy [A] time = 32.6139, size = 187, normalized size = 0.99

$$-\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{11cx^{11}}+\frac{2a\left(c+dx^2\right)^{\frac{3}{2}}(4ad-11bc)}{99c^2x^9}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(4ad\,(4ad-11bc)+33b^2c^2\right)}{231c^3x^7}+\frac{4d\left(c+dx^2\right)^{\frac{3}{2}}\left(4ad\,(4ad-11bc)+33b^2c^2\right)}{1155c^4x^5}-\frac{8d^2\left(c+dx^2\right)^{\frac{3}{2}}\left(4ad\,(4ad-11bc)+33b^2c^2\right)}{3465c^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**12,x)
```

```
[Out] -a**2*(c + d*x**2)**(3/2)/(11*c*x**11) + 2*a*(c + d*x**2)**(3/2)*
(4*a*d - 11*b*c)/(99*c**2*x**9) - (c + d*x**2)**(3/2)*(4*a*d*(4*a
*d - 11*b*c) + 33*b**2*c**2)/(231*c**3*x**7) + 4*d*(c + d*x**2)**
(3/2)*(4*a*d*(4*a*d - 11*b*c) + 33*b**2*c**2)/(1155*c**4*x**5) -
8*d**2*(c + d*x**2)**(3/2)*(4*a*d*(4*a*d - 11*b*c) + 33*b**2*c**2
)/(3465*c**5*x**3)
```

Mathematica [A] time = 0.122385, size = 141, normalized size = 0.75

$$-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(315c^4-280c^3dx^2+240c^2d^2x^4-192cd^3x^6+128d^4x^8\right)+22abcx^2\left(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6\right)\right.}{3465c^5x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -((c + d*x^2)^(3/2)*(33*b^2*c^2*x^4*(15*c^2 - 12*c*d*x^2 + 8*d^2*
x^4) + 22*a*b*c*x^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4 - 16*d^
3*x^6) + a^2*(315*c^4 - 280*c^3*d*x^2 + 240*c^2*d^2*x^4 - 192*c*d
^3*x^6 + 128*d^4*x^8)))/(3465*c^5*x^11)
```

Maple [A] time = 0.012, size = 158, normalized size = 0.8

$$-\frac{128\,a^2d^4x^8-352\,abcd^3x^8+264\,b^2c^2d^2x^8-192\,a^2cd^3x^6+528\,abc^2d^2x^6-396\,b^2c^3dx^6+240\,a^2c^2d^2x^4-660\,abc^3dx^4+49}{3465\,x^{11}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x)
```

```
[Out] -1/3465*(d*x^2+c)^(3/2)*(128*a^2*d^4*x^8-352*a*b*c*d^3*x^8+264*b^
2*c^2*d^2*x^8-192*a^2*c*d^3*x^6+528*a*b*c^2*d^2*x^6-396*b^2*c^3*d
*x^6+240*a^2*c^2*d^2*x^4-660*a*b*c^3*d*x^4+495*b^2*c^4*x^4-280*a^
2*c^3*d*x^2+770*a*b*c^4*x^2+315*a^2*c^4)/x^11/c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.799553, size = 250, normalized size = 1.32

$$-\frac{\left(8\left(33\,b^2c^2d^3-44\,abcd^4+16\,a^2d^5\right)x^{10}-4\left(33\,b^2c^3d^2-44\,abc^2d^3+16\,a^2cd^4\right)x^8+315\,a^2c^5+3\left(33\,b^2c^4d-44\,abc^3d^2+1\right.\right.}{3465\,c^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(33*b^2*c^2*d^3 - 44*a*b*c*d^4 + 16*a^2*d^5)*x^10 - 4*
(33*b^2*c^3*d^2 - 44*a*b*c^2*d^3 + 16*a^2*c*d^4)*x^8 + 315*a^2*c^
5 + 3*(33*b^2*c^4*d - 44*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x^6 + 5*(9
9*b^2*c^5 + 22*a*b*c^4*d - 8*a^2*c^3*d^2)*x^4 + 35*(22*a*b*c^5 +
a^2*c^4*d)*x^2)*sqrt(d*x^2 + c)/(c^5*x^11)
```

Sympy [A] time = 24.0671, size = 1856, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**12,x)
```

```
[Out] -315*a**2*c**9*d**(33/2)*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x*
*10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**
6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1295*a**2*c**8*d**(35/2)
*x**2*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d*
*17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 346
5*c**5*d**20*x**18) - 1990*a**2*c**7*d**(37/2)*x**4*sqrt(c/(d*x**
2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c
**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18)
 - 1358*a**2*c**6*d**(39/2)*x**6*sqrt(c/(d*x**2) + 1)/(3465*c**9*
d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 1
3860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 343*a**2*c**5*d*
*(41/2)*x**8*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*
c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**1
6 + 3465*c**5*d**20*x**18) - 35*a**2*c**4*d**(43/2)*x**10*sqrt(c/
(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*
x**18) - 280*a**2*c**3*d**(45/2)*x**12*sqrt(c/(d*x**2) + 1)/(3465
*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**
14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 560*a**2*c
**2*d**(47/2)*x**14*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 +
 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**
19*x**16 + 3465*c**5*d**20*x**18) - 448*a**2*c*d**(49/2)*x**16*sq
rt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**1
2 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d
**20*x**18) - 128*a**2*d**(51/2)*x**18*sqrt(c/(d*x**2) + 1)/(3465
*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**
14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 70*a*b*c**
7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d
**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 220*a
*b*c**6*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**
14) - 228*a*b*c**5*d**(23/2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*
d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**
4*d**12*x**14) - 80*a*b*c**4*d**(25/2)*x**6*sqrt(c/(d*x**2) + 1)/
(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12
 + 315*c**4*d**12*x**14) + 10*a*b*c**3*d**(27/2)*x**8*sqrt(c/(d*x
**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d
**11*x**12 + 315*c**4*d**12*x**14) + 60*a*b*c**2*d**(29/2)*x**10*
sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 +
 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 80*a*b*c*d**(31/2
)*x**12*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10
*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 32*a*b*d*
*(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 15*
b**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 33*b**2*c**4*d**(11/2)*x
**2*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8
 + 105*c**3*d**6*x**10) - 17*b**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x
**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**
6*x**10) - 3*b**2*c**2*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)/(105*c
**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 12*b*
*2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 21
0*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*b**2*d**(19/2)*x**10*
sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 1
05*c**3*d**6*x**10)
```

GIAC/XCAS [A] time = 0.266392, size = 902, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^12,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(d)*x - sqrt(d*x^2 + c))^16*b^2*d^(7/2) - 8085
```

*(sqrt(d)*x - sqrt(d*x^2 + c))^14*b^2*c*d^(7/2) + 13860*(sqrt(d)*
x - sqrt(d*x^2 + c))^14*a*b*d^(9/2) + 9933*(sqrt(d)*x - sqrt(d*x^
2 + c))^12*b^2*c^2*d^(7/2) - 19404*(sqrt(d)*x - sqrt(d*x^2 + c))^
12*a*b*c*d^(9/2) + 22176*(sqrt(d)*x - sqrt(d*x^2 + c))^12*a^2*d^(
11/2) - 5313*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*c^3*d^(7/2) + 9
24*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c^2*d^(9/2) + 14784*(sqrt
(d)*x - sqrt(d*x^2 + c))^10*a^2*c*d^(11/2) + 2805*(sqrt(d)*x - sq
rt(d*x^2 + c))^8*b^2*c^4*d^(7/2) - 660*(sqrt(d)*x - sqrt(d*x^2 +
c))^8*a*b*c^3*d^(9/2) + 5280*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*
c^2*d^(11/2) - 3135*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^5*d^(7/
2) + 7260*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^4*d^(9/2) - 2640*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^3*d^(11/2) + 1815*(sqrt(d)*
x - sqrt(d*x^2 + c))^4*b^2*c^6*d^(7/2) - 2420*(sqrt(d)*x - sqrt(d
*x^2 + c))^4*a*b*c^5*d^(9/2) + 880*(sqrt(d)*x - sqrt(d*x^2 + c))^
4*a^2*c^4*d^(11/2) - 363*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^7*
d^(7/2) + 484*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^6*d^(9/2) - 1
76*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^5*d^(11/2) + 33*b^2*c^8*
d^(7/2) - 44*a*b*c^7*d^(9/2) + 16*a^2*c^6*d^(11/2))/((sqrt(d)*x -
sqrt(d*x^2 + c))^2 - c)^11

3.613 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=281

$$\frac{c^4 \left(24a^2d^2+bc(7bc-24ad)\right) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{9/2}}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1024d^4}+\frac{c^2x^3\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1536d^3}+\frac{x^5\left(c+dx^2\right)^{3/2}\left(24a^2d^2+bc(7bc-24ad)\right)}{192d^2}+\frac{cx^5\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{384d^2}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{120d^2}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

```
[Out] -(c^3*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(102
4*d^4) + (c^2*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^3*Sqrt[c + d*
x^2])/(1536*d^3) + (c*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*Sqr
t[c + d*x^2])/(384*d^2) + ((24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^
5*(c + d*x^2)^(3/2))/(192*d^2) - (b*(7*b*c - 24*a*d)*x^5*(c + d*x
^2)^(5/2))/(120*d^2) + (b^2*x^7*(c + d*x^2)^(5/2))/(12*d) + (c^4*
(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(1024*d^(9/2))
```

Rubi [A] time = 0.669416, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{c^4 \left(24a^2d^2+bc(7bc-24ad)\right) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{9/2}}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1024d^4}+\frac{c^2x^3\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1536d^3}+\frac{1}{192}x^5\left(c+dx^2\right)^{3/2}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)+\frac{cx^5\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{384d^2}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{120d^2}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -(c^3*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(102
4*d^4) + (c^2*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^3*Sqrt[c + d*
x^2])/(1536*d^3) + (c*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*Sqr
t[c + d*x^2])/(384*d^2) + ((24*a^2 + (b*c*(7*b*c - 24*a*d))/d^2)*
x^5*(c + d*x^2)^(3/2))/192 - (b*(7*b*c - 24*a*d)*x^5*(c + d*x^2)^
(5/2))/(120*d^2) + (b^2*x^7*(c + d*x^2)^(5/2))/(12*d) + (c^4*(24*
a^2*d^2 + b*c*(7*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^
2]])/(1024*d^(9/2))
```

Rubi in Sympy [A] time = 49.2156, size = 270, normalized size = 0.96

$$\frac{b^2x^7\left(c+dx^2\right)^{\frac{5}{2}}}{12d}+\frac{bx^5\left(c+dx^2\right)^{\frac{5}{2}}(24ad-7bc)}{120d^2}+\frac{c^4\left(24a^2d^2-bc\,(24ad-7bc)\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{\frac{9}{2}}}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2-bc\,(24ad-7bc)\right)}{1024d^4}+\frac{c^2x^3\sqrt{c+dx^2}\left(24a^2d^2-bc\,(24ad-7bc)\right)}{1536d^3}+\frac{cx^5\sqrt{c+dx^2}\left(24a^2d^2-bc\,(24ad-7bc)\right)}{384d^2}+\frac{x^5\left(c+dx^2\right)^{\frac{3}{2}}\left(24a^2d^2-bc\,(24ad-7bc)\right)}{192d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*x**7*(c + d*x**2)**(5/2)/(12*d) + b*x**5*(c + d*x**2)**(5/2)
*(24*a*d - 7*b*c)/(120*d**2) + c**4*(24*a**2*d**2 - b*c*(24*a*d -
```

```
 7*b*c))*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(1024*d**(9/2)) - c**3
*x*sqrt(c + d*x**2)*(24*a**2*d**2 - b*c*(24*a*d - 7*b*c))/(1024*d
**4) + c**2*x**3*sqrt(c + d*x**2)*(24*a**2*d**2 - b*c*(24*a*d - 7
*b*c))/(1536*d**3) + c*x**5*sqrt(c + d*x**2)*(24*a**2*d**2 - b*c*
(24*a*d - 7*b*c))/(384*d**2) + x**5*(c + d*x**2)**(3/2)*(24*a**2*
d**2 - b*c*(24*a*d - 7*b*c))/(192*d**2)
```

Mathematica [A] time = 0.244492, size = 225, normalized size = 0.8

$$\frac{15c^4\left(24a^2d^2-24abcd+7b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}x\sqrt{c+dx^2}\left(120a^2d^2\left(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6\right)+2\cdots\right.}{1\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(120*a^2*d^2*(-3*c^3 + 2*c^2*d*x^2 + 2
4*c*d^2*x^4 + 16*d^3*x^6) + 24*a*b*d*(15*c^4 - 10*c^3*d*x^2 + 8*c
^2*d^2*x^4 + 176*c*d^3*x^6 + 128*d^4*x^8) + b^2*(-105*c^5 + 70*c^
4*d*x^2 - 56*c^3*d^2*x^4 + 48*c^2*d^3*x^6 + 1664*c*d^4*x^8 + 1280
*d^5*x^10)) + 15*c^4*(7*b^2*c^2 - 24*a*b*c*d + 24*a^2*d^2)*Log[d*
x + Sqrt[d]*Sqrt[c + d*x^2]])/(15360*d^(9/2))
```

Maple [A] time = 0.03, size = 389, normalized size = 1.4

$$\frac{a^2x^3}{8\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2cx}{16\,d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2c^2x}{64\,d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3\,a^2c^3x}{128\,d^2}\sqrt{dx^2+c}$$
$$+\frac{3\,a^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{b^2x^7}{12\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{7\,b^2cx^5}{120\,d^2}\left(dx^2+c\right)^{\frac{5}{2}}$$
$$+\frac{7\,b^2c^2x^3}{192\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{7\,xb^2c^3}{384\,d^4}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{7\,b^2c^4x}{1536\,d^4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{7\,b^2c^5x}{1024\,d^4}\sqrt{dx^2+c}$$
$$+\frac{7\,b^2c^6}{1024}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}}+\frac{abx^5}{5\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{abcx^3}{8\,d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{abc^2x}{16\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}$$
$$-\frac{abc^3x}{64\,d^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{3\,abc^4x}{128\,d^3}\sqrt{dx^2+c}-\frac{3\,abc^5}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*a^2*x^3*(d*x^2+c)^(5/2)/d-1/16*a^2*c/d^2*x*(d*x^2+c)^(5/2)+1/
64*a^2*c^2/d^2*x*(d*x^2+c)^(3/2)+3/128*a^2*c^3/d^2*x*(d*x^2+c)^(1
/2)+3/128*a^2*c^4/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/12*b^2*
x^7*(d*x^2+c)^(5/2)/d-7/120*b^2*c/d^2*x^5*(d*x^2+c)^(5/2)+7/192*b
^2*c^2/d^3*x^3*(d*x^2+c)^(5/2)-7/384*b^2*c^3/d^4*x*(d*x^2+c)^(5/2
)+7/1536*b^2*c^4/d^4*x*(d*x^2+c)^(3/2)+7/1024*b^2*c^5/d^4*x*(d*x^
2+c)^(1/2)+7/1024*b^2*c^6/d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1
/5*a*b*x^5*(d*x^2+c)^(5/2)/d-1/8*a*b*c/d^2*x^3*(d*x^2+c)^(5/2)+1/
16*a*b*c^2/d^3*x*(d*x^2+c)^(5/2)-1/64*a*b*c^3/d^3*x*(d*x^2+c)^(3/
2)-3/128*a*b*c^4/d^3*x*(d*x^2+c)^(1/2)-3/128*a*b*c^5/d^(7/2)*ln(x
*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.83707, size = 1, normalized size = 0.

$$\left[\frac{2\left(1280\,b^2d^5x^{11}+128\left(13\,b^2cd^4+24\,abd^5\right)x^9+48\left(b^2c^2d^3+88\,abcd^4+40\,a^2d^5\right)x^7-8\left(7\,b^2c^3d^2-24\,abc^2d^3-360\,a^2cd^4\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30720*(2*(1280*b^2*d^5*x^11 + 128*(13*b^2*c*d^4 + 24*a*b*d^5)*
x^9 + 48*(b^2*c^2*d^3 + 88*a*b*c*d^4 + 40*a^2*d^5)*x^7 - 8*(7*b^2
*c^3*d^2 - 24*a*b*c^2*d^3 - 360*a^2*c*d^4)*x^5 + 10*(7*b^2*c^4*d
- 24*a*b*c^3*d^2 + 24*a^2*c^2*d^3)*x^3 - 15*(7*b^2*c^5 - 24*a*b*c
^4*d + 24*a^2*c^3*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 15*(7*b^2*c^6
 - 24*a*b*c^5*d + 24*a^2*c^4*d^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2
*d*x^2 + c)*sqrt(d)))/d^(9/2), 1/15360*((1280*b^2*d^5*x^11 + 128*
(13*b^2*c*d^4 + 24*a*b*d^5)*x^9 + 48*(b^2*c^2*d^3 + 88*a*b*c*d^4
+ 40*a^2*d^5)*x^7 - 8*(7*b^2*c^3*d^2 - 24*a*b*c^2*d^3 - 360*a^2*c
*d^4)*x^5 + 10*(7*b^2*c^4*d - 24*a*b*c^3*d^2 + 24*a^2*c^2*d^3)*x^
3 - 15*(7*b^2*c^5 - 24*a*b*c^4*d + 24*a^2*c^3*d^2)*x)*sqrt(d*x^2
+ c)*sqrt(-d) + 15*(7*b^2*c^6 - 24*a*b*c^5*d + 24*a^2*c^4*d^2)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.246887, size = 355, normalized size = 1.26

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2dx^2+\frac{13\,b^2cd^{10}+24\,abd^{11}}{d^{10}}\right)x^2+\frac{3\left(b^2c^2d^9+88\,abcd^{10}+40\,a^2d^{11}\right)}{d^{10}}\right)x^2-\frac{7\,b^2c^3d^8-24\,abc^2d^9-3}{d^{10}}\right.\right.\right.$$
$$-\frac{\left(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{1024\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d*x^2 + (13*b^2*c*d^10 + 24*a*b*d^11)
/d^10)*x^2 + 3*(b^2*c^2*d^9 + 88*a*b*c*d^10 + 40*a^2*d^11)/d^10)*
x^2 - (7*b^2*c^3*d^8 - 24*a*b*c^2*d^9 - 360*a^2*c*d^10)/d^10)*x^2
 + 5*(7*b^2*c^4*d^7 - 24*a*b*c^3*d^8 + 24*a^2*c^2*d^9)/d^10)*x^2
- 15*(7*b^2*c^5*d^6 - 24*a*b*c^4*d^7 + 24*a^2*c^3*d^8)/d^10)*sqrt
(d*x^2 + c)*x - 1/1024*(7*b^2*c^6 - 24*a*b*c^5*d + 24*a^2*c^4*d^2
)*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.614 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^4) + ((b*c - a*d)*(3*b*
c - a*d)*(c + d*x^2)^(7/2))/(7*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x
^2)^(9/2))/(9*d^4) + (b^2*(c + d*x^2)^(11/2))/(11*d^4)
```

Rubi [A] time = 0.263099, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^4) + ((b*c - a*d)*(3*b*
c - a*d)*(c + d*x^2)^(7/2))/(7*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x
^2)^(9/2))/(9*d^4) + (b^2*(c + d*x^2)^(11/2))/(11*d^4)
```

Rubi in Sympy [A] time = 32.1589, size = 100, normalized size = 0.88

$$\frac{b^2\left(c+dx^2\right)^{\frac{11}{2}}}{11d^4}+\frac{b\left(c+dx^2\right)^{\frac{9}{2}}(2ad-3bc)}{9d^4}-\frac{c\left(c+dx^2\right)^{\frac{5}{2}}(ad-bc)^2}{5d^4}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}(ad-3bc)(ad-bc)}{7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*(c + d*x**2)**(11/2)/(11*d**4) + b*(c + d*x**2)**(9/2)*(2*a*
d - 3*b*c)/(9*d**4) - c*(c + d*x**2)**(5/2)*(a*d - b*c)**2/(5*d**
4) + (c + d*x**2)**(7/2)*(a*d - 3*b*c)*(a*d - b*c)/(7*d**4)
```

Mathematica [A] time = 0.132883, size = 100, normalized size = 0.88

$$\frac{\left(c+dx^2\right)^{5/2}\left(99a^2d^2\left(5dx^2-2c\right)+22abd\left(8c^2-20cdx^2+35d^2x^4\right)-3b^2\left(16c^3-40c^2dx^2+70cd^2x^4-105d^3x^6\right)\right)}{3465d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(99*a^2*d^2*(-2*c + 5*d*x^2) + 22*a*b*d*(8*c^2
 - 20*c*d*x^2 + 35*d^2*x^4) - 3*b^2*(16*c^3 - 40*c^2*d*x^2 + 70*c
*d^2*x^4 - 105*d^3*x^6)))/(3465*d^4)
```

Maple [A] time = 0.012, size = 108, normalized size = 1.

$$-\frac{-315\,b^2x^6d^3-770\,abd^3x^4+210\,b^2cd^2x^4-495\,a^2d^3x^2+440\,abcd^2x^2-120\,b^2c^2dx^2+198\,a^2cd^2-176\,abc^2d+48\,b^2c^3}{3465\,d^4}(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3465*(d*x^2+c)^(5/2)*(-315*b^2*d^3*x^6-770*a*b*d^3*x^4+210*b^2
*c*d^2*x^4-495*a^2*d^3*x^2+440*a*b*c*d^2*x^2-120*b^2*c^2*d*x^2+19
8*a^2*c*d^2-176*a*b*c^2*d+48*b^2*c^3)/d^4
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.222047, size = 242, normalized size = 2.12

$$\frac{\left(315\,b^2d^5x^{10} + 70\left(6\,b^2cd^4 + 11\,abd^5\right)x^8 - 48\,b^2c^5 + 176\,abc^4d - 198\,a^2c^3d^2 + 5\left(3\,b^2c^2d^3 + 220\,abcd^4 + 99\,a^2d^5\right)x^6 - 6\left(3\,b\right.}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 70*(6*b^2*c*d^4 + 11*a*b*d^5)*x^8 - 48
*b^2*c^5 + 176*a*b*c^4*d - 198*a^2*c^3*d^2 + 5*(3*b^2*c^2*d^3 + 2
20*a*b*c*d^4 + 99*a^2*d^5)*x^6 - 6*(3*b^2*c^3*d^2 - 11*a*b*c^2*d^
3 - 132*a^2*c*d^4)*x^4 + (24*b^2*c^4*d - 88*a*b*c^3*d^2 + 99*a^2*
c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A]   time = 13.352, size = 384, normalized size = 3.37

$$\begin{cases} -\frac{2a^2c^3\sqrt{c+dx^2}}{35d^2} + \frac{a^2c^2x^2\sqrt{c+dx^2}}{35d} + \frac{8a^2cx^4\sqrt{c+dx^2}}{35} + \frac{a^2dx^6\sqrt{c+dx^2}}{7} + \frac{16abc^4\sqrt{c+dx^2}}{315d^3} - \frac{8abc^3x^2\sqrt{c+dx^2}}{315d^2} + \frac{2abc^2x^4\sqrt{c+dx^2}}{105d} + \frac{20abcx^6\sqrt{c+dx^2}}{63} \\ c^{\frac{3}{2}}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a**2*c**3*sqrt(c + d*x**2)/(35*d**2) + a**2*c**2*x*
*2*sqrt(c + d*x**2)/(35*d) + 8*a**2*c*x**4*sqrt(c + d*x**2)/35 +
a**2*d*x**6*sqrt(c + d*x**2)/7 + 16*a*b*c**4*sqrt(c + d*x**2)/(31
5*d**3) - 8*a*b*c**3*x**2*sqrt(c + d*x**2)/(315*d**2) + 2*a*b*c**
2*x**4*sqrt(c + d*x**2)/(105*d) + 20*a*b*c*x**6*sqrt(c + d*x**2)/
63 + 2*a*b*d*x**8*sqrt(c + d*x**2)/9 - 16*b**2*c**5*sqrt(c + d*x*
*2)/(1155*d**4) + 8*b**2*c**4*x**2*sqrt(c + d*x**2)/(1155*d**3) -
 2*b**2*c**3*x**4*sqrt(c + d*x**2)/(385*d**2) + b**2*c**2*x**6*sq
rt(c + d*x**2)/(231*d) + 4*b**2*c*x**8*sqrt(c + d*x**2)/33 + b**2
*d*x**10*sqrt(c + d*x**2)/11, Ne(d, 0)), (c**(3/2)*(a**2*x**4/4 +
 a*b*x**6/3 + b**2*x**8/8), True))
```

GIAC/XCAS [A] time = 0.238603, size = 437, normalized size = 3.83

$$\frac{\frac{231\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)a^2c}{d}+\frac{66\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)abc}{d^2}+\frac{33\left(15\left(dx^2+c\right)^{\frac{7}{2}}-42\left(dx^2+c\right)^{\frac{5}{2}}c+35\left(dx^2+c\right)^{\frac{3}{2}}c^2\right)a^2}{d}+ 11 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/3465*(231*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c)^(3/2)*c)*a^2*c/d
 + 66*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2
+ c)^(3/2)*c^2)*a*b*c/d^2 + 33*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2
+ c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*a^2/d + 11*(35*(d*x^2 +
c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 -
105*(d*x^2 + c)^(3/2)*c^3)*b^2*c/d^3 + 22*(35*(d*x^2 + c)^(9/2) -
 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2
 + c)^(3/2)*c^3)*a*b/d^2 + (315*(d*x^2 + c)^(11/2) - 1540*(d*x^2
+ c)^(9/2)*c + 2970*(d*x^2 + c)^(7/2)*c^2 - 2772*(d*x^2 + c)^(5/2
)*c^3 + 1155*(d*x^2 + c)^(3/2)*c^4)*b^2/d^3)/d
```

3.615 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx$

Optimal. Leaf size=235

$$-\frac{c^3 \left(16a^2d^2+3bc(bc-4ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{c^2x\sqrt{c+dx^2}\left(16a^2d^2+3bc(bc-4ad)\right)}{256d^3}+\frac{x^3\left(c+dx^2\right)^{3/2}\left(16a^2d^2+3bc(bc-4ad)\right)}{96d^2}+\frac{cx^3\sqrt{c+dx^2}\left(16a^2d^2+3bc(bc-4ad)\right)}{128d^2}-\frac{bx^3\left(c+dx^2\right)^{5/2}(bc-4ad)}{16d^2}+\frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}$$

```
[Out] (c^2*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x*Sqrt[c + d*x^2])/(256*d
^3) + (c*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*Sqrt[c + d*x^2])/
(128*d^2) + ((16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*(c + d*x^2)^(
3/2))/(96*d^2) - (b*(b*c - 4*a*d)*x^3*(c + d*x^2)^(5/2))/(16*d^2)
 + (b^2*x^5*(c + d*x^2)^(5/2))/(10*d) - (c^3*(16*a^2*d^2 + 3*b*c*
(b*c - 4*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(256*d^(7/2)
)
```

Rubi [A] time = 0.565347, antiderivative size = 232, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{c^3 \left(16a^2d^2+3bc(bc-4ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{c^2x\sqrt{c+dx^2}\left(16a^2d^2+3bc(bc-4ad)\right)}{256d^3}+\frac{1}{96}x^3\left(c+dx^2\right)^{3/2}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)+\frac{cx^3\sqrt{c+dx^2}\left(16a^2d^2+3bc(bc-4ad)\right)}{128d^2}-\frac{bx^3\left(c+dx^2\right)^{5/2}(bc-4ad)}{16d^2}+\frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c^2*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x*Sqrt[c + d*x^2])/(256*d
^3) + (c*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*Sqrt[c + d*x^2])/
(128*d^2) + ((16*a^2 + (3*b*c*(b*c - 4*a*d))/d^2)*x^3*(c + d*x^2)
^(3/2))/96 - (b*(b*c - 4*a*d)*x^3*(c + d*x^2)^(5/2))/(16*d^2) + (
b^2*x^5*(c + d*x^2)^(5/2))/(10*d) - (c^3*(16*a^2*d^2 + 3*b*c*(b*c
 - 4*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(256*d^(7/2))
```

Rubi in Sympy [A] time = 47.126, size = 224, normalized size = 0.95

$$\frac{b^2x^5\left(c+dx^2\right)^{\frac{5}{2}}}{10d}+\frac{bx^3\left(c+dx^2\right)^{\frac{5}{2}}(4ad-bc)}{16d^2}-\frac{c^3\left(16a^2d^2-3bc\left(4ad-bc\right)\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{\frac{7}{2}}}+\frac{c^2x\sqrt{c+dx^2}\left(16a^2d^2-3bc\left(4ad-bc\right)\right)}{256d^3}+\frac{cx^3\sqrt{c+dx^2}\left(16a^2d^2-3bc\left(4ad-bc\right)\right)}{128d^2}+\frac{x^3\left(c+dx^2\right)^{\frac{3}{2}}\left(16a^2d^2-3bc\left(4ad-bc\right)\right)}{96d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*x**5*(c + d*x**2)**(5/2)/(10*d) + b*x**3*(c + d*x**2)**(5/2)
*(4*a*d - b*c)/(16*d**2) - c**3*(16*a**2*d**2 - 3*b*c*(4*a*d - b*
c))*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(256*d**(7/2)) + c**2*x*sqr
t(c + d*x**2)*(16*a**2*d**2 - 3*b*c*(4*a*d - b*c))/(256*d**3) + c
```

```
*x**3*sqrt(c + d*x**2)*(16*a**2*d**2 - 3*b*c*(4*a*d - b*c))/(128*
d**2) + x**3*(c + d*x**2)**(3/2)*(16*a**2*d**2 - 3*b*c*(4*a*d - b
*c))/(96*d**2)
```

Mathematica [A] time = 0.205001, size = 193, normalized size = 0.82

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(80a^2d^2\left(3c^2+14cdx^2+8d^2x^4\right)+60abd\left(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6\right)+3b^2\left(15c^4-10c^3dx^2+8c^2d^2\right.\right.}{3840d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(3*c^2 + 14*c*d*x^2 + 8*d^
2*x^4) + 60*a*b*d*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x
^6) + 3*b^2*(15*c^4 - 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d^3*x^
6 + 128*d^4*x^8)) - 15*c^3*(3*b^2*c^2 - 12*a*b*c*d + 16*a^2*d^2)*
Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(3840*d^(7/2))
```

Maple [A] time = 0.015, size = 321, normalized size = 1.4

$$\frac{a^2x}{6d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2cx}{24d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{a^2c^2x}{16d}\sqrt{dx^2+c}-\frac{a^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}$$
$$+\frac{b^2x^5}{10d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{b^2cx^3}{16d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2c^2x}{32d^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{xb^2c^3}{128d^3}\left(dx^2+c\right)^{\frac{3}{2}}$$
$$-\frac{3b^2c^4x}{256d^3}\sqrt{dx^2+c}-\frac{3b^2c^5}{256}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}+\frac{abx^3}{4d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{abcx}{8d^2}\left(dx^2+c\right)^{\frac{5}{2}}$$
$$+\frac{abc^2x}{32d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3abc^3x}{64d^2}\sqrt{dx^2+c}+\frac{3abc^4}{64}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/6*a^2*x*(d*x^2+c)^(5/2)/d-1/24*a^2*c/d*x*(d*x^2+c)^(3/2)-1/16*a
^2*c^2/d*x*(d*x^2+c)^(1/2)-1/16*a^2*c^3/d^(3/2)*ln(x*d^(1/2)+(d*x
^2+c)^(1/2))+1/10*b^2*x^5*(d*x^2+c)^(5/2)/d-1/16*b^2*c/d^2*x^3*(d
*x^2+c)^(5/2)+1/32*b^2*c^2/d^3*x*(d*x^2+c)^(5/2)-1/128*b^2*c^3/d^
3*x*(d*x^2+c)^(3/2)-3/256*b^2*c^4/d^3*x*(d*x^2+c)^(1/2)-3/256*b^2
*c^5/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*a*b*x^3*(d*x^2+c)^
(5/2)/d-1/8*a*b*c/d^2*x*(d*x^2+c)^(5/2)+1/32*a*b*c^2/d^2*x*(d*x^2
+c)^(3/2)+3/64*a*b*c^3/d^2*x*(d*x^2+c)^(1/2)+3/64*a*b*c^4/d^(5/2)
*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.497441, size = 1, normalized size = 0.

$$\left[\frac{2\left(384\,b^2d^4x^9+48\left(11\,b^2cd^3+20\,abd^4\right)x^7+8\left(3\,b^2c^2d^2+180\,abcd^3+80\,a^2d^4\right)x^5-10\left(3\,b^2c^3d-12\,abc^2d^2-112\,a^2cd^3\right)x\right.}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(2*(384*b^2*d^4*x^9 + 48*(11*b^2*c*d^3 + 20*a*b*d^4)*x^7
+ 8*(3*b^2*c^2*d^2 + 180*a*b*c*d^3 + 80*a^2*d^4)*x^5 - 10*(3*b^2*
c^3*d - 12*a*b*c^2*d^2 - 112*a^2*c*d^3)*x^3 + 15*(3*b^2*c^4 - 12*
a*b*c^3*d + 16*a^2*c^2*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 15*(3*b^
2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*log(2*sqrt(d*x^2 + c)*d*x
- (2*d*x^2 + c)*sqrt(d)))/d^(7/2), 1/3840*((384*b^2*d^4*x^9 + 48*
(11*b^2*c*d^3 + 20*a*b*d^4)*x^7 + 8*(3*b^2*c^2*d^2 + 180*a*b*c*d^
3 + 80*a^2*d^4)*x^5 - 10*(3*b^2*c^3*d - 12*a*b*c^2*d^2 - 112*a^2*
c*d^3)*x^3 + 15*(3*b^2*c^4 - 12*a*b*c^3*d + 16*a^2*c^2*d^2)*x)*sq
rt(d*x^2 + c)*sqrt(-d) - 15*(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^
3*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^3)]
```

Sympy [A] time = 138.387, size = 505, normalized size = 2.15

$$\frac{a^2c^{\frac{5}{2}}x}{16d\sqrt{1+\frac{dx^2}{c}}}+\frac{17a^2c^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{11a^2\sqrt{c}dx^5}{24\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16d^{\frac{3}{2}}}+\frac{a^2d^2x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{3abc^{\frac{7}{2}}x}{64d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x^3}{64d\sqrt{1+\frac{dx^2}{c}}}+\frac{13abc^{\frac{3}{2}}x^5}{32\sqrt{1+\frac{dx^2}{c}}}+\frac{5ab\sqrt{c}dx^7}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{64d^{\frac{5}{2}}}$$
$$+\frac{abd^2x^9}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^{\frac{9}{2}}x}{256d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^{\frac{7}{2}}x^3}{256d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x^5}{640d\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{23b^2c^{\frac{3}{2}}x^7}{160\sqrt{1+\frac{dx^2}{c}}}+\frac{19b^2\sqrt{c}dx^9}{80\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^5\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{256d^{\frac{7}{2}}}+\frac{b^2d^2x^{11}}{10\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*a**2*c**(3/2)*x**3
/(48*sqrt(1 + d*x**2/c)) + 11*a**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*
x**2/c)) - a**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + a**
2*d**2*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c)) - 3*a*b*c**(7/2)*x/(64
*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)*x**3/(64*d*sqrt(1 + d*x*
*2/c)) + 13*a*b*c**(3/2)*x**5/(32*sqrt(1 + d*x**2/c)) + 5*a*b*sqr
t(c)*d*x**7/(8*sqrt(1 + d*x**2/c)) + 3*a*b*c**4*asinh(sqrt(d)*x/s
qrt(c))/(64*d**(5/2)) + a*b*d**2*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/
c)) + 3*b**2*c**(9/2)*x/(256*d**3*sqrt(1 + d*x**2/c)) + b**2*c**(
7/2)*x**3/(256*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x**5/(640
*d*sqrt(1 + d*x**2/c)) + 23*b**2*c**(3/2)*x**7/(160*sqrt(1 + d*x*
*2/c)) + 19*b**2*sqrt(c)*d*x**9/(80*sqrt(1 + d*x**2/c)) - 3*b**2*
c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(7/2)) + b**2*d**2*x**11/(1
0*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.229666, size = 296, normalized size = 1.26

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2dx^2+\frac{11\,b^2cd^8+20\,abd^9}{d^8}\right)x^2+\frac{3\,b^2c^2d^7+180\,abcd^8+80\,a^2d^9}{d^8}\right)x^2-\frac{5\left(3\,b^2c^3d^6-12\,abc^2d^7-112\,a^2cd\right.}{d^8}\right.\right.$$
$$+\frac{\left(3\,b^2c^5-12\,abc^4d+16\,a^2c^3d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d*x^2 + (11*b^2*c*d^8 + 20*a*b*d^9)/d^8)*x
^2 + (3*b^2*c^2*d^7 + 180*a*b*c*d^8 + 80*a^2*d^9)/d^8)*x^2 - 5*(3
*b^2*c^3*d^6 - 12*a*b*c^2*d^7 - 112*a^2*c*d^8)/d^8)*x^2 + 15*(3*b
^2*c^4*d^5 - 12*a*b*c^3*d^6 + 16*a^2*c^2*d^7)/d^8)*sqrt(d*x^2 + c
)*x + 1/256*(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*ln(abs(-s
qrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.616 $\int x \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (2*b*(b*c - a*d)*(c +
 d*x^2)^(7/2))/(7*d^3) + (b^2*(c + d*x^2)^(9/2))/(9*d^3)
```

Rubi [A] time = 0.160311, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (2*b*(b*c - a*d)*(c +
 d*x^2)^(7/2))/(7*d^3) + (b^2*(c + d*x^2)^(9/2))/(9*d^3)
```

Rubi in Sympy [A] time = 23.1785, size = 66, normalized size = 0.86

$$\frac{b^2\left(c+dx^2\right)^{\frac{9}{2}}}{9d^3}+\frac{2b\left(c+dx^2\right)^{\frac{7}{2}}(ad-bc)}{7d^3}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}(ad-bc)^2}{5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*(c + d*x**2)**(9/2)/(9*d**3) + 2*b*(c + d*x**2)**(7/2)*(a*d
- b*c)/(7*d**3) + (c + d*x**2)**(5/2)*(a*d - b*c)**2/(5*d**3)
```

Mathematica [A] time = 0.0812274, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{5/2}\left(63a^2d^2+18abd\left(5dx^2-2c\right)+b^2\left(8c^2-20cdx^2+35d^2x^4\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x^2) + b^2*
(8*c^2 - 20*c*d*x^2 + 35*d^2*x^4)))/(315*d^3)
```

Maple [A] time = 0.008, size = 69, normalized size = 0.9

$$\frac{35\,b^2d^2x^4+90\,abd^2x^2-20\,b^2cdx^2+63\,a^2d^2-36\,cabd+8\,b^2c^2}{315\,d^3}\left(dx^2+c\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/315*(d*x^2+c)^(5/2)*(35*b^2*d^2*x^4+90*a*b*d^2*x^2-20*b^2*c*d*x
^2+63*a^2*d^2-36*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217362, size = 190, normalized size = 2.47

$$\frac{\left(35\,b^2d^4x^8 + 10\,\left(5\,b^2cd^3 + 9\,abd^4\right)x^6 + 8\,b^2c^4 - 36\,abc^3d + 63\,a^2c^2d^2 + 3\,\left(b^2c^2d^2 + 48\,abcd^3 + 21\,a^2d^4\right)x^4 - 2\,\left(2\,b^2c^3d - 9\,a\right.}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 10*(5*b^2*c*d^3 + 9*a*b*d^4)*x^6 + 8*b^2*
c^4 - 36*a*b*c^3*d + 63*a^2*c^2*d^2 + 3*(b^2*c^2*d^2 + 48*a*b*c*d
^3 + 21*a^2*d^4)*x^4 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*
d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 7.62694, size = 303, normalized size = 3.94

$$\begin{cases} \frac{a^2c^2\sqrt{c+dx^2}}{5d} + \frac{2a^2cx^2\sqrt{c+dx^2}}{5} + \frac{a^2dx^4\sqrt{c+dx^2}}{5} - \frac{4abc^3\sqrt{c+dx^2}}{35d^2} + \frac{2abc^2x^2\sqrt{c+dx^2}}{35d} + \frac{16abcx^4\sqrt{c+dx^2}}{35} + \frac{2abdx^6\sqrt{c+dx^2}}{7} + \frac{8b^2c^4\sqrt{c+dx^2}}{315d^3} - \frac{4b}{} \\ c^{\frac{3}{2}}\left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((a**2*c**2*sqrt(c + d*x**2)/(5*d) + 2*a**2*c*x**2*sqrt(
c + d*x**2)/5 + a**2*d*x**4*sqrt(c + d*x**2)/5 - 4*a*b*c**3*sqrt(
c + d*x**2)/(35*d**2) + 2*a*b*c**2*x**2*sqrt(c + d*x**2)/(35*d) +
 16*a*b*c*x**4*sqrt(c + d*x**2)/35 + 2*a*b*d*x**6*sqrt(c + d*x**2
)/7 + 8*b**2*c**4*sqrt(c + d*x**2)/(315*d**3) - 4*b**2*c**3*x**2*
sqrt(c + d*x**2)/(315*d**2) + b**2*c**2*x**4*sqrt(c + d*x**2)/(10
5*d) + 10*b**2*c*x**6*sqrt(c + d*x**2)/63 + b**2*d*x**8*sqrt(c +
d*x**2)/9, Ne(d, 0)), (c**(3/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*
x**6/6), True))
```

GIAC/XCAS [A] time = 0.232292, size = 315, normalized size = 4.09

$$\frac{105\,\left(dx^2 + c\right)^{\frac{3}{2}}a^2c + 21\,\left(3\,\left(dx^2 + c\right)^{\frac{5}{2}} - 5\,\left(dx^2 + c\right)^{\frac{3}{2}}c\right)a^2 + \frac{42\left(3\left(dx^2+c\right)^{\frac{5}{2}} - 5\left(dx^2+c\right)^{\frac{3}{2}}c\right)abc}{d} + \frac{3\left(15\left(dx^2+c\right)^{\frac{7}{2}} - 42\left(dx^2+c\right)^{\frac{5}{2}}c + 35\left(dx^2+\right.\right.}{d^2}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/315*(105*(d*x^2 + c)^(3/2)*a^2*c + 21*(3*(d*x^2 + c)^(5/2) - 5*
(d*x^2 + c)^(3/2)*c)*a^2 + 42*(3*(d*x^2 + c)^(5/2) - 5*(d*x^2 + c
)^(3/2)*c)*a*b*c/d + 3*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/
2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*b^2*c/d^2 + 6*(15*(d*x^2 + c)^(7
/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*a*b/d +
(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)
^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c^3)*b^2/d^2)/d
```

3.617 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx$

Optimal. Leaf size=196

$$\frac{x\left(c+dx^2\right)^{3/2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{192d^2}+\frac{cx\sqrt{c+dx^2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{128d^2}+\frac{c^2\left(48a^2d^2-16abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{5/2}}-\frac{bx\left(c+dx^2\right)^{5/2}(3bc-10ad)}{48d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}$$

```
[Out] (c*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*Sqrt[c + d*x^2])/(128*
d^2) + ((3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*(c + d*x^2)^(3/2)
)/(192*d^2) - (b*(3*b*c - 10*a*d)*x*(c + d*x^2)^(5/2))/(48*d^2) +
 (b*x*(a + b*x^2)*(c + d*x^2)^(5/2))/(8*d) + (c^2*(3*b^2*c^2 - 16
*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128
*d^(5/2))
```

Rubi [A] time = 0.270342, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{x\left(c+dx^2\right)^{3/2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{192d^2}+\frac{cx\sqrt{c+dx^2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{128d^2}+\frac{c^2\left(48a^2d^2-16abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{5/2}}-\frac{bx\left(c+dx^2\right)^{5/2}(3bc-10ad)}{48d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*Sqrt[c + d*x^2])/(128*
d^2) + ((3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*(c + d*x^2)^(3/2)
)/(192*d^2) - (b*(3*b*c - 10*a*d)*x*(c + d*x^2)^(5/2))/(48*d^2) +
 (b*x*(a + b*x^2)*(c + d*x^2)^(5/2))/(8*d) + (c^2*(3*b^2*c^2 - 16
*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128
*d^(5/2))
```

Rubi in Sympy [A] time = 27.3776, size = 192, normalized size = 0.98

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}{8d}+\frac{bx\left(c+dx^2\right)^{\frac{5}{2}}(10ad-3bc)}{48d^2}+\frac{c^2\left(48a^2d^2-16abcd+3b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{\frac{5}{2}}}+\frac{cx\sqrt{c+dx^2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{128d^2}+\frac{x\left(c+dx^2\right)^{\frac{3}{2}}\left(48a^2d^2-16abcd+3b^2c^2\right)}{192d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] b*x*(a + b*x**2)*(c + d*x**2)**(5/2)/(8*d) + b*x*(c + d*x**2)**(5
/2)*(10*a*d - 3*b*c)/(48*d**2) + c**2*(48*a**2*d**2 - 16*a*b*c*d
+ 3*b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(128*d**(5/2)) +
 c*x*sqrt(c + d*x**2)*(48*a**2*d**2 - 16*a*b*c*d + 3*b**2*c**2)/(
128*d**2) + x*(c + d*x**2)**(3/2)*(48*a**2*d**2 - 16*a*b*c*d + 3*
```

```
b**2*c**2)/(192*d**2)
```

Mathematica [A] time = 0.163647, size = 159, normalized size = 0.81

$$\frac{3c^2\left(48a^2d^2-16abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2\left(5c+2dx^2\right)+16abd\left(3c^2+14cdx^2+8d^2x^4\right.\right.}{384d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(5*c + 2*d*x^2) + 16*a*b*d
*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4) + b^2*(-9*c^3 + 6*c^2*d*x^2 + 7
2*c*d^2*x^4 + 48*d^3*x^6)) + 3*c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a
^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(384*d^(5/2))
```

Maple [A] time = 0.011, size = 249, normalized size = 1.3

$$\frac{a^2x}{4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3a^2cx}{8}\sqrt{dx^2+c}+\frac{3a^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}+\frac{b^2x^3}{8d}\left(dx^2+c\right)^{\frac{5}{2}}$$
$$-\frac{b^2cx}{16d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2c^2x}{64d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3xb^2c^3}{128d^2}\sqrt{dx^2+c}+\frac{3b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}$$
$$+\frac{abx}{3d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{abcx}{12d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{abc^2x}{8d}\sqrt{dx^2+c}-\frac{abc^3}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4*a^2*x*(d*x^2+c)^(3/2)+3/8*a^2*c*x*(d*x^2+c)^(1/2)+3/8*a^2*c^2
/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/8*b^2*x^3*(d*x^2+c)^(5/2
)/d-1/16*b^2*c/d^2*x*(d*x^2+c)^(5/2)+1/64*b^2*c^2/d^2*x*(d*x^2+c)
^(3/2)+3/128*b^2*c^3/d^2*x*(d*x^2+c)^(1/2)+3/128*b^2*c^4/d^(5/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x*(d*x^2+c)^(5/2)/d-1/12*a*
b*c/d*x*(d*x^2+c)^(3/2)-1/8*a*b*c^2/d*x*(d*x^2+c)^(1/2)-1/8*a*b*c
^3/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.327499, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48b^2d^3x^7+8\left(9b^2cd^2+16abd^3\right)x^5+2\left(3b^2c^2d+112abcd^2+48a^2d^3\right)x^3-3\left(3b^2c^3-16abc^2d-80a^2cd^2\right)x\right)\sqrt{dx^2+}}{768d^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^2*d^3*x^7 + 8*(9*b^2*c*d^2 + 16*a*b*d^3)*x^5 + 2*
(3*b^2*c^2*d + 112*a*b*c*d^2 + 48*a^2*d^3)*x^3 - 3*(3*b^2*c^3 - 1
6*a*b*c^2*d - 80*a^2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(3*b^2
*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*log(-2*sqrt(d*x^2 + c)*d*x
- (2*d*x^2 + c)*sqrt(d)))/d^(5/2), 1/384*((48*b^2*d^3*x^7 + 8*(9*
b^2*c*d^2 + 16*a*b*d^3)*x^5 + 2*(3*b^2*c^2*d + 112*a*b*c*d^2 + 48
*a^2*d^3)*x^3 - 3*(3*b^2*c^3 - 16*a*b*c^2*d - 80*a^2*c*d^2)*x)*sq
rt(d*x^2 + c)*sqrt(-d) + 3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2
*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^2)]
```

Sympy [A] time = 88.1589, size = 440, normalized size = 2.24

$$\frac{a^2c^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}dx^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{a^2d^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^{\frac{5}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{17abc^{\frac{3}{2}}x^3}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{11ab\sqrt{c}dx^5}{12\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{abd^2x^7}{3\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^{\frac{7}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{b^2c^{\frac{5}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{13b^2c^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2\sqrt{c}dx^7}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}}+\frac{b^2d^2x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + a**2*c**(3/2)*x/(8*sqrt(1
+ d*x**2/c)) + 3*a**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*a
**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*sqrt(d)) + a**2*d**2*x**5/(4
*sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*c**(5/2)*x/(8*d*sqrt(1 + d*x**
2/c)) + 17*a*b*c**(3/2)*x**3/(24*sqrt(1 + d*x**2/c)) + 11*a*b*sqr
t(c)*d*x**5/(12*sqrt(1 + d*x**2/c)) - a*b*c**3*asinh(sqrt(d)*x/sq
rt(c))/(8*d**(3/2)) + a*b*d**2*x**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)
) - 3*b**2*c**(7/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/
2)*x**3/(128*d*sqrt(1 + d*x**2/c)) + 13*b**2*c**(3/2)*x**5/(64*sq
rt(1 + d*x**2/c)) + 5*b**2*sqrt(c)*d*x**7/(16*sqrt(1 + d*x**2/c))
 + 3*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(5/2)) + b**2*d**
2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.235794, size = 236, normalized size = 1.2

$$\frac{1}{384}\left(2\left(4\left(6\,b^2dx^2+\frac{9\,b^2cd^6+16\,abd^7}{d^6}\right)x^2+\frac{3\,b^2c^2d^5+112\,abcd^6+48\,a^2d^7}{d^6}\right)x^2-\frac{3\left(3\,b^2c^3d^4-16\,abc^2d^5-80\,a^2cd^6\right)}{d^6}\right)\sqrt{\;}$$
$$-\frac{\left(3\,b^2c^4-16\,abc^3d+48\,a^2c^2d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (9*b^2*c*d^6 + 16*a*b*d^7)/d^6)*x^2 +
(3*b^2*c^2*d^5 + 112*a*b*c*d^6 + 48*a^2*d^7)/d^6)*x^2 - 3*(3*b^2*
c^3*d^4 - 16*a*b*c^2*d^5 - 80*a^2*c*d^6)/d^6)*sqrt(d*x^2 + c)*x -
 1/128*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*ln(abs(-sqrt(d
)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.618 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x}\,dx$

Optimal. Leaf size=111

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

```
[Out] a^2*c*Sqrt[c + d*x^2] + (a^2*(c + d*x^2)^(3/2))/3 - (b*(b*c - 2*a
*d)*(c + d*x^2)^(5/2))/(5*d^2) + (b^2*(c + d*x^2)^(7/2))/(7*d^2)
- a^2*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rubi [A] time = 0.253041, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] a^2*c*Sqrt[c + d*x^2] + (a^2*(c + d*x^2)^(3/2))/3 - (b*(b*c - 2*a
*d)*(c + d*x^2)^(5/2))/(5*d^2) + (b^2*(c + d*x^2)^(7/2))/(7*d^2)
- a^2*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rubi in Sympy [A] time = 27.487, size = 99, normalized size = 0.89

$$-a^2c^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2c\sqrt{c+dx^2}+\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{3}+\frac{b^2\left(c+dx^2\right)^{\frac{7}{2}}}{7d^2}+\frac{b\left(c+dx^2\right)^{\frac{5}{2}}(2ad-bc)}{5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x,x)
```

```
[Out] -a**2*c**(3/2)*atanh(sqrt(c + d*x**2)/sqrt(c)) + a**2*c*sqrt(c +
d*x**2) + a**2*(c + d*x**2)**(3/2)/3 + b**2*(c + d*x**2)**(7/2)/(
7*d**2) + b*(c + d*x**2)**(5/2)*(2*a*d - b*c)/(5*d**2)
```

Mathematica [A] time = 0.21513, size = 116, normalized size = 1.05

$$\frac{\sqrt{c+dx^2}\left(35a^2d^2\left(4c+dx^2\right)+42abd\left(c+dx^2\right)^2-3b^2\left(2c-5dx^2\right)\left(c+dx^2\right)^2\right)}{105d^2}-a^2c^{3/2}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+a^2c^{3/2}\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(42*a*b*d*(c + d*x^2)^2 - 3*b^2*(2*c - 5*d*x^2)*
(c + d*x^2)^2 + 35*a^2*d^2*(4*c + d*x^2)))/(105*d^2) + a^2*c^(3/2
)*Log[x] - a^2*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]]
```

Maple [A] time = 0.015, size = 115, normalized size = 1.

$$\frac{a^2}{3}\left(dx^2+c\right)^{\frac{3}{2}}-a^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{\frac{3}{2}}+a^2c\sqrt{dx^2+c}+\frac{b^2x^2}{7d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{2b^2c}{35d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{2ab}{5d}\left(dx^2+c\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x)
```

```
[Out] 1/3*a^2*(d*x^2+c)^(3/2)-a^2*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
*c^(3/2)+a^2*c*(d*x^2+c)^(1/2)+1/7*b^2*x^2*(d*x^2+c)^(5/2)/d-2/35
*b^2*c/d^2*(d*x^2+c)^(5/2)+2/5*a*b*(d*x^2+c)^(5/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241282, size = 1, normalized size = 0.01

$$\left[\frac{105\,a^2c^{\frac{3}{2}}d^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\left(4\,b^2cd^2+7\,abd^3\right)x^4+\left(3\,b^2c^2d+\right.\right.}{210\,d^2}\right.$$
$$-\frac{105\,a^2\sqrt{-c}cd^2\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\left(4\,b^2cd^2+7\,abd^3\right)x^4+\left(3\,b^2c^2d+84\right.\right.}{105\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^2*c^(3/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(
c) + 2*c)/x^2) + 2*(15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 1
40*a^2*c*d^2 + 6*(4*b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 8
4*a*b*c*d^2 + 35*a^2*d^3)*x^2)*sqrt(d*x^2 + c))/d^2, -1/105*(105*
a^2*sqrt(-c)*c*d^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (15*b^2
*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6*(4*b^2*c*
d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*
x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A] time = 26.9036, size = 172, normalized size = 1.55

$$-a^2c^2\left(\begin{cases}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for } -c>0\\ \frac{\operatorname{acoth}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for } -c<0\wedge c<c+dx^2\\ \frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for } c>c+dx^2\wedge -c<0\end{cases}\right)+a^2c\sqrt{c+dx^2}+\frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{3}+\frac{b^2\left(c+dx^2\right)^{\frac{7}{2}}}{7d^2}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}\left(4abd-2b^2c\right)}{10d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x,x)
```

```
[Out] -a**2*c**2*Piecewise((-atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c),
-c > 0), (acoth(sqrt(c + d*x**2)/sqrt(c))/sqrt(c), (-c < 0) & (c
< c + d*x**2)), (atanh(sqrt(c + d*x**2)/sqrt(c))/sqrt(c), (-c < 0
) & (c > c + d*x**2))) + a**2*c*sqrt(c + d*x**2) + a**2*(c + d*x*
*2)**(3/2)/3 + b**2*(c + d*x**2)**(7/2)/(7*d**2) + (c + d*x**2)**
(5/2)*(4*a*b*d - 2*b**2*c)/(10*d**2)
```

GIAC/XCAS [A] time = 0.240303, size = 163, normalized size = 1.47

$$\frac{a^2c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2d^{12} - 21\left(dx^2+c\right)^{\frac{5}{2}}b^2cd^{12} + 42\left(dx^2+c\right)^{\frac{5}{2}}abd^{13} + 35\left(dx^2+c\right)^{\frac{3}{2}}a^2d^{14} + 105\sqrt{dx^2+c}a^2cd^{14}}{105\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/105*(15*(d*
x^2 + c)^(7/2)*b^2*d^12 - 21*(d*x^2 + c)^(5/2)*b^2*c*d^12 + 42*(d
*x^2 + c)^(5/2)*a*b*d^13 + 35*(d*x^2 + c)^(3/2)*a^2*d^14 + 105*sq
rt(d*x^2 + c)*a^2*c*d^14)/d^14
```

3.619 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=175

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}-\frac{c\left(b^2c^2-12ad(2ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\left(c+dx^2\right)^{3/2}\left(b^2c^2-12ad(2ad+bc)\right)}{24cd}-\frac{x\sqrt{c+dx^2}\left(b^2c^2-12ad(2ad+bc)\right)}{16d}+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}$$

```
[Out] -((b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*Sqrt[c + d*x^2])/(16*d) - ((
b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c*d) - (
a^2*(c + d*x^2)^(5/2))/(c*x) + (b^2*x*(c + d*x^2)^(5/2))/(6*d) -
(c*(b^2*c^2 - 12*a*d*(b*c + 2*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(16*d^(3/2))
```

Rubi [A] time = 0.29686, antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{1}{24}x\left(c+dx^2\right)^{3/2}\left(\frac{24a^2d}{c}+12ab-\frac{b^2c}{d}\right)-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}-\frac{c\left(b^2c^2-12ad(2ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\sqrt{c+dx^2}\left(b^2c^2-12ad(2ad+bc)\right)}{16d}+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] -((b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*Sqrt[c + d*x^2])/(16*d) + ((
12*a*b - (b^2*c)/d + (24*a^2*d)/c)*x*(c + d*x^2)^(3/2))/24 - (a^2
*(c + d*x^2)^(5/2))/(c*x) + (b^2*x*(c + d*x^2)^(5/2))/(6*d) - (c*
(b^2*c^2 - 12*a*d*(b*c + 2*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(16*d^(3/2))
```

Rubi in Sympy [A] time = 28.0679, size = 155, normalized size = 0.89

$$-\frac{a^2\left(c+dx^2\right)^{\frac{5}{2}}}{cx}+\frac{b^2x\left(c+dx^2\right)^{\frac{5}{2}}}{6d}-\frac{c\left(-12ad\left(2ad+bc\right)+b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{\frac{3}{2}}}-\frac{x\sqrt{c+dx^2}\left(-12ad\left(2ad+bc\right)+b^2c^2\right)}{16d}-\frac{x\left(c+dx^2\right)^{\frac{3}{2}}\left(-12ad\left(2ad+bc\right)+b^2c^2\right)}{24cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**2,x)
```

```
[Out] -a**2*(c + d*x**2)**(5/2)/(c*x) + b**2*x*(c + d*x**2)**(5/2)/(6*d
) - c*(-12*a*d*(2*a*d + b*c) + b**2*c**2)*atanh(sqrt(d)*x/sqrt(c
+ d*x**2))/(16*d**(3/2)) - x*sqrt(c + d*x**2)*(-12*a*d*(2*a*d + b
*c) + b**2*c**2)/(16*d) - x*(c + d*x**2)**(3/2)*(-12*a*d*(2*a*d +
 b*c) + b**2*c**2)/(24*c*d)
```

Mathematica [A] time = 0.23786, size = 135, normalized size = 0.77

$$\sqrt{c+dx^2}\left(\frac{x\left(8a^2d^2+20abcd+b^2c^2\right)}{16d}-\frac{a^2c}{x}+\frac{1}{24}bx^3(12ad+7bc)+\frac{1}{6}b^2dx^5\right)-\frac{c\left(-24a^2d^2-12abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{16d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-((a^2*c)/x) + ((b^2*c^2 + 20*a*b*c*d + 8*a^2*d^
2)*x)/(16*d) + (b*(7*b*c + 12*a*d)*x^3)/24 + (b^2*d*x^5)/6) - (c*
(b^2*c^2 - 12*a*b*c*d - 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*
x^2]])/(16*d^(3/2))
```

Maple [A] time = 0.016, size = 221, normalized size = 1.3

$$\frac{b^2x}{6\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{b^2cx}{24\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{b^2c^2x}{16\,d}\sqrt{dx^2+c}-\frac{b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}-\frac{a^2}{cx}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2dx}{c}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3\,a^2dx}{2}\sqrt{dx^2+c}+\frac{3\,a^2c}{2}\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)+\frac{abx}{2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3\,abcx}{4}\sqrt{dx^2+c}+\frac{3\,abc^2}{4}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x)
```

```
[Out] 1/6*b^2*x*(d*x^2+c)^(5/2)/d-1/24*b^2*c/d*x*(d*x^2+c)^(3/2)-1/16*b
^2*c^2/d*x*(d*x^2+c)^(1/2)-1/16*b^2*c^3/d^(3/2)*ln(x*d^(1/2)+(d*x
^2+c)^(1/2))-a^2*(d*x^2+c)^(5/2)/c/x+a^2*d/c*x*(d*x^2+c)^(3/2)+3/
2*a^2*d*x*(d*x^2+c)^(1/2)+3/2*a^2*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))+1/2*a*b*x*(d*x^2+c)^(3/2)+3/4*a*b*c*x*(d*x^2+c)^(1/2)+3/
4*a*b*c^2/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280115, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^2c^3-12\,abc^2d-24\,a^2cd^2\right)x\log\left(-2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)-2\left(8\,b^2d^2x^6+2\left(7\,b^2cd+12\,abd^2\right)x^4-48\,a^2cd\right.}{96\,d^{\frac{3}{2}}x}\right.$$

$$-\frac{3\left(b^2c^3-12\,abc^2d-24\,a^2cd^2\right)x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(8\,b^2d^2x^6+2\left(7\,b^2cd+12\,abd^2\right)x^4-48\,a^2cd+3\left(b^2c^2+20\,abcd+8\,a\right.}{48\sqrt{-d}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*x*log(-2*sqrt(d
*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)) - 2*(8*b^2*d^2*x^6 + 2*(7*
b^2*c*d + 12*a*b*d^2)*x^4 - 48*a^2*c*d + 3*(b^2*c^2 + 20*a*b*c*d
+ 8*a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(d))/(d^(3/2)*x), -1/48*(3*
(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*x*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) - (8*b^2*d^2*x^6 + 2*(7*b^2*c*d + 12*a*b*d^2)*x^4 - 4
8*a^2*c*d + 3*(b^2*c^2 + 20*a*b*c*d + 8*a^2*d^2)*x^2)*sqrt(d*x^2
+ c)*sqrt(-d))/(sqrt(-d)*d*x)]
```

Sympy [A] time = 56.6956, size = 367, normalized size = 2.1

$$- \frac{a^{2} c^{\frac{3}{2}}}{x \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{a^{2} \sqrt{c} d x \sqrt{1 + \frac{d x^{2}}{c}}}{2} - \frac{a^{2} \sqrt{c} d x}{\sqrt{1 + \frac{d x^{2}}{c}}} + \frac{3 a^{2} c \sqrt{d} \operatorname{asinh}{\left(\frac{\sqrt{d} x}{\sqrt{c}} \right)}}{2} + a b c^{\frac{3}{2}} x \sqrt{1 + \frac{d x^{2}}{c}} + \frac{a b c^{\frac{3}{2}} x}{4 \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{3 a b \sqrt{c} d x^{3}}{4 \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{3 a b c^{2} \operatorname{asinh}{\left(\frac{\sqrt{d} x}{\sqrt{c}} \right)}}{4 \sqrt{d}} + \frac{a b d^{2} x^{5}}{2 \sqrt{c} \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{b^{2} c^{\frac{5}{2}} x}{16 d \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{17 b^{2} c^{\frac{3}{2}} x^{3}}{48 \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{11 b^{2} \sqrt{c} d x^{5}}{24 \sqrt{1 + \frac{d x^{2}}{c}}} - \frac{b^{2} c^{3} \operatorname{asinh}{\left(\frac{\sqrt{d} x}{\sqrt{c}} \right)}}{16 d^{\frac{3}{2}}} + \frac{b^{2} d^{2} x^{7}}{6 \sqrt{c} \sqrt{1 + \frac{d x^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**2,x)
```

```
[Out] -a**2*c**(3/2)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d*x*sqrt(1 +
 d*x**2/c)/2 - a**2*sqrt(c)*d*x/sqrt(1 + d*x**2/c) + 3*a**2*c*sqr
t(d)*asinh(sqrt(d)*x/sqrt(c))/2 + a*b*c**(3/2)*x*sqrt(1 + d*x**2/
c) + a*b*c**(3/2)*x/(4*sqrt(1 + d*x**2/c)) + 3*a*b*sqrt(c)*d*x**3
/(4*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*
sqrt(d)) + a*b*d**2*x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) + b**2*c*
*(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*b**2*c**(3/2)*x**3/(48*sq
rt(1 + d*x**2/c)) + 11*b**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)
) - b**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + b**2*d**2*
x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.249414, size = 234, normalized size = 1.34

$$\frac{2 \, a^{2} c^{2} \sqrt{d}}{{\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2} - c} + \frac{1}{48} \, {\left(2 \, {\left(4 \, b^{2} d x^{2} + \frac{7 \, b^{2} c d^{4} + 12 \, a b d^{5}}{d^{4}}\right)} x^{2} + \frac{3 \, {\left(b^{2} c^{2} d^{3} + 20 \, a b c d^{4} + 8 \, a^{2} d^{5}\right)}}{d^{4}}\right)} \sqrt{d x^{2} + c} x + \frac{{\left(b^{2} c^{3} \sqrt{d} - 12 \, a b c^{2} d^{\frac{3}{2}} - 24 \, a^{2} c d^{\frac{5}{2}}\right)} \ln\left({\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2}\right)}{32 \, d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/48*(2
*(4*b^2*d*x^2 + (7*b^2*c*d^4 + 12*a*b*d^5)/d^4)*x^2 + 3*(b^2*c^2*
d^3 + 20*a*b*c*d^4 + 8*a^2*d^5)/d^4)*sqrt(d*x^2 + c)*x + 1/32*(b^
2*c^3*sqrt(d) - 12*a*b*c^2*d^(3/2) - 24*a^2*c*d^(5/2))*ln((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/d^2
```

3.620 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=136

$$-\frac{a^2(c+dx^2)^{5/2}}{2cx^2}+\frac{a(c+dx^2)^{3/2}(3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}(3ad+4bc)-\frac{1}{2}a\sqrt{c}(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2(c+dx^2)^{5/2}}{5d}$$

```
[Out] (a*(4*b*c + 3*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 3*a*d)*(c + d
*x^2)^(3/2))/(6*c) + (b^2*(c + d*x^2)^(5/2))/(5*d) - (a^2*(c + d*
x^2)^(5/2))/(2*c*x^2) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.314524, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{a^2(c+dx^2)^{5/2}}{2cx^2}+\frac{a(c+dx^2)^{3/2}(3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}(3ad+4bc)-\frac{1}{2}a\sqrt{c}(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2(c+dx^2)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (a*(4*b*c + 3*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 3*a*d)*(c + d
*x^2)^(3/2))/(6*c) + (b^2*(c + d*x^2)^(5/2))/(5*d) - (a^2*(c + d*
x^2)^(5/2))/(2*c*x^2) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/2
```

Rubi in Sympy [A] time = 27.9572, size = 121, normalized size = 0.89

$$-\frac{a^2(c+dx^2)^{\frac{5}{2}}}{2cx^2}-\frac{a\sqrt{c}(3ad+4bc)\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2}+\frac{a\sqrt{c+dx^2}(3ad+4bc)}{2}+\frac{a(c+dx^2)^{\frac{3}{2}}(3ad+4bc)}{6c}+\frac{b^2(c+dx^2)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**3,x)
```

```
[Out] -a**2*(c + d*x**2)**(5/2)/(2*c*x**2) - a*sqrt(c)*(3*a*d + 4*b*c)*
atanh(sqrt(c + d*x**2)/sqrt(c))/2 + a*sqrt(c + d*x**2)*(3*a*d + 4
*b*c)/2 + a*(c + d*x**2)**(3/2)*(3*a*d + 4*b*c)/(6*c) + b**2*(c +
 d*x**2)**(5/2)/(5*d)
```

Mathematica [A] time = 0.270848, size = 128, normalized size = 0.94

$$\frac{1}{30}\left(\frac{\sqrt{c+dx^2}\left(-15a^2d(c-2dx^2)+20abdx^2(4c+dx^2)+6b^2x^2(c+dx^2)^2\right)}{dx^2}-15a\sqrt{c}(3ad+4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+15a\sqrt{c}\log(x)(3ad+4bc)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-15*a^2*d*(c - 2*d*x^2) + 6*b^2*x^2*(c + d*x^2
)^2 + 20*a*b*d*x^2*(4*c + d*x^2)))/(d*x^2) + 15*a*Sqrt[c]*(4*b*c
+ 3*a*d)*Log[x] - 15*a*Sqrt[c]*(4*b*c + 3*a*d)*Log[c + Sqrt[c]*Sq
rt[c + d*x^2]])/30
```

Maple [A] time = 0.016, size = 161, normalized size = 1.2

$$\frac{b^2}{5d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2}{2cx^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d}{2c}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{3a^2d}{2}\sqrt{c}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)+\frac{3a^2d}{2}\sqrt{dx^2+c}+\frac{2ab}{3}\left(dx^2+c\right)^{\frac{3}{2}}-2ab\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)c^{3/2}+2ab\sqrt{dx^2+c}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x)
```

```
[Out] 1/5*b^2*(d*x^2+c)^(5/2)/d-1/2*a^2*(d*x^2+c)^(5/2)/c/x^2+1/2*a^2*d
/c*(d*x^2+c)^(3/2)-3/2*a^2*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^
(1/2))/x)+3/2*a^2*d*(d*x^2+c)^(1/2)+2/3*a*b*(d*x^2+c)^(3/2)-2*a*b
*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*c^(3/2)+2*a*b*(d*x^2+c)^(1
/2)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23984, size = 1, normalized size = 0.01

$$\left[\frac{15\left(4abcd+3a^2d^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(6b^2d^2x^6+4\left(3b^2cd+5abd^2\right)x^4-15a^2cd+2\left(3b^2c^2+40abcd+\right.\right.}{60dx^2}\right.$$
$$-\frac{15\left(4abcd+3a^2d^2\right)\sqrt{-c}x^2\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(6b^2d^2x^6+4\left(3b^2cd+5abd^2\right)x^4-15a^2cd+2\left(3b^2c^2+40abcd+15\right.\right.}{30dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(6*b^2*d^2*x^6 + 4*(3*b^2*c*
d + 5*a*b*d^2)*x^4 - 15*a^2*c*d + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*
a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2), -1/30*(15*(4*a*b*c*d + 3*
a^2*d^2)*sqrt(-c)*x^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (6*b
^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 - 15*a^2*c*d + 2*(3*b^
2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

Sympy [A] time = 47.0053, size = 303, normalized size = 2.23

$$-\frac{3a^2\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2}-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{a^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-2abc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{2abc^2}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abc\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}+2abd\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for } d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2c\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for } d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2d\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**3,x)
```

```
[Out] -3*a**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c*sqrt(d)*s
qrt(c/(d*x**2) + 1)/(2*x) + a**2*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1
)) + a**2*d**(3/2)*x/sqrt(c/(d*x**2) + 1) - 2*a*b*c**(3/2)*asinh(
sqrt(c)/(sqrt(d)*x)) + 2*a*b*c**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)
) + 2*a*b*c*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + 2*a*b*d*Piecewise((s
qrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) + b*
*2*c*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(
3*d), True)) + b**2*d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**
2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne
(d, 0)), (sqrt(c)*x**4/4, True))
```

GIAC/XCAS [A] time = 0.232335, size = 170, normalized size = 1.25

$$\frac{6\left(dx^2+c\right)^{\frac{5}{2}}b^2+20\left(dx^2+c\right)^{\frac{3}{2}}abd+60\sqrt{dx^2+c}abcd+30\sqrt{dx^2+c}a^2d^2-\frac{15\sqrt{dx^2+c}a^2cd}{x^2}+\frac{15\left(4abc^2d+3a^2cd^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(d*x^2 + c)^(5/2)*b^2 + 20*(d*x^2 + c)^(3/2)*a*b*d + 60*s
qrt(d*x^2 + c)*a*b*c*d + 30*sqrt(d*x^2 + c)*a^2*d^2 - 15*sqrt(d*x
^2 + c)*a^2*c*d/x^2 + 15*(4*a*b*c^2*d + 3*a^2*c*d^2)*arctan(sqrt(
d*x^2 + c)/sqrt(-c))/sqrt(-c))/d
```

3.621 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=184

$$-\frac{a^2 (c+dx^2)^{5/2}}{3cx^3}+\frac{x (c+dx^2)^{3/2} (8ad(ad+3bc)+3b^2c^2)}{12c^2}+\frac{x\sqrt{c+dx^2} (8ad(ad+3bc)+3b^2c^2)}{8c}+\frac{(8ad(ad+3bc)+3b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8\sqrt{d}}-\frac{2a (c+dx^2)^{5/2} (ad+3bc)}{3c^2x}$$

```
[Out] ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + ((3
*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^2) - (
a^2*(c + d*x^2)^(5/2))/(3*c*x^3) - (2*a*(3*b*c + a*d)*(c + d*x^2)
^(5/2))/(3*c^2*x) + ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*ArcTanh[(S
qrt[d]*x)/Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

Rubi [A] time = 0.344844, antiderivative size = 181, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a^2 (c+dx^2)^{5/2}}{3cx^3}+\frac{1}{12}x (c+dx^2)^{3/2} \left(\frac{8ad(ad+3bc)}{c^2}+3b^2\right)+\frac{x\sqrt{c+dx^2} (8ad(ad+3bc)+3b^2c^2)}{8c}+\frac{(8ad(ad+3bc)+3b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8\sqrt{d}}-\frac{2a (c+dx^2)^{5/2} (ad+3bc)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + ((3
*b^2 + (8*a*d*(3*b*c + a*d))/c^2)*x*(c + d*x^2)^(3/2))/12 - (a^2*
(c + d*x^2)^(5/2))/(3*c*x^3) - (2*a*(3*b*c + a*d)*(c + d*x^2)^(5/
2))/(3*c^2*x) + ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

Rubi in Sympy [A] time = 28.3801, size = 170, normalized size = 0.92

$$-\frac{a^2 (c+dx^2)^{\frac{5}{2}}}{3cx^3}-\frac{2a (c+dx^2)^{\frac{5}{2}} (ad+3bc)}{3c^2x}+\frac{(8ad (ad+3bc)+3b^2c^2) \operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8\sqrt{d}}+\frac{x\sqrt{c+dx^2} \left(ad (ad+3bc)+\frac{3b^2c^2}{8}\right)}{c}+\frac{x (c+dx^2)^{\frac{3}{2}} (8ad (ad+3bc)+3b^2c^2)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**4,x)
```

```
[Out] -a**2*(c + d*x**2)**(5/2)/(3*c*x**3) - 2*a*(c + d*x**2)**(5/2)*(a
*d + 3*b*c)/(3*c**2*x) + (8*a*d*(a*d + 3*b*c) + 3*b**2*c**2)*atan
h(sqrt(d)*x/sqrt(c + d*x**2))/(8*sqrt(d)) + x*sqrt(c + d*x**2)*(a
*d*(a*d + 3*b*c) + 3*b**2*c**2/8)/c + x*(c + d*x**2)**(3/2)*(8*a*
d*(a*d + 3*b*c) + 3*b**2*c**2)/(12*c**2)
```

Mathematica [A] time = 0.152712, size = 118, normalized size = 0.64

$$\frac{1}{24}\left(\frac{3(8a^2d^2+24abcd+3b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+\frac{\sqrt{c+dx^2}\,(-8a^2c+3bx^4(8ad+5bc)-16ax^2(2ad+3bc)+6b^2dx^6)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*c - 16*a*(3*b*c + 2*a*d)*x^2 + 3*b*(5*b
*c + 8*a*d)*x^4 + 6*b^2*d*x^6))/x^3 + (3*(3*b^2*c^2 + 24*a*b*c*d
+ 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/24
```

Maple [A] time = 0.018, size = 241, normalized size = 1.3

$$\frac{xb^2}{4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3\,b^2cx}{8}\sqrt{dx^2+c}+\frac{3\,b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}-\frac{a^2}{3\,cx^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{2\,a^2d}{3\,c^2x}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{2\,a^2d^2x}{3\,c^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d^2x}{c}\sqrt{dx^2+c}+a^2d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)-2\,\frac{ab\left(dx^2+c\right)^{5/2}}{cx}+2\,\frac{abdx\left(dx^2+c\right)^{3/2}}{c}+3\,abdx\sqrt{dx^2+c}+3\,ab\sqrt{d}c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x)
```

```
[Out] 1/4*x*b^2*(d*x^2+c)^(3/2)+3/8*b^2*c*x*(d*x^2+c)^(1/2)+3/8*b^2*c^2
/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/3*a^2*(d*x^2+c)^(5/2)/c/
x^3-2/3*a^2*d/c^2/x*(d*x^2+c)^(5/2)+2/3*a^2*d^2/c^2*x*(d*x^2+c)^(
3/2)+a^2*d^2/c*x*(d*x^2+c)^(1/2)+a^2*d^(3/2)*ln(x*d^(1/2)+(d*x^2+
c)^(1/2))-2*a*b/c/x*(d*x^2+c)^(5/2)+2*a*b*d/c*x*(d*x^2+c)^(3/2)+3
*a*b*d*x*(d*x^2+c)^(1/2)+3*a*b*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259439, size = 1, normalized size = 0.01

$$\left[\frac{3\left(3\,b^2c^2+24\,abcd+8\,a^2d^2\right)x^3\log\left(-2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+2\left(6\,b^2dx^6+3\left(5\,b^2c+8\,abd\right)x^4-8\,a^2c-16\,(3\,ab\right.}{48\sqrt{d}x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*x^3*log(-2*sqrt(d*x
^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)) + 2*(6*b^2*d*x^6 + 3*(5*b^2*
c + 8*a*b*d)*x^4 - 8*a^2*c - 16*(3*a*b*c + 2*a^2*d)*x^2)*sqrt(d*x
^2 + c)*sqrt(d))/(sqrt(d)*x^3), 1/24*(3*(3*b^2*c^2 + 24*a*b*c*d +
 8*a^2*d^2)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b^2*d*x^6
 + 3*(5*b^2*c + 8*a*b*d)*x^4 - 8*a^2*c - 16*(3*a*b*c + 2*a^2*d)*x
^2)*sqrt(d*x^2 + c)*sqrt(-d))/(sqrt(-d)*x^3)]
```

Sympy [A] time = 33.9108, size = 352, normalized size = 1.91

$$-\frac{a^2\sqrt{c}d}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+a^2d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{a^2d^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}dx\sqrt{1+\frac{dx^2}{c}}-\frac{2ab\sqrt{c}dx}{\sqrt{1+\frac{dx^2}{c}}}+3abc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)+\frac{b^2c^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{b^2c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2\sqrt{c}dx^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{b^2d^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**4,x)
```

```
[Out] -a**2*sqrt(c)*d/(x*sqrt(1 + d*x**2/c)) - a**2*c*sqrt(d)*sqrt(c/(d
*x**2) + 1)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + a**
2*d**(3/2)*asinh(sqrt(d)*x/sqrt(c)) - a**2*d**2*x/(sqrt(c)*sqrt(1
 + d*x**2/c)) - 2*a*b*c**(3/2)/(x*sqrt(1 + d*x**2/c)) + a*b*sqrt(
c)*d*x*sqrt(1 + d*x**2/c) - 2*a*b*sqrt(c)*d*x/sqrt(1 + d*x**2/c)
+ 3*a*b*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + b**2*c**(3/2)*x*sqrt
(1 + d*x**2/c)/2 + b**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c)) + 3*b**
2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*b**2*c**2*asinh(sqrt(
d)*x/sqrt(c))/(8*sqrt(d)) + b**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*
x**2/c))
```

GIAC/XCAS [A] time = 0.249167, size = 354, normalized size = 1.92

$$\frac{1}{8}\left(2\,b^2dx^2+\frac{5\,b^2cd^2+8\,abd^3}{d^2}\right)\sqrt{dx^2+c}x-\frac{\left(3\,b^2c^2\sqrt{d}+24\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,d}+\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc^2\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2cd^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^3\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2c^2\cdots\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d*x^2 + (5*b^2*c*d^2 + 8*a*b*d^3)/d^2)*sqrt(d*x^2 + c)
*x - 1/16*(3*b^2*c^2*sqrt(d) + 24*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*
ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d + 4/3*(3*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*b*c^2*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4
*a^2*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^3*sqrt(d
) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^2*d^(3/2) + 3*a*b*c^4
*sqrt(d) + 2*a^2*c^3*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 -
c)^3
```

3.622 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{a^2(c+dx^2)^{5/2}}{4cx^4}+\frac{(c+dx^2)^{3/2}(3ad(ad+8bc)+8b^2c^2)}{24c^2}+\frac{\sqrt{c+dx^2}\,(3ad(ad+8bc)+8b^2c^2)}{8c}-\frac{(3ad(ad+8bc)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{a(c+dx^2)^{5/2}(ad+8bc)}{8c^2x^2}$$

```
[Out] ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*Sqrt[c + d*x^2])/(8*c) + ((8*b
^2*c^2 + 3*a*d*(8*b*c + a*d))*(c + d*x^2)^(3/2))/(24*c^2) - (a^2*
(c + d*x^2)^(5/2))/(4*c*x^4) - (a*(8*b*c + a*d)*(c + d*x^2)^(5/2)
)/(8*c^2*x^2) - ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rubi [A] time = 0.49698, antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^2(c+dx^2)^{5/2}}{4cx^4}+\frac{1}{24}(c+dx^2)^{3/2}\left(\frac{3ad(ad+8bc)}{c^2}+8b^2\right)+\frac{\sqrt{c+dx^2}\,(3ad(ad+8bc)+8b^2c^2)}{8c}-\frac{(3ad(ad+8bc)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{a(c+dx^2)^{5/2}(ad+8bc)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*Sqrt[c + d*x^2])/(8*c) + ((8*b
^2 + (3*a*d*(8*b*c + a*d))/c^2)*(c + d*x^2)^(3/2))/24 - (a^2*(c +
 d*x^2)^(5/2))/(4*c*x^4) - (a*(8*b*c + a*d)*(c + d*x^2)^(5/2))/(8
*c^2*x^2) - ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*ArcTanh[Sqrt[c + d
*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rubi in Sympy [A] time = 32.8158, size = 167, normalized size = 0.92

$$-\frac{a^2(c+dx^2)^{\frac{5}{2}}}{4cx^4}-\frac{a(c+dx^2)^{\frac{5}{2}}(ad+8bc)}{8c^2x^2}+\frac{\sqrt{c+dx^2}\,(3ad(ad+8bc)+8b^2c^2)}{8c}+\frac{(c+dx^2)^{\frac{3}{2}}(3ad(ad+8bc)+8b^2c^2)}{24c^2}-\frac{(3ad(ad+8bc)+8b^2c^2)\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**5,x)
```

```
[Out] -a**2*(c + d*x**2)**(5/2)/(4*c*x**4) - a*(c + d*x**2)**(5/2)*(a*d
 + 8*b*c)/(8*c**2*x**2) + sqrt(c + d*x**2)*(3*a*d*(a*d + 8*b*c) +
 8*b**2*c**2)/(8*c) + (c + d*x**2)**(3/2)*(3*a*d*(a*d + 8*b*c) +
8*b**2*c**2)/(24*c**2) - (3*a*d*(a*d + 8*b*c) + 8*b**2*c**2)*atan
h(sqrt(c + d*x**2)/sqrt(c))/(8*sqrt(c))
```

Mathematica [A] time = 0.283613, size = 150, normalized size = 0.83

$$\frac{1}{24}\left(-\frac{3\left(3a^2d^2+24abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{\sqrt{c}}+\frac{3\log(x)\left(3a^2d^2+24abcd+8b^2c^2\right)}{\sqrt{c}}+\frac{\sqrt{c+dx^2}\left(-3a^2\left(2c+5dx^2\right)-24abx^2\left(c-2dx^2\right)+8b^2x^4\left(4c+dx^2\right)\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-24*a*b*x^2*(c - 2*d*x^2) + 8*b^2*x^4*(4*c + d
*x^2) - 3*a^2*(2*c + 5*d*x^2)))/x^4 + (3*(8*b^2*c^2 + 24*a*b*c*d
+ 3*a^2*d^2)*Log[x])/Sqrt[c] - (3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2
*d^2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/Sqrt[c])/24
```

Maple [A] time = 0.018, size = 256, normalized size = 1.4

$$-\frac{a^2}{4\,cx^4}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2d}{8\,c^2x^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d^2}{8\,c^2}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{3\,a^2d^2}{8}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{3\,a^2d^2}{8\,c}\sqrt{dx^2+c}+\frac{b^2}{3}\left(dx^2+c\right)^{\frac{3}{2}}-b^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{\frac{3}{2}}+b^2\sqrt{dx^2+c}c-\frac{ab}{cx^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{abd}{c}\left(dx^2+c\right)^{\frac{3}{2}}-3\,abd\sqrt{c}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)+3\,abd\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(5/2)/c/x^4-1/8*a^2*d/c^2/x^2*(d*x^2+c)^(5/2)+
1/8*a^2*d^2/c^2*(d*x^2+c)^(3/2)-3/8*a^2*d^2/c^(1/2)*ln((2*c+2*c^(
1/2)*(d*x^2+c)^(1/2))/x)+3/8*a^2*d^2/c*(d*x^2+c)^(1/2)+1/3*b^2*(d
*x^2+c)^(3/2)-b^2*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*c^(3/2)+b
^2*(d*x^2+c)^(1/2)*c-a*b/c/x^2*(d*x^2+c)^(5/2)+a*b*d/c*(d*x^2+c)^
(3/2)-3*a*b*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+3*a*b
*d*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242509, size = 1, normalized size = 0.01

$$\left[\frac{3\left(8\,b^2c^2+24\,abcd+3\,a^2d^2\right)x^4\log\left(-\frac{(dx^2+2\,c)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)+2\left(8\,b^2dx^6+16\left(2\,b^2c+3\,abd\right)x^4-6\,a^2c-3\left(8\,abc+5\,a^2d\right)\right.}{48\,\sqrt{c}x^4}\right.$$

$$-\frac{3\left(8\,b^2c^2+24\,abcd+3\,a^2d^2\right)x^4\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(8\,b^2dx^6+16\left(2\,b^2c+3\,abd\right)x^4-6\,a^2c-3\left(8\,abc+5\,a^2d\right)x^2\right)\sqrt{dx^2+c}}{24\,\sqrt{-c}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*x^4*log(-((d*x^2 +
2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2) + 2*(8*b^2*d*x^6 + 16*(2
*b^2*c + 3*a*b*d)*x^4 - 6*a^2*c - 3*(8*a*b*c + 5*a^2*d)*x^2)*sqrt
(d*x^2 + c)*sqrt(c))/(sqrt(c)*x^4), -1/24*(3*(8*b^2*c^2 + 24*a*b*
c*d + 3*a^2*d^2)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (8*b^2*d*
x^6 + 16*(2*b^2*c + 3*a*b*d)*x^4 - 6*a^2*c - 3*(8*a*b*c + 5*a^2*d
)*x^2)*sqrt(d*x^2 + c)*sqrt(-c))/(sqrt(-c)*x^4)]
```

Sympy [A] time = 89.0402, size = 332, normalized size = 1.83

$$-\frac{a^2c^2}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2c\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{a^2d^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8\sqrt{c}}$$
$$-3ab\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)-\frac{abc\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{2abc\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}$$
$$-b^2c^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{b^2c^2}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2c\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}+b^2d\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for } d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**5,x)
```

```
[Out] -a**2*c**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c*sqrt(
d)/(8*x**3*sqrt(c/(d*x**2) + 1)) - a**2*d**(3/2)*sqrt(c/(d*x**2)
+ 1)/(2*x) - a**2*d**(3/2)/(8*x*sqrt(c/(d*x**2) + 1)) - 3*a**2*d*
*2*asinh(sqrt(c)/(sqrt(d)*x))/(8*sqrt(c)) - 3*a*b*sqrt(c)*d*asinh
(sqrt(c)/(sqrt(d)*x)) - a*b*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/x + 2*
a*b*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a*b*d**(3/2)*x/sqrt(c/
(d*x**2) + 1) - b**2*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + b**2*c
**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c*sqrt(d)*x/sqrt(c/(d
*x**2) + 1) + b**2*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c +
d*x**2)**(3/2)/(3*d), True))
```

GIAC/XCAS [A] time = 0.242229, size = 246, normalized size = 1.36

$$\frac{8\left(dx^2+c\right)^{\frac{3}{2}}b^2d+24\,\sqrt{dx^2+c}b^2cd+48\,\sqrt{dx^2+c}abd^2+\frac{3\left(8\,b^2c^2d+24\,abcd^2+3\,a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3\left(8\left(dx^2+c\right)^{\frac{3}{2}}abcd^2-8\,\sqrt{dx^2+c}cal\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(d*x^2 + c)^(3/2)*b^2*d + 24*sqrt(d*x^2 + c)*b^2*c*d + 48
*sqrt(d*x^2 + c)*a*b*d^2 + 3*(8*b^2*c^2*d + 24*a*b*c*d^2 + 3*a^2*
d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - 3*(8*(d*x^2 + c)
^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^2 + c)*a*b*c^2*d^2 + 5*(d*x^2 + c)^
(3/2)*a^2*d^3 - 3*sqrt(d*x^2 + c)*a^2*c*d^3)/(d^2*x^4))/d
```

3.623 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{b\left(c+dx^2\right)^{3/2}(4ad+3bc)}{3cx}+\frac{bdx\sqrt{c+dx^2}(4ad+3bc)}{2c}+\frac{1}{2}b\sqrt{d}(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3}$$

```
[Out] (b*d*(3*b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (b*(3*b*c + 4*a*d
)*(c + d*x^2)^(3/2))/(3*c*x) - (a^2*(c + d*x^2)^(5/2))/(5*c*x^5)
- (2*a*b*(c + d*x^2)^(5/2))/(3*c*x^3) + (b*Sqrt[d]*(3*b*c + 4*a*d
)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/2
```

Rubi [A] time = 0.252298, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{b\left(c+dx^2\right)^{3/2}(4ad+3bc)}{3cx}+\frac{bdx\sqrt{c+dx^2}(4ad+3bc)}{2c}+\frac{1}{2}b\sqrt{d}(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] (b*d*(3*b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (b*(3*b*c + 4*a*d
)*(c + d*x^2)^(3/2))/(3*c*x) - (a^2*(c + d*x^2)^(5/2))/(5*c*x^5)
- (2*a*b*(c + d*x^2)^(5/2))/(3*c*x^3) + (b*Sqrt[d]*(3*b*c + 4*a*d
)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/2
```

Rubi in Sympy [A] time = 26.9589, size = 134, normalized size = 0.91

$$-\frac{a^2\left(c+dx^2\right)^{\frac{5}{2}}}{5cx^5}-\frac{2ab\left(c+dx^2\right)^{\frac{5}{2}}}{3cx^3}+\frac{b\sqrt{d}\,(4ad+3bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2}+\frac{bdx\sqrt{c+dx^2}\,(4ad+3bc)}{2c}-\frac{b\left(c+dx^2\right)^{\frac{3}{2}}(4ad+3bc)}{3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**6,x)
```

```
[Out] -a**2*(c + d*x**2)**(5/2)/(5*c*x**5) - 2*a*b*(c + d*x**2)**(5/2)/
(3*c*x**3) + b*sqrt(d)*(4*a*d + 3*b*c)*atanh(sqrt(d)*x/sqrt(c + d
*x**2))/2 + b*d*x*sqrt(c + d*x**2)*(4*a*d + 3*b*c)/(2*c) - b*(c +
 d*x**2)**(3/2)*(4*a*d + 3*b*c)/(3*c*x)
```

Mathematica [A] time = 0.186524, size = 113, normalized size = 0.77

$$\frac{1}{2}b\sqrt{d}(4ad+3bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)-\frac{\sqrt{c+dx^2}\left(6a^2\left(c+dx^2\right)^2+20abcx^2\left(c+4dx^2\right)+15b^2cx^4\left(2c-dx^2\right)\right)}{30cx^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c*x^4*(2*c - d*x^2) + 6*a^2*(c + d*x^2)
^2 + 20*a*b*c*x^2*(c + 4*d*x^2)))/(30*c*x^5) + (b*Sqrt[d]*(3*b*c
+ 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/2
```

Maple [A] time = 0.018, size = 203, normalized size = 1.4

$$-\frac{a^2}{5\,cx^5}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{b^2}{cx}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2dx}{c}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3\,b^2dx}{2}\sqrt{dx^2+c}+\frac{3\,b^2c}{2}\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)-\frac{2\,ab}{3\,cx^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{4\,dab}{3\,c^2x}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{4\,abd^2x}{3\,c^2}\left(dx^2+c\right)^{\frac{3}{2}}+2\,\frac{abd^2x\sqrt{dx^2+c}}{c}+2\,abd^{3/2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(5/2)/c/x^5-b^2/c/x*(d*x^2+c)^(5/2)+b^2*d/c*x*
(d*x^2+c)^(3/2)+3/2*b^2*d*x*(d*x^2+c)^(1/2)+3/2*b^2*d^(1/2)*c*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))-2/3*a*b*(d*x^2+c)^(5/2)/c/x^3-4/3*a*b*
d/c^2/x*(d*x^2+c)^(5/2)+4/3*a*b*d^2/c^2*x*(d*x^2+c)^(3/2)+2*a*b*d
^2/c*x*(d*x^2+c)^(1/2)+2*a*b*d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282195, size = 1, normalized size = 0.01

$$\left[\frac{15\left(3\,b^2c^2+4\,abcd\right)\sqrt{d}x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(15\,b^2cdx^6-2\left(15\,b^2c^2+40\,abcd+3\,a^2d^2\right)x^4-6\,a^2c^2-\right.}{60\,cx^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^5*log(-2*d*x^2 - 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) + 2*(15*b^2*c*d*x^6 - 2*(15*b^2*c^2
+ 40*a*b*c*d + 3*a^2*d^2)*x^4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*
c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^5), 1/30*(15*(3*b^2*c^2 + 4*a*b*c
*d)*sqrt(-d)*x^5*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) + (15*b^2
*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^4 - 6*a^2*c^
2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^5)]
```

Sympy [A] time = 22.9441, size = 304, normalized size = 2.07

$$-\frac{a^{2} c \sqrt{d} \sqrt{\frac{c}{d x^{2}} + 1}}{5 x^{4}} - \frac{2 a^{2} d^{\frac{3}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{5 x^{2}} - \frac{a^{2} d^{\frac{5}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{5 c} - \frac{2 a b \sqrt{c} d}{x \sqrt{1 + \frac{d x^{2}}{c}}} - \frac{2 a b c \sqrt{d} \sqrt{\frac{c}{d x^{2}} + 1}}{3 x^{2}} - \frac{2 a b d^{\frac{3}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{3} + 2 a b d^{\frac{3}{2}} \operatorname{asinh}\left(\frac{\sqrt{d} x}{\sqrt{c}}\right) - \frac{2 a b d^{2} x}{\sqrt{c} \sqrt{1 + \frac{d x^{2}}{c}}} - \frac{b^{2} c^{\frac{3}{2}}}{x \sqrt{1 + \frac{d x^{2}}{c}}} + \frac{b^{2} \sqrt{c} d x \sqrt{1 + \frac{d x^{2}}{c}}}{2} - \frac{b^{2} \sqrt{c} d x}{\sqrt{1 + \frac{d x^{2}}{c}}} + \frac{3 b^{2} c \sqrt{d} \operatorname{asinh}\left(\frac{\sqrt{d} x}{\sqrt{c}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**6,x)
```

```
[Out] -a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a**2*d**(3/2)*s
qrt(c/(d*x**2) + 1)/(5*x**2) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)
/(5*c) - 2*a*b*sqrt(c)*d/(x*sqrt(1 + d*x**2/c)) - 2*a*b*c*sqrt(d)
*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) +
 1)/3 + 2*a*b*d**(3/2)*asinh(sqrt(d)*x/sqrt(c)) - 2*a*b*d**2*x/(s
qrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)/(x*sqrt(1 + d*x**2/c))
 + b**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)/2 - b**2*sqrt(c)*d*x/sqrt(
1 + d*x**2/c) + 3*b**2*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/2
```

GIAC/XCAS [A] time = 0.254954, size = 549, normalized size = 3.73

$$\frac{1}{2} \sqrt{d x^{2} + c} b^{2} d x - \frac{1}{4} \left(3 \, b^{2} c \sqrt{d} + 4 \, a b d^{\frac{3}{2}}\right) \ln\left({\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2}\right) + \frac{2 \, \left(15 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{8} b^{2} c^{2} \sqrt{d} + 60 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{8} a b c d^{\frac{3}{2}} + 15 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{8} a^{2} d^{\frac{5}{2}} - 60 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{6} b^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*d*x - 1/4*(3*b^2*c*sqrt(d) + 4*a*b*d^(3/2
))*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sq
rt(d*x^2 + c))^8*b^2*c^2*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c
))^8*a*b*c*d^(3/2) + 15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/
2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) - 180*(sq
rt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) + 90*(sqrt(d)*x - sq
rt(d*x^2 + c))^4*b^2*c^4*sqrt(d) + 220*(sqrt(d)*x - sqrt(d*x^2 +
c))^4*a*b*c^3*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^
2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) -
140*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) + 15*b^2*c^6*
sqrt(d) + 40*a*b*c^5*d^(3/2) + 3*a^2*c^4*d^(5/2))/((sqrt(d)*x - s
qrt(d*x^2 + c))^2 - c)^5
```

3.624 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2(c+dx^2)^{5/2}}{6cx^6}-\frac{(c+dx^2)^{3/2}(ad(12bc-ad)+24b^2c^2)}{48c^2x^2}+\frac{d\sqrt{c+dx^2}\,(ad(12bc-ad)+24b^2c^2)}{16c^2}-\frac{d\,(ad(12bc-ad)+24b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{24c^2x^4}$$

```
[Out] (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Sqrt[c + d*x^2])/(16*c^2) -
((24*b^2*c^2 + a*d*(12*b*c - a*d))*(c + d*x^2)^(3/2))/(48*c^2*x^2
) - (a^2*(c + d*x^2)^(5/2))/(6*c*x^6) - (a*(12*b*c - a*d)*(c + d*
x^2)^(5/2))/(24*c^2*x^4) - (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*A
rcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rubi [A] time = 0.527798, antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{a^2(c+dx^2)^{5/2}}{6cx^6}-\frac{(c+dx^2)^{3/2}\left(\frac{ad(12bc-ad)}{c^2}+24b^2\right)}{48x^2}+\frac{d\sqrt{c+dx^2}\,(ad(12bc-ad)+24b^2c^2)}{16c^2}-\frac{d\,(ad(12bc-ad)+24b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Sqrt[c + d*x^2])/(16*c^2) -
((24*b^2 + (a*d*(12*b*c - a*d))/c^2)*(c + d*x^2)^(3/2))/(48*x^2)
- (a^2*(c + d*x^2)^(5/2))/(6*c*x^6) - (a*(12*b*c - a*d)*(c + d*x^
2)^(5/2))/(24*c^2*x^4) - (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Arc
Tanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rubi in Sympy [A] time = 34.1998, size = 170, normalized size = 0.91

$$-\frac{a^2(c+dx^2)^{\frac{5}{2}}}{6cx^6}+\frac{a(c+dx^2)^{\frac{5}{2}}(ad-12bc)}{24c^2x^4}+\frac{d\sqrt{c+dx^2}\,(-ad(ad-12bc)+24b^2c^2)}{16c^2}-\frac{(c+dx^2)^{\frac{3}{2}}(-ad(ad-12bc)+24b^2c^2)}{48c^2x^2}-\frac{d\,(-ad(ad-12bc)+24b^2c^2)\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**7,x)
```

```
[Out] -a**2*(c + d*x**2)**(5/2)/(6*c*x**6) + a*(c + d*x**2)**(5/2)*(a*d
 - 12*b*c)/(24*c**2*x**4) + d*sqrt(c + d*x**2)*(-a*d*(a*d - 12*b*
c) + 24*b**2*c**2)/(16*c**2) - (c + d*x**2)**(3/2)*(-a*d*(a*d - 1
2*b*c) + 24*b**2*c**2)/(48*c**2*x**2) - d*(-a*d*(a*d - 12*b*c) +
24*b**2*c**2)*atanh(sqrt(c + d*x**2)/sqrt(c))/(16*c**(3/2))
```

Mathematica [A] time = 0.329896, size = 166, normalized size = 0.89

$$\sqrt{c+dx^2}\left(\frac{-a^2d^2-20abcd-8b^2c^2}{16cx^2}-\frac{a^2c}{6x^6}-\frac{a(7ad+12bc)}{24x^4}+b^2d\right)+\frac{d\,(a^2d^2-12abcd-24b^2c^2)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{16c^{3/2}}-\frac{d\log(x)\,(a^2d^2-12abcd-24b^2c^2)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] (b^2*d - (a^2*c)/(6*x^6) - (a*(12*b*c + 7*a*d))/(24*x^4) + (-8*b^
2*c^2 - 20*a*b*c*d - a^2*d^2)/(16*c*x^2))*Sqrt[c + d*x^2] - (d*(-
24*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*Log[x])/(16*c^(3/2)) + (d*(-24
*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]]
)/(16*c^(3/2))
```

Maple [B] time = 0.02, size = 335, normalized size = 1.8

$$-\frac{a^2}{6\,cx^6}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d}{24\,c^2x^4}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2d^2}{48\,c^3x^2}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{a^2d^3}{48\,c^3}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}-\frac{a^2d^3}{16\,c^2}\sqrt{dx^2+c}-\frac{b^2}{2\,cx^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2d}{2\,c}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{3\,b^2d}{2}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)+\frac{3\,b^2d}{2}\sqrt{dx^2+c}-\frac{ab}{2\,cx^4}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{abd}{4\,c^2x^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{abd^2}{4\,c^2}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{3\,abd^2}{4}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{3\,abd^2}{4\,c}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(5/2)/c/x^6+1/24*a^2*d/c^2/x^4*(d*x^2+c)^(5/2)
+1/48*a^2*d^2/c^3/x^2*(d*x^2+c)^(5/2)-1/48*a^2*d^3/c^3*(d*x^2+c)^
(3/2)+1/16*a^2*d^3/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-
1/16*a^2*d^3/c^2*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(5/2)+1/
2*b^2*d/c*(d*x^2+c)^(3/2)-3/2*b^2*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*
x^2+c)^(1/2))/x)+3/2*b^2*d*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*x^2+c
)^(5/2)-1/4*a*b*d/c^2/x^2*(d*x^2+c)^(5/2)+1/4*a*b*d^2/c^2*(d*x^2+
c)^(3/2)-3/4*a*b*d^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x
)+3/4*a*b*d^2/c*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.239103, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(24\,b^2c^2d+12\,abcd^2-a^2d^3\right)x^6\log\left(-\frac{\left(dx^2+2\,c\right)\sqrt{c}+2\,\sqrt{dx^2+c}c}{x^2}\right)-2\left(48\,b^2cdx^6-3\left(8\,b^2c^2+20\,abcd+a^2d^2\right)x^4-8\,a^2c^2-\right.}{96\,c^{\frac{3}{2}}x^6}\right.$$

$$-\frac{3\left(24\,b^2c^2d+12\,abcd^2-a^2d^3\right)x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(48\,b^2cdx^6-3\left(8\,b^2c^2+20\,abcd+a^2d^2\right)x^4-8\,a^2c^2-2\left(12\,abc^2+7\right.\right.}{48\,\sqrt{-c}cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^2*d^3)*x^6*log(-((d*x^
2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2) - 2*(48*b^2*c*d*x^6
- 3*(8*b^2*c^2 + 20*a*b*c*d + a^2*d^2)*x^4 - 8*a^2*c^2 - 2*(12*a*
b*c^2 + 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c))/(c^(3/2)*x^6), -
1/48*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^2*d^3)*x^6*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) - (48*b^2*c*d*x^6 - 3*(8*b^2*c^2 + 20*a*b*c*d
 + a^2*d^2)*x^4 - 8*a^2*c^2 - 2*(12*a*b*c^2 + 7*a^2*c*d)*x^2)*sqr
t(d*x^2 + c)*sqrt(-c))/(sqrt(-c)*c*x^6)]
```

Sympy [A] time = 134.915, size = 367, normalized size = 1.96

$$-\frac{a^2c^2}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}-\frac{11a^2c\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{17a^2d^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{5}{2}}}{16cx\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{3}{2}}}-\frac{abc^2}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3abc\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x}-\frac{abd^{\frac{3}{2}}}{4x\sqrt{\frac{c}{dx^2}+1}}-\frac{3abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4\sqrt{c}}-\frac{3b^2\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2}-\frac{b^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{b^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**7,x)
```

```
[Out] -a**2*c**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 11*a**2*c*sqrt
(d)/(24*x**5*sqrt(c/(d*x**2) + 1)) - 17*a**2*d**(3/2)/(48*x**3*sq
rt(c/(d*x**2) + 1)) - a**2*d**(5/2)/(16*c*x*sqrt(c/(d*x**2) + 1))
 + a**2*d**3*asinh(sqrt(c)/(sqrt(d)*x))/(16*c**(3/2)) - a*b*c**2/
(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a*b*c*sqrt(d)/(4*x**3*s
qrt(c/(d*x**2) + 1)) - a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/x - a*b*
d**(3/2)/(4*x*sqrt(c/(d*x**2) + 1)) - 3*a*b*d**2*asinh(sqrt(c)/(s
qrt(d)*x))/(4*sqrt(c)) - 3*b**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*
x))/2 - b**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) + b**2*c*sqrt(d
)/(x*sqrt(c/(d*x**2) + 1)) + b**2*d**(3/2)*x/sqrt(c/(d*x**2) + 1)
```

GIAC/XCAS [A] time = 0.241855, size = 350, normalized size = 1.87

$$\frac{48\sqrt{dx^2+c}b^2d^2+\frac{3\left(24b^2c^2d^2+12abcd^3-a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2+60\left(dx^2+c\right)^{\frac{5}{2}}abcd^3\cdots}{\cdots}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(d*x^2 + c)*b^2*d^2 + 3*(24*b^2*c^2*d^2 + 12*a*b*c*d
^3 - a^2*d^4)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (24
*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2
 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 60*(d*x^2 + c)^(5/2)*a*b*c*d^
3 - 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 36*sqrt(d*x^2 + c)*a*b*c^3
*d^3 + 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*(d*x^2 + c)^(3/2)*a^2*c*d^
4 - 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c*d^3*x^6))/d
```

3.625 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^4) + ((b*c - a*d)*(3*b*
c - a*d)*(c + d*x^2)^(9/2))/(9*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x
^2)^(11/2))/(11*d^4) + (b^2*(c + d*x^2)^(13/2))/(13*d^4)
```

Rubi [A] time = 0.265046, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^4) + ((b*c - a*d)*(3*b*
c - a*d)*(c + d*x^2)^(9/2))/(9*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x
^2)^(11/2))/(11*d^4) + (b^2*(c + d*x^2)^(13/2))/(13*d^4)
```

Rubi in Sympy [A] time = 32.6081, size = 100, normalized size = 0.88

$$\frac{b^2\left(c+dx^2\right)^{\frac{13}{2}}}{13d^4}+\frac{b\left(c+dx^2\right)^{\frac{11}{2}}(2ad-3bc)}{11d^4}-\frac{c\left(c+dx^2\right)^{\frac{7}{2}}(ad-bc)^2}{7d^4}+\frac{\left(c+dx^2\right)^{\frac{9}{2}}(ad-3bc)(ad-bc)}{9d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*(c + d*x**2)**(13/2)/(13*d**4) + b*(c + d*x**2)**(11/2)*(2*a
*d - 3*b*c)/(11*d**4) - c*(c + d*x**2)**(7/2)*(a*d - b*c)**2/(7*d
**4) + (c + d*x**2)**(9/2)*(a*d - 3*b*c)*(a*d - b*c)/(9*d**4)
```

Mathematica [A] time = 0.164708, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(143a^2d^2\left(7dx^2-2c\right)+26abd\left(8c^2-28cdx^2+63d^2x^4\right)+b^2\left(-48c^3+168c^2dx^2-378cd^2x^4+693d^3x^6\right)\right)}{9009d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(143*a^2*d^2*(-2*c + 7*d*x^2) + 26*a*b*d*(8*c^
2 - 28*c*d*x^2 + 63*d^2*x^4) + b^2*(-48*c^3 + 168*c^2*d*x^2 - 378
*c*d^2*x^4 + 693*d^3*x^6)))/(9009*d^4)
```

Maple [A] time = 0.011, size = 108, normalized size = 1.

$$-\frac{-693\,b^2x^6d^3-1638\,abd^3x^4+378\,b^2cd^2x^4-1001\,a^2d^3x^2+728\,abcd^2x^2-168\,b^2c^2dx^2+286\,a^2cd^2-208\,abc^2d+48\,b^2c^3}{9009\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] -1/9009*(d*x^2+c)^(7/2)*(-693*b^2*d^3*x^6-1638*a*b*d^3*x^4+378*b^
2*c*d^2*x^4-1001*a^2*d^3*x^2+728*a*b*c*d^2*x^2-168*b^2*c^2*d*x^2+
286*a^2*c*d^2-208*a*b*c^2*d+48*b^2*c^3)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232903, size = 292, normalized size = 2.56

$$\frac{\left(693\, b^2 d^6 x^{12} + 63\left(27\, b^2 c d^5 + 26\, abd^6\right) x^{10} + 7\left(159\, b^2 c^2 d^4 + 598\, abcd^5 + 143\, a^2 d^6\right) x^8 - 48\, b^2 c^6 + 208\, abc^5 d - 286\, a^2 c^4 d^2 + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/9009*(693*b^2*d^6*x^12 + 63*(27*b^2*c*d^5 + 26*a*b*d^6)*x^10 +
7*(159*b^2*c^2*d^4 + 598*a*b*c*d^5 + 143*a^2*d^6)*x^8 - 48*b^2*c^
6 + 208*a*b*c^5*d - 286*a^2*c^4*d^2 + (15*b^2*c^3*d^3 + 2938*a*b*
c^2*d^4 + 2717*a^2*c*d^5)*x^6 - 3*(6*b^2*c^4*d^2 - 26*a*b*c^3*d^3
 - 715*a^2*c^2*d^4)*x^4 + (24*b^2*c^5*d - 104*a*b*c^4*d^2 + 143*a
^2*c^3*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 31.5485, size = 468, normalized size = 4.11

$$\begin{cases} -\frac{2a^2c^4\sqrt{c+dx^2}}{63d^2} + \frac{a^2c^3x^2\sqrt{c+dx^2}}{63d} + \frac{5a^2c^2x^4\sqrt{c+dx^2}}{21} + \frac{19a^2cdx^6\sqrt{c+dx^2}}{63} + \frac{a^2d^2x^8\sqrt{c+dx^2}}{9} + \frac{16abc^5\sqrt{c+dx^2}}{693d^3} - \frac{8abc^4x^2\sqrt{c+dx^2}}{693d^2} + \frac{2abc^3x^4\sqrt{c+dx^2}}{231d} \cdots \\ c^{\frac{5}{2}}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c**4*sqrt(c + d*x**2)/(63*d**2) + a**2*c**3*x*
*2*sqrt(c + d*x**2)/(63*d) + 5*a**2*c**2*x**4*sqrt(c + d*x**2)/21
 + 19*a**2*c*d*x**6*sqrt(c + d*x**2)/63 + a**2*d**2*x**8*sqrt(c +
 d*x**2)/9 + 16*a*b*c**5*sqrt(c + d*x**2)/(693*d**3) - 8*a*b*c**4
*x**2*sqrt(c + d*x**2)/(693*d**2) + 2*a*b*c**3*x**4*sqrt(c + d*x*
*2)/(231*d) + 226*a*b*c**2*x**6*sqrt(c + d*x**2)/693 + 46*a*b*c*d
*x**8*sqrt(c + d*x**2)/99 + 2*a*b*d**2*x**10*sqrt(c + d*x**2)/11
- 16*b**2*c**6*sqrt(c + d*x**2)/(3003*d**4) + 8*b**2*c**5*x**2*sq
rt(c + d*x**2)/(3003*d**3) - 2*b**2*c**4*x**4*sqrt(c + d*x**2)/(1
001*d**2) + 5*b**2*c**3*x**6*sqrt(c + d*x**2)/(3003*d) + 53*b**2*
c**2*x**8*sqrt(c + d*x**2)/429 + 27*b**2*c*d*x**10*sqrt(c + d*x**
2)/143 + b**2*d**2*x**12*sqrt(c + d*x**2)/13, Ne(d, 0)), (c**(5/2
)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8), True))
```

GIAC/XCAS [A] time = 0.239024, size = 1, normalized size = 0.01

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3,x, algorithm="giac")

[Out] Done
```

3.626 $\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=281

$$-\frac{c^4\left(40a^2d^2+bc(5bc-24ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{7/2}}+\frac{c^3x\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{1024d^3}+\frac{c^2x^3\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{512d^2}+\frac{x^3\left(c+dx^2\right)^{5/2}\left(40a^2d^2+bc(5bc-24ad)\right)}{320d^2}+\frac{cx^3\left(c+dx^2\right)^{3/2}\left(40a^2d^2+bc(5bc-24ad)\right)}{384d^2}-\frac{bx^3\left(c+dx^2\right)^{7/2}(5bc-24ad)}{120d^2}+\frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}$$

```
[Out] (c^3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024
*d^3) + (c^2*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*Sqrt[c + d*x
^2])/(512*d^2) + (c*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c +
d*x^2)^(3/2))/(384*d^2) + ((40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^
3*(c + d*x^2)^(5/2))/(320*d^2) - (b*(5*b*c - 24*a*d)*x^3*(c + d*x
^2)^(7/2))/(120*d^2) + (b^2*x^5*(c + d*x^2)^(7/2))/(12*d) - (c^4*
(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(1024*d^(7/2))
```

Rubi [A] time = 0.652653, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{c^4\left(40a^2d^2+bc(5bc-24ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{7/2}}+\frac{c^3x\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{1024d^3}+\frac{c^2x^3\sqrt{c+dx^2}\left(40a^2d^2+bc(5bc-24ad)\right)}{512d^2}+\frac{1}{320}x^3\left(c+dx^2\right)^{5/2}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)+\frac{cx^3\left(c+dx^2\right)^{3/2}\left(40a^2d^2+bc(5bc-24ad)\right)}{384d^2}-\frac{bx^3\left(c+dx^2\right)^{7/2}(5bc-24ad)}{120d^2}+\frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024
*d^3) + (c^2*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*Sqrt[c + d*x
^2])/(512*d^2) + (c*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c +
d*x^2)^(3/2))/(384*d^2) + ((40*a^2 + (b*c*(5*b*c - 24*a*d))/d^2)*
x^3*(c + d*x^2)^(5/2))/320 - (b*(5*b*c - 24*a*d)*x^3*(c + d*x^2)^
(7/2))/(120*d^2) + (b^2*x^5*(c + d*x^2)^(7/2))/(12*d) - (c^4*(40*
a^2*d^2 + b*c*(5*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^
2]])/(1024*d^(7/2))
```

Rubi in Sympy [A] time = 53.6021, size = 270, normalized size = 0.96

$$\frac{b^2x^5\left(c+dx^2\right)^{\frac{7}{2}}}{12d}+\frac{bx^3\left(c+dx^2\right)^{\frac{7}{2}}(24ad-5bc)}{120d^2}-\frac{c^4\left(40a^2d^2-bc\,(24ad-5bc)\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{\frac{7}{2}}}+\frac{c^3x\sqrt{c+dx^2}\left(40a^2d^2-bc\,(24ad-5bc)\right)}{1024d^3}+\frac{c^2x^3\sqrt{c+dx^2}\left(40a^2d^2-bc\,(24ad-5bc)\right)}{512d^2}+\frac{cx^3\left(c+dx^2\right)^{\frac{3}{2}}\left(40a^2d^2-bc\,(24ad-5bc)\right)}{384d^2}+\frac{x^3\left(c+dx^2\right)^{\frac{5}{2}}\left(40a^2d^2-bc\,(24ad-5bc)\right)}{320d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*x**5*(c + d*x**2)**(7/2)/(12*d) + b*x**3*(c + d*x**2)**(7/2)
*(24*a*d - 5*b*c)/(120*d**2) - c**4*(40*a**2*d**2 - b*c*(24*a*d -
```

```
5*b*c))*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(1024*d**(7/2)) + c**3
*x*sqrt(c + d*x**2)*(40*a**2*d**2 - b*c*(24*a*d - 5*b*c))/(1024*d
**3) + c**2*x**3*sqrt(c + d*x**2)*(40*a**2*d**2 - b*c*(24*a*d - 5
*b*c))/(512*d**2) + c*x**3*(c + d*x**2)**(3/2)*(40*a**2*d**2 - b*
c*(24*a*d - 5*b*c))/(384*d**2) + x**3*(c + d*x**2)**(5/2)*(40*a**
2*d**2 - b*c*(24*a*d - 5*b*c))/(320*d**2)
```

Mathematica [A] time = 0.24853, size = 226, normalized size = 0.8

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(40a^2d^2\left(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6\right)+24abd\left(-15c^4+10c^3dx^2+248c^2d^2x^4+336cd^3x^6+128d^4x\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(40*a^2*d^2*(15*c^3 + 118*c^2*d*x^2 +
136*c*d^2*x^4 + 48*d^3*x^6) + 24*a*b*d*(-15*c^4 + 10*c^3*d*x^2 +
248*c^2*d^2*x^4 + 336*c*d^3*x^6 + 128*d^4*x^8) + 5*b^2*(15*c^5 -
10*c^4*d*x^2 + 8*c^3*d^2*x^4 + 432*c^2*d^3*x^6 + 640*c*d^4*x^8 +
256*d^5*x^10)) - 15*c^4*(5*b^2*c^2 - 24*a*b*c*d + 40*a^2*d^2)*Log
[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(15360*d^(7/2))
```

Maple [A] time = 0.023, size = 383, normalized size = 1.4

$$\frac{a^2x}{8d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{a^2cx}{48d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{5a^2c^2x}{192d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5a^2c^3x}{128d}\sqrt{dx^2+c}$$
$$-\frac{5a^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+\frac{b^2x^5}{12d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{b^2cx^3}{24d^2}\left(dx^2+c\right)^{\frac{7}{2}}$$
$$+\frac{b^2c^2x}{64d^3}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{xb^2c^3}{384d^3}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{5b^2c^4x}{1536d^3}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5b^2c^5x}{1024d^3}\sqrt{dx^2+c}$$
$$-\frac{5b^2c^6}{1024}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}+\frac{abx^3}{5d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{3abcx}{40d^2}\left(dx^2+c\right)^{\frac{7}{2}}$$
$$+\frac{abc^2x}{80d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{abc^3x}{64d^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{3abc^4x}{128d^2}\sqrt{dx^2+c}+\frac{3abc^5}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/8*a^2*x*(d*x^2+c)^(7/2)/d-1/48*a^2*c/d*x*(d*x^2+c)^(5/2)-5/192*
a^2*c^2/d*x*(d*x^2+c)^(3/2)-5/128*a^2*c^3/d*x*(d*x^2+c)^(1/2)-5/1
28*a^2*c^4/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/12*b^2*x^5*(d*
x^2+c)^(7/2)/d-1/24*b^2*c/d^2*x^3*(d*x^2+c)^(7/2)+1/64*b^2*c^2/d^
3*x*(d*x^2+c)^(7/2)-1/384*b^2*c^3/d^3*x*(d*x^2+c)^(5/2)-5/1536*b^
2*c^4/d^3*x*(d*x^2+c)^(3/2)-5/1024*b^2*c^5/d^3*x*(d*x^2+c)^(1/2)-
5/1024*b^2*c^6/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/5*a*b*x^3*
(d*x^2+c)^(7/2)/d-3/40*a*b*c/d^2*x*(d*x^2+c)^(7/2)+1/80*a*b*c^2/d
^2*x*(d*x^2+c)^(5/2)+1/64*a*b*c^3/d^2*x*(d*x^2+c)^(3/2)+3/128*a*b
*c^4/d^2*x*(d*x^2+c)^(1/2)+3/128*a*b*c^5/d^(5/2)*ln(x*d^(1/2)+(d*
x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.827388, size = 1, normalized size = 0.

$$\left[\frac{2\left(1280\,b^2d^5x^{11}+128\left(25\,b^2cd^4+24\,abd^5\right)x^9+48\left(45\,b^2c^2d^3+168\,abcd^4+40\,a^2d^5\right)x^7+8\left(5\,b^2c^3d^2+744\,abc^2d^3+680\,a^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/30720*(2*(1280*b^2*d^5*x^11 + 128*(25*b^2*c*d^4 + 24*a*b*d^5)*
x^9 + 48*(45*b^2*c^2*d^3 + 168*a*b*c*d^4 + 40*a^2*d^5)*x^7 + 8*(5
*b^2*c^3*d^2 + 744*a*b*c^2*d^3 + 680*a^2*c*d^4)*x^5 - 10*(5*b^2*c
^4*d - 24*a*b*c^3*d^2 - 472*a^2*c^2*d^3)*x^3 + 15*(5*b^2*c^5 - 24
*a*b*c^4*d + 40*a^2*c^3*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 15*(5*b
^2*c^6 - 24*a*b*c^5*d + 40*a^2*c^4*d^2)*log(2*sqrt(d*x^2 + c)*d*x
 - (2*d*x^2 + c)*sqrt(d)))/d^(7/2), 1/15360*((1280*b^2*d^5*x^11 +
 128*(25*b^2*c*d^4 + 24*a*b*d^5)*x^9 + 48*(45*b^2*c^2*d^3 + 168*a
*b*c*d^4 + 40*a^2*d^5)*x^7 + 8*(5*b^2*c^3*d^2 + 744*a*b*c^2*d^3 +
 680*a^2*c*d^4)*x^5 - 10*(5*b^2*c^4*d - 24*a*b*c^3*d^2 - 472*a^2*
c^2*d^3)*x^3 + 15*(5*b^2*c^5 - 24*a*b*c^4*d + 40*a^2*c^3*d^2)*x)*
sqrt(d*x^2 + c)*sqrt(-d) - 15*(5*b^2*c^6 - 24*a*b*c^5*d + 40*a^2*
c^4*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.241885, size = 358, normalized size = 1.27

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2d^2x^2+\frac{25\,b^2cd^{11}+24\,abd^{12}}{d^{10}}\right)x^2+\frac{3\left(45\,b^2c^2d^{10}+168\,abcd^{11}+40\,a^2d^{12}\right)}{d^{10}}\right)x^2+\frac{5\,b^2c^3d^9+744\,abc^2\cdots}{d^{10}}\cdots\right.\right.\right.$$
$$+\frac{\left(5\,b^2c^6-24\,abc^5d+40\,a^2c^4d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{1024\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^2*x^2 + (25*b^2*c*d^11 + 24*a*b*d^1
2)/d^10)*x^2 + 3*(45*b^2*c^2*d^10 + 168*a*b*c*d^11 + 40*a^2*d^12)
/d^10)*x^2 + (5*b^2*c^3*d^9 + 744*a*b*c^2*d^10 + 680*a^2*c*d^11)/
d^10)*x^2 - 5*(5*b^2*c^4*d^8 - 24*a*b*c^3*d^9 - 472*a^2*c^2*d^10)
/d^10)*x^2 + 15*(5*b^2*c^5*d^7 - 24*a*b*c^4*d^8 + 40*a^2*c^3*d^9)
/d^10)*sqrt(d*x^2 + c)*x + 1/1024*(5*b^2*c^6 - 24*a*b*c^5*d + 40*
a^2*c^4*d^2)*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.627 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (2*b*(b*c - a*d)*(c +
 d*x^2)^(9/2))/(9*d^3) + (b^2*(c + d*x^2)^(11/2))/(11*d^3)
```

Rubi [A] time = 0.160923, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (2*b*(b*c - a*d)*(c +
 d*x^2)^(9/2))/(9*d^3) + (b^2*(c + d*x^2)^(11/2))/(11*d^3)
```

Rubi in Sympy [A] time = 23.5274, size = 66, normalized size = 0.86

$$\frac{b^2\left(c+dx^2\right)^{\frac{11}{2}}}{11d^3}+\frac{2b\left(c+dx^2\right)^{\frac{9}{2}}(ad-bc)}{9d^3}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}(ad-bc)^2}{7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*(c + d*x**2)**(11/2)/(11*d**3) + 2*b*(c + d*x**2)**(9/2)*(a*
d - b*c)/(9*d**3) + (c + d*x**2)**(7/2)*(a*d - b*c)**2/(7*d**3)
```

Mathematica [A] time = 0.104263, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(99a^2d^2+22abd\left(7dx^2-2c\right)+b^2\left(8c^2-28cdx^2+63d^2x^4\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x^2) + b^2*
(8*c^2 - 28*c*d*x^2 + 63*d^2*x^4)))/(693*d^3)
```

Maple [A] time = 0.009, size = 69, normalized size = 0.9

$$\frac{63\,b^2d^2x^4+154\,abd^2x^2-28\,b^2cdx^2+99\,a^2d^2-44\,cabd+8\,b^2c^2}{693\,d^3}\left(dx^2+c\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/693*(d*x^2+c)^(7/2)*(63*b^2*d^2*x^4+154*a*b*d^2*x^2-28*b^2*c*d*
x^2+99*a^2*d^2-44*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232944, size = 240, normalized size = 3.12

$$\frac{\left(63\, b^2 d^5 x^{10} + 7\left(23\, b^2 c d^4 + 22\, a b d^5\right) x^8 + 8\, b^2 c^5 - 44\, a b c^4 d + 99\, a^2 c^3 d^2 + \left(113\, b^2 c^2 d^3 + 418\, a b c d^4 + 99\, a^2 d^5\right) x^6 + 3\left(b^2 c^3 d^2 \right.}{693\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^2*d^5*x^10 + 7*(23*b^2*c*d^4 + 22*a*b*d^5)*x^8 + 8*b^
2*c^5 - 44*a*b*c^4*d + 99*a^2*c^3*d^2 + (113*b^2*c^2*d^3 + 418*a*
b*c*d^4 + 99*a^2*d^5)*x^6 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99
*a^2*c*d^4)*x^4 - (4*b^2*c^4*d - 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3
)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 19.9027, size = 384, normalized size = 4.99

$$\begin{cases} \frac{a^2c^3\sqrt{c+dx^2}}{7d} + \frac{3a^2c^2x^2\sqrt{c+dx^2}}{7} + \frac{3a^2cdx^4\sqrt{c+dx^2}}{7} + \frac{a^2d^2x^6\sqrt{c+dx^2}}{7} - \frac{4abc^4\sqrt{c+dx^2}}{63d^2} + \frac{2abc^3x^2\sqrt{c+dx^2}}{63d} + \frac{10abc^2x^4\sqrt{c+dx^2}}{21} + \frac{38abcdx^6\sqrt{c+dx}}{63} \\ c^{\frac{5}{2}}\left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((a**2*c**3*sqrt(c + d*x**2)/(7*d) + 3*a**2*c**2*x**2*sq
rt(c + d*x**2)/7 + 3*a**2*c*d*x**4*sqrt(c + d*x**2)/7 + a**2*d**2
*x**6*sqrt(c + d*x**2)/7 - 4*a*b*c**4*sqrt(c + d*x**2)/(63*d**2)
+ 2*a*b*c**3*x**2*sqrt(c + d*x**2)/(63*d) + 10*a*b*c**2*x**4*sqrt
(c + d*x**2)/21 + 38*a*b*c*d*x**6*sqrt(c + d*x**2)/63 + 2*a*b*d**
2*x**8*sqrt(c + d*x**2)/9 + 8*b**2*c**5*sqrt(c + d*x**2)/(693*d**
3) - 4*b**2*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + b**2*c**3*x**
4*sqrt(c + d*x**2)/(231*d) + 113*b**2*c**2*x**6*sqrt(c + d*x**2)/
693 + 23*b**2*c*d*x**8*sqrt(c + d*x**2)/99 + b**2*d**2*x**10*sqrt
(c + d*x**2)/11, Ne(d, 0)), (c**(5/2)*(a**2*x**2/2 + a*b*x**4/2 +
 b**2*x**6/6), True))
```

GIAC/XCAS [A] time = 0.237168, size = 564, normalized size = 7.32

$$1155\left(dx^2+c\right)^{\frac{3}{2}}a^2c^2 + 462\left(3\left(dx^2+c\right)^{\frac{5}{2}} - 5\left(dx^2+c\right)^{\frac{3}{2}}c\right)a^2c + \frac{462\left(3\left(dx^2+c\right)^{\frac{5}{2}} - 5\left(dx^2+c\right)^{\frac{3}{2}}c\right)abc^2}{d} + 33\left(15\left(dx^2+c\right)^{\frac{7}{2}} - 42\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x,x, algorithm="giac")
```

```
[Out] 1/3465*(1155*(d*x^2 + c)^(3/2)*a^2*c^2 + 462*(3*(d*x^2 + c)^(5/2)
 - 5*(d*x^2 + c)^(3/2)*c)*a^2*c + 462*(3*(d*x^2 + c)^(5/2) - 5*(d
*x^2 + c)^(3/2)*c)*a*b*c^2/d + 33*(15*(d*x^2 + c)^(7/2) - 42*(d*x
^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*a^2 + 33*(15*(d*x^2 +
 c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c + 35*(d*x^2 + c)^(3/2)*c^2)*b^
2*c^2/d^2 + 132*(15*(d*x^2 + c)^(7/2) - 42*(d*x^2 + c)^(5/2)*c +
35*(d*x^2 + c)^(3/2)*c^2)*a*b*c/d + 22*(35*(d*x^2 + c)^(9/2) - 13
5*(d*x^2 + c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 +
c)^(3/2)*c^3)*b^2*c/d^2 + 22*(35*(d*x^2 + c)^(9/2) - 135*(d*x^2 +
 c)^(7/2)*c + 189*(d*x^2 + c)^(5/2)*c^2 - 105*(d*x^2 + c)^(3/2)*c
^3)*a*b/d + (315*(d*x^2 + c)^(11/2) - 1540*(d*x^2 + c)^(9/2)*c +
2970*(d*x^2 + c)^(7/2)*c^2 - 2772*(d*x^2 + c)^(5/2)*c^3 + 1155*(d
*x^2 + c)^(3/2)*c^4)*b^2/d^2)/d
```

3.628 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} dx$

Optimal. Leaf size=240

$$\frac{x\left(c+dx^2\right)^{5/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{480d^2}+\frac{cx\left(c+dx^2\right)^{3/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{256d^2}+\frac{c^3\left(80a^2d^2-20abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{5/2}}-\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{80d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}{10d}$$

```
[Out] (c^2*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*Sqrt[c + d*x^2])/(25
6*d^2) + (c*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(
3/2))/(384*d^2) + ((3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d
*x^2)^(5/2))/(480*d^2) - (3*b*(b*c - 4*a*d)*x*(c + d*x^2)^(7/2))/
(80*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^(7/2))/(10*d) + (c^3*(3*b
^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*
x^2]])/(256*d^(5/2))
```

Rubi [A] time = 0.328643, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{x\left(c+dx^2\right)^{5/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{480d^2}+\frac{cx\left(c+dx^2\right)^{3/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{256d^2}+\frac{c^3\left(80a^2d^2-20abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{5/2}}-\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{80d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}{10d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^2*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*Sqrt[c + d*x^2])/(25
6*d^2) + (c*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(
3/2))/(384*d^2) + ((3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d
*x^2)^(5/2))/(480*d^2) - (3*b*(b*c - 4*a*d)*x*(c + d*x^2)^(7/2))/
(80*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^(7/2))/(10*d) + (c^3*(3*b
^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*
x^2]])/(256*d^(5/2))
```

Rubi in Sympy [A] time = 32.1846, size = 238, normalized size = 0.99

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{7}{2}}}{10d}+\frac{3bx\left(c+dx^2\right)^{\frac{7}{2}}(4ad-bc)}{80d^2}+\frac{c^3\left(80a^2d^2-20abcd+3b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{\frac{5}{2}}}+\frac{c^2x\sqrt{c+dx^2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{256d^2}+\frac{cx\left(c+dx^2\right)^{\frac{3}{2}}\left(80a^2d^2-20abcd+3b^2c^2\right)}{384d^2}+\frac{x\left(c+dx^2\right)^{\frac{5}{2}}\left(80a^2d^2-20abcd+3b^2c^2\right)}{480d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] b*x*(a + b*x**2)*(c + d*x**2)**(7/2)/(10*d) + 3*b*x*(c + d*x**2)*
*(7/2)*(4*a*d - b*c)/(80*d**2) + c**3*(80*a**2*d**2 - 20*a*b*c*d
+ 3*b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(256*d**(5/2)) +
```

```
c**2*x*sqrt(c + d*x**2)*(80*a**2*d**2 - 20*a*b*c*d + 3*b**2*c**2
)/(256*d**2) + c*x*(c + d*x**2)**(3/2)*(80*a**2*d**2 - 20*a*b*c*d
 + 3*b**2*c**2)/(384*d**2) + x*(c + d*x**2)**(5/2)*(80*a**2*d**2
- 20*a*b*c*d + 3*b**2*c**2)/(480*d**2)
```

Mathematica [A] time = 0.205084, size = 192, normalized size = 0.8

$$\frac{15c^3\left(80a^2d^2-20abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}x\sqrt{c+dx^2}\left(80a^2d^2\left(33c^2+26cdx^2+8d^2x^4\right)+20abd\left(15c^3+\right.\right.}{3840d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(33*c^2 + 26*c*d*x^2 + 8*d
^2*x^4) + 20*a*b*d*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*x^4 + 48*d
^3*x^6) + b^2*(-45*c^4 + 30*c^3*d*x^2 + 744*c^2*d^2*x^4 + 1008*c*
d^3*x^6 + 384*d^4*x^8)) + 15*c^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2
*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(3840*d^(5/2))
```

Maple [A] time = 0.012, size = 308, normalized size = 1.3

$$\frac{a^2x}{6}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,a^2cx}{24}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{5\,a^2c^2x}{16}\sqrt{dx^2+c}+\frac{5\,a^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$
$$+\frac{b^2x^3}{10\,d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{3\,b^2cx}{80\,d^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{b^2c^2x}{160\,d^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{xb^2c^3}{128\,d^2}\left(dx^2+c\right)^{\frac{3}{2}}$$
$$+\frac{3\,b^2c^4x}{256\,d^2}\sqrt{dx^2+c}+\frac{3\,b^2c^5}{256}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{abx}{4\,d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{abcx}{24\,d}\left(dx^2+c\right)^{\frac{5}{2}}$$
$$-\frac{5\,abc^2x}{96\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,abc^3x}{64\,d}\sqrt{dx^2+c}-\frac{5\,abc^4}{64}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/6*a^2*x*(d*x^2+c)^(5/2)+5/24*a^2*c*x*(d*x^2+c)^(3/2)+5/16*a^2*c
^2*x*(d*x^2+c)^(1/2)+5/16*a^2*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^
(1/2))+1/10*b^2*x^3*(d*x^2+c)^(7/2)/d-3/80*b^2*c/d^2*x*(d*x^2+c)^
(7/2)+1/160*b^2*c^2/d^2*x*(d*x^2+c)^(5/2)+1/128*b^2*c^3/d^2*x*(d*
x^2+c)^(3/2)+3/256*b^2*c^4/d^2*x*(d*x^2+c)^(1/2)+3/256*b^2*c^5/d^
(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*a*b*x*(d*x^2+c)^(7/2)/d-1
/24*a*b*c/d*x*(d*x^2+c)^(5/2)-5/96*a*b*c^2/d*x*(d*x^2+c)^(3/2)-5/
64*a*b*c^3/d*x*(d*x^2+c)^(1/2)-5/64*a*b*c^4/d^(3/2)*ln(x*d^(1/2)+
(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.531597, size = 1, normalized size = 0.

$$\left[\frac{2\left(384\,b^2d^4x^9+48\left(21\,b^2cd^3+20\,abd^4\right)x^7+8\left(93\,b^2c^2d^2+340\,abcd^3+80\,a^2d^4\right)x^5+10\left(3\,b^2c^3d+236\,abc^2d^2+208\,a^2cd^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(2*(384*b^2*d^4*x^9 + 48*(21*b^2*c*d^3 + 20*a*b*d^4)*x^7
+ 8*(93*b^2*c^2*d^2 + 340*a*b*c*d^3 + 80*a^2*d^4)*x^5 + 10*(3*b^2
*c^3*d + 236*a*b*c^2*d^2 + 208*a^2*c*d^3)*x^3 - 15*(3*b^2*c^4 - 2
0*a*b*c^3*d - 176*a^2*c^2*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 15*(3
*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*log(-2*sqrt(d*x^2 + c)*
d*x - (2*d*x^2 + c)*sqrt(d)))/d^(5/2), 1/3840*((384*b^2*d^4*x^9 +
 48*(21*b^2*c*d^3 + 20*a*b*d^4)*x^7 + 8*(93*b^2*c^2*d^2 + 340*a*b
*c*d^3 + 80*a^2*d^4)*x^5 + 10*(3*b^2*c^3*d + 236*a*b*c^2*d^2 + 20
8*a^2*c*d^3)*x^3 - 15*(3*b^2*c^4 - 20*a*b*c^3*d - 176*a^2*c^2*d^2
)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 15*(3*b^2*c^5 - 20*a*b*c^4*d + 80
*a^2*c^3*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^2)]
```

Sympy [A] time = 170.432, size = 537, normalized size = 2.24

$$\frac{a^2c^{\frac{5}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{3a^2c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{35a^2c^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{17a^2\sqrt{c}d^2x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{5a^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16\sqrt{d}}$$
$$+\frac{a^2d^3x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^{\frac{7}{2}}x}{64d\sqrt{1+\frac{dx^2}{c}}}+\frac{133abc^{\frac{5}{2}}x^3}{192\sqrt{1+\frac{dx^2}{c}}}+\frac{127abc^{\frac{3}{2}}dx^5}{96\sqrt{1+\frac{dx^2}{c}}}+\frac{23ab\sqrt{c}d^2x^7}{24\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{5abc^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{64d^{\frac{3}{2}}}+\frac{abd^3x^9}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^{\frac{9}{2}}x}{256d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{7}{2}}x^3}{256d\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{129b^2c^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{dx^2}{c}}}+\frac{73b^2c^{\frac{3}{2}}dx^7}{160\sqrt{1+\frac{dx^2}{c}}}+\frac{29b^2\sqrt{c}d^2x^9}{80\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^5\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{256d^{\frac{5}{2}}}+\frac{b^2d^3x^{11}}{10\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] a**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*a**2*c**(5/2)*x/(16*sqrt
(1 + d*x**2/c)) + 35*a**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c))
 + 17*a**2*sqrt(c)*d**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*a**2*c**
3*asinh(sqrt(d)*x/sqrt(c))/(16*sqrt(d)) + a**2*d**3*x**7/(6*sqrt(
c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(7/2)*x/(64*d*sqrt(1 + d*x**2/c
)) + 133*a*b*c**(5/2)*x**3/(192*sqrt(1 + d*x**2/c)) + 127*a*b*c**
(3/2)*d*x**5/(96*sqrt(1 + d*x**2/c)) + 23*a*b*sqrt(c)*d**2*x**7/(
24*sqrt(1 + d*x**2/c)) - 5*a*b*c**4*asinh(sqrt(d)*x/sqrt(c))/(64*
d**(3/2)) + a*b*d**3*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) - 3*b**2
*c**(9/2)*x/(256*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(7/2)*x**3/(2
56*d*sqrt(1 + d*x**2/c)) + 129*b**2*c**(5/2)*x**5/(640*sqrt(1 + d
*x**2/c)) + 73*b**2*c**(3/2)*d*x**7/(160*sqrt(1 + d*x**2/c)) + 29
*b**2*sqrt(c)*d**2*x**9/(80*sqrt(1 + d*x**2/c)) + 3*b**2*c**5*asi
nh(sqrt(d)*x/sqrt(c))/(256*d**(5/2)) + b**2*d**3*x**11/(10*sqrt(c
)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.246865, size = 298, normalized size = 1.24

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{21\,b^2cd^9+20\,abd^{10}}{d^8}\right)x^2+\frac{93\,b^2c^2d^8+340\,abcd^9+80\,a^2d^{10}}{d^8}\right)x^2+\frac{5\left(3\,b^2c^3d^7+236\,abc^2d^8+208\right.}{d^8}\right.\right.$$
$$-\frac{\left(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (21*b^2*c*d^9 + 20*a*b*d^10)/d^8
)*x^2 + (93*b^2*c^2*d^8 + 340*a*b*c*d^9 + 80*a^2*d^10)/d^8)*x^2 +
 5*(3*b^2*c^3*d^7 + 236*a*b*c^2*d^8 + 208*a^2*c*d^9)/d^8)*x^2 - 1
5*(3*b^2*c^4*d^6 - 20*a*b*c^3*d^7 - 176*a^2*c^2*d^8)/d^8)*sqrt(d*
x^2 + c)*x - 1/256*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*ln
(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.629 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2c^2\sqrt{c+dx^2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}-\frac{b\left(c+dx^2\right)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}$$

```
[Out] a^2*c^2*Sqrt[c + d*x^2] + (a^2*c*(c + d*x^2)^(3/2))/3 + (a^2*(c +
 d*x^2)^(5/2))/5 - (b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) +
(b^2*(c + d*x^2)^(9/2))/(9*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]]
```

Rubi [A] time = 0.28269, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2c^2\sqrt{c+dx^2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}-\frac{b\left(c+dx^2\right)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] a^2*c^2*Sqrt[c + d*x^2] + (a^2*c*(c + d*x^2)^(3/2))/3 + (a^2*(c +
 d*x^2)^(5/2))/5 - (b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) +
(b^2*(c + d*x^2)^(9/2))/(9*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]]
```

Rubi in Sympy [A] time = 31.318, size = 117, normalized size = 0.89

$$-a^2c^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2c^2\sqrt{c+dx^2}+\frac{a^2c\left(c+dx^2\right)^{\frac{3}{2}}}{3}+\frac{a^2\left(c+dx^2\right)^{\frac{5}{2}}}{5}+\frac{b^2\left(c+dx^2\right)^{\frac{9}{2}}}{9d^2}+\frac{b\left(c+dx^2\right)^{\frac{7}{2}}(2ad-bc)}{7d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x,x)
```

```
[Out] -a**2*c**(5/2)*atanh(sqrt(c + d*x**2)/sqrt(c)) + a**2*c**2*sqrt(c
 + d*x**2) + a**2*c*(c + d*x**2)**(3/2)/3 + a**2*(c + d*x**2)**(5
/2)/5 + b**2*(c + d*x**2)**(9/2)/(9*d**2) + b*(c + d*x**2)**(7/2)
*(2*a*d - b*c)/(7*d**2)
```

Mathematica [A] time = 0.259324, size = 128, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}\left(21a^2d^2\left(23c^2+11cdx^2+3d^2x^4\right)+90abd\left(c+dx^2\right)^3-5b^2\left(2c-7dx^2\right)\left(c+dx^2\right)^3\right)}{315d^2}-a^2c^{5/2}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+a^2c^{5/2}\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(90*a*b*d*(c + d*x^2)^3 - 5*b^2*(2*c - 7*d*x^2)*
(c + d*x^2)^3 + 21*a^2*d^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(3
15*d^2) + a^2*c^(5/2)*Log[x] - a^2*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c
 + d*x^2]]
```

Maple [A] time = 0.014, size = 132, normalized size = 1.

$$\frac{a^2}{5}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{a^2c}{3}\left(dx^2+c\right)^{\frac{3}{2}}-a^2c^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)+a^2c^2\sqrt{dx^2+c}+\frac{b^2x^2}{9d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{2b^2c}{63d^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{2ab}{7d}\left(dx^2+c\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x)
```

```
[Out] 1/5*a^2*(d*x^2+c)^(5/2)+1/3*a^2*c*(d*x^2+c)^(3/2)-a^2*c^(5/2)*ln(
(2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+a^2*c^2*(d*x^2+c)^(1/2)+1/9*b^
2*x^2*(d*x^2+c)^(7/2)/d-2/63*b^2*c/d^2*(d*x^2+c)^(7/2)+2/7*a*b*(d
*x^2+c)^(7/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.249708, size = 1, normalized size = 0.01

$$\left[\frac{315\,a^2c^{\frac{5}{2}}d^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(35\,b^2d^4x^8+5\left(19\,b^2cd^3+18\,abd^4\right)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\left(25\,\right.\right.}{630\,d^2}\right.$$

$$-\frac{315\,a^2\sqrt{-c}c^2d^2\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(35\,b^2d^4x^8+5\left(19\,b^2cd^3+18\,abd^4\right)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\left(25\,b^2c\right.\right.}{315\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^2*c^(5/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(
c) + 2*c)/x^2) + 2*(35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4
)*x^6 - 10*b^2*c^4 + 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c
^2*d^2 + 90*a*b*c*d^3 + 21*a^2*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*
c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/d^2, -1/315*(315*a
^2*sqrt(-c)*c^2*d^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (35*b^
2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 + 90*a
*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 2
1*a^2*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*
x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A] time = 48.6221, size = 190, normalized size = 1.44

$$-a^{2}c^{3}\left(\begin{cases} -\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for}\: -c > 0 \\ \frac{\operatorname{acoth}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for}\: -c < 0 \wedge c < c + dx^{2} \\ \frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for}\: c > c + dx^{2} \wedge -c < 0 \end{cases}\right) + a^{2}c^{2}\sqrt{c+dx^{2}} + \frac{a^{2}c\left(c+dx^{2}\right)^{\frac{3}{2}}}{3} + \frac{a^{2}\left(c+dx^{2}\right)^{\frac{5}{2}}}{5} + \frac{b^{2}\left(c+dx^{2}\right)^{\frac{9}{2}}}{9d^{2}} + \frac{\left(c+dx^{2}\right)^{\frac{7}{2}}\left(4abd-2b^{2}c\right)}{14d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x,x)
```

```
[Out] -a**2*c**3*Piecewise((-atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c),
-c > 0), (acoth(sqrt(c + d*x**2)/sqrt(c))/sqrt(c), (-c < 0) & (c
< c + d*x**2)), (atanh(sqrt(c + d*x**2)/sqrt(c))/sqrt(c), (-c < 0
) & (c > c + d*x**2))) + a**2*c**2*sqrt(c + d*x**2) + a**2*c*(c +
 d*x**2)**(3/2)/3 + a**2*(c + d*x**2)**(5/2)/5 + b**2*(c + d*x**2
)**(9/2)/(9*d**2) + (c + d*x**2)**(7/2)*(4*a*b*d - 2*b**2*c)/(14*
d**2)
```

GIAC/XCAS [A] time = 0.247865, size = 190, normalized size = 1.44

$$\frac{a^{2}c^{3}\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{35\left(dx^{2}+c\right)^{\frac{9}{2}}b^{2}d^{16} - 45\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}cd^{16} + 90\left(dx^{2}+c\right)^{\frac{7}{2}}abd^{17} + 63\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d^{18} + 105\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cd^{18} + 315\sqrt{dx^{2}}}{315\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/315*(35*(d*
x^2 + c)^(9/2)*b^2*d^16 - 45*(d*x^2 + c)^(7/2)*b^2*c*d^16 + 90*(d
*x^2 + c)^(7/2)*a*b*d^17 + 63*(d*x^2 + c)^(5/2)*a^2*d^18 + 105*(d
*x^2 + c)^(3/2)*a^2*c*d^18 + 315*sqrt(d*x^2 + c)*a^2*c^2*d^18)/d^
18
```

3.630 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=217

$$-\frac{a^2 (c+dx^2)^{7/2}}{cx}-\frac{5c^2 (b^2c^2-16ad(3ad+bc)) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{x (c+dx^2)^{5/2} (b^2c^2-16ad(3ad+bc))}{48cd}-\frac{5x (c+dx^2)^{3/2} (b^2c^2-16ad(3ad+bc))}{192d}-\frac{5cx\sqrt{c+dx^2} (b^2c^2-16ad(3ad+bc))}{128d}+\frac{b^2x (c+dx^2)^{7/2}}{8d}$$

```
[Out] (-5*c*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*Sqrt[c + d*x^2])/(128*d)
 - (5*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(3/2))/(192*
d) - ((b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(5/2))/(48*c
*d) - (a^2*(c + d*x^2)^(7/2))/(c*x) + (b^2*x*(c + d*x^2)^(7/2))/(
8*d) - (5*c^2*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(128*d^(3/2))
```

Rubi [A] time = 0.360751, antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{1}{48}x (c+dx^2)^{5/2} \left(\frac{48a^2d}{c}+16ab-\frac{b^2c}{d}\right)-\frac{a^2 (c+dx^2)^{7/2}}{cx}-\frac{5c^2 (b^2c^2-16ad(3ad+bc)) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{5x (c+dx^2)^{3/2} (b^2c^2-16ad(3ad+bc))}{192d}-\frac{5cx\sqrt{c+dx^2} (b^2c^2-16ad(3ad+bc))}{128d}+\frac{b^2x (c+dx^2)^{7/2}}{8d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] (-5*c*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*Sqrt[c + d*x^2])/(128*d)
 - (5*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(3/2))/(192*
d) + ((16*a*b - (b^2*c)/d + (48*a^2*d)/c)*x*(c + d*x^2)^(5/2))/48
 - (a^2*(c + d*x^2)^(7/2))/(c*x) + (b^2*x*(c + d*x^2)^(7/2))/(8*d
) - (5*c^2*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(128*d^(3/2))
```

Rubi in Sympy [A] time = 32.4377, size = 199, normalized size = 0.92

$$-\frac{a^2 (c+dx^2)^{\frac{7}{2}}}{cx}+\frac{b^2x (c+dx^2)^{\frac{7}{2}}}{8d}-\frac{5c^2 (-16ad (3ad+bc)+b^2c^2) \operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{\frac{3}{2}}}-\frac{5cx\sqrt{c+dx^2} (-16ad (3ad+bc)+b^2c^2)}{128d}-\frac{5x (c+dx^2)^{\frac{3}{2}} (-16ad (3ad+bc)+b^2c^2)}{192d}-\frac{x (c+dx^2)^{\frac{5}{2}} (-16ad (3ad+bc)+b^2c^2)}{48cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**2*(c + d*x**2)**(7/2)/(c*x) + b**2*x*(c + d*x**2)**(7/2)/(8*d
) - 5*c**2*(-16*a*d*(3*a*d + b*c) + b**2*c**2)*atanh(sqrt(d)*x/sq
rt(c + d*x**2))/(128*d**(3/2)) - 5*c*x*sqrt(c + d*x**2)*(-16*a*d*
(3*a*d + b*c) + b**2*c**2)/(128*d) - 5*x*(c + d*x**2)**(3/2)*(-16
```

```
*a*d*(3*a*d + b*c) + b**2*c**2)/(192*d) - x*(c + d*x**2)**(5/2)*(
-16*a*d*(3*a*d + b*c) + b**2*c**2)/(48*c*d)
```

Mathematica [A] time = 0.201584, size = 174, normalized size = 0.8

$$\sqrt{c+dx^2}\left(\frac{1}{192}x^3\left(48a^2d^2+208abcd+59b^2c^2\right)+\frac{cx\left(144a^2d^2+176abcd+5b^2c^2\right)}{128d}-\frac{a^2c^2}{x}+\frac{1}{48}bdx^5(16ad+17bc)+\frac{1}{8}b^2d^2x^7\right)-\frac{5c^2\left(-48a^2d^2-16abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{128d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-((a^2*c^2)/x) + (c*(5*b^2*c^2 + 176*a*b*c*d + 1
44*a^2*d^2)*x)/(128*d) + ((59*b^2*c^2 + 208*a*b*c*d + 48*a^2*d^2)
*x^3)/192 + (b*d*(17*b*c + 16*a*d)*x^5)/48 + (b^2*d^2*x^7)/8) - (
5*c^2*(b^2*c^2 - 16*a*b*c*d - 48*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[
c + d*x^2]])/(128*d^(3/2))
```

Maple [A] time = 0.017, size = 278, normalized size = 1.3

$$\frac{b^2x}{8\,d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{b^2cx}{48\,d}\left(dx^2+c\right)^{\frac{5}{2}}-\frac{5\,b^2c^2x}{192\,d}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,xb^2c^3}{128\,d}\sqrt{dx^2+c}-\frac{5\,b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}-\frac{a^2}{cx}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{a^2dx}{c}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,a^2dx}{4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{15\,a^2cdx}{8}\sqrt{dx^2+c}+\frac{15\,a^2c^2}{8}\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)+\frac{abx}{3}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,abcx}{12}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{5\,abc^2x}{8}\sqrt{dx^2+c}+\frac{5\,abc^3}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x)
```

```
[Out] 1/8*b^2*x*(d*x^2+c)^(7/2)/d-1/48*b^2*c/d*x*(d*x^2+c)^(5/2)-5/192*
b^2*c^2/d*x*(d*x^2+c)^(3/2)-5/128*b^2*c^3/d*x*(d*x^2+c)^(1/2)-5/1
28*b^2*c^4/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-a^2*(d*x^2+c)^(7
/2)/c/x+a^2*d/c*x*(d*x^2+c)^(5/2)+5/4*a^2*d*x*(d*x^2+c)^(3/2)+15/
8*a^2*d*c*x*(d*x^2+c)^(1/2)+15/8*a^2*d^(1/2)*c^2*ln(x*d^(1/2)+(d*
x^2+c)^(1/2))+1/3*a*b*x*(d*x^2+c)^(5/2)+5/12*a*b*c*x*(d*x^2+c)^(3
/2)+5/8*a*b*c^2*x*(d*x^2+c)^(1/2)+5/8*a*b*c^3/d^(1/2)*ln(x*d^(1/2
)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.362638, size = 1, normalized size = 0.

$$\left[-\frac{15\left(b^{2}c^{4}-16\,abc^{3}d-48\,a^{2}c^{2}d^{2}\right)x\log\left(-2\sqrt{dx^{2}+c}dx-\left(2\,dx^{2}+c\right)\sqrt{d}\right)-2\left(48\,b^{2}d^{3}x^{8}+8\left(17\,b^{2}cd^{2}+16\,abd^{3}\right)x^{6}-38\right.}{768\,d^{\frac{3}{2}}x}\right.$$

$$-\frac{15\left(b^{2}c^{4}-16\,abc^{3}d-48\,a^{2}c^{2}d^{2}\right)x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\left(48\,b^{2}d^{3}x^{8}+8\left(17\,b^{2}cd^{2}+16\,abd^{3}\right)x^{6}-384\,a^{2}c^{2}d+2\left(59\,b^{2}c^{2}d+2\right.\right.}{384\sqrt{-d}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*x*log(-2*sq
rt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)) - 2*(48*b^2*d^3*x^8 +
8*(17*b^2*c*d^2 + 16*a*b*d^3)*x^6 - 384*a^2*c^2*d + 2*(59*b^2*c^2
*d + 208*a*b*c*d^2 + 48*a^2*d^3)*x^4 + 3*(5*b^2*c^3 + 176*a*b*c^2
*d + 144*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(d))/(d^(3/2)*x), -1
/384*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*x*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - (48*b^2*d^3*x^8 + 8*(17*b^2*c*d^2 + 16*a
*b*d^3)*x^6 - 384*a^2*c^2*d + 2*(59*b^2*c^2*d + 208*a*b*c*d^2 + 4
8*a^2*d^3)*x^4 + 3*(5*b^2*c^3 + 176*a*b*c^2*d + 144*a^2*c*d^2)*x^
2)*sqrt(d*x^2 + c)*sqrt(-d))/(sqrt(-d)*d*x)]
```

Sympy [A] time = 117.643, size = 496, normalized size = 2.29

$$-\frac{a^{2}c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^{2}}{c}}}+a^{2}c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^{2}}{c}}-\frac{7a^{2}c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3a^{2}\sqrt{c}d^{2}x^{3}}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{15a^{2}c^{2}\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8}$$
$$+\frac{a^{2}d^{3}x^{5}}{4\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+abc^{\frac{5}{2}}x\sqrt{1+\frac{dx^{2}}{c}}+\frac{3abc^{\frac{5}{2}}x}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{35abc^{\frac{3}{2}}dx^{3}}{24\sqrt{1+\frac{dx^{2}}{c}}}+\frac{17ab\sqrt{c}d^{2}x^{5}}{12\sqrt{1+\frac{dx^{2}}{c}}}$$
$$+\frac{5abc^{3}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{abd^{3}x^{7}}{3\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5b^{2}c^{\frac{7}{2}}x}{128d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{133b^{2}c^{\frac{5}{2}}x^{3}}{384\sqrt{1+\frac{dx^{2}}{c}}}$$
$$+\frac{127b^{2}c^{\frac{3}{2}}dx^{5}}{192\sqrt{1+\frac{dx^{2}}{c}}}+\frac{23b^{2}\sqrt{c}d^{2}x^{7}}{48\sqrt{1+\frac{dx^{2}}{c}}}-\frac{5b^{2}c^{4}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{3}{2}}}+\frac{b^{2}d^{3}x^{9}}{8\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + a**2*c**(3/2)*d*x*sqrt(1
+ d*x**2/c) - 7*a**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*a**2
*sqrt(c)*d**2*x**3/(8*sqrt(1 + d*x**2/c)) + 15*a**2*c**2*sqrt(d)*
asinh(sqrt(d)*x/sqrt(c))/8 + a**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d
*x**2/c)) + a*b*c**(5/2)*x*sqrt(1 + d*x**2/c) + 3*a*b*c**(5/2)*x/
(8*sqrt(1 + d*x**2/c)) + 35*a*b*c**(3/2)*d*x**3/(24*sqrt(1 + d*x*
*2/c)) + 17*a*b*sqrt(c)*d**2*x**5/(12*sqrt(1 + d*x**2/c)) + 5*a*b
*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*sqrt(d)) + a*b*d**3*x**7/(3*sqr
t(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d*sqrt(1 + d*x*
*2/c)) + 133*b**2*c**(5/2)*x**3/(384*sqrt(1 + d*x**2/c)) + 127*b*
*2*c**(3/2)*d*x**5/(192*sqrt(1 + d*x**2/c)) + 23*b**2*sqrt(c)*d**
2*x**7/(48*sqrt(1 + d*x**2/c)) - 5*b**2*c**4*asinh(sqrt(d)*x/sqrt
(c))/(128*d**(3/2)) + b**2*d**3*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c
))
```

GIAC/XCAS [A] time = 0.248285, size = 296, normalized size = 1.36

$$\frac{2\,a^2c^3\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{384}\left(2\left(4\left(6\,b^2d^2x^2+\frac{17\,b^2cd^7+16\,abd^8}{d^6}\right)x^2+\frac{59\,b^2c^2d^6+208\,abcd^7+48\,a^2d^8}{d^6}\right)x^2+\frac{3\left(5\,b^2c^3d^5+176\,abc^2d^6+144\,a^2cd\right.}{d^6}\right.$$

$$+\frac{5\left(b^2c^4\sqrt{d}-16\,abc^3d^{\frac{3}{2}}-48\,a^2c^2d^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{256\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^3*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/384*(
2*(4*(6*b^2*d^2*x^2 + (17*b^2*c*d^7 + 16*a*b*d^8)/d^6)*x^2 + (59*
b^2*c^2*d^6 + 208*a*b*c*d^7 + 48*a^2*d^8)/d^6)*x^2 + 3*(5*b^2*c^3
*d^5 + 176*a*b*c^2*d^6 + 144*a^2*c*d^7)/d^6)*sqrt(d*x^2 + c)*x +
5/256*(b^2*c^4*sqrt(d) - 16*a*b*c^3*d^(3/2) - 48*a^2*c^2*d^(5/2))
*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

3.631 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=162

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2}-\frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{a\left(c+dx^2\right)^{5/2}(5ad+4bc)}{10c}+\frac{1}{6}a\left(c+dx^2\right)^{3/2}(5ad+4bc)+\frac{1}{2}ac\sqrt{c+dx^2}(5ad+4bc)+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d}$$

```
[Out] (a*c*(4*b*c + 5*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 5*a*d)*(c +
 d*x^2)^(3/2))/6 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(5/2))/(10*c) +
 (b^2*(c + d*x^2)^(7/2))/(7*d) - (a^2*(c + d*x^2)^(7/2))/(2*c*x^2
) - (a*c^(3/2)*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
2
```

Rubi [A] time = 0.363152, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2}-\frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{a\left(c+dx^2\right)^{5/2}(5ad+4bc)}{10c}+\frac{1}{6}a\left(c+dx^2\right)^{3/2}(5ad+4bc)+\frac{1}{2}ac\sqrt{c+dx^2}(5ad+4bc)+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] (a*c*(4*b*c + 5*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 5*a*d)*(c +
 d*x^2)^(3/2))/6 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(5/2))/(10*c) +
 (b^2*(c + d*x^2)^(7/2))/(7*d) - (a^2*(c + d*x^2)^(7/2))/(2*c*x^2
) - (a*c^(3/2)*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
2
```

Rubi in Sympy [A] time = 31.966, size = 146, normalized size = 0.9

$$-\frac{a^2\left(c+dx^2\right)^{\frac{7}{2}}}{2cx^2}-\frac{ac^{\frac{3}{2}}(5ad+4bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2}+\frac{ac\sqrt{c+dx^2}(5ad+4bc)}{2}+\frac{a\left(c+dx^2\right)^{\frac{3}{2}}(5ad+4bc)}{6}+\frac{a\left(c+dx^2\right)^{\frac{5}{2}}(5ad+4bc)}{10c}+\frac{b^2\left(c+dx^2\right)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -a**2*(c + d*x**2)**(7/2)/(2*c*x**2) - a*c**(3/2)*(5*a*d + 4*b*c)
*atanh(sqrt(c + d*x**2)/sqrt(c))/2 + a*c*sqrt(c + d*x**2)*(5*a*d
+ 4*b*c)/2 + a*(c + d*x**2)**(3/2)*(5*a*d + 4*b*c)/6 + a*(c + d*x
**2)**(5/2)*(5*a*d + 4*b*c)/(10*c) + b**2*(c + d*x**2)**(7/2)/(7*
d)
```

Mathematica [A] time = 0.375689, size = 156, normalized size = 0.96

$$\frac{\sqrt{c+dx^2}\left(35a^2d\left(-3c^2+14cdx^2+2d^2x^4\right)+28abdx^2\left(23c^2+11cdx^2+3d^2x^4\right)+30b^2x^2\left(c+dx^2\right)^3\right)}{210dx^2}-\frac{1}{2}ac^{3/2}(5ad+4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+\frac{1}{2}ac^{3/2}\log(x)(5ad+4bc)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(30*b^2*x^2*(c + d*x^2)^3 + 35*a^2*d*(-3*c^2 + 1
4*c*d*x^2 + 2*d^2*x^4) + 28*a*b*d*x^2*(23*c^2 + 11*c*d*x^2 + 3*d^
2*x^4)))/(210*d*x^2) + (a*c^(3/2)*(4*b*c + 5*a*d)*Log[x])/2 - (a*
c^(3/2)*(4*b*c + 5*a*d)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/2
```

Maple [A] time = 0.018, size = 193, normalized size = 1.2

$$\frac{b^2}{7d}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{a^2}{2cx^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{a^2d}{2c}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5a^2d}{6}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5a^2d}{2}c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)+\frac{5a^2cd}{2}\sqrt{dx^2+c}+\frac{2ab}{5}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{2abc}{3}\left(dx^2+c\right)^{\frac{3}{2}}-2abc^{5/2}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)+2ab\sqrt{dx^2+c}c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x)
```

```
[Out] 1/7*b^2*(d*x^2+c)^(7/2)/d-1/2*a^2*(d*x^2+c)^(7/2)/c/x^2+1/2*a^2*d
/c*(d*x^2+c)^(5/2)+5/6*a^2*d*(d*x^2+c)^(3/2)-5/2*a^2*d*c^(3/2)*ln
((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+5/2*a^2*d*c*(d*x^2+c)^(1/2)+2
/5*a*b*(d*x^2+c)^(5/2)+2/3*a*b*c*(d*x^2+c)^(3/2)-2*a*b*c^(5/2)*ln
((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+2*a*b*(d*x^2+c)^(1/2)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258755, size = 1, normalized size = 0.01

$$\left[\frac{105\left(4abc^2d+5a^2cd^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(30b^2d^3x^8+6\left(15b^2cd^2+14abd^3\right)x^6-105a^2c^2d+2\left(45b^2c^2\right.\right.}{420dx^2}\right.$$

$$-\frac{105\left(4abc^2d+5a^2cd^2\right)\sqrt{-c}x^2\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(30b^2d^3x^8+6\left(15b^2cd^2+14abd^3\right)x^6-105a^2c^2d+2\left(45b^2c^2d+1\right.\right.}{210dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/420*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(c)*x^2*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(30*b^2*d^3*x^8 + 6*(1
5*b^2*c*d^2 + 14*a*b*d^3)*x^6 - 105*a^2*c^2*d + 2*(45*b^2*c^2*d +
 154*a*b*c*d^2 + 35*a^2*d^3)*x^4 + 2*(15*b^2*c^3 + 322*a*b*c^2*d
+ 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2), -1/210*(105*(4*a*
```

```
b*c^2*d + 5*a^2*c*d^2)*sqrt(-c)*x^2*arctan(c/(sqrt(d*x^2 + c)*sqr
t(-c))) - (30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b*d^3)*x^6 - 1
05*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x^4
+ 2*(15*b^2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2
+ c))/(d*x^2)]
```

Sympy [A] time = 71.7866, size = 518, normalized size = 3.2

$$- \frac{5 a^{2} c^{\frac{3}{2}} d \operatorname{asinh}{\left(\frac{\sqrt{c}}{\sqrt{d} x} \right)}}{2} - \frac{a^{2} c^{2} \sqrt{d} \sqrt{\frac{c}{d x^{2}} + 1}}{2 x} + \frac{2 a^{2} c^{2} \sqrt{d}}{x \sqrt{\frac{c}{d x^{2}} + 1}} + \frac{2 a^{2} c d^{\frac{3}{2}} x}{\sqrt{\frac{c}{d x^{2}} + 1}} + a^{2} d^{2} \left(\begin{cases} \frac{\sqrt{c} x^{2}}{2} & \text{for}\: d = 0 \\ \frac{\left(c + d x^{2}\right)^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right) - 2 a b c^{\frac{5}{2}} \operatorname{asinh}{\left(\frac{\sqrt{c}}{\sqrt{d} x} \right)} + \frac{2 a b c^{3}}{\sqrt{d} x \sqrt{\frac{c}{d x^{2}} + 1}} + \frac{2 a b c^{2} \sqrt{d} x}{\sqrt{\frac{c}{d x^{2}} + 1}} + 4 a b c d \left(\begin{cases} \frac{\sqrt{c} x^{2}}{2} & \text{for}\: d = 0 \\ \frac{\left(c + d x^{2}\right)^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right) + 2 a b d^{2} \left(\begin{cases} - \frac{2 c^{2} \sqrt{c + d x^{2}}}{15 d^{2}} + \frac{c x^{2} \sqrt{c + d x^{2}}}{15 d} + \frac{x^{4} \sqrt{c + d x^{2}}}{5} & \text{for}\: d \neq 0 \\ \frac{\sqrt{c} x^{4}}{4} & \text{otherwise} \end{cases}\right) + b^{2} c^{2} \left(\begin{cases} \frac{\sqrt{c} x^{2}}{2} & \text{for}\: d = 0 \\ \frac{\left(c + d x^{2}\right)^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right) + 2 b^{2} c d \left(\begin{cases} - \frac{2 c^{2} \sqrt{c + d x^{2}}}{15 d^{2}} + \frac{c x^{2} \sqrt{c + d x^{2}}}{15 d} + \frac{x^{4} \sqrt{c + d x^{2}}}{5} & \text{for}\: d \neq 0 \\ \frac{\sqrt{c} x^{4}}{4} & \text{otherwise} \end{cases}\right) + b^{2} d^{2} \left(\begin{cases} \frac{8 c^{3} \sqrt{c + d x^{2}}}{105 d^{3}} - \frac{4 c^{2} x^{2} \sqrt{c + d x^{2}}}{105 d^{2}} + \frac{c x^{4} \sqrt{c + d x^{2}}}{35 d} + \frac{x^{6} \sqrt{c + d x^{2}}}{7} & \text{for}\: d \neq 0 \\ \frac{\sqrt{c} x^{6}}{6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -5*a**2*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c**2*sqrt(
d)*sqrt(c/(d*x**2) + 1)/(2*x) + 2*a**2*c**2*sqrt(d)/(x*sqrt(c/(d*
x**2) + 1)) + 2*a**2*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1) + a**2*d**
2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d
), True)) - 2*a*b*c**(5/2)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c**
3/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c**2*sqrt(d)*x/sqrt(c/
(d*x**2) + 1) + 4*a*b*c*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), (
(c + d*x**2)**(3/2)/(3*d), True)) + 2*a*b*d**2*Piecewise((-2*c**2
*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x*
*4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*
c**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(
3*d), True)) + 2*b**2*c*d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15
*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5
, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*d**2*Piecewise((8*c**
3*sqrt(c + d*x**2)/(105*d**3) - 4*c**2*x**2*sqrt(c + d*x**2)/(105
*d**2) + c*x**4*sqrt(c + d*x**2)/(35*d) + x**6*sqrt(c + d*x**2)/7
, Ne(d, 0)), (sqrt(c)*x**6/6, True))
```

GIAC/XCAS [A] time = 0.245608, size = 223, normalized size = 1.38

$$\frac{30 \left(dx^{2}+c\right)^{\frac{7}{2}} b^{2} + 84 \left(dx^{2}+c\right)^{\frac{5}{2}} abd + 140 \left(dx^{2}+c\right)^{\frac{3}{2}} abcd + 420 \sqrt{dx^{2}+c}\, abc^{2} d + 70 \left(dx^{2}+c\right)^{\frac{3}{2}} a^{2} d^{2} + 420 \sqrt{dx^{2}+c}\, a^{2} c d^{2} - \cdots}{210\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/210*(30*(d*x^2 + c)^(7/2)*b^2 + 84*(d*x^2 + c)^(5/2)*a*b*d + 14
0*(d*x^2 + c)^(3/2)*a*b*c*d + 420*sqrt(d*x^2 + c)*a*b*c^2*d + 70*
(d*x^2 + c)^(3/2)*a^2*d^2 + 420*sqrt(d*x^2 + c)*a^2*c*d^2 - 105*s
qrt(d*x^2 + c)*a^2*c^2*d/x^2 + 105*(4*a*b*c^3*d + 5*a^2*c^2*d^2)*
arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c))/d
```

3.632 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=223

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3}+\frac{x\left(c+dx^2\right)^{5/2}\left(4ad(2ad+3bc)+b^2c^2\right)}{6c^2}+\frac{5x\left(c+dx^2\right)^{3/2}\left(4ad(2ad+3bc)+b^2c^2\right)}{24c}+\frac{5}{16}x\sqrt{c+dx^2}\left(4ad(2ad+3bc)+b^2c^2\right)+\frac{5c\left(4ad(2ad+3bc)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16\sqrt{d}}-\frac{2a\left(c+dx^2\right)^{7/2}(2ad+3bc)}{3c^2x}$$

```
[Out] (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*Sqrt[c + d*x^2])/16 + (5*(
b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c) + ((
b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(5/2))/(6*c^2) - (
a^2*(c + d*x^2)^(7/2))/(3*c*x^3) - (2*a*(3*b*c + 2*a*d)*(c + d*x^
2)^(7/2))/(3*c^2*x) + (5*c*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*Sqrt[d])
```

Rubi [A] time = 0.443085, antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3}+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\left(\frac{4ad(2ad+3bc)}{c^2}+b^2\right)+\frac{5x\left(c+dx^2\right)^{3/2}\left(4ad(2ad+3bc)+b^2c^2\right)}{24c}+\frac{5}{16}x\sqrt{c+dx^2}\left(4ad(2ad+3bc)+b^2c^2\right)+\frac{5c\left(4ad(2ad+3bc)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16\sqrt{d}}-\frac{2a\left(c+dx^2\right)^{7/2}(2ad+3bc)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*Sqrt[c + d*x^2])/16 + (5*(
b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c) + ((
b^2 + (4*a*d*(3*b*c + 2*a*d))/c^2)*x*(c + d*x^2)^(5/2))/6 - (a^2*
(c + d*x^2)^(7/2))/(3*c*x^3) - (2*a*(3*b*c + 2*a*d)*(c + d*x^2)^(
7/2))/(3*c^2*x) + (5*c*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*Sqrt[d])
```

Rubi in Sympy [A] time = 32.0122, size = 218, normalized size = 0.98

$$-\frac{a^2\left(c+dx^2\right)^{\frac{7}{2}}}{3cx^3}-\frac{2a\left(c+dx^2\right)^{\frac{7}{2}}(2ad+3bc)}{3c^2x}+\frac{5c\left(4ad\left(2ad+3bc\right)+b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16\sqrt{d}}+x\sqrt{c+dx^2}\left(\frac{5ad\left(2ad+3bc\right)}{4}+\frac{5b^2c^2}{16}\right)+\frac{5x\left(c+dx^2\right)^{\frac{3}{2}}\left(4ad\left(2ad+3bc\right)+b^2c^2\right)}{24c}+\frac{x\left(c+dx^2\right)^{\frac{5}{2}}\left(4ad\left(2ad+3bc\right)+b^2c^2\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -a**2*(c + d*x**2)**(7/2)/(3*c*x**3) - 2*a*(c + d*x**2)**(7/2)*(2
*a*d + 3*b*c)/(3*c**2*x) + 5*c*(4*a*d*(2*a*d + 3*b*c) + b**2*c**2
)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(16*sqrt(d)) + x*sqrt(c + d*x
**2)*(5*a*d*(2*a*d + 3*b*c)/4 + 5*b**2*c**2/16) + 5*x*(c + d*x**2
)**(3/2)*(4*a*d*(2*a*d + 3*b*c) + b**2*c**2)/(24*c) + x*(c + d*x*
*2)**(5/2)*(4*a*d*(2*a*d + 3*b*c) + b**2*c**2)/(6*c**2)
```

Mathematica [A] time = 0.205568, size = 155, normalized size = 0.7

$$\frac{1}{48}\left(\frac{15c\left(8a^2d^2+12abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+\frac{\sqrt{c+dx^2}\left(-8a^2\left(2c^2+14cdx^2-3d^2x^4\right)+12abx^2\left(-8c^2+9cdx^2+2d^2x^4\right)+b^2x^4\left(33c^2+26cdx^2+8d^2x^4\right)\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*(2*c^2 + 14*c*d*x^2 - 3*d^2*x^4) + 12*a
*b*x^2*(-8*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + b^2*x^4*(33*c^2 + 26*c*
d*x^2 + 8*d^2*x^4)))/x^3 + (15*c*(b^2*c^2 + 12*a*b*c*d + 8*a^2*d^
2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/48
```

Maple [A] time = 0.019, size = 298, normalized size = 1.3

$$\frac{xb^2}{6}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,b^2cx}{24}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{5\,b^2c^2x}{16}\sqrt{dx^2+c}+\frac{5\,b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$
$$-\frac{a^2}{3\,cx^3}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{4\,a^2d}{3\,c^2x}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{4\,a^2d^2x}{3\,c^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,a^2d^2x}{3\,c}\left(dx^2+c\right)^{\frac{3}{2}}$$
$$+\frac{5\,a^2d^2x}{2}\sqrt{dx^2+c}+\frac{5\,a^2c}{2}d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)-2\,\frac{ab\left(dx^2+c\right)^{7/2}}{cx}+2\,\frac{abdx\left(dx^2+c\right)^{5/2}}{c}$$
$$+\frac{5\,abdx}{2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{15\,cabdx}{4}\sqrt{dx^2+c}+\frac{15\,abc^2}{4}\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x)
```

```
[Out] 1/6*x*b^2*(d*x^2+c)^(5/2)+5/24*b^2*c*x*(d*x^2+c)^(3/2)+5/16*b^2*c
^2*x*(d*x^2+c)^(1/2)+5/16*b^2*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^
(1/2))-1/3*a^2*(d*x^2+c)^(7/2)/c/x^3-4/3*a^2*d/c^2/x*(d*x^2+c)^(7
/2)+4/3*a^2*d^2/c^2*x*(d*x^2+c)^(5/2)+5/3*a^2*d^2/c*x*(d*x^2+c)^(
3/2)+5/2*a^2*d^2*x*(d*x^2+c)^(1/2)+5/2*a^2*d^(3/2)*c*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))-2*a*b/c/x*(d*x^2+c)^(7/2)+2*a*b*d/c*x*(d*x^2+c)
^(5/2)+5/2*a*b*d*x*(d*x^2+c)^(3/2)+15/4*a*b*d*c*x*(d*x^2+c)^(1/2)
+15/4*a*b*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.354301, size = 1, normalized size = 0.

$$\left[\frac{15\left(b^2c^3+12\,abc^2d+8\,a^2cd^2\right)x^3\log\left(-2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+2\left(8\,b^2d^2x^8+2\left(13\,b^2cd+12\,abd^2\right)x^6+3\left(11\,b^2\right.\right.}{96\,\sqrt{d}x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*x^3*log(-2*sqrt(
d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)) + 2*(8*b^2*d^2*x^8 + 2*(1
3*b^2*c*d + 12*a*b*d^2)*x^6 + 3*(11*b^2*c^2 + 36*a*b*c*d + 8*a^2*
d^2)*x^4 - 16*a^2*c^2 - 16*(6*a*b*c^2 + 7*a^2*c*d)*x^2)*sqrt(d*x^
2 + c)*sqrt(d))/(sqrt(d)*x^3), 1/48*(15*(b^2*c^3 + 12*a*b*c^2*d +
 8*a^2*c*d^2)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^2
*x^8 + 2*(13*b^2*c*d + 12*a*b*d^2)*x^6 + 3*(11*b^2*c^2 + 36*a*b*c
*d + 8*a^2*d^2)*x^4 - 16*a^2*c^2 - 16*(6*a*b*c^2 + 7*a^2*c*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(-d))/(sqrt(-d)*x^3)]
```

Sympy [A] time = 74.2006, size = 490, normalized size = 2.2

$$-\frac{2a^2c^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}}+\frac{a^2\sqrt{c}d^2x\sqrt{1+\frac{dx^2}{c}}}{2}-\frac{2a^2\sqrt{c}d^2x}{\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+\frac{5a^2cd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2}-\frac{2abc^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+2abc^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7abc^{\frac{3}{2}}dx}{4\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}d^2x^3}{4\sqrt{1+\frac{dx^2}{c}}}+\frac{15abc^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{4}+\frac{abd^3x^5}{2\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^{\frac{5}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{3b^2c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{35b^2c^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{17b^2\sqrt{c}d^2x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16\sqrt{d}}+\frac{b^2d^3x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -2*a**2*c**(3/2)*d/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d**2*x*s
qrt(1 + d*x**2/c)/2 - 2*a**2*sqrt(c)*d**2*x/sqrt(1 + d*x**2/c) -
a**2*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*c*d**(3/2)
*sqrt(c/(d*x**2) + 1)/3 + 5*a**2*c*d**(3/2)*asinh(sqrt(d)*x/sqrt(
c))/2 - 2*a*b*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + 2*a*b*c**(3/2)*d*
x*sqrt(1 + d*x**2/c) - 7*a*b*c**(3/2)*d*x/(4*sqrt(1 + d*x**2/c))
+ 3*a*b*sqrt(c)*d**2*x**3/(4*sqrt(1 + d*x**2/c)) + 15*a*b*c**2*sq
rt(d)*asinh(sqrt(d)*x/sqrt(c))/4 + a*b*d**3*x**5/(2*sqrt(c)*sqrt(
1 + d*x**2/c)) + b**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*b**2*c*
*(5/2)*x/(16*sqrt(1 + d*x**2/c)) + 35*b**2*c**(3/2)*d*x**3/(48*sq
rt(1 + d*x**2/c)) + 17*b**2*sqrt(c)*d**2*x**5/(24*sqrt(1 + d*x**2
/c)) + 5*b**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*sqrt(d)) + b**2*d
**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.245588, size = 414, normalized size = 1.86

$$\frac{1}{48}\left(2\left(4\,b^2d^2x^2+\frac{13\,b^2cd^5+12\,abd^6}{d^4}\right)x^2+\frac{3\left(11\,b^2c^2d^4+36\,abcd^5+8\,a^2d^6\right)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{5\left(b^2c^3\sqrt{d}+12\,abc^2d^{\frac{3}{2}}+8\,a^2cd^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{32\,d}+\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc^3\sqrt{d}+9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2c^2d^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^4\sqrt{d}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\cdots\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*d^2*x^2 + (13*b^2*c*d^5 + 12*a*b*d^6)/d^4)*x^2 + 3
*(11*b^2*c^2*d^4 + 36*a*b*c*d^5 + 8*a^2*d^6)/d^4)*sqrt(d*x^2 + c)
*x - 5/32*(b^2*c^3*sqrt(d) + 12*a*b*c^2*d^(3/2) + 8*a^2*c*d^(5/2)
)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d + 2/3*(6*(sqrt(d)*x - sqr
t(d*x^2 + c))^4*a*b*c^3*sqrt(d) + 9*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*a^2*c^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*s
qrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(3/2) + 6*a
*b*c^5*sqrt(d) + 7*a^2*c^4*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c)
)^2 - c)^3
```

3.633 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=222

$$-\frac{a^2(c+dx^2)^{7/2}}{4cx^4}+\frac{(c+dx^2)^{5/2}(5ad(3ad+8bc)+8b^2c^2)}{40c^2}+\frac{(c+dx^2)^{3/2}(5ad(3ad+8bc)+8b^2c^2)}{24c}+\frac{1}{8}\sqrt{c+dx^2}(5ad(3ad+8bc)+8b^2c^2)-\frac{1}{8}\sqrt{c}(5ad(3ad+8bc)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{a(c+dx^2)^{7/2}(3ad+8bc)}{8c^2x^2}$$

```
[Out] ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Sqrt[c + d*x^2])/8 + ((8*b^2
*c^2 + 5*a*d*(8*b*c + 3*a*d))*(c + d*x^2)^(3/2))/(24*c) + ((8*b^2
*c^2 + 5*a*d*(8*b*c + 3*a*d))*(c + d*x^2)^(5/2))/(40*c^2) - (a^2*
(c + d*x^2)^(7/2))/(4*c*x^4) - (a*(8*b*c + 3*a*d)*(c + d*x^2)^(7/
2))/(8*c^2*x^2) - (Sqrt[c]*(8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

Rubi [A] time = 0.594885, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^2(c+dx^2)^{7/2}}{4cx^4}+\frac{1}{40}(c+dx^2)^{5/2}\left(\frac{5ad(3ad+8bc)}{c^2}+8b^2\right)+\frac{(c+dx^2)^{3/2}(5ad(3ad+8bc)+8b^2c^2)}{24c}+\frac{1}{8}\sqrt{c+dx^2}(5ad(3ad+8bc)+8b^2c^2)-\frac{1}{8}\sqrt{c}(5ad(3ad+8bc)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{a(c+dx^2)^{7/2}(3ad+8bc)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Sqrt[c + d*x^2])/8 + ((8*b^2
*c^2 + 5*a*d*(8*b*c + 3*a*d))*(c + d*x^2)^(3/2))/(24*c) + ((8*b^2
 + (5*a*d*(8*b*c + 3*a*d))/c^2)*(c + d*x^2)^(5/2))/40 - (a^2*(c +
 d*x^2)^(7/2))/(4*c*x^4) - (a*(8*b*c + 3*a*d)*(c + d*x^2)^(7/2))/
(8*c^2*x^2) - (Sqrt[c]*(8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

Rubi in Sympy [A] time = 38.0435, size = 207, normalized size = 0.93

$$-\frac{a^2(c+dx^2)^{\frac{7}{2}}}{4cx^4}-\frac{a(c+dx^2)^{\frac{7}{2}}(3ad+8bc)}{8c^2x^2}-\frac{\sqrt{c}(5ad(3ad+8bc)+8b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8}+\sqrt{c+dx^2}\left(\frac{5ad(3ad+8bc)}{8}+b^2c^2\right)+\frac{(c+dx^2)^{\frac{3}{2}}(5ad(3ad+8bc)+8b^2c^2)}{24c}+\frac{(c+dx^2)^{\frac{5}{2}}(5ad(3ad+8bc)+8b^2c^2)}{40c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**5,x)
```

```
[Out] -a**2*(c + d*x**2)**(7/2)/(4*c*x**4) - a*(c + d*x**2)**(7/2)*(3*a
*d + 8*b*c)/(8*c**2*x**2) - sqrt(c)*(5*a*d*(3*a*d + 8*b*c) + 8*b*
*2*c**2)*atanh(sqrt(c + d*x**2)/sqrt(c))/8 + sqrt(c + d*x**2)*(5*
a*d*(3*a*d + 8*b*c)/8 + b**2*c**2) + (c + d*x**2)**(3/2)*(5*a*d*(
3*a*d + 8*b*c) + 8*b**2*c**2)/(24*c) + (c + d*x**2)**(5/2)*(5*a*d
*(3*a*d + 8*b*c) + 8*b**2*c**2)/(40*c**2)
```

Mathematica [A] time = 0.443782, size = 186, normalized size = 0.84

$$-\frac{1}{8}\sqrt{c}\left(15a^2d^2+40abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+\sqrt{c+dx^2}\left(\frac{a^2\left(-2c^2-9cdx^2+8d^2x^4\right)}{8x^4}+\frac{1}{3}ab\left(-\frac{3c^2}{x^2}+14cd+2d^2x^2\right)+\frac{1}{15}b^2\left(23c^2+11cdx^2+3d^2x^4\right)\right)+\frac{1}{8}\sqrt{c}\log(x)\left(15a^2d^2+40abcd+8b^2c^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] Sqrt[c + d*x^2]*((a*b*(14*c*d - (3*c^2)/x^2 + 2*d^2*x^2))/3 + (b^
2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4))/15 + (a^2*(-2*c^2 - 9*c*d*x^
2 + 8*d^2*x^4))/(8*x^4)) + (Sqrt[c]*(8*b^2*c^2 + 40*a*b*c*d + 15*
a^2*d^2)*Log[x])/8 - (Sqrt[c]*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^
2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/8
```

Maple [A] time = 0.018, size = 305, normalized size = 1.4

$$-\frac{a^2}{4\,cx^4}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{3\,a^2d}{8\,c^2x^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{3\,a^2d^2}{8\,c^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,a^2d^2}{8\,c}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{15\,a^2d^2}{8}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)+\frac{15\,a^2d^2}{8}\sqrt{dx^2+c}+\frac{b^2}{5}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{b^2c}{3}\left(dx^2+c\right)^{\frac{3}{2}}-b^2c^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)+b^2\sqrt{dx^2+c}c^2-\frac{ab}{cx^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{abd}{c}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,abd}{3}\left(dx^2+c\right)^{\frac{3}{2}}-5\,abdc^{3/2}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)+5\,abdc\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(7/2)/c/x^4-3/8*a^2*d/c^2/x^2*(d*x^2+c)^(7/2)+
3/8*a^2*d^2/c^2*(d*x^2+c)^(5/2)+5/8*a^2*d^2/c*(d*x^2+c)^(3/2)-15/
8*a^2*d^2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+15/8*a^2*
d^2*(d*x^2+c)^(1/2)+1/5*b^2*(d*x^2+c)^(5/2)+1/3*b^2*c*(d*x^2+c)^(
3/2)-b^2*c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+b^2*(d*x^2
+c)^(1/2)*c^2-a*b/c/x^2*(d*x^2+c)^(7/2)+a*b*d/c*(d*x^2+c)^(5/2)+5
/3*a*b*d*(d*x^2+c)^(3/2)-5*a*b*d*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2
+c)^(1/2))/x)+5*a*b*d*c*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250508, size = 1, normalized size = 0.

$$\left[\frac{15\left(8\,b^2c^2+40\,abcd+15\,a^2d^2\right)\sqrt{c}x^4\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\left(24\,b^2d^2x^8+8\left(11\,b^2cd+10\,abd^2\right)x^6+8\left(23\,b^2c^2+70\,abcd\right.\right.}{240\,x^4}\right.$$

$$-\frac{15\left(8\,b^2c^2+40\,abcd+15\,a^2d^2\right)\sqrt{-c}x^4\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(24\,b^2d^2x^8+8\left(11\,b^2cd+10\,abd^2\right)x^6+8\left(23\,b^2c^2+70\,abcd\right.\right.}{120\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(c)*x^4*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(24*b^2*d^2*x
^8 + 8*(11*b^2*c*d + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*a*b*c*d
 + 15*a^2*d^2)*x^4 - 30*a^2*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)
*sqrt(d*x^2 + c))/x^4, -1/120*(15*(8*b^2*c^2 + 40*a*b*c*d + 15*a^
2*d^2)*sqrt(-c)*x^4*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (24*b^
2*d^2*x^8 + 8*(11*b^2*c*d + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*
a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*
d)*x^2)*sqrt(d*x^2 + c))/x^4]
```

Sympy [A] time = 116.474, size = 473, normalized size = 2.13

$$-\frac{15a^2\sqrt{c}d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8}-\frac{a^2c^3}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2c^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{7a^2cd^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}}$$
$$+\frac{a^2d^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-5abc^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)-\frac{abc^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{4abc^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{4abcd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}$$
$$+2abd^2\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for}\: d=0\\\frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)-b^2c^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)+\frac{b^2c^3}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2c^2\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}$$
$$+2b^2cd\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for}\: d=0\\\frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2d^2\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for}\: d\neq 0\\\frac{\sqrt{c}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**5,x)
```

```
[Out] -15*a**2*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/8 - a**2*c**3/(4
*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c**2*sqrt(d)/(8*x**3
*sqrt(c/(d*x**2) + 1)) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x +
 7*a**2*c*d**(3/2)/(8*x*sqrt(c/(d*x**2) + 1)) + a**2*d**(5/2)*x/s
qrt(c/(d*x**2) + 1) - 5*a*b*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))
 - a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/x + 4*a*b*c**2*sqrt(d)/(
x*sqrt(c/(d*x**2) + 1)) + 4*a*b*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1)
 + 2*a*b*d**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)
**(3/2)/(3*d), True)) - b**2*c**(5/2)*asinh(sqrt(c)/(sqrt(d)*x))
+ b**2*c**3/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c**2*sqrt(d)*
x/sqrt(c/(d*x**2) + 1) + 2*b**2*c*d*Piecewise((sqrt(c)*x**2/2, Eq
(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) + b**2*d**2*Piecewise
((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(1
5*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)
)
```

GIAC/XCAS [A] time = 0.245732, size = 327, normalized size = 1.47

$$\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2d+40\left(dx^2+c\right)^{\frac{3}{2}}b^2cd+120\sqrt{dx^2+c}b^2c^2d+80\left(dx^2+c\right)^{\frac{3}{2}}abd^2+480\sqrt{dx^2+c}abcd^2+120\sqrt{dx^2+c}a^2d^3 -}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/120*(24*(d*x^2 + c)^(5/2)*b^2*d + 40*(d*x^2 + c)^(3/2)*b^2*c*d
+ 120*sqrt(d*x^2 + c)*b^2*c^2*d + 80*(d*x^2 + c)^(3/2)*a*b*d^2 +
480*sqrt(d*x^2 + c)*a*b*c*d^2 + 120*sqrt(d*x^2 + c)*a^2*d^3 + 15*
(8*b^2*c^3*d + 40*a*b*c^2*d^2 + 15*a^2*c*d^3)*arctan(sqrt(d*x^2 +
 c)/sqrt(-c))/sqrt(-c) - 15*(8*(d*x^2 + c)^(3/2)*a*b*c^2*d^2 - 8*
sqrt(d*x^2 + c)*a*b*c^3*d^2 + 9*(d*x^2 + c)^(3/2)*a^2*c*d^3 - 7*s
qrt(d*x^2 + c)*a^2*c^2*d^3)/(d^2*x^4))/d
```

3.634 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=228

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{\left(c+dx^2\right)^{5/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{15c^2x}+\frac{dx\left(c+dx^2\right)^{3/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{12c^2}+\frac{dx\sqrt{c+dx^2}\left(8ad(ad+5bc)+15b^2c^2\right)}{8c}+\frac{1}{8}\sqrt{d}\left(8ad(ad+5bc)+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{15c^2x^3}$$

```
[Out] (d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) +
(d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^
2) - ((15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*(c + d*x^2)^(5/2))/(15*c
^2*x) - (a^2*(c + d*x^2)^(7/2))/(5*c*x^5) - (2*a*(5*b*c + a*d)*(c
 + d*x^2)^(7/2))/(15*c^2*x^3) + (Sqrt[d]*(15*b^2*c^2 + 8*a*d*(5*b
*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/8
```

Rubi [A] time = 0.424608, antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{\left(c+dx^2\right)^{5/2}\left(\frac{8ad(ad+5bc)}{c^2}+15b^2\right)}{15x}+\frac{dx\left(c+dx^2\right)^{3/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{12c^2}+\frac{dx\sqrt{c+dx^2}\left(8ad(ad+5bc)+15b^2c^2\right)}{8c}+\frac{1}{8}\sqrt{d}\left(8ad(ad+5bc)+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] (d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) +
(d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^
2) - ((15*b^2 + (8*a*d*(5*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(15
*x) - (a^2*(c + d*x^2)^(7/2))/(5*c*x^5) - (2*a*(5*b*c + a*d)*(c +
 d*x^2)^(7/2))/(15*c^2*x^3) + (Sqrt[d]*(15*b^2*c^2 + 8*a*d*(5*b*c
 + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/8
```

Rubi in Sympy [A] time = 33.5802, size = 216, normalized size = 0.95

$$-\frac{a^2\left(c+dx^2\right)^{\frac{7}{2}}}{5cx^5}-\frac{2a\left(c+dx^2\right)^{\frac{7}{2}}(ad+5bc)}{15c^2x^3}+\frac{\sqrt{d}\left(8ad\left(ad+5bc\right)+15b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8}+\frac{dx\sqrt{c+dx^2}\left(8ad\left(ad+5bc\right)+15b^2c^2\right)}{8c}+\frac{dx\left(c+dx^2\right)^{\frac{3}{2}}\left(8ad\left(ad+5bc\right)+15b^2c^2\right)}{12c^2}-\frac{\left(c+dx^2\right)^{\frac{5}{2}}\left(8ad\left(ad+5bc\right)+15b^2c^2\right)}{15c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**6,x)
```

```
[Out] -a**2*(c + d*x**2)**(7/2)/(5*c*x**5) - 2*a*(c + d*x**2)**(7/2)*(a
*d + 5*b*c)/(15*c**2*x**3) + sqrt(d)*(8*a*d*(a*d + 5*b*c) + 15*b*
*2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/8 + d*x*sqrt(c + d*x**
2)*(8*a*d*(a*d + 5*b*c) + 15*b**2*c**2)/(8*c) + d*x*(c + d*x**2)*
```

```
*(3/2)*(8*a*d*(a*d + 5*b*c) + 15*b**2*c**2)/(12*c**2) - (c + d*x*
*2)**(5/2)*(8*a*d*(a*d + 5*b*c) + 15*b**2*c**2)/(15*c**2*x)
```

Mathematica [A] time = 0.301412, size = 158, normalized size = 0.69

$$\frac{1}{8}\sqrt{d}\left(8a^2d^2+40abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{c+dx^2}\left(\frac{-23a^2d^2-70abcd-15b^2c^2}{15x}-\frac{a^2c^2}{5x^5}-\frac{ac(11ad+10bc)}{15x^3}+\frac{1}{8}bdx(8ad+9bc)+\frac{1}{4}b^2d^2x^3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] Sqrt[c + d*x^2]*(-(a^2*c^2)/(5*x^5) - (a*c*(10*b*c + 11*a*d))/(15
*x^3) + (-15*b^2*c^2 - 70*a*b*c*d - 23*a^2*d^2)/(15*x) + (b*d*(9*
b*c + 8*a*d)*x)/8 + (b^2*d^2*x^3)/4) + (Sqrt[d]*(15*b^2*c^2 + 40*
a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/8
```

Maple [A] time = 0.024, size = 369, normalized size = 1.6

$$-\frac{a^2}{5\,cx^5}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{2\,a^2d}{15\,c^2x^3}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{8\,a^2d^2}{15\,c^3x}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{8\,a^2d^3x}{15\,c^3}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{2\,a^2d^3x}{3\,c^2}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{a^2d^3x}{c}\sqrt{dx^2+c}+a^2d^{\frac{5}{2}}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)-\frac{b^2}{cx}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{b^2dx}{c}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,b^2dx}{4}\left(dx^2+c\right)^{\frac{3}{2}}+\frac{15\,b^2dcx}{8}\sqrt{dx^2+c}+\frac{15\,b^2c^2}{8}\sqrt{d}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)-\frac{2\,ab}{3\,cx^3}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{8\,abd}{3\,c^2x}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{8\,abd^2x}{3\,c^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{10\,abd^2x}{3\,c}\left(dx^2+c\right)^{\frac{3}{2}}+5\,abd^2x\sqrt{dx^2+c}+5\,abd^{3/2}c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(7/2)/c/x^5-2/15*a^2*d/c^2/x^3*(d*x^2+c)^(7/2)
-8/15*a^2*d^2/c^3/x*(d*x^2+c)^(7/2)+8/15*a^2*d^3/c^3*x*(d*x^2+c)^
(5/2)+2/3*a^2*d^3/c^2*x*(d*x^2+c)^(3/2)+a^2*d^3/c*x*(d*x^2+c)^(1/
2)+a^2*d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-b^2/c/x*(d*x^2+c)^(7
/2)+b^2*d/c*x*(d*x^2+c)^(5/2)+5/4*b^2*d*x*(d*x^2+c)^(3/2)+15/8*b^
2*d*c*x*(d*x^2+c)^(1/2)+15/8*b^2*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+
c)^(1/2))-2/3*a*b/c/x^3*(d*x^2+c)^(7/2)-8/3*a*b*d/c^2/x*(d*x^2+c)
^(7/2)+8/3*a*b*d^2/c^2*x*(d*x^2+c)^(5/2)+10/3*a*b*d^2/c*x*(d*x^2+
c)^(3/2)+5*a*b*d^2*x*(d*x^2+c)^(1/2)+5*a*b*d^(3/2)*c*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.343575, size = 1, normalized size = 0.

$$\left[\frac{15\left(15\,b^2c^2+40\,abcd+8\,a^2d^2\right)\sqrt{d}x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(30\,b^2d^2x^8+15\left(9\,b^2cd+8\,abd^2\right)x^6-8\left(15\,b^2\right.\right.}{240\,x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^5*log(
-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(30*b^2*d^2*x^8 +
 15*(9*b^2*c*d + 8*a*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*b*c*d + 23
*a^2*d^2)*x^4 - 24*a^2*c^2 - 8*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqr
t(d*x^2 + c))/x^5, 1/120*(15*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2
)*sqrt(-d)*x^5*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) + (30*b^2*d
^2*x^8 + 15*(9*b^2*c*d + 8*a*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*b*
c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8*(10*a*b*c^2 + 11*a^2*c*d)*
x^2)*sqrt(d*x^2 + c))/x^5]
```

Sympy [A] time = 48.1252, size = 474, normalized size = 2.08

$$-\frac{a^2\sqrt{c}d^2}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{11a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15x^2}-\frac{8a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15}+a^2d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{a^2d^3x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{4abc^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}d^2x\sqrt{1+\frac{dx^2}{c}}-\frac{4ab\sqrt{c}d^2x}{\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+5abcd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-\frac{b^2c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+b^2c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7b^2c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2\sqrt{c}d^2x^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{15b^2c^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8}+\frac{b^2d^3x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(c)*d**2/(x*sqrt(1 + d*x**2/c)) - a**2*c**2*sqrt(d)*sqr
t(c/(d*x**2) + 1)/(5*x**4) - 11*a**2*c*d**(3/2)*sqrt(c/(d*x**2) +
 1)/(15*x**2) - 8*a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/15 + a**2*d*
*(5/2)*asinh(sqrt(d)*x/sqrt(c)) - a**2*d**3*x/(sqrt(c)*sqrt(1 + d
*x**2/c)) - 4*a*b*c**(3/2)*d/(x*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)
*d**2*x*sqrt(1 + d*x**2/c) - 4*a*b*sqrt(c)*d**2*x/sqrt(1 + d*x**2
/c) - 2*a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*c*
d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*b*c*d**(3/2)*asinh(sqrt(d)*
x/sqrt(c)) - b**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + b**2*c**(3/2)
*d*x*sqrt(1 + d*x**2/c) - 7*b**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/
c)) + 3*b**2*sqrt(c)*d**2*x**3/(8*sqrt(1 + d*x**2/c)) + 15*b**2*c
**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/8 + b**2*d**3*x**5/(4*sqrt(c
)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.25106, size = 689, normalized size = 3.02

$$\frac{1}{8}\left(2\,b^2d^2x^2 + \frac{9\,b^2cd^3 + 8\,abd^4}{d^2}\right)\sqrt{dx^2+c}x - \frac{1}{16}\left(15\,b^2c^2\sqrt{d} + 40\,abcd^{\frac{3}{2}} + 8\,a^2d^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right) + \frac{2\left(15\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 b^2c^3\sqrt{d} + 90\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 abc^2d^{\frac{3}{2}} + 45\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 a^2cd^{\frac{5}{2}} - 60\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d^2*x^2 + (9*b^2*c*d^3 + 8*a*b*d^4)/d^2)*sqrt(d*x^2 +
c)*x - 1/16*(15*b^2*c^2*sqrt(d) + 40*a*b*c*d^(3/2) + 8*a^2*d^(5/2
))*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sq
rt(d*x^2 + c))^8*b^2*c^3*sqrt(d) + 90*(sqrt(d)*x - sqrt(d*x^2 + c
))^8*a*b*c^2*d^(3/2) + 45*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*c*d
^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^4*sqrt(d) - 300
*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(3/2) - 90*(sqrt(d)*x
- sqrt(d*x^2 + c))^6*a^2*c^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2
 + c))^4*b^2*c^5*sqrt(d) + 400*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*
b*c^4*d^(3/2) + 140*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(5/
2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^6*sqrt(d) - 260*(sq
rt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^5*d^(3/2) - 70*(sqrt(d)*x - sq
rt(d*x^2 + c))^2*a^2*c^4*d^(5/2) + 15*b^2*c^7*sqrt(d) + 70*a*b*c^
6*d^(3/2) + 23*a^2*c^5*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2
- c)^5
```

3.635 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=222

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{5/2}\left(ad(ad+12bc)+8b^2c^2\right)}{16c^2x^2}+\frac{5d\left(c+dx^2\right)^{3/2}\left(ad(ad+12bc)+8b^2c^2\right)}{48c^2}+\frac{5d\sqrt{c+dx^2}\left(ad(ad+12bc)+8b^2c^2\right)}{16c}-\frac{5d\left(ad(ad+12bc)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{24c^2x^4}$$

```
[Out] (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*Sqrt[c + d*x^2])/(16*c) + (
5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(3/2))/(48*c^2)
- ((8*b^2*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(5/2))/(16*c^2*x^
2) - (a^2*(c + d*x^2)^(7/2))/(6*c*x^6) - (a*(12*b*c + a*d)*(c + d
*x^2)^(7/2))/(24*c^2*x^4) - (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

Rubi [A] time = 0.619153, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{5/2}\left(\frac{ad(ad+12bc)}{c^2}+8b^2\right)}{16x^2}+\frac{5d\left(c+dx^2\right)^{3/2}\left(ad(ad+12bc)+8b^2c^2\right)}{48c^2}+\frac{5d\sqrt{c+dx^2}\left(ad(ad+12bc)+8b^2c^2\right)}{16c}-\frac{5d\left(ad(ad+12bc)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*Sqrt[c + d*x^2])/(16*c) + (
5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(3/2))/(48*c^2)
- ((8*b^2 + (a*d*(12*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(16*x^2)
 - (a^2*(c + d*x^2)^(7/2))/(6*c*x^6) - (a*(12*b*c + a*d)*(c + d*x
^2)^(7/2))/(24*c^2*x^4) - (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*A
rcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

Rubi in Sympy [A] time = 38.297, size = 211, normalized size = 0.95

$$-\frac{a^2\left(c+dx^2\right)^{\frac{7}{2}}}{6cx^6}-\frac{a\left(c+dx^2\right)^{\frac{7}{2}}(ad+12bc)}{24c^2x^4}+\frac{5d\sqrt{c+dx^2}\left(ad\left(ad+12bc\right)+8b^2c^2\right)}{16c}+\frac{5d\left(c+dx^2\right)^{\frac{3}{2}}\left(ad\left(ad+12bc\right)+8b^2c^2\right)}{48c^2}-\frac{\left(c+dx^2\right)^{\frac{5}{2}}\left(ad\left(ad+12bc\right)+8b^2c^2\right)}{16c^2x^2}-\frac{5d\left(ad\left(ad+12bc\right)+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**7,x)
```

```
[Out] -a**2*(c + d*x**2)**(7/2)/(6*c*x**6) - a*(c + d*x**2)**(7/2)*(a*d
 + 12*b*c)/(24*c**2*x**4) + 5*d*sqrt(c + d*x**2)*(a*d*(a*d + 12*b
*c) + 8*b**2*c**2)/(16*c) + 5*d*(c + d*x**2)**(3/2)*(a*d*(a*d + 1
```

```
2*b*c) + 8*b**2*c**2)/(48*c**2) - (c + d*x**2)**(5/2)*(a*d*(a*d +
 12*b*c) + 8*b**2*c**2)/(16*c**2*x**2) - 5*d*(a*d*(a*d + 12*b*c)
+ 8*b**2*c**2)*atanh(sqrt(c + d*x**2)/sqrt(c))/(16*sqrt(c))
```

Mathematica [A] time = 0.404278, size = 186, normalized size = 0.84

$$\frac{1}{48}\left(-\frac{15d\left(a^2d^2+12abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{\sqrt{c}}-\frac{\sqrt{c+dx^2}\left(a^2\left(8c^2+26cdx^2+33d^2x^4\right)+12abx^2\left(2c^2+9cdx^2-8d^2x^4\right)-8b^2x^4\left(-3c^2+14cdx^2+2d^2x^4\right)\right)}{x^6}+\frac{15d\log(x)\left(a^2d^2+12abcd+8b^2c^2\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] (-((Sqrt[c + d*x^2]*(12*a*b*x^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) -
 8*b^2*x^4*(-3*c^2 + 14*c*d*x^2 + 2*d^2*x^4) + a^2*(8*c^2 + 26*c*
d*x^2 + 33*d^2*x^4)))/x^6) + (15*d*(8*b^2*c^2 + 12*a*b*c*d + a^2*
d^2)*Log[x])/Sqrt[c] - (15*d*(8*b^2*c^2 + 12*a*b*c*d + a^2*d^2)*L
og[c + Sqrt[c]*Sqrt[c + d*x^2]])/Sqrt[c])/48
```

Maple [A] time = 0.023, size = 387, normalized size = 1.7

$$-\frac{a^2}{6\,cx^6}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{a^2d}{24\,c^2x^4}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{a^2d^2}{16\,c^3x^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{a^2d^3}{16\,c^3}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,a^2d^3}{48\,c^2}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{5\,a^2d^3}{16\,c}\sqrt{dx^2+c}-\frac{b^2}{2\,cx^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{b^2d}{2\,c}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,b^2d}{6}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{5\,b^2d}{2}c^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)+\frac{5\,b^2dc}{2}\sqrt{dx^2+c}-\frac{ab}{2\,cx^4}\left(dx^2+c\right)^{\frac{7}{2}}-\frac{3\,abd}{4\,c^2x^2}\left(dx^2+c\right)^{\frac{7}{2}}+\frac{3\,abd^2}{4\,c^2}\left(dx^2+c\right)^{\frac{5}{2}}+\frac{5\,abd^2}{4\,c}\left(dx^2+c\right)^{\frac{3}{2}}-\frac{15\,abd^2}{4}\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)+\frac{15\,abd^2}{4}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(7/2)/c/x^6-1/24*a^2*d/c^2/x^4*(d*x^2+c)^(7/2)
-1/16*a^2*d^2/c^3/x^2*(d*x^2+c)^(7/2)+1/16*a^2*d^3/c^3*(d*x^2+c)^
(5/2)+5/48*a^2*d^3/c^2*(d*x^2+c)^(3/2)-5/16*a^2*d^3/c^(1/2)*ln((2
*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+5/16*a^2*d^3/c*(d*x^2+c)^(1/2)-1
/2*b^2/c/x^2*(d*x^2+c)^(7/2)+1/2*b^2*d/c*(d*x^2+c)^(5/2)+5/6*b^2*
d*(d*x^2+c)^(3/2)-5/2*b^2*d*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(
1/2))/x)+5/2*b^2*d*c*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*x^2+c)^(7/2
)-3/4*a*b*d/c^2/x^2*(d*x^2+c)^(7/2)+3/4*a*b*d^2/c^2*(d*x^2+c)^(5/
2)+5/4*a*b*d^2/c*(d*x^2+c)^(3/2)-15/4*a*b*d^2*c^(1/2)*ln((2*c+2*c
^(1/2)*(d*x^2+c)^(1/2))/x)+15/4*a*b*d^2*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278932, size = 1, normalized size = 0.

$$\left[\frac{15\left(8\,b^2c^2d+12\,abcd^2+a^2d^3\right)x^6\log\left(-\frac{(dx^2+2\,c)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)+2\left(16\,b^2d^2x^8+16\left(7\,b^2cd+6\,abd^2\right)x^6-3\left(8\,b^2c^2+36\,abc\right.\right.}{96\,\sqrt{c}x^6}\right.$$

$$-\frac{15\left(8\,b^2c^2d+12\,abcd^2+a^2d^3\right)x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(16\,b^2d^2x^8+16\left(7\,b^2cd+6\,abd^2\right)x^6-3\left(8\,b^2c^2+36\,abcd+11\,a^2d^2\right)\right.}{48\,\sqrt{-c}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*x^6*log(-((d*x^2
 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2) + 2*(16*b^2*d^2*x^8 +
 16*(7*b^2*c*d + 6*a*b*d^2)*x^6 - 3*(8*b^2*c^2 + 36*a*b*c*d + 11*
a^2*d^2)*x^4 - 8*a^2*c^2 - 2*(12*a*b*c^2 + 13*a^2*c*d)*x^2)*sqrt(
d*x^2 + c)*sqrt(c))/(sqrt(c)*x^6), -1/48*(15*(8*b^2*c^2*d + 12*a*
b*c*d^2 + a^2*d^3)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (16*b^2
*d^2*x^8 + 16*(7*b^2*c*d + 6*a*b*d^2)*x^6 - 3*(8*b^2*c^2 + 36*a*b
*c*d + 11*a^2*d^2)*x^4 - 8*a^2*c^2 - 2*(12*a*b*c^2 + 13*a^2*c*d)*
x^2)*sqrt(d*x^2 + c)*sqrt(-c))/(sqrt(-c)*x^6)]
```

Sympy [A] time = 166.573, size = 468, normalized size = 2.11

$$-\frac{a^2c^3}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}-\frac{17a^2c^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{35a^2cd^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{3a^2d^{\frac{5}{2}}}{16x\sqrt{\frac{c}{dx^2}+1}}$$
$$-\frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16\sqrt{c}}-\frac{15ab\sqrt{c}d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4}-\frac{abc^3}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3abc^2\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}}$$
$$-\frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{7abcd^{\frac{3}{2}}}{4x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abd^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-\frac{5b^2c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2}$$
$$-\frac{b^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{2b^2c^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{2b^2cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}+b^2d^2\left(\begin{cases}\frac{\sqrt{c}x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**7,x)
```

```
[Out] -a**2*c**3/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 17*a**2*c**2*s
qrt(d)/(24*x**5*sqrt(c/(d*x**2) + 1)) - 35*a**2*c*d**(3/2)/(48*x*
*3*sqrt(c/(d*x**2) + 1)) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(2*
x) - 3*a**2*d**(5/2)/(16*x*sqrt(c/(d*x**2) + 1)) - 5*a**2*d**3*as
inh(sqrt(c)/(sqrt(d)*x))/(16*sqrt(c)) - 15*a*b*sqrt(c)*d**2*asinh
(sqrt(c)/(sqrt(d)*x))/4 - a*b*c**3/(2*sqrt(d)*x**5*sqrt(c/(d*x**2
) + 1)) - 3*a*b*c**2*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - 2*a*
b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a*b*c*d**(3/2)/(4*x*sqrt(
c/(d*x**2) + 1)) + 2*a*b*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*b**2
*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - b**2*c**2*sqrt(d)*sqrt
(c/(d*x**2) + 1)/(2*x) + 2*b**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) +
```

```
 1)) + 2*b**2*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1) + b**2*d**2*Piece
wise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True
))
```

GIAC/XCAS [A] time = 0.25007, size = 386, normalized size = 1.74

$$\frac{16\left(dx^2+c\right)^{\frac{3}{2}}b^2d^2+96\sqrt{dx^2+c}b^2cd^2+96\sqrt{dx^2+c}abd^3+\frac{15\left(8b^2c^2d^2+12abcd^3+a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2\right.}{}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(16*(d*x^2 + c)^(3/2)*b^2*d^2 + 96*sqrt(d*x^2 + c)*b^2*c*d^2
 + 96*sqrt(d*x^2 + c)*a*b*d^3 + 15*(8*b^2*c^2*d^2 + 12*a*b*c*d^3
+ a^2*d^4)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - (24*(d*x^2
 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*s
qrt(d*x^2 + c)*b^2*c^4*d^2 + 108*(d*x^2 + c)^(5/2)*a*b*c*d^3 - 19
2*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 84*sqrt(d*x^2 + c)*a*b*c^3*d^3
+ 33*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*d^4 +
 15*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(d^3*x^6))/d
```

3.636 $\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{c^2\left(48a^2d^2+5bc(7bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}-\frac{cx\sqrt{c+dx^2}\left(48a^2d^2+5bc(7bc-16ad)\right)}{128d^4}+\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2+5bc(7bc-16ad)\right)}{192d^3}-\frac{bx^5\sqrt{c+dx^2}(7bc-16ad)}{48d^2}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}$$

```
[Out] -(c*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128
*d^4) + ((48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x^3*Sqrt[c + d*x^2
])/(192*d^3) - (b*(7*b*c - 16*a*d)*x^5*Sqrt[c + d*x^2])/(48*d^2)
+ (b^2*x^7*Sqrt[c + d*x^2])/(8*d) + (c^2*(48*a^2*d^2 + 5*b*c*(7*b
*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(9/2))
```

Rubi [A] time = 0.461947, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{c^2\left(48a^2d^2+5bc(7bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}+\frac{x^3\sqrt{c+dx^2}\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)}{192d}-\frac{cx\sqrt{c+dx^2}\left(48a^2d^2+5bc(7bc-16ad)\right)}{128d^4}-\frac{bx^5\sqrt{c+dx^2}(7bc-16ad)}{48d^2}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -(c*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128
*d^4) + ((48*a^2 + (5*b*c*(7*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x
^2])/(192*d) - (b*(7*b*c - 16*a*d)*x^5*Sqrt[c + d*x^2])/(48*d^2)
+ (b^2*x^7*Sqrt[c + d*x^2])/(8*d) + (c^2*(48*a^2*d^2 + 5*b*c*(7*b
*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(9/2))
```

Rubi in Sympy [A] time = 36.4558, size = 187, normalized size = 0.96

$$\frac{b^2x^7\sqrt{c+dx^2}}{8d}+\frac{bx^5\sqrt{c+dx^2}\,(16ad-7bc)}{48d^2}+\frac{c^2\left(48a^2d^2-5bc\,(16ad-7bc)\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{\frac{9}{2}}}-\frac{cx\sqrt{c+dx^2}\left(48a^2d^2-5bc\,(16ad-7bc)\right)}{128d^4}+\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-5bc\,(16ad-7bc)\right)}{192d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*x**7*sqrt(c + d*x**2)/(8*d) + b*x**5*sqrt(c + d*x**2)*(16*a*
d - 7*b*c)/(48*d**2) + c**2*(48*a**2*d**2 - 5*b*c*(16*a*d - 7*b*c
))*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(128*d**(9/2)) - c*x*sqrt(c
+ d*x**2)*(48*a**2*d**2 - 5*b*c*(16*a*d - 7*b*c))/(128*d**4) + x*
*3*sqrt(c + d*x**2)*(48*a**2*d**2 - 5*b*c*(16*a*d - 7*b*c))/(192*
d**3)
```

Mathematica [A] time = 0.193349, size = 159, normalized size = 0.82

$$\frac{3c^2\left(48a^2d^2-80abcd+35b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2\left(2dx^2-3c\right)+16abd\left(15c^2-10cdx^2+8d\right.\right.}{384d^{9/2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(-3*c + 2*d*x^2) + 16*a*b*
d*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4) + b^2*(-105*c^3 + 70*c^2*d*x^
2 - 56*c*d^2*x^4 + 48*d^3*x^6)) + 3*c^2*(35*b^2*c^2 - 80*a*b*c*d
+ 48*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(384*d^(9/2))
```

Maple [A] time = 0.023, size = 265, normalized size = 1.4

$$\frac{a^2x^3}{4d}\sqrt{dx^2+c}-\frac{3a^2cx}{8d^2}\sqrt{dx^2+c}+\frac{3a^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{b^2x^7}{8d}\sqrt{dx^2+c}-\frac{7b^2cx^5}{48d^2}\sqrt{dx^2+c}+\frac{35b^2c^2x^3}{192d^3}\sqrt{dx^2+c}-\frac{35xb^2c^3}{128d^4}\sqrt{dx^2+c}+\frac{35b^2c^4}{128}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}}+\frac{abx^5}{3d}\sqrt{dx^2+c}-\frac{5abcx^3}{12d^2}\sqrt{dx^2+c}+\frac{5abc^2x}{8d^3}\sqrt{dx^2+c}-\frac{5abc^3}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*a^2*x^3/d*(d*x^2+c)^(1/2)-3/8*a^2*c/d^2*x*(d*x^2+c)^(1/2)+3/8
*a^2*c^2/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/8*b^2*x^7*(d*x^2
+c)^(1/2)/d-7/48*b^2*c/d^2*x^5*(d*x^2+c)^(1/2)+35/192*b^2*c^2/d^3
*x^3*(d*x^2+c)^(1/2)-35/128*b^2*c^3/d^4*x*(d*x^2+c)^(1/2)+35/128*
b^2*c^4/d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x^5/d*(d*x^
2+c)^(1/2)-5/12*a*b*c/d^2*x^3*(d*x^2+c)^(1/2)+5/8*a*b*c^2/d^3*x*(
d*x^2+c)^(1/2)-5/8*a*b*c^3/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^4/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.375325, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48b^2d^3x^7-8\left(7b^2cd^2-16abd^3\right)x^5+2\left(35b^2c^2d-80abcd^2+48a^2d^3\right)x^3-3\left(35b^2c^3-80abc^2d+48a^2cd^2\right)x\right)\sqrt{dx^2}}{768d^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((b*x^2 + a)^2*x^4/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^2*d^3*x^7 - 8*(7*b^2*c*d^2 - 16*a*b*d^3)*x^5 + 2*
(35*b^2*c^2*d - 80*a*b*c*d^2 + 48*a^2*d^3)*x^3 - 3*(35*b^2*c^3 -
80*a*b*c^2*d + 48*a^2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(35*b
^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*log(-2*sqrt(d*x^2 + c)*d*
x - (2*d*x^2 + c)*sqrt(d)))/d^(9/2), 1/384*((48*b^2*d^3*x^7 - 8*(
7*b^2*c*d^2 - 16*a*b*d^3)*x^5 + 2*(35*b^2*c^2*d - 80*a*b*c*d^2 +
48*a^2*d^3)*x^3 - 3*(35*b^2*c^3 - 80*a*b*c^2*d + 48*a^2*c*d^2)*x)
*sqrt(d*x^2 + c)*sqrt(-d) + 3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2
```

```
*c^2*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^4)]
```

Sympy [A] time = 63.9038, size = 422, normalized size = 2.18

$$-\frac{3a^2c^{\frac{3}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2\sqrt{c}x^3}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}}+\frac{a^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^{\frac{5}{2}}x}{8d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^{\frac{3}{2}}x^3}{24d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{ab\sqrt{c}x^5}{12d\sqrt{1+\frac{dx^2}{c}}}-\frac{5abc^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8d^{\frac{7}{2}}}+\frac{abx^7}{3\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{35b^2c^{\frac{7}{2}}x}{128d^4\sqrt{1+\frac{dx^2}{c}}}-\frac{35b^2c^{\frac{5}{2}}x^3}{384d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2c^{\frac{3}{2}}x^5}{192d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2\sqrt{c}x^7}{48d\sqrt{1+\frac{dx^2}{c}}}+\frac{35b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{9}{2}}}+\frac{b^2x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**(3/2)*x/(8*d**2*sqrt(1 + d*x**2/c)) - a**2*sqrt(c)*x**
3/(8*d*sqrt(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(d)*x/sqrt(c))
/(8*d**(5/2)) + a**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*
c**(5/2)*x/(8*d**3*sqrt(1 + d*x**2/c)) + 5*a*b*c**(3/2)*x**3/(24*
d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x**5/(12*d*sqrt(1 + d*x**2
/c)) - 5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(7/2)) + a*b*x**
7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) - 35*b**2*c**(7/2)*x/(128*d**4*s
qrt(1 + d*x**2/c)) - 35*b**2*c**(5/2)*x**3/(384*d**3*sqrt(1 + d*x
**2/c)) + 7*b**2*c**(3/2)*x**5/(192*d**2*sqrt(1 + d*x**2/c)) - b*
*2*sqrt(c)*x**7/(48*d*sqrt(1 + d*x**2/c)) + 35*b**2*c**4*asinh(sq
rt(d)*x/sqrt(c))/(128*d**(9/2)) + b**2*x**9/(8*sqrt(c)*sqrt(1 + d
*x**2/c))
```

GIAC/XCAS [A] time = 0.242375, size = 240, normalized size = 1.24

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,b^2x^2}{d}-\frac{7\,b^2cd^5-16\,abd^6}{d^7}\right)x^2+\frac{35\,b^2c^2d^4-80\,abcd^5+48\,a^2d^6}{d^7}\right)x^2-\frac{3\left(35\,b^2c^3d^3-80\,abc^2d^4+48\,a^2cd^5\right)}{d^7}\right)\sqrt{}-\frac{\left(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2/d - (7*b^2*c*d^5 - 16*a*b*d^6)/d^7)*x^2 +
(35*b^2*c^2*d^4 - 80*a*b*c*d^5 + 48*a^2*d^6)/d^7)*x^2 - 3*(35*b^2
*c^3*d^3 - 80*a*b*c^2*d^4 + 48*a^2*c*d^5)/d^7)*sqrt(d*x^2 + c)*x
- 1/128*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*ln(abs(-sqrt
(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.637 $\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=112

$$-\frac{b(c+dx^2)^{5/2}(3bc-2ad)}{5d^4}+\frac{(c+dx^2)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}(bc-ad)^2}{d^4}+\frac{b^2(c+dx^2)^{7/2}}{7d^4}$$

```
[Out] -((c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4) + ((b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^(3/2))/(3*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^
(5/2))/(5*d^4) + (b^2*(c + d*x^2)^(7/2))/(7*d^4)
```

Rubi [A] time = 0.267724, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{b(c+dx^2)^{5/2}(3bc-2ad)}{5d^4}+\frac{(c+dx^2)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}(bc-ad)^2}{d^4}+\frac{b^2(c+dx^2)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -((c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4) + ((b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^(3/2))/(3*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^
(5/2))/(5*d^4) + (b^2*(c + d*x^2)^(7/2))/(7*d^4)
```

Rubi in Sympy [A] time = 31.9884, size = 99, normalized size = 0.88

$$\frac{b^2(c+dx^2)^{\frac{7}{2}}}{7d^4}+\frac{b(c+dx^2)^{\frac{5}{2}}(2ad-3bc)}{5d^4}-\frac{c\sqrt{c+dx^2}(ad-bc)^2}{d^4}+\frac{(c+dx^2)^{\frac{3}{2}}(ad-3bc)(ad-bc)}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*(c + d*x**2)**(7/2)/(7*d**4) + b*(c + d*x**2)**(5/2)*(2*a*d
- 3*b*c)/(5*d**4) - c*sqrt(c + d*x**2)*(a*d - b*c)**2/d**4 + (c +
 d*x**2)**(3/2)*(a*d - 3*b*c)*(a*d - b*c)/(3*d**4)
```

Mathematica [A] time = 0.0953668, size = 99, normalized size = 0.88

$$\frac{\sqrt{c+dx^2}\left(35a^2d^2(dx^2-2c)+14abd(8c^2-4cdx^2+3d^2x^4)-3b^2(16c^3-8c^2dx^2+6cd^2x^4-5d^3x^6)\right)}{105d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(35*a^2*d^2*(-2*c + d*x^2) + 14*a*b*d*(8*c^2 - 4
*c*d*x^2 + 3*d^2*x^4) - 3*b^2*(16*c^3 - 8*c^2*d*x^2 + 6*c*d^2*x^4
 - 5*d^3*x^6)))/(105*d^4)
```

Maple [A] time = 0.01, size = 108, normalized size = 1.

$$-\frac{-15\,b^2x^6d^3-42\,abd^3x^4+18\,b^2cd^2x^4-35\,a^2d^3x^2+56\,abcd^2x^2-24\,b^2c^2dx^2+70\,a^2cd^2-112\,abc^2d+48\,b^2c^3}{105\,d^4}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/105*(d*x^2+c)^(1/2)*(-15*b^2*d^3*x^6-42*a*b*d^3*x^4+18*b^2*c*d
^2*x^4-35*a^2*d^3*x^2+56*a*b*c*d^2*x^2-24*b^2*c^2*d*x^2+70*a^2*c*
d^2-112*a*b*c^2*d+48*b^2*c^3)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24873, size = 139, normalized size = 1.24

$$\frac{\left(15\,b^2d^3x^6-48\,b^2c^3+112\,abc^2d-70\,a^2cd^2-6\left(3\,b^2cd^2-7\,abd^3\right)x^4+\left(24\,b^2c^2d-56\,abcd^2+35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 - 48*b^2*c^3 + 112*a*b*c^2*d - 70*a^2*c*d^2
 - 6*(3*b^2*c*d^2 - 7*a*b*d^3)*x^4 + (24*b^2*c^2*d - 56*a*b*c*d^2
 + 35*a^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [A] time = 5.10787, size = 240, normalized size = 2.14

$$\begin{cases} -\frac{2a^2c\sqrt{c+dx^2}}{3d^2}+\frac{a^2x^2\sqrt{c+dx^2}}{3d}+\frac{16abc^2\sqrt{c+dx^2}}{15d^3}-\frac{8abcx^2\sqrt{c+dx^2}}{15d^2}+\frac{2abx^4\sqrt{c+dx^2}}{5d}-\frac{16b^2c^3\sqrt{c+dx^2}}{35d^4}+\frac{8b^2c^2x^2\sqrt{c+dx^2}}{35d^3}-\frac{6b^2cx^4\sqrt{c+dx^2}}{35d^2}+b^2 \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{\sqrt{c}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c*sqrt(c + d*x**2)/(3*d**2) + a**2*x**2*sqrt(c
 + d*x**2)/(3*d) + 16*a*b*c**2*sqrt(c + d*x**2)/(15*d**3) - 8*a*b
*c*x**2*sqrt(c + d*x**2)/(15*d**2) + 2*a*b*x**4*sqrt(c + d*x**2)/
(5*d) - 16*b**2*c**3*sqrt(c + d*x**2)/(35*d**4) + 8*b**2*c**2*x**
2*sqrt(c + d*x**2)/(35*d**3) - 6*b**2*c*x**4*sqrt(c + d*x**2)/(35
*d**2) + b**2*x**6*sqrt(c + d*x**2)/(7*d), Ne(d, 0)), ((a**2*x**4
/4 + a*b*x**6/3 + b**2*x**8/8)/sqrt(c), True))
```

GIAC/XCAS [A] time = 0.229019, size = 203, normalized size = 1.81

$$\frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2-63\left(dx^2+c\right)^{\frac{5}{2}}b^2c+105\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2-105\sqrt{dx^2+c}b^2c^3+42\left(dx^2+c\right)^{\frac{5}{2}}abd-140\left(dx^2+c\right)^{\frac{3}{2}}abcd+2}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/105*(15*(d*x^2 + c)^(7/2)*b^2 - 63*(d*x^2 + c)^(5/2)*b^2*c + 10
5*(d*x^2 + c)^(3/2)*b^2*c^2 - 105*sqrt(d*x^2 + c)*b^2*c^3 + 42*(d
*x^2 + c)^(5/2)*a*b*d - 140*(d*x^2 + c)^(3/2)*a*b*c*d + 210*sqrt(
d*x^2 + c)*a*b*c^2*d + 35*(d*x^2 + c)^(3/2)*a^2*d^2 - 105*sqrt(d*
x^2 + c)*a^2*c*d^2)/d^4
```

3.638 $\int \frac{x^2\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{c\left(8a^2d^2+bc(5bc-12ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2+bc(5bc-12ad)\right)}{16d^3}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

```
[Out] ((8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*x*Sqrt[c + d*x^2])/(16*d^3) -
 (b*(5*b*c - 12*a*d)*x^3*Sqrt[c + d*x^2])/(24*d^2) + (b^2*x^5*Sqr
t[c + d*x^2])/(6*d) - (c*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(7/2))
```

Rubi [A] time = 0.389686, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x\sqrt{c+dx^2}\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)}{16d}-\frac{c\left(8a^2d^2+bc(5bc-12ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*a^2 + (b*c*(5*b*c - 12*a*d))/d^2)*x*Sqrt[c + d*x^2])/(16*d) -
 (b*(5*b*c - 12*a*d)*x^3*Sqrt[c + d*x^2])/(24*d^2) + (b^2*x^5*Sqr
t[c + d*x^2])/(6*d) - (c*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(7/2))
```

Rubi in Sympy [A] time = 34.0838, size = 138, normalized size = 0.95

$$\frac{b^2x^5\sqrt{c+dx^2}}{6d}+\frac{bx^3\sqrt{c+dx^2}\,(12ad-5bc)}{24d^2}-\frac{c\left(8a^2d^2-bc\,(12ad-5bc)\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{\frac{7}{2}}}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2-bc\,(12ad-5bc)\right)}{16d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*x**5*sqrt(c + d*x**2)/(6*d) + b*x**3*sqrt(c + d*x**2)*(12*a*
d - 5*b*c)/(24*d**2) - c*(8*a**2*d**2 - b*c*(12*a*d - 5*b*c))*ata
nh(sqrt(d)*x/sqrt(c + d*x**2))/(16*d**(7/2)) + x*sqrt(c + d*x**2)
*(8*a**2*d**2 - b*c*(12*a*d - 5*b*c))/(16*d**3)
```

Mathematica [A] time = 0.138043, size = 125, normalized size = 0.86

$$\frac{\sqrt{d}x\sqrt{c+dx^2}\left(24a^2d^2+12abd\left(2dx^2-3c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)-3c\left(8a^2d^2-12abcd+5b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}\right.}{48d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(-3*c + 2*d*x^2
) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - 3*c*(5*b^2*c^2 - 12*
a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*d^(7
/2))
```

Maple [A] time = 0.014, size = 197, normalized size = 1.4

$$\frac{a^2x}{2\,d}\sqrt{dx^2+c}-\frac{a^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+\frac{b^2x^5}{6\,d}\sqrt{dx^2+c}-\frac{5\,b^2cx^3}{24\,d^2}\sqrt{dx^2+c}+\frac{5\,b^2c^2x}{16\,d^3}\sqrt{dx^2+c}-\frac{5\,b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}+\frac{abx^3}{2\,d}\sqrt{dx^2+c}-\frac{3\,abcx}{4\,d^2}\sqrt{dx^2+c}+\frac{3\,abc^2}{4}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*a^2*x/d*(d*x^2+c)^(1/2)-1/2*a^2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2
+c)^(1/2))+1/6*b^2*x^5*(d*x^2+c)^(1/2)/d-5/24*b^2*c/d^2*x^3*(d*x^
2+c)^(1/2)+5/16*b^2*c^2/d^3*x*(d*x^2+c)^(1/2)-5/16*b^2*c^3/d^(7/2
)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a*b*x^3/d*(d*x^2+c)^(1/2)-3/4
*a*b*c/d^2*x*(d*x^2+c)^(1/2)+3/4*a*b*c^2/d^(5/2)*ln(x*d^(1/2)+(d*
x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29354, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,b^2d^2x^5-2\left(5\,b^2cd-12\,abd^2\right)x^3+3\left(5\,b^2c^2-12\,abcd+8\,a^2d^2\right)x\right)\sqrt{dx^2+c}\sqrt{d}+3\left(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2\right)\log\left(\right.}{96\,d^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*d^2*x^5 - 2*(5*b^2*c*d - 12*a*b*d^2)*x^3 + 3*(5*b
^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(
5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*log(2*sqrt(d*x^2 + c)*d*x
 - (2*d*x^2 + c)*sqrt(d)))/d^(7/2), 1/48*((8*b^2*d^2*x^5 - 2*(5*b
^2*c*d - 12*a*b*d^2)*x^3 + 3*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)
*x)*sqrt(d*x^2 + c)*sqrt(-d) - 3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^
2*c*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^3)]
```

Sympy [A] time = 39.8398, size = 301, normalized size = 2.06

$$\frac{a^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{a^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}-\frac{3abc^{\frac{3}{2}}x}{4d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{ab\sqrt{c}x^3}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^2\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{4d^{\frac{5}{2}}}+\frac{abx^5}{2\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{5}{2}}x}{16d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{3}{2}}x^3}{48d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2\sqrt{c}x^5}{24d\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{16d^{\frac{7}{2}}}+\frac{b^2x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - a**2*c*asinh(sqrt(d)*x/
sqrt(c))/(2*d**(3/2)) - 3*a*b*c**(3/2)*x/(4*d**2*sqrt(1 + d*x**2/
c)) - a*b*sqrt(c)*x**3/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asin
h(sqrt(d)*x/sqrt(c))/(4*d**(5/2)) + a*b*x**5/(2*sqrt(c)*sqrt(1 +
d*x**2/c)) + 5*b**2*c**(5/2)*x/(16*d**3*sqrt(1 + d*x**2/c)) + 5*b
**2*c**(3/2)*x**3/(48*d**2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**
5/(24*d*sqrt(1 + d*x**2/c)) - 5*b**2*c**3*asinh(sqrt(d)*x/sqrt(c)
)/(16*d**(7/2)) + b**2*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.235995, size = 182, normalized size = 1.25

$$\frac{1}{48}\left(2\left(\frac{4\,b^2x^2}{d}-\frac{5\,b^2cd^3-12\,abd^4}{d^5}\right)x^2+\frac{3\left(5\,b^2c^2d^2-12\,abcd^3+8\,a^2d^4\right)}{d^5}\right)\sqrt{dx^2+c}x+\frac{\left(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2/d - (5*b^2*c*d^3 - 12*a*b*d^4)/d^5)*x^2 + 3*(5
*b^2*c^2*d^2 - 12*a*b*c*d^3 + 8*a^2*d^4)/d^5)*sqrt(d*x^2 + c)*x +
 1/16*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*ln(abs(-sqrt(d)*x
+ sqrt(d*x^2 + c)))/d^(7/2)
```

3.639 $\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (2*b*(b*c - a*d)*(c + d*x^2
)^(3/2))/(3*d^3) + (b^2*(c + d*x^2)^(5/2))/(5*d^3)
```

Rubi [A] time = 0.163477, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (2*b*(b*c - a*d)*(c + d*x^2
)^(3/2))/(3*d^3) + (b^2*(c + d*x^2)^(5/2))/(5*d^3)
```

Rubi in Sympy [A] time = 23.1117, size = 65, normalized size = 0.88

$$\frac{b^2\left(c+dx^2\right)^{\frac{5}{2}}}{5d^3}+\frac{2b\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{3d^3}+\frac{\sqrt{c+dx^2}\,(ad-bc)^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] b**2*(c + d*x**2)**(5/2)/(5*d**3) + 2*b*(c + d*x**2)**(3/2)*(a*d
- b*c)/(3*d**3) + sqrt(c + d*x**2)*(a*d - b*c)**2/d**3
```

Mathematica [A] time = 0.0683084, size = 66, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2+10abd\left(dx^2-2c\right)+b^2\left(8c^2-4cdx^2+3d^2x^4\right)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(8*c
^2 - 4*c*d*x^2 + 3*d^2*x^4)))/(15*d^3)
```

Maple [A] time = 0.009, size = 69, normalized size = 0.9

$$\frac{3\,b^2d^2x^4+10\,abd^2x^2-4\,b^2cdx^2+15\,a^2d^2-20\,cabd+8\,b^2c^2}{15\,d^3}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(3*b^2*d^2*x^4+10*a*b*d^2*x^2-4*b^2*c*d*x^2+
15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244498, size = 92, normalized size = 1.24

$$\frac{\left(3\,b^2d^2x^4 + 8\,b^2c^2 - 20\,abcd + 15\,a^2d^2 - 2\left(2\,b^2cd - 5\,abd^2\right)x^2\right)\sqrt{dx^2 + c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*
b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [A] time = 3.10816, size = 158, normalized size = 2.14

$$\begin{cases} \frac{a^2\sqrt{c+dx^2}}{d} - \frac{4abc\sqrt{c+dx^2}}{3d^2} + \frac{2abx^2\sqrt{c+dx^2}}{3d} + \frac{8b^2c^2\sqrt{c+dx^2}}{15d^3} - \frac{4b^2cx^2\sqrt{c+dx^2}}{15d^2} + \frac{b^2x^4\sqrt{c+dx^2}}{5d} & \text{for}\ d \neq 0 \\ \frac{\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*sqrt(c + d*x**2)/d - 4*a*b*c*sqrt(c + d*x**2)/(3*
d**2) + 2*a*b*x**2*sqrt(c + d*x**2)/(3*d) + 8*b**2*c**2*sqrt(c +
d*x**2)/(15*d**3) - 4*b**2*c*x**2*sqrt(c + d*x**2)/(15*d**2) + b*
*2*x**4*sqrt(c + d*x**2)/(5*d), Ne(d, 0)), ((a**2*x**2/2 + a*b*x*
*4/2 + b**2*x**6/6)/sqrt(c), True))
```

GIAC/XCAS [A] time = 0.232259, size = 132, normalized size = 1.78

$$\frac{3\left(dx^2 + c\right)^{\frac{5}{2}}b^2 - 10\left(dx^2 + c\right)^{\frac{3}{2}}b^2c + 15\sqrt{dx^2 + c}b^2c^2 + 10\left(dx^2 + c\right)^{\frac{3}{2}}abd - 30\sqrt{dx^2 + c}abcd + 15\sqrt{dx^2 + c}a^2d^2}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b^2 - 10*(d*x^2 + c)^(3/2)*b^2*c + 15*s
qrt(d*x^2 + c)*b^2*c^2 + 10*(d*x^2 + c)^(3/2)*a*b*d - 30*sqrt(d*x
^2 + c)*a*b*c*d + 15*sqrt(d*x^2 + c)*a^2*d^2)/d^3
```

3.640 $\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=107

$$\frac{(8a^2d^2-8abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}$$

```
[Out] (-3*b*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(8*d^2) + (b*x*(a + b*x^2)
*Sqrt[c + d*x^2])/(4*d) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(5/2))
```

Rubi [A] time = 0.150103, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{(8a^2d^2-8abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*b*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(8*d^2) + (b*x*(a + b*x^2)
*Sqrt[c + d*x^2])/(4*d) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(5/2))
```

Rubi in Sympy [A] time = 20.3841, size = 102, normalized size = 0.95

$$\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}+\frac{3bx\sqrt{c+dx^2}(2ad-bc)}{8d^2}+\frac{(8a^2d^2-8abcd+3b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x*(a + b*x**2)*sqrt(c + d*x**2)/(4*d) + 3*b*x*sqrt(c + d*x**2)*
(2*a*d - b*c)/(8*d**2) + (8*a**2*d**2 - 8*a*b*c*d + 3*b**2*c**2)*
atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*d**(5/2))
```

Mathematica [A] time = 0.088147, size = 91, normalized size = 0.85

$$\frac{(8a^2d^2-8abcd+3b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+b\sqrt{d}x\sqrt{c+dx^2}(8ad-3bc+2bdx^2)}{8d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(-3*b*c + 8*a*d + 2*b*d*x^2) + (3*b^
2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]
)/(8*d^(5/2))
```

Maple [A] time = 0.011, size = 131, normalized size = 1.2

$$a^{2} \ln \left(x\sqrt{d}+\sqrt{d x^{2}+c}\right) \frac{1}{\sqrt{d}}+\frac{b^{2} x^{3}}{4 d} \sqrt{d x^{2}+c}-\frac{3 b^{2} c x}{8 d^{2}} \sqrt{d x^{2}+c} +\frac{3 b^{2} c^{2}}{8} \ln \left(x\sqrt{d}+\sqrt{d x^{2}+c}\right) d^{-\frac{5}{2}}+\frac{a b x}{d} \sqrt{d x^{2}+c}-a b c \ln \left(x\sqrt{d}+\sqrt{d x^{2}+c}\right) d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] a^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)+1/4*b^2*x^3/d*(d*x^2+c)
^(1/2)-3/8*b^2*c/d^2*x*(d*x^2+c)^(1/2)+3/8*b^2*c^2/d^(5/2)*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))+a*b*x/d*(d*x^2+c)^(1/2)-a*b*c/d^(3/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264206, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2 b^{2} d x^{3}-\left(3 b^{2} c-8 a b d\right) x\right) \sqrt{d x^{2}+c} \sqrt{d}+\left(3 b^{2} c^{2}-8 a b c d+8 a^{2} d^{2}\right) \log \left(-2 \sqrt{d x^{2}+c} d x-\left(2 d x^{2}+c\right) \sqrt{d}\right)}{16 d^{\frac{5}{2}}}, \frac{\left(2 b^{2} d x^{3}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b^2*d*x^3 - (3*b^2*c - 8*a*b*d)*x)*sqrt(d*x^2 + c)*sq
rt(d) + (3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*log(-2*sqrt(d*x^2 + c
)*d*x - (2*d*x^2 + c)*sqrt(d)))/d^(5/2), 1/8*((2*b^2*d*x^3 - (3*b
^2*c - 8*a*b*d)*x)*sqrt(d*x^2 + c)*sqrt(-d) + (3*b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*d^
2)]
```

Sympy [A] time = 20.3893, size = 238, normalized size = 2.22

$$a^{2}\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}} \operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\: c > 0 \wedge d < 0 \\ \frac{\sqrt{\frac{c}{d}} \operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\: c > 0 \wedge d > 0 \\ \frac{\sqrt{-\frac{c}{d}} \operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\: d > 0 \wedge c < 0 \end{cases}\right)+\frac{a b \sqrt{c} x \sqrt{1+\frac{d x^{2}}{c}}}{d}-\frac{a b c \operatorname{asinh}\left(\frac{\sqrt{d} x}{\sqrt{c}}\right)}{d^{\frac{3}{2}}} -\frac{3 b^{2} c^{\frac{3}{2}} x}{8 d^{2} \sqrt{1+\frac{d x^{2}}{c}}}-\frac{b^{2} \sqrt{c} x^{3}}{8 d \sqrt{1+\frac{d x^{2}}{c}}}+\frac{3 b^{2} c^{2} \operatorname{asinh}\left(\frac{\sqrt{d} x}{\sqrt{c}}\right)}{8 d^{\frac{5}{2}}}+\frac{b^{2} x^{5}}{4 \sqrt{c} \sqrt{1+\frac{d x^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*Piecewise((sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) &
(d < 0)), (sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0
)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))/sqrt(-c), (d > 0) & (c < 0)))
 + a*b*sqrt(c)*x*sqrt(1 + d*x**2/c)/d - a*b*c*asinh(sqrt(d)*x/sqr
t(c))/d**(3/2) - 3*b**2*c**(3/2)*x/(8*d**2*sqrt(1 + d*x**2/c)) -
b**2*sqrt(c)*x**3/(8*d*sqrt(1 + d*x**2/c)) + 3*b**2*c**2*asinh(sq
rt(d)*x/sqrt(c))/(8*d**(5/2)) + b**2*x**5/(4*sqrt(c)*sqrt(1 + d*x
**2/c))
```

GIAC/XCAS [A] time = 0.242061, size = 123, normalized size = 1.15

$$\frac{1}{8}\left(\frac{2\,b^2x^2}{d} - \frac{3\,b^2cd - 8\,abd^2}{d^3}\right)\sqrt{dx^2 + c}x - \frac{\left(3\,b^2c^2 - 8\,abcd + 8\,a^2d^2\right)\ln\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right)}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*x^2/d - (3*b^2*c*d - 8*a*b*d^2)/d^3)*sqrt(d*x^2 + c)*x
 - 1/8*(3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ln(abs(-sqrt(d)*x + sq
rt(d*x^2 + c)))/d^(5/2)
```

3.641 $\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}(bc-2ad)}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}$$

```
[Out] -((b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2) + (b^2*(c + d*x^2)^(3/2)
)/(3*d^2) - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Rubi [A] time = 0.198181, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}(bc-2ad)}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2) + (b^2*(c + d*x^2)^(3/2)
)/(3*d^2) - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Rubi in Sympy [A] time = 22.4573, size = 66, normalized size = 0.88

$$-\frac{a^2\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{b^2\left(c+dx^2\right)^{\frac{3}{2}}}{3d^2}+\frac{b\sqrt{c+dx^2}\,(2ad-bc)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*atanh(sqrt(c + d*x**2)/sqrt(c))/sqrt(c) + b**2*(c + d*x**2)
**(3/2)/(3*d**2) + b*sqrt(c + d*x**2)*(2*a*d - b*c)/d**2
```

Mathematica [A] time = 0.130006, size = 76, normalized size = 1.01

$$-\frac{a^2\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{\sqrt{c}}+\frac{a^2\log(x)}{\sqrt{c}}+\frac{b\sqrt{c+dx^2}\left(6ad-2bc+bdx^2\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[c + d*x^2]*(-2*b*c + 6*a*d + b*d*x^2))/(3*d^2) + (a^2*Log
[x])/Sqrt[c] - (a^2*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/Sqrt[c]
```

Maple [A] time = 0.015, size = 87, normalized size = 1.2

$$-a^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}+\frac{b^2x^2}{3\,d}\sqrt{dx^2+c}-\frac{2\,b^2c}{3\,d^2}\sqrt{dx^2+c}+2\,\frac{\sqrt{dx^2+c}ab}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x)
```

```
[Out] -a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/3*b^2*x^2/d*
(d*x^2+c)^(1/2)-2/3*b^2*c/d^2*(d*x^2+c)^(1/2)+2*a*b/d*(d*x^2+c)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258776, size = 1, normalized size = 0.01

$$\left[\frac{3\,a^2d^2\log\left(-\frac{(dx^2+2\,c)\,\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)+2\left(b^2dx^2-2\,b^2c+6\,abd\right)\sqrt{dx^2+c}\sqrt{c}}{6\,\sqrt{c}d^2},\right.$$
$$\left.-\frac{3\,a^2d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(b^2dx^2-2\,b^2c+6\,abd\right)\sqrt{dx^2+c}\sqrt{-c}}{3\,\sqrt{-c}d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*d^2*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c
)/x^2) + 2*(b^2*d*x^2 - 2*b^2*c + 6*a*b*d)*sqrt(d*x^2 + c)*sqrt(c
))/(sqrt(c)*d^2), -1/3*(3*a^2*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) - (b^2*d*x^2 - 2*b^2*c + 6*a*b*d)*sqrt(d*x^2 + c)*sqrt(-c))/(sq
rt(-c)*d^2)]
```

Sympy [A] time = 21.7103, size = 167, normalized size = 2.23

$$a^2\left(\begin{cases}\frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\sqrt{c+dx^2}}\right)}{c\sqrt{-\frac{1}{c}}} & \text{for}\ -\frac{1}{c}>0\\ -\frac{\operatorname{acoth}\left(\frac{1}{\sqrt{c+dx^2}\sqrt{\frac{1}{c}}}\right)}{c\sqrt{\frac{1}{c}}} & \text{for}\ -\frac{1}{c}<0\wedge\frac{1}{c}<\frac{1}{c+dx^2}\\ -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{c+dx^2}\sqrt{\frac{1}{c}}}\right)}{c\sqrt{\frac{1}{c}}} & \text{for}\ \frac{1}{c}>\frac{1}{c+dx^2}\wedge-\frac{1}{c}<0\end{cases}\right)+\frac{b^2\left(c+dx^2\right)^{\frac{3}{2}}}{3d^2}+\frac{b\sqrt{c+dx^2}\left(2ad-bc\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*Piecewise((atan(1/(sqrt(-1/c)*sqrt(c + d*x**2)))/(c*sqrt(-1/
c)), -1/c > 0), (-acoth(1/(sqrt(c + d*x**2)*sqrt(1/c)))/(c*sqrt(1
```

```
/c)), (-1/c < 0) & (1/c < 1/(c + d*x**2))), (-atanh(1/(sqrt(c + d
*x**2)*sqrt(1/c)))/(c*sqrt(1/c)), (-1/c < 0) & (1/c > 1/(c + d*x*
*2)))) + b**2*(c + d*x**2)**(3/2)/(3*d**2) + b*sqrt(c + d*x**2)*(
2*a*d - b*c)/d**2
```

GIAC/XCAS [A] time = 0.236444, size = 111, normalized size = 1.48

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\left(dx^2+c\right)^{\frac{3}{2}} b^2 d^4 - 3\sqrt{dx^2+c}\,b^2 c d^4 + 6\sqrt{dx^2+c}\,cabd^5}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/3*((d*x^2 + c)^
(3/2)*b^2*d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 + 6*sqrt(d*x^2 + c)*a
*b*d^5)/d^6
```

3.642 $\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

```
[Out] -((a^2*Sqrt[c + d*x^2])/(c*x)) + (b^2*x*Sqrt[c + d*x^2])/(2*d) -
(b*(b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2)
)
```

Rubi [A] time = 0.14273, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(c*x)) + (b^2*x*Sqrt[c + d*x^2])/(2*d) -
(b*(b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2)
)
```

Rubi in Sympy [A] time = 21.1221, size = 70, normalized size = 0.85

$$-\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}+\frac{b\,(4ad-bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(c + d*x**2)/(c*x) + b**2*x*sqrt(c + d*x**2)/(2*d) + b*
(4*a*d - b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*d**(3/2))
```

Mathematica [A] time = 0.0958954, size = 76, normalized size = 0.93

$$\sqrt{c+dx^2}\left(\frac{b^2x}{2d}-\frac{a^2}{cx}\right)-\frac{b(bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(a^2/(c*x)) + (b^2*x)/(2*d))*Sqrt[c + d*x^2] - (b*(b*c - 4*a*d)
*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Maple [A] time = 0.015, size = 88, normalized size = 1.1

$$\frac{b^2x}{2\,d}\sqrt{dx^2+c}-\frac{b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}-\frac{a^2}{cx}\sqrt{dx^2+c}+2\,\frac{ab\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*b^2*x*(d*x^2+c)^(1/2)/d-1/2*b^2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2
+c)^(1/2))-a^2*(d*x^2+c)^(1/2)/c/x+2*a*b*ln(x*d^(1/2)+(d*x^2+c)^(
1/2))/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257743, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2c^2-4\,abcd\right)x\log\left(-2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)-2\left(b^2cx^2-2\,a^2d\right)\sqrt{dx^2+c}\sqrt{d}}{4\,cd^{\frac{3}{2}}x},\ -\frac{\left(b^2c^2-4\,abcd\right)x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(b^2cx^2-2\,a^2d\right)\sqrt{dx^2+c}\sqrt{-d}}{2\,c\sqrt{-d}dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2 - 4*a*b*c*d)*x*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*
x^2 + c)*sqrt(d)) - 2*(b^2*c*x^2 - 2*a^2*d)*sqrt(d*x^2 + c)*sqrt(
d))/(c*d^(3/2)*x), -1/2*((b^2*c^2 - 4*a*b*c*d)*x*arctan(sqrt(-d)*
x/sqrt(d*x^2 + c)) - (b^2*c*x^2 - 2*a^2*d)*sqrt(d*x^2 + c)*sqrt(-
d))/(c*sqrt(-d)*d*x)]
```

Sympy [A] time = 10.679, size = 155, normalized size = 1.89

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+2ab\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\ d>0\wedge c<0\end{cases}\right)+\frac{b^2\sqrt{c}x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + 2*a*b*Piecewise((sqrt(-c/d
)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asin
h(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*s
qrt(-d/c))/sqrt(-c), (d > 0) & (c < 0))) + b**2*sqrt(c)*x*sqrt(1
+ d*x**2/c)/(2*d) - b**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*d**(3/2))
```

GIAC/XCAS [A] time = 0.248599, size = 126, normalized size = 1.54

$$\frac{\sqrt{dx^2 + c}b^2x}{2\,d} + \frac{2\,a^2\sqrt{d}}{\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 - c} + \frac{\left(b^2c\sqrt{d} - 4\,abd^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x/d + 2*a^2*sqrt(d)/((sqrt(d)*x - sqrt(d*
x^2 + c))^2 - c) + 1/4*(b^2*c*sqrt(d) - 4*a*b*d^(3/2))*ln((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/d^2
```

3.643 $\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

```
[Out] (b^2*Sqrt[c + d*x^2])/d - (a^2*Sqrt[c + d*x^2])/(2*c*x^2) - (a*(4
*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rubi [A] time = 0.226071, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b^2*Sqrt[c + d*x^2])/d - (a^2*Sqrt[c + d*x^2])/(2*c*x^2) - (a*(4
*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rubi in Sympy [A] time = 22.1397, size = 68, normalized size = 0.85

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{a\left(ad-4bc\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(c + d*x**2)/(2*c*x**2) + a*(a*d - 4*b*c)*atanh(sqrt(c
+ d*x**2)/sqrt(c))/(2*c**(3/2)) + b**2*sqrt(c + d*x**2)/d
```

Mathematica [A] time = 0.174317, size = 92, normalized size = 1.15

$$\sqrt{c+dx^2}\left(\frac{b^2}{d}-\frac{a^2}{2cx^2}\right)+\frac{a(ad-4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{2c^{3/2}}-\frac{a\log(x)(ad-4bc)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b^2/d - a^2/(2*c*x^2))*Sqrt[c + d*x^2] - (a*(-4*b*c + a*d)*Log[x
])/(2*c^(3/2)) + (a*(-4*b*c + a*d)*Log[c + Sqrt[c]*Sqrt[c + d*x^2
]])/(2*c^(3/2))
```

Maple [A] time = 0.015, size = 100, normalized size = 1.3

$$\frac{b^2}{d}\sqrt{dx^2+c}-\frac{a^2}{2\,cx^2}\sqrt{dx^2+c}+\frac{a^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}-2\,\frac{ab}{\sqrt{c}}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] b^2*(d*x^2+c)^(1/2)/d-1/2*a^2*(d*x^2+c)^(1/2)/c/x^2+1/2*a^2*d/c^(
3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-2*a*b/c^(1/2)*ln((2*c+
2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260459, size = 1, normalized size = 0.01

$$\left[-\frac{\left(4\,abcd-a^2d^2\right)x^2\log\left(-\frac{\left(dx^2+2\,c\right)\sqrt{c}+2\,\sqrt{dx^2+c}c}{x^2}\right)-2\left(2\,b^2cx^2-a^2d\right)\sqrt{dx^2+c}\sqrt{c}}{4\,c^{\frac{3}{2}}dx^2},\right.$$
$$\left.-\frac{\left(4\,abcd-a^2d^2\right)x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(2\,b^2cx^2-a^2d\right)\sqrt{dx^2+c}\sqrt{-c}}{2\,\sqrt{-c}cdx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b*c*d - a^2*d^2)*x^2*log(-((d*x^2 + 2*c)*sqrt(c) + 2*
sqrt(d*x^2 + c)*c)/x^2) - 2*(2*b^2*c*x^2 - a^2*d)*sqrt(d*x^2 + c)
*sqrt(c))/(c^(3/2)*d*x^2), -1/2*((4*a*b*c*d - a^2*d^2)*x^2*arctan
(sqrt(-c)/sqrt(d*x^2 + c)) - (2*b^2*c*x^2 - a^2*d)*sqrt(d*x^2 + c
)*sqrt(-c))/(sqrt(-c)*c*d*x^2)]
```

Sympy [A] time = 42.8166, size = 99, normalized size = 1.24

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx}+\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2c^{\frac{3}{2}}}-\frac{2ab\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}}+b^2\left(\begin{cases}\frac{x^2}{2\sqrt{c}} & \text{for}\ d=0\\ \frac{\sqrt{c+dx^2}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + a**2*d*asinh(sqrt(c)
/(sqrt(d)*x))/(2*c**(3/2)) - 2*a*b*asinh(sqrt(c)/(sqrt(d)*x))/sqr
t(c) + b**2*Piecewise((x**2/(2*sqrt(c)), Eq(d, 0)), (sqrt(c + d*x
**2)/d, True))
```

GIAC/XCAS [A] time = 0.231535, size = 109, normalized size = 1.36

$$\frac{2\sqrt{dx^2+c}b^2 - \frac{\sqrt{dx^2+c}a^2d}{cx^2} + \frac{(4\,abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b^2 - sqrt(d*x^2 + c)*a^2*d/(c*x^2) + (4*a
*b*c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c))/
d
```

3.644 $\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*c*x^3) - (2*a*(3*b*c - a*d)*Sqrt[c + d*
x^2])/(3*c^2*x) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt
[d]
```

Rubi [A] time = 0.184589, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*c*x^3) - (2*a*(3*b*c - a*d)*Sqrt[c + d*
x^2])/(3*c^2*x) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt
[d]
```

Rubi in Sympy [A] time = 22.214, size = 75, normalized size = 0.89

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}+\frac{2a\sqrt{c+dx^2}\,(ad-3bc)}{3c^2x}+\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(c + d*x**2)/(3*c*x**3) + 2*a*sqrt(c + d*x**2)*(a*d - 3
*b*c)/(3*c**2*x) + b**2*atanh(sqrt(d)*x/sqrt(c + d*x**2))/sqrt(d)
```

Mathematica [A] time = 0.117537, size = 72, normalized size = 0.86

$$\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-\frac{a\sqrt{c+dx^2}\,\left(a\left(c-2dx^2\right)+6bcx^2\right)}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[c + d*x^2]*(6*b*c*x^2 + a*(c - 2*d*x^2)))/(3*c^2*x^3) +
(b^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d]
```

Maple [A] time = 0.015, size = 85, normalized size = 1.

$$b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}-\frac{a^2}{3\,cx^3}\sqrt{dx^2+c}+\frac{2\,a^2d}{3\,c^2x}\sqrt{dx^2+c}-2\,\frac{\sqrt{dx^2+c}ab}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/3*a^2*(d*x^2+c)^(1/2)
/c/x^3+2/3*a^2*d/c^2/x*(d*x^2+c)^(1/2)-2*a*b/c/x*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312928, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^2c^2x^3\log\left(-2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)-2\,\left(a^2c+2\,\left(3\,abc-a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{d}}{6\,c^2\sqrt{d}x^3},\frac{3\,b^2c^2x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(a^2}{3\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^2*c^2*x^3*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sq
rt(d)) - 2*(a^2*c + 2*(3*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt
(d))/(c^2*sqrt(d)*x^3), 1/3*(3*b^2*c^2*x^3*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (a^2*c + 2*(3*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c)*
sqrt(-d))/(c^2*sqrt(-d)*x^3)]
```

Sympy [A] time = 5.53914, size = 158, normalized size = 1.88

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+b^2\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\: c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\: c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\: d>0\wedge c<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 2*a**2*d**(3/2)*s
qrt(c/(d*x**2) + 1)/(3*c**2) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)
/c + b**2*Piecewise((sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c >
0) & (d < 0)), (sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(c), (c > 0) & (
d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))/sqrt(-c), (d > 0) & (c <
 0)))
```

GIAC/XCAS [A] time = 0.252527, size = 211, normalized size = 2.51

$$-\frac{b^2\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\sqrt{d}}+\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ab\sqrt{d}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}+3\,abc^2\sqrt{d}-a^2cd^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] -1/2*b^2*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d) + 4/3*(3*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a*b*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a*b*c*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2
*d^(3/2) + 3*a*b*c^2*sqrt(d) - a^2*c*d^(3/2))/((sqrt(d)*x - sqrt(
d*x^2 + c))^2 - c)^3
```

3.645 $\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{8c^2x^2}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(4*c*x^4) - (a*(8*b*c - 3*a*d)*Sqrt[c + d*
x^2])/(8*c^2*x^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[
Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rubi [A] time = 0.294867, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(4*c*x^4) - (a*(8*b*c - 3*a*d)*Sqrt[c + d*
x^2])/(8*c^2*x^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[
Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rubi in Sympy [A] time = 24.5153, size = 95, normalized size = 0.9

$$-\frac{a^2\sqrt{c+dx^2}}{4cx^4}+\frac{a\sqrt{c+dx^2}\,(3ad-8bc)}{8c^2x^2}-\frac{(ad\,(3ad-8bc)+8b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(c + d*x**2)/(4*c*x**4) + a*sqrt(c + d*x**2)*(3*a*d - 8
*b*c)/(8*c**2*x**2) - (a*d*(3*a*d - 8*b*c) + 8*b**2*c**2)*atanh(s
qrt(c + d*x**2)/sqrt(c))/(8*c**(5/2))
```

Mathematica [A] time = 0.150055, size = 126, normalized size = 1.19

$$\frac{x^4\log(x)\left(3a^2d^2-8abcd+8b^2c^2\right)+x^4\left(-3a^2d^2+8abcd-8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+a\sqrt{c}\sqrt{c+dx^2}\left(-2ac+3adx^2-8\right.}{8c^{5/2}x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c]*Sqrt[c + d*x^2]*(-2*a*c - 8*b*c*x^2 + 3*a*d*x^2) + (8*
b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x^4*Log[x] + (-8*b^2*c^2 + 8*a*b
*c*d - 3*a^2*d^2)*x^4*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/(8*c^(5/2
)*x^4)
```

Maple [A] time = 0.016, size = 157, normalized size = 1.5

$$-\frac{a^2}{4\,c x^4}\sqrt{d x^2+c}+\frac{3\,a^2 d}{8\,c^2 x^2}\sqrt{d x^2+c}-\frac{3\,a^2 d^2}{8}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{d x^2+c}\right)\right)c^{-\frac{5}{2}}$$
$$-b^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{d x^2+c}\right)\right)\frac{1}{\sqrt{c}}-\frac{ab}{c x^2}\sqrt{d x^2+c}+abd\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{d x^2+c}\right)\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(1/2)/c/x^4+3/8*a^2*d/c^2/x^2*(d*x^2+c)^(1/2)-
3/8*a^2*d^2/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b^2/c^(
1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-a*b/c/x^2*(d*x^2+c)^(1
/2)+a*b*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261159, size = 1, normalized size = 0.01

$$\left[\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)x^4\log\left(-\frac{(dx^2+2\,c)\,\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)-2\left(2\,a^2c+\left(8\,abc-3\,a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}}{16\,c^{\frac{5}{2}}x^4},\right.$$
$$\left.-\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)x^4\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(2\,a^2c+\left(8\,abc-3\,a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{-c}}{8\,\sqrt{-c}c^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x^4*log(-((d*x^2 + 2*c
)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2) - 2*(2*a^2*c + (8*a*b*c - 3
*a^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c))/(c^(5/2)*x^4), -1/8*((8*b^2
*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (2*a^2*c + (8*a*b*c - 3*a^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c))
/(sqrt(-c)*c^2*x^4)]
```

Sympy [A] time = 74.4372, size = 178, normalized size = 1.68

$$-\frac{a^2}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2\sqrt{d}}{8cx^3\sqrt{\frac{c}{dx^2}+1}}+\frac{3a^2d^{\frac{3}{2}}}{8c^2x\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8c^{\frac{5}{2}}}$$
$$-\frac{ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{cx}+\frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{c^{\frac{3}{2}}}-\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(8*c*x
**3*sqrt(c/(d*x**2) + 1)) + 3*a**2*d**(3/2)/(8*c**2*x*sqrt(c/(d*x
**2) + 1)) - 3*a**2*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(5/2))
- a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(c*x) + a*b*d*asinh(sqrt(c)/(s
qrt(d)*x))/c**(3/2) - b**2*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c)
```

GIAC/XCAS [A] time = 0.239426, size = 189, normalized size = 1.78

$$\frac{\frac{\left(8\,b^{2}c^{2}d-8\,abcd^{2}+3\,a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^{2}}-\frac{8\,\left(dx^{2}+c\right)^{\frac{3}{2}}abcd^{2}-8\,\sqrt{dx^{2}+c}abc^{2}d^{2}-3\,\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}+5\,\sqrt{dx^{2}+c}a^{2}cd^{3}}{c^{2}d^{2}x^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*((8*b^2*c^2*d - 8*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x^2 +
c)/sqrt(-c))/(sqrt(-c)*c^2) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*
sqrt(d*x^2 + c)*a*b*c^2*d^2 - 3*(d*x^2 + c)^(3/2)*a^2*d^3 + 5*sqr
t(d*x^2 + c)*a^2*c*d^3)/(c^2*d^2*x^4))/d
```

3.646 $\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{\sqrt{c+dx^2}\left(15b^2c^2-4ad(5bc-2ad)\right)}{15c^3x}-\frac{2a\sqrt{c+dx^2}(5bc-2ad)}{15c^2x^3}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*c*x^5) - (2*a*(5*b*c - 2*a*d)*Sqrt[c +
d*x^2])/(15*c^2*x^3) - ((15*b^2*c^2 - 4*a*d*(5*b*c - 2*a*d))*Sqrt
[c + d*x^2])/(15*c^3*x)
```

Rubi [A] time = 0.224181, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\sqrt{c+dx^2}\left(8a^2d^2-20abcd+15b^2c^2\right)}{15c^3x}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a\sqrt{c+dx^2}(5bc-2ad)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*c*x^5) - (2*a*(5*b*c - 2*a*d)*Sqrt[c +
d*x^2])/(15*c^2*x^3) - ((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Sqr
t[c + d*x^2])/(15*c^3*x)
```

Rubi in Sympy [A] time = 22.9235, size = 92, normalized size = 0.93

$$-\frac{a^2\sqrt{c+dx^2}}{5cx^5}+\frac{2a\sqrt{c+dx^2}\,(2ad-5bc)}{15c^2x^3}-\frac{\sqrt{c+dx^2}\left(4ad\,(2ad-5bc)+15b^2c^2\right)}{15c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(c + d*x**2)/(5*c*x**5) + 2*a*sqrt(c + d*x**2)*(2*a*d -
 5*b*c)/(15*c**2*x**3) - sqrt(c + d*x**2)*(4*a*d*(2*a*d - 5*b*c)
+ 15*b**2*c**2)/(15*c**3*x)
```

Mathematica [A] time = 0.0769646, size = 74, normalized size = 0.75

$$-\frac{\sqrt{c+dx^2}\left(a^2\left(3c^2-4cdx^2+8d^2x^4\right)+10abcx^2\left(c-2dx^2\right)+15b^2c^2x^4\right)}{15c^3x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) +
a^2*(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4)))/(15*c^3*x^5)
```

Maple [A] time = 0.01, size = 78, normalized size = 0.8

$$-\frac{8\,x^4a^2d^2-20\,x^4abcd+15\,x^4b^2c^2-4\,x^2a^2cd+10\,ac^2bx^2+3\,a^2c^2}{15\,x^5c^3}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(8*a^2*d^2*x^4-20*a*b*c*d*x^4+15*b^2*c^2*x^
4-4*a^2*c*d*x^2+10*a*b*c^2*x^2+3*a^2*c^2)/x^5/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256527, size = 99, normalized size = 1.

$$-\frac{\left(\left(15\,b^2c^2-20\,abcd+8\,a^2d^2\right)x^4+3\,a^2c^2+2\left(5\,abc^2-2\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^6),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*x^4 + 3*a^2*c^2 + 2*
(5*a*b*c^2 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*x^5)
```

Sympy [A] time = 9.15655, size = 391, normalized size = 3.95

$$-\frac{3a^2c^4d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2a^2c^3d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{3a^2c^2d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{12a^2cd^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{8a^2d^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{4abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**4*d**(9/2)*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 3
0*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 2*a**2*c**3*d**(11/2)*x**
2*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 1
5*c**3*d**6*x**8) - 3*a**2*c**2*d**(13/2)*x**4*sqrt(c/(d*x**2) +
1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) -
12*a**2*c*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4
+ 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 8*a**2*d**(17/2)*x**8*
sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*
c**3*d**6*x**8) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) +
 4*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/(3*c**2) - b**2*sqrt(d)*sqrt
(c/(d*x**2) + 1)/c
```

GIAC/XCAS [A] time = 0.242088, size = 421, normalized size = 4.25

$$2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c\sqrt{d}+60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6abd^{\frac{3}{2}}+90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^2\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^6),x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*sqrt(d) - 60*(sqrt(d
)*x - sqrt(d*x^2 + c))^6*b^2*c*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x
^2 + c))^6*a*b*d^(3/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c
^2*sqrt(d) - 140*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*d^(3/2) +
80*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(5/2) - 60*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 100*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a*b*c^2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*
c*d^(5/2) + 15*b^2*c^4*sqrt(d) - 20*a*b*c^3*d^(3/2) + 8*a^2*c^2*d
^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.647 $\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=151

$$\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{\sqrt{c+dx^2}\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^3x^2}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}(12bc-5ad)}{24c^2x^4}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(6*c*x^6) - (a*(12*b*c - 5*a*d)*Sqrt[c + d
*x^2])/(24*c^2*x^4) - ((8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[
c + d*x^2])/(16*c^3*x^2) + (d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(7/2))
```

Rubi [A] time = 0.399052, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{\sqrt{c+dx^2}\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^3x^2}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}(12bc-5ad)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(6*c*x^6) - (a*(12*b*c - 5*a*d)*Sqrt[c + d
*x^2])/(24*c^2*x^4) - ((8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[
c + d*x^2])/(16*c^3*x^2) + (d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(7/2))
```

Rubi in Sympy [A] time = 29.7328, size = 138, normalized size = 0.91

$$-\frac{a^2\sqrt{c+dx^2}}{6cx^6}+\frac{a\sqrt{c+dx^2}\,(5ad-12bc)}{24c^2x^4}-\frac{\sqrt{c+dx^2}\left(ad\,(5ad-12bc)+8b^2c^2\right)}{16c^3x^2}+\frac{d\left(ad\,(5ad-12bc)+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(c + d*x**2)/(6*c*x**6) + a*sqrt(c + d*x**2)*(5*a*d - 1
2*b*c)/(24*c**2*x**4) - sqrt(c + d*x**2)*(a*d*(5*a*d - 12*b*c) +
8*b**2*c**2)/(16*c**3*x**2) + d*(a*d*(5*a*d - 12*b*c) + 8*b**2*c*
*2)*atanh(sqrt(c + d*x**2)/sqrt(c))/(16*c**(7/2))
```

Mathematica [A] time = 0.224995, size = 168, normalized size = 1.11

$$\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{16c^{7/2}}-\frac{d\log(x)\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^{7/2}}+\sqrt{c+dx^2}\left(\frac{-5a^2d^2+12abcd-8b^2c^2}{16c^3x^2}-\frac{a^2}{6cx^6}+\frac{a(5ad-12bc)}{24c^2x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] (-a^2/(6*c*x^6) + (a*(-12*b*c + 5*a*d))/(24*c^2*x^4) + (-8*b^2*c^
2 + 12*a*b*c*d - 5*a^2*d^2)/(16*c^3*x^2))*Sqrt[c + d*x^2] - (d*(8
*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[x])/(16*c^(7/2)) + (d*(8*b
^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]]
)/(16*c^(7/2))
```

Maple [A] time = 0.019, size = 224, normalized size = 1.5

$$
-\frac{a^2}{6\,cx^6}\sqrt{dx^2+c}+\frac{5\,a^2d}{24\,c^2x^4}\sqrt{dx^2+c}-\frac{5\,a^2d^2}{16\,c^3x^2}\sqrt{dx^2+c}+\frac{5\,a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{7}{2}}
-\frac{b^2}{2\,cx^2}\sqrt{dx^2+c}+\frac{b^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}-\frac{ab}{2\,cx^4}\sqrt{dx^2+c}
+\frac{3\,abd}{4\,c^2x^2}\sqrt{dx^2+c}-\frac{3\,abd^2}{4}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(1/2)/c/x^6+5/24*a^2*d/c^2/x^4*(d*x^2+c)^(1/2)
-5/16*a^2*d^2/c^3/x^2*(d*x^2+c)^(1/2)+5/16*a^2*d^3/c^(7/2)*ln((2*
c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*b^2/c/x^2*(d*x^2+c)^(1/2)+1/2
*b^2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*a*b/c/x^
4*(d*x^2+c)^(1/2)+3/4*a*b*d/c^2/x^2*(d*x^2+c)^(1/2)-3/4*a*b*d^2/c
^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294137, size = 1, normalized size = 0.01

$$
\left[\frac{3\left(8\,b^2c^2d-12\,abcd^2+5\,a^2d^3\right)x^6\log\left(-\frac{\left(dx^2+2\,c\right)\sqrt{c}+2\,\sqrt{dx^2+c}c}{x^2}\right)-2\left(3\left(8\,b^2c^2-12\,abcd+5\,a^2d^2\right)x^4+8\,a^2c^2+2\left(12\,abc^2-\right.\right.}{96\,c^{\frac{7}{2}}x^6}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^7),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*x^6*log(-((d*x^
2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2) - 2*(3*(8*b^2*c^2 -
12*a*b*c*d + 5*a^2*d^2)*x^4 + 8*a^2*c^2 + 2*(12*a*b*c^2 - 5*a^2*c
*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c))/(c^(7/2)*x^6), 1/48*(3*(8*b^2*c
^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 +
 c)) - (3*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*x^4 + 8*a^2*c^2 +
```

```
2*(12*a*b*c^2 - 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c))/(sqrt(-
c)*c^3*x^6)]
```

Sympy [A] time = 130.835, size = 301, normalized size = 1.99

$$-\frac{a^2}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2\sqrt{d}}{24cx^5\sqrt{\frac{c}{dx^2}+1}}-\frac{5a^2d^{\frac{3}{2}}}{48c^2x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{5a^2d^{\frac{5}{2}}}{16c^3x\sqrt{\frac{c}{dx^2}+1}}+\frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{7}{2}}}-\frac{ab}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}}+\frac{ab\sqrt{d}}{4cx^3\sqrt{\frac{c}{dx^2}+1}}+\frac{3abd^{\frac{3}{2}}}{4c^2x\sqrt{\frac{c}{dx^2}+1}}-\frac{3abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4c^{\frac{5}{2}}}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx}+\frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(24*c*
x**5*sqrt(c/(d*x**2) + 1)) - 5*a**2*d**(3/2)/(48*c**2*x**3*sqrt(c
/(d*x**2) + 1)) - 5*a**2*d**(5/2)/(16*c**3*x*sqrt(c/(d*x**2) + 1)
) + 5*a**2*d**3*asinh(sqrt(c)/(sqrt(d)*x))/(16*c**(7/2)) - a*b/(2
*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a*b*sqrt(d)/(4*c*x**3*sqrt(
c/(d*x**2) + 1)) + 3*a*b*d**(3/2)/(4*c**2*x*sqrt(c/(d*x**2) + 1))
 - 3*a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(5/2)) - b**2*sqrt
(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + b**2*d*asinh(sqrt(c)/(sqrt(d)*
x))/(2*c**(3/2))
```

GIAC/XCAS [A] time = 0.234078, size = 325, normalized size = 2.15

$$-\frac{\frac{3\left(8b^2c^2d^2-12abcd^3+5a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2-36\left(dx^2+c\right)^{\frac{5}{2}}abcd^3+96\left(dx^2+c\right)^{\frac{3}{2}}abc^2d^3\cdots}{c^3d^3x^6}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^7),x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 5*a^2*d^4)*arctan(sqrt(d
*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^3) + (24*(d*x^2 + c)^(5/2)*b^2*c^
2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2
*c^4*d^2 - 36*(d*x^2 + c)^(5/2)*a*b*c*d^3 + 96*(d*x^2 + c)^(3/2)*
a*b*c^2*d^3 - 60*sqrt(d*x^2 + c)*a*b*c^3*d^3 + 15*(d*x^2 + c)^(5/
2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*d^4 + 33*sqrt(d*x^2 + c)*
a^2*c^2*d^4)/(c^3*d^3*x^6))/d
```

3.648 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{9/2}}+\frac{x\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}-\frac{x^3\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{24cd^3}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^5\sqrt{c+dx^2}}{6d^2}$$

```
[Out] ((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((35*b^2*c^2 - 60*a
*b*c*d + 24*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^4) - ((35*b^2*c^2 -
 60*a*b*c*d + 24*a^2*d^2)*x^3*Sqrt[c + d*x^2])/(24*c*d^3) + (b^2*
x^5*Sqrt[c + d*x^2])/(6*d^2) - (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a
^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(9/2))
```

Rubi [A] time = 0.416537, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{9/2}}+\frac{x\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}-\frac{x^3\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{24cd^3}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^5\sqrt{c+dx^2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((35*b^2*c^2 - 60*a
*b*c*d + 24*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^4) - ((35*b^2*c^2 -
 60*a*b*c*d + 24*a^2*d^2)*x^3*Sqrt[c + d*x^2])/(24*c*d^3) + (b^2*
x^5*Sqrt[c + d*x^2])/(6*d^2) - (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a
^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(9/2))
```

Rubi in Sympy [A] time = 45.9704, size = 189, normalized size = 0.96

$$\frac{b^2x^5\sqrt{c+dx^2}}{6d^2}-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{\frac{9}{2}}}+\frac{x\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}+\frac{x^5\left(ad-bc\right)^2}{cd^2\sqrt{c+dx^2}}-\frac{x^3\sqrt{c+dx^2}\left(24a^2d^2-60abcd+35b^2c^2\right)}{24cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*x**5*sqrt(c + d*x**2)/(6*d**2) - c*(24*a**2*d**2 - 60*a*b*c*
d + 35*b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(16*d**(9/2))
 + x*sqrt(c + d*x**2)*(24*a**2*d**2 - 60*a*b*c*d + 35*b**2*c**2)/
(16*d**4) + x**5*(a*d - b*c)**2/(c*d**2*sqrt(c + d*x**2)) - x**3*
sqrt(c + d*x**2)*(24*a**2*d**2 - 60*a*b*c*d + 35*b**2*c**2)/(24*c
*d**3)
```

Mathematica [A] time = 0.256648, size = 158, normalized size = 0.8

$$\sqrt{c+dx^2}\left(\frac{x\left(8a^2d^2-28abcd+19b^2c^2\right)}{16d^4}+\frac{cx(bc-ad)^2}{d^4\left(c+dx^2\right)}-\frac{bx^3(11bc-12ad)}{24d^3}+\frac{b^2x^5}{6d^2}\right)-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{16d^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*(((19*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)*x)/(16*d^
4) - (b*(11*b*c - 12*a*d)*x^3)/(24*d^3) + (b^2*x^5)/(6*d^2) + (c*
(b*c - a*d)^2*x)/(d^4*(c + d*x^2))) - (c*(35*b^2*c^2 - 60*a*b*c*d
 + 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(16*d^(9/2))
```

Maple [A] time = 0.025, size = 263, normalized size = 1.3

$$\frac{a^2x^3}{2\,d}\frac{1}{\sqrt{dx^2+c}}+\frac{3\,a^2cx}{2\,d^2}\frac{1}{\sqrt{dx^2+c}}-\frac{3\,a^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{b^2x^7}{6\,d}\frac{1}{\sqrt{dx^2+c}}-\frac{7\,b^2cx^5}{24\,d^2}\frac{1}{\sqrt{dx^2+c}}+\frac{35\,b^2c^2x^3}{48\,d^3}\frac{1}{\sqrt{dx^2+c}}+\frac{35\,xb^2c^3}{16\,d^4}\frac{1}{\sqrt{dx^2+c}}-\frac{35\,b^2c^3}{16}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}}+\frac{abx^5}{2\,d}\frac{1}{\sqrt{dx^2+c}}-\frac{5\,abcx^3}{4\,d^2}\frac{1}{\sqrt{dx^2+c}}-\frac{15\,abc^2x}{4\,d^3}\frac{1}{\sqrt{dx^2+c}}+\frac{15\,abc^2}{4}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*a^2*x^3/d/(d*x^2+c)^(1/2)+3/2*a^2*c/d^2*x/(d*x^2+c)^(1/2)-3/2
*a^2*c/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/6*b^2*x^7/d/(d*x^2
+c)^(1/2)-7/24*b^2*c/d^2*x^5/(d*x^2+c)^(1/2)+35/48*b^2*c^2/d^3*x^
3/(d*x^2+c)^(1/2)+35/16*b^2*c^3/d^4*x/(d*x^2+c)^(1/2)-35/16*b^2*c
^3/d^(9/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*a*b*x^5/d/(d*x^2+c)^
(1/2)-5/4*a*b*c/d^2*x^3/(d*x^2+c)^(1/2)-15/4*a*b*c^2/d^3*x/(d*x^2
+c)^(1/2)+15/4*a*b*c^2/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.31597, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,b^2d^3x^7-2\left(7\,b^2cd^2-12\,abd^3\right)x^5+\left(35\,b^2c^2d-60\,abcd^2+24\,a^2d^3\right)x^3+3\left(35\,b^2c^3-60\,abc^2d+24\,a^2cd^2\right)x\right)\sqrt{dx^2+c}}{96\left(d^5x^2+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*d^3*x^7 - 2*(7*b^2*c*d^2 - 12*a*b*d^3)*x^5 + (35*
b^2*c^2*d - 60*a*b*c*d^2 + 24*a^2*d^3)*x^3 + 3*(35*b^2*c^3 - 60*a
*b*c^2*d + 24*a^2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(35*b^2*c
^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a*b*c^2*d
^2 + 24*a^2*c*d^3)*x^2)*log(2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)
*sqrt(d)))/((d^5*x^2 + c*d^4)*sqrt(d)), 1/48*((8*b^2*d^3*x^7 - 2*
(7*b^2*c*d^2 - 12*a*b*d^3)*x^5 + (35*b^2*c^2*d - 60*a*b*c*d^2 + 2
4*a^2*d^3)*x^3 + 3*(35*b^2*c^3 - 60*a*b*c^2*d + 24*a^2*c*d^2)*x)*
sqrt(d*x^2 + c)*sqrt(-d) - 3*(35*b^2*c^4 - 60*a*b*c^3*d + 24*a^2*
c^2*d^2 + (35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^2*c*d^3)*x^2)*arc
tan(sqrt(-d)*x/sqrt(d*x^2 + c)))/((d^5*x^2 + c*d^4)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a + bx^2\right)^2}{\left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.241186, size = 236, normalized size = 1.2

$$\frac{\left(\left(2\left(\frac{4\,b^2x^2}{d} - \frac{7\,b^2cd^5 - 12\,abd^6}{d^7}\right)x^2 + \frac{35\,b^2c^2d^4 - 60\,abcd^5 + 24\,a^2d^6}{d^7}\right)x^2 + \frac{3\left(35\,b^2c^3d^3 - 60\,abc^2d^4 + 24\,a^2cd^5\right)}{d^7}\right)x}{48\,\sqrt{dx^2 + c}} + \frac{\left(35\,b^2c^3 - 60\,abc^2d + 24\,a^2cd^2\right)\ln\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right)}{16\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*b^2*x^2/d - (7*b^2*c*d^5 - 12*a*b*d^6)/d^7)*x^2 + (35
*b^2*c^2*d^4 - 60*a*b*c*d^5 + 24*a^2*d^6)/d^7)*x^2 + 3*(35*b^2*c^
3*d^3 - 60*a*b*c^2*d^4 + 24*a^2*c*d^5)/d^7)*x/sqrt(d*x^2 + c) + 1
/16*(35*b^2*c^3 - 60*a*b*c^2*d + 24*a^2*c*d^2)*ln(abs(-sqrt(d)*x
+ sqrt(d*x^2 + c)))/d^(9/2)
```

3.649 $\int \frac{x^3 (a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{b (c+dx^2)^{3/2} (3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2 (c+dx^2)^{5/2}}{5d^4}$$

```
[Out] (c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + ((b*c - a*d)*(3*b*c - a
*d)*Sqrt[c + d*x^2])/d^4 - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/
(3*d^4) + (b^2*(c + d*x^2)^(5/2))/(5*d^4)
```

Rubi [A] time = 0.262291, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{b (c+dx^2)^{3/2} (3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2 (c+dx^2)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + ((b*c - a*d)*(3*b*c - a
*d)*Sqrt[c + d*x^2])/d^4 - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/
(3*d^4) + (b^2*(c + d*x^2)^(5/2))/(5*d^4)
```

Rubi in Sympy [A] time = 32.0789, size = 97, normalized size = 0.9

$$\frac{b^2 (c+dx^2)^{\frac{5}{2}}}{5d^4}+\frac{b (c+dx^2)^{\frac{3}{2}} (2ad-3bc)}{3d^4}+\frac{c (ad-bc)^2}{d^4\sqrt{c+dx^2}}+\frac{\sqrt{c+dx^2}\,(ad-3bc)\,(ad-bc)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*(c + d*x**2)**(5/2)/(5*d**4) + b*(c + d*x**2)**(3/2)*(2*a*d
- 3*b*c)/(3*d**4) + c*(a*d - b*c)**2/(d**4*sqrt(c + d*x**2)) + sq
rt(c + d*x**2)*(a*d - 3*b*c)*(a*d - b*c)/d**4
```

Mathematica [A] time = 0.102846, size = 97, normalized size = 0.9

$$\frac{15a^2d^2 (2c+dx^2)+10abd (-8c^2-4cdx^2+d^2x^4)+3b^2 (16c^3+8c^2dx^2-2cd^2x^4+d^3x^6)}{15d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (15*a^2*d^2*(2*c + d*x^2) + 10*a*b*d*(-8*c^2 - 4*c*d*x^2 + d^2*x^
4) + 3*b^2*(16*c^3 + 8*c^2*d*x^2 - 2*c*d^2*x^4 + d^3*x^6))/(15*d^
4*Sqrt[c + d*x^2])
```

Maple [A] time = 0.01, size = 108, normalized size = 1.

$$\frac{3\,b^2x^6d^3+10\,abd^3x^4-6\,b^2cd^2x^4+15\,a^2d^3x^2-40\,abcd^2x^2+24\,b^2c^2dx^2+30\,a^2cd^2-80\,abc^2d+48\,b^2c^3}{15\,d^4}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(3*b^2*d^3*x^6+10*a*b*d^3*x^4-6*b^2*c*d^2*x^4+15*a^2*d^3*x^2
-40*a*b*c*d^2*x^2+24*b^2*c^2*d*x^2+30*a^2*c*d^2-80*a*b*c^2*d+48*b
^2*c^3)/(d*x^2+c)^(1/2)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227164, size = 155, normalized size = 1.44

$$\frac{\left(3\, b^2 d^3 x^6 + 48\, b^2 c^3 - 80\, abc^2 d + 30\, a^2 cd^2 - 2\left(3\, b^2 cd^2 - 5\, abd^3\right) x^4 + \left(24\, b^2 c^2 d - 40\, abcd^2 + 15\, a^2 d^3\right) x^2\right)\sqrt{dx^2 + c}}{15\left(d^5 x^2 + cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^3*x^6 + 48*b^2*c^3 - 80*a*b*c^2*d + 30*a^2*c*d^2 -
2*(3*b^2*c*d^2 - 5*a*b*d^3)*x^4 + (24*b^2*c^2*d - 40*a*b*c*d^2 +
15*a^2*d^3)*x^2)*sqrt(d*x^2 + c)/(d^5*x^2 + c*d^4)
```

Sympy [A] time = 5.67156, size = 236, normalized size = 2.19

$$\begin{cases} \frac{2a^2c}{d^2\sqrt{c+dx^2}} + \frac{a^2x^2}{d\sqrt{c+dx^2}} - \frac{16abc^2}{3d^3\sqrt{c+dx^2}} - \frac{8abcx^2}{3d^2\sqrt{c+dx^2}} + \frac{2abx^4}{3d\sqrt{c+dx^2}} + \frac{16b^2c^3}{5d^4\sqrt{c+dx^2}} + \frac{8b^2c^2x^2}{5d^3\sqrt{c+dx^2}} - \frac{2b^2cx^4}{5d^2\sqrt{c+dx^2}} + \frac{b^2x^6}{5d\sqrt{c+dx^2}} & \text{for}\ d \neq 0 \\ \frac{\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((2*a**2*c/(d**2*sqrt(c + d*x**2)) + a**2*x**2/(d*sqrt(c
 + d*x**2)) - 16*a*b*c**2/(3*d**3*sqrt(c + d*x**2)) - 8*a*b*c*x**
2/(3*d**2*sqrt(c + d*x**2)) + 2*a*b*x**4/(3*d*sqrt(c + d*x**2)) +
 16*b**2*c**3/(5*d**4*sqrt(c + d*x**2)) + 8*b**2*c**2*x**2/(5*d**
3*sqrt(c + d*x**2)) - 2*b**2*c*x**4/(5*d**2*sqrt(c + d*x**2)) + b
**2*x**6/(5*d*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2*x**4/4 + a*b*x
**6/3 + b**2*x**8/8)/c**(3/2), True))
```

GIAC/XCAS [A] time = 0.232378, size = 180, normalized size = 1.67

$$\frac{3\left(dx^2 + c\right)^{\frac{5}{2}} b^2 - 15\left(dx^2 + c\right)^{\frac{3}{2}} b^2 c + 45\sqrt{dx^2 + c}\, b^2 c^2 + 10\left(dx^2 + c\right)^{\frac{3}{2}} abd - 60\sqrt{dx^2 + c}\, abcd + 15\sqrt{dx^2 + c}\, a^2 d^2 + \frac{15\left(b^2 c^3 - 2\right.}{\sqrt{}}}{15\, d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b^2 - 15*(d*x^2 + c)^(3/2)*b^2*c + 45*s
qrt(d*x^2 + c)*b^2*c^2 + 10*(d*x^2 + c)^(3/2)*a*b*d - 60*sqrt(d*x
^2 + c)*a*b*c*d + 15*sqrt(d*x^2 + c)*a^2*d^2 + 15*(b^2*c^3 - 2*a*
b*c^2*d + a^2*c*d^2)/sqrt(d*x^2 + c))/d^4
```

3.650 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{(8a^2d^2-24abcd+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-24abcd+15b^2c^2)}{8cd^3}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}$$

```
[Out] ((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 24*a
*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^3) + (b^2*x^3*Sqrt[
c + d*x^2])/(4*d^2) + ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(7/2))
```

Rubi [A] time = 0.336539, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{(8a^2d^2-24abcd+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-24abcd+15b^2c^2)}{8cd^3}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 24*a
*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^3) + (b^2*x^3*Sqrt[
c + d*x^2])/(4*d^2) + ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(7/2))
```

Rubi in Sympy [A] time = 44.444, size = 143, normalized size = 0.94

$$\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}+\frac{(8a^2d^2-24abcd+15b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{\frac{7}{2}}}+\frac{x^3(ad-bc)^2}{cd^2\sqrt{c+dx^2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-24abcd+15b^2c^2)}{8cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*x**3*sqrt(c + d*x**2)/(4*d**2) + (8*a**2*d**2 - 24*a*b*c*d +
 15*b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*d**(7/2)) + x
**3*(a*d - b*c)**2/(c*d**2*sqrt(c + d*x**2)) - x*sqrt(c + d*x**2)
*(8*a**2*d**2 - 24*a*b*c*d + 15*b**2*c**2)/(8*c*d**3)
```

Mathematica [A] time = 0.18136, size = 124, normalized size = 0.82

$$\frac{(8a^2d^2-24abcd+15b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{7/2}}+\sqrt{c+dx^2}\left(-\frac{x(ad-bc)^2}{d^3(c+dx^2)}-\frac{bx(7bc-8ad)}{8d^3}+\frac{b^2x^3}{4d^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(b*(7*b*c - 8*a*d)*x)/(8*d^3) + (b^2*x^3)/(4*d^
2) - ((-(b*c) + a*d)^2*x)/(d^3*(c + d*x^2))) + ((15*b^2*c^2 - 24*
a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(7/
2))
```

Maple [A] time = 0.015, size = 192, normalized size = 1.3

$$-\frac{a^2x}{d}\frac{1}{\sqrt{dx^2+c}}+a^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}+\frac{b^2x^5}{4d}\frac{1}{\sqrt{dx^2+c}}-\frac{5b^2cx^3}{8d^2}\frac{1}{\sqrt{dx^2+c}}-\frac{15b^2c^2x}{8d^3}\frac{1}{\sqrt{dx^2+c}}+\frac{15b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}+\frac{abx^3}{d}\frac{1}{\sqrt{dx^2+c}}+3\frac{abcx}{d^2\sqrt{dx^2+c}}-3\frac{abc\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -a^2*x/d/(d*x^2+c)^(1/2)+a^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)
)+1/4*b^2*x^5/d/(d*x^2+c)^(1/2)-5/8*b^2*c/d^2*x^3/(d*x^2+c)^(1/2)
-15/8*b^2*c^2/d^3*x/(d*x^2+c)^(1/2)+15/8*b^2*c^2/d^(7/2)*ln(x*d^(
1/2)+(d*x^2+c)^(1/2))+a*b*x^3/d/(d*x^2+c)^(1/2)+3*a*b*c/d^2*x/(d*
x^2+c)^(1/2)-3*a*b*c/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250528, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2b^2d^2x^5-\left(5b^2cd-8abd^2\right)x^3-\left(15b^2c^2-24abcd+8a^2d^2\right)x\right)\sqrt{dx^2+c}\sqrt{d}+\left(15b^2c^3-24abc^2d+8a^2cd^2+\left(15b^2c\right.\right.}{16\left(d^4x^2+cd^3\right)\sqrt{d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b^2*d^2*x^5 - (5*b^2*c*d - 8*a*b*d^2)*x^3 - (15*b^2*c
^2 - 24*a*b*c*d + 8*a^2*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + (15*b^2
*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*d^2
+ 8*a^2*d^3)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt
(d)))/((d^4*x^2 + c*d^3)*sqrt(d)), 1/8*((2*b^2*d^2*x^5 - (5*b^2*c
*d - 8*a*b*d^2)*x^3 - (15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*x)*sq
rt(d*x^2 + c)*sqrt(-d) + (15*b^2*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2
 + (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^3)*x^2)*arctan(sqrt(-d)
*x/sqrt(d*x^2 + c)))/((d^4*x^2 + c*d^3)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + bx^2\right)^2}{\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.238108, size = 177, normalized size = 1.16

$$\frac{\left(\left(\frac{2\,b^2x^2}{d} - \frac{5\,b^2cd^3-8\,abd^4}{d^5}\right)x^2 - \frac{15\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4}{d^5}\right)x}{8\,\sqrt{dx^2+c}} - \frac{\left(15\,b^2c^2 - 24\,abcd + 8\,a^2d^2\right)\ln\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^2*x^2/d - (5*b^2*c*d^3 - 8*a*b*d^4)/d^5)*x^2 - (15*b^2*
c^2*d^2 - 24*a*b*c*d^3 + 8*a^2*d^4)/d^5)*x/sqrt(d*x^2 + c) - 1/8*
(15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ln(abs(-sqrt(d)*x + sqrt(d*
x^2 + c)))/d^(7/2)
```

3.651 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2b\sqrt{c+dx^2}(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^3}$$

```
[Out] -((b*c - a*d)^2/(d^3*Sqrt[c + d*x^2])) - (2*b*(b*c - a*d)*Sqrt[c
+ d*x^2])/d^3 + (b^2*(c + d*x^2)^(3/2))/(3*d^3)
```

Rubi [A] time = 0.158262, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2b\sqrt{c+dx^2}(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -((b*c - a*d)^2/(d^3*Sqrt[c + d*x^2])) - (2*b*(b*c - a*d)*Sqrt[c
+ d*x^2])/d^3 + (b^2*(c + d*x^2)^(3/2))/(3*d^3)
```

Rubi in Sympy [A] time = 23.3565, size = 63, normalized size = 0.86

$$\frac{b^2\left(c+dx^2\right)^{\frac{3}{2}}}{3d^3}+\frac{2b\sqrt{c+dx^2}\,(ad-bc)}{d^3}-\frac{(ad-bc)^2}{d^3\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] b**2*(c + d*x**2)**(3/2)/(3*d**3) + 2*b*sqrt(c + d*x**2)*(a*d - b
*c)/d**3 - (a*d - b*c)**2/(d**3*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.0747515, size = 65, normalized size = 0.89

$$\frac{-3a^2d^2+6abd\left(2c+dx^2\right)+b^2\left(-8c^2-4cdx^2+d^2x^4\right)}{3d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*d^2 + 6*a*b*d*(2*c + d*x^2) + b^2*(-8*c^2 - 4*c*d*x^2 + d
^2*x^4))/(3*d^3*Sqrt[c + d*x^2])
```

Maple [A] time = 0.008, size = 69, normalized size = 1.

$$-\frac{-b^2d^2x^4-6\,abd^2x^2+4\,b^2cdx^2+3\,a^2d^2-12\,cabd+8\,b^2c^2}{3\,d^3}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-b^2*d^2*x^4-6*a*b*d^2*x^2+4*b^2*c*d*x^2+3*a^2*d^2-12*a*b*c
*d+8*b^2*c^2)/(d*x^2+c)^(1/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228579, size = 107, normalized size = 1.47

$$\frac{\left(b^2d^2x^4 - 8\,b^2c^2 + 12\,abcd - 3\,a^2d^2 - 2\left(2\,b^2cd - 3\,abd^2\right)x^2\right)\sqrt{dx^2 + c}}{3\left(d^4x^2 + cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^2*x^4 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*
c*d - 3*a*b*d^2)*x^2)*sqrt(d*x^2 + c)/(d^4*x^2 + c*d^3)
```

Sympy [A] time = 3.54453, size = 155, normalized size = 2.12

$$\begin{cases} -\frac{a^2}{d\sqrt{c+dx^2}} + \frac{4abc}{d^2\sqrt{c+dx^2}} + \frac{2abx^2}{d\sqrt{c+dx^2}} - \frac{8b^2c^2}{3d^3\sqrt{c+dx^2}} - \frac{4b^2cx^2}{3d^2\sqrt{c+dx^2}} + \frac{b^2x^4}{3d\sqrt{c+dx^2}} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-a**2/(d*sqrt(c + d*x**2)) + 4*a*b*c/(d**2*sqrt(c + d*
x**2)) + 2*a*b*x**2/(d*sqrt(c + d*x**2)) - 8*b**2*c**2/(3*d**3*sq
rt(c + d*x**2)) - 4*b**2*c*x**2/(3*d**2*sqrt(c + d*x**2)) + b**2*
x**4/(3*d*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/
2 + b**2*x**6/6)/c**(3/2), True))
```

GIAC/XCAS [A] time = 0.235683, size = 108, normalized size = 1.48

$$\frac{\left(dx^2 + c\right)^{\frac{3}{2}}b^2 - 6\sqrt{dx^2 + c}b^2c + 6\sqrt{dx^2 + c}abd - \frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)}{\sqrt{dx^2+c}}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((d*x^2 + c)^(3/2)*b^2 - 6*sqrt(d*x^2 + c)*b^2*c + 6*sqrt(d*x
^2 + c)*a*b*d - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/sqrt(d*x^2 + c)
)/d^3
```

3.652 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}+\frac{bx\sqrt{c+dx^2}(3bc-2ad)}{2cd^2}-\frac{x\left(a+bx^2\right)(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[Out] -(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(3*b*c
- 2*a*d)*x*Sqrt[c + d*x^2])/(2*c*d^2) - (b*(3*b*c - 4*a*d)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(5/2))
```

Rubi [A] time = 0.142381, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}+\frac{bx\sqrt{c+dx^2}(3bc-2ad)}{2cd^2}-\frac{x\left(a+bx^2\right)(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(3*b*c
- 2*a*d)*x*Sqrt[c + d*x^2])/(2*c*d^2) - (b*(3*b*c - 4*a*d)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(5/2))
```

Rubi in Sympy [A] time = 21.6608, size = 95, normalized size = 0.9

$$\frac{b\left(4ad-3bc\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{\frac{5}{2}}}-\frac{bx\sqrt{c+dx^2}\left(2ad-3bc\right)}{2cd^2}+\frac{x\left(a+bx^2\right)\left(ad-bc\right)}{cd\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] b*(4*a*d - 3*b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*d**(5/2))
- b*x*sqrt(c + d*x**2)*(2*a*d - 3*b*c)/(2*c*d**2) + x*(a + b*x**2
)*(a*d - b*c)/(c*d*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.162037, size = 93, normalized size = 0.88

$$\sqrt{c+dx^2}\left(\frac{x(bc-ad)^2}{cd^2\left(c+dx^2\right)}+\frac{b^2x}{2d^2}\right)-\frac{b(3bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*((b^2*x)/(2*d^2) + ((b*c - a*d)^2*x)/(c*d^2*(c +
d*x^2))) - (b*(3*b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])
/(2*d^(5/2))
```

Maple [A] time = 0.012, size = 123, normalized size = 1.2

$$\frac{a^2x}{c}\frac{1}{\sqrt{dx^2+c}}+\frac{b^2x^3}{2\,d}\frac{1}{\sqrt{dx^2+c}}+\frac{3\,b^2cx}{2\,d^2}\frac{1}{\sqrt{dx^2+c}}-\frac{3\,b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}-2\frac{abx}{d\sqrt{dx^2+c}}+2\frac{ab\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] a^2*x/c/(d*x^2+c)^(1/2)+1/2*b^2*x^3/d/(d*x^2+c)^(1/2)+3/2*b^2*c/d
^2*x/(d*x^2+c)^(1/2)-3/2*b^2*c/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/
2))-2*a*b*x/d/(d*x^2+c)^(1/2)+2*a*b/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.233874, size = 1, normalized size = 0.01

$$\left[\frac{2\left(b^2cdx^3+\left(3\,b^2c^2-4\,abcd+2\,a^2d^2\right)x\right)\sqrt{dx^2+c}\sqrt{d}-\left(3\,b^2c^3-4\,abc^2d+\left(3\,b^2c^2d-4\,abcd^2\right)x^2\right)\log\left(-2\sqrt{dx^2+c}dx-\right.}{4\left(cd^3x^2+c^2d^2\right)\sqrt{d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c*d*x^3 + (3*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2)*x)*sqr
t(d*x^2 + c)*sqrt(d) - (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d -
4*a*b*c*d^2)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt
(d)))/((c*d^3*x^2 + c^2*d^2)*sqrt(d)), 1/2*((b^2*c*d*x^3 + (3*b^2
*c^2 - 4*a*b*c*d + 2*a^2*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) - (3*b^
2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*arctan(sqr
t(-d)*x/sqrt(d*x^2 + c)))/((c*d^3*x^2 + c^2*d^2)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.24809, size = 124, normalized size = 1.17

$$\frac{\left(\frac{b^2x^2}{d} + \frac{3\,b^2c^2d-4\,abcd^2+2\,a^2d^3}{cd^3}\right)x}{2\,\sqrt{dx^2+c}} + \frac{\left(3\,b^2c - 4\,abd\right)\ln\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{2\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2/d + (3*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)/(c*d^3))
*x/sqrt(d*x^2 + c) + 1/2*(3*b^2*c - 4*a*b*d)*ln(abs(-sqrt(d)*x +
sqrt(d*x^2 + c)))/d^(5/2)
```

3.653 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

```
[Out] (b*c - a*d)^2/(c*d^2*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^2
 - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

Rubi [A] time = 0.206644, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c - a*d)^2/(c*d^2*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^2
 - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

Rubi in Sympy [A] time = 36.416, size = 88, normalized size = 1.17

$$\frac{a^2\sqrt{c+dx^2}}{c^2}-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\sqrt{c+dx^2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)+\frac{(ad-bc)^2}{cd^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x/(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*sqrt(c + d*x**2)/c**2 - a**2*atanh(sqrt(c + d*x**2)/sqrt(c))
/c**(3/2) - sqrt(c + d*x**2)*(a**2/c**2 - b**2/d**2) + (a*d - b*c
)**2/(c*d**2*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.190104, size = 88, normalized size = 1.17

$$-\frac{a^2\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{c^{3/2}}+\frac{a^2\log(x)}{c^{3/2}}+\sqrt{c+dx^2}\left(\frac{(bc-ad)^2}{cd^2\left(c+dx^2\right)}+\frac{b^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(b^2/d^2 + (b*c - a*d)^2/(c*d^2*(c + d*x^2))) + (
a^2*Log[x])/c^(3/2) - (a^2*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/c^(3
/2)
```

Maple [A] time = 0.014, size = 102, normalized size = 1.4

$$\frac{a^2}{c}\frac{1}{\sqrt{dx^2+c}}-a^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}+\frac{b^2x^2}{d}\frac{1}{\sqrt{dx^2+c}}+2\,\frac{b^2c}{d^2\sqrt{dx^2+c}}-2\,\frac{ab}{d\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x)
```

```
[Out] a^2/c/(d*x^2+c)^(1/2)-a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/
2))/x)+b^2*x^2/d/(d*x^2+c)^(1/2)+2*b^2*c/d^2/(d*x^2+c)^(1/2)-2*a*
b/d/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237702, size = 1, normalized size = 0.01

$$\left[\frac{2\left(b^2cdx^2+2\,b^2c^2-2\,abcd+a^2d^2\right)\sqrt{dx^2+c}\sqrt{c}+\left(a^2d^3x^2+a^2cd^2\right)\log\left(-\frac{(dx^2+2\,c)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)}{2\left(cd^3x^2+c^2d^2\right)\sqrt{c}},\ \frac{\left(b^2cdx^2+2\,b^2c^2-2\,abc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c*d*x^2 + 2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^
2 + c)*sqrt(c) + (a^2*d^3*x^2 + a^2*c*d^2)*log(-((d*x^2 + 2*c)*sq
rt(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/((c*d^3*x^2 + c^2*d^2)*sqrt(c)
), ((b^2*c*d*x^2 + 2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 +
c)*sqrt(-c) - (a^2*d^3*x^2 + a^2*c*d^2)*arctan(sqrt(-c)/sqrt(d*x^
2 + c)))/((c*d^3*x^2 + c^2*d^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.236193, size = 111, normalized size = 1.48

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{\sqrt{dx^2+c}b^2}{d^2}+\frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) + sqrt(d*x^2 +
c)*b^2/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c*d
^2)
```

3.654 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(b^2c^2-2ad(bc-ad)\right)}{c^2d\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

```
[Out] -(a^2/(c*x*Sqrt[c + d*x^2])) - ((b^2*c^2 - 2*a*d*(b*c - a*d))*x)/
(c^2*d*Sqrt[c + d*x^2]) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/d^(3/2)
```

Rubi [A] time = 0.184729, antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)}{\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(a^2/(c*x*Sqrt[c + d*x^2])) - ((b^2/d - (2*a*(b*c - a*d))/c^2)*x
)/Sqrt[c + d*x^2] + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^
(3/2)
```

Rubi in Sympy [A] time = 21.8182, size = 78, normalized size = 0.86

$$-\frac{a^2}{cx\sqrt{c+dx^2}}+\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{\frac{3}{2}}}-\frac{x\left(2ad\left(ad-bc\right)+b^2c^2\right)}{c^2d\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**2/(c*x*sqrt(c + d*x**2)) + b**2*atanh(sqrt(d)*x/sqrt(c + d*x*
*2))/d**(3/2) - x*(2*a*d*(a*d - b*c) + b**2*c**2)/(c**2*d*sqrt(c
+ d*x**2))
```

Mathematica [A] time = 0.138154, size = 81, normalized size = 0.89

$$\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{3/2}}-\frac{\sqrt{c+dx^2}\left(a^2+\frac{x^2(bc-ad)^2}{d(c+dx^2)}\right)}{c^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[c + d*x^2]*(a^2 + ((b*c - a*d)^2*x^2)/(d*(c + d*x^2))))/(
c^2*x)) + (b^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^(3/2)
```

Maple [A] time = 0.014, size = 99, normalized size = 1.1

$$-\frac{b^2x}{d}\frac{1}{\sqrt{dx^2+c}}+b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}-\frac{a^2}{cx}\frac{1}{\sqrt{dx^2+c}}-2\frac{a^2dx}{c^2\sqrt{dx^2+c}}+2\frac{abx}{c\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2*x/d/(d*x^2+c)^(1/2)+b^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)
)-a^2/c/x/(d*x^2+c)^(1/2)-2*a^2*d/c^2*x/(d*x^2+c)^(1/2)+2*a*b*x/c
/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236213, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(a^2cd+\left(b^2c^2-2\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{d}-\left(b^2c^2dx^3+b^2c^3x\right)\log\left(-2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)}{2\left(c^2d^2x^3+c^3dx\right)\sqrt{d}},\right.$$
$$\left.-\frac{\left(a^2cd+\left(b^2c^2-2\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{-d}-\left(b^2c^2dx^3+b^2c^3x\right)\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{\left(c^2d^2x^3+c^3dx\right)\sqrt{-d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d
*x^2 + c)*sqrt(d) - (b^2*c^2*d*x^3 + b^2*c^3*x)*log(-2*sqrt(d*x^2
 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((c^2*d^2*x^3 + c^3*d*x)*sqrt
(d)), -((a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*
x^2 + c)*sqrt(-d) - (b^2*c^2*d*x^3 + b^2*c^3*x)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)))/((c^2*d^2*x^3 + c^3*d*x)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.24361, size = 140, normalized size = 1.54

$$-\frac{b^2\ln\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{2\,d^{\frac{3}{2}}} + \frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 - c\right)c} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{\sqrt{dx^2 + c}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2) + 2*a^2*sqrt
(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*c) - (b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*x/(sqrt(d*x^2 + c)*c^2*d)
```

3.655 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=103

$$\frac{-\frac{3a^2d}{c} + 4ab - \frac{2b^2c}{d}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} - \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

```
[Out] (4*a*b - (2*b^2*c)/d - (3*a^2*d)/c)/(2*c*Sqrt[c + d*x^2]) - a^2/(
2*c*x^2*Sqrt[c + d*x^2]) - (a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rubi [A] time = 0.283707, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{-\frac{3a^2d}{c} + 4ab - \frac{2b^2c}{d}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} - \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] (4*a*b - (2*b^2*c)/d - (3*a^2*d)/c)/(2*c*Sqrt[c + d*x^2]) - a^2/(
2*c*x^2*Sqrt[c + d*x^2]) - (a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rubi in Sympy [A] time = 24.6581, size = 92, normalized size = 0.89

$$-\frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{a(3ad-4bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{\frac{5}{2}}} - \frac{\frac{ad(3ad-4bc)}{2} + b^2c^2}{c^2d\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**2/(2*c*x**2*sqrt(c + d*x**2)) + a*(3*a*d - 4*b*c)*atanh(sqrt(
c + d*x**2)/sqrt(c))/(2*c**(5/2)) - (a*d*(3*a*d - 4*b*c)/2 + b**2
*c**2)/(c**2*d*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.250312, size = 105, normalized size = 1.02

$$\frac{-\sqrt{c}\sqrt{c+dx^2}\left(\frac{a^2}{x^2} + \frac{2(bc-ad)^2}{d(c+dx^2)}\right) + a(3ad-4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right) - a\log(x)(3ad-4bc)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(Sqrt[c]*Sqrt[c + d*x^2]*(a^2/x^2 + (2*(b*c - a*d)^2)/(d*(c + d
*x^2)))) - a*(-4*b*c + 3*a*d)*Log[x] + a*(-4*b*c + 3*a*d)*Log[c +
 Sqrt[c]*Sqrt[c + d*x^2]])/(2*c^(5/2))
```

Maple [A] time = 0.016, size = 135, normalized size = 1.3

$$-\frac{b^2}{d}\frac{1}{\sqrt{dx^2+c}}-\frac{a^2}{2cx^2}\frac{1}{\sqrt{dx^2+c}}-\frac{3a^2d}{2c^2}\frac{1}{\sqrt{dx^2+c}}+\frac{3a^2d}{2}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}} +2\frac{ab}{c\sqrt{dx^2+c}}-2\frac{ab}{c^{3/2}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2/d/(d*x^2+c)^(1/2)-1/2*a^2/c/x^2/(d*x^2+c)^(1/2)-3/2*a^2*d/c^
2/(d*x^2+c)^(1/2)+3/2*a^2*d/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(
1/2))/x)+2*a*b/c/(d*x^2+c)^(1/2)-2*a*b/c^(3/2)*ln((2*c+2*c^(1/2)*
(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243675, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(a^2cd+\left(2b^2c^2-4abcd+3a^2d^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}+\left(\left(4abcd^2-3a^2d^3\right)x^4+\left(4abc^2d-3a^2cd^2\right)x^2\right)\log\left(-\frac{(dx^2+2c)\sqrt{c}+}{x^2}\right.}{4\left(c^2d^2x^4+c^3dx^2\right)\sqrt{c}}, -\frac{\left(a^2cd+\left(2b^2c^2-4abcd+3a^2d^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{-c}+\left(\left(4abcd^2-3a^2d^3\right)x^4+\left(4abc^2d-3a^2cd^2\right)x^2\right)\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2\left(c^2d^2x^4+c^3dx^2\right)\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^2*c*d + (2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt
(d*x^2 + c)*sqrt(c) + ((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2
*d - 3*a^2*c*d^2)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2
 + c)*c)/x^2))/((c^2*d^2*x^4 + c^3*d*x^2)*sqrt(c)), -1/2*((a^2*c*
d + (2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt
(-c) + ((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*d^
2)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((c^2*d^2*x^4 + c^3*d*x
^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.258964, size = 189, normalized size = 1.83

$$\frac{\left(4\,abc - 3\,a^2 d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}c^2} - \frac{2\left(dx^2+c\right)b^2c^2 - 2\,b^2c^3 - 4\left(dx^2+c\right)abcd + 4\,abc^2d + 3\left(dx^2+c\right)a^2d^2 - 2\,a^2cd^2}{2\left(\left(dx^2+c\right)^{\frac{3}{2}} - \sqrt{dx^2+c}c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c
)*c^2) - 1/2*(2*(d*x^2 + c)*b^2*c^2 - 2*b^2*c^3 - 4*(d*x^2 + c)*a
*b*c*d + 4*a*b*c^2*d + 3*(d*x^2 + c)*a^2*d^2 - 2*a^2*c*d^2)/(((d*
x^2 + c)^(3/2) - sqrt(d*x^2 + c)*c)*c^2*d)
```

3.656 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2}{3cx^3\sqrt{c+dx^2}}+\frac{x\left(3b^2c^2-4ad(3bc-2ad)\right)}{3c^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

```
[Out] -a^2/(3*c*x^3*Sqrt[c + d*x^2]) - (2*a*(3*b*c - 2*a*d))/(3*c^2*x*S
qrt[c + d*x^2]) + ((3*b^2*c^2 - 4*a*d*(3*b*c - 2*a*d))*x)/(3*c^3*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.210615, antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x\left(8a^2d^2-12abcd+3b^2c^2\right)}{3c^3\sqrt{c+dx^2}}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(3*c*x^3*Sqrt[c + d*x^2]) - (2*a*(3*b*c - 2*a*d))/(3*c^2*x*S
qrt[c + d*x^2]) + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x)/(3*c^3
*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 21.0801, size = 90, normalized size = 0.93

$$-\frac{a^2}{3cx^3\sqrt{c+dx^2}}+\frac{2a\,(2ad-3bc)}{3c^2x\sqrt{c+dx^2}}+\frac{x\left(4ad\,(2ad-3bc)+3b^2c^2\right)}{3c^3\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**2/(3*c*x**3*sqrt(c + d*x**2)) + 2*a*(2*a*d - 3*b*c)/(3*c**2*x
*sqrt(c + d*x**2)) + x*(4*a*d*(2*a*d - 3*b*c) + 3*b**2*c**2)/(3*c
**3*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.114484, size = 66, normalized size = 0.68

$$\frac{\sqrt{c+dx^2}\left(-a^2c+ax^2(5ad-6bc)+\frac{3x^4(bc-ad)^2}{c+dx^2}\right)}{3c^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a^2*c) + a*(-6*b*c + 5*a*d)*x^2 + (3*(b*c - a
*d)^2*x^4)/(c + d*x^2)))/(3*c^3*x^3)
```

Maple [A] time = 0.009, size = 77, normalized size = 0.8

$$-\frac{-8\,x^4a^2d^2+12\,x^4abcd-3\,x^4b^2c^2-4\,x^2a^2cd+6\,ac^2bx^2+a^2c^2}{3\,x^3c^3}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-8*a^2*d^2*x^4+12*a*b*c*d*x^4-3*b^2*c^2*x^4-4*a^2*c*d*x^2+6
*a*b*c^2*x^2+a^2*c^2)/(d*x^2+c)^(1/2)/x^3/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235962, size = 115, normalized size = 1.19

$$\frac{\left(\left(3\,b^2c^2 - 12\,abcd + 8\,a^2d^2\right)x^4 - a^2c^2 - 2\left(3\,abc^2 - 2\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{3\left(c^3dx^5 + c^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^4 - a^2*c^2 - 2*(3*a*
b*c^2 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d*x^5 + c^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^4\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.244736, size = 269, normalized size = 2.77

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{\sqrt{dx^2+c}\,c^3} + \frac{2\left(6\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 abc\sqrt{d} - 3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 a^2d^{\frac{3}{2}} - 12\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 abc^2\sqrt{d} + 12\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 a^2c\right.}{3\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c\right)^3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(sqrt(d*x^2 + c)*c^3) + 2/3*(6*
(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sq
rt(d*x^2 + c))^4*a^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a*b*c^2*sqrt(d) + 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/
2) + 6*a*b*c^3*sqrt(d) - 5*a^2*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d
*x^2 + c))^2 - c)^3*c^2)
```

3.657 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}-\frac{(8b^2c^2-3ad(8bc-5ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{8b^2c^2-3ad(8bc-5ad)}{8c^3\sqrt{c+dx^2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

```
[Out] (8*b^2*c^2 - 3*a*d*(8*b*c - 5*a*d))/(8*c^3*Sqrt[c + d*x^2]) - a^2
/(4*c*x^4*Sqrt[c + d*x^2]) - (a*(8*b*c - 5*a*d))/(8*c^2*x^2*Sqrt[
c + d*x^2]) - ((8*b^2*c^2 - 3*a*d*(8*b*c - 5*a*d))*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rubi [A] time = 0.43879, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}+\frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}}-\frac{(8b^2c^2-3ad(8bc-5ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (8*b^2 - (3*a*d*(8*b*c - 5*a*d))/c^2)/(8*c*Sqrt[c + d*x^2]) - a^2
/(4*c*x^4*Sqrt[c + d*x^2]) - (a*(8*b*c - 5*a*d))/(8*c^2*x^2*Sqrt[
c + d*x^2]) - ((8*b^2*c^2 - 3*a*d*(8*b*c - 5*a*d))*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rubi in Sympy [A] time = 29.7308, size = 136, normalized size = 0.94

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}+\frac{a\,(5ad-8bc)}{8c^2x^2\sqrt{c+dx^2}}+\frac{3ad\,(5ad-8bc)+8b^2c^2}{8c^3\sqrt{c+dx^2}}-\frac{(3ad\,(5ad-8bc)+8b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**2/(4*c*x**4*sqrt(c + d*x**2)) + a*(5*a*d - 8*b*c)/(8*c**2*x**
2*sqrt(c + d*x**2)) + (3*a*d*(5*a*d - 8*b*c) + 8*b**2*c**2)/(8*c*
*3*sqrt(c + d*x**2)) - (3*a*d*(5*a*d - 8*b*c) + 8*b**2*c**2)*atan
h(sqrt(c + d*x**2)/sqrt(c))/(8*c**(7/2))
```

Mathematica [A] time = 0.319067, size = 143, normalized size = 0.99

$$\frac{-\left(15a^2d^2-24abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+\log(x)\left(15a^2d^2-24abcd+8b^2c^2\right)+\sqrt{c}\sqrt{c+dx^2}\left(-\frac{2a^2c}{x^4}+\frac{a(7ad-8bc)}{x^2}\right.}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[c]*Sqrt[c + d*x^2]*((-2*a^2*c)/x^4 + (a*(-8*b*c + 7*a*d))/x
^2 + (8*(b*c - a*d)^2)/(c + d*x^2)) + (8*b^2*c^2 - 24*a*b*c*d + 1
5*a^2*d^2)*Log[x] - (8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*Log[c +
```

```
Sqrt[c]*Sqrt[c + d*x^2]])/(8*c^(7/2))
```

Maple [A] time = 0.018, size = 211, normalized size = 1.5

$$-\frac{a^2}{4\,cx^4}\frac{1}{\sqrt{dx^2+c}}+\frac{5\,a^2d}{8\,c^2x^2}\frac{1}{\sqrt{dx^2+c}}+\frac{15\,a^2d^2}{8\,c^3}\frac{1}{\sqrt{dx^2+c}}-\frac{15\,a^2d^2}{8}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{7}{2}}+\frac{b^2}{c}\frac{1}{\sqrt{dx^2+c}}-b^2\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}-\frac{ab}{cx^2}\frac{1}{\sqrt{dx^2+c}}-3\,\frac{abd}{c^2\sqrt{dx^2+c}}+3\,\frac{abd}{c^{5/2}}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4*a^2/c/x^4/(d*x^2+c)^(1/2)+5/8*a^2*d/c^2/x^2/(d*x^2+c)^(1/2)+
15/8*a^2*d^2/c^3/(d*x^2+c)^(1/2)-15/8*a^2*d^2/c^(7/2)*ln((2*c+2*c
^(1/2)*(d*x^2+c)^(1/2))/x)+b^2/c/(d*x^2+c)^(1/2)-b^2/c^(3/2)*ln((
2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-a*b/c/x^2/(d*x^2+c)^(1/2)-3*a*b
*d/c^2/(d*x^2+c)^(1/2)+3*a*b*d/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c
)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242625, size = 1, normalized size = 0.01

$$\left[\frac{2\left(\left(8\,b^2c^2-24\,abcd+15\,a^2d^2\right)x^4-2\,a^2c^2-\left(8\,abc^2-5\,a^2cd\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}+\left(\left(8\,b^2c^2d-24\,abcd^2+15\,a^2d^3\right)x^6+\left(8\,b^2\right.\right.}{16\left(c^3dx^6+c^4x^4\right)\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^5),x, algorithm="fricas")
```

```
[Out] [1/16*(2*((8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*x^4 - 2*a^2*c^2 -
 (8*a*b*c^2 - 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + ((8*b^2*c
^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b*c^2*d
 + 15*a^2*c*d^2)*x^4)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2
+ c)*c)/x^2))/((c^3*d*x^6 + c^4*x^4)*sqrt(c)), 1/8*(((8*b^2*c^2 -
 24*a*b*c*d + 15*a^2*d^2)*x^4 - 2*a^2*c^2 - (8*a*b*c^2 - 5*a^2*c*
d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) - ((8*b^2*c^2*d - 24*a*b*c*d^2 +
 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4)
*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((c^3*d*x^6 + c^4*x^4)*sqrt(-c
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.243229, size = 220, normalized size = 1.52

$$\frac{\left(8\,b^2c^2-24\,abcd+15\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^3}+\frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}c^3}-\frac{8\,\left(dx^2+c\right)^{\frac{3}{2}}abcd-8\,\sqrt{dx^2+c}abc^2d-7\,\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2+9\,\sqrt{dx^2+c}a^2cd^2}{8\,c^3d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*arctan(sqrt(d*x^2 + c)/
sqrt(-c))/(sqrt(-c)*c^3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(
d*x^2 + c)*c^3) - 1/8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2
 + c)*a*b*c^2*d - 7*(d*x^2 + c)^(3/2)*a^2*d^2 + 9*sqrt(d*x^2 + c)
*a^2*c*d^2)/(c^3*d^2*x^4)
```

3.658 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{2dx\left(15b^2c^2-8ad(5bc-3ad)\right)}{15c^4\sqrt{c+dx^2}}-\frac{15b^2c^2-8ad(5bc-3ad)}{15c^3x\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

```
[Out] -a^2/(5*c*x^5*Sqrt[c + d*x^2]) - (2*a*(5*b*c - 3*a*d))/(15*c^2*x^
3*Sqrt[c + d*x^2]) - (15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))/(15*c^3
*x*Sqrt[c + d*x^2]) - (2*d*(15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*x
)/(15*c^4*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.30583, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}}-\frac{2dx\left(15b^2c^2-8ad(5bc-3ad)\right)}{15c^4\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(5*c*x^5*Sqrt[c + d*x^2]) - (2*a*(5*b*c - 3*a*d))/(15*c^2*x^
3*Sqrt[c + d*x^2]) - (15*b^2 - (8*a*d*(5*b*c - 3*a*d))/c^2)/(15*c
*x*Sqrt[c + d*x^2]) - (2*d*(15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*x
)/(15*c^4*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 26.2176, size = 136, normalized size = 0.96

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}+\frac{2a\,(3ad-5bc)}{15c^2x^3\sqrt{c+dx^2}}-\frac{8ad\,(3ad-5bc)+15b^2c^2}{15c^3x\sqrt{c+dx^2}}-\frac{2dx\left(8ad\,(3ad-5bc)+15b^2c^2\right)}{15c^4\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**2/(5*c*x**5*sqrt(c + d*x**2)) + 2*a*(3*a*d - 5*b*c)/(15*c**2*
x**3*sqrt(c + d*x**2)) - (8*a*d*(3*a*d - 5*b*c) + 15*b**2*c**2)/(
15*c**3*x*sqrt(c + d*x**2)) - 2*d*x*(8*a*d*(3*a*d - 5*b*c) + 15*b
**2*c**2)/(15*c**4*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.121269, size = 105, normalized size = 0.74

$$\sqrt{c+dx^2}\left(\frac{-33a^2d^2+50abcd-15b^2c^2}{15c^4x}-\frac{a^2}{5c^2x^5}-\frac{dx(bc-ad)^2}{c^4\left(c+dx^2\right)}+\frac{a(9ad-10bc)}{15c^3x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-a^2/(5*c^2*x^5) + (a*(-10*b*c + 9*a*d))/(15*c^3
*x^3) + (-15*b^2*c^2 + 50*a*b*c*d - 33*a^2*d^2)/(15*c^4*x) - (d*(
b*c - a*d)^2*x)/(c^4*(c + d*x^2)))
```

Maple [A] time = 0.011, size = 117, normalized size = 0.8

$$-\frac{48\,a^{2}d^{3}x^{6}-80\,abcd^{2}x^{6}+30\,b^{2}c^{2}dx^{6}+24\,a^{2}cd^{2}x^{4}-40\,abc^{2}dx^{4}+15\,b^{2}c^{3}x^{4}-6\,a^{2}c^{2}dx^{2}+10\,abc^{3}x^{2}+3\,a^{2}c^{3}}{15\,x^{5}c^{4}}\frac{1}{\sqrt{dx^{2}+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/15*(48*a^2*d^3*x^6-80*a*b*c*d^2*x^6+30*b^2*c^2*d*x^6+24*a^2*c*
d^2*x^4-40*a*b*c^2*d*x^4+15*b^2*c^3*x^4-6*a^2*c^2*d*x^2+10*a*b*c^
3*x^2+3*a^2*c^3)/(d*x^2+c)^(1/2)/x^5/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286789, size = 163, normalized size = 1.16

$$-\frac{\left(2\left(15\,b^{2}c^{2}d-40\,abcd^{2}+24\,a^{2}d^{3}\right)x^{6}+3\,a^{2}c^{3}+\left(15\,b^{2}c^{3}-40\,abc^{2}d+24\,a^{2}cd^{2}\right)x^{4}+2\left(5\,abc^{3}-3\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{15\left(c^{4}dx^{7}+c^{5}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^6),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(15*b^2*c^2*d - 40*a*b*c*d^2 + 24*a^2*d^3)*x^6 + 3*a^2*c
^3 + (15*b^2*c^3 - 40*a*b*c^2*d + 24*a^2*c*d^2)*x^4 + 2*(5*a*b*c^
3 - 3*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c)/(c^4*d*x^7 + c^5*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^{2}\right)^{2}}{x^{6}\left(c+dx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.245448, size = 610, normalized size = 4.33

$$-\frac{\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)x}{\sqrt{dx^{2}+c}c^{4}}$$
$$+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}b^{2}c^{2}\sqrt{d}-30\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}abcd^{\frac{3}{2}}+15\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}a^{2}d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b^{2}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^6),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x/(sqrt(d*x^2 + c)*c^4) + 2/
15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 30*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 15*(sqrt(d)*x - sqrt(d
*x^2 + c))^8*a^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2
*c^3*sqrt(d) + 180*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2
) - 90*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c*d^(5/2) + 90*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 320*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*b*c^3*d^(3/2) + 240*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*s
qrt(d) + 220*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 15
0*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sq
rt(d) - 50*a*b*c^5*d^(3/2) + 33*a^2*c^4*d^(5/2))/(((sqrt(d)*x - s
qrt(d*x^2 + c))^2 - c)^5*c^3)
```

3.659 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{a^2}{6cx^6\sqrt{c+dx^2}}+\frac{d\left(24b^2c^2-5ad(12bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}-\frac{\sqrt{c+dx^2}\left(24b^2c^2-5ad(12bc-7ad)\right)}{16c^4x^2}+\frac{24b^2c^2-5ad(12bc-7ad)}{24c^3x^2\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}$$

```
[Out] -a^2/(6*c*x^6*Sqrt[c + d*x^2]) - (a*(12*b*c - 7*a*d))/(24*c^2*x^4
*Sqrt[c + d*x^2]) + (24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))/(24*c^3
*x^2*Sqrt[c + d*x^2]) - ((24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*Sq
rt[c + d*x^2])/(16*c^4*x^2) + (d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*
a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(9/2))
```

Rubi [A] time = 0.526352, antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{35a^2d^2-60abcd+24b^2c^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{a^2}{6cx^6\sqrt{c+dx^2}}+\frac{d\left(24b^2c^2-5ad(12bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}-\frac{\sqrt{c+dx^2}\left(24b^2c^2-5ad(12bc-7ad)\right)}{16c^4x^2}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(6*c*x^6*Sqrt[c + d*x^2]) - (a*(12*b*c - 7*a*d))/(24*c^2*x^4
*Sqrt[c + d*x^2]) + (24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)/(24*c^
3*x^2*Sqrt[c + d*x^2]) - ((24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*S
qrt[c + d*x^2])/(16*c^4*x^2) + (d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7
*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(9/2))
```

Rubi in Sympy [A] time = 35.4833, size = 184, normalized size = 0.96

$$-\frac{a^2}{6cx^6\sqrt{c+dx^2}}+\frac{a\,(7ad-12bc)}{24c^2x^4\sqrt{c+dx^2}}+\frac{5ad\,(7ad-12bc)+24b^2c^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\left(5ad\,(7ad-12bc)+24b^2c^2\right)}{16c^4x^2}+\frac{d\left(5ad\,(7ad-12bc)+24b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**2/(6*c*x**6*sqrt(c + d*x**2)) + a*(7*a*d - 12*b*c)/(24*c**2*x
**4*sqrt(c + d*x**2)) + (5*a*d*(7*a*d - 12*b*c) + 24*b**2*c**2)/(
24*c**3*x**2*sqrt(c + d*x**2)) - sqrt(c + d*x**2)*(5*a*d*(7*a*d -
 12*b*c) + 24*b**2*c**2)/(16*c**4*x**2) + d*(5*a*d*(7*a*d - 12*b*
c) + 24*b**2*c**2)*atanh(sqrt(c + d*x**2)/sqrt(c))/(16*c**(9/2))
```

Mathematica [A] time = 0.558826, size = 190, normalized size = 0.99

$$\frac{3d\left(35a^2d^2-60abcd+24b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)-3d\log(x)\left(35a^2d^2-60abcd+24b^2c^2\right)-\frac{\sqrt{c}\left(a^2\left(8c^3-14c^2dx^2+35cd^2x^4+1\right.\right.}{}}{48c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[c]*(24*b^2*c^2*x^4*(c + 3*d*x^2) + 12*a*b*c*x^2*(2*c^2 -
 5*c*d*x^2 - 15*d^2*x^4) + a^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x
^4 + 105*d^3*x^6)))/(x^6*Sqrt[c + d*x^2])) - 3*d*(24*b^2*c^2 - 60
*a*b*c*d + 35*a^2*d^2)*Log[x] + 3*d*(24*b^2*c^2 - 60*a*b*c*d + 35
*a^2*d^2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/(48*c^(9/2))
```

Maple [A] time = 0.02, size = 281, normalized size = 1.5

$$-\frac{a^2}{6\,cx^6}\frac{1}{\sqrt{dx^2+c}}+\frac{7\,a^2d}{24\,c^2x^4}\frac{1}{\sqrt{dx^2+c}}-\frac{35\,a^2d^2}{48\,c^3x^2}\frac{1}{\sqrt{dx^2+c}}-\frac{35\,a^2d^3}{16\,c^4}\frac{1}{\sqrt{dx^2+c}}+\frac{35\,a^2d^3}{16}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{9}{2}}-\frac{b^2}{2\,cx^2}\frac{1}{\sqrt{dx^2+c}}-\frac{3\,b^2d}{2\,c^2}\frac{1}{\sqrt{dx^2+c}}+\frac{3\,b^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}-\frac{ab}{2\,cx^4}\frac{1}{\sqrt{dx^2+c}}+\frac{5\,abd}{4\,c^2x^2}\frac{1}{\sqrt{dx^2+c}}+\frac{15\,abd^2}{4\,c^3}\frac{1}{\sqrt{dx^2+c}}-\frac{15\,abd^2}{4}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/6*a^2/c/x^6/(d*x^2+c)^(1/2)+7/24*a^2*d/c^2/x^4/(d*x^2+c)^(1/2)
-35/48*a^2*d^2/c^3/x^2/(d*x^2+c)^(1/2)-35/16*a^2*d^3/c^4/(d*x^2+c
)^(1/2)+35/16*a^2*d^3/c^(9/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/
x)-1/2*b^2/c/x^2/(d*x^2+c)^(1/2)-3/2*b^2*d/c^2/(d*x^2+c)^(1/2)+3/
2*b^2*d/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*a*b/c/x
^4/(d*x^2+c)^(1/2)+5/4*a*b*d/c^2/x^2/(d*x^2+c)^(1/2)+15/4*a*b*d^2
/c^3/(d*x^2+c)^(1/2)-15/4*a*b*d^2/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^
2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247037, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(3\left(24\,b^2c^2d-60\,abcd^2+35\,a^2d^3\right)x^6+8\,a^2c^3+\left(24\,b^2c^3-60\,abc^2d+35\,a^2cd^2\right)x^4+2\left(12\,abc^3-7\,a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{96\left(c^4dx^8+c^5\right.}\right.$$

$$-\frac{\left(3\left(24\,b^2c^2d-60\,abcd^2+35\,a^2d^3\right)x^6+8\,a^2c^3+\left(24\,b^2c^3-60\,abc^2d+35\,a^2cd^2\right)x^4+2\left(12\,abc^3-7\,a^2c^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{-}}{48\left(c^4dx^8+c^5x^6\right)\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^7),x, algorithm="fricas")
```

```
[Out] [-1/96*(2*(3*(24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*x^6 + 8*a
^2*c^3 + (24*b^2*c^3 - 60*a*b*c^2*d + 35*a^2*c*d^2)*x^4 + 2*(12*a
*b*c^3 - 7*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c) - 3*((24*b^2*c
^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d - 60*a*b*
c^2*d^2 + 35*a^2*c*d^3)*x^6)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt
(d*x^2 + c)*c)/x^2))/((c^4*d*x^8 + c^5*x^6)*sqrt(c)), -1/48*((3*(
24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*x^6 + 8*a^2*c^3 + (24*b
^2*c^3 - 60*a*b*c^2*d + 35*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 7*a^2
*c^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) - 3*((24*b^2*c^2*d^2 - 60*a
*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*
a^2*c*d^3)*x^6)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((c^4*d*x^8 + c
^5*x^6)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^7 \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**7*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.240611, size = 360, normalized size = 1.88

$$-\frac{\left(24\, b^2c^2d - 60\, abcd^2 + 35\, a^2d^3\right) \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{16\, \sqrt{-c}c^4} - \frac{b^2c^2d - 2\, abcd^2 + a^2d^3}{\sqrt{dx^2+c}c^4}$$

$$-\frac{24\left(dx^2+c\right)^{\frac{5}{2}} b^2c^2d - 48\left(dx^2+c\right)^{\frac{3}{2}} b^2c^3d + 24\sqrt{dx^2+c}b^2c^4d - 84\left(dx^2+c\right)^{\frac{5}{2}} abcd^2 + 192\left(dx^2+c\right)^{\frac{3}{2}} abc^2d^2 - 108\sqrt{dx^2 \cdots}}{48\, c^4d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^7),x, algorithm="giac")
```

```
[Out] -1/16*(24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*arctan(sqrt(d*x^
2 + c)/sqrt(-c))/(sqrt(-c)*c^4) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)/(sqrt(d*x^2 + c)*c^4) - 1/48*(24*(d*x^2 + c)^(5/2)*b^2*c^2*d
 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d + 24*sqrt(d*x^2 + c)*b^2*c^4*d
- 84*(d*x^2 + c)^(5/2)*a*b*c*d^2 + 192*(d*x^2 + c)^(3/2)*a*b*c^2*
d^2 - 108*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 57*(d*x^2 + c)^(5/2)*a^2*
d^3 - 136*(d*x^2 + c)^(3/2)*a^2*c*d^3 + 87*sqrt(d*x^2 + c)*a^2*c^
2*d^3)/(c^4*d^3*x^6)
```

3.660 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(8a^2d^2-40abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-40abcd+35b^2c^2)}{8cd^4}+\frac{x^3\,(8a^2d^2-40abcd+35b^2c^2)}{12cd^3\sqrt{c+dx^2}}+\frac{x^5(bc-ad)^2}{3cd^2\,(c+dx^2)^{3/2}}+\frac{b^2x^5}{4d^2\sqrt{c+dx^2}}$$

```
[Out] ((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 -
40*a*b*c*d + 8*a^2*d^2)*x^3)/(12*c*d^3*Sqrt[c + d*x^2]) + (b^2*x^
5)/(4*d^2*Sqrt[c + d*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^
2)*x*Sqrt[c + d*x^2])/(8*c*d^4) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a
^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(9/2))
```

Rubi [A] time = 0.415568, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{(8a^2d^2-40abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-40abcd+35b^2c^2)}{8cd^4}+\frac{x^3\,(8a^2d^2-40abcd+35b^2c^2)}{12cd^3\sqrt{c+dx^2}}+\frac{x^5(bc-ad)^2}{3cd^2\,(c+dx^2)^{3/2}}+\frac{b^2x^5}{4d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 -
40*a*b*c*d + 8*a^2*d^2)*x^3)/(12*c*d^3*Sqrt[c + d*x^2]) + (b^2*x^
5)/(4*d^2*Sqrt[c + d*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^
2)*x*Sqrt[c + d*x^2])/(8*c*d^4) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a
^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(9/2))
```

Rubi in Sympy [A] time = 46.5344, size = 190, normalized size = 0.94

$$\frac{b^2x^5}{4d^2\sqrt{c+dx^2}}+\frac{(8a^2d^2-40abcd+35b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{\frac{9}{2}}}+\frac{x^5\,(ad-bc)^2}{3cd^2\,(c+dx^2)^{\frac{3}{2}}}+\frac{x^3\,(8a^2d^2-40abcd+35b^2c^2)}{12cd^3\sqrt{c+dx^2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-40abcd+35b^2c^2)}{8cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*x**5/(4*d**2*sqrt(c + d*x**2)) + (8*a**2*d**2 - 40*a*b*c*d +
 35*b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*d**(9/2)) + x
**5*(a*d - b*c)**2/(3*c*d**2*(c + d*x**2)**(3/2)) + x**3*(8*a**2*
d**2 - 40*a*b*c*d + 35*b**2*c**2)/(12*c*d**3*sqrt(c + d*x**2)) -
x*sqrt(c + d*x**2)*(8*a**2*d**2 - 40*a*b*c*d + 35*b**2*c**2)/(8*c
*d**4)
```

Mathematica [A] time = 0.227294, size = 156, normalized size = 0.77

$$\frac{(8a^2d^2 - 40abcd + 35b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{9/2}} + \frac{x\left(-8a^2d^2\left(3c+4dx^2\right)+8abd\left(15c^2+20cdx^2+3d^2x^4\right)+b^2\left(-\left(105c^3+140c^2dx^2+21cd^2x^4-6d^3x^6\right)\right)\right)}{24d^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(-8*a^2*d^2*(3*c + 4*d*x^2) + 8*a*b*d*(15*c^2 + 20*c*d*x^2 + 3
*d^2*x^4) - b^2*(105*c^3 + 140*c^2*d*x^2 + 21*c*d^2*x^4 - 6*d^3*x
^6)))/(24*d^4*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d + 8*
a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(9/2))
```

Maple [A] time = 0.029, size = 255, normalized size = 1.3

$$-\frac{a^2x^3}{3\,d}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{a^2x}{d^2}\frac{1}{\sqrt{dx^2+c}}+a^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}+\frac{b^2x^7}{4\,d}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{7\,b^2cx^5}{8\,d^2}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{35\,b^2c^2x^3}{24\,d^3}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{35\,b^2c^2x}{8\,d^4}\frac{1}{\sqrt{dx^2+c}}+\frac{35\,b^2c^2}{8}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{9}{2}}+\frac{abx^5}{d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{5\,abcx^3}{3\,d^2}\left(dx^2+c\right)^{-\frac{3}{2}}+5\,\frac{abcx}{d^3\sqrt{dx^2+c}}-5\,\frac{abc\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*a^2*x^3/d/(d*x^2+c)^(3/2)-a^2/d^2*x/(d*x^2+c)^(1/2)+a^2/d^(5
/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*b^2*x^7/d/(d*x^2+c)^(3/2)-7
/8*b^2*c/d^2*x^5/(d*x^2+c)^(3/2)-35/24*b^2*c^2/d^3*x^3/(d*x^2+c)^
(3/2)-35/8*b^2*c^2/d^4*x/(d*x^2+c)^(1/2)+35/8*b^2*c^2/d^(9/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))+a*b*x^5/d/(d*x^2+c)^(3/2)+5/3*a*b*c/d^
2*x^3/(d*x^2+c)^(3/2)+5*a*b*c/d^3*x/(d*x^2+c)^(1/2)-5*a*b*c/d^(7/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.308755, size = 1, normalized size = 0.

$$\left[\frac{2\left(6\,b^2d^3x^7-3\left(7\,b^2cd^2-8\,abd^3\right)x^5-4\left(35\,b^2c^2d-40\,abcd^2+8\,a^2d^3\right)x^3-3\left(35\,b^2c^3-40\,abc^2d+8\,a^2cd^2\right)x\right)\sqrt{dx^2+c}\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*b^2*d^3*x^7 - 3*(7*b^2*c*d^2 - 8*a*b*d^3)*x^5 - 4*(35
*b^2*c^2*d - 40*a*b*c*d^2 + 8*a^2*d^3)*x^3 - 3*(35*b^2*c^3 - 40*a
*b*c^2*d + 8*a^2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(35*b^2*c^
4 - 40*a*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*a*b*c*d^3
 + 8*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^
3)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((d^
6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*sqrt(d)), 1/24*((6*b^2*d^3*x^7 - 3
*(7*b^2*c*d^2 - 8*a*b*d^3)*x^5 - 4*(35*b^2*c^2*d - 40*a*b*c*d^2 +
 8*a^2*d^3)*x^3 - 3*(35*b^2*c^3 - 40*a*b*c^2*d + 8*a^2*c*d^2)*x)*
sqrt(d*x^2 + c)*sqrt(-d) + 3*(35*b^2*c^4 - 40*a*b*c^3*d + 8*a^2*c
^2*d^2 + (35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^4 + 2*(35*
b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^2)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)))/((d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (a + bx^2)^2}{(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.247294, size = 257, normalized size = 1.27

$$\frac{\left(\left(3\left(\frac{2\,b^2x^2}{d}-\frac{7\,b^2c^2d^5-8\,abcd^6}{cd^7}\right)x^2-\frac{4\left(35\,b^2c^3d^4-40\,abc^2d^5+8\,a^2cd^6\right)}{cd^7}\right)x^2-\frac{3\left(35\,b^2c^4d^3-40\,abc^3d^4+8\,a^2c^2d^5\right)}{cd^7}\right)x}{24\,(dx^2+c)^{\frac{3}{2}}}-\frac{\left(35\,b^2c^2-40\,abcd+8\,a^2d^2\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^4/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*b^2*x^2/d - (7*b^2*c^2*d^5 - 8*a*b*c*d^6)/(c*d^7))*x^
2 - 4*(35*b^2*c^3*d^4 - 40*a*b*c^2*d^5 + 8*a^2*c*d^6)/(c*d^7))*x^
2 - 3*(35*b^2*c^4*d^3 - 40*a*b*c^3*d^4 + 8*a^2*c^2*d^5)/(c*d^7))*
x/(d*x^2 + c)^(3/2) - 1/8*(35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*l
n(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.661 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\sqrt{c+dx^2}(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

```
[Out] (c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c
 - a*d))/(d^4*Sqrt[c + d*x^2]) - (b*(3*b*c - 2*a*d)*Sqrt[c + d*x^
2])/d^4 + (b^2*(c + d*x^2)^(3/2))/(3*d^4)
```

Rubi [A] time = 0.266968, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{b\sqrt{c+dx^2}(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c
 - a*d))/(d^4*Sqrt[c + d*x^2]) - (b*(3*b*c - 2*a*d)*Sqrt[c + d*x^
2])/d^4 + (b^2*(c + d*x^2)^(3/2))/(3*d^4)
```

Rubi in Sympy [A] time = 32.5863, size = 97, normalized size = 0.88

$$\frac{b^2(c+dx^2)^{\frac{3}{2}}}{3d^4}+\frac{b\sqrt{c+dx^2}(2ad-3bc)}{d^4}+\frac{c(ad-bc)^2}{3d^4(c+dx^2)^{\frac{3}{2}}}-\frac{(ad-3bc)(ad-bc)}{d^4\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*(c + d*x**2)**(3/2)/(3*d**4) + b*sqrt(c + d*x**2)*(2*a*d - 3
*b*c)/d**4 + c*(a*d - b*c)**2/(3*d**4*(c + d*x**2)**(3/2)) - (a*d
 - 3*b*c)*(a*d - b*c)/(d**4*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.108336, size = 98, normalized size = 0.89

$$\frac{-a^2d^2(2c+3dx^2)+2abd(8c^2+12cdx^2+3d^2x^4)+b^2(-16c^3-24c^2dx^2-6cd^2x^4+d^3x^6)}{3d^4(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2*(2*c + 3*d*x^2)) + 2*a*b*d*(8*c^2 + 12*c*d*x^2 + 3*d^2
*x^4) + b^2*(-16*c^3 - 24*c^2*d*x^2 - 6*c*d^2*x^4 + d^3*x^6))/(3*
d^4*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.01, size = 108, normalized size = 1.

$$-\frac{-b^2x^6d^3-6\,abd^3x^4+6\,b^2cd^2x^4+3\,a^2d^3x^2-24\,abcd^2x^2+24\,b^2c^2dx^2+2\,a^2cd^2-16\,abc^2d+16\,b^2c^3}{3\,d^4}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-b^2*d^3*x^6-6*a*b*d^3*x^4+6*b^2*c*d^2*x^4+3*a^2*d^3*x^2-24
*a*b*c*d^2*x^2+24*b^2*c^2*d*x^2+2*a^2*c*d^2-16*a*b*c^2*d+16*b^2*c
^3)/(d*x^2+c)^(3/2)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220106, size = 167, normalized size = 1.52

$$\frac{\left(b^2d^3x^6-16\,b^2c^3+16\,abc^2d-2\,a^2cd^2-6\left(b^2cd^2-abd^3\right)x^4-3\left(8\,b^2c^2d-8\,abcd^2+a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{3\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^3*x^6 - 16*b^2*c^3 + 16*a*b*c^2*d - 2*a^2*c*d^2 - 6*(b
^2*c*d^2 - a*b*d^3)*x^4 - 3*(8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)
*x^2)*sqrt(d*x^2 + c)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

Sympy [A] time = 6.74306, size = 454, normalized size = 4.13

$$\begin{cases} -\frac{2a^2cd^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{3a^2d^3x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{16abc^2d}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{24abcd^2x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{6abd^3x^4}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c} \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c*d**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**
2*sqrt(c + d*x**2)) - 3*a**2*d**3*x**2/(3*c*d**4*sqrt(c + d*x**2)
 + 3*d**5*x**2*sqrt(c + d*x**2)) + 16*a*b*c**2*d/(3*c*d**4*sqrt(c
 + d*x**2) + 3*d**5*x**2*sqrt(c + d*x**2)) + 24*a*b*c*d**2*x**2/(
3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c + d*x**2)) + 6*a*b
*d**3*x**4/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c + d*x*
*2)) - 16*b**2*c**3/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt
(c + d*x**2)) - 24*b**2*c**2*d*x**2/(3*c*d**4*sqrt(c + d*x**2) +
3*d**5*x**2*sqrt(c + d*x**2)) - 6*b**2*c*d**2*x**4/(3*c*d**4*sqrt
(c + d*x**2) + 3*d**5*x**2*sqrt(c + d*x**2)) + b**2*d**3*x**6/(3*
c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c + d*x**2)), Ne(d, 0)
```

```
), ((a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/c**(5/2), True))
```

GIAC/XCAS [A] time = 0.238662, size = 173, normalized size = 1.57

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2-9\sqrt{dx^2+c}b^2c+6\sqrt{dx^2+c}abd-\frac{9\left(dx^2+c\right)b^2c^2-b^2c^3-12\left(dx^2+c\right)abcd+2\,abc^2d+3\left(dx^2+c\right)a^2d^2-a^2cd^2}{\left(dx^2+c\right)^{\frac{3}{2}}}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^3/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((d*x^2 + c)^(3/2)*b^2 - 9*sqrt(d*x^2 + c)*b^2*c + 6*sqrt(d*x
^2 + c)*a*b*d - (9*(d*x^2 + c)*b^2*c^2 - b^2*c^3 - 12*(d*x^2 + c)
*a*b*c*d + 2*a*b*c^2*d + 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/(d*x^
2 + c)^(3/2))/d^4
```

3.662 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

```
[Out] ((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d
)*x)/(d^3*Sqrt[c + d*x^2]) + (b^2*x*Sqrt[c + d*x^2])/(2*d^3) - (b
*(5*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(7/2)
)
```

Rubi [A] time = 0.32199, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d
)*x)/(d^3*Sqrt[c + d*x^2]) + (b^2*x*Sqrt[c + d*x^2])/(2*d^3) - (b
*(5*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(7/2)
)
```

Rubi in Sympy [A] time = 58.8152, size = 110, normalized size = 0.91

$$\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{2bx\,(ad-bc)}{d^3\sqrt{c+dx^2}}+\frac{b\,(4ad-5bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{\frac{7}{2}}}+\frac{x^3\,(ad-bc)^2}{3cd^2\,(c+dx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*x*sqrt(c + d*x**2)/(2*d**3) - 2*b*x*(a*d - b*c)/(d**3*sqrt(c
 + d*x**2)) + b*(4*a*d - 5*b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))
/(2*d**(7/2)) + x**3*(a*d - b*c)**2/(3*c*d**2*(c + d*x**2)**(3/2)
)
```

Mathematica [A] time = 0.188548, size = 118, normalized size = 0.98

$$\frac{x\left(2a^2d^3x^2-4abcd\left(3c+4dx^2\right)+b^2c\left(15c^2+20cdx^2+3d^2x^4\right)\right)}{6cd^3\left(c+dx^2\right)^{3/2}}+\frac{b(4ad-5bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a^2*d^3*x^2 - 4*a*b*c*d*(3*c + 4*d*x^2) + b^2*c*(15*c^2 + 2
0*c*d*x^2 + 3*d^2*x^4)))/(6*c*d^3*(c + d*x^2)^(3/2)) + (b*(-5*b*c
```

```
+ 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Maple [A] time = 0.015, size = 185, normalized size = 1.5

$$-\frac{a^2x}{3\,d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{a^2x}{3\,cd}\frac{1}{\sqrt{dx^2+c}}+\frac{b^2x^5}{2\,d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{5\,b^2cx^3}{6\,d^2}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{5\,b^2cx}{2\,d^3}\frac{1}{\sqrt{dx^2+c}}-\frac{5\,b^2c}{2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{7}{2}}-\frac{2\,abx^3}{3\,d}\left(dx^2+c\right)^{-\frac{3}{2}}-2\,\frac{abx}{d^2\sqrt{dx^2+c}}+2\,\frac{ab\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*a^2/d*x/(d*x^2+c)^(3/2)+1/3*a^2/c/d*x/(d*x^2+c)^(1/2)+1/2*b^
2*x^5/d/(d*x^2+c)^(3/2)+5/6*b^2*c/d^2*x^3/(d*x^2+c)^(3/2)+5/2*b^2
*c/d^3*x/(d*x^2+c)^(1/2)-5/2*b^2*c/d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)
^(1/2))-2/3*a*b*x^3/d/(d*x^2+c)^(3/2)-2*a*b/d^2*x/(d*x^2+c)^(1/2)
+2*a*b/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25625, size = 1, normalized size = 0.01

$$\left[\frac{2\left(3\,b^2cd^2x^5+2\left(10\,b^2c^2d-8\,abcd^2+a^2d^3\right)x^3+3\left(5\,b^2c^3-4\,abc^2d\right)x\right)\sqrt{dx^2+c}\sqrt{d}-3\left(5\,b^2c^4-4\,abc^3d+\left(5\,b^2c^2d^2-4\right.\right.}{12\left(cd^5x^4+2\,c^2d^4x^2+c^3d^3\right)\sqrt{d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(3*b^2*c*d^2*x^5 + 2*(10*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d
^3)*x^3 + 3*(5*b^2*c^3 - 4*a*b*c^2*d)*x)*sqrt(d*x^2 + c)*sqrt(d)
- 3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d^3)*x^4
+ 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x
 - (2*d*x^2 + c)*sqrt(d)))/((c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)
*sqrt(d)), 1/6*((3*b^2*c*d^2*x^5 + 2*(10*b^2*c^2*d - 8*a*b*c*d^2
+ a^2*d^3)*x^3 + 3*(5*b^2*c^3 - 4*a*b*c^2*d)*x)*sqrt(d*x^2 + c)*s
qrt(-d) - 3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d
^3)*x^4 + 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)))/((c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*sqrt(-d)
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.242582, size = 176, normalized size = 1.45

$$\frac{\left(\left(\frac{3\,b^2x^2}{d} + \frac{2\,(10\,b^2c^2d^3 - 8\,abcd^4 + a^2d^5)}{cd^5}\right)x^2 + \frac{3\,(5\,b^2c^3d^2 - 4\,abc^2d^3)}{cd^5}\right)x}{6\,(dx^2 + c)^{\frac{3}{2}}} + \frac{(5\,b^2c - 4\,abd)\ln\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right)}{2\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x^2/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*b^2*x^2/d + 2*(10*b^2*c^2*d^3 - 8*a*b*c*d^4 + a^2*d^5)/(c
*d^5))*x^2 + 3*(5*b^2*c^3*d^2 - 4*a*b*c^2*d^3)/(c*d^5))*x/(d*x^2
+ c)^(3/2) + 1/2*(5*b^2*c - 4*a*b*d)*ln(abs(-sqrt(d)*x + sqrt(d*x
^2 + c)))/d^(7/2)
```

3.663 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3(c+dx^2)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

```
[Out] -(b*c - a*d)^2/(3*d^3*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d))/(d^3
*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^3
```

Rubi [A] time = 0.158596, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3(c+dx^2)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] -(b*c - a*d)^2/(3*d^3*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d))/(d^3
*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^3
```

Rubi in Sympy [A] time = 23.8012, size = 63, normalized size = 0.88

$$\frac{b^2\sqrt{c+dx^2}}{d^3}-\frac{2b(ad-bc)}{d^3\sqrt{c+dx^2}}-\frac{(ad-bc)^2}{3d^3(c+dx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*sqrt(c + d*x**2)/d**3 - 2*b*(a*d - b*c)/(d**3*sqrt(c + d*x**
2)) - (a*d - b*c)**2/(3*d**3*(c + d*x**2)**(3/2))
```

Mathematica [A] time = 0.0763108, size = 67, normalized size = 0.93

$$\frac{-a^2d^2-2abd\left(2c+3dx^2\right)+b^2\left(8c^2+12cdx^2+3d^2x^4\right)}{3d^3(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(2*c + 3*d*x^2) + b^2*(8*c^2 + 12*c*d*x^2 +
 3*d^2*x^4))/(3*d^3*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.008, size = 68, normalized size = 0.9

$$-\frac{-3\,b^2d^2x^4+6\,abd^2x^2-12\,b^2cdx^2+a^2d^2+4\,cabd-8\,b^2c^2}{3\,d^3}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-3*b^2*d^2*x^4+6*a*b*d^2*x^2-12*b^2*c*d*x^2+a^2*d^2+4*a*b*c
*d-8*b^2*c^2)/(d*x^2+c)^(3/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.216283, size = 123, normalized size = 1.71

$$\frac{\left(3\,b^2d^2x^4 + 8\,b^2c^2 - 4\,abcd - a^2d^2 + 6\,\left(2\,b^2cd - abd^2\right)x^2\right)\sqrt{dx^2 + c}}{3\,\left(d^5x^4 + 2\,cd^4x^2 + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c
*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)/(d^5*x^4 + 2*c*d^4*x^2 + c^2*d
^3)
```

Sympy [A] time = 4.25115, size = 303, normalized size = 4.21

$$\begin{cases} -\frac{a^2d^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} - \frac{4abcd}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} - \frac{6abd^2x^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} + \frac{8b^2c^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} + \frac{12b^2cdx^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c} \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{c^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-a**2*d**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sq
rt(c + d*x**2)) - 4*a*b*c*d/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x
**2*sqrt(c + d*x**2)) - 6*a*b*d**2*x**2/(3*c*d**3*sqrt(c + d*x**2
) + 3*d**4*x**2*sqrt(c + d*x**2)) + 8*b**2*c**2/(3*c*d**3*sqrt(c
+ d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)) + 12*b**2*c*d*x**2/(3*c
*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)) + 3*b**2*d
**2*x**4/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2
)), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(5/2)
, True))
```

GIAC/XCAS [A] time = 0.233521, size = 105, normalized size = 1.46

$$\frac{3\,\sqrt{dx^2 + c}b^2 + \frac{6\,\left(dx^2+c\right)b^2c - b^2c^2 - 6\,\left(dx^2+c\right)abd + 2\,abcd - a^2d^2}{\left(dx^2+c\right)^{\frac{3}{2}}}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*x/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(d*x^2 + c)*b^2 + (6*(d*x^2 + c)*b^2*c - b^2*c^2 - 6*(
d*x^2 + c)*a*b*d + 2*a*b*c*d - a^2*d^2)/(d*x^2 + c)^(3/2))/d^3
```

3.664 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(3*c*d*(c + d*x^2)^(3/2)) - ((b*c -
a*d)*(3*b*c + 2*a*d)*x)/(3*c^2*d^2*Sqrt[c + d*x^2]) + (b^2*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(5/2)
```

Rubi [A] time = 0.136059, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(3*c*d*(c + d*x^2)^(3/2)) - ((b*c -
a*d)*(3*b*c + 2*a*d)*x)/(3*c^2*d^2*Sqrt[c + d*x^2]) + (b^2*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(5/2)
```

Rubi in Sympy [A] time = 22.4313, size = 94, normalized size = 0.9

$$\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{\frac{5}{2}}}+\frac{x\left(a+bx^2\right)(ad-bc)}{3cd\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{x(ad-bc)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] b**2*atanh(sqrt(d)*x/sqrt(c + d*x**2))/d**(5/2) + x*(a + b*x**2)*
(a*d - b*c)/(3*c*d*(c + d*x**2)**(3/2)) + x*(a*d - b*c)*(2*a*d +
3*b*c)/(3*c**2*d**2*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.222373, size = 101, normalized size = 0.96

$$\frac{x\left(a^2d^2\left(3c+2dx^2\right)+2abcd^2x^2-b^2c^2\left(3c+4dx^2\right)\right)}{3c^2d^2\left(c+dx^2\right)^{3/2}}+\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a*b*c*d^2*x^2 + a^2*d^2*(3*c + 2*d*x^2) - b^2*c^2*(3*c + 4*
d*x^2)))/(3*c^2*d^2*(c + d*x^2)^(3/2)) + (b^2*Log[d*x + Sqrt[d]*S
qrt[c + d*x^2]])/d^(5/2)
```

Maple [A] time = 0.012, size = 136, normalized size = 1.3

$$\frac{a^2x}{3c}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{2a^2x}{3c^2}\frac{1}{\sqrt{dx^2+c}}-\frac{b^2x^3}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{b^2x}{d^2}\frac{1}{\sqrt{dx^2+c}}+b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{5}{2}}-\frac{2abx}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{2abx}{3cd}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*a^2*x/c/(d*x^2+c)^(3/2)+2/3*a^2/c^2*x/(d*x^2+c)^(1/2)-1/3*b^2
*x^3/d/(d*x^2+c)^(3/2)-b^2/d^2*x/(d*x^2+c)^(1/2)+b^2/d^(5/2)*ln(x
*d^(1/2)+(d*x^2+c)^(1/2))-2/3*a*b/d*x/(d*x^2+c)^(3/2)+2/3*a*b/c/d
*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235764, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(2\left(2b^2c^2d-abcd^2-a^2d^3\right)x^3+3\left(b^2c^3-a^2cd^2\right)x\right)\sqrt{dx^2+c}\sqrt{d}-3\left(b^2c^2d^2x^4+2b^2c^3dx^2+b^2c^4\right)\log\left(-2\sqrt{dx^2+c}dx\right.}{6\left(c^2d^4x^4+2c^3d^3x^2+c^4d^2\right)\sqrt{d}},\right.$$
$$\left.-\frac{\left(2\left(2b^2c^2d-abcd^2-a^2d^3\right)x^3+3\left(b^2c^3-a^2cd^2\right)x\right)\sqrt{dx^2+c}\sqrt{-d}-3\left(b^2c^2d^2x^4+2b^2c^3dx^2+b^2c^4\right)\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{3\left(c^2d^4x^4+2c^3d^3x^2+c^4d^2\right)\sqrt{-d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(2*(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3 + 3*(b^2*c^3
- a^2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(b^2*c^2*d^2*x^4 + 2*
b^2*c^3*d*x^2 + b^2*c^4)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 +
c)*sqrt(d)))/((c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*sqrt(d)), -
1/3*((2*(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3 + 3*(b^2*c^3 - a^
2*c*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) - 3*(b^2*c^2*d^2*x^4 + 2*b^2
*c^3*d*x^2 + b^2*c^4)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/((c^2*d
^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.237555, size = 142, normalized size = 1.35

$$-\frac{x\left(\frac{2\,(2\,b^2c^2d^2-abcd^3-a^2d^4)\,x^2}{c^2d^3}+\frac{3\,(b^2c^3d-a^2cd^3)}{c^2d^3}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}-\frac{b^2\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^2/(c^2*d^3) + 3
*(b^2*c^3*d - a^2*c*d^3)/(c^2*d^3))/(d*x^2 + c)^(3/2) - b^2*ln(ab
s(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.665 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\,(c+dx^2)^{3/2}}$$

```
[Out] (b*c - a*d)^2/(3*c*d^2*(c + d*x^2)^(3/2)) + (a^2/c^2 - b^2/d^2)/S
qrt[c + d*x^2] - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Rubi [A] time = 0.239524, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b*c - a*d)^2/(3*c*d^2*(c + d*x^2)^(3/2)) + (a^2/c^2 - b^2/d^2)/S
qrt[c + d*x^2] - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Rubi in Sympy [A] time = 40.5884, size = 73, normalized size = 0.83

$$-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}+\frac{(ad-bc)^2}{3cd^2\,(c+dx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x/(d*x**2+c)**(5/2),x)
```

```
[Out] -a**2*atanh(sqrt(c + d*x**2)/sqrt(c))/c**(5/2) + (a**2/c**2 - b**
2/d**2)/sqrt(c + d*x**2) + (a*d - b*c)**2/(3*c*d**2*(c + d*x**2)*
*(3/2))
```

Mathematica [A] time = 0.444759, size = 106, normalized size = 1.2

$$\frac{\frac{\sqrt{c}\left(a^2d^2\left(4c+3dx^2\right)-2abc^2d-b^2c^2\left(2c+3dx^2\right)\right)}{d^2\left(c+dx^2\right)^{3/2}}-3a^2\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+3a^2\log(x)}{3c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[c]*(-2*a*b*c^2*d - b^2*c^2*(2*c + 3*d*x^2) + a^2*d^2*(4*c
+ 3*d*x^2)))/(d^2*(c + d*x^2)^(3/2)) + 3*a^2*Log[x] - 3*a^2*Log[c
 + Sqrt[c]*Sqrt[c + d*x^2]])/(3*c^(5/2))
```

Maple [A] time = 0.014, size = 120, normalized size = 1.4

$$\frac{a^2}{3c}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{a^2}{c^2}\frac{1}{\sqrt{dx^2+c}}-a^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}-\frac{b^2x^2}{d}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{2b^2c}{3d^2}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{2ab}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*a^2/c/(d*x^2+c)^(3/2)+a^2/c^2/(d*x^2+c)^(1/2)-a^2/c^(5/2)*ln(
(2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b^2*x^2/d/(d*x^2+c)^(3/2)-2/3*
b^2*c/d^2/(d*x^2+c)^(3/2)-2/3*a*b/d/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.231119, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(2b^2c^3+2abc^2d-4a^2cd^2+3\left(b^2c^2d-a^2d^3\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}-3\left(a^2d^4x^4+2a^2cd^3x^2+a^2c^2d^2\right)\log\left(-\frac{\left(dx^2+2c\right)\sqrt{c}-2\sqrt{d}}{x^2}\right.}{6\left(c^2d^4x^4+2c^3d^3x^2+c^4d^2\right)\sqrt{c}}\right.$$
$$\left.-\frac{\left(2b^2c^3+2abc^2d-4a^2cd^2+3\left(b^2c^2d-a^2d^3\right)x^2\right)\sqrt{dx^2+c}\sqrt{-c}+3\left(a^2d^4x^4+2a^2cd^3x^2+a^2c^2d^2\right)\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{3\left(c^2d^4x^4+2c^3d^3x^2+c^4d^2\right)\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(2*b^2*c^3 + 2*a*b*c^2*d - 4*a^2*c*d^2 + 3*(b^2*c^2*d -
a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(c) - 3*(a^2*d^4*x^4 + 2*a^2*c*
d^3*x^2 + a^2*c^2*d^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2
 + c)*c)/x^2))/((c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*sqrt(c)),
 -1/3*((2*b^2*c^3 + 2*a*b*c^2*d - 4*a^2*c*d^2 + 3*(b^2*c^2*d - a^
2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 3*(a^2*d^4*x^4 + 2*a^2*c*d
^3*x^2 + a^2*c^2*d^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((c^2*d^4
*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.238783, size = 138, normalized size = 1.57

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2} - \frac{3\left(dx^2+c\right)b^2c^2 - b^2c^3 + 2\,abc^2d - 3\left(dx^2+c\right)a^2d^2 - a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/3*(3*(d*x
^2 + c)*b^2*c^2 - b^2*c^3 + 2*a*b*c^2*d - 3*(d*x^2 + c)*a^2*d^2 -
 a^2*c*d^2)/((d*x^2 + c)^(3/2)*c^2*d^2)
```

3.666 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

```
[Out] -(a^2/(c*x*(c + d*x^2)^(3/2))) + (x*(2*a*(b*c - 2*a*d) + b^2*c*x^
2))/(3*c^2*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^3*Sqrt
[c + d*x^2])
```

Rubi [A] time = 0.146317, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^(3/2))) + (x*(2*a*(b*c - 2*a*d) + b^2*c*x^
2))/(3*c^2*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^3*Sqrt
[c + d*x^2])
```

Rubi in Sympy [A] time = 19.5037, size = 83, normalized size = 0.92

$$-\frac{a^2}{cx\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{4ax\left(2ad-bc\right)}{3c^3\sqrt{c+dx^2}}-\frac{x\left(2a\left(2ad-bc\right)-b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(5/2),x)
```

```
[Out] -a**2/(c*x*(c + d*x**2)**(3/2)) - 4*a*x*(2*a*d - b*c)/(3*c**3*sqr
t(c + d*x**2)) - x*(2*a*(2*a*d - b*c) - b**2*c*x**2)/(3*c**2*(c +
 d*x**2)**(3/2))
```

Mathematica [A] time = 0.0850988, size = 76, normalized size = 0.84

$$\frac{-a^2\left(3c^2+12cdx^2+8d^2x^4\right)+2abcx^2\left(3c+2dx^2\right)+b^2c^2x^4}{3c^3x\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4 + 2*a*b*c*x^2*(3*c + 2*d*x^2) - a^2*(3*c^2 + 12*c*d*
x^2 + 8*d^2*x^4))/(3*c^3*x*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 78, normalized size = 0.9

$$-\frac{8\,x^4a^2d^2-4\,x^4abcd-x^4b^2c^2+12\,x^2a^2cd-6\,ac^2bx^2+3\,a^2c^2}{3\,xc^3}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(8*a^2*d^2*x^4-4*a*b*c*d*x^4-b^2*c^2*x^4+12*a^2*c*d*x^2-6*a*
b*c^2*x^2+3*a^2*c^2)/(d*x^2+c)^(3/2)/x/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230325, size = 124, normalized size = 1.38

$$\frac{\left(\left(b^2c^2 + 4\,abcd - 8\,a^2d^2\right)x^4 - 3\,a^2c^2 + 6\,\left(abc^2 - 2\,a^2cd\right)x^2\right)\sqrt{dx^2 + c}}{3\,(c^3d^2x^5 + 2\,c^4dx^3 + c^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/3*((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*x^4 - 3*a^2*c^2 + 6*(a*b*c
^2 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d^2*x^5 + 2*c^4*d*x^3 +
 c^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2}{x^2\,(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.239057, size = 158, normalized size = 1.76

$$\frac{x\left(\frac{\left(b^2c^4d + 4\,abc^3d^2 - 5\,a^2c^2d^3\right)x^2}{c^5d} + \frac{6\,\left(abc^4d - a^2c^3d^2\right)}{c^5d}\right)}{3\,(dx^2 + c)^{\frac{3}{2}}} + \frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 - c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] 1/3*x*((b^2*c^4*d + 4*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x^2/(c^5*d) +
6*(a*b*c^4*d - a^2*c^3*d^2)/(c^5*d))/(d*x^2 + c)^(3/2) + 2*a^2*sq
rt(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*c^2)
```

3.667 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{-\frac{5a^2d}{c}+4ab-\frac{2b^2c}{d}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}$$

```
[Out] (4*a*b - (2*b^2*c)/d - (5*a^2*d)/c)/(6*c*(c + d*x^2)^(3/2)) - a^2
/(2*c*x^2*(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d))/(2*c^3*Sqrt[c
+ d*x^2]) - (a*(4*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
(2*c^(7/2))
```

Rubi [A] time = 0.335154, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{-\frac{5a^2d}{c}+4ab-\frac{2b^2c}{d}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (4*a*b - (2*b^2*c)/d - (5*a^2*d)/c)/(6*c*(c + d*x^2)^(3/2)) - a^2
/(2*c*x^2*(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d))/(2*c^3*Sqrt[c
+ d*x^2]) - (a*(4*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
(2*c^(7/2))
```

Rubi in Sympy [A] time = 29.4304, size = 121, normalized size = 0.92

$$-\frac{a^2}{2cx^2\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{a\left(5ad-4bc\right)}{2c^3\sqrt{c+dx^2}}+\frac{a\left(5ad-4bc\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{\frac{7}{2}}}-\frac{\frac{ad(5ad-4bc)}{2}+b^2c^2}{3c^2d\left(c+dx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(5/2),x)
```

```
[Out] -a**2/(2*c*x**2*(c + d*x**2)**(3/2)) - a*(5*a*d - 4*b*c)/(2*c**3*
sqrt(c + d*x**2)) + a*(5*a*d - 4*b*c)*atanh(sqrt(c + d*x**2)/sqrt
(c))/(2*c**(7/2)) - (a*d*(5*a*d - 4*b*c)/2 + b**2*c**2)/(3*c**2*d
*(c + d*x**2)**(3/2))
```

Mathematica [A] time = 0.437749, size = 128, normalized size = 0.98

$$\frac{-\sqrt{c}\sqrt{c+dx^2}\left(\frac{3a^2}{x^2}+\frac{12a(ad-bc)}{c+dx^2}+\frac{2c(bc-ad)^2}{d(c+dx^2)^2}\right)+3a(5ad-4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)-3a\log(x)(5ad-4bc)}{6c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(Sqrt[c]*Sqrt[c + d*x^2]*((3*a^2)/x^2 + (2*c*(b*c - a*d)^2)/(d*
(c + d*x^2)^2) + (12*a*(-(b*c) + a*d))/(c + d*x^2))) - 3*a*(-4*b*
c + 5*a*d)*Log[x] + 3*a*(-4*b*c + 5*a*d)*Log[c + Sqrt[c]*Sqrt[c +
```

```
d*x^2]])/(6*c^(7/2))
```

Maple [A] time = 0.018, size = 169, normalized size = 1.3

$$-\frac{b^2}{3d}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{a^2}{2\,cx^2}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{5\,a^2d}{6\,c^2}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{5\,a^2d}{2\,c^3}\frac{1}{\sqrt{dx^2+c}}+\frac{5\,a^2d}{2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{7}{2}}+\frac{2\,ab}{3\,c}\left(dx^2+c\right)^{-\frac{3}{2}}+2\,\frac{ab}{c^2\sqrt{dx^2+c}}-2\,\frac{ab}{c^{5/2}}\ln\left(\frac{2\,c+2\,\sqrt{c}\sqrt{dx^2+c}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b^2/d/(d*x^2+c)^(3/2)-1/2*a^2/c/x^2/(d*x^2+c)^(3/2)-5/6*a^2*
d/c^2/(d*x^2+c)^(3/2)-5/2*a^2*d/c^3/(d*x^2+c)^(1/2)+5/2*a^2*d/c^(
7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+2/3*a*b/c/(d*x^2+c)^(3
/2)+2*a*b/c^2/(d*x^2+c)^(1/2)-2*a*b/c^(5/2)*ln((2*c+2*c^(1/2)*(d*
x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.238608, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(3\,a^2c^2d-3\left(4\,abcd^2-5\,a^2d^3\right)x^4+2\left(b^2c^3-8\,abc^2d+10\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}+3\left(\left(4\,abcd^3-5\,a^2d^4\right)x^6+2\left(4\,abc^2\right.\right.}{12\left(c^3d^3x^6+2\,c^4d^2x^4+c^5dx^2\right)\sqrt{c}}\right.$$

$$-\frac{\left(3\,a^2c^2d-3\left(4\,abcd^2-5\,a^2d^3\right)x^4+2\left(b^2c^3-8\,abc^2d+10\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{-c}+3\left(\left(4\,abcd^3-5\,a^2d^4\right)x^6+2\left(4\,abc^2d\right.\right.}{6\left(c^3d^3x^6+2\,c^4d^2x^4+c^5dx^2\right)\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(3*a^2*c^2*d - 3*(4*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 2*(b^2
*c^3 - 8*a*b*c^2*d + 10*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(c) +
 3*((4*a*b*c*d^3 - 5*a^2*d^4)*x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^
3)*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2)*log(-((d*x^2 + 2*c)*s
qrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/((c^3*d^3*x^6 + 2*c^4*d^2*x^4
 + c^5*d*x^2)*sqrt(c)), -1/6*((3*a^2*c^2*d - 3*(4*a*b*c*d^2 - 5*a
^2*d^3)*x^4 + 2*(b^2*c^3 - 8*a*b*c^2*d + 10*a^2*c*d^2)*x^2)*sqrt(
d*x^2 + c)*sqrt(-c) + 3*((4*a*b*c*d^3 - 5*a^2*d^4)*x^6 + 2*(4*a*b
*c^2*d^2 - 5*a^2*c*d^3)*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2)*
arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((c^3*d^3*x^6 + 2*c^4*d^2*x^4 +
 c^5*d*x^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.245765, size = 173, normalized size = 1.32

$$\frac{\left(4\,abc-5\,a^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}c^3}-\frac{\sqrt{dx^2+c}a^2}{2\,c^3x^2}-\frac{b^2c^3-6\left(dx^2+c\right)abcd-2\,abc^2d+6\left(dx^2+c\right)a^2d^2+a^2cd^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 5*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c
)*c^3) - 1/2*sqrt(d*x^2 + c)*a^2/(c^3*x^2) - 1/3*(b^2*c^3 - 6*(d*
x^2 + c)*a*b*c*d - 2*a*b*c^2*d + 6*(d*x^2 + c)*a^2*d^2 + a^2*c*d^
2)/((d*x^2 + c)^(3/2)*c^3*d)
```

3.668 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{2x\left(b^2c^2-8ad(bc-ad)\right)}{3c^4\sqrt{c+dx^2}}+\frac{x\left(b^2c^2-8ad(bc-ad)\right)}{3c^3\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}$$

```
[Out] -a^2/(3*c*x^3*(c + d*x^2)^(3/2)) - (2*a*(b*c - a*d))/(c^2*x*(c +
d*x^2)^(3/2)) + ((b^2*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^3*(c + d*x
^2)^(3/2)) + (2*(b^2*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^4*Sqrt[c +
d*x^2])
```

Rubi [A] time = 0.32442, antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{x\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{2x\left(b^2c^2-8ad(bc-ad)\right)}{3c^4\sqrt{c+dx^2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] -a^2/(3*c*x^3*(c + d*x^2)^(3/2)) - (2*a*(b*c - a*d))/(c^2*x*(c +
d*x^2)^(3/2)) + ((b^2 - (8*a*d*(b*c - a*d))/c^2)*x)/(3*c*(c + d*x
^2)^(3/2)) + (2*(b^2*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^4*Sqrt[c +
d*x^2])
```

Rubi in Sympy [A] time = 24.8495, size = 117, normalized size = 0.89

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{2a\left(ad-bc\right)}{c^2x\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{x\left(8ad\left(ad-bc\right)+b^2c^2\right)}{3c^3\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{2x\left(8ad\left(ad-bc\right)+b^2c^2\right)}{3c^4\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(5/2),x)
```

```
[Out] -a**2/(3*c*x**3*(c + d*x**2)**(3/2)) + 2*a*(a*d - b*c)/(c**2*x*(c
 + d*x**2)**(3/2)) + x*(8*a*d*(a*d - b*c) + b**2*c**2)/(3*c**3*(c
 + d*x**2)**(3/2)) + 2*x*(8*a*d*(a*d - b*c) + b**2*c**2)/(3*c**4*
sqrt(c + d*x**2))
```

Mathematica [A] time = 0.11441, size = 107, normalized size = 0.82

$$\frac{a^2\left(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6\right)-2abcx^2\left(3c^2+12cdx^2+8d^2x^4\right)+b^2c^2x^4\left(3c+2dx^2\right)}{3c^4x^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4*(3*c + 2*d*x^2) - 2*a*b*c*x^2*(3*c^2 + 12*c*d*x^2 +
8*d^2*x^4) + a^2*(-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6)
)/(3*c^4*x^3*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.01, size = 116, normalized size = 0.9

$$-\frac{-16\,a^{2}d^{3}x^{6}+16\,abc\,d^{2}x^{6}-2\,b^{2}c^{2}d\,x^{6}-24\,a^{2}c\,d^{2}x^{4}+24\,ab\,c^{2}d\,x^{4}-3\,b^{2}c^{3}x^{4}-6\,a^{2}c^{2}d\,x^{2}+6\,ab\,c^{3}x^{2}+a^{2}c^{3}}{3\,x^{3}c^{4}}\left(d\,x^{2}+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-16*a^2*d^3*x^6+16*a*b*c*d^2*x^6-2*b^2*c^2*d*x^6-24*a^2*c*d
^2*x^4+24*a*b*c^2*d*x^4-3*b^2*c^3*x^4-6*a^2*c^2*d*x^2+6*a*b*c^3*x
^2+a^2*c^3)/(d*x^2+c)^(3/2)/x^3/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28059, size = 176, normalized size = 1.34

$$\frac{\left(2\,\left(b^{2}c^{2}d-8\,abcd^{2}+8\,a^{2}d^{3}\right)x^{6}-a^{2}c^{3}+3\,\left(b^{2}c^{3}-8\,abc^{2}d+8\,a^{2}cd^{2}\right)x^{4}-6\,\left(abc^{3}-a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{3\,\left(c^{4}d^{2}x^{7}+2\,c^{5}dx^{5}+c^{6}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(b^2*c^2*d - 8*a*b*c*d^2 + 8*a^2*d^3)*x^6 - a^2*c^3 + 3*(b
^2*c^3 - 8*a*b*c^2*d + 8*a^2*c*d^2)*x^4 - 6*(a*b*c^3 - a^2*c^2*d)
*x^2)*sqrt(d*x^2 + c)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.236542, size = 348, normalized size = 2.66

$$\frac{x\left(\frac{2\,\left(b^{2}c^{5}d^{2}-5\,abc^{4}d^{3}+4\,a^{2}c^{3}d^{4}\right)x^{2}}{c^{7}d}+\frac{3\,\left(b^{2}c^{6}d-4\,abc^{5}d^{2}+3\,a^{2}c^{4}d^{3}\right)}{c^{7}d}\right)}{3\,\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}abc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}d^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abc^{2}\sqrt{d}+9\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}cd^{\frac{3}{2}}\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^2*c^5*d^2 - 5*a*b*c^4*d^3 + 4*a^2*c^3*d^4)*x^2/(c^7*d
) + 3*(b^2*c^6*d - 4*a*b*c^5*d^2 + 3*a^2*c^4*d^3)/(c^7*d))/(d*x^2
 + c)^(3/2) + 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d
) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(3/2) - 6*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 9*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a^2*c*d^(3/2) + 3*a*b*c^3*sqrt(d) - 4*a^2*c^2*d^(3/2))/((
(sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^3)
```

3.669 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{24c^3\left(c+dx^2\right)^{3/2}}-\frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}}$$

```
[Out] (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(24*c^3*(c + d*x^2)^(3/2)) -
a^2/(4*c*x^4*(c + d*x^2)^(3/2)) - (a*(8*b*c - 7*a*d))/(8*c^2*x^2*
(c + d*x^2)^(3/2)) + (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(8*c^4*S
qrt[c + d*x^2]) - ((8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Rubi [A] time = 0.533453, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}+\frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}-\frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (8*b^2 - (5*a*d*(8*b*c - 7*a*d))/c^2)/(24*c*(c + d*x^2)^(3/2)) -
a^2/(4*c*x^4*(c + d*x^2)^(3/2)) - (a*(8*b*c - 7*a*d))/(8*c^2*x^2*
(c + d*x^2)^(3/2)) + (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(8*c^4*S
qrt[c + d*x^2]) - ((8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Rubi in Sympy [A] time = 35.6733, size = 175, normalized size = 0.95

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{a\left(7ad-8bc\right)}{8c^2x^2\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{5ad\left(7ad-8bc\right)+8b^2c^2}{24c^3\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{5ad\left(7ad-8bc\right)+8b^2c^2}{8c^4\sqrt{c+dx^2}}-\frac{\left(5ad\left(7ad-8bc\right)+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(5/2),x)
```

```
[Out] -a**2/(4*c*x**4*(c + d*x**2)**(3/2)) + a*(7*a*d - 8*b*c)/(8*c**2*
x**2*(c + d*x**2)**(3/2)) + (5*a*d*(7*a*d - 8*b*c) + 8*b**2*c**2)
/(24*c**3*(c + d*x**2)**(3/2)) + (5*a*d*(7*a*d - 8*b*c) + 8*b**2*
c**2)/(8*c**4*sqrt(c + d*x**2)) - (5*a*d*(7*a*d - 8*b*c) + 8*b**2
*c**2)*atanh(sqrt(c + d*x**2)/sqrt(c))/(8*c**(9/2))
```

Mathematica [A] time = 0.479322, size = 179, normalized size = 0.97

$$\frac{-3\left(35a^2d^2-40abcd+8b^2c^2\right)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+\sqrt{c}\sqrt{c+dx^2}\left(\frac{24\left(3a^2d^2-4abcd+b^2c^2\right)}{c+dx^2}-\frac{6a^2c}{x^4}+\frac{3a(11ad-8bc)}{x^2}+\frac{8c(bc-ad)^2}{\left(c+dx^2\right)^2}\right)}{24c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c]*Sqrt[c + d*x^2]*((-6*a^2*c)/x^4 + (3*a*(-8*b*c + 11*a*d)
)/x^2 + (8*c*(b*c - a*d)^2)/(c + d*x^2)^2 + (24*(b^2*c^2 - 4*a*b*
c*d + 3*a^2*d^2))/(c + d*x^2)) + 3*(8*b^2*c^2 - 40*a*b*c*d + 35*a
^2*d^2)*Log[x] - 3*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*Log[c +
Sqrt[c]*Sqrt[c + d*x^2]])/(24*c^(9/2))
```

Maple [A] time = 0.02, size = 265, normalized size = 1.4

$$-\frac{a^2}{4cx^4}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{7a^2d}{8c^2x^2}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{35a^2d^2}{24c^3}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{35a^2d^2}{8c^4}\frac{1}{\sqrt{dx^2+c}}-\frac{35a^2d^2}{8}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{9}{2}}+\frac{b^2}{3c}\left(dx^2+c\right)^{-\frac{3}{2}}+\frac{b^2}{c^2}\frac{1}{\sqrt{dx^2+c}}-b^2\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{5}{2}}-\frac{ab}{cx^2}\left(dx^2+c\right)^{-\frac{3}{2}}-\frac{5abd}{3c^2}\left(dx^2+c\right)^{-\frac{3}{2}}-5\frac{abd}{c^3\sqrt{dx^2+c}}+5\frac{abd}{c^{7/2}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/4*a^2/c/x^4/(d*x^2+c)^(3/2)+7/8*a^2*d/c^2/x^2/(d*x^2+c)^(3/2)+
35/24*a^2*d^2/c^3/(d*x^2+c)^(3/2)+35/8*a^2*d^2/c^4/(d*x^2+c)^(1/2
)-35/8*a^2*d^2/c^(9/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/3*
b^2/c/(d*x^2+c)^(3/2)+b^2/c^2/(d*x^2+c)^(1/2)-b^2/c^(5/2)*ln((2*c
+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-a*b/c/x^2/(d*x^2+c)^(3/2)-5/3*a*b*
d/c^2/(d*x^2+c)^(3/2)-5*a*b*d/c^3/(d*x^2+c)^(1/2)+5*a*b*d/c^(7/2)
*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.255123, size = 1, normalized size = 0.01

$$\left[\frac{2\left(3\left(8b^2c^2d-40abcd^2+35a^2d^3\right)x^6-6a^2c^3+4\left(8b^2c^3-40abc^2d+35a^2cd^2\right)x^4-3\left(8abc^3-7a^2c^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{c}}{48(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^5),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(3*(8*b^2*c^2*d - 40*a*b*c*d^2 + 35*a^2*d^3)*x^6 - 6*a^2
*c^3 + 4*(8*b^2*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2)*x^4 - 3*(8*a*b
```

```
*c^3 - 7*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 3*((8*b^2*c^2*
d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d - 40*a*b*c^
2*d^2 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^
2*d^2)*x^4)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^
2))/((c^4*d^2*x^8 + 2*c^5*d*x^6 + c^6*x^4)*sqrt(c)), 1/24*((3*(8*
b^2*c^2*d - 40*a*b*c*d^2 + 35*a^2*d^3)*x^6 - 6*a^2*c^3 + 4*(8*b^2
*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2)*x^4 - 3*(8*a*b*c^3 - 7*a^2*c^
2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) - 3*((8*b^2*c^2*d^2 - 40*a*b*c
*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d - 40*a*b*c^2*d^2 + 35*a^2
*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4)*ar
ctan(sqrt(-c)/sqrt(d*x^2 + c)))/((c^4*d^2*x^8 + 2*c^5*d*x^6 + c^6
*x^4)*sqrt(-c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.252295, size = 284, normalized size = 1.54

$$\frac{\left(8\,b^2c^2 - 40\,abcd + 35\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^4} + \frac{3\left(dx^2+c\right)b^2c^2 + b^2c^3 - 12\left(dx^2+c\right)abcd - 2\,abc^2d + 9\left(dx^2+c\right)a^2d^2 + a^2cd^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c^4} - \frac{8\left(dx^2+c\right)^{\frac{3}{2}}abcd - 8\sqrt{dx^2+c}abc^2d - 11\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2 + 13\sqrt{dx^2+c}a^2cd^2}{8\,c^4d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*arctan(sqrt(d*x^2 + c)/
sqrt(-c))/(sqrt(-c)*c^4) + 1/3*(3*(d*x^2 + c)*b^2*c^2 + b^2*c^3 -
 12*(d*x^2 + c)*a*b*c*d - 2*a*b*c^2*d + 9*(d*x^2 + c)*a^2*d^2 + a
^2*c*d^2)/((d*x^2 + c)^(3/2)*c^4) - 1/8*(8*(d*x^2 + c)^(3/2)*a*b*
c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 11*(d*x^2 + c)^(3/2)*a^2*d^2
+ 13*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^4*d^2*x^4)
```

3.670 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{5cx^5(c+dx^2)^{3/2}}-\frac{8dx(5b^2c^2-4ad(5bc-4ad))}{15c^5\sqrt{c+dx^2}}-\frac{4dx(5b^2c^2-4ad(5bc-4ad))}{15c^4(c+dx^2)^{3/2}}-\frac{5b^2c^2-4ad(5bc-4ad)}{5c^3x(c+dx^2)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}}$$

```
[Out] -a^2/(5*c*x^5*(c + d*x^2)^(3/2)) - (2*a*(5*b*c - 4*a*d))/(15*c^2*
x^3*(c + d*x^2)^(3/2)) - (5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))/(5*c
^3*x*(c + d*x^2)^(3/2)) - (4*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d)
)*x)/(15*c^4*(c + d*x^2)^(3/2)) - (8*d*(5*b^2*c^2 - 4*a*d*(5*b*c
- 4*a*d))*x)/(15*c^5*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.417538, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a^2}{5cx^5(c+dx^2)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}}-\frac{8dx(5b^2c^2-4ad(5bc-4ad))}{15c^5\sqrt{c+dx^2}}-\frac{4dx(5b^2c^2-4ad(5bc-4ad))}{15c^4(c+dx^2)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] -a^2/(5*c*x^5*(c + d*x^2)^(3/2)) - (2*a*(5*b*c - 4*a*d))/(15*c^2*
x^3*(c + d*x^2)^(3/2)) - (5*b^2 - (4*a*d*(5*b*c - 4*a*d))/c^2)/(5
*c*x*(c + d*x^2)^(3/2)) - (4*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d)
)*x)/(15*c^4*(c + d*x^2)^(3/2)) - (8*d*(5*b^2*c^2 - 4*a*d*(5*b*c
- 4*a*d))*x)/(15*c^5*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 30.0054, size = 180, normalized size = 0.98

$$-\frac{a^2}{5cx^5(c+dx^2)^{\frac{3}{2}}}+\frac{2a(4ad-5bc)}{15c^2x^3(c+dx^2)^{\frac{3}{2}}}-\frac{4ad(4ad-5bc)+5b^2c^2}{5c^3x(c+dx^2)^{\frac{3}{2}}}-\frac{4dx(4ad(4ad-5bc)+5b^2c^2)}{15c^4(c+dx^2)^{\frac{3}{2}}}-\frac{8dx(4ad(4ad-5bc)+5b^2c^2)}{15c^5\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(5/2),x)
```

```
[Out] -a**2/(5*c*x**5*(c + d*x**2)**(3/2)) + 2*a*(4*a*d - 5*b*c)/(15*c*
*2*x**3*(c + d*x**2)**(3/2)) - (4*a*d*(4*a*d - 5*b*c) + 5*b**2*c*
*2)/(5*c**3*x*(c + d*x**2)**(3/2)) - 4*d*x*(4*a*d*(4*a*d - 5*b*c)
 + 5*b**2*c**2)/(15*c**4*(c + d*x**2)**(3/2)) - 8*d*x*(4*a*d*(4*a
*d - 5*b*c) + 5*b**2*c**2)/(15*c**5*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.160322, size = 142, normalized size = 0.78

$$\frac{-a^2\left(3c^4-8c^3dx^2+48c^2d^2x^4+192cd^3x^6+128d^4x^8\right)+10abcx^2\left(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6\right)-5b^2c^2x^4\left(3c^2+12cd\right.}{15c^5x^5(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*b^2*c^2*x^4*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 10*a*b*c*x^2*(
-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) - a^2*(3*c^4 - 8*
c^3*d*x^2 + 48*c^2*d^2*x^4 + 192*c*d^3*x^6 + 128*d^4*x^8))/(15*c^
5*x^5*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.011, size = 158, normalized size = 0.9

$$-\frac{128 a^2 d^4 x^8 - 160 abcd^3 x^8 + 40 b^2 c^2 d^2 x^8 + 192 a^2 cd^3 x^6 - 240 abc^2 d^2 x^6 + 60 b^2 c^3 dx^6 + 48 a^2 c^2 d^2 x^4 - 60 abc^3 dx^4 + 15 b^2 c^4}{15 x^5 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/15*(128*a^2*d^4*x^8-160*a*b*c*d^3*x^8+40*b^2*c^2*d^2*x^8+192*a
^2*c*d^3*x^6-240*a*b*c^2*d^2*x^6+60*b^2*c^3*d*x^6+48*a^2*c^2*d^2*
x^4-60*a*b*c^3*d*x^4+15*b^2*c^4*x^4-8*a^2*c^3*d*x^2+10*a*b*c^4*x^
2+3*a^2*c^4)/(d*x^2+c)^(3/2)/x^5/c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.409861, size = 231, normalized size = 1.26

$$-\frac{\left(8 \left(5 b^2 c^2 d^2 - 20 abcd^3 + 16 a^2 d^4\right) x^8 + 12 \left(5 b^2 c^3 d - 20 abc^2 d^2 + 16 a^2 cd^3\right) x^6 + 3 a^2 c^4 + 3 \left(5 b^2 c^4 - 20 abc^3 d + 16 a^2 c^2 d^2\right)}{15 \left(c^5 d^2 x^9 + 2 c^6 dx^7 + c^7 x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^6),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*b^2*c^2*d^2 - 20*a*b*c*d^3 + 16*a^2*d^4)*x^8 + 12*(5*
b^2*c^3*d - 20*a*b*c^2*d^2 + 16*a^2*c*d^3)*x^6 + 3*a^2*c^4 + 3*(5
*b^2*c^4 - 20*a*b*c^3*d + 16*a^2*c^2*d^2)*x^4 + 2*(5*a*b*c^4 - 4*
a^2*c^3*d)*x^2)*sqrt(d*x^2 + c)/(c^5*d^2*x^9 + 2*c^6*d*x^7 + c^7*
x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.252757, size = 687, normalized size = 3.75

$$-\frac{x\left(\frac{\left(5\,b^{2}c^{6}d^{3}-16\,abc^{5}d^{4}+11\,a^{2}c^{4}d^{5}\right)x^{2}}{c^{9}d}+\frac{6\,\left(b^{2}c^{7}d^{2}-3\,abc^{6}d^{3}+2\,a^{2}c^{5}d^{4}\right)}{c^{9}d}\right)}{3\,\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}b^{2}c^{2}\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}abcd^{\frac{3}{2}}+45\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}a^{2}d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b^{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^6),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*b^2*c^6*d^3 - 16*a*b*c^5*d^4 + 11*a^2*c^4*d^5)*x^2/(c^
9*d) + 6*(b^2*c^7*d^2 - 3*a*b*c^6*d^3 + 2*a^2*c^5*d^4)/(c^9*d))/(
d*x^2 + c)^(3/2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c
^2*sqrt(d) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 4
5*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*(sqrt(d)*x - s
qrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 300*(sqrt(d)*x - sqrt(d*x^2 +
 c))^6*a*b*c^2*d^(3/2) - 240*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*
c*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) -
500*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 490*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*b^2*c^5*sqrt(d) + 340*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*b*c^4*d^(3/2) - 320*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d
^(5/2) + 15*b^2*c^6*sqrt(d) - 80*a*b*c^5*d^(3/2) + 73*a^2*c^4*d^(
5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^4)
```

3.671 $\int \frac{x^5}{\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

```
[Out] -((a*x^2)/(b^2*Sqrt[d*x^2])) + x^4/(3*b*Sqrt[d*x^2]) + (a^(3/2)*x
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[d*x^2])
```

Rubi [A] time = 0.066319, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[d*x^2])) + x^4/(3*b*Sqrt[d*x^2]) + (a^(3/2)*x
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[d*x^2])
```

Rubi in Sympy [A] time = 22.3482, size = 70, normalized size = 0.97

$$\frac{a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{\frac{5}{2}}\sqrt{d}}-\frac{a\sqrt{dx^2}}{b^2d}+\frac{\left(dx^2\right)^{\frac{3}{2}}}{3bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] a**(3/2)*atan(sqrt(b)*sqrt(d*x**2)/(sqrt(a)*sqrt(d)))/(b**(5/2)*s
qrt(d)) - a*sqrt(d*x**2)/(b**2*d) + (d*x**2)**(3/2)/(3*b*d**2)
```

Mathematica [A] time = 0.040234, size = 56, normalized size = 0.78

$$\frac{x\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]]))/(3*b^(5/2)*Sqrt[d*x^2])
```

Maple [A] time = 0.01, size = 53, normalized size = 0.7

$$\frac{x}{3\,b^2}\left(\sqrt{ab}x^3b-3\,\sqrt{ab}xa+3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/3*x*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(x*b/(a*b)
^(1/2)))/(d*x^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220001, size = 1, normalized size = 0.01

$$\left[\frac{3\,ad\sqrt{-\frac{a}{bd}}\log\left(\frac{2\,bdx^2\sqrt{-\frac{a}{bd}}+(bx^2-a)\sqrt{dx^2}}{bx^3+ax}\right)+2\left(bx^2-3\,a\right)\sqrt{dx^2}}{6\,b^2d}, \frac{3\,ad\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}}{d\sqrt{\frac{a}{bd}}}\right)+\left(bx^2-3\,a\right)\sqrt{dx^2}}{3\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*d*sqrt(-a/(b*d))*log((2*b*d*x^2*sqrt(-a/(b*d)) + (b*x^2
 - a)*sqrt(d*x^2))/(b*x^3 + a*x)) + 2*(b*x^2 - 3*a)*sqrt(d*x^2))/
(b^2*d), 1/3*(3*a*d*sqrt(a/(b*d))*arctan(sqrt(d*x^2)/(d*sqrt(a/(b
*d)))) + (b*x^2 - 3*a)*sqrt(d*x^2))/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(d*x**2)*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.236454, size = 95, normalized size = 1.32

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b^2}+\frac{\sqrt{dx^2}b^2d^5x^2-3\sqrt{dx^2}abd^5}{3\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + 1/3*(sq
rt(d*x^2)*b^2*d^5*x^2 - 3*sqrt(d*x^2)*a*b*d^5)/(b^3*d^6)
```

3.672 $\int \frac{x^3}{\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=52

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

```
[Out] x^2/(b*Sqrt[d*x^2]) - (Sqrt[a]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^
(3/2)*Sqrt[d*x^2])
```

Rubi [A] time = 0.0446219, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] x^2/(b*Sqrt[d*x^2]) - (Sqrt[a]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^
(3/2)*Sqrt[d*x^2])
```

Rubi in Sympy [A] time = 17.4194, size = 51, normalized size = 0.98

$$-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{\frac{3}{2}}\sqrt{d}}+\frac{\sqrt{dx^2}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] -sqrt(a)*atan(sqrt(b)*sqrt(d*x**2)/(sqrt(a)*sqrt(d)))/(b**(3/2)*s
qrt(d)) + sqrt(d*x**2)/(b*d)
```

Mathematica [A] time = 0.0257385, size = 44, normalized size = 0.85

$$\frac{x\left(\sqrt{b}x-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sq
rt[d*x^2])
```

Maple [A] time = 0.008, size = 38, normalized size = 0.7

$$\frac{x}{b}\left(x\sqrt{ab}-a\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] x*(x*(a*b)^(1/2)-a*arctan(x*b/(a*b)^(1/2)))/(d*x^2)^(1/2)/b/(a*b)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222269, size = 1, normalized size = 0.02

$$\left[\frac{d\sqrt{-\frac{a}{bd}}\log\left(-\frac{2\,bdx^2\sqrt{-\frac{a}{bd}}-(bx^2-a)\sqrt{dx^2}}{bx^3+ax}\right)+2\sqrt{dx^2}}{2\,bd}, -\frac{d\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}}{d\sqrt{\frac{a}{bd}}}\right)-\sqrt{dx^2}}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/(b*d))*log(-(2*b*d*x^2*sqrt(-a/(b*d)) - (b*x^2 -
a)*sqrt(d*x^2))/(b*x^3 + a*x)) + 2*sqrt(d*x^2))/(b*d), -(d*sqrt(a
/(b*d))*arctan(sqrt(d*x^2)/(d*sqrt(a/(b*d)))) - sqrt(d*x^2))/(b*d
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(d*x**2)*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.236749, size = 62, normalized size = 1.19

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b}-\frac{\sqrt{dx^2}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*
x^2)/b)/d
```

3.673 $\int \frac{x}{\sqrt{dx^2}(a+bx^2)} \, dx$

Optimal. Leaf size=34

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Rubi [A] time = 0.0232445, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Rubi in Sympy [A] time = 12.2338, size = 39, normalized size = 1.15

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{a}\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] atan(sqrt(b)*sqrt(d*x**2)/(sqrt(a)*sqrt(d)))/(sqrt(a)*sqrt(b)*sqr
t(d))
```

Mathematica [A] time = 0.012135, size = 34, normalized size = 1.

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Maple [A] time = 0.005, size = 24, normalized size = 0.7

$$x\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)/(d*x^2)^(1/2),x)

[Out] 1/(d*x^2)^(1/2)*x/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.225072, size = 1, normalized size = 0.03

$$\left[-\frac{\sqrt{-abd}\log\left(\frac{2\,abdx^2+\sqrt{-abd}\,(bx^2-a)\,\sqrt{dx^2}}{bx^3+ax}\right)}{2\,abd},\ \frac{\sqrt{abd}\arctan\left(\frac{\sqrt{abd}\sqrt{dx^2}}{ad}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b*d)*log((2*a*b*d*x^2 + sqrt(-a*b*d)*(b*x^2 - a)*sq
rt(d*x^2))/(b*x^3 + a*x))/(a*b*d), sqrt(a*b*d)*arctan(sqrt(a*b*d)
*sqrt(d*x^2)/(a*d))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)/(d*x**2)**(1/2),x)

[Out] Integral(x/(sqrt(d*x**2)*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.23842, size = 31, normalized size = 0.91

$$\frac{\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*sqrt(d*x^2)),x, algorithm="giac")

[Out] arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

3.674 $\int \frac{1}{x\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

```
[Out] -(1/(a*Sqrt[d*x^2])) - (Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a
^(3/2)*Sqrt[d*x^2])
```

Rubi [A] time = 0.0418573, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -(1/(a*Sqrt[d*x^2])) - (Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a
^(3/2)*Sqrt[d*x^2])
```

Rubi in Sympy [A] time = 17.7112, size = 53, normalized size = 1.06

$$-\frac{1}{a\sqrt{dx^2}}-\frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{\frac{3}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] -1/(a*sqrt(d*x**2)) - sqrt(b)*atan(sqrt(b)*sqrt(d*x**2)/(sqrt(a)*
sqrt(d)))/(a**(3/2)*sqrt(d))
```

Mathematica [A] time = 0.0273057, size = 46, normalized size = 0.92

$$-\frac{dx^2\left(\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}\left(dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((d*x^2*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3
/2)*(d*x^2)^(3/2)))
```

Maple [A] time = 0.008, size = 36, normalized size = 0.7

$$-\frac{1}{a}\left(b\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x+\sqrt{ab}\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] -(b*arctan(x*b/(a*b)^(1/2))*x+(a*b)^(1/2))/(d*x^2)^(1/2)/a/(a*b)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24417, size = 1, normalized size = 0.02

$$\left[\frac{dx^2\sqrt{-\frac{b}{ad}}\log\left(-\frac{2\,adx^2\sqrt{-\frac{b}{ad}}-(bx^2-a)\sqrt{dx^2}}{bx^3+ax}\right)-2\sqrt{dx^2}}{2\,adx^2}, -\frac{dx^2\sqrt{\frac{b}{ad}}\arctan\left(\frac{\sqrt{dx^2}b}{ad\sqrt{\frac{b}{ad}}}\right)+\sqrt{dx^2}}{adx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(d*x^2*sqrt(-b/(a*d))*log(-(2*a*d*x^2*sqrt(-b/(a*d)) - (b*x^
2 - a)*sqrt(d*x^2))/(b*x^3 + a*x)) - 2*sqrt(d*x^2))/(a*d*x^2), -(
d*x^2*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*b/(a*d*sqrt(b/(a*d)))) + s
qrt(d*x^2))/(a*d*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(d*x**2)*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.230399, size = 65, normalized size = 1.3

$$-d\left(\frac{b\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}ad}+\frac{1}{\sqrt{dx^2}ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2)*x),x, algorithm="giac")
```

```
[Out] -d*(b*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*a*d) + 1/(sq
rt(d*x^2)*a*d))
```

3.675 $\int \frac{1}{x^3\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal. Leaf size=68

$$\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

```
[Out] b/(a^2*Sqrt[d*x^2]) - 1/(3*a*x^2*Sqrt[d*x^2]) + (b^(3/2)*x*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[d*x^2])
```

Rubi [A] time = 0.0611417, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] b/(a^2*Sqrt[d*x^2]) - 1/(3*a*x^2*Sqrt[d*x^2]) + (b^(3/2)*x*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[d*x^2])
```

Rubi in Sympy [A] time = 23.9533, size = 66, normalized size = 0.97

$$-\frac{d}{3a\left(dx^2\right)^{\frac{3}{2}}}+\frac{b}{a^2\sqrt{dx^2}}+\frac{b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{\frac{5}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] -d/(3*a*(d*x**2)**(3/2)) + b/(a**2*sqrt(d*x**2)) + b**(3/2)*atan(
sqrt(b)*sqrt(d*x**2)/(sqrt(a)*sqrt(d)))/(a**(5/2)*sqrt(d))
```

Mathematica [A] time = 0.0358477, size = 58, normalized size = 0.85

$$\frac{d\left(3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\left(a-3bx^2\right)\right)}{3a^{5/2}\left(dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (d*(-(Sqrt[a]*(a - 3*b*x^2)) + 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/S
qrt[a]]))/(3*a^(5/2)*(d*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.9

$$\frac{1}{3a^2x^2}\left(3b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3+3bx^2\sqrt{ab}-a\sqrt{ab}\right)\frac{1}{\sqrt{dx^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/3/x^2*(3*b^2*arctan(x*b/(a*b)^(1/2))*x^3+3*b*x^2*(a*b)^(1/2)-a*
(a*b)^(1/2))/(d*x^2)^(1/2)/a^2/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245265, size = 1, normalized size = 0.01

$$\left[\frac{3\, bdx^{4} \sqrt{-\frac{b}{ad}} \log\left(\frac{2\, adx^{2} \sqrt{-\frac{b}{ad}} + \left(bx^{2} - a\right) \sqrt{dx^{2}}}{bx^{3} + ax}\right) + 2\, \left(3\, bx^{2} - a\right) \sqrt{dx^{2}}}{6\, a^{2} dx^{4}}, \frac{3\, bdx^{4} \sqrt{\frac{b}{ad}} \arctan\left(\frac{\sqrt{dx^{2}} b}{ad \sqrt{\frac{b}{ad}}}\right) + \left(3\, bx^{2} - a\right) \sqrt{dx^{2}}}{3\, a^{2} dx^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2)*x^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*d*x^4*sqrt(-b/(a*d))*log((2*a*d*x^2*sqrt(-b/(a*d)) + (b
*x^2 - a)*sqrt(d*x^2))/(b*x^3 + a*x)) + 2*(3*b*x^2 - a)*sqrt(d*x^
2))/(a^2*d*x^4), 1/3*(3*b*d*x^4*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*
b/(a*d*sqrt(b/(a*d)))) + (3*b*x^2 - a)*sqrt(d*x^2))/(a^2*d*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \sqrt{dx^{2}} \left(a + bx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(d*x**2)*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.232168, size = 92, normalized size = 1.35

$$\frac{1}{3}\, d^{2} \left(\frac{3\, b^{2} \arctan\left(\frac{\sqrt{dx^{2}} b}{\sqrt{abd}}\right)}{\sqrt{abd} a^{2} d^{2}} + \frac{3\, bdx^{2} - ad}{\sqrt{dx^{2}} a^{2} d^{3} x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2)*x^3),x, algorithm="giac")
```

```
[Out] 1/3*d^2*(3*b^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*a^2
*d^2) + (3*b*d*x^2 - a*d)/(sqrt(d*x^2)*a^2*d^3*x^2))
```

3.676 $\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=157

$$\frac{a^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}-\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{x\sqrt{c+dx^2}(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

```
[Out] ((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2*d) + (x^3*Sqrt[c + d*x^2
])/(4*b) + (a^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/b^3 - ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2
)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3*d^(3/2))
```

Rubi [A] time = 0.607539, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{a^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}-\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{x\sqrt{c+dx^2}(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2*d) + (x^3*Sqrt[c + d*x^2
])/(4*b) + (a^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/b^3 - ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2
)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3*d^(3/2))
```

Rubi in Sympy [A] time = 66.7809, size = 143, normalized size = 0.91

$$-\frac{a^{\frac{3}{2}}\sqrt{ad-bc}\,\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{x\sqrt{c+dx^2}\,(4ad-bc)}{8b^2d}+\frac{\left(8a^2d^2-4abcd-b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] -a**(3/2)*sqrt(a*d - b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c
 + d*x**2)))/b**3 + x**3*sqrt(c + d*x**2)/(4*b) - x*sqrt(c + d*x*
*2)*(4*a*d - b*c)/(8*b**2*d) + (8*a**2*d**2 - 4*a*b*c*d - b**2*c*
*2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*b**3*d**(3/2))
```

Mathematica [A] time = 0.359307, size = 148, normalized size = 0.94

$$\frac{8a^{3/2}d^{3/2}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)-\left(-8a^2d^2+4abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+b\sqrt{d}x\sqrt{c+dx^2}\left(-4ad+bc+2b\right.}{8b^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(b*c - 4*a*d + 2*b*d*x^2) + 8*a^(3/2
)*d^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqr
t[c + d*x^2])] - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*Log[d*x + Sqrt
[d]*Sqrt[c + d*x^2]])/(8*b^3*d^(3/2))
```

Maple [B] time = 0.022, size = 1088, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b*x*(d*x^2+c)^(3/2)/d-1/8/b*c/d*x*(d*x^2+c)^(1/2)-1/8/b*c^2/d
^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/2/b^2*a*x*(d*x^2+c)^(1/2)-
1/2/b^2*a*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2/b^2*a^2/(-a
*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^3*a^2*d^(1/2)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^
3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d-1/2/b^2*a^2/(-a*b)^(1/2)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))*c-1/2/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b
^3*a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))-1/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*d+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.412373, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a*b*c + a^2*d)*a*d^(3/2)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(
(b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d*x^3 + (b^2*c - 4*a*b*d)
*x)*sqrt(d*x^2 + c)*sqrt(d) - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*l
og(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^3*d^(3/2))
, 1/8*(2*sqrt(-a*b*c + a^2*d)*a*sqrt(-d)*d*log(((b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (2*b^2*d*x^3 + (b^2*c - 4*a*b*d
)*x)*sqrt(d*x^2 + c)*sqrt(-d) - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^3*sqrt(-d)*d), -1/16*(8*s
qrt(a*b*c - a^2*d)*a*d^(3/2)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c
)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) - 2*(2*b^2*d*x^3 + (b^
2*c - 4*a*b*d)*x)*sqrt(d*x^2 + c)*sqrt(d) + (b^2*c^2 + 4*a*b*c*d
- 8*a^2*d^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))
/(b^3*d^(3/2)), -1/8*(4*sqrt(a*b*c - a^2*d)*a*sqrt(-d)*d*arctan(-
1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c
)*x)) - (2*b^2*d*x^3 + (b^2*c - 4*a*b*d)*x)*sqrt(d*x^2 + c)*sqrt(
-d) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*arctan(sqrt(-d)*x/sqrt(d*
x^2 + c)))/(b^3*sqrt(-d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.242496, size = 254, normalized size = 1.62

$$\frac{1}{8}\sqrt{dx^2+c}\,x\left(\frac{2\,x^2}{b}+\frac{b^5cd-4\,ab^4d^2}{b^6d^2}\right)-\frac{\left(a^2bc\sqrt{d}-a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,b^3}+\frac{\left(b^2c^2\sqrt{d}+4\,abcd^{\frac{3}{2}}-8\,a^2d^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*x*(2*x^2/b + (b^5*c*d - 4*a*b^4*d^2)/(b^6*d^2
)) - (a^2*b*c*sqrt(d) - a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqr
t(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a
*b*c*d - a^2*d^2)*b^3) + 1/16*(b^2*c^2*sqrt(d) + 4*a*b*c*d^(3/2)
- 8*a^2*d^(5/2))*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^3*d^2)
```

3.677 $\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=88

$$\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

```
[Out] -((a*Sqrt[c + d*x^2])/b^2) + (c + d*x^2)^(3/2)/(3*b*d) + (a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^
(5/2)
```

Rubi [A] time = 0.235172, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] -((a*Sqrt[c + d*x^2])/b^2) + (c + d*x^2)^(3/2)/(3*b*d) + (a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^
(5/2)
```

Rubi in Sympy [A] time = 26.7021, size = 73, normalized size = 0.83

$$-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{a\sqrt{ad-bc}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{5}{2}}}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] -a*sqrt(c + d*x**2)/b**2 + a*sqrt(a*d - b*c)*atan(sqrt(b)*sqrt(c
+ d*x**2)/sqrt(a*d - b*c))/b**(5/2) + (c + d*x**2)**(3/2)/(3*b*d)
```

Mathematica [A] time = 0.188574, size = 85, normalized size = 0.97

$$\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\sqrt{c+dx^2}\left(b\left(c+dx^2\right)-3ad\right)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*a*d + b*(c + d*x^2)))/(3*b^2*d) + (a*Sqrt[b*
c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5
/2)
```

Maple [B] time = 0.02, size = 963, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(3/2)/b/d-1/2*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*a/b^3*d
^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2*a/
b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*c-1/2*a/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*a/b^3*d^(
1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/2*a/b
^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262815, size = 1, normalized size = 0.01

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2+4\,\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\left(bdx^2+bc-3\,ad\right)\sqrt{dx^2+c}}{12\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2
 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*(b*d*x^2 + b*c - 3*a*d)*sqrt(d*x^2 + c))
/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2
*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) + 2*(b*d*x^
2 + b*c - 3*a*d)*sqrt(d*x^2 + c))/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{c + dx^2}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.227211, size = 130, normalized size = 1.48

$$-\frac{\frac{3\left(abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2-3\sqrt{dx^2+c}abd}{b^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/3*(3*(a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - ((d*x^2 + c)^(3/2)*b^2 - 3
*sqrt(d*x^2 + c)*a*b*d)/b^3)/d
```

3.678 $\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^2 + ((b*c - 2*a*d)*A
rcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

Rubi [A] time = 0.270774, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^2 + ((b*c - 2*a*d)*A
rcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

Rubi in Sympy [A] time = 40.5569, size = 99, normalized size = 0.88

$$\frac{\sqrt{a}\sqrt{ad-bc}\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{x\sqrt{c+dx^2}}{2b}-\frac{(2ad-bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] sqrt(a)*sqrt(a*d - b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c +
 d*x**2)))/b**2 + x*sqrt(c + d*x**2)/(2*b) - (2*a*d - b*c)*atanh(
sqrt(d)*x/sqrt(c + d*x**2))/(2*b**2*sqrt(d))
```

Mathematica [A] time = 0.214566, size = 108, normalized size = 0.96

$$\frac{\frac{(bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-2\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+bx\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + ((b*c - 2*a*d)*Log[d*x +
Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^2)
```

Maple [B] time = 0.018, size = 1010, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b+1/2/b*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1
/2))-1/2*a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*a/b^2*d^(1/2)*l
n((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))-1/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2*a/(-a*b)^(
1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x-1/b*(-a*b)^(1/2)))*c+1/2*a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-1/2*a/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))*d-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.331841, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{dx^2+c}b\sqrt{d}x-(bc-2\,ad)\log\left(2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+\sqrt{-abc+a^2d}\sqrt{d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\,(3\,ab\ldots}{\ldots}\right.}{4\,b^2\sqrt{d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*sqrt(d)*x - (b*c - 2*a*d)*log(2*sqrt(d*
x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)) + sqrt(-a*b*c + a^2*d)*sqrt
(d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b
*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(b^2*s
qrt(d)), 1/4*(2*sqrt(d*x^2 + c)*b*sqrt(-d)*x + 2*(b*c - 2*a*d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + sqrt(-a*b*c + a^2*d)*sqrt(-d)*
log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
```

```
^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c +
 a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(b^2*sqrt(
-d)), 1/4*(2*sqrt(d*x^2 + c)*b*sqrt(d)*x + 2*sqrt(a*b*c - a^2*d)*
sqrt(d)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d
)*sqrt(d*x^2 + c)*x)) - (b*c - 2*a*d)*log(2*sqrt(d*x^2 + c)*d*x -
 (2*d*x^2 + c)*sqrt(d)))/(b^2*sqrt(d)), 1/2*(sqrt(d*x^2 + c)*b*sq
rt(-d)*x + (b*c - 2*a*d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + sqr
t(a*b*c - a^2*d)*sqrt(-d)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(
sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(b^2*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c + dx^2}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.24782, size = 185, normalized size = 1.65

$$\frac{\sqrt{dx^2 + cx}}{2\,b} + \frac{\left(abc\sqrt{d} - a^2 d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2 d^2}}\right)}{\sqrt{abcd - a^2 d^2}\, b^2} - \frac{(bc - 2\,ad)\ln\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{4\,b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*x/b + (a*b*c*sqrt(d) - a^2*d^(3/2))*arctan(1/
2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d
- a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2) - 1/4*(b*c - 2*a*d)*ln(
(sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^2*sqrt(d))
```

3.679 $\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(3/2)
```

Rubi [A] time = 0.147658, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(3/2)
```

Rubi in Sympy [A] time = 20.0182, size = 53, normalized size = 0.82

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{ad-bc}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] sqrt(c + d*x**2)/b - sqrt(a*d - b*c)*atan(sqrt(b)*sqrt(c + d*x**2
)/sqrt(a*d - b*c))/b**(3/2)
```

Mathematica [A] time = 0.0516501, size = 65, normalized size = 1.

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(3/2)
```

Maple [B] time = 0.015, size = 936, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1
/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x-1/b*(-a*b)^(1/2)))*a*d-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/2/b*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)-1/2/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x
+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2)))*a*d-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258703, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\sqrt{dx^2+c}}{4\,b},\ -\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}\,b\sqrt{-\frac{bc-ad}{b}}}\right)-2\sqrt{dx^2+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^
2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) + 4*sqrt(d*x^2 + c))/b, -1/2*(sqrt(-(b*c - a*d)/b)*
arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c
- a*d)/b))) - 2*sqrt(d*x^2 + c))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.23859, size = 86, normalized size = 1.32

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}+\frac{\sqrt{dx^2+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b) + sqrt(d*x^2 + c)/b
```

3.680 $\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/b
```

Rubi [A] time = 0.119653, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/b
```

Rubi in Sympy [A] time = 21.162, size = 70, normalized size = 0.86

$$\frac{\sqrt{d}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}-\frac{\sqrt{ad-bc}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] sqrt(d)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/b - sqrt(a*d - b*c)*ata
nh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*b)
```

Mathematica [A] time = 0.0543789, size = 84, normalized size = 1.04

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(Sqrt[a]*b) + (Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])
/b
```

Maple [B] time = 0.014, size = 948, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*d^(1/2)/b*ln((d*(-a*b)^(
1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/(
-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))
)*c-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*d^(1/2)/b*ln((-d*(-a
*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-
1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d+1/2/(-a*b)^(1/2)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275759, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\cdots}{b^2x^4+2\,abx^2+a^2}\right.}{4\,b}\right.$$

$$-\frac{\sqrt{\frac{bc-ad}{a}}\arctan\left(-\frac{(bc-2\,ad)x^2-ac}{2\,\sqrt{dx^2+c}ax\sqrt{\frac{bc-ad}{a}}}\right)-\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)}{2\,b},\frac{2\,\sqrt{-d}\arctan\left(\frac{dx}{\sqrt{dx^2+c}\sqrt{-d}}\right)-\sqrt{\frac{bc-ad}{a}}\operatorname{arc}\cdots}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
 sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4
+ a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c -
 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2
*a*b*x^2 + a^2)))/b, 1/4*(4*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*
sqrt(-d))) + sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*
x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/
```

```
(b^2*x^4 + 2*a*b*x^2 + a^2)))/b, -1/2*(sqrt((b*c - a*d)/a)*arctan
(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c -
a*d)/a))) - sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c))/b, 1/2*(2*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - s
qrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*
x^2 + c)*a*x*sqrt((b*c - a*d)/a))))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.2437, size = 151, normalized size = 1.86

$$-\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b}-\frac{\sqrt{d}\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 +
c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a
^2*d^2)*b) - 1/2*sqrt(d)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b
```

3.681 $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

```
[Out] -((Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a) + (Sqrt[b*c - a*d
]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b])
```

Rubi [A] time = 0.207051, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a) + (Sqrt[b*c - a*d
]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b])
```

Rubi in Sympy [A] time = 26.017, size = 66, normalized size = 0.82

$$-\frac{\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{\sqrt{ad-bc}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x/(b*x**2+a),x)
```

```
[Out] -sqrt(c)*atanh(sqrt(c + d*x**2)/sqrt(c))/a + sqrt(a*d - b*c)*atan
(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*sqrt(b))
```

Mathematica [C] time = 0.602429, size = 229, normalized size = 2.86

$$\frac{\frac{\sqrt{bc-ad}\left(\log\left(-\frac{2a\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{3/2}}\right)+\log\left(-\frac{2a\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{3/2}}\right)\right)}{\sqrt{b}}-2\sqrt{c}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+2\sqrt{c}\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[c]*Log[x] - 2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c + d*x^2]] +
(Sqrt[b*c - a*d]*(Log[(-2*a*Sqrt[b]*(Sqrt[b]*c - I*Sqrt[a]*d*x +
Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(3/2)*(I*Sqrt[a] +
 Sqrt[b]*x))] + Log[(-2*a*Sqrt[b]*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sq
rt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(3/2)*((-I)*Sqrt[a]
+ Sqrt[b]*x))]))/Sqrt[b])/(2*a)
```

Maple [B] time = 0.017, size = 984, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x/(b*x^2+a),x)
```

```
[Out] -1/a*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a*(d*x^2+c)^
(1/2)-1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*d^(1/2)*(-a*b)^(1/2)/b*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/2/a*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+1/2/a*d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+
(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2)))*d+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 0.317225, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)}{4\,a},\right.$$
$$-\frac{4\sqrt{-c}\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,a},\ \sqrt{-\ldots}$$
$$\left.-\frac{2\sqrt{-c}\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}\,b\sqrt{-\frac{bc-ad}{b}}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^
2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) + 2*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2))/a, -1/4*(4*sqrt(-c)*arctan(c/(sqrt(d*x^2 + c)*sqrt(
-c))) - sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*
b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*
a*b*x^2 + a^2)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2
 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) + sqrt(
c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, -1/2*(
2*sqrt(-c)*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - sqrt(-(b*c - a*
d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(
-(b*c - a*d)/b))))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/x/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.232818, size = 117, normalized size = 1.46

$$-d\left(\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] -d*((b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(s
qrt(-b^2*c + a*b*d)*a*d) - c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*
sqrt(-c)*d))
```

3.682 $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

Rubi [A] time = 0.145741, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

Rubi in Sympy [A] time = 23.2525, size = 56, normalized size = 0.8

$$-\frac{\sqrt{c+dx^2}}{ax}+\frac{\sqrt{ad-bc}\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(c + d*x**2)/(a*x) + sqrt(a*d - b*c)*atanh(x*sqrt(a*d - b*c)
/(sqrt(a)*sqrt(c + d*x**2)))/a**(3/2)
```

Mathematica [A] time = 0.121519, size = 71, normalized size = 1.01

$$-\frac{\frac{x\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}+\frac{\sqrt{c+dx^2}}{a}}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c + d*x^2]/a + (Sqrt[b*c - a*d]*x*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(3/2))/x)
```

Maple [B] time = 0.024, size = 1017, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/c/x*(d*x^2+c)^(3/2)+1/a*d/c*x*(d*x^2+c)^(1/2)+1/a*d^(1/2)*ln
(x*d^(1/2)+(d*x^2+c)^(1/2))-1/2*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)-1/2/a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2
*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/2*b/a/(-a*b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)-1/2/a*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/
2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*d-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2), x)
```

Fricas [A] time = 0.259548, size = 1, normalized size = 0.01

$$\left[\frac{x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)\,x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)\,x^2+4\,(a^2cx-(abc-2\,a^2d)\,x^3)\,\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,\sqrt{dx^2+c}}{4\,ax}, \frac{x\sqrt{\frac{bc-ad}{a}}\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (
a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c))/(a*x), 1/2*(x*sqrt((
b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 +
 c)*a*x*sqrt((b*c - a*d)/a))) - 2*sqrt(d*x^2 + c))/(a*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{x^2\,(a + bx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.78806, size = 158, normalized size = 2.26

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a}+\frac{2\,c\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] (b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c
))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^
2*d^2)*a) + 2*c*sqrt(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a)
```

3.683 $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=113

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

```
[Out] -Sqrt[c + d*x^2]/(2*a*x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/(2*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/a^2
```

Rubi [A] time = 0.361708, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*a*x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/(2*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/a^2
```

Rubi in Sympy [A] time = 46.9083, size = 97, normalized size = 0.86

$$-\frac{\sqrt{c+dx^2}}{2ax^2}-\frac{\sqrt{b}\sqrt{ad-bc}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2}-\frac{\left(\frac{ad}{2}-bc\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a),x)
```

```
[Out] -sqrt(c + d*x**2)/(2*a*x**2) - sqrt(b)*sqrt(a*d - b*c)*atan(sqrt(
b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/a**2 - (a*d/2 - b*c)*atanh(s
qrt(c + d*x**2)/sqrt(c))/(a**2*sqrt(c))
```

Mathematica [C] time = 1.64559, size = 281, normalized size = 2.49

$$-\frac{\sqrt{b}\sqrt{bc-ad}\log\left(\frac{2a^2\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\sqrt{b}\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{3/2}}\right)+\sqrt{b}\sqrt{bc-ad}\log\left(\frac{2a^2\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\sqrt{b}\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{3/2}}\right)+\frac{(ad-2bc)\log\left(\sqrt{c}\sqrt{c+dx\right.}}{\sqrt{c}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] -((a*Sqrt[c + d*x^2])/x^2 + ((2*b*c - a*d)*Log[x])/Sqrt[c] + ((-2
*b*c + a*d)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/Sqrt[c] + Sqrt[b]*S
qrt[b*c - a*d]*Log[(2*a^2*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c -
 a*d]*Sqrt[c + d*x^2]))/(Sqrt[b]*(b*c - a*d)^(3/2)*(I*Sqrt[a] + S
qrt[b]*x))] + Sqrt[b]*Sqrt[b*c - a*d]*Log[(2*a^2*(Sqrt[b]*c + I*S
qrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(Sqrt[b]*(b*c - a*
```

```
d)^(3/2)*((-I)*Sqrt[a] + Sqrt[b]*x))])/(2*a^2)
```

Maple [B] time = 0.018, size = 1054, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2/a/c/x^2*(d*x^2+c)^(3/2)-1/2/a*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d
*x^2+c)^(1/2))/x)+1/2/a*d/c*(d*x^2+c)^(1/2)+b/a^2*c^(1/2)*ln((2*c
+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a^2*(d*x^2+c)^(1/2)+1/2*b/a^2*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)+1/2/a^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)
/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/a/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))*d-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/2*b/a^2*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-1/2/a^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b
*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/a/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*d-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3), x)
```

Fricas [A] time = 0.340166, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{b^2c - abd}\sqrt{c}x^2 \log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)\,x^2-4\,(bdx^2+2\,bc-ad)\,\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) - (2\,bc - ad)x^2 \log\left(-\frac{(dx \cdots}{\cdots}\right) \cdots}{4\,a^2\sqrt{c}x^2}\right.$$

$$-\frac{2\,\sqrt{-b^2c + abd}\sqrt{c}x^2 \arctan\left(\frac{bdx^2+2\,bc-ad}{2\,\sqrt{-b^2c+abd}\sqrt{dx^2+c}}\right) + (2\,bc - ad)x^2 \log\left(-\frac{(dx^2+2\,c)\,\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right) + 2\,\sqrt{dx^2 + c}a\sqrt{c}}{4\,a^2\sqrt{c}x^2},$$

$$\left.-\frac{\sqrt{-b^2c + abd}\sqrt{-c}x^2 \arctan\left(\frac{bdx^2+2\,bc-ad}{2\,\sqrt{-b^2c+abd}\sqrt{dx^2+c}}\right) - (2\,bc - ad)x^2 \arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right) + \sqrt{dx^2 + c}a\sqrt{-c}}{2\,a^2\sqrt{-c}x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*sqrt(c)*x^2*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*
x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - (2*b*c - a*d)*x^2*log(-((d*x^2 + 2*c)*sqrt(
c) - 2*sqrt(d*x^2 + c)*c)/x^2) - 2*sqrt(d*x^2 + c)*a*sqrt(c))/(a^
2*sqrt(c)*x^2), 1/4*(2*(2*b*c - a*d)*x^2*arctan(sqrt(-c)/sqrt(d*x
^2 + c)) + sqrt(b^2*c - a*b*d)*sqrt(-c)*x^2*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4
*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^
2*x^4 + 2*a*b*x^2 + a^2)) - 2*sqrt(d*x^2 + c)*a*sqrt(-c))/(a^2*sq
rt(-c)*x^2), -1/4*(2*sqrt(-b^2*c + a*b*d)*sqrt(c)*x^2*arctan(1/2*
(b*d*x^2 + 2*b*c - a*d)/(sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c))) +
 (2*b*c - a*d)*x^2*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c
)*c)/x^2) + 2*sqrt(d*x^2 + c)*a*sqrt(c))/(a^2*sqrt(c)*x^2), -1/2*
(sqrt(-b^2*c + a*b*d)*sqrt(-c)*x^2*arctan(1/2*(b*d*x^2 + 2*b*c -
a*d)/(sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c))) - (2*b*c - a*d)*x^2*
arctan(sqrt(-c)/sqrt(d*x^2 + c)) + sqrt(d*x^2 + c)*a*sqrt(-c))/(a
^2*sqrt(-c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.25255, size = 163, normalized size = 1.44

$$\frac{1}{2}\,d^2\left(\frac{2\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}-\frac{\sqrt{dx^2+c}}{ad^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*(b^2*c - a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c - a*d)*arctan(sq
rt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) - sqrt(d*x^2 + c)/(a*d
^2*x^2))
```

3.684 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=105

$$\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}(3bc-ad)}{3a^2cx}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

```
[Out] -Sqrt[c + d*x^2]/(3*a*x^3) + ((3*b*c - a*d)*Sqrt[c + d*x^2])/(3*a
^2*c*x) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/a^(5/2)
```

Rubi [A] time = 0.352539, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}(3bc-ad)}{3a^2cx}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*a*x^3) + ((3*b*c - a*d)*Sqrt[c + d*x^2])/(3*a
^2*c*x) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/a^(5/2)
```

Rubi in Sympy [A] time = 54.96, size = 90, normalized size = 0.86

$$-\frac{\sqrt{c+dx^2}}{3ax^3}-\frac{\sqrt{c+dx^2}\,(ad-3bc)}{3a^2cx}-\frac{b\sqrt{ad-bc}\,\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(c + d*x**2)/(3*a*x**3) - sqrt(c + d*x**2)*(a*d - 3*b*c)/(3*
a**2*c*x) - b*sqrt(a*d - b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sq
rt(c + d*x**2)))/a**(5/2)
```

Mathematica [A] time = 0.195097, size = 93, normalized size = 0.89

$$\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\left(3bcx^2-a\left(c+dx^2\right)\right)}{3a^2cx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + d*x^2)))/(3*a^2*c*x^3) + (b*
Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/a^(5/2)
```

Maple [B] time = 0.021, size = 1059, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x)
```

```
[Out] -1/3/a/c/x^3*(d*x^2+c)^(3/2)+b/a^2/c/x*(d*x^2+c)^(3/2)-b/a^2*d/c*
x*(d*x^2+c)^(1/2)-b/a^2*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2
*b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b/a^2*d^(1/2)*ln((d
*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d-1/2*b^2/a^2/(-a*b)^(1/
2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))*c-1/2*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+1/2*b/a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*d+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4), x)
```

Fricas [A] time = 0.310627, size = 1, normalized size = 0.01

$$\left[\frac{3\,bcx^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left((3\,bc-ad)x^2-\right.}{12\,a^2cx^3}\right.$$

$$\left.-\frac{3\,bcx^3\sqrt{\frac{bc-ad}{a}}\arctan\left(-\frac{(bc-2\,ad)x^2-ac}{2\sqrt{dx^2+c}\,ax\sqrt{\frac{bc-ad}{a}}}\right)-2\left((3\,bc-ad)x^2-ac\right)\sqrt{dx^2+c}}{6\,a^2cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b*c*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^
2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/
a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((3*b*c - a*d)*x^2 - a*c)*sq
rt(d*x^2 + c))/(a^2*c*x^3), -1/6*(3*b*c*x^3*sqrt((b*c - a*d)/a)*a
rctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b
*c - a*d)/a))) - 2*((3*b*c - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^
2*c*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^4\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.734108, size = 290, normalized size = 2.76

$$-\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2}-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}-ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] -(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d
 - a^2*d^2)*a^2) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqr
t(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(
3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2)
```

3.685 $\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=210

$$\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2-10abcd+b^2c^2\right)}{16b^3d}+\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

```
[Out] ((b^2*c^2 - 10*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3*d)
 + ((7*b*c - 6*a*d)*x^3*Sqrt[c + d*x^2])/(24*b^2) + (d*x^5*Sqrt[c
 + d*x^2])/(6*b) + (a^(3/2)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^4 - ((b*c - 2*a*d)*(b^2*c^2
 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
16*b^4*d^(3/2))
```

Rubi [A] time = 1.01949, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2-10abcd+b^2c^2\right)}{16b^3d}+\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b^2*c^2 - 10*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3*d)
 + ((7*b*c - 6*a*d)*x^3*Sqrt[c + d*x^2])/(24*b^2) + (d*x^5*Sqrt[c
 + d*x^2])/(6*b) + (a^(3/2)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^4 - ((b*c - 2*a*d)*(b^2*c^2
 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
16*b^4*d^(3/2))
```

Rubi in Sympy [A] time = 135.424, size = 197, normalized size = 0.94

$$\frac{a^{\frac{3}{2}}(ad-bc)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}+\frac{dx^5\sqrt{c+dx^2}}{6b}-\frac{x^3\sqrt{c+dx^2}(6ad-7bc)}{24b^2}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2-10abcd+b^2c^2\right)}{16b^3d}-\frac{(2ad-bc)\left(8a^2d^2-8abcd-b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] a**(3/2)*(a*d - b*c)**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt
(c + d*x**2)))/b**4 + d*x**5*sqrt(c + d*x**2)/(6*b) - x**3*sqrt(c
 + d*x**2)*(6*a*d - 7*b*c)/(24*b**2) + x*sqrt(c + d*x**2)*(8*a**2
*d**2 - 10*a*b*c*d + b**2*c**2)/(16*b**3*d) - (2*a*d - b*c)*(8*a*
*2*d**2 - 8*a*b*c*d - b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2)
)/(16*b**4*d**(3/2))
```

Mathematica [A] time = 0.380343, size = 196, normalized size = 0.93

$$\frac{48a^{3/2}d^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+b\sqrt{d}x\sqrt{c+dx^2}\left(24a^2d^2-6abd\left(5c+2dx^2\right)+b^2\left(3c^2+14cdx^2+8d^2x^4\right)\right)-3\,(1}{48b^4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(5*c + 2*d*x^2
) + b^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4)) + 48*a^(3/2)*d^(3/2)*(b
*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])] - 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*
Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*b^4*d^(3/2))
```

Maple [B] time = 0.022, size = 2081, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/2/b^3*a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2/b^2*a^2/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))*c^2+1/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))*d^2-1/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-1/2/b^2*a^2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+3/4/b^3*a^2*d^(1/2)*ln((-d*(-a*b)
^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1
/2/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/4/b^3*a^2*d*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x+3/4/b^3*a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/b^3*a^3/(-a*b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2/b^2*a^2/(-a*b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*c-3/8/b^2*a*c^2/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1
/24/b*c/d*x*(d*x^2+c)^(3/2)-1/16/b*c^2/d*x*(d*x^2+c)^(1/2)-3/8/b^
2*a*c*x*(d*x^2+c)^(1/2)+1/4/b^3*a^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/6/b
^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/b^4*a^3*d^(3/2)*ln((-
d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))+1/6/b^2*a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/b^4*a^3*d^(
3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+1/6/b*x*(d*x^2+c)^(5/2)/d-1/16/b*c^3/d^(3/2)*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))-1/4/b^2*a*x*(d*x^2+c)^(3/2)-1/b^3*a^3/(-a
*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2)))*d*c+1/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))*d*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.46672, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/96*(24*(a*b*c*d - a^2*d^2)*sqrt(-a*b*c + a^2*d)*sqrt(d)*log((
(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(8*b^3*d^2*x
^5 + 2*(7*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(b^3*c^2 - 10*a*b^2*c*d
+ 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(b^3*c^3 + 6*a*b^2*
c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*log(2*sqrt(d*x^2 + c)*d*x -
(2*d*x^2 + c)*sqrt(d)))/(b^4*d^(3/2)), -1/48*(12*(a*b*c*d - a^2*d
^2)*sqrt(-a*b*c + a^2*d)*sqrt(-d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c -
 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) - (8*b^3*d^2*x^5 + 2*(7*b^3*c*d - 6*a*b^2*
d^2)*x^3 + 3*(b^3*c^2 - 10*a*b^2*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2
 + c)*sqrt(-d) + 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16
*a^3*d^3)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^4*sqrt(-d)*d), -
1/96*(48*(a*b*c*d - a^2*d^2)*sqrt(a*b*c - a^2*d)*sqrt(d)*arctan(-
1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c
)*x)) - 2*(8*b^3*d^2*x^5 + 2*(7*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(b
^3*c^2 - 10*a*b^2*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) -
 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*log(2*
sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^4*d^(3/2)), -1/4
8*(24*(a*b*c*d - a^2*d^2)*sqrt(a*b*c - a^2*d)*sqrt(-d)*arctan(-1/
2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*
x)) - (8*b^3*d^2*x^5 + 2*(7*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(b^3*c
^2 - 10*a*b^2*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 3*
(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*arctan(sq
rt(-d)*x/sqrt(d*x^2 + c)))/(b^4*sqrt(-d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (c + dx^2)^{\frac{3}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.257921, size = 358, normalized size = 1.7

$$\frac{1}{48}\left(2\left(\frac{4\,dx^2}{b}+\frac{7\,b^9cd^4-6\,ab^8d^5}{b^{10}d^4}\right)x^2+\frac{3\left(b^9c^2d^3-10\,ab^8cd^4+8\,a^2b^7d^5\right)}{b^{10}d^4}\right)\sqrt{dx^2+c}x-\frac{\left(a^2b^2c^2\sqrt{d}-2\,a^3bcd^{\frac{3}{2}}+a^4d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b^4}+\frac{\left(b^3c^3\sqrt{d}+6\,ab^2c^2d^{\frac{3}{2}}-24\,a^2bcd^{\frac{5}{2}}+16\,a^3d^{\frac{7}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{32\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d*x^2/b + (7*b^9*c*d^4 - 6*a*b^8*d^5)/(b^10*d^4))*x^2
+ 3*(b^9*c^2*d^3 - 10*a*b^8*c*d^4 + 8*a^2*b^7*d^5)/(b^10*d^4))*sq
rt(d*x^2 + c)*x - (a^2*b^2*c^2*sqrt(d) - 2*a^3*b*c*d^(3/2) + a^4*
d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*
a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^4) + 1/3
2*(b^3*c^3*sqrt(d) + 6*a*b^2*c^2*d^(3/2) - 24*a^2*b*c*d^(5/2) + 1
6*a^3*d^(7/2))*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*d^2)
```

3.686 $\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=115

$$\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{a\sqrt{c+dx^2}(bc-ad)}{b^3}-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}$$

```
[Out] -((a*(b*c - a*d)*Sqrt[c + d*x^2])/b^3) - (a*(c + d*x^2)^(3/2))/(3
*b^2) + (c + d*x^2)^(5/2)/(5*b*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi [A] time = 0.296371, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{a\sqrt{c+dx^2}(bc-ad)}{b^3}-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*Sqrt[c + d*x^2])/b^3) - (a*(c + d*x^2)^(3/2))/(3
*b^2) + (c + d*x^2)^(5/2)/(5*b*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi in Sympy [A] time = 34.3607, size = 97, normalized size = 0.84

$$-\frac{a\left(c+dx^2\right)^{\frac{3}{2}}}{3b^2}+\frac{a\sqrt{c+dx^2}\,(ad-bc)}{b^3}-\frac{a\,(ad-bc)^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{7}{2}}}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}}{5bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(3/2)/(3*b**2) + a*sqrt(c + d*x**2)*(a*d - b*c)/
b**3 - a*(a*d - b*c)**(3/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*
d - b*c))/b**(7/2) + (c + d*x**2)**(5/2)/(5*b*d)
```

Mathematica [A] time = 0.205964, size = 108, normalized size = 0.94

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2-5abd\left(4c+dx^2\right)+3b^2\left(c+dx^2\right)^2\right)}{15b^3d}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 3*b^2*(c + d*x^2)^2 - 5*a*b*d*(4*c
 + d*x^2)))/(15*b^3*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Maple [B] time = 0.019, size = 1897, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/5*(d*x^2+c)^(5/2)/b/d-1/6*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*a/b^3*d
*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4*a/b^3*d^(1/2)*(-a*b)^(1/2
)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))*c+1/2*a^2/b^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2*a/b^2*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*c+1/2*a^2/b^4*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)
/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a^3/b
^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))*d^2-a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+1/2*a/b^2/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*c^2-1/6*a/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*a/b^3*d*(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln((-d
*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))*c+1/2*a^2/b^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2*a/b^2*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*c-1/2*a^2/b^4*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a^3/b^4/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-
a*b)^(1/2)))*d^2-a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+1/2*a/b^2/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257989, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(abcd-a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}-4\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\left(3\,b^{2}d^{2}x^{4}+\right.}{60\,b^{3}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x
^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*
x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c -
 a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*b^2*d^2*x^4 + 3*b^2
*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^2)*sqr
t(d*x^2 + c))/(b^3*d), 1/30*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c -
a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqr
t(-(b*c - a*d)/b))) + 2*(3*b^2*d^2*x^4 + 3*b^2*c^2 - 20*a*b*c*d +
 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^3*
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\left(c+dx^{2}\right)^{\frac{3}{2}}}{a+bx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.239244, size = 204, normalized size = 1.77

$$-\frac{\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{3}}+\frac{3\left(dx^{2}+c\right)^{\frac{5}{2}}b^{4}d^{4}-5\left(dx^{2}+c\right)^{\frac{3}{2}}ab^{3}d^{5}-15\sqrt{dx^{2}+c}ab^{3}cd^{5}+15\sqrt{dx^{2}+c}a^{2}b^{2}d^{6}}{15\,b^{5}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^2 + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 +
c)^(5/2)*b^4*d^4 - 5*(d*x^2 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^2
+ c)*a*b^3*c*d^5 + 15*sqrt(d*x^2 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

3.687 $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=158

$$\frac{(8a^2d^2-12abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}-\frac{\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}(5bc-4ad)}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

```
[Out] ((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*Sqrt[c + d*x
^2])/(4*b) - (Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^3 + ((3*b^2*c^2 - 12*a*b*c*d + 8*
a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```

Rubi [A] time = 0.652622, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{(8a^2d^2-12abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}-\frac{\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}(5bc-4ad)}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*Sqrt[c + d*x
^2])/(4*b) - (Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^3 + ((3*b^2*c^2 - 12*a*b*c*d + 8*
a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```

Rubi in Sympy [A] time = 84.2162, size = 146, normalized size = 0.92

$$-\frac{\sqrt{a}\,(ad-bc)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{dx^3\sqrt{c+dx^2}}{4b}-\frac{x\sqrt{c+dx^2}\,(4ad-5bc)}{8b^2}+\frac{(8a^2d^2-12abcd+3b^2c^2)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] -sqrt(a)*(a*d - b*c)**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt
(c + d*x**2)))/b**3 + d*x**3*sqrt(c + d*x**2)/(4*b) - x*sqrt(c +
d*x**2)*(4*a*d - 5*b*c)/(8*b**2) + (8*a**2*d**2 - 12*a*b*c*d + 3*
b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*b**3*sqrt(d))
```

Mathematica [A] time = 0.28518, size = 139, normalized size = 0.88

$$\frac{\frac{(8a^2d^2-12abcd+3b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+bx\sqrt{c+dx^2}\left(-4ad+5bc+2bdx^2\right)-8\sqrt{a}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(5*b*c - 4*a*d + 2*b*d*x^2) - 8*Sqrt[a]*(b*c
 - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)] + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt
[c + d*x^2]])/Sqrt[d])/(8*b^3)
```

Maple [B] time = 0.018, size = 1973, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b*x*(d*x^2+c)^(3/2)+3/8/b*c*x*(d*x^2+c)^(1/2)+3/8/b*c^2/d^(1/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/6*a/(-a*b)^(1/2)/b*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)-1/4*a/b^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4*a/b^2*d^(1/2)*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))*c+1/2*a^2/(-a*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2*a/(-a*b
)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2*a^2/b^3*d^(3/2)*ln((d*(-a*b)^
(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*
a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-a^2/(-a*b)^(1/2)/b^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))*d*c+1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/6*a/(
-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*a/b^2*d*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-3/4*a/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d
)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2*a^2/(-a*b)^(1/2)/b^2*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*d+1/2*a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2*a^2
/b^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2
)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))-1/2*a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d
^2+a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/2*a/(-a*b)^(1/2)/b/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.786378, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt(-a*b*c + a^2*d)*(b*c - a*d)*sqrt(d)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(2*b^2*d*x^3 + (5*b^2
*c - 4*a*b*d)*x)*sqrt(d*x^2 + c)*sqrt(d) - (3*b^2*c^2 - 12*a*b*c*
d + 8*a^2*d^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)
))/(b^3*sqrt(d)), -1/8*(2*sqrt(-a*b*c + a^2*d)*(b*c - a*d)*sqrt(-
d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*
c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^2
*d*x^3 + (5*b^2*c - 4*a*b*d)*x)*sqrt(d*x^2 + c)*sqrt(-d) - (3*b^2
*c^2 - 12*a*b*c*d + 8*a^2*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
)/(b^3*sqrt(-d)), 1/16*(8*sqrt(a*b*c - a^2*d)*(b*c - a*d)*sqrt(d)
*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(
d*x^2 + c)*x)) + 2*(2*b^2*d*x^3 + (5*b^2*c - 4*a*b*d)*x)*sqrt(d*x
^2 + c)*sqrt(d) + (3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*log(-2*sqr
t(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^3*sqrt(d)), 1/8*(4*
sqrt(a*b*c - a^2*d)*(b*c - a*d)*sqrt(-d)*arctan(-1/2*((b*c - 2*a*
d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) + (2*b^2*d
*x^3 + (5*b^2*c - 4*a*b*d)*x)*sqrt(d*x^2 + c)*sqrt(-d) + (3*b^2*c
^2 - 12*a*b*c*d + 8*a^2*d^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/
(b^3*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.243154, size = 267, normalized size = 1.69

$$\frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,dx^2}{b}+\frac{5\,b^5cd^2-4\,ab^4d^3}{b^6d^2}\right)x+\frac{\left(ab^2c^2\sqrt{d}-2\,a^2bcd^{\frac{3}{2}}+a^3d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,b^3}-\frac{\left(3\,b^2c^2-12\,abcd+8\,a^2d^2\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d*x^2/b + (5*b^5*c*d^2 - 4*a*b^4*d^3)/(b^6
*d^2))*x + (a*b^2*c^2*sqrt(d) - 2*a^2*b*c*d^(3/2) + a^3*d^(5/2))*
arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt
(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) - 1/16*(3*b^2*
c^2 - 12*a*b*c*d + 8*a^2*d^2)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)
/(b^3*sqrt(d))
```

3.688 $\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\sqrt{c+dx^2}(bc-ad)}{b^2}+\frac{(c+dx^2)^{3/2}}{3b}$$

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (c + d*x^2)^(3/2)/(3*b) - ((b
*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]
])/b^(5/2)
```

Rubi [A] time = 0.19507, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\sqrt{c+dx^2}(bc-ad)}{b^2}+\frac{(c+dx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (c + d*x^2)^(3/2)/(3*b) - ((b
*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]
])/b^(5/2)
```

Rubi in Sympy [A] time = 26.2274, size = 75, normalized size = 0.82

$$\frac{(c+dx^2)^{\frac{3}{2}}}{3b}-\frac{\sqrt{c+dx^2}\,(ad-bc)}{b^2}+\frac{(ad-bc)^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(3/2)/(3*b) - sqrt(c + d*x**2)*(a*d - b*c)/b**2 + (
a*d - b*c)**(3/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/
b**(5/2)
```

Mathematica [A] time = 0.150504, size = 83, normalized size = 0.91

$$\frac{\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)}{3b^2}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2))/(3*b^2) - ((b*c - a*d
)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/
2)
```

Maple [B] time = 0.017, size = 1856, normalized size = 20.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/6/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^2*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x+3/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-
a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/b^2*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*a*d+1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^3*d^(3/2)*(-a*b)^(
1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*a-1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2+1/b^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))*a*d*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/6/b*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/
4/b^2*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b^2*d^(1/2)*(-a*b)
^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))*c-1/2/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d+1/2/b*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*c+1/2/b^3*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+
(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a-1/2/b^3/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(
-a*b)^(1/2)))*a^2*d^2+1/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c-1/2/b/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271029, size = 1, normalized size = 0.01

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)\,x^2+4\,(b^2dx^2+2\,b^2c-abd)\,\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(bdx^2+4\,bc-3\,c}{12\,b^2}\right.$$

$$\left.-\frac{3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\,\sqrt{dx^2+c}b\sqrt{-\frac{bc-ad}{b}}}\right)-2\,(bdx^2+4\,bc-3\,ad)\,\sqrt{dx^2+c}}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^
2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b*d*x^2 + 4*b*c - 3*a*d)*sqrt(
d*x^2 + c))/b^2, -1/6*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/
b))) - 2*(b*d*x^2 + 4*b*c - 3*a*d)*sqrt(d*x^2 + c))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.23479, size = 151, normalized size = 1.66

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2+3\,\sqrt{dx^2+c}b^2c-3\,\sqrt{dx^2+c}abd}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)
*b^2 + 3*sqrt(d*x^2 + c)*b^2*c - 3*sqrt(d*x^2 + c)*a*b*d)/b^3
```

3.689 $\int \frac{(c+dx^2)^{3/2}}{a+bx^2} dx$

Optimal. Leaf size=113

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2}+\frac{\sqrt{d}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^2) + (Sqrt[d]*(
3*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rubi [A] time = 0.251784, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2}+\frac{\sqrt{d}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^2) + (Sqrt[d]*(
3*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rubi in Sympy [A] time = 44.3367, size = 102, normalized size = 0.9

$$\frac{dx\sqrt{c+dx^2}}{2b}-\frac{\sqrt{d}\,(2ad-3bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{(ad-bc)^{\frac{3}{2}}\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] d*x*sqrt(c + d*x**2)/(2*b) - sqrt(d)*(2*a*d - 3*b*c)*atanh(sqrt(d
)*x/sqrt(c + d*x**2))/(2*b**2) + (a*d - b*c)**(3/2)*atanh(x*sqrt(
a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*b**2)
```

Mathematica [A] time = 0.284517, size = 110, normalized size = 0.97

$$\frac{\sqrt{d}(3bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}}+bdx\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2] + (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(3*b*c - 2*
a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.016, size = 1875, normalized size = 16.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/6/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*d/b*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x+3/4/b*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))*c-1/2/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*a*d+1/2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*ln((d
*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))*a-1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2+1/(-a*b)^(1/2)
/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))*a*d*c-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/6
/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*d/b*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3
/4/b*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2
)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))*c+1/2/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a
*d-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*ln((-d*
(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))*a+1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2-1/(-a*b)^(1/2)/
b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))*a*d*c+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.424702, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{dx^2+c}bdx-(3bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2}{\right.}{4b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*
x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*sqrt(-(b*c -
 a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^
2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x + 2*(3*b*c - 2*a*d)*sqrt(-
d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - (b*c - a*d)*sqrt(-(b*
c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*
x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(b*c - a*d)*sqrt((b
*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 +
c)*a*x*sqrt((b*c - a*d)/a))) - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x
^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/b^2, 1/2*(sqrt(d*x^2 + c)*
b*d*x + (3*b*c - 2*a*d)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt
(-d))) - (b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*
d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.251766, size = 205, normalized size = 1.81

$$\frac{\sqrt{dx^2+c}\,dx}{2b}-\frac{\left(3bc\sqrt{d}-2ad^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4b^2}-\frac{\left(b^2c^2\sqrt{d}-2abcd^{\frac{3}{2}}+a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d*x/b - 1/4*(3*b*c*sqrt(d) - 2*a*d^(3/2))*ln(
(sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 - (b^2*c^2*sqrt(d) - 2*a*b*c
*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*
d^2)*b^2)
```

3.690 $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=96

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

```
[Out] (d*Sqrt[c + d*x^2])/b - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/a + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(a*b^(3/2))
```

Rubi [A] time = 0.344164, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2])/b - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/a + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(a*b^(3/2))
```

Rubi in Sympy [A] time = 41.8312, size = 80, normalized size = 0.83

$$\frac{d\sqrt{c+dx^2}}{b}-\frac{c^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}-\frac{(ad-bc)^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{ab^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x/(b*x**2+a),x)
```

```
[Out] d*sqrt(c + d*x**2)/b - c**(3/2)*atanh(sqrt(c + d*x**2)/sqrt(c))/a
 - (a*d - b*c)**(3/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*
c))/(a*b**(3/2))
```

Mathematica [C] time = 0.799252, size = 271, normalized size = 2.82

$$\frac{(bc-ad)^{3/2}\log\left(-\frac{2ab^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{5/2}}\right)+(bc-ad)^{3/2}\log\left(-\frac{2ab^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{5/2}}\right)+2a\sqrt{b}d\sqrt{c+d}}{2ab^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (2*a*Sqrt[b]*d*Sqrt[c + d*x^2] + 2*b^(3/2)*c^(3/2)*Log[x] - 2*b^(
3/2)*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]] + (b*c - a*d)^(3/2)
*Log[(-2*a*b^(3/2)*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*S
qrt[c + d*x^2]))/((b*c - a*d)^(5/2)*(I*Sqrt[a] + Sqrt[b]*x))] + (
b*c - a*d)^(3/2)*Log[(-2*a*b^(3/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + S
qrt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(5/2)*((-I)*Sqrt[a]
```

```
+ Sqrt[b]*x))])/(2*a*b^(3/2))
```

Maple [B] time = 0.018, size = 1919, normalized size = 20.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x/(b*x^2+a),x)
```

```
[Out] 1/3/a*(d*x^2+c)^(3/2)-1/a*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/
2))/x)+1/a*(d*x^2+c)^(1/2)*c-1/6/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a*d*(-
a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a/b*d^(1/2)*(-a*b)^(1/2)*l
n((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))*c+1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/2/a*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c
+1/2/b^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^
(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*a/b^2/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*
d^2-1/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))*d*c+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/6/a*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+1/4/a*d*(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/a
/b*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1
/2/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-
a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*d^2-1/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+1/2/a/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 0.552479, size = 1, normalized size = 0.01

$$\left[\frac{2\,bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+4\,\sqrt{dx^2+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2\cdots}{b^2x^4+2\,abx^2+a^2}\right.}{4\,ab}\right.$$

$$-\frac{4\,b\sqrt{-c}c\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-4\,\sqrt{dx^2+c}ad+(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\cdots}{b^2x^4+2\,abx^2+a^2}\right.}{4\,ab}$$

$$\left.-\frac{2\,b\sqrt{-c}c\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-2\,\sqrt{dx^2+c}ad-(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\,\sqrt{dx^2+c}b\sqrt{-\frac{bc-ad}{b}}}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 4*sqrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)
*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b), -1/4*(
4*b*sqrt(-c)*c*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - 4*sqrt(d*x^
2 + c)*a*d + (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 -
4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/
b))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b), 1/2*(b*c^(3/2)*log(-(d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*
d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c
- a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))))/(a*b), -1/2*(2*
b*sqrt(-c)*c*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - 2*sqrt(d*x^2
+ c)*a*d - (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 +
 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))))/(a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/x/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.239995, size = 158, normalized size = 1.65

$$d\left(\frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}+\frac{\sqrt{dx^2+c}}{b}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}abd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] d*(c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d) + sqrt(d*x
^2 + c)/b - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c
)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b*d))
```

3.691 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=102

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rubi [A] time = 0.260762, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rubi in Sympy [A] time = 42.1378, size = 85, normalized size = 0.83

$$\frac{d^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}-\frac{c\sqrt{c+dx^2}}{ax}-\frac{(ad-bc)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a),x)
```

```
[Out] d**(3/2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/b - c*sqrt(c + d*x**2)
/(a*x) - (a*d - b*c)**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt
(c + d*x**2)))/(a**(3/2)*b)
```

Mathematica [A] time = 0.213369, size = 105, normalized size = 1.03

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*Lo
g[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/b
```

Maple [B] time = 0.019, size = 1956, normalized size = 19.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/c/x*(d*x^2+c)^(5/2)+1/a*d/c*x*(d*x^2+c)^(3/2)+3/2/a*d*x*(d*x
^2+c)^(1/2)+3/2/a*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-1/6*b/a
/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a*d*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3
/4/a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))*c+1/2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/
2*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b*d^(3/2)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1
/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-1/(-a*b)^(1/2)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))*d*c+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/6*b/a/(-a*b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a*d^
(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))*c-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2*b/a/
(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/b*d^(3/2)*ln((-d*(-a*b)^(1
/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b*
a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2+1/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
*d*c-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2), x)
```

Fricas [A] time = 0.380138, size = 1, normalized size = 0.01

$$\left[\frac{2\,ad^{\frac{3}{2}}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)\,x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)\,x^2-4\,(a^2cx-(}{b^2x^4+2\,abx^2+a^2}\right.}{4\,abx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*d^(3/2)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) - (b*c - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(
a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d
)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c)*b*c)/(a*b*
x), 1/4*(4*a*sqrt(-d)*d*x*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d)))
- (b*c - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2
*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c)*b*c)/(a*b*x),
 1/2*(a*d^(3/2)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 + (b*c - a*d)*x*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x
^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) - 2*sqrt(d*x
^2 + c)*b*c)/(a*b*x), 1/2*(2*a*sqrt(-d)*d*x*arctan(d*x/(sqrt(d*x^
2 + c)*sqrt(-d))) + (b*c - a*d)*x*sqrt((b*c - a*d)/a)*arctan(-1/2
*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/
a))) - 2*sqrt(d*x^2 + c)*b*c)/(a*b*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.241393, size = 220, normalized size = 2.16

$$-\frac{d^{\frac{3}{2}}\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b}+\frac{2\,c^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}+\frac{\left(b^2c^2\sqrt{d}-2\,abcd^{\frac{3}{2}}+a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b + 2*c^2*sqrt(d
)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a) + (b^2*c^2*sqrt(d) -
2*a*b*c*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d
 - a^2*d^2)*a*b)
```

3.692 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=114

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

```
[Out] -(c*Sqrt[c + d*x^2])/(2*a*x^2) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2) - ((b*c - a*d)^(3/2)*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b])
```

Rubi [A] time = 0.411838, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -(c*Sqrt[c + d*x^2])/(2*a*x^2) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2) - ((b*c - a*d)^(3/2)*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b])
```

Rubi in Sympy [A] time = 46.4144, size = 100, normalized size = 0.88

$$-\frac{c\sqrt{c+dx^2}}{2ax^2}-\frac{\sqrt{c}\,(3ad-2bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{(ad-bc)^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a),x)
```

```
[Out] -c*sqrt(c + d*x**2)/(2*a*x**2) - sqrt(c)*(3*a*d - 2*b*c)*atanh(sq
rt(c + d*x**2)/sqrt(c))/(2*a**2) + (a*d - b*c)**(3/2)*atan(sqrt(b
)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a**2*sqrt(b))
```

Mathematica [C] time = 1.33947, size = 284, normalized size = 2.49

$$-\frac{\frac{(bc-ad)^{3/2}\log\left(\frac{2a^2\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{5/2}}\right)}{\sqrt{b}}+\frac{(bc-ad)^{3/2}\log\left(\frac{2a^2\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{5/2}}\right)}{\sqrt{b}}-\sqrt{c}(2bc-3ad)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right.}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -((a*c*Sqrt[c + d*x^2])/x^2 + Sqrt[c]*(2*b*c - 3*a*d)*Log[x] - Sq
rt[c]*(2*b*c - 3*a*d)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]] + ((b*c -
a*d)^(3/2)*Log[(2*a^2*Sqrt[b]*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b
*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(5/2)*(I*Sqrt[a] + Sqrt[
b]*x))])/Sqrt[b] + ((b*c - a*d)^(3/2)*Log[(2*a^2*Sqrt[b]*(Sqrt[b]
```

```
*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*
d)^(5/2)*((-I)*Sqrt[a] + Sqrt[b]*x))])/Sqrt[b])/(2*a^2)
```

Maple [B] time = 0.02, size = 2003, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x)
```

```
[Out] 3/4/a^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/2/a*d*(d*x^2+c)^(1/2)+1
/6*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/3*b/a^
2*(d*x^2+c)^(3/2)-1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/6*b/a^2*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)-1/2/b/a*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1
/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))*d*c-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/4/a^2*d*(-a*b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x-3/4/a^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2
)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b/
a*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-1/2*b/a^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))*c^2+1/4/a^2*d*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x+1/2*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-3/2/a
*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+b/a^2*c^(3/2)*ln
((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a^2*(d*x^2+c)^(1/2)*c-1/2/a
/c/x^2*(d*x^2+c)^(5/2)+1/2/a*d/c*(d*x^2+c)^(3/2)+1/2*b/a^2*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^{\frac{3}{2}}}{(bx^2 + a)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3), x)
```

Fricas [A] time = 0.581458, size = 1, normalized size = 0.01

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+(2\,bc-3\,ad)\sqrt{c}x^2\,l}{4\,a^2x^2}\right.$$

$$-\frac{2\,(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}b\sqrt{-\frac{bc-ad}{b}}}\right)+(2\,bc-3\,ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\sqrt{dx^2+c}ac}{4\,a^2x^2},\ \frac{(2\,bc-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3),x, algorithm="fricas")

[Out] [-1/4*((b*c - a*d)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*
(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b)
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 +
c)*a*c)/(a^2*x^2), 1/4*(2*(2*b*c - 3*a*d)*sqrt(-c)*x^2*arctan(c/(
sqrt(d*x^2 + c)*sqrt(-c))) - (b*c - a*d)*x^2*sqrt((b*c - a*d)/b)*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
 - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 +
c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*sqrt(d*x
^2 + c)*a*c)/(a^2*x^2), -1/4*(2*(b*c - a*d)*x^2*sqrt(-(b*c - a*d)
/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(
b*c - a*d)/b))) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^
2), 1/2*((2*b*c - 3*a*d)*sqrt(-c)*x^2*arctan(c/(sqrt(d*x^2 + c)*s
qrt(-c))) - (b*c - a*d)*x^2*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*
x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) - sq
rt(d*x^2 + c)*a*c)/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a),x)

[Out] Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.238803, size = 182, normalized size = 1.6

$$\frac{1}{2}\,d^2\left(\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{\left(2\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}-\frac{\sqrt{dx^2+c}c}{ad^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)
*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c^
2 - 3*a*c*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2)
- sqrt(d*x^2 + c)*c/(a*d^2*x^2))
```

3.693 $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=102

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}(3bc-4ad)}{3a^2x}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

```
[Out] -(c*Sqrt[c + d*x^2])/(3*a*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d*x^2]
)/(3*a^2*x) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/a^(5/2)
```

Rubi [A] time = 0.372934, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}(3bc-4ad)}{3a^2x}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -(c*Sqrt[c + d*x^2])/(3*a*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d*x^2]
)/(3*a^2*x) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/a^(5/2)
```

Rubi in Sympy [A] time = 47.7446, size = 88, normalized size = 0.86

$$-\frac{c\sqrt{c+dx^2}}{3ax^3}-\frac{\sqrt{c+dx^2}\,(4ad-3bc)}{3a^2x}+\frac{(ad-bc)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a),x)
```

```
[Out] -c*sqrt(c + d*x**2)/(3*a*x**3) - sqrt(c + d*x**2)*(4*a*d - 3*b*c)
/(3*a**2*x) + (a*d - b*c)**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)
*sqrt(c + d*x**2)))/a**(5/2)
```

Mathematica [A] time = 0.180249, size = 90, normalized size = 0.88

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\left(3bcx^2-a\left(c+4dx^2\right)\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + 4*d*x^2)))/(3*a^2*x^3) + ((b
*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/a^(5/2)
```

Maple [B] time = 0.021, size = 2089, normalized size = 20.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x)
```

```
[Out] 3/4*b/a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))*c-b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*
c+b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-1/2/a*d^(3/2)*ln((-d*(-a*b
)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/
a*d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2*b/a/(-a*b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*d-1/2*b^2/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-
3/2*b/a^2*d*x*(d*x^2+c)^(1/2)-3/2*b/a^2*d^(1/2)*c*ln(x*d^(1/2)+(d
*x^2+c)^(1/2))-2/3/a*d/c^2/x*(d*x^2+c)^(5/2)+2/3/a*d^2/c^2*x*(d*x
^2+c)^(3/2)+1/a*d^2/c*x*(d*x^2+c)^(1/2)+b/a^2/c/x*(d*x^2+c)^(5/2)
+1/4*b/a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*b/a^2*d^(1/2)*ln((d*(-a*b)
^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1
/2*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2*b^2/a^2/(-a*b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*c+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*
c^2-b/a^2*d/c*x*(d*x^2+c)^(3/2)-1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))*c^2-1/3/a/c/x^3*(d*x^2+c)^(5/2)+1/6*b^2/a^2/(-a*b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-1/6*b^2/
a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/2/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))*d^2-1/2/a*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*b/a^2*d*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{(bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4), x)
```

Fricas [A] time = 0.287334, size = 1, normalized size = 0.01

$$\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)\,x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)\,x^2+4\,(a^2cx-(abc-2\,a^2d)\,x^3)\,\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,((3\,bc-\right.$$

$$\left.-\frac{3\,(bc-ad)x^3\sqrt{\frac{bc-ad}{a}}\arctan\left(-\frac{(bc-2\,ad)x^2-ac}{2\,\sqrt{dx^2+c}ax\sqrt{\frac{bc-ad}{a}}}\right)-2\,\left((3\,bc-4\,ad)x^2-ac\right)\sqrt{dx^2+c}}{6\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^
2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((3*b*c - 4*a*d)*x^
2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3), -1/6*(3*(b*c - a*d)*x^3*sqrt
((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2
 + c)*a*x*sqrt((b*c - a*d)/a))) - 2*((3*b*c - 4*a*d)*x^2 - a*c)*s
qrt(d*x^2 + c))/(a^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.643643, size = 346, normalized size = 3.39

$$-\frac{\left(b^2c^2\sqrt{d}-2\,abcd^{\frac{3}{2}}+a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2}$$

$$-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc^2\sqrt{d}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4acd^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^3\sqrt{d}+6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ac^2d^{\frac{3}{2}}\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((s
qrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2
*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2) - 2/3*(3*(sqrt(d)*x - sqrt(d
```

```
*x^2 + c))^4*b*c^2*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*
c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^3*sqrt(d) + 6*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^2*d^(3/2) + 3*b*c^4*sqrt(d) -
4*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2)
```

3.694 $\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=291

$$\frac{a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^5}+\frac{x^3\sqrt{c+dx^2}\,(48a^2d^2-104abcd+59b^2c^2)}{192b^3}+\frac{x\sqrt{c+dx^2}\,(-64a^3d^3+144a^2bcd^2-88ab^2c^2d+5b^3c^3)}{128b^4d}-\frac{(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128b^5d^{3/2}}+\frac{dx^5\sqrt{c+dx^2}(11bc-8ad)}{48b^2}+\frac{dx^5\,(c+dx^2)^{3/2}}{8b}$$

```
[Out] ((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^3)*x*Sq
rt[c + d*x^2])/(128*b^4*d) + ((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*
d^2)*x^3*Sqrt[c + d*x^2])/(192*b^3) + (d*(11*b*c - 8*a*d)*x^5*Sqr
t[c + d*x^2])/(48*b^2) + (d*x^5*(c + d*x^2)^(3/2))/(8*b) + (a^(3/
2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/b^5 - ((5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d
^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(128*b^5*d^(3/2))
```

Rubi [A] time = 1.43642, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^5}+\frac{x^3\sqrt{c+dx^2}\,(48a^2d^2-104abcd+59b^2c^2)}{192b^3}+\frac{x\sqrt{c+dx^2}\,(-64a^3d^3+144a^2bcd^2-88ab^2c^2d+5b^3c^3)}{128b^4d}-\frac{(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128b^5d^{3/2}}+\frac{dx^5\sqrt{c+dx^2}(11bc-8ad)}{48b^2}+\frac{dx^5\,(c+dx^2)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```


```
[Out] ((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^3)*x*Sq
rt[c + d*x^2])/(128*b^4*d) + ((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*
d^2)*x^3*Sqrt[c + d*x^2])/(192*b^3) + (d*(11*b*c - 8*a*d)*x^5*Sqr
t[c + d*x^2])/(48*b^2) + (d*x^5*(c + d*x^2)^(3/2))/(8*b) + (a^(3/
2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/b^5 - ((5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d
^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(128*b^5*d^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.411166, size = 247, normalized size = 0.85

$$\frac{384a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\frac{bx\sqrt{c+dx^2}\left(-192a^3d^3+48a^2bd^2\left(9c+2dx^2\right)-8ab^2d\left(33c^2+26cdx^2+8d^2x^4\right)+b^3\left(15c^3+118c^2dx^2+136cd^2x^4+\right.\right.}{d}}{384b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(-192*a^3*d^3 + 48*a^2*b*d^2*(9*c + 2*d*x^2
) - 8*a*b^2*d*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^3*(15*c^3 + 1
18*c^2*d*x^2 + 136*c*d^2*x^4 + 48*d^3*x^6)))/d + 384*a^(3/2)*(b*c
 - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)] + (3*(-5*b^4*c^4 - 40*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 - 320*
a^3*b*c*d^3 + 128*a^4*d^4)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^
(3/2))/(384*b^5)
```

Maple [B] time = 0.032, size = 3373, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/6/b^3*a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/6/b^2*a^2/(-a*b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/4/b^4*a^3*d^2*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-5/16/b^2*a*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/8/b^3*a
^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)*x-5/4/b^4*a^3*d^(3/2)*ln((d*(-a*b)^(1/2)
/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b^4
*a^4/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/2/b^2*a^2/(-a*b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*c^2-1/48/b*c/d*x*(d*x^2+c)^(5/2)-5/24/b^2*a
*c*x*(d*x^2+c)^(3/2)+15/16/b^3*a^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+
(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/6/b^3*
a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/6/b^2*a^2/(-a*b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*c-1/4/b^4*a^3*d^2*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/b^
4*a^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))*c-5/16/b^2*a*c^2*x*(d*x^2+c)^(1/2)-1/2/b^
4*a^4/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-1/2/b^2*a^2/(-a*b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*c^2-5/128/b*c^3/d*x*(d*x^2+c)^(1/2)-1/2/b^
2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/8/b*x*(d*x^2+c)^(7/2)/d-
5/128/b*c^4/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/2/b^5*a^4*d^(
5/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+7/16/b^3*a^2*d*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/b^3*a^3/(-
a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+7/16/b^3*a^2*d*c*((x+1/b*(-a*b
```

```
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x+1/b^3*a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-1/2/b^5*a
^5/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))*c^3+1/2/b^5*a^5/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3-5/19
2/b*c^2/d*x*(d*x^2+c)^(3/2)-1/6/b^2*a*x*(d*x^2+c)^(5/2)+1/8/b^3*a
^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)*x+15/16/b^3*a^2*d^(1/2)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/1
0/b^2*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/2/b^5*a^4*d^(5/2)*ln
((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))+1/10/b^2*a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/2/b^4*a^4
/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c+3/2/b^3*a^3/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*d*c^2+3/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c-
3/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.06747, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/768*(192*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-a*b*c +
 a^2*d)*sqrt(d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a
^2)) + 2*(48*b^4*d^3*x^7 + 8*(17*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + 2
*(59*b^4*c^2*d - 104*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(5*b^4
```

```
*c^3 - 88*a*b^3*c^2*d + 144*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x)*sqrt
(d*x^2 + c)*sqrt(d) - 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2
*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*log(-2*sqrt(d*x^2 + c)*
d*x - (2*d*x^2 + c)*sqrt(d)))/(b^5*d^(3/2)), 1/384*(96*(a*b^2*c^2
*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-a*b*c + a^2*d)*sqrt(-d)*log((
(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (48*b^4*d^3*x^
7 + 8*(17*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + 2*(59*b^4*c^2*d - 104*a*
b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(5*b^4*c^3 - 88*a*b^3*c^2*d +
 144*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x)*sqrt(d*x^2 + c)*sqrt(-d) -
3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c
*d^3 - 128*a^4*d^4)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^5*sqrt
(-d)*d), -1/768*(384*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt
(a*b*c - a^2*d)*sqrt(d)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sq
rt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) - 2*(48*b^4*d^3*x^7 + 8*(17
*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + 2*(59*b^4*c^2*d - 104*a*b^3*c*d^2
 + 48*a^2*b^2*d^3)*x^3 + 3*(5*b^4*c^3 - 88*a*b^3*c^2*d + 144*a^2*
b^2*c*d^2 - 64*a^3*b*d^3)*x)*sqrt(d*x^2 + c)*sqrt(d) + 3*(5*b^4*c
^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128
*a^4*d^4)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b
^5*d^(3/2)), -1/384*(192*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*
sqrt(a*b*c - a^2*d)*sqrt(-d)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c
)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) - (48*b^4*d^3*x^7 + 8*
(17*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + 2*(59*b^4*c^2*d - 104*a*b^3*c*
d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(5*b^4*c^3 - 88*a*b^3*c^2*d + 144*a
^2*b^2*c*d^2 - 64*a^3*b*d^3)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 3*(5*b
^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 -
 128*a^4*d^4)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^5*sqrt(-d)*d
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(c + dx^2\right)^{\frac{5}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.260082, size = 483, normalized size = 1.66

$$\frac{1}{384}\left(2\left(4\left(\frac{6 d^2 x^2}{b}+\frac{17 b^{14} c d^7-8 a b^{13} d^8}{b^{15} d^6}\right) x^2+\frac{59 b^{14} c^2 d^6-104 a b^{13} c d^7+48 a^2 b^{12} d^8}{b^{15} d^6}\right) x^2+\frac{3\left(5 b^{14} c^3 d^5-88 a b^{13} c^2 d^6+1\right.}{b^{15} d^6}\right.$$
$$-\frac{\left(a^2 b^3 c^3 \sqrt{d}-3 a^3 b^2 c^2 d^{\frac{3}{2}}+3 a^4 b c d^{\frac{5}{2}}-a^5 d^{\frac{7}{2}}\right) \arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\, b^5}$$
$$+\frac{\left(5 b^4 c^4 \sqrt{d}+40 a b^3 c^3 d^{\frac{3}{2}}-240 a^2 b^2 c^2 d^{\frac{5}{2}}+320 a^3 b c d^{\frac{7}{2}}-128 a^4 d^{\frac{9}{2}}\right) \ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{256 b^5 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^2*x^2/b + (17*b^14*c*d^7 - 8*a*b^13*d^8)/(b^15*d
^6))*x^2 + (59*b^14*c^2*d^6 - 104*a*b^13*c*d^7 + 48*a^2*b^12*d^8)
/(b^15*d^6))*x^2 + 3*(5*b^14*c^3*d^5 - 88*a*b^13*c^2*d^6 + 144*a^
2*b^12*c*d^7 - 64*a^3*b^11*d^8)/(b^15*d^6))*sqrt(d*x^2 + c)*x - (
a^2*b^3*c^3*sqrt(d) - 3*a^3*b^2*c^2*d^(3/2) + 3*a^4*b*c*d^(5/2) -
```

```
 a^5*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
 + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^5)
+ 1/256*(5*b^4*c^4*sqrt(d) + 40*a*b^3*c^3*d^(3/2) - 240*a^2*b^2*c
^2*d^(5/2) + 320*a^3*b*c*d^(7/2) - 128*a^4*d^(9/2))*ln((sqrt(d)*x
 - sqrt(d*x^2 + c))^2)/(b^5*d^2)
```

3.695 $\int \frac{x^3 (c+dx^2)^{5/2}}{a+bx^2} dx$

Optimal. Leaf size=144

$$\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}-\frac{a\sqrt{c+dx^2}(bc-ad)^2}{b^4}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

```
[Out] -((a*(b*c - a*d)^2*Sqrt[c + d*x^2])/b^4) - (a*(b*c - a*d)*(c + d*
x^2)^(3/2))/(3*b^3) - (a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)
^(7/2)/(7*b*d) + (a*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/b^(9/2)
```

Rubi [A] time = 0.377182, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}-\frac{a\sqrt{c+dx^2}(bc-ad)^2}{b^4}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```


```
[Out] -((a*(b*c - a*d)^2*Sqrt[c + d*x^2])/b^4) - (a*(b*c - a*d)*(c + d*
x^2)^(3/2))/(3*b^3) - (a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)
^(7/2)/(7*b*d) + (a*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/b^(9/2)
```

Rubi in Sympy [A] time = 43.9815, size = 122, normalized size = 0.85

$$-\frac{a\left(c+dx^2\right)^{\frac{5}{2}}}{5b^2}+\frac{a\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{3b^3}-\frac{a\sqrt{c+dx^2}\,(ad-bc)^2}{b^4}+\frac{a\,(ad-bc)^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{9}{2}}}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}}{7bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(5/2)/(5*b**2) + a*(c + d*x**2)**(3/2)*(a*d - b*
c)/(3*b**3) - a*sqrt(c + d*x**2)*(a*d - b*c)**2/b**4 + a*(a*d - b
*c)**(5/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/b**(9/2
) + (c + d*x**2)**(7/2)/(7*b*d)
```

Mathematica [C] time = 0.674259, size = 298, normalized size = 2.07

$$\frac{105ad(bc-ad)^{5/2}\log\left(-\frac{2b^{9/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(a\sqrt{b}x+ia^{3/2}\right)(bc-ad)^{7/2}}\right)+105ad(bc-ad)^{5/2}\log\left(-\frac{2b^{9/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(a\sqrt{b}x-ia^{3/2}\right)(bc-ad)^{7/2}}\right)+2\sqrt{\cdots}}{210b^{9/2}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[c + d*x^2]*(-105*a^3*d^3 + 15*b^3*(c + d*x^2)^3 +
 35*a^2*b*d^2*(7*c + d*x^2) - 7*a*b^2*d*(23*c^2 + 11*c*d*x^2 + 3*
d^2*x^4)) + 105*a*d*(b*c - a*d)^(5/2)*Log[(-2*b^(9/2)*(Sqrt[b]*c
- I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^
(7/2)*(I*a^(3/2) + a*Sqrt[b]*x))] + 105*a*d*(b*c - a*d)^(5/2)*Log
[(-2*b^(9/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c
+ d*x^2]))/((b*c - a*d)^(7/2)*((-I)*a^(3/2) + a*Sqrt[b]*x))])/(21
0*b^(9/2)*d)
```

Maple [B] time = 0.022, size = 3127, normalized size = 21.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -3/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2-1/8*a/b^3*d*(-a*b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)*x-15/16*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/
2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x+1/b*(-a*b)^(1/2)))*c^3+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+a^2/b^3
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*d*c+a^2/b^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2*a^3/
b^5*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2
))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^4/b^5/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^
3-1/10*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/10*a/b^2*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1
/8*a/b^3*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16*a/b^3*d^(1/2)
*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))*c^2-1/4*a^2/b^4*d^2*(-a*b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x-5/4*a^2/b^4*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2
)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/2*a^
3/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))*d^2*c-1/2*a^3/b^5*d^(5/2)*(-a*b)^(1/2)*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))-1/2*a^4/b^5/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3+1/4*a^2/b^4*d^2*(-a*b)^(1/2)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x+5/4*a^2/b^4*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a
*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*
c+3/2*a^3/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
```

```
^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c-3/2*a^2/b^3/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d
*c^2+1/7*(d*x^2+c)^(7/2)/b/d+1/6*a^2/b^3*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/
6*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2*a^3/b^4*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2
-1/2*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/6*a/b^2*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*
c-1/2*a^3/b^4*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-1/2*a/b^2*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*c^2+1/6*a^2/b^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-7/16*a/b^3*d*(-a*b)^(1/2
)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x+7/16*a/b^3*d*(-a*b)^(1/2)*c*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294001, size = 1, normalized size = 0.01

$$\left[\frac{105\left(ab^2c^2d - 2a^2bcd^2 + a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right) + \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/420*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt((b*c - a
*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*
b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d
*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(
15*b^3*d^3*x^6 + 15*b^3*c^3 - 161*a*b^2*c^2*d + 245*a^2*b*c*d^2 -
 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*d^3)*x^4 + (45*b^3*c^2*d
 - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 + c))/(b^4*d),
1/210*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-(b*c - a
*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt
(-(b*c - a*d)/b))) + 2*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 161*a*b^2*c
^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*
d^3)*x^4 + (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sq
rt(d*x^2 + c))/(b^4*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(c + dx^2\right)^{\frac{5}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.248863, size = 308, normalized size = 2.14

$$-\frac{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^4} + \frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^6d^6 - 21\left(dx^2+c\right)^{\frac{5}{2}}ab^5d^7 - 35\left(dx^2+c\right)^{\frac{3}{2}}ab^5cd^7 - 105\sqrt{dx^2+c}ab^5c^2d^7 + 35\left(dx^2+c\right)^{\frac{3}{2}}a^2b^4d^8 + 210\sqrt{dx^2}}{105\,b^7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(s
qrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4)
 + 1/105*(15*(d*x^2 + c)^(7/2)*b^6*d^6 - 21*(d*x^2 + c)^(5/2)*a*b
^5*d^7 - 35*(d*x^2 + c)^(3/2)*a*b^5*c*d^7 - 105*sqrt(d*x^2 + c)*a
*b^5*c^2*d^7 + 35*(d*x^2 + c)^(3/2)*a^2*b^4*d^8 + 210*sqrt(d*x^2
+ c)*a^2*b^4*c*d^8 - 105*sqrt(d*x^2 + c)*a^3*b^3*d^9)/(b^7*d^7)
```

3.696 $\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=217

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}-\frac{\sqrt{a}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

```
[Out] ((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3
) + (d*(3*b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*(c +
 d*x^2)^(3/2))/(6*b) - (Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^4 + ((5*b^3*c^3 - 30*a*
b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(16*b^4*Sqrt[d])
```

Rubi [A] time = 1.00248, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}-\frac{\sqrt{a}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3
) + (d*(3*b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*(c +
 d*x^2)^(3/2))/(6*b) - (Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^4 + ((5*b^3*c^3 - 30*a*
b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(16*b^4*Sqrt[d])
```

Rubi in Sympy [A] time = 126.215, size = 207, normalized size = 0.95

$$\frac{\sqrt{a}\left(ad-bc\right)^{\frac{5}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}+\frac{dx^3\left(c+dx^2\right)^{\frac{3}{2}}}{6b}-\frac{dx^3\sqrt{c+dx^2}\left(2ad-3bc\right)}{8b^2}+\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}-\frac{\left(16a^3d^3-40a^2bcd^2+30ab^2c^2d-5b^3c^3\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] sqrt(a)*(a*d - b*c)**(5/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(
c + d*x**2)))/b**4 + d*x**3*(c + d*x**2)**(3/2)/(6*b) - d*x**3*sq
rt(c + d*x**2)*(2*a*d - 3*b*c)/(8*b**2) + x*sqrt(c + d*x**2)*(8*a
**2*d**2 - 18*a*b*c*d + 11*b**2*c**2)/(16*b**3) - (16*a**3*d**3 -
 40*a**2*b*c*d**2 + 30*a*b**2*c**2*d - 5*b**3*c**3)*atanh(sqrt(d)
*x/sqrt(c + d*x**2))/(16*b**4*sqrt(d))
```

Mathematica [A] time = 0.244738, size = 187, normalized size = 0.86

$$\frac{bx\sqrt{c+dx^2}\left(24a^2d^2-6abd\left(9c+2dx^2\right)+b^2\left(33c^2+26cdx^2+8d^2x^4\right)\right)+\frac{3\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}}{48b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(9*c + 2*d*x^2) + b^2*
(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4)) - 48*Sqrt[a]*(b*c - a*d)^(5/2)
*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + (3*(5*b^
3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*Log[d*x + S
qrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^4)
```

Maple [B] time = 0.021, size = 3235, normalized size = 14.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/2*a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-15/16*a/b^2*d^(1/2)
*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))*c^2+1/6*a^2/(-a*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/6*
a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/2*a^3/(-a*b)^(1/2)/b^3*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*d^2+1/2*a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c
^2-1/8*a/b^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-15/16*a/b^2*d^(1/2)*ln((-d*(
-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)*c^2-1/6*a^2/(-a*b)^(1/2)/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/6*a/(-a*b)
^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)*c+1/4*a^2/b^3*d^2*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)*x+5/4*a^2/b^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/8*a/b^2*d*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)*x+1/6/b*x*(d*x^2+c)^(5/2)-a^2/(-a*b)^(1/2)/b^2*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*d*c+1/2*a^4/(-a*b)^(1/2)/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3-1/2*a/(-a*b
)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2)))*c^3-1/2*a^3/(-a*b)^(1/2)/b^3*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*d^2-1/10*a/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/2*a^3/b
^4*d^(5/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))+5/24/b*c*x*(d*x^2+c)^(3/2)+5/16/b*c^2*x*(d*x^
2+c)^(1/2)+5/16/b*c^3/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/10*
a/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/2*a^3/b^4*d^(5/2)*ln((-d*(
```

```
-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)+1/4*a^2/b^3*d^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*a^2/b^3*d^(3/2)*ln((d
*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))*c-7/16*a/b^2*d*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-7/16*a/b^2*d*c*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x+a^2/(-a*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-1/2*a
^4/(-a*b)^(1/2)/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^3+1/2*a/(-a*b)^(1/2)/b/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*c^3-3/2*a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+3/
2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^2+3/2*a^3/(-a*b)^(1/2)
/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))*d^2*c-3/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))*d*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.01344, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/96*(24*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b*c + a^2*d)*sq
rt(d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3
*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a
*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(
8*b^3*d^2*x^5 + 2*(13*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(11*b^3*c^2
- 18*a*b^2*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(5*b
^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*log(2*sqrt
(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^4*sqrt(d)), 1/48*(12
*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b*c + a^2*d)*sqrt(-d)*lo
g(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a
^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (8*b^3*d^2*
```

```
x^5 + 2*(13*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(11*b^3*c^2 - 18*a*b^2
*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 3*(5*b^3*c^3 -
30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)))/(b^4*sqrt(-d)), 1/96*(48*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(a*b*c - a^2*d)*sqrt(d)*arctan(-1/2*((b*c - 2*a*d)*x
^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) + 2*(8*b^3*d^2
*x^5 + 2*(13*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(11*b^3*c^2 - 18*a*b^
2*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(5*b^3*c^3 -
30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*log(2*sqrt(d*x^2 +
c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^4*sqrt(d)), 1/48*(24*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c - a^2*d)*sqrt(-d)*arctan(-1/2*
((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)
) + (8*b^3*d^2*x^5 + 2*(13*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(11*b^3
*c^2 - 18*a*b^2*c*d + 8*a^2*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) +
3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*arct
an(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^4*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(c + dx^2\right)^{\frac{5}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.263947, size = 373, normalized size = 1.72

$$\frac{1}{48}\left(2\left(\frac{4\,d^2x^2}{b} + \frac{13\,b^9cd^5 - 6\,ab^8d^6}{b^{10}d^4}\right)x^2 + \frac{3\left(11\,b^9c^2d^4 - 18\,ab^8cd^5 + 8\,a^2b^7d^6\right)}{b^{10}d^4}\right)\sqrt{dx^2 + c}x + \frac{\left(ab^3c^3\sqrt{d} - 3\,a^2b^2c^2d^{\frac{3}{2}} + 3\,a^3bcd^{\frac{5}{2}} - a^4d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}b^4} - \frac{\left(5\,b^3c^3 - 30\,ab^2c^2d + 40\,a^2bcd^2 - 16\,a^3d^3\right)\ln\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{32\,b^4\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2/b + (13*b^9*c*d^5 - 6*a*b^8*d^6)/(b^10*d^4))*x
^2 + 3*(11*b^9*c^2*d^4 - 18*a*b^8*c*d^5 + 8*a^2*b^7*d^6)/(b^10*d^
4))*sqrt(d*x^2 + c)*x + (a*b^3*c^3*sqrt(d) - 3*a^2*b^2*c^2*d^(3/2
) + 3*a^3*b*c*d^(5/2) - a^4*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqr
t(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a
*b*c*d - a^2*d^2)*b^4) - 1/32*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^
2*b*c*d^2 - 16*a^3*d^3)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*
sqrt(d))
```

3.697 $\int \frac{x\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\sqrt{c+dx^2}(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5b}$$

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/b^3 + ((b*c - a*d)*(c + d*x^2)^(3
/2))/(3*b^2) + (c + d*x^2)^(5/2)/(5*b) - ((b*c - a*d)^(5/2)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi [A] time = 0.25741, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\sqrt{c+dx^2}(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/b^3 + ((b*c - a*d)*(c + d*x^2)^(3
/2))/(3*b^2) + (c + d*x^2)^(5/2)/(5*b) - ((b*c - a*d)^(5/2)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi in Sympy [A] time = 34.9982, size = 99, normalized size = 0.83

$$\frac{\left(c+dx^2\right)^{\frac{5}{2}}}{5b}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{3b^2}+\frac{\sqrt{c+dx^2}\,(ad-bc)^2}{b^3}-\frac{(ad-bc)^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(5/2)/(5*b) - (c + d*x**2)**(3/2)*(a*d - b*c)/(3*b*
*2) + sqrt(c + d*x**2)*(a*d - b*c)**2/b**3 - (a*d - b*c)**(5/2)*a
tan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/b**(7/2)
```

Mathematica [C] time = 0.570494, size = 268, normalized size = 2.25

$$\frac{2\sqrt{b}\sqrt{c+dx^2}\left(15a^2d^2-5abd\left(7c+dx^2\right)+b^2\left(23c^2+11cdx^2+3d^2x^4\right)\right)-15(bc-ad)^{5/2}\log\left(\frac{2b^{7/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}\right.}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{7/2}}\right.}{30b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[c + d*x^2]*(15*a^2*d^2 - 5*a*b*d*(7*c + d*x^2) +
b^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)) - 15*(b*c - a*d)^(5/2)*Log
[(2*b^(7/2)*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c +
 d*x^2]))/((b*c - a*d)^(7/2)*(I*Sqrt[a] + Sqrt[b]*x))] - 15*(b*c
- a*d)^(5/2)*Log[(2*b^(7/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c
 - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(7/2)*((-I)*Sqrt[a] + Sqrt
```

```
[b]*x))])/(30*b^(7/2))
```

Maple [B] time = 0.019, size = 3078, normalized size = 25.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 1/6/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)*c+1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/6/b*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*c+1/2/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+15/16/b^2*d^(1
/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d*c-15/
16/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2-1/b^2*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*a*d*c-1/2/b^4*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a^2+1/2/b^4/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*a^3*d^3+1/2/b^4*d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2
)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a^2+1/2/
b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*a^3*d^3+5/4/b^3*d^(3/2)*(-a*b)^(1/2)*ln((-d*(
-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)*a*c-3/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2*c-5/4/b^3*d^(3/2)*(-a*b)^(1
/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))*a*c-3/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^2*c+3/2/b^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2)))*a*d*c^2+7/16/b^2*d*(-a*b)^(1/2)*c*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
-1/4/b^3*d^2*(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*a+1/8/b^2*d*(-a*b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/10/b*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/10/b*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(5/2)-1/6/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*a*d+1/2/b^3*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*a^2*d^2-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3+3/2/b^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
```

```
1/2)))*a*d*c^2-7/16/b^2*d*(-a*b)^(1/2)*c*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/
4/b^3*d^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*a-1/8/b^2*d*(-a*b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)*x-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3-1/6/b^2*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)*a*d+1/2/b^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a^2*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286611, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right) + 4\left(3\,b^{\cdots}\right.}{60\,b^3}\right.$$

$$-\frac{15\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}\,b\sqrt{-\frac{bc-ad}{b}}}\right) - 2\left(3\,b^2d^2x^4 + 23\,b^2c^2 - 35\,abcd + 15\,a^2d^2 + \left(11\,b^2cd - 5\cdots\right.\right.}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*
sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b^2*d^2*
x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*
d^2)*x^2)*sqrt(d*x^2 + c))/b^3, -1/30*(15*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/
(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*(3*b^2*d^2*x^4 + 23
*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x^2
)*sqrt(d*x^2 + c))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c + dx^2\right)^{\frac{5}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.236429, size = 248, normalized size = 2.08

$$\frac{\left(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{3}} + \frac{3\left(dx^{2}+c\right)^{\frac{5}{2}}b^{4} + 5\left(dx^{2}+c\right)^{\frac{3}{2}}b^{4}c + 15\sqrt{dx^{2}+c}b^{4}c^{2} - 5\left(dx^{2}+c\right)^{\frac{3}{2}}ab^{3}d - 30\sqrt{dx^{2}+c}ab^{3}cd + 15\sqrt{dx^{2}+c}a^{2}b^{2}d^{2}}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d
*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/
15*(3*(d*x^2 + c)^(5/2)*b^4 + 5*(d*x^2 + c)^(3/2)*b^4*c + 15*sqrt
(d*x^2 + c)*b^4*c^2 - 5*(d*x^2 + c)^(3/2)*a*b^3*d - 30*sqrt(d*x^2
 + c)*a*b^3*c*d + 15*sqrt(d*x^2 + c)*a^2*b^2*d^2)/b^5
```

3.698 $\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^3}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{8b^2}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

```
[Out] (d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x*(c + d*x^2)^
(3/2))/(4*b) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^3) + (Sqrt[d]*(15*b^2*c^2 - 20
*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^
3)
```

Rubi [A] time = 0.463559, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^3}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{8b^2}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x*(c + d*x^2)^
(3/2))/(4*b) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^3) + (Sqrt[d]*(15*b^2*c^2 - 20
*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^
3)
```

Rubi in Sympy [A] time = 70.2407, size = 146, normalized size = 0.94

$$\frac{dx\left(c+dx^2\right)^{\frac{3}{2}}}{4b}-\frac{dx\sqrt{c+dx^2}\left(4ad-7bc\right)}{8b^2}+\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3}-\frac{(ad-bc)^{\frac{5}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] d*x*(c + d*x**2)**(3/2)/(4*b) - d*x*sqrt(c + d*x**2)*(4*a*d - 7*b
*c)/(8*b**2) + sqrt(d)*(8*a**2*d**2 - 20*a*b*c*d + 15*b**2*c**2)*
atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*b**3) - (a*d - b*c)**(5/2)*a
tanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*b**3)
```

Mathematica [A] time = 0.178762, size = 140, normalized size = 0.9

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bdx\sqrt{c+dx^2}\left(-4ad+9bc+2bdx^2\right)+\frac{8(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}}}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2]*(9*b*c - 4*a*d + 2*b*d*x^2) + (8*(b*c - a*
d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/S
qrt[a] + Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Log[d*x +
Sqrt[d]*Sqrt[c + d*x^2]])/(8*b^3)
```

Maple [B] time = 0.018, size = 3101, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 7/16*d/b*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2/(-a*b)^(1/2)/b^2*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*a^2*d^2+7/16*d/b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/6/(-a*b)^(1/2
)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)*a*d-1/4/b^2*d^2*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*a-5
/4/b^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))*a*c-1/4/b^2*d^2*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*a-5
/4/b^2*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))*a*c-1/2/(-a*b)^(1/2)/b^2*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*a^2*d^2+3/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2*c-3/2/(
-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c^2+3/2/(-a*b)^(1/2)/b/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))*a*d*c^2-1/2/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^3*d^3+1/(-
a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d*c-1/10/(-a*b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(5/2)+1/10/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/2/(-a*b)^(1/2
)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2)))*a^2*d^2*c+1/2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*c^2+1/6/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/6/(-a*b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*c-1/2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/(
-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d*c+1/2/(-a*b)^(1/2)/b^3/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*a^3*d^3+1/6/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-
```

```
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*a*d+1/
8*d/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)*x+1/8*d/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16/b
*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))*c^2+1/2/b^3*d^(5/2)*ln((-d*(-a*b)^(1/2)/b+(x+1
/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a^2+1/2/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))*c^3-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+15/
16/b*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))*c^2+1/2/b^3*d^(5/2)*ln((d*(-a*b)^(1/2)/b+(x
-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34159, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2
 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 4*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a
*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^
2)*x)*sqrt(d*x^2 + c))/b^3, 1/8*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2
*d^2)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) + 2*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (2*b^2*d^2*x^3 + (9*b^
2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/16*(8*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d
)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) - (15*b^2
*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^
2 + c)*sqrt(d)*x - c) - 2*(2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2
)*x)*sqrt(d*x^2 + c))/b^3, 1/8*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*
d^2)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - 4*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2
*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) + (2*
b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2), x)
```

GIAC/XCAS [A] time = 0.252312, size = 290, normalized size = 1.86

$$\frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,d^2x^2}{b}+\frac{9\,b^5cd^3-4\,ab^4d^4}{b^6d^2}\right)x - \frac{\left(15\,b^2c^2\sqrt{d}-20\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^3} - \frac{\left(b^3c^3\sqrt{d}-3\,ab^2c^2d^{\frac{3}{2}}+3\,a^2bcd^{\frac{5}{2}}-a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d^2*x^2/b + (9*b^5*c*d^3 - 4*a*b^4*d^4)/(b
^6*d^2))*x - 1/16*(15*b^2*c^2*sqrt(d) - 20*a*b*c*d^(3/2) + 8*a^2*
d^(5/2))*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^3 - (b^3*c^3*sqrt(
d) - 3*a*b^2*c^2*d^(3/2) + 3*a^2*b*c*d^(5/2) - a^3*d^(7/2))*arcta
n(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*
c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3)
```

3.699 $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}+\frac{d\sqrt{c+dx^2}(2bc-ad)}{b^2}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}$$

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (d*(c + d*x^2)^(3/2))/(3*
b) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^
(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(5
/2))
```

Rubi [A] time = 0.551436, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}+\frac{d\sqrt{c+dx^2}(2bc-ad)}{b^2}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (d*(c + d*x^2)^(3/2))/(3*
b) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^
(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(5
/2))
```

Rubi in Sympy [A] time = 63.0279, size = 105, normalized size = 0.85

$$\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{3b}-\frac{d\sqrt{c+dx^2}\,(ad-2bc)}{b^2}-\frac{c^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{(ad-bc)^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{ab^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x/(b*x**2+a),x)
```

```
[Out] d*(c + d*x**2)**(3/2)/(3*b) - d*sqrt(c + d*x**2)*(a*d - 2*b*c)/b*
*2 - c**(5/2)*atanh(sqrt(c + d*x**2)/sqrt(c))/a + (a*d - b*c)**(5
/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*b**(5/2))
```

Mathematica [C] time = 0.555474, size = 288, normalized size = 2.32

$$\frac{3(bc-ad)^{5/2}\log\left(-\frac{2ab^{5/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{7/2}}\right)+3(bc-ad)^{5/2}\log\left(-\frac{2ab^{5/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{7/2}}\right)+2a\sqrt{b}d\sqrt{c+}}{6ab^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (2*a*Sqrt[b]*d*Sqrt[c + d*x^2]*(7*b*c - 3*a*d + b*d*x^2) + 6*b^(5
/2)*c^(5/2)*Log[x] - 6*b^(5/2)*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c + d
*x^2]] + 3*(b*c - a*d)^(5/2)*Log[(-2*a*b^(5/2)*(Sqrt[b]*c - I*Sqr
t[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(7/2)*(
```

```
I*Sqrt[a] + Sqrt[b]*x))] + 3*(b*c - a*d)^(5/2)*Log[(-2*a*b^(5/2)*
(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((
b*c - a*d)^(7/2)*((-I)*Sqrt[a] + Sqrt[b]*x))])/(6*a*b^(5/2))
```

Maple [B] time = 0.021, size = 3148, normalized size = 25.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x/(b*x^2+a),x)
```

```
[Out] 5/4/b^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4/b^2*d^2*(-a*b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x+3/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c+1/8/a*d*(-a
*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16/a/b*d^(1/2)*(-a*b)^(1/2)*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))*c^2+3/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c-1/8/a*d*(-a*b)^(1/2
)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)*x-15/16/a/b*d^(1/2)*(-a*b)^(1/2)*ln((d*(-
a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
*c^2-7/16/a*d*(-a*b)^(1/2)/b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/6/b*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)*d-1/6/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2/a*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*c^2+1/a*(d*x^2+c)^(1/2)*c^2+1/3/a*c*(d*x^2+c)^(3/2)-1/a*c^(5
/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/6/b*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)*d-1/6/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2/a*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/
2*a/b^3*d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-1/4/b^2*d^2*(-a*b)^(1/2)*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x-5/4/b^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b
+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2*a/b^3
*d^(5/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3-3
/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))*d*c^2-1/2*a/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/b*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
```

```
-(a*d-b*c)/b)^(1/2)*d*c+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3-1/2*a/b^2*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*d^2+1/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2/a/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*c^3-1/10/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/10/a*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/5
/a*(d*x^2+c)^(5/2)+7/16/a*d*(-a*b)^(1/2)/b*c*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 1.3427, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/12*(6*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*
c)/x^2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*l
og((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
- 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c
)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*d^2*
x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2), -1/12*(12*
b^2*sqrt(-c)*c^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - 3*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/
b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3
*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2), 1/6*(3*b^2*c^(5/2)*log(-(d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 3*(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a
*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) + 2*(a*b*d^2*x^2 +
7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2), -1/6*(6*b^2*sqrt
(-c)*c^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - 3*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c
 - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*(a*b*d^2*x^
2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{x(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.235674, size = 227, normalized size = 1.83

$$\frac{1}{3}\left(\frac{3\,c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2+6\,\sqrt{dx^2+c}b^2c-3\,\sqrt{dx^2+c}abd}{b^3}-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\right.}{\sqrt{-b^2c+abd}ab^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/3*(3*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d) + ((d*
x^2 + c)^(3/2)*b^2 + 6*sqrt(d*x^2 + c)*b^2*c - 3*sqrt(d*x^2 + c)*
a*b*d)/b^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*a*b^2*d))*d
```

3.700 $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}$$

```
[Out] (d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2
))/(a*x) - ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]
*Sqrt[c + d*x^2])])/(a^(3/2)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rubi [A] time = 0.499407, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2
))/(a*x) - ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]
*Sqrt[c + d*x^2])])/(a^(3/2)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rubi in Sympy [A] time = 72.7701, size = 128, normalized size = 0.88

$$-\frac{d^{\frac{3}{2}}(2ad-5bc)\,\text{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}-\frac{c\left(c+dx^2\right)^{\frac{3}{2}}}{ax}+\frac{dx\sqrt{c+dx^2}\,(ad+2bc)}{2ab}+\frac{(ad-bc)^{\frac{5}{2}}\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a),x)
```

```
[Out] -d**(3/2)*(2*a*d - 5*b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*b*
*2) - c*(c + d*x**2)**(3/2)/(a*x) + d*x*sqrt(c + d*x**2)*(a*d + 2
*b*c)/(2*a*b) + (a*d - b*c)**(5/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(
a)*sqrt(c + d*x**2)))/(a**(3/2)*b**2)
```

Mathematica [A] time = 0.148711, size = 132, normalized size = 0.91

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^2}+\sqrt{c+dx^2}\left(\frac{d^2x}{2b}-\frac{c^2}{ax}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (-(c^2/(a*x)) + (d^2*x)/(2*b))*Sqrt[c + d*x^2] - ((b*c - a*d)^(5/
2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2
)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^
```

```
2]])/(2*b^2)
```

Maple [B] time = 0.022, size = 3191, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x)
```

```
[Out] -7/16/a*d*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/6*b/a/(-a*b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)*c-1/2/b*a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2-7/16/a*d
*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*x+1/6*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
*c+1/2/b*a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/2*b/a/(-a*b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*c^2+3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c^
2+1/a/d/c*x*(d*x^2+c)^(5/2)+15/8/a*d*c*x*(d*x^2+c)^(1/2)-1/2*b/a/
(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/6/(-a*b)^(1/2)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)*d-1/6/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-3/2/(-a*b)^(1/2
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1
/b*(-a*b)^(1/2)))*d*c^2+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*
d^3-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3-1/2/b^2*a^2/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))*d^3+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3
+1/10*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/8/a*d*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)*x-15/16/a*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/4/b*d^2*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x+5/4/b*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*d*c-1/2/b^2*a*d^(5/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/a/c/x*(d*x^2+c)^(7/2)+5/4/a*d*
x*(d*x^2+c)^(3/2)+15/8/a*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2)
)-1/10*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/8/a*d*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)*x-15/16/a*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+1/4/b*d^2*((x-1/b*(-a*b)^(1
```

```
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x+5/4/b*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
d*c-1/2/b^2*a*d^(5/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d
)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))*d^2*c+3/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{(bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2), x)
```

Fricas [A] time = 0.91137, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((5*a*b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*
x^2 + c)*sqrt(d)*x - c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(
-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2
*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x
^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*
x), 1/4*(2*(5*a*b*c*d - 2*a^2*d^2)*sqrt(-d)*x*arctan(d*x/(sqrt(d*
x^2 + c)*sqrt(-d))) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*
c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*
x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) + 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x),
1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt((b*c - a*d)/a)*arct
an(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c
- a*d)/a))) - (5*a*b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt
(d*x^2 + c))/(a*b^2*x), 1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(-d)*x*a
rctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*x*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c
)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) + (a*b*d^2*x^2 - 2*b
^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.244544, size = 279, normalized size = 1.92

$$\frac{\sqrt{dx^2+c}\,d^2x}{2\,b}+\frac{2\,c^3\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^2}+\frac{\left(b^3c^3\sqrt{d}-3\,ab^2c^2d^{\frac{3}{2}}+3\,a^2bcd^{\frac{5}{2}}-a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d^2*x/b + 2*c^3*sqrt(d)/(((sqrt(d)*x - sqrt(d
*x^2 + c))^2 - c)*a) - 1/4*(5*b*c*d^(3/2) - 2*a*d^(5/2))*ln((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/b^2 + (b^3*c^3*sqrt(d) - 3*a*b^2*c^2*
d^(3/2) + 3*a^2*b*c*d^(5/2) - a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(
sqrt(a*b*c*d - a^2*d^2)*a*b^2)
```

3.701 $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=144

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}(2ad+bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2}$$

```
[Out] (d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))
/(2*a*x^2) + (c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqr
t[c]])/(2*a^2) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/(a^2*b^(3/2))
```

Rubi [A] time = 0.638936, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}(2ad+bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))
/(2*a*x^2) + (c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqr
t[c]])/(2*a^2) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/(a^2*b^(3/2))
```

Rubi in Sympy [A] time = 69.7412, size = 126, normalized size = 0.88

$$-\frac{c\left(c+dx^2\right)^{\frac{3}{2}}}{2ax^2}+\frac{d\sqrt{c+dx^2}\,(2ad+bc)}{2ab}-\frac{c^{\frac{3}{2}}\,(5ad-2bc)\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{(ad-bc)^{\frac{5}{2}}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a),x)
```

```
[Out] -c*(c + d*x**2)**(3/2)/(2*a*x**2) + d*sqrt(c + d*x**2)*(2*a*d + b
*c)/(2*a*b) - c**(3/2)*(5*a*d - 2*b*c)*atanh(sqrt(c + d*x**2)/sqr
t(c))/(2*a**2) - (a*d - b*c)**(5/2)*atan(sqrt(b)*sqrt(c + d*x**2)
/sqrt(a*d - b*c))/(a**2*b**(3/2))
```

Mathematica [C] time = 0.654135, size = 311, normalized size = 2.16

$$\frac{1}{2}\left(-\frac{(bc-ad)^{5/2}\log\left(\frac{2a^2b^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{7/2}}\right)}{a^2b^{3/2}}-\frac{(bc-ad)^{5/2}\log\left(\frac{2a^2b^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{7/2}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^2}+\frac{c^{3/2}\log(x)(5ad-2bc)}{a^2}+2\sqrt{c+dx^2}\left(\frac{d^2}{b}-\frac{c^2}{2ax^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (2*(d^2/b - c^2/(2*a*x^2))*Sqrt[c + d*x^2] + (c^(3/2)*(-2*b*c + 5
*a*d)*Log[x])/a^2 + (c^(3/2)*(2*b*c - 5*a*d)*Log[c + Sqrt[c]*Sqrt
[c + d*x^2]])/a^2 - ((b*c - a*d)^(5/2)*Log[(2*a^2*b^(3/2)*(Sqrt[b
]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((b*c - a
*d)^(7/2)*(I*Sqrt[a] + Sqrt[b]*x))])/(a^2*b^(3/2)) - ((b*c - a*d)
^(5/2)*Log[(2*a^2*b^(3/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c -
 a*d]*Sqrt[c + d*x^2]))/((b*c - a*d)^(7/2)*((-I)*Sqrt[a] + Sqrt[b
]*x))])/(a^2*b^(3/2)))/2
```

Maple [B] time = 0.022, size = 3247, normalized size = 22.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x)
```

```
[Out] 1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2*c+3/2/a/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))*d*c^2-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3-1/8/a^2*d*(-a*
b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-15/16/a^2*d^(1/2)*(-a*b)^(1/2)*ln(
(-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))*c^2+1/2/b^2*a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3+1/8/a^2*d*(-a*b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*x+1/10*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/6/a*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)*d-7/16/a^2*d*(-a*b)^(1/2)*c*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/
4/b/a*d^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/b/a*d^(3/2)*(-a*
b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))*c+5/4/b/a*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1
/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+7/16
/a^2*d*(-a*b)^(1/2)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b/a*d^2*(-a*b)^(1
/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x-3/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+3/2/a/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))*d*c^2-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
```

```
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3-1/3*b/a^2*c*(d*x^2
+c)^(3/2)+b/a^2*c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a
^2*(d*x^2+c)^(1/2)*c^2+1/6*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2/a/c/x^
2*(d*x^2+c)^(7/2)+1/2/a*d/c*(d*x^2+c)^(5/2)-5/2/a*d*c^(3/2)*ln((2
*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+5/2/a*d*c*(d*x^2+c)^(1/2)+1/6*b/
a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)*c-1/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c-1/a*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*d*c+1/2*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-1/2/b^2*d^(5/2
)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))+1/2*b/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2+1/2/b^
2*d^(5/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/1
0*b/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(5/2)-1/6/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/2/b*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*d^2+5/6/a*d*(d*x^2+c)^(3/2)-1/5*b/a^2*(d*x^2+c)^
(5/2)+15/16/a^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3), x)
```

Fricas [A] time = 1.50896, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*d)/b)*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*
sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^2*c^2 -
5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) +
2*c)/x^2) + 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x
^2), 1/4*(2*(2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(c/(sqrt(d
*x^2 + c)*sqrt(-c))) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((
b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2
+ 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)
*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2), -1/
4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/b)*arc
tan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a
*d)/b))) + (2*b^2*c^2 - 5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sq
rt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^2*d^2*x^2 - a*b*c^2)*s
qrt(d*x^2 + c))/(a^2*b*x^2), 1/2*((2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c
```

```
)*x^2*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c
- a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) + (2*a^2*d^2*x^2
 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.241429, size = 231, normalized size = 1.6

$$\frac{1}{2}\,d^2\left(\frac{2\sqrt{dx^2+c}}{b}-\frac{\sqrt{dx^2+c}\,c^2}{ad^2x^2}-\frac{\left(2\,bc^3-5\,ac^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}+\frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2+}}{\sqrt{-b^2c+}}\right.}{\sqrt{-b^2c+abd}a^2bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*sqrt(d*x^2 + c)/b - sqrt(d*x^2 + c)*c^2/(a*d^2*x^2) -
(2*b*c^3 - 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(
-c)*d^2) + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*a^2*b*d^2))
```

3.702 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=130

$$\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{c\sqrt{c+dx^2}(bc-2ad)}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] (c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*x) - (c*(c + d*x^2)^(3/2))
/(3*a*x^3) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(5/2)*b) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]])/b
```

Rubi [A] time = 0.521142, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{c\sqrt{c+dx^2}(bc-2ad)}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*x) - (c*(c + d*x^2)^(3/2))
/(3*a*x^3) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(5/2)*b) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]])/b
```

Rubi in Sympy [A] time = 72.4753, size = 114, normalized size = 0.88

$$\frac{d^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}-\frac{c\left(c+dx^2\right)^{\frac{3}{2}}}{3ax^3}-\frac{c\sqrt{c+dx^2}\,(2ad-bc)}{a^2x}-\frac{(ad-bc)^{\frac{5}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a),x)
```

```
[Out] d**(5/2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/b - c*(c + d*x**2)**(3
/2)/(3*a*x**3) - c*sqrt(c + d*x**2)*(2*a*d - b*c)/(a**2*x) - (a*d
 - b*c)**(5/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2))
)/(a**(5/2)*b)
```

Mathematica [A] time = 0.170605, size = 125, normalized size = 0.96

$$\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{c\sqrt{c+dx^2}\left(3bcx^2-a\left(c+7dx^2\right)\right)}{3a^2x^3}+\frac{d^{5/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + 7*d*x^2)))/(3*a^2*x^3) + (
(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*
x^2])])/(a^(5/2)*b) + (d^(5/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]
)/b
```

Maple [B] time = 0.023, size = 3346, normalized size = 25.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x)
```

```
[Out] -4/3/a*d/c^2/x*(d*x^2+c)^(7/2)-5/4*b/a^2*d*x*(d*x^2+c)^(3/2)-15/8
*b/a^2*d^(1/2)*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+b/a^2/c/x*(d*x^2
+c)^(7/2)+1/8*b/a^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16*b/a^2*d^(1/2)*l
n((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))*c^2+1/6*b/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d-1/6*b^2/a
^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/2*b^2/a^2/(-a*b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*c^2+3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2*c+4/3/
a*d^2/c^2*x*(d*x^2+c)^(5/2)+7/16*b/a^2*d*c*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-
b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d*c+1/2/b*a/(-a*b)^(1/2)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))*d^3-1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/2
*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^3+b/a/(-a*b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*d*c-1/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^3+1/2/(-
a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/2/b*d^(5/2)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/(-a
*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d^2+1/2/b*d^(5/2)*ln((-d*(-a*b)^(1/
2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2*b^2
/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-3/2/(-a*b)^(1/2)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2)))*d^2*c+1/8*b/a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+15/16*b/a^2*d^(
1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c^2-1/6*b/a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*d+1/6*
b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-1/10*b^2/a^2/(-a*b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(5/2)-1/4/a*d^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/a*d^(
3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))*c+1/10*b^2/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/
```

```
4/a*d^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/a*d^(3/2)*ln((d*(-a*b)^(1/2)/b+
(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/3/a/c/x^
3*(d*x^2+c)^(7/2)+5/2/a*d^2*x*(d*x^2+c)^(1/2)+5/2/a*d^(3/2)*c*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))-b/a^2*d/c*x*(d*x^2+c)^(5/2)+7/16*b/a^2
*d*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*x+5/3/a*d^2/c*x*(d*x^2+c)^(3/2)+3/2*b/a/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c^2-3/2*b/a/(-a*b)^(1/2)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))*d*c^2-15/8*b/a^2*d*c*x*(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{(bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4), x)
```

Fricas [A] time = 0.699507, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^2*d^(5/2)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)
*x - c) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)
/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqr
t(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) -
 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*
b*x^3), 1/12*(12*a^2*sqrt(-d)*d^2*x^3*arctan(d*x/(sqrt(d*x^2 + c)
*sqrt(-d))) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)
*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) - 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(
a^2*b*x^3), 1/6*(3*a^2*d^(5/2)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 +
c)*sqrt(d)*x - c) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt((b
*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 +
c)*a*x*sqrt((b*c - a*d)/a))) - 2*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*
d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), 1/6*(6*a^2*sqrt(-d)*d^2*x^3
*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - 3*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^3*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2
 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) - 2*(a*b*c^2 -
 (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^4\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.252304, size = 410, normalized size = 3.15

$$-\frac{d^{\frac{5}{2}}\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b}-\frac{\left(b^3c^3\sqrt{d}-3\,ab^2c^2d^{\frac{3}{2}}+3\,a^2bcd^{\frac{5}{2}}-a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2b}-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc^3\sqrt{d}-9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ac^2d^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^4\sqrt{d}+12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ac^3d\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] -1/2*d^(5/2)*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b - (b^3*c^3*sqr
t(d) - 3*a*b^2*c^2*d^(3/2) + 3*a^2*b*c*d^(5/2) - a^3*d^(7/2))*arc
tan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*
b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*b) - 2/3*(3*(sqrt(
d)*x - sqrt(d*x^2 + c))^4*b*c^3*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*a*c^2*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^
4*sqrt(d) + 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^3*d^(3/2) + 3*
b*c^5*sqrt(d) - 7*a*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^
2 - c)^3*a^2)
```

3.703 $\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(ad+bc)}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}$$

```
[Out] -(((b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2)) + (c + d*x^2)^(3/2)/(3
*b*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(b^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.285799, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(ad+bc)}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2)) + (c + d*x^2)^(3/2)/(3
*b*d^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(b^(5/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 34.0468, size = 85, normalized size = 0.85

$$\frac{a^2\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{5}{2}}\sqrt{ad-bc}}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3bd^2}-\frac{\sqrt{c+dx^2}\,(ad+bc)}{b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(b**(5/2)*sqr
t(a*d - b*c)) + (c + d*x**2)**(3/2)/(3*b*d**2) - sqrt(c + d*x**2)
*(a*d + b*c)/(b**2*d**2)
```

Mathematica [A] time = 0.159616, size = 89, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(-3ad-2bc+bdx^2\right)}{3b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b*c - 3*a*d + b*d*x^2))/(3*b^2*d^2) - (a^2*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[
b*c - a*d])
```

Maple [B] time = 0.02, size = 362, normalized size = 3.6

$$\frac{x^2}{3\,bd}\sqrt{dx^2+c}-\frac{2\,c}{3\,bd^2}\sqrt{dx^2+c}-\frac{a}{b^2d}\sqrt{dx^2+c}$$
$$-\frac{a^2}{2\,b^3}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$
$$-\frac{a^2}{2\,b^3}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3/b*x^2/d*(d*x^2+c)^(1/2)-2/3/b*c/d^2*(d*x^2+c)^(1/2)-1/b^2*a/d
*(d*x^2+c)^(1/2)-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*a^2/b^3/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256928, size = 1, normalized size = 0.01

$$\left[\frac{3\,a^2d^2\log\left(\frac{\left(b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2\right)\sqrt{b^2c-abd}-4\,\left(2\,b^3c^2-3\,ab^2cd+a^2bd^2+\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\left(bdx^2-2\right.}{12\,\sqrt{b^2c-abd}b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^2*d^2*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d) - 4*(2*b
^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^2)*sqr
t(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b*d*x^2 - 2*b*c -
 3*a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(sqrt(b^2*c - a*b*d)
*b^2*d^2), 1/6*(3*a^2*d^2*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqr
t(-b^2*c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))) + 2*(b*d*x^2
 - 2*b*c - 3*a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c))/(sqrt(-b^
2*c + a*b*d)*b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.239251, size = 142, normalized size = 1.42

$$\frac{a^{2} \arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d^{4}-3\sqrt{dx^{2}+c}b^{2}cd^{4}-3\sqrt{dx^{2}+c}abd^{5}}{3\,b^{3}d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
 a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^2 + c)
*b^2*c*d^4 - 3*sqrt(d*x^2 + c)*a*b*d^5)/(b^3*d^6)
```

3.704 $\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.193159, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 21.6863, size = 56, normalized size = 0.82

$$-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{3}{2}}\sqrt{ad-bc}}+\frac{\sqrt{c+dx^2}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(b**(3/2)*sqrt(
a*d - b*c)) + sqrt(c + d*x**2)/(b*d)
```

Mathematica [A] time = 0.0708919, size = 68, normalized size = 1.

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.017, size = 318, normalized size = 4.7

$$\frac{1}{bd}\sqrt{dx^2+c} + \frac{a}{2b^2}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right. + \frac{a}{2b^2}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)/b/d+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2*a/b^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267235, size = 1, normalized size = 0.01

$$\left[\frac{ad\log\left(\frac{\left(b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2\right)\sqrt{b^2c-abd}+4\left(2\,b^3c^2-3\,ab^2cd+a^2bd^2+\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\sqrt{b^2c-abd}\sqrt{dx^2+c}}{4\sqrt{b^2c-abd}bd}, -\frac{ad\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}}{2\left(b^2c-abd\right)\sqrt{dx^2+c}}\right)-2\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\sqrt{-b^2c+abd}bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2
*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d) + 4*(2*b^3*c^2
- 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(b^2*c - a*b*d)*sqrt(
d*x^2 + c))/(sqrt(b^2*c - a*b*d)*b*d), -1/2*(a*d*arctan(-1/2*(b*d
*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*
x^2 + c))) - 2*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c))/(sqrt(-b^2*c
 + a*b*d)*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.23375, size = 86, normalized size = 1.26

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^2+c}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) - sqrt(d*x^2 + c)/b)/d
```

3.705 $\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqr
t[b*c - a*d]))
```

Rubi [A] time = 0.113835, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqr
t[b*c - a*d]))
```

Rubi in Sympy [A] time = 15.8001, size = 41, normalized size = 0.84

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(sqrt(b)*sqrt(a*d
- b*c))
```

Mathematica [A] time = 0.0352749, size = 49, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqr
t[b*c - a*d]))
```

Maple [B] time = 0.015, size = 300, normalized size = 6.1

$$-\frac{1}{2b}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$
$$-\frac{1}{2b}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x-1/b*(-a*b)^(1/2)))-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244582, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{\left(b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2\right)\sqrt{b^2c-abd}-4\left(2\,b^3c^2-3\,ab^2cd+a^2bd^2+\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\sqrt{b^2c-abd}}, \frac{\arctan\left(-\frac{\left(bdx^2+2\,bc\right.}{2\left(b^2c-\right.}\right.}{2\sqrt{-b^2c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b
^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d) - 4*(2*b^3*c^2 - 3*a
*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c)
)/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(b^2*c - a*b*d), 1/2*arctan(-1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)/((b^2*c - a*b*d)*
sqrt(d*x^2 + c)))/sqrt(-b^2*c + a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.235547, size = 53, normalized size = 1.08

$$\frac{\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*
d)
```

3.706 $\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])) + (Sqrt[b]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.214718, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])) + (Sqrt[b]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 25.2733, size = 68, normalized size = 0.85

$$-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a\sqrt{ad-bc}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(b)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*sqrt(a
*d - b*c)) - atanh(sqrt(c + d*x**2)/sqrt(c))/(a*sqrt(c))
```

Mathematica [C] time = 0.527885, size = 229, normalized size = 2.86

$$\frac{\frac{\sqrt{b}\left(\log\left(-\frac{2a\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\sqrt{b}\left(\sqrt{b}x+i\sqrt{a}\right)\sqrt{bc-ad}}\right)+\log\left(-\frac{2a\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\sqrt{b}\left(\sqrt{b}x-i\sqrt{a}\right)\sqrt{bc-ad}}\right)\right)}{\sqrt{bc-ad}}-\frac{2\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{\sqrt{c}}+\frac{2\log(x)}{\sqrt{c}}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((2*Log[x])/Sqrt[c] - (2*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/Sqrt[c
] + (Sqrt[b]*(Log[(-2*a*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a
*d]*Sqrt[c + d*x^2]))/(Sqrt[b]*Sqrt[b*c - a*d]*(I*Sqrt[a] + Sqrt[
b]*x))] + Log[(-2*a*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*
Sqrt[c + d*x^2]))/(Sqrt[b]*Sqrt[b*c - a*d]*((-I)*Sqrt[a] + Sqrt[b
]*x))]))/Sqrt[b*c - a*d])/(2*a)
```

Maple [B] time = 0.017, size = 331, normalized size = 4.1

$$-\frac{1}{a}\ln\Big(\frac{1}{x}\big(2c+2\sqrt{c}\sqrt{dx^2+c}\big)\Big)\frac{1}{\sqrt{c}}$$
$$+\frac{1}{2a}\ln\Bigg(1\bigg(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\Big(x-\frac{\sqrt{-ab}}{b}\Big)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\Big(x-\frac{\sqrt{-ab}}{b}\Big)^2 d+2\frac{d\sqrt{-ab}}{b}\Big(x-\frac{\sqrt{-ab}}{b}\Big)-\frac{ad-bc}{b}}\bigg)\Big($$
$$+\frac{1}{2a}\ln\Bigg(1\bigg(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\Big(x+\frac{\sqrt{-ab}}{b}\Big)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\Big(x+\frac{\sqrt{-ab}}{b}\Big)^2 d-2\frac{d\sqrt{-ab}}{b}\Big(x+\frac{\sqrt{-ab}}{b}\Big)-\frac{ad-bc}{b}}\bigg)\Big(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/a/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2/a/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x), x)
```

Fricas [A] time = 0.317367, size = 1, normalized size = 0.01

$$\Bigg[\frac{\sqrt{c}\sqrt{\frac{b}{bc-ad}}\log\Big(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)\,x^2+4\,(2\,b^2c^2-3\,abcd+a^2d^2+(b^2cd-abd^2)\,x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\Big)+2\log\Big(-\frac{(dx^2+}{}}{4\,a\sqrt{c}}$$

$$-\frac{\sqrt{c}\sqrt{-\frac{b}{bc-ad}}\arctan\Big(-\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}\Big)-\log\Big(-\frac{(dx^2+2\,c)\sqrt{c}-2\sqrt{dx^2+c}c}{x^2}\Big)}{2\,a\sqrt{c}},$$

$$-\frac{\sqrt{-c}\sqrt{-\frac{b}{bc-ad}}\arctan\Big(-\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}\Big)+2\arctan\Big(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\Big)}{2\,a\sqrt{-c}}\Bigg]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^
2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c
```

```
)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*log(-((d*
x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/(a*sqrt(c)), 1/4*
(sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 -
3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sq
rt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*arctan(sqrt(-
c)/sqrt(d*x^2 + c)))/(a*sqrt(-c)), -1/2*(sqrt(c)*sqrt(-b/(b*c - a
*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c -
a*d)*sqrt(-b/(b*c - a*d)))) - log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqr
t(d*x^2 + c)*c)/x^2))/(a*sqrt(c)), -1/2*(sqrt(-c)*sqrt(-b/(b*c -
a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c -
 a*d)*sqrt(-b/(b*c - a*d)))) + 2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
)/(a*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.228524, size = 107, normalized size = 1.34

$$-d\left(\frac{b\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ad}-\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -d*(b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*a*d) - arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.707 $\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

```
[Out] -Sqrt[c + d*x^2]/(2*a*c*x^2) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]])/(2*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.362275, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*a*c*x^2) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]])/(2*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 47.8614, size = 97, normalized size = 0.84

$$-\frac{\sqrt{c+dx^2}}{2acx^2}+\frac{b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2\sqrt{ad-bc}}+\frac{\left(\frac{ad}{2}+bc\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(c + d*x**2)/(2*a*c*x**2) + b**(3/2)*atan(sqrt(b)*sqrt(c + d
*x**2)/sqrt(a*d - b*c))/(a**2*sqrt(a*d - b*c)) + (a*d/2 + b*c)*at
anh(sqrt(c + d*x**2)/sqrt(c))/(a**2*c**(3/2))
```

Mathematica [C] time = 0.668216, size = 292, normalized size = 2.54

$$\frac{-\frac{\sqrt{c}\left(b^{3/2}cx^2\log\left(\frac{2a^2\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{b^{3/2}\left(\sqrt{b}x+i\sqrt{a}\right)\sqrt{bc-ad}}\right)+b^{3/2}cx^2\log\left(\frac{2a^2\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{b^{3/2}\left(\sqrt{b}x-i\sqrt{a}\right)\sqrt{bc-ad}}\right)+a\sqrt{c+dx^2}\sqrt{bc-ad}\right)}{x^2\sqrt{bc-ad}}+(ad+2bc)\log\left(\sqrt{c}\sqrt{c+dx\right.}}{2a^2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((2*b*c + a*d)*Log[x]) + (2*b*c + a*d)*Log[c + Sqrt[c]*Sqrt[c +
 d*x^2]] - (Sqrt[c]*(a*Sqrt[b*c - a*d]*Sqrt[c + d*x^2] + b^(3/2)*
c*x^2*Log[(2*a^2*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqr
t[c + d*x^2]))/(b^(3/2)*Sqrt[b*c - a*d]*(I*Sqrt[a] + Sqrt[b]*x))]
 + b^(3/2)*c*x^2*Log[(2*a^2*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c
 - a*d]*Sqrt[c + d*x^2]))/(b^(3/2)*Sqrt[b*c - a*d]*((-I)*Sqrt[a]
```

```
+ Sqrt[b]*x))]))/(Sqrt[b*c - a*d]*x^2))/(2*a^2*c^(3/2))
```

Maple [B] time = 0.02, size = 385, normalized size = 3.4

$$-\frac{1}{2\,ac x^2}\sqrt{dx^2+c}+\frac{d}{2\,a}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}+\frac{b}{a^2}\ln\left(\frac{1}{x}\left(2\,c+2\,\sqrt{c}\sqrt{dx^2+c}\right)\right)\frac{1}{\sqrt{c}}$$
$$-\frac{b}{2\,a^2}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$
$$-\frac{b}{2\,a^2}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(d*x^2+c)^(1/2)/a/c/x^2+1/2/a*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d
*x^2+c)^(1/2))/x)+b/a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2)
)/x)-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3), x)
```

Fricas [A] time = 0.394214, size = 1, normalized size = 0.01

$$\left[\frac{bc^{\frac{3}{2}}x^2\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+(2\,bc+ad}{4\,a^2c^{\frac{3}{2}}x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c^(3/2)*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*
c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*
b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x
^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (2*b*
c + a*d)*x^2*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x
^2) - 2*sqrt(d*x^2 + c)*a*sqrt(c))/(a^2*c^(3/2)*x^2), 1/4*(b*sqrt
(-c)*c*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a
```

```
*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 -
 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*s
qrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b*c + a*d
)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*sqrt(d*x^2 + c)*a*sqrt
(-c))/(a^2*sqrt(-c)*c*x^2), 1/4*(2*b*c^(3/2)*x^2*sqrt(-b/(b*c - a
*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c -
a*d)*sqrt(-b/(b*c - a*d)))) + (2*b*c + a*d)*x^2*log(-((d*x^2 + 2*
c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2) - 2*sqrt(d*x^2 + c)*a*sqrt
(c))/(a^2*c^(3/2)*x^2), 1/2*(b*sqrt(-c)*c*x^2*sqrt(-b/(b*c - a*d)
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d
)*sqrt(-b/(b*c - a*d)))) + (2*b*c + a*d)*x^2*arctan(sqrt(-c)/sqrt
(d*x^2 + c)) - sqrt(d*x^2 + c)*a*sqrt(-c))/(a^2*sqrt(-c)*c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + bx^2) \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.248291, size = 159, normalized size = 1.38

$$\frac{1}{2}\, d^2 \left(\frac{2\, b^2 \arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c + abd}a^2 d^2} - \frac{(2\, bc + ad) \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2 \sqrt{-c}cd^2} - \frac{\sqrt{dx^2 + c}}{acd^2 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sq
rt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c + a*d)*arctan(sqrt(d*x^2 + c
)/sqrt(-c))/(a^2*sqrt(-c)*c*d^2) - sqrt(d*x^2 + c)/(a*c*d^2*x^2))
```

3.708 $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*
d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

Rubi [A] time = 0.279249, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*
d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

Rubi in Sympy [A] time = 39.6118, size = 100, normalized size = 0.88

$$\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{ad-bc}}+\frac{x\sqrt{c+dx^2}}{2bd}-\frac{(2ad+bc)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(b**
2*sqrt(a*d - b*c)) + x*sqrt(c + d*x**2)/(2*b*d) - (2*a*d + b*c)*a
tanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*b**2*d**(3/2))
```

Mathematica [A] time = 0.201706, size = 112, normalized size = 0.98

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}-\frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{3/2}}+\frac{bx\sqrt{c+dx^2}}{d}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[
d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^(3/2))/(2*b^2)
```

Maple [B] time = 0.019, size = 386, normalized size = 3.4

$$\frac{x}{2bd}\sqrt{dx^2+c}-\frac{c}{2b}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^{-\frac{3}{2}}-\frac{a}{b^2}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$
$$-\frac{a^2}{2b^2}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$
$$+\frac{a^2}{2b^2}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b/d-1/2/b*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^
(1/2))-1/b^2*a*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/2/b^2*a^2/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.384246, size = 1, normalized size = 0.01

$$\left[\frac{ad^{\frac{3}{2}}\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+2\sqrt{dx}}{4\,b^2d^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d^(3/2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^
2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(
d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2
*sqrt(d*x^2 + c)*b*sqrt(d)*x + (b*c + 2*a*d)*log(2*sqrt(d*x^2 + c
)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^2*d^(3/2)), 1/4*(a*sqrt(-d)*d*
sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
 a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*
d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(
-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)
*b*sqrt(-d)*x - 2*(b*c + 2*a*d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
))/(b^2*sqrt(-d)*d), 1/4*(2*a*d^(3/2)*sqrt(a/(b*c - a*d))*arctan(
```

```
1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt
(a/(b*c - a*d)))) + 2*sqrt(d*x^2 + c)*b*sqrt(d)*x + (b*c + 2*a*d)
*log(2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b^2*d^(3/2)
), 1/2*(a*sqrt(-d)*d*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d
)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d))))
 + sqrt(d*x^2 + c)*b*sqrt(-d)*x - (b*c + 2*a*d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)))/(b^2*sqrt(-d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.250187, size = 184, normalized size = 1.61

$$-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b^2}+\frac{\sqrt{dx^2+c}x}{2\,bd}+\frac{\left(bc\sqrt{d}+2\,ad^{\frac{3}{2}}\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2) +
 1/2*sqrt(d*x^2 + c)*x/(b*d) + 1/4*(b*c*sqrt(d) + 2*a*d^(3/2))*ln
((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^2*d^2)
```

3.709 $\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])
/(b*Sqrt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*S
qrt[d])
```

Rubi [A] time = 0.155655, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])
/(b*Sqrt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*S
qrt[d])
```

Rubi in Sympy [A] time = 23.7468, size = 70, normalized size = 0.85

$$-\frac{\sqrt{a}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{ad-bc}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(b*s
qrt(a*d - b*c)) + atanh(sqrt(d)*x/sqrt(c + d*x**2))/(b*sqrt(d))
```

Mathematica [A] time = 0.0846598, size = 85, normalized size = 1.04

$$\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])
/(b*Sqrt[b*c - a*d])) + Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]/(b*Sqr
t[d])
```

Maple [B] time = 0.015, size = 337, normalized size = 4.1

$$\frac{1}{b}\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\frac{1}{\sqrt{d}}$$
$$+\frac{a}{2b}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(\right.\right.$$
$$-\frac{a}{2b}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/b*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)+1/2*a/(-a*b)^(1/2)/b/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.355237, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{d}\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)\,x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)\,x^2-4\,\left((b^2c^2-3\,abcd+2\,a^2d^2)\,x^3-(abc^2-a^2cd)\,x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+2\log}{4\,b\sqrt{d}}\right.$$
$$-\frac{\sqrt{d}\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\,\sqrt{dx^2+c}(bc-ad)x\sqrt{\frac{a}{bc-ad}}}\right)-\log\left(-2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)}{2\,b\sqrt{d}},$$
$$\left.-\frac{\sqrt{-d}\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\,\sqrt{dx^2+c}(bc-ad)x\sqrt{\frac{a}{bc-ad}}}\right)-2\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{2\,b\sqrt{-d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(d)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*
x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*l
```

```
og(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b*sqrt(d)),
1/4*(sqrt(-d)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*
x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b*sqrt(-d)), -1/2*(sqrt(d)*sq
rt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^
2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - log(-2*sqrt(d*x^2 +
c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(b*sqrt(d)), -1/2*(sqrt(-d)*sqrt
(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2
+ c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - 2*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)))/(b*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.263037, size = 138, normalized size = 1.68

$$\frac{a\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}b}-\frac{\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] a*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2
*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b) - 1/2*
ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b*sqrt(d))
```

3.710 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[b*c - a*d])
```

Rubi [A] time = 0.057907, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[b*c - a*d])
```

Rubi in Sympy [A] time = 11.1516, size = 42, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*sqrt
(a*d - b*c))
```

Mathematica [A] time = 0.0375766, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[b*c - a*d])
```

Maple [B] time = 0.006, size = 306, normalized size = 6.2

$$-\frac{1}{2}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x\right.\right.$$
$$+\frac{1}{2}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302111, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{\left(\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}\right)\sqrt{-abc+a^{2}d}+4\left(\left(ab^{2}c^{2}-3\,a^{2}bcd+2\,a^{3}d^{2}\right)x^{3}-\left(a^{2}bc^{2}-a^{3}cd\right)x\right)\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{4\sqrt{-abc+a^{2}d}},\ \frac{\arctan\left(\frac{(bc-}{2\sqrt{abc}}\right.}{2\sqrt{abc}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3
*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 -
 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x
^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(-a*b*c + a^2*d), 1/2*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x
^2 + c)*x))/sqrt(a*b*c - a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+bx^{2}\right)\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.247413, size = 95, normalized size = 1.94

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*
a*d)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.711 $\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}} dx$

Optimal. Leaf size=74

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.154837, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 25.1114, size = 61, normalized size = 0.82

$$-\frac{\sqrt{c+dx^2}}{acx}-\frac{b\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{3}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(c + d*x**2)/(a*c*x) - b*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sq
rt(c + d*x**2)))/(a**(3/2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.0800713, size = 74, normalized size = 1.

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.019, size = 334, normalized size = 4.5

$$-\frac{1}{acx}\sqrt{dx^2+c}$$
$$+\frac{b}{2a}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right(\right.$$
$$-\frac{b}{2a}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -(d*x^2+c)^(1/2)/a/c/x+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*b
/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [A] time = 0.324314, size = 1, normalized size = 0.01

$$\left[\frac{bcx\log\left(\frac{((b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2)\sqrt{-abc+a^2d}-4\,((ab^2c^2-3\,a^2bcd+2\,a^3d^2)x^3-(a^2bc^2-a^3cd)x)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{-abc}}{4\sqrt{-abc+a^2d}dacx}\right.$$
$$\left.-\frac{bcx\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\sqrt{abc-a^2d}\sqrt{dx^2+cx}}\right)+2\sqrt{abc-a^2d}\sqrt{dx^2+c}}{2\sqrt{abc-a^2d}dacx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*x*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^
2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*s
qrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-a*b*c + a^
2*d)*sqrt(d*x^2 + c))/(sqrt(-a*b*c + a^2*d)*a*c*x), -1/2*(b*c*x*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x
^2 + c)*x)) + 2*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(sqrt(a*b*c
- a^2*d)*a*c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.242706, size = 150, normalized size = 2.03

$$d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ad}+\frac{2}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] d^(3/2)*(b*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c +
2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2
/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a*d))
```

3.712 $\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

```
[Out] -Sqrt[c + d*x^2]/(3*a*c*x^3) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/
(3*a^2*c^2*x) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(a^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.373361, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*a*c*x^3) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/
(3*a^2*c^2*x) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(a^(5/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 58.2618, size = 95, normalized size = 0.86

$$-\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{\sqrt{c+dx^2}\,(2ad+3bc)}{3a^2c^2x}+\frac{b^2\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{5}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(c + d*x**2)/(3*a*c*x**3) + sqrt(c + d*x**2)*(2*a*d + 3*b*c)
/(3*a**2*c**2*x) + b**2*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c +
 d*x**2)))/(a**(5/2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.17042, size = 96, normalized size = 0.87

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\left(-ac+2adx^2+3bcx^2\right)}{3a^2c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 + 2*a*d*x^2))/(3*a^2*c^2*x^3
) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
a^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.021, size = 379, normalized size = 3.5

$$-\frac{1}{3\,ac x^3}\sqrt{dx^2+c}+\frac{2\,d}{3\,ac^2x}\sqrt{dx^2+c}+\frac{b}{a^2cx}\sqrt{dx^2+c}$$
$$-\frac{b^2}{2\,a^2}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$
$$+\frac{b^2}{2\,a^2}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(d*x^2+c)^(1/2)/a/c/x^3+2/3/a*d/c^2/x*(d*x^2+c)^(1/2)+b/a^2/
c/x*(d*x^2+c)^(1/2)-1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2*
b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [A] time = 0.324493, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^2c^2x^3\log\left(\frac{\left((b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2\right)\sqrt{-abc+a^2d}+4\left((ab^2c^2-3\,a^2bcd+2\,a^3d^2)x^3-(a^2bc^2-a^3cd)x\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\sqrt{-}}{12\sqrt{-abc+a^2d}a^2c^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b^2*c^2*x^3*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4
+ a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) +
 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*
c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(-a
*b*c + a^2*d)*((3*b*c + 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(sqrt(
-a*b*c + a^2*d)*a^2*c^2*x^3), 1/6*(3*b^2*c^2*x^3*arctan(1/2*((b*c
 - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) + 2
*sqrt(a*b*c - a^2*d)*((3*b*c + 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))
/(sqrt(a*b*c - a^2*d)*a^2*c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + bx^2) \sqrt{c + dx^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.739166, size = 263, normalized size = 2.39

$$-\frac{1}{3} d^{\frac{5}{2}} \left(\frac{3 b^2 \arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 b - bc + 2 ad}{2 \sqrt{abcd - a^2 d^2}} \right)}{\sqrt{abcd - a^2 d^2} a^2 d^2} + \frac{2 \left(3 \left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 b - 6 \left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 bc - 6 \left(\sqrt{d}x - \sqrt{dx^2 + c}\right)\right.}{\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 - c\right)^3 a^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] -1/3*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)
*a^2*d^2) + 2*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
 + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^
2*d^2))
```

3.713 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

```
[Out] -((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a^(3/2)*ArcTan[(Sqrt[
b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) +
 ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Rubi [A] time = 0.310914, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a^(3/2)*ArcTan[(Sqrt[
b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) +
 ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Rubi in Sympy [A] time = 42.8231, size = 92, normalized size = 0.84

$$-\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\left(ad-bc\right)^{\frac{3}{2}}}+\frac{cx}{d\sqrt{c+dx^2}\left(ad-bc\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{bd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -a**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(b*
(a*d - b*c)**(3/2)) + c*x/(d*sqrt(c + d*x**2)*(a*d - b*c)) + atan
h(sqrt(d)*x/sqrt(c + d*x**2))/(b*d**(3/2))
```

Mathematica [A] time = 0.2704, size = 111, normalized size = 1.02

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}+\frac{cx}{d\sqrt{c+dx^2}(ad-bc)}+\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x)/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[
b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) +
 Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Maple [B] time = 0.021, size = 720, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/b*x/d/(d*x^2+c)^(1/2)+1/b/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)
)-1/b^2*a*x/c/(d*x^2+c)^(1/2)-1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)+1/2/b^2*a^2/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
+1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2/b*a^2/(-a*b
)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^2*a^2/(a*d-b*c)/c/((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x*d-1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.466243, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*b*c*sqrt(d)*x + (a*d^2*x^2 + a*c*d)*sqrt
(d)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x
^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*
b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*s
qrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2
+ c)*sqrt(d)))/((b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)*x^
2)*sqrt(d)), -1/4*(4*sqrt(d*x^2 + c)*b*c*sqrt(-d)*x + (a*d^2*x^2
+ a*c*d)*sqrt(-d)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqr
t(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) -
 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan(sqrt(-d)*x/sqrt(d
*x^2 + c)))/((b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)*x^2)*
sqrt(-d)), -1/2*(2*sqrt(d*x^2 + c)*b*c*sqrt(d)*x - (a*d^2*x^2 + a
*c*d)*sqrt(d)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - (b*c
^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2
*d*x^2 + c)*sqrt(d)))/((b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*
```

```
d^3)*x^2)*sqrt(d)), -1/2*(2*sqrt(d*x^2 + c)*b*c*sqrt(-d)*x - (a*d
^2*x^2 + a*c*d)*sqrt(-d)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2
*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d
)))) - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)))/((b^2*c^2*d - a*b*c*d^2 + (b^2*c*d^2 - a*b*d^3)
*x^2)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.256757, size = 201, normalized size = 1.84

$$-\frac{b^2cx}{(b^3cd-ab^2d^2)\sqrt{dx^2+c}}-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(b^2c-abd)}-\frac{\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,bd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] -b^2*c*x/((b^3*c*d - a*b^2*d^2)*sqrt(d*x^2 + c)) - a^2*sqrt(d)*ar
ctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a
*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b^2*c - a*b*d)) - 1/
2*ln((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b*d^(3/2))
```

3.714 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -(c/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.20655, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(c/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 24.3372, size = 63, normalized size = 0.82

$$\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{\frac{3}{2}}}+\frac{c}{d\sqrt{c+dx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] a*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(sqrt(b)*(a*d -
b*c)**(3/2)) + c/(d*sqrt(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.128109, size = 78, normalized size = 1.01

$$\frac{\frac{c}{d\sqrt{c+dx^2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}}{ad-bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c/(d*Sqrt[c + d*x^2]) - (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqr
t[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(-(b*c) + a*d)
```

Maple [B] time = 0.018, size = 653, normalized size = 8.5

$$-\frac{1}{bd}\frac{1}{\sqrt{dx^2+c}}+\frac{a}{2\,(ad-bc)\,b}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{axd}{2\,b^2\,(ad-bc)\,c}\sqrt{-ab}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{a}{2\,(ad-bc)\,b}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\right.\right.$$
$$+\frac{a}{2\,(ad-bc)\,b}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{axd}{2\,b^2\,(ad-bc)\,c}\sqrt{-ab}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{a}{2\,(ad-bc)\,b}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/b/d/(d*x^2+c)^(1/2)+1/2*a/b/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*
a/b^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*a/b/(a
*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))+1/2*a/b/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2
*a/b^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*a/b/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.254401, size = 1, normalized size = 0.01

$$\left[-\frac{4\,\sqrt{b^2c-abd}\sqrt{dx^2+c}c+\left(ad^2x^2+acd\right)\log\left(\frac{\left(b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2\right)\sqrt{b^2c-abd}-4\,\left(2\,b^3c^2-3\,ab^2cd+a^2bd^2+(\right.}{b^2x^4+2\,abx^2+a^2}\right.}{4\,(bc^2d-acd^2+(bcd^2-ad^3)x^2)\sqrt{b^2c-abd}}\right.$$
$$\left.-\frac{2\,\sqrt{-b^2c+abd}\sqrt{dx^2+c}c+\left(ad^2x^2+acd\right)\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}}{2\,(b^2c-abd)\sqrt{dx^2+c}}\right)}{2\,(bc^2d-acd^2+(bcd^2-ad^3)x^2)\sqrt{-b^2c+abd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c)*c + (a*d^2*x^2 + a*c
*d)*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^
2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d) - 4*(2*b^3*c^2 - 3*a*
b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((b*c^2*d - a*c*d^2 + (b*c*d^2 - a
*d^3)*x^2)*sqrt(b^2*c - a*b*d)), -1/2*(2*sqrt(-b^2*c + a*b*d)*sqr
t(d*x^2 + c)*c + (a*d^2*x^2 + a*c*d)*arctan(-1/2*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(-b^2*c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))))/
((b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*sqrt(-b^2*c + a*b*d)
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.228596, size = 105, normalized size = 1.36

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{c}{\sqrt{dx^2+c}\,(bc-ad)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*(b*c - a*d)) + c/(sqrt(d*x^2 + c)*(b*c - a*d)))/d
```

3.715 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Rubi [A] time = 0.147628, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Rubi in Sympy [A] time = 24.0822, size = 61, normalized size = 0.82

$$\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(ad-bc)^{\frac{3}{2}}}-\frac{x}{\sqrt{c+dx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] sqrt(a)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(a*d
- b*c)**(3/2) - x/(sqrt(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.132593, size = 74, normalized size = 1.

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Maple [B] time = 0.017, size = 653, normalized size = 8.8

$$\frac{x}{bc}\frac{1}{\sqrt{dx^2+c}}+\frac{a}{2\,ad-2\,bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{adx}{2\,(ad-bc)\,bc}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{a}{2\,ad-2\,bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}}\right.\right.$$
$$-\frac{a}{2\,ad-2\,bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{adx}{2\,(ad-bc)\,bc}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{a}{2\,ad-2\,bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/b*x/c/(d*x^2+c)^(1/2)+1/2*a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-1/2*a/b/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*a/(-a*b)^
(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*a/(-a*b)^(1/2)/(a*d-b*c)/((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-1/2*a/b/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*a
/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.3067, size = 1, normalized size = 0.01

$$\left[-\frac{\left(dx^2+c\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)},\ -\frac{\left(dx^2+c\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\,\sqrt{dx^2+c}(bc-ad)x\sqrt{\frac{a}{bc-ad}}}\right)-2\,\sqrt{dx^2+c}x}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sq
rt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 4*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2), -1/
2*((d*x^2 + c)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2
- a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - 2*s
qrt(d*x^2 + c)*x)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.234531, size = 138, normalized size = 1.86

$$\frac{a\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)}+\frac{x}{\sqrt{dx^2+c}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] a*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2
*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*
d)) + x/(sqrt(d*x^2 + c)*(b*c - a*d))
```

3.716 $\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rubi [A] time = 0.155797, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rubi in Sympy [A] time = 21.1594, size = 61, normalized size = 0.85

$$-\frac{\sqrt{b}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{(ad-bc)^{\frac{3}{2}}}-\frac{1}{\sqrt{c+dx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -sqrt(b)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*d - b*
c)**(3/2) - 1/(sqrt(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.119809, size = 72, normalized size = 1.

$$\frac{1}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Maple [B] time = 0.017, size = 618, normalized size = 8.6

$$-\frac{1}{2\,ad-2\,bc}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{dx}{2\,(ad-bc)\,bc}\sqrt{-ab}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{1}{2\,ad-2\,bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}}\right.\right.$$
$$-\frac{1}{2\,ad-2\,bc}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{dx}{2\,(ad-bc)\,bc}\sqrt{-ab}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{1}{2\,ad-2\,bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b*(-a*b)^(1/2)/(a*d-b*c)/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d+1/2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/(a*d-b
*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)-1/2/b*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*x*d+1/2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262217, size = 1, normalized size = 0.01

$$\left[-\frac{\left(dx^{2}+c\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\,\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\,\left(2\,b^{2}c^{2}-3\,abcd+a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}\sqrt{\frac{b}{bc-ad}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\,\sqrt{d}}{4\,\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b
^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^
2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt
(d*x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2), 1/2*((d*x^2 +
 c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqr
t(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) + 2*sqrt(d*x^2 +
c))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.233527, size = 96, normalized size = 1.33

$$\frac{b\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{1}{\sqrt{dx^2+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*(b*c - a*d)) + 1/(sqrt(d*x^2 + c)*(b*c - a*d))
```

3.717 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.113777, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 20.0658, size = 66, normalized size = 0.84

$$\frac{dx}{c\sqrt{c+dx^2}\,(ad-bc)}-\frac{b\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d*x/(c*sqrt(c + d*x**2)*(a*d - b*c)) - b*atanh(x*sqrt(a*d - b*c)/
(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.128053, size = 78, normalized size = 0.99

$$\frac{dx}{c\sqrt{c+dx^2}(ad-bc)}+\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (b*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Maple [B] time = 0., size = 628, normalized size = 8.

$$-\frac{b}{2\,ad-2\,bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}+\frac{dx}{(2\,ad-2\,bc)\,c}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}+\frac{b}{2\,ad-2\,bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}}\right.\right.+\frac{b}{2\,ad-2\,bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}+\frac{dx}{(2\,ad-2\,bc)\,c}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}-\frac{b}{2\,ad-2\,bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*
c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.331636, size = 1, normalized size = 0.01

$$\left[-\frac{4\,\sqrt{-abc+a^{2}d}\sqrt{dx^{2}+c}dx+\left(bcdx^{2}+bc^{2}\right)\log\left(\frac{\left((b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})\,x^{4}+a^{2}c^{2}-2\,(3\,abc^{2}-4\,a^{2}cd)\,x^{2}\right)\sqrt{-abc+a^{2}d}-4\,\left((ab^{2}c^{2}-3\,a^{2}bcd+2\right.}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right.}{4\,(bc^{3}-ac^{2}d+(bc^{2}d-acd^{2})x^{2})\sqrt{-abc+a^{2}d}},\ -\frac{2\,\sqrt{abc-a^{2}d}\sqrt{dx^{2}+c}dx-\left(bcdx^{2}+bc^{2}\right)\arctan\left(\frac{(bc-2\,ad)x^{2}-ac}{2\,\sqrt{abc-a^{2}d}\sqrt{dx^{2}+cx}}\right)}{2\,(bc^{3}-ac^{2}d+(bc^{2}d-acd^{2})x^{2})\sqrt{abc-a^{2}d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*d*x + (b*c*d*x^2 +
b*c^2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2
 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((b*c^3 - a*c^2*d + (b*c
^2*d - a*c*d^2)*x^2)*sqrt(-a*b*c + a^2*d)), -1/2*(2*sqrt(a*b*c -
a^2*d)*sqrt(d*x^2 + c)*d*x - (b*c*d*x^2 + b*c^2)*arctan(1/2*((b*c
 - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((
b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.238732, size = 146, normalized size = 1.85

$$-\frac{b\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)}-\frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c +
2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a
*d)) - d*x/((b*c^2 - a*c*d)*sqrt(d*x^2 + c))
```

3.718 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]]/(a*c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(a*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.346962, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]]/(a*c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(a*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 48.0304, size = 87, normalized size = 0.81

$$\frac{d}{c\sqrt{c+dx^2}\,(ad-bc)}+\frac{b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a\,(ad-bc)^{\frac{3}{2}}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d/(c*sqrt(c + d*x**2)*(a*d - b*c)) + b**(3/2)*atan(sqrt(b)*sqrt(c
 + d*x**2)/sqrt(a*d - b*c))/(a*(a*d - b*c)**(3/2)) - atanh(sqrt(c
 + d*x**2)/sqrt(c))/(a*c**(3/2))
```

Mathematica [C] time = 2.32741, size = 316, normalized size = 2.95

$$\frac{\log(x)}{ac^{3/2}}+\frac{1}{2}\left(\frac{b^{3/2}\log\left(-\frac{2a\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{3/2}\left(\sqrt{b}x+i\sqrt{a}\right)}\right)}{a(bc-ad)^{3/2}}\right.$$
$$+\frac{b^{3/2}\log\left(-\frac{2a\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{3/2}\left(\sqrt{b}x-i\sqrt{a}\right)}\right)}{a(bc-ad)^{3/2}}$$
$$\left.+\frac{2d}{c\sqrt{c+dx^2}(ad-bc)}-\frac{2\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{ac^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] Log[x]/(a*c^(3/2)) + ((2*d)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) -
(2*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/(a*c^(3/2)) + (b^(3/2)*Log[(
-2*a*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqrt[b*c - a*d]*x +
 b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(3/2)*(I*Sqrt[a]
+ Sqrt[b]*x))])/(a*(b*c - a*d)^(3/2)) + (b^(3/2)*Log[(-2*a*(Sqrt[
b]*c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c
 + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(3/2)*((-I)*Sqrt[a] + Sqrt[b
]*x))])/(a*(b*c - a*d)^(3/2)))/2
```

Maple [B] time = 0.018, size = 681, normalized size = 6.4

$$\frac{1}{ac}\frac{1}{\sqrt{dx^2+c}}-\frac{1}{a}\ln\left(\frac{1}{x}\left(2c+2\sqrt{c}\sqrt{dx^2+c}\right)\right)c^{-\frac{3}{2}}$$
$$+\frac{b}{2a(ad-bc)}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{dx}{2a(ad-bc)c}\sqrt{-ab}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{b}{2a(ad-bc)}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{a}{}}\right.\right.$$
$$+\frac{b}{2a(ad-bc)}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{dx}{2a(ad-bc)c}\sqrt{-ab}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{b}{2a(ad-bc)}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{a}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/a/c/(d*x^2+c)^(1/2)-1/a/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/
2))/x)+1/2/a/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*(-a*b)^(1/2)/(
a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a/(a*d-b*c)*b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))+1/2/a/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a*(-a*b)^(1/2)/(a*d
-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a/(a*d-b*c)*b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x), x)
```

Fricas [A] time = 0.526308, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*a*sqrt(c)*d + (b*c*d*x^2 + b*c^2)*sqrt(c
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*
d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*
c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(b*c^2 - a*c*d + (b*c
*d - a*d^2)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*
c)/x^2))/((a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - a^2*c*d^2)*x^2)*sqr
t(c)), -1/4*(4*sqrt(d*x^2 + c)*a*sqrt(-c)*d + (b*c*d*x^2 + b*c^2)
*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 -
3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sq
rt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b*c^2 - a*c*
d + (b*c*d - a*d^2)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((a*b*
c^3 - a^2*c^2*d + (a*b*c^2*d - a^2*c*d^2)*x^2)*sqrt(-c)), -1/2*(2
*sqrt(d*x^2 + c)*a*sqrt(c)*d + (b*c*d*x^2 + b*c^2)*sqrt(c)*sqrt(-
b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 +
c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - (b*c^2 - a*c*d + (b*c*d -
 a*d^2)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x
^2))/((a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - a^2*c*d^2)*x^2)*sqrt(c)
), -1/2*(2*sqrt(d*x^2 + c)*a*sqrt(-c)*d + (b*c*d*x^2 + b*c^2)*sqr
t(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(s
qrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) + 2*(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((a*
b*c^3 - a^2*c^2*d + (a*b*c^2*d - a^2*c*d^2)*x^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.232371, size = 158, normalized size = 1.48

$$-\left(\frac{b^{2}\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(abcd-a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{1}{\left(bc^{2}-acd\right)\sqrt{dx^{2}+c}}-\frac{\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}cd}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c*d -
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/((b*c^2 - a*c*d)*sqrt(d*x^2 +
c)) - arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*c*d))*d
```

3.719 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) - ((b*c - 2*a*d)*Sqrt[c +
d*x^2])/(a*c^2*(b*c - a*d)*x) - (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.36288, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) - ((b*c - 2*a*d)*Sqrt[c +
d*x^2])/(a*c^2*(b*c - a*d)*x) - (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 58.2331, size = 100, normalized size = 0.81

$$\frac{d}{cx\sqrt{c+dx^2}\,(ad-bc)}-\frac{\sqrt{c+dx^2}\,(2ad-bc)}{ac^2x\,(ad-bc)}+\frac{b^2\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{3}{2}}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d/(c*x*sqrt(c + d*x**2)*(a*d - b*c)) - sqrt(c + d*x**2)*(2*a*d -
b*c)/(a*c**2*x*(a*d - b*c)) + b**2*atanh(x*sqrt(a*d - b*c)/(sqrt(
a)*sqrt(c + d*x**2)))/(a**(3/2)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.366333, size = 102, normalized size = 0.82

$$\frac{\frac{d^2x^2}{bc-ad}-\frac{c+dx^2}{a}}{c^2x\sqrt{c+dx^2}}-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d^2*x^2)/(b*c - a*d) - (c + d*x^2)/a)/(c^2*x*Sqrt[c + d*x^2]) -
 (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(
3/2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.02, size = 695, normalized size = 5.6

$$-\frac{1}{acx}\frac{1}{\sqrt{dx^2+c}}-2\frac{dx}{ac^2\sqrt{dx^2+c}}+\frac{b^2}{2a(ad-bc)}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{bxd}{2a(ad-bc)c}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{b^2}{2a(ad-bc)}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\right.\right.\right.$$
$$-\frac{b^2}{2a(ad-bc)}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{bxd}{2a(ad-bc)c}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{b^2}{2a(ad-bc)}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/a/c/x/(d*x^2+c)^(1/2)-2/a*d/c^2*x/(d*x^2+c)^(1/2)+1/2*b^2/a/(-
a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/a/(a*d-b*c)/c/((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x*d-1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))-1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/a/
(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^2/a/(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2), x)
```

Fricas [A] time = 0.358749, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{-abc+a^2d}\left(bc^2-acd+\left(bcd-2\,ad^2\right)x^2\right)\sqrt{dx^2+c}+\left(b^2c^2dx^3+b^2c^3x\right)\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)\right.}{}\right.}{4\left(\left(abc^3d-a^2c^2d^2\right)x^3+\left(abc^4-a^2c^3d\right)x\right)\sqrt{-abc+a^2d}}\right.$$

$$\left.-\frac{2\sqrt{abc-a^2d}\left(bc^2-acd+\left(bcd-2\,ad^2\right)x^2\right)\sqrt{dx^2+c}+\left(b^2c^2dx^3+b^2c^3x\right)\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\sqrt{abc-a^2d}\sqrt{dx^2+cx}}\right)}{2\left(\left(abc^3d-a^2c^2d^2\right)x^3+\left(abc^4-a^2c^3d\right)x\right)\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a*b*c + a^2*d)*(b*c^2 - a*c*d + (b*c*d - 2*a*d^2)*
x^2)*sqrt(d*x^2 + c) + (b^2*c^2*d*x^3 + b^2*c^3*x)*log((((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*
d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^
3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 +
 2*a*b*x^2 + a^2)))/(((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 -
a^2*c^3*d)*x)*sqrt(-a*b*c + a^2*d)), -1/2*(2*sqrt(a*b*c - a^2*d)*
(b*c^2 - a*c*d + (b*c*d - 2*a*d^2)*x^2)*sqrt(d*x^2 + c) + (b^2*c^
2*d*x^3 + b^2*c^3*x)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a
*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a*b*c^3*d - a^2*c^2*d^2)*x^
3 + (a*b*c^4 - a^2*c^3*d)*x)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.720 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

```
[Out] -(d*(b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[c + d*x^2]) - 1/(2*a
*c*x^2*Sqrt[c + d*x^2]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]])/(2*a^2*c^(5/2)) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.646642, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d*(b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[c + d*x^2]) - 1/(2*a
*c*x^2*Sqrt[c + d*x^2]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]])/(2*a^2*c^(5/2)) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(a^2*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 77.492, size = 136, normalized size = 0.87

$$-\frac{1}{2acx^2\sqrt{c+dx^2}}-\frac{d\,(3ad-bc)}{2ac^2\sqrt{c+dx^2}\,(ad-bc)}-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2\,(ad-bc)^{\frac{3}{2}}}+\frac{(3ad+2bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -1/(2*a*c*x**2*sqrt(c + d*x**2)) - d*(3*a*d - b*c)/(2*a*c**2*sqrt
(c + d*x**2)*(a*d - b*c)) - b**(5/2)*atan(sqrt(b)*sqrt(c + d*x**2
)/sqrt(a*d - b*c))/(a**2*(a*d - b*c)**(3/2)) + (3*a*d + 2*b*c)*at
anh(sqrt(c + d*x**2)/sqrt(c))/(2*a**2*c**(5/2))
```

Mathematica [C] time = 4.38667, size = 355, normalized size = 2.28

$$\frac{1}{2}\left(-\frac{b^{5/2}\log\left(\frac{2a^2\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{5/2}\left(\sqrt{b}x+i\sqrt{a}\right)}\right)}{a^2(bc-ad)^{3/2}}-\frac{b^{5/2}\log\left(\frac{2a^2\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{5/2}\left(\sqrt{b}x-i\sqrt{a}\right)}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^2c^{5/2}}-\frac{\log(x)(3ad+2bc)}{a^2c^{5/2}}+\frac{\frac{2d^2}{bc-ad}-\frac{\frac{c}{x^2}+d}{a}}{c^2\sqrt{c+dx^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (((2*d^2)/(b*c - a*d) - (d + c/x^2)/a)/(c^2*Sqrt[c + d*x^2]) - ((
2*b*c + 3*a*d)*Log[x])/(a^2*c^(5/2)) + ((2*b*c + 3*a*d)*Log[c + S
qrt[c]*Sqrt[c + d*x^2]])/(a^2*c^(5/2)) - (b^(5/2)*Log[(2*a^2*(Sqr
t[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt
[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(5/2)*(I*Sqrt[a] + Sqrt[b]
*x))])/(a^2*(b*c - a*d)^(3/2)) - (b^(5/2)*Log[(2*a^2*(Sqrt[b]*c*S
qrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c + d*x
^2] - a*d*Sqrt[c + d*x^2]))/(b^(5/2)*((-I)*Sqrt[a] + Sqrt[b]*x))]
)/(a^2*(b*c - a*d)^(3/2)))/2
```

Maple [B] time = 0.021, size = 763, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/a/c/x^2/(d*x^2+c)^(1/2)-3/2/a*d/c^2/(d*x^2+c)^(1/2)+3/2/a*d/
c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b/a^2/c/(d*x^2+c)^(
1/2)+b/a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2*b^2/
a^2/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b/a^2*(-a*b)^(1/2)/(a*d-b*c
)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^2/a^2/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
-1/2*b^2/a^2/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/a^2*(-a*b)^(1/2)
/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^2/a^2/(a*d-b*c)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{3}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3), x)
```

Fricas [A] time = 0.793519, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2*d*x^4 + b^2*c^3*x^2)*sqrt(c)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
 - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d
 - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
2*a*b*x^2 + a^2)) + 2*(a*b*c^2 - a^2*c*d + (a*b*c*d - 3*a^2*d^2)*
x^2)*sqrt(d*x^2 + c)*sqrt(c) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*
d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*log(-((d*x^
2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/(((a^2*b*c^3*d - a^
3*c^2*d^2)*x^4 + (a^2*b*c^4 - a^3*c^3*d)*x^2)*sqrt(c)), -1/4*((b^
2*c^2*d*x^4 + b^2*c^3*x^2)*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*
d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*
d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 2*(a*b*c^2 - a^2*c*d + (a*b*c*d - 3*a^2*d^2)*x^2)*sqrt
(d*x^2 + c)*sqrt(-c) - 2*((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x
^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*arctan(sqrt(-c)/s
qrt(d*x^2 + c)))/(((a^2*b*c^3*d - a^3*c^2*d^2)*x^4 + (a^2*b*c^4 -
 a^3*c^3*d)*x^2)*sqrt(-c)), 1/4*(2*(b^2*c^2*d*x^4 + b^2*c^3*x^2)*
sqrt(c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/
(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(a*b*c^2
- a^2*c*d + (a*b*c*d - 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(c) +
((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2
*d - 3*a^2*c*d^2)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2
 + c)*c)/x^2))/(((a^2*b*c^3*d - a^3*c^2*d^2)*x^4 + (a^2*b*c^4 - a
^3*c^3*d)*x^2)*sqrt(c)), 1/2*((b^2*c^2*d*x^4 + b^2*c^3*x^2)*sqrt(
-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqr
t(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - (a*b*c^2 - a^2*
c*d + (a*b*c*d - 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + ((2*b
^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d -
3*a^2*c*d^2)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(((a^2*b*c^3*
d - a^3*c^2*d^2)*x^4 + (a^2*b*c^4 - a^3*c^3*d)*x^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.248932, size = 248, normalized size = 1.59

$$\frac{1}{2}\left(\frac{2\,b^3\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\left(dx^2+c\right)bc-3\left(dx^2+c\right)ad+2\,acd}{\left(abc^3d-a^2c^2d^2\right)\left(\left(dx^2+c\right)^{\frac{3}{2}}-\sqrt{dx^2+c}c\right)}-\frac{\left(2\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2d^2}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - ((d*x^2 + c)*b*c - 3*(d
*x^2 + c)*a*d + 2*a*c*d)/((a*b*c^3*d - a^2*c^2*d^2)*((d*x^2 + c)^
(3/2) - sqrt(d*x^2 + c)*c)) - (2*b*c + 3*a*d)*arctan(sqrt(d*x^2 +
 c)/sqrt(-c))/(a^2*sqrt(-c)*c^2*d^2))*d^2
```

3.721 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) - ((b*c - 4*a*d)*Sqrt[c
+ d*x^2])/(3*a*c^2*(b*c - a*d)*x^3) + ((3*b*c - 4*a*d)*(b*c + 2*a
*d)*Sqrt[c + d*x^2])/(3*a^2*c^3*(b*c - a*d)*x) + (b^3*ArcTan[(Sqr
t[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^
(3/2))
```

Rubi [A] time = 0.685407, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) - ((b*c - 4*a*d)*Sqrt[c
+ d*x^2])/(3*a*c^2*(b*c - a*d)*x^3) + ((3*b*c - 4*a*d)*(b*c + 2*a
*d)*Sqrt[c + d*x^2])/(3*a^2*c^3*(b*c - a*d)*x) + (b^3*ArcTan[(Sqr
t[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^
(3/2))
```

Rubi in Sympy [A] time = 96.1181, size = 151, normalized size = 0.86

$$\frac{d}{cx^3\sqrt{c+dx^2}\,(ad-bc)}-\frac{\sqrt{c+dx^2}\,(4ad-bc)}{3ac^2x^3\,(ad-bc)}+\frac{\sqrt{c+dx^2}\,(2ad+bc)\,(4ad-3bc)}{3a^2c^3x\,(ad-bc)}-\frac{b^3\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{5}{2}}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d/(c*x**3*sqrt(c + d*x**2)*(a*d - b*c)) - sqrt(c + d*x**2)*(4*a*d
 - b*c)/(3*a*c**2*x**3*(a*d - b*c)) + sqrt(c + d*x**2)*(2*a*d + b
*c)*(4*a*d - 3*b*c)/(3*a**2*c**3*x*(a*d - b*c)) - b**3*atanh(x*sq
rt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(a**(5/2)*(a*d - b*c)**
(3/2))
```

Mathematica [A] time = 0.36402, size = 124, normalized size = 0.7

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\left(\frac{x^2(5ad+3bc)}{a^2}+\frac{3d^3x^4}{(c+dx^2)(ad-bc)}-\frac{c}{a}\right)}{3c^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(c/a) + ((3*b*c + 5*a*d)*x^2)/a^2 + (3*d^3*x^4
)/((-(b*c) + a*d)*(c + d*x^2))))/(3*c^3*x^3) + (b^3*ArcTan[(Sqrt[
b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3
/2))
```

Maple [B] time = 0.025, size = 762, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3/a/c/x^3/(d*x^2+c)^(1/2)+4/3/a*d/c^2/x/(d*x^2+c)^(1/2)+8/3/a*
d^2/c^3*x/(d*x^2+c)^(1/2)+b/a^2/c/x/(d*x^2+c)^(1/2)+2*b/a^2*d/c^2
*x/(d*x^2+c)^(1/2)-1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+1/2*b^2/a^2/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*b^3/
a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2*b^3/a^2/(-a*b)^(1/
2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b^2/a^2/(a*d-b*c)/c/((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x*d-1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4), x)
```

Fricas [A] time = 0.478106, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(abc^3 - a^2c^2d - \left(3\,b^2c^2d + 2\,abcd^2 - 8\,a^2d^3\right)x^4 - \left(3\,b^2c^3 + abc^2d - 4\,a^2cd^2\right)x^2\right)\sqrt{-abc + a^2d}\sqrt{dx^2 + c} + 3\left(b^3c^3dx^5 + \right.}{12\left((a^2bc^4d - a^3c^3d^2)x^5 + (a^2bc^5\right.}\right.$$

$$-\frac{2\left(abc^3 - a^2c^2d - \left(3\,b^2c^2d + 2\,abcd^2 - 8\,a^2d^3\right)x^4 - \left(3\,b^2c^3 + abc^2d - 4\,a^2cd^2\right)x^2\right)\sqrt{abc - a^2d}\sqrt{dx^2 + c} - 3\left(b^3c^3dx^5 + b^3c\right.}{6\left((a^2bc^4d - a^3c^3d^2)x^5 + (a^2bc^5 - a^3c^4d)x^3\right)\sqrt{abc - a^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(a*b*c^3 - a^2*c^2*d - (3*b^2*c^2*d + 2*a*b*c*d^2 - 8*a
^2*d^3)*x^4 - (3*b^2*c^3 + a*b*c^2*d - 4*a^2*c*d^2)*x^2)*sqrt(-a*
```

```
b*c + a^2*d)*sqrt(d*x^2 + c) + 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*lo
g((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^
2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*
b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c)
)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(((a^2*b*c^4*d - a^3*c^3*d^2)*x^5
 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*sqrt(-a*b*c + a^2*d)), -1/6*(2*(a
*b*c^3 - a^2*c^2*d - (3*b^2*c^2*d + 2*a*b*c*d^2 - 8*a^2*d^3)*x^4
- (3*b^2*c^3 + a*b*c^2*d - 4*a^2*c*d^2)*x^2)*sqrt(a*b*c - a^2*d)*
sqrt(d*x^2 + c) - 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*arctan(1/2*((b*
c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(
((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*s
qrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 1.89573, size = 373, normalized size = 2.12

$$-\frac{b^3\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bc-a^3d\right)\sqrt{abcd-a^2d^2}}-\frac{d^3x}{\left(bc^4-ac^3d\right)\sqrt{dx^2+c}}-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc^2\sqrt{d}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 acd^{\frac{3}{2}}+\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3 a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] -b^3*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a*b*c*d
 - a^2*d^2)) - d^3*x/((b*c^4 - a*c^3*d)*sqrt(d*x^2 + c)) - 2/3*(3
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqrt(d)*x - sqr
t(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c
^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b
*c^3*sqrt(d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2
 - c)^3*a^2*c^2)
```

3.722 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}(bc-ad)^2}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -(c*x)/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((b*c - 4*a*d)*x)/(3
*d*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.325697, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}(bc-ad)^2}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(c*x)/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((b*c - 4*a*d)*x)/(3
*d*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rubi in Sympy [A] time = 51.3509, size = 99, normalized size = 0.85

$$\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(ad-bc)^{\frac{5}{2}}}+\frac{cx}{3d\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}-\frac{x\left(4ad-bc\right)}{3d\sqrt{c+dx^2}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] a**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(a*d
 - b*c)**(5/2) + c*x/(3*d*(c + d*x**2)**(3/2)*(a*d - b*c)) - x*(4
*a*d - b*c)/(3*d*sqrt(c + d*x**2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.324499, size = 94, normalized size = 0.8

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{-3acx-4adx^3+bcx^3}{3\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a*c*x + b*c*x^3 - 4*a*d*x^3)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/
2)) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(b*c - a*d)^(5/2)
```

Maple [B] time = 0.029, size = 1207, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/b/d*x/(d*x^2+c)^(3/2)+1/3/b/c/d*x/(d*x^2+c)^(1/2)-1/3/b^2*a*
x/c/(d*x^2+c)^(3/2)-2/3/b^2*a/c^2*x/(d*x^2+c)^(1/2)-1/6/b*a^2/(-a
*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/b^2*a^2*d/(a*d-b*c)/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)*x+1/3/b^2*a^2*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)*x+1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/
b*a^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*a^2/(-a*b)^(1/2)/
(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))+1/6/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)+1/6/b^2*a^2*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1
/3/b^2*a^2*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*a^2/(-a*b)^(
1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/b*a^2/(a*d-b*c)^2/c/((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x*d+1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.571423, size = 1, normalized size = 0.01

$$\left[\frac{3\left(ad^2x^4+2\,acdx^2+ac^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{d}}{b^2x^4+2\,abx^2+a^2}\right.}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(-a/(b*c - a*d))*l
og(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^
2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (
```

```
a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) + 4*((b*c - 4*a*d)*x^3 - 3*a*c*x)*sqrt(d*
x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*x^2), 1/6*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(a/(b*c -
 a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c
 - a*d)*x*sqrt(a/(b*c - a*d)))) + 2*((b*c - 4*a*d)*x^3 - 3*a*c*x)
*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2
+ a^2*c*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.25077, size = 410, normalized size = 3.5

$$-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{(b^3c^4d-6\,ab^2c^3d^2+9\,a^2bc^2d^3-4\,a^3cd^4)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}-\frac{3\,(ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] -a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(a*b*c*d - a^2*d^2)) + 1/3*((b^3*c^4*d - 6*a*b^2*c^3*d^2 +
9*a^2*b*c^2*d^3 - 4*a^3*c*d^4)*x^2/(b^4*c^5*d - 4*a*b^3*c^4*d^2 +
 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5) - 3*(a*b^2*c^4*
d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)/(b^4*c^5*d - 4*a*b^3*c^4*d^2 +
 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2 + c)^
(3/2)
```

3.723 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a}{\sqrt{c+dx^2}(bc-ad)^2}-\frac{c}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] -c/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - a/((b*c - a*d)^2*Sqrt[c
+ d*x^2]) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
 - a*d]])/(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.263603, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{a}{\sqrt{c+dx^2}(bc-ad)^2}-\frac{c}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -c/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - a/((b*c - a*d)^2*Sqrt[c
+ d*x^2]) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
 - a*d]])/(b*c - a*d)^(5/2)
```

Rubi in Sympy [A] time = 31.6656, size = 85, normalized size = 0.83

$$-\frac{a\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{(ad-bc)^{\frac{5}{2}}}-\frac{a}{\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{c}{3d\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] -a*sqrt(b)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*d -
b*c)**(5/2) - a/(sqrt(c + d*x**2)*(a*d - b*c)**2) + c/(3*d*(c + d
*x**2)**(3/2)*(a*d - b*c))
```

Mathematica [A] time = 0.316848, size = 99, normalized size = 0.96

$$\frac{-ad\left(2c+3dx^2\right)-bc^2}{3d\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c^2) - a*d*(2*c + 3*d*x^2))/(3*d*(b*c - a*d)^2*(c + d*x^2)^(
3/2)) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/(b*c - a*d)^(5/2)
```

Maple [B] time = 0.02, size = 1123, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/b/d/(d*x^2+c)^(3/2)+1/6*a/b/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/
6*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*a/b^
2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*a/(a*d-b
*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/2*a/b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x*d+1/2*a/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/6*a/b/(a*
d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)+1/6*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*x+1/3*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x-1/2*a/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*a/b/(a
*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*a/(a*d-b*c)^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279152, size = 1, normalized size = 0.01

$$\left[\frac{3\left(ad^3x^4 + 2\,acd^2x^2 + ac^2d\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+}}{b^2x^4+2\,abx^2+a^2}\right.}{12\left(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^2d^3 - 2\,abcd^4 + a^2d^5)x^4 + 2\left(b^2c^3d^2 - 2\,abc^2d^3 + \right.\right.}\right.$$

$$\left.-\frac{3\left(ad^3x^4 + 2\,acd^2x^2 + ac^2d\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{bdx^2+2\,bc-ad}{2\,\sqrt{dx^2+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}\right) + 2\left(3\,ad^2x^2 + bc^2 + 2\,acd\right)\sqrt{dx^2+c}}{6\left(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^2d^3 - 2\,abcd^4 + a^2d^5)x^4 + 2\left(b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(b/(b*c - a*d)
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c
*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - 4*(3*a*d^2*x^2 + b*c^2 + 2*a*c*d)*sqrt(d*x^
2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 -
 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^
2*c*d^4)*x^2), -1/6*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt
(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2
+ c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) + 2*(3*a*d^2*x^2 + b*c^2
+ 2*a*c*d)*sqrt(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*
d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2
- 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.244476, size = 171, normalized size = 1.66

$$-\frac{\frac{3\,abd\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{bc^2+3\,(dx^2+c)\,ad-acd}{(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (b*c^2 + 3*(
d*x^2 + c)*a*d - a*c*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 +
 c)^(3/2)))/d
```

3.724 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}(bc-ad)^2}+\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\sqrt{a}b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*b*c + a*d)*x)/(3*c*(b*c
 - a*d)^2*Sqrt[c + d*x^2]) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.261409, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}(bc-ad)^2}+\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\sqrt{a}b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*b*c + a*d)*x)/(3*c*(b*c
 - a*d)^2*Sqrt[c + d*x^2]) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rubi in Sympy [A] time = 48.3616, size = 97, normalized size = 0.84

$$-\frac{\sqrt{a}b\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(ad-bc)^{\frac{5}{2}}}-\frac{x}{3(c+dx^2)^{\frac{3}{2}}(ad-bc)}+\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] -sqrt(a)*b*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(a
*d - b*c)**(5/2) - x/(3*(c + d*x**2)**(3/2)*(a*d - b*c)) + x*(a*d
 + 2*b*c)/(3*c*sqrt(c + d*x**2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.333755, size = 103, normalized size = 0.9

$$\frac{ad^2x^3+bcx\left(3c+2dx^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{a}b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (a*d^2*x^3 + b*c*x*(3*c + 2*d*x^2))/(3*c*(b*c - a*d)^2*(c + d*x^2
)^(3/2)) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(b*c - a*d)^(5/2)
```

Maple [B] time = 0.02, size = 1134, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3/b*x/c/(d*x^2+c)^(3/2)+2/3/b/c^2*x/(d*x^2+c)^(1/2)+1/6*a/(-a*b
)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6*a/b*d/(a*d-b*c)/c/((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)*x-1/3*a/b*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/
2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*a/(a*d-b*c
)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))-1/6*a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/
6*a/b*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*a/b*d/(a*d-b*c)/c^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x+1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)+1/2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*a/(-a*
b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.502157, size = 1, normalized size = 0.01

$$\left[\frac{3\,\left(bcd^2x^4 + 2\,bc^2dx^2 + bc^3\right)\sqrt{-\frac{a}{bc-ad}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\,\left(3\,abc^2-4\,a^2cd\right)x^2-4\,\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)}{b^2x^4+2\,abx^2+a^2}\right.}{12\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^4 + 2\,(b^2c^4d - 2\,abc^3d}\right.$$

$$\left.-\frac{3\,\left(bcd^2x^4 + 2\,bc^2dx^2 + bc^3\right)\sqrt{\frac{a}{bc-ad}}\,\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\,\sqrt{dx^2+c}(bc-ad)x\sqrt{\frac{a}{bc-ad}}}\right) - 2\,\left(3\,bc^2x + \left(2\,bcd + ad^2\right)x^3\right)\sqrt{dx^2+c}}{6\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^4 + 2\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3)*sqrt(-a/(b*c - a*d
))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3
 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*c^2*x + (2*b*c*d + a*d^2)*x^
3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c
^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^
3*d^2 + a^2*c^2*d^3)*x^2), -1/6*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 +
 b*c^3)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/
(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - 2*(3*b*c^2
*x + (2*b*c*d + a*d^2)*x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4
*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4
+ 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.239251, size = 393, normalized size = 3.42

$$\frac{ab\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{(2\,b^3c^3d^2-3\,ab^2c^2d^3+a^3d^5)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}+\frac{3\,(b^3c^4d-2\,ab^2c^3d^2+a^2bc^2d^3)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] a*b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c +
 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*sqrt(a*b*c*d - a^2*d^2)) + 1/3*((2*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3
 + a^3*d^5)*x^2/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3
- 4*a^3*b*c^2*d^4 + a^4*c*d^5) + 3*(b^3*c^4*d - 2*a*b^2*c^3*d^2 +
 a^2*b*c^2*d^3)/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3
- 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2 + c)^(3/2)
```

3.725 $\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}(bc-ad)^2}+\frac{1}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] 1/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + b/((b*c - a*d)^2*Sqrt[c + d
*x^2]) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*
d]])/(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.190559, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}(bc-ad)^2}+\frac{1}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] 1/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + b/((b*c - a*d)^2*Sqrt[c + d
*x^2]) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*
d]])/(b*c - a*d)^(5/2)
```

Rubi in Sympy [A] time = 28.7243, size = 82, normalized size = 0.84

$$\frac{b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{(ad-bc)^{\frac{5}{2}}}+\frac{b}{\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{1}{3\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] b**(3/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*d - b*
c)**(5/2) + b/(sqrt(c + d*x**2)*(a*d - b*c)**2) - 1/(3*(c + d*x**
2)**(3/2)*(a*d - b*c))
```

Mathematica [A] time = 0.243286, size = 91, normalized size = 0.93

$$\frac{-ad+4bc+3bdx^2}{3\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (4*b*c - a*d + 3*b*d*x^2)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (
b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c
- a*d)^(5/2)
```

Maple [B] time = 0.018, size = 1086, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/b*d*(-a*b)^(1/2)/(a*d-b*c)
/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)*x+1/3/b*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x+1/2*b/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/(a*d-b
*c)^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b/(a*d-b*c)^2/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))-1/6/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6/b*d*(-a*b)^(1
/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3/b*d*(-a*b)^(1/2)/(a*d-b
*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2*b/(a*d-b*c)^2/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+1/2/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b/(
a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271907, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bd^2x^4+2\,bcdx^2+bc^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\cdots}}{b^2x^4+2\,abx^2+a^2}\cdots}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2c\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(b/(b*c - a*d))*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d -
 a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 4*(3*b*d*x^2 + 4*b*c - a*d)*sqrt(d*x^2 + c))/(b
^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 +
```

```
 a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1
/6*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(-b/(b*c - a*d))*arct
an(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt
(-b/(b*c - a*d)))) + 2*(3*b*d*x^2 + 4*b*c - a*d)*sqrt(d*x^2 + c))
/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^
3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.232301, size = 159, normalized size = 1.62

$$\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{3\left(dx^2+c\right)b+bc-ad}{3\,(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(3*(d*x^2 + c)*b +
 b*c - a*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2))
```

3.726 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}}-\frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{dx}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(5*b*c - 2*a*d)*x
)/(3*c^2*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b^2*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.270984, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}}-\frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{dx}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(5*b*c - 2*a*d)*x
)/(3*c^2*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b^2*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 55.9503, size = 107, normalized size = 0.88

$$\frac{dx}{3c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{dx\,(2ad-5bc)}{3c^2\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{b^2\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d*x/(3*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*x*(2*a*d - 5*b*c)/(
3*c**2*sqrt(c + d*x**2)*(a*d - b*c)**2) + b**2*atanh(x*sqrt(a*d -
 b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.285055, size = 111, normalized size = 0.91

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}}+\frac{dx\left(ad\left(3c+2dx^2\right)-bc\left(6c+5dx^2\right)\right)}{3c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*x*(a*d*(3*c + 2*d*x^2) - b*c*(6*c + 5*d*x^2)))/(3*c^2*(b*c - a
*d)^2*(c + d*x^2)^(3/2)) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.019, size = 1086, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6*d/(a*d-b*c)
/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)*x+1/3*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x+1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b/(a*d
-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1
/b*(-a*b)^(1/2)))+1/6/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)+1/6*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*d/(a*d-b*c)/c^2/((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x-1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-1/2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/(-a*b)^(1/
2)*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.564618, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(\left(5\,bcd^2-2\,ad^3\right)x^3+3\left(2\,bc^2d-acd^2\right)x\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}-3\left(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4\right)\log\left(\frac{\left((b^2c^2-8\,abcd+8\cdots\right.}{\cdots}\right.}{12\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d\cdots}\right.$$

$$-\frac{2\left(\left(5\,bcd^2-2\,ad^3\right)x^3+3\left(2\,bc^2d-acd^2\right)x\right)\sqrt{abc-a^2d}\sqrt{dx^2+c}-3\left(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4\right)\arctan\left(\frac{(bc-2\,ad)x^2-\cdots}{2\,\sqrt{abc-a^2d}\sqrt{dx\cdots}}\right)}{6\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x^2)\sqrt{abc-a^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*((5*b*c*d^2 - 2*a*d^3)*x^3 + 3*(2*b*c^2*d - a*c*d^2)*x)
*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - 3*(b^2*c^2*d^2*x^4 + 2*b^
2*c^3*d*x^2 + b^2*c^4)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
```

```
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d)
 + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^
3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((b^2*c^
6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^
2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
*sqrt(-a*b*c + a^2*d)), -1/6*(2*((5*b*c*d^2 - 2*a*d^3)*x^3 + 3*(2
*b*c^2*d - a*c*d^2)*x)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) - 3*(b
^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*arctan(1/2*((b*c - 2*
a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((b^2*c
^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a
^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2
)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.242996, size = 433, normalized size = 3.55

$$-\frac{b^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}}-\frac{\left(\frac{(5\,b^3c^3d^3-12\,ab^2c^2d^4+9\,a^2bcd^5-2\,a^3d^6)x^2}{b^4c^6d-4\,ab^3c^5d^2+6\,a^2b^2c^4d^3-4\,a^3bc^3d^4+a^4c^2d^5}+\frac{3\,(2\,b^3c^4d^2-5\,ab^2c^3d^3+4\,a^2bc^2d^4-a^3cd^5)}{b^4c^6d-4\,ab^3c^5d^2+6\,a^2b^2c^4d^3-4\,a^3bc^3d^4+a^4c^2d^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] -b^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(a*b*c*d - a^2*d^2)) - 1/3*((5*b^3*c^3*d^3 - 12*a*b^2*c^2*d
^4 + 9*a^2*b*c*d^5 - 2*a^3*d^6)*x^2/(b^4*c^6*d - 4*a*b^3*c^5*d^2
+ 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 + a^4*c^2*d^5) + 3*(2*b^3*c
^4*d^2 - 5*a*b^2*c^3*d^3 + 4*a^2*b*c^2*d^4 - a^3*c*d^5)/(b^4*c^6*
d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 + a^4*c
^2*d^5))*x/(d*x^2 + c)^(3/2)
```

3.727 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

```
[Out] -d/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d))/(c^2*(
b*c - a*d)^2*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/
(a*c^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
 - a*d]])/(a*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.588484, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d))/(c^2*(
b*c - a*d)^2*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/
(a*c^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
 - a*d]])/(a*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 76.2553, size = 121, normalized size = 0.83

$$\frac{d}{3c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d\,(ad-2bc)}{c^2\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a\,(ad-bc)^{\frac{5}{2}}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d/(3*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(a*d - 2*b*c)/(c**2*s
qrt(c + d*x**2)*(a*d - b*c)**2) - b**(5/2)*atan(sqrt(b)*sqrt(c +
d*x**2)/sqrt(a*d - b*c))/(a*(a*d - b*c)**(5/2)) - atanh(sqrt(c +
d*x**2)/sqrt(c))/(a*c**(5/2))
```

Mathematica [C] time = 1.81088, size = 365, normalized size = 2.52

$$\frac{1}{6}\left(\frac{3b^{5/2}\log\left(-\frac{2a(bc-ad)\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^3x+i\sqrt{a}b^{5/2}}\right)}{a(bc-ad)^{5/2}}+\frac{3b^{5/2}\log\left(-\frac{2a(bc-ad)\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^3x-i\sqrt{a}b^{5/2}}\right)}{a(bc-ad)^{5/2}}+\frac{6d(ad-2bc)}{c^2\sqrt{c+dx^2}(bc-ad)^2}+\frac{2d}{c\left(c+dx^2\right)^{3/2}(ad-bc)}-\frac{6\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{ac^{5/2}}+\frac{6\log(x)}{ac^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((2*d)/(c*(-(b*c) + a*d)*(c + d*x^2)^(3/2)) + (6*d*(-2*b*c + a*d)
)/(c^2*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (6*Log[x])/(a*c^(5/2)) -
(6*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/(a*c^(5/2)) + (3*b^(5/2)*Log
[(-2*a*(b*c - a*d)*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqrt[
b*c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(I*Sqr
t[a]*b^(5/2) + b^3*x)])/(a*(b*c - a*d)^(5/2)) + (3*b^(5/2)*Log[(-
2*a*(b*c - a*d)*(Sqrt[b]*c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c
 - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/((-I)*Sqr
t[a]*b^(5/2) + b^3*x)])/(a*(b*c - a*d)^(5/2)))/6
```

Maple [B] time = 0.019, size = 1186, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3/a/c/(d*x^2+c)^(3/2)+1/a/c^2/(d*x^2+c)^(1/2)-1/a/c^(5/2)*ln((2
*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/6/a/(a*d-b*c)*b/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)-1/6/a*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3/
a*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/a*b^2/(a
*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x*d+1/2/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/6
/a/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/a*d*(-a*b)^(1/2)/(a*d-b*c)
/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)*x+1/3/a*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x-1/2/a*b^2/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a*
b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/a*b^2/(a
*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x), x)
```

Fricas [A] time = 1.21486, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(c)*sq
rt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d +
a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c -
a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(7*a*b*c^2*d - 4*a^2*c*d^
2 + 3*(2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 6*(b
^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*lo
g(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/((a*b^2*c^
6 - 2*a^2*b*c^5*d + a^3*c^4*d^2 + (a*b^2*c^4*d^2 - 2*a^2*b*c^3*d^
3 + a^3*c^2*d^4)*x^4 + 2*(a*b^2*c^5*d - 2*a^2*b*c^4*d^2 + a^3*c^3
*d^3)*x^2)*sqrt(c)), 1/12*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
 b^2*c^4)*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b
^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^
2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(7*a
*b*c^2*d - 4*a^2*c*d^2 + 3*(2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(d*x^
2 + c)*sqrt(-c) - 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*
c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((a*b^2*c^
6 - 2*a^2*b*c^5*d + a^3*c^4*d^2 + (a*b^2*c^4*d^2 - 2*a^2*b*c^3*d^
3 + a^3*c^2*d^4)*x^4 + 2*(a*b^2*c^5*d - 2*a^2*b*c^4*d^2 + a^3*c^3
*d^3)*x^2)*sqrt(-c)), -1/6*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2
+ b^2*c^4)*sqrt(c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*
b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) +
2*(7*a*b*c^2*d - 4*a^2*c*d^2 + 3*(2*a*b*c*d^2 - a^2*d^3)*x^2)*sqr
t(d*x^2 + c)*sqrt(c) - 3*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c
^2*d^2 + a^2*c*d^3)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x
^2 + c)*c)/x^2))/((a*b^2*c^6 - 2*a^2*b*c^5*d + a^3*c^4*d^2 + (a*b
^2*c^4*d^2 - 2*a^2*b*c^3*d^3 + a^3*c^2*d^4)*x^4 + 2*(a*b^2*c^5*d
- 2*a^2*b*c^4*d^2 + a^3*c^3*d^3)*x^2)*sqrt(c)), -1/6*(3*(b^2*c^2*
d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-c)*sqrt(-b/(b*c - a*d)
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d
)*sqrt(-b/(b*c - a*d)))) + 2*(7*a*b*c^2*d - 4*a^2*c*d^2 + 3*(2*a*
b*c*d^2 - a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 6*(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*arctan(sqrt(
-c)/sqrt(d*x^2 + c)))/((a*b^2*c^6 - 2*a^2*b*c^5*d + a^3*c^4*d^2 +
 (a*b^2*c^4*d^2 - 2*a^2*b*c^3*d^3 + a^3*c^2*d^4)*x^4 + 2*(a*b^2*c
^5*d - 2*a^2*b*c^4*d^2 + a^3*c^3*d^3)*x^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.236478, size = 242, normalized size = 1.67

$$-\frac{1}{3}\left(\frac{3\,b^3\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(ab^2c^2d-2\,a^2bcd^2+a^3d^3)\sqrt{-b^2c+abd}}+\frac{6\left(dx^2+c\right)bc+bc^2-3\left(dx^2+c\right)ad-acd}{(b^2c^4-2\,abc^3d+a^2c^2d^2)(dx^2+c)^{\frac{3}{2}}}-\frac{3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}c^2d}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b^
2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b^2*c + a*b*d)) + (6*(d*
x^2 + c)*b*c + b*c^2 - 3*(d*x^2 + c)*a*d - a*c*d)/((b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^(3/2)) - 3*arctan(sqrt(d*x^2
+ c)/sqrt(-c))/(a*sqrt(-c)*c^2*d))*d
```

3.728 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -d/(3*c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) - (d*(7*b*c - 4*a*d))/(3
*c^2*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) - ((b*c - 4*a*d)*(3*b*c - 2
*a*d)*Sqrt[c + d*x^2])/(3*a*c^3*(b*c - a*d)^2*x) - (b^3*ArcTan[(S
qrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d
)^(5/2))
```

Rubi [A] time = 0.686313, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) - (d*(7*b*c - 4*a*d))/(3
*c^2*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) - ((b*c - 4*a*d)*(3*b*c - 2
*a*d)*Sqrt[c + d*x^2])/(3*a*c^3*(b*c - a*d)^2*x) - (b^3*ArcTan[(S
qrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d
)^(5/2))
```

Rubi in Sympy [A] time = 99.8846, size = 153, normalized size = 0.86

$$\frac{d}{3cx\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d(4ad-7bc)}{3c^2x\sqrt{c+dx^2}(ad-bc)^2}-\frac{\sqrt{c+dx^2}(2ad-3bc)(4ad-bc)}{3ac^3x(ad-bc)^2}-\frac{b^3\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{3}{2}}(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d/(3*c*x*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(4*a*d - 7*b*c)/(3*
c**2*x*sqrt(c + d*x**2)*(a*d - b*c)**2) - sqrt(c + d*x**2)*(2*a*d
 - 3*b*c)*(4*a*d - b*c)/(3*a*c**3*x*(a*d - b*c)**2) - b**3*atanh(
x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(a**(3/2)*(a*d - b*
c)**(5/2))
```

Mathematica [A] time = 0.434817, size = 143, normalized size = 0.8

$$\frac{\sqrt{c+dx^2}\left(\frac{d^2x^2(8bc-5ad)}{(c+dx^2)(bc-ad)^2}+\frac{cd^2x^2}{(c+dx^2)^2(bc-ad)}-\frac{3}{a}\right)}{3c^3x}-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3/a + (c*d^2*x^2)/((b*c - a*d)*(c + d*x^2)^2)
+ (d^2*(8*b*c - 5*a*d)*x^2)/((b*c - a*d)^2*(c + d*x^2))))/(3*c^3*
x) - (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(a^(3/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.025, size = 1192, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/a/c/x/(d*x^2+c)^(3/2)-4/3/a*d/c^2*x/(d*x^2+c)^(3/2)-8/3/a*d/c^
3*x/(d*x^2+c)^(1/2)+1/6*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(3/2)-1/6*b/a*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/3*b/a*d/(a*
d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)
^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+1/2*b^2/a/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)*x*d+1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/6*b^2/
a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/6*b/a*d/(a*d-b*c)/
c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)*x-1/3*b/a*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x+1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b
^2/a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^3/a/(-a*b)^(1/2)
/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{5}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2), x)
```

Fricas [A] time = 0.67363, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(3b^2c^4 - 6abc^3d + 3a^2c^2d^2 + \left(3b^2c^2d^2 - 14abcd^3 + 8a^2d^4\right)x^4 + 3\left(2b^2c^3d - 7abc^2d^2 + 4a^2cd^3\right)x^2\right)\sqrt{-abc + a^2d}\sqrt{dx}}{12\left((ab^2c^5d^2 - 2a^2bc^4d^3 + a^3c^3d^4)x^5 + 2(ab^2c\right.}\right.$$

$$-\frac{2\left(3b^2c^4 - 6abc^3d + 3a^2c^2d^2 + \left(3b^2c^2d^2 - 14abcd^3 + 8a^2d^4\right)x^4 + 3\left(2b^2c^3d - 7abc^2d^2 + 4a^2cd^3\right)x^2\right)\sqrt{abc - a^2d}\sqrt{dx^2}}{6\left((ab^2c^5d^2 - 2a^2bc^4d^3 + a^3c^3d^4)x^5 + 2(ab^2c^6d - 2a^2bc^5d^2 + a^3c^4d^3)x^3 + (ab^2c^7 - 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(3*b^2*c^4 - 6*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d
^2 - 14*a*b*c*d^3 + 8*a^2*d^4)*x^4 + 3*(2*b^2*c^3*d - 7*a*b*c^2*d
^2 + 4*a^2*c*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - 3*(
b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*log((((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x
^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^
2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a
*b*x^2 + a^2)))/(((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)
*x^5 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^3 + (a*b
^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*x)*sqrt(-a*b*c + a^2*d)), -
1/6*(2*(3*b^2*c^4 - 6*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2
- 14*a*b*c*d^3 + 8*a^2*d^4)*x^4 + 3*(2*b^2*c^3*d - 7*a*b*c^2*d^2
+ 4*a^2*c*d^3)*x^2)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) + 3*(b^3*
c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*arctan(1/2*((b*c - 2*a
*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a*b^2
*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^5 + 2*(a*b^2*c^6*d -
2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^3 + (a*b^2*c^7 - 2*a^2*b*c^6*d +
 a^3*c^5*d^2)*x)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a + bx^2\right)\left(c + dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.729 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}(bc-ad)^2}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}}$$

```
[Out] -(d*(3*b*c - 5*a*d))/(6*a*c^2*(b*c - a*d)*(c + d*x^2)^(3/2)) - 1/
(2*a*c*x^2*(c + d*x^2)^(3/2)) - (d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d
^2))/(2*a*c^3*(b*c - a*d)^2*Sqrt[c + d*x^2]) + ((2*b*c + 5*a*d)*A
rcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c^(7/2)) - (b^(7/2)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*(b*c - a*d)^(
5/2))
```

Rubi [A] time = 0.948776, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}(bc-ad)^2}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*(3*b*c - 5*a*d))/(6*a*c^2*(b*c - a*d)*(c + d*x^2)^(3/2)) - 1/
(2*a*c*x^2*(c + d*x^2)^(3/2)) - (d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d
^2))/(2*a*c^3*(b*c - a*d)^2*Sqrt[c + d*x^2]) + ((2*b*c + 5*a*d)*A
rcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c^(7/2)) - (b^(7/2)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*(b*c - a*d)^(
5/2))
```

Rubi in Sympy [A] time = 119.089, size = 189, normalized size = 0.9

$$-\frac{1}{2acx^2\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{d\left(5ad-3bc\right)}{6ac^2\left(c+dx^2\right)^{\frac{3}{2}}\left(ad-bc\right)}-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}\left(ad-bc\right)^2}+\frac{b^{\frac{7}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2\left(ad-bc\right)^{\frac{5}{2}}}+\frac{\left(5ad+2bc\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] -1/(2*a*c*x**2*(c + d*x**2)**(3/2)) - d*(5*a*d - 3*b*c)/(6*a*c**2
*(c + d*x**2)**(3/2)*(a*d - b*c)) - d*(5*a**2*d**2 - 8*a*b*c*d +
b**2*c**2)/(2*a*c**3*sqrt(c + d*x**2)*(a*d - b*c)**2) + b**(7/2)*
atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a**2*(a*d - b*c)*
*(5/2)) + (5*a*d + 2*b*c)*atanh(sqrt(c + d*x**2)/sqrt(c))/(2*a**2
*c**(7/2))
```

Mathematica [C] time = 3.13942, size = 409, normalized size = 1.94

$$\frac{1}{2}\left(-\frac{b^{7/2}\log\left(\frac{2a^2(bc-ad)\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^4x+i\sqrt{a}b^{7/2}}\right)}{a^2(bc-ad)^{5/2}} - \frac{b^{7/2}\log\left(\frac{2a^2(bc-ad)\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^4x-i\sqrt{a}b^{7/2}}\right)}{a^2(bc-ad)^{5/2}} + \frac{(5ad+2bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^2c^{7/2}} - \frac{\log(x)(5ad+2bc)}{a^2c^{7/2}} + \frac{\sqrt{c+dx^2}\left(\frac{6d^2(3bc-2ad)}{(c+dx^2)(bc-ad)^2}+\frac{2cd^2}{(c+dx^2)^2(bc-ad)}-\frac{3}{ax^2}\right)}{3c^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-3/(a*x^2) + (2*c*d^2)/((b*c - a*d)*(c + d*x^2
)^2) + (6*d^2*(3*b*c - 2*a*d))/((b*c - a*d)^2*(c + d*x^2))))/(3*c
^3) - ((2*b*c + 5*a*d)*Log[x])/(a^2*c^(7/2)) + ((2*b*c + 5*a*d)*L
og[c + Sqrt[c]*Sqrt[c + d*x^2]])/(a^2*c^(7/2)) - (b^(7/2)*Log[(2*
a^2*(b*c - a*d)*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqrt[b*c
 - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(I*Sqrt[a
]*b^(7/2) + b^4*x)])/(a^2*(b*c - a*d)^(5/2)) - (b^(7/2)*Log[(2*a^
2*(b*c - a*d)*(Sqrt[b]*c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c -
 a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/((-I)*Sqrt[
a]*b^(7/2) + b^4*x)])/(a^2*(b*c - a*d)^(5/2)))/2
```

Maple [B] time = 0.022, size = 1289, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/2/a/c/x^2/(d*x^2+c)^(3/2)-5/6/a*d/c^2/(d*x^2+c)^(3/2)-5/2/a*d/
c^3/(d*x^2+c)^(1/2)+5/2/a*d/c^(7/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(
1/2))/x)-1/3*b/a^2/c/(d*x^2+c)^(3/2)-b/a^2/c^2/(d*x^2+c)^(1/2)+b/
a^2/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/6*b^2/a^2/(a*
d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)+1/6*b/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*x+1/3*b/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x+1/2*b^3/a^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*
b^2/a^2/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^
3/a^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/6*b^2/a^2/(a*d-b*c)/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)-1/6*b/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)*x-1/3*b/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

```
+1/2*b^3/a^2/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2*b^2/a^2/(a*d-b*c
)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2*b^3/a^2/(a*d-b*c)^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3), x)
```

Fricas [A] time = 2.31912, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*c^3*d^2*x^6 + 2*b^3*c^4*d*x^4 + b^3*c^5*x^2)*sqrt(c
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*
d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*
c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(3*a*b^2*c^4 - 6*a^2*
b*c^3*d + 3*a^3*c^2*d^2 + 3*(a*b^2*c^2*d^2 - 8*a^2*b*c*d^3 + 5*a^
3*d^4)*x^4 + 2*(3*a*b^2*c^3*d - 16*a^2*b*c^2*d^2 + 10*a^3*c*d^3)*
x^2)*sqrt(d*x^2 + c)*sqrt(c) + 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3
- 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2
 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^2*c^4*d
- 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*log(-((d*x^2 + 2*c)*sqrt(
c) + 2*sqrt(d*x^2 + c)*c)/x^2))/(((a^2*b^2*c^5*d^2 - 2*a^3*b*c^4*
d^3 + a^4*c^3*d^4)*x^6 + 2*(a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4
*c^4*d^3)*x^4 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2)*
sqrt(c)), 1/12*(3*(b^3*c^3*d^2*x^6 + 2*b^3*c^4*d*x^4 + b^3*c^5*x^
2)*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2
- 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*
sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(3*a*b^2*c^
4 - 6*a^2*b*c^3*d + 3*a^3*c^2*d^2 + 3*(a*b^2*c^2*d^2 - 8*a^2*b*c*
d^3 + 5*a^3*d^4)*x^4 + 2*(3*a*b^2*c^3*d - 16*a^2*b*c^2*d^2 + 10*a
^3*c*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 6*((2*b^3*c^3*d^2 + a*b
^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*
b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a
*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*arctan(sqrt(-c
)/sqrt(d*x^2 + c)))/(((a^2*b^2*c^5*d^2 - 2*a^3*b*c^4*d^3 + a^4*c^
3*d^4)*x^6 + 2*(a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*d^3)*x^
4 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2)*sqrt(-c)), 1
/12*(6*(b^3*c^3*d^2*x^6 + 2*b^3*c^4*d*x^4 + b^3*c^5*x^2)*sqrt(c)*
sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*
x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(3*a*b^2*c^4 - 6*
a^2*b*c^3*d + 3*a^3*c^2*d^2 + 3*(a*b^2*c^2*d^2 - 8*a^2*b*c*d^3 +
5*a^3*d^4)*x^4 + 2*(3*a*b^2*c^3*d - 16*a^2*b*c^2*d^2 + 10*a^3*c*d
^3)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 3*((2*b^3*c^3*d^2 + a*b^2*c^2*
d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*b^2*c^3
*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^2*c^
4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*log(-((d*x^2 + 2*c)*s
```

```
qrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/(((a^2*b^2*c^5*d^2 - 2*a^3*b*
c^4*d^3 + a^4*c^3*d^4)*x^6 + 2*(a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 +
 a^4*c^4*d^3)*x^4 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x
^2)*sqrt(c)), 1/6*(3*(b^3*c^3*d^2*x^6 + 2*b^3*c^4*d*x^4 + b^3*c^5
*x^2)*sqrt(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c
- a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - (3*a
*b^2*c^4 - 6*a^2*b*c^3*d + 3*a^3*c^2*d^2 + 3*(a*b^2*c^2*d^2 - 8*a
^2*b*c*d^3 + 5*a^3*d^4)*x^4 + 2*(3*a*b^2*c^3*d - 16*a^2*b*c^2*d^2
 + 10*a^3*c*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 3*((2*b^3*c^3*d^
2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4
*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*
c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)))/(((a^2*b^2*c^5*d^2 - 2*a^3*b*c^4*d^3 +
 a^4*c^3*d^4)*x^6 + 2*(a^2*b^2*c^6*d - 2*a^3*b*c^5*d^2 + a^4*c^4*
d^3)*x^4 + (a^2*b^2*c^7 - 2*a^3*b*c^6*d + a^4*c^5*d^2)*x^2)*sqrt(
-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.248098, size = 294, normalized size = 1.39

$$\frac{1}{6}\left(\frac{6\,b^4\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2b^2c^2d^2-2\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}+\frac{2\left(9\left(dx^2+c\right)bc+bc^2-6\left(dx^2+c\right)ad-acd\right)}{\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{3\,(2\,bc+5\,ad)\arctan\left(\cdots\right)}{a^2\sqrt{-c}c^3d^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^4*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
^2*c^2*d^2 - 2*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)) + 2*(
9*(d*x^2 + c)*b*c + b*c^2 - 6*(d*x^2 + c)*a*d - a*c*d)/((b^2*c^5
- 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^(3/2)) - 3*(2*b*c + 5*a*
d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^3*d^2) - 3*sq
rt(d*x^2 + c)/(a*c^3*d^2*x^2))*d^2
```

3.730 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=245

$$\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}+\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3(bc-ad)^2}-\frac{d(3bc-2ad)}{c^2x^3\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -d/(3*c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) - (d*(3*b*c - 2*a*d))/
(c^2*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) - ((b^2*c^2 - 12*a*b*c*d
+ 8*a^2*d^2)*Sqrt[c + d*x^2])/(3*a*c^3*(b*c - a*d)^2*x^3) + ((b*c
 - 2*a*d)*(3*b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*Sqrt[c + d*x^2])/(3
*a^2*c^4*(b*c - a*d)^2*x) + (b^4*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 1.08825, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}+\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3(bc-ad)^2}-\frac{d(3bc-2ad)}{c^2x^3\sqrt{c+dx^2}(bc-ad)^2}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) - (d*(3*b*c - 2*a*d))/
(c^2*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) - ((b^2*c^2 - 12*a*b*c*d
+ 8*a^2*d^2)*Sqrt[c + d*x^2])/(3*a*c^3*(b*c - a*d)^2*x^3) + ((b*c
 - 2*a*d)*(3*b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*Sqrt[c + d*x^2])/(3
*a^2*c^4*(b*c - a*d)^2*x) + (b^4*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 174.2, size = 224, normalized size = 0.91

$$\frac{d}{3cx^3\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d\,(2ad-3bc)}{c^2x^3\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3\,(ad-bc)^2}+\frac{\sqrt{c+dx^2}\,(2ad-bc)\left(8a^2d^2-8abcd-3b^2c^2\right)}{3a^2c^4x\,(ad-bc)^2}+\frac{b^4\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{\frac{5}{2}}\,(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d/(3*c*x**3*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(2*a*d - 3*b*c)/
(c**2*x**3*sqrt(c + d*x**2)*(a*d - b*c)**2) - sqrt(c + d*x**2)*(8
*a**2*d**2 - 12*a*b*c*d + b**2*c**2)/(3*a*c**3*x**3*(a*d - b*c)**
2) + sqrt(c + d*x**2)*(2*a*d - b*c)*(8*a**2*d**2 - 8*a*b*c*d - 3*
b**2*c**2)/(3*a**2*c**4*x*(a*d - b*c)**2) + b**4*atanh(x*sqrt(a*d
 - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(a**(5/2)*(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.706967, size = 160, normalized size = 0.65

$$\frac{b^4 \tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}+\frac{\sqrt{c+dx^2}\left(\frac{x^2(8ad+3bc)}{a^2}+\frac{d^3x^4(8ad-11bc)}{(c+dx^2)(bc-ad)^2}-\frac{cd^3x^4}{(c+dx^2)^2(bc-ad)}-\frac{c}{a}\right)}{3c^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(c/a) + ((3*b*c + 8*a*d)*x^2)/a^2 - (c*d^3*x^4
)/((b*c - a*d)*(c + d*x^2)^2) + (d^3*(-11*b*c + 8*a*d)*x^4)/((b*c
 - a*d)^2*(c + d*x^2))))/(3*c^4*x^3) + (b^4*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.027, size = 1285, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/a/c/x^3/(d*x^2+c)^(3/2)+2/a*d/c^2/x/(d*x^2+c)^(3/2)+8/3/a*d^
2/c^3*x/(d*x^2+c)^(3/2)+16/3/a*d^2/c^4*x/(d*x^2+c)^(1/2)+b/a^2/c/
x/(d*x^2+c)^(3/2)+4/3*b/a^2*d/c^2*x/(d*x^2+c)^(3/2)+8/3*b/a^2*d/c
^3*x/(d*x^2+c)^(1/2)-1/6*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)+1/6*b^2/a^2*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b^
2/a^2*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2*b^4/a^2/(-a*b)^(1
/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2*b^3/a^2/(a*d-b*c)^2/c/((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x*d-1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)
^(1/2)))+1/6*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1
/6*b^2/a^2*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b^2/a^2*d/(a*d
-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c
)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)-1/2*b^3/a^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x*d+1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4), x)
```

Fricas [A] time = 1.02971, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2 - (3*b^3*c^3*d
^2 + 2*a*b^2*c^2*d^3 - 24*a^2*b*c*d^4 + 16*a^3*d^5)*x^6 - 3*(2*b^
3*c^4*d + a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 8*a^3*c*d^4)*x^4 - 3
*(b^3*c^5 - 3*a^2*b*c^3*d^2 + 2*a^3*c^2*d^3)*x^2)*sqrt(-a*b*c + a
^2*d)*sqrt(d*x^2 + c) - 3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^
4*c^6*x^3)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2
*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sq
rt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(((a^2*b^2*c^6*d^2 -
 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^7 + 2*(a^2*b^2*c^7*d - 2*a^3*b*
c^6*d^2 + a^4*c^5*d^3)*x^5 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c
^6*d^2)*x^3)*sqrt(-a*b*c + a^2*d)), -1/6*(2*(a*b^2*c^5 - 2*a^2*b*
c^4*d + a^3*c^3*d^2 - (3*b^3*c^3*d^2 + 2*a*b^2*c^2*d^3 - 24*a^2*b
*c*d^4 + 16*a^3*d^5)*x^6 - 3*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 12*a^
2*b*c^2*d^3 + 8*a^3*c*d^4)*x^4 - 3*(b^3*c^5 - 3*a^2*b*c^3*d^2 + 2
*a^3*c^2*d^3)*x^2)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) - 3*(b^4*c
^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*arctan(1/2*((b*c - 2*
a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a^2*
b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^7 + 2*(a^2*b^2*c^7
*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^5 + (a^2*b^2*c^8 - 2*a^3*b*
c^7*d + a^4*c^6*d^2)*x^3)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.731 $\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{a}(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

```
[Out] (x*Sqrt[c + d*x^2])/b^2 - (x^3*Sqrt[c + d*x^2])/(2*b*(a + b*x^2))
 - (Sqrt[a]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*b^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3*Sqrt[d])
```

Rubi [A] time = 0.489636, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{\sqrt{a}(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/b^2 - (x^3*Sqrt[c + d*x^2])/(2*b*(a + b*x^2))
 - (Sqrt[a]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*b^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3*Sqrt[d])
```

Rubi in Sympy [A] time = 65.8972, size = 134, normalized size = 0.89

$$\frac{\sqrt{a}\,(4ad-3bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{ad-bc}}-\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{x\sqrt{c+dx^2}}{b^2}-\frac{(4ad-bc)\,\text{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] sqrt(a)*(4*a*d - 3*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c +
 d*x**2)))/(2*b**3*sqrt(a*d - b*c)) - x**3*sqrt(c + d*x**2)/(2*b*
(a + b*x**2)) + x*sqrt(c + d*x**2)/b**2 - (4*a*d - b*c)*atanh(sqr
t(d)*x/sqrt(c + d*x**2))/(2*b**3*sqrt(d))
```

Mathematica [A] time = 0.293502, size = 134, normalized size = 0.89

$$\frac{\frac{bx(2a+bx^2)\sqrt{c+dx^2}}{a+bx^2}+\frac{(bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+\frac{\sqrt{a}(4ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*(2*a + b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + (Sqrt[a]*(-3*b
*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/Sqrt[b*c - a*d] + ((b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x
```

```
^2]])/Sqrt[d])/(2*b^3)
```

Maple [B] time = 0.024, size = 2615, normalized size = 17.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b^2+1/2/b^2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))-1/4*a/b^2/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
3/2)+1/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*a
^2/b^3*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^4*d^2*(-a*b)^(1/2)/(a
*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))-1/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2)))*c+1/4*a/b^2*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*a/b^2
*d^(1/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4*a/b^2/(a*d-b*c)/(x+1/b*(-a*b)^(
1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)-1/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-1/4*a^2/b^3*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)
/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*a^2/b
^4*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4*a/b^3*d*(-a*b)^
(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4*a/b^2*d/(a*d-b*c)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x+1/4*a/b^2*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x
+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/4/b^2*a/(-
a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b^3*a*d^(1/2)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4/b^3
*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2)))*c+3/4/b^2*a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b^3*a*
d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))+3/4/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d-3/4/
b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
```

```
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+cx^4}}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.395816, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*sqrt
(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d +
2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(
b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*x^3 + 2*a*b*x)
*sqrt(d*x^2 + c)*sqrt(d) + 2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)
*x^2)*log(2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((b^4*x
^2 + a*b^3)*sqrt(d)), -1/8*((3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b
*d)*x^2)*sqrt(-d)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqr
t(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) -
 4*(b^2*x^3 + 2*a*b*x)*sqrt(d*x^2 + c)*sqrt(-d) - 4*(a*b*c - 4*a^
2*d + (b^2*c - 4*a*b*d)*x^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/
((b^4*x^2 + a*b^3)*sqrt(-d)), -1/4*((3*a*b*c - 4*a^2*d + (3*b^2*c
 - 4*a*b*d)*x^2)*sqrt(d)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2
*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d
)))) - 2*(b^2*x^3 + 2*a*b*x)*sqrt(d*x^2 + c)*sqrt(d) + (a*b*c - 4
*a^2*d + (b^2*c - 4*a*b*d)*x^2)*log(2*sqrt(d*x^2 + c)*d*x - (2*d*
x^2 + c)*sqrt(d)))/((b^4*x^2 + a*b^3)*sqrt(d)), -1/4*((3*a*b*c -
4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*sqrt(a/(b*c - a*d))*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*
x*sqrt(a/(b*c - a*d)))) - 2*(b^2*x^3 + 2*a*b*x)*sqrt(d*x^2 + c)*s
qrt(-d) - 2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)))/((b^4*x^2 + a*b^3)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.609319, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2,x, algorithm="giac")

[Out] sage0*x
```

3.732 $\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(2bc-3ad)}{2b^2(bc-ad)}+\frac{a\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*b^2*(b*c - a*d)) + (a*(c + d
*x^2)^(3/2))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - 3*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[
b*c - a*d])
```

Rubi [A] time = 0.296521, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(2bc-3ad)}{2b^2(bc-ad)}+\frac{a\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*b^2*(b*c - a*d)) + (a*(c + d
*x^2)^(3/2))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - 3*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[
b*c - a*d])
```

Rubi in Sympy [A] time = 35.1264, size = 110, normalized size = 0.81

$$-\frac{a\left(c+dx^2\right)^{\frac{3}{2}}}{2b\left(a+bx^2\right)(ad-bc)}+\frac{\sqrt{c+dx^2}\left(\frac{3ad}{2}-bc\right)}{b^2\,(ad-bc)}-\frac{\left(\frac{3ad}{2}-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{5}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] -a*(c + d*x**2)**(3/2)/(2*b*(a + b*x**2)*(a*d - b*c)) + sqrt(c +
d*x**2)*(3*a*d/2 - b*c)/(b**2*(a*d - b*c)) - (3*a*d/2 - b*c)*atan
(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(b**(5/2)*sqrt(a*d - b
*c))
```

Mathematica [A] time = 0.157149, size = 91, normalized size = 0.67

$$\frac{1}{2}\left(\frac{\left(\frac{a}{a+bx^2}+2\right)\sqrt{c+dx^2}}{b^2}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(2 + a/(a + b*x^2)))/b^2 - ((2*b*c - 3*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*
```

```
c - a*d]))/2
```

Maple [B] time = 0.021, size = 2543, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b^3*d^(1/2)*(-a*b)^(1/2)*ln((d*(-
a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
+1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/2/b^2*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)-1/2/b^3*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2
)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/b^3/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/b^2*(-a*b)^(1/2
)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^2*a*d/
(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^3*(-a*b)^(1/2)*d^(3/2)*a/(a*d
-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))+1/4/b^3*a^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b^2*
a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x-1/4/b^2*(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))*c-1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+1/4/b^2*a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b^3*(
-a*b)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^3*a^2*d^2/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))-1/4/b^2*a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/b^
2*(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/b^2*(-a*b)^(
1/2)*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274448, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{b^2c-abd}\left(2\,bx^2+3\,a\right)\sqrt{dx^2+c}-\left(2\,abc-3\,a^2d+\left(2\,b^2c-3\,abd\right)x^2\right)\log\left(\frac{\left(b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2\right)\cdots}{b^2\cdots}\right)}{8\left(b^3x^2+ab^2\right)\sqrt{b^2c-abd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b^2*c - a*b*d)*(2*b*x^2 + 3*a)*sqrt(d*x^2 + c) - (2*
a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*log(((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sq
rt(b^2*c - a*b*d) + 4*(2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3
*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^
2)))/((b^3*x^2 + a*b^2)*sqrt(b^2*c - a*b*d)), 1/4*(2*sqrt(-b^2*c
+ a*b*d)*(2*b*x^2 + 3*a)*sqrt(d*x^2 + c) + (2*a*b*c - 3*a^2*d + (
2*b^2*c - 3*a*b*d)*x^2)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(
-b^2*c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))))/((b^3*x^2 + a
*b^2)*sqrt(-b^2*c + a*b*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.230687, size = 150, normalized size = 1.1

$$\frac{\frac{\sqrt{dx^2+c}ad^2}{\left(\left(dx^2+c\right)b-bc+ad\right)b^2}+\frac{2\sqrt{dx^2+c}d}{b^2}+\frac{\left(2\,bcd-3\,ad^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x^2 + c)*a*d^2/(((d*x^2 + c)*b - b*c + a*d)*b^2) + 2*
sqrt(d*x^2 + c)*d/b^2 + (2*b*c*d - 3*a*d^2)*arctan(sqrt(d*x^2 + c
)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2))/d
```

3.733 $\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=120

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] -(x*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(S
qrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^2*Sqrt
[b*c - a*d]) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rubi [A] time = 0.256267, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -(x*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(S
qrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^2*Sqrt
[b*c - a*d]) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rubi in Sympy [A] time = 39.6842, size = 105, normalized size = 0.88

$$-\frac{x\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{\sqrt{d}\,\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}-\frac{(2ad-bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] -x*sqrt(c + d*x**2)/(2*b*(a + b*x**2)) + sqrt(d)*atanh(sqrt(d)*x/
sqrt(c + d*x**2))/b**2 - (2*a*d - b*c)*atanh(x*sqrt(a*d - b*c)/(s
qrt(a)*sqrt(c + d*x**2)))/(2*sqrt(a)*b**2*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.133627, size = 118, normalized size = 0.98

$$\frac{-\frac{bx\sqrt{c+dx^2}}{a+bx^2}+\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}+2\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2])/(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*Sqrt[b*c -
a*d]) + 2*Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.02, size = 2547, normalized size = 21.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/b^2*
d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(3/2)*a/(a
*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))-1/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))*a+1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4/b*d/(
a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b*d^(1/2)/(a*d-b*c)*ln((d*(-a*
b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
+1/4/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^2
*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(3/2)*a/(
a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))+1/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))*a-1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4/b*d
/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b*d^(1/2)/(a*d-b*c)*ln((-d*(
-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)*c+1/4/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*ln((d*
(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))+1/4/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*d-1/4/(-a*b)^(1/2)/b/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))*c-1/4/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^
(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d+1/4/(-a*
b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}x^2}{(bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.342458, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*b*x - 4*sqrt(-a*b*c
 + a^2*d)*(b*x^2 + a)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) + (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*log((((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d
+ 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)))/((b^3*x^2 + a*b^2)*sqrt(-a*b*c + a^2*d)
), -1/8*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*b*x - 8*sqrt(-a*b
*c + a^2*d)*(b*x^2 + a)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt
(-d))) + (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*log((((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*
d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^
3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 +
 2*a*b*x^2 + a^2)))/((b^3*x^2 + a*b^2)*sqrt(-a*b*c + a^2*d)), -1/
4*(2*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*b*x - 2*sqrt(a*b*c - a^2
*d)*(b*x^2 + a)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) - (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*((b
*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/
((b^3*x^2 + a*b^2)*sqrt(a*b*c - a^2*d)), -1/4*(2*sqrt(a*b*c - a^2
*d)*sqrt(d*x^2 + c)*b*x - 4*sqrt(a*b*c - a^2*d)*(b*x^2 + a)*sqrt(
-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - (a*b*c - 2*a^2*d + (
b^2*c - 2*a*b*d)*x^2)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(
a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((b^3*x^2 + a*b^2)*sqrt(a*b*c
 - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c + dx^2}}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.568549, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2,x, algorithm="giac")

[Out] sage0*x
```

3.734 $\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\,(a+bx^2)}$$

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.161995, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 20.6286, size = 65, normalized size = 0.81

$$-\frac{\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{d\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2b^{\frac{3}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(c + d*x**2)/(2*b*(a + b*x**2)) + d*atan(sqrt(b)*sqrt(c + d*
x**2)/sqrt(a*d - b*c))/(2*b**(3/2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.0854687, size = 80, normalized size = 1.

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.017, size = 1617, normalized size = 20.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(3/2)-1/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*(-a*b)^(1/2)/b^
2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))+1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4*(-a*b)^(1/2)/a/b*d/(a
*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b
*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))*c+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2)
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*(
-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-
a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*a/b^2*d^2/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))+1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4*(-a*b)
^(1/2)/a/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4*(-a*b)^(1/2)/a/b
*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/
d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248809, size = 1, normalized size = 0.01

$$\left[\frac{\left(bdx^2 + ad\right)\log\left(\frac{\left(b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2\right)\sqrt{b^2c-abd}-4\left(2\,b^3c^2-3\,ab^2cd+a^2bd^2+\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) - 4\sqrt{b^2}}{8\left(b^2x^2+ab\right)\sqrt{b^2c-abd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((b*d*x^2 + a*d)*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d) - 4
*(2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^2
```

```
)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(b^2*c -
a*b*d)*sqrt(d*x^2 + c))/((b^2*x^2 + a*b)*sqrt(b^2*c - a*b*d)), 1/
4*((b*d*x^2 + a*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*
c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))) - 2*sqrt(-b^2*c + a
*b*d)*sqrt(d*x^2 + c))/((b^2*x^2 + a*b)*sqrt(-b^2*c + a*b*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c + dx^2}}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.238177, size = 107, normalized size = 1.34

$$\frac{1}{2}\, d\left(\frac{\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c + abd}b} - \frac{\sqrt{dx^2 + c}}{((dx^2 + c)b - bc + ad)b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*(arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b) - sqrt(d*x^2 + c)/(((d*x^2 + c)*b - b*c + a*d)*b))
```

3.735 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0982831, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 18.5389, size = 68, normalized size = 0.83

$$\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}+\frac{c\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] x*sqrt(c + d*x**2)/(2*a*(a + b*x**2)) + c*atanh(x*sqrt(a*d - b*c)
/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(3/2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.149047, size = 82, normalized size = 1.

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.02, size = 2559, normalized size = 31.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a/b
*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b*d^(3/2)/(a*d-
b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+1/4/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-
1/4/a/b*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4/a*d/(a*d-b
*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x+1/4/a*d^(1/2)/(a*d-b*c)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4/
a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a/b*d*(-
a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b*d^(3/2)/(a*d-b*c)*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))-1/4/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/
a/b*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/a*d/(a*d-b*c)*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x+1/4/a*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)
/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4/(-a
*b)^(1/2)/a*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d^(1/2)/b*ln((d*(-a*b)^(1/2
)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*
b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*d-1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
-1/4/(-a*b)^(1/2)/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d^(1/2)/b*ln((-d*(-
a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/4/(-a*b)^(1/2)/a/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.304556, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{-abc+a^2d}\sqrt{dx^2+c}\,x+\left(bcx^2+ac\right)\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2\right)\sqrt{-abc+a^2d}+4\left(\left(ab^2c^2-3\,a^2bcd+2\,a^3d^2\right)x\right.}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(abx^2+a^2\right)\sqrt{-abc+a^2d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2 + a*c)*
log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^
2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 +
c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a*b*x^2 + a^2)*sqrt(-a*b*c +
a^2*d)), 1/4*(2*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2
+ a*c)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*
sqrt(d*x^2 + c)*x)))/((a*b*x^2 + a^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 2.17038, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.736 $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=119

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a(a+bx^2)}$$

```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x
^2]/Sqrt[c]])/a^2 + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
2])/Sqrt[b*c - a*d]])/(2*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.341224, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x
^2]/Sqrt[c]])/a^2 + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
2])/Sqrt[b*c - a*d]])/(2*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 39.6491, size = 99, normalized size = 0.83

$$\frac{\sqrt{c+dx^2}}{2a(a+bx^2)}-\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\left(\frac{ad}{2}-bc\right)\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2\sqrt{b}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x/(b*x**2+a)**2,x)
```

```
[Out] sqrt(c + d*x**2)/(2*a*(a + b*x**2)) - sqrt(c)*atanh(sqrt(c + d*x*
*2)/sqrt(c))/a**2 + (a*d/2 - b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/s
qrt(a*d - b*c))/(a**2*sqrt(b)*sqrt(a*d - b*c))
```

Mathematica [C] time = 1.02889, size = 313, normalized size = 2.63

$$\frac{\frac{(2bc-ad)\log\left(-\frac{4a^2\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)\sqrt{bc-ad}(2bc-ad)}\right)}{\sqrt{b}\sqrt{bc-ad}}+\frac{(2bc-ad)\log\left(-\frac{4a^2\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)\sqrt{bc-ad}(2bc-ad)}\right)}{\sqrt{b}\sqrt{bc-ad}}+\frac{2a\sqrt{c+dx^2}}{a+bx^2}-4\sqrt{c}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((2*a*Sqrt[c + d*x^2])/(a + b*x^2) + 4*Sqrt[c]*Log[x] - 4*Sqrt[c]
*Log[c + Sqrt[c]*Sqrt[c + d*x^2]] + ((2*b*c - a*d)*Log[(-4*a^2*Sq
rt[b]*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2
]))/(Sqrt[b*c - a*d]*(2*b*c - a*d)*(I*Sqrt[a] + Sqrt[b]*x))])/(Sq
rt[b]*Sqrt[b*c - a*d]) + ((2*b*c - a*d)*Log[(-4*a^2*Sqrt[b]*(Sqrt
[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(Sqrt[b
```

```
*c - a*d]*(2*b*c - a*d)*((-I)*Sqrt[a] + Sqrt[b]*x))])/(Sqrt[b]*Sq
rt[b*c - a*d]))/(4*a^2)
```

Maple [B] time = 0.023, size = 2585, normalized size = 21.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x)
```

```
[Out] -1/a^2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a^2*(d*x^2
+c)^(1/2)-1/2/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a^2*d^(1/2)*(-a*b)^(1/2
)/b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))-1/2/a/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/2/a^2/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-
1/2/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a^2*d^(1/2)*(-a*b)^(1/2)/b*ln((-d
*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))-1/2/a/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4/(-a*b
)^(1/2)/a/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/
4/a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/(-a*b)^(1/2)*d^(3/2)/(a*d
-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))+1/4*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/a*d/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))*c+1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*x+1/4/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*ln((d*(-a*b)^(
1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4
/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)+1/4/a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/(-a*b)^(1/2)*d^(3/2
)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))+1/4*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a*d
/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x-1/4/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*ln((-d*
(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
```

```
))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x), x)
```

Fricas [A] time = 0.360969, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b^2*c - a*b*d)*(b*x^2 + a)*sqrt(c)*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*sqrt(b^2*c - a*b*d)*sqrt(d
*x^2 + c)*a - (2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*log(((b^2
*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2)*sqrt(b^2*c - a*b*d) - 4*(2*b^3*c^2 - 3*a*b^2*c*d + a^2
*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^2*x^4 + 2
*a*b*x^2 + a^2)))/((a^2*b*x^2 + a^3)*sqrt(b^2*c - a*b*d)), -1/8*(
8*sqrt(b^2*c - a*b*d)*(b*x^2 + a)*sqrt(-c)*arctan(c/(sqrt(d*x^2 +
 c)*sqrt(-c))) - 4*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c)*a + (2*a*b
*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*log(((b^2*d^2*x^4 + 8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*
c - a*b*d) - 4*(2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d -
a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((
a^2*b*x^2 + a^3)*sqrt(b^2*c - a*b*d)), 1/4*(2*sqrt(-b^2*c + a*b*d
)*(b*x^2 + a)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2) + 2*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)*a - (2*a*b*c -
a^2*d + (2*b^2*c - a*b*d)*x^2)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d
)*sqrt(-b^2*c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))))/((a^2*
b*x^2 + a^3)*sqrt(-b^2*c + a*b*d)), -1/4*(4*sqrt(-b^2*c + a*b*d)*
(b*x^2 + a)*sqrt(-c)*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - 2*sqr
t(-b^2*c + a*b*d)*sqrt(d*x^2 + c)*a + (2*a*b*c - a^2*d + (2*b^2*c
 - a*b*d)*x^2)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c +
a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))))/((a^2*b*x^2 + a^3)*sqr
t(-b^2*c + a*b*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{x\,(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.249757, size = 170, normalized size = 1.43

$$\frac{1}{2}\,d^2\left(\frac{\sqrt{dx^2+c}}{((dx^2+c)b-bc+ad)ad}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(sqrt(d*x^2 + c)/(((d*x^2 + c)*b - b*c + a*d)*a*d) - (2*b
*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2*d^2) + 2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^
2*sqrt(-c)*d^2))
```

3.737 $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=113

$$-\frac{(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}$$

```
[Out] (-3*Sqrt[c + d*x^2])/(2*a^2*x) + Sqrt[c + d*x^2]/(2*a*x*(a + b*x^
2)) - ((3*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.335033, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-3*Sqrt[c + d*x^2])/(2*a^2*x) + Sqrt[c + d*x^2]/(2*a*x*(a + b*x^
2)) - ((3*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 42.7179, size = 95, normalized size = 0.84

$$\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{(2ad-3bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{5}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(c + d*x**2)/(2*a*x*(a + b*x**2)) - 3*sqrt(c + d*x**2)/(2*a**
2*x) + (2*a*d - 3*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c +
d*x**2)))/(2*a**(5/2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.126896, size = 101, normalized size = 0.89

$$\frac{(2ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\left(-\frac{bx}{2a^2\,(a+bx^2)}-\frac{1}{a^2x}\right)\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2))) + ((-3
*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.025, size = 2618, normalized size = 23.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] 3/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+1/4/a^2/(a*d-b*c)*b/(x+1/b*(
-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a*d^2*(-a*b)^(1/2)/(a*d-b*c)
/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))-1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))*c-1/4/a*d^2*(-a*b)^(1/2)/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/a^2*d
*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/a^2*d^(1/2)*ln(x*d^(1
/2)+(d*x^2+c)^(1/2))-3/4/a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4/a^2*d^(1/2)*ln(
(-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))-3/4*b/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b/a^2/(-a*b
)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^
(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/a^
2/c/x*(d*x^2+c)^(3/2)+1/4/a*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)
/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/a^2*d
/(a*d-b*c)*b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*
c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+1/a^2*d/c*x*(d*x^2+c)^(1/2)-3/4/a/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))*d+1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)-1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-1/4/a^2*d^(1/2)/(a*d-b*c)*b*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4*b/a^2/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))*c-1/4/a^2*d/(a*d-b*c)*b*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x-1/4/a^2*d^(1/2)/(a*d-b*c)*b*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^
(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/4*b/a^2/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A] time = 0.335551, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{-abc + a^2d}\left(3\,bx^2 + 2\,a\right)\sqrt{dx^2 + c} + \left(\left(3\,b^2c - 2\,abd\right)x^3 + \left(3\,abc - 2\,a^2d\right)x\right)\log\left(\frac{\left(\left(b^2c^2 - 8\,abcd + 8\,a^2d^2\right)x^4 + a^2c^2 - 2\,\left(3\,abc^2 - 4\right.\right.}{}\right.}{8\left(a^2bx^3 + a^3x\right)\sqrt{-abc + a^2d}}, -\frac{2\sqrt{abc - a^2d}\left(3\,bx^2 + 2\,a\right)\sqrt{dx^2 + c} + \left(\left(3\,b^2c - 2\,abd\right)x^3 + \left(3\,abc - 2\,a^2d\right)x\right)\arctan\left(\frac{(bc - 2\,ad)x^2 - ac}{2\sqrt{abc - a^2d}\sqrt{dx^2 + c}x}\right)}{4\left(a^2bx^3 + a^3x\right)\sqrt{abc - a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(-a*b*c + a^2*d)*(3*b*x^2 + 2*a)*sqrt(d*x^2 + c) + (
(3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*log((((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)
*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*
d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2
*a*b*x^2 + a^2)))/((a^2*b*x^3 + a^3*x)*sqrt(-a*b*c + a^2*d)), -1/
4*(2*sqrt(a*b*c - a^2*d)*(3*b*x^2 + 2*a)*sqrt(d*x^2 + c) + ((3*b^
2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*arctan(1/2*((b*c - 2*
a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((a^2*b
*x^3 + a^3*x)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{x^2 \left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 2.9383, size = 4, normalized size = 0.04

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.738 $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{b\sqrt{c+dx^2}}{a^2(a+bx^2)}-\frac{\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

```
[Out] -((b*Sqrt[c + d*x^2])/(a^2*(a + b*x^2))) - Sqrt[c + d*x^2]/(2*a*x
^2*(a + b*x^2)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/(2*a^3*Sqrt[c]) - (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.618938, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{b\sqrt{c+dx^2}}{a^2(a+bx^2)}-\frac{\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((b*Sqrt[c + d*x^2])/(a^2*(a + b*x^2))) - Sqrt[c + d*x^2]/(2*a*x
^2*(a + b*x^2)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/(2*a^3*Sqrt[c]) - (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 61.2622, size = 133, normalized size = 0.84

$$\frac{\sqrt{c+dx^2}}{2ax^2(a+bx^2)}-\frac{\sqrt{c+dx^2}}{a^2x^2}-\frac{\sqrt{b}(3ad-4bc)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^3\sqrt{ad-bc}}-\frac{(ad-4bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] sqrt(c + d*x**2)/(2*a*x**2*(a + b*x**2)) - sqrt(c + d*x**2)/(a**2
*x**2) - sqrt(b)*(3*a*d - 4*b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sq
rt(a*d - b*c))/(2*a**3*sqrt(a*d - b*c)) - (a*d - 4*b*c)*atanh(sqr
t(c + d*x**2)/sqrt(c))/(2*a**3*sqrt(c))
```

Mathematica [C] time = 1.98927, size = 343, normalized size = 2.16

$$-\frac{\frac{\sqrt{b}(4bc-3ad)\log\left(\frac{4a^3\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\sqrt{b}\left(\sqrt{b}x+i\sqrt{a}\right)(4bc-3ad)\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\sqrt{b}(4bc-3ad)\log\left(\frac{4ia^3\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\sqrt{b}\left(\sqrt{a}+i\sqrt{b}x\right)(4bc-3ad)\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{2a\left(a+2bx^2\right)\sqrt{c+dx^2}}{x^2(a+bx^2)}-\frac{2(4bc-ad)\log\left(\sqrt{c}\cdots\right.}{\sqrt{c}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((2*a*(a + 2*b*x^2)*Sqrt[c + d*x^2])/(x^2*(a + b*x^2)) + (2*(4*b
*c - a*d)*Log[x])/Sqrt[c] - (2*(4*b*c - a*d)*Log[c + Sqrt[c]*Sqrt
[c + d*x^2]])/Sqrt[c] + (Sqrt[b]*(4*b*c - 3*a*d)*Log[(4*a^3*(Sqrt
```

```
[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(Sqrt[b
]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*(I*Sqrt[a] + Sqrt[b]*x))])/Sqrt
[b*c - a*d] + (Sqrt[b]*(4*b*c - 3*a*d)*Log[((4*I)*a^3*(Sqrt[b]*c
+ I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(Sqrt[b]*(4*b
*c - 3*a*d)*Sqrt[b*c - a*d]*(Sqrt[a] + I*Sqrt[b]*x))])/Sqrt[b*c -
 a*d])/(4*a^3)
```

Maple [B] time = 0.025, size = 2669, normalized size = 16.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/4*b/a^2*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/a*d^2/(a*d-b*c)/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))+1/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4*b^2/a^2
/(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*b^2/a^2/(-a*b
)^(1/2)*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/
2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4*b/a^2*d/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*c-1/4*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x-1/4*b^2/a^2/(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/a^
3*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d+1/4*b^2/a
^2/(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)+1/4*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*b^2/a^2/(-a
*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*
b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/a^3*d^(1/2)*(-a*b)
^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))+1/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d+2*b/a^3*c^(1/2)*ln((2*c+
2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2/a^2/c/x^2*(d*x^2+c)^(3/2)-1/2/a
^2*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2/a^2*d/c*(d
*x^2+c)^(1/2)+b/a^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+b/a^3*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-2*b/a^3*(d*x^2+c)^(1/2)-1/4/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-b/a^
3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
```

```
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))*c-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4*b/a^2*d/(a*d-b*c)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A] time = 0.441971, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(c
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*
d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*
c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b*x^2 + a^2)*sqr
t(d*x^2 + c)*sqrt(c) + 2*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*
d)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/
((a^3*b*x^4 + a^4*x^2)*sqrt(c)), -1/8*(((4*b^2*c - 3*a*b*d)*x^4 +
 (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*d
^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2
)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
+ a^2)) + 4*(2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(-c) - 4*((4*b^
2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*arctan(sqrt(-c)/sqrt(d*
x^2 + c)))/((a^3*b*x^4 + a^4*x^2)*sqrt(-c)), 1/4*(((4*b^2*c - 3*a
*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(c)*sqrt(-b/(b*c - a*d))
*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)
*sqrt(-b/(b*c - a*d)))) - 2*(2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqr
t(c) - ((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*log(-((d*x
^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/((a^3*b*x^4 + a^4*
x^2)*sqrt(c)), 1/4*(((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d
)*x^2)*sqrt(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c
 - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(
2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(-c) + 2*((4*b^2*c - a*b*d)*
x^4 + (4*a*b*c - a^2*d)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((
a^3*b*x^4 + a^4*x^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^3\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.243367, size = 258, normalized size = 1.62

$$-\frac{1}{2}\,d^3\left(\frac{2\,(dx^2+c)^{\frac{3}{2}}b-2\,\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad}{\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)a^2d^2}-\frac{\left(4\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^3d^3}+\frac{(4\,bc-ad)\arc}{a^3\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/2*d^3*((2*(d*x^2 + c)^(3/2)*b - 2*sqrt(d*x^2 + c)*b*c + sqrt(d
*x^2 + c)*a*d)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d
*x^2 + c)*a*d - a*c*d)*a^2*d^2) - (4*b^2*c - 3*a*b*d)*arctan(sqrt
(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3*d^3
) + (4*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*
d^3))
```

3.739 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=147

$$\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(15bc-2ad)}{6a^3cx}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

```
[Out] (-5*Sqrt[c + d*x^2])/(6*a^2*x^3) + ((15*b*c - 2*a*d)*Sqrt[c + d*x
^2])/(6*a^3*c*x) + Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + (b*(5*
b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])
])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.59739, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}(15bc-2ad)}{6a^3cx}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^2])/(6*a^2*x^3) + ((15*b*c - 2*a*d)*Sqrt[c + d*x
^2])/(6*a^3*c*x) + Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + (b*(5*
b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])
])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 77.589, size = 129, normalized size = 0.88

$$\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}-\frac{\sqrt{c+dx^2}\,(2ad-15bc)}{6a^3cx}-\frac{b\,(4ad-5bc)\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{7}{2}}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(c + d*x**2)/(2*a*x**3*(a + b*x**2)) - 5*sqrt(c + d*x**2)/(6*
a**2*x**3) - sqrt(c + d*x**2)*(2*a*d - 15*b*c)/(6*a**3*c*x) - b*(
4*a*d - 5*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2))
)/(2*a**(7/2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.24808, size = 120, normalized size = 0.82

$$\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\left(3bx^2\left(\frac{bx^2}{a+bx^2}+4\right)-\frac{2a(c+dx^2)}{c}\right)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*((-2*a*(c + d*x^2))/c + 3*b*x^2*(4 + (b*x^2)/(a
+ b*x^2))))/(6*a^3*x^3) + (b*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.026, size = 2667, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/4*b/a^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*b^2/a^3/(a*d-b*c)/(x+1/
b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*b/a^2*d^(3/2)/(a*d-b*c)*l
n((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))+5/4*b^2/a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4*b/a^3*d
^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))-5/4*b^2/a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4*
b/a^3*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))-1/3/a^2/c/x^3*(d*x^2+c)^(3/2)-2*b/a^3*d^(
1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*b^2/a^3*d/(a*d-b*c)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*x+1/4*b^2/a^3*d^(1/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/
b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4*b/a^
3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-1/4*b/a^3*d*(-a*b)^(
1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-2*b/a^3*d/c*x*(d*x^2+c)^(1/2)-1/
4/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4*b^2/a^3*d/(a
*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4*b^2/a^3/(a*d-b*c)/(x-1/b*(-a*b)
^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)+2*b/a^3/c/x*(d*x^2+c)^(3/2)+1/4*b^2/a
^3*d^(1/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d
)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+5/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(
1/2)))*d-5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+1/4*b/a^3*d*(
-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b/a^3*d*(-a*b)^(1/2
)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))*d+5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+1/4/a^2*d^2*(-
a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4), x)
```

Fricas [A] time = 0.385319, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(15\,b^2c-2\,abd\right)x^4-2\,a^2c+2\left(5\,abc-a^2d\right)x^2\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}-3\left(\left(5\,b^3c^2-4\,ab^2cd\right)x^5+\left(5\,ab^2c^2-4\,a^2bcd\right)x\right.}{24\left(a^3bcx^5+a^4cx^3\right)\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/24*(4*((15*b^2*c - 2*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - a^2*d
)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - 3*((5*b^3*c^2 - 4*a
*b^2*c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*log((((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)
*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*
d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2
*a*b*x^2 + a^2)))/((a^3*b*c*x^5 + a^4*c*x^3)*sqrt(-a*b*c + a^2*d)
), 1/12*(2*((15*b^2*c - 2*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - a^2
*d)*x^2)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) + 3*((5*b^3*c^2 - 4*
a*b^2*c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*arctan(1/2*((b*
c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(
(a^3*b*c*x^5 + a^4*c*x^3)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 7.38974, size = 487, normalized size = 3.31

$$-\frac{\left(5\,b^{2}c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}a^{3}}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abd^{\frac{3}{2}}-b^{2}c^{2}\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)a^{3}}-\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}\sqrt{d}+6\,bc^{3}\sqrt{d}-ac^{2}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c*sqrt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - s
qrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c
*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^
2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b
*c^2)*a^3) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) -
 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sq
rt(d*x^2 + c))^2*b*c^2*sqrt(d) + 6*b*c^3*sqrt(d) - a*c^2*d^(3/2))
/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

3.740 $\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=197

$$\frac{3(8a^2d^2-8abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}-\frac{3\sqrt{a}(bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^4}+\frac{3x\sqrt{c+dx^2}(3bc-4ad)}{8b^3}-\frac{x^3(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}$$

```
[Out] (3*(3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x^3*Sqrt[c +
 d*x^2])/(4*b^2) - (x^3*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) - (3
*Sqrt[a]*(b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^4) + (3*(b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^4*Sqrt[d])
```

Rubi [A] time = 0.869854, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{3(8a^2d^2-8abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}-\frac{3\sqrt{a}(bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^4}+\frac{3x\sqrt{c+dx^2}(3bc-4ad)}{8b^3}-\frac{x^3(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*(3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x^3*Sqrt[c +
 d*x^2])/(4*b^2) - (x^3*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) - (3
*Sqrt[a]*(b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^4) + (3*(b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^4*Sqrt[d])
```

Rubi in Sympy [A] time = 104.804, size = 187, normalized size = 0.95

$$-\frac{3\sqrt{a}\sqrt{ad-bc}\,(2ad-bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^4}-\frac{x^3\left(c+dx^2\right)^{\frac{3}{2}}}{2b\left(a+bx^2\right)}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{3x\sqrt{c+dx^2}\,(4ad-3bc)}{8b^3}+\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] -3*sqrt(a)*sqrt(a*d - b*c)*(2*a*d - b*c)*atanh(x*sqrt(a*d - b*c)/
(sqrt(a)*sqrt(c + d*x**2)))/(2*b**4) - x**3*(c + d*x**2)**(3/2)/(
2*b*(a + b*x**2)) + 3*d*x**3*sqrt(c + d*x**2)/(4*b**2) - 3*x*sqrt
(c + d*x**2)*(4*a*d - 3*b*c)/(8*b**3) + 3*(8*a**2*d**2 - 8*a*b*c*
d + b**2*c**2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(8*b**4*sqrt(d))
```

Mathematica [A] time = 0.28295, size = 192, normalized size = 0.97

$$\frac{\frac{3(8a^2d^2-8abcd+b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-\frac{12\sqrt{a}(2a^2d^2-3abcd+b^2c^2)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}+\frac{b\sqrt{c+dx^2}\left(-12a^2dx+ab(9cx-6dx^3)+b^2x^3(5c+2dx^2)\right)}{a+bx^2}}{8b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*Sqrt[c + d*x^2]*(-12*a^2*d*x + b^2*x^3*(5*c + 2*d*x^2) + a*b*
(9*c*x - 6*d*x^3)))/(a + b*x^2) - (12*Sqrt[a]*(b^2*c^2 - 3*a*b*c*
d + 2*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/Sqrt[b*c - a*d] + (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Log[d
*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(8*b^4)
```

Maple [B] time = 0.026, size = 4795, normalized size = 24.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 3/2*a^2/b^4*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-1/4*a/b^
2/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4/b^2*a/(-
a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/8/b^2*c^2/d^(1/2)*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))+3/8/b^2*c*x*(d*x^2+c)^(1/2)+1/4/b^2*a/(-a*b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)+3/4/b^4*a^2*d^(3/2)*ln((-d*(-a*b)^(1/2)/b
+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b^2*a/(
-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b^4*a^2*d^(3/2)*ln((d*(-a*b)^
(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4*
a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/4*a^3/b^5*
d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/8*a/b^2*d/(a*d-b*c)*
c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^
2+1/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/8*a^2/
b^3*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8*a^2/b^3*d^(3/2)/(a*d-
b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c-3/4*a^2/b^4*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)+1/4*a/b^2*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8*a/b^2*d
^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)
/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))*c^2-1/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/
8*a^2/b^3*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8*a^2/b^3*d^(3/2)
/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))*c+3/4*a^2/b^4*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x
```

```
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+1/4*a/b^2*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8*
a/b^2*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4*a/b^3*d*(-a*b)^(1/2)/(a
*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))*c^2-3/2*a^2/b^4*d^2*(-a*b)^(1/2)/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))*c+3/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))*c^2+3/4/b^4*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2-3/4/b^4*a^
3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2-3/2/b^3*a^2/(-a*b)^(1/2)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*d*c+3/2/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+3/4*a
/b^3*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4*a^3/b^5*d
^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/8*a/b^2*d/(a*d-b*c)*c
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x-1/4*a/b^2/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(5/2)+3/4*a^3/b^4*d^(5/2)/(a*d-b*c)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/8/b^3
*a*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/b^3*a*d^(1/2)*ln((d*(-a*b)^(1/2)/b
+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/b^3*a
^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-3/4/b^2*a/(-a*b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*c-3/8/b^3*a*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/b^3*a*d^(
1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))*c-3/4/b^3*a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+3
/4*a^3/b^4*d^(5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^
(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2*x*(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}} x^4}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.553655, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c +
 a^2*d)*sqrt(d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a
^2)) - 2*(2*b^3*d*x^5 + (5*b^3*c - 6*a*b^2*d)*x^3 + 3*(3*a*b^2*c
- 4*a^2*b*d)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(a*b^2*c^2 - 8*a^2*b*
c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*log(
-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((b^5*x^2 + a*b^
4)*sqrt(d)), -1/8*(3*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sq
rt(-a*b*c + a^2*d)*sqrt(-d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2
*a*b*x^2 + a^2)) - (2*b^3*d*x^5 + (5*b^3*c - 6*a*b^2*d)*x^3 + 3*(
3*a*b^2*c - 4*a^2*b*d)*x)*sqrt(d*x^2 + c)*sqrt(-d) - 3*(a*b^2*c^2
 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2
)*x^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/((b^5*x^2 + a*b^4)*sqr
t(-d)), 1/16*(12*sqrt(a*b*c - a^2*d)*(a*b*c - 2*a^2*d + (b^2*c -
2*a*b*d)*x^2)*sqrt(d)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt
(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) + 2*(2*b^3*d*x^5 + (5*b^3*c -
 6*a*b^2*d)*x^3 + 3*(3*a*b^2*c - 4*a^2*b*d)*x)*sqrt(d*x^2 + c)*sq
rt(d) + 3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b
^2*c*d + 8*a^2*b*d^2)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2
+ c)*sqrt(d)))/((b^5*x^2 + a*b^4)*sqrt(d)), 1/8*(6*sqrt(a*b*c - a
^2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-d)*arctan(-
1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c
)*x)) + (2*b^3*d*x^5 + (5*b^3*c - 6*a*b^2*d)*x^3 + 3*(3*a*b^2*c -
 4*a^2*b*d)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 3*(a*b^2*c^2 - 8*a^2*b*
c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*arct
an(sqrt(-d)*x/sqrt(d*x^2 + c)))/((b^5*x^2 + a*b^4)*sqrt(-d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(c + dx^2\right)^{\frac{3}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.725661, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.741 $\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{\sqrt{c+dx^2}(2bc-5ad)}{2b^3}+\frac{(c+dx^2)^{3/2}(2bc-5ad)}{6b^2(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{2b(a+bx^2)(bc-ad)}$$

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^2])/(2*b^3) + ((2*b*c - 5*a*d)*(c +
 d*x^2)^(3/2))/(6*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(5/2))/(2*b*(
b*c - a*d)*(a + b*x^2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Rubi [A] time = 0.365929, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{\sqrt{c+dx^2}(2bc-5ad)}{2b^3}+\frac{(c+dx^2)^{3/2}(2bc-5ad)}{6b^2(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{2b(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^2])/(2*b^3) + ((2*b*c - 5*a*d)*(c +
 d*x^2)^(3/2))/(6*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(5/2))/(2*b*(
b*c - a*d)*(a + b*x^2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Rubi in Sympy [A] time = 42.8085, size = 136, normalized size = 0.83

$$-\frac{a(c+dx^2)^{\frac{5}{2}}}{2b(a+bx^2)(ad-bc)}+\frac{(c+dx^2)^{\frac{3}{2}}\left(\frac{5ad}{2}-bc\right)}{3b^2(ad-bc)}-\frac{\sqrt{c+dx^2}\left(\frac{5ad}{2}-bc\right)}{b^3}+\frac{\sqrt{ad-bc}\left(\frac{5ad}{2}-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] -a*(c + d*x**2)**(5/2)/(2*b*(a + b*x**2)*(a*d - b*c)) + (c + d*x*
*2)**(3/2)*(5*a*d/2 - b*c)/(3*b**2*(a*d - b*c)) - sqrt(c + d*x**2
)*(5*a*d/2 - b*c)/b**3 + sqrt(a*d - b*c)*(5*a*d/2 - b*c)*atan(sqr
t(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/b**(7/2)
```

Mathematica [A] time = 0.287113, size = 115, normalized size = 0.71

$$\frac{\sqrt{c+dx^2}\left(-\frac{3a(ad-bc)}{a+bx^2}-12ad+8bc+2bdx^2\right)}{6b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(8*b*c - 12*a*d + 2*b*d*x^2 - (3*a*(-(b*c) + a*d
))/(a + b*x^2))))/(6*b^3) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Maple [B] time = 0.024, size = 4673, normalized size = 28.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/4/b^2*a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/b^3/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a
*d*c+1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8/b
^2*(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4/b^2*a*d/(a*d-
b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*c+3/4/b^4*(-a*b)^(1/2)*d^(5/2)*a^2/(a*d-
b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))-3/4/b^4*a^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/4/b^2*
a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4/b^4*(-a*b)^(1/2)*d^(5/2)*
a^2/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2
)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))-3/4/b^4*a^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-
1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8/b^2*(-
a*b)^(1/2)*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/b^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
*c^2-1/2/b^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-
1/2/b^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d+1/6/b^2*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/6/b
^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)+1/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*d*c+3/2/b^3*a^2*d^
2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*c-3/4/b^3*a^2*d^2/(a*d-b*c)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)-1/4/b^3*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b^3*d
^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)
```

```
/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/2/b^4*d^(3/2)*(-a*b)^(1/2)*ln((
-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))*a-1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2*d^2+1/4/b^3*d*(-a*b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)*x+3/4/b^3*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/
2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/2/b
^4*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a-1/2/b^4/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d^
2-1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(5/2)-3/4/b^3*a^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*(-
a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4
/b^2*a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/2/b^2*((x+1/b*(-a*b)^(1/
2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)*c+1/2/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4/b^2*a*d/(a*d-b*c)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(
1/2)))*c^2-3/8/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-3/8/b^3*(-a*b)^(1/2)*d^2*a/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8
/b^3*(-a*b)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/2/b^3*a^2*d^2
/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c-3/4/b^2*a*d/(a*d-b*c)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)*c^2+3/8/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/
8/b^3*(-a*b)^(1/2)*d^2*a/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8/b^3*
(-a*b)^(1/2)*d^(3/2)*a/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279546, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(2\,abc-5\,a^{2}d+\left(2\,b^{2}c-5\,abd\right)x^{2}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)\cdots}{24\left(b^{4}x^{2}+ab^{3}\right)}\right.$$

$$-\frac{3\left(2\,abc-5\,a^{2}d+\left(2\,b^{2}c-5\,abd\right)x^{2}\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^{2}+2\,bc-ad}{2\,\sqrt{dx^{2}+c}b\sqrt{-\frac{bc-ad}{b}}}\right)-2\left(2\,b^{2}dx^{4}+11\,abc-15\,a^{2}d+2\left(4\,b^{2}c-5\,a\cdots\right.\right.}{12\left(b^{4}x^{2}+ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(2*a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt((b*c
 - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2
*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sq
rt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) -
 4*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d + 2*(4*b^2*c - 5*a*b*d)*x^2
)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/12*(3*(2*a*b*c - 5*a^2*d
 + (2*b^2*c - 5*a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*
x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*
(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d + 2*(4*b^2*c - 5*a*b*d)*x^2)*s
qrt(d*x^2 + c))/(b^4*x^2 + a*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\left(c+dx^{2}\right)^{\frac{3}{2}}}{\left(a+bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.226692, size = 234, normalized size = 1.44

$$\frac{\left(2\,b^{2}c^{2}-7\,abcd+5\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{2\,\sqrt{-b^{2}c+abd}b^{3}}+\frac{\sqrt{dx^{2}+c}abcd-\sqrt{dx^{2}+c}a^{2}d^{2}}{2\left((dx^{2}+c)b-bc+ad\right)b^{3}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{4}+3\,\sqrt{dx^{2}+c}b^{4}c-6\,\sqrt{dx^{2}+c}ab^{3}d}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/
sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/2*(sqrt(d*x^
2 + c)*a*b*c*d - sqrt(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c +
 a*d)*b^3) + 1/3*((d*x^2 + c)^(3/2)*b^4 + 3*sqrt(d*x^2 + c)*b^4*c
 - 6*sqrt(d*x^2 + c)*a*b^3*d)/b^6
```

3.742 $\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=149

$$\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3}+\frac{\sqrt{d}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

```
[Out] (d*x*Sqrt[c + d*x^2])/b^2 - (x*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2
)) + ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^3) + (Sqrt[d]*(3*b*c - 4*a
*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)
```

Rubi [A] time = 0.439977, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3}+\frac{\sqrt{d}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/b^2 - (x*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2
)) + ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^3) + (Sqrt[d]*(3*b*c - 4*a
*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)
```

Rubi in Sympy [A] time = 62.468, size = 134, normalized size = 0.9

$$-\frac{x(c+dx^2)^{\frac{3}{2}}}{2b(a+bx^2)}+\frac{dx\sqrt{c+dx^2}}{b^2}-\frac{\sqrt{d}(4ad-3bc)\,\text{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}+\frac{\sqrt{ad-bc}(4ad-bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] -x*(c + d*x**2)**(3/2)/(2*b*(a + b*x**2)) + d*x*sqrt(c + d*x**2)/
b**2 - sqrt(d)*(4*a*d - 3*b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/
(2*b**3) + sqrt(a*d - b*c)*(4*a*d - b*c)*atanh(x*sqrt(a*d - b*c)/
(sqrt(a)*sqrt(c + d*x**2)))/(2*sqrt(a)*b**3)
```

Mathematica [A] time = 0.268624, size = 155, normalized size = 1.04

$$\frac{\frac{(4a^2d^2-5abcd+b^2c^2)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}(b(c-dx^2)-2ad)}{a+bx^2}+\sqrt{d}(3bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2]*(-2*a*d + b*(c - d*x^2)))/(a + b*x^2)) +
((b^2*c^2 - 5*a*b*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sq
```

```
rt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*Sqrt[b*c - a*d]) + Sqrt[d]*(3*b
*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^3)
```

Maple [B] time = 0.023, size = 4685, normalized size = 31.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+3/4/b^3*d^2*(-a*b)^(1/2)/(
a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*a+3/8/b^2*d^2*a/(a*d-b*c)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*x-3/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
c-3/8/b*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/(-a*b)^(1/2)/b^3/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))*a^2*d^2+9/8/b^2*d^(3/2)*a/(a*d-b*c)*ln((-d*(-a*b)
^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3
/4/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a+3/4/b^2*d
*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/8/b*d/(a*d-b*c)*c
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x-1/4/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2*d
^2-3/2/b^3*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a*c+1/8/b^2
*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*x+3/8/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x
+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4/(-a*b)^(
1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4/b^3*d^(3/2)*ln((-d*(-a*b)^(1/2)/
b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a+1/4/b/(a
*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4/b/(a*d-b*c)/
(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/8/b^2*d*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)*x+3/8/b^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/b*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*c-1/4/b^3*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/
2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*a+3/8/b^2*d^2*a/(a*d-b*c)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*x+9/8/b^2*d^(3/2)*a/(a*d-b*c)*ln((d*(-a*b)^(1/2)
/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/2/b^3
*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a*c-3/4/b^4*d^3*(-a*b
)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a
```

```
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*a^2-3/4/b^2*d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))*c^2-3/8/b*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*
b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1
/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/4/b^3*d^(5/
2)*a^2/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*
x-1/4/(-a*b)^(1/2)/b^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d-1/4/(-a*b)^(1/2)/b
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))*c^2-3/4/b^3*d^(5/2)*a^2/(a*d-b*c)*ln((d*(-a*b)^(
1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/b
*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8/b*d^(1/2)/(a*d-b*c)*c^2*ln
((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))-1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/(
-a*b)^(1/2)/b^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*a*d-1/12/(-a*b)^(1/2)/b*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+1/12/(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/2/(-a*b)
^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2)))*a*d*c-1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2)))*a*d*c+3/4/b^4*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*a^2
+3/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}x^2}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.402141, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*lo
g(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c - 4*a^2*d
+ (b^2*c - 4*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 - (b^2*c
- 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), 1/8*(4*(3*a*b*c
 - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(d*x/(sqrt(d
*x^2 + c)*sqrt(-d))) - (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*
sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
 a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c -
2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 4*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2
+ c))/(b^4*x^2 + a*b^3), -1/4*((a*b*c - 4*a^2*d + (b^2*c - 4*a*b*
d)*x^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)
/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) + (3*a*b*c - 4*a^2*d
+ (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 +
c)*sqrt(d)*x - c) - 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^
2 + c))/(b^4*x^2 + a*b^3), 1/4*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c -
 4*a*b*d)*x^2)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) -
(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arc
tan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c
 - a*d)/a))) + 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c
))/(b^4*x^2 + a*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(c + dx^2\right)^{\frac{3}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.616383, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.743 $\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=99

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

```
[Out] (3*d*Sqrt[c + d*x^2])/(2*b^2) - (c + d*x^2)^(3/2)/(2*b*(a + b*x^2
)) - (3*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[
b*c - a*d]])/(2*b^(5/2))
```

Rubi [A] time = 0.202665, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x^2])/(2*b^2) - (c + d*x^2)^(3/2)/(2*b*(a + b*x^2
)) - (3*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[
b*c - a*d]])/(2*b^(5/2))
```

Rubi in Sympy [A] time = 25.619, size = 85, normalized size = 0.86

$$-\frac{(c+dx^2)^{\frac{3}{2}}}{2b(a+bx^2)}+\frac{3d\sqrt{c+dx^2}}{2b^2}-\frac{3d\sqrt{ad-bc}\,\text{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**(3/2)/(2*b*(a + b*x**2)) + 3*d*sqrt(c + d*x**2)/(2
*b**2) - 3*d*sqrt(a*d - b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a
*d - b*c))/(2*b**(5/2))
```

Mathematica [A] time = 0.21133, size = 96, normalized size = 0.97

$$\frac{1}{2}\sqrt{c+dx^2}\left(\frac{ad-bc}{b^2(a+bx^2)}+\frac{2d}{b^2}\right)-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*((2*d)/b^2 + (-(b*c) + a*d)/(b^2*(a + b*x^2))))/
2 - (3*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*b^(5/2))
```

Maple [B] time = 0.018, size = 2821, normalized size = 28.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 3/4*a/b^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d/(a*d-b*c)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*c-3/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4*(-
a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4*a^2/b^3*d^3/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))+3/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-3/8*(
-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/8*(-a*b)^(1
/2)/b^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8*(-a*b)^(1/2)/b^2*
d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^
(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4*(-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-
b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+3/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/8*(-a*b
)^(1/2)/a/b*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/b*d/(a*d-b*c)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b*
d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x
-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4*(-a*b)^(1/2)/a/b/(a*d-
b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/8*(-a*b)^(1/2)/b^
2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8*(-a*b)^(1/2)/b^2*d^(3/2
)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))*c+3/4*a/b^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8*(-a
*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/8*(-a*b)^(1/2)/a/
b*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)*x-3/2*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-
3/2*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278714, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bdx^2+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(2bdx^2-bc+3\right.}{8\left(b^3x^2+ab^2\right)}, -\frac{3\left(bdx^2+ad\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2bc-ad}{2\sqrt{dx^2+c}b\sqrt{-\frac{bc-ad}{b}}}\right)-2\left(2bdx^2-bc+3ad\right)\sqrt{dx^2+c}}{4\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d*x^2 + a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4
*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*b*d*x^2 - b*c + 3*a*d)*sqr
t(d*x^2 + c))/(b^3*x^2 + a*b^2), -1/4*(3*(b*d*x^2 + a*d)*sqrt(-(b
*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*
b*sqrt(-(b*c - a*d)/b))) - 2*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2
 + c))/(b^3*x^2 + a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.273867, size = 161, normalized size = 1.63

$$\frac{1}{2}d\left(\frac{3\left(bc-ad\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{2\sqrt{dx^2+c}}{b^2}-\frac{\sqrt{dx^2+c}bc-\sqrt{dx^2+c}ad}{((dx^2+c)b-bc+ad)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*(3*(b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d
))/(sqrt(-b^2*c + a*b*d)*b^2) + 2*sqrt(d*x^2 + c)/b^2 - (sqrt(d*x
^2 + c)*b*c - sqrt(d*x^2 + c)*a*d)/(((d*x^2 + c)*b - b*c + a*d)*b
^2))
```

3.744 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{bc-ad}(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab\,(a+bx^2)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + (Sqrt[b*c -
 a*d]*(b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(2*a^(3/2)*b^2) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/b^2
```

Rubi [A] time = 0.253693, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\sqrt{bc-ad}(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab\,(a+bx^2)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + (Sqrt[b*c -
 a*d]*(b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(2*a^(3/2)*b^2) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/b^2
```

Rubi in Sympy [A] time = 42.5119, size = 114, normalized size = 0.87

$$\frac{d^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}-\frac{x\sqrt{c+dx^2}\,(ad-bc)}{2ab\,(a+bx^2)}-\frac{\sqrt{ad-bc}\,(2ad+bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] d**(3/2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/b**2 - x*sqrt(c + d*x*
*2)*(a*d - b*c)/(2*a*b*(a + b*x**2)) - sqrt(a*d - b*c)*(2*a*d + b
*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(3/
2)*b**2)
```

Mathematica [A] time = 0.259246, size = 141, normalized size = 1.08

$$\frac{\frac{(-2a^2d^2+abcd+b^2c^2)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}+\frac{bx\sqrt{c+dx^2}(bc-ad)}{a(a+bx^2)}+2d^{3/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*(b*c - a*d)*x*Sqrt[c + d*x^2])/(a*(a + b*x^2)) + ((b^2*c^2 +
a*b*c*d - 2*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]) + 2*d^(3/2)*Log[d*x + Sqrt[d
```

```
]*Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] time = 0.021, size = 4689, normalized size = 35.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)^2,x)

[Out] -1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-1/4/(-a
*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/4/(-a*b)^(1/2)/a*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*c+1/4/(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-1/4/(-a*b)^(1/
2)/a*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*c-1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(5/2)-1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4/a*d
/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8/a*d^(1/2)/(a*d-b*c)*c^2*ln((
d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))+1/8/a*d/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/8/b*d^2/(a*d-b*c)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x-9/8/b*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b
*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/4/b^2*d^2*(-a*
b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*a/b^2*d^(5/2)/(a*d-b*c
)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))-3/8/b*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a/b*d*(-
a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/8/a*d/b*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)*x+1/4/a/b*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/8/
a*d/(a*d-b*c)*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/a/b*d*(-a*b)^(1/2)/(a*d
-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)+3/8/a*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-1/4/(-a*b)^(1/2)*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+1/2/(-a*b)^(1/2)/b/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))*d*c+1/4/(-a*b)^(1/2)*a/b^2/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2-1/
2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+1/12/(-a*b)^(1/2)/a*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(3/2)-1/4/b^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
```

```
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/12/(-a*b)^(1/2)/a*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(3/2)-1/4/b^2*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4/a/b*d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))*c^2-9/8/b*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(
1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4
/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*a/b^2*d^(
5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))+1/4/a*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+
3/8/a/b*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(
1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^
2+3/8/a*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)
^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/8/a/b*d^(1/2)*ln((d*(-a
*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*
c-3/4/a/b*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+3/4*a/b^
3*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/2/b^2*d^2*(-a*b)^(
1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+3/4/a/b*d*(-a*b)^(1/2)/(a*d-b*c)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*c-3/4*a/b^3*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))+3/2/b^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.373678, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x + 4*(a*b*d*x^2 + a^2*d)
*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c
 + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^
2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)
*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*x^2 +
 a^2*b^2), 1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x + 8*(a*b*d*x^
2 + a^2*d)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) + (a*b
*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 +
c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*x^2
 + a^2*b^2), 1/4*(2*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x - (a*b*c +
2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*
((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a
))) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 +
 c)*sqrt(d)*x - c))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2*c - a*b*d)
*sqrt(d*x^2 + c)*x + 4*(a*b*d*x^2 + a^2*d)*sqrt(-d)*arctan(d*x/(s
qrt(d*x^2 + c)*sqrt(-d))) - (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*
x^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(s
qrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^{\frac{3}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.673748, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.745 $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}(bc-ad)}{2ab\,(a+bx^2)}$$

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) - (c^(3/2)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2 + (Sqrt[b*c - a*d]*(2*b*c + a*d
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(3
/2))
```

Rubi [A] time = 0.410609, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) - (c^(3/2)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2 + (Sqrt[b*c - a*d]*(2*b*c + a*d
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(3
/2))
```

Rubi in Sympy [A] time = 45.2006, size = 109, normalized size = 0.84

$$-\frac{\sqrt{c+dx^2}\,(ad-bc)}{2ab\,(a+bx^2)}-\frac{c^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{ad-bc}\,(ad+2bc)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(c + d*x**2)*(a*d - b*c)/(2*a*b*(a + b*x**2)) - c**(3/2)*ata
nh(sqrt(c + d*x**2)/sqrt(c))/a**2 + sqrt(a*d - b*c)*(a*d + 2*b*c)
*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*a**2*b**(3/2))
```

Mathematica [C] time = 0.634531, size = 381, normalized size = 2.95

$$\frac{\frac{(-a^2d^2-abcd+2b^2c^2)\log\left(-\frac{4a^2b^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)\sqrt{bc-ad}\left(-a^2d^2-abcd+2b^2c^2\right)}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{(-a^2d^2-abcd+2b^2c^2)\log\left(-\frac{4ia^2b^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{a}+i\sqrt{b}x\right)\sqrt{bc-ad}\left(-a^2d^2-abcd+2b^2c^2\right)}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2a\sqrt{c+dx^2}(bc}{b(a+bx^2}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((2*a*(b*c - a*d)*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + 4*c^(3/2)*Lo
g[x] - 4*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]] + ((2*b^2*c^2 -
 a*b*c*d - a^2*d^2)*Log[(-4*a^2*b^(3/2)*(Sqrt[b]*c - I*Sqrt[a]*d*
x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(Sqrt[b*c - a*d]*(2*b^2*c^2
```

```
 - a*b*c*d - a^2*d^2)*(I*Sqrt[a] + Sqrt[b]*x))])/(b^(3/2)*Sqrt[b*
c - a*d]) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[((-4*I)*a^2*b^(3
/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2])
)/(Sqrt[b*c - a*d]*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(Sqrt[a] + I*S
qrt[b]*x))])/(b^(3/2)*Sqrt[b*c - a*d]))/(4*a^2)
```

Maple [B] time = 0.024, size = 4718, normalized size = 36.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x)
```

```
[Out] 3/8/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2/a^2/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))*c^2+1/2/a/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2/b^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))*d^2+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4/a*d/(a*d-b*c)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)-3/4*d^2/(a*d-b*c)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+9/8/(-a*b)^(1/2
)*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)
/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/a*d/(a*d-b*c)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*c+1/2/a/b^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b
*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/8/(-a*b)^(1/2)*d
^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/(-a*b)^(1/2)*d^(3/2)/(a*d-
b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c+3/4/a*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4/(-a*b
)^(1/2)/a/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/
4/a^2/b*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(
1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/a/b/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c
+3/8/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4/a*d
/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/4*d^2/(a*d-b*c)/b*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)+1/2/a/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^2-1
/6/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)-1/6/a^2*((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/3/a^2*(d
*x^2+c)^(3/2)+1/4/a^2*d*(-a*b)^(1/2)/b*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/
a^2/b*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/a/b/(-(a*d-b*c)/b)^(1/2)
```

```
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c-
1/4/a^2*d*(-a*b)^(1/2)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/(-a*b)^(1/2)*a
*d^(5/2)/b/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)
/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4*a*d^3/(a*d-b*c)/b^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))+3/2*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-3/4/a*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))*c^2+1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2
))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(5/2)-3/4/(-a*b)^(1/2)*a*d^(5/2)/b/(a*d-b*c)*ln(
(-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))-3/4*a*d^3/(a*d-b*c)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/2*d^2/(a*d-b*c)/
b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))*c-3/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/2/a^2
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*c-1/2/a^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/a^2*c^(3/2
)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/a^2*(d*x^2+c)^(1/2)*c-3
/8/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/a/b^2
*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*
((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)*x-3/8/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*c^2*
ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))+1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+
3/8/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*c^2*ln((d*(-a*b)^(1/2)/b+(
x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x), x)
```

Fricas [A] time = 0.545829, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [1/8*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/
b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*
c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2
 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*
c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) +
2*c)/x^2) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3
*b), -1/8*(8*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(c/(sqrt(d*x^2 +
c)*sqrt(-c))) - (2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b
*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*
sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 - 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*
((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*a
rctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c -
 a*d)/b))) + 2*(b^2*c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c - a^2*d)*sqrt(d*x^2 + c)
)/(a^2*b^2*x^2 + a^3*b), -1/4*(4*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arc
tan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (2*a*b*c + a^2*d + (2*b^2*c +
 a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a
*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*(a*b*c - a^2*d)
*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.221111, size = 223, normalized size = 1.73

$$\frac{1}{2}\,d^2\left(\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}+\frac{\sqrt{dx^2+c}bc-\sqrt{dx^2+c}ad}{((dx^2+c)b-bc+ad)abd}-\frac{\left(2\,b^2c^2-abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2bd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2
) + (sqrt(d*x^2 + c)*b*c - sqrt(d*x^2 + c)*a*d)/(((d*x^2 + c)*b -
 b*c + a*d)*a*b*d) - (2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(sqrt(
d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b*d^
2))
```

3.746 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\,(a+bx^2)}$$

```
[Out] -((3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*Sqrt[
c + d*x^2])/(2*a*b*x*(a + b*x^2)) - (3*c*Sqrt[b*c - a*d]*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2))
```

Rubi [A] time = 0.388203, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -((3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*Sqrt[
c + d*x^2])/(2*a*b*x*(a + b*x^2)) - (3*c*Sqrt[b*c - a*d]*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 59.32, size = 105, normalized size = 0.82

$$-\frac{\sqrt{c+dx^2}\,(ad-bc)}{2abx\,(a+bx^2)}+\frac{\sqrt{c+dx^2}\,(ad-3bc)}{2a^2bx}+\frac{3c\sqrt{ad-bc}\,\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(c + d*x**2)*(a*d - b*c)/(2*a*b*x*(a + b*x**2)) + sqrt(c + d
*x**2)*(a*d - 3*b*c)/(2*a**2*b*x) + 3*c*sqrt(a*d - b*c)*atanh(x*s
qrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(5/2))
```

Mathematica [A] time = 0.205031, size = 100, normalized size = 0.78

$$\frac{\sqrt{c+dx^2}\left(\frac{x^2(ad-bc)}{a+bx^2}-2c\right)}{2a^2x}-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*c + ((-(b*c) + a*d)*x^2)/(a + b*x^2)))/(2*a^
2*x) - (3*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*a^(5/2))
```

Maple [B] time = 0.026, size = 4764, normalized size = 37.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] 1/4/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+1/4/
a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*b/a^2/(-a*b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)-3/8/a^2*d*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/a^
2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))*c-3/4/a/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+3/
4/b/a*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))-1/4*b/a^2/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)-3/8/a^2*d*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8/a^2*d^(1/2)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+3/4/
a/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d+3/4/b/a*d^(3/2)*ln((d*(-a*b)
^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+3/4
/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-3/8/a^2*d/(a*d-
b*c)*b*c*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*c^2-3/8/a^2*d/(a*d-b*c)*b*c*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+
3/2/a^2*d^(1/2)*c*ln(x*d^(1/2)+(d*x^2+c)^(1/2))-3/4*d^(5/2)/b/(a*
d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))-3/4*d^(5/2)/b/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/a^2/c/x*(d*x^
2+c)^(5/2)+3/2/a^2*d*x*(d*x^2+c)^(1/2)-3/4/a*d^2*(-a*b)^(1/2)/(a*
d-b*c)/b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/2/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d
*c-3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/4/a^2*d/(a*d-b*c)
*b*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)*x-3/8/a^2*d^(1/2)/(a*d-b*c)*b*c^2*ln((-d*(
-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)-3/2/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+3/4*b/a^2/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))*c^2-3/2/a*d^2*(-a*b)^(1/2)/(a*d-b*c)/b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
```

```
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))*c+3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4
*d^3*(-a*b)^(1/2)/(a*d-b*c)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/2/a*d^2*(-a*b)^
(1/2)/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+3/4/a*d^2*(-a*b)^(1/2)/(a*d-b
*c)/b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)*c+3/4*d^3*(-a*b)^(1/2)/(a*d-b*c)/b^2/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))
)-1/4/a^2*d/(a*d-b*c)*b*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8/a^2*d^(1/2)/(a*
d-b*c)*b*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)
+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))+3/8/a*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9
/8/a*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))
*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-3/4*b/a^2/(-a*b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*c+3/4/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+3/4*b/a^2/(
-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/4/b/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
)))*d^2+1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5
/2)-1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/8/a*d^
2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+9/8/a*d^(3/2)/(a*d-b*c)*ln((d*(
-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)*c+1/a^2*d/c*x*(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A] time = 0.280924, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bcx^3+acx\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(\left(3\,bc-\right.\right.}{8\left(a^2bx^3+a^3x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*x^3 + a*c*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^
2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((3*b*c - a*d)*x^2
+ 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*x^3 + a^3*x), 1/4*(3*(b*c*x^3 +
a*c*x)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/
(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) - 2*((3*b*c - a*d)*x^2
 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*x^3 + a^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 2.62483, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.747 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{\sqrt{c+dx^2}(2bc-ad)}{2a^2(a+bx^2)}-\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*(a + b*x^2)) - (c*Sqrt[c
+ d*x^2])/(2*a*x^2*(a + b*x^2)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a
*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sq
rt[b])
```

Rubi [A] time = 0.695402, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{\sqrt{c+dx^2}(2bc-ad)}{2a^2(a+bx^2)}-\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*(a + b*x^2)) - (c*Sqrt[c
+ d*x^2])/(2*a*x^2*(a + b*x^2)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a
*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sq
rt[b])
```

Rubi in Sympy [A] time = 83.3755, size = 153, normalized size = 0.9

$$-\frac{\sqrt{c+dx^2}\,(ad-bc)}{2abx^2\,(a+bx^2)}+\frac{\sqrt{c+dx^2}\,(ad-2bc)}{2a^2bx^2}-\frac{\sqrt{c}\,(3ad-4bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}+\frac{(ad-4bc)\sqrt{ad-bc}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^3\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(c + d*x**2)*(a*d - b*c)/(2*a*b*x**2*(a + b*x**2)) + sqrt(c
+ d*x**2)*(a*d - 2*b*c)/(2*a**2*b*x**2) - sqrt(c)*(3*a*d - 4*b*c)
*atanh(sqrt(c + d*x**2)/sqrt(c))/(2*a**3) + (a*d - 4*b*c)*sqrt(a*
d - b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*a**3*s
qrt(b))
```

Mathematica [C] time = 0.838654, size = 405, normalized size = 2.38

$$-\frac{\frac{(a^2d^2-5abcd+4b^2c^2)\log\left(\frac{4a^3\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)\sqrt{bc-ad}\left(a^2d^2-5abcd+4b^2c^2\right)}\right)}{\sqrt{b}\sqrt{bc-ad}}+\frac{(a^2d^2-5abcd+4b^2c^2)\log\left(\frac{4ia^3\sqrt{b}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{a}+i\sqrt{b}x\right)\sqrt{bc-ad}\left(a^2d^2-5abcd+4b^2c^2\right)}\right)}{\sqrt{b}\sqrt{bc-ad}}+\frac{2a\sqrt{c+dx^2}(a(c-}{x^2(a+}}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((2*a*Sqrt[c + d*x^2]*(2*b*c*x^2 + a*(c - d*x^2)))/(x^2*(a + b*x
^2)) + 2*Sqrt[c]*(4*b*c - 3*a*d)*Log[x] - 2*Sqrt[c]*(4*b*c - 3*a*
d)*Log[c + Sqrt[c]*Sqrt[c + d*x^2]] + ((4*b^2*c^2 - 5*a*b*c*d + a
^2*d^2)*Log[(4*a^3*Sqrt[b]*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c
- a*d]*Sqrt[c + d*x^2]))/(Sqrt[b*c - a*d]*(4*b^2*c^2 - 5*a*b*c*d
+ a^2*d^2)*(I*Sqrt[a] + Sqrt[b]*x))]])/(Sqrt[b]*Sqrt[b*c - a*d]) +
 ((4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*Log[((4*I)*a^3*Sqrt[b]*(Sqrt[
b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(Sqrt[b*
c - a*d]*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*(Sqrt[a] + I*Sqrt[b]*x
))])/(Sqrt[b]*Sqrt[b*c - a*d]))/(4*a^3)
```

Maple [B]   time = 0.026, size = 4820, normalized size = 28.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(5/2)-3/4*b/a^2*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-3/2/a*d^2
/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))*c+1/2/a^3*d*(-a*b)^(1/2)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x+3/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+1/4*b^2/a^2/(-
a*b)^(1/2)*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+3/8*b^2/a^2/(-a*b)^(
1/2)*d^(1/2)/(a*d-b*c)*c^2*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/8*b/a/(-a*b)^(1/2)*d^2/(a*
d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(
x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/4*b/a^2*d/(a*d-b*c)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)*c-3/2/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c+3/8*b^2/
a^2/(-a*b)^(1/2)*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/a*d^2/(a
*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/(-a*b)^(1/2)*d^(5/2)/(a*d-b*c)*ln
((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))-1/2/a^2/c/x^2*(d*x^2+c)^(5/2)+b/a^3*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c
+3/4/a*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/(-a*b)^(1/2)*d^(5/2)
/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))+b/a^3*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+1/2/a^2*d/c*(d*x^
2+c)^(3/2)-3/8*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*c*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x-3/2/a^2*d*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+2*
b/a^3*c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-2*b/a^3*(d*x^
2+c)^(1/2)*c+1/3*b/a^3*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
```

```
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/a^2*((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*d+3/2/a^2*d*(d*x^2+c)^(1/2)+1/3*b/a^3*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/a
^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*d-2/3*b/a^3*(d*x^2+c)^(3/2)+3/2/a^3*d^(1/2
)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/
2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))*c+3/8*b/a/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x+9/8*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln((d*(-
a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
*c+3/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-1/4*b^2/a^2/(-a*b)^(
1/2)*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-3/8*b^2/a^2/(-a*b)^(1/2)*d
^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)
/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))-9/8*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b
*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))*c-1/b/a^2*d^(3/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b
+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/b/a/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))*d^2+2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c-b/a^3/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^
2+1/b/a^2*d^(3/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/b/a/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*d^
2+2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x+1/b*(-a*b)^(1/2)))*d*c-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-1/4*b/a^2*d
/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b*d^3/(a*d-b*c)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))
)-1/4*b/a^2*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/b*d^3/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))-1/2/a^3*d*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/2/a
^3*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2)
)*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{3}{2}}}{\left(bx^2 + a\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A] time = 0.523474, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt((b*c
- a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*
(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqr
t(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(c)*log
(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*c + (2*
a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), 1/8*(
4*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)*ar
ctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - ((4*b^2*c - a*b*d)*x^4 + (4*
a*b*c - a^2*d)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*
c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^
2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a^2*c + (2*a*b*c - a^2*d)*x^2)*s
qrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c - a*b*d)*x
^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*
x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) + ((
4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(c)*log(-(d
*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c + (2*a*b*
c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), 1/4*(2*((
4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)*arctan
(c/(sqrt(d*x^2 + c)*sqrt(-c))) - ((4*b^2*c - a*b*d)*x^4 + (4*a*b*
c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c
- a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*(a^2*c + (2*
a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.224614, size = 300, normalized size = 1.76

$$-\frac{1}{2}\,d^{3}\left(\frac{2\,(dx^2+c)^{\frac{3}{2}}bc-2\,\sqrt{dx^2+c}bc^2-(dx^2+c)^{\frac{3}{2}}ad+2\,\sqrt{dx^2+c}acd}{\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)a^2d^2}-\frac{(4\,b^2c^2-5\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^3d^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/2*d^3*((2*(d*x^2 + c)^(3/2)*b*c - 2*sqrt(d*x^2 + c)*b*c^2 - (d
*x^2 + c)^(3/2)*a*d + 2*sqrt(d*x^2 + c)*a*c*d)/(((d*x^2 + c)^2*b
- 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)*a^2*d^2) -
 (4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3*d^3) + (4*b*c^2 - 3*a*
c*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*d^3))
```

3.748 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=166

$$\frac{(5bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}(15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3\,(a+bx^2)}$$

```
[Out] -((5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b*c - 11*
a*d)*Sqrt[c + d*x^2])/(6*a^3*x) + ((b*c - a*d)*Sqrt[c + d*x^2])/(
2*a*b*x^3*(a + b*x^2)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Rubi [A] time = 0.702527, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{(5bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}(15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -((5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b*c - 11*
a*d)*Sqrt[c + d*x^2])/(6*a^3*x) + ((b*c - a*d)*Sqrt[c + d*x^2])/(
2*a*b*x^3*(a + b*x^2)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Rubi in Sympy [A] time = 105.109, size = 144, normalized size = 0.87

$$-\frac{\sqrt{c+dx^2}\,(ad-bc)}{2abx^3\,(a+bx^2)}+\frac{\sqrt{c+dx^2}\,(3ad-5bc)}{6a^2bx^3}-\frac{\sqrt{c+dx^2}\,(11ad-15bc)}{6a^3x}+\frac{\sqrt{ad-bc}\,(2ad-5bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(c + d*x**2)*(a*d - b*c)/(2*a*b*x**3*(a + b*x**2)) + sqrt(c
+ d*x**2)*(3*a*d - 5*b*c)/(6*a**2*b*x**3) - sqrt(c + d*x**2)*(11*
a*d - 15*b*c)/(6*a**3*x) + sqrt(a*d - b*c)*(2*a*d - 5*b*c)*atanh(
x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(7/2))
```

Mathematica [A] time = 0.222728, size = 131, normalized size = 0.79

$$\frac{(5bc-2ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}\left(-2a^2\left(c+4dx^2\right)+abx^2\left(10c-11dx^2\right)+15b^2cx^4\right)}{6a^3x^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(10*c - 11*d*x^2) - 2*a^
2*(c + 4*d*x^2)))/(6*a^3*x^3*(a + b*x^2)) + ((5*b*c - 2*a*d)*Sqrt
```

```
[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(2*a^(7/2))
```

Maple [B] time = 0.029, size = 4908, normalized size = 29.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -3/8*b/a^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-9/8*b/a^2*d^(3/2)/
(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))*c+1/4*b^2/a^3*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2)
)^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*
x+3/8*b^2/a^3*d^(1/2)/(a*d-b*c)*c^2*ln((d*(-a*b)^(1/2)/b+(x-1/b*(
-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4*b^2/a^3*d/(a*d-b
*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)*x+3/8*b^2/a^3*d^(1/2)/(a*d-b*c)*c^2*ln((-
d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))+5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2-2*b/a^3*d/c*x*(
d*x^2+c)^(3/2)-5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2-5/4/a
^2*d^(3/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))-5/4/a^2*d^(3/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/a^2*d^(3/2)*ln(x*
d^(1/2)+(d*x^2+c)^(1/2))-2/3/a^2*d/c^2/x*(d*x^2+c)^(5/2)+2/3/a^2*
d^2/c^2*x*(d*x^2+c)^(3/2)+1/a^2*d^2/c*x*(d*x^2+c)^(1/2)-1/4*b^2/a
^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3/4/a^2*d^2
*(-a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/8*b/a^3*d*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x+15/8*b/a^3*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-5/4*b/a^2/(-a*b)^(1/2)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*d+5/4*b^2/a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*c-5/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d^2+2*b/a^3/c/x*(d*x^2+c)
^(5/2)-3*b/a^3*d*x*(d*x^2+c)^(1/2)-3*b/a^3*d^(1/2)*c*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))+5/8*b/a^3*d*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+15/8*b/a^3*d^
(1/2)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))*c+5/4*b/a^2/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*d-5/
4*b^2/a^3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c+5/4/a/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*
(-a*b)^(1/2)))*d^2-3/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
```

```
2)))*c^2+3/4/a*d^(5/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a
*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/3/a^2/c/x^3*(d*x^2+c
)^(5/2)+5/12*b^2/a^3/(-a*b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/12*b^2/a^
3/(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/4/a*d^(5/2)/(a*d-b*c)*ln((d*
(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))+3/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c^2+5/2*b/a
^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*d*c+3/2/a^2*d^2*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2)))*c+3/8*b^2/a^3*d/(a*d-b*c)*c*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)*x+3/8*b^2/a^3*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/2*b/a^2/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2)))*d*c+3/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*
c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*c-3/4/b/a*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2)))+3/4/b/a*d^3*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/2
/a^2*d^2*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-3/4*b/a^3*d*(-a
*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-1/4*b^2/a^3/(a*d-b*c)/(
x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3/4/a^2*d^2*(-a*b)^(1/2)/
(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/8*b/a^2*d^2/(a*d-b*c)*((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x-9/8*b/a^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1
/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c-1/4*b/a^3*d*(-
a*b)^(1/2)/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*b/a^3*d*(-a*b)^(1/2)
/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4), x)
```

Fricas [A] time = 0.360474, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(\left(5\,b^2c-2\,abd\right)x^5+\left(5\,abc-2\,a^2d\right)x^3\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)}{b^2x^4+2\,abx^2+a^2}\right.}{24\left(a^3bx^5+a^4x^3\right)}\right.$$

$$-\frac{3\left(\left(5\,b^2c-2\,abd\right)x^5+\left(5\,abc-2\,a^2d\right)x^3\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(-\frac{(bc-2\,ad)x^2-ac}{2\sqrt{dx^2+c}ax\sqrt{\frac{bc-ad}{a}}}\right)-2\left(\left(15\,b^2c-11\,abd\right)x^4-2\,a^2c+2\left(5\,ab\right.\right.}{12\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqr
t(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b
*x^2 + a^2)) - 4*((15*b^2*c - 11*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*
c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3), -1/12*(
3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt((b*c -
 a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a
*x*sqrt((b*c - a*d)/a))) - 2*((15*b^2*c - 11*a*b*d)*x^4 - 2*a^2*c
 + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x
^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 6.31468, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.749 $\int \frac{x^4 (c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=258

$$\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}+\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}-\frac{\sqrt{a}(3bc-8ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^5}+\frac{dx^3\sqrt{c+dx^2}(7bc-8ad)}{8b^3}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

```
[Out] ((19*b^2*c^2 - 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^
4) + (d*(7*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^3) + (2*d*x^3*(
c + d*x^2)^(3/2))/(3*b^2) - (x^3*(c + d*x^2)^(5/2))/(2*b*(a + b*x
^2)) - (Sqrt[a]*(3*b*c - 8*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^5) + ((5*b^3*c^3 - 6
0*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(16*b^5*Sqrt[d])
```

Rubi [A] time = 1.18009, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}+\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}-\frac{\sqrt{a}(3bc-8ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^5}+\frac{dx^3\sqrt{c+dx^2}(7bc-8ad)}{8b^3}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```



```
[Out] ((19*b^2*c^2 - 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^
4) + (d*(7*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^3) + (2*d*x^3*(
c + d*x^2)^(3/2))/(3*b^2) - (x^3*(c + d*x^2)^(5/2))/(2*b*(a + b*x
^2)) - (Sqrt[a]*(3*b*c - 8*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^5) + ((5*b^3*c^3 - 6
0*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(16*b^5*Sqrt[d])
```

Rubi in Sympy [A] time = 148.015, size = 246, normalized size = 0.95

$$\frac{\sqrt{a}\left(ad-bc\right)^{\frac{3}{2}}\left(8ad-3bc\right)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^5}-\frac{x^3\left(c+dx^2\right)^{\frac{5}{2}}}{2b\left(a+bx^2\right)}+\frac{2dx^3\left(c+dx^2\right)^{\frac{3}{2}}}{3b^2}-\frac{dx^3\sqrt{c+dx^2}\left(8ad-7bc\right)}{8b^3}+\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}-\frac{\left(64a^3d^3-120a^2bcd^2+60ab^2c^2d-5b^3c^3\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] sqrt(a)*(a*d - b*c)**(3/2)*(8*a*d - 3*b*c)*atanh(x*sqrt(a*d - b*c
)/(sqrt(a)*sqrt(c + d*x**2)))/(2*b**5) - x**3*(c + d*x**2)**(5/2)
/(2*b*(a + b*x**2)) + 2*d*x**3*(c + d*x**2)**(3/2)/(3*b**2) - d*x
**3*sqrt(c + d*x**2)*(8*a*d - 7*b*c)/(8*b**3) + x*sqrt(c + d*x**2
)*(32*a**2*d**2 - 52*a*b*c*d + 19*b**2*c**2)/(16*b**4) - (64*a**3
*d**3 - 120*a**2*b*c*d**2 + 60*a*b**2*c**2*d - 5*b**3*c**3)*atanh
(sqrt(d)*x/sqrt(c + d*x**2))/(16*b**5*sqrt(d))
```

Mathematica [A] time = 0.576895, size = 219, normalized size = 0.85

$$\frac{bx\sqrt{c+dx^2}\left(72a^2d^2+2bdx^2(13bc-12ad)+\frac{24a(bc-ad)^2}{a+bx^2}-108abcd+33b^2c^2+8b^2d^2x^4\right)+\frac{3\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right.}{\sqrt{d}}}{48b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(33*b^2*c^2 - 108*a*b*c*d + 72*a^2*d^2 + 2*b
*d*(13*b*c - 12*a*d)*x^2 + 8*b^2*d^2*x^4 + (24*a*(b*c - a*d)^2)/(
a + b*x^2)) + 24*Sqrt[a]*(b*c - a*d)^(3/2)*(-3*b*c + 8*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + (3*(5*b^3*c^3
- 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*Log[d*x + Sqrt[d
]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^5)
```

Maple [B] time = 0.041, size = 7611, normalized size = 29.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}x^4}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A] time = 3.03263, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(12*(3*a*b^2*c^2 - 11*a^2*b*c*d + 8*a^3*d^2 + (3*b^3*c^2 -
11*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d)*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^
2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(8*b^4*d^2
*x^7 + 2*(13*b^4*c*d - 8*a*b^3*d^2)*x^5 + (33*b^4*c^2 - 82*a*b^3*
c*d + 48*a^2*b^2*d^2)*x^3 + 3*(19*a*b^3*c^2 - 52*a^2*b^2*c*d + 32
*a^3*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(5*a*b^3*c^3 - 60*a^2*
b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^3*
c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*log(2*sqrt(d*x^2 +
 c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((b^6*x^2 + a*b^5)*sqrt(d)), 1/
48*(6*(3*a*b^2*c^2 - 11*a^2*b*c*d + 8*a^3*d^2 + (3*b^3*c^2 - 11*a
*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(-d)*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d
)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (8*b^4*d^2*x^7
+ 2*(13*b^4*c*d - 8*a*b^3*d^2)*x^5 + (33*b^4*c^2 - 82*a*b^3*c*d +
 48*a^2*b^2*d^2)*x^3 + 3*(19*a*b^3*c^2 - 52*a^2*b^2*c*d + 32*a^3*
b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 3*(5*a*b^3*c^3 - 60*a^2*b^2*
c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*
d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)))/((b^6*x^2 + a*b^5)*sqrt(-d)), 1/96*(24*(3*a*b^2*c^2
 - 11*a^2*b*c*d + 8*a^3*d^2 + (3*b^3*c^2 - 11*a*b^2*c*d + 8*a^2*b
*d^2)*x^2)*sqrt(a*b*c - a^2*d)*sqrt(d)*arctan(-1/2*((b*c - 2*a*d)
*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)) + 2*(8*b^4*d
^2*x^7 + 2*(13*b^4*c*d - 8*a*b^3*d^2)*x^5 + (33*b^4*c^2 - 82*a*b^
3*c*d + 48*a^2*b^2*d^2)*x^3 + 3*(19*a*b^3*c^2 - 52*a^2*b^2*c*d +
32*a^3*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(5*a*b^3*c^3 - 60*a^
2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^
3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*log(2*sqrt(d*x^2
 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((b^6*x^2 + a*b^5)*sqrt(d)),
1/48*(12*(3*a*b^2*c^2 - 11*a^2*b*c*d + 8*a^3*d^2 + (3*b^3*c^2 - 1
1*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(a*b*c - a^2*d)*sqrt(-d)*arct
an(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2
 + c)*x)) + (8*b^4*d^2*x^7 + 2*(13*b^4*c*d - 8*a*b^3*d^2)*x^5 + (
33*b^4*c^2 - 82*a*b^3*c*d + 48*a^2*b^2*d^2)*x^3 + 3*(19*a*b^3*c^2
 - 52*a^2*b^2*c*d + 32*a^3*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(-d) + 3
*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 +
 (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*
x^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/((b^6*x^2 + a*b^5)*sqrt(
-d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.581435, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.750 $\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=198

$$-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{9/2}}+\frac{\sqrt{c+dx^2}(2bc-7ad)(bc-ad)}{2b^4}+\frac{(c+dx^2)^{3/2}(2bc-7ad)}{6b^3}+\frac{(c+dx^2)^{5/2}(2bc-7ad)}{10b^2(bc-ad)}+\frac{a(c+dx^2)^{7/2}}{2b(a+bx^2)(bc-ad)}$$

```
[Out] ((2*b*c - 7*a*d)*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^4) + ((2*b*c -
 7*a*d)*(c + d*x^2)^(3/2))/(6*b^3) + ((2*b*c - 7*a*d)*(c + d*x^2)
^(5/2))/(10*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(7/2))/(2*b*(b*c -
a*d)*(a + b*x^2)) - ((2*b*c - 7*a*d)*(b*c - a*d)^(3/2)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(9/2))
```

Rubi [A] time = 0.463654, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{9/2}}+\frac{\sqrt{c+dx^2}(2bc-7ad)(bc-ad)}{2b^4}+\frac{(c+dx^2)^{3/2}(2bc-7ad)}{6b^3}+\frac{(c+dx^2)^{5/2}(2bc-7ad)}{10b^2(bc-ad)}+\frac{a(c+dx^2)^{7/2}}{2b(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 7*a*d)*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^4) + ((2*b*c -
 7*a*d)*(c + d*x^2)^(3/2))/(6*b^3) + ((2*b*c - 7*a*d)*(c + d*x^2)
^(5/2))/(10*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(7/2))/(2*b*(b*c -
a*d)*(a + b*x^2)) - ((2*b*c - 7*a*d)*(b*c - a*d)^(3/2)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(9/2))
```

Rubi in Sympy [A] time = 53.1518, size = 170, normalized size = 0.86

$$-\frac{a\left(c+dx^2\right)^{\frac{7}{2}}}{2b\left(a+bx^2\right)(ad-bc)}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}\left(\frac{7ad}{2}-bc\right)}{5b^2(ad-bc)}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(\frac{7ad}{2}-bc\right)}{3b^3}+\frac{\sqrt{c+dx^2}(ad-bc)(7ad-2bc)}{2b^4}-\frac{(ad-bc)^{\frac{3}{2}}\left(\frac{7ad}{2}-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] -a*(c + d*x**2)**(7/2)/(2*b*(a + b*x**2)*(a*d - b*c)) + (c + d*x*
*2)**(5/2)*(7*a*d/2 - b*c)/(5*b**2*(a*d - b*c)) - (c + d*x**2)**(
3/2)*(7*a*d/2 - b*c)/(3*b**3) + sqrt(c + d*x**2)*(a*d - b*c)*(7*a
*d - 2*b*c)/(2*b**4) - (a*d - b*c)**(3/2)*(7*a*d/2 - b*c)*atan(sq
rt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/b**(9/2)
```

Mathematica [C] time = 0.92877, size = 332, normalized size = 1.68

$$\frac{2\sqrt{b}\sqrt{c+dx^2}\left(90a^2d^2+2bdx^2(11bc-10ad)+\frac{15a(bc-ad)^2}{a+bx^2}-140abcd+46b^2c^2+6b^2d^2x^4\right)-15(2bc-7ad)(bc-ad)^{3/2}\log}{60b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (2*Sqrt[b]*Sqrt[c + d*x^2]*(46*b^2*c^2 - 140*a*b*c*d + 90*a^2*d^2
 + 2*b*d*(11*b*c - 10*a*d)*x^2 + 6*b^2*d^2*x^4 + (15*a*(b*c - a*d
)^2)/(a + b*x^2)) - 15*(2*b*c - 7*a*d)*(b*c - a*d)^(3/2)*Log[(4*b
^(9/2)*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^
2]))/((2*b*c - 7*a*d)*(b*c - a*d)^(5/2)*(I*Sqrt[a] + Sqrt[b]*x))]
 - 15*(2*b*c - 7*a*d)*(b*c - a*d)^(3/2)*Log[(4*b^(9/2)*(Sqrt[b]*c
 + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/((2*b*c - 7*
a*d)*(b*c - a*d)^(5/2)*((-I)*Sqrt[a] + Sqrt[b]*x))])/(60*b^(9/2))
```

Maple [B] time = 0.027, size = 7443, normalized size = 37.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)*x^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304577, size = 1, normalized size = 0.01

$$\left[\frac{15\left(2\,ab^2c^2 - 9\,a^2bcd + 7\,a^3d^2 + \left(2\,b^3c^2 - 9\,ab^2cd + 7\,a^2bd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\,(}{b^2x^4+2\,abx^2+a^2}\right.}{}\right.$$

$$-\frac{15\left(2\,ab^2c^2 - 9\,a^2bcd + 7\,a^3d^2 + \left(2\,b^3c^2 - 9\,ab^2cd + 7\,a^2bd^2\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\,\sqrt{dx^2+c}b\sqrt{-\frac{bc-ad}{b}}}\right) - 2\left(6\,b^3d^2x^6 + 61\,a\right.}{60\left(b^5x^2 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)*x^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/120*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d + 7*a^3*d^2 + (2*b^3*c^2 -
9*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c
- a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(6*b^3*d^2*x^6 + 61*a
*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3*c*d - 7*a*b^2*
d^2)*x^4 + 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)*sqrt
(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/60*(15*(2*a*b^2*c^2 - 9*a^2*b*
```

```
c*d + 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sq
rt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2
 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*(6*b^3*d^2*x^6 + 61*a*b^2*c^2
- 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3*c*d - 7*a*b^2*d^2)*x^4
+ 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)*sqrt(d*x^2 +
c))/(b^5*x^2 + a*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.22858, size = 356, normalized size = 1.8

$$\frac{\left(2\,b^{3}c^{3}-11\,ab^{2}c^{2}d+16\,a^{2}bcd^{2}-7\,a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{2\sqrt{-b^{2}c+abd}b^{4}}+\frac{\sqrt{dx^{2}+c}ab^{2}c^{2}d-2\sqrt{dx^{2}+c}a^{2}bcd^{2}+\sqrt{dx^{2}+c}a^{3}d^{3}}{2\left((dx^{2}+c)b-bc+ad\right)b^{4}}+\frac{3\left(dx^{2}+c\right)^{\frac{5}{2}}b^{8}+5\left(dx^{2}+c\right)^{\frac{3}{2}}b^{8}c+15\sqrt{dx^{2}+c}b^{8}c^{2}-10\left(dx^{2}+c\right)^{\frac{3}{2}}ab^{7}d-60\sqrt{dx^{2}+c}ab^{7}cd+45\sqrt{dx^{2}+c}a^{2}b^{6}d^{2}}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*c^3 - 11*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 7*a^3*d^3)*arc
tan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b^4) + 1/2*(sqrt(d*x^2 + c)*a*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a^2*
b*c*d^2 + sqrt(d*x^2 + c)*a^3*d^3)/(((d*x^2 + c)*b - b*c + a*d)*b
^4) + 1/15*(3*(d*x^2 + c)^(5/2)*b^8 + 5*(d*x^2 + c)^(3/2)*b^8*c +
 15*sqrt(d*x^2 + c)*b^8*c^2 - 10*(d*x^2 + c)^(3/2)*a*b^7*d - 60*s
qrt(d*x^2 + c)*a*b^7*c*d + 45*sqrt(d*x^2 + c)*a^2*b^6*d^2)/b^10
```

3.751 $\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^4}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{8b^3}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}$$

```
[Out] (d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x*(c + d*x
^2)^(3/2))/(4*b^2) - (x*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) + ((
b*c - 6*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a
]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^4) + (Sqrt[d]*(15*b^2*c^2 - 40*
a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^
4)
```

Rubi [A] time = 0.634916, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^4}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{8b^3}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x*(c + d*x
^2)^(3/2))/(4*b^2) - (x*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) + ((
b*c - 6*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a
]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^4) + (Sqrt[d]*(15*b^2*c^2 - 40*
a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^
4)
```

Rubi in Sympy [A] time = 94.1015, size = 182, normalized size = 0.93

$$-\frac{x\left(c+dx^2\right)^{\frac{5}{2}}}{2b\left(a+bx^2\right)}+\frac{3dx\left(c+dx^2\right)^{\frac{3}{2}}}{4b^2}-\frac{dx\sqrt{c+dx^2}\left(12ad-11bc\right)}{8b^3}+\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4}-\frac{\left(ad-bc\right)^{\frac{3}{2}}\left(6ad-bc\right)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] -x*(c + d*x**2)**(5/2)/(2*b*(a + b*x**2)) + 3*d*x*(c + d*x**2)**(
3/2)/(4*b**2) - d*x*sqrt(c + d*x**2)*(12*a*d - 11*b*c)/(8*b**3) +
 sqrt(d)*(24*a**2*d**2 - 40*a*b*c*d + 15*b**2*c**2)*atanh(sqrt(d)
*x/sqrt(c + d*x**2))/(8*b**4) - (a*d - b*c)**(3/2)*(6*a*d - b*c)*
atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*sqrt(a)*b*
*4)
```

Mathematica [A] time = 0.374174, size = 173, normalized size = 0.89

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bx\sqrt{c+dx^2}\left(-\frac{4(bc-ad)^2}{a+bx^2}+d(9bc-8ad)+2bd^2x^2\right)+\frac{4(bc-6ad)(bc-}{}}{8b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(d*(9*b*c - 8*a*d) + 2*b*d^2*x^2 - (4*(b*c -
 a*d)^2)/(a + b*x^2)) + (4*(b*c - 6*a*d)*(b*c - a*d)^(3/2)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d
]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c
 + d*x^2]])/(8*b^4)
```

Maple [B] time = 0.026, size = 7459, normalized size = 38.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}x^2}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A] time = 1.42925, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 -
40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) + 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2
 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(
d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2
*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 -
17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4
), 1/8*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 -
 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(d*x/(sqrt(d*x^
2 + c)*sqrt(-d))) + (a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c
```

```
^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c
)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (2*b^3*d^2
*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*
d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/16*(4
*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d +
6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*
x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) - (15*a*b^2
*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 2
4*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d
)*x - c) - 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (
4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5
*x^2 + a*b^4), 1/8*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (
15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(d*
x/(sqrt(d*x^2 + c)*sqrt(-d))) - 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^
3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*
d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*
sqrt((b*c - a*d)/a))) + (2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d
^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2
 + c))/(b^5*x^2 + a*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.557498, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.752 $\int \frac{x\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=126

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{5d\sqrt{c+dx^2}(bc-ad)}{2b^3}-\frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d\left(c+dx^2\right)^{3/2}}{6b^2}$$

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^3) + (5*d*(c + d*x^2)^(3/2
))/(6*b^2) - (c + d*x^2)^(5/2)/(2*b*(a + b*x^2)) - (5*d*(b*c - a*
d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b
^(7/2))
```

Rubi [A] time = 0.266577, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{5d\sqrt{c+dx^2}(bc-ad)}{2b^3}-\frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d\left(c+dx^2\right)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^3) + (5*d*(c + d*x^2)^(3/2
))/(6*b^2) - (c + d*x^2)^(5/2)/(2*b*(a + b*x^2)) - (5*d*(b*c - a*
d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b
^(7/2))
```

Rubi in Sympy [A] time = 33.1669, size = 110, normalized size = 0.87

$$-\frac{\left(c+dx^2\right)^{\frac{5}{2}}}{2b\left(a+bx^2\right)}+\frac{5d\left(c+dx^2\right)^{\frac{3}{2}}}{6b^2}-\frac{5d\sqrt{c+dx^2}\left(ad-bc\right)}{2b^3}+\frac{5d\left(ad-bc\right)^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**(5/2)/(2*b*(a + b*x**2)) + 5*d*(c + d*x**2)**(3/2)
/(6*b**2) - 5*d*sqrt(c + d*x**2)*(a*d - b*c)/(2*b**3) + 5*d*(a*d
- b*c)**(3/2)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*b
**(7/2))
```

Mathematica [C] time = 0.674033, size = 289, normalized size = 2.29

$$\frac{-15d(bc-ad)^{3/2}\log\left(\frac{4b^{7/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{5d\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{5/2}}\right)-15d(bc-ad)^{3/2}\log\left(\frac{4b^{7/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{5d\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{5/2}}\right)-\frac{2\sqrt{b}\sqrt{c+dx^2}}{}}{12b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((-2*Sqrt[b]*Sqrt[c + d*x^2]*(3*(b*c - a*d)^2 + 2*d*(-7*b*c + 6*a
*d)*(a + b*x^2) - 2*b*d^2*x^2*(a + b*x^2)))/(a + b*x^2) - 15*d*(b
*c - a*d)^(3/2)*Log[(4*b^(7/2)*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[
```

```
b*c - a*d]*Sqrt[c + d*x^2]))/(5*d*(b*c - a*d)^(5/2)*(I*Sqrt[a] +
Sqrt[b]*x))] - 15*d*(b*c - a*d)^(3/2)*Log[(4*b^(7/2)*(Sqrt[b]*c +
 I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))/(5*d*(b*c - a*
d)^(5/2)*((-I)*Sqrt[a] + Sqrt[b]*x))])/(12*b^(7/2))
```

Maple [B] time = 0.023, size = 4363, normalized size = 34.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] 5/12*a/b^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/12/b*d/(a*d-b*c)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*c+5/12*a/b^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
-5/12/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*c-5/4*a^2/b^3*d^3/(a*d-b*
c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)-5/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-
5/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c^2-5/4*a^2/b^3*d^3/(a*d-b*
c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+5/16*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)*x-5/16*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)*x-75/32*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln((d*(-a*b)^(1
/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+5/
2*a/b^2*d^2/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*c-5/4*(-a*b)^(1/2)*a^2/
b^4*d^(7/2)/(a*d-b*c)*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d
)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))-5/4*a^3/b^4*d^4/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))+5/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^3+1/4*(-a*b)^(1/2)/a/
b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(7/2)+5/16*(-a*b)^
(1/2)/b^2*d^2/(a*d-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+75/32*(-a*b)^(1/2)/
b^2*d^(3/2)/(a*d-b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*
d)/d^(1/2)+((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c^2+5/2*a/b^2*d^2/(a*d-b*c)*((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*c+5/4*(-a*b)^(1/2)*a^2/b^4*d^(7/2)/(a*d-b*c)*ln((-
d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))-5/4*a^3/b^4*d^4/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+5/4/b*d/(a*d-b*c)/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))*c^3-1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^
(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(7/2)-1/4/b*d/(a*d-b*c)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-
1/4/b*d/(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+35/32*(-a*b)^(1/2)/b^2*d^2/
(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
```

```
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/8*(-a*b)^(1/2)*a/b^3*d^3/(a*d
-b*c)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x-25/8*(-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-
b*c)*ln((-d*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))*c+15/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c-15/
4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c^2+1/4*(-a*b)^(1/2)/a/b*d/
(a*d-b*c)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(5/2)*x+15/32*(-a*b)^(1/2)/a/b*d^(1/2)/(a
*d-b*c)*c^3*ln((d*(-a*b)^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))-35/32*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*c*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x+5/8*(-a*b)^(1/2)*a/b^3*d^3/(a*d-b*c)*((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x+25/8*(-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln((d*(-a*b)
^(1/2)/b+(x-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))*c+1
5/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-15/4*a/b^2*d^2/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
+1/b*(-a*b)^(1/2)))*c^2-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(5/2)*x-15/32*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^3*ln((-d
*(-a*b)^(1/2)/b+(x+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))+15/32*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c^2*((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-
5/16*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-15/32*(
-a*b)^(1/2)/a/b*d/(a*d-b*c)*c^2*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286367, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-}{b}}}{b^2x^4+2\,abx^2+a^2}\right.}{24\left(b^4x^2+ab^3\right)}\right.$$

$$-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}\,b\sqrt{-\frac{bc-ad}{b}}}\right)-2\left(2\,b^2d^2x^4-3\,b^2c^2+20\,abcd-15\,a^2d^2+2\,(\right.}{12\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt((b*
c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*s
qrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 4*(2*b^2*d^2*x^4 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b
^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/1
2*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(-(b*c -
a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqr
t(-(b*c - a*d)/b))) - 2*(2*b^2*d^2*x^4 - 3*b^2*c^2 + 20*a*b*c*d -
 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^
4*x^2 + a*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c+dx^2\right)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.227573, size = 257, normalized size = 2.04

$$\frac{1}{6}d\left(\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3}-\frac{3\left(\sqrt{dx^2+c}b^2c^2-2\sqrt{dx^2+c}abcd+\sqrt{dx^2+c}a^2d^2\right)}{((dx^2+c)b-bc+ad)b^3}+\frac{2\left(\left(dx^2+c\right)^{\frac{3}{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)*x/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 3*(sqrt(d*x^
2 + c)*b^2*c^2 - 2*sqrt(d*x^2 + c)*a*b*c*d + sqrt(d*x^2 + c)*a^2*
d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^3) + 2*((d*x^2 + c)^(3/2)*b^4
 + 6*sqrt(d*x^2 + c)*b^4*c - 6*sqrt(d*x^2 + c)*a*b^3*d)/b^6)
```

3.753 $\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] -(d*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(
c + d*x^2)^(3/2))/(2*a*b*(a + b*x^2)) + ((b*c - a*d)^(3/2)*(b*c +
 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*a^(3/2)*b^3) + (d^(3/2)*(5*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(2*b^3)
```

Rubi [A] time = 0.527996, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -(d*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(
c + d*x^2)^(3/2))/(2*a*b*(a + b*x^2)) + ((b*c - a*d)^(3/2)*(b*c +
 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*a^(3/2)*b^3) + (d^(3/2)*(5*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(2*b^3)
```

Rubi in Sympy [A] time = 74.451, size = 155, normalized size = 0.89

$$-\frac{d^{\frac{3}{2}}(4ad-5bc)\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{2ab\left(a+bx^2\right)}+\frac{dx\sqrt{c+dx^2}\,(2ad-bc)}{2ab^2}+\frac{(ad-bc)^{\frac{3}{2}}(4ad+bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] -d**(3/2)*(4*a*d - 5*b*c)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*b*
*3) - x*(c + d*x**2)**(3/2)*(a*d - b*c)/(2*a*b*(a + b*x**2)) + d*
x*sqrt(c + d*x**2)*(2*a*d - b*c)/(2*a*b**2) + (a*d - b*c)**(3/2)*
(4*a*d + b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))
/(2*a**(3/2)*b**3)
```

Mathematica [A] time = 0.325655, size = 143, normalized size = 0.82

$$\frac{\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}+d^{3/2}(5bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bx\sqrt{c+dx^2}\left(\frac{(bc-ad)^2}{a\left(a+bx^2\right)}+d^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(d^2 + (b*c - a*d)^2/(a*(a + b*x^2))) + ((b*
c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/a^(3/2) + d^(3/2)*(5*b*c - 4*a*d)*Log[d*x + Sq
rt[d]*Sqrt[c + d*x^2]])/(2*b^3)
```

Maple [B] time = 0.025, size = 7451, normalized size = 42.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^{\frac{5}{2}}}{(bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2, x)
```

Fricas [A] time = 0.985829, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x
^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*
b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c
*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a
*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3)
, 1/8*(4*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x
^2)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - (a*b^2*c^2
+ 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)
*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^
4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d
 + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/4*(
(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4
*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x
^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) + (5*a^2*b*c
*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*
d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(a*b^2*d^2*x^3 + (b^
```

```
3*c^2 - 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2
 + a^2*b^3), 1/4*(2*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a
^2*b*d^2)*x^2)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) -
(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4
*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x
^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) + 2*(a*b^2*d
^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c)
)/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^{\frac{5}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.558972, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.754 $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=160

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] -(d*(b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*(c +
d*x^2)^(3/2))/(2*a*b*(a + b*x^2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x
^2]/Sqrt[c]])/a^2 + ((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(5/2))
```

Rubi [A] time = 0.627101, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] -(d*(b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*(c +
d*x^2)^(3/2))/(2*a*b*(a + b*x^2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x
^2]/Sqrt[c]])/a^2 + ((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(5/2))
```

Rubi in Sympy [A] time = 68.9213, size = 138, normalized size = 0.86

$$-\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{2ab\left(a+bx^2\right)}+\frac{d\sqrt{c+dx^2}\,(3ad-bc)}{2ab^2}-\frac{c^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{(ad-bc)^{\frac{3}{2}}(3ad+2bc)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**(3/2)*(a*d - b*c)/(2*a*b*(a + b*x**2)) + d*sqrt(c
+ d*x**2)*(3*a*d - b*c)/(2*a*b**2) - c**(5/2)*atanh(sqrt(c + d*x*
*2)/sqrt(c))/a**2 - (a*d - b*c)**(3/2)*(3*a*d + 2*b*c)*atan(sqrt(
b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*a**2*b**(5/2))
```

Mathematica [C] time = 0.803669, size = 344, normalized size = 2.15

$$\frac{1}{4}\left(\frac{(bc-ad)^{3/2}(3ad+2bc)\log\left(-\frac{4a^2b^{5/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{5/2}(3ad+2bc)}\right)}{a^2b^{5/2}}+\frac{(bc-ad)^{3/2}(3ad+2bc)\log\left(-\frac{4a^2b^{5/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{5/2}(3ad+2bc)}\right)}{a^2b^{5/2}}-\frac{4c^{5/2}\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^2}+\frac{4c^{5/2}\log(x)}{a^2}+\frac{2\sqrt{c+dx^2}\left(\frac{(bc-ad)^2}{a(a+bx^2)}+2d^2\right)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((2*Sqrt[c + d*x^2]*(2*d^2 + (b*c - a*d)^2/(a*(a + b*x^2))))/b^2
+ (4*c^(5/2)*Log[x])/a^2 - (4*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c + d*
x^2]])/a^2 + ((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*Log[(-4*a^2*b^(5/
2)*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2]))
/((b*c - a*d)^(5/2)*(2*b*c + 3*a*d)*(I*Sqrt[a] + Sqrt[b]*x))])/(a
^2*b^(5/2)) + ((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*Log[(-4*a^2*b^(5
/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*x^2])
)/((b*c - a*d)^(5/2)*(2*b*c + 3*a*d)*((-I)*Sqrt[a] + Sqrt[b]*x))]
)/(a^2*b^(5/2)))/4
```

Maple [B] time = 0.03, size = 7477, normalized size = 46.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x), x)
```

Fricas [A] time = 1.54751, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*
c*d - 3*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4
*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^3*c^2*x^2 + a*b^2*c^2)*sqr
t(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(2*a
^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 +
c))/(a^2*b^3*x^2 + a^3*b^2), -1/8*(8*(b^3*c^2*x^2 + a*b^2*c^2)*sq
rt(-c)*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) + (2*a*b^2*c^2 + a^2*
b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sq
rt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*
b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 +
a^2)) - 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)
*sqrt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/4*((2*a*b^2*c^2 + a^
2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*
sqrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x
^2 + c)*b*sqrt(-(b*c - a*d)/b))) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*sq
rt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*
a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 +
 c))/(a^2*b^3*x^2 + a^3*b^2), -1/4*(4*(b^3*c^2*x^2 + a*b^2*c^2)*s
qrt(-c)*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (2*a*b^2*c^2 + a^2
*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*s
qrt(-(b*c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^
2 + c)*b*sqrt(-(b*c - a*d)/b))) - 2*(2*a^2*b*d^2*x^2 + a*b^2*c^2
- 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^{2}\right)^{\frac{5}{2}}}{x\left(a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.233599, size = 290, normalized size = 1.81

$$\frac{1}{2}\, d^{2}\left(\frac{2\, c^{3} \arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{a^{2} \sqrt{-c} d^{2}} + \frac{2\, \sqrt{dx^{2} + c}}{b^{2}} + \frac{\sqrt{dx^{2} + c} b^{2} c^{2} - 2\, \sqrt{dx^{2} + c} abcd + \sqrt{dx^{2} + c} a^{2} d^{2}}{((dx^{2} + c)b - bc + ad)ab^{2} d} - \frac{\left(2\, b^{3} c^{3} - ab^{2} c^{2} d - 4\, a^{2} bcd^{2}\right.}{\sqrt{-b^{2} c + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*d^2*(2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2
) + 2*sqrt(d*x^2 + c)/b^2 + (sqrt(d*x^2 + c)*b^2*c^2 - 2*sqrt(d*x
^2 + c)*a*b*c*d + sqrt(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c
+ a*d)*a*b^2*d) - (2*b^3*c^3 - a*b^2*c^2*d - 4*a^2*b*c*d^2 + 3*a^
3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a^2*b^2*d^2))
```

3.755 $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\,(a+bx^2)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] -(c*(3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*(c
+ d*x^2)^(3/2))/(2*a*b*x*(a + b*x^2)) - ((b*c - a*d)^(3/2)*(3*b*c
 + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(2*a^(5/2)*b^2) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/
b^2
```

Rubi [A] time = 0.537071, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\,(a+bx^2)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c*(3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*(c
+ d*x^2)^(3/2))/(2*a*b*x*(a + b*x^2)) - ((b*c - a*d)^(3/2)*(3*b*c
 + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(2*a^(5/2)*b^2) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/
b^2
```

Rubi in Sympy [A] time = 73.8086, size = 144, normalized size = 0.86

$$\frac{d^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}-\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{2abx\,(a+bx^2)}+\frac{c\sqrt{c+dx^2}\,(ad-3bc)}{2a^2bx}-\frac{(ad-bc)^{\frac{3}{2}}\,(2ad+3bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] d**(5/2)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/b**2 - (c + d*x**2)**(
3/2)*(a*d - b*c)/(2*a*b*x*(a + b*x**2)) + c*sqrt(c + d*x**2)*(a*d
 - 3*b*c)/(2*a**2*b*x) - (a*d - b*c)**(3/2)*(2*a*d + 3*b*c)*atanh
(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(5/2)*b**2)
```

Mathematica [A] time = 0.244004, size = 150, normalized size = 0.89

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}+\sqrt{c+dx^2}\left(-\frac{x(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{a^2x}\right)+\frac{d^{5/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(c^2/(a^2*x)) - ((b*c - a*d)^2*x)/(2*a^2*b*(a +
 b*x^2))) - ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*L
og[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/b^2
```

Maple [B] time = 0.035, size = 7529, normalized size = 44.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A] time = 0.711887, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(d)*x - c) - ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^
2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c - a*
d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3
*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 - 4*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sq
rt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), 1/8*(8*(a^2*b*d^2*x^3 +
 a^3*d^2*x)*sqrt(-d)*arctan(d*x/(sqrt(d*x^2 + c)*sqrt(-d))) - ((3
*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*
```

```
d - 2*a^3*d^2)*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a
^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)
/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a*b^2*c^2 + (3*b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^
3*b^2*x), 1/4*(((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*
b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt((b*c - a*d)/a)*arctan(-1
/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d
)/a))) + 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^
2*x), 1/4*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d)*arctan(d*x/(sqr
t(d*x^2 + c)*sqrt(-d))) + ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*
x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt((b*c - a*d)/a
)*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*a*x*sqrt
((b*c - a*d)/a))) - 2*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.602174, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2),x, algorithm="giac")

[Out] sage0*x
```

3.756 $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=180

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}(bc-ad)(2bc-ad)}{2a^2b\,(a+bx^2)}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\,(a+bx^2)}$$

```
[Out] -((b*c - a*d)*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*(a + b*x^2)
) - (c*(c + d*x^2)^(3/2))/(2*a*x^2*(a + b*x^2)) + (c^(3/2)*(4*b*c
 - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3) - ((b*c - a*d
)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*a^3*b^(3/2))
```

Rubi [A] time = 0.719622, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}(bc-ad)(2bc-ad)}{2a^2b\,(a+bx^2)}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```


```
[Out] -((b*c - a*d)*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*(a + b*x^2)
) - (c*(c + d*x^2)^(3/2))/(2*a*x^2*(a + b*x^2)) + (c^(3/2)*(4*b*c
 - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3) - ((b*c - a*d
)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*a^3*b^(3/2))
```

Rubi in Sympy [A] time = 72.7551, size = 155, normalized size = 0.86

$$-\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{2abx^2\,(a+bx^2)}+\frac{c\sqrt{c+dx^2}\,(ad-2bc)}{2a^2bx^2}-\frac{c^{\frac{3}{2}}\,(5ad-4bc)\,\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}+\frac{(ad-bc)^{\frac{3}{2}}\,(ad+4bc)\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**(3/2)*(a*d - b*c)/(2*a*b*x**2*(a + b*x**2)) + c*sq
rt(c + d*x**2)*(a*d - 2*b*c)/(2*a**2*b*x**2) - c**(3/2)*(5*a*d -
4*b*c)*atanh(sqrt(c + d*x**2)/sqrt(c))/(2*a**3) + (a*d - b*c)**(3
/2)*(a*d + 4*b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/
(2*a**3*b**(3/2))
```

Mathematica [C] time = 1.18527, size = 349, normalized size = 1.94

$$-\frac{\frac{(bc-ad)^{3/2}(ad+4bc)\log\left(\frac{4a^3b^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}-i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x+i\sqrt{a}\right)(bc-ad)^{5/2}(ad+4bc)}\right)}{b^{3/2}}+\frac{(bc-ad)^{3/2}(ad+4bc)\log\left(\frac{4a^3b^{3/2}\left(\sqrt{c+dx^2}\sqrt{bc-ad}+i\sqrt{a}dx+\sqrt{b}c\right)}{\left(\sqrt{b}x-i\sqrt{a}\right)(bc-ad)^{5/2}(ad+4bc)}\right)}{b^{3/2}}-2c^{3/2}(4bc-5ad)\,\mathrm{l}}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -(2*a*Sqrt[c + d*x^2]*(c^2/x^2 + (b*c - a*d)^2/(b*(a + b*x^2))) +
 2*c^(3/2)*(4*b*c - 5*a*d)*Log[x] - 2*c^(3/2)*(4*b*c - 5*a*d)*Log
[c + Sqrt[c]*Sqrt[c + d*x^2]] + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*
Log[(4*a^3*b^(3/2)*(Sqrt[b]*c - I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*S
qrt[c + d*x^2]))/((b*c - a*d)^(5/2)*(4*b*c + a*d)*(I*Sqrt[a] + Sq
rt[b]*x))])/b^(3/2) + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*Log[(4*a^3
*b^(3/2)*(Sqrt[b]*c + I*Sqrt[a]*d*x + Sqrt[b*c - a*d]*Sqrt[c + d*
x^2]))/((b*c - a*d)^(5/2)*(4*b*c + a*d)*((-I)*Sqrt[a] + Sqrt[b]*x
))])/b^(3/2))/(4*a^3)
```

Maple [B] time = 0.031, size = 7590, normalized size = 42.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^{\frac{5}{2}}}{(bx^2 + a)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A] time = 1.64854, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^
4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c -
a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*((4*b^3*c^2 - 5*a*b^2*c
*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b*c^2 + (2*a*b^2*c
^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 +
a^4*b*x^2), 1/8*(4*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2
- 5*a^2*b*c*d)*x^2)*sqrt(-c)*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c)))
 - ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*
a^2*b*c*d - a^3*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 +
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)
/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a^2*b*c^2 + (2*a*b^2*c^2 -
 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*
```

```
b*x^2), -1/4*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b
^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b*sqrt(-(b*c - a*d)/
b))) + ((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*
d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^
2) + 2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sq
rt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2), 1/4*(2*((4*b^3*c^2 - 5*
a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(-c)*arctan
(c/(sqrt(d*x^2 + c)*sqrt(-c))) - ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-(b*
c - a*d)/b)*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*b
*sqrt(-(b*c - a*d)/b))) - 2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c
*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.238189, size = 390, normalized size = 2.17

$$-\frac{1}{2}d^3\left(\frac{\left(4bc^3-5ac^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^3\sqrt{-c}d^3}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2-2\sqrt{dx^2+c}b^2c^3-2\left(dx^2+c\right)^{\frac{3}{2}}abcd+3\sqrt{dx^2+c}abc^2d+(dx}{\left(\left(dx^2+c\right)^2b-2\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-acd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] -1/2*d^3*((4*b*c^3 - 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(a^3*sqrt(-c)*d^3) + (2*(d*x^2 + c)^(3/2)*b^2*c^2 - 2*sqrt(d*x^2
+ c)*b^2*c^3 - 2*(d*x^2 + c)^(3/2)*a*b*c*d + 3*sqrt(d*x^2 + c)*a*
b*c^2*d + (d*x^2 + c)^(3/2)*a^2*d^2 - sqrt(d*x^2 + c)*a^2*c*d^2)/
(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d -
 a*c*d)*a^2*b*d^2) - (4*b^3*c^3 - 7*a*b^2*c^2*d + 2*a^2*b*c*d^2 +
 a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^3*b*d^3))
```

3.757 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=176

$$\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}-\frac{c\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

```
[Out] -(c*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b^2*c^2
 - 20*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*b*x) + ((b*c -
 a*d)*(c + d*x^2)^(3/2))/(2*a*b*x^3*(a + b*x^2)) + (5*c*(b*c - a*
d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*a^(7/2))
```

Rubi [A] time = 0.701997, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}-\frac{c\sqrt{c+dx^2}(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -(c*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b^2*c^2
 - 20*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*b*x) + ((b*c -
 a*d)*(c + d*x^2)^(3/2))/(2*a*b*x^3*(a + b*x^2)) + (5*c*(b*c - a*
d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*a^(7/2))
```

Rubi in Sympy [A] time = 106.82, size = 156, normalized size = 0.89

$$-\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}{2abx^3\left(a+bx^2\right)}+\frac{c\sqrt{c+dx^2}\,(3ad-5bc)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}+\frac{5c\,(ad-bc)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -(c + d*x**2)**(3/2)*(a*d - b*c)/(2*a*b*x**3*(a + b*x**2)) + c*sq
rt(c + d*x**2)*(3*a*d - 5*b*c)/(6*a**2*b*x**3) + sqrt(c + d*x**2)
*(3*a**2*d**2 - 20*a*b*c*d + 15*b**2*c**2)/(6*a**3*b*x) + 5*c*(a*
d - b*c)**(3/2)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)
))/(2*a**(7/2))
```

Mathematica [A] time = 0.276535, size = 121, normalized size = 0.69

$$\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}\left(2cx^2(6bc-7ad)+\frac{3x^4(bc-ad)^2}{a+bx^2}-2ac^2\right)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*c^2 + 2*c*(6*b*c - 7*a*d)*x^2 + (3*(b*c -
a*d)^2*x^4)/(a + b*x^2)))/(6*a^3*x^3) + (5*c*(b*c - a*d)^(3/2)*Ar
cTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Maple [B] time = 0.032, size = 7705, normalized size = 43.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{5}{2}}}{\left(bx^2 + a\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4), x)
```

Fricas [A] time = 0.355838, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(\left(b^2c^2 - abcd\right)x^5 + \left(abc^2 - a^2cd\right)x^3\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{\cdots}}{b^2x^4+2\,abx^2+a^2}\right.\cdots}{24\left(a^3bx^5 + a^4x^3\right)}\right.$$

$$-\frac{15\left(\left(b^2c^2 - abcd\right)x^5 + \left(abc^2 - a^2cd\right)x^3\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(-\frac{(bc-2\,ad)x^2-ac}{2\sqrt{dx^2+c}ax\sqrt{\frac{bc-ad}{a}}}\right) - 2\left(\left(15\,b^2c^2 - 20\,abcd + 3\,a^2d^2\right)x^4 - 2\,a^2c\cdots\right.}{12\left(a^3bx^5 + a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a^2*c*d)*x^3)*sq
rt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*
a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) - 4*((15*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*
a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*
x^5 + a^4*x^3), -1/12*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a
^2*c*d)*x^3)*sqrt((b*c - a*d)/a)*arctan(-1/2*((b*c - 2*a*d)*x^2 -
 a*c)/(sqrt(d*x^2 + c)*a*x*sqrt((b*c - a*d)/a))) - 2*((15*b^2*c^2
 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2
*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 4.99204, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.758 $\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3
*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(2*b^2*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^
2]]/(b^2*Sqrt[d])
```

Rubi [A] time = 0.325883, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3
*b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(2*b^2*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^
2]]/(b^2*Sqrt[d])
```

Rubi in Sympy [A] time = 43.118, size = 116, normalized size = 0.88

$$-\frac{\sqrt{a}\,(2ad-3bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2\,(ad-bc)^{\frac{3}{2}}}-\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(ad-bc)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*(2*a*d - 3*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c
+ d*x**2)))/(2*b**2*(a*d - b*c)**(3/2)) - a*x*sqrt(c + d*x**2)/(2
*b*(a + b*x**2)*(a*d - b*c)) + atanh(sqrt(d)*x/sqrt(c + d*x**2))/
(b**2*sqrt(d))
```

Mathematica [A] time = 0.318985, size = 129, normalized size = 0.98

$$\frac{\frac{abx\sqrt{c+dx^2}}{(a+bx^2)(bc-ad)}+\frac{\sqrt{a}(2ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}+\frac{2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (Sqrt[a]*(-3
*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(b*c - a*d)^(3/2) + (2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sq
```

```
rt[d])/(2*b^2)
```

Maple [B] time = 0.021, size = 846, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/4*a/b^2/(a*d-b*c)/(
x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*a/b^3*d*(-a*b)^(1/2)/
(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))-1/4*a/b^2/(a*d-b*c)/(x+1/b*(-a*b)^(1/2)
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-1/4*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2)))+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-3/4/b^2*a/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [A] time = 0.614859, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*sqrt(d)*x + (3*a*b*c - 2*a^2*d + (3*b
^2*c - 2*a*b*d)*x^2)*sqrt(d)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)
*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*
c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)) + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*log(-2*sqrt(d
*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/((a*b^3*c - a^2*b^2*d + (
b^4*c - a*b^3*d)*x^2)*sqrt(d)), 1/8*(4*sqrt(d*x^2 + c)*a*b*sqrt(-
d)*x + (3*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*x^2)*sqrt(-d)*sqr
t(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
```

```
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d +
 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/
(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 8*(a*b*c - a^2*d + (
b^2*c - a*b*d)*x^2)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/((a*b^3*c
 - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2)*sqrt(-d)), 1/4*(2*sqrt(d*x^
2 + c)*a*b*sqrt(d)*x - (3*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*x
^2)*sqrt(d)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 - a
*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) + 2*(a*b
*c - a^2*d + (b^2*c - a*b*d)*x^2)*log(-2*sqrt(d*x^2 + c)*d*x - (2
*d*x^2 + c)*sqrt(d)))/((a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x
^2)*sqrt(d)), 1/4*(2*sqrt(d*x^2 + c)*a*b*sqrt(-d)*x - (3*a*b*c -
2*a^2*d + (3*b^2*c - 2*a*b*d)*x^2)*sqrt(-d)*sqrt(a/(b*c - a*d))*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*
x*sqrt(a/(b*c - a*d)))) + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/((a*b^3*c - a^2*b^2*d + (b^4
*c - a*b^3*d)*x^2)*sqrt(-d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.571823, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.759 $\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*
(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.245129, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{a\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*
(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 26.5652, size = 78, normalized size = 0.79

$$-\frac{a\sqrt{c+dx^2}}{2b(a+bx^2)(ad-bc)}+\frac{\left(\frac{ad}{2}-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{b^{\frac{3}{2}}(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*sqrt(c + d*x**2)/(2*b*(a + b*x**2)*(a*d - b*c)) + (a*d/2 - b*c
)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(b**(3/2)*(a*d -
 b*c)**(3/2))
```

Mathematica [A] time = 0.126695, size = 99, normalized size = 1.

$$\frac{a\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*
(b*c - a*d)^(3/2))
```

Maple [B] time = 0.019, size = 807, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/b^2*(-a*b)^(
1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*a
*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/
b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268286, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{b^2c - abd}\sqrt{dx^2 + c}a + \left(2\,abc - a^2d + \left(2\,b^2c - abd\right)x^2\right)\log\left(\frac{\left(b^2d^2x^4 + 8\,b^2c^2 - 8\,abcd + a^2d^2 + 2\left(4\,b^2cd - 3\,abd^2\right)x^2\right)\sqrt{b^2c - abd} - 4\left(2\,b^3c\right.}{b^2x^4 + 2\,abx^2 + a^2}\right.}{8\left(ab^2c - a^2bd + (b^3c - ab^2d)x^2\right)\sqrt{b^2c - abd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c)*a + (2*a*b*c - a^2*d
+ (2*b^2*c - a*b*d)*x^2)*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d)
- 4*(2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*
x^2)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a*b^2*c - a
^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(b^2*c - a*b*d)), 1/4*(2*sqrt
(-b^2*c + a*b*d)*sqrt(d*x^2 + c)*a + (2*a*b*c - a^2*d + (2*b^2*c
- a*b*d)*x^2)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a
*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 + c))))/((a*b^2*c - a^2*b*d + (
b^3*c - a*b^2*d)*x^2)*sqrt(-b^2*c + a*b*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.236052, size = 157, normalized size = 1.59

$$\frac{\frac{\sqrt{dx^2+c}ad^2}{(b^2c-abd)((dx^2+c)b-bc+ad)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x^2 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^2 + c)*b - b*c
+ a*d)) + (2*b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
+ a*b*d))/((b^2*c - a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.760 $\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

```
[Out] -(x*Sqrt[c + d*x^2])/(2*(b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqr
t[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d
)^(3/2))
```

Rubi [A] time = 0.15905, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(x*Sqrt[c + d*x^2])/(2*(b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqr
t[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d
)^(3/2))
```

Rubi in Sympy [A] time = 24.4845, size = 73, normalized size = 0.82

$$\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(ad-bc)}-\frac{c\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] x*sqrt(c + d*x**2)/(2*(a + b*x**2)*(a*d - b*c)) - c*atanh(x*sqrt(
a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*sqrt(a)*(a*d - b*c)**(3
/2))
```

Mathematica [A] time = 0.151971, size = 86, normalized size = 0.97

$$\frac{\frac{x\sqrt{c+dx^2}}{a+bx^2}-\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}}{2ad-2bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((x*Sqrt[c + d*x^2])/(a + b*x^2) - (c*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*Sqrt[b*c - a*d]))/(-2*b*c +
2*a*d)
```

Maple [B] time = 0.019, size = 817, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b^2*
d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/b/(a*d-b*c)/(x+1/b*(
-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d*(-a*b)^(1/2)/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1
/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [A] time = 0.341371, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{-abc+a^2d}\sqrt{dx^2+c}x+\left(bcx^2+ac\right)\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2\right)\sqrt{-abc+a^2d}-4\left(\left(ab^2c^2-3\,a^2bcd+2\,a^3d^2\cdots\right.\right.}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-abc+a^2d}},\right.$$
$$\left.-\frac{2\sqrt{abc-a^2d}\sqrt{dx^2+c}x-\left(bcx^2+ac\right)\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\sqrt{abc-a^2d}\sqrt{dx^2+c}x}\right)}{4\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2 + a*c)
*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a
^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 +
 c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a*b*c - a^2*d + (b^2*c - a*b
*d)*x^2)*sqrt(-a*b*c + a^2*d)), -1/4*(2*sqrt(a*b*c - a^2*d)*sqrt(
d*x^2 + c)*x - (b*c*x^2 + a*c)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*
```

```
c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((a*b*c - a^2*d + (b
^2*c - a*b*d)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 2.00856, size = 4, normalized size = 0.04

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.761 $\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\,(a+bx^2)\,(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^2]/(2*(b*c - a*d)*(a + b*x^2)) + (d*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2)
)
```

Rubi [A] time = 0.171883, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*(b*c - a*d)*(a + b*x^2)) + (d*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2)
)
```

Rubi in Sympy [A] time = 21.7303, size = 70, normalized size = 0.8

$$\frac{\sqrt{c+dx^2}}{2\,(a+bx^2)\,(ad-bc)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2\sqrt{b}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c + d*x**2)/(2*(a + b*x**2)*(a*d - b*c)) + d*atan(sqrt(b)*sq
rt(c + d*x**2)/sqrt(a*d - b*c))/(2*sqrt(b)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.148276, size = 84, normalized size = 0.97

$$\frac{\frac{\sqrt{c+dx^2}}{a+bx^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}}{2ad-2bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]/(a + b*x^2) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(-2*b*c + 2*a*d)
```

Maple [B] time = 0.017, size = 513, normalized size = 5.9

$$-\frac{1}{4ab(ad-bc)}\sqrt{-ab}\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\left(x-\frac{1}{b}\sqrt{-ab}\right)^{-1}$$
$$-\frac{d}{4(ad-bc)b}\ln\left(1\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\right.\right.\right.$$
$$+\frac{1}{4ab(ad-bc)}\sqrt{-ab}\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\left(x+\frac{1}{b}\sqrt{-ab}\right)^{-1}$$
$$-\frac{d}{4(ad-bc)b}\ln\left(1\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4*(-a*b)^(1/2)/a/b/(a
*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b*d/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257694, size = 1, normalized size = 0.01

$$\left[-\frac{(bdx^2+ad)\log\left(\frac{(b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2)\sqrt{b^2c-abd}-4(2b^3c^2-3ab^2cd+a^2bd^2+(b^3cd-ab^2d^2)x^2)\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+4\sqrt{b}}{8(abc-a^2d+(b^2c-abd)x^2)\sqrt{b^2c-abd}}\right.$$
$$\left.-\frac{(bdx^2+ad)\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{-b^2c+abd}}{2(b^2c-abd)\sqrt{dx^2+c}}\right)+2\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{4(abc-a^2d+(b^2c-abd)x^2)\sqrt{-b^2c+abd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^2 + a*d)*log(((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d
+ a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c - a*b*d) -
```

```
4*(2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^
2)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(b^2*c -
 a*b*d)*sqrt(d*x^2 + c))/((a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*s
qrt(b^2*c - a*b*d)), -1/4*((b*d*x^2 + a*d)*arctan(-1/2*(b*d*x^2 +
 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)/((b^2*c - a*b*d)*sqrt(d*x^2 +
c))) + 2*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c))/((a*b*c - a^2*d +
(b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.238996, size = 124, normalized size = 1.43

$$-\frac{1}{2}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{\sqrt{dx^2+c}}{\left((dx^2+c)b-bc+ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -1/2*d*(arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2
*c + a*b*d)*(b*c - a*d)) + sqrt(d*x^2 + c)/(((d*x^2 + c)*b - b*c
+ a*d)*(b*c - a*d)))
```

3.762 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a(a+bx^2)(bc-ad)}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.147401, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
3/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 21.7432, size = 83, normalized size = 0.83

$$-\frac{bx\sqrt{c+dx^2}}{2a(a+bx^2)(ad-bc)}+\frac{(2ad-bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*x*sqrt(c + d*x**2)/(2*a*(a + b*x**2)*(a*d - b*c)) + (2*a*d - b
*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(3/
2)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.174678, size = 100, normalized size = 1.

$$\frac{\frac{\sqrt{a}bx\sqrt{c+dx^2}}{(a+bx^2)(bc-ad)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[a]*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + ((b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
b*c - a*d)^(3/2))/(2*a^(3/2))
```

Maple [B] time = 0.008, size = 823, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b/a
*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/a/(a*d-b*c)/(x+1/b*
(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))-1/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/a/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [A] time = 0.365965, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{-abc+a^2d}\sqrt{dx^2+c}\,bx+\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2\right)\sqrt{-abc+a^2d}+4}{b^2x^4+2\,abx^2+a}\right.}{8\left(a^2bc-a^3d+(ab^2c-a^2bd)x^2\right)\sqrt{-abc+a^2d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*b*x + (a*b*c - 2*a^2
*d + (b^2*c - 2*a*b*d)*x^2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a
^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2
 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a
^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-a*b*c + a^2*d)),
1/4*(2*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*b*x + (a*b*c - 2*a^2*d
 + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(s
qrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((a^2*b*c - a^3*d + (a*b^
2*c - a^2*b*d)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.240779, size = 304, normalized size = 3.04

$$-\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d
^2)^(3/2) + 2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*d - b*c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sq
rt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2)))
```

3.763 $\int \frac{1}{x(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) - ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]]/(a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d
)^(3/2))
```

Rubi [A] time = 0.393794, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) - ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]]/(a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d
)^(3/2))
```

Rubi in Sympy [A] time = 51.5562, size = 110, normalized size = 0.85

$$-\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(ad-bc)}-\frac{\sqrt{b}\left(\frac{3ad}{2}-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{a^2\left(ad-bc\right)^{\frac{3}{2}}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*sqrt(c + d*x**2)/(2*a*(a + b*x**2)*(a*d - b*c)) - sqrt(b)*(3*a
*d/2 - b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a**2*
(a*d - b*c)**(3/2)) - atanh(sqrt(c + d*x**2)/sqrt(c))/(a**2*sqrt(
c))
```

Mathematica [C] time = 1.19668, size = 360, normalized size = 2.77

$$\frac{\frac{\sqrt{b}(2bc-3ad)\log\left(-\frac{4ia^2\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{\sqrt{b}\left(\sqrt{a}+i\sqrt{b}x\right)(2bc-3ad)}\right)}{(bc-ad)^{3/2}}+\frac{\sqrt{b}(2bc-3ad)\log\left(\frac{4a^2\left(i\sqrt{a}dx\sqrt{bc-ad}-\sqrt{b}c\sqrt{bc-ad}+ad\sqrt{c+dx^2}-bc\sqrt{c+dx^2}\right)}{\sqrt{b}\left(\sqrt{b}x+i\sqrt{a}\right)(2bc-3ad)}\right)}{(bc-ad)^{3/2}}-\frac{2\ldots}{(a+\ldots}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-2*a*b*Sqrt[c + d*x^2])/((-(b*c) + a*d)*(a + b*x^2)) + (4*Log[x
])/Sqrt[c] - (4*Log[c + Sqrt[c]*Sqrt[c + d*x^2]])/Sqrt[c] + (Sqrt
[b]*(2*b*c - 3*a*d)*Log[((-4*I)*a^2*(Sqrt[b]*c*Sqrt[b*c - a*d] +
```

```
I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c
+ d*x^2]))/(Sqrt[b]*(2*b*c - 3*a*d)*(Sqrt[a] + I*Sqrt[b]*x))])/(b
*c - a*d)^(3/2) + (Sqrt[b]*(2*b*c - 3*a*d)*Log[(4*a^2*(-(Sqrt[b]*
c*Sqrt[b*c - a*d]) + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x - b*c*Sqrt[c +
 d*x^2] + a*d*Sqrt[c + d*x^2]))/(Sqrt[b]*(2*b*c - 3*a*d)*(I*Sqrt[
a] + Sqrt[b]*x))])/(b*c - a*d)^(3/2))/(4*a^2)
```

Maple [B] time = 0.021, size = 838, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/2/a^2/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/(-a*b)^(1/2)/(a*d-b*c)*
b/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/a*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))+1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x), x)
```

Fricas [A] time = 0.728419, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*sqrt(c) + (2*a*b*c - 3*a^2*d + (2*b^2
*c - 3*a*b*d)*x^2)*sqrt(c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 +
 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
+ 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*s
qrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*log(-((d*x^2 + 2*c)*sqr
```

```
t(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/((a^3*b*c - a^4*d + (a^2*b^2*c
- a^3*b*d)*x^2)*sqrt(c)), 1/8*(4*sqrt(d*x^2 + c)*a*b*sqrt(-c) + (
2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(-c)*sqrt(b/(b*c
 - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(
4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b
^2*x^4 + 2*a*b*x^2 + a^2)) - 8*(a*b*c - a^2*d + (b^2*c - a*b*d)*x
^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((a^3*b*c - a^4*d + (a^2*b^
2*c - a^3*b*d)*x^2)*sqrt(-c)), 1/4*(2*sqrt(d*x^2 + c)*a*b*sqrt(c)
 - (2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(c)*sqrt(-b/
(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) + 2*(a*b*c - a^2*d + (b^2*c -
 a*b*d)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x
^2))/((a^3*b*c - a^4*d + (a^2*b^2*c - a^3*b*d)*x^2)*sqrt(c)), 1/4
*(2*sqrt(d*x^2 + c)*a*b*sqrt(-c) - (2*a*b*c - 3*a^2*d + (2*b^2*c
- 3*a*b*d)*x^2)*sqrt(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^
2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)
))) - 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*arctan(sqrt(-c)/sqr
t(d*x^2 + c)))/((a^3*b*c - a^4*d + (a^2*b^2*c - a^3*b*d)*x^2)*sqr
t(-c))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.232729, size = 207, normalized size = 1.59

$$-\frac{1}{2}\,d^2\left(\frac{\left(2\,b^2c-3\,abd\right)\,\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx^2+c}b}{\left(abcd-a^2d^2\right)\left(\left(dx^2+c\right)b-bc+ad\right)}-\frac{2\,\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -1/2*d^2*((2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*
c + a*b*d))/((a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt
(d*x^2 + c)*b/((a*b*c*d - a^2*d^2)*((d*x^2 + c)*b - b*c + a*d)) -
 2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2))
```

3.764 $\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=147

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(3bc-2ad)}{2a^2cx(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax\,(a+bx^2)\,(bc-ad)}$$

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*x) + (b*S
qrt[c + d*x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) - (b*(3*b*c - 4*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.412581, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(3bc-2ad)}{2a^2cx(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*x) + (b*S
qrt[c + d*x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) - (b*(3*b*c - 4*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
5/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 61.0116, size = 124, normalized size = 0.84

$$-\frac{b\sqrt{c+dx^2}}{2ax\,(a+bx^2)\,(ad-bc)}-\frac{\sqrt{c+dx^2}\,(2ad-3bc)}{2a^2cx\,(ad-bc)}-\frac{b\,(4ad-3bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{5}{2}}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*sqrt(c + d*x**2)/(2*a*x*(a + b*x**2)*(a*d - b*c)) - sqrt(c + d
*x**2)*(2*a*d - 3*b*c)/(2*a**2*c*x*(a*d - b*c)) - b*(4*a*d - 3*b*
c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(5/2
)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.267638, size = 116, normalized size = 0.79

$$\frac{\sqrt{c+dx^2}\left(\frac{b^2x^2}{(a+bx^2)(ad-bc)}-\frac{2}{c}\right)}{2a^2x}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2/c + (b^2*x^2)/((-(b*c) + a*d)*(a + b*x^2))))
/(2*a^2*x) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.024, size = 841, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/a^2/c/x*(d*x^2+c)^(1/2)+1/4/a^2/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2
))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)-1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(
1/2)))+1/4/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+1/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/4*b/a^2/(-a
*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))-3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [A] time = 0.404023, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(2\,abc-2\,a^2d+\left(3\,b^2c-2\,abd\right)x^2\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}-\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^3+\left(3\,ab^2c^2-4\,a^2bcd\right)x\right)\log\left(\frac{((b^2c^2-\cdots}{\cdots}\right.}{8\,((a^2b^2c^2-a^3bcd)x^3+(a^3bc^2-a^4cd)x\cdots}\right.$$

$$-\frac{2\left(2\,abc-2\,a^2d+\left(3\,b^2c-2\,abd\right)x^2\right)\sqrt{abc-a^2d}\sqrt{dx^2+c}+\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^3+\left(3\,ab^2c^2-4\,a^2bcd\right)x\right)\arctan\left(\frac{(bc-\cdots}{2\sqrt{abc\cdots}}\right.}{4\,((a^2b^2c^2-a^3bcd)x^3+(a^3bc^2-a^4cd)x)\sqrt{abc-a^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*
c + a^2*d)*sqrt(d*x^2 + c) - ((3*b^3*c^2 - 4*a*b^2*c*d)*x^3 + (3*
a*b^2*c^2 - 4*a^2*b*c*d)*x)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a
^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2
 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(((
```

```
a^2*b^2*c^2 - a^3*b*c*d)*x^3 + (a^3*b*c^2 - a^4*c*d)*x)*sqrt(-a*b
*c + a^2*d)), -1/4*(2*(2*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*x^
2)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) + ((3*b^3*c^2 - 4*a*b^2*c*
d)*x^3 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a^2*b^2*
c^2 - a^3*b*c*d)*x^3 + (a^3*b*c^2 - a^4*c*d)*x)*sqrt(a*b*c - a^2*
d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 2.1349, size = 535, normalized size = 3.64

$$\frac{1}{2}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^{2}c-4\,abd\right)\,\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{2}c-4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}abd-6\left(\sqrt{d}x\right.\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}x-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*
x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d
^2 - a^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x - sqrt(d
*x^2 + c))^4*b^2*c - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*d - 6*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2 + 14*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a*b*c*d - 8*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^2 +
 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

3.765 $\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}(2bc-ad)}{2a^2c\,(a+bx^2)\,(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\,(a+bx^2)}$$

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*(a + b*x^
2)) - Sqrt[c + d*x^2]/(2*a*c*x^2*(a + b*x^2)) + ((4*b*c + a*d)*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(3/2)) - (b^(3/2)*(4*b*c
 - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*
a^3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.676969, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}(2bc-ad)}{2a^2c\,(a+bx^2)\,(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*(a + b*x^
2)) - Sqrt[c + d*x^2]/(2*a*c*x^2*(a + b*x^2)) + ((4*b*c + a*d)*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(3/2)) - (b^(3/2)*(4*b*c
 - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*
a^3*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 80.6159, size = 158, normalized size = 0.85

$$-\frac{\sqrt{c+dx^2}}{2acx^2\,(a+bx^2)}-\frac{b\sqrt{c+dx^2}\,(ad-2bc)}{2a^2c\,(a+bx^2)\,(ad-bc)}+\frac{b^{\frac{3}{2}}\,(5ad-4bc)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^3\,(ad-bc)^{\frac{3}{2}}}+\frac{(ad+4bc)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(c + d*x**2)/(2*a*c*x**2*(a + b*x**2)) - b*sqrt(c + d*x**2)*
(a*d - 2*b*c)/(2*a**2*c*(a + b*x**2)*(a*d - b*c)) + b**(3/2)*(5*a
*d - 4*b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*a**
3*(a*d - b*c)**(3/2)) + (a*d + 4*b*c)*atanh(sqrt(c + d*x**2)/sqrt
(c))/(2*a**3*c**(3/2))
```

Mathematica [C] time = 2.11906, size = 387, normalized size = 2.09

$$-\frac{\frac{b^{3/2}(4bc-5ad)\log\left(\frac{4a^3\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{3/2}\left(\sqrt{b}x+i\sqrt{a}\right)(4bc-5ad)}\right)}{(bc-ad)^{3/2}}+\frac{b^{3/2}(4bc-5ad)\log\left(\frac{4ia^3\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{3/2}\left(\sqrt{a}+i\sqrt{b}x\right)(4bc-5ad)}\right)}{(bc-ad)^{3/2}}-}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(-2*a*Sqrt[c + d*x^2]*(-(1/(c*x^2)) + b^2/((-(b*c) + a*d)*(a + b
*x^2))) + (2*(4*b*c + a*d)*Log[x])/c^(3/2) - (2*(4*b*c + a*d)*Log
[c + Sqrt[c]*Sqrt[c + d*x^2]])/c^(3/2) + (b^(3/2)*(4*b*c - 5*a*d)
*Log[(4*a^3*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqrt[b*c - a
*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(3/2)*(4*b
*c - 5*a*d)*(I*Sqrt[a] + Sqrt[b]*x))])/(b*c - a*d)^(3/2) + (b^(3/
2)*(4*b*c - 5*a*d)*Log[((4*I)*a^3*(Sqrt[b]*c*Sqrt[b*c - a*d] + I*
Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c +
d*x^2]))/(b^(3/2)*(4*b*c - 5*a*d)*(Sqrt[a] + I*Sqrt[b]*x))])/(b*c
 - a*d)^(3/2))/(4*a^3)
```

Maple [B] time = 0.023, size = 899, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/a^2/c/x^2*(d*x^2+c)^(1/2)+1/2/a^2*d/c^(3/2)*ln((2*c+2*c^(1/2
)*(d*x^2+c)^(1/2))/x)-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-b/a^3/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
+2*b/a^3/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4*b^2/a^
2/(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-1/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4*b^2/a^2/(-a*b)^(1/2)/(
a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4*b/a^2*d/(a*
d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3), x)
```

Fricas [A] time = 1.22973, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d
)*x^2)*sqrt(c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c
^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a^2*b*c
 - a^3*d + (2*a*b^2*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 2
*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^
2*b*c*d - a^3*d^2)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^
2 + c)*c)/x^2))/(((a^3*b^2*c^2 - a^4*b*c*d)*x^4 + (a^4*b*c^2 - a^
5*c*d)*x^2)*sqrt(c)), 1/8*(((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*
b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2
*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)) - 4*(a^2*b*c - a^3*d + (2*a*b^2*c - a^2*b*d)*x^2)*sqrt(
d*x^2 + c)*sqrt(-c) + 4*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^
4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*arctan(sqrt(-c)/sq
rt(d*x^2 + c)))/(((a^3*b^2*c^2 - a^4*b*c*d)*x^4 + (a^4*b*c^2 - a^
5*c*d)*x^2)*sqrt(-c)), 1/4*(((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a
*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)*sqrt(-b/(b*c - a*d))*arctan(
-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b
/(b*c - a*d)))) - 2*(a^2*b*c - a^3*d + (2*a*b^2*c - a^2*b*d)*x^2)
*sqrt(d*x^2 + c)*sqrt(c) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)
*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*log(-((d*x^2 +
2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/(((a^3*b^2*c^2 - a^4*b*
c*d)*x^4 + (a^4*b*c^2 - a^5*c*d)*x^2)*sqrt(c)), 1/4*(((4*b^3*c^2
- 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(-c)*sq
rt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^
2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(a^2*b*c - a^3*d +
(2*a*b^2*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 2*((4*b^3*c
^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d -
a^3*d^2)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(((a^3*b^2*c^2 -
a^4*b*c*d)*x^4 + (a^4*b*c^2 - a^5*c*d)*x^2)*sqrt(-c))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.244442, size = 362, normalized size = 1.96

$$\frac{1}{2}\,d^3\left(\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{2\,\left(dx^2+c\right)^{\frac{3}{2}}b^2c-2\,\sqrt{dx^2+c}b^2c^2-\left(dx^2+c\right)^{\frac{3}{2}}abd+2\,\sqrt{dx^2+c}abcd-\sqrt{dx^2}}{\left(a^2bc^2d^2-a^3cd^3\right)\left(\left(dx^2+c\right)^2b-2\,\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-acd\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^3*((4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2
*c + a*b*d))/((a^3*b*c*d^3 - a^4*d^4)*sqrt(-b^2*c + a*b*d)) - (2*
(d*x^2 + c)^(3/2)*b^2*c - 2*sqrt(d*x^2 + c)*b^2*c^2 - (d*x^2 + c)
^(3/2)*a*b*d + 2*sqrt(d*x^2 + c)*a*b*c*d - sqrt(d*x^2 + c)*a^2*d^
2)/((a^2*b*c^2*d^2 - a^3*c*d^3)*((d*x^2 + c)^2*b - 2*(d*x^2 + c)*
b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - (4*b*c + a*d)*arctan(sq
rt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c*d^3))
```

3.766 $\int \frac{1}{x^4(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{\sqrt{c+dx^2}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{6a^3c^2x(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^2*c*(b*c - a*d)*x^3) + ((
15*b^2*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*c^2*(
b*c - a*d)*x) + (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x
^2)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.741834, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{\sqrt{c+dx^2}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{6a^3c^2x(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^2*c*(b*c - a*d)*x^3) + ((
15*b^2*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*c^2*(
b*c - a*d)*x) + (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x
^2)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 128.861, size = 180, normalized size = 0.87

$$-\frac{b\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)(ad-bc)}-\frac{\sqrt{c+dx^2}\,(2ad-5bc)}{6a^2cx^3\,(ad-bc)}+\frac{\sqrt{c+dx^2}\left(4a^2d^2+8abcd-15b^2c^2\right)}{6a^3c^2x\,(ad-bc)}+\frac{b^2\,(6ad-5bc)\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{7}{2}}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*sqrt(c + d*x**2)/(2*a*x**3*(a + b*x**2)*(a*d - b*c)) - sqrt(c
+ d*x**2)*(2*a*d - 5*b*c)/(6*a**2*c*x**3*(a*d - b*c)) + sqrt(c +
d*x**2)*(4*a**2*d**2 + 8*a*b*c*d - 15*b**2*c**2)/(6*a**3*c**2*x*(
a*d - b*c)) + b**2*(6*a*d - 5*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(
a)*sqrt(c + d*x**2)))/(2*a**(7/2)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.410323, size = 136, normalized size = 0.66

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\left(\frac{3b^3x^4}{(a+bx^2)(bc-ad)}+\frac{4x^2(ad+3bc)}{c^2}-\frac{2a}{c}\right)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*((-2*a)/c + (4*(3*b*c + a*d)*x^2)/c^2 + (3*b^3*x
^4)/((b*c - a*d)*(a + b*x^2))))/(6*a^3*x^3) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/
2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.026, size = 893, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3/a^2/c/x^3*(d*x^2+c)^(1/2)+2/3/a^2*d/c^2/x*(d*x^2+c)^(1/2)+2*
b/a^3/c/x*(d*x^2+c)^(1/2)-1/4*b^2/a^3/(a*d-b*c)/(x-1/b*(-a*b)^(1/
2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2)))-1/4*b^2/a^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-1/4*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/4*b^
2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [A] time = 0.558448, size = 1, normalized size = 0.

$$\left[-\frac{4\left(2a^2bc^2-2a^3cd-\left(15b^3c^2-8ab^2cd-4a^2bd^2\right)x^4-2\left(5ab^2c^2-3a^2bcd-2a^3d^2\right)x^2\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}-3\left(\left(5\right.\right.}{24\left(\left(a^3b^2c^3\right.\right.}\right.$$

$$-\frac{2\left(2a^2bc^2-2a^3cd-\left(15b^3c^2-8ab^2cd-4a^2bd^2\right)x^4-2\left(5ab^2c^2-3a^2bcd-2a^3d^2\right)x^2\right)\sqrt{abc-a^2d}\sqrt{dx^2+c}-3\left(\left(5b^4\right.\right.}{12\left(\left(a^3b^2c^3-a^4bc^2d\right)x^5+\left(a^4bc^3-a^5c^2d\right)x^3\right)\sqrt{abc-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(2*a^2*b*c^2 - 2*a^3*c*d - (15*b^3*c^2 - 8*a*b^2*c*d -
4*a^2*b*d^2)*x^4 - 2*(5*a*b^2*c^2 - 3*a^2*b*c*d - 2*a^3*d^2)*x^2)
*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - 3*((5*b^4*c^3 - 6*a*b^3*c
^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^3)*log((((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)
*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*
d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2
*a*b*x^2 + a^2)))/(((a^3*b^2*c^3 - a^4*b*c^2*d)*x^5 + (a^4*b*c^3
- a^5*c^2*d)*x^3)*sqrt(-a*b*c + a^2*d)), -1/12*(2*(2*a^2*b*c^2 -
2*a^3*c*d - (15*b^3*c^2 - 8*a*b^2*c*d - 4*a^2*b*d^2)*x^4 - 2*(5*a
*b^2*c^2 - 3*a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(a*b*c - a^2*d)*sqrt
(d*x^2 + c) - 3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 -
 6*a^2*b^2*c^2*d)*x^3)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt
(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a^3*b^2*c^3 - a^4*b*c^2*d
)*x^5 + (a^4*b*c^3 - a^5*c^2*d)*x^3)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 4.63135, size = 506, normalized size = 2.46

$$-\frac{1}{6}\,d^{\frac{7}{2}}\left(\frac{3\left(5\,b^{3}c-6\,ab^{2}d\right)\,\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{(a^{3}bcd^{3}-a^{4}d^{4})\sqrt{abcd-a^{2}d^{2}}}+\frac{6\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{3}c-2\left(\sqrt{d}x-\sqrt{dx\right.}\right.}{(a^{3}bcd^{3}-a^{4}d^{4})\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] -1/6*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(1/2*((sqrt(d)*x - sq
rt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*
b*c*d^3 - a^4*d^4)*sqrt(a*b*c*d - a^2*d^2)) + 6*((sqrt(d)*x - sqr
t(d*x^2 + c))^2*b^3*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*d
 - b^3*c^2)/((a^3*b*c*d^3 - a^4*d^4)*((sqrt(d)*x - sqrt(d*x^2 + c
))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*d + b*c^2)) + 8*(3*(sqrt(d)*x - sqrt(d*x^2 +
c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 3*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x - sqrt(d*x
^2 + c))^2 - c)^3*a^3*d^3))
```

3.767 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3\sqrt{a}c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

```
[Out] ((2*b*c + a*d)*x)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a*x)/(2*
b*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(5
/2))
```

Rubi [A] time = 0.28252, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3\sqrt{a}c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*x)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a*x)/(2*
b*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(5
/2))
```

Rubi in Sympy [A] time = 52.0721, size = 110, normalized size = 0.85

$$-\frac{3\sqrt{a}c\,\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\left(ad-bc\right)^{\frac{5}{2}}}-\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\left(ad-bc\right)}+\frac{x\left(ad+2bc\right)}{2b\sqrt{c+dx^2}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -3*sqrt(a)*c*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/
(2*(a*d - b*c)**(5/2)) - a*x/(2*b*(a + b*x**2)*sqrt(c + d*x**2)*(
a*d - b*c)) + x*(a*d + 2*b*c)/(2*b*sqrt(c + d*x**2)*(a*d - b*c)**
2)
```

Mathematica [A] time = 0.435764, size = 106, normalized size = 0.82

$$\frac{1}{2}\left(\frac{3acx+adx^3+2bcx^3}{\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)^2}-\frac{3\sqrt{a}c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((3*a*c*x + 2*b*c*x^3 + a*d*x^3)/((b*c - a*d)^2*(a + b*x^2)*Sqrt[
c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
```

```
rt[c + d*x^2])])/(b*c - a*d)^(5/2))/2
```

Maple [B] time = 0.032, size = 1498, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/b^2*x/c/(d*x^2+c)^(1/2)-1/4*a/b^2/(a*d-b*c)/(x-1/b*(-a*b)^(1/2)
)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+3/4*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+3/4*a^2/b^2*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
-3/4*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-5/4*a/b^2/(
a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4*a/b^2/(a*d-b*c)/(x+1/b*(-a
*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)
^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+3/4*a^2/b^2*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*x+3/4*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/
4*a/b^2/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+3/4/b*a/(-a*b)^(1/2)/
(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))-3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/
4/b*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [A] time = 0.50435, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bcdx^4+ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2c\right.\right.}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2\right.\right.},\ -\frac{3\left(bcdx^4+ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\sqrt{dx^2+c}(bc-ad)x\sqrt{\frac{a}{bc-ad}}}\right)-2\left((2\,bc+ad)x^3+3\,acx\right)\sqrt{dx^2+c}}{4\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(-a/(b*c -
a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3
*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*
x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d))
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((2*b*c + a*d)*x^3 + 3*a*c*x)*
sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^
2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a
^2*b*c*d^2 + a^3*d^3)*x^2), -1/4*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 +
 a*c*d)*x^2)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 -
a*c)/(sqrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - 2*((2
*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*
c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 3.34081, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.768 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{a}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{5/2}}$$

```
[Out] (2*b*c + a*d)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + a/(2*b*(b*c -
 a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(5/
2))
```

Rubi [A] time = 0.30845, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{a}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + a/(2*b*(b*c -
 a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(5/
2))
```

Rubi in Sympy [A] time = 36.7018, size = 107, normalized size = 0.8

$$-\frac{a}{2b(a+bx^2)\sqrt{c+dx^2}\,(ad-bc)}+\frac{\frac{ad}{2}+bc}{b\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{\left(\frac{ad}{2}+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -a/(2*b*(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)) + (a*d/2 + b*c
)/(b*sqrt(c + d*x**2)*(a*d - b*c)**2) + (a*d/2 + b*c)*atan(sqrt(b
)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(sqrt(b)*(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.28481, size = 111, normalized size = 0.83

$$\frac{1}{2}\left(\frac{3ac+adx^2+2bcx^2}{(a+bx^2)\sqrt{c+dx^2}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((3*a*c + 2*b*c*x^2 + a*d*x^2)/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c
+ d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(5/2)))/2
```

Maple [B] time = 0.02, size = 1456, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/b/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/b^2*(-a*b)^(1/2)/(a*d-b*c)
/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/b/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/b/
(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/b^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)*x*d+1/2/b/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/b^2*(-a*b
)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b*
a*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b^2*(-a*b)^(1/2)*d^2*a/(a
*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b*a*d/(a*d-b*c)^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(
1/2)))-1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+3/4/b*a*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b^2*(
-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/b*a*d/(
a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.323456, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{b^2c - abd}\left((2\,bc + ad)x^2 + 3\,ac\right)\sqrt{dx^2 + c} + \left(\left(2\,b^2cd + abd^2\right)x^4 + 2\,abc^2 + a^2cd + \left(2\,b^2c^2 + 3\,abcd + a^2d^2\right)x^2\right)\log\left(\frac{(b^2d^2 \cdots}{\cdots}\right.}{8\left(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^4 + (b^3c^3 - \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b^2*c - a*b*d)*((2*b*c + a*d)*x^2 + 3*a*c)*sqrt(d*x^
2 + c) + ((2*b^2*c*d + a*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^
2*c^2 + 3*a*b*c*d + a^2*d^2)*x^2)*log(((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2)*sqrt(b^2*c
- a*b*d) - 4*(2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*
b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a*
b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^2)*sqrt(b^2*c - a*b*d)), 1/4*(2*sqrt(-b^2*c + a*b*d)*((2*b*c
+ a*d)*x^2 + 3*a*c)*sqrt(d*x^2 + c) + ((2*b^2*c*d + a*b*d^2)*x^4
+ 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^2)*ar
ctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)/((b^2*c -
a*b*d)*sqrt(d*x^2 + c))))/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2
*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-b^2*c + a*b*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.236015, size = 244, normalized size = 1.82

$$\frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{2\,(dx^2+c)\,bcd-2\,bc^2d+(dx^2+c)\,ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^2+c)^{\frac{3}{2}}\,b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] 1/2*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b
*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*
(d*x^2 + c)*b*c*d - 2*b*c^2*d + (d*x^2 + c)*a*d^2 + 2*a*c*d^2)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2
+ c)*b*c + sqrt(d*x^2 + c)*a*d)))/d
```

3.769 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}(bc-ad)^2}+\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{5/2}}$$

```
[Out] (-3*d*x)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - x/(2*(b*c - a*d)*(a
+ b*x^2)*Sqrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.271164, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}(bc-ad)^2}+\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d*x)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - x/(2*(b*c - a*d)*(a
+ b*x^2)*Sqrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 45.0008, size = 104, normalized size = 0.85

$$-\frac{3dx}{2\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)}+\frac{\left(ad+\frac{bc}{2}\right)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -3*d*x/(2*sqrt(c + d*x**2)*(a*d - b*c)**2) + x/(2*(a + b*x**2)*sq
rt(c + d*x**2)*(a*d - b*c)) + (a*d + b*c/2)*atanh(x*sqrt(a*d - b*
c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.246196, size = 110, normalized size = 0.89

$$\frac{1}{2}\left(\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}}-\frac{x\left(2ad+b\left(c+3dx^2\right)\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-((x*(2*a*d + b*(c + 3*d*x^2)))/((b*c - a*d)^2*(a + b*x^2)*Sqrt[
c + d*x^2])) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]
*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2)))/2
```

Maple [B] time = 0.022, size = 1453, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d*
(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d^2*a/(a*d-b*c
)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))+3/4/b/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/4/b/(a*d-
b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/b*d*(-a*b)^(1/2
)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/b*d^2*a/(a*d-b*c)^2/c/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x-3/4/b*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
+3/4/b/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4/(-a*b)^(1/2)/(a*d-
b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+1/4/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
))+1/4/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/(-a*b)^(1/2
)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [A] time = 0.594638, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(3\,bdx^3+(bc+2\,ad)x\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}-\left(\left(b^2cd+2\,abd^2\right)x^4+abc^2+2\,a^2cd+\left(b^2c^2+3\,abcd+2\,a^2d^2\right)x^2\right)\log\left(\right.}{8\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3}\right.$$

$$-\frac{2\left(3\,bdx^3+(bc+2\,ad)x\right)\sqrt{abc-a^2d}\sqrt{dx^2+c}-\left(\left(b^2cd+2\,abd^2\right)x^4+abc^2+2\,a^2cd+\left(b^2c^2+3\,abcd+2\,a^2d^2\right)x^2\right)\arctan}{4\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)\sqrt{abc-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(3*b*d*x^3 + (b*c + 2*a*d)*x)*sqrt(-a*b*c + a^2*d)*sqrt(
d*x^2 + c) - ((b^2*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (
b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^2)*log((((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(
-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 -
(a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
a^2)))/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a
*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^
2 + a^3*d^3)*x^2)*sqrt(-a*b*c + a^2*d)), -1/4*(2*(3*b*d*x^3 + (b*
c + 2*a*d)*x)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) - ((b^2*c*d + 2
*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*c*d + 2*a^
2*d^2)*x^2)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^
2*d)*sqrt(d*x^2 + c)*x)))/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2
*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 3.35368, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.770 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{3d}{2\sqrt{c+dx^2}(bc-ad)^2}-\frac{1}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

```
[Out] (-3*d)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - 1/(2*(b*c - a*d)*(a +
b*x^2)*Sqrt[c + d*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.223024, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{3d}{2\sqrt{c+dx^2}(bc-ad)^2}-\frac{1}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - 1/(2*(b*c - a*d)*(a +
b*x^2)*Sqrt[c + d*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 29.2105, size = 97, normalized size = 0.86

$$-\frac{3\sqrt{b}d\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2\left(ad-bc\right)^{\frac{5}{2}}}-\frac{3d}{2\sqrt{c+dx^2}\left(ad-bc\right)^2}+\frac{1}{2\left(a+bx^2\right)\sqrt{c+dx^2}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -3*sqrt(b)*d*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*(a
*d - b*c)**(5/2)) - 3*d/(2*sqrt(c + d*x**2)*(a*d - b*c)**2) + 1/(
2*(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.320424, size = 102, normalized size = 0.9

$$\frac{1}{2}\left(\frac{-2ad-b\left(c+3dx^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)^2}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((-2*a*d - b*(c + 3*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d
*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(b*c - a*d)^(5/2))/2
```

Maple [B] time = 0.019, size = 989, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-3/4*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*(-a*b)^(1/2)/b*
d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*d/(a*d-b*c)^2/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)
^(1/2)))-1/2*(-a*b)^(1/2)/a/b/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-3/4*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*(-a*b)^(1/2)/b*
d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*d/(a*d-b*c)^2/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))+1/2*(-a*b)^(1/2)/a/b/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281814, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bd^2x^4+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\cdots}{b^2x^4+2\,abx^2+a^2}\cdots\right.}{8\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2\cdots\right.\right.}\right.$$

$$\left.-\frac{3\left(bd^2x^4+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{bdx^2+2\,bc-ad}{2\sqrt{dx^2+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}\right)+2\left(3\,bdx^2+bc+2\,ad\right)\sqrt{dx^2+c}}{4\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(b/(b*c - a
*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^
2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^
```

```
2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) - 4*(3*b*d*x^2 + b*c + 2*a*d)*sqrt(d*x^2 +
 c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^
2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
 a^3*d^3)*x^2), -1/4*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)
*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d
*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) + 2*(3*b*d*x^2 + b*c
 + 2*a*d)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2
*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.222084, size = 203, normalized size = 1.8

$$-\frac{1}{2}\,d\left(\frac{3\,b\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{3\left(dx^2+c\right)b-2\,bc+2\,ad}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/2*d*(3*b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (3*(d*x^2 + c)
*b - 2*b*c + 2*a*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^2 + c)
^(3/2)*b - sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*a*d)))
```

3.771 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}+\frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] (d*(b*c + 2*a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b*x)
/(2*a*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)
*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2
)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.28875, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}+\frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b*x)
/(2*a*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)
*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2
)*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 55.4798, size = 121, normalized size = 0.85

$$-\frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}\,(ad-bc)}+\frac{dx\,(2ad+bc)}{2ac\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{b\,(4ad-bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}\,(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -b*x/(2*a*(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)) + d*x*(2*a*d
 + b*c)/(2*a*c*sqrt(c + d*x**2)*(a*d - b*c)**2) - b*(4*a*d - b*c)
*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(3/2)*
(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.372684, size = 120, normalized size = 0.85

$$\frac{1}{2}\left(\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}+\frac{x\sqrt{c+dx^2}\left(\frac{b^2}{a^2+abx^2}+\frac{2d^2}{c^2+cdx^2}\right)}{(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((x*Sqrt[c + d*x^2]*(b^2/(a^2 + a*b*x^2) + (2*d^2)/(c^2 + c*d*x^2
)))/(b*c - a*d)^2 + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(
```

```
Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2)))/2
```

Maple [B] time = 0.02, size = 1461, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a*d
*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*d^2/(a*d-b*c)^2
/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)*x-3/4/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)
^(1/2)))-1/4/a/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4/a/(a*d-b*c
)/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a*d*(-a*b)^(1/2)/(
a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*x+3/4/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/
(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4/a/(-a*b)^(1/2)/(a*d-b*c)*
b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))+1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/a/(-a*
b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [A] time = 0.668301, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(b^2cd+2\,abd^2\right)x^3+\left(b^2c^2+2\,a^2d^2\right)x\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}-\left(ab^2c^3-4\,a^2bc^2d+\left(b^3c^2d-4\,ab^2cd^2\right)x^4+\left(b^3c^3-3\,a\right.\right.}{8\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+\left(ab^3c^3d-2\,a^2b^2c^2d^2+a^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*((b^2*c*d + 2*a*b*d^2)*x^3 + (b^2*c^2 + 2*a^2*d^2)*x)*sqr
t(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - (a*b^2*c^3 - 4*a^2*b*c^2*d +
(b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^
2*b*c*d^2)*x^2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((
a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*
x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 +
 a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 +
a^4*c*d^3)*x^2)*sqrt(-a*b*c + a^2*d)), 1/4*(2*((b^2*c*d + 2*a*b*d
^2)*x^3 + (b^2*c^2 + 2*a^2*d^2)*x)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2
 + c) + (a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^2)*arctan(1/2*(
(b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x))
)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*
a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d -
 a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 3.29811, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.772 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + b/(2*a*
(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]]/(a^2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.654345, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + b/(2*a*
(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]]/(a^2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 82.1787, size = 144, normalized size = 0.85

$$-\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)}+\frac{d\,(2ad+bc)}{2ac\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{b^{\frac{3}{2}}\,(5ad-2bc)\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^2\,(ad-bc)^{\frac{5}{2}}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -b/(2*a*(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)) + d*(2*a*d + b
*c)/(2*a*c*sqrt(c + d*x**2)*(a*d - b*c)**2) + b**(3/2)*(5*a*d - 2
*b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*a**2*(a*d
 - b*c)**(5/2)) - atanh(sqrt(c + d*x**2)/sqrt(c))/(a**2*c**(3/2))
```

Mathematica [C] time = 1.8413, size = 406, normalized size = 2.39

$$\frac{1}{4}\left(\frac{b^{3/2}(2bc-5ad)\log\left(-\frac{4a^2(bc-ad)\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{3/2}\left(\sqrt{b}x+i\sqrt{a}\right)(2bc-5ad)}\right)}{a^2(bc-ad)^{5/2}}+\frac{b^{3/2}(2bc-5ad)\log\left(-\frac{4a^2(bc-ad)\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{3/2}\left(\sqrt{b}x-i\sqrt{a}\right)(2bc-5ad)}\right)}{a^2(bc-ad)^{5/2}}+\frac{2\sqrt{c+dx^2}\left(\frac{b^2}{a^2+abx^2}+\frac{2d^2}{c^2+cdx^2}\right)}{(bc-ad)^2}-\frac{4\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^2c^{3/2}}+\frac{4\log(x)}{a^2c^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((2*Sqrt[c + d*x^2]*(b^2/(a^2 + a*b*x^2) + (2*d^2)/(c^2 + c*d*x^2
)))/(b*c - a*d)^2 + (4*Log[x])/(a^2*c^(3/2)) - (4*Log[c + Sqrt[c]
*Sqrt[c + d*x^2]])/(a^2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*Log[(
-4*a^2*(b*c - a*d)*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqrt[
b*c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(3/
2)*(2*b*c - 5*a*d)*(I*Sqrt[a] + Sqrt[b]*x))])/(a^2*(b*c - a*d)^(5
/2)) + (b^(3/2)*(2*b*c - 5*a*d)*Log[(-4*a^2*(b*c - a*d)*(Sqrt[b]*
c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c +
d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(3/2)*(2*b*c - 5*a*d)*((-I)*Sqr
t[a] + Sqrt[b]*x))])/(a^2*(b*c - a*d)^(5/2)))/4
```

Maple [B] time = 0.023, size = 1672, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/a^2/c/(d*x^2+c)^(1/2)-1/a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)
^(1/2))/x)+1/2/a^2/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/2/a^2*(-a*b)
^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a^2/(a*d-b*c)*b/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))+1/2/a^2/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/a^2*(-a
*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/a^2/(a*d-b*c)*b
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))-1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/
2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+3/4/a*d/(a*d-b*c)^2*b/((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+3/4/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a
*d/(a*d-b*c)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/(-a*b)^(1/2)/a/(a*d-b*c)*b/c
```

```
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d+1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+1/b*(-
a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a*d/(a*d-b*c)^2*b/((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)-3/4/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
*x-3/4/a*d/(a*d-b*c)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/2/(-a*b)^(1/2)/a/(a*d-
b*c)*b/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x), x)
```

Fricas [A] time = 2.84598, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^3 - 5*a^2*b*c^2*d + (2*b^3*c^2*d - 5*a*b^2*c*d^
2)*x^4 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(c)
*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d
 + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c
 - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c^2 + 2*a^3*d^2
 + (a*b^2*c*d + 2*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(c) - 4*(a*
b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/x^2))/(
(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 + (a^2*b^3*c^3*d - 2*a
^3*b^2*c^2*d^2 + a^4*b*c*d^3)*x^4 + (a^2*b^3*c^4 - a^3*b^2*c^3*d
- a^4*b*c^2*d^2 + a^5*c*d^3)*x^2)*sqrt(c)), -1/8*((2*a*b^2*c^3 -
5*a^2*b*c^2*d + (2*b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (2*b^3*c^3 -
3*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-c)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
 - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d
 - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
2*a*b*x^2 + a^2)) - 4*(a*b^2*c^2 + 2*a^3*d^2 + (a*b^2*c*d + 2*a^2
*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 8*(a*b^2*c^3 - 2*a^2*b*c^
2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*arctan(sqrt(-
c)/sqrt(d*x^2 + c)))/((a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2
+ (a^2*b^3*c^3*d - 2*a^3*b^2*c^2*d^2 + a^4*b*c*d^3)*x^4 + (a^2*b^
3*c^4 - a^3*b^2*c^3*d - a^4*b*c^2*d^2 + a^5*c*d^3)*x^2)*sqrt(-c))
, -1/4*((2*a*b^2*c^3 - 5*a^2*b*c^2*d + (2*b^3*c^2*d - 5*a*b^2*c*d
^2)*x^4 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(c
)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(
d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(a*b^2*c^2 + 2*
a^3*d^2 + (a*b^2*c*d + 2*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(c)
- 2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2
*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
```

```
a^3*d^3)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)*c)/
x^2))/((a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 + (a^2*b^3*c^3*
d - 2*a^3*b^2*c^2*d^2 + a^4*b*c*d^3)*x^4 + (a^2*b^3*c^4 - a^3*b^2
*c^3*d - a^4*b*c^2*d^2 + a^5*c*d^3)*x^2)*sqrt(c)), -1/4*((2*a*b^2
*c^3 - 5*a^2*b*c^2*d + (2*b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (2*b^3
*c^3 - 3*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-c)*sqrt(-b/(b*c
- a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c
 - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(a*b^2*c^2 + 2*a^3*d^2 + (a*b^
2*c*d + 2*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 4*(a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b
*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*
arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((a^3*b^2*c^4 - 2*a^4*b*c^3*d +
 a^5*c^2*d^2 + (a^2*b^3*c^3*d - 2*a^3*b^2*c^2*d^2 + a^4*b*c*d^3)*
x^4 + (a^2*b^3*c^4 - a^3*b^2*c^3*d - a^4*b*c^2*d^2 + a^5*c*d^3)*x
^2)*sqrt(-c))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.236842, size = 319, normalized size = 1.88

$$-\frac{1}{2}\,d^2\left(\frac{\left(2\,b^3c-5\,ab^2d\right)\,\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2b^2c^2d^2-2\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{\left(dx^2+c\right)b^2c+2\left(dx^2+c\right)abd-2\,abcd+2\,a^2d^2}{\left(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3\right)\left(\left(dx^2+c\right)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x),x, algorithm="giac")
```

```
[Out] -1/2*d^2*((2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^
2*c + a*b*d))/((a^2*b^2*c^2*d^2 - 2*a^3*b*c*d^3 + a^4*d^4)*sqrt(-
b^2*c + a*b*d)) - ((d*x^2 + c)*b^2*c + 2*(d*x^2 + c)*a*b*d - 2*a*
b*c*d + 2*a^2*d^2)/((a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*(
(d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*a*d))
 - 2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c*d^2))
```

3.773 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx\sqrt{c+dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) + b/(2*
a*(b*c - a*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 4*a*
b*c*d + 4*a^2*d^2)*Sqrt[c + d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*x) -
 (3*b^2*(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.776525, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) + b/(2*
a*(b*c - a*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 4*a*
b*c*d + 4*a^2*d^2)*Sqrt[c + d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*x) -
 (3*b^2*(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 135.396, size = 178, normalized size = 0.87

$$-\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)}+\frac{d\,(2ad+bc)}{2acx\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x\,(ad-bc)^2}+\frac{3b^2\,(2ad-bc)\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{5}{2}}\,(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -b/(2*a*x*(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)) + d*(2*a*d +
 b*c)/(2*a*c*x*sqrt(c + d*x**2)*(a*d - b*c)**2) - sqrt(c + d*x**2
)*(4*a**2*d**2 - 4*a*b*c*d + 3*b**2*c**2)/(2*a**2*c**2*x*(a*d - b
*c)**2) + 3*b**2*(2*a*d - b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*s
qrt(c + d*x**2)))/(2*a**(5/2)*(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.611781, size = 145, normalized size = 0.71

$$\sqrt{c+dx^2}\left(-\frac{\frac{b^3x}{2(a+bx^2)(bc-ad)^2}+\frac{1}{c^2x}}{a^2}-\frac{d^3x}{c^2\left(c+dx^2\right)(bc-ad)^2}\right)-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-((d^3*x)/(c^2*(b*c - a*d)^2*(c + d*x^2))) - (1/
(c^2*x) + (b^3*x)/(2*(b*c - a*d)^2*(a + b*x^2)))/a^2) - (3*b^2*(b
*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(2*a^(5/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.026, size = 1524, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/a^2/c/x/(d*x^2+c)^(1/2)-2/a^2*d/c^2*x/(d*x^2+c)^(1/2)+1/4/a^2/
(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a^2*d*(-
a*b)^(1/2)/(a*d-b*c)^2*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a*d^2*b/(a*d-b*c
)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)^2*b/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))-1/4/a^2/(a*d-b*c)*b/c/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/4
/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4/a^2
*d*(-a*b)^(1/2)/(a*d-b*c)^2*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4/a*d^2*b/(a*
d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-3/4/a^2*d*(-a*b)^(1/2)/(a*d-b*c)
^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))-1/4/a^2/(a*d-b*c)*b/c/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*
d+3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/
a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-3/4*b^2/a^2/(-a*b)^(1/
2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2), x)
```

Fricas [A] time = 0.771603, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2 + (3*b^3*c^2*
d - 4*a*b^2*c*d^2 + 4*a^2*b*d^3)*x^4 + (3*b^3*c^3 - 2*a*b^2*c^2*d
 - 2*a^2*b*c*d^2 + 4*a^3*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^
2 + c) + 3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^4*c^4 - a*b^3*
c^3*d - 2*a^2*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d)*x)
*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a
^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 +
 c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(((a^2*b^3*c^4*d - 2*a^3*b^2*c
^3*d^2 + a^4*b*c^2*d^3)*x^5 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*
b*c^3*d^2 + a^5*c^2*d^3)*x^3 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5
*c^3*d^2)*x)*sqrt(-a*b*c + a^2*d)), -1/4*(2*(2*a*b^2*c^3 - 4*a^2*
b*c^2*d + 2*a^3*c*d^2 + (3*b^3*c^2*d - 4*a*b^2*c*d^2 + 4*a^2*b*d^
3)*x^4 + (3*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c*d^2 + 4*a^3*d^3)*
x^2)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) + 3*((b^4*c^3*d - 2*a*b^
3*c^2*d^2)*x^5 + (b^4*c^4 - a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2)*x^3
+ (a*b^3*c^4 - 2*a^2*b^2*c^3*d)*x)*arctan(1/2*((b*c - 2*a*d)*x^2
- a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a^2*b^3*c^4*d
 - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^5 + (a^2*b^3*c^5 - a^3*b^
2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3 + (a^3*b^2*c^5 - 2*a^4
*b*c^4*d + a^5*c^3*d^2)*x)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.774 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}$$

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(2*a^2*c^2*(b*c - a*d)^2
*Sqrt[c + d*x^2]) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b
*x^2)*Sqrt[c + d*x^2]) - 1/(2*a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2]
) + ((4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(
5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(5/2))
```

Rubi [A] time = 1.02358, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(2*a^2*c^2*(b*c - a*d)^2
*Sqrt[c + d*x^2]) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b
*x^2)*Sqrt[c + d*x^2]) - 1/(2*a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2]
) + ((4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(
5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 123.639, size = 216, normalized size = 0.9

$$-\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{b\left(ad-2bc\right)}{2a^2c\left(a+bx^2\right)\sqrt{c+dx^2}\left(ad-bc\right)}-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}\left(ad-bc\right)^2}-\frac{b^{\frac{5}{2}}\left(7ad-4bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^3\left(ad-bc\right)^{\frac{5}{2}}}+\frac{\left(3ad+4bc\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] -1/(2*a*c*x**2*(a + b*x**2)*sqrt(c + d*x**2)) - b*(a*d - 2*b*c)/(
2*a**2*c*(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)) - d*(3*a**2*d
**2 - 2*a*b*c*d + 2*b**2*c**2)/(2*a**2*c**2*sqrt(c + d*x**2)*(a*d
 - b*c)**2) - b**(5/2)*(7*a*d - 4*b*c)*atan(sqrt(b)*sqrt(c + d*x*
*2)/sqrt(a*d - b*c))/(2*a**3*(a*d - b*c)**(5/2)) + (3*a*d + 4*b*c
)*atanh(sqrt(c + d*x**2)/sqrt(c))/(2*a**3*c**(5/2))
```

Mathematica [C] time = 3.81412, size = 451, normalized size = 1.87

$$\frac{1}{4}\left(-\frac{b^{5/2}(4bc-7ad)\log\left(\frac{4a^3(bc-ad)\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{5/2}\left(\sqrt{b}x+i\sqrt{a}\right)(4bc-7ad)}\right)}{a^3(bc-ad)^{5/2}} - \frac{b^{5/2}(4bc-7ad)\log\left(\frac{4a^3(bc-ad)\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{5/2}\left(\sqrt{b}x-i\sqrt{a}\right)(4bc-7ad)}\right)}{a^3(bc-ad)^{5/2}} + \frac{2(3ad+4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^3c^{5/2}} - \frac{2\log(x)(3ad+4bc)}{a^3c^{5/2}} + 4\sqrt{c+dx^2}\left(\frac{-\frac{b^3}{2(a+bx^2)(bc-ad)^2}-\frac{1}{2c^2x^2}}{a^2}-\frac{d^3}{c^2(c+dx^2)(bc-ad)^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (4*Sqrt[c + d*x^2]*(-(d^3/(c^2*(b*c - a*d)^2*(c + d*x^2))) + (-1/
(2*c^2*x^2) - b^3/(2*(b*c - a*d)^2*(a + b*x^2)))/a^2) - (2*(4*b*c
 + 3*a*d)*Log[x])/(a^3*c^(5/2)) + (2*(4*b*c + 3*a*d)*Log[c + Sqrt
[c]*Sqrt[c + d*x^2]])/(a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*Lo
g[(4*a^3*(b*c - a*d)*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a]*d*Sqr
t[b*c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(
5/2)*(4*b*c - 7*a*d)*(I*Sqrt[a] + Sqrt[b]*x))])/(a^3*(b*c - a*d)^
(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*Log[(4*a^3*(b*c - a*d)*(Sqrt[b]
*c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c +
 d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(5/2)*(4*b*c - 7*a*d)*((-I)*Sq
rt[a] + Sqrt[b]*x))])/(a^3*(b*c - a*d)^(5/2)))/4
```

Maple [B] time = 0.025, size = 1778, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/a^2/c/x^2/(d*x^2+c)^(1/2)-3/2/a^2*d/c^2/(d*x^2+c)^(1/2)+3/2/
a^2*d/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-b^2/a^3/(a*d-
b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+b/a^3*(-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x*d+b^2/a^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-b^2/a^3/(a*d-b*c
)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-b/a^3*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)*x*d+b^2/a^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-2*b/a^3/c/(d*x^2+c)
^(1/2)+2*b/a^3/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4*
b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-3/4*b^2/a^2*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/a/(-
```

```
a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*b^2/a^2*d/
(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2)))+1/2*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/c/
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)*x*d-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1/b
*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^2*d/(a*d-b*c)^2/((x+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)+3/4*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x+3/4*b^2/a^2*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/2*b^2/a
^2/(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3), x)
```

Fricas [A] time = 5.38081, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^6 + (4*b^4*c^4 - 3*a*b^
3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^4 + (4*a*b^3*c^4 - 7*a^2*b^2*c^3*d
)*x^2)*sqrt(c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c
^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b^2
*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^2*d - 2*a^2*b^2*c*d
^2 + 3*a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^
2 + 3*a^4*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(c) - 2*((4*b^4*c^3*d - 5
*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^4*c^4
- a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4
+ (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)
*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/((
(a^3*b^3*c^4*d - 2*a^4*b^2*c^3*d^2 + a^5*b*c^2*d^3)*x^6 + (a^3*b^
3*c^5 - a^4*b^2*c^4*d - a^5*b*c^3*d^2 + a^6*c^2*d^3)*x^4 + (a^4*b
^2*c^5 - 2*a^5*b*c^4*d + a^6*c^3*d^2)*x^2)*sqrt(c)), -1/8*(((4*b^
4*c^3*d - 7*a*b^3*c^2*d^2)*x^6 + (4*b^4*c^4 - 3*a*b^3*c^3*d - 7*a
^2*b^2*c^2*d^2)*x^4 + (4*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x^2)*sqrt(-
c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c
*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b
*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b^2*c^3 - 2*a^3
*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^2*d - 2*a^2*b^2*c*d^2 + 3*a^3*b
*d^3)*x^4 + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^
3)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) - 4*((4*b^4*c^3*d - 5*a*b^3*c^2*
d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^4*c^4 - a*b^3*c^3
```

```
*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4*a*b^3*
c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*arcta
n(sqrt(-c)/sqrt(d*x^2 + c)))/(((a^3*b^3*c^4*d - 2*a^4*b^2*c^3*d^2
 + a^5*b*c^2*d^3)*x^6 + (a^3*b^3*c^5 - a^4*b^2*c^4*d - a^5*b*c^3*
d^2 + a^6*c^2*d^3)*x^4 + (a^4*b^2*c^5 - 2*a^5*b*c^4*d + a^6*c^3*d
^2)*x^2)*sqrt(-c)), 1/4*(((4*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^6 + (
4*b^4*c^4 - 3*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^4 + (4*a*b^3*c^4
 - 7*a^2*b^2*c^3*d)*x^2)*sqrt(c)*sqrt(-b/(b*c - a*d))*arctan(-1/2
*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*
c - a*d)))) - 2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b
^3*c^2*d - 2*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - a^
2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(
c) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*
d^4)*x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c
*d^3 + 3*a^4*d^4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*
c^2*d^2 + 3*a^4*c*d^3)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(
d*x^2 + c)*c)/x^2))/(((a^3*b^3*c^4*d - 2*a^4*b^2*c^3*d^2 + a^5*b*
c^2*d^3)*x^6 + (a^3*b^3*c^5 - a^4*b^2*c^4*d - a^5*b*c^3*d^2 + a^6
*c^2*d^3)*x^4 + (a^4*b^2*c^5 - 2*a^5*b*c^4*d + a^6*c^3*d^2)*x^2)*
sqrt(c)), 1/4*(((4*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^6 + (4*b^4*c^4
- 3*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^4 + (4*a*b^3*c^4 - 7*a^2*b
^2*c^3*d)*x^2)*sqrt(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2
 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))
)) - 2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^2*d
- 2*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - a^2*b^2*c^2
*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 2*(
(4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x
^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 +
 3*a^4*d^4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^
2 + 3*a^4*c*d^3)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(((a^3*b^
3*c^4*d - 2*a^4*b^2*c^3*d^2 + a^5*b*c^2*d^3)*x^6 + (a^3*b^3*c^5 -
 a^4*b^2*c^4*d - a^5*b*c^3*d^2 + a^6*c^2*d^3)*x^4 + (a^4*b^2*c^5
- 2*a^5*b*c^4*d + a^6*c^3*d^2)*x^2)*sqrt(-c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.238508, size = 510, normalized size = 2.12

$$\frac{1}{2}\,d^3\left(\frac{\left(4\,b^4c - 7\,ab^3d\right)\,\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3b^2c^2d^3 - 2\,a^4bcd^4 + a^5d^5\right)\sqrt{-b^2c+abd}} - \frac{2\left(dx^2+c\right)^2b^3c^2 - 2\left(dx^2+c\right)b^3c^3 - 2\left(dx^2+c\right)^2ab^2cd + 3\left(dx^2+c\right)ab^2\cdots}{\left(a^2b^2c^4d^2 - 2\,a^3bc^3d^3 + a^4c^2d^4\right)\left(\left(dx^2+c\right)^{\frac{5}{2}}b - 2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*d^3*((4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2
*c + a*b*d))/((a^3*b^2*c^2*d^3 - 2*a^4*b*c*d^4 + a^5*d^5)*sqrt(-b
^2*c + a*b*d)) - (2*(d*x^2 + c)^2*b^3*c^2 - 2*(d*x^2 + c)*b^3*c^3
 - 2*(d*x^2 + c)^2*a*b^2*c*d + 3*(d*x^2 + c)*a*b^2*c^2*d + 3*(d*x
^2 + c)^2*a^2*b*d^2 - 7*(d*x^2 + c)*a^2*b*c*d^2 + 2*a^2*b*c^2*d^2
 + 3*(d*x^2 + c)*a^3*d^3 - 2*a^3*c*d^3)/((a^2*b^2*c^4*d^2 - 2*a^3
*b*c^3*d^3 + a^4*c^2*d^4)*((d*x^2 + c)^(5/2)*b - 2*(d*x^2 + c)^(3
/2)*b*c + sqrt(d*x^2 + c)*b*c^2 + (d*x^2 + c)^(3/2)*a*d - sqrt(d*
x^2 + c)*a*c*d)) - (4*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c
))/(a^3*sqrt(-c)*c^2*d^3))
```

3.775 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{6a^3c^3x(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx^3\sqrt{c+dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) + b/(
2*a*(b*c - a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 -
4*a*b*c*d + 8*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*
x^3) + ((15*b^3*c^3 - 14*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3
)*Sqrt[c + d*x^2])/(6*a^3*c^3*(b*c - a*d)^2*x) + (b^3*(5*b*c - 8*
a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^
(7/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 1.16252, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{6a^3c^3x(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx^3\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) + b/(
2*a*(b*c - a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 -
4*a*b*c*d + 8*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*
x^3) + ((15*b^3*c^3 - 14*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3
)*Sqrt[c + d*x^2])/(6*a^3*c^3*(b*c - a*d)^2*x) + (b^3*(5*b*c - 8*
a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^
(7/2)*(b*c - a*d)^(5/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.745059, size = 167, normalized size = 0.6

$$\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}+\sqrt{c+dx^2}\left(\frac{\frac{b^4x}{2(a+bx^2)(bc-ad)^2}+\frac{2b}{c^2x}}{a^3}-\frac{c-5dx^2}{3a^2c^3x^3}+\frac{d^4x}{c^3\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(c - 5*d*x^2)/(3*a^2*c^3*x^3) + (d^4*x)/(c^3*(b
*c - a*d)^2*(c + d*x^2)) + ((2*b)/(c^2*x) + (b^4*x)/(2*(b*c - a*d
)^2*(a + b*x^2)))/a^3) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

Maple [B] time = 0.028, size = 1608, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3/a^2/c/x^3/(d*x^2+c)^(1/2)+4/3/a^2*d/c^2/x/(d*x^2+c)^(1/2)+8/
3/a^2*d^2/c^3*x/(d*x^2+c)^(1/2)+2*b/a^3/c/x/(d*x^2+c)^(1/2)+4*b/a
^3*d/c^2*x/(d*x^2+c)^(1/2)-1/4*b^2/a^3/(a*d-b*c)/(x-1/b*(-a*b)^(1
/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)*x-3/4*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/4*
b^2/a^3/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4*b^2/a^3/(a*d-b*c)
/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^3*d*(-a*b)^(1
/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2*d^2/(a*d-b*c)^2/c
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x+3/4*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-
a*b)^(1/2)))+3/4*b^2/a^3/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-5/4*
b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4*b^3/a^3/(-
a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+5/4*b^3/a^3/(-a*b)^(1/2)/(a*
d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-
a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^2 (dx^2 + c)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4), x)
```

Fricas [A] time = 1.25841, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(2*a^2*b^2*c^4 - 4*a^3*b*c^3*d + 2*a^4*c^2*d^2 - (15*b^
4*c^3*d - 14*a*b^3*c^2*d^2 - 8*a^2*b^2*c*d^3 + 16*a^3*b*d^4)*x^6
- (15*b^4*c^4 - 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 16*a^4*d^4)*
x^4 - 2*(5*a*b^3*c^4 - 6*a^2*b^2*c^3*d - 3*a^3*b*c^2*d^2 + 4*a^4*
c*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) + 3*((5*b^5*c^4*
d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d - 8*a^2*b^3
*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*log((((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*
c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*
a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*
b*c^3*d^3)*x^7 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a
^6*c^3*d^3)*x^5 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3
)*sqrt(-a*b*c + a^2*d)), -1/12*(2*(2*a^2*b^2*c^4 - 4*a^3*b*c^3*d
+ 2*a^4*c^2*d^2 - (15*b^4*c^3*d - 14*a*b^3*c^2*d^2 - 8*a^2*b^2*c*
d^3 + 16*a^3*b*d^4)*x^6 - (15*b^4*c^4 - 4*a*b^3*c^3*d - 18*a^2*b^
2*c^2*d^2 + 16*a^4*d^4)*x^4 - 2*(5*a*b^3*c^4 - 6*a^2*b^2*c^3*d -
3*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2)*sqrt(a*b*c - a^2*d)*sqrt(d*x^
2 + c) - 3*((5*b^5*c^4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*
a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c
^4*d)*x^3)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2
*d)*sqrt(d*x^2 + c)*x)))/(((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a
^5*b*c^3*d^3)*x^7 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2
+ a^6*c^3*d^3)*x^5 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*
x^3)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 9.97954, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.776 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

```
[Out] ((2*b*c + 3*a*d)*x)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (a*x)
/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b*c + 11*a
*d)*x)/(6*(b*c - a*d)^3*Sqrt[c + d*x^2]) - (Sqrt[a]*(3*b*c + 2*a*
d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c
 - a*d)^(7/2))
```

Rubi [A] time = 0.578457, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((2*b*c + 3*a*d)*x)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (a*x)
/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b*c + 11*a
*d)*x)/(6*(b*c - a*d)^3*Sqrt[c + d*x^2]) - (Sqrt[a]*(3*b*c + 2*a*
d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c
 - a*d)^(7/2))
```

Rubi in Sympy [A] time = 88.8406, size = 151, normalized size = 0.87

$$\frac{\sqrt{a}\left(2ad+3bc\right)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\left(ad-bc\right)^{\frac{7}{2}}}-\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\left(ad-bc\right)}-\frac{x\left(11ad+4bc\right)}{6\sqrt{c+dx^2}\left(ad-bc\right)^3}+\frac{x\left(3ad+2bc\right)}{6b\left(c+dx^2\right)^{\frac{3}{2}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] sqrt(a)*(2*a*d + 3*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c +
 d*x**2)))/(2*(a*d - b*c)**(7/2)) - a*x/(2*b*(a + b*x**2)*(c + d*
x**2)**(3/2)*(a*d - b*c)) - x*(11*a*d + 4*b*c)/(6*sqrt(c + d*x**2
)*(a*d - b*c)**3) + x*(3*a*d + 2*b*c)/(6*b*(c + d*x**2)**(3/2)*(a
*d - b*c)**2)
```

Mathematica [A] time = 0.450762, size = 158, normalized size = 0.91

$$\sqrt{c+dx^2}\left(\frac{abx}{2\left(a+bx^2\right)(bc-ad)^3}+\frac{2x(2ad+bc)}{3\left(c+dx^2\right)(bc-ad)^3}+\frac{cx}{3\left(c+dx^2\right)^2(bc-ad)^2}\right)-\frac{\sqrt{a}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*((a*b*x)/(2*(b*c - a*d)^3*(a + b*x^2)) + (c*x)/(3
*(b*c - a*d)^2*(c + d*x^2)^2) + (2*(b*c + 2*a*d)*x)/(3*(b*c - a*d
)^3*(c + d*x^2))) - (Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.036, size = 2463, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/3/b^2*x/c/(d
*x^2+c)^(3/2)+2/3/b^2/c^2*x/(d*x^2+c)^(1/2)-3/4*a/(-a*b)^(1/2)/(a
*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))-1/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-3/
4*a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4*a/b^2/(a*d-b*c)/
(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4*a/b^2/(a*d-b*c)/(x+1
/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b*a/(-a*b)^(1/2)/(a*d-b*
c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)+5/12*a^2/b^2*d^2/(a*d-b*c)^2/c/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)*x+5/6*a^2/b^2*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-
5/4*a^2/b*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*a/b*d*(-a*b
)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+3/4/b*a/(a*d-b*c)^2/c/((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x*d-7/12*a/b^2*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-7
/6*a/b^2*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/12*a^2/b^2*d^2/(
a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/6*a^2/b^2*d^2/(a*d-b*c)^2/c^
2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x-5/4*a^2/b*d^2/(a*d-b*c)^3/c/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x+5/12*a/b^2*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)-5/4*a/b*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/12*a/b
^2*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*a/b*d*(-a*b
)^(1/2)/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*a/b*d*(-a*b)^(1/2)/(a
*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2)))-7/12*a/b^2*d/(a*d-b*c)/c/((x+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
```

```
(3/2)*x-7/6*a/b^2*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4/b*a/(
a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [A] time = 1.03612, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*
d + (6*b^2*c^2*d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*
a*b*c^2*d + 4*a^2*c*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2
*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x
^2 + a^2)) - 4*((4*b^2*c*d + 11*a*b*d^2)*x^5 + 2*(3*b^2*c^2 + 8*a
*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x)*sqrt(d*x^2
 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d
^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5
)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*
b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d
^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), -1/12*(3*((3*b^2*c*d^2
+ 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2*c^2*d + 7*a*b
*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*d^2)
*x^2)*sqrt(a/(b*c - a*d))*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(s
qrt(d*x^2 + c)*(b*c - a*d)*x*sqrt(a/(b*c - a*d)))) - 2*((4*b^2*c*
d + 11*a*b*d^2)*x^5 + 2*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x^3 +
 3*(3*a*b*c^2 + 2*a^2*c*d)*x)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2
*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b
^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*
a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 +
(b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*
a^4*c*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 5.01781, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.777 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{a}{2b\,(a+bx^2)\,(c+dx^2)^{3/2}\,(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}(bc-ad)^3}+\frac{3ad+2bc}{6b\,(c+dx^2)^{3/2}\,(bc-ad)^2}-\frac{\sqrt{b}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

```
[Out] (2*b*c + 3*a*d)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + a/(2*b*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (2*b*c + 3*a*d)/(2*(b*
c - a*d)^3*Sqrt[c + d*x^2]) - (Sqrt[b]*(2*b*c + 3*a*d)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.492164, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{a}{2b\,(a+bx^2)\,(c+dx^2)^{3/2}\,(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}(bc-ad)^3}+\frac{3ad+2bc}{6b\,(c+dx^2)^{3/2}\,(bc-ad)^2}-\frac{\sqrt{b}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*b*c + 3*a*d)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + a/(2*b*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (2*b*c + 3*a*d)/(2*(b*
c - a*d)^3*Sqrt[c + d*x^2]) - (Sqrt[b]*(2*b*c + 3*a*d)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 47.5891, size = 141, normalized size = 0.83

$$-\frac{a}{2b\,(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}\,(ad-bc)}-\frac{\sqrt{b}\left(\frac{3ad}{2}+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{(ad-bc)^{\frac{7}{2}}}-\frac{\frac{3ad}{2}+bc}{\sqrt{c+dx^2}\,(ad-bc)^3}+\frac{\frac{3ad}{2}+bc}{3b\,(c+dx^2)^{\frac{3}{2}}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] -a/(2*b*(a + b*x**2)*(c + d*x**2)**(3/2)*(a*d - b*c)) - sqrt(b)*(
3*a*d/2 + b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(a*
d - b*c)**(7/2) - (3*a*d/2 + b*c)/(sqrt(c + d*x**2)*(a*d - b*c)**
3) + (3*a*d/2 + b*c)/(3*b*(c + d*x**2)**(3/2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.731788, size = 137, normalized size = 0.81

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^2}\left(\frac{6(ad+bc)}{c+dx^2}+\frac{2c(bc-ad)}{(c+dx^2)^2}+\frac{3ab}{a+bx^2}\right)}{(bc-ad)^3}-\frac{3\sqrt{b}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[c + d*x^2]*((3*a*b)/(a + b*x^2) + (2*c*(b*c - a*d))/(c + d
*x^2)^2 + (6*(b*c + a*d))/(c + d*x^2)))/(b*c - a*d)^3 - (3*Sqrt[b
]*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*
d]])/(b*c - a*d)^(7/2))/6
```

Maple [B] time = 0.023, size = 2400, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/12/b*a*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/b^2*(-a*b)^(1/2)/(
a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/2/(a*d-b*c)^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/b/(a
*d-b*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/b^2*d*(-a*b)
^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/b^2*d*(-a*b)^(1/2)/(a*
d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/b/(a*d-b*c)^2*(-a*b)^(1/2)/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d-1/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x-5/4*a*d/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*a*d/
(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/6/b/(a*d-b*c)/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)-1/2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/6/b/(a*d-b*c)/((x-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)-1/2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-5/12/b^2*(-a*b)^(1/2)*d^2*a
/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/4*a*d/(a*d-b*c)^3/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))+5/12/b*a*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*a*d/(a*d-b
*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))-1/4/b^2*(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*b
)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/b^2*d*(-a*b)^(1/2)/(a*d-b*c)/c/(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)*x+5/4/b*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^3/c/((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x-5/6/b^2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c^2/((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x+5/6/b^2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c^2/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)*x-5/4/b*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^3/c/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x+5/12/b^2*(-a*b)^(1/2)*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
```

```
/b)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.405653, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="fricas")

[Out] [-1/24*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*
d + (4*b^2*c^2*d + 8*a*b*c*d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*
a*b*c^2*d + 6*a^2*c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^
4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2
)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) - 4*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2 + 4*a^2*c*d +
2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^
3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b
^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^
4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b
*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/12*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*
x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2*c^2*d + 8*a*b*c*d^2 + 3*a^
2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*d^2)*x^2)*sqrt(-b
/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c
)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) + 2*(3*(2*b^2*c*d + 3*a*b*d^
2)*x^4 + 11*a*b*c^2 + 4*a^2*c*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^
2*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3
*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2
*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*
a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c
^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.237834, size = 351, normalized size = 2.06

$$\frac{\frac{3\sqrt{dx^2+c}abd^2}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx^2+c)b-bc+ad)}+\frac{3\left(2\,b^2cd+3\,abd^2\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}+\frac{2\left(3\left(dx^2+c\right)bcd+bc^2d+3\left(dx^2+c\right)ad^2-acd^2\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx^2+c)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(d*x^2 + c)*a*b*d^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*((d*x^2 + c)*b - b*c + a*d)) + 3*(2*b^2*c*d +
3*a*b*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d
)) + 2*(3*(d*x^2 + c)*b*c*d + b*c^2*d + 3*(d*x^2 + c)*a*d^2 - a*c
*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2
 + c)^(3/2)))/d
```

3.778 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}(bc-ad)^3}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}-\frac{5dx}{6(c+dx^2)^{3/2}(bc-ad)^2}+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

```
[Out] (-5*d*x)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - x/(2*(b*c - a*d)*(
a + b*x^2)*(c + d*x^2)^(3/2)) - (d*(13*b*c + 2*a*d)*x)/(6*c*(b*c
- a*d)^3*Sqrt[c + d*x^2]) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.444429, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}(bc-ad)^3}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}-\frac{5dx}{6(c+dx^2)^{3/2}(bc-ad)^2}+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d*x)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - x/(2*(b*c - a*d)*(
a + b*x^2)*(c + d*x^2)^(3/2)) - (d*(13*b*c + 2*a*d)*x)/(6*c*(b*c
- a*d)^3*Sqrt[c + d*x^2]) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 79.0366, size = 143, normalized size = 0.88

$$-\frac{5dx}{6(c+dx^2)^{\frac{3}{2}}(ad-bc)^2}+\frac{x}{2(a+bx^2)(c+dx^2)^{\frac{3}{2}}(ad-bc)}+\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}(ad-bc)^3}-\frac{b(4ad+bc)\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(ad-bc)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] -5*d*x/(6*(c + d*x**2)**(3/2)*(a*d - b*c)**2) + x/(2*(a + b*x**2)
*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*x*(2*a*d + 13*b*c)/(6*c*sqr
t(c + d*x**2)*(a*d - b*c)**3) - b*(4*a*d + b*c)*atanh(x*sqrt(a*d
- b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*sqrt(a)*(a*d - b*c)**(7/2))
```

Mathematica [A] time = 0.383999, size = 163, normalized size = 1.

$$\sqrt{c+dx^2}\left(-\frac{b^2x}{2(a+bx^2)(bc-ad)^3}-\frac{dx(ad+5bc)}{3c(c+dx^2)(bc-ad)^3}-\frac{dx}{3(c+dx^2)^2(bc-ad)^2}\right)+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(b^2*x)/(2*(b*c - a*d)^3*(a + b*x^2)) - (d*x)/(
3*(b*c - a*d)^2*(c + d*x^2)^2) - (d*(5*b*c + a*d)*x)/(3*c*(b*c -
a*d)^3*(c + d*x^2))) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.024, size = 2369, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/4/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+5/12/b*d*(-a*b)^(1/2)/
(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(
-a*b)^(1/2)))-1/4/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/4/(-a*b
)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/12/(-a*b)^(1/2)/(a*d-b*c)
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)-1/12/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)-5/12/b*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/12/b*
d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/6/b*d/(a*d-b*c)/c^2/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x+5/4*d^2*a/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*
d^2*a/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/12/b*d/(a*d-b*c)/c/((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)*x+5/6/b*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1
/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/12/b*d^2*a/(a*d-b
*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5/6/b*d^2*a/(a*d-b*c)^2/c^2/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x-5/12/b*d^2*a/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-5
/6/b*d^2*a/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*d*(-a*b)^(1/
2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)-1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+1/4/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*d
*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b/(a*d-b*c)/(x+
1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*d*(-a*b)^(1/2)/(a*d-b*c
```

```
)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [A] time = 1.18564, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*((13*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 2*(9*b^2*c^2*d + 5*a*
b*c*d^2 + a^2*d^3)*x^3 + 3*(b^2*c^3 + 4*a*b*c^2*d)*x)*sqrt(-a*b*c
 + a^2*d)*sqrt(d*x^2 + c) + 3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c
^2*d^2 + 4*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*
a^2*b*c*d^3)*x^4 + (b^3*c^4 + 6*a*b^2*c^3*d + 8*a^2*b*c^2*d^2)*x^
2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3
*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a*b^3*c^6 - 3*a^2*b^2*c^5*
d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^
3 + 3*a^2*b^2*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3
*c^4*d^2 + 3*a^2*b^2*c^3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (
b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a
^4*c^2*d^4)*x^2)*sqrt(-a*b*c + a^2*d)), -1/12*(2*((13*b^2*c*d^2 +
 2*a*b*d^3)*x^5 + 2*(9*b^2*c^2*d + 5*a*b*c*d^2 + a^2*d^3)*x^3 + 3
*(b^2*c^3 + 4*a*b*c^2*d)*x)*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c) -
 3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*d^3)*x^6
 + (2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4
 + 6*a*b^2*c^3*d + 8*a^2*b*c^2*d^2)*x^2)*arctan(1/2*((b*c - 2*a*d
)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((a*b^3*c^
6 - 3*a^2*b^2*c^5*d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^
2 - 3*a*b^3*c^3*d^3 + 3*a^2*b^2*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b
^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^2*c^3*d^3 + a^3*b*c^2*d^4 -
a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 + 5*a
^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 4.49879, size = 803, normalized size = 4.93

$$-\frac{\left(\frac{\left(5\,b^{4}c^{4}d^{3}-14\,ab^{3}c^{3}d^{4}+12\,a^{2}b^{2}c^{2}d^{5}-2\,a^{3}bcd^{6}-a^{4}d^{7}\right)x^{2}}{b^{6}c^{7}d-6\,ab^{5}c^{6}d^{2}+15\,a^{2}b^{4}c^{5}d^{3}-20\,a^{3}b^{3}c^{4}d^{4}+15\,a^{4}b^{2}c^{3}d^{5}-6\,a^{5}bc^{2}d^{6}+a^{6}cd^{7}}+\frac{6\left(b^{4}c^{5}d^{2}-3\,ab^{3}c^{4}d^{3}+3\,a^{2}b^{2}c^{3}d^{4}-a^{3}bc^{2}d^{5}\right)}{b^{6}c^{7}d-6\,ab^{5}c^{6}d^{2}+15\,a^{2}b^{4}c^{5}d^{3}-20\,a^{3}b^{3}c^{4}d^{4}+15\,a^{4}b^{2}c^{3}d^{5}-6\,a^{5}bc^{2}d^{6}+a^{6}cd^{7}}\right)x}{3\,(dx^{2}+c)^{\frac{3}{2}}} - \frac{\left(b^{2}c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\sqrt{abcd-a^{2}d^{2}}} + \frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abd^{\frac{3}{2}}-b^{2}c^{2}\sqrt{d}}{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] -1/3*((5*b^4*c^4*d^3 - 14*a*b^3*c^3*d^4 + 12*a^2*b^2*c^2*d^5 - 2*
a^3*b*c*d^6 - a^4*d^7)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*
b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c
^2*d^6 + a^6*c*d^7) + 6*(b^4*c^5*d^2 - 3*a*b^3*c^4*d^3 + 3*a^2*b^
2*c^3*d^4 - a^3*b*c^2*d^5)/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*
b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c
^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) - 1/2*(b^2*c*sqrt(d) + 4
*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
 + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + ((sqrt(d)*x - s
qrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))
^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))
^2*a*d + b*c^2))
```

3.779 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (-5*d)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - 1/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^(3/2)) - (5*b*d)/(2*(b*c - a*d)^3*Sqrt[c + d
*x^2]) + (5*b^(3/2)*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.273968, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - 1/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^(3/2)) - (5*b*d)/(2*(b*c - a*d)^3*Sqrt[c + d
*x^2]) + (5*b^(3/2)*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 38.6956, size = 122, normalized size = 0.87

$$\frac{5b^{\frac{3}{2}}d\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2\left(ad-bc\right)^{\frac{7}{2}}}+\frac{5bd}{2\sqrt{c+dx^2}\left(ad-bc\right)^3}-\frac{5d}{6\left(c+dx^2\right)^{\frac{3}{2}}\left(ad-bc\right)^2}+\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] 5*b**(3/2)*d*atan(sqrt(b)*sqrt(c + d*x**2)/sqrt(a*d - b*c))/(2*(a
*d - b*c)**(7/2)) + 5*b*d/(2*sqrt(c + d*x**2)*(a*d - b*c)**3) - 5
*d/(6*(c + d*x**2)**(3/2)*(a*d - b*c)**2) + 1/(2*(a + b*x**2)*(c
+ d*x**2)**(3/2)*(a*d - b*c))
```

Mathematica [A] time = 0.353241, size = 137, normalized size = 0.98

$$\frac{2a^2d^2-2abd\left(7c+5dx^2\right)+b^2\left(-\left(3c^2+20cdx^2+15d^2x^4\right)\right)}{6\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^3}+\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 2*a*b*d*(7*c + 5*d*x^2) - b^2*(3*c^2 + 20*c*d*x^2 +
15*d^2*x^4))/(6*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (5
*b^(3/2)*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2
*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.02, size = 1639, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(3/2)-5/12*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/12*(-a*b)^(1/2)/
b*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/6*(-a*b)^(1/2)/b*d^2/
(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*b*d/(a*d-b*c)^3/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)-5/4*(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)*x-5/4*b*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/3*(-a*b)^(1/2)/a/b*d/(
a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-2/3*(-a*b)^(1/2)/a/b*d/(a*d-b*c)
/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*x+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+1/b*
(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/12*d/(a*d-b*c)^2/((x+1/b*(-a*
b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(3/2)-5/12*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
*x-5/6*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2
*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5
/4*b*d/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4*(-a*b)^(1/2)*d^2/(a*d-
b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b*d/(a*d-b*c)^3/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)+1/3*(-a*b)^(1/2)/a/b*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+2/3*(-
a*b)^(1/2)/a/b*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.335559, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4
 + (b^2*c^2*d + 2*a*b*c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^
2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^
2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)
*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) + 4*(15*b^2*d^2*x^4 + 3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 +
 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*
b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5
*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 +
 (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2
*a^4*c*d^4)*x^2), -1/12*(15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d
^2 + a*b*d^3)*x^4 + (b^2*c^2*d + 2*a*b*c*d^2)*x^2)*sqrt(-b/(b*c -
 a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c
- a*d)*sqrt(-b/(b*c - a*d)))) + 2*(15*b^2*d^2*x^4 + 3*b^2*c^2 + 1
4*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2
+ c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^
3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^
2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.228184, size = 301, normalized size = 2.15

$$-\frac{1}{6}\left(\frac{15\,b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}+\frac{3\,\sqrt{dx^2+c}b^2}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx^2+c)b-bc+ad)}+\frac{}{(b^3c^3-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] -1/6*(15*b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b
*d)) + 3*sqrt(d*x^2 + c)*b^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*((d*x^2 + c)*b - b*c + a*d)) + 2*(6*(d*x^2 + c)*
b + b*c - a*d)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*(d*x^2 + c)^(3/2)))*d
```

3.780 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=201

$$\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{dx(2ad+3bc)}{6ac\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d)*x)/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (
b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*
c^2 + 16*a*b*c*d - 4*a^2*d^2)*x)/(6*a*c^2*(b*c - a*d)^3*Sqrt[c +
d*x^2]) + (b^2*(b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.543394, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{dx(2ad+3bc)}{6ac\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*x)/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (
b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*
c^2 + 16*a*b*c*d - 4*a^2*d^2)*x)/(6*a*c^2*(b*c - a*d)^3*Sqrt[c +
d*x^2]) + (b^2*(b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 117.102, size = 178, normalized size = 0.89

$$-\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{dx\,(2ad+3bc)}{6ac\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)^2}+\frac{dx\left(4a^2d^2-16abcd-3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}\,(ad-bc)^3}+\frac{b^2\,(6ad-bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}\,(ad-bc)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] -b*x/(2*a*(a + b*x**2)*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*x*(2*
a*d + 3*b*c)/(6*a*c*(c + d*x**2)**(3/2)*(a*d - b*c)**2) + d*x*(4*
a**2*d**2 - 16*a*b*c*d - 3*b**2*c**2)/(6*a*c**2*sqrt(c + d*x**2)*
(a*d - b*c)**3) + b**2*(6*a*d - b*c)*atanh(x*sqrt(a*d - b*c)/(sqr
t(a)*sqrt(c + d*x**2)))/(2*a**(3/2)*(a*d - b*c)**(7/2))
```

Mathematica [A] time = 0.736565, size = 170, normalized size = 0.85

$$\frac{1}{6}\left(\frac{3b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{7/2}}+x\sqrt{c+dx^2}\left(-\frac{3b^3}{a\,(a+bx^2)\,(ad-bc)^3}+\frac{4d^2(4bc-ad)}{c^2\,(c+dx^2)\,(bc-ad)^3}+\frac{2d^2}{c\,(c+dx^2)^2\,(bc-ad)^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*((-3*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (
2*d^2)/(c*(b*c - a*d)^2*(c + d*x^2)^2) + (4*d^2*(4*b*c - a*d))/(c
^2*(b*c - a*d)^3*(c + d*x^2))) + (3*b^2*(b*c - 6*a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^
(7/2)))/6
```

Maple [B] time = 0.024, size = 2405, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/12*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/12*d^2/(a*d-b*c)^2
/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)*x+5/12/a*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+1/12/(-a*b)^(1/2)/a/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2
))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
-1/4/a*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/4/a*b*d*(-a*b)^(1/2)
/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/4*b*d^2/(a*d-b*c)^3/c/((x+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x-1/4/a*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-1/2/a*d/(
a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/2/a*d/(a*d-b*c)/c^2/((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x+1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-
5/4/a*b*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-5/12/a*d*(
-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/12/(-a*b)^(1/2)/a/(
a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2
))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)-1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)-1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
-5/4*b*d^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/6*d^2/(a*d-b*c)^
2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x-1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
```

```
a*d-b*c)/b)^(1/2)+5/6*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1
/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/a*b/(a*
d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-5/4/a*b*d*(-a*b)^(1/2)/(a*d-b*
c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))+5/4/a*b*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(
1/2)))-1/4/a*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [A] time = 1.40255, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] [1/24*(4*((3*b^3*c^2*d^2 + 16*a*b^2*c*d^3 - 4*a^2*b*d^4)*x^5 + 2*
(3*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3 - 2*a^3*d^4)*x^3 +
 3*(b^3*c^4 + 6*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x)*sqrt(-a*b*c + a^2
*d)*sqrt(d*x^2 + c) + 3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (b^4*c^3*d
^2 - 6*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4*a*b^3*c^4*d - 12*a^2*b^2*c^3*d
^2)*x^2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c
^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt
(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a^2*b^3*c^7 - 3*a^3*
b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^
2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4
*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 -
a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d^2
 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)*sqrt(-a*b*c + a^2*d)), 1
/12*(2*((3*b^3*c^2*d^2 + 16*a*b^2*c*d^3 - 4*a^2*b*d^4)*x^5 + 2*(3
*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3 - 2*a^3*d^4)*x^3 + 3
*(b^3*c^4 + 6*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x)*sqrt(a*b*c - a^2*d)
*sqrt(d*x^2 + c) + 3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (b^4*c^3*d^2
- 6*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^2*
b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)
*x^2)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*s
qrt(d*x^2 + c)*x)))/((a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5
*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b
^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*
d^2 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b
^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*
a^5*c^3*d^4)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 4.86797, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.781 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}+\frac{d\left(-2a^2d^2+6abcd+b^2c^2\right)}{2ac^2\sqrt{c+dx^2}(bc-ad)^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ac\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + b/(
2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 6*
a*b*c*d - 2*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) - A
rcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c -
 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^
2*(b*c - a*d)^(7/2))
```

Rubi [A] time = 1.02758, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}+\frac{d\left(-2a^2d^2+6abcd+b^2c^2\right)}{2ac^2\sqrt{c+dx^2}(bc-ad)^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ac\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + b/(
2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 6*
a*b*c*d - 2*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) - A
rcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c -
 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^
2*(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 124.364, size = 197, normalized size = 0.88

$$-\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d(2ad+3bc)}{6ac\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)^2}+\frac{d\left(2a^2d^2-6abcd-b^2c^2\right)}{2ac^2\sqrt{c+dx^2}(ad-bc)^3}-\frac{b^{\frac{5}{2}}(7ad-2bc)\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2a^2(ad-bc)^{\frac{7}{2}}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] -b/(2*a*(a + b*x**2)*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(2*a*d
+ 3*b*c)/(6*a*c*(c + d*x**2)**(3/2)*(a*d - b*c)**2) + d*(2*a**2*d
**2 - 6*a*b*c*d - b**2*c**2)/(2*a*c**2*sqrt(c + d*x**2)*(a*d - b*
c)**3) - b**(5/2)*(7*a*d - 2*b*c)*atan(sqrt(b)*sqrt(c + d*x**2)/s
qrt(a*d - b*c))/(2*a**2*(a*d - b*c)**(7/2)) - atanh(sqrt(c + d*x*
*2)/sqrt(c))/(a**2*c**(5/2))
```

Mathematica [C] time = 2.41167, size = 461, normalized size = 2.05

$$\frac{b^{5/2}(2bc-7ad)\log\left(-\frac{4a^2(bc-ad)^2\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{5/2}\left(\sqrt{b}x-i\sqrt{a}\right)(2bc-7ad)}\right)}{4a^2(bc-ad)^{7/2}}+\frac{b^{5/2}(2bc-7ad)\log\left(\frac{4a^2(bc-ad)^2\left(i\sqrt{a}dx\sqrt{bc-ad}-\sqrt{b}c\sqrt{bc-ad}+ad\sqrt{c+dx^2}-bc\sqrt{c+dx^2}\right)}{b^{5/2}\left(\sqrt{b}x+i\sqrt{a}\right)(2bc-7ad)}\right)}{4a^2(bc-ad)^{7/2}}-\frac{\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^2c^{5/2}}+\frac{\log(x)}{a^2c^{5/2}}+\sqrt{c+dx^2}\left(-\frac{b^3}{2a(a+bx^2)(ad-bc)^3}+\frac{d^2(3bc-ad)}{c^2(c+dx^2)(bc-ad)^3}+\frac{d^2}{3c(c+dx^2)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-b^3/(2*a*(-(b*c) + a*d)^3*(a + b*x^2)) + d^2/(3
*c*(b*c - a*d)^2*(c + d*x^2)^2) + (d^2*(3*b*c - a*d))/(c^2*(b*c -
 a*d)^3*(c + d*x^2))) + Log[x]/(a^2*c^(5/2)) - Log[c + Sqrt[c]*Sq
rt[c + d*x^2]]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c - 7*a*d)*Log[(-4*a
^2*(b*c - a*d)^2*(Sqrt[b]*c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*
c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(5/2)
*(2*b*c - 7*a*d)*((-I)*Sqrt[a] + Sqrt[b]*x))])/(4*a^2*(b*c - a*d)
^(7/2)) + (b^(5/2)*(2*b*c - 7*a*d)*Log[(4*a^2*(b*c - a*d)^2*(-(Sq
rt[b]*c*Sqrt[b*c - a*d]) + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x - b*c*Sq
rt[c + d*x^2] + a*d*Sqrt[c + d*x^2]))/(b^(5/2)*(2*b*c - 7*a*d)*(I
*Sqrt[a] + Sqrt[b]*x))])/(4*a^2*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.026, size = 2837, normalized size = 12.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/12/a*d/(a*d-b*c)^2*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4/a*d/(a*d-b*c)^3*b^
2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+5/12/a*d/(a*d-b*c)^2*b/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-
5/4/a*d/(a*d-b*c)^3*b^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/6/a^2/(a*d-b*c)*b/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)-1/2/a^2*b^2/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/
6/a^2/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-1/2/a^2*b^2/(a*d-b*c)^2/((x
+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+1/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)*x-1/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c/((x-1/b*(-a*b)^(1/2))
^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x
-2/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/3/a
^2*d*(-a*b)^(1/2)/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-1/6/a^2*d*(
-a*b)^(1/2)/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/4/(-a*b)^(1/2)/a/
(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4/a*d/(a*d
-b*c)^3*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^
```

```
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x-1/b
*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4/a*d/(a*d-b*c)^3*b^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b
)^(1/2)))+1/3/a^2/c/(d*x^2+c)^(3/2)+1/a^2/c^2/(d*x^2+c)^(1/2)-1/a
^2/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)-1/2/a^2*b/(a*d-b
*c)^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+1/2/a^2*b^2/(a*d-b*c
)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2
*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2)))+5/12/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)*x+5/6/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c^2/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)*x-5/12/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)*x-5/6/(-a*b)^(1/2)*d^2*b/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*
x+5/4/(-a*b)^(1/2)*d^2*b^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/
4/(-a*b)^(1/2)*d^2*b^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/6/a^
2*d*(-a*b)^(1/2)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3/a^2*d*(-a*
b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2/a^2*b/(a*d-b*c)^
2*(-a*b)^(1/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+2/3/(-a*b)^(1/2)/a*d/(a*
d-b*c)*b/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+1/2/a^2*b^2/(a*d-b*c)^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x), x)
```

Fricas [A] time = 7.74842, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(2*a*b^3*c^5 - 7*a^2*b^2*c^4*d + (2*b^4*c^3*d^2 - 7*a*b^
3*c^2*d^3)*x^6 + (4*b^4*c^4*d - 12*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*
d^3)*x^4 + (2*b^4*c^5 - 3*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*
sqrt(c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*
a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt
(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*b^3*c^4 +
```

```
20*a^3*b*c^2*d^2 - 8*a^4*c*d^3 + 3*(a*b^3*c^2*d^2 + 6*a^2*b^2*c*d
^3 - 2*a^3*b*d^4)*x^4 + 2*(3*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 + 5
*a^3*b*c*d^3 - 3*a^4*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 12*(a*b^
3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b
^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^
4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b
*c^2*d^3 - 2*a^4*c*d^4)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt
(d*x^2 + c)*c)/x^2))/((a^3*b^3*c^7 - 3*a^4*b^2*c^6*d + 3*a^5*b*c^
5*d^2 - a^6*c^4*d^3 + (a^2*b^4*c^5*d^2 - 3*a^3*b^3*c^4*d^3 + 3*a^
4*b^2*c^3*d^4 - a^5*b*c^2*d^5)*x^6 + (2*a^2*b^4*c^6*d - 5*a^3*b^3
*c^5*d^2 + 3*a^4*b^2*c^4*d^3 + a^5*b*c^3*d^4 - a^6*c^2*d^5)*x^4 +
 (a^2*b^4*c^7 - a^3*b^3*c^6*d - 3*a^4*b^2*c^5*d^2 + 5*a^5*b*c^4*d
^3 - 2*a^6*c^3*d^4)*x^2)*sqrt(c)), 1/24*(3*(2*a*b^3*c^5 - 7*a^2*b
^2*c^4*d + (2*b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (4*b^4*c^4*d -
 12*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (2*b^4*c^5 - 3*a*b^3
*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-c)*sqrt(b/(b*c - a*d))*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d -
 a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 4*(3*a*b^3*c^4 + 20*a^3*b*c^2*d^2 - 8*a^4*c*d^3
 + 3*(a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^4 + 2*(3*a
*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2)
*sqrt(d*x^2 + c)*sqrt(-c) - 24*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a
^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a
^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 +
3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3
*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*
arctan(sqrt(-c)/sqrt(d*x^2 + c)))/((a^3*b^3*c^7 - 3*a^4*b^2*c^6*d
 + 3*a^5*b*c^5*d^2 - a^6*c^4*d^3 + (a^2*b^4*c^5*d^2 - 3*a^3*b^3*c
^4*d^3 + 3*a^4*b^2*c^3*d^4 - a^5*b*c^2*d^5)*x^6 + (2*a^2*b^4*c^6*
d - 5*a^3*b^3*c^5*d^2 + 3*a^4*b^2*c^4*d^3 + a^5*b*c^3*d^4 - a^6*c
^2*d^5)*x^4 + (a^2*b^4*c^7 - a^3*b^3*c^6*d - 3*a^4*b^2*c^5*d^2 +
5*a^5*b*c^4*d^3 - 2*a^6*c^3*d^4)*x^2)*sqrt(-c)), -1/12*(3*(2*a*b^
3*c^5 - 7*a^2*b^2*c^4*d + (2*b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 +
 (4*b^4*c^4*d - 12*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (2*b^
4*c^5 - 3*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(c)*sqrt(-b/
(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(3*a*b^3*c^4 + 20*a^3*b*c
^2*d^2 - 8*a^4*c*d^3 + 3*(a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - 2*a^3
*b*d^4)*x^4 + 2*(3*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 + 5*a^3*b*c*d
^3 - 3*a^4*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(c) - 6*(a*b^3*c^5 - 3*a
^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a
*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d -
5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4
+ (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 -
2*a^4*c*d^4)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^2 + c)
*c)/x^2))/((a^3*b^3*c^7 - 3*a^4*b^2*c^6*d + 3*a^5*b*c^5*d^2 - a^6
*c^4*d^3 + (a^2*b^4*c^5*d^2 - 3*a^3*b^3*c^4*d^3 + 3*a^4*b^2*c^3*d
^4 - a^5*b*c^2*d^5)*x^6 + (2*a^2*b^4*c^6*d - 5*a^3*b^3*c^5*d^2 +
3*a^4*b^2*c^4*d^3 + a^5*b*c^3*d^4 - a^6*c^2*d^5)*x^4 + (a^2*b^4*c
^7 - a^3*b^3*c^6*d - 3*a^4*b^2*c^5*d^2 + 5*a^5*b*c^4*d^3 - 2*a^6*
c^3*d^4)*x^2)*sqrt(c)), -1/12*(3*(2*a*b^3*c^5 - 7*a^2*b^2*c^4*d +
 (2*b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (4*b^4*c^4*d - 12*a*b^3*
c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (2*b^4*c^5 - 3*a*b^3*c^4*d - 1
4*a^2*b^2*c^3*d^2)*x^2)*sqrt(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2
*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*
c - a*d)))) - 2*(3*a*b^3*c^4 + 20*a^3*b*c^2*d^2 - 8*a^4*c*d^3 + 3
*(a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^4 + 2*(3*a*b^3
*c^3*d + 10*a^2*b^2*c^2*d^2 + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2)*sqr
t(d*x^2 + c)*sqrt(-c) + 12*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b
*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b
^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^
2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4
*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*arct
an(sqrt(-c)/sqrt(d*x^2 + c)))/((a^3*b^3*c^7 - 3*a^4*b^2*c^6*d + 3
*a^5*b*c^5*d^2 - a^6*c^4*d^3 + (a^2*b^4*c^5*d^2 - 3*a^3*b^3*c^4*d
^3 + 3*a^4*b^2*c^3*d^4 - a^5*b*c^2*d^5)*x^6 + (2*a^2*b^4*c^6*d -
5*a^3*b^3*c^5*d^2 + 3*a^4*b^2*c^4*d^3 + a^5*b*c^3*d^4 - a^6*c^2*d
^5)*x^4 + (a^2*b^4*c^7 - a^3*b^3*c^6*d - 3*a^4*b^2*c^5*d^2 + 5*a^
5*b*c^4*d^3 - 2*a^6*c^3*d^4)*x^2)*sqrt(-c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.239995, size = 410, normalized size = 1.82

$$\frac{1}{6}\left(\frac{3\sqrt{dx^2+c}\,b^3}{(ab^3c^3d-3\,a^2b^2c^2d^2+3\,a^3bcd^3-a^4d^4)((dx^2+c)b-bc+ad)}-\frac{3\left(2\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(a^2b^3c^3d^2-3\,a^3b^2c^2d^3+3\,a^4bcd^4-a^5d^5)\sqrt{-b^2c+abd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(d*x^2 + c)*b^3/((a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 3*
a^3*b*c*d^3 - a^4*d^4)*((d*x^2 + c)*b - b*c + a*d)) - 3*(2*b^4*c
- 7*a*b^3*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b^3*c^3*d^2 - 3*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - a^5*d^5)*sqrt(
-b^2*c + a*b*d)) + 2*(9*(d*x^2 + c)*b*c + b*c^2 - 3*(d*x^2 + c)*a
*d - a*c*d)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2
*d^3)*(d*x^2 + c)^(3/2)) + 6*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^
2*sqrt(-c)*c^2*d^2))*d^2
```

3.782 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{6ac^2x\sqrt{c+dx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3\right)}{6a^2c^3x(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x*(c + d*x^2)^(3/2)) + b
/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^
2 + 20*a*b*c*d - 8*a^2*d^2))/(6*a*c^2*(b*c - a*d)^3*x*Sqrt[c + d*
x^2]) - ((9*b^3*c^3 - 18*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^
3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*x) - (b^3*(3*b*c - 8
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a
^(5/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 1.26694, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{6ac^2x\sqrt{c+dx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3\right)}{6a^2c^3x(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x*(c + d*x^2)^(3/2)) + b
/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^
2 + 20*a*b*c*d - 8*a^2*d^2))/(6*a*c^2*(b*c - a*d)^3*x*Sqrt[c + d*
x^2]) - ((9*b^3*c^3 - 18*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^
3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*x) - (b^3*(3*b*c - 8
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a
^(5/2)*(b*c - a*d)^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.910351, size = 188, normalized size = 0.67

$$\sqrt{c+dx^2}\left(\frac{b^4x}{2a^2\left(a+bx^2\right)(ad-bc)^3}-\frac{1}{a^2c^3x}+\frac{d^3x(5ad-11bc)}{3c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3x}{3c^2\left(c+dx^2\right)^2(bc-ad)^2}\right)-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(1/(a^2*c^3*x)) + (b^4*x)/(2*a^2*(-(b*c) + a*d)
^3*(a + b*x^2)) - (d^3*x)/(3*c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (
d^3*(-11*b*c + 5*a*d)*x)/(3*c^3*(b*c - a*d)^3*(c + d*x^2))) - (b^
3*(3*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*
x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

Maple [B] time = 0.029, size = 2513, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/a^2/c/x/(d*x^2+c)^(3/2)-3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+1/b*(-a
*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)-5/12/a*d^2*b/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/6/a^2
*d/(a*d-b*c)*b/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+3/4*b^2/a^2/(a*d-b*c)^2/
c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)*x*d+3/4*b^2/a^2/(a*d-b*c)^2/c/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)*x*d-5/6/a*d^2*b/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/
a*d^2*b^2/(a*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4/a^2*d*(-a*b)^(1
/2)/(a*d-b*c)^3*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/12/a^2*d/(a*d-b*c)*b/c/((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)*x-5/12/a*d^2*b/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*
x-5/6/a*d^2*b/(a*d-b*c)^2/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/a*d^2*b^2
/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4/a^2*d*(-a*b)^(1/2)/(a*d-
b*c)^3*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2)))+1/12/a^2*d/(a*d-b*c)*b/c/((x+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)*x+1/6/a^2*d/(a*d-b*c)*b/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/12/a^2
*d*(-a*b)^(1/2)/(a*d-b*c)^2*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4/a^2*d*(-a*b
)^(1/2)/(a*d-b*c)^3*b^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/12/a^2*d*(-a*b)^(1/
2)/(a*d-b*c)^2*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4/a^2*d*(-a*b)^(1/2)/(a*d-
b*c)^3*b^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)
```

```
^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))-3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))+3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1
/4/a^2/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4*b
^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1/4/a^2/(a*d-b*
c)*b/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-4/3/a^2*d/c^2*x/(d*
x^2+c)^(3/2)-8/3/a^2*d/c^3*x/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2), x)
```

Fricas [A] time = 1.76424, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(6*a*b^3*c^5 - 18*a^2*b^2*c^4*d + 18*a^3*b*c^3*d^2 - 6*
a^4*c^2*d^3 + (9*b^4*c^3*d^2 - 18*a*b^3*c^2*d^3 + 40*a^2*b^2*c*d^
4 - 16*a^3*b*d^5)*x^6 + 2*(9*b^4*c^4*d - 15*a*b^3*c^3*d^2 + 21*a^
2*b^2*c^2*d^3 + 8*a^3*b*c*d^4 - 8*a^4*d^5)*x^4 + 3*(3*b^4*c^5 - 2
*a*b^3*c^4*d - 6*a^2*b^2*c^3*d^2 + 18*a^3*b*c^2*d^3 - 8*a^4*c*d^4
)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - 3*((3*b^5*c^4*d^2 -
 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*c^4*d^2 - 8*a^2*b
^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2
)*x^3 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*log((((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*
sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x
^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^
4*d^4 - a^5*b*c^3*d^5)*x^7 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2
 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^5 + (a^2*b^
4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a
^6*c^4*d^4)*x^3 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^
2 - a^6*c^5*d^3)*x)*sqrt(-a*b*c + a^2*d)), -1/12*(2*(6*a*b^3*c^5
- 18*a^2*b^2*c^4*d + 18*a^3*b*c^3*d^2 - 6*a^4*c^2*d^3 + (9*b^4*c^
3*d^2 - 18*a*b^3*c^2*d^3 + 40*a^2*b^2*c*d^4 - 16*a^3*b*d^5)*x^6 +
 2*(9*b^4*c^4*d - 15*a*b^3*c^3*d^2 + 21*a^2*b^2*c^2*d^3 + 8*a^3*b
*c*d^4 - 8*a^4*d^5)*x^4 + 3*(3*b^4*c^5 - 2*a*b^3*c^4*d - 6*a^2*b^
2*c^3*d^2 + 18*a^3*b*c^2*d^3 - 8*a^4*c*d^4)*x^2)*sqrt(a*b*c - a^2
*d)*sqrt(d*x^2 + c) + 3*((3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 +
(6*b^5*c^5*d - 13*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3)*x^5 + (3*b^5
*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2)*x^3 + (3*a*b^4*c^6 - 8
*a^2*b^3*c^5*d)*x)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b
*c - a^2*d)*sqrt(d*x^2 + c)*x)))/(((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c
^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^7 + (2*a^2*b^4*c^7*
```

```
d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c
^3*d^5)*x^5 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 +
5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^3 + (a^3*b^3*c^8 - 3*a^4*b^2*c
^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3)*x)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 7.08523, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.783 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}}-\frac{d\left(5a^2d^2-6abcd+6b^2c^2\right)}{6a^2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{2a^2c^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}$$

```
[Out] -(d*(6*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2))/(6*a^2*c^2*(b*c - a*d)^2
*(c + d*x^2)^(3/2)) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a +
 b*x^2)*(c + d*x^2)^(3/2)) - 1/(2*a*c*x^2*(a + b*x^2)*(c + d*x^2)
^(3/2)) - (d*(2*b*c - a*d)*(b^2*c^2 - a*b*c*d + 5*a^2*d^2))/(2*a^
2*c^3*(b*c - a*d)^3*Sqrt[c + d*x^2]) + ((4*b*c + 5*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(7/2)) - (b^(7/2)*(4*b*c - 9*a*
d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*
c - a*d)^(7/2))
```

Rubi [A] time = 1.46975, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}}-\frac{d\left(5a^2d^2-6abcd+6b^2c^2\right)}{6a^2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{2a^2c^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*(6*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2))/(6*a^2*c^2*(b*c - a*d)^2
*(c + d*x^2)^(3/2)) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a +
 b*x^2)*(c + d*x^2)^(3/2)) - 1/(2*a*c*x^2*(a + b*x^2)*(c + d*x^2)
^(3/2)) - (d*(2*b*c - a*d)*(b^2*c^2 - a*b*c*d + 5*a^2*d^2))/(2*a^
2*c^3*(b*c - a*d)^3*Sqrt[c + d*x^2]) + ((4*b*c + 5*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(7/2)) - (b^(7/2)*(4*b*c - 9*a*
d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*
c - a*d)^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 6.3297, size = 489, normalized size = 1.61

$$\frac{1}{4}\left(-\frac{b^{7/2}(4bc-9ad)\log\left(\frac{4a^3(bc-ad)^2\left(-i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{7/2}\left(\sqrt{b}x+i\sqrt{a}\right)(4bc-9ad)}\right)}{a^3(bc-ad)^{7/2}}\right.$$
$$-\frac{b^{7/2}(4bc-9ad)\log\left(\frac{4a^3(bc-ad)^2\left(i\sqrt{a}dx\sqrt{bc-ad}+\sqrt{b}c\sqrt{bc-ad}-ad\sqrt{c+dx^2}+bc\sqrt{c+dx^2}\right)}{b^{7/2}\left(\sqrt{b}x-i\sqrt{a}\right)(4bc-9ad)}\right)}{a^3(bc-ad)^{7/2}}$$
$$+\frac{2(5ad+4bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)}{a^3c^{7/2}}-\frac{2\log(x)(5ad+4bc)}{a^3c^{7/2}}$$
$$\left.+\frac{2}{3}\sqrt{c+dx^2}\left(\frac{3b^4}{a^2(a+bx^2)(ad-bc)^3}-\frac{3}{a^2c^3x^2}+\frac{12d^3(ad-2bc)}{c^3(c+dx^2)(bc-ad)^3}-\frac{2d^3}{c^2(c+dx^2)^2(bc-ad)^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((2*Sqrt[c + d*x^2]*(-3/(a^2*c^3*x^2) + (3*b^4)/(a^2*(-(b*c) + a*
d)^3*(a + b*x^2)) - (2*d^3)/(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (
12*d^3*(-2*b*c + a*d))/(c^3*(b*c - a*d)^3*(c + d*x^2))))/3 - (2*(
4*b*c + 5*a*d)*Log[x])/(a^3*c^(7/2)) + (2*(4*b*c + 5*a*d)*Log[c +
 Sqrt[c]*Sqrt[c + d*x^2]])/(a^3*c^(7/2)) - (b^(7/2)*(4*b*c - 9*a*
d)*Log[(4*a^3*(b*c - a*d)^2*(Sqrt[b]*c*Sqrt[b*c - a*d] - I*Sqrt[a
]*d*Sqrt[b*c - a*d]*x + b*c*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]
))/(b^(7/2)*(4*b*c - 9*a*d)*(I*Sqrt[a] + Sqrt[b]*x))])/(a^3*(b*c
- a*d)^(7/2)) - (b^(7/2)*(4*b*c - 9*a*d)*Log[(4*a^3*(b*c - a*d)^2
*(Sqrt[b]*c*Sqrt[b*c - a*d] + I*Sqrt[a]*d*Sqrt[b*c - a*d]*x + b*c
*Sqrt[c + d*x^2] - a*d*Sqrt[c + d*x^2]))/(b^(7/2)*(4*b*c - 9*a*d)
*((-I)*Sqrt[a] + Sqrt[b]*x))])/(a^3*(b*c - a*d)^(7/2)))/4
```

Maple [B] time = 0.028, size = 2980, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b^2/a^3/(a*d-b*c)/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+b^3/a^3/(a*d-b*c)^2/((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)-1/2/a^2/c/x^2/(d*x^2+c)^(3/2)-5/6/a^2*d/c^2/(d*
x^2+c)^(3/2)-1/3*b^2/a^3/(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+b^3/a^3/(a
*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2/3*b/a^3/c/(d*x^2+c)^(3/2)-2*b/a^3
/c^2/(d*x^2+c)^(1/2)+2*b/a^3/c^(5/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^
(1/2))/x)-5/2/a^2*d/c^3/(d*x^2+c)^(1/2)+5/2/a^2*d/c^(7/2)*ln((2*c
+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)
/(x-1/b*(-a*b)^(1/2))/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*b^3/a^2*d/(a*d-b*c)
^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*
d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+1/b*(-a*
b)^(1/2))/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*b^3/a^2*d/(a*d-b*c)^3/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
```

```
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))-5/12*b^2/a^2*d/(a*d-b*c)^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*b^3/a^2
*d/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-b^3/a^3/(a*d-b*c)^2/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))-b^3/a^3/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+5/4*b^3/a^2*d/(a*d-b*c)
^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+5/6*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c^2
/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/(
(x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)*x-1/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)*x-2/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)*x-5/12*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(3/2)*x-5/6*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c^2/((x-1/b*
(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)*x+5/4*b^3/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x-1/b*(-
a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)*x-5/12*b^2/a^2*d/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/12*b
^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/3*b^2
/a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+2/3*b^2/a^2
/(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-2/3*b/a^3*d*(
-a*b)^(1/2)/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+b^2/a^3/(a*d-b*c)
^2*(-a*b)^(1/2)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/3*b/a^3*d*(-a*b)^(1/2
)/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x-b^2/a^3/(a*d-b*c)^2*(-a*b)^(1
/2)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)*x*d+2/3*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/
c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)*x+1/3*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/c/((
x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3), x)
```

Fricas [A] time = 14.2661, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3),x, algorithm="fricas")

[Out] [1/24*(3*((4*b^5*c^4*d^2 - 9*a*b^4*c^3*d^3)*x^8 + (8*b^5*c^5*d - 14*a*b^4*c^4*d^2 - 9*a^2*b^3*c^3*d^3)*x^6 + (4*b^5*c^6 - a*b^4*c^5*d - 18*a^2*b^3*c^4*d^2)*x^4 + (4*a*b^4*c^6 - 9*a^2*b^3*c^5*d)*x^2)*sqrt(c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a^2*b^3*c^5 - 9*a^3*b^2*c^4*d + 9*a^4*b*c^3*d^2 - 3*a^5*c^2*d^3 + 3*(2*a*b^4*c^3*d^2 - 3*a^2*b^3*c^2*d^3 + 11*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 + (12*a*b^4*c^4*d - 15*a^2*b^3*c^3*d^2 + 35*a^3*b^2*c^2*d^3 + 13*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 + (6*a*b^4*c^5 - 3*a^2*b^3*c^4*d - 9*a^3*b^2*c^3*d^2 + 41*a^4*b*c^2*d^3 - 20*a^5*c*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 6*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/(((a^3*b^4*c^6*d^2 - 3*a^4*b^3*c^5*d^3 + 3*a^5*b^2*c^4*d^4 - a^6*b*c^3*d^5)*x^8 + (2*a^3*b^4*c^7*d - 5*a^4*b^3*c^6*d^2 + 3*a^5*b^2*c^5*d^3 + a^6*b*c^4*d^4 - a^7*c^3*d^5)*x^6 + (a^3*b^4*c^8 - a^4*b^3*c^7*d - 3*a^5*b^2*c^6*d^2 + 5*a^6*b*c^5*d^3 - 2*a^7*c^4*d^4)*x^4 + (a^4*b^3*c^8 - 3*a^5*b^2*c^7*d + 3*a^6*b*c^6*d^2 - a^7*c^5*d^3)*x^2)*sqrt(c)), 1/24*(3*((4*b^5*c^4*d^2 - 9*a*b^4*c^3*d^3)*x^8 + (8*b^5*c^5*d - 14*a*b^4*c^4*d^2 - 9*a^2*b^3*c^3*d^3)*x^6 + (4*b^5*c^6 - a*b^4*c^5*d - 18*a^2*b^3*c^4*d^2)*x^4 + (4*a*b^4*c^6 - 9*a^2*b^3*c^5*d)*x^2)*sqrt(-c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a^2*b^3*c^5 - 9*a^3*b^2*c^4*d + 9*a^4*b*c^3*d^2 - 3*a^5*c^2*d^3 + 3*(2*a*b^4*c^3*d^2 - 3*a^2*b^3*c^2*d^3 + 11*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 + (12*a*b^4*c^4*d - 15*a^2*b^3*c^3*d^2 + 35*a^3*b^2*c^2*d^3 + 13*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 + (6*a*b^4*c^5 - 3*a^2*b^3*c^4*d - 9*a^3*b^2*c^3*d^2 + 41*a^4*b*c^2*d^3 - 20*a^5*c*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(-c) + 12*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(((a^3*b^4*c^6*d^2 - 3*a^4*b^3*c^5*d^3 + 3*a^5*b^2*c^4*d^4 - a^6*b*c^3*d^5)*x^8 + (2*a^3*b^4*c^7*d - 5*a^4*b^3*c^6*d^2 + 3*a^5*b^2*c^5*d^3 + a^6*b*c^4*d^4 - a^7*c^3*d^5)*x^6 + (a^3*b^4*c^8 - a^4*b^3*c^7*d - 3*a^5*b^2*c^6*d^2 + 5*a^6*b*c^5*d^3 - 2*a^7*c^4*d^4)*x^4 + (a^4*b^3*c^8 - 3*a^5*b^2*c^7*d + 3*a^6*b*c^6*d^2 - a^7*c^5*d^3)*x^2)*sqrt(-c)), 1/12*(3*((4*b^5*c^4*d^2 - 9*a*b^4*c^3*d^3)*x^8 + (8*b^5*c^5*d - 14*a*b^4*c^4*d^2 - 9*a^2*b^3*c^3*d^3)*x^6 + (4*b^5*c^6 - a*b^4*c^5*d - 18*a^2*b^3*c^4*d^2)*x^4 + (4*a*b^4*c^6 - 9*a^2*b^3*c^5*d)*x^2)*sqrt(c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(3*a^2*b^3*c^5 - 9*a^3*b^2*c^4*d + 9*a^4*b*c^3*d^2 - 3*a^5*c^2*d^3 + 3*(2*a*b^4*c^3*d^2 - 3*a^2*b^3*c^2*d^3 + 11*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 + (12*a*b^4*c^4*d - 15*a^2*b^3*c^3*d^2 + 35*a^3*b^2*c^2*d^3 + 13*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 + (6*a*b^4*c^5 - 3*a^2*b^3*c^4*d - 9*a^3*b^2*c^3*d^2 + 41*a^4*b*c^2*d^3 - 20*a^5*c*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(c) + 3*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*log(-((d*x^2 + 2*c)*sqrt(c) + 2*sqrt(d*x^2 + c)*c)/x^2))/(((a^3*b^4*c^6*d^2 - 3*a^4*b^3*c^5*d^3 + 3*a^5*b^2*c^4*d^4 - a^6*b*c^3*d^5)*x^8 + (2*a^3*b^4*c^7*d - 5*a^4*b^3*c^6*d^2 + 3*a^5*b^2*c^5*d^3 + a^6*b*c^4*d^4 - a^7*c^3*d^5)*x^6 + (a^3*b^4*c^8 - a^4*b^3*c^7*d - 3*a^5*b^2*c^6*d^2 + 5*a^6*b*c^5*d^3 - 2*a^7*c^4*d^4)*x^4 + (a^4*b^3*c^8 -

```
3*a^5*b^2*c^7*d + 3*a^6*b*c^6*d^2 - a^7*c^5*d^3)*x^2)*sqrt(c)), 1
/12*(3*((4*b^5*c^4*d^2 - 9*a*b^4*c^3*d^3)*x^8 + (8*b^5*c^5*d - 14
*a*b^4*c^4*d^2 - 9*a^2*b^3*c^3*d^3)*x^6 + (4*b^5*c^6 - a*b^4*c^5*
d - 18*a^2*b^3*c^4*d^2)*x^4 + (4*a*b^4*c^6 - 9*a^2*b^3*c^5*d)*x^2
)*sqrt(-c)*sqrt(-b/(b*c - a*d))*arctan(-1/2*(b*d*x^2 + 2*b*c - a*
d)/(sqrt(d*x^2 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d)))) - 2*(3*a^2
*b^3*c^5 - 9*a^3*b^2*c^4*d + 9*a^4*b*c^3*d^2 - 3*a^5*c^2*d^3 + 3*
(2*a*b^4*c^3*d^2 - 3*a^2*b^3*c^2*d^3 + 11*a^3*b^2*c*d^4 - 5*a^4*b
*d^5)*x^6 + (12*a*b^4*c^4*d - 15*a^2*b^3*c^3*d^2 + 35*a^3*b^2*c^2
*d^3 + 13*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 + (6*a*b^4*c^5 - 3*a^2*b^
3*c^4*d - 9*a^3*b^2*c^3*d^2 + 41*a^4*b*c^2*d^3 - 20*a^5*c*d^4)*x^
2)*sqrt(d*x^2 + c)*sqrt(-c) + 6*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3
 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b
^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2
*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d -
17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^
5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2
 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*arctan(sqrt(-c)/sqrt(d*
x^2 + c)))/(((a^3*b^4*c^6*d^2 - 3*a^4*b^3*c^5*d^3 + 3*a^5*b^2*c^4
*d^4 - a^6*b*c^3*d^5)*x^8 + (2*a^3*b^4*c^7*d - 5*a^4*b^3*c^6*d^2
+ 3*a^5*b^2*c^5*d^3 + a^6*b*c^4*d^4 - a^7*c^3*d^5)*x^6 + (a^3*b^4
*c^8 - a^4*b^3*c^7*d - 3*a^5*b^2*c^6*d^2 + 5*a^6*b*c^5*d^3 - 2*a^
7*c^4*d^4)*x^4 + (a^4*b^3*c^8 - 3*a^5*b^2*c^7*d + 3*a^6*b*c^6*d^2
 - a^7*c^5*d^3)*x^2)*sqrt(-c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.258918, size = 684, normalized size = 2.25

$$\frac{1}{6}\, d^3\left(\frac{3\left(4\, b^5 c - 9\, a b^4 d\right) \arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3 b^3 c^3 d^3 - 3\, a^4 b^2 c^2 d^4 + 3\, a^5 b c d^5 - a^6 d^6\right)\sqrt{-b^2 c + abd}} - \frac{3\left(2\left(dx^2+c\right)^{\frac{3}{2}} b^4 c^3 - 2\sqrt{dx^2+c}\, b^4 c^4 - 3\left(dx^2+c\right)^{\frac{3}{2}} a b^3 c^2 d\right.}{\left(a^2 b^3 c^6 d^2 - 3\, a^3 b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] 1/6*d^3*(3*(4*b^5*c - 9*a*b^4*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b
^2*c + a*b*d))/((a^3*b^3*c^3*d^3 - 3*a^4*b^2*c^2*d^4 + 3*a^5*b*c*
d^5 - a^6*d^6)*sqrt(-b^2*c + a*b*d)) - 3*(2*(d*x^2 + c)^(3/2)*b^4
*c^3 - 2*sqrt(d*x^2 + c)*b^4*c^4 - 3*(d*x^2 + c)^(3/2)*a*b^3*c^2*
d + 4*sqrt(d*x^2 + c)*a*b^3*c^3*d + 3*(d*x^2 + c)^(3/2)*a^2*b^2*c
*d^2 - 6*sqrt(d*x^2 + c)*a^2*b^2*c^2*d^2 - (d*x^2 + c)^(3/2)*a^3*
b*d^3 + 4*sqrt(d*x^2 + c)*a^3*b*c*d^3 - sqrt(d*x^2 + c)*a^4*d^4)/
((a^2*b^3*c^6*d^2 - 3*a^3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3
*d^5)*((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*
a*d - a*c*d)) - 2*(12*(d*x^2 + c)*b*c + b*c^2 - 6*(d*x^2 + c)*a*d
 - a*c*d)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d
^3)*(d*x^2 + c)^(3/2)) - 3*(4*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)
/sqrt(-c))/(a^3*sqrt(-c)*c^3*d^3))
```

3.784 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}+\frac{d\left(-4a^2d^2+8abcd+b^2c^2\right)}{2ac^2x^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{6a^2c^3x^3(bc-ad)^3}+\frac{\sqrt{c+dx^2}\left(-32a^4d^4+64a^3bcd^3-12a^2b^2c^2d^2-20ab^3c^3d+15b^4c^4\right)}{6a^3c^4x(bc-ad)^3}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx^3(c+dx^2)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^(3/2)) +
 b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*
c^2 + 8*a*b*c*d - 4*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*x^3*Sqrt[c +
 d*x^2]) - ((5*b^3*c^3 - 6*a*b^2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*
d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*x^3) + ((15*b^4*c^
4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b*c*d^3 - 32*a^4
*d^4)*Sqrt[c + d*x^2])/(6*a^3*c^4*(b*c - a*d)^3*x) + (5*b^4*(b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*a^(7/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 1.66449, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}+\frac{d\left(-4a^2d^2+8abcd+b^2c^2\right)}{2ac^2x^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{6a^2c^3x^3(bc-ad)^3}+\frac{\sqrt{c+dx^2}\left(-32a^4d^4+64a^3bcd^3-12a^2b^2c^2d^2-20ab^3c^3d+15b^4c^4\right)}{6a^3c^4x(bc-ad)^3}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx^3(c+dx^2)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^(3/2)) +
 b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*
c^2 + 8*a*b*c*d - 4*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*x^3*Sqrt[c +
 d*x^2]) - ((5*b^3*c^3 - 6*a*b^2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*
d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*x^3) + ((15*b^4*c^
4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b*c*d^3 - 32*a^4
*d^4)*Sqrt[c + d*x^2])/(6*a^3*c^4*(b*c - a*d)^3*x) + (5*b^4*(b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*a^(7/2)*(b*c - a*d)^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.13437, size = 210, normalized size = 0.58

$$\frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}+\frac{\sqrt{c+dx^2}\left(-\frac{3b^5x^4}{a^3(a+bx^2)(ad-bc)^3}+\frac{4x^2(4ad+3bc)}{a^3c^4}-\frac{2}{a^2c^3}+\frac{4d^4x^4(7bc-4ad)}{c^4(c+dx^2)(bc-ad)^3}+\frac{2d^4x^4}{c^3(c+dx^2)^2(bc-ad)^2}\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2/(a^2*c^3) + (4*(3*b*c + 4*a*d)*x^2)/(a^3*c^4
) - (3*b^5*x^4)/(a^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^4*x^4)/
(c^3*(b*c - a*d)^2*(c + d*x^2)^2) + (4*d^4*(7*b*c - 4*a*d)*x^4)/(
c^4*(b*c - a*d)^3*(c + d*x^2))))/(6*x^3) + (5*b^4*(b*c - 2*a*d)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*
(b*c - a*d)^(7/2))
```

Maple [B] time = 0.032, size = 2623, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 2/a^2*d/c^2/x/(d*x^2+c)^(3/2)+8/3/a^2*d^2/c^3*x/(d*x^2+c)^(3/2)+1
6/3/a^2*d^2/c^4*x/(d*x^2+c)^(1/2)+2*b/a^3/c/x/(d*x^2+c)^(3/2)-5/1
2*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/((x+1/b*(-a*b)^(1/2))^2*d-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5/4*b^3/
a^3*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+8/3*b/a^3*d/c^
2*x/(d*x^2+c)^(3/2)+16/3*b/a^3*d/c^3*x/(d*x^2+c)^(1/2)-5/4*b^3/a^
3*d*(-a*b)^(1/2)/(a*d-b*c)^3/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+5/4*b^4/a^3/(-a*
b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-5/4*b^4/a^3/(-a*b)^(1/2)/(a*
d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))+5/12*b^2/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^2/
((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-1/4*b^2/a^3/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))/((
x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)-1/4*b^2/a^3/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))/((x-
1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)+1/6*b^2/a^3*d/(a*d-b*c)/c^2/((x+1/b*(-a*b)^(1/2))
^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
-5/4*b^3/a^3/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-5/4*b^3/a^3/(a
*d-b*c)^2/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d+5/12*b^2/a^2*d^2/(a*d-b*c)^2/
c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)*x+5/6*b^2/a^2*d^2/(a*d-b*c)^2/c^2/((x+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)*x-5/4*b^3/a^2*d^2/(a*d-b*c)^3/c/((x+1/b*(-a*b)^(1/2))^
2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-
5/4*b^3/a^3*d*(-a*b)^(1/2)/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
```

```
c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/12*b^2/a
^3*d/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/12*b^3/a^3/(-a*b)^(1/2)/
(a*d-b*c)/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)^
2/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-5/12*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)+5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)^2/((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)+5/12*b^2/a^2*d^2/(a*d-b*c)^2/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+5/6*b^2
/a^2*d^2/(a*d-b*c)^2/c^2/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a^2*d^2/(a
*d-b*c)^3/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x+5/4*b^3/a^3*d*(-a*b)^(1/2)/(a*d
-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x-1/b*(-a*b)^(1/2)))-1/3/a^2/c/x^3/(d*x^2+c)^(3/2)+1/12*b^2/a^
3*d/(a*d-b*c)/c/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)*x+1/6*b^2/a^3*d/(a*d-b*c)/c^2
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4), x)
```

Fricas [A] time = 3.27575, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(2*a^2*b^3*c^6 - 6*a^3*b^2*c^5*d + 6*a^4*b*c^4*d^2 - 2*
a^5*c^3*d^3 - (15*b^5*c^4*d^2 - 20*a*b^4*c^3*d^3 - 12*a^2*b^3*c^2
*d^4 + 64*a^3*b^2*c*d^5 - 32*a^4*b*d^6)*x^8 - 2*(15*b^5*c^5*d - 1
5*a*b^4*c^4*d^2 - 19*a^2*b^3*c^3*d^3 + 42*a^3*b^2*c^2*d^4 + 8*a^4
*b*c*d^5 - 16*a^5*d^6)*x^6 - 3*(5*b^5*c^6 - 14*a^2*b^3*c^4*d^2 +
2*a^3*b^2*c^3*d^3 + 28*a^4*b*c^2*d^4 - 16*a^5*c*d^5)*x^4 - 2*(5*a
*b^4*c^6 - 9*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 13*a^4*b*c^3*d^3
 - 6*a^5*c^2*d^4)*x^2)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) - 15*
((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5*c^5
*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5
 + (a*b^5*c^7 - 2*a^2*b^4*c^6*d)*x^3)*log((((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(
-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 -
(a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
a^2)))/(((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4
 - a^6*b*c^4*d^5)*x^9 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*
a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^7 + (a^3*b^4*c^9
 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^
5*d^4)*x^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a
^7*c^6*d^3)*x^3)*sqrt(-a*b*c + a^2*d)), -1/12*(2*(2*a^2*b^3*c^6 -
```

```
 6*a^3*b^2*c^5*d + 6*a^4*b*c^4*d^2 - 2*a^5*c^3*d^3 - (15*b^5*c^4*
d^2 - 20*a*b^4*c^3*d^3 - 12*a^2*b^3*c^2*d^4 + 64*a^3*b^2*c*d^5 -
32*a^4*b*d^6)*x^8 - 2*(15*b^5*c^5*d - 15*a*b^4*c^4*d^2 - 19*a^2*b
^3*c^3*d^3 + 42*a^3*b^2*c^2*d^4 + 8*a^4*b*c*d^5 - 16*a^5*d^6)*x^6
 - 3*(5*b^5*c^6 - 14*a^2*b^3*c^4*d^2 + 2*a^3*b^2*c^3*d^3 + 28*a^4
*b*c^2*d^4 - 16*a^5*c*d^5)*x^4 - 2*(5*a*b^4*c^6 - 9*a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 13*a^4*b*c^3*d^3 - 6*a^5*c^2*d^4)*x^2)*sqr
t(a*b*c - a^2*d)*sqrt(d*x^2 + c) - 15*((b^6*c^5*d^2 - 2*a*b^5*c^4
*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5*c^5*d^2 - 2*a^2*b^4*c^4*d^3)*x
^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*b^5*c^7 - 2*a^2*b^4*c
^6*d)*x^3)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2
*d)*sqrt(d*x^2 + c)*x)))/(((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 +
 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^9 + (2*a^3*b^4*c^8*d - 5*a^
4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*
x^7 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*
c^6*d^3 - 2*a^7*c^5*d^4)*x^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3
*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.785 $\int (ex)^{3/2}\sqrt{a+bx^2}\,(A+Bx^2)\,dx$

Optimal. Leaf size=212

$$-\frac{2a^{7/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{4ae\sqrt{ex}\sqrt{a+bx^2}(11Ab-5aB)}{231b^2}+\frac{2(ex)^{5/2}\sqrt{a+bx^2}(11Ab-5aB)}{77be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

```
[Out] (4*a*(11*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (2
*(11*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*B*(e
*x)^(5/2)*(a + b*x^2)^(3/2))/(11*b*e) - (2*a^(7/4)*(11*A*b - 5*a*
B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.378458, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{2a^{7/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{4ae\sqrt{ex}\sqrt{a+bx^2}(11Ab-5aB)}{231b^2}+\frac{2(ex)^{5/2}\sqrt{a+bx^2}(11Ab-5aB)}{77be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (4*a*(11*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (2
*(11*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*B*(e
*x)^(5/2)*(a + b*x^2)^(3/2))/(11*b*e) - (2*a^(7/4)*(11*A*b - 5*a*
B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 34.7358, size = 199, normalized size = 0.94

$$\frac{2B\,(ex)^{\frac{5}{2}}\left(a+bx^2\right)^{\frac{3}{2}}}{11be}-\frac{2a^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(11Ab-5Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231b^{\frac{9}{4}}\sqrt{a+bx^2}}+\frac{4ae\sqrt{ex}\sqrt{a+bx^2}\,(11Ab-5Ba)}{231b^2}+\frac{2\,(ex)^{\frac{5}{2}}\sqrt{a+bx^2}\,(11Ab-5Ba)}{77be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] 2*B*(e*x)**(5/2)*(a + b*x**2)**(3/2)/(11*b*e) - 2*a**(7/4)*e**(3/
2)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)
*x)*(11*A*b - 5*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/
4)*sqrt(e))), 1/2)/(231*b**(9/4)*sqrt(a + b*x**2)) + 4*a*e*sqrt(e
*x)*sqrt(a + b*x**2)*(11*A*b - 5*B*a)/(231*b**2) + 2*(e*x)**(5/2)
*sqrt(a + b*x**2)*(11*A*b - 5*B*a)/(77*b*e)
```

Mathematica [C] time = 0.498514, size = 159, normalized size = 0.75

$$\frac{2e\sqrt{ex}\left(-\left(a+bx^2\right)\left(10a^2B-2ab\left(11A+3Bx^2\right)-3b^2x^2\left(11A+7Bx^2\right)\right)+\frac{2ia^2\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(5aB-11Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{231b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*(-((a + b*x^2)*(10*a^2*B - 2*a*b*(11*A + 3*B*x^2)
- 3*b^2*x^2*(11*A + 7*B*x^2))) + ((2*I)*a^2*(-11*A*b + 5*a*B)*Sqr
t[1 + a/(b*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqr
t[b]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(231*b^2*Sqrt[a
+ b*x^2])
```

Maple [A] time = 0.059, size = 276, normalized size = 1.3

$$-\frac{2e}{231xb^3}\sqrt{ex}\left(-21Bx^7b^4+11A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{2}\sqrt{-ab}a^2b-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] -2/231*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-21*B*x^7*b^4+11*A*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*(-a*b)^(1/2)*a^2*b-33*A
*x^5*b^4-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*(-a*
b)^(1/2)*a^3-27*B*x^5*a*b^3-55*A*x^3*a*b^3+4*B*x^3*a^2*b^2-22*A*x
*a^2*b^2+10*B*x*a^3*b)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(Bx^2+A\right)\sqrt{bx^2+a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bex^3+Aex\right)\sqrt{bx^2+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Sympy [A] time = 134.715, size = 97, normalized size = 0.46

$$\frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,),
 b*x**2*exp_polar(I*pi)/a)/(2*gamma(9/4)) + B*sqrt(a)*e**(3/2)*x*
*(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2+A\right)\sqrt{bx^2+a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2), x)
```

3.786 $\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=337

$$\frac{2a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} - \frac{4a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} + \frac{4a\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2(ex)^{3/2}\sqrt{a+bx^2}(3Ab-aB)}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

```
[Out] (2*(3*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*b*e) + (4*a*(3*
A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt
[b]*x)) + (2*B*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*b*e) - (4*a^(5/4
)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^
(1/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]) + (2*a^(5/4)*
(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1
/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.634259, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} - \frac{4a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} + \frac{4a\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2(ex)^{3/2}\sqrt{a+bx^2}(3Ab-aB)}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*(3*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*b*e) + (4*a*(3*
A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt
[b]*x)) + (2*B*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*b*e) - (4*a^(5/4
)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^
(1/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]) + (2*a^(5/4)*
(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1
/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 62.874, size = 308, normalized size = 0.91

$$\frac{2B(ex)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{2}}}{9be}-\frac{4a^{\frac{5}{4}}\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}\sqrt{a+bx^2}}+\frac{2a^{\frac{5}{4}}\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}\sqrt{a+bx^2}}+\frac{4a\sqrt{ex}\sqrt{a+bx^2}(3Ab-Ba)}{15b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(ex)^{\frac{3}{2}}\sqrt{a+bx^2}(3Ab-Ba)}{15be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(e*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] 2*B*(e*x)**(3/2)*(a + b*x**2)**(3/2)/(9*b*e) - 4*a**(5/4)*sqrt(e)
*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x
)*(3*A*b - B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(15*b**(7/4)*sqrt(a + b*x**2)) + 2*a**(5/4)*sqrt(e)
*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x
)*(3*A*b - B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(15*b**(7/4)*sqrt(a + b*x**2)) + 4*a*sqrt(e*x)*sqrt
(a + b*x**2)*(3*A*b - B*a)/(15*b**(3/2)*(sqrt(a) + sqrt(b)*x)) +
2*(e*x)**(3/2)*sqrt(a + b*x**2)*(3*A*b - B*a)/(15*b*e)
```

Mathematica [C] time = 1.66605, size = 234, normalized size = 0.69

$$\frac{2e\left(bx^2\left(a+bx^2\right)\left(2aB+9Ab+5bBx^2\right)-\frac{6a(aB-3Ab)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a+bx^2\right)+\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}E\left(i\sinh^{-1}\right.\right.}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}}{45b^2\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*e*(b*x^2*(a + b*x^2)*(9*A*b + 2*a*B + 5*b*B*x^2) - (6*a*(-3*A*
b + a*B)*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*x^2) - Sqrt[a]*Sqrt[b]
*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[a])
/Sqrt[b]]/Sqrt[x]], -1] + Sqrt[a]*Sqrt[b]*Sqrt[1 + a/(b*x^2)]*x^(
3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])
)/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(45*b^2*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.047, size = 414, normalized size = 1.2

$$\frac{2}{45\,b^2x}\sqrt{ex}\left(5\,Bx^6b^3+18\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2b-9\,A\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] 2/45*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(5*B*x^6*b^3+18*A*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
```

```
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-9*A*((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-6*B*((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2),1/2*2^(1/2))*a^3+3*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*a^3+9*A*x^4*b^3+7*B*x^4*a*b^2+9*A*x^2*a*b^2+2*B*x^
2*a^2*b)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

Sympy [A] time = 9.64481, size = 95, normalized size = 0.28

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e\,\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x*
*2*exp_polar(I*pi)/a)/(2*e*gamma(7/4)) + B*sqrt(a)*(e*x)**(7/2)*g
amma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(
2*e**3*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx^2 + A)\sqrt{bx^2 + a}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

3.787 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=176

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(7Ab-aB)}{21be}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be}$$

```
[Out] (2*(7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b*e) + (2*B*Sqrt[
e*x]*(a + b*x^2)^(3/2))/(7*b*e) + (2*a^(3/4)*(7*A*b - a*B)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(
5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.292616, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(7Ab-aB)}{21be}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*(7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b*e) + (2*B*Sqrt[
e*x]*(a + b*x^2)^(3/2))/(7*b*e) + (2*a^(3/4)*(7*A*b - a*B)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(
5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 27.9703, size = 158, normalized size = 0.9

$$\frac{2B\sqrt{ex}\left(a+bx^2\right)^{\frac{3}{2}}}{7be}+\frac{2a^{\frac{3}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21b^{\frac{5}{4}}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}\,(7Ab-Ba)}{21be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] 2*B*sqrt(e*x)*(a + b*x**2)**(3/2)/(7*b*e) + 2*a**(3/4)*sqrt((a +
b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(7*A*b -
B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/
2)/(21*b**(5/4)*sqrt(e)*sqrt(a + b*x**2)) + 2*sqrt(e*x)*sqrt(a +
b*x**2)*(7*A*b - B*a)/(21*b*e)
```

Mathematica [C] time = 0.360139, size = 132, normalized size = 0.75

$$\frac{2x\left(\left(a+bx^2\right)\left(2aB+7Ab+3bBx^2\right)-\frac{2ia\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(aB-7Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{21b\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*x*((a + b*x^2)*(7*A*b + 2*a*B + 3*b*B*x^2) - ((2*I)*a*(-7*A*b
+ a*B)*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sq
rt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(21*b*
Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.037, size = 246, normalized size = 1.4

$$\frac{2}{21\,b^2}\left(7\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}ab-B\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x)
```

```
[Out] 2/21/(b*x^2+a)^(1/2)*(7*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*(-a*b)^(1/2)*a*b-B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^
(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*(-a*b)^(1/2)*a^2+3*B*x^5*b^3+7*A*x^3*b^3+5*B*x^3*a*b^2
+7*A*x*a*b^2+2*B*x*a^2*b)/(e*x)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

Sympy [A] time = 10.7972, size = 97, normalized size = 0.55

$$\frac{A\sqrt{a}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{B\sqrt{a}x^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*sqrt(e)*gamma(5/4)) + B*sqrt(a)*x**(5/2)*gamm
a(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sq
rt(e)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

3.788 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=333

$$\frac{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}} + \frac{2(ex)^{3/2}\sqrt{a+bx^2}(aB+5Ab)}{5ae^3} + \frac{4\sqrt{ex}\sqrt{a+bx^2}(aB+5Ab)}{5\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

```
[Out] (2*(5*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^3) + (4*(5*A
*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*Sqrt[b]*e^2*(Sqrt[a] + Sq
rt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(a*e*Sqrt[e*x]) - (4*a^(1/4)*
(5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt
[e])], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (2*a^(1/4)*(5*
A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.636198, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}} + \frac{2(ex)^{3/2}\sqrt{a+bx^2}(aB+5Ab)}{5ae^3} + \frac{4\sqrt{ex}\sqrt{a+bx^2}(aB+5Ab)}{5\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (2*(5*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^3) + (4*(5*A
*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*Sqrt[b]*e^2*(Sqrt[a] + Sq
rt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(a*e*Sqrt[e*x]) - (4*a^(1/4)*
(5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt
[e])], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (2*a^(1/4)*(5*
A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 63.9852, size = 309, normalized size = 0.93

$$-\frac{2A\left(a+bx^2\right)^{\frac{3}{2}}}{ae\sqrt{ex}}-\frac{4\sqrt[4]{a}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(5Ab+Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}+\frac{2\sqrt[4]{a}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(5Ab+Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}+\frac{4\sqrt{ex}\sqrt{a+bx^2}(5Ab+Ba)}{5\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(ex)^{\frac{3}{2}}\sqrt{a+bx^2}(5Ab+Ba)}{5ae^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(3/2)/(a*e*sqrt(e*x)) - 4*a**(1/4)*sqrt((a + b
*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(5*A*b + B
*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2
)/(5*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + 2*a**(1/4)*sqrt((a + b
*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(5*A*b + B
*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2
)/(5*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + 4*sqrt(e*x)*sqrt(a + b
*x**2)*(5*A*b + B*a)/(5*sqrt(b)*e**2*(sqrt(a) + sqrt(b)*x)) + 2*(
e*x)**(3/2)*sqrt(a + b*x**2)*(5*A*b + B*a)/(5*a*e**3)
```

Mathematica [C] time = 1.15575, size = 186, normalized size = 0.56

$$\frac{x^{3/2}\left(\frac{2\sqrt{a+bx^2}(Bx^2-5A)}{\sqrt{x}}-\frac{4x(aB+5Ab)\left(-\sqrt{x}\left(\frac{a}{x^2}+b\right)+\frac{ia\sqrt{\frac{a}{bx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}}\right)}{b\sqrt{a+bx^2}}\right)}{5(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (x^(3/2)*((2*Sqrt[a + b*x^2]*(-5*A + B*x^2))/Sqrt[x] - (4*(5*A*b
+ a*B)*x*(-((b + a/x^2)*Sqrt[x]) + (I*a*Sqrt[1 + a/(b*x^2)]*(Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] - Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/((I*Sqrt[a
])/Sqrt[b])^(3/2)))/(b*Sqrt[a + b*x^2])))/(5*(e*x)^(3/2))
```

Maple [A] time = 0.049, size = 391, normalized size = 1.2

$$\frac{2}{5\,be}\left(10\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x)
```

```
[Out] 2/5*(10*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-5*A
*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b+2*B*((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-B*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+b^2*B*x^4-5*A*x^2*b^2+B*x^2*a*b-
5*a*b*A)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}}{\sqrt{ex}ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/(sqrt(e*x)*e*x), x)
```

Sympy [A] time = 12.7345, size = 100, normalized size = 0.3

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)} + \frac{B\sqrt{a}x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] A*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + B*sqrt(a)*x**(3/2)*g
amma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2
*e**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2), x)
```

3.789 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{3ae^3}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

```
[Out] (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*a*e^3) - (2*A*(a + b
*x^2)^(3/2))/(3*a*e*(e*x)^(3/2)) + (2*(A*b + a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(1/
4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.288693, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{3ae^3}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*a*e^3) - (2*A*(a + b
*x^2)^(3/2))/(3*a*e*(e*x)^(3/2)) + (2*(A*b + a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(1/
4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 28.8323, size = 156, normalized size = 0.91

$$-\frac{2A\left(a+bx^2\right)^{\frac{3}{2}}}{3ae\,(ex)^{\frac{3}{2}}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}\,(Ab+Ba)}{3ae^3}+\frac{2\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}e^{\frac{5}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(3/2)/(3*a*e*(e*x)**(3/2)) + 2*sqrt(e*x)*sqrt(
a + b*x**2)*(A*b + B*a)/(3*a*e**3) + 2*sqrt((a + b*x**2)/(sqrt(a)
 + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b + B*a)*elliptic_f(2*
atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(3*a**(1/4)*b**
(1/4)*e**(5/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.365106, size = 120, normalized size = 0.7

$$\frac{2x\left(\left(a+bx^2\right)\left(Bx^2-A\right)+\frac{2ix^{5/2}\sqrt{\frac{a}{bx^2}+1}(aB+Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{3(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*x*((a + b*x^2)*(-A + B*x^2) + ((2*I)*(A*b + a*B)*Sqrt[1 + a/(b
*x^2)]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt
[x]], -1])/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(3*(e*x)^(5/2)*Sqrt[a + b*
x^2])
```

Maple [A] time = 0.041, size = 234, normalized size = 1.4

$$\frac{2}{3\,xe^2b}\left(A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{1\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}EllipticF\left(\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*x*b+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*x*a+b^2*B*x^4-A*x^2*b^2+B*x^2*a*b-a*b*A)/e
^2/(e*x)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{\sqrt{ex}e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/(sqrt(e*x)*e^2*x^2), x)
```

Sympy [A] time = 65.5552, size = 100, normalized size = 0.58

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\,\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{a}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] A*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(a)*sqrt(x)*g
amma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2
*e**(5/2)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2), x)
```

3.790 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=338

$$\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}+\frac{4\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(5aB+Ab)}{5ae^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\sqrt{a+bx^2}(5aB+Ab)}{5ae^3\sqrt{ex}}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*(A*b + 5*a*B)*Sqrt[a + b*x^2])/(5*a*e^3*Sqrt[e*x]) + (4*Sqrt[
b]*(A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a*e^4*(Sqrt[a] + S
qrt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(5*a*e*(e*x)^(5/2)) - (4*b^(
1/4)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2]) + (2*b^(1/4
)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.646433, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}+\frac{4\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(5aB+Ab)}{5ae^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\sqrt{a+bx^2}(5aB+Ab)}{5ae^3\sqrt{ex}}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```


```
[Out] (-2*(A*b + 5*a*B)*Sqrt[a + b*x^2])/(5*a*e^3*Sqrt[e*x]) + (4*Sqrt[
b]*(A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a*e^4*(Sqrt[a] + S
qrt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(5*a*e*(e*x)^(5/2)) - (4*b^(
1/4)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2]) + (2*b^(1/4
)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 65.6255, size = 313, normalized size = 0.93

$$-\frac{2A\left(a+bx^2\right)^{\frac{3}{2}}}{5ae\left(ex\right)^{\frac{5}{2}}}+\frac{4\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}\left(Ab+5Ba\right)}{5ae^4\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\sqrt{a+bx^2}\left(Ab+5Ba\right)}{5ae^3\sqrt{ex}}-\frac{4\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(Ab+5Ba\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{a+bx^2}}+\frac{2\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(Ab+5Ba\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(3/2)/(5*a*e*(e*x)**(5/2)) + 4*sqrt(b)*sqrt(e*
x)*sqrt(a + b*x**2)*(A*b + 5*B*a)/(5*a*e**4*(sqrt(a) + sqrt(b)*x)
) - 2*sqrt(a + b*x**2)*(A*b + 5*B*a)/(5*a*e**3*sqrt(e*x)) - 4*b**
(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt
(b)*x)*(A*b + 5*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/
4)*sqrt(e))), 1/2)/(5*a**(3/4)*e**(7/2)*sqrt(a + b*x**2)) + 2*b**
(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt
(b)*x)*(A*b + 5*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/
4)*sqrt(e))), 1/2)/(5*a**(3/4)*e**(7/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.895357, size = 217, normalized size = 0.64

$$\frac{x\left(-2\sqrt{a}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a+bx^2\right)\left(A-5Bx^2\right)+4\sqrt{b}x^{7/2}\sqrt{\frac{a}{bx^2}+1}(5aB+Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-4\sqrt{b}x^{7/2}\sqrt{\frac{a}{bx^2}+1}(5aB+A}{5\sqrt{a}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(ex)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (x*(-2*Sqrt[a]*Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*x^2)*(A - 5*B*x^2
) - 4*Sqrt[b]*(A*b + 5*a*B)*Sqrt[1 + a/(b*x^2)]*x^(7/2)*EllipticE
[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] + 4*Sqrt[b]*(A
*b + 5*a*B)*Sqrt[1 + a/(b*x^2)]*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[
(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(5*Sqrt[a]*Sqrt[(I*Sqrt[a])/
Sqrt[b]]*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.049, size = 417, normalized size = 1.2

$$\frac{2}{5x^2e^3a}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*
a*b-A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
```

```
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b+10*
B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-5*B*((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-2*A*b^2*x^4-
5*B*x^4*a*b-3*a*A*b*x^2-5*B*x^2*a^2-A*a^2)/(b*x^2+a)^(1/2)/e^3/(e
*x)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}}{\sqrt{ex}e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/(sqrt(e*x)*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2), x)
```

3.791 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(Ab-7aB)}{21ax^{3/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

```
[Out] (2*(A*b - 7*a*B)*Sqrt[a + b*x^2])/(21*a*x^(3/2)) - (2*A*(a + b*x^
2)^(3/2))/(7*a*x^(7/2)) - (2*b^(3/4)*(A*b - 7*a*B)*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*a^(5/4)*Sqrt[a + b*x^2]
)
```

Rubi [A] time = 0.229706, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(Ab-7aB)}{21ax^{3/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

```
[Out] (2*(A*b - 7*a*B)*Sqrt[a + b*x^2])/(21*a*x^(3/2)) - (2*A*(a + b*x^
2)^(3/2))/(7*a*x^(7/2)) - (2*b^(3/4)*(A*b - 7*a*B)*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*a^(5/4)*Sqrt[a + b*x^2]
)
```

Rubi in Sympy [A] time = 20.3476, size = 139, normalized size = 0.91

$$-\frac{2A\left(a+bx^2\right)^{\frac{3}{2}}}{7ax^{\frac{7}{2}}}+\frac{2\sqrt{a+bx^2}\,(Ab-7Ba)}{21ax^{\frac{3}{2}}}-\frac{2b^{\frac{3}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-7Ba)\,F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{21a^{\frac{5}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(9/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(3/2)/(7*a*x**(7/2)) + 2*sqrt(a + b*x**2)*(A*b
 - 7*B*a)/(21*a*x**(3/2)) - 2*b**(3/4)*sqrt((a + b*x**2)/(sqrt(a)
 + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b - 7*B*a)*elliptic_f(
2*atan(b**(1/4)*sqrt(x)/a**(1/4)), 1/2)/(21*a**(5/4)*sqrt(a + b*x
**2))
```

Mathematica [C] time = 0.247867, size = 139, normalized size = 0.91

$$\sqrt{a+bx^2}\left(-\frac{2(7aB+2Ab)}{21ax^{3/2}}-\frac{2A}{7x^{7/2}}\right)+\frac{4ibx\sqrt{\frac{a}{bx^2}+1}(7aB-Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{21a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

```
[Out] ((-2*A)/(7*x^(7/2)) - (2*(2*A*b + 7*a*B))/(21*a*x^(3/2)))*Sqrt[a
+ b*x^2] + (((4*I)/21)*b*(-(A*b) + 7*a*B)*Sqrt[1 + a/(b*x^2)]*x*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/(a*Sq
rt[(I*Sqrt[a])/Sqrt[b]]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.058, size = 242, normalized size = 1.6

$$-\frac{2}{21\,a}\left(A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{1\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}EllipticF\left(\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/21/(b*x^2+a)^(1/2)*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*x^3*b-7*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*(-a*b)^(1/2)*x^3*a+2*A*b^2*x^4+7*B*x^4*a*b+5*a*A*b*
x^2+7*B*x^2*a^2+3*A*a^2)/x^(7/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{bx^2+a}}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

3.792 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}+\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}-\frac{4b^{3/2}\sqrt{x}\sqrt{a+bx^2}(Ab-3aB)}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4b\sqrt{a+bx^2}(Ab-3aB)}{15a^2\sqrt{x}}+\frac{2\sqrt{a+bx^2}(Ab-3aB)}{15ax^{5/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}}$$

```
[Out] (2*(A*b - 3*a*B)*Sqrt[a + b*x^2])/(15*a*x^(5/2)) + (4*b*(A*b - 3*
a*B)*Sqrt[a + b*x^2])/(15*a^2*Sqrt[x]) - (4*b^(3/2)*(A*b - 3*a*B)
*Sqrt[x]*Sqrt[a + b*x^2])/(15*a^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(
a + b*x^2)^(3/2))/(9*a*x^(9/2)) + (4*b^(5/4)*(A*b - 3*a*B)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a
+ b*x^2]) - (2*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[x])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.533446, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}+\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}-\frac{4b^{3/2}\sqrt{x}\sqrt{a+bx^2}(Ab-3aB)}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4b\sqrt{a+bx^2}(Ab-3aB)}{15a^2\sqrt{x}}+\frac{2\sqrt{a+bx^2}(Ab-3aB)}{15ax^{5/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```


```
[Out] (2*(A*b - 3*a*B)*Sqrt[a + b*x^2])/(15*a*x^(5/2)) + (4*b*(A*b - 3*
a*B)*Sqrt[a + b*x^2])/(15*a^2*Sqrt[x]) - (4*b^(3/2)*(A*b - 3*a*B)
*Sqrt[x]*Sqrt[a + b*x^2])/(15*a^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(
a + b*x^2)^(3/2))/(9*a*x^(9/2)) + (4*b^(5/4)*(A*b - 3*a*B)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a
+ b*x^2]) - (2*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[x])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 50.707, size = 309, normalized size = 0.93

$$-\frac{2A\left(a+bx^2\right)^{\frac{3}{2}}}{9ax^{\frac{9}{2}}}+\frac{2\sqrt{a+bx^2}\,(Ab-3Ba)}{15ax^{\frac{5}{2}}}-\frac{4b^{\frac{3}{2}}\sqrt{x}\sqrt{a+bx^2}\,(Ab-3Ba)}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4b\sqrt{a+bx^2}\,(Ab-3Ba)}{15a^2\sqrt{x}}+\frac{4b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-3Ba)\,E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15a^{\frac{7}{4}}\sqrt{a+bx^2}}-\frac{2b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-3Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15a^{\frac{7}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(11/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(3/2)/(9*a*x**(9/2)) + 2*sqrt(a + b*x**2)*(A*b
 - 3*B*a)/(15*a*x**(5/2)) - 4*b**(3/2)*sqrt(x)*sqrt(a + b*x**2)*(
A*b - 3*B*a)/(15*a**2*(sqrt(a) + sqrt(b)*x)) + 4*b*sqrt(a + b*x**
2)*(A*b - 3*B*a)/(15*a**2*sqrt(x)) + 4*b**(5/4)*sqrt((a + b*x**2)
/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b - 3*B*a)*el
liptic_e(2*atan(b**(1/4)*sqrt(x)/a**(1/4)), 1/2)/(15*a**(7/4)*sqr
t(a + b*x**2)) - 2*b**(5/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*
x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b - 3*B*a)*elliptic_f(2*atan(b**(
1/4)*sqrt(x)/a**(1/4)), 1/2)/(15*a**(7/4)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.442362, size = 237, normalized size = 0.72

$$-\frac{2\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\left(a+bx^2\right)\left(a^2\left(5A+9Bx^2\right)+2abx^2\left(A+9Bx^2\right)-6Ab^2x^4\right)+6\sqrt{a}b^{3/2}x^5\sqrt{\frac{bx^2}{a}+1}(3aB-Ab)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}\ldots}{\sqrt{a}}}\right.\right.\right.}{45a^2x^{9/2}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

```
[Out] (-2*(Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(a + b*x^2)*(-6*A*b^2*x^4 + 2*a*
b*x^2*(A + 9*B*x^2) + a^2*(5*A + 9*B*x^2)) - 6*Sqrt[a]*b^(3/2)*(-
(A*b) + 3*a*B)*x^5*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[b]*x)/Sqrt[a]]], -1] + 6*Sqrt[a]*b^(3/2)*(-(A*b) + 3*a*B)*
x^5*Sqrt[1 + (b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sq
rt[a]]], -1]))/(45*a^2*x^(9/2)*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*Sqrt[a
 + b*x^2])
```

Maple [A] time = 0.064, size = 439, normalized size = 1.3

$$-\frac{2}{45\,a^2}\left(6\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathit{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^4ab^2-3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-b\ldots}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x)
```

```
[Out] -2/45*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*
b^2-3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
```

```
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b^2
-18*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*b+9
*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*b-6*A*
x^6*b^3+18*B*x^6*a*b^2-4*A*x^4*a*b^2+27*B*x^4*a^2*b+7*A*x^2*a^2*b
+9*B*x^2*a^3+5*A*a^3)/(b*x^2+a)^(1/2)/x^(9/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

3.793 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-11aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231a^{9/4}\sqrt{a+bx^2}}+\frac{4b\sqrt{a+bx^2}(5Ab-11aB)}{231a^2x^{3/2}}+\frac{2\sqrt{a+bx^2}(5Ab-11aB)}{77ax^{7/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}}$$

```
[Out] (2*(5*A*b - 11*a*B)*Sqrt[a + b*x^2])/(77*a*x^(7/2)) + (4*b*(5*A*b
 - 11*a*B)*Sqrt[a + b*x^2])/(231*a^2*x^(3/2)) - (2*A*(a + b*x^2)^
(3/2))/(11*a*x^(11/2)) + (2*b^(7/4)*(5*A*b - 11*a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*a^(9/4)*Sqrt[a + b*x
^2])
```

Rubi [A] time = 0.278419, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-11aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231a^{9/4}\sqrt{a+bx^2}}+\frac{4b\sqrt{a+bx^2}(5Ab-11aB)}{231a^2x^{3/2}}+\frac{2\sqrt{a+bx^2}(5Ab-11aB)}{77ax^{7/2}}-\frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

```
[Out] (2*(5*A*b - 11*a*B)*Sqrt[a + b*x^2])/(77*a*x^(7/2)) + (4*b*(5*A*b
 - 11*a*B)*Sqrt[a + b*x^2])/(231*a^2*x^(3/2)) - (2*A*(a + b*x^2)^
(3/2))/(11*a*x^(11/2)) + (2*b^(7/4)*(5*A*b - 11*a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*a^(9/4)*Sqrt[a + b*x
^2])
```

Rubi in Sympy [A] time = 25.4306, size = 177, normalized size = 0.95

$$-\frac{2A\left(a+bx^2\right)^{\frac{3}{2}}}{11ax^{\frac{11}{2}}}+\frac{2\sqrt{a+bx^2}\,(5Ab-11Ba)}{77ax^{\frac{7}{2}}}+\frac{4b\sqrt{a+bx^2}\,(5Ab-11Ba)}{231a^2x^{\frac{3}{2}}}+\frac{2b^{\frac{7}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(5Ab-11Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{231a^{\frac{9}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(13/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(3/2)/(11*a*x**(11/2)) + 2*sqrt(a + b*x**2)*(5
*A*b - 11*B*a)/(77*a*x**(7/2)) + 4*b*sqrt(a + b*x**2)*(5*A*b - 11
*B*a)/(231*a**2*x**(3/2)) + 2*b**(7/4)*sqrt((a + b*x**2)/(sqrt(a)
 + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(5*A*b - 11*B*a)*elliptic
_f(2*atan(b**(1/4)*sqrt(x)/a**(1/4)), 1/2)/(231*a**(9/4)*sqrt(a +
 b*x**2))
```

Mathematica [C] time = 0.319543, size = 163, normalized size = 0.87

$$\sqrt{a+bx^2}\left(-\frac{4b(11aB-5Ab)}{231a^2x^{3/2}}-\frac{2(11aB+2Ab)}{77ax^{7/2}}-\frac{2A}{11x^{11/2}}\right)-\frac{4ib^2x\sqrt{\frac{a}{bx^2}+1}(11aB-5Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{231a^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

```
[Out] ((-2*A)/(11*x^(11/2)) - (2*(2*A*b + 11*a*B))/(77*a*x^(7/2)) - (4*
b*(-5*A*b + 11*a*B))/(231*a^2*x^(3/2)))*Sqrt[a + b*x^2] - (((4*I)
/231)*b^2*(-5*A*b + 11*a*B)*Sqrt[1 + a/(b*x^2)]*x*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/(a^2*Sqrt[(I*Sqrt[a
])/Sqrt[b]]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.046, size = 270, normalized size = 1.4

$$\frac{2}{231\,a^2}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^5b^2-11\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x)
```

```
[Out] 2/231/(b*x^2+a)^(1/2)*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*x^5*b^2-11*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^5*a*b+10*A*x^6*b^3-22*B*x^6*a*b
^2+4*A*x^4*a*b^2-55*B*x^4*a^2*b-27*A*x^2*a^2*b-33*B*x^2*a^3-21*A*
a^3)/x^(11/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{bx^2 + a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

3.794 $\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=252

$$-\frac{4a^{11/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2e\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{231b^2}+\frac{4a(ex)^{5/2}\sqrt{a+bx^2}(3Ab-aB)}{77be}+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}(3Ab-aB)}{33be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

```
[Out] (8*a^2*(3*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*
a*(3*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*(3*A*b
 - a*B)*(e*x)^(5/2)*(a + b*x^2)^(3/2))/(33*b*e) + (2*B*(e*x)^(5/2
)*(a + b*x^2)^(5/2))/(15*b*e) - (4*a^(11/4)*(3*A*b - a*B)*e^(3/2)
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/
(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.431151, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{4a^{11/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2e\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{231b^2}+\frac{4a(ex)^{5/2}\sqrt{a+bx^2}(3Ab-aB)}{77be}+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}(3Ab-aB)}{33be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (8*a^2*(3*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*
a*(3*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*(3*A*b
 - a*B)*(e*x)^(5/2)*(a + b*x^2)^(3/2))/(33*b*e) + (2*B*(e*x)^(5/2
)*(a + b*x^2)^(5/2))/(15*b*e) - (4*a^(11/4)*(3*A*b - a*B)*e^(3/2)
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/
(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 41.8079, size = 230, normalized size = 0.91

$$\frac{2B\left(ex\right)^{\frac{5}{2}}\left(a+bx^2\right)^{\frac{5}{2}}}{15be}-\frac{4a^{\frac{11}{4}}e^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(3Ab-Ba\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231b^{\frac{9}{4}}\sqrt{a+bx^2}}+\frac{8a^2e\sqrt{ex}\sqrt{a+bx^2}\left(3Ab-Ba\right)}{231b^2}+\frac{4a\left(ex\right)^{\frac{5}{2}}\sqrt{a+bx^2}\left(3Ab-Ba\right)}{77be}+\frac{2\left(ex\right)^{\frac{5}{2}}\left(a+bx^2\right)^{\frac{3}{2}}\left(3Ab-Ba\right)}{33be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] 2*B*(e*x)**(5/2)*(a + b*x**2)**(5/2)/(15*b*e) - 4*a**(11/4)*e**(3
/2)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b
```

```
)*x)*(3*A*b - B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)
*sqrt(e))), 1/2)/(231*b**(9/4)*sqrt(a + b*x**2)) + 8*a**2*e*sqrt(
e*x)*sqrt(a + b*x**2)*(3*A*b - B*a)/(231*b**2) + 4*a*(e*x)**(5/2)
*sqrt(a + b*x**2)*(3*A*b - B*a)/(77*b*e) + 2*(e*x)**(5/2)*(a + b*
x**2)**(3/2)*(3*A*b - B*a)/(33*b*e)
```

Mathematica [C] time = 0.585068, size = 178, normalized size = 0.71

$$\frac{2e\sqrt{ex}\left(-\left(a+bx^2\right)\left(20a^3B-12a^2b\left(5A+Bx^2\right)-ab^2x^2\left(195A+119Bx^2\right)-7b^3x^4\left(15A+11Bx^2\right)\right)+\frac{20ia^3\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(aB-3Ab\cdots}{\sqrt{\cdots}}\right.}{1155b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*(-((a + b*x^2)*(20*a^3*B - 12*a^2*b*(5*A + B*x^2)
- 7*b^3*x^4*(15*A + 11*B*x^2) - a*b^2*x^2*(195*A + 119*B*x^2))) +
 ((20*I)*a^3*(-3*A*b + a*B)*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[
a])/Sqrt[b]]))/(1155*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.037, size = 300, normalized size = 1.2

$$-\frac{2e}{1155xb^3}\sqrt{ex}\left(-77b^5Bx^9-105Ax^7b^5-196Bx^7ab^4+30A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{\cdots}}{\sqrt{-\cdots}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] -2/1155*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-77*b^5*B*x^9-105*A*x^7*
b^5-196*B*x^7*a*b^4+30*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*a^3*b-300*A*x^5*a*b^4-10*B*((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^4-131*B*x^5*a^2*b^3
-255*A*x^3*a^2*b^3+8*B*x^3*a^3*b^2-60*A*x*a^3*b^2+20*B*x*a^4*b)/b
^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbex^5 + (Ba + Ab)ex^3 + Aaex\right)\sqrt{bx^2 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*e*x^5 + (B*a + A*b)*e*x^3 + A*a*e*x)*sqrt(b*x^2 + a
)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2 + A\right)\left(bx^2 + a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2), x)
```

3.795 $\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=377

$$\frac{4a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(13Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(13Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}+\frac{8a^2\sqrt{ex}\sqrt{a+bx^2}(13Ab-3aB)}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}(13Ab-3aB)}{117be}+\frac{4a(ex)^{3/2}\sqrt{a+bx^2}(13Ab-3aB)}{195be}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

```
[Out] (4*a*(13*A*b - 3*a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(195*b*e) + (8
*a^2*(13*A*b - 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(195*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x)) + (2*(13*A*b - 3*a*B)*(e*x)^(3/2)*(a + b*x^2)
^(3/2))/(117*b*e) + (2*B*(e*x)^(3/2)*(a + b*x^2)^(5/2))/(13*b*e)
- (8*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^2]
) + (4*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^
2])
```

Rubi [A] time = 0.717556, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{4a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(13Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(13Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}+\frac{8a^2\sqrt{ex}\sqrt{a+bx^2}(13Ab-3aB)}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}(13Ab-3aB)}{117be}+\frac{4a(ex)^{3/2}\sqrt{a+bx^2}(13Ab-3aB)}{195be}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (4*a*(13*A*b - 3*a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(195*b*e) + (8
*a^2*(13*A*b - 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(195*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x)) + (2*(13*A*b - 3*a*B)*(e*x)^(3/2)*(a + b*x^2)
^(3/2))/(117*b*e) + (2*B*(e*x)^(3/2)*(a + b*x^2)^(5/2))/(13*b*e)
- (8*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^2]
) + (4*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^
2])
```

Rubi in Sympy [A] time = 72.7116, size = 352, normalized size = 0.93

$$\frac{2B(ex)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{5}{2}}}{13be}-\frac{8a^{\frac{9}{4}}\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(13Ab-3Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195b^{\frac{7}{4}}\sqrt{a+bx^2}}+\frac{4a^{\frac{9}{4}}\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(13Ab-3Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195b^{\frac{7}{4}}\sqrt{a+bx^2}}+\frac{8a^2\sqrt{ex}\sqrt{a+bx^2}(13Ab-3Ba)}{195b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4a(ex)^{\frac{3}{2}}\sqrt{a+bx^2}(13Ab-3Ba)}{195be}+\frac{2(ex)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{2}}(13Ab-3Ba)}{117be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)*(e*x)**(1/2),x)
```

```
[Out] 2*B*(e*x)**(3/2)*(a + b*x**2)**(5/2)/(13*b*e) - 8*a**(9/4)*sqrt(e
)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*
x)*(13*A*b - 3*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4
)*sqrt(e))), 1/2)/(195*b**(7/4)*sqrt(a + b*x**2)) + 4*a**(9/4)*sq
rt(e)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt
(b)*x)*(13*A*b - 3*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**
(1/4)*sqrt(e))), 1/2)/(195*b**(7/4)*sqrt(a + b*x**2)) + 8*a**2*sq
rt(e*x)*sqrt(a + b*x**2)*(13*A*b - 3*B*a)/(195*b**(3/2)*(sqrt(a)
+ sqrt(b)*x)) + 4*a*(e*x)**(3/2)*sqrt(a + b*x**2)*(13*A*b - 3*B*a
)/(195*b*e) + 2*(e*x)**(3/2)*(a + b*x**2)**(3/2)*(13*A*b - 3*B*a)
/(117*b*e)
```

Mathematica [C] time = 1.46976, size = 214, normalized size = 0.57

$$\frac{2\sqrt{x}\sqrt{ex}\left(b\sqrt{x}\left(a+bx^2\right)\left(12a^2B+ab\left(143A+75Bx^2\right)+5b^2x^2\left(13A+9Bx^2\right)\right)+12a^2(3aB-13Ab)\left(-\sqrt{x}\left(\frac{a}{x^2}+b\right)+\frac{ia\sqrt{\frac{a}{bx^2}}\cdots}{\cdots}\right)\right)}{585b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[e*x]*(b*Sqrt[x]*(a + b*x^2)*(12*a^2*B + 5*b^2*x^2
*(13*A + 9*B*x^2) + a*b*(143*A + 75*B*x^2)) + 12*a^2*(-13*A*b + 3
*a*B)*(-((b + a/x^2)*Sqrt[x]) + (I*a*Sqrt[1 + a/(b*x^2)]*(Ellipti
cE[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] - EllipticF[
I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/((I*Sqrt[a])/
Sqrt[b])^(3/2))))/(585*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.037, size = 438, normalized size = 1.2

$$\frac{2}{585\,b^2x}\sqrt{ex}\left(45\,Bx^8b^4+65\,Ax^6b^4+120\,Bx^6ab^3+156\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x)
```

```
[Out] 2/585/(b*x^2+a)^(1/2)*(e*x)^(1/2)/b^2*(45*B*x^8*b^4+65*A*x^6*b^4+
120*B*x^6*a*b^3+156*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(
1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a^3*b-78*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)
*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a
^3*b-36*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4+18*
B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4+208*A*x^4*a
*b^3+87*B*x^4*a^2*b^2+143*A*x^2*a^2*b^2+12*B*x^2*a^3*b)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbx^4+(Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(e
*x), x)
```

Sympy [A] time = 57.8709, size = 197, normalized size = 0.52

$$\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e\,\Gamma\left(\frac{7}{4}\right)}+\frac{A\sqrt{a}b\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\,\Gamma\left(\frac{11}{4}\right)}+\frac{Ba^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\,\Gamma\left(\frac{11}{4}\right)}+\frac{B\sqrt{a}b\,(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^5\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x
**2*exp_polar(I*pi)/a)/(2*e*gamma(7/4)) + A*sqrt(a)*b*(e*x)**(7/2
)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**3*gamma(11/4)) + B*a**(3/2)*(e*x)**(7/2)*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**3*gamma(11
/4)) + B*sqrt(a)*b*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4),
(15/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**5*gamma(15/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^2 + A\right)\left(bx^2 + a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x), x)
```

3.796 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=214

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(11Ab-aB)}{77be}+\frac{4a\sqrt{ex}\sqrt{a+bx^2}(11Ab-aB)}{77be}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be}$$

```
[Out] (4*a*(11*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(77*b*e) + (2*(11*
A*b - a*B)*Sqrt[e*x]*(a + b*x^2)^(3/2))/(77*b*e) + (2*B*Sqrt[e*x]
*(a + b*x^2)^(5/2))/(11*b*e) + (4*a^(7/4)*(11*A*b - a*B)*(Sqrt[a]
 + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(77*b^(5/
4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.358399, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(11Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(11Ab-aB)}{77be}+\frac{4a\sqrt{ex}\sqrt{a+bx^2}(11Ab-aB)}{77be}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{5/2}}{11be}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```


```
[Out] (4*a*(11*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(77*b*e) + (2*(11*
A*b - a*B)*Sqrt[e*x]*(a + b*x^2)^(3/2))/(77*b*e) + (2*B*Sqrt[e*x]
*(a + b*x^2)^(5/2))/(11*b*e) + (4*a^(7/4)*(11*A*b - a*B)*(Sqrt[a]
 + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(77*b^(5/
4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 34.239, size = 192, normalized size = 0.9

$$\frac{2B\sqrt{ex}\left(a+bx^2\right)^{\frac{5}{2}}}{11be}+\frac{4a^{\frac{7}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(11Ab-Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{77b^{\frac{5}{4}}\sqrt{e}\sqrt{a+bx^2}}+\frac{4a\sqrt{ex}\sqrt{a+bx^2}\,(11Ab-Ba)}{77be}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{\frac{3}{2}}(11Ab-Ba)}{77be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(1/2),x)
```

```
[Out] 2*B*sqrt(e*x)*(a + b*x**2)**(5/2)/(11*b*e) + 4*a**(7/4)*sqrt((a +
 b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(11*A*b
- B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))),
1/2)/(77*b**(5/4)*sqrt(e)*sqrt(a + b*x**2)) + 4*a*sqrt(e*x)*sqrt(
a + b*x**2)*(11*A*b - B*a)/(77*b*e) + 2*sqrt(e*x)*(a + b*x**2)**(
3/2)*(11*A*b - B*a)/(77*b*e)
```

Mathematica [C] time = 0.408146, size = 155, normalized size = 0.72

$$\frac{2x\left(\left(a+bx^2\right)\left(4a^2B+ab\left(33A+13Bx^2\right)+b^2x^2\left(11A+7Bx^2\right)\right)-\frac{4ia^2\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(aB-11Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{77b\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*x*((a + b*x^2)*(4*a^2*B + b^2*x^2*(11*A + 7*B*x^2) + a*b*(33*A
 + 13*B*x^2)) - ((4*I)*a^2*(-11*A*b + a*B)*Sqrt[1 + a/(b*x^2)]*Sq
rt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]
)/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(77*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.022, size = 272, normalized size = 1.3

$$\frac{2}{77\,b^2}\left(7\,Bx^7b^4+22\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{2}\sqrt{-ab}a^2b+11\,Ax^5b^4-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x)
```

```
[Out] 2/77/(b*x^2+a)^(1/2)*(7*B*x^7*b^4+22*A*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*2^(1/2)*(-a*b)^(1/2)*a^2*b+11*A*x^5*b^4-2*B*((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*(-a*b)^(1/2)*a^3+20*B*x^
5*a*b^3+44*A*x^3*a*b^3+17*B*x^3*a^2*b^2+33*A*x*a^2*b^2+4*B*x*a^3*
b)/b^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^4+(Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}}{\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/sqrt(e
*x), x)
```

Sympy [A] time = 51.0508, size = 199, normalized size = 0.93

$$\frac{Aa^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{A\sqrt{a}bx^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{B\sqrt{a}bx^{\frac{9}{2}}\,\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*sqrt(e)*gamma(5/4)) + A*sqrt(a)*b*x**(5/2)*g
amma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2
*sqrt(e)*gamma(9/4)) + B*a**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2
, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4))
+ B*sqrt(a)*b*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x
**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x), x)
```

3.797 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{4a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{8a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}} + \frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}(aB+9Ab)}{9ae^3} + \frac{4(ex)^{3/2}\sqrt{a+bx^2}(aB+9Ab)}{15e^3} + \frac{8a\sqrt{ex}\sqrt{a+bx^2}(aB+9Ab)}{15\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

```
[Out] (4*(9*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*e^3) + (8*a*(9*
A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*Sqrt[b]*e^2*(Sqrt[a] +
Sqrt[b]*x)) + (2*(9*A*b + a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*
a*e^3) - (2*A*(a + b*x^2)^(5/2))/(a*e*Sqrt[e*x]) - (8*a^(5/4)*(9*
A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (4*a^(5/4)*(9*A*
b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b
]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])]
, 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.72001, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{4a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{8a^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+9Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}} + \frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}(aB+9Ab)}{9ae^3} + \frac{4(ex)^{3/2}\sqrt{a+bx^2}(aB+9Ab)}{15e^3} + \frac{8a\sqrt{ex}\sqrt{a+bx^2}(aB+9Ab)}{15\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```


```
[Out] (4*(9*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*e^3) + (8*a*(9*
A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*Sqrt[b]*e^2*(Sqrt[a] +
Sqrt[b]*x)) + (2*(9*A*b + a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*
a*e^3) - (2*A*(a + b*x^2)^(5/2))/(a*e*Sqrt[e*x]) - (8*a^(5/4)*(9*
A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (4*a^(5/4)*(9*A*
b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b
]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])]
, 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 73.9658, size = 343, normalized size = 0.93

$$-\frac{2A\left(a+bx^2\right)^{\frac{5}{2}}}{ae\sqrt{ex}}-\frac{8a^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(9Ab+Ba)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}+\frac{4a^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(9Ab+Ba)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}+\frac{8a\sqrt{ex}\sqrt{a+bx^2}(9Ab+Ba)}{15\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4(ex)^{\frac{3}{2}}\sqrt{a+bx^2}(9Ab+Ba)}{15e^3}+\frac{2(ex)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{2}}(9Ab+Ba)}{9ae^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(3/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(5/2)/(a*e*sqrt(e*x)) - 8*a**(5/4)*sqrt((a + b
*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(9*A*b + B
*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2
)/(15*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + 4*a**(5/4)*sqrt((a +
b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(9*A*b +
B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/
2)/(15*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + 8*a*sqrt(e*x)*sqrt(a
 + b*x**2)*(9*A*b + B*a)/(15*sqrt(b)*e**2*(sqrt(a) + sqrt(b)*x))
+ 4*(e*x)**(3/2)*sqrt(a + b*x**2)*(9*A*b + B*a)/(15*e**3) + 2*(e*
x)**(3/2)*(a + b*x**2)**(3/2)*(9*A*b + B*a)/(9*a*e**3)
```

Mathematica [C] time = 1.10047, size = 206, normalized size = 0.56

$$\frac{x^{3/2}\left(\frac{2\sqrt{a+bx^2}\left(-45aA+11aBx^2+9Abx^2+5bBx^4\right)}{3\sqrt{x}}-\frac{8ax(aB+9Ab)\left(-\sqrt{x}\left(\frac{a}{x^2}+b\right)+\frac{ia\sqrt{\frac{a}{bx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}}\right)}{b\sqrt{a+bx^2}}\right)}{15(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (x^(3/2)*((2*Sqrt[a + b*x^2]*(-45*a*A + 9*A*b*x^2 + 11*a*B*x^2 +
5*b*B*x^4))/(3*Sqrt[x]) - (8*a*(9*A*b + a*B)*x*(-((b + a/x^2)*Sqr
t[x]) + (I*a*Sqrt[1 + a/(b*x^2)]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqr
t[a])/Sqrt[b]]/Sqrt[x]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a
])/Sqrt[b]]/Sqrt[x]], -1]))/((I*Sqrt[a])/Sqrt[b])^(3/2)))/(b*Sqrt
[a + b*x^2])))/(15*(e*x)^(3/2))
```

Maple [A] time = 0.027, size = 421, normalized size = 1.2

$$\frac{2}{45\,be}\left(5\,Bx^6b^3+108\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)a^2b-54\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x)
```

```
[Out] 2/45*(5*B*x^6*b^3+108*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*a^2*b-54*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
*a^2*b+12*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ell
ipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-6
*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+9*A*x^4*b^
3+16*B*x^4*a*b^2-36*A*x^2*a*b^2+11*B*x^2*a^2*b-45*A*a^2*b)/(b*x^2
+a)^(1/2)/b/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bbx^4 + (Ba + Ab)x^2 + Aa)\sqrt{bx^2 + a}}{\sqrt{ex}ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(sqrt(
e*x)*e*x), x)
```

Sympy [A] time = 68.5735, size = 202, normalized size = 0.55

$$\frac{Aa^{\frac{3}{2}}\,\Gamma\left(-\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)} + \frac{A\sqrt{a}bx^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}bx^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_pol
ar(I*pi)/a)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + A*sqrt(a)*b*x**(3/2
```

```
)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)
/(2*e**(3/2)*gamma(7/4)) + B*a**(3/2)*x**(3/2)*gamma(3/4)*hyper((
-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7
/4)) + B*sqrt(a)*b*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,)
, b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2), x)
```

3.798 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}} \, dx$

Optimal. Leaf size=210

$$\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3aB+7Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(3aB+7Ab)}{21ae^3}+\frac{4\sqrt{ex}\sqrt{a+bx^2}(3aB+7Ab)}{21e^3}-\frac{2A\left(a+bx^2\right)^{5/2}}{3ae(ex)^{3/2}}$$

```
[Out] (4*(7*A*b + 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*e^3) + (2*(7*A*
b + 3*a*B)*Sqrt[e*x]*(a + b*x^2)^(3/2))/(21*a*e^3) - (2*A*(a + b*
x^2)^(5/2))/(3*a*e*(e*x)^(3/2)) + (4*a^(3/4)*(7*A*b + 3*a*B)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b
^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.361701, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3aB+7Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{3/2}(3aB+7Ab)}{21ae^3}+\frac{4\sqrt{ex}\sqrt{a+bx^2}(3aB+7Ab)}{21e^3}-\frac{2A\left(a+bx^2\right)^{5/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (4*(7*A*b + 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*e^3) + (2*(7*A*
b + 3*a*B)*Sqrt[e*x]*(a + b*x^2)^(3/2))/(21*a*e^3) - (2*A*(a + b*
x^2)^(5/2))/(3*a*e*(e*x)^(3/2)) + (4*a^(3/4)*(7*A*b + 3*a*B)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b
^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 35.2711, size = 197, normalized size = 0.94

$$-\frac{2A\left(a+bx^2\right)^{\frac{5}{2}}}{3ae\,(ex)^{\frac{3}{2}}}+\frac{4a^{\frac{3}{4}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab+3Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21\sqrt[4]{b}e^{\frac{5}{2}}\sqrt{a+bx^2}}+\frac{4\sqrt{ex}\sqrt{a+bx^2}\,(7Ab+3Ba)}{21e^3}+\frac{2\sqrt{ex}\left(a+bx^2\right)^{\frac{3}{2}}(7Ab+3Ba)}{21ae^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(5/2)/(3*a*e*(e*x)**(3/2)) + 4*a**(3/4)*sqrt((
a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(7*A*
b + 3*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e)
)), 1/2)/(21*b**(1/4)*e**(5/2)*sqrt(a + b*x**2)) + 4*sqrt(e*x)*sq
rt(a + b*x**2)*(7*A*b + 3*B*a)/(21*e**3) + 2*sqrt(e*x)*(a + b*x**
2)**(3/2)*(7*A*b + 3*B*a)/(21*a*e**3)
```

Mathematica [C] time = 0.390099, size = 140, normalized size = 0.67

$$\frac{2x\left(\left(a+bx^2\right)\left(-7aA+9aBx^2+7Abx^2+3bBx^4\right)+\frac{4iax^{5/2}\sqrt{\frac{a}{bx^2}+1}(3aB+7Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{21(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*x*((a + b*x^2)*(-7*a*A + 7*A*b*x^2 + 9*a*B*x^2 + 3*b*B*x^4) +
((4*I)*a*(7*A*b + 3*a*B)*Sqrt[1 + a/(b*x^2)]*x^(5/2)*EllipticF[I*
ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[a])
/Sqrt[b]]))/(21*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.025, size = 255, normalized size = 1.2

$$\frac{2}{21\,bxe^2}\left(14\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}xab+6\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x)
```

```
[Out] 2/21/(b*x^2+a)^(1/2)/x*(14*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1
/2*2^(1/2))*(-a*b)^(1/2)*x*a*b+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a^2+3*B*x^6*b^3+7*A*x^4*b^3+12*B
*x^4*a*b^2+9*B*x^2*a^2*b-7*A*a^2*b)/b/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^4+(Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}}{\sqrt{ex}e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(sqrt(
e*x)*e^2*x^2), x)
```

Sympy [A] time = 127.336, size = 202, normalized size = 0.96

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{A\sqrt{a}b\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{B\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(5/2),x)
```

```
[Out] A*a**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_pol
ar(I*pi)/a)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + A*sqrt(a)*b*sqrt(x
)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)
/(2*e**(5/2)*gamma(5/4)) + B*a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-
1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/
4)) + B*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,),
b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)\left(bx^2+a\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2), x)
```

3.799 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=365

$$\frac{12\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}} - \frac{24\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}} + \frac{12b(ex)^{3/2}\sqrt{a+bx^2}(aB+Ab)}{5ae^5} + \frac{24\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{5e^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\left(a+bx^2\right)^{3/2}(aB+Ab)}{ae^3\sqrt{ex}} - \frac{2A\left(a+bx^2\right)^{5/2}}{5ae(ex)^{5/2}}$$

```
[Out] (12*b*(A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^5) + (24*Sq
rt[b]*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*e^4*(Sqrt[a] + Sq
rt[b]*x)) - (2*(A*b + a*B)*(a + b*x^2)^(3/2))/(a*e^3*Sqrt[e*x]) -
 (2*A*(a + b*x^2)^(5/2))/(5*a*e*(e*x)^(5/2)) - (24*a^(1/4)*b^(1/4
)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt
[e])], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(1/4)*(A
*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])
], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.701946, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{12\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}} - \frac{24\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}} + \frac{12b(ex)^{3/2}\sqrt{a+bx^2}(aB+Ab)}{5ae^5} + \frac{24\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{5e^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\left(a+bx^2\right)^{3/2}(aB+Ab)}{ae^3\sqrt{ex}} - \frac{2A\left(a+bx^2\right)^{5/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```


```
[Out] (12*b*(A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^5) + (24*Sq
rt[b]*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*e^4*(Sqrt[a] + Sq
rt[b]*x)) - (2*(A*b + a*B)*(a + b*x^2)^(3/2))/(a*e^3*Sqrt[e*x]) -
 (2*A*(a + b*x^2)^(5/2))/(5*a*e*(e*x)^(5/2)) - (24*a^(1/4)*b^(1/4
)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt
[e])], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(1/4)*(A
*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])
], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 75.0598, size = 337, normalized size = 0.92

$$-\frac{2A\left(a+bx^2\right)^{\frac{5}{2}}}{5ae\left(ex\right)^{\frac{5}{2}}}-\frac{24\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(Ab+Ba\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5e^{\frac{7}{2}}\sqrt{a+bx^2}}+\frac{12\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(Ab+Ba\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5e^{\frac{7}{2}}\sqrt{a+bx^2}}+\frac{24\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}\left(Ab+Ba\right)}{5e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{12b\left(ex\right)^{\frac{3}{2}}\sqrt{a+bx^2}\left(Ab+Ba\right)}{5ae^5}-\frac{2\left(a+bx^2\right)^{\frac{3}{2}}\left(Ab+Ba\right)}{ae^3\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(7/2),x)
```

```
[Out] -2*A*(a + b*x**2)**(5/2)/(5*a*e*(e*x)**(5/2)) - 24*a**(1/4)*b**(1
/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b
)*x)*(A*b + B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*s
qrt(e))), 1/2)/(5*e**(7/2)*sqrt(a + b*x**2)) + 12*a**(1/4)*b**(1/
4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)
*x)*(A*b + B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(5*e**(7/2)*sqrt(a + b*x**2)) + 24*sqrt(b)*sqrt(e*x
)*sqrt(a + b*x**2)*(A*b + B*a)/(5*e**4*(sqrt(a) + sqrt(b)*x)) + 1
2*b*(e*x)**(3/2)*sqrt(a + b*x**2)*(A*b + B*a)/(5*a*e**5) - 2*(a +
 b*x**2)**(3/2)*(A*b + B*a)/(a*e**3*sqrt(e*x))
```

Mathematica [C] time = 1.02817, size = 232, normalized size = 0.64

$$\frac{x\left(24\sqrt{a}\sqrt{b}x^{7/2}\sqrt{\frac{a}{bx^2}+1}(aB+Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-24\sqrt{a}\sqrt{b}x^{7/2}\sqrt{\frac{a}{bx^2}+1}(aB+Ab)E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)+2}{5\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(ex)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (x*(2*Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*x^2)*(-(a*A) + 5*A*b*x^2 +
 7*a*B*x^2 + b*B*x^4) - 24*Sqrt[a]*Sqrt[b]*(A*b + a*B)*Sqrt[1 + a
/(b*x^2)]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/S
qrt[x]], -1] + 24*Sqrt[a]*Sqrt[b]*(A*b + a*B)*Sqrt[1 + a/(b*x^2)]
*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]],
-1]))/(5*Sqrt[(I*Sqrt[a])/Sqrt[b]]*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.027, size = 422, normalized size = 1.2

$$\frac{2}{5x^2e^3}\left(12A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(12*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*E
llipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2
```

```
*a*b-6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b+
12*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-6*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2+B*b^2*x^6
-7*A*b^2*x^4-4*B*x^4*a*b-8*a*A*b*x^2-5*B*x^2*a^2-A*a^2)/(b*x^2+a)
^(1/2)/e^3/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bbx^4 + (Ba + Ab)x^2 + Aa)\sqrt{bx^2 + a}}{\sqrt{ex}e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")

[Out] integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(sqrt(
e*x)*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(7/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(bx^2 + a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2), x)
```

3.800 $\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=338

$$-\frac{a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}+\frac{2a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}-\frac{2ae^2\sqrt{ex}\sqrt{a+bx^2}(9Ab-7aB)}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2e(ex)^{3/2}\sqrt{a+bx^2}(9Ab-7aB)}{45b^2}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

```
[Out] (2*(9*A*b - 7*a*B)*e*(e*x)^(3/2)*Sqrt[a + b*x^2])/(45*b^2) + (2*B
*(e*x)^(7/2)*Sqrt[a + b*x^2])/(9*b*e) - (2*a*(9*A*b - 7*a*B)*e^2*
Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + (
2*a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a +
 b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt
[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2]) -
(a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a +
b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.649639, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}+\frac{2a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(9Ab-7aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}-\frac{2ae^2\sqrt{ex}\sqrt{a+bx^2}(9Ab-7aB)}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2e(ex)^{3/2}\sqrt{a+bx^2}(9Ab-7aB)}{45b^2}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```


```
[Out] (2*(9*A*b - 7*a*B)*e*(e*x)^(3/2)*Sqrt[a + b*x^2])/(45*b^2) + (2*B
*(e*x)^(7/2)*Sqrt[a + b*x^2])/(9*b*e) - (2*a*(9*A*b - 7*a*B)*e^2*
Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + (
2*a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a +
 b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt
[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2]) -
(a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a +
b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 64.9764, size = 318, normalized size = 0.94

$$\frac{2B(ex)^{\frac{7}{2}}\sqrt{a+bx^2}}{9be}+\frac{2a^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(9Ab-7Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{11}{4}}\sqrt{a+bx^2}}-\frac{a^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(9Ab-7Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{11}{4}}\sqrt{a+bx^2}}-\frac{2ae^2\sqrt{ex}\sqrt{a+bx^2}(9Ab-7Ba)}{15b^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2e(ex)^{\frac{3}{2}}\sqrt{a+bx^2}(9Ab-7Ba)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] 2*B*(e*x)**(7/2)*sqrt(a + b*x**2)/(9*b*e) + 2*a**(5/4)*e**(5/2)*s
qrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*
(9*A*b - 7*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(15*b**(11/4)*sqrt(a + b*x**2)) - a**(5/4)*e**(5/2)
*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x
)*(9*A*b - 7*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*
sqrt(e))), 1/2)/(15*b**(11/4)*sqrt(a + b*x**2)) - 2*a*e**2*sqrt(e
*x)*sqrt(a + b*x**2)*(9*A*b - 7*B*a)/(15*b**(5/2)*(sqrt(a) + sqrt
(b)*x)) + 2*e*(e*x)**(3/2)*sqrt(a + b*x**2)*(9*A*b - 7*B*a)/(45*b
**2)
```

Mathematica [C] time = 1.51681, size = 237, normalized size = 0.7

$$\frac{2(ex)^{5/2}\left(bx^2\left(a+bx^2\right)\left(-7aB+9Ab+5bBx^2\right)+\frac{3a(7aB-9Ab)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a+bx^2\right)+\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+}\right.}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right.}{45b^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*(e*x)^(5/2)*(b*x^2*(a + b*x^2)*(9*A*b - 7*a*B + 5*b*B*x^2) + (
3*a*(-9*A*b + 7*a*B)*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*x^2) - Sqr
t[a]*Sqrt[b]*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt
[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] + Sqrt[a]*Sqrt[b]*Sqrt[1 + a/
(b*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sq
rt[x]], -1]))/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(45*b^3*x^3*Sqrt[a + b*
x^2])
```

Maple [A] time = 0.041, size = 417, normalized size = 1.2

$$-\frac{e^2}{45\,xb^3}\sqrt{ex}\left(-10\,Bx^6b^3+54\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)a^2b-27\,A\sqrt{\frac{b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/45/x*e^2*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^3*(-10*B*x^6*b^3+54*A*(
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
```

```
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-27*A*((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-42*B*((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+21*B*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))*a^3-18*A*x^4*b^3+4*B*x^4*a*b^2-18*A*x^
2*a*b^2+14*B*x^2*a^2*b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{5}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Be^2x^4 + Ae^2x^2)\sqrt{ex}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^4 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{5}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a), x)
```

3.801 $\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=174

$$-\frac{a^{3/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}+\frac{2e\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{21b^2}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

```
[Out] (2*(7*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*B*(
e*x)^(5/2)*Sqrt[a + b*x^2])/(7*b*e) - (a^(3/4)*(7*A*b - 5*a*B)*e^
(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.293975, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{a^{3/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}+\frac{2e\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{21b^2}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*(7*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*B*(
e*x)^(5/2)*Sqrt[a + b*x^2])/(7*b*e) - (a^(3/4)*(7*A*b - 5*a*B)*e^
(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 28.6141, size = 162, normalized size = 0.93

$$\frac{2B(ex)^{\frac{5}{2}}\sqrt{a+bx^2}}{7be}-\frac{a^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-5Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21b^{\frac{9}{4}}\sqrt{a+bx^2}}+\frac{2e\sqrt{ex}\sqrt{a+bx^2}\,(7Ab-5Ba)}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] 2*B*(e*x)**(5/2)*sqrt(a + b*x**2)/(7*b*e) - a**(3/4)*e**(3/2)*sqr
t((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(7
*A*b - 5*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt
(e))), 1/2)/(21*b**(9/4)*sqrt(a + b*x**2)) + 2*e*sqrt(e*x)*sqrt(a
 + b*x**2)*(7*A*b - 5*B*a)/(21*b**2)
```

Mathematica [C] time = 0.430906, size = 134, normalized size = 0.77

$$\frac{2e\sqrt{ex}\left(-\left(a+bx^2\right)\left(5aB-7Ab-3bBx^2\right)+\frac{ia\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(5aB-7Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{21b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*e*Sqrt[e*x]*(-((a + b*x^2)*(-7*A*b + 5*a*B - 3*b*B*x^2)) + (I*
a*(-7*A*b + 5*a*B)*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*EllipticF[I*ArcSin
h[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[a])/Sqrt[
b]]))/(21*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.039, size = 250, normalized size = 1.4

$$-\frac{e}{21\,xb^3}\sqrt{ex}\left(7\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}ab-5\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/21*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(7*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b-5*B*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2-6*B*x^5*b^3-14*A*x
^3*b^3+4*B*x^3*a*b^2-14*A*x*a*b^2+10*B*x*a^2*b)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)(ex)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^3+Aex\right)\sqrt{ex}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Sympy [A] time = 75.1032, size = 94, normalized size = 0.54

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*ga
mma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*
sqrt(a)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)(ex)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a), x)
```

3.802 $\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=299

$$\frac{2\sqrt{ex}\sqrt{a+bx^2}(5Ab-3aB)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

```
[Out] (2*B*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*b*e) + (2*(5*A*b - 3*a*B)*Sq
rt[e*x]*Sqrt[a + b*x^2])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (2*a
^(1/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*
x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2]) + (a^(1
/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])
/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.546146, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{2\sqrt{ex}\sqrt{a+bx^2}(5Ab-3aB)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*B*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*b*e) + (2*(5*A*b - 3*a*B)*Sq
rt[e*x]*Sqrt[a + b*x^2])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (2*a
^(1/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*
x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2]) + (a^(1
/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])
/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 53.9259, size = 277, normalized size = 0.93

$$\frac{2B\left(ex\right)^{\frac{3}{2}}\sqrt{a+bx^2}}{5be}-\frac{2\sqrt[4]{a}\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(5Ab-3Ba\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(5Ab-3Ba\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}\left(5Ab-3Ba\right)}{5b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] 2*B*(e*x)**(3/2)*sqrt(a + b*x**2)/(5*b*e) - 2*a**(1/4)*sqrt(e)*sq
rt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(
5*A*b - 3*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqr
t(e))), 1/2)/(5*b**(7/4)*sqrt(a + b*x**2)) + a**(1/4)*sqrt(e)*sqr
t((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(5
*A*b - 3*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt
(e))), 1/2)/(5*b**(7/4)*sqrt(a + b*x**2)) + 2*sqrt(e*x)*sqrt(a +
b*x**2)*(5*A*b - 3*B*a)/(5*b**(3/2)*(sqrt(a) + sqrt(b)*x))
```

Mathematica [C] time = 1.09915, size = 181, normalized size = 0.61

$$\frac{2\sqrt{ex}\left(Bx^{3/2}\sqrt{a+bx^2}-\frac{x(5Ab-3aB)\left(-\sqrt{x}\left(\frac{a}{x^2}+b\right)+\frac{ia\sqrt{\frac{a}{bx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}}\right)}{b\sqrt{a+bx^2}}\right)}{5b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*Sqrt[e*x]*(B*x^(3/2)*Sqrt[a + b*x^2] - ((5*A*b - 3*a*B)*x*(-((
b + a/x^2)*Sqrt[x]) + (I*a*Sqrt[1 + a/(b*x^2)]*(EllipticE[I*ArcSi
nh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] - EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/((I*Sqrt[a])/Sqrt[b])^(
3/2)))/(b*Sqrt[a + b*x^2])))/(5*b*Sqrt[x])
```

Maple [A] time = 0.022, size = 379, normalized size = 1.3

$$\frac{1}{5\,b^2x}\sqrt{ex}\left(10\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(10*A*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))*a*b-5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*a*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*a^2+3*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*
EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^
2+2*b^2*B*x^4+2*B*x^2*a*b)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\sqrt{ex}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^{2}+A\right)\sqrt{ex}}{\sqrt{bx^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

Sympy [A] time = 8.37777, size = 92, normalized size = 0.31

$$\frac{A\left(ex\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}e\,\Gamma\left(\frac{7}{4}\right)}+\frac{B\left(ex\right)^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}e^{3}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*(e*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*e*gamma(7/4)) + B*(e*x)**(7/2)*gamma(7/4)
*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*
e**3*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^{2}+A\right)\sqrt{ex}}{\sqrt{bx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

3.803 $\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=139

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

```
[Out] (2*B*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*e) + ((3*A*b - a*B)*(Sqrt[a]
 + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4
)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.234162, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*B*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*e) + ((3*A*b - a*B)*(Sqrt[a]
 + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4
)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 22.1353, size = 124, normalized size = 0.89

$$\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}+\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-Ba)F\left(2\,\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{a}b^{\frac{5}{4}}\sqrt{e}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] 2*B*sqrt(e*x)*sqrt(a + b*x**2)/(3*b*e) + sqrt((a + b*x**2)/(sqrt(
a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(3*A*b - B*a)*elliptic_
f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(3*a**(1/4)
*b**(5/4)*sqrt(e)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.470076, size = 116, normalized size = 0.83

$$\frac{2x\left(B\left(a+bx^2\right)-\frac{i\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(aB-3Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{3b\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*x*(B*(a + b*x^2) - (I*(-3*A*b + a*B)*Sqrt[1 + a/(b*x^2)]*Sqrt[
x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/S
qrt[(I*Sqrt[a])/Sqrt[b]]))/(3*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.023, size = 214, normalized size = 1.5

$$\frac{1}{3b^2}\left(3A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}b-B\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3/(b*x^2+a)^(1/2)*(3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*b-B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*a+2*b^2*B*x^3+2*B*x*a*b)/(e*x)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx^2 + A}{\sqrt{bx^2 + a}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

Sympy [A] time = 7.68379, size = 94, normalized size = 0.68

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hy
per((1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*sqrt
(e)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

3.804 $\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=290

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{a\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

```
[Out] (-2*A*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*(A*b + a*B)*Sqrt[e*x]
*Sqrt[a + b*x^2])/(a*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b
 + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])],
 1/2])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((A*b + a*B)*(
Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a
^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.552984, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{a\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```


```
[Out] (-2*A*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*(A*b + a*B)*Sqrt[e*x]
*Sqrt[a + b*x^2])/(a*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b
 + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])],
 1/2])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((A*b + a*B)*(
Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a
^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 55.4627, size = 265, normalized size = 0.91

$$-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}\,(Ab+Ba)}{a\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+Ba)\,E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}+\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(a + b*x**2)/(a*e*sqrt(e*x)) + 2*sqrt(e*x)*sqrt(a + b*x*
*2)*(A*b + B*a)/(a*sqrt(b)*e**2*(sqrt(a) + sqrt(b)*x)) - 2*sqrt((
a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b
+ B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))),
1/2)/(a**(3/4)*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + sqrt((a + b*
x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b + B*a)
*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(
a**(3/4)*b**(3/4)*e**(3/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.412106, size = 193, normalized size = 0.67

$$\frac{x\left(2\sqrt{a}x\sqrt{\frac{bx^2}{a}+1}(aB+Ab)E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)-2\left(\sqrt{a}x\sqrt{\frac{bx^2}{a}+1}(aB+Ab)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)+A\sqrt{b}\sqrt{\frac{i}{}}\right.\right.}{a\sqrt{b}(ex)^{3/2}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (x*(2*Sqrt[a]*(A*b + a*B)*x*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSi
nh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] - 2*(A*Sqrt[b]*Sqrt[(I*Sqrt[
b]*x)/Sqrt[a]]*(a + b*x^2) + Sqrt[a]*(A*b + a*B)*x*Sqrt[1 + (b*x^
2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1])))/(a
*Sqrt[b]*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.026, size = 378, normalized size = 1.3

$$\frac{1}{bea}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\mathit{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-A*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b+2*B*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
```

```
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-B*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^2-2*A*x^2*b^2-2*a*b*A)/(b*x^2+a)^(1/2)/b/
e/(e*x)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{bx^2 + a}\sqrt{ex}ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)*e*x), x)
```

Sympy [A] time = 18.3657, size = 97, normalized size = 0.33

$$\frac{A\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)} + \frac{Bx^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*sqrt(a)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)
*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e
**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)), x)
```

3.805 $\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*A*Sqrt[a + b*x^2])/(3*a*e*(e*x)^(3/2)) - ((A*b - 3*a*B)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(
5/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.238926, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^2])/(3*a*e*(e*x)^(3/2)) - ((A*b - 3*a*B)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(
5/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 23.1752, size = 126, normalized size = 0.91

$$-\frac{2A\sqrt{a+bx^2}}{3ae\left(ex\right)^{\frac{3}{2}}}-\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(Ab-3Ba\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3a^{\frac{5}{4}}\sqrt[4]{b}e^{\frac{5}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(a + b*x**2)/(3*a*e*(e*x)**(3/2)) - sqrt((a + b*x**2)/(s
qrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b - 3*B*a)*ellip
tic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(3*a**(
5/4)*b**(1/4)*e**(5/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.437697, size = 118, normalized size = 0.86

$$\frac{2x\left(-A\left(a+bx^2\right)+\frac{ix^{5/2}\sqrt{\frac{a}{bx^2}+1}(3aB-Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{3a(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*x*(-(A*(a + b*x^2)) + (I*(-(A*b) + 3*a*B)*Sqrt[1 + a/(b*x^2)]*
x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -
1])/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(3*a*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.025, size = 223, normalized size = 1.6

$$-\frac{1}{3\,abxe^{2}}\left(A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{1\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}EllipticF\left(\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/(b*x^2+a)^(1/2)/x*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*(-a*b)^(1/2)*x*b-3*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*(-a*b)^(1/2)*x*a+2*A*x^2*b^2+2*a*b*A)/b/a/e^2/(e*x)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{bx^2 + a}\sqrt{ex}e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)*e^2*x^2), x)
```

Sympy [A] time = 151.268, size = 97, normalized size = 0.7

$$\frac{A\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{5}{2}}x^{\frac{3}{2}}\,\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*sqrt(a)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)
*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e
**(5/2)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)), x)
```

3.806 $\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=342

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}+\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^3\sqrt{ex}}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*A*Sqrt[a + b*x^2])/(5*a*e*(e*x)^(5/2)) + (2*(3*A*b - 5*a*B)*S
qrt[a + b*x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*Sqrt[b]*(3*A*b - 5*a*B
)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a^2*e^4*(Sqrt[a] + Sqrt[b]*x)) +
(2*b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/
(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2]) - (
b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^
(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.638127, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}+\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2\sqrt{a+bx^2}(3Ab-5aB)}{5a^2e^3\sqrt{ex}}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```


```
[Out] (-2*A*Sqrt[a + b*x^2])/(5*a*e*(e*x)^(5/2)) + (2*(3*A*b - 5*a*B)*S
qrt[a + b*x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*Sqrt[b]*(3*A*b - 5*a*B
)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a^2*e^4*(Sqrt[a] + Sqrt[b]*x)) +
(2*b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/
(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2]) - (
b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^
(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 68.6799, size = 321, normalized size = 0.94

$$-\frac{2A\sqrt{a+bx^2}}{5ae\,(ex)^{\frac{5}{2}}}-\frac{2\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}\,(3Ab-5Ba)}{5a^2e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2\sqrt{a+bx^2}\,(3Ab-5Ba)}{5a^2e^3\sqrt{ex}}+\frac{2\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-5Ba)\,E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{7}{4}}e^{\frac{7}{2}}\sqrt{a+bx^2}}-\frac{\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-5Ba)\,F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{7}{4}}e^{\frac{7}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(a + b*x**2)/(5*a*e*(e*x)**(5/2)) - 2*sqrt(b)*sqrt(e*x)*
sqrt(a + b*x**2)*(3*A*b - 5*B*a)/(5*a**2*e**4*(sqrt(a) + sqrt(b)*
x)) + 2*sqrt(a + b*x**2)*(3*A*b - 5*B*a)/(5*a**2*e**3*sqrt(e*x))
+ 2*b**(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a)
 + sqrt(b)*x)*(3*A*b - 5*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x
)/(a**(1/4)*sqrt(e))), 1/2)/(5*a**(7/4)*e**(7/2)*sqrt(a + b*x**2)
) - b**(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a)
 + sqrt(b)*x)*(3*A*b - 5*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x
)/(a**(1/4)*sqrt(e))), 1/2)/(5*a**(7/4)*e**(7/2)*sqrt(a + b*x**2)
)
```

Mathematica [C] time = 0.486475, size = 221, normalized size = 0.65

$$\frac{x\left(2\sqrt{a}\sqrt{b}x^3\sqrt{\frac{bx^2}{a}+1}(5aB-3Ab)E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)-2\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\left(a+bx^2\right)\left(a\left(A+5Bx^2\right)-3Abx^2\right)+\sqrt{a}\sqrt{b}x^3\sqrt{\right.\right.}{5a^2(ex)^{7/2}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (x*(2*Sqrt[a]*Sqrt[b]*(-3*A*b + 5*a*B)*x^3*Sqrt[1 + (b*x^2)/a]*El
lipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] - 2*(Sqrt[(I*
Sqrt[b]*x)/Sqrt[a]]*(a + b*x^2)*(-3*A*b*x^2 + a*(A + 5*B*x^2)) +
Sqrt[a]*Sqrt[b]*(-3*A*b + 5*a*B)*x^3*Sqrt[1 + (b*x^2)/a]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1])))/(5*a^2*Sqrt[(I*S
qrt[b]*x)/Sqrt[a]]*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.027, size = 417, normalized size = 1.2

$$-\frac{1}{5\,x^2e^3a^2}\left(6\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/5/x^2*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*E
llipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*a*b-3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(
```

```
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-
10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2+5*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-6*A*b^2*x
^4+10*B*x^4*a*b-4*a*A*b*x^2+10*B*x^2*a^2+2*A*a^2)/(b*x^2+a)^(1/2)
/e^3/(e*x)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{bx^2 + a}\sqrt{ex}e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{bx^2 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)), x)
```

3.807 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{5a^{3/4}e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-9aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{a+bx^2}}+\frac{5e^3\sqrt{ex}\sqrt{a+bx^2}(7Ab-9aB)}{21b^3}-\frac{e(ex)^{5/2}(7Ab-9aB)}{7b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

```
[Out] -((7*A*b - 9*a*B)*e*(e*x)^(5/2))/(7*b^2*Sqrt[a + b*x^2]) + (2*B*(
e*x)^(9/2))/(7*b*e*Sqrt[a + b*x^2]) + (5*(7*A*b - 9*a*B)*e^3*Sqrt
[e*x]*Sqrt[a + b*x^2])/(21*b^3) - (5*a^(3/4)*(7*A*b - 9*a*B)*e^(7
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2
])/(42*b^(13/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.363669, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{5a^{3/4}e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-9aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{a+bx^2}}+\frac{5e^3\sqrt{ex}\sqrt{a+bx^2}(7Ab-9aB)}{21b^3}-\frac{e(ex)^{5/2}(7Ab-9aB)}{7b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((7*A*b - 9*a*B)*e*(e*x)^(5/2))/(7*b^2*Sqrt[a + b*x^2]) + (2*B*(
e*x)^(9/2))/(7*b*e*Sqrt[a + b*x^2]) + (5*(7*A*b - 9*a*B)*e^3*Sqrt
[e*x]*Sqrt[a + b*x^2])/(21*b^3) - (5*a^(3/4)*(7*A*b - 9*a*B)*e^(7
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2
])/(42*b^(13/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 36.77, size = 199, normalized size = 0.94

$$\frac{2B\left(ex\right)^{\frac{9}{2}}}{7be\sqrt{a+bx^2}}-\frac{5a^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-9Ba)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{42b^{\frac{13}{4}}\sqrt{a+bx^2}}-\frac{e\left(ex\right)^{\frac{5}{2}}(7Ab-9Ba)}{7b^2\sqrt{a+bx^2}}+\frac{5e^3\sqrt{ex}\sqrt{a+bx^2}\,(7Ab-9Ba)}{21b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*B*(e*x)**(9/2)/(7*b*e*sqrt(a + b*x**2)) - 5*a**(3/4)*e**(7/2)*s
qrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*
(7*A*b - 9*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(42*b**(13/4)*sqrt(a + b*x**2)) - e*(e*x)**(5/2)*(7
*A*b - 9*B*a)/(7*b**2*sqrt(a + b*x**2)) + 5*e**3*sqrt(e*x)*sqrt(a
 + b*x**2)*(7*A*b - 9*B*a)/(21*b**3)
```

Mathematica [C] time = 0.29038, size = 168, normalized size = 0.8

$$\frac{e^3\sqrt{ex}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-45a^2B+ab\left(35A-18Bx^2\right)+2b^2x^2\left(7A+3Bx^2\right)\right)-5ia\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(7Ab-9aB)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{21b^3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(-45*a^2*B + a*b*(35*A
- 18*B*x^2) + 2*b^2*x^2*(7*A + 3*B*x^2)) - (5*I)*a*(7*A*b - 9*a*B
)*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a]
)/Sqrt[b]]/Sqrt[x]], -1]))/(21*Sqrt[(I*Sqrt[a])/Sqrt[b]]*b^3*Sqrt
[a + b*x^2])
```

Maple [A] time = 0.053, size = 252, normalized size = 1.2

$$-\frac{e^3}{42\,xb^4}\sqrt{ex}\left(35\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}ab-45\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/42*e^3/x*(e*x)^(1/2)*(35*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*(-a*b)^(1/2)*a*b-45*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2-12*B*x^5*b^3-28*A*x^3*b^3+36*B
*x^3*a*b^2-70*A*x*a*b^2+90*B*x*a^2*b)/(b*x^2+a)^(1/2)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)(ex)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^3x^5+Ae^3x^3\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^3*x^5 + A*e^3*x^3)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

3.808 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=337

$$\frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}} - \frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-7aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{a+bx^2}} + \frac{3e^2\sqrt{ex}\sqrt{a+bx^2}(5Ab-7aB)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{e(ex)^{3/2}(5Ab-7aB)}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

```
[Out] -((5*A*b - 7*a*B)*e*(e*x)^(3/2))/(5*b^2*Sqrt[a + b*x^2]) + (2*B*(
e*x)^(7/2))/(5*b*e*Sqrt[a + b*x^2]) + (3*(5*A*b - 7*a*B)*e^2*Sqrt
[e*x]*Sqrt[a + b*x^2])/(5*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(
1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(11/4)*Sqrt[a + b*x^2]) + (3*a^(
1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(10*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.645411, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}} - \frac{3\sqrt[4]{a}e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-7aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{a+bx^2}} + \frac{3e^2\sqrt{ex}\sqrt{a+bx^2}(5Ab-7aB)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{e(ex)^{3/2}(5Ab-7aB)}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```


```
[Out] -((5*A*b - 7*a*B)*e*(e*x)^(3/2))/(5*b^2*Sqrt[a + b*x^2]) + (2*B*(
e*x)^(7/2))/(5*b*e*Sqrt[a + b*x^2]) + (3*(5*A*b - 7*a*B)*e^2*Sqrt
[e*x]*Sqrt[a + b*x^2])/(5*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(
1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(11/4)*Sqrt[a + b*x^2]) + (3*a^(
1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(10*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 65.6036, size = 316, normalized size = 0.94

$$\frac{2B(ex)^{\frac{7}{2}}}{5be\sqrt{a+bx^2}} - \frac{3\sqrt[4]{a}e^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(5Ab-7Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{11}{4}}\sqrt{a+bx^2}} + \frac{3\sqrt[4]{a}e^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(5Ab-7Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{10b^{\frac{11}{4}}\sqrt{a+bx^2}} - \frac{e(ex)^{\frac{3}{2}}(5Ab-7Ba)}{5b^2\sqrt{a+bx^2}} + \frac{3e^2\sqrt{ex}\sqrt{a+bx^2}(5Ab-7Ba)}{5b^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*B*(e*x)**(7/2)/(5*b*e*sqrt(a + b*x**2)) - 3*a**(1/4)*e**(5/2)*s
qrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*
(5*A*b - 7*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(5*b**(11/4)*sqrt(a + b*x**2)) + 3*a**(1/4)*e**(5/2
)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*
x)*(5*A*b - 7*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)
*sqrt(e))), 1/2)/(10*b**(11/4)*sqrt(a + b*x**2)) - e*(e*x)**(3/2)
*(5*A*b - 7*B*a)/(5*b**2*sqrt(a + b*x**2)) + 3*e**2*sqrt(e*x)*sqr
t(a + b*x**2)*(5*A*b - 7*B*a)/(5*b**(5/2)*(sqrt(a) + sqrt(b)*x))
```

Mathematica [C] time = 1.44467, size = 229, normalized size = 0.68

$$\frac{(ex)^{5/2}\left(bx^2\left(7aB-5Ab+2bBx^2\right)+\frac{3(5Ab-7aB)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a+bx^2\right)+\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right.\right.\right.}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}}{5b^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] ((e*x)^(5/2)*(b*x^2*(-5*A*b + 7*a*B + 2*b*B*x^2) + (3*(5*A*b - 7*
a*B)*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*x^2) - Sqrt[a]*Sqrt[b]*Sqr
t[1 + a/(b*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqr
t[b]]/Sqrt[x]], -1] + Sqrt[a]*Sqrt[b]*Sqrt[1 + a/(b*x^2)]*x^(3/2)
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/Sq
rt[(I*Sqrt[a])/Sqrt[b]]))/(5*b^3*x^3*Sqrt[a + b*x^2])
```

Maple [A] time = 0.05, size = 391, normalized size = 1.2

$$\frac{e^2}{10\,xb^3}\sqrt{ex}\left(30\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-15\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/10/x*e^2*(e*x)^(1/2)*(30*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1
```

```
/2*2^(1/2))*a*b-15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^
(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*a*b-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+
21*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+4*b^2*B*
x^4-10*A*x^2*b^2+14*B*x^2*a*b)/(b*x^2+a)^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Be^2x^4 + Ae^2x^2)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^4 + A*e^2*x^2)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

3.809 $\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(3Ab-5aB)}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

```
[Out] -((3*A*b - 5*a*B)*e*Sqrt[e*x])/(3*b^2*Sqrt[a + b*x^2]) + (2*B*(e*
x)^(5/2))/(3*b*e*Sqrt[a + b*x^2]) + ((3*A*b - 5*a*B)*e^(3/2)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^
(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.298376, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(3Ab-5aB)}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((3*A*b - 5*a*B)*e*Sqrt[e*x])/(3*b^2*Sqrt[a + b*x^2]) + (2*B*(e*
x)^(5/2))/(3*b*e*Sqrt[a + b*x^2]) + ((3*A*b - 5*a*B)*e^(3/2)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^
(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 29.3885, size = 160, normalized size = 0.92

$$\frac{2B\left(ex\right)^{\frac{5}{2}}}{3be\sqrt{a+bx^2}}-\frac{e\sqrt{ex}\left(3Ab-5Ba\right)}{3b^2\sqrt{a+bx^2}}+\frac{e^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(3Ab-5Ba\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{6\sqrt[4]{a}b^{\frac{9}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*B*(e*x)**(5/2)/(3*b*e*sqrt(a + b*x**2)) - e*sqrt(e*x)*(3*A*b -
5*B*a)/(3*b**2*sqrt(a + b*x**2)) + e**(3/2)*sqrt((a + b*x**2)/(sq
rt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(3*A*b - 5*B*a)*elli
ptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(6*a**
(1/4)*b**(9/4)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.230373, size = 143, normalized size = 0.82

$$\frac{e\sqrt{ex}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(5aB-3Ab+2bBx^2\right)+i\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(3Ab-5aB)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3b^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (e*Sqrt[e*x]*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(-3*A*b + 5*a*B + 2*b*B*x
^2) + I*(3*A*b - 5*a*B)*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*EllipticF[I*A
rcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(3*Sqrt[(I*Sqrt[
a])/Sqrt[b]]*b^2*Sqrt[a + b*x^2])
```

Maple [A] time = 0.027, size = 225, normalized size = 1.3

$$\frac{e}{6\,xb^{3}}\sqrt{ex}\left(3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-ab}b-5\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*e/x*(e*x)^(1/2)*(3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*b-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a+4*b^2*B*x^3-6*A*x*b^2+10*B*x*a*b)/(b*x^2
+a)^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^{2}+A\right)\left(ex\right)^{\frac{3}{2}}}{\left(bx^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^{3}+Aex\right)\sqrt{ex}}{\left(bx^{2}+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.810 $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(Ab-3aB)}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(a*b*e*Sqrt[a + b*x^2]) - ((A*b - 3*a*B
)*Sqrt[e*x]*Sqrt[a + b*x^2])/(a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) +
((A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(
1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(7/4)*Sqrt[a + b*x^2]) - ((A*b -
 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(2*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.583796, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(Ab-3aB)}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```


```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(a*b*e*Sqrt[a + b*x^2]) - ((A*b - 3*a*B
)*Sqrt[e*x]*Sqrt[a + b*x^2])/(a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) +
((A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(
1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(7/4)*Sqrt[a + b*x^2]) - ((A*b -
 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(2*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 56.7525, size = 270, normalized size = 0.9

$$\frac{(ex)^{\frac{3}{2}}(Ab-Ba)}{abe\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(Ab-3Ba)}{ab^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-3Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{a+bx^2}}-\frac{\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-3Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] (e*x)**(3/2)*(A*b - B*a)/(a*b*e*sqrt(a + b*x**2)) - sqrt(e*x)*sqr
t(a + b*x**2)*(A*b - 3*B*a)/(a*b**(3/2)*(sqrt(a) + sqrt(b)*x)) +
sqrt(e)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sq
rt(b)*x)*(A*b - 3*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(
1/4)*sqrt(e))), 1/2)/(a**(3/4)*b**(7/4)*sqrt(a + b*x**2)) - sqrt(
e)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)
*x)*(A*b - 3*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*
sqrt(e))), 1/2)/(2*a**(3/4)*b**(7/4)*sqrt(a + b*x**2))
```

Mathematica [C] time = 1.01997, size = 216, normalized size = 0.72

$$\frac{ie\left(\sqrt{a}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(3aB-Ab+2bBx^2\right)-\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}(Ab-3aB)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)+\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}(Ab-3aB)E\left(\right.\right.}{b^{5/2}\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (I*e*(Sqrt[a]*Sqrt[(I*Sqrt[a])/Sqrt[b]]*(-(A*b) + 3*a*B + 2*b*B*x
^2) + Sqrt[b]*(A*b - 3*a*B)*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticE
[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] - Sqrt[b]*(A*b
 - 3*a*B)*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(((I*Sqrt[a])/Sqrt[b])^(3/2)*b
^(5/2)*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A] time = 0.026, size = 382, normalized size = 1.3

$$-\frac{1}{2b^2xa}\sqrt{ex}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)ab-A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(e*x)^(1/2)*(2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(
1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a*b-A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-6*
```

```
B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+3*B*((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*A*x^2*b^2+2*B*x^2*
a*b)/(b*x^2+a)^(1/2)/b^2/x/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

Sympy [A] time = 46.2766, size = 94, normalized size = 0.31

$$\frac{A\sqrt{e}x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{e}x^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*a**(3/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gam
ma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(3/2)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

3.811 $\int \frac{A+Bx^2}{\sqrt{ex}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(a*b*e*Sqrt[a + b*x^2]) + ((A*b + a*B)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*
a^(5/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.247807, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(a*b*e*Sqrt[a + b*x^2]) + ((A*b + a*B)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*
a^(5/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 23.7532, size = 126, normalized size = 0.88

$$\frac{\sqrt{ex}\,(Ab-Ba)}{abe\sqrt{a+bx^2}}+\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+Ba)\,F\left(2\,\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{5}{4}}b^{\frac{5}{4}}\sqrt{e}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] sqrt(e*x)*(A*b - B*a)/(a*b*e*sqrt(a + b*x**2)) + sqrt((a + b*x**2
)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b + B*a)*ell
iptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(2*a*
*(5/4)*b**(5/4)*sqrt(e)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.154984, size = 133, normalized size = 0.92

$$\frac{ix^{3/2}\sqrt{\frac{a}{bx^2}+1}(aB+Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)+x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(Ab-aB)}{ab\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[a])/Sqrt[b]]*(A*b - a*B)*x + I*(A*b + a*B)*Sqrt[1 +
 a/(b*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]
/Sqrt[x]], -1])/(a*Sqrt[(I*Sqrt[a])/Sqrt[b]]*b*Sqrt[e*x]*Sqrt[a +
 b*x^2])
```

Maple [A] time = 0.029, size = 213, normalized size = 1.5

$$\frac{1}{2\,ab^2}\left(A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{1\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}EllipticF\left(\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-abl}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 1/2*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/
2)*b+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/
2)*a+2*A*x*b^2-2*B*x*a*b)/(b*x^2+a)^(1/2)/a/(e*x)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{(bx^2+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2+A}{(bx^2+a)^{\frac{3}{2}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

Sympy [A] time = 71.0387, size = 94, normalized size = 0.65

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*h
yper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*sq
rt(e)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

3.812 $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=333

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{(ex)^{3/2}(3Ab-aB)}{a^2e^3\sqrt{a+bx^2}} + \frac{\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{a^2\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^2]) - ((3*A*b - a*B)*(e*x)^(3/
2))/(a^2*e^3*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^2])/(a^2*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((3*A*b - a*B)
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/
(a^(7/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7
/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.642021, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{(ex)^{3/2}(3Ab-aB)}{a^2e^3\sqrt{a+bx^2}} + \frac{\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{a^2\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```


```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^2]) - ((3*A*b - a*B)*(e*x)^(3/
2))/(a^2*e^3*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^2])/(a^2*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((3*A*b - a*B)
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/
(a^(7/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7
/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 66.8392, size = 303, normalized size = 0.91

$$-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(ex)^{\frac{3}{2}}(3Ab-Ba)}{a^2e^3\sqrt{a+bx^2}}+\frac{\sqrt{ex}\sqrt{a+bx^2}(3Ab-Ba)}{a^2\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{7}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}+\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(3Ab-Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{7}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] -2*A/(a*e*sqrt(e*x)*sqrt(a + b*x**2)) - (e*x)**(3/2)*(3*A*b - B*a
)/(a**2*e**3*sqrt(a + b*x**2)) + sqrt(e*x)*sqrt(a + b*x**2)*(3*A*
b - B*a)/(a**2*sqrt(b)*e**2*(sqrt(a) + sqrt(b)*x)) - sqrt((a + b*
x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(3*A*b - B*
a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)
/(a**(7/4)*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + sqrt((a + b*x**2
)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(3*A*b - B*a)*e
lliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(2*
a**(7/4)*b**(3/4)*e**(3/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.400567, size = 202, normalized size = 0.61

$$\frac{x\left(\sqrt{b}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\left(-2aA+aBx^2-3Abx^2\right)+\sqrt{a}x\sqrt{\frac{bx^2}{a}+1}(aB-3Ab)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)-\sqrt{a}x\sqrt{\frac{bx^2}{a}+1}(aB-3Ab)E\right.}{a^2\sqrt{b}(ex)^{3/2}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(Sqrt[b]*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(-2*a*A - 3*A*b*x^2 + a*B
*x^2) - Sqrt[a]*(-3*A*b + a*B)*x*Sqrt[1 + (b*x^2)/a]*EllipticE[I*
ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] + Sqrt[a]*(-3*A*b + a*B
)*x*Sqrt[1 + (b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sq
rt[a]]], -1]))/(a^2*Sqrt[b]*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(e*x)^(3/
2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.031, size = 386, normalized size = 1.2

$$\frac{1}{2\,bea^2}\left(6\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)ab-3\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-3*A*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x
```

```
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-2*B*((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+B*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*A*x^2*b^2+2*B*x^2*a*b-4*a*b*A)/
(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}} (ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{(bex^3 + aex)\sqrt{bx^2 + a}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b*e*x^3 + a*e*x)*sqrt(b*x^2 + a)*sqrt(e*x)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}} (ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.813 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{\sqrt{ex}(5Ab-3aB)}{3a^2e^3\sqrt{a+bx^2}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*Sqr
t[e*x])/(3*a^2*e^3*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*(Sqrt[a] +
 Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^(9/4)*
b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.303116, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{\sqrt{ex}(5Ab-3aB)}{3a^2e^3\sqrt{a+bx^2}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*Sqr
t[e*x])/(3*a^2*e^3*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*(Sqrt[a] +
 Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^(9/4)*
b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 30.44, size = 163, normalized size = 0.93

$$-\frac{2A}{3ae\,(ex)^{\frac{3}{2}}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\,(5Ab-3Ba)}{3a^2e^3\sqrt{a+bx^2}}-\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(5Ab-3Ba)\,F\left(2\,\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{6a^{\frac{9}{4}}\sqrt[4]{b}e^{\frac{5}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] -2*A/(3*a*e*(e*x)**(3/2)*sqrt(a + b*x**2)) - sqrt(e*x)*(5*A*b - 3
*B*a)/(3*a**2*e**3*sqrt(a + b*x**2)) - sqrt((a + b*x**2)/(sqrt(a)
 + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(5*A*b - 3*B*a)*elliptic_
f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(6*a**(9/4)
*b**(1/4)*e**(5/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.223779, size = 146, normalized size = 0.83

$$\frac{x\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-2aA+3aBx^2-5Abx^2\right)-ix^{5/2}\sqrt{\frac{a}{bx^2}+1}(5Ab-3aB)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3a^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(-2*a*A - 5*A*b*x^2 + 3*a*B*x^2) -
I*(5*A*b - 3*a*B)*Sqrt[1 + a/(b*x^2)]*x^(5/2)*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(3*a^2*Sqrt[(I*Sqrt[a]
)/Sqrt[b]]*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.032, size = 232, normalized size = 1.3

$$-\frac{1}{6\,bxa^2e^2}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-ab}xb-3\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6/x*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ell
ipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b
)^(1/2)*x*b-3*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)
*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(
-a*b)^(1/2)*x*a+10*A*x^2*b^2-6*B*x^2*a*b+4*a*b*A)/(b*x^2+a)^(1/2)
/b/a^2/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{(be^2x^4 + ae^2x^2)\sqrt{bx^2 + a}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b*e^2*x^4 + a*e^2*x^2)*sqrt(b*x^2 + a)*sqr
t(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}} (ex)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)), x)
```

3.814 $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=379

$$-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10a^{11/4}e^{7/2}\sqrt{a+bx^2}}+\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{11/4}e^{7/2}\sqrt{a+bx^2}}-\frac{3\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{3\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^3\sqrt{ex}}-\frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^2]) - (7*A*b - 5*a*B)/(5*a
^2*e^3*Sqrt[e*x]*Sqrt[a + b*x^2]) + (3*(7*A*b - 5*a*B)*Sqrt[a + b
*x^2])/(5*a^3*e^3*Sqrt[e*x]) - (3*Sqrt[b]*(7*A*b - 5*a*B)*Sqrt[e*
x]*Sqrt[a + b*x^2])/(5*a^3*e^4*(Sqrt[a] + Sqrt[b]*x)) + (3*b^(1/4
)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(5*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2]) - (3*b^(1/4
)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(10*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.72855, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10a^{11/4}e^{7/2}\sqrt{a+bx^2}}+\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-5aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{11/4}e^{7/2}\sqrt{a+bx^2}}-\frac{3\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{3\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^3\sqrt{ex}}-\frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^2]) - (7*A*b - 5*a*B)/(5*a
^2*e^3*Sqrt[e*x]*Sqrt[a + b*x^2]) + (3*(7*A*b - 5*a*B)*Sqrt[a + b
*x^2])/(5*a^3*e^3*Sqrt[e*x]) - (3*Sqrt[b]*(7*A*b - 5*a*B)*Sqrt[e*
x]*Sqrt[a + b*x^2])/(5*a^3*e^4*(Sqrt[a] + Sqrt[b]*x)) + (3*b^(1/4
)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(5*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2]) - (3*b^(1/4
)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(10*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 79.6542, size = 359, normalized size = 0.95

$$-\frac{2A}{5ae(ex)^{\frac{5}{2}}\sqrt{a+bx^2}}-\frac{7Ab-5Ba}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}-\frac{3\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(7Ab-5Ba)}{5a^3e^4\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{3\sqrt{a+bx^2}(7Ab-5Ba)}{5a^3e^3\sqrt{ex}}+\frac{3\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-5Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{11}{4}}e^{\frac{7}{2}}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{b}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-5Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{10a^{\frac{11}{4}}e^{\frac{7}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] -2*A/(5*a*e*(e*x)**(5/2)*sqrt(a + b*x**2)) - (7*A*b - 5*B*a)/(5*a
**2*e**3*sqrt(e*x)*sqrt(a + b*x**2)) - 3*sqrt(b)*sqrt(e*x)*sqrt(a
 + b*x**2)*(7*A*b - 5*B*a)/(5*a**3*e**4*(sqrt(a) + sqrt(b)*x)) +
3*sqrt(a + b*x**2)*(7*A*b - 5*B*a)/(5*a**3*e**3*sqrt(e*x)) + 3*b*
*(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqr
t(b)*x)*(7*A*b - 5*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**
(1/4)*sqrt(e))), 1/2)/(5*a**(11/4)*e**(7/2)*sqrt(a + b*x**2)) - 3
*b**(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) +
sqrt(b)*x)*(7*A*b - 5*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(
a**(1/4)*sqrt(e))), 1/2)/(10*a**(11/4)*e**(7/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.626032, size = 233, normalized size = 0.61

$$\frac{x\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\left(-2a^2\left(A+5Bx^2\right)+abx^2\left(14A-15Bx^2\right)+21Ab^2x^4\right)-3\sqrt{a}\sqrt{b}x^3\sqrt{\frac{bx^2}{a}+1}(5aB-7Ab)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-\right.}{5a^3(ex)^{7/2}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(21*A*b^2*x^4 + a*b*x^2*(14*A - 1
5*B*x^2) - 2*a^2*(A + 5*B*x^2)) + 3*Sqrt[a]*Sqrt[b]*(-7*A*b + 5*a
*B)*x^3*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x
)/Sqrt[a]]], -1] - 3*Sqrt[a]*Sqrt[b]*(-7*A*b + 5*a*B)*x^3*Sqrt[1
+ (b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1
]))/(5*a^3*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(e*x)^(7/2)*Sqrt[a + b*x^2
])
```

Maple [A] time = 0.033, size = 417, normalized size = 1.1

$$-\frac{1}{10x^2e^3a^3}\left(42A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticE\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^2ab-21A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/10/x^2*(42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)
*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x
```

```
^2*a*b-21*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ell
ipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b-30*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2+1
5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2-42*A*
b^2*x^4+30*B*x^4*a*b-28*a*A*b*x^2+20*B*x^2*a^2+4*A*a^2)/(b*x^2+a)
^(1/2)/e^3/(e*x)^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{(be^3x^5 + ae^3x^3)\sqrt{bx^2 + a}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b*e^3*x^5 + a*e^3*x^3)*sqrt(b*x^2 + a)*sqr
t(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.815 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{5e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}b^{13/4}\sqrt{a+bx^2}} - \frac{5e^3\sqrt{ex}(Ab-3aB)}{6b^3\sqrt{a+bx^2}} - \frac{e(ex)^{5/2}(Ab-3aB)}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}}$$

```
[Out] -((A*b - 3*a*B)*e*(e*x)^(5/2))/(3*b^2*(a + b*x^2)^(3/2)) + (2*B*(
e*x)^(9/2))/(3*b*e*(a + b*x^2)^(3/2)) - (5*(A*b - 3*a*B)*e^3*Sqrt
[e*x])/(6*b^3*Sqrt[a + b*x^2]) + (5*(A*b - 3*a*B)*e^(7/2)*(Sqrt[a
] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(1
/4)*b^(13/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.358899, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{5e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-3aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}b^{13/4}\sqrt{a+bx^2}} - \frac{5e^3\sqrt{ex}(Ab-3aB)}{6b^3\sqrt{a+bx^2}} - \frac{e(ex)^{5/2}(Ab-3aB)}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -((A*b - 3*a*B)*e*(e*x)^(5/2))/(3*b^2*(a + b*x^2)^(3/2)) + (2*B*(
e*x)^(9/2))/(3*b*e*(a + b*x^2)^(3/2)) - (5*(A*b - 3*a*B)*e^3*Sqrt
[e*x])/(6*b^3*Sqrt[a + b*x^2]) + (5*(A*b - 3*a*B)*e^(7/2)*(Sqrt[a
] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(1
/4)*b^(13/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 38.1814, size = 194, normalized size = 0.93

$$\frac{2B\,(ex)^{\frac{9}{2}}}{3be\left(a+bx^2\right)^{\frac{3}{2}}} - \frac{e\,(ex)^{\frac{5}{2}}\,(Ab-3Ba)}{3b^2\left(a+bx^2\right)^{\frac{3}{2}}} - \frac{5e^3\sqrt{ex}\,(Ab-3Ba)}{6b^3\sqrt{a+bx^2}} + \frac{5e^{\frac{7}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-3Ba)\,F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12\sqrt[4]{a}b^{\frac{13}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] 2*B*(e*x)**(9/2)/(3*b*e*(a + b*x**2)**(3/2)) - e*(e*x)**(5/2)*(A*
b - 3*B*a)/(3*b**2*(a + b*x**2)**(3/2)) - 5*e**3*sqrt(e*x)*(A*b -
 3*B*a)/(6*b**3*sqrt(a + b*x**2)) + 5*e**(7/2)*sqrt((a + b*x**2)/
(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b - 3*B*a)*ell
iptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(12*a
**(1/4)*b**(13/4)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.392782, size = 163, normalized size = 0.78

$$\frac{(ex)^{7/2}\left(\frac{\sqrt{x}\left(15a^2B+a\left(21bBx^2-5Ab\right)+b^2x^2\left(4Bx^2-7A\right)\right)}{b^3(a+bx^2)}+\frac{5ix\sqrt{\frac{a}{bx^2}+1}(Ab-3aB)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{b^3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{6x^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(7/2)*((Sqrt[x]*(15*a^2*B + b^2*x^2*(-7*A + 4*B*x^2) + a*(
-5*A*b + 21*b*B*x^2)))/(b^3*(a + b*x^2)) + ((5*I)*(A*b - 3*a*B)*S
qrt[1 + a/(b*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]
]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[a])/Sqrt[b]]*b^3)))/(6*x^(7/2)*Sqr
t[a + b*x^2])
```

Maple [B] time = 0.056, size = 439, normalized size = 2.1

$$\frac{e^3}{12\,xb^4}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^2b^2-15\,B\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^
(1/2)*x^2*b^2-15*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*(-a*b)^(1/2)*x^2*a*b+5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a*b-15*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+8*B*x^5*b^3-14*A*x^3*b^3+42*B*x^3
*a*b^2-10*A*x*a*b^2+30*B*x*a^2*b)*e^3/x*(e*x)^(1/2)/b^4/(b*x^2+a)
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^3x^5 + Ae^3x^3\right)\sqrt{ex}}{(b^2x^4 + 2\,abx^2 + a^2)\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^3*x^5 + A*e^3*x^3)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2
+ a^2)*sqrt(b*x^2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

3.816 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^2}}+\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{11/4}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}(Ab-7aB)}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{e(ex)^{3/2}(Ab-7aB)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b - 7
*a*B)*e*(e*x)^(3/2))/(6*a*b^2*Sqrt[a + b*x^2]) - ((A*b - 7*a*B)*e
^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(2*a*b^(5/2)*(Sqrt[a] + Sqrt[b]*x))
 + ((A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2]) - (
(A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1
/4)*Sqrt[e])], 1/2])/(4*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.66431, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^2}}+\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(Ab-7aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{11/4}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}(Ab-7aB)}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{e(ex)^{3/2}(Ab-7aB)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```



```
[Out] ((A*b - a*B)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b - 7
*a*B)*e*(e*x)^(3/2))/(6*a*b^2*Sqrt[a + b*x^2]) - ((A*b - 7*a*B)*e
^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(2*a*b^(5/2)*(Sqrt[a] + Sqrt[b]*x))
 + ((A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2]) - (
(A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1
/4)*Sqrt[e])], 1/2])/(4*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 69.7875, size = 313, normalized size = 0.9

$$\frac{(ex)^{\frac{7}{2}}(Ab-Ba)}{3abe(a+bx^2)^{\frac{3}{2}}}+\frac{e(ex)^{\frac{3}{2}}(Ab-7Ba)}{6ab^2\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}(Ab-7Ba)}{2ab^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{e^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-7Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{11}{4}}\sqrt{a+bx^2}}-\frac{e^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab-7Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{3}{4}}b^{\frac{11}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] (e*x)**(7/2)*(A*b - B*a)/(3*a*b*e*(a + b*x**2)**(3/2)) + e*(e*x)*
*(3/2)*(A*b - 7*B*a)/(6*a*b**2*sqrt(a + b*x**2)) - e**2*sqrt(e*x)
*sqrt(a + b*x**2)*(A*b - 7*B*a)/(2*a*b**(5/2)*(sqrt(a) + sqrt(b)*
x)) + e**(5/2)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(
a) + sqrt(b)*x)*(A*b - 7*B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x
)/(a**(1/4)*sqrt(e))), 1/2)/(2*a**(3/4)*b**(11/4)*sqrt(a + b*x**2
)) - e**(5/2)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a
) + sqrt(b)*x)*(A*b - 7*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)
/(a**(1/4)*sqrt(e))), 1/2)/(4*a**(3/4)*b**(11/4)*sqrt(a + b*x**2)
)
```

Mathematica [C] time = 1.02553, size = 249, normalized size = 0.71

$$\frac{(ex)^{5/2}\left(bx^2\left(-7a^2B+ab\left(A-9Bx^2\right)+3Ab^2x^2\right)-\frac{3(a+bx^2)(Ab-7aB)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(a+bx^2)+\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{a}\sqrt{b}x^{3/2}\ldots\right.}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\ldots}{6ab^3x^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(5/2)*(b*x^2*(-7*a^2*B + 3*A*b^2*x^2 + a*b*(A - 9*B*x^2))
- (3*(A*b - 7*a*B)*(a + b*x^2)*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*
x^2) - Sqrt[a]*Sqrt[b]*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticE[I*Ar
cSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] + Sqrt[a]*Sqrt[b]*S
qrt[1 + a/(b*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/S
qrt[b]]/Sqrt[x]], -1]))/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(6*a*b^3*x^3*
(a + b*x^2)^(3/2))
```

Maple [B] time = 0.064, size = 767, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*
```

```
b^2-3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2
-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+2
1*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+6*A
*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-3*A*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-42*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+21*B*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*a^3-6*A*x^4*b^3+18*B*x^4*a*b^2-2*A*x^
2*a*b^2+14*B*x^2*a^2*b)/x*e^2*(e*x)^(1/2)/b^3/a/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Be^2x^4 + Ae^2x^2)\sqrt{ex}}{(b^2x^4 + 2\,abx^2 + a^2)\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^4 + A*e^2*x^2)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2
+ a^2)*sqrt(b*x^2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

3.817 $\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(5aB+Ab)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/2)) - ((A*b + 5
*a*B)*e*Sqrt[e*x])/(6*a*b^2*Sqrt[a + b*x^2]) + ((A*b + 5*a*B)*e^(
3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/
2])/(12*a^(5/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.306688, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(5aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}(5aB+Ab)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/2)) - ((A*b + 5
*a*B)*e*Sqrt[e*x])/(6*a*b^2*Sqrt[a + b*x^2]) + ((A*b + 5*a*B)*e^(
3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/
2])/(12*a^(5/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 32.0885, size = 163, normalized size = 0.88

$$\frac{(ex)^{\frac{5}{2}}\,(Ab-Ba)}{3abe\,(a+bx^2)^{\frac{3}{2}}}-\frac{e\sqrt{ex}\,(Ab+5Ba)}{6ab^2\sqrt{a+bx^2}}+\frac{e^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+5Ba)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12a^{\frac{5}{4}}b^{\frac{9}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] (e*x)**(5/2)*(A*b - B*a)/(3*a*b*e*(a + b*x**2)**(3/2)) - e*sqrt(e
*x)*(A*b + 5*B*a)/(6*a*b**2*sqrt(a + b*x**2)) + e**(3/2)*sqrt((a
+ b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(A*b +
5*B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))),
1/2)/(12*a**(5/4)*b**(9/4)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.319532, size = 163, normalized size = 0.88

$$\frac{e\sqrt{ex}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-5a^2B-ab\left(A+7Bx^2\right)+Ab^2x^2\right)+i\sqrt{x}\sqrt{\frac{a}{bx^2}+1}\left(a+bx^2\right)(5aB+Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{6ab^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[e*x]*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(-5*a^2*B + A*b^2*x^2 - a
*b*(A + 7*B*x^2)) + I*(A*b + 5*a*B)*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*(
a + b*x^2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]]
, -1]))/(6*a*Sqrt[(I*Sqrt[a])/Sqrt[b]]*b^2*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.028, size = 429, normalized size = 2.3

$$\frac{e}{12\,axb^3}\left(A\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{1\left(-bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}EllipticF\left(\sqrt{1\left(bx+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1
/2)*x^2*b^2+5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)
*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(
-a*b)^(1/2)*x^2*a*b+A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(
1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*(-a*b)^(1/2)*a*b+5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*(-a*b)^(1/2)*a^2+2*A*x^3*b^3-14*B*x^3*a*b^2-2*A*x*a*b^2-
10*B*x*a^2*b)*e/x*(e*x)^(1/2)/a/b^3/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^3+Aex\right)\sqrt{ex}}{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^3 + A*e*x)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2 + a^2)
*sqrt(b*x^2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.818 $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=344

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{(ex)^{3/2}(aB+Ab)}{2a^2be\sqrt{a+bx^2}}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + a
*B)*(e*x)^(3/2))/(2*a^2*b*e*Sqrt[a + b*x^2]) - ((A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^2])/(2*a^2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A
*b + a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(2*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2]) - ((A*b + a*
B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(4*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.641502, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+Ab)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{(ex)^{3/2}(aB+Ab)}{2a^2be\sqrt{a+bx^2}}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + a
*B)*(e*x)^(3/2))/(2*a^2*b*e*Sqrt[a + b*x^2]) - ((A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^2])/(2*a^2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A
*b + a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(2*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2]) - ((A*b + a*
B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(4*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 67.1822, size = 304, normalized size = 0.88

$$\frac{(ex)^{\frac{3}{2}}(Ab-Ba)}{3abe(a+bx^2)^{\frac{3}{2}}}+\frac{(ex)^{\frac{3}{2}}(Ab+Ba)}{2a^2be\sqrt{a+bx^2}}-\frac{\sqrt{ex}\sqrt{a+bx^2}(Ab+Ba)}{2a^2b^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+Ba)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{7}{4}}b^{\frac{7}{4}}\sqrt{a+bx^2}}-\frac{\sqrt{e}\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(Ab+Ba)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{7}{4}}b^{\frac{7}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] (e*x)**(3/2)*(A*b - B*a)/(3*a*b*e*(a + b*x**2)**(3/2)) + (e*x)**(
3/2)*(A*b + B*a)/(2*a**2*b*e*sqrt(a + b*x**2)) - sqrt(e*x)*sqrt(a
 + b*x**2)*(A*b + B*a)/(2*a**2*b**(3/2)*(sqrt(a) + sqrt(b)*x)) +
sqrt(e)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sq
rt(b)*x)*(A*b + B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/
4)*sqrt(e))), 1/2)/(2*a**(7/4)*b**(7/4)*sqrt(a + b*x**2)) - sqrt(
e)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)
*x)*(A*b + B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sq
rt(e))), 1/2)/(4*a**(7/4)*b**(7/4)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.940555, size = 247, normalized size = 0.72

$$\frac{e\left(bx^2\left(a^2B+ab\left(5A+3Bx^2\right)+3Ab^2x^2\right)-\frac{3\left(a+bx^2\right)(aB+Ab)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a+bx^2\right)+\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{a}\sqrt{b}x^{3/2}\sqrt{\frac{a}{bx^2}+1}E\cdots\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{6a^2b^2\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (e*(b*x^2*(a^2*B + 3*A*b^2*x^2 + a*b*(5*A + 3*B*x^2)) - (3*(A*b +
 a*B)*(a + b*x^2)*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(a + b*x^2) - Sqrt[a
]*Sqrt[b]*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] + Sqrt[a]*Sqrt[b]*Sqrt[1 + a/(b*
x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[
x]], -1]))/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(6*a^2*b^2*Sqrt[e*x]*(a +
b*x^2)^(3/2))
```

Maple [B] time = 0.027, size = 764, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*
b^2-3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
```

```
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2
+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b-3*
B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+6*A*(
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-3*A*((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b+6*B*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-3*B*((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2),1/2*2^(1/2))*a^3-6*A*x^4*b^3-6*B*x^4*a*b^2-10*A*x^2*a*
b^2-2*B*x^2*a^2*b)*(e*x)^(1/2)/b^2/a^2/x/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{ex}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^2 + A)\sqrt{ex}}{(b^2x^4 + 2\,abx^2 + a^2)\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(
b*x^2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\sqrt{ex}}{(bx^2 + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2), x)
```

3.819 $\int \frac{A+Bx^2}{\sqrt{ex}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{9/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(aB+5Ab)}{6a^2be\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/2)) + ((5*A*b + a
*B)*Sqrt[e*x])/(6*a^2*b*e*Sqrt[a + b*x^2]) + ((5*A*b + a*B)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^
(9/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.312689, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(aB+5Ab)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{9/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(aB+5Ab)}{6a^2be\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```


```
[Out] ((A*b - a*B)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/2)) + ((5*A*b + a
*B)*Sqrt[e*x])/(6*a^2*b*e*Sqrt[a + b*x^2]) + ((5*A*b + a*B)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^
(9/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 31.2919, size = 163, normalized size = 0.87

$$\frac{\sqrt{ex}\left(Ab-Ba\right)}{3abe\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{\sqrt{ex}\left(5Ab+Ba\right)}{6a^2be\sqrt{a+bx^2}}+\frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(5Ab+Ba\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12a^{\frac{9}{4}}b^{\frac{5}{4}}\sqrt{e}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(b*x**2+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] sqrt(e*x)*(A*b - B*a)/(3*a*b*e*(a + b*x**2)**(3/2)) + sqrt(e*x)*(
5*A*b + B*a)/(6*a**2*b*e*sqrt(a + b*x**2)) + sqrt((a + b*x**2)/(s
qrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(5*A*b + B*a)*ellip
tic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(12*a**
(9/4)*b**(5/4)*sqrt(e)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.254897, size = 164, normalized size = 0.88

$$\frac{x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(a^2(-B)+ab\left(7A+Bx^2\right)+5Ab^2x^2\right)+ix^{3/2}\sqrt{\frac{a}{bx^2}+1}\left(a+bx^2\right)(aB+5Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{6a^2b\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[a])/Sqrt[b]]*x*(-(a^2*B) + 5*A*b^2*x^2 + a*b*(7*A +
 B*x^2)) + I*(5*A*b + a*B)*Sqrt[1 + a/(b*x^2)]*x^(3/2)*(a + b*x^2
)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/(6
*a^2*Sqrt[(I*Sqrt[a])/Sqrt[b]]*b*Sqrt[e*x]*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.032, size = 425, normalized size = 2.3

$$\frac{1}{12\,a^2b^2}\left(5\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^2b^2+B\sqrt{1\left(bx+\sqrt{-ab}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x)
```

```
[Out] 1/12*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^
(1/2)*x^2*b^2+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)
*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(
-a*b)^(1/2)*x^2*a*b+5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*(-a*b)^(1/2)*a*b+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*(-a*b)^(1/2)*a^2+10*A*x^3*b^3+2*B*x^3*a*b^2+14*A*x*a*b^2
-2*B*x*a^2*b)/(e*x)^(1/2)/a^2/b^2/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx^2+A}{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)
*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(b*x**2+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)), x)
```

3.820 $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=377

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{(ex)^{3/2}(7Ab-aB)}{2a^3e^3\sqrt{a+bx^2}} + \frac{\sqrt{ex}\sqrt{a+bx^2}(7Ab-aB)}{2a^3\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{(ex)^{3/2}(7Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(
3/2))/(3*a^2*e^3*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))
/(2*a^3*e^3*Sqrt[a + b*x^2]) + ((7*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
b*x^2])/(2*a^3*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((7*A*b - a*B
)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])
/(2*a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((7*A*b - a*B)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*
a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.718684, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(7Ab-aB)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} - \frac{(ex)^{3/2}(7Ab-aB)}{2a^3e^3\sqrt{a+bx^2}} + \frac{\sqrt{ex}\sqrt{a+bx^2}(7Ab-aB)}{2a^3\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{(ex)^{3/2}(7Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```


```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(
3/2))/(3*a^2*e^3*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))
/(2*a^3*e^3*Sqrt[a + b*x^2]) + ((7*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
b*x^2])/(2*a^3*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((7*A*b - a*B
)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])
/(2*a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((7*A*b - a*B)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*
a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 78.1897, size = 342, normalized size = 0.91

$$-\frac{2A}{ae\sqrt{ex}\,(a+bx^2)^{\frac{3}{2}}}-\frac{(ex)^{\frac{3}{2}}\,(7Ab-Ba)}{3a^2e^3\,(a+bx^2)^{\frac{3}{2}}}-\frac{(ex)^{\frac{3}{2}}\,(7Ab-Ba)}{2a^3e^3\sqrt{a+bx^2}}+\frac{\sqrt{ex}\sqrt{a+bx^2}\,(7Ab-Ba)}{2a^3\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-Ba)\,E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{11}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}} + \frac{\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)(7Ab-Ba)\,F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{11}{4}}b^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] -2*A/(a*e*sqrt(e*x)*(a + b*x**2)**(3/2)) - (e*x)**(3/2)*(7*A*b -
B*a)/(3*a**2*e**3*(a + b*x**2)**(3/2)) - (e*x)**(3/2)*(7*A*b - B*
a)/(2*a**3*e**3*sqrt(a + b*x**2)) + sqrt(e*x)*sqrt(a + b*x**2)*(7
*A*b - B*a)/(2*a**3*sqrt(b)*e**2*(sqrt(a) + sqrt(b)*x)) - sqrt((a
 + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(7*A*b
 - B*a)*elliptic_e(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))),
 1/2)/(2*a**(11/4)*b**(3/4)*e**(3/2)*sqrt(a + b*x**2)) + sqrt((a
+ b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(7*A*b
- B*a)*elliptic_f(2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))),
1/2)/(4*a**(11/4)*b**(3/4)*e**(3/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.539367, size = 182, normalized size = 0.48

$$\frac{x\left(\frac{a^2(5Bx^2-12A)+a(3bBx^4-35Abx^2)-21Ab^2x^4}{a+bx^2}+\frac{3ia\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{bx^2}{a}+1}(aB-7Ab)\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)\right)}{b}\right)}{6a^3(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (x*((-21*A*b^2*x^4 + a^2*(-12*A + 5*B*x^2) + a*(-35*A*b*x^2 + 3*b
*B*x^4))/(a + b*x^2) + ((3*I)*a*(-7*A*b + a*B)*Sqrt[(I*Sqrt[b]*x)
/Sqrt[a]]*Sqrt[1 + (b*x^2)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b
]*x)/Sqrt[a]]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt
[a]]], -1]))/b))/(6*a^3*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.034, size = 771, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*
b^2-21*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^
2-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
```

```
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+3
*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b+42*A
*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-21*A*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-6*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+3*B*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))*a^3-42*A*x^4*b^3+6*B*x^4*a*b^2-70*A*x^
2*a*b^2+10*B*x^2*a^2*b-24*A*a^2*b)/b/a^3/e/(e*x)^(1/2)/(b*x^2+a)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{5}{2}} (ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{(b^2ex^5 + 2\,abex^3 + a^2ex)\sqrt{bx^2 + a}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(b*
x^2 + a)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{5}{2}} (ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.821 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{13/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{ex}(3Ab-aB)}{6a^3e^3\sqrt{a+bx^2}}-\frac{\sqrt{ex}(3Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/2)) - ((3*A*b - a*B)*Sqr
t[e*x])/(3*a^2*e^3*(a + b*x^2)^(3/2)) - (5*(3*A*b - a*B)*Sqrt[e*x
])/(6*a^3*e^3*Sqrt[a + b*x^2]) - (5*(3*A*b - a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(13/4)*b^(
1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.368943, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}(3Ab-aB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{13/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{ex}(3Ab-aB)}{6a^3e^3\sqrt{a+bx^2}}-\frac{\sqrt{ex}(3Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/2)) - ((3*A*b - a*B)*Sqr
t[e*x])/(3*a^2*e^3*(a + b*x^2)^(3/2)) - (5*(3*A*b - a*B)*Sqrt[e*x
])/(6*a^3*e^3*Sqrt[a + b*x^2]) - (5*(3*A*b - a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(13/4)*b^(
1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 37.8319, size = 197, normalized size = 0.92

$$-\frac{2A}{3ae\left(ex\right)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{\sqrt{ex}\left(3Ab-Ba\right)}{3a^2e^3\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{5\sqrt{ex}\left(3Ab-Ba\right)}{6a^3e^3\sqrt{a+bx^2}}-\frac{5\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\left(\sqrt{a}+\sqrt{b}x\right)\left(3Ab-Ba\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12a^{\frac{13}{4}}\sqrt[4]{b}e^{\frac{5}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] -2*A/(3*a*e*(e*x)**(3/2)*(a + b*x**2)**(3/2)) - sqrt(e*x)*(3*A*b
- B*a)/(3*a**2*e**3*(a + b*x**2)**(3/2)) - 5*sqrt(e*x)*(3*A*b - B
*a)/(6*a**3*e**3*sqrt(a + b*x**2)) - 5*sqrt((a + b*x**2)/(sqrt(a)
 + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*(3*A*b - B*a)*elliptic_f(
2*atan(b**(1/4)*sqrt(e*x)/(a**(1/4)*sqrt(e))), 1/2)/(12*a**(13/4)
*b**(1/4)*e**(5/2)*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.446226, size = 166, normalized size = 0.78

$$\frac{x^{5/2}\left(\frac{a^2(7Bx^2-4A)+a(5bBx^4-21Abx^2)-15Ab^2x^4}{a^3x^{3/2}(a+bx^2)}+\frac{5ix\sqrt{\frac{a}{bx^2}+1}(aB-3Ab)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{a^3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{6(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (x^(5/2)*((-15*A*b^2*x^4 + a^2*(-4*A + 7*B*x^2) + a*(-21*A*b*x^2
+ 5*b*B*x^4))/(a^3*x^(3/2)*(a + b*x^2)) + ((5*I)*(-3*A*b + a*B)*S
qrt[1 + a/(b*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]
]/Sqrt[x]], -1])/(a^3*Sqrt[(I*Sqrt[a])/Sqrt[b]])))/(6*(e*x)^(5/2)
*Sqrt[a + b*x^2])
```

Maple [B] time = 0.035, size = 446, normalized size = 2.1

$$-\frac{1}{12\,xe^2a^3b}\left(15\,A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}EllipticF\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^3b^2-5\,B\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ell
ipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b
)^(1/2)*x^3*b^2-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(
1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*(-a*b)^(1/2)*x^3*a*b+15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1
/2*2^(1/2))*(-a*b)^(1/2)*x*a*b-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a^2+30*A*x^4*b^3-10*B*x^4*a*b^2+
42*A*x^2*a*b^2-14*B*x^2*a^2*b+8*A*a^2*b)/x/e^2/(e*x)^(1/2)/a^3/b/
(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2+A}{(b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2)\sqrt{bx^2+a}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)
*sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{(bx^2 + a)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)), x)
```

3.822 $\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=288

$$-\frac{2c^{7/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(11a^2d^2+bc(3bc-10ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2(ex)^{5/2}\sqrt{c+dx^2}\left(11a^2d^2+bc(3bc-10ad)\right)}{77d^2e}+\frac{4ce\sqrt{ex}\sqrt{c+dx^2}\left(11a^2d^2+bc(3bc-10ad)\right)}{231d^3}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{3/2}(3bc-10ad)}{55d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

```
[Out] (4*c*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x
^2])/(231*d^3) + (2*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*(e*x)^(5/
2)*Sqrt[c + d*x^2])/(77*d^2*e) - (2*b*(3*b*c - 10*a*d)*(e*x)^(5/2
)*(c + d*x^2)^(3/2))/(55*d^2*e) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^
(3/2))/(15*d*e^3) - (2*c^(7/4)*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d)
)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], 1/2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.688208, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{2c^{7/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(11a^2d^2+bc(3bc-10ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2(ex)^{5/2}\sqrt{c+dx^2}\left(11a^2d^2+bc(3bc-10ad)\right)}{77d^2e}+\frac{4ce\sqrt{ex}\sqrt{c+dx^2}\left(11a^2d^2+bc(3bc-10ad)\right)}{231d^3}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{3/2}(3bc-10ad)}{55d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```



```
[Out] (4*c*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x
^2])/(231*d^3) + (2*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*(e*x)^(5/
2)*Sqrt[c + d*x^2])/(77*d^2*e) - (2*b*(3*b*c - 10*a*d)*(e*x)^(5/2
)*(c + d*x^2)^(3/2))/(55*d^2*e) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^
(3/2))/(15*d*e^3) - (2*c^(7/4)*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d)
)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], 1/2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 57.6151, size = 277, normalized size = 0.96

$$\frac{2b^2(ex)^{\frac{9}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}{15de^3}+\frac{2b(ex)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{3}{2}}(10ad-3bc)}{55d^2e}-\frac{2c^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(11a^2d^2-bc(10ad-3bc)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231d^{\frac{13}{4}}\sqrt{c+dx^2}}+\frac{4ce\sqrt{ex}\sqrt{c+dx^2}\left(11a^2d^2-bc(10ad-3bc)\right)}{231d^3}+\frac{2(ex)^{\frac{5}{2}}\sqrt{c+dx^2}\left(11a^2d^2-bc(10ad-3bc)\right)}{77d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(9/2)*(c + d*x**2)**(3/2)/(15*d*e**3) + 2*b*(e*x)**
(5/2)*(c + d*x**2)**(3/2)*(10*a*d - 3*b*c)/(55*d**2*e) - 2*c**(7/
4)*e**(3/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c)
+ sqrt(d)*x)*(11*a**2*d**2 - b*c*(10*a*d - 3*b*c))*elliptic_f(2*a
tan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(231*d**(13/4)*s
qrt(c + d*x**2)) + 4*c*e*sqrt(e*x)*sqrt(c + d*x**2)*(11*a**2*d**2
 - b*c*(10*a*d - 3*b*c))/(231*d**3) + 2*(e*x)**(5/2)*sqrt(c + d*x
**2)*(11*a**2*d**2 - b*c*(10*a*d - 3*b*c))/(77*d**2*e)
```

Mathematica [C] time = 0.446055, size = 225, normalized size = 0.78

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)\left(55a^2d^2(2c+3dx^2)+10abd(-10c^2+6cdx^2+21d^2x^4)+b^2(30c^3-18c^2dx^2+14cd^2x^4+77d^3x^6)\right)}{5d^3}-\frac{4ic^2x\sqrt{\frac{c}{dx^2}+1}\left(11a^2d^2-10abcd+3b^2c^2\right.}{d^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{231x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(55*a^2*d^2*(2*c + 3*d*x^2)
+ 10*a*b*d*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4) + b^2*(30*c^3 - 18*
c^2*d*x^2 + 14*c*d^2*x^4 + 77*d^3*x^6)))/(5*d^3) - ((4*I)*c^2*(3*
b^2*c^2 - 10*a*b*c*d + 11*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqr
t[c])/Sqrt[d]]*d^3)))/(231*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.095, size = 448, normalized size = 1.6

$$-\frac{2e}{1155\,xd^4}\sqrt{ex}\left(-77\,x^9b^2d^5-210\,x^7abd^5-91\,x^7b^2cd^4+55\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-c}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] -2/1155*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-77*x^9*b^2*d^5-210*x^7*
a*b*d^5-91*x^7*b^2*c*d^4+55*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*a^2*c^2*d^2-50*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^3*d+15*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^4-165*x^5*a^2*d
^5-270*x^5*a*b*c*d^4+4*x^5*b^2*c^2*d^3-275*x^3*a^2*c*d^4+40*x^3*a
*b*c^2*d^3-12*x^3*b^2*c^3*d^2-110*x*a^2*c^2*d^3+100*x*a*b*c^3*d^2
-30*x*b^2*c^4*d)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^2\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2ex^5 + 2\,abex^3 + a^2ex\right)\sqrt{dx^2 + c}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(d*x^2 + c)*sqrt
(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^2 \sqrt{dx^2 + c}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2), x)
```

3.823 $\int \sqrt{ex} \left(a+bx^2\right)^2 \sqrt{c+dx^2} \, dx$

Optimal. Leaf size=425

$$\frac{2c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}} - \frac{4c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}} + \frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(39a^2d^2+bc(7bc-26ad)\right)}{195d^2e} + \frac{4c\sqrt{ex}\sqrt{c+dx^2}\left(39a^2d^2+bc(7bc-26ad)\right)}{195d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{117d^2e} + \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

```
[Out] (2*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2
])/(195*d^2*e) + (4*c*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e*
x]*Sqrt[c + d*x^2])/(195*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*b*(7
*b*c - 26*a*d)*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(117*d^2*e) + (2*b^
2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*d*e^3) - (4*c^(5/4)*(39*a^2*
d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2])
 + (2*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c
] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(
11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.98075, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}} - \frac{4c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}} + \frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(39a^2d^2+bc(7bc-26ad)\right)}{195d^2e} + \frac{4c\sqrt{ex}\sqrt{c+dx^2}\left(39a^2d^2+bc(7bc-26ad)\right)}{195d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{117d^2e} + \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```


```
[Out] (2*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2
])/(195*d^2*e) + (4*c*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e*
x]*Sqrt[c + d*x^2])/(195*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*b*(7
*b*c - 26*a*d)*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(117*d^2*e) + (2*b^
2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*d*e^3) - (4*c^(5/4)*(39*a^2*
d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2])
 + (2*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c
] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(
11/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 89.4098, size = 405, normalized size = 0.95

$$\frac{2b^2(ex)^{\frac{7}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}{13de^3}+\frac{2b(ex)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}(26ad-7bc)}{117d^2e}-\frac{4c^{\frac{5}{4}}\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(39a^2d^2-bc(26ad-7bc)\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195d^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{2c^{\frac{5}{4}}\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(39a^2d^2-bc(26ad-7bc)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195d^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{4c\sqrt{ex}\sqrt{c+dx^2}\left(39a^2d^2-bc(26ad-7bc)\right)}{195d^{\frac{5}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\left(39a^2d^2-bc(26ad-7bc)\right)}{195d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(e*x)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(7/2)*(c + d*x**2)**(3/2)/(13*d*e**3) + 2*b*(e*x)**
(3/2)*(c + d*x**2)**(3/2)*(26*a*d - 7*b*c)/(117*d**2*e) - 4*c**(5
/4)*sqrt(e)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c)
+ sqrt(d)*x)*(39*a**2*d**2 - b*c*(26*a*d - 7*b*c))*elliptic_e(2*a
tan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(195*d**(11/4)*s
qrt(c + d*x**2)) + 2*c**(5/4)*sqrt(e)*sqrt((c + d*x**2)/(sqrt(c)
+ sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(39*a**2*d**2 - b*c*(26*a*
d - 7*b*c))*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e
))), 1/2)/(195*d**(11/4)*sqrt(c + d*x**2)) + 4*c*sqrt(e*x)*sqrt(c
 + d*x**2)*(39*a**2*d**2 - b*c*(26*a*d - 7*b*c))/(195*d**(5/2)*(s
qrt(c) + sqrt(d)*x)) + 2*(e*x)**(3/2)*sqrt(c + d*x**2)*(39*a**2*d
**2 - b*c*(26*a*d - 7*b*c))/(195*d**2*e)
```

Mathematica [C] time = 1.24173, size = 282, normalized size = 0.66

$$\frac{2e\left(dx^2\left(c+dx^2\right)\left(117a^2d^2+26abd\left(2c+5dx^2\right)+b^2\left(-14c^2+10cdx^2+45d^2x^4\right)\right)+\frac{6c\left(39a^2d^2-26abcd+7b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)+\sqrt{c}\right.}{}\right.}{585d^3\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (2*e*(d*x^2*(c + d*x^2)*(117*a^2*d^2 + 26*a*b*d*(2*c + 5*d*x^2) +
 b^2*(-14*c^2 + 10*c*d*x^2 + 45*d^2*x^4)) + (6*c*(7*b^2*c^2 - 26*
a*b*c*d + 39*a^2*d^2)*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(c + d*x^2) - Sq
rt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] + Sqrt[c]*Sqrt[d]*Sqrt[1 + c
/(d*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/S
qrt[x]], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(585*d^3*Sqrt[e*x]*Sqr
t[c + d*x^2])
```

Maple [A] time = 0.065, size = 658, normalized size = 1.6

$$\frac{2}{585\,d^3x}\sqrt{ex}\left(45\,x^8b^2d^4+130\,x^6abd^4+55\,x^6b^2cd^3+234\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 2/585*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^3*(45*x^8*b^2*d^4+130*x^6*a*b
*d^4+55*x^6*b^2*c*d^3+234*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*c^2*d^2-156*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a*b*c^3*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^
(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*b^2*c^4-117*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
^2*c^2*d^2+78*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b
*c^3*d-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4
+117*x^4*a^2*d^4+182*x^4*a*b*c*d^3-4*x^4*b^2*c^2*d^2+117*x^2*a^2*
c*d^3+52*x^2*a*b*c^2*d^2-14*x^2*b^2*c^3*d)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2\sqrt{dx^2+c}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)*sqrt(e*x), x
)
```

Sympy [A] time = 23.6811, size = 148, normalized size = 0.35

$$\frac{a^2\sqrt{c}\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e\,\Gamma\left(\frac{7}{4}\right)}+\frac{ab\sqrt{c}\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^3\,\Gamma\left(\frac{11}{4}\right)}+\frac{b^2\sqrt{c}\,(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^5\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(e*x)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d
*x**2*exp_polar(I*pi)/c)/(2*e*gamma(7/4)) + a*b*sqrt(c)*(e*x)**(7
/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)
/c)/(e**3*gamma(11/4)) + b**2*sqrt(c)*(e*x)**(11/2)*gamma(11/4)*h
yper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**5*gam
ma(15/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^2 \sqrt{dx^2 + c}\sqrt{ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

3.824 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$

Optimal. Leaf size=244

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-22abcd+5b^2c^2\right)}{231d^2e} + \frac{2c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-22abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}} - \frac{2b\sqrt{ex}\left(c+dx^2\right)^{3/2}(5bc-22ad)}{77d^2e} + \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

```
[Out] (2*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2
])/(231*d^2*e) - (2*b*(5*b*c - 22*a*d)*Sqrt[e*x]*(c + d*x^2)^(3/2
))/(77*d^2*e) + (2*b^2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e^3)
+ (2*c^(3/4)*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqr
t[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(9/4)*Sq
rt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.513392, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-22abcd+5b^2c^2\right)}{231d^2e} + \frac{2c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-22abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}} - \frac{2b\sqrt{ex}\left(c+dx^2\right)^{3/2}(5bc-22ad)}{77d^2e} + \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```


```
[Out] (2*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2
])/(231*d^2*e) - (2*b*(5*b*c - 22*a*d)*Sqrt[e*x]*(c + d*x^2)^(3/2
))/(77*d^2*e) + (2*b^2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e^3)
+ (2*c^(3/4)*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqr
t[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(9/4)*Sq
rt[e]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 48.9388, size = 228, normalized size = 0.93

$$\frac{2b^2(ex)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}{11de^3} + \frac{2b\sqrt{ex}\left(c+dx^2\right)^{\frac{3}{2}}(22ad-5bc)}{77d^2e} + \frac{2c^{\frac{3}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(77a^2d^2-bc(22ad-5bc)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231d^{\frac{9}{4}}\sqrt{e}\sqrt{c+dx^2}} + \frac{2\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-bc(22ad-5bc)\right)}{231d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(5/2)*(c + d*x**2)**(3/2)/(11*d*e**3) + 2*b*sqrt(e*
x)*(c + d*x**2)**(3/2)*(22*a*d - 5*b*c)/(77*d**2*e) + 2*c**(3/4)*
sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)
*(77*a**2*d**2 - b*c*(22*a*d - 5*b*c))*elliptic_f(2*atan(d**(1/4)
*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(231*d**(9/4)*sqrt(e)*sqrt(c
 + d*x**2)) + 2*sqrt(e*x)*sqrt(c + d*x**2)*(77*a**2*d**2 - b*c*(2
2*a*d - 5*b*c))/(231*d**2*e)
```

Mathematica [C] time = 0.315699, size = 189, normalized size = 0.77

$$\frac{\sqrt{x}\left(\frac{2\sqrt{x}(c+dx^2)\left(77a^2d^2+22abd(2c+3dx^2)+b^2(-10c^2+6cdx^2+21d^2x^4)\right)}{d^2}+\frac{4icx\sqrt{\frac{c}{dx^2}+1}\left(77a^2d^2-22abcd+5b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{d^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```

```
[Out] (Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(2*c + 3*
d*x^2) + b^2*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4)))/d^2 + ((4*I)*c*
(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*
Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.041, size = 401, normalized size = 1.6

$$\frac{2}{231\,d^3}\left(21\,x^7b^2d^4+77\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)a^2cd^2-22\,\sqrt{-cd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)*(21*x^7*b^2*d^4+77*(-c*d)^(1/2)*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-22*(-c*d)^(1/2)*((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+5*(-c*d)^
(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+66*
x^5*a*b*d^4+27*x^5*b^2*c*d^3+77*x^3*a^2*d^4+110*x^3*a*b*c*d^3-4*x
^3*b^2*c^2*d^2+77*x*a^2*c*d^3+44*x*a*b*c^2*d^2-10*x*b^2*c^3*d)/(e
*x)^(1/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{dx^2 + c}}{\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/sqrt(e*x), x
)
```

Sympy [A] time = 26.9265, size = 150, normalized size = 0.61

$$\frac{a^2\sqrt{c}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)} + \frac{ab\sqrt{c}x^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)} + \frac{b^2\sqrt{c}x^{\frac{9}{2}}\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2
*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a*b*sqrt(c)*x**(5/2)
*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/
(sqrt(e)*gamma(9/4)) + b**2*sqrt(c)*x**(9/2)*gamma(9/4)*hyper((-1
/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(13/
4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2\sqrt{dx^2 + c}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

3.825 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}-\frac{2\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(b^2c^2-3ad(5ad+2bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(b^2c^2-3ad(5ad+2bc)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-3ad(5ad+2bc)\right)}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(b^2c^2-3ad(5ad+2bc)\right)}{15cde^3}+\frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3}$$

```
[Out] (-2*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2]
)/(15*c*d*e^3) - (4*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*Sqrt[e*x]*S
qrt[c + d*x^2])/(15*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(
c + d*x^2)^(3/2))/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2
)^(3/2))/(9*d*e^3) + (4*c^(1/4)*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))
*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*
EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (2*c^(1/4)*(b^2*c^2 - 3*a*
d*(2*b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c
] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.946531, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}-\frac{2\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(b^2c^2-3ad(5ad+2bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(b^2c^2-3ad(5ad+2bc)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-3ad(5ad+2bc)\right)}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(b^2c^2-3ad(5ad+2bc)\right)}{15cde^3}+\frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

```
[Out] (-2*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2]
)/(15*c*d*e^3) - (4*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*Sqrt[e*x]*S
qrt[c + d*x^2])/(15*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(
c + d*x^2)^(3/2))/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2
)^(3/2))/(9*d*e^3) + (4*c^(1/4)*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))
*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*
EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (2*c^(1/4)*(b^2*c^2 - 3*a*
```

```
d*(2*b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c
] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 94.0651, size = 396, normalized size = 0.94

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{3}{2}}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}{9de^3}+\frac{4\sqrt[4]{c}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-3ad\left(5ad+2bc\right)+b^2c^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}-\frac{2\sqrt[4]{c}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-3ad\left(5ad+2bc\right)+b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(-3ad\left(5ad+2bc\right)+b^2c^2\right)}{15d^{\frac{3}{2}}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\left(-3ad\left(5ad+2bc\right)+b^2c^2\right)}{15cde^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(c*e*sqrt(e*x)) + 2*b**2*(e*x)**(3/2)
*(c + d*x**2)**(3/2)/(9*d*e**3) + 4*c**(1/4)*sqrt((c + d*x**2)/(s
qrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-3*a*d*(5*a*d + 2*
b*c) + b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*
sqrt(e))), 1/2)/(15*d**(7/4)*e**(3/2)*sqrt(c + d*x**2)) - 2*c**(1
/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d
)*x)*(-3*a*d*(5*a*d + 2*b*c) + b**2*c**2)*elliptic_f(2*atan(d**(1
/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(15*d**(7/4)*e**(3/2)*sqr
t(c + d*x**2)) - 4*sqrt(e*x)*sqrt(c + d*x**2)*(-3*a*d*(5*a*d + 2*
b*c) + b**2*c**2)/(15*d**(3/2)*e**2*(sqrt(c) + sqrt(d)*x)) - 2*(e
*x)**(3/2)*sqrt(c + d*x**2)*(-3*a*d*(5*a*d + 2*b*c) + b**2*c**2)/
(15*c*d*e**3)
```

Mathematica [C] time = 0.851718, size = 260, normalized size = 0.62

$$\frac{2x\left(d\left(c+dx^2\right)\left(-45a^2d+18abdx^2+b^2x^2\left(2c+5dx^2\right)\right)-\frac{6\left(-15a^2d^2-6abcd+b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)+\sqrt{c}\sqrt{d}x^{3/2}\sqrt{\frac{c}{dx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{45d^2(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

```
[Out] (2*x*(d*(c + d*x^2)*(-45*a^2*d + 18*a*b*d*x^2 + b^2*x^2*(2*c + 5*
d*x^2)) - (6*(b^2*c^2 - 6*a*b*c*d - 15*a^2*d^2)*(Sqrt[(I*Sqrt[c])
/Sqrt[d]]*(c + d*x^2) - Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/
2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] +
Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]
]))/(45*d^2*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.055, size = 624, normalized size = 1.5

$$\frac{2}{45\,e d^{2}}\left(5\,x^{6}b^{2}d^{3}+90\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)a^{2}cd^{2}+36\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x)
```

```
[Out] 2/45*(5*x^6*b^2*d^3+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a^2*c*d^2+36*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*c^3-45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2
-18*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+3*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+18*x^4*a*b
*d^3+7*x^4*b^2*c*d^2-45*x^2*a^2*d^3+18*x^2*a*b*c*d^2+2*x^2*b^2*c^
2*d-45*a^2*c*d^2)/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{2}\sqrt{dx^{2}+c}}{\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{dx^{2}+c}}{\sqrt{ex}ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/(sqrt(e*x)*e
*x), x)
```

Sympy [A] time = 30.7029, size = 153, normalized size = 0.36

$$\frac{a^{2}\sqrt{c}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{ab\sqrt{c}x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)}+\frac{b^{2}\sqrt{c}x^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*sqrt(c)*x**(3
/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/
c)/(e**(3/2)*gamma(7/4)) + b**2*sqrt(c)*x**(7/2)*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamm
a(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2), x)
```

3.826 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}}-\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(b^2c^2-7ad(ad+2bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-7ad(ad+2bc)\right)}{21cde^3}+\frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7de^3}$$

```
[Out] (-2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(2
1*c*d*e^3) - (2*a^2*(c + d*x^2)^(3/2))/(3*c*e*(e*x)^(3/2)) + (2*b
^2*Sqrt[e*x]*(c + d*x^2)^(3/2))/(7*d*e^3) - (2*(b^2*c^2 - 7*a*d*(
2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], 1/2])/(21*c^(1/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.506539, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{3ce(ex)^{3/2}}-\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(b^2c^2-7ad(ad+2bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-7ad(ad+2bc)\right)}{21cde^3}+\frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

```
[Out] (-2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(2
1*c*d*e^3) - (2*a^2*(c + d*x^2)^(3/2))/(3*c*e*(e*x)^(3/2)) + (2*b
^2*Sqrt[e*x]*(c + d*x^2)^(3/2))/(7*d*e^3) - (2*(b^2*c^2 - 7*a*d*(
2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], 1/2])/(21*c^(1/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 51.0397, size = 216, normalized size = 0.92

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{3}{2}}}{3ce\left(ex\right)^{\frac{3}{2}}}+\frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{\frac{3}{2}}}{7de^3}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(-7ad\left(ad+2bc\right)+b^2c^2\right)}{21cde^3}-\frac{2\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-7ad\left(ad+2bc\right)+b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(3*c*e*(e*x)**(3/2)) + 2*b**2*sqrt(e*
x)*(c + d*x**2)**(3/2)/(7*d*e**3) - 2*sqrt(e*x)*sqrt(c + d*x**2)*
(-7*a*d*(a*d + 2*b*c) + b**2*c**2)/(21*c*d*e**3) - 2*sqrt((c + d*
x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-7*a*d*(a*
d + 2*b*c) + b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**
(1/4)*sqrt(e))), 1/2)/(21*c**(1/4)*d**(5/4)*e**(5/2)*sqrt(c + d*x
**2))
```

Mathematica [C] time = 0.294712, size = 171, normalized size = 0.73

$$\frac{x^{5/2}\left(\frac{2(c+dx^2)\left(-7a^2d+14abdx^2+b^2x^2(2c+3dx^2)\right)}{dx^{3/2}}+\frac{4ix\sqrt{\frac{c}{dx^2}+1}\left(7a^2d^2+14abcd-b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

```
[Out] (x^(5/2)*((2*(c + d*x^2)*(-7*a^2*d + 14*a*b*d*x^2 + b^2*x^2*(2*c
+ 3*d*x^2)))/(d*x^(3/2)) + ((4*I)*(-(b^2*c^2) + 14*a*b*c*d + 7*a^
2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])
/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))/(21*(e*x
)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.046, size = 383, normalized size = 1.6

$$\frac{2}{21\,xe^2d^2}\left(7\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}xa^2d^2+14\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] 2/21/(d*x^2+c)^(1/2)/x*(7*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*(-c*d)^(1/2)*x*a^2*d^2+14*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*a*b*c*d-((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*b^2*c^2+3*x^6*b^2*d^3+14*x
^4*a*b*d^3+5*x^4*b^2*c*d^2-7*x^2*a^2*d^3+14*x^2*a*b*c*d^2+2*x^2*b
^2*c^2*d-7*a^2*c*d^2)/e^2/(e*x)^(1/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)\sqrt{dx^2+c}}{\sqrt{ex}e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/(sqrt(e*x)*e
^2*x^2), x)
```

Sympy [A] time = 83.4005, size = 153, normalized size = 0.65

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{ab\sqrt{c}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{b^2\sqrt{c}x^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(c)*sqrt
(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/
c)/(e**(5/2)*gamma(5/4)) + b**2*sqrt(c)*x**(5/2)*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamma
(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2), x)
```

3.827 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(ad(ad+10bc)+b^2c^2\right)}{5c^2e^5}+\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(ad(ad+10bc)+b^2c^2\right)}{5c\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(ad+10bc)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{4\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(ad+10bc)+b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{5c^2e^3\sqrt{ex}}$$

```
[Out] (2*(b^2*c^2 + a*d*(10*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5
*c^2*e^5) + (4*(b^2*c^2 + a*d*(10*b*c + a*d))*Sqrt[e*x]*Sqrt[c +
d*x^2])/(5*c*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x
^2)^(3/2))/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c + a*d)*(c + d*x^2)^
(3/2))/(5*c^2*e^3*Sqrt[e*x]) - (4*(b^2*c^2 + a*d*(10*b*c + a*d))*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (2*(b^2*c^2 + a*d*(1
0*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], 1/2])/(5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.963847, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(ad(ad+10bc)+b^2c^2\right)}{5c^2e^5}+\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(ad(ad+10bc)+b^2c^2\right)}{5c\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(ad+10bc)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{4\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(ad+10bc)+b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{5c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```


```
[Out] (2*(b^2*c^2 + a*d*(10*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5
*c^2*e^5) + (4*(b^2*c^2 + a*d*(10*b*c + a*d))*Sqrt[e*x]*Sqrt[c +
d*x^2])/(5*c*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x
^2)^(3/2))/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c + a*d)*(c + d*x^2)^
(3/2))/(5*c^2*e^3*Sqrt[e*x]) - (4*(b^2*c^2 + a*d*(10*b*c + a*d))*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (2*(b^2*c^2 + a*d*(1
```

```
0*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], 1/2])/(5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 94.1369, size = 394, normalized size = 0.94

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{3}{2}}}{5ce\left(ex\right)^{\frac{5}{2}}}-\frac{2a\left(c+dx^2\right)^{\frac{3}{2}}\left(ad+10bc\right)}{5c^2e^3\sqrt{ex}}+\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(ad\left(ad+10bc\right)+b^2c^2\right)}{5c\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\left(ex\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(ad\left(ad+10bc\right)+b^2c^2\right)}{5c^2e^5}-\frac{4\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\left(ad+10bc\right)+b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}+\frac{2\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\left(ad+10bc\right)+b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(5*c*e*(e*x)**(5/2)) - 2*a*(c + d*x**
2)**(3/2)*(a*d + 10*b*c)/(5*c**2*e**3*sqrt(e*x)) + 4*sqrt(e*x)*sq
rt(c + d*x**2)*(a*d*(a*d + 10*b*c) + b**2*c**2)/(5*c*sqrt(d)*e**4
*(sqrt(c) + sqrt(d)*x)) + 2*(e*x)**(3/2)*sqrt(c + d*x**2)*(a*d*(a
*d + 10*b*c) + b**2*c**2)/(5*c**2*e**5) - 4*sqrt((c + d*x**2)/(sq
rt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(a*d*(a*d + 10*b*c)
+ b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(
e))), 1/2)/(5*c**(3/4)*d**(3/4)*e**(7/2)*sqrt(c + d*x**2)) + 2*sq
rt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(
a*d*(a*d + 10*b*c) + b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e
*x)/(c**(1/4)*sqrt(e))), 1/2)/(5*c**(3/4)*d**(3/4)*e**(7/2)*sqrt(
c + d*x**2))
```

Mathematica [C] time = 1.40011, size = 226, normalized size = 0.54

$$\frac{x^{7/2}\left(\frac{2\sqrt{c+dx^2}\left(-a^2\left(c+2dx^2\right)-10abcx^2+b^2cx^4\right)}{cx^{5/2}}-\frac{4x\left(a^2d^2+10abcd+b^2c^2\right)\left(-\sqrt{x}\left(\frac{c}{x^2}+d\right)+\frac{ic\sqrt{\frac{c}{dx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{c}}{\sqrt{d}}\right)^{3/2}}\right)}{cd\sqrt{c+dx^2}}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```

```
[Out] (x^(7/2)*((2*Sqrt[c + d*x^2]*(-10*a*b*c*x^2 + b^2*c*x^4 - a^2*(c
+ 2*d*x^2)))/(c*x^(5/2)) - (4*(b^2*c^2 + 10*a*b*c*d + a^2*d^2)*x*
(-((d + c/x^2)*Sqrt[x]) + (I*c*Sqrt[1 + c/(d*x^2)]*(EllipticE[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] - EllipticF[I*ArcS
inh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/((I*Sqrt[c])/Sqrt[d
])^(3/2)))/(c*d*Sqrt[c + d*x^2])))/(5*(e*x)^(7/2))
```

Maple [A] time = 0.074, size = 648, normalized size = 1.5

$$\frac{2}{5\,dx^2e^3c}\left(2\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2+20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^
2*c*d^2+20*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*
b*c^2*d+2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2
*c^3-((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^
2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*
d-((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+x^6
*b^2*c*d^2-2*x^4*a^2*d^3-10*x^4*a*b*c*d^2+x^4*b^2*c^2*d-3*x^2*a^2
*c*d^2-10*x^2*a*b*c^2*d-a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*x)^(1
/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{\sqrt{ex}e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/(sqrt(e*x)*e
^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2), x)
```

3.828 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}+\frac{2\sqrt{x}\sqrt{c+dx^2}\left(ad(14bc-ad)+7b^2c^2\right)}{21c^2}+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(14bc-ad)+7b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt[4]{d}\sqrt{c+dx^2}}-\frac{2a\left(c+dx^2\right)^{3/2}(14bc-ad)}{21c^2x^{3/2}}$$

```
[Out] (2*(7*b^2*c^2 + a*d*(14*b*c - a*d))*Sqrt[x]*Sqrt[c + d*x^2])/(21*
c^2) - (2*a^2*(c + d*x^2)^(3/2))/(7*c*x^(7/2)) - (2*a*(14*b*c - a
*d)*(c + d*x^2)^(3/2))/(21*c^2*x^(3/2)) + (2*(7*b^2*c^2 + a*d*(14
*b*c - a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/
(21*c^(5/4)*d^(1/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.432012, antiderivative size = 210, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}+\frac{2}{21}\sqrt{x}\sqrt{c+dx^2}\left(\frac{ad(14bc-ad)}{c^2}+7b^2\right)+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(14bc-ad)+7b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt[4]{d}\sqrt{c+dx^2}}-\frac{2a\left(c+dx^2\right)^{3/2}(14bc-ad)}{21c^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

```
[Out] (2*(7*b^2 + (a*d*(14*b*c - a*d))/c^2)*Sqrt[x]*Sqrt[c + d*x^2])/21
 - (2*a^2*(c + d*x^2)^(3/2))/(7*c*x^(7/2)) - (2*a*(14*b*c - a*d)*
(c + d*x^2)^(3/2))/(21*c^2*x^(3/2)) + (2*(7*b^2*c^2 + a*d*(14*b*c
 - a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d
]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(21*
c^(5/4)*d^(1/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 37.7216, size = 199, normalized size = 0.93

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{3}{2}}}{7cx^{\frac{7}{2}}}+\frac{2a\left(c+dx^2\right)^{\frac{3}{2}}(ad-14bc)}{21c^2x^{\frac{3}{2}}}+\frac{2\sqrt{x}\sqrt{c+dx^2}\left(-ad\left(ad-14bc\right)+7b^2c^2\right)}{21c^2}+\frac{2\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-ad\left(ad-14bc\right)+7b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)\middle|\frac{1}{2}\right)}{21c^{\frac{5}{4}}\sqrt[4]{d}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(9/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(7*c*x**(7/2)) + 2*a*(c + d*x**2)**(3
/2)*(a*d - 14*b*c)/(21*c**2*x**(3/2)) + 2*sqrt(x)*sqrt(c + d*x**2
)*(-a*d*(a*d - 14*b*c) + 7*b**2*c**2)/(21*c**2) + 2*sqrt((c + d*x
```

```
**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-a*d*(a*d -
 14*b*c) + 7*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(x)/c**(1/
4)), 1/2)/(21*c**(5/4)*d**(1/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.32464, size = 160, normalized size = 0.75

$$\frac{2\left(\left(c+dx^2\right)\left(-a^2\left(3c+2dx^2\right)-14abcx^2+7b^2cx^4\right)+\frac{2ix^{9/2}\sqrt{\frac{c}{dx^2}+1}\left(-a^2d^2+14abcd+7b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21cx^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

```
[Out] (2*((c + d*x^2)*(-14*a*b*c*x^2 + 7*b^2*c*x^4 - a^2*(3*c + 2*d*x^2
)) + ((2*I)*(7*b^2*c^2 + 14*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)
]*x^(9/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
 -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(21*c*x^(7/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.079, size = 385, normalized size = 1.8

$$-\frac{2}{21\,cd}\left(\sqrt{1\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{2}\sqrt{1\left(-dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{-dx\frac{1}{\sqrt{-cd}}}EllipticF\left(\sqrt{1\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x)
```

```
[Out] -2/21/(d*x^2+c)^(1/2)*(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*(-c*d)^(1/2)*x^3*a^2*d^2-14*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*a*b*c*d-7*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*b^2*c^2-7*x^6*b^2*c*d
^2+2*x^4*a^2*d^3+14*x^4*a*b*c*d^2-7*x^4*b^2*c^2*d+5*x^2*a^2*c*d^2
+14*x^2*a*b*c^2*d+3*a^2*c^2*d)/x^(7/2)/c/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{dx^2 + c}}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2\sqrt{dx^2 + c}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

3.829 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$

Optimal. Leaf size=386

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}-\frac{2\sqrt{c+dx^2}\left(ad(6bc-ad)+15b^2c^2\right)}{15c^2\sqrt{x}}+\frac{4\sqrt{d}\sqrt{x}\sqrt{c+dx^2}\left(ad(6bc-ad)+15b^2c^2\right)}{15c^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-ad)+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{4\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-ad)+15b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{2a\left(c+dx^2\right)^{3/2}(6bc-ad)}{15c^2x^{5/2}}$$

```
[Out] (-2*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[c + d*x^2])/(15*c^2*Sqr
t[x]) + (4*Sqrt[d]*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[x]*Sqrt[
c + d*x^2])/(15*c^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(
3/2))/(9*c*x^(9/2)) - (2*a*(6*b*c - a*d)*(c + d*x^2)^(3/2))/(15*c
^2*x^(5/2)) - (4*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c
] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Elliptic
E[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c +
 d*x^2]) + (2*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c + d*
x^2])
```

Rubi [A] time = 0.768105, antiderivative size = 383, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}-\frac{2\sqrt{c+dx^2}\left(\frac{ad(6bc-ad)}{c^2}+15b^2\right)}{15\sqrt{x}}+\frac{4\sqrt{d}\sqrt{x}\sqrt{c+dx^2}\left(ad(6bc-ad)+15b^2c^2\right)}{15c^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-ad)+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{4\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-ad)+15b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{2a\left(c+dx^2\right)^{3/2}(6bc-ad)}{15c^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

```
[Out] (-2*(15*b^2 + (a*d*(6*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/(15*Sqrt[
x]) + (4*Sqrt[d]*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[x]*Sqrt[c
+ d*x^2])/(15*c^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/
2))/(9*c*x^(9/2)) - (2*a*(6*b*c - a*d)*(c + d*x^2)^(3/2))/(15*c^2
*x^(5/2)) - (4*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c]
+ Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[
2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c + d
*x^2]) + (2*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c] + S
qrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*A
rcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c + d*x^
```

```
2])
```

Rubi in Sympy [A] time = 64.9998, size = 360, normalized size = 0.93

$$-\frac{2a^2(c+dx^2)^{\frac{3}{2}}}{9cx^{\frac{9}{2}}}+\frac{2a(c+dx^2)^{\frac{3}{2}}(ad-6bc)}{15c^2x^{\frac{5}{2}}}+\frac{4\sqrt{d}\sqrt{x}\sqrt{c+dx^2}\,(-ad(ad-6bc)+15b^2c^2)}{15c^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2\sqrt{c+dx^2}\,(-ad(ad-6bc)+15b^2c^2)}{15c^2\sqrt{x}}-\frac{4\sqrt[4]{d}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)(-ad(ad-6bc)+15b^2c^2)\,E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{7}{4}}\sqrt{c+dx^2}}+\frac{2\sqrt[4]{d}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)(-ad(ad-6bc)+15b^2c^2)\,F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{7}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(11/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(9*c*x**(9/2)) + 2*a*(c + d*x**2)**(3
/2)*(a*d - 6*b*c)/(15*c**2*x**(5/2)) + 4*sqrt(d)*sqrt(x)*sqrt(c +
 d*x**2)*(-a*d*(a*d - 6*b*c) + 15*b**2*c**2)/(15*c**2*(sqrt(c) +
sqrt(d)*x)) - 2*sqrt(c + d*x**2)*(-a*d*(a*d - 6*b*c) + 15*b**2*c*
*2)/(15*c**2*sqrt(x)) - 4*d**(1/4)*sqrt((c + d*x**2)/(sqrt(c) + s
qrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-a*d*(a*d - 6*b*c) + 15*b**2
*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(x)/c**(1/4)), 1/2)/(15*c**
(7/4)*sqrt(c + d*x**2)) + 2*d**(1/4)*sqrt((c + d*x**2)/(sqrt(c) +
 sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-a*d*(a*d - 6*b*c) + 15*b*
*2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(x)/c**(1/4)), 1/2)/(15*c
**(7/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.845727, size = 283, normalized size = 0.73

$$\frac{2\left((-c-dx^2)\left(3x^4\left(-2a^2d^2+12abcd+15b^2c^2\right)+5a^2c^2+2acx^2(ad+9bc)\right)+\frac{6x^4\left(-a^2d^2+6abcd+15b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(c+dx^2)+\sqrt{c}\sqrt{d}x^{3/2}\cdots\right.}{\cdots}\right)}{45c^2x^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

```
[Out] (2*((-c - d*x^2)*(5*a^2*c^2 + 2*a*c*(9*b*c + a*d)*x^2 + 3*(15*b^2
*c^2 + 12*a*b*c*d - 2*a^2*d^2)*x^4) + (6*(15*b^2*c^2 + 6*a*b*c*d
- a^2*d^2)*x^4*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(c + d*x^2) - Sqrt[c]*S
qrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I*Sq
rt[c])/Sqrt[d]]/Sqrt[x]], -1] + Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2
)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]]
, -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(45*c^2*x^(9/2)*Sqrt[c + d*x^
2])
```

Maple [A] time = 0.084, size = 659, normalized size = 1.7

$$-\frac{2}{45\,c^2}\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\,EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^4a^2cd^2-36\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx\cdots}{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x)
```

```
[Out] -2/45*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*
c*d^2-36*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b*
c^2*d-90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*
c^3-3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^2*c*d
^2+18*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b*c^2
*d+45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c^3
-6*x^6*a^2*d^3+36*x^6*a*b*c*d^2+45*x^6*b^2*c^2*d-4*x^4*a^2*c*d^2+
54*x^4*a*b*c^2*d+45*x^4*b^2*c^3+7*x^2*a^2*c^2*d+18*x^2*a*b*c^3+5*
a^2*c^3)/(d*x^2+c)^(1/2)/x^(9/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

3.830 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{2\sqrt{c+dx^2}\,(5a^2d^2-22abcd+77b^2c^2)}{231c^2x^{3/2}}+\frac{2d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(5a^2d^2-22abcd+77b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{c+dx^2}}-\frac{2a^2\,(c+dx^2)^{3/2}}{11cx^{11/2}}-\frac{2a\,(c+dx^2)^{3/2}\,(22bc-5ad)}{77c^2x^{7/2}}$$

```
[Out] (-2*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*Sqrt[c + d*x^2])/(231*c
^2*x^(3/2)) - (2*a^2*(c + d*x^2)^(3/2))/(11*c*x^(11/2)) - (2*a*(2
2*b*c - 5*a*d)*(c + d*x^2)^(3/2))/(77*c^2*x^(7/2)) + (2*d^(3/4)*(
77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*S
qrt[x])/c^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.455355, antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{2d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(5a^2d^2-22abcd+77b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{c+dx^2}}-\frac{2a^2\,(c+dx^2)^{3/2}}{11cx^{11/2}}-\frac{2\sqrt{c+dx^2}\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)}{231x^{3/2}}-\frac{2a\,(c+dx^2)^{3/2}\,(22bc-5ad)}{77c^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

```
[Out] (-2*(77*b^2 - (a*d*(22*b*c - 5*a*d))/c^2)*Sqrt[c + d*x^2])/(231*x
^(3/2)) - (2*a^2*(c + d*x^2)^(3/2))/(11*c*x^(11/2)) - (2*a*(22*b*
c - 5*a*d)*(c + d*x^2)^(3/2))/(77*c^2*x^(7/2)) + (2*d^(3/4)*(77*b
^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c +
d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
x])/c^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 37.8611, size = 204, normalized size = 0.94

$$-\frac{2a^2\,(c+dx^2)^{\frac{3}{2}}}{11cx^{\frac{11}{2}}}+\frac{2a\,(c+dx^2)^{\frac{3}{2}}\,(5ad-22bc)}{77c^2x^{\frac{7}{2}}}-\frac{2\sqrt{c+dx^2}\,(ad\,(5ad-22bc)+77b^2c^2)}{231c^2x^{\frac{3}{2}}}+\frac{2d^{\frac{3}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)(ad\,(5ad-22bc)+77b^2c^2)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)\middle|\frac{1}{2}\right)}{231c^{\frac{9}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(13/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(11*c*x**(11/2)) + 2*a*(c + d*x**2)**
(3/2)*(5*a*d - 22*b*c)/(77*c**2*x**(7/2)) - 2*sqrt(c + d*x**2)*(a
*d*(5*a*d - 22*b*c) + 77*b**2*c**2)/(231*c**2*x**(3/2)) + 2*d**(3
/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d
)*x)*(a*d*(5*a*d - 22*b*c) + 77*b**2*c**2)*elliptic_f(2*atan(d**(
```

```
1/4)*sqrt(x)/c**(1/4)), 1/2)/(231*c**(9/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.244865, size = 187, normalized size = 0.86

$$-\frac{2\sqrt{c+dx^2}\left(a^2\left(21c^2+6cdx^2-10d^2x^4\right)+22abcx^2\left(3c+2dx^2\right)+77b^2c^2x^4\right)}{231c^2x^{11/2}}+\frac{4idx\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2-22abcd+77b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{231c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[c + d*x^2]*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c + 2*d*x^2
) + a^2*(21*c^2 + 6*c*d*x^2 - 10*d^2*x^4)))/(231*c^2*x^(11/2)) +
(((4*I)/231)*d*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(
d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
 -1])/(c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.049, size = 403, normalized size = 1.9

$$\frac{2}{231\,c^2}\left(5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^5a^2d^2-22\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*(-c*d)^(1/2)*x^5*a^2*d^2-22*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*a*b*c*d+77*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*b^2*c^2+10*x^6*a^2
*d^3-44*x^6*a*b*c*d^2-77*x^6*b^2*c^2*d+4*x^4*a^2*c*d^2-110*x^4*a*
b*c^2*d-77*x^4*b^2*c^3-27*x^2*a^2*c^2*d-66*x^2*a*b*c^3-21*a^2*c^3
)/x^(11/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{dx^2 + c}}{x^{\frac{13}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2\sqrt{dx^2 + c}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

3.831 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx$

Optimal. Leaf size=441

$$-\frac{2\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^2x^{5/2}}+\frac{2d^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-26abcd+39b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-26abcd+39b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^3\sqrt{x}}+\frac{4d^{3/2}\sqrt{x}\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^3\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{117c^2x^{9/2}}$$

```
[Out] (-2*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(195*c
^2*x^(5/2)) - (4*d*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c +
 d*x^2])/(195*c^3*Sqrt[x]) + (4*d^(3/2)*(39*b^2*c^2 - 26*a*b*c*d
+ 7*a^2*d^2)*Sqrt[x]*Sqrt[c + d*x^2])/(195*c^3*(Sqrt[c] + Sqrt[d]
*x)) - (2*a^2*(c + d*x^2)^(3/2))/(13*c*x^(13/2)) - (2*a*(26*b*c -
 7*a*d)*(c + d*x^2)^(3/2))/(117*c^2*x^(9/2)) - (4*d^(5/4)*(39*b^2
*c^2 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*
x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x]
)/c^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2]) + (2*d^(5/4)*(39
*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
+ d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqr
t[x])/c^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.887275, antiderivative size = 437, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{2d^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-26abcd+39b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-26abcd+39b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^3\sqrt{x}}+\frac{4d^{3/2}\sqrt{x}\sqrt{c+dx^2}\left(7a^2d^2-26abcd+39b^2c^2\right)}{195c^3\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}-\frac{2\sqrt{c+dx^2}\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)}{195x^{5/2}}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{117c^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

```
[Out] (-2*(39*b^2 - (a*d*(26*b*c - 7*a*d))/c^2)*Sqrt[c + d*x^2])/(195*x
^(5/2)) - (4*d*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x
^2])/(195*c^3*Sqrt[x]) + (4*d^(3/2)*(39*b^2*c^2 - 26*a*b*c*d + 7*
a^2*d^2)*Sqrt[x]*Sqrt[c + d*x^2])/(195*c^3*(Sqrt[c] + Sqrt[d]*x))
 - (2*a^2*(c + d*x^2)^(3/2))/(13*c*x^(13/2)) - (2*a*(26*b*c - 7*a
*d)*(c + d*x^2)^(3/2))/(117*c^2*x^(9/2)) - (4*d^(5/4)*(39*b^2*c^2
 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)
/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^
(1/4)], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2]) + (2*d^(5/4)*(39*b^2
*c^2 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*
```

```
x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x]
)/c^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 74.4561, size = 415, normalized size = 0.94

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{3}{2}}}{13cx^{\frac{13}{2}}}+\frac{2a\left(c+dx^2\right)^{\frac{3}{2}}\left(7ad-26bc\right)}{117c^2x^{\frac{9}{2}}}-\frac{2\sqrt{c+dx^2}\left(ad\left(7ad-26bc\right)+39b^2c^2\right)}{195c^2x^{\frac{5}{2}}}+\frac{4d^{\frac{3}{2}}\sqrt{x}\sqrt{c+dx^2}\left(ad\left(7ad-26bc\right)+39b^2c^2\right)}{195c^3\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{4d\sqrt{c+dx^2}\left(ad\left(7ad-26bc\right)+39b^2c^2\right)}{195c^3\sqrt{x}}-\frac{4d^{\frac{5}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\left(7ad-26bc\right)+39b^2c^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)\middle|\frac{1}{2}\right)}{195c^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{2d^{\frac{5}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\left(7ad-26bc\right)+39b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)\middle|\frac{1}{2}\right)}{195c^{\frac{11}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(15/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(3/2)/(13*c*x**(13/2)) + 2*a*(c + d*x**2)**
(3/2)*(7*a*d - 26*b*c)/(117*c**2*x**(9/2)) - 2*sqrt(c + d*x**2)*(
a*d*(7*a*d - 26*b*c) + 39*b**2*c**2)/(195*c**2*x**(5/2)) + 4*d**(
3/2)*sqrt(x)*sqrt(c + d*x**2)*(a*d*(7*a*d - 26*b*c) + 39*b**2*c**
2)/(195*c**3*(sqrt(c) + sqrt(d)*x)) - 4*d*sqrt(c + d*x**2)*(a*d*(
7*a*d - 26*b*c) + 39*b**2*c**2)/(195*c**3*sqrt(x)) - 4*d**(5/4)*s
qrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*
(a*d*(7*a*d - 26*b*c) + 39*b**2*c**2)*elliptic_e(2*atan(d**(1/4)*
sqrt(x)/c**(1/4)), 1/2)/(195*c**(11/4)*sqrt(c + d*x**2)) + 2*d**(
5/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(
d)*x)*(a*d*(7*a*d - 26*b*c) + 39*b**2*c**2)*elliptic_f(2*atan(d**
(1/4)*sqrt(x)/c**(1/4)), 1/2)/(195*c**(11/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.701818, size = 241, normalized size = 0.55

$$\frac{2\left(\left(-c-dx^2\right)\left(a^2\left(45c^3+10c^2dx^2-14cd^2x^4+42d^3x^6\right)+26abcx^2\left(5c^2+2cdx^2-6d^2x^4\right)+117b^2c^2x^4\left(c+2dx^2\right)\right)+\frac{6id^2x^8}{\cdots}\right.}{585c^3x^{13/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

```
[Out] (2*((-c - d*x^2)*(117*b^2*c^2*x^4*(c + 2*d*x^2) + 26*a*b*c*x^2*(5
*c^2 + 2*c*d*x^2 - 6*d^2*x^4) + a^2*(45*c^3 + 10*c^2*d*x^2 - 14*c
*d^2*x^4 + 42*d^3*x^6)) + ((6*I)*d^2*(39*b^2*c^2 - 26*a*b*c*d + 7
*a^2*d^2)*x^8*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[(I*Sq
rt[d]*x)/Sqrt[c]]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)/
Sqrt[c]]], -1]))/((I*Sqrt[d]*x)/Sqrt[c])^(3/2)))/(585*c^3*x^(13/2
)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.054, size = 706, normalized size = 1.6

$$\frac{2}{585\,c^3}\left(42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^6a^2cd^3-156\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x)
```

```
[Out] 2/585*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6*a^2
*c*d^3-156*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6*a*
b*c^2*d^2+234*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6
*b^2*c^3*d-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6
*a^2*c*d^3+78*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^6
*a*b*c^2*d^2-117*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*
x^6*b^2*c^3*d-42*x^8*a^2*d^4+156*x^8*a*b*c*d^3-234*x^8*b^2*c^2*d^
2-28*x^6*a^2*c*d^3+104*x^6*a*b*c^2*d^2-351*x^6*b^2*c^3*d+4*x^4*a^
2*c^2*d^2-182*x^4*a*b*c^3*d-117*x^4*b^2*c^4-55*x^2*a^2*c^3*d-130*
x^2*a*b*c^4-45*a^2*c^4)/(d*x^2+c)^(1/2)/x^(13/2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{dx^2+c}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{dx^2+c}}{x^{\frac{15}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x^2 + c)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

3.832 $\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=530

$$-\frac{4c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(51a^2d^2+bc(11bc-42ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}+\frac{8c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(51a^2d^2+bc(11bc-42ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}-\frac{8c^3e^2\sqrt{ex}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{3315d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{8c^2e(ex)^{3/2}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{9945d^3}+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}\left(51a^2d^2+bc(11bc-42ad)\right)}{663d^2e}+\frac{4c(ex)^{7/2}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{1989d^2e}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5/2}(11bc-42ad)}{357d^2e}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

```
[Out] (8*c^2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e*(e*x)^(3/2)*Sqrt[c
+ d*x^2])/(9945*d^3) + (4*c*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*
(e*x)^(7/2)*Sqrt[c + d*x^2])/(1989*d^2*e) - (8*c^3*(51*a^2*d^2 +
b*c*(11*b*c - 42*a*d))*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(3315*d^(7/
2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d
))*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(663*d^2*e) - (2*b*(11*b*c - 42
*a*d)*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(357*d^2*e) + (2*b^2*(e*x)^(
11/2)*(c + d*x^2)^(5/2))/(21*d*e^3) + (8*c^(13/4)*(51*a^2*d^2 + b
*c*(11*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x
^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(15/4)*Sqrt[c + d*x^2]) - (4
*c^(13/4)*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(15
/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.28858, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{4c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(51a^2d^2+bc(11bc-42ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}+\frac{8c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(51a^2d^2+bc(11bc-42ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}-\frac{8c^3e^2\sqrt{ex}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{3315d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{8c^2e(ex)^{3/2}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{9945d^3}+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}\left(51a^2d^2+bc(11bc-42ad)\right)}{663d^2e}+\frac{4c(ex)^{7/2}\sqrt{c+dx^2}\left(51a^2d^2+bc(11bc-42ad)\right)}{1989d^2e}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5/2}(11bc-42ad)}{357d^2e}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (8*c^2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e*(e*x)^(3/2)*Sqrt[c
+ d*x^2])/(9945*d^3) + (4*c*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*
(e*x)^(7/2)*Sqrt[c + d*x^2])/(1989*d^2*e) - (8*c^3*(51*a^2*d^2 +
b*c*(11*b*c - 42*a*d))*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(3315*d^(7/
2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d
))*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(663*d^2*e) - (2*b*(11*b*c - 42
*a*d)*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(357*d^2*e) + (2*b^2*(e*x)^(
11/2)*(c + d*x^2)^(5/2))/(21*d*e^3) + (8*c^(13/4)*(51*a^2*d^2 + b
*c*(11*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x
^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(15/4)*Sqrt[c + d*x^2]) - (4
*c^(13/4)*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(15
/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 119.482, size = 510, normalized size = 0.96

$$\frac{2b^2(ex)^{\frac{11}{2}}\left(c+dx^2\right)^{\frac{5}{2}}}{21de^3}+\frac{2b(ex)^{\frac{7}{2}}\left(c+dx^2\right)^{\frac{5}{2}}(42ad-11bc)}{357d^2e}+\frac{8c^{\frac{13}{4}}e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(51a^2d^2-bc(42ad-11bc)\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3315d^{\frac{15}{4}}\sqrt{c+dx^2}}-\frac{4c^{\frac{13}{4}}e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(51a^2d^2-bc(42ad-11bc)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3315d^{\frac{15}{4}}\sqrt{c+dx^2}}-\frac{8c^3e^2\sqrt{ex}\sqrt{c+dx^2}\left(51a^2d^2-bc(42ad-11bc)\right)}{3315d^{\frac{7}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{8c^2e(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\left(51a^2d^2-bc(42ad-11bc)\right)}{9945d^3}+\frac{4c(ex)^{\frac{7}{2}}\sqrt{c+dx^2}\left(51a^2d^2-bc(42ad-11bc)\right)}{1989d^2e}+\frac{2(ex)^{\frac{7}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\left(51a^2d^2-bc(42ad-11bc)\right)}{663d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] 2*b**2*(e*x)**(11/2)*(c + d*x**2)**(5/2)/(21*d*e**3) + 2*b*(e*x)*
*(7/2)*(c + d*x**2)**(5/2)*(42*a*d - 11*b*c)/(357*d**2*e) + 8*c**
(13/4)*e**(5/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt
(c) + sqrt(d)*x)*(51*a**2*d**2 - b*c*(42*a*d - 11*b*c))*elliptic_
e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(3315*d**(1
5/4)*sqrt(c + d*x**2)) - 4*c**(13/4)*e**(5/2)*sqrt((c + d*x**2)/(
sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(51*a**2*d**2 - b*
c*(42*a*d - 11*b*c))*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/
4)*sqrt(e))), 1/2)/(3315*d**(15/4)*sqrt(c + d*x**2)) - 8*c**3*e**
2*sqrt(e*x)*sqrt(c + d*x**2)*(51*a**2*d**2 - b*c*(42*a*d - 11*b*c
))/(3315*d**(7/2)*(sqrt(c) + sqrt(d)*x)) + 8*c**2*e*(e*x)**(3/2)*
sqrt(c + d*x**2)*(51*a**2*d**2 - b*c*(42*a*d - 11*b*c))/(9945*d**
3) + 4*c*(e*x)**(7/2)*sqrt(c + d*x**2)*(51*a**2*d**2 - b*c*(42*a*
d - 11*b*c))/(1989*d**2*e) + 2*(e*x)**(7/2)*(c + d*x**2)**(3/2)*(
51*a**2*d**2 - b*c*(42*a*d - 11*b*c))/(663*d**2*e)
```

Mathematica [C] time = 2.04367, size = 304, normalized size = 0.57

$$2(ex)^{5/2}\left(d\sqrt{x}\left(c+dx^2\right)\left(357a^2d^2\left(4c^2+25cdx^2+15d^2x^4\right)+42abd\left(-28c^3+20c^2dx^2+285cd^2x^4+195d^3x^6\right)+b^2\left(308c^4-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (2*(e*x)^(5/2)*(d*Sqrt[x]*(c + d*x^2)*(357*a^2*d^2*(4*c^2 + 25*c*
d*x^2 + 15*d^2*x^4) + 42*a*b*d*(-28*c^3 + 20*c^2*d*x^2 + 285*c*d^
2*x^4 + 195*d^3*x^6) + b^2*(308*c^4 - 220*c^3*d*x^2 + 180*c^2*d^2
*x^4 + 4485*c*d^3*x^6 + 3315*d^4*x^8)) + 84*c^3*(11*b^2*c^2 - 42*
a*b*c*d + 51*a^2*d^2)*(-((d + c/x^2)*Sqrt[x]) + (I*c*Sqrt[1 + c/(
d*x^2)]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
-1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]
))/((I*Sqrt[c])/Sqrt[d])^(3/2))))/(69615*d^4*x^(3/2)*Sqrt[c + d*x
^2])
```

Maple [A] time = 0.054, size = 743, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -2/69615/x*e^2*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^4*(-3315*x^12*b^2*d^
6-8190*x^10*a*b*d^6-7800*x^10*b^2*c*d^5-5355*x^8*a^2*d^6-20160*x^
8*a*b*c*d^5-4665*x^8*b^2*c^2*d^4-14280*x^6*a^2*c*d^5-12810*x^6*a*
b*c^2*d^4+40*x^6*b^2*c^3*d^3+4284*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*a^2*c^4*d^2-3528*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*a*b*c^5*d+924*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*b^2*c^6-2142*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*a^2*c^4*d^2+1764*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*a*b*c^5*d-462*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*b^2*c^6-10353*x^4*a^2*c^2*d^4+336*x^4*a*b*c^3*d^3-88*x^4*b
^2*c^4*d^2-1428*x^2*a^2*c^3*d^3+1176*x^2*a*b*c^4*d^2-308*x^2*b^2*
c^5*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2de^2x^8 + \left(b^2c + 2\,abd\right)e^2x^6 + a^2ce^2x^2 + \left(2\,abc + a^2d\right)e^2x^4\right)\sqrt{dx^2 + c}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*e^2*x^8 + (b^2*c + 2*a*b*d)*e^2*x^6 + a^2*c*e^2*x
^2 + (2*a*b*c + a^2*d)*e^2*x^4)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.450555, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] Done
```

3.833 $\int (ex)^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=340

$$-\frac{4c^{11/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(57a^2d^2+bc(9bc-38ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4389d^{13/4}\sqrt{c+dx^2}}+\frac{8c^2e\sqrt{ex}\sqrt{c+dx^2}\left(57a^2d^2+bc(9bc-38ad)\right)}{4389d^3}+\frac{2(ex)^{5/2}\left(c+dx^2\right)^{3/2}\left(57a^2d^2+bc(9bc-38ad)\right)}{627d^2e}+\frac{4c(ex)^{5/2}\sqrt{c+dx^2}\left(57a^2d^2+bc(9bc-38ad)\right)}{1463d^2e}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}(9bc-38ad)}{285d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

```
[Out] (8*c^2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e*Sqrt[e*x]*Sqrt[c + d
*x^2])/(4389*d^3) + (4*c*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x
)^(5/2)*Sqrt[c + d*x^2])/(1463*d^2*e) + (2*(57*a^2*d^2 + b*c*(9*b
*c - 38*a*d))*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(627*d^2*e) - (2*b*(
9*b*c - 38*a*d)*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(285*d^2*e) + (2*b
^2*(e*x)^(9/2)*(c + d*x^2)^(5/2))/(19*d*e^3) - (4*c^(11/4)*(57*a^
2*d^2 + b*c*(9*b*c - 38*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[
(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4389*d^(13/4)*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.779461, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{4c^{11/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(57a^2d^2+bc(9bc-38ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4389d^{13/4}\sqrt{c+dx^2}}+\frac{8c^2e\sqrt{ex}\sqrt{c+dx^2}\left(57a^2d^2+bc(9bc-38ad)\right)}{4389d^3}+\frac{2(ex)^{5/2}\left(c+dx^2\right)^{3/2}\left(57a^2d^2+bc(9bc-38ad)\right)}{627d^2e}+\frac{4c(ex)^{5/2}\sqrt{c+dx^2}\left(57a^2d^2+bc(9bc-38ad)\right)}{1463d^2e}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}(9bc-38ad)}{285d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (8*c^2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e*Sqrt[e*x]*Sqrt[c + d
*x^2])/(4389*d^3) + (4*c*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x
)^(5/2)*Sqrt[c + d*x^2])/(1463*d^2*e) + (2*(57*a^2*d^2 + b*c*(9*b
*c - 38*a*d))*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(627*d^2*e) - (2*b*(
9*b*c - 38*a*d)*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(285*d^2*e) + (2*b
^2*(e*x)^(9/2)*(c + d*x^2)^(5/2))/(19*d*e^3) - (4*c^(11/4)*(57*a^
2*d^2 + b*c*(9*b*c - 38*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[
(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4389*d^(13/4)*Sqrt[c + d*x^
2])
```

Rubi in Sympy [A] time = 67.0829, size = 328, normalized size = 0.96

$$\frac{2b^2(ex)^{\frac{9}{2}}\left(c+dx^2\right)^{\frac{5}{2}}}{19de^3}+\frac{2b(ex)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{5}{2}}(38ad-9bc)}{285d^2e}-\frac{4c^{\frac{11}{4}}e^{\frac{3}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(57a^2d^2-bc(38ad-9bc)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{4389d^{\frac{13}{4}}\sqrt{c+dx^2}}+\frac{8c^2e\sqrt{ex}\sqrt{c+dx^2}\left(57a^2d^2-bc(38ad-9bc)\right)}{4389d^3}+\frac{4c(ex)^{\frac{5}{2}}\sqrt{c+dx^2}\left(57a^2d^2-bc(38ad-9bc)\right)}{1463d^2e}+\frac{2(ex)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\left(57a^2d^2-bc(38ad-9bc)\right)}{627d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] 2*b**2*(e*x)**(9/2)*(c + d*x**2)**(5/2)/(19*d*e**3) + 2*b*(e*x)**
(5/2)*(c + d*x**2)**(5/2)*(38*a*d - 9*b*c)/(285*d**2*e) - 4*c**(1
1/4)*e**(3/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c
) + sqrt(d)*x)*(57*a**2*d**2 - b*c*(38*a*d - 9*b*c))*elliptic_f(2
*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(4389*d**(13/4
)*sqrt(c + d*x**2)) + 8*c**2*e*sqrt(e*x)*sqrt(c + d*x**2)*(57*a**
2*d**2 - b*c*(38*a*d - 9*b*c))/(4389*d**3) + 4*c*(e*x)**(5/2)*sqr
t(c + d*x**2)*(57*a**2*d**2 - b*c*(38*a*d - 9*b*c))/(1463*d**2*e)
 + 2*(e*x)**(5/2)*(c + d*x**2)**(3/2)*(57*a**2*d**2 - b*c*(38*a*d
 - 9*b*c))/(627*d**2*e)
```

Mathematica [C] time = 0.442885, size = 259, normalized size = 0.76

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)\left(285a^2d^2\left(4c^2+13cdx^2+7d^2x^4\right)+38abd\left(-20c^3+12c^2dx^2+119cd^2x^4+77d^3x^6\right)+3b^2\left(60c^4-36c^3dx^2+28c^2d^2x^4+539cd^3x^6+385d^4x^8\right)\right)}{5d^3}-\right.}{4389x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(285*a^2*d^2*(4*c^2 + 13*c*d
*x^2 + 7*d^2*x^4) + 38*a*b*d*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*
x^4 + 77*d^3*x^6) + 3*b^2*(60*c^4 - 36*c^3*d*x^2 + 28*c^2*d^2*x^4
 + 539*c*d^3*x^6 + 385*d^4*x^8)))/(5*d^3) - ((8*I)*c^3*(9*b^2*c^2
 - 38*a*b*c*d + 57*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/S
qrt[d]]*d^3)))/(4389*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.041, size = 489, normalized size = 1.4

$$-\frac{2e}{21945\,xd^4}\sqrt{ex}\left(-1155\,x^{11}b^2d^6-2926\,x^9abd^6-2772\,x^9b^2cd^5-1995\,x^7a^2d^6-7448\,x^7abcd^5-1701\,x^7b^2c^2d^4+570\sqrt{\frac{dx}{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -2/21945*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1155*x^11*b^2*d^6-2926
*x^9*a*b*d^6-2772*x^9*b^2*c*d^5-1995*x^7*a^2*d^6-7448*x^7*a*b*c*d
```

```
^5-1701*x^7*b^2*c^2*d^4+570*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*a^2*c^3*d^2-380*((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^4*d+90*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^5-5700*x^5*a^2
*c*d^5-4978*x^5*a*b*c^2*d^4+24*x^5*b^2*c^3*d^3-4845*x^3*a^2*c^2*d
^4+304*x^3*a*b*c^3*d^3-72*x^3*b^2*c^4*d^2-1140*x*a^2*c^3*d^3+760*
x*a*b*c^4*d^2-180*x*b^2*c^5*d)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dex^7+\left(b^2c+2\,abd\right)ex^5+a^2cex+\left(2\,abc+a^2d\right)ex^3\right)\sqrt{dx^2+c}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*e*x^7 + (b^2*c + 2*a*b*d)*e*x^5 + a^2*c*e*x + (2*
a*b*c + a^2*d)*e*x^3)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.339373, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] Done
```

3.834 $\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=482

$$\frac{4c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(221a^2d^2+3bc(7bc-34ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}} - \frac{8c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(221a^2d^2+3bc(7bc-34ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}} + \frac{8c^2\sqrt{ex}\sqrt{c+dx^2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{3315d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{1989d^2e} + \frac{4c(ex)^{3/2}\sqrt{c+dx^2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{3315d^2e} - \frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{221d^2e} + \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

```
[Out] (4*c*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*Sqrt[c +
d*x^2])/(3315*d^2*e) + (8*c^2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*
d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(3315*d^(5/2)*(Sqrt[c] + Sqrt[d]*x
)) + (2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*(c + d
*x^2)^(3/2))/(1989*d^2*e) - (2*b*(7*b*c - 34*a*d)*(e*x)^(3/2)*(c
+ d*x^2)^(5/2))/(221*d^2*e) + (2*b^2*(e*x)^(7/2)*(c + d*x^2)^(5/2
))/(17*d*e^3) - (8*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))
*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d
]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, 1/2])/(3315*d^(11/4)*Sqrt[c + d*x^2]) + (4*c^(9/4)*(221*a^2*d^2
 + 3*b*c*(7*b*c - 34*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
+ d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.11401, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{4c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(221a^2d^2+3bc(7bc-34ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}} - \frac{8c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(221a^2d^2+3bc(7bc-34ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}} + \frac{8c^2\sqrt{ex}\sqrt{c+dx^2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{3315d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{1989d^2e} + \frac{4c(ex)^{3/2}\sqrt{c+dx^2}\left(221a^2d^2+3bc(7bc-34ad)\right)}{3315d^2e} - \frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{221d^2e} + \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (4*c*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*Sqrt[c +
d*x^2])/(3315*d^2*e) + (8*c^2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*
```

```
d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(3315*d^(5/2)*(Sqrt[c] + Sqrt[d]*x
)) + (2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*(c + d
*x^2)^(3/2))/(1989*d^2*e) - (2*b*(7*b*c - 34*a*d)*(e*x)^(3/2)*(c
+ d*x^2)^(5/2))/(221*d^2*e) + (2*b^2*(e*x)^(7/2)*(c + d*x^2)^(5/2
))/(17*d*e^3) - (8*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))
*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d
]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, 1/2])/(3315*d^(11/4)*Sqrt[c + d*x^2]) + (4*c^(9/4)*(221*a^2*d^2
 + 3*b*c*(7*b*c - 34*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
+ d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 102.842, size = 464, normalized size = 0.96

$$\frac{2b^2(ex)^{\frac{7}{2}}\left(c+dx^2\right)^{\frac{5}{2}}}{17de^3}+\frac{2b(ex)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{5}{2}}(34ad-7bc)}{221d^2e}-\frac{8c^{\frac{9}{4}}\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(221a^2d^2-3bc(34ad-7bc)\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3315d^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{4c^{\frac{9}{4}}\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(221a^2d^2-3bc(34ad-7bc)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3315d^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{8c^2\sqrt{ex}\sqrt{c+dx^2}\left(221a^2d^2-3bc(34ad-7bc)\right)}{3315d^{\frac{5}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{4c(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\left(221a^2d^2-3bc(34ad-7bc)\right)}{3315d^2e}+\frac{2(ex)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\left(221a^2d^2-3bc(34ad-7bc)\right)}{1989d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)*(e*x)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(7/2)*(c + d*x**2)**(5/2)/(17*d*e**3) + 2*b*(e*x)**
(3/2)*(c + d*x**2)**(5/2)*(34*a*d - 7*b*c)/(221*d**2*e) - 8*c**(9
/4)*sqrt(e)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c)
+ sqrt(d)*x)*(221*a**2*d**2 - 3*b*c*(34*a*d - 7*b*c))*elliptic_e(
2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(3315*d**(11/
4)*sqrt(c + d*x**2)) + 4*c**(9/4)*sqrt(e)*sqrt((c + d*x**2)/(sqrt
(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(221*a**2*d**2 - 3*b*c
*(34*a*d - 7*b*c))*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)
*sqrt(e))), 1/2)/(3315*d**(11/4)*sqrt(c + d*x**2)) + 8*c**2*sqrt(
e*x)*sqrt(c + d*x**2)*(221*a**2*d**2 - 3*b*c*(34*a*d - 7*b*c))/(3
315*d**(5/2)*(sqrt(c) + sqrt(d)*x)) + 4*c*(e*x)**(3/2)*sqrt(c + d
*x**2)*(221*a**2*d**2 - 3*b*c*(34*a*d - 7*b*c))/(3315*d**2*e) + 2
*(e*x)**(3/2)*(c + d*x**2)**(3/2)*(221*a**2*d**2 - 3*b*c*(34*a*d
- 7*b*c))/(1989*d**2*e)
```

Mathematica [C] time = 1.46705, size = 316, normalized size = 0.66

$$\frac{2e\left(dx^2\left(c+dx^2\right)\left(221a^2d^2\left(11c+5dx^2\right)+102abd\left(4c^2+25cdx^2+15d^2x^4\right)+b^2\left(-84c^3+60c^2dx^2+855cd^2x^4+585d^3x^6\right)\right)\right.}{9945d^3\sqrt{ex}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (2*e*(d*x^2*(c + d*x^2)*(221*a^2*d^2*(11*c + 5*d*x^2) + 102*a*b*d
*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4) + b^2*(-84*c^3 + 60*c^2*d*x^2
+ 855*c*d^2*x^4 + 585*d^3*x^6)) + (12*c^2*(21*b^2*c^2 - 102*a*b*c
*d + 221*a^2*d^2)*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(c + d*x^2) - Sqrt[c
]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] + Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*
x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[
x]], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(9945*d^3*Sqrt[e*x]*Sqrt[c
 + d*x^2])
```

Maple [A] time = 0.025, size = 699, normalized size = 1.5

$$\frac{2}{9945\,d^3x}\sqrt{ex}\left(585\,x^{10}b^2d^5+1530\,x^8abd^5+1440\,x^8b^2cd^4+1105\,x^6a^2d^5+4080\,x^6abcd^4+915\,x^6b^2c^2d^3+2652\sqrt{\frac{dx+\sqrt{-c}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x)
```

```
[Out] 2/9945/(d*x^2+c)^(1/2)*(e*x)^(1/2)/d^3*(585*x^10*b^2*d^5+1530*x^8
*a*b*d^5+1440*x^8*b^2*c*d^4+1105*x^6*a^2*d^5+4080*x^6*a*b*c*d^4+9
15*x^6*b^2*c^2*d^3+2652*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a^2*c^3*d^2-1224*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b*c^4*d+252*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^
(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*b^2*c^5-1326*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a^2*c^3*d^2+612*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
*b*c^4*d-126*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*El
lipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*
c^5+3536*x^4*a^2*c*d^4+2958*x^4*a*b*c^2*d^3-24*x^4*b^2*c^3*d^2+24
31*x^2*a^2*c^2*d^3+408*x^2*a*b*c^3*d^2-84*x^2*b^2*c^4*d)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c +
a^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

Sympy [A] time = 138.9, size = 304, normalized size = 0.63

$$\frac{a^{2}c^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2e\,\Gamma\left(\frac{7}{4}\right)}+\frac{a^{2}\sqrt{c}d(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2e^{3}\,\Gamma\left(\frac{11}{4}\right)}+\frac{abc^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{e^{3}\,\Gamma\left(\frac{11}{4}\right)}+\frac{ab\sqrt{c}d(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{e^{5}\,\Gamma\left(\frac{15}{4}\right)}+\frac{b^{2}c^{\frac{3}{2}}(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2e^{5}\,\Gamma\left(\frac{15}{4}\right)}+\frac{b^{2}\sqrt{c}d(ex)^{\frac{15}{2}}\,\Gamma\left(\frac{15}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{15}{4}\\\frac{19}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2e^{7}\,\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)*(e*x)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
d*x**2*exp_polar(I*pi)/c)/(2*e*gamma(7/4)) + a**2*sqrt(c)*d*(e*x)
**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I
*pi)/c)/(2*e**3*gamma(11/4)) + a*b*c**(3/2)*(e*x)**(7/2)*gamma(7/
4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(e**3*ga
mma(11/4)) + a*b*sqrt(c)*d*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2,
 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(e**5*gamma(15/4)) + b
**2*c**(3/2)*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,
), d*x**2*exp_polar(I*pi)/c)/(2*e**5*gamma(15/4)) + b**2*sqrt(c)*
d*(e*x)**(15/2)*gamma(15/4)*hyper((-1/2, 15/4), (19/4,), d*x**2*e
xp_polar(I*pi)/c)/(2*e**7*gamma(19/4))
```

GIAC/XCAS [A] time = 0.432528, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x),x, algorithm="giac")
```

```
[Out] Done
```

3.835 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}} \, dx$

Optimal. Leaf size=286

$$\frac{4c^{7/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(33a^2d^2+bc(bc-6ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}} + \frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}\left(33a^2d^2+bc(bc-6ad)\right)}{231d^2e} + \frac{4c\sqrt{ex}\sqrt{c+dx^2}\left(33a^2d^2+bc(bc-6ad)\right)}{231d^2e} - \frac{2b\sqrt{ex}\left(c+dx^2\right)^{5/2}(bc-6ad)}{33d^2e} + \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

```
[Out] (4*c*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/
(231*d^2*e) + (2*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*(c +
d*x^2)^(3/2))/(231*d^2*e) - (2*b*(b*c - 6*a*d)*Sqrt[e*x]*(c + d*x
^2)^(5/2))/(33*d^2*e) + (2*b^2*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15
*d*e^3) + (4*c^(7/4)*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*(Sqrt[c] +
Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(9/4)
*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.658576, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{4c^{7/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(33a^2d^2+bc(bc-6ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}} + \frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}\left(33a^2d^2+bc(bc-6ad)\right)}{231d^2e} + \frac{4c\sqrt{ex}\sqrt{c+dx^2}\left(33a^2d^2+bc(bc-6ad)\right)}{231d^2e} - \frac{2b\sqrt{ex}\left(c+dx^2\right)^{5/2}(bc-6ad)}{33d^2e} + \frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

```
[Out] (4*c*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/
(231*d^2*e) + (2*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*(c +
d*x^2)^(3/2))/(231*d^2*e) - (2*b*(b*c - 6*a*d)*Sqrt[e*x]*(c + d*x
^2)^(5/2))/(33*d^2*e) + (2*b^2*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15
*d*e^3) + (4*c^(7/4)*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*(Sqrt[c] +
Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(9/4)
*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 56.9428, size = 270, normalized size = 0.94

$$\frac{2b^2(ex)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{5}{2}}}{15de^3} + \frac{2b\sqrt{ex}\left(c+dx^2\right)^{\frac{5}{2}}(6ad-bc)}{33d^2e} + \frac{4c^{\frac{7}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(33a^2d^2-bc\left(6ad-bc\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231d^{\frac{9}{4}}\sqrt{e}\sqrt{c+dx^2}} + \frac{4c\sqrt{ex}\sqrt{c+dx^2}\left(33a^2d^2-bc\left(6ad-bc\right)\right)}{231d^2e} + \frac{2\sqrt{ex}\left(c+dx^2\right)^{\frac{3}{2}}\left(33a^2d^2-bc\left(6ad-bc\right)\right)}{231d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(5/2)*(c + d*x**2)**(5/2)/(15*d*e**3) + 2*b*sqrt(e*
x)*(c + d*x**2)**(5/2)*(6*a*d - b*c)/(33*d**2*e) + 4*c**(7/4)*sqr
t((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(3
3*a**2*d**2 - b*c*(6*a*d - b*c))*elliptic_f(2*atan(d**(1/4)*sqrt(
e*x)/(c**(1/4)*sqrt(e))), 1/2)/(231*d**(9/4)*sqrt(e)*sqrt(c + d*x
**2)) + 4*c*sqrt(e*x)*sqrt(c + d*x**2)*(33*a**2*d**2 - b*c*(6*a*d
 - b*c))/(231*d**2*e) + 2*sqrt(e*x)*(c + d*x**2)**(3/2)*(33*a**2*
d**2 - b*c*(6*a*d - b*c))/(231*d**2*e)
```

Mathematica [C] time = 0.378363, size = 223, normalized size = 0.78

$$\frac{\sqrt{x}\left(\frac{2\sqrt{x}(c+dx^2)\left(165a^2d^2(3c+dx^2)+30abd(4c^2+13cdx^2+7d^2x^4)+b^2(-20c^3+12c^2dx^2+119cd^2x^4+77d^3x^6)\right)}{5d^2}+\frac{8ic^2x\sqrt{\frac{c}{dx^2}+1}(33a^2d^2-6abcd+b^2c^2)F\left(i \sin\right.}{d^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{231\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

```
[Out] (Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(165*a^2*d^2*(3*c + d*x^2) + 30*
a*b*d*(4*c^2 + 13*c*d*x^2 + 7*d^2*x^4) + b^2*(-20*c^3 + 12*c^2*d*
x^2 + 119*c*d^2*x^4 + 77*d^3*x^6)))/(5*d^2) + ((8*I)*c^2*(b^2*c^2
 - 6*a*b*c*d + 33*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcS
inh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sq
rt[d]]*d^2)))/(231*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 444, normalized size = 1.6

$$\frac{2}{1155\,d^3}\left(77\,x^9b^2d^5+210\,x^7abd^5+196\,x^7b^2cd^4+330\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 2/1155/(d*x^2+c)^(1/2)*(77*x^9*b^2*d^5+210*x^7*a*b*d^5+196*x^7*b^
2*c*d^4+330*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d
)^(1/2)*a^2*c^2*d^2-60*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*(-c*d)^(1/2)*a*b*c^3*d+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^4+165*x^5*a^2*d^5+600*x^5*a*b*c
*d^4+131*x^5*b^2*c^2*d^3+660*x^3*a^2*c*d^4+510*x^3*a*b*c^2*d^3-8*
x^3*b^2*c^3*d^2+495*x*a^2*c^2*d^3+120*x*a*b*c^3*d^2-20*x*b^2*c^4*
d)/d^3/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2dx^6 + \left(b^2c + 2\,abd\right)x^4 + a^2c + \left(2\,abc + a^2d\right)x^2\right)\sqrt{dx^2 + c}}{\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c +
a^2*d)*x^2)*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

Sympy [A] time = 150.981, size = 306, normalized size = 1.07

$$\frac{a^2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)} + \frac{a^2\sqrt{c}dx^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)} + \frac{abc^{\frac{3}{2}}x^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)} + \frac{ab\sqrt{c}dx^{\frac{9}{2}}\,\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)} + \frac{b^2c^{\frac{3}{2}}x^{\frac{9}{2}}\,\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)} + \frac{b^2\sqrt{c}dx^{\frac{13}{2}}\,\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**
2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a**2*sqrt(c)*d*x**(
5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)
/c)/(2*sqrt(e)*gamma(9/4)) + a*b*c**(3/2)*x**(5/2)*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(
9/4)) + a*b*sqrt(c)*d*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/
4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(13/4)) + b**2*c**(3
/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_po
lar(I*pi)/c)/(2*sqrt(e)*gamma(13/4)) + b**2*sqrt(c)*d*x**(13/2)*g
amma(13/4)*hyper((-1/2, 13/4), (17/4,), d*x**2*exp_polar(I*pi)/c)
/(2*sqrt(e)*gamma(17/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2\left(dx^2 + c\right)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x), x)
```

3.836 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=476

$$-\frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}}-\frac{8c\sqrt{ex}\sqrt{c+dx^2}\,(3b^2c^2-13ad(9ad+2bc))}{195d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2(ex)^{3/2}(c+dx^2)^{3/2}(3b^2c^2-13ad(9ad+2bc))}{117cde^3}-\frac{4(ex)^{3/2}\sqrt{c+dx^2}\,(3b^2c^2-13ad(9ad+2bc))}{195de^3}-\frac{4c^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-13ad(9ad+2bc))\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{8c^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-13ad(9ad+2bc))\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}(c+dx^2)^{5/2}}{13de^3}$$

```
[Out] (-4*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*Sqrt[c + d*x
^2])/(195*d*e^3) - (8*c*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*Sqrt
[e*x]*Sqrt[c + d*x^2])/(195*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) -
(2*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*(c + d*x^2)^(
3/2))/(117*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(c*e*Sqrt[e*x]) +
 (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(13*d*e^3) + (8*c^(5/4)*(3
*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
+ d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d
*x^2]) - (4*c^(5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c]
 + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF
[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7
/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.08154, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}}-\frac{8c\sqrt{ex}\sqrt{c+dx^2}\,(3b^2c^2-13ad(9ad+2bc))}{195d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2(ex)^{3/2}(c+dx^2)^{3/2}(3b^2c^2-13ad(9ad+2bc))}{117cde^3}-\frac{4(ex)^{3/2}\sqrt{c+dx^2}\,(3b^2c^2-13ad(9ad+2bc))}{195de^3}-\frac{4c^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-13ad(9ad+2bc))\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{8c^{5/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-13ad(9ad+2bc))\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}(c+dx^2)^{5/2}}{13de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

```
[Out] (-4*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*Sqrt[c + d*x
^2])/(195*d*e^3) - (8*c*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*Sqrt
[e*x]*Sqrt[c + d*x^2])/(195*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) -
(2*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*(c + d*x^2)^(
3/2))/(117*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(c*e*Sqrt[e*x]) +
 (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(13*d*e^3) + (8*c^(5/4)*(3
*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
```

```
+ d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d
*x^2]) - (4*c^(5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c]
 + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF
[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7
/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 105.974, size = 454, normalized size = 0.95

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{5}{2}}}{ce\sqrt{ex}}+\frac{2b^2\left(ex\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{5}{2}}}{13de^3}+\frac{8c^{\frac{5}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-13ad\left(9ad+2bc\right)+3b^2c^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}-\frac{4c^{\frac{5}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-13ad\left(9ad+2bc\right)+3b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}-\frac{8c\sqrt{ex}\sqrt{c+dx^2}\left(-13ad\left(9ad+2bc\right)+3b^2c^2\right)}{195d^{\frac{3}{2}}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{4\left(ex\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(-13ad\left(9ad+2bc\right)+3b^2c^2\right)}{195de^3}-\frac{2\left(ex\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\left(-13ad\left(9ad+2bc\right)+3b^2c^2\right)}{117cde^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(3/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(5/2)/(c*e*sqrt(e*x)) + 2*b**2*(e*x)**(3/2)
*(c + d*x**2)**(5/2)/(13*d*e**3) + 8*c**(5/4)*sqrt((c + d*x**2)/(
sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-13*a*d*(9*a*d +
2*b*c) + 3*b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1
/4)*sqrt(e))), 1/2)/(195*d**(7/4)*e**(3/2)*sqrt(c + d*x**2)) - 4*
c**(5/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + s
qrt(d)*x)*(-13*a*d*(9*a*d + 2*b*c) + 3*b**2*c**2)*elliptic_f(2*at
an(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(195*d**(7/4)*e**
(3/2)*sqrt(c + d*x**2)) - 8*c*sqrt(e*x)*sqrt(c + d*x**2)*(-13*a*d
*(9*a*d + 2*b*c) + 3*b**2*c**2)/(195*d**(3/2)*e**2*(sqrt(c) + sqr
t(d)*x)) - 4*(e*x)**(3/2)*sqrt(c + d*x**2)*(-13*a*d*(9*a*d + 2*b*
c) + 3*b**2*c**2)/(195*d*e**3) - 2*(e*x)**(3/2)*(c + d*x**2)**(3/
2)*(-13*a*d*(9*a*d + 2*b*c) + 3*b**2*c**2)/(117*c*d*e**3)
```

Mathematica [C] time = 1.65051, size = 261, normalized size = 0.55

$$\frac{x^{3/2}\left(\frac{2\sqrt{c+dx^2}\left(117a^2d\left(dx^2-5c\right)+26abdx^2\left(11c+5dx^2\right)+3b^2x^2\left(4c^2+25cdx^2+15d^2x^4\right)\right)}{3d\sqrt{x}}-\frac{8cx\left(117a^2d^2+26abcd-3b^2c^2\right)\left(-\sqrt{x}\left(\frac{c}{x^2}+d\right)+\frac{ic\sqrt{\frac{c}{dx^2}+1}\left(E\left(i\sinh^{-}\ldots\right.\right.}{\ldots}\right)}{d^2\sqrt{c+dx^2}}\right)}{195(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

```
[Out] (x^(3/2)*((2*Sqrt[c + d*x^2]*(117*a^2*d*(-5*c + d*x^2) + 26*a*b*d
*x^2*(11*c + 5*d*x^2) + 3*b^2*x^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^
```

```
4)))/(3*d*Sqrt[x]) - (8*c*(-3*b^2*c^2 + 26*a*b*c*d + 117*a^2*d^2)
*x*(-((d + c/x^2)*Sqrt[x]) + (I*c*Sqrt[1 + c/(d*x^2)]*(EllipticE[
I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] - EllipticF[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/((I*Sqrt[c])/Sqr
t[d])^(3/2)))/(d^2*Sqrt[c + d*x^2])))/(195*(e*x)^(3/2))
```

Maple [A] time = 0.032, size = 669, normalized size = 1.4

$$\frac{2}{585\,ed^2}\left(45\,x^8b^2d^4+130\,x^6abd^4+120\,x^6b^2cd^3+1404\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\,EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x)
```

```
[Out] 2/585*(45*x^8*b^2*d^4+130*x^6*a*b*d^4+120*x^6*b^2*c*d^3+1404*((d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+312*((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-36*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-702*((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-156*((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d+18*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4+117*x^4*a^2*d^4+416*x^4*a*b*
c*d^3+87*x^4*b^2*c^2*d^2-468*x^2*a^2*c*d^3+286*x^2*a*b*c^2*d^2+12
*x^2*b^2*c^3*d-585*a^2*c^2*d^2)/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}}{\sqrt{ex}ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c +
a^2*d)*x^2)*sqrt(d*x^2 + c)/(sqrt(e*x)*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2), x)
```

3.837 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} dx$

Optimal. Leaf size=288

$$-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}-\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}\left(3b^2c^2-11ad(7ad+6bc)\right)}{231cde^3}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(3b^2c^2-11ad(7ad+6bc)\right)}{231de^3}-\frac{4c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-11ad(7ad+6bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{5/4}e^{5/2}\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de^3}$$

```
[Out] (-4*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2
])/(231*d*e^3) - (2*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x
]*(c + d*x^2)^(3/2))/(231*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(3
*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*(c + d*x^2)^(5/2))/(11*d*e^3
) - (4*c^(3/4)*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*(Sqrt[c] + Sq
rt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(5/4)*e
^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.622613, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}-\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}\left(3b^2c^2-11ad(7ad+6bc)\right)}{231cde^3}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(3b^2c^2-11ad(7ad+6bc)\right)}{231de^3}-\frac{4c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3b^2c^2-11ad(7ad+6bc)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{5/4}e^{5/2}\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```


```
[Out] (-4*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2
])/(231*d*e^3) - (2*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x
]*(c + d*x^2)^(3/2))/(231*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(3
*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*(c + d*x^2)^(5/2))/(11*d*e^3
) - (4*c^(3/4)*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*(Sqrt[c] + Sq
rt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(5/4)*e
^(5/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 59.478, size = 272, normalized size = 0.94

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{5}{2}}}{3ce\left(ex\right)^{\frac{3}{2}}}+\frac{2b^2\sqrt{ex}\left(c+dx^2\right)^{\frac{5}{2}}}{11de^3}-\frac{4c^{\frac{3}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-11ad\left(7ad+6bc\right)+3b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{c+dx^2}}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\left(-11ad\left(7ad+6bc\right)+3b^2c^2\right)}{231de^3}-\frac{2\sqrt{ex}\left(c+dx^2\right)^{\frac{3}{2}}\left(-11ad\left(7ad+6bc\right)+3b^2c^2\right)}{231cde^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(5/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(5/2)/(3*c*e*(e*x)**(3/2)) + 2*b**2*sqrt(e*
x)*(c + d*x**2)**(5/2)/(11*d*e**3) - 4*c**(3/4)*sqrt((c + d*x**2)
/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-11*a*d*(7*a*d
+ 6*b*c) + 3*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**
(1/4)*sqrt(e))), 1/2)/(231*d**(5/4)*e**(5/2)*sqrt(c + d*x**2)) -
4*sqrt(e*x)*sqrt(c + d*x**2)*(-11*a*d*(7*a*d + 6*b*c) + 3*b**2*c*
*2)/(231*d*e**3) - 2*sqrt(e*x)*(c + d*x**2)**(3/2)*(-11*a*d*(7*a*
d + 6*b*c) + 3*b**2*c**2)/(231*c*d*e**3)
```

Mathematica [C] time = 0.366136, size = 202, normalized size = 0.7

$$\frac{x^{5/2}\left(\frac{2\left(c+dx^2\right)\left(77a^2d\left(dx^2-c\right)+66abdx^2\left(3c+dx^2\right)+3b^2x^2\left(4c^2+13cdx^2+7d^2x^4\right)\right)}{dx^{3/2}}+\frac{8icx\sqrt{\frac{c}{dx^2}+1}\left(77a^2d^2+66abcd-3b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```

```
[Out] (x^(5/2)*((2*(c + d*x^2)*(77*a^2*d*(-c + d*x^2) + 66*a*b*d*x^2*(3
*c + d*x^2) + 3*b^2*x^2*(4*c^2 + 13*c*d*x^2 + 7*d^2*x^4)))/(d*x^(
3/2)) + ((8*I)*c*(-3*b^2*c^2 + 66*a*b*c*d + 77*a^2*d^2)*Sqrt[1 +
c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x
]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))/(231*(e*x)^(5/2)*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.028, size = 415, normalized size = 1.4

$$\frac{2}{231\,xd^2e^2}\left(21\,x^8b^2d^4+154\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)xa^2cd^2+132\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)/x*(21*x^8*b^2*d^4+154*(-c*d)^(1/2)*((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^2*c*d^2+132*(-c*d)^(1
/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*b*c^2*d-6
*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*b
^2*c^3+66*x^6*a*b*d^4+60*x^6*b^2*c*d^3+77*x^4*a^2*d^4+264*x^4*a*b
*c*d^3+51*x^4*b^2*c^2*d^2+198*x^2*a*b*c^2*d^2+12*x^2*b^2*c^3*d-77
*a^2*c^2*d^2)/d^2/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2dx^6 + \left(b^2c + 2\,abd\right)x^4 + a^2c + \left(2\,abc + a^2d\right)x^2\right)\sqrt{dx^2 + c}}{\sqrt{ex}e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c +
a^2*d)*x^2)*sqrt(d*x^2 + c)/(sqrt(e*x)*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2\left(dx^2 + c\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2), x)
```

3.838 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=468

$$-\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(9ad(ad+2bc)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{8\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(9ad(ad+2bc)+b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{2(ex)^{3/2}(c+dx^2)^{3/2}\left(9ad(ad+2bc)+b^2c^2\right)}{9c^2e^5}+\frac{4(ex)^{3/2}\sqrt{c+dx^2}\left(9ad(ad+2bc)+b^2c^2\right)}{15ce^5}+\frac{8\sqrt{ex}\sqrt{c+dx^2}\left(9ad(ad+2bc)+b^2c^2\right)}{15\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a(c+dx^2)^{5/2}(ad+2bc)}{c^2e^3\sqrt{ex}}$$

```
[Out] (4*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(
15*c*e^5) + (8*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c +
 d*x^2])/(15*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) + (2*(b^2*c^2 + 9
*a*d*(2*b*c + a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*c^2*e^5) -
(2*a^2*(c + d*x^2)^(5/2))/(5*c*e*(e*x)^(5/2)) - (2*a*(2*b*c + a*d
)*(c + d*x^2)^(5/2))/(c^2*e^3*Sqrt[e*x]) - (8*c^(1/4)*(b^2*c^2 +
9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt
[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (4*c^(
1/4)*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c +
d*x^2])
```

Rubi [A] time = 1.07944, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(9ad(ad+2bc)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{8\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(9ad(ad+2bc)+b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{2(ex)^{3/2}(c+dx^2)^{3/2}\left(9ad(ad+2bc)+b^2c^2\right)}{9c^2e^5}+\frac{4(ex)^{3/2}\sqrt{c+dx^2}\left(9ad(ad+2bc)+b^2c^2\right)}{15ce^5}+\frac{8\sqrt{ex}\sqrt{c+dx^2}\left(9ad(ad+2bc)+b^2c^2\right)}{15\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a(c+dx^2)^{5/2}(ad+2bc)}{c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```



```
[Out] (4*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(
15*c*e^5) + (8*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c +
 d*x^2])/(15*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) + (2*(b^2*c^2 + 9
*a*d*(2*b*c + a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*c^2*e^5) -
(2*a^2*(c + d*x^2)^(5/2))/(5*c*e*(e*x)^(5/2)) - (2*a*(2*b*c + a*d
)*(c + d*x^2)^(5/2))/(c^2*e^3*Sqrt[e*x]) - (8*c^(1/4)*(b^2*c^2 +
```

```
9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt
[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (4*c^(
1/4)*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c +
d*x^2])
```

Rubi in Sympy [A] time = 105.925, size = 444, normalized size = 0.95

$$-\frac{2a^2\left(c+dx^2\right)^{\frac{5}{2}}}{5ce\left(ex\right)^{\frac{5}{2}}}-\frac{2a\left(c+dx^2\right)^{\frac{5}{2}}\left(ad+2bc\right)}{c^2e^3\sqrt{ex}}-\frac{8\sqrt[4]{c}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(9ad\left(ad+2bc\right)+b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}+\frac{4\sqrt[4]{c}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(9ad\left(ad+2bc\right)+b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}+\frac{8\sqrt{ex}\sqrt{c+dx^2}\left(9ad\left(ad+2bc\right)+b^2c^2\right)}{15\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{4\left(ex\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(9ad\left(ad+2bc\right)+b^2c^2\right)}{15ce^5}+\frac{2\left(ex\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\left(9ad\left(ad+2bc\right)+b^2c^2\right)}{9c^2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(7/2),x)
```

```
[Out] -2*a**2*(c + d*x**2)**(5/2)/(5*c*e*(e*x)**(5/2)) - 2*a*(c + d*x**
2)**(5/2)*(a*d + 2*b*c)/(c**2*e**3*sqrt(e*x)) - 8*c**(1/4)*sqrt((
c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(9*a*
d*(a*d + 2*b*c) + b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)
/(c**(1/4)*sqrt(e))), 1/2)/(15*d**(3/4)*e**(7/2)*sqrt(c + d*x**2)
) + 4*c**(1/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(
c) + sqrt(d)*x)*(9*a*d*(a*d + 2*b*c) + b**2*c**2)*elliptic_f(2*at
an(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(15*d**(3/4)*e**(
7/2)*sqrt(c + d*x**2)) + 8*sqrt(e*x)*sqrt(c + d*x**2)*(9*a*d*(a*d
 + 2*b*c) + b**2*c**2)/(15*sqrt(d)*e**4*(sqrt(c) + sqrt(d)*x)) +
4*(e*x)**(3/2)*sqrt(c + d*x**2)*(9*a*d*(a*d + 2*b*c) + b**2*c**2)
/(15*c*e**5) + 2*(e*x)**(3/2)*(c + d*x**2)**(3/2)*(9*a*d*(a*d + 2
*b*c) + b**2*c**2)/(9*c**2*e**5)
```

Mathematica [C] time = 1.51636, size = 240, normalized size = 0.51

$$\frac{x^{7/2}\left(\frac{2\sqrt{c+dx^2}\left(-9a^2\left(c+7dx^2\right)+18abx^2\left(dx^2-5c\right)+b^2x^4\left(11c+5dx^2\right)\right)}{3x^{5/2}}-\frac{8x\left(9a^2d^2+18abcd+b^2c^2\right)\left(-\sqrt{x}\left(\frac{c}{x^2}+d\right)+\frac{ic\sqrt{\frac{c}{dx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\cdots\right.\right.}{\left(\frac{i\sqrt{c}}{\sqrt{d}}\right)^{3/2}}\cdots\right)}{d\sqrt{c+dx^2}}\cdots\right)}{15(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```

```
[Out] (x^(7/2)*((2*Sqrt[c + d*x^2]*(18*a*b*x^2*(-5*c + d*x^2) + b^2*x^4
*(11*c + 5*d*x^2) - 9*a^2*(c + 7*d*x^2)))/(3*x^(5/2)) - (8*(b^2*c
^2 + 18*a*b*c*d + 9*a^2*d^2)*x*(-((d + c/x^2)*Sqrt[x]) + (I*c*Sqr
t[1 + c/(d*x^2)]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/S
qrt[x]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt
[x]], -1]))/((I*Sqrt[c])/Sqrt[d])^(3/2)))/(d*Sqrt[c + d*x^2])))/(
15*(e*x)^(7/2))
```

Maple [A] time = 0.031, size = 668, normalized size = 1.4

$$\frac{2}{45\,dx^2e^3}\left(5\,b^2d^3x^8+108\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2+216\sqrt{\frac{dx+}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x)
```

```
[Out] 2/45/x^2*(5*b^2*d^3*x^8+108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*x^2*a^2*c*d^2+216*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*x^2*a*b*c^2*d+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*x^2*b^2*c^3-54*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/
2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*a^2*c*d^2-108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*x^2*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/
2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*b^2*c^3+18*x^6*a*b*d^3+16*x^6*b^2*c*d^2-63*x^4*a^2*
d^3-72*x^4*a*b*c*d^2+11*x^4*b^2*c^2*d-72*x^2*a^2*c*d^2-90*x^2*a*b
*c^2*d-9*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2\right)\sqrt{dx^2+c}}{\sqrt{ex}e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c +
a^2*d)*x^2)*sqrt(d*x^2 + c)/(sqrt(e*x)*e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2), x)
```

3.839 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=430

$$-\frac{c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(117a^2d^2+7bc(11bc-26ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}+\frac{2c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(117a^2d^2+7bc(11bc-26ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}-\frac{2ce^2\sqrt{ex}\sqrt{c+dx^2}\left(117a^2d^2+7bc(11bc-26ad)\right)}{195d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2e(ex)^{3/2}\sqrt{c+dx^2}\left(117a^2d^2+7bc(11bc-26ad)\right)}{585d^3}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{117d^2e}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

```
[Out] (2*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e*(e*x)^(3/2)*Sqrt[c +
 d*x^2])/(585*d^3) - (2*b*(11*b*c - 26*a*d)*(e*x)^(7/2)*Sqrt[c +
d*x^2])/(117*d^2*e) + (2*b^2*(e*x)^(11/2)*Sqrt[c + d*x^2])/(13*d*
e^3) - (2*c*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^2*Sqrt[e*x]
*Sqrt[c + d*x^2])/(195*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*c^(5/4
)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*(Sqrt[c] + Sqrt
[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcT
an[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sq
rt[c + d*x^2]) - (c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))
*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d
]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, 1/2])/(195*d^(15/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.990731, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(117a^2d^2+7bc(11bc-26ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}+\frac{2c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(117a^2d^2+7bc(11bc-26ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}-\frac{2ce^2\sqrt{ex}\sqrt{c+dx^2}\left(117a^2d^2+7bc(11bc-26ad)\right)}{195d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2e(ex)^{3/2}\sqrt{c+dx^2}\left(117a^2d^2+7bc(11bc-26ad)\right)}{585d^3}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{117d^2e}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e*(e*x)^(3/2)*Sqrt[c +
 d*x^2])/(585*d^3) - (2*b*(11*b*c - 26*a*d)*(e*x)^(7/2)*Sqrt[c +
d*x^2])/(117*d^2*e) + (2*b^2*(e*x)^(11/2)*Sqrt[c + d*x^2])/(13*d*
e^3) - (2*c*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^2*Sqrt[e*x]
*Sqrt[c + d*x^2])/(195*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*c^(5/4
)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*(Sqrt[c] + Sqrt
[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcT
an[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sq
```

```
rt[c + d*x^2]) - (c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))
*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d
]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, 1/2])/(195*d^(15/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 92.2903, size = 413, normalized size = 0.96

$$\frac{2b^2(ex)^{\frac{11}{2}}\sqrt{c+dx^2}}{13de^3}+\frac{2b(ex)^{\frac{7}{2}}\sqrt{c+dx^2}\,(26ad-11bc)}{117d^2e}+\frac{2c^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(117a^2d^2-7bc(26ad-11bc)\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195d^{\frac{15}{4}}\sqrt{c+dx^2}}-\frac{c^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(117a^2d^2-7bc(26ad-11bc)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{195d^{\frac{15}{4}}\sqrt{c+dx^2}}-\frac{2ce^2\sqrt{ex}\sqrt{c+dx^2}\left(117a^2d^2-7bc(26ad-11bc)\right)}{195d^{\frac{7}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2e(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\left(117a^2d^2-7bc(26ad-11bc)\right)}{585d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(11/2)*sqrt(c + d*x**2)/(13*d*e**3) + 2*b*(e*x)**(7
/2)*sqrt(c + d*x**2)*(26*a*d - 11*b*c)/(117*d**2*e) + 2*c**(5/4)*
e**(5/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + s
qrt(d)*x)*(117*a**2*d**2 - 7*b*c*(26*a*d - 11*b*c))*elliptic_e(2*
atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(195*d**(15/4)*
sqrt(c + d*x**2)) - c**(5/4)*e**(5/2)*sqrt((c + d*x**2)/(sqrt(c)
+ sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(117*a**2*d**2 - 7*b*c*(26
*a*d - 11*b*c))*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sq
rt(e))), 1/2)/(195*d**(15/4)*sqrt(c + d*x**2)) - 2*c*e**2*sqrt(e*
x)*sqrt(c + d*x**2)*(117*a**2*d**2 - 7*b*c*(26*a*d - 11*b*c))/(19
5*d**(7/2)*(sqrt(c) + sqrt(d)*x)) + 2*e*(e*x)**(3/2)*sqrt(c + d*x
**2)*(117*a**2*d**2 - 7*b*c*(26*a*d - 11*b*c))/(585*d**3)
```

Mathematica [C] time = 1.74581, size = 237, normalized size = 0.55

$$\frac{2(ex)^{5/2}\left(d\sqrt{x}\left(c+dx^2\right)\left(117a^2d^2+26abd\left(5dx^2-7c\right)+b^2\left(77c^2-55cdx^2+45d^2x^4\right)\right)+3c\left(117a^2d^2-182abcd+77b^2c^2\right)\right.}{585d^4x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*(e*x)^(5/2)*(d*Sqrt[x]*(c + d*x^2)*(117*a^2*d^2 + 26*a*b*d*(-7
*c + 5*d*x^2) + b^2*(77*c^2 - 55*c*d*x^2 + 45*d^2*x^4)) + 3*c*(77
*b^2*c^2 - 182*a*b*c*d + 117*a^2*d^2)*(-((d + c/x^2)*Sqrt[x]) + (
I*c*Sqrt[1 + c/(d*x^2)]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqr
t[d]]/Sqrt[x]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d
]]/Sqrt[x]], -1]))/((I*Sqrt[c])/Sqrt[d])^(3/2))))/(585*d^4*x^(3/2
)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.044, size = 661, normalized size = 1.5

$$-\frac{e^{2}}{585\,x d^{4}}\sqrt{ex}\left(-90\,x^{8}b^{2}d^{4}-260\,x^{6}abd^{4}+20\,x^{6}b^{2}cd^{3}+702\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-c}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/585/x*e^2*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^4*(-90*x^8*b^2*d^4-260
*x^6*a*b*d^4+20*x^6*b^2*c*d^3+702*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*c^2*d^2-1092*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a*b*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)
^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*b^2*c^4-351*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*c^2*d^2+546*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a*b*c^3*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*b^2*c^4-234*x^4*a^2*d^4+104*x^4*a*b*c*d^3-44*x^4*b^2*c^2*d^2
-234*x^2*a^2*c*d^3+364*x^2*a*b*c^2*d^2-154*x^2*b^2*c^3*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{2}(ex)^{\frac{5}{2}}}{\sqrt{dx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2}e^{2}x^{6}+2\,abe^{2}x^{4}+a^{2}e^{2}x^{2}\right)\sqrt{ex}}{\sqrt{dx^{2}+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)*sqrt(e*x)/sq
rt(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2 (ex)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c), x)
```

3.840 $\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=240

$$-\frac{c^{3/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2+5bc(9bc-22ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2e\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2+5bc(9bc-22ad)\right)}{231d^3}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}(9bc-22ad)}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

```
[Out] (2*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x
^2])/(231*d^3) - (2*b*(9*b*c - 22*a*d)*(e*x)^(5/2)*Sqrt[c + d*x^2
])/(77*d^2*e) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d*e^3) -
(c^(3/4)*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e^(3/2)*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(13/
4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.552012, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{c^{3/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2+5bc(9bc-22ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2e\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2+5bc(9bc-22ad)\right)}{231d^3}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}(9bc-22ad)}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x
^2])/(231*d^3) - (2*b*(9*b*c - 22*a*d)*(e*x)^(5/2)*Sqrt[c + d*x^2
])/(77*d^2*e) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d*e^3) -
(c^(3/4)*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e^(3/2)*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(13/
4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 49.5247, size = 230, normalized size = 0.96

$$\frac{2b^2(ex)^{\frac{9}{2}}\sqrt{c+dx^2}}{11de^3}+\frac{2b(ex)^{\frac{5}{2}}\sqrt{c+dx^2}(22ad-9bc)}{77d^2e}-\frac{c^{\frac{3}{4}}e^{\frac{3}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(77a^2d^2-5bc(22ad-9bc)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231d^{\frac{13}{4}}\sqrt{c+dx^2}}+\frac{2e\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-5bc(22ad-9bc)\right)}{231d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(9/2)*sqrt(c + d*x**2)/(11*d*e**3) + 2*b*(e*x)**(5/
2)*sqrt(c + d*x**2)*(22*a*d - 9*b*c)/(77*d**2*e) - c**(3/4)*e**(3
/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d
)*x)*(77*a**2*d**2 - 5*b*c*(22*a*d - 9*b*c))*elliptic_f(2*atan(d*
```

```
*(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(231*d**(13/4)*sqrt(c
+ d*x**2)) + 2*e*sqrt(e*x)*sqrt(c + d*x**2)*(77*a**2*d**2 - 5*b*c
*(22*a*d - 9*b*c))/(231*d**3)
```

Mathematica [C] time = 0.345803, size = 190, normalized size = 0.79

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)\left(77a^2d^2+22abd(3dx^2-5c)+3b^2(15c^2-9cdx^2+7d^2x^4)\right)}{d^3}-\frac{2icx\sqrt{\frac{c}{dx^2}+1}(77a^2d^2-110abcd+45b^2c^2)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{d^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(-5*c
 + 3*d*x^2) + 3*b^2*(15*c^2 - 9*c*d*x^2 + 7*d^2*x^4)))/d^3 - ((2*
I)*c*(45*b^2*c^2 - 110*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*
x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(S
qrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.044, size = 405, normalized size = 1.7

$$-\frac{e}{231\,xd^4}\sqrt{ex}\left(-42\,x^7b^2d^4+77\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)a^2cd^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/231*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-42*x^7*b^2*d^4+77*(-c*d)
^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-
110*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a*b*c^2*d+45*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*b^2*c^3-132*x^5*a*b*d^4+12*x^5*b^2*c*d^3-154*x^3*a^2*d^4
+88*x^3*a*b*c*d^3-36*x^3*b^2*c^2*d^2-154*x*a^2*c*d^3+220*x*a*b*c^
2*d^2-90*x*b^2*c^3*d)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2ex^5 + 2\,abex^3 + a^2ex\right)\sqrt{ex}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(e*x)/sqrt(d*x^2
 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 (ex)^{\frac{3}{2}}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c), x)
```

3.841 $\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=375

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(15a^2d^2+bc(7bc-18ad)\right)}{15d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}} - \frac{2\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}} - \frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

```
[Out] (-2*b*(7*b*c - 18*a*d)*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*d^2*e) +
(2*b^2*(e*x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e^3) + (2*(15*a^2*d^2 +
b*c*(7*b*c - 18*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(5/2)*(Sqr
t[c] + Sqrt[d]*x)) - (2*c^(1/4)*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d
))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt
[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2]) + (c^(1/4)*(15*a^2*d^2 +
b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x
^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.863554, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(15a^2d^2+bc(7bc-18ad)\right)}{15d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}} - \frac{2\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}} - \frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```


```
[Out] (-2*b*(7*b*c - 18*a*d)*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*d^2*e) +
(2*b^2*(e*x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e^3) + (2*(15*a^2*d^2 +
b*c*(7*b*c - 18*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(5/2)*(Sqr
t[c] + Sqrt[d]*x)) - (2*c^(1/4)*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d
))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt
[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2]) + (c^(1/4)*(15*a^2*d^2 +
b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x
^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 78.5934, size = 354, normalized size = 0.94

$$\frac{2b^2(ex)^{\frac{7}{2}}\sqrt{c+dx^2}}{9de^3}+\frac{2b(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\,(18ad-7bc)}{45d^2e}-\frac{2\sqrt[4]{c}\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(15a^2d^2-bc(18ad-7bc)\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(15a^2d^2-bc(18ad-7bc)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{11}{4}}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(15a^2d^2-bc(18ad-7bc)\right)}{15d^{\frac{5}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(7/2)*sqrt(c + d*x**2)/(9*d*e**3) + 2*b*(e*x)**(3/2
)*sqrt(c + d*x**2)*(18*a*d - 7*b*c)/(45*d**2*e) - 2*c**(1/4)*sqrt
(e)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d
)*x)*(15*a**2*d**2 - b*c*(18*a*d - 7*b*c))*elliptic_e(2*atan(d**(
1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(15*d**(11/4)*sqrt(c + d
*x**2)) + c**(1/4)*sqrt(e)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x
)**2)*(sqrt(c) + sqrt(d)*x)*(15*a**2*d**2 - b*c*(18*a*d - 7*b*c))
*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(
15*d**(11/4)*sqrt(c + d*x**2)) + 2*sqrt(e*x)*sqrt(c + d*x**2)*(15
*a**2*d**2 - b*c*(18*a*d - 7*b*c))/(15*d**(5/2)*(sqrt(c) + sqrt(d
)*x))
```

Mathematica [C] time = 1.0749, size = 249, normalized size = 0.66

$$\frac{2e\left(bdx^2\left(c+dx^2\right)\left(18ad-7bc+5bdx^2\right)+\frac{3\left(15a^2d^2-18abcd+7b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)+\sqrt{c}\sqrt{d}x^{3/2}\sqrt{\frac{c}{dx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{c}\sqrt{d}x^{3/2}\sqrt{\cdots}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{45d^3\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*e*(b*d*x^2*(c + d*x^2)*(-7*b*c + 18*a*d + 5*b*d*x^2) + (3*(7*b
^2*c^2 - 18*a*b*c*d + 15*a^2*d^2)*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(c +
 d*x^2) - Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticE[I
*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] + Sqrt[c]*Sqrt[d
]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]
)/Sqrt[d]]/Sqrt[x]], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(45*d^3*Sq
rt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.027, size = 604, normalized size = 1.6

$$\frac{1}{45\,d^3x}\sqrt{ex}\left(10\,x^6b^2d^3+90\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-108\sqrt{\frac{dx+\cdots}{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/45*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^3*(10*x^6*b^2*d^3+90*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-108*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+42*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-45*((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+54*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^2*d-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*b^2*c^3+36*x^4*a*b*d^3-4*x^4*b^2*c*d^2+36*x^2*a
*b*c*d^2-14*x^2*b^2*c^2*d)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ex}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(e*x)/sqrt(d*x^2 + c), x
)
```

Sympy [A] time = 16.9715, size = 143, normalized size = 0.38

$$\frac{a^2(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e\,\Gamma\left(\frac{7}{4}\right)}+\frac{ab(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^3\,\Gamma\left(\frac{11}{4}\right)}+\frac{b^2(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^5\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*(e*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp
_polar(I*pi)/c)/(2*sqrt(c)*e*gamma(7/4)) + a*b*(e*x)**(7/2)*gamma
(7/4)*hyper((1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(
c)*e**3*gamma(11/4)) + b**2*(e*x)**(11/2)*gamma(11/4)*hyper((1/2,
```

```
 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(c)*e**5*gamma(
15/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

3.842 $\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-14abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{21d^2e}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

```
[Out] (-2*b*(5*b*c - 14*a*d)*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*d^2*e) + (2
*b^2*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d*e^3) + ((5*b^2*c^2 - 14*a*
b*c*d + 21*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[
c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.413634, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-14abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{21d^2e}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*b*(5*b*c - 14*a*d)*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*d^2*e) + (2
*b^2*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d*e^3) + ((5*b^2*c^2 - 14*a*
b*c*d + 21*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[
c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 41.0705, size = 178, normalized size = 0.92

$$\frac{2b^2(ex)^{\frac{5}{2}}\sqrt{c+dx^2}}{7de^3}+\frac{2b\sqrt{ex}\sqrt{c+dx^2}\,(14ad-5bc)}{21d^2e}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(21a^2d^2-bc\,(14ad-5bc)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21\sqrt[4]{c}d^{\frac{9}{4}}\sqrt{e}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] 2*b**2*(e*x)**(5/2)*sqrt(c + d*x**2)/(7*d*e**3) + 2*b*sqrt(e*x)*s
qrt(c + d*x**2)*(14*a*d - 5*b*c)/(21*d**2*e) + sqrt((c + d*x**2)/
(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(21*a**2*d**2 - b
*c*(14*a*d - 5*b*c))*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/
4)*sqrt(e))), 1/2)/(21*c**(1/4)*d**(9/4)*sqrt(e)*sqrt(c + d*x**2)
)
```

Mathematica [C] time = 0.373638, size = 148, normalized size = 0.77

$$\frac{2x\left(-b\left(c+dx^2\right)\left(-14ad+5bc-3bdx^2\right)+\frac{i\sqrt{x}\sqrt{\frac{c}{dx^2}+1}\left(21a^2d^2-14abcd+5b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21d^2\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*x*(-(b*(c + d*x^2)*(5*b*c - 14*a*d - 3*b*d*x^2)) + (I*(5*b^2*c
^2 - 14*a*b*c*d + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqr
t[c])/Sqrt[d]]))/(21*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.028, size = 350, normalized size = 1.8

$$\frac{1}{21\,d^3}\left(21\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2d^2-14\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/21/(d*x^2+c)^(1/2)*(21*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^2*d^2-14*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+5*(-c*d)^(1/2)*((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+6*x^5*b^2*d^3+28*x^3*a*
b*d^3-4*x^3*b^2*c*d^2+28*x*a*b*c*d^2-10*x*b^2*c^2*d)/(e*x)^(1/2)/
d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\sqrt{dx^2+c}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)),
 x)
```

Sympy [A] time = 18.3917, size = 144, normalized size = 0.75

$$\frac{a^2\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{abx^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{b^2x^{\frac{9}{2}}\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(c)*sqrt(e)*gamma(5/4)) + a*b*x**(5/2)*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*s
qrt(e)*gamma(9/4)) + b**2*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (
13/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(c)*sqrt(e)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

3.843 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=372

$$-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\,(3b^2c^2-5ad(ad+2bc))}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-5ad(ad+2bc))\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-5ad(ad+2bc))\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*Sqr
t[c + d*x^2])/(5*d*e^3) - (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*Sq
rt[e*x]*Sqrt[c + d*x^2])/(5*c*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x))
+ (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d^(7/4)*e^(3/2)*
Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[c] +
Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.826278, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\,(3b^2c^2-5ad(ad+2bc))}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-5ad(ad+2bc))\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(3b^2c^2-5ad(ad+2bc))\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*Sqr
t[c + d*x^2])/(5*d*e^3) - (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*Sq
rt[e*x]*Sqrt[c + d*x^2])/(5*c*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x))
+ (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d^(7/4)*e^(3/2)*
Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[c] +
Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 81.2749, size = 347, normalized size = 0.93

$$-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{\frac{3}{2}}\sqrt{c+dx^2}}{5de^3}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(-5ad(ad+2bc)+3b^2c^2\right)}{5cd^{\frac{3}{2}}e^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-5ad(ad+2bc)+3b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}-\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-5ad(ad+2bc)+3b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*a**2*sqrt(c + d*x**2)/(c*e*sqrt(e*x)) + 2*b**2*(e*x)**(3/2)*sq
rt(c + d*x**2)/(5*d*e**3) - 2*sqrt(e*x)*sqrt(c + d*x**2)*(-5*a*d*
(a*d + 2*b*c) + 3*b**2*c**2)/(5*c*d**(3/2)*e**2*(sqrt(c) + sqrt(d
)*x)) + 2*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) +
sqrt(d)*x)*(-5*a*d*(a*d + 2*b*c) + 3*b**2*c**2)*elliptic_e(2*atan
(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(5*c**(3/4)*d**(7/4
)*e**(3/2)*sqrt(c + d*x**2)) - sqrt((c + d*x**2)/(sqrt(c) + sqrt(
d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-5*a*d*(a*d + 2*b*c) + 3*b**2*c*
*2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2
)/(5*c**(3/4)*d**(7/4)*e**(3/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.70417, size = 200, normalized size = 0.54

$$\frac{2x\left(d\left(c+dx^2\right)\left(b^2cx^2-5a^2d\right)+x^{3/2}\left(-5a^2d^2-10abcd+3b^2c^2\right)\left(-\sqrt{x}\left(\frac{c}{x^2}+d\right)+\frac{ic\sqrt{\frac{c}{dx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\cdots\right.\right.\right.}{\left(\frac{i\sqrt{c}}{\sqrt{d}}\right)^{3/2}}\right.\right.}{5cd^2(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*x*(d*(-5*a^2*d + b^2*c*x^2)*(c + d*x^2) + (3*b^2*c^2 - 10*a*b*
c*d - 5*a^2*d^2)*x^(3/2)*(-((d + c/x^2)*Sqrt[x]) + (I*c*Sqrt[1 +
c/(d*x^2)]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]
], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
-1]))/((I*Sqrt[c])/Sqrt[d])^(3/2))))/(5*c*d^2*(e*x)^(3/2)*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.032, size = 595, normalized size = 1.6

$$\frac{1}{5ed^2c}\left(10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2+20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\cdots}{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/5*(10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
```

```
cE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2
+20*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-6*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-5*((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-10*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+3*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+2*x^4*b^2*c*d^2-10*x^2*a^2
*d^3+2*x^2*b^2*c^2*d-10*a^2*c*d^2)/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1
/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\sqrt{dx^2+c}\sqrt{ex}ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)*e
*x), x)
```

Sympy [A] time = 25.1243, size = 148, normalized size = 0.4

$$\frac{a^2\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{abx^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)}+\frac{b^2x^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{3}{2}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x**2*exp_polar(I*pi
)/c)/(2*sqrt(c)*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*x**(3/2)*gamma
(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c
)*e**(3/2)*gamma(7/4)) + b**2*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4
), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(c)*e**(3/2)*gamma(1
```

```
1/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)), x)
```

3.844 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(a^2d^2-6abcd+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*S
qrt[c + d*x^2])/(3*d*e^3) - ((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(Sqr
t[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^
(5/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.390037, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(a^2d^2-6abcd+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*S
qrt[c + d*x^2])/(3*d*e^3) - ((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(Sqr
t[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^
(5/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 42.289, size = 165, normalized size = 0.9

$$-\frac{2a^2\sqrt{c+dx^2}}{3ce\,(ex)^{\frac{3}{2}}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}-\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\,(ad-6bc)+b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{3c^{\frac{5}{4}}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*a**2*sqrt(c + d*x**2)/(3*c*e*(e*x)**(3/2)) + 2*b**2*sqrt(e*x)*
sqrt(c + d*x**2)/(3*d*e**3) - sqrt((c + d*x**2)/(sqrt(c) + sqrt(d
)*x)**2)*(sqrt(c) + sqrt(d)*x)*(a*d*(a*d - 6*b*c) + b**2*c**2)*el
liptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(3*c
**(5/4)*d**(5/4)*e**(5/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.288025, size = 165, normalized size = 0.9

$$\frac{x\left(2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)\left(b^2cx^2-a^2d\right)-2ix^{5/2}\sqrt{\frac{c}{dx^2}+1}\left(a^2d^2-6abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3cd\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*(2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-(a^2*d) + b^2*c*x^2)*(c + d*x^2
) - (2*I)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(
5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])
)/(3*c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.029, size = 352, normalized size = 1.9

$$-\frac{1}{3\,cxe^2d^2}\left(\sqrt{1\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{2}\sqrt{1\left(-dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{-dx\frac{1}{\sqrt{-cd}}}EllipticF\left(\sqrt{1\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3/(d*x^2+c)^(1/2)/x*(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*(-c*d)^(1/2)*x*a^2*d^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*(-c*d)^(1/2)*x*a*b*c*d+((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x*b^2*c^2-2*x^4*b^2*c*d^2+2*x^2
*a^2*d^3-2*x^2*b^2*c^2*d+2*a^2*c*d^2)/c/e^2/(e*x)^(1/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\sqrt{dx^2+c}\sqrt{ex}e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)*e
^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)), x)
```

3.845 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=387

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\,(-3a^2d^2+10abcd+5b^2c^2)}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(-3a^2d^2+10abcd+5b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(-3a^2d^2+10abcd+5b^2c^2)\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{5c^2e^3\sqrt{ex}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c - 3*a
*d)*Sqrt[c + d*x^2])/(5*c^2*e^3*Sqrt[e*x]) + (2*(5*b^2*c^2 + 10*a
*b*c*d - 3*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^2*Sqrt[d]*e^4
*(Sqrt[c] + Sqrt[d]*x)) - (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)
*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*
EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + ((5*b^2*c^2 + 10*a*
b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c
] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.837622, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{ex}\sqrt{c+dx^2}\,(-3a^2d^2+10abcd+5b^2c^2)}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(-3a^2d^2+10abcd+5b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(-3a^2d^2+10abcd+5b^2c^2)\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{5c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c - 3*a
*d)*Sqrt[c + d*x^2])/(5*c^2*e^3*Sqrt[e*x]) + (2*(5*b^2*c^2 + 10*a
*b*c*d - 3*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^2*Sqrt[d]*e^4
*(Sqrt[c] + Sqrt[d]*x)) - (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)
*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*
EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + ((5*b^2*c^2 + 10*a*
b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c
] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 84.3162, size = 360, normalized size = 0.93

$$-\frac{2a^2\sqrt{c+dx^2}}{5ce\,(ex)^{\frac{5}{2}}}+\frac{2a\sqrt{c+dx^2}\,(3ad-10bc)}{5c^2e^3\sqrt{ex}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\,\left(-ad\,(3ad-10bc)+5b^2c^2\right)}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}$$
$$-\frac{2\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-ad\,(3ad-10bc)+5b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-ad\,(3ad-10bc)+5b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*a**2*sqrt(c + d*x**2)/(5*c*e*(e*x)**(5/2)) + 2*a*sqrt(c + d*x*
*2)*(3*a*d - 10*b*c)/(5*c**2*e**3*sqrt(e*x)) + 2*sqrt(e*x)*sqrt(c
 + d*x**2)*(-a*d*(3*a*d - 10*b*c) + 5*b**2*c**2)/(5*c**2*sqrt(d)*
e**4*(sqrt(c) + sqrt(d)*x)) - 2*sqrt((c + d*x**2)/(sqrt(c) + sqrt
(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-a*d*(3*a*d - 10*b*c) + 5*b**2*
c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1
/2)/(5*c**(7/4)*d**(3/4)*e**(7/2)*sqrt(c + d*x**2)) + sqrt((c + d
*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(-a*d*(3*a
*d - 10*b*c) + 5*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/
(c**(1/4)*sqrt(e))), 1/2)/(5*c**(7/4)*d**(3/4)*e**(7/2)*sqrt(c +
d*x**2))
```

Mathematica [C] time = 1.43669, size = 217, normalized size = 0.56

$$\frac{x^{7/2}\left(-\frac{2a\sqrt{c+dx^2}\left(a\left(c-3dx^2\right)+10bcx^2\right)}{c^2x^{5/2}}-\frac{2x\left(-3a^2d^2+10abcd+5b^2c^2\right)\left(-\sqrt{x}\left(\frac{c}{x^2}+d\right)+\frac{ic\sqrt{\frac{c}{dx^2}+1}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{c}}{\sqrt{d}}\right)^{3/2}}\right)}{c^2d\sqrt{c+dx^2}}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(7/2)*((-2*a*Sqrt[c + d*x^2]*(10*b*c*x^2 + a*(c - 3*d*x^2)))/(
c^2*x^(5/2)) - (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*x*(-((d +
c/x^2)*Sqrt[x]) + (I*c*Sqrt[1 + c/(d*x^2)]*(EllipticE[I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] - EllipticF[I*ArcSinh[Sqrt
[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/((I*Sqrt[c])/Sqrt[d])^(3/2)
))/(c^2*d*Sqrt[c + d*x^2])))/(5*(e*x)^(7/2))
```

Maple [A] time = 0.03, size = 626, normalized size = 1.6

$$-\frac{1}{5\,dx^2e^3c^2}\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2-20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/5/x^2*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
^2*c*d^2-20*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b*c^2*d-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b
^2*c^3-3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*
c*d^2+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d+5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c
^3-6*x^4*a^2*d^3+20*x^4*a*b*c*d^2-4*x^2*a^2*c*d^2+20*x^2*a*b*c^2*
d+2*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*x)^(1/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\sqrt{dx^2+c}\sqrt{ex}e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)*e
^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{\sqrt{dx^2 + c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)), x)
```

3.846 $\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-14abcd+21b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21c^{9/4}\sqrt[4]{d}e^{9/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}-\frac{2a\sqrt{c+dx^2}(14bc-5ad)}{21c^2e^3(ex)^{3/2}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) - (2*a*(14*b*c - 5*a
*d)*Sqrt[c + d*x^2])/(21*c^2*e^3*(e*x)^(3/2)) + ((21*b^2*c^2 - 14
*a*b*c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqr
t[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(21*c^(9/4)*d^(1/4)*e^(9/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.432334, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-14abcd+21b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21c^{9/4}\sqrt[4]{d}e^{9/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}-\frac{2a\sqrt{c+dx^2}(14bc-5ad)}{21c^2e^3(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) - (2*a*(14*b*c - 5*a
*d)*Sqrt[c + d*x^2])/(21*c^2*e^3*(e*x)^(3/2)) + ((21*b^2*c^2 - 14
*a*b*c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqr
t[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(21*c^(9/4)*d^(1/4)*e^(9/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 42.6298, size = 178, normalized size = 0.92

$$-\frac{2a^2\sqrt{c+dx^2}}{7ce\,(ex)^{\frac{7}{2}}}+\frac{2a\sqrt{c+dx^2}\,(5ad-14bc)}{21c^2e^3\,(ex)^{\frac{3}{2}}}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\,(5ad-14bc)+21b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{21c^{\frac{9}{4}}\sqrt[4]{d}e^{\frac{9}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*a**2*sqrt(c + d*x**2)/(7*c*e*(e*x)**(7/2)) + 2*a*sqrt(c + d*x*
*2)*(5*a*d - 14*b*c)/(21*c**2*e**3*(e*x)**(3/2)) + sqrt((c + d*x*
*2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(a*d*(5*a*d -
 14*b*c) + 21*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c*
*(1/4)*sqrt(e))), 1/2)/(21*c**(9/4)*d**(1/4)*e**(9/2)*sqrt(c + d*
x**2))
```

Mathematica [C] time = 0.303104, size = 159, normalized size = 0.82

$$\frac{x^{9/2}\left(\frac{2a(c+dx^2)(-3ac+5adx^2-14bcx^2)}{c^2x^{7/2}}+\frac{2ix\sqrt{\frac{c}{dx^2}+1}(5a^2d^2-14abcd+21b^2c^2)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21(ex)^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(9/2)*((2*a*(c + d*x^2)*(-3*a*c - 14*b*c*x^2 + 5*a*d*x^2))/(c^
2*x^(7/2)) + ((2*I)*(21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*Sqrt[1
+ c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt
[x]], -1])/(c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]])))/(21*(e*x)^(9/2)*Sqrt
[c + d*x^2])
```

Maple [A] time = 0.051, size = 370, normalized size = 1.9

$$\frac{1}{21\,x^3dc^2e^4}\left(5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^3a^2d^2-14\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/21/(d*x^2+c)^(1/2)/x^3*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*x^3*a^2*d^2-14*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*a*b*c*d+21*((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^3*b^2*c^2+10*x^4*
a^2*d^3-28*x^4*a*b*c*d^2+4*x^2*a^2*c*d^2-28*x^2*a*b*c^2*d-6*a^2*c
^2*d)/d/c^2/e^4/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\sqrt{dx^2+c}\sqrt{ex}e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)*e
^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{\sqrt{dx^2 + c}\,(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)), x)
```

3.847 $\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=438

$$\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-18abcd+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}} - \frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-18abcd+15b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}} + \frac{2\sqrt{d}\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-18abcd+15b^2c^2\right)}{15c^3e^6\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2\sqrt{c+dx^2}\left(7a^2d^2-18abcd+15b^2c^2\right)}{15c^3e^5\sqrt{ex}} - \frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a\sqrt{c+dx^2}(18bc-7ad)}{45c^2e^3(ex)^{5/2}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(9*c*e*(e*x)^(9/2)) - (2*a*(18*b*c - 7*a
*d)*Sqrt[c + d*x^2])/(45*c^2*e^3*(e*x)^(5/2)) - (2*(15*b^2*c^2 -
18*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(15*c^3*e^5*Sqrt[e*x]) +
 (2*Sqrt[d]*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*Sqrt[e*x]*Sqrt[
c + d*x^2])/(15*c^3*e^6*(Sqrt[c] + Sqrt[d]*x)) - (2*d^(1/4)*(15*b
^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c +
d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*
x^2]) + (d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4
)*e^(11/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.980644, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-18abcd+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}} - \frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-18abcd+15b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}} + \frac{2\sqrt{d}\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-18abcd+15b^2c^2\right)}{15c^3e^6\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2\sqrt{c+dx^2}\left(7a^2d^2-18abcd+15b^2c^2\right)}{15c^3e^5\sqrt{ex}} - \frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a\sqrt{c+dx^2}(18bc-7ad)}{45c^2e^3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(9*c*e*(e*x)^(9/2)) - (2*a*(18*b*c - 7*a
*d)*Sqrt[c + d*x^2])/(45*c^2*e^3*(e*x)^(5/2)) - (2*(15*b^2*c^2 -
18*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(15*c^3*e^5*Sqrt[e*x]) +
 (2*Sqrt[d]*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*Sqrt[e*x]*Sqrt[
c + d*x^2])/(15*c^3*e^6*(Sqrt[c] + Sqrt[d]*x)) - (2*d^(1/4)*(15*b
^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c +
d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*
x^2]) + (d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] +
 Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4
)*e^(11/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 98.4659, size = 410, normalized size = 0.94

$$-\frac{2a^2\sqrt{c+dx^2}}{9ce\,(ex)^{\frac{9}{2}}}+\frac{2a\sqrt{c+dx^2}\,(7ad-18bc)}{45c^2e^3\,(ex)^{\frac{5}{2}}}+\frac{2\sqrt{d}\sqrt{ex}\sqrt{c+dx^2}\,\left(ad\,(7ad-18bc)+15b^2c^2\right)}{15c^3e^6\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2\sqrt{c+dx^2}\,\left(ad\,(7ad-18bc)+15b^2c^2\right)}{15c^3e^5\sqrt{ex}}-\frac{2\sqrt[4]{d}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\,(7ad-18bc)+15b^2c^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{11}{4}}e^{\frac{11}{2}}\sqrt{c+dx^2}}+\frac{\sqrt[4]{d}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\,(7ad-18bc)+15b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{11}{4}}e^{\frac{11}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(11/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*a**2*sqrt(c + d*x**2)/(9*c*e*(e*x)**(9/2)) + 2*a*sqrt(c + d*x*
*2)*(7*a*d - 18*b*c)/(45*c**2*e**3*(e*x)**(5/2)) + 2*sqrt(d)*sqrt
(e*x)*sqrt(c + d*x**2)*(a*d*(7*a*d - 18*b*c) + 15*b**2*c**2)/(15*
c**3*e**6*(sqrt(c) + sqrt(d)*x)) - 2*sqrt(c + d*x**2)*(a*d*(7*a*d
 - 18*b*c) + 15*b**2*c**2)/(15*c**3*e**5*sqrt(e*x)) - 2*d**(1/4)*
sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)
*(a*d*(7*a*d - 18*b*c) + 15*b**2*c**2)*elliptic_e(2*atan(d**(1/4)
*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(15*c**(11/4)*e**(11/2)*sqrt
(c + d*x**2)) + d**(1/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)*
*2)*(sqrt(c) + sqrt(d)*x)*(a*d*(7*a*d - 18*b*c) + 15*b**2*c**2)*e
lliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(15
*c**(11/4)*e**(11/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.694552, size = 288, normalized size = 0.66

$$\frac{\sqrt{ex}\left(-6\sqrt{c}\sqrt{d}x^5\sqrt{\frac{dx^2}{c}+1}\left(7a^2d^2-18abcd+15b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{d}x}{\sqrt{c}}}\right)\middle|-1\right)+6\sqrt{c}\sqrt{d}x^5\sqrt{\frac{dx^2}{c}+1}\left(7a^2d^2-18abcd+\right.\right.}{45c^3e^6x^5\sqrt{\frac{i\sqrt{}}{}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[e*x]*(-2*Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]*(c + d*x^2)*(45*b^2*c^
2*x^4 + 18*a*b*c*x^2*(c - 3*d*x^2) + a^2*(5*c^2 - 7*c*d*x^2 + 21*
d^2*x^4)) + 6*Sqrt[c]*Sqrt[d]*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^
2)*x^5*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)
/Sqrt[c]]], -1] - 6*Sqrt[c]*Sqrt[d]*(15*b^2*c^2 - 18*a*b*c*d + 7*
a^2*d^2)*x^5*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[d]*x)/Sqrt[c]]], -1]))/(45*c^3*e^6*x^5*Sqrt[(I*Sqrt[d]*x)/Sqrt[c
]]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.055, size = 667, normalized size = 1.5

$$\frac{1}{45\,x^4e^5c^3}\left(42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\mathit{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^4a^2cd^2-108\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/45/x^4*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*El
lipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*
a^2*c*d^2-108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4
*a*b*c^2*d+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4
*b^2*c^3-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a
^2*c*d^2+54*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a
*b*c^2*d-45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b
^2*c^3-42*x^6*a^2*d^3+108*x^6*a*b*c*d^2-90*x^6*b^2*c^2*d-28*x^4*a
^2*c*d^2+72*x^4*a*b*c^2*d-90*x^4*b^2*c^3+4*x^2*a^2*c^2*d-36*x^2*a
*b*c^3-10*a^2*c^3)/(d*x^2+c)^(1/2)/e^5/(e*x)^(1/2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2}{\sqrt{dx^2 + c}\,(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^4 + 2\,abx^2 + a^2}{\sqrt{dx^2 + c}\sqrt{ex}e^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)*e
^5*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(11/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)), x)
```

3.848 $\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77b^2c^2-5ad(22bc-9ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231c^{13/4}e^{13/2}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\left(77b^2c^2-5ad(22bc-9ad)\right)}{231c^3e^5(ex)^{3/2}}-\frac{2a\sqrt{c+dx^2}(22bc-9ad)}{77c^2e^3(ex)^{7/2}}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(11*c*e*(e*x)^(11/2)) - (2*a*(22*b*c - 9
*a*d)*Sqrt[c + d*x^2])/(77*c^2*e^3*(e*x)^(7/2)) - (2*(77*b^2*c^2
- 5*a*d*(22*b*c - 9*a*d))*Sqrt[c + d*x^2])/(231*c^3*e^5*(e*x)^(3/
2)) - (d^(3/4)*(77*b^2*c^2 - 5*a*d*(22*b*c - 9*a*d))*(Sqrt[c] + S
qrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*A
rcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*c^(13/4)
*e^(13/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.573652, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77b^2c^2-5ad(22bc-9ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231c^{13/4}e^{13/2}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\left(77b^2c^2-5ad(22bc-9ad)\right)}{231c^3e^5(ex)^{3/2}}-\frac{2a\sqrt{c+dx^2}(22bc-9ad)}{77c^2e^3(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(11*c*e*(e*x)^(11/2)) - (2*a*(22*b*c - 9
*a*d)*Sqrt[c + d*x^2])/(77*c^2*e^3*(e*x)^(7/2)) - (2*(77*b^2*c^2
- 5*a*d*(22*b*c - 9*a*d))*Sqrt[c + d*x^2])/(231*c^3*e^5*(e*x)^(3/
2)) - (d^(3/4)*(77*b^2*c^2 - 5*a*d*(22*b*c - 9*a*d))*(Sqrt[c] + S
qrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*A
rcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*c^(13/4)
*e^(13/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 52.4674, size = 231, normalized size = 0.95

$$-\frac{2a^2\sqrt{c+dx^2}}{11ce\,(ex)^{\frac{11}{2}}}+\frac{2a\sqrt{c+dx^2}\,(9ad-22bc)}{77c^2e^3\,(ex)^{\frac{7}{2}}}-\frac{2\sqrt{c+dx^2}\,\left(5ad\,(9ad-22bc)+77b^2c^2\right)}{231c^3e^5\,(ex)^{\frac{3}{2}}}-\frac{d^{\frac{3}{4}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(5ad\,(9ad-22bc)+77b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{231c^{\frac{13}{4}}e^{\frac{13}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(13/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*a**2*sqrt(c + d*x**2)/(11*c*e*(e*x)**(11/2)) + 2*a*sqrt(c + d*
x**2)*(9*a*d - 22*b*c)/(77*c**2*e**3*(e*x)**(7/2)) - 2*sqrt(c + d
*x**2)*(5*a*d*(9*a*d - 22*b*c) + 77*b**2*c**2)/(231*c**3*e**5*(e*
x)**(3/2)) - d**(3/4)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)
*(sqrt(c) + sqrt(d)*x)*(5*a*d*(9*a*d - 22*b*c) + 77*b**2*c**2)*el
liptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(231
*c**(13/4)*e**(13/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.376315, size = 196, normalized size = 0.81

$$\frac{x^{13/2}\left(-\frac{2\left(c+dx^2\right)\left(3a^2\left(7c^2-9cdx^2+15d^2x^4\right)+22abcx^2\left(3c-5dx^2\right)+77b^2c^2x^4\right)}{c^3x^{11/2}}-\frac{2idx\sqrt{\frac{c}{dx^2}+1}\left(45a^2d^2-110abcd+77b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231(ex)^{13/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(13/2)*((-2*(c + d*x^2)*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c -
5*d*x^2) + 3*a^2*(7*c^2 - 9*c*d*x^2 + 15*d^2*x^4)))/(c^3*x^(11/2)
) - ((2*I)*d*(77*b^2*c^2 - 110*a*b*c*d + 45*a^2*d^2)*Sqrt[1 + c/(
d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
 -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]])))/(231*(e*x)^(13/2)*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.051, size = 411, normalized size = 1.7

$$-\frac{1}{231\,x^5c^3e^6}\left(45\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}x^5a^2d^2-110\sqrt{\frac{dx+\sqrt{-}}{\sqrt{-cd}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/231/(d*x^2+c)^(1/2)/x^5*(45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)
^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*(-c*d)^(1/2)*x^5*a^2*d^2-110*((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*a*b*c*d+77*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*x^5*b^2*c^2+90*
x^6*a^2*d^3-220*x^6*a*b*c*d^2+154*x^6*b^2*c^2*d+36*x^4*a^2*c*d^2-
88*x^4*a*b*c^2*d+154*x^4*b^2*c^3-12*x^2*a^2*c^2*d+132*x^2*a*b*c^3
+42*a^2*c^3)/c^3/e^6/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4 + 2\,abx^2 + a^2}{\sqrt{dx^2 + c}\sqrt{ex}e^6x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/(sqrt(d*x^2 + c)*sqrt(e*x)*e
^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(13/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{\sqrt{dx^2 + c}\,(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)), x)
```

3.849 $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\,(77a^2d^2-198abcd+117b^2c^2)}{231d^4}-\frac{e(ex)^{5/2}\sqrt{c+dx^2}\,(77a^2d^2-198abcd+117b^2c^2)}{77cd^3}-\frac{5c^{3/4}e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(77a^2d^2-198abcd+117b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{462d^{17/4}\sqrt{c+dx^2}}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (5*(117*b
^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])
/(231*d^4) - ((117*b^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e*(e*x)^(5
/2)*Sqrt[c + d*x^2])/(77*c*d^3) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x
^2])/(11*d^2*e) - (5*c^(3/4)*(117*b^2*c^2 - 198*a*b*c*d + 77*a^2*
d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], 1/2])/(462*d^(17/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.617181, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\,(77a^2d^2-198abcd+117b^2c^2)}{231d^4}-\frac{e(ex)^{5/2}\sqrt{c+dx^2}\,(77a^2d^2-198abcd+117b^2c^2)}{77cd^3}-\frac{5c^{3/4}e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(77a^2d^2-198abcd+117b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{462d^{17/4}\sqrt{c+dx^2}}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (5*(117*b
^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])
/(231*d^4) - ((117*b^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e*(e*x)^(5
/2)*Sqrt[c + d*x^2])/(77*c*d^3) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x
^2])/(11*d^2*e) - (5*c^(3/4)*(117*b^2*c^2 - 198*a*b*c*d + 77*a^2*
d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], 1/2])/(462*d^(17/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 69.6021, size = 282, normalized size = 0.95

$$\frac{2b^2(ex)^{\frac{9}{2}}\sqrt{c+dx^2}}{11d^2e} - \frac{5c^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(77a^2d^2-198abcd+117b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{462d^{\frac{17}{4}}\sqrt{c+dx^2}} + \frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-198abcd+117b^2c^2\right)}{231d^4} + \frac{(ex)^{\frac{9}{2}}(ad-bc)^2}{cd^2e\sqrt{c+dx^2}} - \frac{e(ex)^{\frac{5}{2}}\sqrt{c+dx^2}\left(77a^2d^2-198abcd+117b^2c^2\right)}{77cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] 2*b**2*(e*x)**(9/2)*sqrt(c + d*x**2)/(11*d**2*e) - 5*c**(3/4)*e**
(7/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt
(d)*x)*(77*a**2*d**2 - 198*a*b*c*d + 117*b**2*c**2)*elliptic_f(2*
atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(462*d**(17/4)*
sqrt(c + d*x**2)) + 5*e**3*sqrt(e*x)*sqrt(c + d*x**2)*(77*a**2*d*
*2 - 198*a*b*c*d + 117*b**2*c**2)/(231*d**4) + (e*x)**(9/2)*(a*d
- b*c)**2/(c*d**2*e*sqrt(c + d*x**2)) - e*(e*x)**(5/2)*sqrt(c + d
*x**2)*(77*a**2*d**2 - 198*a*b*c*d + 117*b**2*c**2)/(77*c*d**3)
```

Mathematica [C] time = 0.434476, size = 226, normalized size = 0.76

$$\frac{e^3\sqrt{ex}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(77a^2d^2\left(5c+2dx^2\right)+66abd\left(-15c^2-6cdx^2+2d^2x^4\right)+3b^2\left(195c^3+78c^2dx^2-26cd^2x^4+14d^3x^6\right)\right)-5ic\sqrt{}\right.}{231d^4\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(77*a^2*d^2*(5*c + 2*d*
x^2) + 66*a*b*d*(-15*c^2 - 6*c*d*x^2 + 2*d^2*x^4) + 3*b^2*(195*c^
3 + 78*c^2*d*x^2 - 26*c*d^2*x^4 + 14*d^3*x^6)) - (5*I)*c*(117*b^2
*c^2 - 198*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*Elli
pticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(231*Sq
rt[(I*Sqrt[c])/Sqrt[d]]*d^4*Sqrt[c + d*x^2])
```

Maple [A] time = 0.056, size = 407, normalized size = 1.4

$$-\frac{e^3}{462\,xd^5}\sqrt{ex}\left(-84\,x^7b^2d^4+385\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/462*e^3/x*(e*x)^(1/2)*(-84*x^7*b^2*d^4+385*(-c*d)^(1/2)*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-990*(-c*d)^(1
/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
```

```
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+585
*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*c^3-264*x^5*a*b*d^4+156*x^5*b^2*c*d^3-308*x^3*a^2*d^4+792*x^3*a*
b*c*d^3-468*x^3*b^2*c^2*d^2-770*x*a^2*c*d^3+1980*x*a*b*c^2*d^2-11
70*x*b^2*c^3*d)/(d*x^2+c)^(1/2)/d^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2(ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2e^3x^7+2\,abe^3x^5+a^2e^3x^3)\sqrt{ex}}{(dx^2+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^3*x^7 + 2*a*b*e^3*x^5 + a^2*e^3*x^3)*sqrt(e*x)/(d
*x^2 + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2(ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.850 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=436

$$\frac{\sqrt[4]{c}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(45a^2d^2-126abcd+77b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{30d^{15/4}\sqrt{c+dx^2}} - \frac{\sqrt[4]{c}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(45a^2d^2-126abcd+77b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{15/4}\sqrt{c+dx^2}} + \frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{15d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{e(ex)^{3/2}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{45cd^3} + \frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*
c^2 - 126*a*b*c*d + 45*a^2*d^2)*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(4
5*c*d^3) + (2*b^2*(e*x)^(7/2)*Sqrt[c + d*x^2])/(9*d^2*e) + ((77*b
^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])
/(15*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) - (c^(1/4)*(77*b^2*c^2 - 126*
a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x
^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(15/4)*Sqrt[c + d*x^2]) + (c^(
1/4)*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + S
qrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*A
rcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(30*d^(15/4)*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.911696, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt[4]{c}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(45a^2d^2-126abcd+77b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{30d^{15/4}\sqrt{c+dx^2}} - \frac{\sqrt[4]{c}e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(45a^2d^2-126abcd+77b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{15/4}\sqrt{c+dx^2}} + \frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{15d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{e(ex)^{3/2}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{45cd^3} + \frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*
c^2 - 126*a*b*c*d + 45*a^2*d^2)*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(4
5*c*d^3) + (2*b^2*(e*x)^(7/2)*Sqrt[c + d*x^2])/(9*d^2*e) + ((77*b
^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])
/(15*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) - (c^(1/4)*(77*b^2*c^2 - 126*
a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x
^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(15/4)*Sqrt[c + d*x^2]) + (c^(
1/4)*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + S
qrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*A
rcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(30*d^(15/4)*
Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 104.712, size = 410, normalized size = 0.94

$$\frac{2b^2(ex)^{\frac{7}{2}}\sqrt{c+dx^2}}{9d^2e} - \frac{\sqrt[4]{c}e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(45a^2d^2-126abcd+77b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{15d^{\frac{15}{4}}\sqrt{c+dx^2}} + \frac{\sqrt[4]{c}e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(45a^2d^2-126abcd+77b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{30d^{\frac{15}{4}}\sqrt{c+dx^2}} + \frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{15d^{\frac{7}{2}}\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{(ex)^{\frac{7}{2}}(ad-bc)^2}{cd^2e\sqrt{c+dx^2}} - \frac{e(ex)^{\frac{3}{2}}\sqrt{c+dx^2}\left(45a^2d^2-126abcd+77b^2c^2\right)}{45cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] 2*b**2*(e*x)**(7/2)*sqrt(c + d*x**2)/(9*d**2*e) - c**(1/4)*e**(5/
2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)
*x)*(45*a**2*d**2 - 126*a*b*c*d + 77*b**2*c**2)*elliptic_e(2*atan
(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(15*d**(15/4)*sqrt(
c + d*x**2)) + c**(1/4)*e**(5/2)*sqrt((c + d*x**2)/(sqrt(c) + sqr
t(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(45*a**2*d**2 - 126*a*b*c*d + 7
7*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(
e))), 1/2)/(30*d**(15/4)*sqrt(c + d*x**2)) + e**2*sqrt(e*x)*sqrt(
c + d*x**2)*(45*a**2*d**2 - 126*a*b*c*d + 77*b**2*c**2)/(15*d**(7
/2)*(sqrt(c) + sqrt(d)*x)) + (e*x)**(7/2)*(a*d - b*c)**2/(c*d**2*
e*sqrt(c + d*x**2)) - e*(e*x)**(3/2)*sqrt(c + d*x**2)*(45*a**2*d*
*2 - 126*a*b*c*d + 77*b**2*c**2)/(45*c*d**3)
```

Mathematica [C] time = 1.16269, size = 276, normalized size = 0.63

$$\frac{(ex)^{5/2}\left(dx^2\left(-45a^2d^2+18abd\left(7c+2dx^2\right)+b^2\left(-77c^2-22cdx^2+10d^2x^4\right)\right)+\frac{3\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)+\sqrt{c}\sqrt{d}x^{3/\cdots}\right.}{\cdots}\right)}{45d^4x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((e*x)^(5/2)*(d*x^2*(-45*a^2*d^2 + 18*a*b*d*(7*c + 2*d*x^2) + b^2
*(-77*c^2 - 22*c*d*x^2 + 10*d^2*x^4)) + (3*(77*b^2*c^2 - 126*a*b*
c*d + 45*a^2*d^2)*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(c + d*x^2) - Sqrt[c
]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] + Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*
x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[
x]], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(45*d^4*x^3*Sqrt[c + d*x^2
])
```

Maple [A] time = 0.056, size = 618, normalized size = 1.4

$$\frac{e^2}{90\,xd^4}\sqrt{ex}\left(20\,x^6b^2d^3+270\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-756\sqrt{\frac{dx}{\cdots}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/90/x*e^2*(e*x)^(1/2)*(20*x^6*b^2*d^3+270*((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-756*((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2),1/2*2^(1/2))*b^2*c^3-135*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*c*d^2+378*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*b*c^2*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*b^2*c^3+72*x^4*a*b*d^3-44*x^4*b^2*c*d^2-90*x^2*a^2*d^3+2
52*x^2*a*b*c*d^2-154*x^2*b^2*c^2*d)/(d*x^2+c)^(1/2)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2 (ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2)\sqrt{ex}}{(dx^2+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)*sqrt(e*x)/(d
*x^2 + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.851 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=245

$$\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-70abcd+45b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{42\sqrt[4]{c}d^{13/4}\sqrt{c+dx^2}}-\frac{e\sqrt{ex}\sqrt{c+dx^2}\left(21a^2d^2-70abcd+45b^2c^2\right)}{21cd^3}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((45*b^2*
c^2 - 70*a*b*c*d + 21*a^2*d^2)*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c
*d^3) + (2*b^2*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d^2*e) + ((45*b^2*
c^2 - 70*a*b*c*d + 21*a^2*d^2)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(42*c^(1/4)*d^(13/4)*Sqrt[c
 + d*x^2])
```

Rubi [A] time = 0.52173, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(21a^2d^2-70abcd+45b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{42\sqrt[4]{c}d^{13/4}\sqrt{c+dx^2}}-\frac{e\sqrt{ex}\sqrt{c+dx^2}\left(21a^2d^2-70abcd+45b^2c^2\right)}{21cd^3}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((45*b^2*
c^2 - 70*a*b*c*d + 21*a^2*d^2)*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c
*d^3) + (2*b^2*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d^2*e) + ((45*b^2*
c^2 - 70*a*b*c*d + 21*a^2*d^2)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(42*c^(1/4)*d^(13/4)*Sqrt[c
 + d*x^2])
```

Rubi in Sympy [A] time = 60.1852, size = 228, normalized size = 0.93

$$\frac{2b^2(ex)^{\frac{5}{2}}\sqrt{c+dx^2}}{7d^2e}+\frac{(ex)^{\frac{5}{2}}(ad-bc)^2}{cd^2e\sqrt{c+dx^2}}-\frac{e\sqrt{ex}\sqrt{c+dx^2}\left(21a^2d^2-70abcd+45b^2c^2\right)}{21cd^3}+\frac{e^{\frac{3}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(21a^2d^2-70abcd+45b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{42\sqrt[4]{c}d^{\frac{13}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] 2*b**2*(e*x)**(5/2)*sqrt(c + d*x**2)/(7*d**2*e) + (e*x)**(5/2)*(a
*d - b*c)**2/(c*d**2*e*sqrt(c + d*x**2)) - e*sqrt(e*x)*sqrt(c + d
*x**2)*(21*a**2*d**2 - 70*a*b*c*d + 45*b**2*c**2)/(21*c*d**3) + e
**(3/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sq
rt(d)*x)*(21*a**2*d**2 - 70*a*b*c*d + 45*b**2*c**2)*elliptic_f(2*
atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(42*c**(1/4)*d*
```

```
*(13/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.328986, size = 191, normalized size = 0.78

$$\frac{e\sqrt{ex}\left(i\sqrt{x}\sqrt{\frac{c}{dx^2}+1}\left(21a^2d^2-70abcd+45b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(-21a^2d^2+14abd\left(5c+2dx^2\right)-3b^2\left(1\right.\right.}{21d^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-21*a^2*d^2 + 14*a*b*d*(
5*c + 2*d*x^2) - 3*b^2*(15*c^2 + 6*c*d*x^2 - 2*d^2*x^4)) + I*(45*
b^2*c^2 - 70*a*b*c*d + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(21*S
qrt[(I*Sqrt[c])/Sqrt[d]]*d^3*Sqrt[c + d*x^2])
```

Maple [A] time = 0.032, size = 363, normalized size = 1.5

$$\frac{e}{42\,xd^4}\sqrt{ex}\left(21\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2d^2-70\sqrt{-cd}\sqrt{\frac{dx+\sqrt{}}{\sqrt{-c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/42*e/x*(e*x)^(1/2)*(21*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^2*d^2-70*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+45*(-c*d)^(1/2)*((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+12*x^5*b^2*d^3+56*x^3*
a*b*d^3-36*x^3*b^2*c*d^2-42*x*a^2*d^3+140*x*a*b*c*d^2-90*x*b^2*c^
2*d)/(d*x^2+c)^(1/2)/d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2ex^5+2\,abex^3+a^2ex\right)\sqrt{ex}}{\left(dx^2+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(e*x)/(d*x^2 + c
)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 (ex)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.852 $\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=384

$$-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-30abcd+21b^2c^2\right)}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-30abcd+21b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{11/4}\sqrt{c+dx^2}}+\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-30abcd+21b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{11/4}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*(e
*x)^(3/2)*Sqrt[c + d*x^2])/(5*d^2*e) - ((21*b^2*c^2 - 30*a*b*c*d
+ 5*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c*d^(5/2)*(Sqrt[c] + S
qrt[d]*x)) + ((21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*(Sqrt
[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Ellipt
icE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(
3/4)*d^(11/4)*Sqrt[c + d*x^2]) - ((21*b^2*c^2 - 30*a*b*c*d + 5*a^
2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(10*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.801673, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-30abcd+21b^2c^2\right)}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-30abcd+21b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{11/4}\sqrt{c+dx^2}}+\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2-30abcd+21b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{11/4}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*(e
*x)^(3/2)*Sqrt[c + d*x^2])/(5*d^2*e) - ((21*b^2*c^2 - 30*a*b*c*d
+ 5*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c*d^(5/2)*(Sqrt[c] + S
qrt[d]*x)) + ((21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*(Sqrt
[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Ellipt
icE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(
3/4)*d^(11/4)*Sqrt[c + d*x^2]) - ((21*b^2*c^2 - 30*a*b*c*d + 5*a^
2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(10*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 91.0302, size = 357, normalized size = 0.93

$$\frac{2b^2(ex)^{\frac{3}{2}}\sqrt{c+dx^2}}{5d^2e}+\frac{(ex)^{\frac{3}{2}}(ad-bc)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-30abcd+21b^2c^2\right)}{5cd^{\frac{5}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(5a^2d^2-30abcd+21b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}d^{\frac{11}{4}}\sqrt{c+dx^2}}-\frac{\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(5a^2d^2-30abcd+21b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{10c^{\frac{3}{4}}d^{\frac{11}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] 2*b**2*(e*x)**(3/2)*sqrt(c + d*x**2)/(5*d**2*e) + (e*x)**(3/2)*(a
*d - b*c)**2/(c*d**2*e*sqrt(c + d*x**2)) - sqrt(e*x)*sqrt(c + d*x
**2)*(5*a**2*d**2 - 30*a*b*c*d + 21*b**2*c**2)/(5*c*d**(5/2)*(sqr
t(c) + sqrt(d)*x)) + sqrt(e)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)
*x)**2)*(sqrt(c) + sqrt(d)*x)*(5*a**2*d**2 - 30*a*b*c*d + 21*b**2
*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))),
1/2)/(5*c**(3/4)*d**(11/4)*sqrt(c + d*x**2)) - sqrt(e)*sqrt((c +
d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(5*a**2*d
**2 - 30*a*b*c*d + 21*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(
e*x)/(c**(1/4)*sqrt(e))), 1/2)/(10*c**(3/4)*d**(11/4)*sqrt(c + d*
x**2))
```

Mathematica [C] time = 0.878989, size = 244, normalized size = 0.64

$$\frac{e\left(dx^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(5(bc-ad)^2+2b^2c\left(c+dx^2\right)\right)-\left(5a^2d^2-30abcd+21b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)+\sqrt{c}\sqrt{d}x^{3/2}\sqrt{\frac{c}{dx^2}+1}\left(F\left(i\sinh\right.\right.\right.\right.}{5cd^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*x^2*(5*(b*c - a*d)^2 + 2*b^2*c*(c
 + d*x^2)) - (21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*(Sqrt[(I*Sqrt[
c])/Sqrt[d]]*(c + d*x^2) + Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^
(3/2)*(-EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -
1] + EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])
)))/(5*c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.031, size = 597, normalized size = 1.6

$$-\frac{1}{10\,d^3xc}\sqrt{ex}\left(10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-60\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/10*(e*x)^(1/2)*(10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
```

```
^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a^2*c*d^2-60*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/
2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a*b*c^2*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*c^3-5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+
30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-21*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-4*x^4*b^2*
c*d^2-10*x^2*a^2*d^3+20*x^2*a*b*c*d^2-14*x^2*b^2*c^2*d)/(d*x^2+c)
^(1/2)/d^3/x/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ex}}{\left(dx^2+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(e*x)/(d*x^2 + c)^(3/2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(e*x)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2), x)
```

3.853 $\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-3a^2d^2-6abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{5/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e}$$

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*Sqrt
[e*x]*Sqrt[c + d*x^2])/(3*d^2*e) - ((5*b^2*c^2 - 6*a*b*c*d - 3*a^
2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
1/2])/(6*c^(5/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.420626, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-3a^2d^2-6abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{5/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*Sqrt
[e*x]*Sqrt[c + d*x^2])/(3*d^2*e) - ((5*b^2*c^2 - 6*a*b*c*d - 3*a^
2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
1/2])/(6*c^(5/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 51.1573, size = 177, normalized size = 0.92

$$\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e}+\frac{\sqrt{ex}\,(ad-bc)^2}{cd^2e\sqrt{c+dx^2}}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(3a^2d^2+6abcd-5b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{6c^{\frac{5}{4}}d^{\frac{9}{4}}\sqrt{e}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] 2*b**2*sqrt(e*x)*sqrt(c + d*x**2)/(3*d**2*e) + sqrt(e*x)*(a*d - b
*c)**2/(c*d**2*e*sqrt(c + d*x**2)) + sqrt((c + d*x**2)/(sqrt(c) +
 sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(3*a**2*d**2 + 6*a*b*c*d -
5*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(
e))), 1/2)/(6*c**(5/4)*d**(9/4)*sqrt(e)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.248336, size = 174, normalized size = 0.9

$$\frac{ix^{3/2}\sqrt{\frac{c}{dx^2}+1}\left(3a^2d^2+6abcd-5b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)+x\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(3a^2d^2-6abcd+b^2c\left(5c+2dx^2\right)\right)}{3cd^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[d]]*x*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(5*c
 + 2*d*x^2)) + I*(-5*b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + c/
(d*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sq
rt[x]], -1])/(3*c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2*Sqrt[e*x]*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.033, size = 341, normalized size = 1.8

$$\frac{1}{6\,cd^3}\left(3\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2d^2+6\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 1/6*(3*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(
1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*d^2+6*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b*c*d-5*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*b^2*c^2+4*x^3*b^2*c*d^2+6*x*a^2*d^3-12*x*a*b*c*d^2
+10*x*b^2*c^2*d)/(d*x^2+c)^(1/2)/c/(e*x)^(1/2)/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d*x^2 + c)^(3/2)*sqrt(e*x)
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(sqrt(e*x)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.854 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=393

$$-\frac{(ex)^{3/2}\left(3a^2d^2-2abcd+b^2c^2\right)}{c^2de^3\sqrt{c+dx^2}}+\frac{\sqrt{ex}\sqrt{c+dx^2}\left(3a^2d^2-2abcd+3b^2c^2\right)}{c^2d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3a^2d^2-2abcd+3b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3a^2d^2-2abcd+3b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*Sqrt[c + d*x^2]) - ((b^2*c^2 - 2*a*b*c*d
+ 3*a^2*d^2)*(e*x)^(3/2))/(c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c
^2 - 2*a*b*c*d + 3*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(c^2*d^(3/
2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - ((3*b^2*c^2 - 2*a*b*c*d + 3*a^2*d
^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) + ((3*b^2*c^2 - 2*a*
b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c
] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(2*c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.861488, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{(ex)^{3/2}\left(3a^2d^2-2abcd+b^2c^2\right)}{c^2de^3\sqrt{c+dx^2}}+\frac{\sqrt{ex}\sqrt{c+dx^2}\left(3a^2d^2-2abcd+3b^2c^2\right)}{c^2d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3a^2d^2-2abcd+3b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(3a^2d^2-2abcd+3b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*Sqrt[c + d*x^2]) - ((b^2*c^2 - 2*a*b*c*d
+ 3*a^2*d^2)*(e*x)^(3/2))/(c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c
^2 - 2*a*b*c*d + 3*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(c^2*d^(3/
2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - ((3*b^2*c^2 - 2*a*b*c*d + 3*a^2*d
^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) + ((3*b^2*c^2 - 2*a*
b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c
] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(2*c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 84.9013, size = 359, normalized size = 0.91

$$-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}-\frac{(ex)^{\frac{3}{2}}\left(ad\left(3ad-2bc\right)+b^2c^2\right)}{c^2de^3\sqrt{c+dx^2}}+\frac{\sqrt{ex}\sqrt{c+dx^2}\left(ad\left(3ad-2bc\right)+3b^2c^2\right)}{c^2d^{\frac{3}{2}}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\left(3ad-2bc\right)+3b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{7}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(ad\left(3ad-2bc\right)+3b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2c^{\frac{7}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] -2*a**2/(c*e*sqrt(e*x)*sqrt(c + d*x**2)) - (e*x)**(3/2)*(a*d*(3*a
*d - 2*b*c) + b**2*c**2)/(c**2*d*e**3*sqrt(c + d*x**2)) + sqrt(e*
x)*sqrt(c + d*x**2)*(a*d*(3*a*d - 2*b*c) + 3*b**2*c**2)/(c**2*d**
(3/2)*e**2*(sqrt(c) + sqrt(d)*x)) - sqrt((c + d*x**2)/(sqrt(c) +
sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(a*d*(3*a*d - 2*b*c) + 3*b**
2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))),
 1/2)/(c**(7/4)*d**(7/4)*e**(3/2)*sqrt(c + d*x**2)) + sqrt((c + d
*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(a*d*(3*a*
d - 2*b*c) + 3*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c
**(1/4)*sqrt(e))), 1/2)/(2*c**(7/4)*d**(7/4)*e**(3/2)*sqrt(c + d*
x**2))
```

Mathematica [C] time = 0.568719, size = 250, normalized size = 0.64

$$\frac{x\left(-\sqrt{c}x\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-2abcd+3b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{d}x}{\sqrt{c}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-2abcd+3b^2c^2\right)E\left(i\sinh\right.\right.}{c^2d^{3/2}(ex)^{3/2}\sqrt{\frac{i\sqrt{d}x}{\sqrt{c}}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(-(Sqrt[d]*Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]*(b^2*c^2*x^2 - 2*a*b*c*
d*x^2 + a^2*d*(2*c + 3*d*x^2))) + Sqrt[c]*(3*b^2*c^2 - 2*a*b*c*d
+ 3*a^2*d^2)*x*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(I*Sq
rt[d]*x)/Sqrt[c]]], -1] - Sqrt[c]*(3*b^2*c^2 - 2*a*b*c*d + 3*a^2*
d^2)*x*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)
/Sqrt[c]]], -1]))/(c^2*d^(3/2)*Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]*(e*x)^
(3/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.034, size = 594, normalized size = 1.5

$$\frac{1}{2\,d^2ec^2}\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)a^2cd^2-4\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\right.}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
```

```
E(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-
4*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+6*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-3*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+2*((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-3*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-6*x^2*a^2*d^3+4*x^2*a*b*c*d^2
-2*x^2*b^2*c^2*d-4*a^2*c*d^2)/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)/c
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{(dex^3+cex)\sqrt{dx^2+c}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d*e*x^3 + c*e*x)*sqrt(d*x^
2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.855 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{ex}\left(5a^2d^2-6abcd+3b^2c^2\right)}{3c^2de^3\sqrt{c+dx^2}}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-5ad)+3b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 6*a*
b*c*d + 5*a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*Sqrt[c + d*x^2]) + ((3
*b^2*c^2 + a*d*(6*b*c - 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(9/4)*d^(5/4)*e^(5/2)*Sqrt[c
+ d*x^2])
```

Rubi [A] time = 0.472997, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\sqrt{ex}\left(5a^2d^2-6abcd+3b^2c^2\right)}{3c^2de^3\sqrt{c+dx^2}}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(6bc-5ad)+3b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 6*a*
b*c*d + 5*a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*Sqrt[c + d*x^2]) + ((3
*b^2*c^2 + a*d*(6*b*c - 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(9/4)*d^(5/4)*e^(5/2)*Sqrt[c
+ d*x^2])
```

Rubi in Sympy [A] time = 45.4871, size = 189, normalized size = 0.91

$$-\frac{2a^2}{3ce\,(ex)^{\frac{3}{2}}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\left(ad\,(5ad-6bc)+3b^2c^2\right)}{3c^2de^3\sqrt{c+dx^2}}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-ad\,(5ad-6bc)+3b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{6c^{\frac{9}{4}}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] -2*a**2/(3*c*e*(e*x)**(3/2)*sqrt(c + d*x**2)) - sqrt(e*x)*(a*d*(5
*a*d - 6*b*c) + 3*b**2*c**2)/(3*c**2*d*e**3*sqrt(c + d*x**2)) + s
qrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*
(-a*d*(5*a*d - 6*b*c) + 3*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*s
qrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(6*c**(9/4)*d**(5/4)*e**(5/2)*
sqrt(c + d*x**2))
```

Mathematica [C] time = 0.297353, size = 181, normalized size = 0.87

$$\frac{x\left(-ix^{5/2}\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2-6abcd-3b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)-\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(a^2d\left(2c+5dx^2\right)-6abcdx^2+3b^2c^2x^2\right)\right)}{3c^2d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(-(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(3*b^2*c^2*x^2 - 6*a*b*c*d*x^2 +
a^2*d*(2*c + 5*d*x^2))) - I*(-3*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*
Sqrt[1 + c/(d*x^2)]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/
Sqrt[d]]/Sqrt[x]], -1]))/(3*c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*(e*x)
^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] time = 0.036, size = 353, normalized size = 1.7

$$-\frac{1}{6\,xc^2e^2d^2}\left(5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)\sqrt{-cd}xa^2d^2-6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/6/x*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^
(1/2)*x*a^2*d^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/
2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))
*(-c*d)^(1/2)*x*a*b*c*d-3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*(-c*d)^(1/2)*x*b^2*c^2+10*x^2*a^2*d^3-12*x^2*a*b*c*d^2+6
*x^2*b^2*c^2*d+4*a^2*c*d^2)/(d*x^2+c)^(1/2)/c^2/e^2/(e*x)^(1/2)/d
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2}{(dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{(de^2x^4+ce^2x^2)\sqrt{dx^2+c}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d*e^2*x^4 + c*e^2*x^2)*sqr
t(d*x^2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{3}{2}}\left(ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.856 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5b^2c^2-3ad(10bc-7ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5b^2c^2-3ad(10bc-7ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\left(5b^2c^2-3ad(10bc-7ad)\right)}{5c^3e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5b^2c^2-3ad(10bc-7ad)\right)}{5c^3\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}}$$

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*Sqrt[c + d*x^2]) - (2*a*(10*b*c - 7*a
*d))/(5*c^2*e^3*Sqrt[e*x]*Sqrt[c + d*x^2]) + ((5*b^2*c^2 - 3*a*d*
(10*b*c - 7*a*d))*(e*x)^(3/2))/(5*c^3*e^5*Sqrt[c + d*x^2]) - ((5*
b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c
^3*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) + ((5*b^2*c^2 - 3*a*d*(10*b
*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], 1/2])/(5*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) - ((5*b^
2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[
c + d*x^2])
```

Rubi [A] time = 1.01458, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5b^2c^2-3ad(10bc-7ad)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5b^2c^2-3ad(10bc-7ad)\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\left(5b^2c^2-3ad(10bc-7ad)\right)}{5c^3e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(5b^2c^2-3ad(10bc-7ad)\right)}{5c^3\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*Sqrt[c + d*x^2]) - (2*a*(10*b*c - 7*a
*d))/(5*c^2*e^3*Sqrt[e*x]*Sqrt[c + d*x^2]) + ((5*b^2*c^2 - 3*a*d*
(10*b*c - 7*a*d))*(e*x)^(3/2))/(5*c^3*e^5*Sqrt[c + d*x^2]) - ((5*
b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c
^3*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) + ((5*b^2*c^2 - 3*a*d*(10*b
*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
```

```
e])], 1/2])/(5*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) - ((5*b^
2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[
c + d*x^2])
```

Rubi in Sympy [A] time = 96.8345, size = 411, normalized size = 0.95

$$-\frac{2a^2}{5ce\,(ex)^{\frac{5}{2}}\sqrt{c+dx^2}}+\frac{2a\,(7ad-10bc)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}}+\frac{(ex)^{\frac{3}{2}}\left(3ad\,(7ad-10bc)+5b^2c^2\right)}{5c^3e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(3ad\,(7ad-10bc)+5b^2c^2\right)}{5c^3\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(3ad\,(7ad-10bc)+5b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{11}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}-\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(3ad\,(7ad-10bc)+5b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{10c^{\frac{11}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] -2*a**2/(5*c*e*(e*x)**(5/2)*sqrt(c + d*x**2)) + 2*a*(7*a*d - 10*b
*c)/(5*c**2*e**3*sqrt(e*x)*sqrt(c + d*x**2)) + (e*x)**(3/2)*(3*a*
d*(7*a*d - 10*b*c) + 5*b**2*c**2)/(5*c**3*e**5*sqrt(c + d*x**2))
- sqrt(e*x)*sqrt(c + d*x**2)*(3*a*d*(7*a*d - 10*b*c) + 5*b**2*c**
2)/(5*c**3*sqrt(d)*e**4*(sqrt(c) + sqrt(d)*x)) + sqrt((c + d*x**2
)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(3*a*d*(7*a*d -
 10*b*c) + 5*b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**
(1/4)*sqrt(e))), 1/2)/(5*c**(11/4)*d**(3/4)*e**(7/2)*sqrt(c + d*x
**2)) - sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sq
rt(d)*x)*(3*a*d*(7*a*d - 10*b*c) + 5*b**2*c**2)*elliptic_f(2*atan
(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(10*c**(11/4)*d**(3
/4)*e**(7/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.574978, size = 277, normalized size = 0.64

$$\frac{i\left(\sqrt{d}\sqrt{\frac{i\sqrt{d}x}{\sqrt{c}}}\left(a^2\left(-2c^2+14cdx^2+21d^2x^4\right)-10abcx^2\left(2c+3dx^2\right)+5b^2c^2x^4\right)+\sqrt{c}x^3\sqrt{\frac{dx^2}{c}+1}\left(21a^2d^2-30abcd+5b^2c^2\right)\right.}{5c^{7/2}e^2(ex)^{3/2}\left(\frac{i\sqrt{d}x}{\sqrt{c}}\right)^{3/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((I/5)*(Sqrt[d]*Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]*(5*b^2*c^2*x^4 - 10*a
*b*c*x^2*(2*c + 3*d*x^2) + a^2*(-2*c^2 + 14*c*d*x^2 + 21*d^2*x^4)
) - Sqrt[c]*(5*b^2*c^2 - 30*a*b*c*d + 21*a^2*d^2)*x^3*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]], -1] +
Sqrt[c]*(5*b^2*c^2 - 30*a*b*c*d + 21*a^2*d^2)*x^3*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]], -1]))/(c^(
7/2)*e^2*((I*Sqrt[d]*x)/Sqrt[c])^(3/2)*(e*x)^(3/2)*Sqrt[c + d*x^2
])
```

Maple [A] time = 0.036, size = 638, normalized size = 1.5

$$-\frac{1}{10\,dx^2e^3c^3}\left(42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}EllipticE\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\sqrt{2}\right)x^2a^2cd^2-60\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/10/x^2*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*a^2*c*d^2-60*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*a*b*c^2*d+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*b^2*c^3-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
^2*c*d^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b*c^2*d-5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^
2*c^3-42*x^4*a^2*d^3+60*x^4*a*b*c*d^2-10*x^4*b^2*c^2*d-28*x^2*a^2
*c*d^2+40*x^2*a*b*c^2*d+4*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*x)^
(1/2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{(de^3x^5+ce^3x^3)\sqrt{dx^2+c}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d*e^3*x^5 + c*e^3*x^3)*sqr
t(d*x^2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^2}{(dx^2 + c)^{\frac{3}{2}} (ex)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)), x)
```

3.857 $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{5e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-42abcd+39b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{84\sqrt[4]{c}d^{17/4}\sqrt{c+dx^2}} - \frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-42abcd+39b^2c^2\right)}{42cd^4} + \frac{e(ex)^{5/2}\left(7a^2d^2-42abcd+39b^2c^2\right)}{14cd^3\sqrt{c+dx^2}} + \frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((39*
b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*e*(e*x)^(5/2))/(14*c*d^3*Sqrt[c
 + d*x^2]) + (2*b^2*(e*x)^(9/2))/(7*d^2*e*Sqrt[c + d*x^2]) - (5*(
39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2
])/(42*c*d^4) + (5*(39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*e^(7/2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
84*c^(1/4)*d^(17/4)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.610062, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{5e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(7a^2d^2-42abcd+39b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{84\sqrt[4]{c}d^{17/4}\sqrt{c+dx^2}} - \frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-42abcd+39b^2c^2\right)}{42cd^4} + \frac{e(ex)^{5/2}\left(7a^2d^2-42abcd+39b^2c^2\right)}{14cd^3\sqrt{c+dx^2}} + \frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((39*
b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*e*(e*x)^(5/2))/(14*c*d^3*Sqrt[c
 + d*x^2]) + (2*b^2*(e*x)^(9/2))/(7*d^2*e*Sqrt[c + d*x^2]) - (5*(
39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2
])/(42*c*d^4) + (5*(39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*e^(7/2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
84*c^(1/4)*d^(17/4)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 71.672, size = 286, normalized size = 0.95

$$\frac{2b^2(ex)^{\frac{9}{2}}}{7d^2e\sqrt{c+dx^2}} + \frac{(ex)^{\frac{9}{2}}(ad-bc)^2}{3cd^2e\left(c+dx^2\right)^{\frac{3}{2}}} + \frac{e(ex)^{\frac{5}{2}}\left(7a^2d^2-42abcd+39b^2c^2\right)}{14cd^3\sqrt{c+dx^2}} - \frac{5e^3\sqrt{ex}\sqrt{c+dx^2}\left(7a^2d^2-42abcd+39b^2c^2\right)}{42cd^4} + \frac{5e^{\frac{7}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(7a^2d^2-42abcd+39b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{84\sqrt[4]{c}d^{\frac{17}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] 2*b**2*(e*x)**(9/2)/(7*d**2*e*sqrt(c + d*x**2)) + (e*x)**(9/2)*(a
*d - b*c)**2/(3*c*d**2*e*(c + d*x**2)**(3/2)) + e*(e*x)**(5/2)*(7
*a**2*d**2 - 42*a*b*c*d + 39*b**2*c**2)/(14*c*d**3*sqrt(c + d*x**
2)) - 5*e**3*sqrt(e*x)*sqrt(c + d*x**2)*(7*a**2*d**2 - 42*a*b*c*d
 + 39*b**2*c**2)/(42*c*d**4) + 5*e**(7/2)*sqrt((c + d*x**2)/(sqrt
(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(7*a**2*d**2 - 42*a*b*
c*d + 39*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4
)*sqrt(e))), 1/2)/(84*c**(1/4)*d**(17/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.476266, size = 222, normalized size = 0.74

$$\frac{(ex)^{7/2}\left(\frac{\sqrt{x}\left(-7a^2d^2\left(5c+7dx^2\right)+14abd\left(15c^2+21cdx^2+4d^2x^4\right)+b^2\left(-\left(195c^3+273c^2dx^2+52cd^2x^4-12d^3x^6\right)\right)\right)}{d^4\left(c+dx^2\right)}+\frac{5ix\sqrt{\frac{c}{dx^2}+1}\left(7a^2d^2-42abcd+39b^2c^2\right)F\left(i\sin\ldots\right.}{d^4\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{42x^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(7/2)*((Sqrt[x]*(-7*a^2*d^2*(5*c + 7*d*x^2) + 14*a*b*d*(15
*c^2 + 21*c*d*x^2 + 4*d^2*x^4) - b^2*(195*c^3 + 273*c^2*d*x^2 + 5
2*c*d^2*x^4 - 12*d^3*x^6)))/(d^4*(c + d*x^2)) + ((5*I)*(39*b^2*c^
2 - 42*a*b*c*d + 7*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/S
qrt[d]]*d^4)))/(42*x^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.062, size = 696, normalized size = 2.3

$$\frac{e^3}{84\,xd^5}\left(35\,EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\sqrt{-cd}x^2a^2d^3-210\,EllipticF\left(\sqrt{\frac{dx\ldots}{\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/84*(35*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*(-c*d)^(
1/2)*x^2*a^2*d^3-210*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*(-c*d)^(1/2)*x^2*a*b*c*d^2+195*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*
d)^(1/2)*d)^(1/2)*(-c*d)^(1/2)*x^2*b^2*c^2*d+24*x^7*b^2*d^4+35*(-
c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*
d^2-210*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^
(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a*b*c^2*d+195*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)
^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*b^2*c^3+112*x^5*a*b*d^4-104*x^5*b^2*c*d^3-98*x^3*a^
2*d^4+588*x^3*a*b*c*d^3-546*x^3*b^2*c^2*d^2-70*x*a^2*c*d^3+420*x*
a*b*c^2*d^2-390*x*b^2*c^3*d)*e^3/x*(e*x)^(1/2)/d^5/(d*x^2+c)^(3/2
```

)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2e^3x^7+2\,abe^3x^5+a^2e^3x^3\right)\sqrt{ex}}{(d^2x^4+2\,cdx^2+c^2)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e^3*x^7 + 2*a*b*e^3*x^5 + a^2*e^3*x^3)*sqrt(e*x)/((
d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2), x)
```

3.858 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=442

$$-\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\,(5a^2d^2-70abcd+77b^2c^2)}{10cd^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{e(ex)^{3/2}\,(5a^2d^2-70abcd+77b^2c^2)}{30cd^3\sqrt{c+dx^2}}-\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(5a^2d^2-70abcd+77b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{20c^{3/4}d^{15/4}\sqrt{c+dx^2}}+\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(5a^2d^2-70abcd+77b^2c^2)\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{15/4}\sqrt{c+dx^2}}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}+\frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((77*
b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e*(e*x)^(3/2))/(30*c*d^3*Sqrt[c
 + d*x^2]) + (2*b^2*(e*x)^(7/2))/(5*d^2*e*Sqrt[c + d*x^2]) - ((77
*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])
/(10*c*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + ((77*b^2*c^2 - 70*a*b*c*d
 + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqr
t[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(10*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2]) - ((77
*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(20*c^(3/4)*d^(15/4)*Sq
rt[c + d*x^2])
```

Rubi [A] time = 0.923458, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\,(5a^2d^2-70abcd+77b^2c^2)}{10cd^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{e(ex)^{3/2}\,(5a^2d^2-70abcd+77b^2c^2)}{30cd^3\sqrt{c+dx^2}}-\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(5a^2d^2-70abcd+77b^2c^2)\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{20c^{3/4}d^{15/4}\sqrt{c+dx^2}}+\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,(5a^2d^2-70abcd+77b^2c^2)\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{15/4}\sqrt{c+dx^2}}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}+\frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((77*
b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e*(e*x)^(3/2))/(30*c*d^3*Sqrt[c
 + d*x^2]) + (2*b^2*(e*x)^(7/2))/(5*d^2*e*Sqrt[c + d*x^2]) - ((77
*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])
/(10*c*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + ((77*b^2*c^2 - 70*a*b*c*d
 + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqr
t[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(10*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2]) - ((77
*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(20*c^(3/4)*d^(15/4)*Sq
rt[c + d*x^2])
```

Rubi in Sympy [A] time = 106.014, size = 413, normalized size = 0.93

$$\frac{2b^2(ex)^{\frac{7}{2}}}{5d^2e\sqrt{c+dx^2}}+\frac{(ex)^{\frac{7}{2}}(ad-bc)^2}{3cd^2e(c+dx^2)^{\frac{3}{2}}}+\frac{e(ex)^{\frac{3}{2}}(5a^2d^2-70abcd+77b^2c^2)}{30cd^3\sqrt{c+dx^2}}-\frac{e^2\sqrt{ex}\sqrt{c+dx^2}(5a^2d^2-70abcd+77b^2c^2)}{10cd^{\frac{7}{2}}(\sqrt{c}+\sqrt{d}x)}+\frac{e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}(\sqrt{c}+\sqrt{d}x)(5a^2d^2-70abcd+77b^2c^2)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{10c^{\frac{3}{4}}d^{\frac{15}{4}}\sqrt{c+dx^2}}-\frac{e^{\frac{5}{2}}\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}(\sqrt{c}+\sqrt{d}x)(5a^2d^2-70abcd+77b^2c^2)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{20c^{\frac{3}{4}}d^{\frac{15}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] 2*b**2*(e*x)**(7/2)/(5*d**2*e*sqrt(c + d*x**2)) + (e*x)**(7/2)*(a
*d - b*c)**2/(3*c*d**2*e*(c + d*x**2)**(3/2)) + e*(e*x)**(3/2)*(5
*a**2*d**2 - 70*a*b*c*d + 77*b**2*c**2)/(30*c*d**3*sqrt(c + d*x**
2)) - e**2*sqrt(e*x)*sqrt(c + d*x**2)*(5*a**2*d**2 - 70*a*b*c*d +
 77*b**2*c**2)/(10*c*d**(7/2)*(sqrt(c) + sqrt(d)*x)) + e**(5/2)*s
qrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*
(5*a**2*d**2 - 70*a*b*c*d + 77*b**2*c**2)*elliptic_e(2*atan(d**(1
/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(10*c**(3/4)*d**(15/4)*sq
rt(c + d*x**2)) - e**(5/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x
)**2)*(sqrt(c) + sqrt(d)*x)*(5*a**2*d**2 - 70*a*b*c*d + 77*b**2*c
**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/
2)/(20*c**(3/4)*d**(15/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.37026, size = 298, normalized size = 0.67

$$\frac{(ex)^{5/2}\left(-dx^2\left(-5a^2d^2\left(c+3dx^2\right)+10abcd\left(7c+9dx^2\right)+b^2(-c)\left(77c^2+99cdx^2+12d^2x^4\right)\right)-\frac{3\left(c+dx^2\right)\left(5a^2d^2-70abcd+77b^2c^2\right)}{\cdots}\right.}{30cd^4x^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(5/2)*(-(d*x^2*(-5*a^2*d^2*(c + 3*d*x^2) + 10*a*b*c*d*(7*c
 + 9*d*x^2) - b^2*c*(77*c^2 + 99*c*d*x^2 + 12*d^2*x^4))) - (3*(77
*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*(c + d*x^2)*(Sqrt[(I*Sqrt[c])/
Sqrt[d]]*(c + d*x^2) - Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2
)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1] + S
qrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*EllipticF[I*ArcSinh[Sq
rt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/Sqrt[(I*Sqrt[c])/Sqrt[d]]
))/(30*c*d^4*x^3*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.059, size = 1191, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)

[Out] -1/60*(30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3-420*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d^2+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^3+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d^2-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3*d-24*x^6*b^2*c*d^3+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-420*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-30*x^4*a^2*d^4+180*x^4*a*b*c*d^3-198*x^4*b^2*c^2*d^2-10*x^2*a^2*c*d^3+140*x^2*a*b*c^2*d^2-154*x^2*b^2*c^3*d)/x*e^2*(e*x)^(1/2)/d^4/c/(d*x^2+c)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2 (ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2\right)\sqrt{ex}}{(d^2x^4+2\,cdx^2+c^2)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2),x, algorithm="fricas")

[Out] integral((b^2*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2)*sqrt(e*x)/((

```
d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 (ex)^{\frac{5}{2}}}{(dx^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2), x)
```

3.859 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=248

$$\frac{e\sqrt{ex}\left(-a^2d^2-10abcd+15b^2c^2\right)}{6cd^3\sqrt{c+dx^2}}-\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-10abcd+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{5/4}d^{13/4}\sqrt{c+dx^2}}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}+\frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((15*
b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e*Sqrt[e*x])/(6*c*d^3*Sqrt[c + d*
x^2]) + (2*b^2*(e*x)^(5/2))/(3*d^2*e*Sqrt[c + d*x^2]) - ((15*b^2*
c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(5/4)*d^(13/4)*Sqrt[c +
d*x^2])
```

Rubi [A] time = 0.506208, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{e\sqrt{ex}\left(-a^2d^2-10abcd+15b^2c^2\right)}{6cd^3\sqrt{c+dx^2}}-\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-10abcd+15b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{5/4}d^{13/4}\sqrt{c+dx^2}}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}+\frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((15*
b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e*Sqrt[e*x])/(6*c*d^3*Sqrt[c + d*
x^2]) + (2*b^2*(e*x)^(5/2))/(3*d^2*e*Sqrt[c + d*x^2]) - ((15*b^2*
c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(5/4)*d^(13/4)*Sqrt[c +
d*x^2])
```

Rubi in Sympy [A] time = 61.5478, size = 226, normalized size = 0.91

$$\frac{2b^2(ex)^{\frac{5}{2}}}{3d^2e\sqrt{c+dx^2}}+\frac{(ex)^{\frac{5}{2}}(ad-bc)^2}{3cd^2e\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{e\sqrt{ex}\left(a^2d^2+10abcd-15b^2c^2\right)}{6cd^3\sqrt{c+dx^2}}+\frac{e^{\frac{3}{2}}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(a^2d^2+10abcd-15b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12c^{\frac{5}{4}}d^{\frac{13}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] 2*b**2*(e*x)**(5/2)/(3*d**2*e*sqrt(c + d*x**2)) + (e*x)**(5/2)*(a
*d - b*c)**2/(3*c*d**2*e*(c + d*x**2)**(3/2)) - e*sqrt(e*x)*(a**2
```

```
*d**2 + 10*a*b*c*d - 15*b**2*c**2)/(6*c*d**3*sqrt(c + d*x**2)) +
e**(3/2)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + s
qrt(d)*x)*(a**2*d**2 + 10*a*b*c*d - 15*b**2*c**2)*elliptic_f(2*at
an(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(12*c**(5/4)*d**(
13/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.423975, size = 204, normalized size = 0.82

$$\frac{(ex)^{3/2}\left(\frac{\sqrt{x}\left(a^2d^2\left(dx^2-c\right)-2abcd\left(5c+7dx^2\right)+b^2c\left(15c^2+21cdx^2+4d^2x^4\right)\right)}{cd^3\left(c+dx^2\right)}+\frac{ix\sqrt{\frac{c}{dx^2}+1}\left(a^2d^2+10abcd-15b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{cd^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{6x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(3/2)*((Sqrt[x]*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(5*c + 7
*d*x^2) + b^2*c*(15*c^2 + 21*c*d*x^2 + 4*d^2*x^4)))/(c*d^3*(c + d
*x^2)) + (I*(-15*b^2*c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^
2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]
)/(c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(6*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.032, size = 674, normalized size = 2.7

$$\frac{e}{12\,cxd^4}\left(EllipticF\left(\sqrt{1\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{1\left(dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{2}\sqrt{1\left(-dx+\sqrt{-cd}\right)\frac{1}{\sqrt{-cd}}}\sqrt{-dx\frac{1}{\sqrt{-cd}}}\sqrt{-cdx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12*(EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*(-c*d)^(1/2
)*x^2*a^2*d^3+10*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
(-c*d)^(1/2)*x^2*a*b*c*d^2-15*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*(-c*d)^(1/2)*x^2*b^2*c^2*d+(-c*d)^(1/2)*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+10*(-c*d)^(1/2)*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-15*(-c*d)^
(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+8*x
^5*b^2*c*d^3+2*x^3*a^2*d^4-28*x^3*a*b*c*d^3+42*x^3*b^2*c^2*d^2-2*
x*a^2*c*d^3-20*x*a*b*c^2*d^2+30*x*b^2*c^3*d)*e/x*(e*x)^(1/2)/c/d^
4/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2ex^5 + 2\,abex^3 + a^2ex\right)\sqrt{ex}}{(d^2x^4 + 2\,cdx^2 + c^2)\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*sqrt(e*x)/((d^2*x^4
+ 2*c*d*x^2 + c^2)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2 (ex)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.860 $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=403

$$\frac{\sqrt{ex}\sqrt{c+dx^2}\left(-a^2d^2-2abcd+7b^2c^2\right)}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-2abcd+7b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{7/4}d^{11/4}\sqrt{c+dx^2}}-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-2abcd+7b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{11/4}\sqrt{c+dx^2}}-\frac{(ex)^{3/2}(ad+3bc)(bc-ad)}{2c^2d^2e\sqrt{c+dx^2}}+\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c
 - a*d)*(3*b*c + a*d)*(e*x)^(3/2))/(2*c^2*d^2*e*Sqrt[c + d*x^2])
+ ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(
2*c^2*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - ((7*b^2*c^2 - 2*a*b*c*d -
a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], 1/2])/(2*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2]) + ((7*b^2*c^
2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c +
d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4*c^(7/4)*d^(11/4)*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.821614, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{\sqrt{ex}\sqrt{c+dx^2}\left(-a^2d^2-2abcd+7b^2c^2\right)}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-2abcd+7b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{7/4}d^{11/4}\sqrt{c+dx^2}}-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(-a^2d^2-2abcd+7b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{11/4}\sqrt{c+dx^2}}-\frac{(ex)^{3/2}(ad+3bc)(bc-ad)}{2c^2d^2e\sqrt{c+dx^2}}+\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c
 - a*d)*(3*b*c + a*d)*(e*x)^(3/2))/(2*c^2*d^2*e*Sqrt[c + d*x^2])
+ ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(
2*c^2*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - ((7*b^2*c^2 - 2*a*b*c*d -
a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], 1/2])/(2*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2]) + ((7*b^2*c^
2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c +
d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4*c^(7/4)*d^(11/4)*Sqrt[c + d*x^
2])
```

Rubi in Sympy [A] time = 93.8246, size = 369, normalized size = 0.92

$$\frac{(ex)^{\frac{3}{2}}(ad-bc)^2}{3cd^2e(c+dx^2)^{\frac{3}{2}}}+\frac{(ex)^{\frac{3}{2}}(ad-bc)(ad+3bc)}{2c^2d^2e\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(a^2d^2+2abcd-7b^2c^2\right)}{2c^2d^{\frac{5}{2}}\left(\sqrt{c}+\sqrt{d}x\right)}$$
$$+\frac{\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(a^2d^2+2abcd-7b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2c^{\frac{7}{4}}d^{\frac{11}{4}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{e}\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(a^2d^2+2abcd-7b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{4c^{\frac{7}{4}}d^{\frac{11}{4}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] (e*x)**(3/2)*(a*d - b*c)**2/(3*c*d**2*e*(c + d*x**2)**(3/2)) + (e
*x)**(3/2)*(a*d - b*c)*(a*d + 3*b*c)/(2*c**2*d**2*e*sqrt(c + d*x*
*2)) - sqrt(e*x)*sqrt(c + d*x**2)*(a**2*d**2 + 2*a*b*c*d - 7*b**2
*c**2)/(2*c**2*d**(5/2)*(sqrt(c) + sqrt(d)*x)) + sqrt(e)*sqrt((c
+ d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(a**2*d
**2 + 2*a*b*c*d - 7*b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt(e*
x)/(c**(1/4)*sqrt(e))), 1/2)/(2*c**(7/4)*d**(11/4)*sqrt(c + d*x**
2)) - sqrt(e)*sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c
) + sqrt(d)*x)*(a**2*d**2 + 2*a*b*c*d - 7*b**2*c**2)*elliptic_f(2
*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(4*c**(7/4)*d*
*(11/4)*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.33442, size = 281, normalized size = 0.7

$$\frac{e\left(dx^2\left(2c(bc-ad)^2-3\left(c+dx^2\right)\left(-a^2d^2-2abcd+3b^2c^2\right)\right)+\frac{3\left(c+dx^2\right)\left(-a^2d^2-2abcd+7b^2c^2\right)\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(c+dx^2\right)+\sqrt{c}\sqrt{d}x^{3/2}\sqrt{\frac{c}{dx^2}+1}F\left(i\,\mathrm{si}\cdots\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\cdots\right)}{6c^2d^3\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (e*(d*x^2*(2*c*(b*c - a*d)^2 - 3*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2
)*(c + d*x^2)) + (3*(7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*(c + d*x^2)
*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(c + d*x^2) - Sqrt[c]*Sqrt[d]*Sqrt[1
+ c/(d*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]
]/Sqrt[x]], -1] + Sqrt[c]*Sqrt[d]*Sqrt[1 + c/(d*x^2)]*x^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/Sqrt[(
I*Sqrt[c])/Sqrt[d]]))/(6*c^2*d^3*Sqrt[e*x]*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.033, size = 1176, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/12*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
```

```
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*
c*d^3+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d^2-42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^
2*c^3*d-3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2
*c*d^3-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d^2+21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^
2*c^3*d+6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2
*d^2+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d
-42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-3*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-6*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d+21*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-6*x^4*a^2*d^4-12*x^
4*a*b*c*d^3+18*x^4*b^2*c^2*d^2-10*x^2*a^2*c*d^3-4*x^2*a*b*c^2*d^2
+14*x^2*b^2*c^3*d)*(e*x)^(1/2)/d^3/c^2/x/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ex}}{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(e*x)/((d^2*x^4 + 2*c*d*
x^2 + c^2)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2\sqrt{ex}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2), x)
```

3.861 $\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2+2abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{9/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{\sqrt{ex}(5ad+7bc)(bc-ad)}{6c^2d^2e\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c -
 a*d)*(7*b*c + 5*a*d)*Sqrt[e*x])/(6*c^2*d^2*e*Sqrt[c + d*x^2]) +
((5*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(9/4)*d^(9/4)*Sqrt[e]*S
qrt[c + d*x^2])
```

Rubi [A] time = 0.437618, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5a^2d^2+2abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{9/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}-\frac{\sqrt{ex}(5ad+7bc)(bc-ad)}{6c^2d^2e\sqrt{c+dx^2}}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c -
 a*d)*(7*b*c + 5*a*d)*Sqrt[e*x])/(6*c^2*d^2*e*Sqrt[c + d*x^2]) +
((5*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(9/4)*d^(9/4)*Sqrt[e]*S
qrt[c + d*x^2])
```

Rubi in Sympy [A] time = 53.821, size = 194, normalized size = 0.91

$$\frac{\sqrt{ex}\left(ad-bc\right)^2}{3cd^2e\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{\sqrt{ex}\left(ad-bc\right)\left(5ad+7bc\right)}{6c^2d^2e\sqrt{c+dx^2}}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(5a^2d^2+2abcd+5b^2c^2\right)F\left(2\,\text{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12c^{\frac{9}{4}}d^{\frac{9}{4}}\sqrt{e}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] sqrt(e*x)*(a*d - b*c)**2/(3*c*d**2*e*(c + d*x**2)**(3/2)) + sqrt(
e*x)*(a*d - b*c)*(5*a*d + 7*b*c)/(6*c**2*d**2*e*sqrt(c + d*x**2))
 + sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)
*x)*(5*a**2*d**2 + 2*a*b*c*d + 5*b**2*c**2)*elliptic_f(2*atan(d**
(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(12*c**(9/4)*d**(9/4)*s
qrt(e)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.449948, size = 169, normalized size = 0.79

$$\frac{x\left(\frac{i\sqrt{x}\sqrt{\frac{c}{dx^2}+1}\left(5a^2d^2+2abcd+5b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}+5a^2d^2+\frac{2c(bc-ad)^2}{c+dx^2}+2abcd-7b^2c^2\right)}{6c^2d^2\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(-7*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2 + (2*c*(b*c - a*d)^2)/(c +
 d*x^2) + (I*(5*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^
2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]
], -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(6*c^2*d^2*Sqrt[e*x]*Sqrt[c +
 d*x^2])
```

Maple [B] time = 0.035, size = 660, normalized size = 3.1

$$\frac{1}{12\,c^2d^3}\left(5\,EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},1/2\,\sqrt{2}\right)\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{dx}{\sqrt{-cd}}}\sqrt{-cd}x^2a^2d^3+2\,EllipticF\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x)
```

```
[Out] 1/12*(5*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*(-c*d)^(1
/2)*x^2*a^2*d^3+2*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*(-c*d)^(1/2)*x^2*a*b*c*d^2+5*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*(-c*d)^(1/2)*x^2*b^2*c^2*d+5*(-c*d)^(1/2)*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+2*(-c*d)^(1/2)*((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+5*(-c*d)^
(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+10*
x^3*a^2*d^4+4*x^3*a*b*c*d^3-14*x^3*b^2*c^2*d^2+14*x*a^2*c*d^3-4*x
*a*b*c^2*d^2-10*x*b^2*c^3*d)/(e*x)^(1/2)/c^2/d^3/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4 + 2\,abx^2 + a^2}{(d^2x^4 + 2\,cdx^2 + c^2)\sqrt{dx^2 + c}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d^2*x^4 + 2*c*d*x^2 + c^2)
*sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{(dx^2 + c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.862 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=442

$$-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(2bc-7ad)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(2bc-7ad)+b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(ad(2bc-7ad)+b^2c^2\right)}{2c^3d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{(ex)^{3/2}\left(ad(2bc-7ad)+b^2c^2\right)}{2c^3de^3\sqrt{c+dx^2}}$$

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*
d + 7*a^2*d^2)*(e*x)^(3/2))/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b
^2*c^2 + a*d*(2*b*c - 7*a*d))*(e*x)^(3/2))/(2*c^3*d*e^3*Sqrt[c +
d*x^2]) - ((b^2*c^2 + a*d*(2*b*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x
^2])/(2*c^3*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) + ((b^2*c^2 + a*d*
(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], 1/2])/(2*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((
b^2*c^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*
x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], 1/2])/(4*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c
+ d*x^2])
```

Rubi [A] time = 1.01156, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(2bc-7ad)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(ad(2bc-7ad)+b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(ad(2bc-7ad)+b^2c^2\right)}{2c^3d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{(ex)^{3/2}\left(ad(2bc-7ad)+b^2c^2\right)}{2c^3de^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```


```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*
d + 7*a^2*d^2)*(e*x)^(3/2))/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b
^2*c^2 + a*d*(2*b*c - 7*a*d))*(e*x)^(3/2))/(2*c^3*d*e^3*Sqrt[c +
d*x^2]) - ((b^2*c^2 + a*d*(2*b*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x
^2])/(2*c^3*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) + ((b^2*c^2 + a*d*
(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], 1/2])/(2*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((
b^2*c^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*
x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], 1/2])/(4*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c
+ d*x^2])
```

Rubi in Sympy [A] time = 100.651, size = 406, normalized size = 0.92

$$-\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{\frac{3}{2}}} - \frac{(ex)^{\frac{3}{2}}\left(ad\,(7ad-2bc)+b^2c^2\right)}{3c^2de^3\,(c+dx^2)^{\frac{3}{2}}} + \frac{(ex)^{\frac{3}{2}}\left(-ad\,(7ad-2bc)+b^2c^2\right)}{2c^3de^3\sqrt{c+dx^2}} - \frac{\sqrt{ex}\sqrt{c+dx^2}\left(-ad\,(7ad-2bc)+b^2c^2\right)}{2c^3d^{\frac{3}{2}}e^2\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-ad\,(7ad-2bc)+b^2c^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{2c^{\frac{11}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}} - \frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-ad\,(7ad-2bc)+b^2c^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{4c^{\frac{11}{4}}d^{\frac{7}{4}}e^{\frac{3}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] -2*a**2/(c*e*sqrt(e*x)*(c + d*x**2)**(3/2)) - (e*x)**(3/2)*(a*d*(
7*a*d - 2*b*c) + b**2*c**2)/(3*c**2*d*e**3*(c + d*x**2)**(3/2)) +
 (e*x)**(3/2)*(-a*d*(7*a*d - 2*b*c) + b**2*c**2)/(2*c**3*d*e**3*s
qrt(c + d*x**2)) - sqrt(e*x)*sqrt(c + d*x**2)*(-a*d*(7*a*d - 2*b*
c) + b**2*c**2)/(2*c**3*d**(3/2)*e**2*(sqrt(c) + sqrt(d)*x)) + sq
rt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(
-a*d*(7*a*d - 2*b*c) + b**2*c**2)*elliptic_e(2*atan(d**(1/4)*sqrt
(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(2*c**(11/4)*d**(7/4)*e**(3/2)*sq
rt(c + d*x**2)) - sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(sq
rt(c) + sqrt(d)*x)*(-a*d*(7*a*d - 2*b*c) + b**2*c**2)*elliptic_f(
2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(4*c**(11/4)*
d**(7/4)*e**(3/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.798618, size = 222, normalized size = 0.5

$$\frac{x\left(\frac{a^2(-d)\left(12c^2+35cdx^2+21d^2x^4\right)+2abcdx^2\left(5c+3dx^2\right)+b^2c^2x^2\left(c+3dx^2\right)}{c+dx^2} - \frac{3ix^2\sqrt{\frac{dx^2}{c}+1}\left(-7a^2d^2+2abcd+b^2c^2\right)\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{d}x}{\sqrt{c}}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\right.\right.\right.}{\left(\frac{i\sqrt{d}x}{\sqrt{c}}\right)^{3/2}}\right)}{6c^3d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*((b^2*c^2*x^2*(c + 3*d*x^2) + 2*a*b*c*d*x^2*(5*c + 3*d*x^2) -
a^2*d*(12*c^2 + 35*c*d*x^2 + 21*d^2*x^4))/(c + d*x^2) - ((3*I)*(b
^2*c^2 + 2*a*b*c*d - 7*a^2*d^2)*x^2*Sqrt[1 + (d*x^2)/c]*(Elliptic
E[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]], -1] - EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]], -1]))/((I*Sqrt[d]*x)/Sqrt[c])^(3
/2)))/(6*c^3*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.039, size = 1187, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*
c*d^3-12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2
*c^3*d-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2
*c*d^3+6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*
c^2*d^2+3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2
*c^3*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2
*d^2-12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d
-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-21*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+6*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d+3*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-42*x^4*a^2*d^4+12*x^
4*a*b*c*d^3+6*x^4*b^2*c^2*d^2-70*x^2*a^2*c*d^3+20*x^2*a*b*c^2*d^2
+2*x^2*b^2*c^3*d-24*a^2*c^2*d^2)/d^2/c^3/e/(e*x)^(1/2)/(d*x^2+c)^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{\left(d^2ex^5+2\,cdex^3+c^2ex\right)\sqrt{dx^2+c}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d^2*e*x^5 + 2*c*d*e*x^3 +
c^2*e*x)*sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{\left(dx^2 + c\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.863 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{\sqrt{ex}\left(3a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5ad(2bc-3ad)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{13/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}+\frac{\sqrt{ex}\left(5ad(2bc-3ad)+b^2c^2\right)}{6c^3de^3\sqrt{c+dx^2}}$$

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*
b*c*d + 3*a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + (
(b^2*c^2 + 5*a*d*(2*b*c - 3*a*d))*Sqrt[e*x])/(6*c^3*d*e^3*Sqrt[c
+ d*x^2]) + ((b^2*c^2 + 5*a*d*(2*b*c - 3*a*d))*(Sqrt[c] + Sqrt[d]
*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(13/4)*d^(5/4
)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.582222, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$-\frac{\sqrt{ex}\left(3a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(5ad(2bc-3ad)+b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{13/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}+\frac{\sqrt{ex}\left(5ad(2bc-3ad)+b^2c^2\right)}{6c^3de^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*
b*c*d + 3*a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + (
(b^2*c^2 + 5*a*d*(2*b*c - 3*a*d))*Sqrt[e*x])/(6*c^3*d*e^3*Sqrt[c
+ d*x^2]) + ((b^2*c^2 + 5*a*d*(2*b*c - 3*a*d))*(Sqrt[c] + Sqrt[d]
*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(13/4)*d^(5/4
)*e^(5/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 56.4105, size = 236, normalized size = 0.91

$$-\frac{2a^2}{3ce\left(ex\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{\sqrt{ex}\left(ad\left(3ad-2bc\right)+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{\frac{3}{2}}}+\frac{\sqrt{ex}\left(-5ad\left(3ad-2bc\right)+b^2c^2\right)}{6c^3de^3\sqrt{c+dx^2}}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(-5ad\left(3ad-2bc\right)+b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{12c^{\frac{13}{4}}d^{\frac{5}{4}}e^{\frac{5}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] -2*a**2/(3*c*e*(e*x)**(3/2)*(c + d*x**2)**(3/2)) - sqrt(e*x)*(a*d
*(3*a*d - 2*b*c) + b**2*c**2)/(3*c**2*d*e**3*(c + d*x**2)**(3/2))
```

```
 + sqrt(e*x)*(-5*a*d*(3*a*d - 2*b*c) + b**2*c**2)/(6*c**3*d*e**3*
sqrt(c + d*x**2)) + sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(
sqrt(c) + sqrt(d)*x)*(-5*a*d*(3*a*d - 2*b*c) + b**2*c**2)*ellipti
c_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(12*c**(1
3/4)*d**(5/4)*e**(5/2)*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.448213, size = 211, normalized size = 0.82

$$\frac{x^{5/2}\left(\frac{a^2(-d)\left(4c^2+21cdx^2+15d^2x^4\right)+2abcdx^2\left(7c+5dx^2\right)+b^2c^2x^2\left(dx^2-c\right)}{c^3dx^{3/2}\left(c+dx^2\right)}+\frac{ix\sqrt{\frac{c}{dx^2}+1}\left(-15a^2d^2+10abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^3d\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{6(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^(5/2)*((b^2*c^2*x^2*(-c + d*x^2) + 2*a*b*c*d*x^2*(7*c + 5*d*x^
2) - a^2*d*(4*c^2 + 21*c*d*x^2 + 15*d^2*x^4))/(c^3*d*x^(3/2)*(c +
 d*x^2)) + (I*(b^2*c^2 + 10*a*b*c*d - 15*a^2*d^2)*Sqrt[1 + c/(d*x
^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1
])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))/(6*(e*x)^(5/2)*Sqrt[c + d*
x^2])
```

Maple [B] time = 0.037, size = 686, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/12*(15*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*x^3*a^2*d^3-10*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^3*a*b*c*d^2-(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*x^3*b^2*c^2*d+15*(-c*d)^(1/2)*((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a^2*c*d^2-10*(-c*d)^(1/
2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*a*b*c^2*d-(-
c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x*b^2*
c^3+30*x^4*a^2*d^4-20*x^4*a*b*c*d^3-2*x^4*b^2*c^2*d^2+42*x^2*a^2*
c*d^3-28*x^2*a*b*c^2*d^2+2*x^2*b^2*c^3*d+8*a^2*c^2*d^2)/x/e^2/(e*
x)^(1/2)/c^3/d^2/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4 + 2\,abx^2 + a^2}{(d^2e^2x^6 + 2\,cde^2x^4 + c^2e^2x^2)\sqrt{dx^2 + c}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d^2*e^2*x^6 + 2*c*d*e^2*x^
4 + c^2*e^2*x^2)*sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{(dx^2 + c)^{\frac{5}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.864 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=489

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{20c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{15c^3e^5\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}-\frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*(c + d*x^2)^(3/2)) - (2*a*(10*b*c - 1
1*a*d))/(5*c^2*e^3*Sqrt[e*x]*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 7
0*a*b*c*d + 77*a^2*d^2)*(e*x)^(3/2))/(15*c^3*e^5*(c + d*x^2)^(3/2
)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(e*x)^(3/2))/(10*c^4*
e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*Sqr
t[e*x]*Sqrt[c + d*x^2])/(10*c^4*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)
) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(15/4)*d^(3/4)*e^
(7/2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(
Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2
0*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.08812, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{20c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{15c^3e^5\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}-\frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```



```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*(c + d*x^2)^(3/2)) - (2*a*(10*b*c - 1
1*a*d))/(5*c^2*e^3*Sqrt[e*x]*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 7
0*a*b*c*d + 77*a^2*d^2)*(e*x)^(3/2))/(15*c^3*e^5*(c + d*x^2)^(3/2
)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(e*x)^(3/2))/(10*c^4*
e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*Sqr
t[e*x]*Sqrt[c + d*x^2])/(10*c^4*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)
) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(15/4)*d^(3/4)*e^
(7/2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(
Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
```

```
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2
0*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 110.417, size = 461, normalized size = 0.94

$$-\frac{2a^2}{5ce\,(ex)^{\frac{5}{2}}\,(c+dx^2)^{\frac{3}{2}}}+\frac{2a\,(11ad-10bc)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{\frac{3}{2}}}+\frac{(ex)^{\frac{3}{2}}\left(7ad\,(11ad-10bc)+5b^2c^2\right)}{15c^3e^5\,(c+dx^2)^{\frac{3}{2}}}+\frac{(ex)^{\frac{3}{2}}\left(7ad\,(11ad-10bc)+5b^2c^2\right)}{10c^4e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(7ad\,(11ad-10bc)+5b^2c^2\right)}{10c^4\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(7ad\,(11ad-10bc)+5b^2c^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{10c^{\frac{15}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}-\frac{\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\left(\sqrt{c}+\sqrt{d}x\right)\left(7ad\,(11ad-10bc)+5b^2c^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{20c^{\frac{15}{4}}d^{\frac{3}{4}}e^{\frac{7}{2}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] -2*a**2/(5*c*e*(e*x)**(5/2)*(c + d*x**2)**(3/2)) + 2*a*(11*a*d -
10*b*c)/(5*c**2*e**3*sqrt(e*x)*(c + d*x**2)**(3/2)) + (e*x)**(3/2
)*(7*a*d*(11*a*d - 10*b*c) + 5*b**2*c**2)/(15*c**3*e**5*(c + d*x*
*2)**(3/2)) + (e*x)**(3/2)*(7*a*d*(11*a*d - 10*b*c) + 5*b**2*c**2
)/(10*c**4*e**5*sqrt(c + d*x**2)) - sqrt(e*x)*sqrt(c + d*x**2)*(7
*a*d*(11*a*d - 10*b*c) + 5*b**2*c**2)/(10*c**4*sqrt(d)*e**4*(sqrt
(c) + sqrt(d)*x)) + sqrt((c + d*x**2)/(sqrt(c) + sqrt(d)*x)**2)*(
sqrt(c) + sqrt(d)*x)*(7*a*d*(11*a*d - 10*b*c) + 5*b**2*c**2)*elli
ptic_e(2*atan(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), 1/2)/(10*c*
*(15/4)*d**(3/4)*e**(7/2)*sqrt(c + d*x**2)) - sqrt((c + d*x**2)/(
sqrt(c) + sqrt(d)*x)**2)*(sqrt(c) + sqrt(d)*x)*(7*a*d*(11*a*d - 1
0*b*c) + 5*b**2*c**2)*elliptic_f(2*atan(d**(1/4)*sqrt(e*x)/(c**(1
/4)*sqrt(e))), 1/2)/(20*c**(15/4)*d**(3/4)*e**(7/2)*sqrt(c + d*x*
*2))
```

Mathematica [C] time = 0.824256, size = 246, normalized size = 0.5

$$\frac{x\left(\frac{a^2\left(-12c^3+132c^2dx^2+385cd^2x^4+231d^3x^6\right)-10abcx^2\left(12c^2+35cdx^2+21d^2x^4\right)+5b^2c^2x^4\left(5c+3dx^2\right)}{c+dx^2}+\frac{3icx^2\sqrt{\frac{i\sqrt{d}x}{\sqrt{c}}}\sqrt{\frac{dx^2}{c}+1}\left(77a^2d^2-70abcd+5b^2c^2\right)\left(E\left(i\right.\right.}{d}\right.}{30c^4(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*((5*b^2*c^2*x^4*(5*c + 3*d*x^2) - 10*a*b*c*x^2*(12*c^2 + 35*c*
d*x^2 + 21*d^2*x^4) + a^2*(-12*c^3 + 132*c^2*d*x^2 + 385*c*d^2*x^
4 + 231*d^3*x^6))/(c + d*x^2) + ((3*I)*c*(5*b^2*c^2 - 70*a*b*c*d
+ 77*a^2*d^2)*x^2*Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]*Sqrt[1 + (d*x^2)/c]
*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]], -1] - Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[d]*x)/Sqrt[c]]], -1]))/d))/(30*c^4*(e*x
)^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.04, size = 1231, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/60*(462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a^
2*c*d^3-420*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ell
ipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a
*b*c^2*d^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^4
*b^2*c^3*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^
4*a^2*c*d^3+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*x
^4*a*b*c^2*d^2-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/
2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))
*x^4*b^2*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*a^2*c^2*d^2-420*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*a*b*c^3*d+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*b^2*c^4-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^
(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a^2*c^2*d^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b*c^3*d-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*b^2*c^4-462*x^6*a^2*d^4+420*x^6*a*b*c*d^3-30*x^6*b^2*c
^2*d^2-770*x^4*a^2*c*d^3+700*x^4*a*b*c^2*d^2-50*x^4*b^2*c^3*d-264
*x^2*a^2*c^2*d^2+240*x^2*a*b*c^3*d+24*a^2*c^3*d)/x^2/d/c^4/e^3/(e
*x)^(1/2)/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+2\,abx^2+a^2}{(d^2e^3x^7+2\,cde^3x^5+c^2e^3x^3)\sqrt{dx^2+c}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)/((d^2*e^3*x^7 + 2*c*d*e^3*x^
5 + c^2*e^3*x^3)*sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^2}{\left(dx^2 + c\right)^{\frac{5}{2}} \left(ex\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)), x)
```

3.865 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{21b^2d}-\frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b}$$

```
[Out] (2*(2*b*c - 7*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2*d) - (2
*e*(e*x)^(5/2)*Sqrt[c - d*x^2])/(7*b) - (2*c^(1/4)*(2*b^2*c^2 + 1
4*a*b*c*d - 21*a^2*d^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(21*b^3*d^(5/4)*
Sqrt[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c -
d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.05913, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{21b^2d}-\frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```


```
[Out] (2*(2*b*c - 7*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2*d) - (2
*e*(e*x)^(5/2)*Sqrt[c - d*x^2])/(7*b) - (2*c^(1/4)*(2*b^2*c^2 + 1
4*a*b*c*d - 21*a^2*d^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(21*b^3*d^(5/4)*
Sqrt[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c -
d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.74449, size = 382, normalized size = 1.03

$$\frac{2(ex)^{7/2}\left(-\frac{9acx^2\left(21a^2d^2-14abcd-2b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{25a^2c^2(7ad}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right.\right.}}{105b^2dx^3\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*(e*x)^(7/2)*(5*(c - d*x^2)*(2*b*c - 7*a*d - 3*b*d*x^2) + (25*a
^2*c^2*(-2*b*c + 7*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*
x^2)/a])/((a - b*x^2)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c
, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c,
 (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a
]))) - (9*a*c*(-2*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*x^2*AppellF1
[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(9*a*c*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Appe
llF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4,
 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(105*b^2*d*x^3*Sqrt[c -
 d*x^2])
```

Maple [B] time = 0.097, size = 1479, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/42*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2/d*(42*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-70*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^
2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^
(1/2)+24*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*a*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*(c*d)^(1/2)+4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-21*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
```

```
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*(c*d)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-2
1*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+21*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*(c*d)^(1/2)+12*x^5*a*b^2*d^4*(a*b)^(1/2)-12*x^5*b^3*
c*d^3*(a*b)^(1/2)+28*x^3*a^2*b*d^4*(a*b)^(1/2)-48*x^3*a*b^2*c*d^3
*(a*b)^(1/2)+20*x^3*b^3*c^2*d^2*(a*b)^(1/2)-28*x*a^2*b*c*d^3*(a*b
)^(1/2)+36*x*a*b^2*c^2*d^2*(a*b)^(1/2)-8*x*b^3*c^3*d*(a*b)^(1/2))
/x/(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/
2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a),x, algorithm="maxima")

[Out] -integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a), x)
```

3.866 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=414

$$-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b}$$

```
[Out] (-2*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b) - (2*c^(3/4)*(2*b*c - 5*
a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) + (
2*c^(3/4)*(2*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^2*d^(3/4)
*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4
)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1
- (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*
Sqrt[c - d*x^2])
```

Rubi [A] time = 2.19546, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```


```
[Out] (-2*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b) - (2*c^(3/4)*(2*b*c - 5*
a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) + (
2*c^(3/4)*(2*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^2*d^(3/4)
*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4
)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1
- (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*
```

```
Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.20133, size = 418, normalized size = 1.01

$$\frac{2e(ex)^{3/2}\left(\frac{14x^2(a-bx^2)(c-dx^2)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11ac(7ac-2adx^2-9bcx^2+7bdx^4)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)} - \right.}{35b(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*((-49*a^2*c^2*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)
/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x
^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x
^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))
+ (11*a*c*(7*a*c - 9*b*c*x^2 - 2*a*d*x^2 + 7*b*d*x^4)*AppellF1[7/
4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 14*x^2*(a - b*x^2)*(c -
d*x^2)*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a]
+ a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))/(11*a*
c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*
c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*Appell
F1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))))/(35*b*(-a + b*x^
2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.065, size = 1491, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/10*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(5*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2
*b*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-5
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1
/2)*(c*d)^(1/2)*a*b*c*d-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
```

```
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*b*c*d^2+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*
c^2*d-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3+10*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-14*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d+4*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^3*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2-5*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2
)*a*b*c*d+4*x^4*a*b^2*d^3-4*x^4*b^3*c*d^2-4*x^2*a*b^2*c*d^2+4*x^2
*b^3*c^2*d)/x/b^2/(d*x^2-c)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^
(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2 + c}\,(ex)^{\frac{5}{2}}}{bx^2 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.867 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=315

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b}$$

```
[Out] (-2*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b) - (2*c^(1/4)*(2*b*c - 3*a*
d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(3*b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^
(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[
b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c
- a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.37871, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```


```
[Out] (-2*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b) - (2*c^(1/4)*(2*b*c - 3*a*
d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(3*b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^
(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[
b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c
- a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.653605, size = 418, normalized size = 1.33

$$\frac{2e\sqrt{ex}\left(\frac{10x^2(a-bx^2)(c-dx^2)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9ac(5ac-2adx^2-7bcx^2+5bdx^4)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{\cdots}{2x^2(2\cdots}\right.}{15b(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*((-25*a^2*c^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c
, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2
)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)
/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (9
*a*c*(5*a*c - 7*b*c*x^2 - 2*a*d*x^2 + 5*b*d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 10*x^2*(a - b*x^2)*(c - d*x^2)
*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*A
ppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(9*a*c*AppellF
1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2
, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(15*b*(-a + b*x^2)*Sqrt[c -
d*x^2])
```

Maple [B] time = 0.026, size = 1286, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/6*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b*(6*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c
*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-10*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+4*Elli
pticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
b^2*c^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*
b)^(1/2)*(c*d)^(1/2)*a^2*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d+3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d-3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
```

```
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^
(1/2)*a^2*d^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d+3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d+4*x^3*a*b*d^3*(a*b)^(1/2)-
4*x^3*b^2*c*d^2*(a*b)^(1/2)-4*x*a*b*c*d^2*(a*b)^(1/2)+4*x*b^2*c^2
*d*(a*b)^(1/2))/x/(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2
)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.868 $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=365

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}$$

```
[Out] (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) -
 (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])
- (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
 + (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.52213, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) -
 (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])
- (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
 + (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.345066, size = 164, normalized size = 0.45

$$-\frac{14acx\sqrt{ex}\sqrt{c-dx^2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3\left(a-bx^2\right)\left(2x^2\left(adF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2bcF_1\left(\frac{7}{4};-\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)-7acF_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (-14*a*c*x*Sqrt[e*x]*Sqrt[c - d*x^2]*AppellF1[3/4, -1/2, 1, 7/4,
(d*x^2)/c, (b*x^2)/a])/(3*(a - b*x^2)*(-7*a*c*AppellF1[3/4, -1/2,
 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(-2*b*c*AppellF1[7/4, -1/2
, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 1/2, 1, 11/4
, (d*x^2)/c, (b*x^2)/a])))
```

Maple [B] time = 0.024, size = 701, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x)
```

```
[Out] 1/2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*d*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-
(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
a*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2+(c*d)^(1/2)*
(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c+Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+(c*d)^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c-4*EllipticE(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+4*EllipticE(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+2*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-2*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2)/b/x/(d*
x^2-c)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex}\sqrt{c - dx^2}}{-a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2 + c}\sqrt{ex}}{bx^2 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a), x)
```

3.869 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)}\,dx$

Optimal. Leaf size=283

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\sqrt{c-dx^2}}$$

```
[Out] (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[e]*Sqrt[c - d*x^2]) +
 (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(
b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.990688, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[e]*Sqrt[c - d*x^2]) +
 (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(
b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 164.329, size = 255, normalized size = 0.9

$$\frac{2\sqrt[4]{c}d^{\frac{3}{4}}\sqrt{1-\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(ad-bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(ad-bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

```
[Out] 2*c**(1/4)*d**(3/4)*sqrt(1 - d*x**2/c)*elliptic_f(asin(d**(1/4)*s
qrt(e*x)/(c**(1/4)*sqrt(e))), -1)/(b*sqrt(e)*sqrt(c - d*x**2)) -
```

```
c**(1/4)*sqrt(1 - d*x**2/c)*(a*d - b*c)*elliptic_pi(-sqrt(b)*sqrt
(c)/(sqrt(a)*sqrt(d)), asin(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))
), -1)/(a*b*d**(1/4)*sqrt(e)*sqrt(c - d*x**2)) - c**(1/4)*sqrt(1
- d*x**2/c)*(a*d - b*c)*elliptic_pi(sqrt(b)*sqrt(c)/(sqrt(a)*sqrt
(d)), asin(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), -1)/(a*b*d**(1
/4)*sqrt(e)*sqrt(c - d*x**2))
```

Mathematica [C] time = 0.343848, size = 162, normalized size = 0.57

$$-\frac{10acx\sqrt{c-dx^2}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{\sqrt{ex}\left(a-bx^2\right)\left(2x^2\left(adF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2bcF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)-5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (-10*a*c*x*Sqrt[c - d*x^2]*AppellF1[1/4, -1/2, 1, 5/4, (d*x^2)/c,
 (b*x^2)/a])/(Sqrt[e*x]*(a - b*x^2)*(-5*a*c*AppellF1[1/4, -1/2, 1
, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(-2*b*c*AppellF1[5/4, -1/2,
2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 1/2, 1, 9/4, (d
*x^2)/c, (b*x^2)/a])))
```

Maple [B] time = 0.046, size = 651, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x)
```

```
[Out] 1/2*(-d*x^2+c)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(c*d)^(1/2)*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*d^2*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*b*c*d*(a*b)^(1/2)+EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*a*b*d*(c*d)^(1/2)-EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*a*d^2*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*b^2*c*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*b*c*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*
b*d*(c*d)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d^2*(a
*b)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*(c*d)^(1
/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c*d*(a*b)^(1/2))/(
e*x)^(1/2)/(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((
c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)),x, algorithm="maxima")

[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c - dx^2}}{-a\sqrt{ex} + bx^2\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)/(e*x)**(1/2),x)

[Out] -Integral(sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2 + c}}{(bx^2 - a)\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)),x, algorithm="giac")

[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.870 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*Sqrt[1
- (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*S
qrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*Sqr
t[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.04357, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*Sqrt[1
- (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*S
qrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*Sqr
t[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C]   time = 0.923681, size = 337, normalized size = 0.86

$$\frac{2x\left(\frac{49cx^2(bc-2ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}-\frac{33bcdx^4F_1\left(\frac{7}{4}\right.}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\right.\right.}\right.}{21(ex)^{3/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (2*x*((-21*(c - d*x^2))/a + (49*c*(b*c - 2*a*d)*x^2*AppellF1[3/4,
 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(7*a*c*AppellF1
[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[
7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2,
 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) - (33*b*c*d*x^4*AppellF1[7/4,
1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((-a + b*x^2)*(11*a*c*Appell
F1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Appell
F1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4,
 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(21*(e*x)^(3/2)*Sqrt[c
- d*x^2])
```

Maple [B]   time = 0.057, size = 1274, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x)
```

```
[Out] 1/2*(-d*x^2+c)^(1/2)*d*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
E(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+2*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*(c*d)^(1/2)*a*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*b*c
+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*d-((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*b*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-((
```

```
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d-((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*b^2*c^2+4*a*b*d^2*x^2-4*b^2*c*d*x^2-4*c*a*b*d+4*b^2*c
^2)/e/(e*x)^(1/2)/(d*x^2-c)/a/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d
)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c-dx^2}}{-a\,(ex)^{\frac{3}{2}}+bx^2\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.871 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx$

Optimal. Leaf size=308

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*Sqr
t[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(3*a*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*
d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2
)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.35995, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*Sqr
t[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(3*a*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*
d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2
)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.03683, size = 338, normalized size = 1.1

$$\frac{2x\left(\frac{25cx^2(3bc-2ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{9bcdx^4F_1\left(\frac{5}{4};\frac{1}{2},1\right.}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\right.\right.\right.}\right.}{15(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (2*x*((-5*(c - d*x^2))/a + (25*c*(3*b*c - 2*a*d)*x^2*AppellF1[1/4
, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(5*a*c*AppellF
1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a]))) + (9*b*c*d*x^4*AppellF1[5/4, 1/
2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/((-a + b*x^2)*(9*a*c*AppellF1[5
/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/
4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1
, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(15*(e*x)^(5/2)*Sqrt[c - d*x^2
])
```

Maple [B] time = 0.05, size = 1167, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x)
```

```
[Out] 1/6*(-d*x^2+c)^(1/2)*b*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*
d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*x*a*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*d*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
```

```
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b*c*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*x^
2*a*d^2*(a*b)^(1/2)-4*x^2*b*c*d*(a*b)^(1/2)-4*a*c*d*(a*b)^(1/2)+4
*b*c^2*(a*b)^(1/2))/x/a/e^2/(e*x)^(1/2)/(d*x^2-c)/(a*b)^(1/2)/((a
*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2 + c}}{(bx^2 - a)\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2 + c}}{(bx^2 - a)\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.872 $\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}(a-bx^2)}\,dx$

Optimal. Leaf size=457

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5bc-2ad)}{5a^2ce^3\sqrt{ex}}-\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 2*a*d)*Sqr
t[c - d*x^2])/(5*a^2*c*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c - 2*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(5*a^2*c^(1/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d
^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(1/4)*e^(7/2)*Sq
rt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*S
qrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c
]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqr
t[c - d*x^2])
```

Rubi [A] time = 2.7909, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5bc-2ad)}{5a^2ce^3\sqrt{ex}}-\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 2*a*d)*Sqr
t[c - d*x^2])/(5*a^2*c*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c - 2*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(5*a^2*c^(1/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d
^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(1/4)*e^(7/2)*Sq
rt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*S
qrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c
```

```
]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqr
t[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.37521, size = 381, normalized size = 0.83

$$\frac{2x\left(\frac{49ax^4\left(2a^2d^2-10abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{33abdx^6(5bc-2ad\cdots}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\cdots\right.\right.}\right.}{105a^2(ex)^{7/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (2*x*((-21*(c - d*x^2)*(5*b*c*x^2 + a*(c - 2*d*x^2)))/c + (49*a*(
5*b^2*c^2 - 10*a*b*c*d + 2*a^2*d^2)*x^4*AppellF1[3/4, 1/2, 1, 7/4
, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1
, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2,
 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d
*x^2)/c, (b*x^2)/a]))) + (33*a*b*d*(5*b*c - 2*a*d)*x^6*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(11*a*c*App
ellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*App
ellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11
/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(105*a^2*(e*x)^(7/2)
*Sqrt[c - d*x^2])
```

Maple [B] time = 0.058, size = 1553, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x)
```

```
[Out] -1/10*(-d*x^2+c)^(1/2)*b*d*(8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*x^2*a^2*c*d^2-28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x
^2*a*b*c^2*d+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*
c^3-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2+14*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d-10*((d*x+(c
```

```
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+5*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*(c*d)^(1/2)*x^2*b*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(
c*d)^(1/2)*x^2*a*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*x^2*b*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b*c^2*d+5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*x^2*b^2*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b*c^2*d+5*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*x^2*b^2*c^3+8*x^4*a^2*d^3-28*x^4*a*b*c*d^2+20*x^4*
b^2*c^2*d-12*x^2*a^2*c*d^2+32*x^2*a*b*c^2*d-20*x^2*b^2*c^3+4*a^2*
c^2*d-4*a*b*c^3)/x^2/e^3/(e*x)^(1/2)/(d*x^2-c)/a^2/((a*b)^(1/2)*d
+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.873 $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=485

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}(11bc-9ad)}{45b^2}+\frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be}$$

```
[Out] (-2*(11*b*c - 9*a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(45*b^2) + (2
*d*(e*x)^(7/2)*Sqrt[c - d*x^2])/(9*b*e) - (2*c^(3/4)*(4*b^2*c^2 -
 21*a*b*c*d + 15*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4
)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*
d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2]) -
(Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Ellipt
icPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2])
 + (Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Ell
ipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.94196, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}(11bc-9ad)}{45b^2}+\frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*(11*b*c - 9*a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(45*b^2) + (2
*d*(e*x)^(7/2)*Sqrt[c - d*x^2])/(9*b*e) - (2*c^(3/4)*(4*b^2*c^2 -
 21*a*b*c*d + 15*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4
)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*
```

```
d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2]) -
(Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Ellipt
icPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2])
 + (Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Ell
ipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.17683, size = 378, normalized size = 0.78

$$\frac{2e(ex)^{3/2}\left(\frac{33acx^2(15a^2d^2-21abcd+4b^2c^2)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{49a^2c^2(9}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},b\right.\right.\right.}\right.}{315b^2\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*(-7*(c - d*x^2)*(11*b*c - 9*a*d - 5*b*d*x^2) + (
49*a^2*c^2*(-11*b*c + 9*a*d)*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c
, (b*x^2)/a])/((-a + b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*
x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*
x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (
b*x^2)/a]))) + (33*a*c*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*d^2)*x^2*
AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(
11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*
(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*A
ppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(315*b^2*S
qrt[c - d*x^2])
```

Maple [B] time = 0.05, size = 2183, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x)
```

```
[Out] -1/90*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*(90*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-90*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+45*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
```

```
(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3*d
+45*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*d-180*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a^3*b*c*d^3+432*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*b^2*c^2*d^2-300*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a*b^3*c^3*d+90*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*
b*c*d^3-216*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*
d^2+150*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3*d+48*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^4-24*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^4-100*x^4*a*b^3*c*d^3-36*x^2*a^2
*b^2*c*d^3+80*x^2*a*b^3*c^2*d^2+20*x^6*a*b^3*d^4-20*x^6*b^4*c*d^3
+36*x^4*a^2*b^2*d^4+64*x^4*b^4*c^2*d^2-44*x^2*b^4*c^3*d-45*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d+45*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d+45*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1
/2)*a^3*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d
)^(1/2)-90*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b
^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+45*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^
(1/2)*a^3*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c
*d)^(1/2)-90*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2
*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))/x/b^3/(d*x^2-c
)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2 + c\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}}{bx^2 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.874 $\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{21b^2}+\frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be}$$

```
[Out] (-2*(9*b*c - 7*a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2) + (2*d*
(e*x)^(5/2)*Sqrt[c - d*x^2])/(7*b*e) - (2*c^(1/4)*(12*b^2*c^2 - 3
5*a*b*c*d + 21*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(21*b^3*d^(1/4)*
Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c -
d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.05172, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{21b^2}+\frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*(9*b*c - 7*a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2) + (2*d*
(e*x)^(5/2)*Sqrt[c - d*x^2])/(7*b*e) - (2*c^(1/4)*(12*b^2*c^2 - 3
5*a*b*c*d + 21*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(21*b^3*d^(1/4)*
Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c -
d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.17214, size = 378, normalized size = 1.02

$$\frac{2e\sqrt{ex}\left(\frac{9acx^2(21a^2d^2-35abcd+12b^2c^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{25a^2c^2(7ad-9b}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+ac\right.\right.}\right.}{105b^2\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*(-5*(c - d*x^2)*(9*b*c - 7*a*d - 3*b*d*x^2) + (25*
a^2*c^2*(-9*b*c + 7*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b
*x^2)/a])/((-a + b*x^2)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)
/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/
c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)
/a]))) + (9*a*c*(12*b^2*c^2 - 35*a*b*c*d + 21*a^2*d^2)*x^2*Appell
F1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(9*a*c*A
ppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Ap
pellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/
4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(105*b^2*Sqrt[c - d*x
^2])
```

Maple [B] time = 0.032, size = 1920, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x)
```

```
[Out] -1/42*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2*(42*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-112*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*
b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1
/2)+94*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*(c*d)^(1/2)-24*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^3*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-42*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+42*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
```

```
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*(c*d)^(1/2)+21*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3*d-21
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d-21*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*2^(1/2)*a^3*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*(c*d)^(1/2)+42*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2
^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+42*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-21*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*a*b^3*c^3*d-21*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*
d)^(1/2)*a*b^2*c^2*d+12*x^5*a*b^2*d^4*(a*b)^(1/2)-12*x^5*b^3*c*d^
3*(a*b)^(1/2)+28*x^3*a^2*b*d^4*(a*b)^(1/2)-76*x^3*a*b^2*c*d^3*(a*
b)^(1/2)+48*x^3*b^3*c^2*d^2*(a*b)^(1/2)-28*x*a^2*b*c*d^3*(a*b)^(1
/2)+64*x*a*b^2*c^2*d^2*(a*b)^(1/2)-36*x*b^3*c^3*d*(a*b)^(1/2))/x/
(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*
b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.875 $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=421

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be}$$

```
[Out] (2*d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b*e) + (2*c^(3/4)*d^(1/4)*(7
*b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) -
(2*c^(3/4)*d^(1/4)*(7*b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*El
lipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^
2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (d*x
^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(
1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(
1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.25314, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```


```
[Out] (2*d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b*e) + (2*c^(3/4)*d^(1/4)*(7
*b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) -
(2*c^(3/4)*d^(1/4)*(7*b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*El
lipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*b^
2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (d*x
^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(
1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[
```

```
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(
1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)*(e*x)**(1/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.810788, size = 427, normalized size = 1.01

$$\frac{2x\sqrt{ex}\left(\frac{49ac^2(3ad-5bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{-42dx^2\left(a-bx^2\right)\left(c-dx^2\right)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right.}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\right.\right.}\right.}{105b\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*((49*a*c^2*(-5*b*c + 3*a*d)*AppellF1[3/4, 1/2, 1,
7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*
x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*
x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (
b*x^2)/a])) + (-33*a*c*d*(7*a*c - 14*b*c*x^2 - 2*a*d*x^2 + 7*b*d*
x^4)*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] - 42*d*x^2
*(a - b*x^2)*(c - d*x^2)*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x
^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (
b*x^2)/a]))/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2
)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^
2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))
)/(105*b*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.03, size = 1927, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x)
```

```
[Out] -1/10*(-d*x^2+c)^(1/2)*(e*x)^(1/2)*d*(5*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*
b*c*d^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-10
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(
```

```
1/2)*(c*d)^(1/2)*a*b*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^3*c^3-5*(c*d)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2+5*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*a^2*b*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^
2*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-10*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^3+5
*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-20*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+48*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d-28*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*b^3*c^3+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a^2*b*c*d^2-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c
^2*d+14*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3+4*x^4*a*
b^2*d^3-4*x^4*b^3*c*d^2-4*x^2*a*b^2*c*d^2+4*x^2*b^3*c^2*d)/b^2/x/
(d*x^2-c)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2
)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a),x, algorithm="maxima")

[Out] -integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\,dx-\int \left(-\frac{dx^2\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(3/2)*(e*x)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integr
al(-d*x**2*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{bx^2-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a), x)
```

3.876 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)}\,dx$

Optimal. Leaf size=328

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be}$$

```
[Out] (2*d*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*e) + (2*c^(1/4)*d^(3/4)*(5*b
*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(3*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (c
^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*
(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.43485, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*d*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*e) + (2*c^(1/4)*d^(3/4)*(5*b
*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(3*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (c
^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*
(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.810809, size = 425, normalized size = 1.3

$$\frac{2x\left(\frac{25ac^2(ad-3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{d\left(-10x^2(a-bx^2)(c-dx^2)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\right.\right.}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+c\right.}\right.}{15b\sqrt{ex}\,(bx^2-a)\,\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*x*((25*a*c^2*(-3*b*c + a*d)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)
/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x
^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^
2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) +
(d*(-9*a*c*(5*a*c - 10*b*c*x^2 - 2*a*d*x^2 + 5*b*d*x^4)*AppellF1[
5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 10*x^2*(a - b*x^2)*(c -
 d*x^2)*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a]
+ a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))/(9*a*c
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*
AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[
9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(15*b*Sqrt[e*x]*(-a
+ b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.033, size = 1721, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x)
```

```
[Out] -1/6*(-d*x^2+c)^(1/2)/b*d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-16*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c
*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+10*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+3*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(
c*d)^(1/2)*a^2*d^2-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d+6*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d+3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2)
)*b^3*c^3-3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
```

```
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2-
3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(
1/2)*(c*d)^(1/2)*a^2*d^2+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d+6*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d-3*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*b^3*c^3-3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^
2*c^2+4*x^3*a*b*d^3*(a*b)^(1/2)-4*x^3*b^2*c*d^2*(a*b)^(1/2)-4*x*a
*b*c*d^2*(a*b)^(1/2)+4*x*b^2*c^2*d*(a*b)^(1/2))/(e*x)^(1/2)/(d*x^
2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*
b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx-\int \left(-\frac{dx^2\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(3/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x
) - Integral(-d*x**2*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt
```

```
(e*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^{2}+c\right)^{\frac{3}{2}}}{\left(bx^{2}-a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.877 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$

Optimal. Leaf size=417

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}}$$

```
[Out] (-2*c*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(b*c
+ a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(a*b*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4
)*d^(1/4)*(b*c + a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b*e^(3/2)*Sqrt[c - d*x^
2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d
*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d
*x^2])
```

Rubi [A] time = 2.18939, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (-2*c*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(b*c
+ a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(a*b*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4
)*d^(1/4)*(b*c + a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b*e^(3/2)*Sqrt[c - d*x^
2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d
*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d
```

```
*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.792125, size = 436, normalized size = 1.05

$$\frac{2cx\left(\frac{33a\left(a\left(-7c^2+7cdx^2+d^2x^4\right)+bcx^2\left(7c-6dx^2\right)\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-42x^2\left(a-bx^2\right)\left(c-dx^2\right)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{a\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}\right)}{21(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (2*c*x*((49*c*(b*c - 3*a*d)*x^2*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2
)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*
x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*
x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))
 + (33*a*(b*c*x^2*(7*c - 6*d*x^2) + a*(-7*c^2 + 7*c*d*x^2 + d^2*x
^4))*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] - 42*x^2*(
a - b*x^2)*(c - d*x^2)*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2
)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*
x^2)/a]))/(a*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x
^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))
)))/(21*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.036, size = 1754, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x)
```

```
[Out] -1/2*(-d*x^2+c)^(1/2)*d*(2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*b*c*d^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3
+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2
)*(c*d)^(1/2)*a^2*d^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
```

```
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2*d+2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d+((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*b^3*c^3-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2+(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*
(c*d)^(1/2)*a^2*d^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a*b*c*d+((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*b^3*c^3+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-4*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+4*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3-4*x^2*a*b^2*c*d^2+4*x^2*b^3
*c^2*d+4*a*c^2*d*b^2-4*c^3*b^3)/b/e/(e*x)^(1/2)/(d*x^2-c)/a/((a*b
)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2 + c\right)^{\frac{3}{2}}}{\left(bx^2 - a\right)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.878 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*c*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*(
b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*b*e^(5/2)*Sqrt[c - d*x^2]) + (
c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)
*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.54627, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (-2*c*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*(
b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*b*e^(5/2)*Sqrt[c - d*x^2]) + (
c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)
*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.76395, size = 438, normalized size = 1.33

$$\frac{2cx\left(\frac{9a\left(a\left(-5c^2+5cdx^2+3d^2x^4\right)+bcx^2\left(5c-6dx^2\right)\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-10x^2\left(a-bx^2\right)\left(c-dx^2\right)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{a\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}\right.}{15(ex)^{5/2}\left(a-bx^2\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (2*c*x*((25*c*(3*b*c - 5*a*d)*x^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x
^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (
b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b
*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))
 + (9*a*(b*c*x^2*(5*c - 6*d*x^2) + a*(-5*c^2 + 5*c*d*x^2 + 3*d^2*
x^4))*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 10*x^2*(
a - b*x^2)*(c - d*x^2)*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)
/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^
2)/a]))/(a*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a
] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a
] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(1
5*(e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.035, size = 1740, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x)
```

```
[Out] -1/6*(-d*x^2+c)^(1/2)*d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*d^2*(c*d)^(1/2)*(a*b)^(1/2)*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-8*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a*b*c*d*(c*d)^(1/2)*
(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*
(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d^2*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*x*a^2*d^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)-6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*2^(1/2)*x*a*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+6*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b*c*d*(c*d)^(
1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b^3*c^3*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*x*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d
```

```
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x*a^2*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*d^2*(c*d)^(1/2)*(a*b)^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+6*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)+6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*x*a*b*c*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^3*c
^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-4*x^2*a*b*c*d^2*(a*b)^(1/2)+4*x^2
*b^2*c^2*d*(a*b)^(1/2)+4*a*b*c^2*d*(a*b)^(1/2)-4*b^2*c^3*(a*b)^(1
/2))/x/a/e^2/(e*x)^(1/2)/(d*x^2-c)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*
d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)\left(ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.879 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx$

Optimal. Leaf size=459

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5bc-7ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*c*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 7*a*d)*S
qrt[c - d*x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(5*b*c
 - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*e^(7/2)*Sqrt[c - d*x^2]) + (2*c
^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*e^(7/2)*Sq
rt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)
*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)
*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.92088, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5bc-7ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```


```
[Out] (-2*c*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 7*a*d)*S
qrt[c - d*x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(5*b*c
 - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*e^(7/2)*Sqrt[c - d*x^2]) + (2*c
^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*e^(7/2)*Sq
rt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)
*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*El
```

```
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)
*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.32863, size = 380, normalized size = 0.83

$$\frac{2x\left(\frac{49acx^4\left(12a^2d^2-15abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{33abcdx^6(5bc-7ad}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\right.\right.\right.}}{105a^2(ex)^{7/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (2*x*(-21*(c - d*x^2)*(5*b*c*x^2 + a*(c - 7*d*x^2)) + (49*a*c*(5*
b^2*c^2 - 15*a*b*c*d + 12*a^2*d^2)*x^4*AppellF1[3/4, 1/2, 1, 7/4,
 (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1,
 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2,
11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*
x^2)/c, (b*x^2)/a]))) + (33*a*b*c*d*(5*b*c - 7*a*d)*x^6*AppellF1[
7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(11*a*c*Ap
pellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Ap
pellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[1
1/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(105*a^2*(e*x)^(7/2
)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 2028, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x)
```

```
[Out] -1/10*(-d*x^2+c)^(1/2)*d*(-4*a*b^2*c^3+5*(c*d)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*x^2*b^2*c^2+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*x^2*a^2*b*c*d^2-5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*x^2*a^2*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a^2*b*c*d^2-10*(
```

```
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a^
2*b*c*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c^2*d-5*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^2*c^2+5*(c*d)^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*x^2*a^2*d^2+20*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*b^3*c^3+4*a^2*b*c^2*d-48*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d-14*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a^2*b*c*d^2+24*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*x^2*a*b^2*c^2*d+10*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*x^2*a*b*c*d-10*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*x^2*a*b*c*d-20*b^3*c^3*x^2+28*x^4*a^2*b*d^3+20*x^4*b^3*c^2*
d-32*a^2*b*c*d^2*x^2-48*x^4*a*b^2*c*d^2+52*a*b^2*c^2*d*x^2-10*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3+5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*x^2*b^3*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^3)/x^2/e^3/(
e*x)^(1/2)/(d*x^2-c)/a^2/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/
2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] -integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(3/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-dx^2 + c\right)^{\frac{3}{2}}}{\left(bx^2 - a\right)\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.880 $\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=305

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd}$$

```
[Out] (2*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*d) - (2*c^(1/4)*(b*c + 3*a
*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(3*b^2*d^(5/4)*Sqrt[c - d*x^2]) + (a
*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*S
qrt[c - d*x^2])
```

Rubi [A] time = 1.32492, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{2\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*d) - (2*c^(1/4)*(b*c + 3*a
*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(3*b^2*d^(5/4)*Sqrt[c - d*x^2]) + (a
*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*S
qrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.731736, size = 423, normalized size = 1.39

$$\frac{2(ex)^{7/2}\left(\frac{25a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{-10x^2(a-bx^2)(c-dx^2)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+a\cdots}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\cdots}\cdots\right.}{15bdx^3(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*(e*x)^(7/2)*((25*a^2*c^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
 (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)
/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/
a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (-9
*a*c*(5*a*c - 4*b*c*x^2 - 2*a*d*x^2 + 5*b*d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 10*x^2*(a - b*x^2)*(c - d*x^2)
*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*A
ppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(9*a*c*AppellF
1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2
, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(15*b*d*x^3*(-a + b*x^2)*Sqr
t[c - d*x^2])
```

Maple [B] time = 0.052, size = 853, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/6/b/d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*2^(1/2)*a^2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2
)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^
2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-
3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(
1/2)*(c*d)^(1/2)*a^2*d^2+4*x^3*a*b*d^3*(a*b)^(1/2)-4*x^3*b^2*c*d^
2*(a*b)^(1/2)-4*x*a*b*c*d^2*(a*b)^(1/2)+4*x*b^2*c^2*d*(a*b)^(1/2)
)*(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d
)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.881 $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}$$

```
[Out] (-2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2])
+ (2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2])
 - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]
*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.47099, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367

$$-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```


```
[Out] (-2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2])
+ (2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2])
 - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]
*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.223809, size = 165, normalized size = 0.47

$$-\frac{22acx(ex)^{5/2}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{7\left(bx^2-a\right)\sqrt{c-dx^2}\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-22*a*c*x*(e*x)^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*
x^2)/a])/(7*(-a + b*x^2)*Sqrt[c - d*x^2]*(11*a*c*AppellF1[7/4, 1/
2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1
/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 1
5/4, (d*x^2)/c, (b*x^2)/a])))
```

Maple [A] time = 0.049, size = 472, normalized size = 1.4

$$\frac{\sqrt{2}e^2}{2\,x\left(dx^2-c\right)b}\left(4\,EllipticE\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},1/2\,\sqrt{2}\right)abcd-4\,EllipticE\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},1/2\,\sqrt{2}\right)b^2c^2-2\,EllipticF\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a*b*c*d-4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*b^2*c^2-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a*b*c*d+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*b^2*c^2+(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*a*d-(c*d)^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*a*b*c*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b
*c*d)*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x^2+c)^(1/
2)*e^2*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d
+(c*d)^(1/2)*b)/(d*x^2-c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{5}{2}}}{(bx^2 - a)\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.882 $\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=261

$$\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] (-2*c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])
+ (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[
c - d*x^2])
```

Rubi [A] time = 1.00198, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])
+ (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[
c - d*x^2])
```

Rubi in Sympy [A] time = 156.888, size = 238, normalized size = 0.91

$$-\frac{2\sqrt[4]{c}e^{\frac{3}{2}}\sqrt{1-\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{\frac{3}{2}}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{\frac{3}{2}}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -2*c**(1/4)*e**(3/2)*sqrt(1 - d*x**2/c)*elliptic_f(asin(d**(1/4)*
sqrt(e*x)/(c**(1/4)*sqrt(e))), -1)/(b*d**(1/4)*sqrt(c - d*x**2))
```

```
+ c**(1/4)*e**(3/2)*sqrt(1 - d*x**2/c)*elliptic_pi(-sqrt(b)*sqrt(
c)/(sqrt(a)*sqrt(d)), asin(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e)))
, -1)/(b*d**(1/4)*sqrt(c - d*x**2)) + c**(1/4)*e**(3/2)*sqrt(1 -
d*x**2/c)*elliptic_pi(sqrt(b)*sqrt(c)/(sqrt(a)*sqrt(d)), asin(d**
(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), -1)/(b*d**(1/4)*sqrt(c - d*x
**2))
```

Mathematica [C] time = 0.229992, size = 165, normalized size = 0.63

$$-\frac{18acx(ex)^{3/2}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{5\left(bx^2-a\right)\sqrt{c-dx^2}\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-18*a*c*x*(e*x)^(3/2)*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x
^2)/a])/(5*(-a + b*x^2)*Sqrt[c - d*x^2]*(9*a*c*AppellF1[5/4, 1/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2,
2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4,
(d*x^2)/c, (b*x^2)/a])))
```

Maple [B] time = 0.034, size = 417, normalized size = 1.6

$$-\frac{\sqrt{2}e}{2\,x\,(dx^2-c)}\left(EllipticPi\left(\sqrt{1\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},b\sqrt{cd}\left(\sqrt{abd}+\sqrt{cdb}\right)^{-1},\frac{\sqrt{2}}{2}\right)ab\sqrt{cd}-EllipticPi\left(\sqrt{1\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*(c*d)^(1/2)-El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*d*(a*b)^(1/2)-EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*(c*d)^(1/2)-EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a*d*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*d*(a*b)^(1/2)-2*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*(a*b)^(1/2
))*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(-d*x
^2+c)^(1/2)*e*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^
(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral((e*x)**(3/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x
**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.883 $\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

```
[Out] -((c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])) + (c^(1/4)
*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.810686, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] -((c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])) + (c^(1/4)
*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 101.271, size = 187, normalized size = 0.92

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -c**(1/4)*sqrt(e)*sqrt(1 - d*x**2/c)*elliptic_pi(-sqrt(b)*sqrt(c)
/(sqrt(a)*sqrt(d)), asin(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))),
-1)/(sqrt(a)*sqrt(b)*d**(1/4)*sqrt(c - d*x**2)) + c**(1/4)*sqrt(e
)*sqrt(1 - d*x**2/c)*elliptic_pi(sqrt(b)*sqrt(c)/(sqrt(a)*sqrt(d)
), asin(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), -1)/(sqrt(a)*sqrt
(b)*d**(1/4)*sqrt(c - d*x**2))
```

Mathematica [C] time = 0.223889, size = 165, normalized size = 0.81

$$-\frac{14acx\sqrt{ex}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3\left(bx^2-a\right)\sqrt{c-dx^2}\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-14*a*c*x*Sqrt[e*x]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2
)/a])/(3*(-a + b*x^2)*Sqrt[c - d*x^2]*(7*a*c*AppellF1[3/4, 1/2, 1
, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2,
 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d
*x^2)/c, (b*x^2)/a])))
```

Maple [B] time = 0.032, size = 337, normalized size = 1.7

$$-\frac{d\sqrt{2}}{2\,x\,(dx^2-c)}\left(EllipticPi\left(\sqrt{1\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},b\sqrt{cd}\left(\sqrt{abd}+\sqrt{cdb}\right)^{-1},\frac{\sqrt{2}}{2}\right)bc-\sqrt{cd}\sqrt{ab}EllipticPi\left(\sqrt{1\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{c}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b*c-(c*d)^(1/2)*(a
*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))+EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*b*c+(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2)))*d*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x^2+c)^(1/2)*(e*x)^(1/2)/((c*d)^(1/2)*b-(a*b)^(
1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/x/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{ex}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{ex}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(sqrt(e*x)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{ex}}{(bx^2 - a)\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.884 $\int \frac{1}{\sqrt{ex}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

```
[Out] (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(a*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*Sqrt[e]*Sqrt[c
- d*x^2])
```

Rubi [A] time = 0.7443, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(a*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*Sqrt[e]*Sqrt[c
- d*x^2])
```

Rubi in Sympy [A] time = 116.801, size = 170, normalized size = 0.9

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\operatorname{asin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)/(e*x)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] c**(1/4)*sqrt(1 - d*x**2/c)*elliptic_pi(-sqrt(b)*sqrt(c)/(sqrt(a)
*sqrt(d)), asin(d**(1/4)*sqrt(e*x)/(c**(1/4)*sqrt(e))), -1)/(a*d*
*(1/4)*sqrt(e)*sqrt(c - d*x**2)) + c**(1/4)*sqrt(1 - d*x**2/c)*el
liptic_pi(sqrt(b)*sqrt(c)/(sqrt(a)*sqrt(d)), asin(d**(1/4)*sqrt(e
*x)/(c**(1/4)*sqrt(e))), -1)/(a*d**(1/4)*sqrt(e)*sqrt(c - d*x**2)
)
```

Mathematica [C] time = 0.231564, size = 163, normalized size = 0.87

$$-\frac{10acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{\sqrt{ex}\left(bx^2-a\right)\sqrt{c-dx^2}\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-10*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(Sqr
t[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2]*(5*a*c*AppellF1[1/4, 1/2, 1,
5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9
/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2
)/c, (b*x^2)/a])))
```

Maple [B] time = 0.038, size = 346, normalized size = 1.8

$$-\frac{\sqrt{2}b}{2\,dx^2-2\,c}\left(\sqrt{cd}EllipticPi\left(\sqrt{1\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},b\sqrt{cd}\left(\sqrt{abd}+\sqrt{cdb}\right)^{-1},\frac{\sqrt{2}}{2}\right)b-\sqrt{cd}EllipticPi\left(\sqrt{1\left(dx+\sqrt{cd}\right)\frac{1}{\sqrt{cd}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*((c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b-(c*d
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b-(a*b)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*d-(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*d)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*b*(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(
1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)/(e*x)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-a\sqrt{ex}\sqrt{c-dx^2}+bx^2\sqrt{ex}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)/(e*x)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(1/(-a*sqrt(e*x)*sqrt(c - d*x**2) + b*x**2*sqrt(e*x)*sqr
t(c - d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.885 $\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=379

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(a*c*e*Sqrt[e*x]) - (2*d^(1/4)*Sqrt[1 - (d*x
^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*Sqrt[1 - (d*
x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(
1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*e^(3/2)*S
qrt[c - d*x^2])
```

Rubi [A] time = 1.94727, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(a*c*e*Sqrt[e*x]) - (2*d^(1/4)*Sqrt[1 - (d*x
^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*Sqrt[1 - (d*
x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(
1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*e^(3/2)*S
qrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.632381, size = 338, normalized size = 0.89

$$\frac{2x\left(\frac{49x^2(bc-ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}-\frac{33bdx^4F_1\left(\frac{7}{4};\right.}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\right.\right.}\right.}{21(ex)^{3/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*((-21*(c - d*x^2))/(a*c) + (49*(b*c - a*d)*x^2*AppellF1[3/4,
 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(7*a*c*AppellF1
[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[
7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2,
 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) - (33*b*d*x^4*AppellF1[7/4, 1/
2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((-a + b*x^2)*(11*a*c*AppellF1
[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3
/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(21*(e*x)^(3/2)*Sqrt[c -
d*x^2])
```

Maple [B] time = 0.041, size = 835, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*b*c^2-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2+(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*c-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*c*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2+2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d-2*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*b*c^2-4*x^2*a*d^2+4*x^2*b*c*d+4*a*c*d-4*b*c^2)
*d*b*(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*
d+(c*d)^(1/2)*b)/a/c/(d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2 - a)\sqrt{-dx^2 + c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.886 $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=297

$$\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*c*e*(e*x)^(3/2)) + (2*d^(3/4)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(3*a*c^(3/4)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*
e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.30622, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*c*e*(e*x)^(3/2)) + (2*d^(3/4)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(3*a*c^(3/4)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*
e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.561811, size = 338, normalized size = 1.14

$$\frac{2x\left(\frac{25x^2(ad+3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{9bdx^4F_1\left(\frac{5}{4};\frac{1}{2},1 \ldots\right.}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\ldots\right.\right.\right.}\right.}{15(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*((-5*(c - d*x^2))/(a*c) + (25*(3*b*c + a*d)*x^2*AppellF1[1/4
, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(5*a*c*AppellF
1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a]))) + (9*b*d*x^4*AppellF1[5/4, 1/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a])/((-a + b*x^2)*(9*a*c*AppellF1[5/4
, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4,
 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1,
13/4, (d*x^2)/c, (b*x^2)/a])))))/(15*(e*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.04, size = 740, normalized size = 2.5

$$\frac{bd}{6\,cxa\,(dx^2-c)\,e^2}\left(2\,EllipticF\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},1/2\,\sqrt{2}\right)\sqrt{2}xad\sqrt{ab}\sqrt{cd}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}-2\,EllipticF\left(\sqrt{\frac{dx}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*b*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*2^(1/2)*x*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b*c*(a*b)^(1/2)*(c*
d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)+4*x^2*a*d^2*(a*b)^(1/2)-4*x^2*b*c*d*(a*b)^(1/2)-4*a*c*d*(a*b)^(
1/2)+4*b*c^2*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/x/c/a/((c*d)^(1/2)*b-(
a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(d*x^2-c)
/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2 - a)\sqrt{-dx^2 + c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.887 $\int \frac{1}{(ex)^{7/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=444

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(3ad+5bc)}{5a^2c^2e^3\sqrt{ex}}-\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}}$$

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*c*e*(e*x)^(5/2)) - (2*(5*b*c + 3*a*d)*S
qrt[c - d*x^2])/(5*a^2*c^2*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c + 3
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) +
(2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(5/4)*e^(7/2
)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c -
d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.64767, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(3ad+5bc)}{5a^2c^2e^3\sqrt{ex}}-\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```


```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*c*e*(e*x)^(5/2)) - (2*(5*b*c + 3*a*d)*S
qrt[c - d*x^2])/(5*a^2*c^2*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c + 3
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) +
(2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(5/4)*e^(7/2
)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c -
d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
```

```
*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 1.35509, size = 383, normalized size = 0.86

$$\frac{2x\left(\frac{49acx^4\left(-3a^2d^2-5abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{\left(a-bx^2\right)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{33abcdx^6(3ad+5bc\cdots}{\left(a-bx^2\right)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\cdots\right.\right.}\cdots\right.}{105a^2c^2(ex)^{7/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]

[Out] (2*x*(-21*(c - d*x^2)*(5*b*c*x^2 + a*(c + 3*d*x^2)) + (49*a*c*(5*
b^2*c^2 - 5*a*b*c*d - 3*a^2*d^2)*x^4*AppellF1[3/4, 1/2, 1, 7/4, (
d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7
/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11
/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^
2)/c, (b*x^2)/a]))) + (33*a*b*c*d*(5*b*c + 3*a*d)*x^6*AppellF1[7/
4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(11*a*c*Appe
llF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Appe
llF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/
4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(105*a^2*c^2*(e*x)^(7
/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.043, size = 1109, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x)

[Out] -1/10*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^2*c^3-5*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*(c*d)^(1/2)*x^2*b*c^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^2*
c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b*c^2-12*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)
```

```
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*c*d^2-8*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b*c^2*d+20*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*x^2*b^2*c^3+6*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*x^2*a^2*c*d^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b*c^2*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
x^2*b^2*c^3-12*x^4*a^2*d^3-8*x^4*a*b*c*d^2+20*x^4*b^2*c^2*d+8*x^2
*a^2*c*d^2+12*x^2*a*b*c^2*d-20*x^2*b^2*c^3+4*a^2*c^2*d-4*a*b*c^3)
*b*d*(-d*x^2+c)^(1/2)/x^2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1
/2)*d+(c*d)^(1/2)*b)/a^2/c^2/(d*x^2-c)/e^3/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)),x, algorithm="giac")

[Out] integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)), x)
```

3.888 $\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}} dx$

Optimal. Leaf size=444

$$-\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3bc-2ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}\sqrt{c-dx^2}(bc-ad)}-\frac{ce^3(ex)^{3/2}}{d\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((c*e^3*(e*x)^(3/2))/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)
*(3*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*
Sqrt[c - d*x^2]) - (c^(3/4)*(3*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (a^(3/2)*c^(1/4)*e^
(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]
*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a^(3/2)*c^(1/4)*e
^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^
(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.20844, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3bc-2ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}\sqrt{c-dx^2}(bc-ad)}-\frac{ce^3(ex)^{3/2}}{d\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((c*e^3*(e*x)^(3/2))/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)
*(3*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*
Sqrt[c - d*x^2]) - (c^(3/4)*(3*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (a^(3/2)*c^(1/4)*e^
(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]
*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a^(3/2)*c^(1/4)*e
^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
```

```
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^
(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(9/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.721983, size = 424, normalized size = 0.95

$$\frac{c(ex)^{9/2}\left(\frac{49a^2cF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{11a(7ac-2adx^2-4bcx^2)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c}\right.}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\right.\right.\right.}\right.}{7dx^3\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (c*(e*x)^(9/2)*((49*a^2*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (
b*x^2)/a])/((-a + b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2
)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2
)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x
^2)/a]))) + (11*a*(7*a*c - 4*b*c*x^2 - 2*a*d*x^2)*AppellF1[7/4, 1
/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] - 14*x^2*(-a + b*x^2)*(2*b*c*A
ppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[
11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))/((a - b*x^2)*(11*a*c*
AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*
AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1
[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(7*d*(-(b*c) + a*
d)*x^3*Sqrt[c - d*x^2])
```

Maple [B] time = 0.066, size = 1041, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-10*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d+6*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3-2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*b*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*c^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
```

```
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*
c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-2*x^2*a*b^2*c*d^2+2
*x^2*b^3*c^2*d)*(-d*x^2+c)^(1/2)*e^4*(e*x)^(1/2)/x/d/((c*d)^(1/2)
*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*d-b*c)/(d*x^2-
c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{9}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.889 $\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=338

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{5/4}\sqrt{c-dx^2}(bc-ad)}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{ce^3\sqrt{ex}}{d\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((c*e^3*Sqrt[e*x])/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(1/4)*(
b*c - 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(5/4)*(b*c - a*d)*Sqrt
[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[
-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.43327, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{5/4}\sqrt{c-dx^2}(bc-ad)}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{ce^3\sqrt{ex}}{d\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((c*e^3*Sqrt[e*x])/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(1/4)*(
b*c - 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(5/4)*(b*c - a*d)*Sqrt
[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[
-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.767085, size = 424, normalized size = 1.25

$$\frac{c(ex)^{7/2}\left(\frac{25a^2cF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{9a(5ac-2adx^2-4bcx^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\cdots\right.}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c}\cdots\right.\right.\right.}\cdots\right.}{5dx^3\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (c*(e*x)^(7/2)*((25*a^2*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (
b*x^2)/a])/((-a + b*x^2)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2
)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)
/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2
)/a]))) + (9*a*(5*a*c - 4*b*c*x^2 - 2*a*d*x^2)*AppellF1[5/4, 1/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 10*x^2*(-a + b*x^2)*(2*b*c*Appel
lF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4,
3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/((a - b*x^2)*(9*a*c*AppellF
1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2
, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(5*d*(-(b*c) + a*d)*x^3*Sqr
t[c - d*x^2])
```

Maple [B] time = 0.061, size = 826, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*a^2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)-3*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*2^(1/2)*a*b*c*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)+EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d
^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)*a^2*d^2-((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1
/2)*a^2*d^2+2*x*a*b*c*d^2*(a*b)^(1/2)-2*x*b^2*c^2*d*(a*b)^(1/2))*
(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/
((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(a*d-b*c)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.890 $\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=414

$$-\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{e(ex)^{3/2}}{\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((e*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3
/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2
]) + (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.99355, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{e(ex)^{3/2}}{\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((e*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3
/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2
]) + (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.523401, size = 327, normalized size = 0.79

$$\frac{e(ex)^{3/2}\left(\frac{49a^2cF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{11abcx\cdots}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\cdots\right.\right.}\right.}{7\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*(e*x)^(3/2)*(7 + (49*a^2*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/
c, (b*x^2)/a])/((-a + b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d
*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d
*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c,
(b*x^2)/a]))) + (11*a*b*c*x^2*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)
/c, (b*x^2)/a])/((a - b*x^2)*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4,
(d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4,
 (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)
/c, (b*x^2)/a])))))/(7*(-(b*c) + a*d)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 839, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2)
)*(c*d)^(1/2)*a*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*(c*d)^(1/2)*a*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c
^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+2*a*b*d^2*x^2-2*b^2*c*d*x^2)*
(-d*x^2+c)^(1/2)*e^2*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/
((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*d-b*c)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.891 $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{e\sqrt{ex}}{\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((e*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) - (c^(1/4)*e^(3/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c -
 a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.23651, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{e\sqrt{ex}}{\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((e*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) - (c^(1/4)*e^(3/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c -
 a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.48502, size = 328, normalized size = 1.04

$$\frac{e\sqrt{ex}\left(\frac{25a^2cF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{9abcx^2F_1\left(\frac{5}{4}\right.}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF\right.\right.}\right.}{5\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*Sqrt[e*x]*(5 + (25*a^2*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
 (b*x^2)/a])/((-a + b*x^2)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x
^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^
2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x
^2)/a]))) + (9*a*b*c*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b
*x^2)/a])/((-a + b*x^2)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)
/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)
/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^
2)/a])))))/(5*(-(b*c) + a*d)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.039, size = 704, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*b*(EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a*d*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)-EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*2^(1/2)*b*c*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b*c*d-((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*(c*d)^(1/2)*a*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d-((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*d+2*x*a*d^2*(a*b)^(1/2)-2*
x*b*c*d*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e*(e*x)^(1/2)/x/((c*d)^(1/2
)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(a*d
-b*c)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate((e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(ex)^{\frac{3}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] -Integral((e*x)**(3/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c -
 d*x**2) + b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-(e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.892 $\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=420

$$-\frac{d(ex)^{3/2}}{ce\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((d*(e*x)^(3/2))/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) + (d^(1/4)*S
qrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (
d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*
x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-
((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d
*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x
^2])
```

Rubi [A] time = 2.06945, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{d(ex)^{3/2}}{ce\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```


```
[Out] -((d*(e*x)^(3/2))/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) + (d^(1/4)*S
qrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (
d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*
x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-
((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d
*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x
^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.798072, size = 356, normalized size = 0.85

$$\frac{x\sqrt{ex}\left(\frac{33abdx^2F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)(ad-bc)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}-\frac{\cdots}{(a-bx^2)(ad-bc)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\cdots\right.\right.\right.}\right.}{21\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[e*x]*((-21*d)/(b*c^2 - a*c*d) - (49*a*(2*b*c + a*d)*Appel
lF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((-(b*c) + a*d)*(a -
 b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] +
 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] +
 a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) + (33*a
*b*d*x^2*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((-(b
*c) + a*d)*(a - b*x^2)*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2
)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^
2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b
*x^2)/a])))))/(21*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 830, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*b*c^2-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2+(c*d)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*c+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2-((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*b*c^2+2*x^2*a*d^2-2*x^2*b*c*d)*d*b*(-d*x^2+c)^(1/2)
*(e*x)^(1/2)/c/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)
^(1/2)*b)/(a*d-b*c)/(d*x^2-c)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex}}{(bx^2 - a)(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{ex}}{(bx^2 - a)(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.893 $\int \frac{1}{\sqrt{ex}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=328

$$-\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}-\frac{d\sqrt{ex}}{ce\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -((d*Sqrt[e*x])/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) - (d^(3/4)*Sqr
t[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(c^(3/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*
c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b
*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.3048, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}-\frac{d\sqrt{ex}}{ce\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((d*Sqrt[e*x])/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) - (d^(3/4)*Sqr
t[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(c^(3/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*
c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b
*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)/(-d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.764581, size = 357, normalized size = 1.09

$$\frac{x\left(\frac{9abdx^2F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)(ad-bc)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{25a(ad}{(a-bx^2)(ad-bc)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c}\right.\right.\right.}\right.}{5\sqrt{ex}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*((-5*d)/(b*c^2 - a*c*d) + (25*a*(-2*b*c + a*d)*AppellF1[1/4, 1
/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/((-(b*c) + a*d)*(a - b*x^2)*(5
*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*
b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*Appell
F1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))) + (9*a*b*d*x^2*Appe
llF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/((-(b*c) + a*d)*(-a
 + b*x^2)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]
 + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a]
 + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(5*
Sqrt[e*x]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.046, size = 708, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] 1/2*b*d*(EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*a*d*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)-EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*2^(1/2)*b*c*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^2-((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*(c*d)^(1/2)*b*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b*c+2*x*a*d^2*(a*b)^(1/2)-
2*x*b*c*d*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/c/((c*d)^(1/2)*b-(a*b)^(1
/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(a*d-b*c)/(d*x^2
-c)/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ac\sqrt{ex}\sqrt{c-dx^2}+adx^2\sqrt{ex}\sqrt{c-dx^2}+bcx^2\sqrt{ex}\sqrt{c-dx^2}-bdx^4\sqrt{ex}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)/(-d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] -Integral(1/(-a*c*sqrt(e*x)*sqrt(c - d*x**2) + a*d*x**2*sqrt(e*x)
*sqrt(c - d*x**2) + b*c*x**2*sqrt(e*x)*sqrt(c - d*x**2) - b*d*x**
4*sqrt(e*x)*sqrt(c - d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.894 $\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=493

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(2bc-3ad)}{ac^2e\sqrt{ex}(bc-ad)}-\frac{d}{ce\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -(d/(c*(b*c - a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])) - ((2*b*c - 3*a*
d)*Sqrt[c - d*x^2])/(a*c^2*(b*c - a*d)*e*Sqrt[e*x]) - (d^(1/4)*(2
*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqr
t[c - d*x^2]) + (d^(1/4)*(2*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(5
/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(
1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/
4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.76648, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(2bc-3ad)}{ac^2e\sqrt{ex}(bc-ad)}-\frac{d}{ce\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])) - ((2*b*c - 3*a*
d)*Sqrt[c - d*x^2])/(a*c^2*(b*c - a*d)*e*Sqrt[e*x]) - (d^(1/4)*(2
*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[
```

```
e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqr
t[c - d*x^2]) + (d^(1/4)*(2*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(5
/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(
1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/
4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.8487, size = 401, normalized size = 0.81

$$\frac{x\left(\frac{49cx^2(3a^2d^2-2abcd+2b^2c^2)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)(bc-ad)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{3\left(\frac{11bcdx^4(3ad\cdots}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+\cdots\right.\right.}\right.}{\cdots}\right.}{21c^2(ex)^{3/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*((49*c*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x^2*AppellF1[3/4, 1
/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((b*c - a*d)*(a - b*x^2)*(7*a*
c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c
*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1
[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) + (3*((14*b*c*(c - d
*x^2))/a + 7*d*(-2*c + 3*d*x^2) + (11*b*c*d*(-2*b*c + 3*a*d)*x^4*
AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/((a - b*x^2)*(
11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*
(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*A
ppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(-(b*c) +
a*d)))/(21*c^2*(e*x)^(3/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.043, size = 1058, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*b^2*c^3-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
```

```
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*b*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^3+(c*d)^(1/2)*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2+6*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a^2*c*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^2*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c^3+6*x^2*a^2*d^3-10*x^2*a*b*c*d^2+4*x^2*b^2
*c^2*d-4*a^2*c*d^2+8*a*b*c^2*d-4*b^2*c^3)*b*d*(-d*x^2+c)^(1/2)/c^
2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*
d-b*c)/a/(d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2 - a)(-dx^2 + c)^{\frac{3}{2}} (ex)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2 - a)(-dx^2 + c)^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.895 $\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=397

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac^2e(ex)^{3/2}(bc-ad)}-\frac{d}{ce(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -(d/(c*(b*c - a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])) - ((2*b*c - 5*
a*d)*Sqrt[c - d*x^2])/(3*a*c^2*(b*c - a*d)*e*(e*x)^(3/2)) + (d^(3
/4)*(2*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*c^(7/4)*(b*c - a*d)*e^(
5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sq
rt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x
^2])
```

Rubi [A] time = 2.11053, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac^2e(ex)^{3/2}(bc-ad)}-\frac{d}{ce(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```



```
[Out] -(d/(c*(b*c - a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])) - ((2*b*c - 5*
a*d)*Sqrt[c - d*x^2])/(3*a*c^2*(b*c - a*d)*e*(e*x)^(3/2)) + (d^(3
/4)*(2*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*c^(7/4)*(b*c - a*d)*e^(
5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sq
rt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x
^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.60533, size = 413, normalized size = 1.04

$$\frac{x\left(-\frac{25cx^2\left(-5a^2d^2+2abcd+6b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)(bc-ad)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{9bcdx^4(}{(bx^2-a)(bc-ad)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx}{c}\right.\right.\right.}\right.}{15c^2(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*((10*b*c*(c - d*x^2) + 5*a*d*(-2*c + 5*d*x^2))/(a*(-(b*c) + a*
d)) - (25*c*(6*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x^2*AppellF1[1/4,
 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/((b*c - a*d)*(-a + b*x^2)*(5
*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*
b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*Appell
F1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))) + (9*b*c*d*(2*b*c -
 5*a*d)*x^4*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/((b
*c - a*d)*(-a + b*x^2)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/
c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/
c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2
)/a])))))/(15*c^2*(e*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.046, size = 896, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/6*b*d*(3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^
3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticF(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*d^2*(c*d)^(1/2)*(a
*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+7*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a*b*c*d*(c
*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-2*Elli
pticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
x*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*b^3*c^
3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*2^(1/2)*x*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-10*x^2*a^2*d^3*(a*b)^(1/2)+14*x^2*
a*b*c*d^2*(a*b)^(1/2)-4*x^2*b^2*c^2*d*(a*b)^(1/2)+4*a^2*c*d^2*(a*
b)^(1/2)-8*a*b*c^2*d*(a*b)^(1/2)+4*b^2*c^3*(a*b)^(1/2))*(-d*x^2+c
)^(1/2)/x/c^2/a/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d
)^(1/2)*b)/(a*b)^(1/2)/(a*d-b*c)/(d*x^2-c)/e^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.896 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=362

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2}$$

```
[Out] (7*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*(e*x)^(5/2)*Sqrt[c
 - d*x^2])/(2*b*(a - b*x^2)) + (c^(1/4)*(8*b*c - 21*a*d)*e^(7/2)*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(6*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 7
*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.85079, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)} + \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (7*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*(e*x)^(5/2)*Sqrt[c
 - d*x^2])/(2*b*(a - b*x^2)) + (c^(1/4)*(8*b*c - 21*a*d)*e^(7/2)*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(6*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 7
*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.788213, size = 426, normalized size = 1.18

$$\frac{(ex)^{7/2}\left(\frac{175a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{-10x^2(7a-4bx^2)(c-dx^2)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+\ldots}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\ldots\right.\right.}\right.}{30b^2x^3\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] ((e*x)^(7/2)*((175*a^2*c^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
(b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/
a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a
] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (-9*
a*c*(7*a*(5*c - 2*d*x^2) + 4*b*x^2*(-7*c + 5*d*x^2))*AppellF1[5/4
, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 10*x^2*(7*a - 4*b*x^2)*(c
- d*x^2)*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a]
 + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(9*a*c
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*
AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[
9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(30*b^2*x^3*(-a + b*
x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.067, size = 2561, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/24*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2*(16*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-16*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)
*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)
^(1/2)-16*x^5*b^3*c*d^2*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)+28*x*a^2*b*c*d^2*(a*b)^(1/2)-28*x*a*b^2*c^2*d*(a*b)^(1/2)-58
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/
2)*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2
```

```
)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*
(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x
^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*
b)^(1/2)-28*x^3*a^2*b*d^3*(a*b)^(1/2)+16*x^3*b^3*c^2*d*(a*b)^(1/2
)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d
^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-2
1*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+42*E
llipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*(a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^
(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c
*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5
8*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^
(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*
(a*b)^(1/2)+16*x^5*a*b^2*d^3*(a*b)^(1/2)+12*x^3*a*b^2*c*d^2*(a*b)
^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^
2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-42*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-21*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(
1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2))/x/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)
*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{(bx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2+c}\sqrt{ex}e^3x^3}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 + c)*sqrt(e*x)*e^3*x^3/(b^2*x^4 - 2*a*b*x^2
+ a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{7}{2}}}{(bx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.897 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=413

$$\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} - \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}$$

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (5*c^(3/4)*d^
(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (5*c^(3/
4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (c^
(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]
) - (c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*
x^2])
```

Rubi [A] time = 2.04258, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} - \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```


```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (5*c^(3/4)*d^
(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (5*c^(3/
4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (c^
(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]
) - (c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*
x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.362822, size = 318, normalized size = 0.77

$$\frac{e(ex)^{3/2}\left(\frac{49ac^2F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{55acdx^2F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\ldots\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\ldots\right)\ldots\right.}\ldots\right)}{14b\,(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*c + 7*d*x^2 + (49*a*c^2*AppellF1[3/4, 1/2, 1,
7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*
x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*
x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (
b*x^2)/a])) - (55*a*c*d*x^2*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c
, (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x
^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*
x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])
)))/(14*b*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.064, size = 2542, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(20*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2)
)*(c*d)^(1/2)*x^2*b^2*c-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^
(1/2)*x^2*b^2*c+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+5*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)
^(1/2)*a*b*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c
-4*x^2*b^3*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
```

```
1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2
*a*b*d+4*x^2*a*b^2*c*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2-3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*x^2*b^3*c^2-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*
b^2*c^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+3*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-20
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+10*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d-4*x^4*a*b^2*d^2+
4*x^4*b^3*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*
a*b*d+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-10*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+5*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*(c*d)^(1/2)*a^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*a^2*b*c*d)/x/b^2/(d*x^2-c)/(b*x^2-a)/((a*b)^(1/2)
*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}{(bx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}\,(ex)^{\frac{5}{2}}}{(bx^2 - a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.898 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=328

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b(a-bx^2)}-\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}$$

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (3*c^(1/4)*d^(3
/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(
b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c -
 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.31309, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b(a-bx^2)}-\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (3*c^(1/4)*d^(3
/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(
b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c -
 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.37544, size = 318, normalized size = 0.97

$$\frac{e\sqrt{ex}\left(\frac{25ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{27acdx^2F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+5dx^2-5c\right)}{10b\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*(-5*c + 5*d*x^2 + (25*a*c^2*AppellF1[1/4, 1/2, 1, 5/
4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^
2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2
)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^
2)/a])) - (27*a*c*d*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*
x^2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]
+ 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a]
+ a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(10*b
*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.027, size = 2255, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b*d*(6*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-6*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^
2*c*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+
3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)
^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*(c*d)^(1/2)*x^2*a*b*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^
(1/2)*x^2*a*b*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
```

```
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-6*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+6*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(
1/2)*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*(c*d)^(1/2)*a*b*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c
-4*x^3*a*b*d^2*(a*b)^(1/2)+4*x^3*b^2*c*d*(a*b)^(1/2)+4*x*a*b*c*d*
(a*b)^(1/2)-4*x*b^2*c^2*(a*b)^(1/2))/x/(d*x^2-c)/(b*x^2-a)/(a*b)^
(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{(bx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}{(bx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.899 $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2} dx$

Optimal. Leaf size=417

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}+\frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae(a-bx^2)}$$

```
[Out] ((e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) - (c^(3/4)*d^(1
/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*Sqrt[c - d*x^2]) + (c^(3/4)*d
^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*Sqrt[c - d*x^2]) - (c^(1/4
)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(
1/4)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.02688, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}+\frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```


```
[Out] ((e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) - (c^(3/4)*d^(1
/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*Sqrt[c - d*x^2]) + (c^(3/4)*d
^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*Sqrt[c - d*x^2]) - (c^(1/4
)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(
1/4)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.304875, size = 317, normalized size = 0.76

$$\frac{x\sqrt{ex}\left(-\frac{49c^2F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{33cdx^2F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx}{c}\cdots\right.}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx}{c}\cdots\right.\right.}\cdots\right.}{42\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (x*Sqrt[e*x]*((-21*(c - d*x^2))/a - (49*c^2*AppellF1[3/4, 1/2, 1,
 7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d
*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d
*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c,
(b*x^2)/a])) - (33*c*d*x^2*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c,
 (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x
^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))
))/(42*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.028, size = 2534, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/8*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(4*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*x^2*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)
^(1/2)*x^2*b^2*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2
*b^2*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^
2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-((
```

```
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-4*x^2*b^3*c^2-((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+4*x^2*a*b^2*
c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*x^2*b^3*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-4*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+2*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^
2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*x^2*a*b^2*c*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*x^2*a*b^2*c*d-4*x^4*a*b^2*d^2+4*x^4*b^3*c*d+((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+4*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*b*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d)/b/x/(d*x^2-c)/a/(
b*x^2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)
*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}\sqrt{ex}}{(bx^2 - a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}\sqrt{ex}}{(bx^2 - a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.900 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

Optimal. Leaf size=335

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\,(a-bx^2)}$$

```
[Out] (Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) + (c^(1/4)*d^(3/4
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*
b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*a^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c -
 a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.23237, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) + (c^(1/4)*d^(3/4
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*
b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*a^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c -
 a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.314595, size = 317, normalized size = 0.95

$$\frac{x\left(-\frac{75c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{9cdx^2F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}\right)}{10\sqrt{ex}\,(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (x*((-5*(c - d*x^2))/a - (75*c^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^
2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b
*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*
x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))
+ (9*c*d*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(9
*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*
b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*Appel
lF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(10*Sqrt[e*x]*(-
a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.037, size = 2251, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(-d*x^2+c)^(1/2)*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b*d*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^2*c*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*x^2*a*b^2*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^
2*a*b*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2
*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
```

```
1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
(c*d)^(1/2)*x^2*b^2*c-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*
c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(
1/2)*a*b*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+4*x^3*a*b*d^2*(a
*b)^(1/2)-4*x^3*b^2*c*d*(a*b)^(1/2)-4*x*a*b*c*d*(a*b)^(1/2)+4*x*b
^2*c^2*(a*b)^(1/2))/a/(e*x)^(1/2)/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)
/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.901 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=444

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}}+\frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)}$$

```
[Out] (-5*Sqrt[c - d*x^2])/(2*a^2*e*Sqrt[e*x]) + Sqrt[c - d*x^2]/(2*a*e
*Sqrt[e*x]*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*
EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
a^2*e^(3/2)*Sqrt[c - d*x^2]) + (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2
)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a*d)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]))
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*
Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b*c - 3*a*
d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5
/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.52903, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}}+\frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```


```
[Out] (-5*Sqrt[c - d*x^2])/(2*a^2*e*Sqrt[e*x]) + Sqrt[c - d*x^2]/(2*a*e
*Sqrt[e*x]*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*
EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
a^2*e^(3/2)*Sqrt[c - d*x^2]) + (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2
)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a*d)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]))
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*
Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b*c - 3*a*
d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5
```

```
/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.00702, size = 340, normalized size = 0.77

$$\frac{x\left(\frac{49acx^2(5bc-8ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{165abcdx^4F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}\right.}{42a^2(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(21*(4*a - 5*b*x^2)*(-c + d*x^2) + (49*a*c*(5*b*c - 8*a*d)*x^2
*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF
1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1
[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2
, 1, 11/4, (d*x^2)/c, (b*x^2)/a])) + (165*a*b*c*d*x^4*AppellF1[7/
4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2
, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/
2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15
/4, (d*x^2)/c, (b*x^2)/a]))))/(42*a^2*(e*x)^(3/2)*(a - b*x^2)*Sqr
t[c - d*x^2])
```

Maple [B] time = 0.041, size = 2568, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(-d*x^2+c)^(1/2)*d*(-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*b^3*c^2-16*a*b^2*c^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(
c*d)^(1/2)*x^2*b^2*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*d+3*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
```

```
^(1/2))*x^2*a*b^2*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1
/2)*a*b*c+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+16
*a^2*b*c*d+20*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d
)^(1/2)*x^2*a*b*d-4*x^2*a*b^2*c*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*x^2*b^3*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2-5*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*x^2*b^3*c^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b^2*c^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*
b^2*c^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+5*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
*b^2*c^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
E(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*
d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d-16*x^
2*a^2*b*d^2+20*x^4*a*b^2*d^2-20*x^4*b^3*c*d+3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-20*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*b*c*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*
d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d)/e/(e*x)^
(1/2)/(d*x^2-c)/a^2/(b*x^2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d
)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2 (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2 (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.902 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=355

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}}-\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}}+\frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}(a-bx^2)}$$

```
[Out] (-7*Sqrt[c - d*x^2])/(6*a^2*e*(e*x)^(3/2)) + Sqrt[c - d*x^2]/(2*a
*e*(e*x)^(3/2)*(a - b*x^2)) + (7*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)
/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(6*a^2*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/
4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*e^(5/
2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.77868, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}}-\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}}+\frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (-7*Sqrt[c - d*x^2])/(6*a^2*e*(e*x)^(3/2)) + Sqrt[c - d*x^2]/(2*a
*e*(e*x)^(3/2)*(a - b*x^2)) + (7*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)
/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(6*a^2*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/
4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*e^(5/
2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.971014, size = 361, normalized size = 1.02

$$\frac{x\left(\frac{25acx^2(21bc-8ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{63abcdx^4F_1\left(\frac{5}{4};\frac{1}{2},1\right.}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2}\right.\right.\right.}\right.}{30a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*((5*(4*a - 7*b*x^2)*(-c + d*x^2))/(a - b*x^2) + (25*a*c*(21*b*
c - 8*a*d)*x^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/
((a - b*x^2)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)
/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/
a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))) + (6
3*a*b*c*d*x^4*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(
(-a + b*x^2)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)
/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)
/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/
(30*a^2*(e*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 2316, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/24*(-d*x^2+c)^(1/2)*b*d*(-28*x^4*b^2*c*d*(a*b)^(1/2)-14*Ellipti
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3
*b^2*c*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*b^2*
c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*b^2*c*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x^3*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^
2*c*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-
14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2
^(1/2)*x*a^2*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(
a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*
a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+15*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*d*(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+15*EllipticPi(((d*x+(c*d)^(
```

```
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^
2*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-
16*x^2*a^2*d^2*(a*b)^(1/2)+28*x^2*b^2*c^2*(a*b)^(1/2)+16*a^2*c*d*
(a*b)^(1/2)-16*a*b*c^2*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)+28*x^4*a*b*d^2*(a*b)^(1/2)+14*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a*b*d*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-15*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*b*d*(c*d)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-15*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*x^3*a*b*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+14*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a*b*c*(c*d)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-21*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b*c*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x*a*b*c*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*2^(1/2)*x*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-12*x^2*a*b*c*d*(a*b)^(1/2)+21*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2))/x/a^2/e^2/(e*x)^(1/2)/(d*x^2-c)/(b
*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-
(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2 (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}}{(bx^2 - a)^2 (ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.903 $\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=429

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{42b^4\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^4\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^4\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e^3\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{42b^3}+\frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2}$$

```
[Out] ((57*b*c - 77*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(42*b^3) - (11*
d*e*(e*x)^(5/2)*Sqrt[c - d*x^2])/(14*b^2) + (e*(e*x)^(5/2)*(c - d
*x^2)^(3/2))/(2*b*(a - b*x^2)) + (c^(1/4)*(48*b^2*c^2 - 259*a*b*c
*d + 231*a^2*d^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(42*b^4*d^(1/4)*Sqrt[c
 - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4
)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d
^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.54322, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{42b^4\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^4\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^4\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e^3\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{42b^3}+\frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```


```
[Out] ((57*b*c - 77*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(42*b^3) - (11*
d*e*(e*x)^(5/2)*Sqrt[c - d*x^2])/(14*b^2) + (e*(e*x)^(5/2)*(c - d
*x^2)^(3/2))/(2*b*(a - b*x^2)) + (c^(1/4)*(48*b^2*c^2 - 259*a*b*c
*d + 231*a^2*d^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(42*b^4*d^(1/4)*Sqrt[c
 - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4
)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d
^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.06209, size = 392, normalized size = 0.91

$$\frac{(ex)^{7/2}\left(\frac{9acx^2\left(231a^2d^2-259abcd+48b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{25a^2c^2(77ad-57bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\ldots\right.}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\ldots\right.\right.}\right.}{210b^3x^3\left(bx^2-a\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((e*x)^(7/2)*(5*(c - d*x^2)*(77*a^2*d - 12*b^2*x^2*(-3*c + d*x^2)
 - a*b*(57*c + 44*d*x^2)) - (25*a^2*c^2*(-57*b*c + 77*a*d)*Appell
F1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4,
1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1
/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4
, (d*x^2)/c, (b*x^2)/a])) + (9*a*c*(48*b^2*c^2 - 259*a*b*c*d + 23
1*a^2*d^2)*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/
(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(
2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*App
ellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(210*b^3*x^3*(
-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.042, size = 3790, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/168*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^3*(-48*x^7*b^4*c*d^3*(a*
b)^(1/2)+176*x^5*a^2*b^2*d^4*(a*b)^(1/2)+192*x^5*b^4*c^2*d^2*(a*b
)^(1/2)-308*x^3*a^3*b*d^4*(a*b)^(1/2)-144*x^3*b^4*c^3*d*(a*b)^(1/
2)+360*x^3*a^2*b^2*c*d^3*(a*b)^(1/2)+92*x^3*a*b^3*c^2*d^2*(a*b)^(
1/2)+308*x*a^3*b*c*d^3*(a*b)^(1/2)-536*x*a^2*b^2*c^2*d^2*(a*b)^(1
/2)+228*x*a*b^3*c^3*d*(a*b)^(1/2)+336*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-980*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+614*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)
*x^2*a*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x^2*a*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
```

*(a*b)^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*x^2*a*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*x^2*a*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+96*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^
3*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)-96*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*2^(1/2)*x^2*b^4*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)-368*x^5*a*b^3*c*d^3*(a*b)^(1/2)+48*x^7*a*b^3*d^4
*(a*b)^(1/2)+105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*b^2*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
a*b)^(1/2)+462*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*2^(1/2)*x^2*a^3*b*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(a*b)^(1/2)-231*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*x^2*a^3*b*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)+980*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-614*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-336*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+105*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-336*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-231*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b*d^3*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+336*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^3*c^2*d^2*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)+336*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*a^4*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)+336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*2^(1/2)*a^3*b^2*c^2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-1
05*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^3*d
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+231*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*a^4*b*c*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^
4*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(

```
1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)-336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b^2*c^
2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+105*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^3*d*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)-462*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*a^4*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(a*b)^(1/2)-231*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*a^4*b*c*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
+231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b^2
*c*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-336*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^3*c^2*d^2*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)+105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*2^(1/2)*x^2*a*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-
231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b^2*
c*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2))/x/(d*x^2-c)/(a*b)^(1/
2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(b
*x^2-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(de^3x^5-ce^3x^3\right)\sqrt{-dx^2+c}\sqrt{ex}}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*e^3*x^5 - c*e^3*x^3)*sqrt(-d*x^2 + c)*sqrt(e*x)/(b^2
*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.904 $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=485

$$\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(11bc-15ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}} - \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(11bc-15ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2}$$

```
[Out] (-9*d*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(10*b^2) + (e*(e*x)^(3/2)*(c
 - d*x^2)^(3/2))/(2*b*(a - b*x^2)) - (3*c^(3/4)*d^(1/4)*(11*b*c -
 15*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c
^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(10*b^3
*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*
e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*Sqrt[a]*b^(7/2)*d^(1/4)*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b^2*c^
2 - 4*a*b*c*d + 3*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(7/2)*d^(1/4)*Sqrt[c - d*x^
2])
```

Rubi [A] time = 2.7314, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(11bc-15ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}} - \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(11bc-15ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (-9*d*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(10*b^2) + (e*(e*x)^(3/2)*(c
 - d*x^2)^(3/2))/(2*b*(a - b*x^2)) - (3*c^(3/4)*d^(1/4)*(11*b*c -
 15*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sq
```

```
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c
^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(10*b^3
*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*
e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*Sqrt[a]*b^(7/2)*d^(1/4)*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b^2*c^
2 - 4*a*b*c*d + 3*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(7/2)*d^(1/4)*Sqrt[c - d*x^
2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.966034, size = 353, normalized size = 0.73

$$\frac{(ex)^{5/2}\left(-\frac{49ac^2(9ad-5bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{33acdx^2(15ad-11bc)F_1\left(\frac{7}{4};\frac{1}{2}\right.}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\right.\right.}\right.}{70b^2\left(bx^3-ax\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((e*x)^(5/2)*(-7*(c - d*x^2)*(5*b*c - 9*a*d + 4*b*d*x^2) - (49*a*
c^2*(-5*b*c + 9*a*d)*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2
)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2
*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a
*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])) + (33*a*c*
d*(-11*b*c + 15*a*d)*x^2*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (
b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)
/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2
)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))))
/(70*b^2*Sqrt[c - d*x^2]*(-(a*x) + b*x^3))
```

Maple [B] time = 0.038, size = 3886, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/40/x*e^2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(132*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3+36*x^4*a^2*b^2*d^3+4*x^4*b^4*
c^2*d-16*x^6*a*b^3*d^3+16*x^6*b^4*c*d^2-40*x^4*a*b^3*c*d^2-45*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
```

/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-180*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2+312*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d+90*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2-156*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d-20*x^2*b^4*c^3-36*x^2*a^2*b^2*c*d^2+56*x^2*a*b^3*c^2*d-60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+15*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2-15*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+180*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-312*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d-90*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+156*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d-15*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+15*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-66*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3-132*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3+66*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c

```
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3+15*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*a*b^3*c^3+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+45*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-45*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+60*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-60*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-60*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*
b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-60*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*
b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+45*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))/b^3/(d*x^2-c)/((a*b)^(1/2)*d
+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(b*x^2-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{(bx^2-a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.905 $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2} dx$

Optimal. Leaf size=381

$$-\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2}$$

```
[Out] (-7*d*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*Sqrt[e*x]*(c - d*
x^2)^(3/2))/(2*b*(a - b*x^2)) - (c^(1/4)*d^(3/4)*(17*b*c - 21*a*d
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(6*b^3*Sqrt[c - d*x^2]) - (c^(1/4)*(b*
c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1
/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.83662, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (-7*d*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*Sqrt[e*x]*(c - d*
x^2)^(3/2))/(2*b*(a - b*x^2)) - (c^(1/4)*d^(3/4)*(17*b*c - 21*a*d
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(6*b^3*Sqrt[c - d*x^2]) - (c^(1/4)*(b*
c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1
/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.996288, size = 353, normalized size = 0.93

$$\frac{(ex)^{3/2}\left(-\frac{25ac^2(7ad-3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{9acdx^2(21ad-17bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4}\right.}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\right.\right.}\right.}{30b^2\left(bx^3-ax\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((e*x)^(3/2)*(-5*(c - d*x^2)*(3*b*c - 7*a*d + 4*b*d*x^2) - (25*a*
c^2*(-3*b*c + 7*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2
)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2
*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*
d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (9*a*c*d*(
-17*b*c + 21*a*d)*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^
2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] +
2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] +
a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(30*b^2
*Sqrt[c - d*x^2]*(-(a*x) + b*x^3))
```

Maple [B] time = 0.038, size = 3466, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/24*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^2*d*(-34*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^2*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+34*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b
)^(1/2)+16*x^5*b^3*c*d^2*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)+24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)-28*x*a^2*b*c*d^2*(a*b)^(1/2)+40*x*a*b^2*c^2*d*(a*b)^(1/2)+7
```

```
6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^
(1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1
/2)*(a*b)^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/
2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*(a*b)^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a
*b)^(1/2)+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*
c^2+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-3*
(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-3*(c*d)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1
/2))*x^2*b^4*c^3+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*a*b^3*c^3-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+28*x^3*a^2*b
*d^3*(a*b)^(1/2)-4*x^3*b^3*c^2*d*(a*b)^(1/2)+21*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-42*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-76*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-16*x^5*a*b
^2*d^3*(a*b)^(1/2)-12*x*b^3*c^3*(a*b)^(1/2)-24*x^3*a*b^2*c*d^2*(a
*b)^(1/2)+24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2
*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+42*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(
```

```
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)*(a*b)^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)*(a*b)^(1/2))/x/(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1
/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2,x, algorithm="giac")

[Out] integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.906 $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=474

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}+\frac{(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}{2abe\left(a-bx^2\right)}$$

```
[Out] ((b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) -
 (c^(3/4)*d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
ticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2
*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b^2*c^2 + 4*a*b*
c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) + (
c^(1/4)*(b^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4
)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.29544, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}+\frac{(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}{2abe\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) -
 (c^(3/4)*d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
ticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2
*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b^2*c^2 + 4*a*b*
c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) + (
c^(1/4)*(b^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2
```

```
)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4
)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.692986, size = 428, normalized size = 0.9

$$\frac{x\sqrt{ex}\left(\frac{42x^2(dx^2-c)(bc-ad)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+33ac\left(ad(7c-2dx^2)+bc(6dx^2-7c)\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)} - \frac{\cdots}{2x\cdots}\right.}{42b(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (x*Sqrt[e*x]*((-49*c^2*(b*c + 3*a*d)*AppellF1[3/4, 1/2, 1, 7/4, (
d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c
, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c
, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)
/a])) + (33*a*c*(a*d*(7*c - 2*d*x^2) + b*c*(-7*c + 6*d*x^2))*Appe
llF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 42*(b*c - a*d)*x^
2*(-c + d*x^2)*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*
x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])
)/(a*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] +
2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] +
 a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(42*
b*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.037, size = 3858, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*d*(-4*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*x^2*b^4*c^3-4*x^4*a^2*b^2*d^3-4*x^4*b^4*c^2*d+8*x^
4*a*b^3*c*d^2+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-4*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
```

(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3
*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+20*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2-24*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a^2*b^2*c^2*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a^3*b*c*d^2+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^
2*b^2*c^2*d+4*x^2*b^4*c^3+4*x^2*a^2*b^2*c*d^2-8*x^2*a*b^3*c^2*d+4
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^
(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2-(c*d)^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-20*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+24*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*x^2*a^2*b^2*c*d^2-12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*x^2*a*b^3*c^2*d-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*a*b^2*c^2+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*
c^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*x^2*b^4*c^3-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+4*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-2*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^
3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)

```
^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3-5*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(
1/2)*(a*b)^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*
b)^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b
^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2
)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^
(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*
c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a
^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/
2))/b^2/x/(d*x^2-c)/a/(b*x^2-a)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c
*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}}{\left(bx^2-a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2 + c\right)^{\frac{3}{2}} \sqrt{ex}}{\left(bx^2 - a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2,x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.907 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

Optimal. Leaf size=366

$$\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{2ab^2\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{2abe\,(a-bx^2)}$$

```
[Out] ((b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) + (
c^(1/4)*d^(3/4)*(b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[e]*S
qrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^(1/4)*
Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^
(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.48969, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(ad+bc)(bc-ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{2ab^2\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{2abe\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) + (
c^(1/4)*d^(3/4)*(b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[e]*S
qrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^(1/4)*
Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^
(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C]   time = 0.647606, size = 428, normalized size = 1.17

$$\frac{x\left(\frac{10x^2(dx^2-c)(bc-ad)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9ac\left(ad(5c-2dx^2)+bc(6dx^2-5c)\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}-\frac{\cdots}{2x^2\left(2bcF_1\left(\cdots\right.\right.}\right.}{10b\sqrt{ex}\,(bx^2-a)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (x*((-25*c^2*(3*b*c + a*d)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
(b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/
a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a
] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (9*a
*c*(a*d*(5*c - 2*d*x^2) + b*c*(-5*c + 6*d*x^2))*AppellF1[5/4, 1/2
, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 10*(b*c - a*d)*x^2*(-c + d*x^2)
*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*A
ppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(a*(9*a*c*Appe
llF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Appel
lF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4,
3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(10*b*Sqrt[e*x]*(-a + b*
x^2)*Sqrt[c - d*x^2])
```

Maple [B]   time = 0.04, size = 2531, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(-d*x^2+c)^(1/2)/b*d*(-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)+4*x*a^2*b*c*d^2*(a*b)^(1/2)-8*x*a*b^2*c^2*d*(a*b)^(1/2)-4*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*
(a*b)^(1/2)+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^
3*c^2+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-
3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^
```

```
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-3*(c*d)^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*x^2*b^4*c^3+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*a*b^3*c^3-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3-4*x^3*a^2*
b*d^3*(a*b)^(1/2)-4*x^3*b^3*c^2*d*(a*b)^(1/2)-3*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+6*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b
*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)
+4*x*b^3*c^3*(a*b)^(1/2)+8*x^3*a*b^2*c*d^2*(a*b)^(1/2)-6*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*
d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-
3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(c*d)^(1/2)*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))/a/(e*x)^(1/2)/
(d*x^2-c)/(b*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c
*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.908 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=519

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}} - \frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt{c-dx^2}(5bc-ad)}{2a^2be\sqrt{ex}} - \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}} + \frac{\sqrt{c-dx^2}(bc-ad)}{2abe\sqrt{ex}\,(a-bx^2)}$$

```
[Out] -((5*b*c - a*d)*Sqrt[c - d*x^2])/(2*a^2*b*e*Sqrt[e*x]) + ((b*c -
a*d)*Sqrt[c - d*x^2])/(2*a*b*e*Sqrt[e*x]*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*(5*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*b*e^(3/2)*Sqrt[c -
d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*
b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d - a^
2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^(5/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5
*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c -
 d*x^2])
```

Rubi [A] time = 2.98566, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}} - \frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt{c-dx^2}(5bc-ad)}{2a^2be\sqrt{ex}} - \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}} + \frac{\sqrt{c-dx^2}(bc-ad)}{2abe\sqrt{ex}\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```


```
[Out] -((5*b*c - a*d)*Sqrt[c - d*x^2])/(2*a^2*b*e*Sqrt[e*x]) + ((b*c -
a*d)*Sqrt[c - d*x^2])/(2*a*b*e*Sqrt[e*x]*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*(5*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*b*e^(3/2)*Sqrt[c -
d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*
b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d - a^
2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
```

```
(4*a^(5/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5
*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c -
 d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.22922, size = 454, normalized size = 0.87

$$\frac{x\left(\frac{42x^2\left(c-dx^2\right)\left(5bcx^2-a\left(4c+dx^2\right)\right)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)-33ac\left(a\left(28c^2-21cdx^2-6d^2x^4\right)+5bcx^2\left(6dx^2-7c\right)\right)F_1\left(\frac{7}{4};\frac{1}{2},1\right.}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}\right.}{42a^2(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*((49*a*c^2*(5*b*c - 9*a*d)*x^2*AppellF1[3/4, 1/2, 1, 7/4, (d*x
^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (
b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (
b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]
)) + (-33*a*c*(5*b*c*x^2*(-7*c + 6*d*x^2) + a*(28*c^2 - 21*c*d*x^
2 - 6*d^2*x^4))*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]
 + 42*x^2*(c - d*x^2)*(5*b*c*x^2 - a*(4*c + d*x^2))*(2*b*c*Appell
F1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4,
 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))/(11*a*c*AppellF1[7/4, 1/2,
 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2
, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/
4, (d*x^2)/c, (b*x^2)/a]))))/(42*a^2*(e*x)^(3/2)*(a - b*x^2)*Sqrt
[c - d*x^2])
```

Maple [B] time = 0.049, size = 3879, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x)
```

```
[Out] 1/8*(-d*x^2+c)^(1/2)*d*(-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*b^4*c^3-4*x^4*a^2*b^2*d^3-20*x^4*b^4*c^2*d+24*x^4*a*b^3*c
*d^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2
*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^
(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
```

c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^3*b*c*d^2-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a^2*b^2*c^2*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*
d^2+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d+16*
a^2*b^2*c^2*d+20*x^2*b^4*c^3-16*a*b^3*c^3-12*x^2*a^2*b^2*c*d^2-8*
x^2*a*b^3*c^2*d-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2
)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*
(a*b)^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^
2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b
)^(1/2)+5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^
2-5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-4*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+24*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-12*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d-5*(c*d)^(1/2)*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+5*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*a*b^2*c^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*x^2*b^4*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^3-5*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*x^2*b^4*c^3+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b^3*c^3-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3
+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d

```
)^(1/2)*b),1/2*2^(1/2))*a*b^3*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^
3-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b
*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(c*d)^(1/2)*(a*b)^(1/2))/b/e/(e*x)^(1/2)/(d*x^2-c)/a^2/(b*x^2-a)
/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2 (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.909 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=412

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2be^{5/2}\sqrt{c-dx^2}}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{6a^2be(ex)^{3/2}}+\frac{\sqrt{c-dx^2}(bc-ad)}{2abe(ex)^{3/2}(a-bx^2)}$$

```
[Out] -((7*b*c - 3*a*d)*Sqrt[c - d*x^2])/(6*a^2*b*e*(e*x)^(3/2)) + ((b*
c - a*d)*Sqrt[c - d*x^2])/(2*a*b*e*(e*x)^(3/2)*(a - b*x^2)) + (c^
(1/4)*d^(3/4)*(7*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*b*e^(5/2)*S
qrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4)*e^
(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4
)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.16732, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2be^{5/2}\sqrt{c-dx^2}}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{6a^2be(ex)^{3/2}}+\frac{\sqrt{c-dx^2}(bc-ad)}{2abe(ex)^{3/2}(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] -((7*b*c - 3*a*d)*Sqrt[c - d*x^2])/(6*a^2*b*e*(e*x)^(3/2)) + ((b*
c - a*d)*Sqrt[c - d*x^2])/(2*a*b*e*(e*x)^(3/2)*(a - b*x^2)) + (c^
(1/4)*d^(3/4)*(7*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*b*e^(5/2)*S
qrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4)*e^
(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4
)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.20109, size = 453, normalized size = 1.1

$$\frac{x\left(\frac{10x^2(c-dx^2)(-4ac-3adx^2+7bcx^2)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)-9ac\left(a\left(20c^2-5cdx^2-18d^2x^4\right)+7bcx^2\left(6dx^2-5c\right)\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\right.}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}\right.}{30a^2(ex)^{5/2}\left(a-bx^2\right)\sqrt{c-dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*((25*a*c^2*(21*b*c - 17*a*d)*x^2*AppellF1[1/4, 1/2, 1, 5/4, (d
*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
 (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c,
(b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]
)) + (-9*a*c*(7*b*c*x^2*(-5*c + 6*d*x^2) + a*(20*c^2 - 5*c*d*x^2
- 18*d^2*x^4))*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] +
 10*x^2*(c - d*x^2)*(-4*a*c + 7*b*c*x^2 - 3*a*d*x^2)*(2*b*c*Appel
lF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4,
3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(9*a*c*AppellF1[5/4, 1/2, 1
, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2,
 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d
*x^2)/c, (b*x^2)/a]))))/(30*a^2*(e*x)^(5/2)*(a - b*x^2)*Sqrt[c -
d*x^2])
```

Maple [B] time = 0.043, size = 3484, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x)
```

```
[Out] -1/24*(-d*x^2+c)^(1/2)*d*(21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*x*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)*(a*b)^(1/2)-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*x^2*a*b^2*c^2*d*(a*b)^(1/2
)-40*x^4*a*b^2*c*d^2*(a*b)^(1/2)-16*a^2*b*c^2*d*(a*b)^(1/2)+28*x^
4*b^3*c^2*d*(a*b)^(1/2)-28*x^2*b^3*c^3*(a*b)^(1/2)+4*x^2*a^2*b*c*
d^2*(a*b)^(1/2)+12*x^4*a^2*b*d^3*(a*b)^(1/2)+21*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*b^4*c^3*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*x^3*b^4*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+6*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a^2
*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
```

```
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/
2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a^2*b*d
^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*a^2*b*d^2*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-24*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+20*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*b*
c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+
24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*b^2*c*d
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*a*b^2*c*d*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-20*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)
*x^3*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*
a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^
(1/2)+16*a*b^2*c^3*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*2^(1/2)*x*a*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^3*c^3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*2^(1/2)*x^3*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)+24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*
a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+14*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^3*b*c*d^2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*x^3*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*
b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^3*d^2*((d*x+(c*d)^(1/2))/(c*
```

```
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+24*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^3*d^2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-24*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*2^(1/2)*x*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(c*d)^(1/2)*(a*b)^(1/2))/x/a^2/e^2/(e*x)^(1/2)/(d*x^2-c)/(b
*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-
(a*b)^(1/2)*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2 (ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2-a\right)^2 (ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.910 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=484

$$\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)} + \frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)} + \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)(bc-ad)}$$

```
[Out] (a*e^3*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2))
 + (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/
4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) +
(Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1
/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.29223, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)} + \frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4bc-5ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}(bc-ad)} + \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```



```
[Out] (a*e^3*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2))
 + (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/
4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) +
(Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*
```

```
Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1
/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.587322, size = 414, normalized size = 0.86

$$\frac{a(ex)^{9/2}\left(\frac{49ac^2F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{14x^2(dx^2-c)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4}\right.\right.}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+ad\right.}\right.}{14bx^3\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (a*(e*x)^(9/2)*((49*a*c^2*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (
b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a
] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a
] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])) + (11
*c*(-7*a*c + 4*b*c*x^2 + 2*a*d*x^2)*AppellF1[7/4, 1/2, 1, 11/4, (
d*x^2)/c, (b*x^2)/a] + 14*x^2*(-c + d*x^2)*(2*b*c*AppellF1[11/4,
1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1,
15/4, (d*x^2)/c, (b*x^2)/a]))/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4,
 (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4
, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2
)/c, (b*x^2)/a]))))/(14*b*(b*c - a*d)*x^3*(-a + b*x^2)*Sqrt[c - d
*x^2])
```

Maple [B] time = 0.051, size = 2956, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*(-16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-4*x
^4*a^2*b^2*d^3+4*x^4*a*b^3*c*d^2+5*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a
*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(
```

```
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+20
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2-36*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d-10*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2+18*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*b^2*c^2*d+4*x^2*a^2*b^2*c*d^2-4*x^2*a*b^3*c^
2*d+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*
c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-
7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-20*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+36*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+10*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2-18*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+8*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3+16*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a*b^3*c^3-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b^3*c^3-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^
2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b
)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^
2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1
/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+5*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/
```

```
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-5*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*
e^4*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c
*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)/(d*x^2-c)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{(bx^2 - a)^2 \sqrt{-dx^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{(bx^2 - a)^2 \sqrt{-dx^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.911 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=376

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(4bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} + \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b\,(a-bx^2)\,(bc-ad)}$$

```
[Out] (a*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) +
 (c^(1/4)*(4*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(1/4)
*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a*d)*e^(7/2)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^2*
d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a*d)*e
^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*
b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.46624, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(4bc-3ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} + \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b\,(a-bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```


```
[Out] (a*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) +
 (c^(1/4)*(4*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(1/4)
*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a*d)*e^(7/2)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^2*
d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a*d)*e
^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*
b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.597887, size = 414, normalized size = 1.1

$$\frac{a(ex)^{7/2}\left(\frac{25ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{10x^2\left(dx^2-c\right)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},\ldots\right.\right.}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\ldots\right.\right.}\right.}{10bx^3\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (a*(e*x)^(7/2)*((25*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (
b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a]
 + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (-9*c
*(5*a*c - 4*b*c*x^2 - 2*a*d*x^2)*AppellF1[5/4, 1/2, 1, 9/4, (d*x^
2)/c, (b*x^2)/a] + 10*x^2*(-c + d*x^2)*(2*b*c*AppellF1[9/4, 1/2,
2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4,
(d*x^2)/c, (b*x^2)/a]))/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)
/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)
/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^
2)/a]))))/(10*b*(b*c - a*d)*x^3*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.044, size = 2520, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8/b*(8*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)*(a*b)^(1/2)-8*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a
*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+4*
x*a^2*b*c*d^2*(a*b)^(1/2)-4*x*a*b^2*c^2*d*(a*b)^(1/2)-14*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*
a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^
(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^
2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2
```

```
)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*
d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*
x^3*a^2*b*d^3*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c
*d)^(1/2)*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2)+14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*x^3*a*b^2*c*d^2*(a*b)^(1/2)-5*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a
^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/
2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2
*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*
d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))
*(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)/(d
*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2 - a)^2 \sqrt{-dx^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.912 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=460

$$\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} + \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)} - \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)} + \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(a-bx^2)(bc-ad)}$$

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) - (c^
(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c -
d*x^2]) + (c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*(b*c - a
*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)
*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - a*d)*e^
(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*S
qrt[a]*b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.18056, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} - \frac{\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)} + \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)} - \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)} + \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(a-bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```


```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) - (c^
(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c -
d*x^2]) + (c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*(b*c - a
*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)
*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - a*d)*e^
(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
```

```
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*S
qrt[a]*b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.410571, size = 325, normalized size = 0.71

$$\frac{e(ex)^{3/2}\left(-\frac{49ac^2F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{11acdx^2F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4}\cdots\right.}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\cdots\right.\right.}\cdots\right)}{14\left(bx^2-a\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*(e*x)^(3/2)*(7*c - 7*d*x^2 - (49*a*c^2*AppellF1[3/4, 1/2, 1, 7
/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x
^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x
^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b
*x^2)/a])) + (11*a*c*d*x^2*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c,
 (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x
^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))
))/(14*(-(b*c) + a*d)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.039, size = 2548, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
x^2*a*b^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d-3*((d*x+(c*d)^(1
```

/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*(c*d)^(1/2)*a*b*c-4*x^2*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*(c*d)^(1/2)*x^2*a*b*d+4*x^2*a*b^2*c*d-2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3
c^2-3((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-4*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*a*b^2*c^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*a*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a
*b^2*c*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d
-4*x^4*a*b^2*d^2+4*x^4*b^3*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(
c*d)^(1/2)*x^2*a*b*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
^2*b*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*a^2*b*c*d)*d*(-d*x^2+c)^(1/2)*e^2*(e*x)^(1/2
)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(
b*x^2-a)/(a*d-b*c)/(d*x^2-c)/b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.913 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=363

$$\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(a-bx^2)(bc-ad)}$$

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) + (c^(1
/4)*d^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*
x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c -
 d*x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c -
 d*x^2])
```

Rubi [A] time = 1.39739, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(a-bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) + (c^(1
/4)*d^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*
x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c -
 d*x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c -
 d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.385612, size = 325, normalized size = 0.9

$$\frac{e\sqrt{ex}\left(-\frac{25ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{9acdx^2F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\cdots\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\cdots\right.\right.}\cdots\right)}{10\left(bx^2-a\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*Sqrt[e*x]*(5*c - 5*d*x^2 - (25*a*c^2*AppellF1[1/4, 1/2, 1, 5/4
, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2
)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)
/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2
)/a])) - (9*a*c*d*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^
2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] +
2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] +
a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(10*(-(
b*c) + a*d)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.038, size = 2258, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*2^(1/2)*x^2*a*b*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*(c*d)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^2*c*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c*
d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*x^2*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(
1/2)*x^2*b^2*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d-((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
```

```
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*
b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-2*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^
(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*a*b*c*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*(c*d)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(
c*d)^(1/2)*a*b*c+4*x^3*a*b*d^2*(a*b)^(1/2)-4*x^3*b^2*c*d*(a*b)^(1
/2)-4*x*a*b*c*d*(a*b)^(1/2)+4*x*b^2*c^2*(a*b)^(1/2))*(-d*x^2+c)^(
1/2)*e*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d
+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.914 $\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=464

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{c-dx^2}(bc-ad)}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{c-dx^2}(bc-ad)}+\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)(bc-ad)}$$

```
[Out] (b*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) -
 (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[
c - d*x^2]) + (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellipt
icF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*(b*c
 - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*Sqrt[e]*Sqrt[1
- (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqr
t[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - 3*a*d
)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^(3/2)*Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.15716, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{c-dx^2}(bc-ad)}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{c-dx^2}(bc-ad)}+\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (b*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) -
 (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[
c - d*x^2]) + (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellipt
icF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*(b*c
 - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*Sqrt[e]*Sqrt[1
- (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqr
t[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - 3*a*d
)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
```

```
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^(3/2)*Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.518092, size = 335, normalized size = 0.72

$$\frac{x\sqrt{ex}\left(\frac{33bcdx^2F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{49c(bc-4ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\right.}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\right.\right.}\right.}{42\left(bx^2-a\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*Sqrt[e*x]*((21*b*(c - d*x^2))/a + (49*c*(b*c - 4*a*d)*AppellF1
[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/
2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2
, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4
, (d*x^2)/c, (b*x^2)/a])) + (33*b*c*d*x^2*AppellF1[7/4, 1/2, 1, 1
1/4, (d*x^2)/c, (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (
d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4,
(d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/
c, (b*x^2)/a]))))/(42*(-(b*c) + a*d)*(-a + b*x^2)*Sqrt[c - d*x^2]
)
```

Maple [B] time = 0.037, size = 2545, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
x^2*a*b^2*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
```

```
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d-((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*(c*d)^(1/2)*a*b*c+4*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*(c*d)^(1/2)*x^2*a*b*d-4*x^2*a*b^2*c*d+2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c
^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a*b^2*c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
*b^2*c^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d
-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c*d+4*x^4*a
*b^2*d^2-4*x^4*b^3*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(
1/2)*x^2*a*b*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ell
ipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c
*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-
3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d)*d*(-d*x^2+c)^(1/2)*(e*x)^(1/2)/
((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^
2-a)/a/(a*d-b*c)/x/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.915 $\int \frac{1}{\sqrt{ex}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=367

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)(bc-ad)}$$

```
[Out] (b*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) + (
c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[e]*Sqrt[c
- d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt[e]*
Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt
[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.36463, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{e}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (b*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) + (
c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[e]*Sqrt[c
- d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt[e]*
Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt
[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.520566, size = 336, normalized size = 0.92

$$\frac{x\left(-\frac{9bcdx^2F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{25c(3bc-4ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}\right)}{10\sqrt{ex}\,(bx^2-a)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*((5*b*(c - d*x^2))/a + (25*c*(3*b*c - 4*a*d)*AppellF1[1/4, 1/2
, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4,
 (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c
, (b*x^2)/a])) - (9*b*c*d*x^2*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/
c, (b*x^2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^
2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^
2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))
/(10*(-(b*c) + a*d)*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.043, size = 2266, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*b*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*2^(1/2)*x^2*a*b*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*(c*d)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^2*c*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^
2*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*(c*d)^(1/2)*x^2*b^2*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d-5*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
```

```
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^
(1/2)*x^2*b^2*c-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*2^(1/2)*a^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*(c*d)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d+
5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+3*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/
2)*a*b*c+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+5*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+4*x^3*a*b*d^2*
(a*b)^(1/2)-4*x^3*b^2*c*d*(a*b)^(1/2)-4*x*a*b*c*d*(a*b)^(1/2)+4*x
*b^2*c^2*(a*b)^(1/2))*(-d*x^2+c)^(1/2)/a/((c*d)^(1/2)*b-(a*b)^(1/
2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*
c)/(d*x^2-c)/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 - a)^2 \sqrt{-dx^2 + c}\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.916 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=535

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(5bc-4ad)}{2a^2ce\sqrt{ex}(bc-ad)}+\frac{b\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)\,(bc-ad)}$$

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c - d*x^2])/(2*a^2*c*(b*c - a*d)*e*Sqrt[e*
x]) + (b*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2
)) - (d^(1/4)*(5*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*
c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b*c - 4*a*d)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
- (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2
)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b
*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.91156, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(5bc-4ad)}{2a^2ce\sqrt{ex}(bc-ad)}+\frac{b\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c - d*x^2])/(2*a^2*c*(b*c - a*d)*e*Sqrt[e*
x]) + (b*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2
)) - (d^(1/4)*(5*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSi
```

```
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*
c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b*c - 4*a*d)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
- (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2
)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b
*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.59127, size = 390, normalized size = 0.73

$$\frac{x\left(-\frac{49ax^2\left(4a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{21\left(c-dx^2\right)\left(4a^2d-4ab\left(c+dx^2\right)+5b^2cx^2\right)}{c}+\frac{\cdots}{2x^2\left(2bcF_1\left(\frac{11}{4}\cdots\right.\right.}\right.}{42a^2(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*((-21*(c - d*x^2)*(4*a^2*d + 5*b^2*c*x^2 - 4*a*b*(c + d*x^2)))
/c - (49*a*(5*b^2*c^2 - 12*a*b*c*d + 4*a^2*d^2)*x^2*AppellF1[3/4,
 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1,
 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2,
11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*
x^2)/c, (b*x^2)/a])) + (33*a*b*d*(-5*b*c + 4*a*d)*x^4*AppellF1[7/
4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2
, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/
2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15
/4, (d*x^2)/c, (b*x^2)/a]))))/(42*a^2*(-(b*c) + a*d)*(e*x)^(3/2)*
(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.051, size = 2982, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(-16*a*b^2*c^3+16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
^3*c*d^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elliptic
E(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3-8*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
```

/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2-10*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*
b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^3-5*(c*d)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*x^2*b^2*c^2-16*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*x^2*a^2*b*c*d^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^
2*d+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c^2*d+5*(c*d)^(1/2)*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*x^2*b^2*c^2-16*a^3*c*d^2-20*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3+32*a^2*b*c^2*d+36*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d+8*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*c*d^2-18*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d+16*a^3*d^3*x^2-7*(c*d)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b*c*d+7*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b*c*d+20*b^3*c^3*x^2-16*x^4*
a^2*b*d^3-20*x^4*b^3*c^2*d+18*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a^2*b*c^2*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c^2*d-7*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*a^2*b*c^2*d+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(
1/2)*a^2*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c
^2-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d+5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((

```
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c^2-16*a^2*b*c*d^2*x^
2+36*x^4*a*b^2*c*d^2-36*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a^2*b*c^2*d-20*a*b^2*c^2*d*x^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*b^3*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3-5*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*x^2*b^3*c^3)*b*d*(-d*x^2+c)^(1/2)/c/((c*d)^(1/2)*b
-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)
/a^2/(d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.917 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=429

$$\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-9ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-9ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{3/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{6a^2ce(ex)^{3/2}(bc-ad)}+\frac{b\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)(bc-ad)}$$

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c - d*x^2])/(6*a^2*c*(b*c - a*d)*e*(e*x)^(
3/2)) + (b*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b
*x^2)) + (d^(3/4)*(7*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(3/4)
*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*c - 9*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^
3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*
c - 9*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*a^3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.12532, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-9ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-9ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{3/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{6a^2ce(ex)^{3/2}(bc-ad)}+\frac{b\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c - d*x^2])/(6*a^2*c*(b*c - a*d)*e*(e*x)^(
3/2)) + (b*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b
*x^2)) + (d^(3/4)*(7*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(3/4)
*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*c - 9*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^
3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*
c - 9*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*a^3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.47613, size = 390, normalized size = 0.91

$$\frac{x\left(\frac{25ax^2(4a^2d^2+20abcd-21b^2c^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{5(c-dx^2)(4a^2d-4ab(c+dx^2)+7b^2cx^2)}{c}+\frac{\cdots}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\cdots\right.\right.}\right.}{30a^2(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*((-5*(c - d*x^2)*(4*a^2*d + 7*b^2*c*x^2 - 4*a*b*(c + d*x^2)))/
c + (25*a*(-21*b^2*c^2 + 20*a*b*c*d + 4*a^2*d^2)*x^2*AppellF1[1/4
, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2,
 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x
^2)/c, (b*x^2)/a])) + (9*a*b*d*(7*b*c - 4*a*d)*x^4*AppellF1[5/4,
1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1,
9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 1
3/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x
^2)/c, (b*x^2)/a]))))/(30*a^2*(-(b*c) + a*d)*(e*x)^(5/2)*(a - b*x
^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.048, size = 2622, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/24*b*d*(21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a
*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(
1/2)-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*x*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d
)^(1/2)*(a*b)^(1/2)+28*x^2*a*b^2*c^2*d*(a*b)^(1/2)-44*x^4*a*b^2*c
*d^2*(a*b)^(1/2)-32*a^2*b*c^2*d*(a*b)^(1/2)+28*x^4*b^3*c^2*d*(a*b
)^(1/2)-28*x^2*b^3*c^3*(a*b)^(1/2)+16*x^2*a^2*b*c*d^2*(a*b)^(1/2)
+16*x^4*a^2*b*d^3*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*
2^(1/2))*2^(1/2)*x^3*b^4*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^4*c^3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+8*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a^2*b*d^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
```

```
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a*b^2*c^2*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-27*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+22*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c*d*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+27*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*b^2*c*d*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+27*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*a*b^2*c*d*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-22*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a*
b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1
/2)-27*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b*c
*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+1
6*a*b^2*c^3*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2
))*2^(1/2)*x*a*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+21*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^3*c^3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+16*a^3*c*d^2*(a*b)^(1/2)-16*x^2*a^
3*d^3*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(
1/2)*x^3*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*(a*b)^(1/2)+27*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x^3*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-21*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^2*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+14*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*b^3*c^
2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-27
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*a*b^3*c^2*d
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+27*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)
*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)-27*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
x*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-8*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^3*
d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))
*(-d*x^2+c)^(1/2)/x/c/a^2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1
/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)/(d*x^2-c)/e^
2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^2 \sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 - a)^2 \sqrt{-dx^2 + c}\,(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.918 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=529

$$\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}\sqrt{c-dx^2}(bc-ad)^2}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}\sqrt{c-dx^2}(bc-ad)^2}+\frac{e^3(ex)^{3/2}(ad+2bc)}{2b\sqrt{c-dx^2}(bc-ad)^2}+\frac{ae^3(ex)^{3/2}}{2b(a-bx^2)\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] ((2*b*c + a*d)*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2
]) + (a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*
x^2]) - (c^(3/4)*(2*b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*Ellipt
icE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3
/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (c^(3/4)*(2*b*c + a*d)*e^(9/
2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) +
 (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2
*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/
4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.94995, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}\sqrt{c-dx^2}(bc-ad)^2}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}\sqrt{c-dx^2}(bc-ad)^2}+\frac{e^3(ex)^{3/2}(ad+2bc)}{2b\sqrt{c-dx^2}(bc-ad)^2}+\frac{ae^3(ex)^{3/2}}{2b(a-bx^2)\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] ((2*b*c + a*d)*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2
]) + (a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*
```

```
x^2]) - (c^(3/4)*(2*b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*Ellipt
icE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3
/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (c^(3/4)*(2*b*c + a*d)*e^(9/
2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) +
 (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2
*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/
4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.984728, size = 432, normalized size = 0.82

$$\frac{(ex)^{9/2}\left(\frac{147a^2c^2F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bx^2-a)\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{33ac(7ac-2adx^2-4bcx^2)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c}\right.}{(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},\right.\right.\right.}\right.}{14x^3\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((e*x)^(9/2)*((147*a^2*c^2*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c,
(b*x^2)/a])/((-a + b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^
2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^
2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*
x^2)/a]))) + (33*a*c*(7*a*c - 4*b*c*x^2 - 2*a*d*x^2)*AppellF1[7/4
, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] - 14*x^2*(-3*a*c + 2*b*c*x^
2 + a*d*x^2)*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^
2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))/
((a - b*x^2)*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x
^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])
)))/(14*(b*c - a*d)^2*x^3*Sqrt[c - d*x^2])
```

Maple [B] time = 0.046, size = 2964, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/8*(8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-4*x^
4*a^2*b^2*d^3+8*x^4*b^4*c^2*d-4*x^4*a*b^3*c*d^2+EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
```

)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2+4
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d-2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^2-2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d+12*x^2*a^2*b^2*c*d^2-12*x^2
*a*b^3*c^2*d+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x
^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*
b)^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a
*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(
1/2)-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2
-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d+2*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c*d^2+2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^2*d-4*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^3-8*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*a*b^3*c^3+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^3*c^3-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a
*b)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^
2*b*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1
/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))

```
*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+7*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2))*(-d*x^2+c)^(1/2)*e^4*(e*x
)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2
)*b)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="giac")

[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.919 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=420

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+5bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+5bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{e^3\sqrt{ex}(ad+2bc)}{2b\sqrt{c-dx^2}(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] ((2*b*c + a*d)*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2])
 + (a*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]
) + (c^(1/4)*(2*b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(1/4)*
(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d^(
1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d
^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.82892, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+5bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+5bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{e^3\sqrt{ex}(ad+2bc)}{2b\sqrt{c-dx^2}(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] ((2*b*c + a*d)*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2])
 + (a*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]
) + (c^(1/4)*(2*b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(1/4)*
(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d^(
1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d
^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.929618, size = 422, normalized size = 1.

$$\frac{(ex)^{7/2}\left(\frac{75a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{10x^2\left(-3ac+adx^2+2bcx^2\right)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+\right.}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+\right.}\right.}{10x^3\left(bx^2-a\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((e*x)^(7/2)*((75*a^2*c^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (
b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a]
 + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (-27*
a*c*(5*a*c - 4*b*c*x^2 - 2*a*d*x^2)*AppellF1[5/4, 1/2, 1, 9/4, (d
*x^2)/c, (b*x^2)/a] + 10*x^2*(-3*a*c + 2*b*c*x^2 + a*d*x^2)*(2*b*
c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF
1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(9*a*c*AppellF1[5/4,
 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4,
1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 1
3/4, (d*x^2)/c, (b*x^2)/a]))))/(10*(b*c - a*d)^2*x^3*(-a + b*x^2)
*Sqrt[c - d*x^2])
```

Maple [B] time = 0.046, size = 2530, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] -1/8*(4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d
)^(1/2)*(a*b)^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^
3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c*d^2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+12*x*a^
```

```
2*b*c*d^2*(a*b)^(1/2)-12*x*a*b^2*c^2*d*(a*b)^(1/2)-2*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^
2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2
)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*
d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+5*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c*d*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x^3*
a^2*b*d^3*(a*b)^(1/2)+8*x^3*b^3*c^2*d*(a*b)^(1/2)+EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c
*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-5*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-4*x^3*a*b^2*c*d
^2*(a*b)^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)+2*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^
(1/2)*(a*b)^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1
/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2
^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2))*(-d*x^2+c)^(1/2)*e^3*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*
b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(
a*d-b*c)^2/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2 - a)^2(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.920 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=485

$$\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3de(ex)^{3/2}}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{e(ex)^{3/2}}{2(a-bx^2)\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] (3*d*e*(e*x)^(3/2))/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*(e*x)^
(3/2))/(2*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (3*c^(3/4)*d
^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2])
 + (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*(b*c - a*d)^2*S
qrt[c - d*x^2]) + (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1
/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b*c + a*d)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt
[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.70152, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{3\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3de(ex)^{3/2}}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{e(ex)^{3/2}}{2(a-bx^2)\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```



```
[Out] (3*d*e*(e*x)^(3/2))/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*(e*x)^
(3/2))/(2*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (3*c^(3/4)*d
^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2])
 + (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*(b*c - a*d)^2*S
qrt[c - d*x^2]) + (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1
/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b*c + a*d)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
```

```
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt
[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.01224, size = 339, normalized size = 0.7

$$\frac{e(ex)^{3/2}\left(\frac{33abcdx^2F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{49ac(2ad+bc)F_1\left(\frac{3}{4};\frac{1}{2},1\right.}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\right.\right.}\right.}{14\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*(e*x)^(3/2)*(7*(b*c + 2*a*d - 3*b*d*x^2) - (49*a*c*(b*c + 2*a*
d)*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/(7*a*c*Appel
lF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Appell
F1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3
/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])) + (33*a*b*c*d*x^2*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/
2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1
/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 1
5/4, (d*x^2)/c, (b*x^2)/a]))))/(14*(b*c - a*d)^2*(a - b*x^2)*Sqrt
[c - d*x^2])
```

Maple [B] time = 0.044, size = 2561, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*(-12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2-3*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+3*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*x^2*a*b^2*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
```

```
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d+3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-3*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+4*x^2*b^3*c^2-3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+4*x^2*a*b^2*c*d+6*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))
*x^2*b^3*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2+12*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-6*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*a*b^2*c^2+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*a*b^2*c*d-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2
*a*b^2*c*d-8*x^2*a^2*b*d^2+12*x^4*a*b^2*d^2-12*x^4*b^3*c*d+3*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d-12*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+6*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c
*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^
2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b
*c*d)*d*(-d*x^2+c)^(1/2)*e^2*(e*x)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(
1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)^2/(d*x^
2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2 - a)^2(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2 - a)^2(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.921 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=391

$$\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3de\sqrt{ex}}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{e\sqrt{ex}}{2\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] (3*d*e*Sqrt[e*x])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*Sqrt[e*x
])/(2*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (3*c^(1/4)*d^(3/
4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (
c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]
) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2])
```

Rubi [A] time = 1.71217, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3de\sqrt{ex}}{2\sqrt{c-dx^2}(bc-ad)^2}+\frac{e\sqrt{ex}}{2\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] (3*d*e*Sqrt[e*x])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*Sqrt[e*x
])/(2*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (3*c^(1/4)*d^(3/
4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (
c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]
) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.815966, size = 340, normalized size = 0.87

$$\frac{(ex)^{3/2}\left(\frac{27abcdx^2F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{25ac(2ad+bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c}\right.}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\right.\right.}\right.}{10\left(bx^3-ax\right)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((e*x)^(3/2)*(-5*(b*c + 2*a*d - 3*b*d*x^2) + (25*a*c*(b*c + 2*a*d
)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*Appell
F1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF
1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2
, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) + (27*a*b*c*d*x^2*AppellF1[5/4,
 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1,
 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2,
13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*
x^2)/c, (b*x^2)/a]))))/(10*(b*c - a*d)^2*Sqrt[c - d*x^2]*(-(a*x)
+ b*x^3))
```

Maple [B] time = 0.043, size = 2277, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*b*d*(-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c-
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^2*c+5*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),
1/2*2^(1/2))*x^2*a*b^2*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b^2*c*d+(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2
)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1
```

```
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*(c*d)^(1/2)*x^2*a*b*d+12*x^3*a*b*d^2*(a*b)^(1/2)-8*x*a^2*d^
2*(a*b)^(1/2)-12*x^3*b^2*c*d*(a*b)^(1/2)+((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2
*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*
b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+6*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b*d*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-6*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^2*
c*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+6*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a*b*c*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^
(1/2)-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b*d+5*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/
2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-5*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2
*b*c*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+4*x*b^2*c^2*(a*b)^(1/2)+4
*x*a*b*c*d*(a*b)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*(c*d)^(1/2))*(-d*x^2+c)^(1/2)*e*(e*x)^(1/2)/x/
((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(a*b)
^(1/2)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2 - a)^2(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="giac")

[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.922 $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{d(ex)^{3/2}(2ad+bc)}{2ace\sqrt{c-dx^2}(bc-ad)^2}+\frac{b(ex)^{3/2}}{2ae(a-bx^2)\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d)*(e*x)^(3/2))/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x
^2]) + (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*
x^2]) - (d^(1/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellipt
icE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1
/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (d^(1/4)*(b*c + 2*a*d)*Sqrt[
e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) -
 (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2
*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/
4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.95987, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^2}+\frac{d(ex)^{3/2}(2ad+bc)}{2ace\sqrt{c-dx^2}(bc-ad)^2}+\frac{b(ex)^{3/2}}{2ae(a-bx^2)\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```



```
[Out] (d*(b*c + 2*a*d)*(e*x)^(3/2))/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x
^2]) + (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*
x^2]) - (d^(1/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellipt
icE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1
/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (d^(1/4)*(b*c + 2*a*d)*Sqrt[
e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) -
 (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
```

```
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2
*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/
4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.32507, size = 482, normalized size = 0.91

$$\frac{x\sqrt{ex}\left(\frac{33ac\left(14a^2d^2-12abd^2x^2+b^2c\left(7c-6dx^2\right)\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-42x^2\left(-2a^2d^2+2abd^2x^2+b^2c\left(dx^2-c\right)\right)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1\right.\right.}{ac(a-bx^2)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}\right.}{42\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[e*x]*((-49*(b^2*c^2 - 8*a*b*c*d - 2*a^2*d^2)*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((-a + b*x^2)*(7*a*c*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF
1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/
2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) + (33*a*c*(14*a^2*d^2 - 12*a
*b*d^2*x^2 + b^2*c*(7*c - 6*d*x^2))*AppellF1[7/4, 1/2, 1, 11/4, (
d*x^2)/c, (b*x^2)/a] - 42*x^2*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c
*(-c + d*x^2))*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*
x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])
)/(a*c*(a - b*x^2)*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c,
 (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c
, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2
)/a])))))/(42*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Maple [B] time = 0.049, size = 2950, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*(-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2+4*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3+4*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
```

```
*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b^2*c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^3+((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*a*b^2*c^3-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
x^2*b^2*c^2+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipt
icE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*
c*d^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^2*c^2*d+7*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*x^2*a*b^2*c^2*d+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x
^2*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^3*c^3
-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d-4*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b*c*d^2+2*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^2*c^2*d-8*a^3*d^3*x^2-7*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b*c*d+7*(c*d)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a*b*c*d+4*b^3*c^3*x^2+8
*x^4*a^2*b*d^3-4*x^4*b^3*c^2*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*b*c^2*d-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b*c^2*d-7*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*a^2*b*c^2*d+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d
)^(1/2)*a^2*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a*b*
c^2-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d+((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c^2+8*a^2*b*c*d^2*x^2-
4*x^4*a*b^2*c*d^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
```

```
EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*
b*c^2*d-4*a*b^2*c^2*d*x^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*b^3*c^3-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)
^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^3-((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*x^2*b^3*c^3)*d*b*(-d*x^2+c)^(1/2)*(e*x)^(1/2)/c/((c*d)^(1/2)
*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/a/(a*d-
b*c)^2/x/(d*x^2-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.923 $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=426

$$\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b\sqrt{ex}}{2ae\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d\sqrt{ex}(2ad+bc)}{2ace\sqrt{c-dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d)*Sqrt[e*x])/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2
]) + (b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]
) + (d^(3/4)*(b*c + 2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(3/4)*(b*c - a
*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/
4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*d^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 1.91928, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{3b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{3/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b\sqrt{ex}}{2ae\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d\sqrt{ex}(2ad+bc)}{2ace\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```


```
[Out] (d*(b*c + 2*a*d)*Sqrt[e*x])/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2
]) + (b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]
) + (d^(3/4)*(b*c + 2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(3/4)*(b*c - a
*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/
4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*d^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.32074, size = 472, normalized size = 1.11

$$\frac{x\left(\frac{25(2a^2d^2-8abcd+3b^2c^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{9ac(10a^2d^2-12abd^2x^2+b^2c(5c-6dx^2))F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{ac\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\cdots\right.\right.\right.}\right.}{10\sqrt{ex}\,(a-bx^2)\sqrt{c-dx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*((25*(3*b^2*c^2 - 8*a*b*c*d + 2*a^2*d^2)*AppellF1[1/4, 1/2, 1,
 5/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d
*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*
x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b
*x^2)/a])) + (9*a*c*(10*a^2*d^2 - 12*a*b*d^2*x^2 + b^2*c*(5*c - 6
*d*x^2))*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 10*x^
2*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2))*(2*b*c*Appell
F1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3
/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(a*c*(9*a*c*AppellF1[5/4, 1/
2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2
, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4
, (d*x^2)/c, (b*x^2)/a])))))/(10*(b*c - a*d)^2*Sqrt[e*x]*(a - b*x
^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.056, size = 2554, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*b*d*(-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*2^(1/2)*x^2*b^3*c^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(c*d)^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*
a*b^3*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-8*x*a^3*d^3*(a*b
)^(1/2)+8*x*a^2*b*c*d^2*(a*b)^(1/2)-4*x*a*b^2*c^2*d*(a*b)^(1/2)-2
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/
2)*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
```

```
*x^2*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2
*a*b^2*c*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)
^(1/2)+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^3*c^2
+3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^3*c^2-3*(c*
d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^2*c^2-3*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2)
)*x^2*b^4*c^3+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*b^4*c^3+3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^
(1/2))*a*b^3*c^3-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+8*x^3*a^2*b*d^3
*(a*b)^(1/2)-4*x^3*b^3*c^2*d*(a*b)^(1/2)+4*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*d^2*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(
1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+4*x*b^3*c^3*
(a*b)^(1/2)-4*x^3*a*b^2*c*d^2*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*a^3*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*
(c*d)^(1/2)*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2)
)*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2))*(-d
*x^2+c)^(1/2)/c/a/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)*d+(c
*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)/(e*x)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 - a)^2(-dx^2 + c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.924 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=628

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(6a^2d^2-8abcd+5b^2c^2\right)}{2a^2c^2e\sqrt{ex}(bc-ad)^2}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b}{2ae\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ace\sqrt{ex}\sqrt{c-dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*Sqrt[c - d*x^2
]) + b/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2])
- ((5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*Sqrt[c - d*x^2])/(2*a^2*c^
2*(b*c - a*d)^2*e*Sqrt[e*x]) - (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d +
6*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*
Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*S
qrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x
^2]) - (b^(3/2)*c^(1/4)*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^
2*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(5*b*c - 11*a*d)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d
^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi **[A]** time = 3.76504, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(6a^2d^2-8abcd+5b^2c^2\right)}{2a^2c^2e\sqrt{ex}(bc-ad)^2}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b}{2ae\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ace\sqrt{ex}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*Sqrt[c - d*x^2
]) + b/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2])
- ((5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*Sqrt[c - d*x^2])/(2*a^2*c^
2*(b*c - a*d)^2*e*Sqrt[e*x]) - (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d +
6*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*
Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*S
qrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x
^2]) - (b^(3/2)*c^(1/4)*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^
2*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(5*b*c - 11*a*d)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d
^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.55671, size = 476, normalized size = 0.76

$$\frac{x\left(\frac{33abcdx^4\left(6a^2d^2-8abcd+5b^2c^2\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{49acx^2\left(-6a^3d^3+8a^2bcd^2-16ab^2c^2d+5b^3c^3\right)F_1}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right.}\right.}{42a^2c^2(ex)^{3/2}\,(a}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(-21*(2*a^3*d^2*(2*c - 3*d*x^2) - 5*b^3*c^2*x^2*(c - d*x^2) +
4*a*b^2*c*(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d
*x^2 + 3*d^2*x^4)) + (49*a*c*(5*b^3*c^3 - 16*a*b^2*c^2*d + 8*a^2*
b*c*d^2 - 6*a^3*d^3)*x^2*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b
*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a]
 + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a]
 + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])) + (33*
a*b*c*d*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*x^4*AppellF1[7/4, 1/2
, 1, 11/4, (d*x^2)/c, (b*x^2)/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 1
1/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2,
15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d
*x^2)/c, (b*x^2)/a]))))/(42*a^2*c^2*(b*c - a*d)^2*(e*x)^(3/2)*(a
- b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.053, size = 3385, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*(12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^4*c*d^3-10*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^4+5*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(
1/2))*a*b^3*c^4-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b
^4*c^4+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^4*c^4-5*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*x^2*b^4*c^4+5*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*a*b^3*c^4
-24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^4*c*d^3+20*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^4-5*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*x^2*b^4*c^4-11*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*a^2
*b*c^2*d+11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*b*c^
2*d+56*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c^2*d^2-52*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^3*d-28*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c^2*d^2+26*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^3*d-11*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*
a^2*b^2*c^3*d+16*a^4*c*d^3-16*a*b^3*c^4-24*x^2*a^4*d^4+20*x^2*b^4
*c^4+11*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*x^2*a*b^2*c^
2*d-11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a
*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b^2*c^2
*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b
)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^3+11*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*x^2*a*b^3*c^3*d+5*EllipticPi(((d*x+(c*d)^(1/
```

```
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*(c*d)^(1/2)*a*b^2*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(
c*d)^(1/2)*a*b^2*c^3-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/
2)*x^2*b^3*c^3+11*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*x^2*a*b^3*c^3*d+24*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^3*b*c*d^3-56*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2+52*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b^3*c^3*d-12*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*a^3*b*c*d^3+28*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2-26*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*
d/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*x^2*a*b^3*c^3*d+52*x^4*a*b^3*c^2*d^2+40*x^2*a^3*
b*c*d^3-36*x^2*a*b^3*c^3*d-11*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*a^2*b^2*c^3*d+
24*x^4*a^3*b*d^4-20*x^4*b^4*c^3*d-48*a^3*b*c^2*d^2+48*a^2*b^2*c^3
*d-56*x^4*a^2*b^2*c*d^3)*d*b*(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b
)^(1/2)*d)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/(b*x^2-a)/(a*d-b*c)^2/a^
2/c^2/(d*x^2-c)/e/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="maxima"
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="fricas"
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.925 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=512

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-13ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-13ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(10a^2d^2-8abcd+7b^2c^2\right)}{6a^2c^2e(ex)^{3/2}(bc-ad)^2}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b}{2ae(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ace(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*Sqrt[c - d*x
^2]) + b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^
2]) - ((7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*Sqrt[c - d*x^2])/(6*a
^2*c^2*(b*c - a*d)^2*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^2*c^2 - 8*a*b
*c*d + 10*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(7/4)*(b*c - a*d)^2*
e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*
(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^3*d^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.85036, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-13ad)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-13ad)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(10a^2d^2-8abcd+7b^2c^2\right)}{6a^2c^2e(ex)^{3/2}(bc-ad)^2}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b}{2ae(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ace(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*Sqrt[c - d*x
^2]) + b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^
2]) - ((7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*Sqrt[c - d*x^2])/(6*a
^2*c^2*(b*c - a*d)^2*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^2*c^2 - 8*a*b
*c*d + 10*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(7/4)*(b*c - a*d)^2*
e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*
(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
```

```
^3*d^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)

[Out] Timed out
```

Mathematica [C] time = 2.48863, size = 476, normalized size = 0.93

$$\frac{x\left(-\frac{9abcdx^4(10a^2d^2-8abcd+7b^2c^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{25acx^2(10a^3d^3-8a^2bcd^2-32ab^2c^2d+21b^3c^3)F_1\left(\cdots\right.}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\cdots}\right.}{30a^2c^2(ex)^{5/2}(a-\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]

[Out] (x*(-5*(2*a^3*d^2*(2*c - 5*d*x^2) - 7*b^3*c^2*x^2*(c - d*x^2) + 4
*a*b^2*c*(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*
x^2 + 5*d^2*x^4)) + (25*a*c*(21*b^3*c^3 - 32*a*b^2*c^2*d - 8*a^2*
b*c*d^2 + 10*a^3*d^3)*x^2*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (
b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a]
 + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])) - (9*a*
b*c*d*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*x^4*AppellF1[5/4, 1/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4,
 (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4,
 (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/
c, (b*x^2)/a]))))/(30*a^2*c^2*(b*c - a*d)^2*(e*x)^(5/2)*(a - b*x^
2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.06, size = 2871, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x)

[Out] 1/24*b*d*(-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^
3*b^5*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-72*x^4*a^2*b^2*c*d
^3*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*x^3*b^5*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-40*x^2*a^4*d^4
*(a*b)^(1/2)+28*x^2*b^4*c^4*(a*b)^(1/2)-36*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a^2*b^2*c*d^2*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+30*E
llipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*x^3*a*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
```

2)*(a*b)^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2
)*x^3*a*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
d)^(1/2))^(1/2)(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*x^3*a*b^3*c^2*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)+36*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*x*a^3*b*c*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)-30*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+39*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(
c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b^2*c^2*d*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+20*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^3*a^3*b*d
^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+1
4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^
(1/2)*x*a*b^3*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c
d)^(1/2))^(1/2)(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)
*x*a*b^3*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*
b^3*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)+40*x^4*a^3*b*d^4*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*2^(1/2)*x^3*a*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^3*b
^4*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*a*b^4*
c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^3*b^4*c^3*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-20*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*a^4*d^3*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-39*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*
d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x*a^2*b^3*c^3*d*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*x*a^2*b^3*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
2^(1/2)*x^3*b^4*c^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/
2)*x*a*b^4*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-21*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x*a*b^4*c^4*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)+16*a^4*c*d^3*(a*b)^(1/2)-16*a*b^3*c^4*(a*b

```
)^(1/2)+60*x^4*a*b^3*c^2*d^2*(a*b)^(1/2)+56*x^2*a^3*b*c*d^3*(a*b)
^(1/2)-44*x^2*a*b^3*c^3*d*(a*b)^(1/2)-48*a^3*b*c^2*d^2*(a*b)^(1/2
)+48*a^2*b^2*c^3*d*(a*b)^(1/2)-28*x^4*b^4*c^3*d*(a*b)^(1/2))*(-d*
x^2+c)^(1/2)/x/c^2/a^2/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((a*b)^(1/2)
*d+(c*d)^(1/2)*b)/(a*b)^(1/2)/(b*x^2-a)/(a*d-b*c)^2/(d*x^2-c)/e^2
/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="maxima'

[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="fricas'

[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.926 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=568

$$\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3} - \frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3} + \frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3ad+7bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3ad+7bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} + \frac{e^3(ex)^{3/2}(4ad+bc)}{2\sqrt{c-dx^2}(bc-ad)^3} + \frac{e^3(ex)^{3/2}(3ad+2bc)}{6b\,(c-dx^2)^{3/2}\,(bc-ad)^2} + \frac{ae^3(ex)^{3/2}}{2b\,(a-bx^2)\,(c-dx^2)^{3/2}\,(bc-ad)}$$

```
[Out] ((2*b*c + 3*a*d)*e^3*(e*x)^(3/2))/(6*b*(b*c - a*d)^2*(c - d*x^2)^
(3/2)) + (a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*
x^2)^(3/2)) + ((b*c + 4*a*d)*e^3*(e*x)^(3/2))/(2*(b*c - a*d)^3*Sq
rt[c - d*x^2]) - (c^(3/4)*(b*c + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/
c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (c^(3/4)*(b*c + 4*a*d
)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^
2]) + (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c
]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[b]*d^(1/4)*(b*c -
 a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt
[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 3.70999, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3} - \frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3} + \frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3ad+7bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3ad+7bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} + \frac{e^3(ex)^{3/2}(4ad+bc)}{2\sqrt{c-dx^2}(bc-ad)^3} + \frac{e^3(ex)^{3/2}(3ad+2bc)}{6b\,(c-dx^2)^{3/2}\,(bc-ad)^2} + \frac{ae^3(ex)^{3/2}}{2b\,(a-bx^2)\,(c-dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] ((2*b*c + 3*a*d)*e^3*(e*x)^(3/2))/(6*b*(b*c - a*d)^2*(c - d*x^2)^
(3/2)) + (a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*
```

```
x^2)^(3/2)) + ((b*c + 4*a*d)*e^3*(e*x)^(3/2))/(2*(b*c - a*d)^3*Sq
rt[c - d*x^2]) - (c^(3/4)*(b*c + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/
c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (c^(3/4)*(b*c + 4*a*d
)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^
2]) + (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c
]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[b]*d^(1/4)*(b*c -
 a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt
[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.48643, size = 522, normalized size = 0.92

$$\frac{(ex)^{9/2}\left(\frac{14x^2\left(a^2d\left(7c-9dx^2\right)+4ab\left(2c^2-4cdx^2+3d^2x^4\right)+b^2cx^2\left(3dx^2-5c\right)\right)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11ac\left(7a^2d\left(7c-9dx^2 \cdots\right.\right.}{\left(dx^2-c\right)\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1 \cdots\right.\right.}\right.}{42x^3\left(a-bx \cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] ((e*x)^(9/2)*((49*a^2*c*(8*b*c + 7*a*d)*AppellF1[3/4, 1/2, 1, 7/4
, (d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2
)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2
)/c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x
^2)/a])) + (11*a*c*(7*a^2*d*(7*c - 9*d*x^2) + 2*b^2*c*x^2*(-16*c
+ 9*d*x^2) + 4*a*b*(14*c^2 - 25*c*d*x^2 + 18*d^2*x^4))*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 14*x^2*(a^2*d*(7*c - 9*
d*x^2) + b^2*c*x^2*(-5*c + 3*d*x^2) + 4*a*b*(2*c^2 - 4*c*d*x^2 +
3*d^2*x^4))*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2
)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))/(
(-c + d*x^2)*(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a] + 2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x
^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))
)))/(42*(-(b*c) + a*d)^3*x^3*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.09, size = 5126, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.927 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=454

$$\frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} + \frac{5e^3\sqrt{ex}(2ad+bc)}{6\sqrt{c-dx^2}(bc-ad)^3} + \frac{e^3\sqrt{ex}(3ad+2bc)}{6b\left(c-dx^2\right)^{3/2}(bc-ad)^2} + \frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] ((2*b*c + 3*a*d)*e^3*Sqrt[e*x])/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3
/2)) + (a*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)
^(3/2)) + (5*(b*c + 2*a*d)*e^3*Sqrt[e*x])/(6*(b*c - a*d)^3*Sqrt[c
 - d*x^2]) + (5*c^(1/4)*(b*c + 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*d)*
e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*
d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.2747, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{5\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} + \frac{5e^3\sqrt{ex}(2ad+bc)}{6\sqrt{c-dx^2}(bc-ad)^3} + \frac{e^3\sqrt{ex}(3ad+2bc)}{6b\left(c-dx^2\right)^{3/2}(bc-ad)^2} + \frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] ((2*b*c + 3*a*d)*e^3*Sqrt[e*x])/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3
/2)) + (a*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)
^(3/2)) + (5*(b*c + 2*a*d)*e^3*Sqrt[e*x])/(6*(b*c - a*d)^3*Sqrt[c
 - d*x^2]) + (5*c^(1/4)*(b*c + 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*d)*
e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*
d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.43422, size = 520, normalized size = 1.15

$$\frac{(ex)^{7/2}\left(\frac{2x^2\left(a^2d\left(5c-7dx^2\right)+2ab\left(5c^2-8cdx^2+5d^2x^4\right)+b^2cx^2\left(5dx^2-7c\right)\right)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9ac\left(a^2d\left(5c-7dx^2\right)+2a\right.}{\left(dx^2-c\right)\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx}{c}\right.\right.}\right.}{6x^3\left(a-bx^2\right)\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] ((e*x)^(7/2)*((25*a^2*c*(2*b*c + a*d)*AppellF1[1/4, 1/2, 1, 5/4,
(d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/
c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c
, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/
a])) + (9*a*c*(a^2*d*(5*c - 7*d*x^2) + 2*b^2*c*x^2*(-4*c + 3*d*x^
2) + 2*a*b*(5*c^2 - 9*c*d*x^2 + 6*d^2*x^4))*AppellF1[5/4, 1/2, 1,
 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(a^2*d*(5*c - 7*d*x^2) + b^2*
c*x^2*(-7*c + 5*d*x^2) + 2*a*b*(5*c^2 - 8*c*d*x^2 + 5*d^2*x^4))*(
2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*App
ellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))/((-c + d*x^2)*(9
*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*
b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*Appel
lF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(6*(-(b*c) + a*
d)^3*x^3*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.082, size = 4403, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] 1/24*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*b*(36*x^3*a^2*b*c*d^3*(a*b)
^(1/2)-36*x^3*a*b^2*c^2*d^2*(a*b)^(1/2)-20*x*a^2*b*c^2*d^2*(a*b)^
(1/2)+40*x*a*b^2*c^3*d*(a*b)^(1/2)-20*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a^2*b*d^3*(-x*d/(c*
d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+10*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b^
3*c^2*d*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a^2*b*c^2*d*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^3*b*c^2*d^2+15*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
```

)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*a^3*b*c^2*d^2+15*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)
*(c*d)^(1/2)*a^2*b*c^2*d+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
2^(1/2)((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d
)^(1/2)*a^2*b*c^2*d+20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^3*d^3*(-x*d/(c*d)^(1/2))^(1/2)*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+10*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a*b^2*c*d^2*(-x*d/(
c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+10*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*
a^2*b*c*d^2*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)-20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*2^(1/2)*x^2*a*b^2*c^2*d*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c
*d)^(1/2)*x^2*a^2*b*c*d^2-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*
d)^(1/2)*x^2*a^2*b*c*d^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
2^(1/2)((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b^2*c^3*d+20
*x^5*b^3*c^2*d^2*(a*b)^(1/2)-28*x^3*b^3*c^3*d*(a*b)^(1/2)-20*x*a^
3*c*d^3*(a*b)^(1/2)+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*x^4*a*b^2*c*d^2+28*x^3*a^3*d^4*(a*b)^(1/2)-15*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*(c*d)^(1/2)*x^2*a*b^2*c^2*d-15*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*
b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b^2*c^2*d+20*x^5*a*b^2*c*d^3*(a
b)^(1/2)+15((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^3*c^3*d-15*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*x^2*a*b^3*c^3*d+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*a^2*b^2*c^3*d-10
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*x^2*b^3*c^3*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)-20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*a^3*c*d^2*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2
)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^3*(-x*d/(c*d)^(1/2))^(1/
2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+15*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x
*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^
4*a*b^3*c^2*d^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*

```
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^2*a^3*b*c*d^3-30*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a^
3*b*c*d^3+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1
/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2+15*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*
d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^3*c*d^2+15*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^3*c*d^2-15*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*(c*d)^(1/2)*x^2*a^3*d^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(
1/2))*(c*d)^(1/2)*x^2*a^3*d^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^
(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^4*a^2*b^2
*c*d^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^4*a*b^3*c^2*d^2+15*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*x^4*a^2*b^2*c*d^3+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*
d)^(1/2)*x^4*a^2*b*d^3+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^
(1/2)*x^4*a^2*b*d^3-40*x^5*a^2*b*d^4*(a*b)^(1/2)+15*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1
/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^4*a*b^2*c*d^2)/x/(a*d-b*c)^3/(b
*x^2-a)/(a*b)^(1/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-
(a*b)^(1/2)*d)/(d*x^2-c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.928 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=551

$$\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt{b}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(7ad+3bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{b}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(7ad+3bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{de(ex)^{3/2}(ad+4bc)}{2c\sqrt{c-dx^2}(bc-ad)^3}+\frac{5de(ex)^{3/2}}{6\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{e(ex)^{3/2}}{2\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] (5*d*e*(e*x)^(3/2))/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*(e*x
)^(3/2))/(2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(4*b*
c + a*d)*e*(e*x)^(3/2))/(2*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (d^
(1/4)*(4*b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*c^(1/4)*(b*c - a*
d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(4*b*c + a*d)*e^(5/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/
4)*(3*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt
[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x
^2]) - (Sqrt[b]*c^(1/4)*(3*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/
c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4)*(b*c - a
*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 3.56544, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt{b}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(7ad+3bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{b}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}(7ad+3bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{de(ex)^{3/2}(ad+4bc)}{2c\sqrt{c-dx^2}(bc-ad)^3}+\frac{5de(ex)^{3/2}}{6\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{e(ex)^{3/2}}{2\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (5*d*e*(e*x)^(3/2))/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*(e*x
)^(3/2))/(2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(4*b*
```

```
c + a*d)*e*(e*x)^(3/2))/(2*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (d^
(1/4)*(4*b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*c^(1/4)*(b*c - a*
d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(4*b*c + a*d)*e^(5/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/
4)*(3*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt
[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x
^2]) - (Sqrt[b]*c^(1/4)*(3*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/
c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4)*(b*c - a
*d)^3*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.87978, size = 568, normalized size = 1.03

$$\frac{e(ex)^{3/2}\left(\frac{49a\left(a^2d^2+11abcd+3b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(a-bx^2)(ad-bc)^3\left(2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+7acF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}+\frac{-14x^2\left(a^2d^2\left(c-3dx^2\right)+abd\left(11c^2\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (e*(e*x)^(3/2)*((49*a*(3*b^2*c^2 + 11*a*b*c*d + a^2*d^2)*AppellF1
[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((-(b*c) + a*d)^3*(a -
b*x^2)*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] +
2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] +
a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) + (-11*a
*c*(7*a^2*d^2*(c - 3*d*x^2) + a*b*d*(77*c^2 - 67*c*d*x^2 + 18*d^2
*x^4) + b^2*c*(21*c^2 - 107*c*d*x^2 + 72*d^2*x^4))*AppellF1[7/4,
1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] - 14*x^2*(a^2*d^2*(c - 3*d*x^
2) + a*b*d*(11*c^2 - 10*c*d*x^2 + 3*d^2*x^4) + b^2*c*(3*c^2 - 17*
c*d*x^2 + 12*d^2*x^4))*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2
)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*
x^2)/a]))/(c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)*(11*a*c*Appel
lF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*Appel
lF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4
, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(42*Sqrt[c - d*x^2])
```

Maple [B] time = 0.063, size = 5078, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.929 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=447

$$\frac{d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+14bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6c^{3/4}\sqrt{c-dx^2}(bc-ad)^3} - \frac{b\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(9ad+bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{b\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(9ad+bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} + \frac{de\sqrt{ex}(ad+14bc)}{6c\sqrt{c-dx^2}(bc-ad)^3} + \frac{5de\sqrt{ex}}{6\left(c-dx^2\right)^{3/2}(bc-ad)^2} + \frac{e\sqrt{ex}}{2\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] (5*d*e*Sqrt[e*x])/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*Sqrt[e
*x])/(2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(14*b*c +
 a*d)*e*Sqrt[e*x])/(6*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(3/4)
*(14*b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*c^(3/4)*(b*c - a*d)^3
*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c -
 a*d)^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b
*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.36827, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+14bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6c^{3/4}\sqrt{c-dx^2}(bc-ad)^3} - \frac{b\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(9ad+bc)\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} - \frac{b\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(9ad+bc)\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3} + \frac{de\sqrt{ex}(ad+14bc)}{6c\sqrt{c-dx^2}(bc-ad)^3} + \frac{5de\sqrt{ex}}{6\left(c-dx^2\right)^{3/2}(bc-ad)^2} + \frac{e\sqrt{ex}}{2\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] (5*d*e*Sqrt[e*x])/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*Sqrt[e
*x])/(2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(14*b*c +
 a*d)*e*Sqrt[e*x])/(6*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(3/4)
*(14*b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*c^(3/4)*(b*c - a*d)^3
*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c -
 a*d)^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b
*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.646, size = 547, normalized size = 1.22

$$\frac{(ex)^{3/2}\left(\frac{25a(-a^2d^2+13abcd+3b^2c^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{9ac(5a^2d^2(c+dx^2)+abd(-65c^2+51cdx^2-6d^2x^4)+b^2c(-}{}\right.}{30\,(bx}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] ((e*x)^(3/2)*((25*a*(3*b^2*c^2 + 13*a*b*c*d - a^2*d^2)*AppellF1[1
/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2,
 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2,
2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d
*x^2)/c, (b*x^2)/a])) + (9*a*c*(5*a^2*d^2*(c + d*x^2) + b^2*c*(-1
5*c^2 + 109*c*d*x^2 - 84*d^2*x^4) + a*b*d*(-65*c^2 + 51*c*d*x^2 -
 6*d^2*x^4))*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] - 1
0*x^2*(-(a^2*d^2*(c + d*x^2)) + a*b*d*(13*c^2 - 10*c*d*x^2 + d^2*
x^4) + b^2*c*(3*c^2 - 19*c*d*x^2 + 14*d^2*x^4))*(2*b*c*AppellF1[9
/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2,
1, 13/4, (d*x^2)/c, (b*x^2)/a]))/(c*(c - d*x^2)*(9*a*c*AppellF1[5
/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/
4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1
, 13/4, (d*x^2)/c, (b*x^2)/a])))))/(30*(b*c - a*d)^3*Sqrt[c - d*x
^2]*(-(a*x) + b*x^3))
```

Maple [B] time = 0.06, size = 4403, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] -1/24*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)*b*d*(-36*x^3*a^2*b*c*d^3*(a*
b)^(1/2)-36*x^3*a*b^2*c^2*d^2*(a*b)^(1/2)+56*x*a^2*b*c^2*d^2*(a*b
)^(1/2)-40*x*a*b^2*c^3*d*(a*b)^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a*b^3
*c^4-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+
(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*b^4*c^4-3*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*a*
b^3*c^4-12*x*b^3*c^4*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a^2*b*d^3*(-x*d/(c*d)^
(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
```

-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b^3*c^2*d*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+26*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c^2*d*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*x^2*b^4*c^4-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*b^3*c^2*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4*b^3*c^2*d*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-27*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*a^2*b*c^2*d-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*a^2*b*c^2*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^3*d^3*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+26*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*a*b^2*c*d^2*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b*c*d^2*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^2*c^2*d*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*b*c*d^2+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a^2*b*c*d^2+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*a^2*b^2*c^3*d-56*x^5*b^3*c^2*d^2*(a*b)^(1/2)+76*x^3*b^3*c^3*d*(a*b)^(1/2)-4*x*a^3*c*d^3*(a*b)^(1/2)-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*x^4*a*b^2*c*d^2-4*x^3*a^3*d^4*(a*b)^(1/2)+30*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*x^2*a*b^2*c^2*d+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*a*b^2*c^2*d+52*x^5*a*b^2*c*d^3*(a*b)^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*(c*d)^(1/2)*x^2*b^3*c^3-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^2*a*b^3*c^3*d-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)

```
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*a*b^2*c^3-3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b
),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^3+3*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*(c*d)^(1/2)*x^2*b^3*c^3+30*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*x^2*a*b^3*
c^3*d-27*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-x*d/(c*d)^(1/2))^(1/2)*a^2*b^2*c^3*d+28*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^3*c^3*(-
x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+2*Elli
pticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
a^3*c*d^2*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)-28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*a*b^2*c^3*(-x*d/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*x^4*a*b^3*c^2*d^2+27*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*x^2*a^2*b^2*c^2*d^2-27*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x
^2*a^2*b^2*c^2*d^2+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*x^4*a*b^3*c^2*d^2+3*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*
b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*b^4*c^3*d*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*2^(1/2)*x^4*b^4*c^3*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(
1/2)+4*x^5*a^2*b*d^4*(a*b)^(1/2)-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/
2))*(c*d)^(1/2)*x^4*a*b^2*c*d^2)/x/(a*d-b*c)^3/(b*x^2-a)/(a*b)^(1
/2)/((a*b)^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(
d*x^2-c)^2/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="giac")

[Out] integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.930 $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=625

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-11ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-11ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{2ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b(ex)^{3/2}}{2ae\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{6ace\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d)*(e*x)^(3/2))/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2
)^(3/2)) + (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*
x^2)^(3/2)) + (d*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*(e*x)^(3/2))/(2*
a*c^2*(b*c - a*d)^3*e*Sqrt[c - d*x^2]) - (d^(1/4)*(b^2*c^2 + 5*a*
b*c*d - a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d
)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*S
qrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2
]) - (b^(3/2)*c^(1/4)*(b*c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a
*d)^3*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(b*c - 11*a*d)*Sqrt[e]*
Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi [A] time = 3.87844, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-11ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-11ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{2ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b(ex)^{3/2}}{2ae\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{6ace\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*(e*x)^(3/2))/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2
)^(3/2)) + (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*
x^2)^(3/2)) + (d*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*(e*x)^(3/2))/(2*
a*c^2*(b*c - a*d)^3*e*Sqrt[c - d*x^2]) - (d^(1/4)*(b^2*c^2 + 5*a*
b*c*d - a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d
)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*S
qrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2
]) - (b^(3/2)*c^(1/4)*(b*c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a
*d)^3*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(b*c - 11*a*d)*Sqrt[e]*
Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.42885, size = 626, normalized size = 1.

$$x\sqrt{ex}\left(\frac{14x^2\left(a^3d^3\left(3dx^2-5c\right)+a^2bd^2\left(17c^2-10cdx^2-3d^2x^4\right)+ab^2cd^2x^2\left(15dx^2-17c\right)+3b^3c^2\left(c-dx^2\right)^2\right)\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\right.\right.}{a(dx^2-c)(ad-bc)^3\left(2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*Sqrt[e*x]*((49*c*(b^3*c^3 - 12*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a
^3*d^3)*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/((b*c -
 a*d)^3*(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] +
 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/c, (b*x^2)/a] +
 a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))) + (-11*
a*c*(2*a*b^2*c*d^2*x^2*(52*c - 45*d*x^2) + 7*a^3*d^3*(5*c - 3*d*x
^2) - 3*b^3*c^2*(7*c^2 - 13*c*d*x^2 + 6*d^2*x^4) + a^2*b*d^2*(-11
9*c^2 + 73*c*d*x^2 + 18*d^2*x^4))*AppellF1[7/4, 1/2, 1, 11/4, (d*
x^2)/c, (b*x^2)/a] + 14*x^2*(3*b^3*c^2*(c - d*x^2)^2 + a^3*d^3*(-
5*c + 3*d*x^2) + a*b^2*c*d^2*x^2*(-17*c + 15*d*x^2) + a^2*b*d^2*(
17*c^2 - 10*c*d*x^2 - 3*d^2*x^4))*(2*b*c*AppellF1[11/4, 1/2, 2, 1
5/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*
x^2)/c, (b*x^2)/a]))/(a*(-(b*c) + a*d)^3*(-c + d*x^2)*(11*a*c*App
ellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*App
ellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[11
/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a])))))/(42*c^2*(a - b*x^2)*
Sqrt[c - d*x^2])
```

Maple [B] time = 0.066, size = 5689, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.931 $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=514

$$\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{6ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6ac^{7/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-13ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-13ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b\sqrt{ex}}{2ae\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d\sqrt{ex}(2ad+3bc)}{6ace\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d)*Sqrt[e*x])/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^
(3/2)) + (b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)
^(3/2)) + (d*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[e*x])/(6*a
*c^2*(b*c - a*d)^3*e*Sqrt[c - d*x^2]) + (d^(3/4)*(3*b^2*c^2 + 17*
a*b*c*d - 5*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a*c^(7/4)*(b*c - a*d)^3*
Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*
(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*d^(1/4)*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi [A] time = 2.48185, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{6ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6ac^{7/4}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-13ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(3bc-13ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b\sqrt{ex}}{2ae\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d\sqrt{ex}(2ad+3bc)}{6ace\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] (d*(3*b*c + 2*a*d)*Sqrt[e*x])/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^
(3/2)) + (b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)
^(3/2)) + (d*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[e*x])/(6*a
*c^2*(b*c - a*d)^3*e*Sqrt[c - d*x^2]) + (d^(3/4)*(3*b^2*c^2 + 17*
a*b*c*d - 5*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a*c^(7/4)*(b*c - a*d)^3*
```

```
Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*
(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13
*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*d^(1/4)*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.79476, size = 629, normalized size = 1.22

$$x\left(\frac{10x^2\left(a^3d^3(5dx^2-7c)+a^2bd^2(19c^2-10cdx^2-5d^2x^4)+ab^2cd^2x^2(17dx^2-19c)+3b^3c^2(c-dx^2)^2\right)\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)-}{a(dx^2-c)(ad-bc)^3\left(2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((25*c*(9*b^3*c^3 - 36*a*b^2*c^2*d + 17*a^2*b*c*d^2 - 5*a^3*d^
3)*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/((b*c - a*d)
^3*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 2*x^
2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c, (b*x^2)/a] + a*d*A
ppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))) + (-9*a*c*(2*a
*b^2*c*d^2*x^2*(56*c - 51*d*x^2) + 5*a^3*d^3*(7*c - 5*d*x^2) - 3*
b^3*c^2*(5*c^2 - 11*c*d*x^2 + 6*d^2*x^4) + 5*a^2*b*d^2*(-19*c^2 +
 9*c*d*x^2 + 6*d^2*x^4))*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b
*x^2)/a] + 10*x^2*(3*b^3*c^2*(c - d*x^2)^2 + a^3*d^3*(-7*c + 5*d*
x^2) + a*b^2*c*d^2*x^2*(-19*c + 17*d*x^2) + a^2*b*d^2*(19*c^2 - 1
0*c*d*x^2 - 5*d^2*x^4))*(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2
)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x
^2)/a]))/(a*(-(b*c) + a*d)^3*(-c + d*x^2)*(9*a*c*AppellF1[5/4, 1/
2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[9/4, 1/2
, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*AppellF1[9/4, 3/2, 1, 13/4
, (d*x^2)/c, (b*x^2)/a])))))/(30*c^2*Sqrt[e*x]*(a - b*x^2)*Sqrt[c
 - d*x^2])
```

Maple [B] time = 0.067, size = 4776, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] -1/24*(-d*x^2+c)^(1/2)*b*d*(-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*2^(1/2)*a^2*b^2*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)*(a*b)^(1/2)+44*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
```

1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*b*c^2*d^2*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+20*x^3*a^4*d^5*(a*b)^(1/2)
-12*x*b^4*c^5*(a*b)^(1/2)-16*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c^2*d^2*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+34*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3
*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/
2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^4*a*b^3*
c^2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1
/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b
^3*c^3*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/
2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+30*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^4*c^4*d*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*2^(1/2)*x^2*b^4*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d
/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(
1/2)*(a*b)^(1/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*2^(1/2)*x^2*a^4*d^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b^4*c^4*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d
)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*a*b^3*c^4*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2
^(1/2))*2^(1/2)*x^4*b^5*c^4*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4*b^5*c^4*
d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2
)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^4*d*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a
^2*b^3*c^4*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(
1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-20*x^5*a^3*b*d^
5*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*
x^2*b^4*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b
)^(1/2)-10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*2^(1/2)*a^4*c*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)*(a*b)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*2^(1/2)*a*b^3*c^4*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1
/2*2^(1/2))*2^(1/2)*x^4*a*b^4*c^3*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4

```
*a*b^4*c^3*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-39*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)
*d+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^3*c^3*d^2*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)-30*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/2
*2^(1/2))*2^(1/2)*x^2*a*b^4*c^4*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^2*b^5*
c^5*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/
2)*b),1/2*2^(1/2))*2^(1/2)*a*b^4*c^5*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4
*c^5*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)-39*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(
1/2)*b),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^3*d*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-28*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^3*d*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-10*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^
4*a^3*b*d^4*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1
/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b
)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*2^(1/2)*x^4*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(c*d)^(1/2)*(a*b)^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*x^4*b^4*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d
)^(1/2)*(a*b)^(1/2)-56*x^5*a*b^3*c^2*d^3*(a*b)^(1/2)-60*x^3*a^3*b
*c*d^4*(a*b)^(1/2)-36*x^3*a^2*b^2*c^2*d^3*(a*b)^(1/2)+52*x^3*a*b^
3*c^3*d^2*(a*b)^(1/2)+104*x*a^3*b*c^2*d^3*(a*b)^(1/2)-76*x*a^2*b^
2*c^3*d^2*(a*b)^(1/2)+12*x*a*b^3*c^4*d*(a*b)^(1/2)+9*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d
+(c*d)^(1/2)*b),1/2*2^(1/2))*2^(1/2)*x^2*b^5*c^5*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)+88*x^5*a^2*b^2*c*d^4*(a*b)^(1/2)-39*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*x^4*a*b^3*c^2*d^2*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+44*Ellipti
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4
*a^2*b^2*c*d^3*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)
^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(
a*b)^(1/2)-28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*2^(1/2)*x^4*a*b^3*c^2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b),1/
2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c^2*d^2*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+30*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((a*b)^(1/2)*d+(c*d)^(1/2)*b)
,1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*x^2*a^2*b^2*c^2*d^2*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+30*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*x^2*a*b^3*c^3*d*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-34*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a^3*b*c*d^3*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-x*d/(c*d)^(1/2))^(1/2)*((d*x+(
```

```
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-12*x^5*b^4
*c^3*d^2*(a*b)^(1/2)+24*x^3*b^4*c^4*d*(a*b)^(1/2)-28*x*a^4*c*d^4*
(a*b)^(1/2))/(e*x)^(1/2)/(a*d-b*c)^3/(b*x^2-a)/(a*b)^(1/2)/((a*b)
^(1/2)*d+(c*d)^(1/2)*b)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(d*x^2-c)^2
/a/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.932 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=735

$$-\frac{5b^{5/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{5b^{5/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d\left(-7a^2d^2+19abcd+3b^2c^2\right)}{6ac^2e\sqrt{ex}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)}{2a^2c^3e\sqrt{ex}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b}{2ae\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ace\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*(c - d*x^2)^
(3/2)) + b/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(
3/2)) + (d*(3*b^2*c^2 + 19*a*b*c*d - 7*a^2*d^2))/(6*a*c^2*(b*c -
a*d)^3*e*Sqrt[e*x]*Sqrt[c - d*x^2]) - ((5*b^3*c^3 - 12*a*b^2*c^2*
d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[c - d*x^2])/(2*a^2*c^3*(b*c
- a*d)^3*e*Sqrt[e*x]) - (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*c^2*d + 19
*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c -
a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*
c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/
4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) - (5*b^(5/2)*c^(1/4)*(b
*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) +
(5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*S
qrt[c - d*x^2])
```

Rubi [A] time = 4.75141, antiderivative size = 735, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$-\frac{5b^{5/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{5b^{5/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d\left(-7a^2d^2+19abcd+3b^2c^2\right)}{6ac^2e\sqrt{ex}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)}{2a^2c^3e\sqrt{ex}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\Big|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b}{2ae\sqrt{ex}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ace\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*(c - d*x^2)^
(3/2)) + b/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(
3/2)) + (d*(3*b^2*c^2 + 19*a*b*c*d - 7*a^2*d^2))/(6*a*c^2*(b*c -
a*d)^3*e*Sqrt[e*x]*Sqrt[c - d*x^2]) - ((5*b^3*c^3 - 12*a*b^2*c^2*
d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[c - d*x^2])/(2*a^2*c^3*(b*c
- a*d)^3*e*Sqrt[e*x]) - (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*c^2*d + 19
*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c -
a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*
c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/
4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) - (5*b^(5/2)*c^(1/4)*(b
*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) +
(5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*S
qrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.59308, size = 582, normalized size = 0.79

$$\frac{x\left(\frac{33abcdx^4\left(7a^3d^3-19a^2bcd^2+12ab^2c^2d-5b^3c^3\right)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{11}{4};\frac{1}{2},2;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{11}{4};\frac{3}{2},1;\frac{15}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+11acF_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}-\frac{49acx^2\left(7a^4d^4-19a^3bcd^3+12a^2b^2c^2d^2-20ab^3c^3d+5b\right.}{2x^2\left(2bcF_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right.}\right.}{42}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((-7*(15*b^4*c^3*x^2*(c - d*x^2)^2 - 12*a*b^3*c^2*(c - d*x^2)^
2*(c + 3*d*x^2) + a^4*d^3*(12*c^2 - 35*c*d*x^2 + 21*d^2*x^4) - a^
3*b*d^2*(36*c^3 - 83*c^2*d*x^2 + 22*c*d^2*x^4 + 21*d^3*x^6) + a^2
*b^2*c*d*(36*c^3 - 36*c^2*d*x^2 - 59*c*d^2*x^4 + 57*d^3*x^6)))/(c
 - d*x^2) - (49*a*c*(5*b^4*c^4 - 20*a*b^3*c^3*d + 12*a^2*b^2*c^2*
d^2 - 19*a^3*b*c*d^3 + 7*a^4*d^4)*x^2*AppellF1[3/4, 1/2, 1, 7/4,
(d*x^2)/c, (b*x^2)/a])/(7*a*c*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/
c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[7/4, 1/2, 2, 11/4, (d*x^2)/
c, (b*x^2)/a] + a*d*AppellF1[7/4, 3/2, 1, 11/4, (d*x^2)/c, (b*x^2
)/a])) + (33*a*b*c*d*(-5*b^3*c^3 + 12*a*b^2*c^2*d - 19*a^2*b*c*d^
2 + 7*a^3*d^3)*x^4*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)
/a])/(11*a*c*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a] +
2*x^2*(2*b*c*AppellF1[11/4, 1/2, 2, 15/4, (d*x^2)/c, (b*x^2)/a] +
 a*d*AppellF1[11/4, 3/2, 1, 15/4, (d*x^2)/c, (b*x^2)/a]))))/(42*a
^2*c^3*(-(b*c) + a*d)^3*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.079, size = 6334, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 - a)^2 (-dx^2 + c)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)),x, algorithm="maxima'
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)),x, algorithm="fricas'
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.2428, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.933 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=606

$$\frac{b^3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-17ad)\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-17ad)\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d\left(-3a^2d^2+7abcd+b^2c^2\right)}{2ac^2e(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)}{6a^2c^3e(ex)^{3/2}(bc-ad)^3}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{11/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b}{2ae(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ace(ex)^{3/2}\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*(c - d*x^2
)^(3/2)) + b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*(c - d*x^
2)^(3/2)) + (d*(b^2*c^2 + 7*a*b*c*d - 3*a^2*d^2))/(2*a*c^2*(b*c -
 a*d)^3*e*(e*x)^(3/2)*Sqrt[c - d*x^2]) - ((7*b^3*c^3 - 12*a*b^2*c
^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[c - d*x^2])/(6*a^2*c^3*(
b*c - a*d)^3*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^3*c^3 - 12*a*b^2*c^2*
d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(11/4)
*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 1
7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1
/4)*(7*b*c - 17*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi [A] time = 3.82499, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{b^3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-17ad)\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-17ad)\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d\left(-3a^2d^2+7abcd+b^2c^2\right)}{2ac^2e(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)}{6a^2c^3e(ex)^{3/2}(bc-ad)^3}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{11/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b}{2ae(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ace(ex)^{3/2}\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```


```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*(c - d*x^2
)^(3/2)) + b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*(c - d*x^
2)^(3/2)) + (d*(b^2*c^2 + 7*a*b*c*d - 3*a^2*d^2))/(2*a*c^2*(b*c -
 a*d)^3*e*(e*x)^(3/2)*Sqrt[c - d*x^2]) - ((7*b^3*c^3 - 12*a*b^2*c
```

```
^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[c - d*x^2])/(6*a^2*c^3*(
b*c - a*d)^3*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^3*c^3 - 12*a*b^2*c^2*
d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(11/4)
*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 1
7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1
/4)*(7*b*c - 17*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.75623, size = 582, normalized size = 0.96

$$\frac{x\left(\frac{9abcdx^4(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{2x^2\left(2bcF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}+\frac{25acx^2(15a^4d^4-35a^3bcd^3+12a^2b^2c^2d^2+44ab^3c^3d-21b^4c^4}{2x^2\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)+5}\right.}{30a^2c^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((-5*(7*b^4*c^3*x^2*(c - d*x^2)^2 - 4*a*b^3*c^2*(c - d*x^2)^2*
(c + 3*d*x^2) + a^4*d^3*(4*c^2 - 21*c*d*x^2 + 15*d^2*x^4) - a^3*b
*d^2*(12*c^3 - 45*c^2*d*x^2 + 14*c*d^2*x^4 + 15*d^3*x^6) + a^2*b^
2*c*d*(12*c^3 - 12*c^2*d*x^2 - 37*c*d^2*x^4 + 35*d^3*x^6)))/(c -
d*x^2) + (25*a*c*(-21*b^4*c^4 + 44*a*b^3*c^3*d + 12*a^2*b^2*c^2*d
^2 - 35*a^3*b*c*d^3 + 15*a^4*d^4)*x^2*AppellF1[1/4, 1/2, 1, 5/4,
(d*x^2)/c, (b*x^2)/a])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/
c, (b*x^2)/a] + 2*x^2*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, (d*x^2)/c
, (b*x^2)/a] + a*d*AppellF1[5/4, 3/2, 1, 9/4, (d*x^2)/c, (b*x^2)/
a])) + (9*a*b*c*d*(7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 -
15*a^3*d^3)*x^4*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])
/(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a] + 2*x^2*
(2*b*c*AppellF1[9/4, 1/2, 2, 13/4, (d*x^2)/c, (b*x^2)/a] + a*d*Ap
pellF1[9/4, 3/2, 1, 13/4, (d*x^2)/c, (b*x^2)/a]))))/(30*a^2*c^3*(
-(b*c) + a*d)^3*(e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] time = 0.079, size = 5248, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.25053, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.934 $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{24b^2d^2}+\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd}$$

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(16*b^2*d^3) - ((5*b*c + 3*a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x
^2])/(24*b^2*d^2) + (x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(6*b*
d) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*b^(5/2)*d^(7
/2))
```

Rubi [A] time = 0.597901, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{24b^2d^2}+\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(16*b^2*d^3) - ((5*b*c + 3*a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x
^2])/(24*b^2*d^2) + (x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(6*b*
d) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*b^(5/2)*d^(7
/2))
```

Rubi in Sympy [A] time = 42.9904, size = 194, normalized size = 0.93

$$\frac{x^2\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{6bd}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\,(3ad+5bc)}{24b^2d^2}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}+\frac{(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{\frac{5}{2}}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x**2*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(6*b*d) - (a + b*x**2)*
*(3/2)*sqrt(c + d*x**2)*(3*a*d + 5*b*c)/(24*b**2*d**2) + sqrt(a +
 b*x**2)*sqrt(c + d*x**2)*(a**2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(
16*b**2*d**3) + (a*d - b*c)*(a**2*d**2 + 2*a*b*c*d + 5*b**2*c**2)
*atanh(sqrt(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(c + d*x**2)))/(16*b
**(5/2)*d**(7/2))
```

Mathematica [A] time = 0.183472, size = 174, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-3a^2d^2+2abd\left(dx^2-2c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)}{48b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{32b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*
x^2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*b^2*d^3) - ((b
*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[b*c + a*d + 2*b*d
*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(32*b^
(5/2)*d^(7/2))
```

Maple [B] time = 0.072, size = 532, normalized size = 2.6

$$\frac{1}{96\,b^2d^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(16\,x^4b^2d^2\sqrt{bd}\sqrt{bdx^4+adx^2+cx^2b+ac}+4\,\sqrt{bdx^4+adx^2+cx^2b+ac}x^2abd^2\sqrt{bd}-20\,\sqrt{bdx^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*x^4*b^2*d^2*(b*d)^(1/2)*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*d^3
+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+9*ln(1/2*(2*b*d*x^2+2*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*c^2*a*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*a^2*d^2*(b*d)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/b^2/d^3/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^5/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274168, size = 1, normalized size = 0.

$$\left[\frac{4\left(8\,b^2d^2x^4+15\,b^2c^2-4\,abcd-3\,a^2d^2-2\left(5\,b^2cd-abd^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}-3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a\right.}{192\,\sqrt{bd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^5/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/192*(4*(8*b^2*d^2*x^4 + 15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2 - 2
*(5*b^2*c*d - a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
b*d) - 3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*log(
4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*
d + a*b*d^2)*x^2)*sqrt(b*d)))/(sqrt(b*d)*b^2*d^3), 1/96*(2*(8*b^2
*d^2*x^4 + 15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2 - 2*(5*b^2*c*d - a*
b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) - 3*(5*b^3
*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*(2*b*d*x
^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d))
)/(sqrt(-b*d)*b^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.253463, size = 304, normalized size = 1.45

$$\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{bd}-\frac{5\,b^3cd^3+7\,ab^2d^4}{b^3d^5}\right)+\frac{3\left(5\,b^4c^2d^2+2\,ab^3cd^3+a^2b^2d^4\right)}{b^3d^5}\right)+\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-\cdots\right.}{\cdots}}{48\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^5/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b
*x^2 + a)*(4*(b*x^2 + a)/(b*d) - (5*b^3*c*d^3 + 7*a*b^2*d^4)/(b^3
*d^5)) + 3*(5*b^4*c^2*d^2 + 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(b^3*d^5
)) + 3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*ln(abs
(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*d^3))/(b*abs(b))
```

3.935 $\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+3bc)}{8bd^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd}$$

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b*d^2) + ((a
+ b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*b*d) + ((b*c - a*d)*(3*b*c + a
*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])
/(8*b^(3/2)*d^(5/2))
```

Rubi [A] time = 0.321787, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+3bc)}{8bd^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b*d^2) + ((a
+ b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*b*d) + ((b*c - a*d)*(3*b*c + a
*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])
/(8*b^(3/2)*d^(5/2))
```

Rubi in Sympy [A] time = 27.8596, size = 119, normalized size = 0.87

$$\frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{4bd}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+3bc)}{8bd^2}-\frac{(ad-bc)\,(ad+3bc)\,\text{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8b^{\frac{3}{2}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] (a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(4*b*d) - sqrt(a + b*x**2)*s
qrt(c + d*x**2)*(a*d + 3*b*c)/(8*b*d**2) - (a*d - b*c)*(a*d + 3*b
*c)*atanh(sqrt(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(8
*b**(3/2)*d**(5/2))
```

Mathematica [A] time = 0.125631, size = 126, normalized size = 0.92

$$\frac{1}{16}\left(\frac{(bc-ad)(ad+3bc)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{b^{3/2}d^{5/2}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(ad-3bc+2bdx^2\right)}{bd^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + a*d + 2*b*d*x^2))/(
b*d^2) + ((b*c - a*d)*(3*b*c + a*d)*Log[b*c + a*d + 2*b*d*x^2 + 2
*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(b^(3/2)*d^(5/
2)))/16
```

Maple [B] time = 0.02, size = 339, normalized size = 2.5

$$-\frac{1}{16\,d^2b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-4\sqrt{bdx^4+adx^2+cx^2b+acx}^2db\sqrt{bd}+\ln\left(\frac{1}{2}\left(2\,bdx^2+2\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-4*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*x^2*d*b*(b*d)^(1/2)+ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*d^2
+2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a*d*b-3*b^2*ln(1/2*(2*b*d*x^2+2*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*c^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d*(b*d)^(1/2)+6*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c*b*(b*d)^(1/2))/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/d^2/b/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*x^3/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263239, size = 1, normalized size = 0.01

$$\left[\frac{4\left(2\,bdx^2-3\,bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}-\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\log\left(-4\left(2\,b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2+\right.}{32\sqrt{bd}bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*x^3/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/32*(4*(2*b*d*x^2 - 3*b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
)*sqrt(b*d) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log(-4*(2*b^2*d^2
*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^
2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)
*x^2)*sqrt(b*d)))/(sqrt(b*d)*b*d^2), 1/16*(2*(2*b*d*x^2 - 3*b*c +
 a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) + (3*b^2*c^2 - 2
*a*b*c*d - a^2*d^2)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)
/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/(sqrt(-b*d)*b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.247937, size = 207, normalized size = 1.51

$$\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(\frac{2\,(bx^2+a)}{bd}-\frac{3\,b^2cd+abd^2}{b^2d^3}\right)-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\ln\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}d^2}}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^3/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*
x^2 + a)/(b*d) - (3*b^2*c*d + a*b*d^2)/(b^2*d^3)) - (3*b^2*c^2 -
2*a*b*c*d - a^2*d^2)*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2))/abs(b)
```

3.936 $\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}$$

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^
(3/2))
```

Rubi [A] time = 0.177337, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^
(3/2))
```

Rubi in Sympy [A] time = 18.1603, size = 73, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}+\frac{(ad-bc)\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(a + b*x**2)*sqrt(c + d*x**2)/(2*d) + (a*d - b*c)*atanh(sqrt(
b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(2*sqrt(b)*d**(3/
2))
```

Mathematica [A] time = 0.086989, size = 101, normalized size = 1.17

$$\frac{(ad-bc)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{4\sqrt{b}d^{3/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) + ((-(b*c) + a*d)*Log[b*c
 + a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]])/(4*Sqrt[b]*d^(3/2))
```

Maple [B] time = 0.014, size = 198, normalized size = 2.3

$$\frac{1}{4d}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(a\ln\left(\frac{1}{2}\left(2bdx^2+2\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)d-b\ln\left(\frac{1}{2}\left(2bdx^2+2\sqrt{bdx^4\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(a*ln(1/2*(2*b*d*x^2+2*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d
-b*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250387, size = 1, normalized size = 0.01

$$\left[-\frac{(bc-ad)\log\left(4\left(2b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}+\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2\right)\sqrt{bd}\cdots}{8\sqrt{bd}d},\right.$$
$$\left.-\frac{(bc-ad)\arctan\left(\frac{(2bdx^2+bc+ad)\sqrt{-bd}}{2\sqrt{bx^2+a}\sqrt{dx^2+c}bd}\right)-2\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{4\sqrt{-bd}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*log(4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d)) - 4*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c)*sqrt(b*d))/(sqrt(b*d)*d), -1/4*((b*c - a*d
)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)*b*d)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b
*d))/(sqrt(-b*d)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.249572, size = 143, normalized size = 1.66

$$\frac{b\left(\frac{(bc-ad)\ln\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}}{bd}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/2*b*((b*c - a*d)*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)/(b*d))/abs(b)
```

3.937 $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])])/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sq
rt[b]*Sqrt[c + d*x^2])])/Sqrt[d]
```

Rubi [A] time = 0.27373, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])])/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sq
rt[b]*Sqrt[c + d*x^2])])/Sqrt[d]
```

Rubi in Sympy [A] time = 26.6491, size = 83, normalized size = 0.9

$$-\frac{\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2)
))/sqrt(c) + sqrt(b)*atanh(sqrt(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt
(a + b*x**2)))/sqrt(d)
```

Mathematica [C] time = 0.554556, size = 238, normalized size = 2.59

$$\frac{5a\left(a+bx^2\right)^{3/2}(bc-ad)F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};\frac{d(bx^2+a)}{ad-bc},\frac{bx^2}{a}+1\right)}{3bx^2\sqrt{c+dx^2}\left(5a(bc-ad)F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};\frac{d(bx^2+a)}{ad-bc},\frac{bx^2}{a}+1\right)-(a+bx^2)\left((2ad-2bc)F_1\left(\frac{5}{2};\frac{1}{2},2;\frac{7}{2};\frac{d(bx^2+a)}{ad-bc},\frac{bx^2}{a}+1\right)+adF_1\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (5*a*(b*c - a*d)*(a + b*x^2)^(3/2)*AppellF1[3/2, 1/2, 1, 5/2, (d*
(a + b*x^2))/(-(b*c) + a*d), 1 + (b*x^2)/a])/(3*b*x^2*Sqrt[c + d*
x^2]*(5*a*(b*c - a*d)*AppellF1[3/2, 1/2, 1, 5/2, (d*(a + b*x^2))/
(-(b*c) + a*d), 1 + (b*x^2)/a] - (a + b*x^2)*((-2*b*c + 2*a*d)*Ap
pellF1[5/2, 1/2, 2, 7/2, (d*(a + b*x^2))/(-(b*c) + a*d), 1 + (b*x
^2)/a] + a*d*AppellF1[5/2, 3/2, 1, 7/2, (d*(a + b*x^2))/(-(b*c) +
```

```
a*d), 1 + (b*x^2)/a])))
```

Maple [B] time = 0.036, size = 177, normalized size = 1.9

$$\frac{1}{2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)b\sqrt{ac}-a\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*(
a*c)^(1/2)-a*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.356338, size = 1, normalized size = 0.01

$$\left[\frac{1}{4}\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2\right.\right.$$
$$\left.+4\left(2\,bd^2x^2+bcd+ad^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{a}{c}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(2\,ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right),\ \frac{1}{2}$$
$$+\frac{1}{4}\sqrt{\frac{a}{c}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(2\,ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right),$$
$$-\frac{1}{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{(bc+ad)x^2+2\,ac}{2\,\sqrt{bx^2+a}\sqrt{dx^2+c}\,c\sqrt{-\frac{a}{c}}}\right)+\frac{1}{4}\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd\right.$$
$$\left.+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bd^2x^2+bcd+ad^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right),$$
$$-\frac{1}{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{(bc+ad)x^2+2\,ac}{2\,\sqrt{bx^2+a}\sqrt{dx^2+c}\,c\sqrt{-\frac{a}{c}}}\right)+\frac{1}{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{2\,bdx^2+bc+ad}{2\,\sqrt{bx^2+a}\sqrt{dx^2+c}\,d\sqrt{-\frac{b}{d}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqr
```

```
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 1/4*sqrt(a/c)*log(((b^2
*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*
d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(a/c))/x^4), 1/2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*sqrt(-b/d))) + 1/4
*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 +
 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), -1/2*sqrt(-a/c)*ar
ctan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*c*sqrt(-a/c))) + 1/4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6
*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 +
 b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)), -1/2*
sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)*c*sqrt(-a/c))) + 1/2*sqrt(-b/d)*arctan(1/2*(2*b*d
*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*sqrt(-b/d)))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{x\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.249875, size = 208, normalized size = 2.26

$$-\frac{b^2\left(\frac{2\sqrt{bd}\,a\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,b}+\frac{\sqrt{bd}\ln\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{bd}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -1/2*b^2*(2*sqrt(b*d)*a*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(
-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b) + sqrt(b*d)*ln((sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(b*d))/abs(
b)
```

3.938 $\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - ((b*c - a*d)*ArcTa
nh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[
a]*c^(3/2))
```

Rubi [A] time = 0.251465, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - ((b*c - a*d)*ArcTa
nh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[
a]*c^(3/2))
```

Rubi in Sympy [A] time = 20.945, size = 76, normalized size = 0.85

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}+\frac{(ad-bc)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2)*sqrt(c + d*x**2)/(2*c*x**2) + (a*d - b*c)*atanh
(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2)))/(2*sqrt(a)*
c**(3/2))
```

Mathematica [C] time = 0.420173, size = 188, normalized size = 2.11

$$\frac{\frac{2bdx^4(bc-ad)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}-\left(a+bx^2\right)\left(c+dx^2\right)}{2cx^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a + b*x^2)*(c + d*x^2)) + (2*b*d*(b*c - a*d)*x^4*AppellF1[1,
1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))])/(-4*b*d*x^2*AppellF1[1,
 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))] + b*c*AppellF1[2, 1/2,
3/2, 3, -(a/(b*x^2)), -(c/(d*x^2))] + a*d*AppellF1[2, 3/2, 1/2, 3
, -(a/(b*x^2)), -(c/(d*x^2))]))/(2*c*x^2*Sqrt[a + b*x^2]*Sqrt[c +
```

```
d*x^2])
```

Maple [B] time = 0.043, size = 207, normalized size = 2.3

$$\frac{1}{4\,cx^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+2\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2\,ac\right)\right)x^2ad-\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(ln((a*d*x^2+c*x^2*b+2*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a*d-l
n((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)+2*a*c)/x^2)*x^2*b*c-2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^2/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278143, size = 1, normalized size = 0.01

$$\left[-\frac{(bc-ad)x^2\log\left(\frac{4\left(2\,a^2c^2+\left(abc^2+a^2cd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}+\left(\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2\right)\sqrt{ac}}{x^4}\right)+4\sqrt{bx^2+a}\sqrt{dx^2+c}}{8\sqrt{ac}cx^2}\right.$$

$$\left.-\frac{(bc-ad)x^2\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{-ac}}{2\sqrt{bx^2+a}\sqrt{dx^2+cac}}\right)+2\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-ac}}{4\sqrt{-ac}cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*x^2*log((4*(2*a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + ((b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(a*c))/x^4)
 + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/(sqrt(a*c)*c*x^2)
, -1/4*((b*c - a*d)*x^2*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt
(-a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)) + 2*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c)*sqrt(-a*c))/(sqrt(-a*c)*c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**3*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.939 $\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=143

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

```
[Out] ((b*c + 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a*c^2*x^2) - (
(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*a*c*x^4) + ((b*c - a*d)*(b*
c + 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])])/(8*a^(3/2)*c^(5/2))
```

Rubi [A] time = 0.376479, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*c + 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a*c^2*x^2) - (
(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*a*c*x^4) + ((b*c - a*d)*(b*
c + 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])])/(8*a^(3/2)*c^(5/2))
```

Rubi in Sympy [A] time = 29.4279, size = 126, normalized size = 0.88

$$-\frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{4acx^4}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3ad+bc)}{8ac^2x^2}-\frac{(ad-bc)\,(3ad+bc)\,\mathrm{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{\frac{3}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(4*a*c*x**4) + sqrt(a + b*x
**2)*sqrt(c + d*x**2)*(3*a*d + b*c)/(8*a*c**2*x**2) - (a*d - b*c)
*(3*a*d + b*c)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d
*x**2)))/(8*a**(3/2)*c**(5/2))
```

Mathematica [C] time = 0.387784, size = 224, normalized size = 1.57

$$\frac{\frac{2bdx^6\left(3a^2d^2-2abcd-b^2c^2\right)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}+\left(a+bx^2\right)\left(c+dx^2\right)\left(-2ac+3adx^2-bcx^2\right)}{8ac^2x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a + b*x^2)*(c + d*x^2)*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (2*b*d*
(-(b^2*c^2) - 2*a*b*c*d + 3*a^2*d^2)*x^6*AppellF1[1, 1/2, 1/2, 2,
 -(a/(b*x^2)), -(c/(d*x^2))])/(-4*b*d*x^2*AppellF1[1, 1/2, 1/2, 2
```

```
, -(a/(b*x^2)), -(c/(d*x^2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -(a/
(b*x^2)), -(c/(d*x^2))] + a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*x^2
)), -(c/(d*x^2))]))/(8*a*c^2*x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.04, size = 355, normalized size = 2.5

$$-\frac{1}{16\,c^2ax^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\ln\left(\frac{adx^2+cx^2b+2\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2\,ac}{x^2}\right)x^4a^2d^2-2\ln\left(\frac{adx^2+cx^2b+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2*(3*ln((a*d*x^2+c*x^2*
b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x
^4*a^2*d^2-2*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a*b*c*d-ln((a*d*x^2+c*x^2*b+2*(
a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*b^
2*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)+2
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c*a*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/x^4/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.334684, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)x^4\log\left(-\frac{4\left(2\,a^2c^2+\left(abc^2+a^2cd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}-\left(\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2\right)\sqrt{ac}}{x^4}\right)+4\left(\right.}{32\,\sqrt{ac}ac^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*log(-(4*(2*a^2*c^2
+ (a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) - ((b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c
*d)*x^2)*sqrt(a*c))/x^4) + 4*((b*c - 3*a*d)*x^2 + 2*a*c)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/(sqrt(a*c)*a*c^2*x^4), 1/16*((
b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)) - 2*((
b*c - 3*a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a
*c))/(sqrt(-a*c)*a*c^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**5*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.940 $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=343

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{15b^2d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{15bd^2}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*d
^2*Sqrt[c + d*x^2]) - ((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*b*d^2) + (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(
5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^
(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.855569, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)}{15b^2d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{15bd^2}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```


```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*d
^2*Sqrt[c + d*x^2]) - ((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*b*d^2) + (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(
5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^
(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 99.3325, size = 308, normalized size = 0.9

$$-\frac{a^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-4bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15b^{\frac{3}{2}}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{\sqrt{a}\sqrt{c+dx^2}\left(2a^2d^2+3abcd-8b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15b^{\frac{3}{2}}d^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-4bc)}{15bd^2}-\frac{x\sqrt{c+dx^2}\left(2a^2d^2+3abcd-8b^2c^2\right)}{15bd^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**(3/2)*sqrt(c + d*x**2)*(a*d - 4*b*c)*elliptic_f(atan(sqrt(b)*
x/sqrt(a)), -a*d/(b*c) + 1)/(15*b**(3/2)*d**2*sqrt(a*(c + d*x**2)
/(c*(a + b*x**2)))*sqrt(a + b*x**2)) + sqrt(a)*sqrt(c + d*x**2)*(
2*a**2*d**2 + 3*a*b*c*d - 8*b**2*c**2)*elliptic_e(atan(sqrt(b)*x/
sqrt(a)), -a*d/(b*c) + 1)/(15*b**(3/2)*d**3*sqrt(a*(c + d*x**2)/(
c*(a + b*x**2)))*sqrt(a + b*x**2)) + x**3*sqrt(a + b*x**2)*sqrt(c
 + d*x**2)/(5*d) + x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d - 4*b
*c)/(15*b*d**2) - x*sqrt(c + d*x**2)*(2*a**2*d**2 + 3*a*b*c*d - 8
*b**2*c**2)/(15*b*d**3*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.759512, size = 246, normalized size = 0.72

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+7abcd-8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2+3abcd-8b^2c^2\right)E\left(\right.}{15bd^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + a*d + 3*b*d*x^2)
 + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(15*b*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.029, size = 526, normalized size = 1.5

$$\frac{1}{15\,d^3\left(bdx^4+adx^2+cx^2b+ac\right)b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+4\sqrt{-\frac{b}{a}}x^5abd^3-\sqrt{-\frac{b}{a}}x^5b^2cd^2+\sqrt{-\frac{b}{a}}x^3a^2d^3-4\sqrt{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+
4*(-b/a)^(1/2)*x^5*a*b*d^3-(-b/a)^(1/2)*x^5*b^2*c*d^2+(-b/a)^(1/2
)*x^3*a^2*d^3-4*(-b/a)^(1/2)*x^3*b^2*c^2*d+((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c
*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-2*((
```

```
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*b^2*c^3+(-b/a)^(1/2)*x*a^2*c*d^2-4*(-b/a)^(1/2)*x*a*b*c^2
*d)/d^3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/b/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

3.941 $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=259

$$-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{x\sqrt{a+bx^2}(2bc-ad)}{3bd\sqrt{c+dx^2}}$$

```
[Out] -((2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b*d*Sqrt[c + d*x^2]) + (x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.495099, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{x\sqrt{a+bx^2}(2bc-ad)}{3bd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -((2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b*d*Sqrt[c + d*x^2]) + (x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 63.0175, size = 219, normalized size = 0.85

$$-\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3d^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\sqrt{c}\sqrt{a+bx^2}\,(ad-2bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3bd^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\,(ad-2bc)}{3bd\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -c**(3/2)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1
- b*c/(a*d))/(3*d**(3/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sq
rt(c + d*x**2)) + x*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(3*d) - sqr
t(c)*sqrt(a + b*x**2)*(a*d - 2*b*c)*elliptic_e(atan(sqrt(d)*x/sqr
t(c)), 1 - b*c/(a*d))/(3*b*d**(3/2)*sqrt(c*(a + b*x**2)/(a*(c + d
*x**2)))*sqrt(c + d*x**2)) + x*sqrt(a + b*x**2)*(a*d - 2*b*c)/(3*
```

```
b*d*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.439861, size = 199, normalized size = 0.77

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-2bc)E\left(i\right.}{3d^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(-2*b*c + a*d)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.025, size = 335, normalized size = 1.3

$$-\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac\right)d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-\sqrt{-\frac{b}{a}}x^5bd^2-\sqrt{-\frac{b}{a}}x^3ad^2-\sqrt{-\frac{b}{a}}x^3bcd+2\,ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*x^5*b*d^2-(-b
/a)^(1/2)*x^3*a*d^2-(-b/a)^(1/2)*x^3*b*c*d+2*a*c*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*b*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+2*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*b*c^2-(-b/a)^(1/2)*x*a*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^
2/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+ax^2}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}\,x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

3.942 $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=232

$$\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt
[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.433286, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt
[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 57.4794, size = 197, normalized size = 0.85

$$\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] d*x*sqrt(a + b*x**2)/(c*sqrt(c + d*x**2)) - sqrt(a + b*x**2)*sqrt
(c + d*x**2)/(c*x) - sqrt(d)*sqrt(a + b*x**2)*elliptic_e(atan(sqr
t(d)*x/sqrt(c)), 1 - b*c/(a*d))/(sqrt(c)*sqrt(c*(a + b*x**2)/(a*(
c + d*x**2)))*sqrt(c + d*x**2)) + b*sqrt(c)*sqrt(a + b*x**2)*elli
ptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(a*sqrt(d)*sqrt(c*
```

```
(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.477754, size = 111, normalized size = 0.48

$$\frac{\frac{bcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}-\left(a+bx^2\right)\left(c+dx^2\right)}{cx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a + b*x^2)*(c + d*x^2)) + (b*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[
-(b/a)])/(c*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.038, size = 168, normalized size = 0.7

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,cx}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-\sqrt{-\frac{b}{a}}x^4bd+bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}xEllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-\sqrt{-\frac{b}{a}}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*x^4*b*d+b*c*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))-(-b/a)^(1/2)*x^2*a*d-(-b/a)^(1/2)*x^2*b*c-(-b/a)^(1/
2)*a*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c/x/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{x^2\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**2*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

3.943 $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=307

$$-\frac{\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3ac^2x}+\frac{dx\sqrt{a+bx^2}(bc-2ad)}{3ac^2\sqrt{c+dx^2}}-\frac{b\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*a*c^2*Sqrt[c + d*x^2]) - (
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.771586, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{3ac^2x}+\frac{dx\sqrt{a+bx^2}(bc-2ad)}{3ac^2\sqrt{c+dx^2}}-\frac{b\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*a*c^2*Sqrt[c + d*x^2]) - (
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 98.6877, size = 269, normalized size = 0.88

$$-\frac{\sqrt{a}\sqrt{b}d\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3c^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{bx\sqrt{c+dx^2}\,(2ad-bc)}{3ac^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(2ad-bc)}{3ac^2x}+\frac{\sqrt{b}\sqrt{c+dx^2}\,(2ad-bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3\sqrt{a}c^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*d*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqr
t(a)), -a*d/(b*c) + 1)/(3*c**2*sqrt(a*(c + d*x**2)/(c*(a + b*x**2
```

```
)))*sqrt(a + b*x**2)) - sqrt(a + b*x**2)*sqrt(c + d*x**2)/(3*c*x*
*3) - b*x*sqrt(c + d*x**2)*(2*a*d - b*c)/(3*a*c**2*sqrt(a + b*x**
2)) + sqrt(a + b*x**2)*sqrt(c + d*x**2)*(2*a*d - b*c)/(3*a*c**2*x
) + sqrt(b)*sqrt(c + d*x**2)*(2*a*d - b*c)*elliptic_e(atan(sqrt(b
)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*sqrt(a)*c**2*sqrt(a*(c + d*x**2)
/(c*(a + b*x**2)))*sqrt(a + b*x**2))
```

Mathematica [C] time = 1.88978, size = 228, normalized size = 0.74

$$\frac{-\frac{(a+bx^2)(c+dx^2)(ac-2adx^2+bcx^2)}{a} + icx^3\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(bc-ad)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) + icx^3\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}}{3c^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((a + b*x^2)*(c + d*x^2)*(a*c + b*c*x^2 - 2*a*d*x^2))/a) + I*S
qrt[b/a]*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*Sqrt[b/
a]*c*(b*c - a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*c^2*x^3*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.038, size = 418, normalized size = 1.4

$$\frac{1}{(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac)\,c^2x^3a}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\sqrt{-\frac{b}{a}}x^6abd^2-\sqrt{-\frac{b}{a}}x^6b^2cd+bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathit{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*x^6*a*b*d^2-(
-b/a)^(1/2)*x^6*b^2*c*d+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c-((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*x^3*b^2*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b*c*d+((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
x^3*b^2*c^2+2*(-b/a)^(1/2)*x^4*a^2*d^2-(-b/a)^(1/2)*x^4*b^2*c^2+(
-b/a)^(1/2)*x^2*a^2*c*d-2*(-b/a)^(1/2)*x^2*a*b*c^2-(-b/a)^(1/2)*a
^2*c^2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/x^3/a/(-b/a)^(1/2)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**4*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

3.944 $\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=276

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)}{128b^2d^4}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(3a^2d^2+10abcd+35b^2c^2\right)}{192b^2d^3}+\frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{5/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{48b^2d^2}+\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd}$$

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(128*b^2*d^4) + ((35*b^2*c^2 + 10*a*b*c*d + 3
*a^2*d^2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(192*b^2*d^3) - ((7*
b*c + 3*a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(48*b^2*d^2) + (x
^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(8*b*d) + ((b*c - a*d)^2*(3
5*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x
^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(5/2)*d^(9/2))
```

Rubi [A] time = 0.763458, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)}{128b^2d^4}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(3a^2d^2+10abcd+35b^2c^2\right)}{192b^2d^3}+\frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{5/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{48b^2d^2}+\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(128*b^2*d^4) + ((35*b^2*c^2 + 10*a*b*c*d + 3
*a^2*d^2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(192*b^2*d^3) - ((7*
b*c + 3*a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(48*b^2*d^2) + (x
^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(8*b*d) + ((b*c - a*d)^2*(3
5*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x
^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(5/2)*d^(9/2))
```

Rubi in Sympy [A] time = 61.6349, size = 260, normalized size = 0.94

$$\frac{x^2\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}{8bd}-\frac{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}\,(3ad+7bc)}{48b^2d^2}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(3a^2d^2+10abcd+35b^2c^2\right)}{192b^2d^3}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)\left(3a^2d^2+10abcd+35b^2c^2\right)}{128b^2d^4}+\frac{(ad-bc)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{\frac{5}{2}}d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x**2*(a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(8*b*d) - (a + b*x**2)*
*(5/2)*sqrt(c + d*x**2)*(3*a*d + 7*b*c)/(48*b**2*d**2) + (a + b*x
**2)**(3/2)*sqrt(c + d*x**2)*(3*a**2*d**2 + 10*a*b*c*d + 35*b**2*
c**2)/(192*b**2*d**3) + sqrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d -
b*c)*(3*a**2*d**2 + 10*a*b*c*d + 35*b**2*c**2)/(128*b**2*d**4) +
(a*d - b*c)**2*(3*a**2*d**2 + 10*a*b*c*d + 35*b**2*c**2)*atanh(sq
rt(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(c + d*x**2)))/(128*b**(5/2)*
d**(9/2))
```

Mathematica [A] time = 0.261225, size = 224, normalized size = 0.81

$$\frac{3(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)-2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(9a^3\right.}{768b^{5/2}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(9*a^3*d^3 +
3*a^2*b*d^2*(5*c - 2*d*x^2) + a*b^2*d*(-145*c^2 + 92*c*d*x^2 - 72
*d^2*x^4) + b^3*(105*c^3 - 70*c^2*d*x^2 + 56*c*d^2*x^4 - 48*d^3*x
^6)) + 3*(b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Log[
b*c + a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2]])/(768*b^(5/2)*d^(9/2))
```

Maple [B] time = 0.046, size = 770, normalized size = 2.8

$$\frac{1}{768\,b^2d^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(96\,x^6b^3d^3\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+144\,x^4ab^2d^3\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}-112\,x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(96*x^6*b^3*d^3*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+144*x^4*a*b^2*d^3*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-112*x^4*b^3*c*d^2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+12*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2)-184*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*x^2*a*c*b^2*d^2*(b*d)^(1/2)+140*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^2*c^2*b^3*d*(b*d)^(1/2)+9*ln(1/2*(2*b*d*x^2+2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1
/2))*a^4*d^4+12*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*c*b*d^3+54*ln(1/2*(2
*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*
c)/(b*d)^(1/2))*a^2*c^2*b^2*d^2-180*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3*
a*b^3*d+105*b^4*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4-18*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*a^3*d^3*(b*d)^(1/2)-30*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*a^2*c*b*d^2*(b*d)^(1/2)+290*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*a*c^2*b^2*d*(b*d)^(1/2)-210*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*c^3*b^3*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/b^2/d^4/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^5/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300149, size = 1, normalized size = 0.

$$\left[\frac{4\left(48\,b^3d^3x^6 - 105\,b^3c^3 + 145\,ab^2c^2d - 15\,a^2bcd^2 - 9\,a^3d^3 - 8\left(7\,b^3cd^2 - 9\,ab^2d^3\right)x^4 + 2\left(35\,b^3c^2d - 46\,ab^2cd^2 + 3\,a^2bd^3\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^5/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/1536*(4*(48*b^3*d^3*x^6 - 105*b^3*c^3 + 145*a*b^2*c^2*d - 15*a
^2*b*c*d^2 - 9*a^3*d^3 - 8*(7*b^3*c*d^2 - 9*a*b^2*d^3)*x^4 + 2*(3
5*b^3*c^2*d - 46*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^2)*sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)*sqrt(b*d) + 3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a
^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*log(4*(2*b^2*d^2*x^2
+ b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*d^2
*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2)
*sqrt(b*d)))/(sqrt(b*d)*b^2*d^4), 1/768*(2*(48*b^3*d^3*x^6 - 105*
b^3*c^3 + 145*a*b^2*c^2*d - 15*a^2*b*c*d^2 - 9*a^3*d^3 - 8*(7*b^3
*c*d^2 - 9*a*b^2*d^3)*x^4 + 2*(35*b^3*c^2*d - 46*a*b^2*c*d^2 + 3*
a^2*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) + 3*(3
5*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 +
 3*a^4*d^4)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/(sqrt(-b*d)*b^2*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(a + bx^2\right)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.25367, size = 410, normalized size = 1.49

$$\frac{\sqrt{b^2c + (bx^2 + a)bd - abd}\sqrt{bx^2 + a}\left(2\left(bx^2 + a\right)\left(4\left(bx^2 + a\right)\left(\frac{6\left(bx^2+a\right)}{bd} - \frac{7\,b^3cd^5+9\,ab^2d^6}{b^3d^7}\right) + \frac{35\,b^4c^2d^4+10\,ab^3cd^5+3\,a^2b^2d^6}{b^3d^7}\right) - \frac{3\,(35}{}\right.}{384\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^5/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(
b*x^2 + a)*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b*d) - (7*b^3*c*d^5 + 9
*a*b^2*d^6)/(b^3*d^7)) + (35*b^4*c^2*d^4 + 10*a*b^3*c*d^5 + 3*a^2
*b^2*d^6)/(b^3*d^7)) - 3*(35*b^5*c^3*d^3 - 25*a*b^4*c^2*d^4 - 7*a
^2*b^3*c*d^5 - 3*a^3*b^2*d^6)/(b^3*d^7)) - 3*(35*b^4*c^4 - 60*a*b
```

```
^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*ln(abs
(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*d^4))/(b*abs(b))
```

3.945 $\int \frac{x^3 (a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)(ad+5bc)}{16bd^3} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(ad+5bc)}{24bd^2}+\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6bd}$$

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b
*d^3) - ((5*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b*d
^2) + ((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)^
2*(5*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
 + d*x^2])])/(16*b^(3/2)*d^(7/2))
```

Rubi [A] time = 0.429235, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)(ad+5bc)}{16bd^3} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(ad+5bc)}{24bd^2}+\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b
*d^3) - ((5*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b*d
^2) + ((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)^
2*(5*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
 + d*x^2])])/(16*b^(3/2)*d^(7/2))
```

Rubi in Sympy [A] time = 39.467, size = 163, normalized size = 0.87

$$\frac{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}{6bd}-\frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad+5bc)}{24bd^2} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)\,(ad+5bc)}{16bd^3}-\frac{(ad-bc)^2\,(ad+5bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{\frac{3}{2}}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] (a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(6*b*d) - (a + b*x**2)**(3/2
)*sqrt(c + d*x**2)*(a*d + 5*b*c)/(24*b*d**2) - sqrt(a + b*x**2)*s
qrt(c + d*x**2)*(a*d - b*c)*(a*d + 5*b*c)/(16*b*d**3) - (a*d - b*
c)**2*(a*d + 5*b*c)*atanh(sqrt(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(
c + d*x**2)))/(16*b**(3/2)*d**(7/2))
```

Mathematica [A] time = 0.153012, size = 163, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(3a^2d^2+2abd\left(7dx^2-11c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)}{48bd^3} - \frac{(bc-ad)^2(ad+5bc)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{32b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*
d*x^2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*b*d^3) - ((b
*c - a*d)^2*(5*b*c + a*d)*Log[b*c + a*d + 2*b*d*x^2 + 2*Sqrt[b]*S
qrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(32*b^(3/2)*d^(7/2))
```

Maple [B] time = 0.024, size = 532, normalized size = 2.8

$$-\frac{1}{96\,d^3b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-16\,x^4b^2d^2\sqrt{bd}\sqrt{bdx^4+adx^2+cx^2b+ac}-28\,\sqrt{bdx^4+adx^2+cx^2b+ac}x^2abd^2\sqrt{bd}+20\,\sqrt{bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-16*x^4*b^2*d^2*(b*d)^(1/2
)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-28*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)+20*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*
d^3+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2-27*ln(1/2*(2*b*d*x^2+2
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*c^2*a*b^2*d+15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*a^2*d^2*(b*d)^(1/2)+44*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)-30*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/d^3/b/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x^3/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273407, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,b^2d^2x^4+15\,b^2c^2-22\,abcd+3\,a^2d^2-2\left(5\,b^2cd-7\,abd^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}+3\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2\right.}{192\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x^3/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/192*(4*(8*b^2*d^2*x^4 + 15*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2 -
2*(5*b^2*c*d - 7*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sq
rt(b*d) + 3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)
*log(-4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(
```

```
d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(
b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d)))/(sqrt(b*d)*b*d^3), 1/96*(2*(8
*b^2*d^2*x^4 + 15*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2 - 2*(5*b^2*c*d
 - 7*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) - 3
*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c)*b*d)))/(sqrt(-b*d)*b*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.260105, size = 293, normalized size = 1.57

$$\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{bd}-\frac{5\,b^2cd^3+abd^4}{b^2d^5}\right)+\frac{3\left(5\,b^3c^2d^2-4\,ab^2cd^3-a^2bd^4\right)}{b^2d^5}\right)+\frac{3\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2\right.}{}}{48\left|b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x^3/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b
*x^2 + a)*(4*(b*x^2 + a)/(b*d) - (5*b^2*c*d^3 + a*b*d^4)/(b^2*d^5
)) + 3*(5*b^3*c^2*d^2 - 4*a*b^2*c*d^3 - a^2*b*d^4)/(b^2*d^5)) + 3
*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*ln(abs(-sq
rt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))
/(sqrt(b*d)*d^3))/abs(b)
```

3.946 $\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=125

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8\sqrt{b}d^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8d^2}+\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d}$$

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + ((a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*d) + (3*(b*c - a*d)^2*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*Sqrt[b]*d^
(5/2))
```

Rubi [A] time = 0.24393, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8\sqrt{b}d^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8d^2}+\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + ((a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*d) + (3*(b*c - a*d)^2*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*Sqrt[b]*d^
(5/2))
```

Rubi in Sympy [A] time = 25.9904, size = 110, normalized size = 0.88

$$\frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{4d}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)}{8d^2}+\frac{3\,(ad-bc)^2\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] (a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(4*d) + 3*sqrt(a + b*x**2)*s
qrt(c + d*x**2)*(a*d - b*c)/(8*d**2) + 3*(a*d - b*c)**2*atanh(sqr
t(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(8*sqrt(b)*d**(
5/2))
```

Mathematica [A] time = 0.0925797, size = 119, normalized size = 0.95

$$\frac{3(bc-ad)^2\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{16\sqrt{b}d^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(5ad-3bc+2bdx^2)}{8d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 5*a*d + 2*b*d*x^2))/(8
*d^2) + (3*(b*c - a*d)^2*Log[b*c + a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sq
rt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(16*Sqrt[b]*d^(5/2))
```

Maple [B] time = 0.018, size = 337, normalized size = 2.7

$$\frac{1}{16\,d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(4\sqrt{bdx^4+adx^2+cx^2b+acx^2}db\sqrt{bd}+3\ln\left(1/2\,\frac{2\,bdx^2+2\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*x^2*d*b*(b*d)^(1/2)+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*d^2
-6*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a*d*b+3*b^2*ln(1/2*(2*b*d*x^2+2*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*c^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d*(b*d)^(1/2)-6*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c*b*(b*d)^(1/2))/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)/d^2/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2834, size = 1, normalized size = 0.01

$$\left[\frac{4\left(2\,bdx^2-3\,bc+5\,ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}+3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(4\left(2\,b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2+}\right.}{32\sqrt{bd}d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/32*(4*(2*b*d*x^2 - 3*b*c + 5*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c)*sqrt(b*d) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(4*(2*b^2*d^
2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2
)*x^2)*sqrt(b*d)))/(sqrt(b*d)*d^2), 1/16*(2*(2*b*d*x^2 - 3*b*c +
5*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) + 3*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d
)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/(sqrt(-b*d)*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.247332, size = 201, normalized size = 1.61

$$\frac{\left(\sqrt{b^2c + (bx^2 + a)bd - abd}\sqrt{bx^2 + a}\left(\frac{2\,(bx^2+a)}{bd} - \frac{3\,(bcd-ad^2)}{bd^3}\right) - \frac{3\,(b^2c^2-2\,abcd+a^2d^2)\ln\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}d^2}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*
x^2 + a)/(b*d) - 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2))*b/abs(b)
```

3.947 $\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=133

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}$$

```
[Out] (b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - (a^(3/2)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (Sqrt
[b]*(b*c - 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt
[c + d*x^2])])/(2*d^(3/2))
```

Rubi [A] time = 0.439869, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - (a^(3/2)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (Sqrt
[b]*(b*c - 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt
[c + d*x^2])])/(2*d^(3/2))
```

Rubi in Sympy [A] time = 42.3388, size = 119, normalized size = 0.89

$$-\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\,(3ad-bc)\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2d^{\frac{3}{2}}}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**(3/2)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2
)))/sqrt(c) + sqrt(b)*(3*a*d - b*c)*atanh(sqrt(b)*sqrt(c + d*x**2
)/(sqrt(d)*sqrt(a + b*x**2)))/(2*d**(3/2)) + b*sqrt(a + b*x**2)*s
qrt(c + d*x**2)/(2*d)
```

Mathematica [C] time = 0.998953, size = 400, normalized size = 3.01

$$\frac{b\left(\frac{4a^2dx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}+\frac{x^2(a+bx^2)(c+dx^2)\left(adF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(2;\right.\right.}{d\left(x^2\left(adF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+b\right.\right.}\right.}{2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*((4*a^2*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^
2))])/(-4*b*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x
```

```
^2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -(a/(b*x^2)), -(c/(d*x^2))]
+ a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*x^2)), -(c/(d*x^2))]) + (-2
*a*c*(2*a*c + b*c*x^2 + 5*a*d*x^2 + 2*b*d*x^4)*AppellF1[1, 1/2, 1
/2, 2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(a + b*x^2)*(c + d*x^2)*
(a*d*AppellF1[2, 1/2, 3/2, 3, -((b*x^2)/a), -((d*x^2)/c)] + b*c*A
ppellF1[2, 3/2, 1/2, 3, -((b*x^2)/a), -((d*x^2)/c)]))/(d*(-4*a*c*
AppellF1[1, 1/2, 1/2, 2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(a*d*A
ppellF1[2, 1/2, 3/2, 3, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF
1[2, 3/2, 1/2, 3, -((b*x^2)/a), -((d*x^2)/c)]))))))/(2*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.021, size = 287, normalized size = 2.2

$$\frac{1}{4d}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\,a\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)b\sqrt{acd}-cb^2\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bd}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*a*ln(1/2*(2*b*d*x^2+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*b*(a*c)^(1/2)*d-c*b^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)-2*a^2
*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)+2*a*c)/x^2)*(b*d)^(1/2)*d+2*b*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/(b*d)^(1/2)/(a*c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.888444, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c
*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (b*c -
 3*a*d)*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*b)/d, 1/4*(a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c
^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/
c))/x^4) - (b*c - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c +
 a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*sqrt(-b/d))) + 2*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*b)/d, -1/8*(4*a*d*sqrt(-a/c)*arctan(1/2*
((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*sqrt
```

```
(-a/c))) + (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 +
6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2
+ b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) - 4*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, -1/4*(2*a*d*sqrt(-a/c)*arcta
n(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*
c*sqrt(-a/c))) + (b*c - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*sqrt(-b/d))) - 2*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{x\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.250013, size = 284, normalized size = 2.14

$$-\frac{\frac{4\sqrt{bd}a^2\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}b}-\frac{2\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}}{bd}-\frac{\left(\sqrt{bd}bc-3\sqrt{bd}ad\right)\ln\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(b\cdots}{bd^2}}{4\left|b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -1/4*(4*sqrt(b*d)*a^2*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a
*b*c*d)*b))/(sqrt(-a*b*c*d)*b) - 2*sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d)*sqrt(b*x^2 + a)/(b*d) - (sqrt(b*d)*b*c - 3*sqrt(b*d)*a*d)
*ln((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2)/(b*d^2))*b^2/abs(b)
```

3.948 $\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

```
[Out] -(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - (Sqrt[a]*(3*b*c
- a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(2*c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqr
t[b]*Sqrt[c + d*x^2])])/Sqrt[d]
```

Rubi [A] time = 0.440704, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - (Sqrt[a]*(3*b*c
- a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(2*c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqr
t[b]*Sqrt[c + d*x^2])])/Sqrt[d]
```

Rubi in Sympy [A] time = 42.9618, size = 122, normalized size = 0.9

$$\frac{\sqrt{a}\,(ad-3bc)\,\mathrm{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{\frac{3}{2}}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}+\frac{b^{\frac{3}{2}}\,\mathrm{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(a)*(a*d - 3*b*c)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqr
t(c + d*x**2)))/(2*c**(3/2)) - a*sqrt(a + b*x**2)*sqrt(c + d*x**2
)/(2*c*x**2) + b**(3/2)*atanh(sqrt(d)*sqrt(a + b*x**2)/(sqrt(b)*s
qrt(c + d*x**2)))/sqrt(d)
```

Mathematica [C] time = 0.512971, size = 327, normalized size = 2.4

$$\frac{a\left(-\frac{4b^2c^2x^4F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(adF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-4acF_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}+\frac{2bdx^4(3bc-ad)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\dots\right.}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},\dots\right.}\right.}{2cx^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*(-((a + b*x^2)*(c + d*x^2)) + (2*b*d*(3*b*c - a*d)*x^4*AppellF
1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))])/(-4*b*d*x^2*Appell
```

```
F1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))] + b*c*AppellF1[2,
1/2, 3/2, 3, -(a/(b*x^2)), -(c/(d*x^2))] + a*d*AppellF1[2, 3/2, 1
/2, 3, -(a/(b*x^2)), -(c/(d*x^2))]) - (4*b^2*c^2*x^4*AppellF1[1,
1/2, 1/2, 2, -((b*x^2)/a), -((d*x^2)/c)])/(-4*a*c*AppellF1[1, 1/2
, 1/2, 2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(a*d*AppellF1[2, 1/2,
 3/2, 3, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[2, 3/2, 1/2,
3, -((b*x^2)/a), -((d*x^2)/c)]))))/(2*c*x^2*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])
```

Maple [B] time = 0.023, size = 298, normalized size = 2.2

$$\frac{1}{4\,cx^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\,\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^2b^2c\sqrt{ac}+\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(2*ln(1/2*(2*b*d*x^2+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*x^2*b^2*c*(a*c)^(1/2)+ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a^2*d*(b*d)^(1/2)-3*l
n((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)+2*a*c)/x^2)*x^2*a*b*c*(b*d)^(1/2)-2*a*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/x^2/(a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.745405, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d
 + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d +
 a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (3*b*c - a*d
)*x^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*
c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), 1/8*(4*b*c*x^2*sqrt(-b/d)*arcta
n(1/2*(2*b*d*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*
sqrt(-b/d))) - (3*b*c - a*d)*x^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*
c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*(2
*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqr
t(a/c))/x^4) - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), 1/4*
(b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (3*b*c - a*d)*x^2*s
```

```
qrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*c*sqrt(-a/c))) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*a)/(c*x^2), 1/4*(2*b*c*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*
c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*sqrt(-b/d))) - (3*b*c
 - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*c*sqrt(-a/c))) - 2*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*a)/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{x^3\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**3*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.602651, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.949 $\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8c^2x^2}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - ((
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (3*(b*c - a*d)^2*Ar
cTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*Sq
rt[a]*c^(5/2))
```

Rubi [A] time = 0.349063, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}{8c^2x^2}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - ((
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (3*(b*c - a*d)^2*Ar
cTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*Sq
rt[a]*c^(5/2))
```

Rubi in Sympy [A] time = 29.0505, size = 117, normalized size = 0.89

$$-\frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{4cx^4}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)}{8c^2x^2}-\frac{3\,(ad-bc)^2\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(4*c*x**4) + 3*sqrt(a + b*x
**2)*sqrt(c + d*x**2)*(a*d - b*c)/(8*c**2*x**2) - 3*(a*d - b*c)**
2*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2)))/(8*s
qrt(a)*c**(5/2))
```

Mathematica [C] time = 0.42053, size = 208, normalized size = 1.59

$$\frac{\frac{6bdx^6(bc-ad)^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}+\left(a+bx^2\right)\left(c+dx^2\right)\left(-2ac+3adx^2-5bcx^2\right)}{8c^2x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a + b*x^2)*(c + d*x^2)*(-2*a*c - 5*b*c*x^2 + 3*a*d*x^2) + (6*b*
d*(b*c - a*d)^2*x^6*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d
```

```
*x^2))])/(-4*b*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(
d*x^2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -(a/(b*x^2)), -(c/(d*x^2)
)] + a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*x^2)), -(c/(d*x^2))]))/(
8*c^2*x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.029, size = 352, normalized size = 2.7

$$-\frac{1}{16\,c^2x^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\,\ln\left(\frac{adx^2+cx^2b+2\,\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2\,ac}{x^2}\right)x^4a^2d^2-6\,\ln\left(\frac{adx^2+cx^2b+2\,\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(3*ln((a*d*x^2+c*x^2*b+
2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4
*a^2*d^2-6*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a*b*c*d+3*ln((a*d*x^2+c*x^2*b+2*(
a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*b^
2*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)+1
0*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c*a*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)/x^4/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.339389, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)x^4\log\left(-\frac{4\left(2\,a^2c^2+\left(abc^2+a^2cd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}-\left(\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2\right)\sqrt{ac}}{x^4}\right)-4\left(\left(5\cdots\right.\right.}{32\,\sqrt{ac}c^2x^4}\right.$$

$$\left.-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)x^4\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{-ac}}{2\,\sqrt{bx^2+a}\sqrt{dx^2+c}ac}\right)+2\left((5\,bc-3\,ad)x^2+2\,ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-ac}}{16\,\sqrt{-ac}c^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(-(4*(2*a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) - ((b^2
*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*
d)*x^2)*sqrt(a*c))/x^4) - 4*((5*b*c - 3*a*d)*x^2 + 2*a*c)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/(sqrt(a*c)*c^2*x^4), -1/16*(3
*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)) + 2*((
5*b*c - 3*a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
-a*c))/(sqrt(-a*c)*c^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/x**5/(d*x**2+c)**(1/2),x)

[Out] Integral((a + b*x**2)**(3/2)/(x**5*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^5),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.950 $\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=429

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2-11abcd+8b^2c^2\right)}{35bd^3}-\frac{2x\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)}{35b^2d^3\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{35d^2}+\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

```
[Out] (-2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*x*Sqrt[a + b*
x^2])/(35*b^2*d^3*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 11*a*b*c*d + a
^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b*d^3) - (2*(3*b*c
 - 4*a*d)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d^2) + (b*x^5*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (2*Sqrt[c]*(2*b*c - a*d)
*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(7/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(8*b^
2*c^2 - 11*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b*d^(7/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.40105, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2-11abcd+8b^2c^2\right)}{35bd^3}-\frac{2x\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)}{35b^2d^3\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{35d^2}+\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```


```
[Out] (-2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*x*Sqrt[a + b*
x^2])/(35*b^2*d^3*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 11*a*b*c*d + a
^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b*d^3) - (2*(3*b*c
 - 4*a*d)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d^2) + (b*x^5*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (2*Sqrt[c]*(2*b*c - a*d)
*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(7/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(8*b^
2*c^2 - 11*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b*d^(7/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 162.498, size = 398, normalized size = 0.93

$$\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}+\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(4ad-3bc)}{35d^2}-\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{35bd^{\frac{7}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2-11abcd+8b^2c^2\right)}{35bd^3}+\frac{2\sqrt{c}\sqrt{a+bx^2}\,(ad-2bc)\left(a^2d^2+4abcd-4b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{35b^2d^{\frac{7}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{2x\sqrt{a+bx^2}\,(ad-2bc)\left(a^2d^2+4abcd-4b^2c^2\right)}{35b^2d^3\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x**5*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(7*d) + 2*x**3*sqrt(a +
b*x**2)*sqrt(c + d*x**2)*(4*a*d - 3*b*c)/(35*d**2) - c**(3/2)*sqr
t(a + b*x**2)*(a**2*d**2 - 11*a*b*c*d + 8*b**2*c**2)*elliptic_f(a
tan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(35*b*d**(7/2)*sqrt(c*(a +
 b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)) + x*sqrt(a + b*x**2)
*sqrt(c + d*x**2)*(a**2*d**2 - 11*a*b*c*d + 8*b**2*c**2)/(35*b*d*
*3) + 2*sqrt(c)*sqrt(a + b*x**2)*(a*d - 2*b*c)*(a**2*d**2 + 4*a*b
*c*d - 4*b**2*c**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(
a*d))/(35*b**2*d**(7/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqr
t(c + d*x**2)) - 2*x*sqrt(a + b*x**2)*(a*d - 2*b*c)*(a**2*d**2 +
4*a*b*c*d - 4*b**2*c**2)/(35*b**2*d**3*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.09909, size = 305, normalized size = 0.71

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(a^2d^2+abd\left(8dx^2-11c\right)+b^2\left(8c^2-6cdx^2+5d^2x^4\right)\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^3d^3+15a^2bcd^2\right.}{35bd^4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(-11*c +
8*d*x^2) + b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(8*b^3*
c^3 - 12*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] - I*c*(16*b^3*c^3 - 32*a*b^2*c^2*d + 15*a^2*b*c*d^2 + a^3*d^
3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

Maple [A] time = 0.029, size = 782, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/35*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(5*(-b/a)^(1/2)*x^9*b^3*d^4+
13*(-b/a)^(1/2)*x^7*a*b^2*d^4-(-b/a)^(1/2)*x^7*b^3*c*d^3+9*(-b/a)
^(1/2)*x^5*a^2*b*d^4-4*(-b/a)^(1/2)*x^5*a*b^2*c*d^3+2*(-b/a)^(1/2
)*x^5*b^3*c^2*d^2+(-b/a)^(1/2)*x^3*a^3*d^4-2*(-b/a)^(1/2)*x^3*a^2
*b*c*d^3-9*(-b/a)^(1/2)*x^3*a*b^2*c^2*d^2+8*(-b/a)^(1/2)*x^3*b^3*
c^3*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+15*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^
2-32*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d+16*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-2*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^3*c*d^3-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^2+24*((b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*a*b^2*c^3*d-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4+(-b/a)^(1/2
)*x*a^3*c*d^3-11*(-b/a)^(1/2)*x*a^2*b*c^2*d^2+8*(-b/a)^(1/2)*x*a*
b^2*c^3*d)/b/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^6+ax^4\right)\sqrt{bx^2+a}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^6 + a*x^4)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

3.951 $\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=335

$$-\frac{\sqrt{c}\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(-\frac{3a^2d}{b}+13ac-\frac{8bc^2}{d}\right)}{15d\sqrt{c+dx^2}}+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{15d^2}+\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

```
[Out] -((13*a*c - (8*b*c^2)/d - (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*d*S
qrt[c + d*x^2]) - (2*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*d^2) + (b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5
/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c
^(3/2)*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.929073, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{\sqrt{c}\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(-\frac{3a^2d}{b}+13ac-\frac{8bc^2}{d}\right)}{15d\sqrt{c+dx^2}}+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{15d^2}+\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```


```
[Out] -((13*a*c - (8*b*c^2)/d - (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*d*S
qrt[c + d*x^2]) - (2*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*d^2) + (b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5
/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c
^(3/2)*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 106.902, size = 309, normalized size = 0.92

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} - \frac{2c^{\frac{3}{2}}\sqrt{a+bx^2}\,(3ad-2bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{15d^{\frac{5}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3ad-2bc)}{15d^2} - \frac{\sqrt{c}\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{15bd^{\frac{5}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)}{15bd^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x**3*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(5*d) - 2*c**(3/2)*sqrt(
a + b*x**2)*(3*a*d - 2*b*c)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1
 - b*c/(a*d))/(15*d**(5/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*
sqrt(c + d*x**2)) + 2*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(3*a*d
- 2*b*c)/(15*d**2) - sqrt(c)*sqrt(a + b*x**2)*(3*a**2*d**2 - 13*a
*b*c*d + 8*b**2*c**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c
/(a*d))/(15*b*d**(5/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt
(c + d*x**2)) + x*sqrt(a + b*x**2)*(3*a**2*d**2 - 13*a*b*c*d + 8*
b**2*c**2)/(15*b*d**2*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.763775, size = 245, normalized size = 0.73

$$\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(9a^2d^2-17abcd+8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-13abcd+8b^2c^2\right)}{15d^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 6*a*d + 3*b*d*x^
2) - I*c*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] + I*c*(8*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(15*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.027, size = 544, normalized size = 1.6

$$-\frac{1}{15\,d^3\left(bdx^4+adx^2+cx^2b+ac\right)}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-3\sqrt{-\frac{b}{a}}x^7b^2d^3-9\sqrt{-\frac{b}{a}}x^5abd^3+\sqrt{-\frac{b}{a}}x^5b^2cd^2-6\sqrt{-\frac{b}{a}}x^3a^2d^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*(-b/a)^(1/2)*x^7*b^2*d^
3-9*(-b/a)^(1/2)*x^5*a*b*d^3+(-b/a)^(1/2)*x^5*b^2*c*d^2-6*(-b/a)^
(1/2)*x^3*a^2*d^3-5*(-b/a)^(1/2)*x^3*a*b*c*d^2+4*(-b/a)^(1/2)*x^3
*b^2*c^2*d+9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
```

```
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-17*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c
^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+13*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-6*(-b/a)^(1/
2)*x*a^2*c*d^2+4*(-b/a)^(1/2)*x*a*b*c^2*d)/d^3/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4+ax^2)\sqrt{bx^2+a}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c),x, algorithm="fricas")

[Out] integral((b*x^4 + a*x^2)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(x**2*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

3.952 $\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=244

$$-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{x\sqrt{a+bx^2}(ad+bc)}{c\sqrt{c+dx^2}}+\frac{2b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (a*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(c*x) - ((b*c + a*d)*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2
*b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.484514, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{x\sqrt{a+bx^2}(ad+bc)}{c\sqrt{c+dx^2}}+\frac{2b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (a*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(c*x) - ((b*c + a*d)*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2
*b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 63.9405, size = 211, normalized size = 0.86

$$-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{2b\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\,(ad+bc)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\,(ad+bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(c*x) + 2*b*sqrt(c)*sqrt(a +
 b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(sqrt
(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)) + x*s
```

```
qrt(a + b*x**2)*(a*d + b*c)/(c*sqrt(c + d*x**2)) - sqrt(a + b*x**
2)*(a*d + b*c)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))
/(sqrt(c)*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c +
d*x**2))
```

Mathematica [C] time = 0.451526, size = 206, normalized size = 0.84

$$\frac{-ad\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}{cdx\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)) - I*b*c*(b*c + a*d)*x*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*x*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(Sqrt[b/a]*c*d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.025, size = 352, normalized size = 1.4

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,cxd}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-\sqrt{-\frac{b}{a}}x^4abd^2+\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)xabcd-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*x^4*a*b*d^2+((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*x*a*b*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*b^2*c^2+((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
x*a*b*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*x*b^2*c^2-(-b/a)^(1/2)*x^2*a^2*d^2-(-b
/a)^(1/2)*x^2*a*b*c*d-(-b/a)^(1/2)*a^2*c*d)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)/c/x/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{x^2\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**2*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx^2 + c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.953 $\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=311

$$-\frac{2\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{3c^2x}+\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3c^2\sqrt{c+dx^2}}+\frac{b\sqrt{a+bx^2}(3bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x^2]) - (
a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - (2*(2*b*c - a*d)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c^2*x) - (2*Sqrt[d]*(2*b*c - a
*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (b*(3*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.824706, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{2\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{3c^2x}+\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3c^2\sqrt{c+dx^2}}+\frac{b\sqrt{a+bx^2}(3bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x^2]) - (
a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - (2*(2*b*c - a*d)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c^2*x) - (2*Sqrt[d]*(2*b*c - a
*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (b*(3*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 106.261, size = 279, normalized size = 0.9

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,(ad-3bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3c^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,(ad-2bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3c^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{2bx\sqrt{c+dx^2}\,(ad-2bc)}{3c^2\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-2bc)}{3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*(a*d - 3*b*c)*elliptic_f(atan(s
qrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*c**2*sqrt(a*(c + d*x**2)/(c
*(a + b*x**2)))*sqrt(a + b*x**2)) + 2*sqrt(a)*sqrt(b)*sqrt(c + d*
x**2)*(a*d - 2*b*c)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c
) + 1)/(3*c**2*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x
**2)) - a*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(3*c*x**3) - 2*b*x*sq
rt(c + d*x**2)*(a*d - 2*b*c)/(3*c**2*sqrt(a + b*x**2)) + 2*sqrt(a
 + b*x**2)*sqrt(c + d*x**2)*(a*d - 2*b*c)/(3*c**2*x)
```

Mathematica [C] time = 0.605334, size = 227, normalized size = 0.73

$$\frac{\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(-ac+2adx^2-4bcx^2\right)-ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ibcx^3\sqrt{\frac{bx^2}{a}}}{3c^2x^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 4*b*c*x^2 + 2*a*d*x^
2) + (2*I)*b*c*(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(
-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*x^3*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.027, size = 433, normalized size = 1.4

$$\frac{1}{(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac)\,c^2x^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\sqrt{-\frac{b}{a}}x^6abd^2-4\sqrt{-\frac{b}{a}}x^6b^2cd+bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}Elliptic\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*x^6*a*b*d^2-4
*(-b/a)^(1/2)*x^6*b^2*c*d+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c-((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*x^3*b^2*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b*c*d+4*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*x^3*b^2*c^2+2*(-b/a)^(1/2)*x^4*a^2*d^2-3*(-b/a)^(1/2)*x^4*a*b
*c*d-4*(-b/a)^(1/2)*x^4*b^2*c^2+(-b/a)^(1/2)*x^2*a^2*c*d-5*(-b/a)
^(1/2)*x^2*a*b*c^2-(-b/a)^(1/2)*a^2*c^2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)/c^2/x^3/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**4*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.954 $\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}} dx$

Optimal. Leaf size=340

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)}{256b^2d^5}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)}{384b^2d^4}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(3a^2d^2+14abcd+63b^2c^2\right)}{480b^2d^3}-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{256b^{5/2}d^{11/2}}-\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{80b^2d^2}+\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd}$$

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(256*b^2*d^5) - ((b*c - a*d)*(63*b^2*c^2 + 1
4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(384*b^
2*d^4) + ((63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^(5/2)
*Sqrt[c + d*x^2])/(480*b^2*d^3) - (3*(3*b*c + a*d)*(a + b*x^2)^(7
/2)*Sqrt[c + d*x^2])/(80*b^2*d^2) + (x^2*(a + b*x^2)^(7/2)*Sqrt[c
 + d*x^2])/(10*b*d) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3
*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x
^2])])/(256*b^(5/2)*d^(11/2))
```

Rubi [A] time = 0.930944, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)}{256b^2d^5}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)}{384b^2d^4}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(3a^2d^2+14abcd+63b^2c^2\right)}{480b^2d^3}-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{256b^{5/2}d^{11/2}}-\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{80b^2d^2}+\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```


```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(256*b^2*d^5) - ((b*c - a*d)*(63*b^2*c^2 + 1
4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(384*b^
2*d^4) + ((63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^(5/2)
*Sqrt[c + d*x^2])/(480*b^2*d^3) - (3*(3*b*c + a*d)*(a + b*x^2)^(7
/2)*Sqrt[c + d*x^2])/(80*b^2*d^2) + (x^2*(a + b*x^2)^(7/2)*Sqrt[c
 + d*x^2])/(10*b*d) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3
*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x
^2])])/(256*b^(5/2)*d^(11/2))
```

Rubi in Sympy [A] time = 81.7891, size = 323, normalized size = 0.95

$$\frac{x^2\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}{10bd}-\frac{3\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}\,(ad+3bc)}{80b^2d^2}+\frac{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}\left(3a^2d^2+14abcd+63b^2c^2\right)}{480b^2d^3}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-bc)\left(3a^2d^2+14abcd+63b^2c^2\right)}{384b^2d^4}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)^2\left(3a^2d^2+14abcd+63b^2c^2\right)}{256b^2d^5}+\frac{(ad-bc)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{256b^{\frac{5}{2}}d^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x**2*(a + b*x**2)**(7/2)*sqrt(c + d*x**2)/(10*b*d) - 3*(a + b*x**
2)**(7/2)*sqrt(c + d*x**2)*(a*d + 3*b*c)/(80*b**2*d**2) + (a + b*
x**2)**(5/2)*sqrt(c + d*x**2)*(3*a**2*d**2 + 14*a*b*c*d + 63*b**2
*c**2)/(480*b**2*d**3) + (a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(a*
d - b*c)*(3*a**2*d**2 + 14*a*b*c*d + 63*b**2*c**2)/(384*b**2*d**4
) + sqrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)**2*(3*a**2*d**2
 + 14*a*b*c*d + 63*b**2*c**2)/(256*b**2*d**5) + (a*d - b*c)**3*(3
*a**2*d**2 + 14*a*b*c*d + 63*b**2*c**2)*atanh(sqrt(d)*sqrt(a + b*
x**2)/(sqrt(b)*sqrt(c + d*x**2)))/(256*b**(5/2)*d**(11/2))
```

Mathematica [A] time = 0.305848, size = 274, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-45a^4d^4+30a^3bd^3\left(dx^2-3c\right)+2a^2b^2d^2\left(782c^2-481cdx^2+372d^2x^4\right)+2ab^3d\left(-1155c^3+749c^2dx^2-\right.\right.}{3840b^2d^5}-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{512b^{5/2}d^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-45*a^4*d^4 + 30*a^3*b*d^3*(-3*
c + d*x^2) + 2*a^2*b^2*d^2*(782*c^2 - 481*c*d*x^2 + 372*d^2*x^4)
+ 2*a*b^3*d*(-1155*c^3 + 749*c^2*d*x^2 - 592*c*d^2*x^4 + 504*d^3*
x^6) + b^4*(945*c^4 - 630*c^3*d*x^2 + 504*c^2*d^2*x^4 - 432*c*d^3
*x^6 + 384*d^4*x^8)))/(3840*b^2*d^5) - ((b*c - a*d)^3*(63*b^2*c^2
 + 14*a*b*c*d + 3*a^2*d^2)*Log[b*c + a*d + 2*b*d*x^2 + 2*Sqrt[b]*
Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(512*b^(5/2)*d^(11/2))
```

Maple [B] time = 0.05, size = 1054, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/7680*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(768*x^8*b^4*d^4*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+2016*x^6*a*b^3*d^4*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-864*x^6*b^4*c*d^3*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+1488*x^4*a^2*b^2*d^4*(b
```

```
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-2368*x^4*a*b^3*c*d^
3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+1008*x^4*b^4*c^
2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+60*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^3*d^4*(b*d)^(1/2)*b-1924*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^2*c*d^3*(b*d)^(1/2)*b^2+2996*
b^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*c^2*d^2*(b*d)^(1/2)
-1260*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c^3*b^4*d*(b*d)^(1/
2)+45*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^5*d^5+75*ln(1/2*(2*b*d*x^2+2*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*a^4*c*d^4*b+450*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*c^2*d^3*b^2-2250*b
^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c^3*d^2+2625*b^4*ln(1/2*(2*b*d*x^
2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d
)^(1/2))*c^4*a*d-945*b^5*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^5-90*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^4*d^4*(b*d)^(1/2)-180*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*a^3*c*d^3*(b*d)^(1/2)*b+3128*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*a^2*c^2*d^2*(b*d)^(1/2)*b^2-4620*b^3*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c^3*d*(b*d)^(1/2)+1890*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*c^4*b^4*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)/d^5/(b*d)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^5/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.3067, size = 1, normalized size = 0.

$$\left[\frac{4\left(384\,b^4d^4x^8+945\,b^4c^4-2310\,ab^3c^3d+1564\,a^2b^2c^2d^2-90\,a^3bcd^3-45\,a^4d^4-144\left(3\,b^4cd^3-7\,ab^3d^4\right)x^6+8\left(63\,b^4c^2d^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^5/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/15360*(4*(384*b^4*d^4*x^8 + 945*b^4*c^4 - 2310*a*b^3*c^3*d + 1
564*a^2*b^2*c^2*d^2 - 90*a^3*b*c*d^3 - 45*a^4*d^4 - 144*(3*b^4*c*
d^3 - 7*a*b^3*d^4)*x^6 + 8*(63*b^4*c^2*d^2 - 148*a*b^3*c*d^3 + 93
*a^2*b^2*d^4)*x^4 - 2*(315*b^4*c^3*d - 749*a*b^3*c^2*d^2 + 481*a^
2*b^2*c*d^3 - 15*a^3*b*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*
sqrt(b*d) - 15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^
2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*log(4*(2*b^2*
d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8
*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d
^2)*x^2)*sqrt(b*d)))/(sqrt(b*d)*b^2*d^5), 1/7680*(2*(384*b^4*d^4*
x^8 + 945*b^4*c^4 - 2310*a*b^3*c^3*d + 1564*a^2*b^2*c^2*d^2 - 90*
a^3*b*c*d^3 - 45*a^4*d^4 - 144*(3*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 +
8*(63*b^4*c^2*d^2 - 148*a*b^3*c*d^3 + 93*a^2*b^2*d^4)*x^4 - 2*(31
5*b^4*c^3*d - 749*a*b^3*c^2*d^2 + 481*a^2*b^2*c*d^3 - 15*a^3*b*d^
4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) - 15*(63*b^5*c
^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 -
 5*a^4*b*c*d^4 - 3*a^5*d^5)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sq
rt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/(sqrt(-b*d)*b^2*
d^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265059, size = 536, normalized size = 1.58

$$\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\Bigg(2\left(bx^2+a\right)\Bigg(4\left(bx^2+a\right)\Bigg(6\left(bx^2+a\right)\left(\frac{8\left(bx^2+a\right)}{bd}-\frac{9\,b^3cd^7+11\,ab^2d^8}{b^3d^9}\right)+\frac{63\,b^4c^2d^6+14\,ab^3cd^7+3\,a}{b^3d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*x^5/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/3840*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*
(b*x^2 + a)*(4*(b*x^2 + a)*(6*(b*x^2 + a)*(8*(b*x^2 + a)/(b*d) -
(9*b^3*c*d^7 + 11*a*b^2*d^8)/(b^3*d^9)) + (63*b^4*c^2*d^6 + 14*a*
b^3*c*d^7 + 3*a^2*b^2*d^8)/(b^3*d^9)) - 5*(63*b^5*c^3*d^5 - 49*a*
b^4*c^2*d^6 - 11*a^2*b^3*c*d^7 - 3*a^3*b^2*d^8)/(b^3*d^9)) + 15*(
63*b^6*c^4*d^4 - 112*a*b^5*c^3*d^5 + 38*a^2*b^4*c^2*d^6 + 8*a^3*b
^3*c*d^7 + 3*a^4*b^2*d^8)/(b^3*d^9)) + 15*(63*b^5*c^5 - 175*a*b^4
*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4
 - 3*a^5*d^5)*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^5))/(b*abs(b))
```

3.955 $\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=237

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2(ad+7bc)}{128bd^4}+\frac{5(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)(ad+7bc)}{192bd^3}-\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}(ad+7bc)}{48bd^2}+\frac{(a+bx^2)^{7/2}\sqrt{c+dx^2}}{8bd}$$

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/
(128*b*d^4) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt
[c + d*x^2])/(192*b*d^3) - ((7*b*c + a*d)*(a + b*x^2)^(5/2)*Sqrt[
c + d*x^2])/(48*b*d^2) + ((a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(8*b
*d) + (5*(b*c - a*d)^3*(7*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(3/2)*d^(9/2))
```

Rubi [A] time = 0.536146, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2(ad+7bc)}{128bd^4}+\frac{5(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)(ad+7bc)}{192bd^3}-\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}(ad+7bc)}{48bd^2}+\frac{(a+bx^2)^{7/2}\sqrt{c+dx^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/
(128*b*d^4) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt
[c + d*x^2])/(192*b*d^3) - ((7*b*c + a*d)*(a + b*x^2)^(5/2)*Sqrt[
c + d*x^2])/(48*b*d^2) + ((a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(8*b
*d) + (5*(b*c - a*d)^3*(7*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(3/2)*d^(9/2))
```

Rubi in Sympy [A] time = 54.3256, size = 212, normalized size = 0.89

$$\frac{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}{8bd}-\frac{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}\,(ad+7bc)}{48bd^2}-\frac{5\,(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-bc)\,(ad+7bc)}{192bd^3}-\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)^2\,(ad+7bc)}{128bd^4}-\frac{5\,(ad-bc)^3\,(ad+7bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{\frac{3}{2}}d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] (a + b*x**2)**(7/2)*sqrt(c + d*x**2)/(8*b*d) - (a + b*x**2)**(5/2
)*sqrt(c + d*x**2)*(a*d + 7*b*c)/(48*b*d**2) - 5*(a + b*x**2)**(3
/2)*sqrt(c + d*x**2)*(a*d - b*c)*(a*d + 7*b*c)/(192*b*d**3) - 5*s
qrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c)**2*(a*d + 7*b*c)/(12
8*b*d**4) - 5*(a*d - b*c)**3*(a*d + 7*b*c)*atanh(sqrt(d)*sqrt(a +
 b*x**2)/(sqrt(b)*sqrt(c + d*x**2)))/(128*b**(3/2)*d**(9/2))
```

Mathematica [A] time = 0.207506, size = 204, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^3d^3+a^2bd^2\left(118dx^2-191c\right)+ab^2d\left(265c^2-172cdx^2+136d^2x^4\right)+b^3\left(-105c^3+70c^2dx^2-56cd^2x\right.\right.}{384bd^4}$$
$$+\frac{5(ad+7bc)(bc-ad)^3\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{256b^{3/2}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(15*a^3*d^3 + a^2*b*d^2*(-191*c
+ 118*d*x^2) + a*b^2*d*(265*c^2 - 172*c*d*x^2 + 136*d^2*x^4) + b^
3*(-105*c^3 + 70*c^2*d*x^2 - 56*c*d^2*x^4 + 48*d^3*x^6)))/(384*b*
d^4) + (5*(b*c - a*d)^3*(7*b*c + a*d)*Log[b*c + a*d + 2*b*d*x^2 +
 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(256*b^(3/2)
*d^(9/2))
```

Maple [B] time = 0.027, size = 770, normalized size = 3.3

$$-\frac{1}{768\,d^4b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-96\,x^6b^3d^3\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}-272\,x^4ab^2d^3\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+112\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-96*x^6*b^3*d^3*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-272*x^4*a*b^2*d^3*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+112*x^4*b^3*c*d^2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-236*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2)+344*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^2*a*c*b^2*d^2*(b*d)^(1/2)-140*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*x^2*c^2*b^3*d*(b*d)^(1/2)+15*ln(1/2*(2*b*d*x^
2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d
)^(1/2))*a^4*d^4+60*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*c*b*d^3-270*ln(1
/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a
*d+b*c)/(b*d)^(1/2))*a^2*c^2*b^2*d^2+300*ln(1/2*(2*b*d*x^2+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*c^3*a*b^3*d-105*b^4*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4-30*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*a^3*d^3*(b*d)^(1/2)+382*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*a^2*c*b*d^2*(b*d)^(1/2)-530*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*a*c^2*b^2*d*(b*d)^(1/2)+210*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*c^3*b^3*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/d^4/(b*d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^3/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296138, size = 1, normalized size = 0.

$$\left[\frac{4\left(48\,b^3d^3x^6 - 105\,b^3c^3 + 265\,ab^2c^2d - 191\,a^2bcd^2 + 15\,a^3d^3 - 8\left(7\,b^3cd^2 - 17\,ab^2d^3\right)x^4 + 2\left(35\,b^3c^2d - 86\,ab^2cd^2 + 59\,a^2b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^3/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/1536*(4*(48*b^3*d^3*x^6 - 105*b^3*c^3 + 265*a*b^2*c^2*d - 191*
a^2*b*c*d^2 + 15*a^3*d^3 - 8*(7*b^3*c*d^2 - 17*a*b^2*d^3)*x^4 + 2
*(35*b^3*c^2*d - 86*a*b^2*c*d^2 + 59*a^2*b*d^3)*x^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(b*d) - 15*(7*b^4*c^4 - 20*a*b^3*c^3*d +
18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*log(-4*(2*b^2*d^2*x
^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*
d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x
^2)*sqrt(b*d)))/(sqrt(b*d)*b*d^4), 1/768*(2*(48*b^3*d^3*x^6 - 105
*b^3*c^3 + 265*a*b^2*c^2*d - 191*a^2*b*c*d^2 + 15*a^3*d^3 - 8*(7*
b^3*c*d^2 - 17*a*b^2*d^3)*x^4 + 2*(35*b^3*c^2*d - 86*a*b^2*c*d^2
+ 59*a^2*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) +
 15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 - a^4*d^4)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/(sqrt(-b*d)*b*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.255627, size = 397, normalized size = 1.68

$$\frac{\sqrt{b^2c + (bx^2+a)bd - abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(4\left(bx^2+a\right)\left(\frac{6\left(bx^2+a\right)}{bd} - \frac{7\,b^2cd^5+abd^6}{b^2d^7}\right) + \frac{5\left(7\,b^3c^2d^4-6\,ab^2cd^5-a^2bd^6\right)}{b^2d^7}\right) - \frac{15\left(7\,b^4\right.}{}\right.}{384\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^3/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(
b*x^2 + a)*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b*d) - (7*b^2*c*d^5 + a
*b*d^6)/(b^2*d^7)) + 5*(7*b^3*c^2*d^4 - 6*a*b^2*c*d^5 - a^2*b*d^6
)/(b^2*d^7)) - 15*(7*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4 + 5*a^2*b^2*c
*d^5 + a^3*b*d^6)/(b^2*d^7)) - 15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 1
8*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*ln(abs(-sqrt(b*x^2 +
 a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d
)*d^4))/abs(b)
```

3.956 $\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=164

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}d^{7/2}}+\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}{24d^2}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d}$$

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*d^3) - (5*(
b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*d^2) + ((a + b*
x^2)^(5/2)*Sqrt[c + d*x^2])/(6*d) - (5*(b*c - a*d)^3*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*Sqrt[b]*d^(
7/2))
```

Rubi [A] time = 0.33064, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}d^{7/2}}+\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}{24d^2}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*d^3) - (5*(
b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*d^2) + ((a + b*
x^2)^(5/2)*Sqrt[c + d*x^2])/(6*d) - (5*(b*c - a*d)^3*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*Sqrt[b]*d^(
7/2))
```

Rubi in Sympy [A] time = 35.8391, size = 146, normalized size = 0.89

$$\frac{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}{6d}+\frac{5\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-bc)}{24d^2}+\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)^2}{16d^3}+\frac{5\,(ad-bc)^3\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16\sqrt{b}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] (a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(6*d) + 5*(a + b*x**2)**(3/2
)*sqrt(c + d*x**2)*(a*d - b*c)/(24*d**2) + 5*sqrt(a + b*x**2)*sqr
t(c + d*x**2)*(a*d - b*c)**2/(16*d**3) + 5*(a*d - b*c)**3*atanh(s
qrt(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(16*sqrt(b)*d
**(7/2))
```

Mathematica [A] time = 0.141231, size = 152, normalized size = 0.93

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(33a^2d^2+2abd\left(13dx^2-20c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)}{48d^3}-\frac{5(bc-ad)^3\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{32\sqrt{b}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(33*a^2*d^2 + 2*a*b*d*(-20*c + 1
3*d*x^2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*d^3) - (5*
(b*c - a*d)^3*Log[b*c + a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]])/(32*Sqrt[b]*d^(7/2))
```

Maple [B] time = 0.022, size = 529, normalized size = 3.2

$$\frac{1}{96\,d^3}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(16\,x^4b^2d^2\sqrt{bd}\sqrt{bdx^4+adx^2+cx^2b+ac}+52\,\sqrt{bdx^4+adx^2+cx^2b+ac}x^2abd^2\sqrt{bd}-20\,\sqrt{bdx^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*x^4*b^2*d^2*(b*d)^(1/2)*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+52*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+15*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*d
^3-45*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+45*ln(1/2*(2*b*d*x^2+2
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*c^2*a*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+66*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^2*d^2*(b*d)^(1/2)-80*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d^3/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*x/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271252, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,b^2d^2x^4+15\,b^2c^2-40\,abcd+33\,a^2d^2-2\left(5\,b^2cd-13\,abd^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}-15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd\right.}{192}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*x/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/192*(4*(8*b^2*d^2*x^4 + 15*b^2*c^2 - 40*a*b*c*d + 33*a^2*d^2 -
 2*(5*b^2*c*d - 13*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*
sqrt(b*d) - 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(
```

```
d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(
b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d)))/(sqrt(b*d)*d^3), 1/96*(2*(8*b
^2*d^2*x^4 + 15*b^2*c^2 - 40*a*b*c*d + 33*a^2*d^2 - 2*(5*b^2*c*d
- 13*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) - 1
5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*
(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*b*d)))/(sqrt(-b*d)*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.249203, size = 284, normalized size = 1.73

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{bd}-\frac{5\left(bcd^3-ad^4\right)}{bd^5}\right)+\frac{15\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)}{bd^5}\right)+\frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2b\cdots\right.}{\cdots}\right.}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*x/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b
*x^2 + a)*(4*(b*x^2 + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 1
5*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*ln(abs(-sqrt(b*x^2 + a)*
sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^
3))*b/abs(b)
```

3.957 $\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{8d^2}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

```
[Out] -(b*(3*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + (b
*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*d) - (a^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (Sqrt
[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a
 + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*d^(5/2))
```

Rubi [A] time = 0.674128, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{8d^2}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -(b*(3*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + (b
*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*d) - (a^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (Sqrt
[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a
 + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*d^(5/2))
```

Rubi in Sympy [A] time = 65.934, size = 173, normalized size = 0.93

$$-\frac{a^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8d^{\frac{5}{2}}}+\frac{b\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{4d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\left(7ad-3bc\right)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**(5/2)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2
)))/sqrt(c) + sqrt(b)*(15*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2)*a
tanh(sqrt(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(8*d**(
5/2)) + b*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(4*d) + b*sqrt(a +
 b*x**2)*sqrt(c + d*x**2)*(7*a*d - 3*b*c)/(8*d**2)
```

Mathematica [C] time = 0.944195, size = 357, normalized size = 1.91

$$\frac{\frac{8a^3bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}+\frac{b\left(\left(a+bx^2\right)\left(c+dx^2\right)\left(9ad-3bc+2bdx^2\right)-\frac{2acx^2}{x^2\left(adF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{bx^2}{a}\right.\right.}\right.}{2c}}{4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] ((8*a^3*b*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2
))])/(-4*b*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^
2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -(a/(b*x^2)), -(c/(d*x^2))] +
 a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*x^2)), -(c/(d*x^2))]) + (b*(
(a + b*x^2)*(c + d*x^2)*(-3*b*c + 9*a*d + 2*b*d*x^2) - (2*a*c*(3*
b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*x^2*AppellF1[1, 1/2, 1/2, 2, -
((b*x^2)/a), -((d*x^2)/c)])/(-4*a*c*AppellF1[1, 1/2, 1/2, 2, -((b
*x^2)/a), -((d*x^2)/c)] + x^2*(a*d*AppellF1[2, 1/2, 3/2, 3, -((b*
x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[2, 3/2, 1/2, 3, -((b*x^2)/a
), -((d*x^2)/c)]))))/(2*d^2))/(4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.026, size = 446, normalized size = 2.4

$$\frac{1}{16\,d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(4\,b^2\sqrt{bdx^4+adx^2+cx^2b+ac}x^2d\sqrt{bd}\sqrt{ac}+15\,b\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*b^2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*x^2*d*(b*d)^(1/2)*(a*c)^(1/2)+15*b*ln(1/2*(2*b*d*x
^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*
d)^(1/2))*a^2*d^2*(a*c)^(1/2)-10*b^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a
*d*(a*c)^(1/2)+3*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2*(a*c)^(1/2)-8*a
^3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+2*a*c)/x^2)*d^2*(b*d)^(1/2)+18*b*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*a*d*(b*d)^(1/2)*(a*c)^(1/2)-6*b^2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*c*(b*d)^(1/2)*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)/d^2/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73185, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/32*(8*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2
+ a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (
3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(b/d)*log(8*b^2*d^2*x^4
```

```
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(
2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt
(b/d)) + 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c))/d^2, 1/16*(4*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(
2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sq
rt(a/c))/x^4) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*
arctan(1/2*(2*b*d*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*d*sqrt(-b/d))) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c))/d^2, -1/32*(16*a^2*d^2*sqrt(-a/c)*arctan(
1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*
sqrt(-a/c))) - (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(b/d)*lo
g(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*
b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c)*sqrt(b/d)) - 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c))/d^2, -1/16*(8*a^2*d^2*sqrt(-a/c)*arc
tan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c
)*c*sqrt(-a/c))) - (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/
d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2
 + c)*d*sqrt(-b/d))) - 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^{2}\right)^{\frac{5}{2}}}{x\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.262097, size = 354, normalized size = 1.89

$$-\frac{\left(\frac{16\sqrt{bd}a^{3}\arctan\left(-\frac{b^{2}c+abd-\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}b}-2\sqrt{b^{2}c+(bx^{2}+a)bd-abd}\sqrt{bx^{2}+a}\left(\frac{2\left(bx^{2}+a\right)}{bd}-\frac{3b^{2}cd-7abd^{2}}{b^{2}d^{3}}\right)+\right.}{16|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -1/16*(16*sqrt(b*d)*a^3*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(
-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b) - 2*sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(b*d) - (3*b^2*c*d - 7*a
*b*d^2)/(b^2*d^3)) + (3*sqrt(b*d)*b^2*c^2 - 10*sqrt(b*d)*a*b*c*d
+ 15*sqrt(b*d)*a^2*d^2)*ln((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^2)/(b*d^3))*b^2/abs(b)
```

3.958 $\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{2cd}$$

```
[Out] (b*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*d) - (a*(a +
 b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*c*x^2) - (a^(3/2)*(5*b*c - a*d)
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*c^(3/2)) - (b^(3/2)*(b*c - 5*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^
2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/2))
```

Rubi [A] time = 0.683332, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{2cd}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*d) - (a*(a +
 b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*c*x^2) - (a^(3/2)*(5*b*c - a*d)
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*c^(3/2)) - (b^(3/2)*(b*c - 5*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^
2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/2))
```

Rubi in Sympy [A] time = 71.3697, size = 167, normalized size = 0.89

$$\frac{a^{\frac{3}{2}}\left(ad-5bc\right)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{\frac{3}{2}}}-\frac{a\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{2cx^2}+\frac{b^{\frac{3}{2}}\left(5ad-bc\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{\frac{3}{2}}}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\left(ad+bc\right)}{2cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] a**(3/2)*(a*d - 5*b*c)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sq
rt(c + d*x**2)))/(2*c**(3/2)) - a*(a + b*x**2)**(3/2)*sqrt(c + d*
x**2)/(2*c*x**2) + b**(3/2)*(5*a*d - b*c)*atanh(sqrt(d)*sqrt(a +
b*x**2)/(sqrt(b)*sqrt(c + d*x**2)))/(2*d**(3/2)) + b*sqrt(a + b*x
**2)*sqrt(c + d*x**2)*(a*d + b*c)/(2*c*d)
```

Mathematica [C] time = 0.836098, size = 358, normalized size = 1.91

$$\frac{\frac{2a^2bd^2x^4(5bc-ad)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}+\left(a+bx^2\right)\left(c+dx^2\right)\left(b^2cx^2-a^2d\right)-\frac{\cdots}{x^2\left(adF_1\left(2;\frac{1}{2},\cdots\right.\right.}}{2cdx^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a + b*x^2)*(-(a^2*d) + b^2*c*x^2)*(c + d*x^2) + (2*a^2*b*d^2*(5
*b*c - a*d)*x^4*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2
))])/(-4*b*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^
2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -(a/(b*x^2)), -(c/(d*x^2))] +
 a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*x^2)), -(c/(d*x^2))]) - (2*a
*b^2*c^2*(-(b*c) + 5*a*d)*x^4*AppellF1[1, 1/2, 1/2, 2, -((b*x^2)/
a), -((d*x^2)/c)])/(-4*a*c*AppellF1[1, 1/2, 1/2, 2, -((b*x^2)/a),
 -((d*x^2)/c)] + x^2*(a*d*AppellF1[2, 1/2, 3/2, 3, -((b*x^2)/a),
-((d*x^2)/c)] + b*c*AppellF1[2, 3/2, 1/2, 3, -((b*x^2)/a), -((d*x
^2)/c)])))/(2*c*d*x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.025, size = 423, normalized size = 2.3

$$\frac{1}{4\,c x^2 d}\sqrt{b x^2+a}\sqrt{d x^2+c}\left(\ln\left(\frac{1}{x^2}\left(a d x^2+c x^2 b+2\,\sqrt{ac}\sqrt{b d x^4+a d x^2+c x^2 b+ac}+2\,ac\right)\right)x^2 a^3 d^2\sqrt{bd}-5\,\ln\left(\frac{a d x^2+c x^2 b}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(ln((a*d*x^2+c*x^2*b+2*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a^3*d
^2*(b*d)^(1/2)-5*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a^2*b*c*d*(b*d)^(1/2)+5*ln(
1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*x^2*a*b^2*c*d*(a*c)^(1/2)-ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)
^(1/2))*x^2*b^3*c^2*(a*c)^(1/2)+2*x^2*b^2*c*(a*c)^(1/2)*(b*d)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-2*a^2*d*(a*c)^(1/2)*(b*d)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/x^2/(a*c)^(1/2)/(b*d)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12796, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d
)) + (5*a*b*c*d - a^2*d^2)*x^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*
d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*(2*a
```

```
*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
a/c))/x^4) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
))/(c*d*x^2), -1/8*(2*(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan
(1/2*(2*b*d*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*s
qrt(-b/d))) + (5*a*b*c*d - a^2*d^2)*x^2*sqrt(a/c)*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2
 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c)*sqrt(a/c))/x^4) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/(c*d*x^2), -1/8*(2*(5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/
c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x
^2 + c)*c*sqrt(-a/c))) + (b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(
8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*
d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(b/d)) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c))/(c*d*x^2), -1/4*((5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/c
)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)*c*sqrt(-a/c))) + (b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arct
an(1/2*(2*b*d*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d
*sqrt(-b/d))) - 2*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c))/(c*d*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^{\frac{5}{2}}}{x^3\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**3*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.619278, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.959 $\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{8c^2x^2}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

```
[Out] -(a*(7*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2)
- (a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (Sqrt[a]*(15*
b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2
])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*c^(5/2)) + (b^(5/2)*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt[d]
```

Rubi [A] time = 0.635704, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{8c^2x^2}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*(7*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2)
- (a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (Sqrt[a]*(15*
b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2
])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*c^(5/2)) + (b^(5/2)*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt[d]
```

Rubi in Sympy [A] time = 64.1813, size = 180, normalized size = 0.94

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{\frac{5}{2}}}-\frac{a\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{4cx^4}+\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}\left(3ad-7bc\right)}{8c^2x^2}+\frac{b^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*(3*a**2*d**2 - 10*a*b*c*d + 15*b**2*c**2)*atanh(sqrt(c)*
sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2)))/(8*c**(5/2)) - a*(a
+ b*x**2)**(3/2)*sqrt(c + d*x**2)/(4*c*x**4) + a*sqrt(a + b*x**2)
*sqrt(c + d*x**2)*(3*a*d - 7*b*c)/(8*c**2*x**2) + b**(5/2)*atanh(
sqrt(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(c + d*x**2)))/sqrt(d)
```

Mathematica [C] time = 0.929212, size = 359, normalized size = 1.87

$$\frac{a\left(\frac{2bdx^6\left(3a^2d^2-10abcd+15b^2c^2\right)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}-\frac{16b^3c^3x^6F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{bx^2}{a},-\right.}{x^2\left(adF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{bx^2}{a},-\frac{dx\right.}{c}\right.}\right.}{8c^2x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*((a + b*x^2)*(c + d*x^2)*(-2*a*c - 9*b*c*x^2 + 3*a*d*x^2) + (2
*b*d*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^6*AppellF1[1, 1/2, 1
/2, 2, -(a/(b*x^2)), -(c/(d*x^2))])/(-4*b*d*x^2*AppellF1[1, 1/2,
1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))] + b*c*AppellF1[2, 1/2, 3/2, 3
, -(a/(b*x^2)), -(c/(d*x^2))] + a*d*AppellF1[2, 3/2, 1/2, 3, -(a/
(b*x^2)), -(c/(d*x^2))]) - (16*b^3*c^3*x^6*AppellF1[1, 1/2, 1/2,
2, -((b*x^2)/a), -((d*x^2)/c)])/(-4*a*c*AppellF1[1, 1/2, 1/2, 2,
-((b*x^2)/a), -((d*x^2)/c)] + x^2*(a*d*AppellF1[2, 1/2, 3/2, 3, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[2, 3/2, 1/2, 3, -((b*x^
2)/a), -((d*x^2)/c)]))))/(8*c^2*x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Maple [B] time = 0.024, size = 464, normalized size = 2.4

$$-\frac{1}{16\,c^2x^4}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\,\ln\left(\frac{adx^2+cx^2b+2\,\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2\,ac}{x^2}\right)x^4a^3d^2\sqrt{bd}-10\,\ln\left(\frac{adx^2+cx^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(3*ln((a*d*x^2+c*x^2*b+
2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4
*a^3*d^2*(b*d)^(1/2)-10*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*b*c*d*(b*d)^(1/2
)+15*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+2*a*c)/x^2)*x^4*a*b^2*c^2*(b*d)^(1/2)-8*ln(1/2*(2*b*d*x
^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*
d)^(1/2))*x^4*b^3*c^2*(a*c)^(1/2)-6*x^2*a^2*d*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(a*c)^(1/2)*(b*d)^(1/2)+18*x^2*a*b*c*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*(b*d)^(1/2)+4*a^2*c*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)/(a*c)^(1/2)/x^4/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78177, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] [1/32*(8*b^2*c^2*x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*
b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*
c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (15*b^2
```

```
*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*
(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*s
qrt(a/c))/x^4) - 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c))/(c^2*x^4), 1/32*(16*b^2*c^2*x^4*sqrt(-b/d)
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c)*d*sqrt(-b/d))) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sq
rt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*
(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*(2*a^2*c + 3*(3*a*
b*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), 1/1
6*(4*b^2*c^2*x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d
+ a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (15*b^2*c^2
 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b*c + a*d)
*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*sqrt(-a/c))) - 2
*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c))/(c^2*x^4), 1/16*(8*b^2*c^2*x^4*sqrt(-b/d)*arctan(1/2*(2*b*d*
x^2 + b*c + a*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d*sqrt(-b/d)))
- (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*sqr
t(-a/c))) - 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/(c^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**5*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 1.88474, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.960 $\int \frac{x^4\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=553

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{315d^3}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{315bd^4}+\frac{c^{3/2}\sqrt{a+bx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315bd^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(-10a^4d^4-25a^3bcd^3+243a^2b^2c^2d^2-328ab^3c^3d+128b^4c^4\right)}{315b^2d^4\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-10a^4d^4-25a^3bcd^3+243a^2b^2c^2d^2-328ab^3c^3d+128b^4c^4\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^2d^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{63d^2}+\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}$$

```
[Out] ((128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^3*b*
c*d^3 - 10*a^4*d^4)*x*Sqrt[a + b*x^2])/(315*b^2*d^4*Sqrt[c + d*x^
2]) - ((64*b^3*c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^
3)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(315*b*d^4) + ((48*b^2*c^2
- 115*a*b*c*d + 75*a^2*d^2)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/
(315*d^3) - (4*b*(2*b*c - 3*a*d)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(63*d^2) + (b*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*d) -
 (Sqrt[c]*(128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 -
25*a^3*b*c*d^3 - 10*a^4*d^4)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(315*b^2*d^(9/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(64*b^3*c^
3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(315*b
*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.85435, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{315d^3}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{315bd^4}+\frac{c^{3/2}\sqrt{a+bx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315bd^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(-10a^4d^4-25a^3bcd^3+243a^2b^2c^2d^2-328ab^3c^3d+128b^4c^4\right)}{315b^2d^4\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-10a^4d^4-25a^3bcd^3+243a^2b^2c^2d^2-328ab^3c^3d+128b^4c^4\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^2d^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{63d^2}+\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^3*b*
c*d^3 - 10*a^4*d^4)*x*Sqrt[a + b*x^2])/(315*b^2*d^4*Sqrt[c + d*x^
2]) - ((64*b^3*c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^
```

```
3)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(315*b*d^4) + ((48*b^2*c^2
- 115*a*b*c*d + 75*a^2*d^2)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/
(315*d^3) - (4*b*(2*b*c - 3*a*d)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(63*d^2) + (b*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*d) -
 (Sqrt[c]*(128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 -
25*a^3*b*c*d^3 - 10*a^4*d^4)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(315*b^2*d^(9/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(64*b^3*c^
3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(315*b
*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 3.05645, size = 379, normalized size = 0.69

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(5a^3d^3+15a^2bd^2\left(5dx^2-7c\right)+ab^2d\left(156c^2-115cdx^2+95d^2x^4\right)+b^3\left(-64c^3+48c^2dx^2-40cc\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]

[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(5*a^3*d^3 + 15*a^2*b*d^2*
(-7*c + 5*d*x^2) + a*b^2*d*(156*c^2 - 115*c*d*x^2 + 95*d^2*x^4) +
 b^3*(-64*c^3 + 48*c^2*d*x^2 - 40*c*d^2*x^4 + 35*d^3*x^6)) + I*c*
(-128*b^4*c^4 + 328*a*b^3*c^3*d - 243*a^2*b^2*c^2*d^2 + 25*a^3*b*
c*d^3 + 10*a^4*d^4)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-128*b^4*c^4 + 3
92*a*b^3*c^3*d - 399*a^2*b^2*c^2*d^2 + 130*a^3*b*c*d^3 + 5*a^4*d^
4)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)])/(315*b*Sqrt[b/a]*d^5*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])
```

Maple [A] time = 0.037, size = 1047, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)

[Out] 1/315*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-25*(-b/a)^(1/2)*x^7*a*b^3
*c*d^4+128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^5-64*(-b/a)^(1/2)*x^3*b^4*c^4*d
+5*(-b/a)^(1/2)*x*a^4*c*d^4+130*(-b/a)^(1/2)*x^9*a*b^3*d^5-5*(-b/
a)^(1/2)*x^9*b^4*c*d^4+170*(-b/a)^(1/2)*x^7*a^2*b^2*d^5+8*(-b/a)^
(1/2)*x^7*b^4*c^2*d^3+80*(-b/a)^(1/2)*x^5*a^3*b*d^5-16*(-b/a)^(1/
2)*x^5*b^4*c^3*d^2-50*(-b/a)^(1/2)*x^5*a^2*b^2*c*d^4+49*(-b/a)^(1
/2)*x^5*a*b^3*c^2*d^3-25*(-b/a)^(1/2)*x^3*a^3*b*c*d^4-64*(-b/a)^(
```

```
1/2)*x^3*a^2*b^2*c^2*d^3+140*(-b/a)^(1/2)*x^3*a*b^3*c^3*d^2-105*(
-b/a)^(1/2)*x*a^3*b*c^2*d^3+156*(-b/a)^(1/2)*x*a^2*b^2*c^3*d^2-64
*(-b/a)^(1/2)*x*a*b^3*c^4*d+5*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^4-10*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^4*c*d^4-25*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d^3+243*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^2*b^2*c^3*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^5-328*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b^3*c^4*d+130*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d^3-399*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^2*b^2*c^3*d^2+392*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^4*d+
35*(-b/a)^(1/2)*x^11*b^4*d^5+5*(-b/a)^(1/2)*x^3*a^4*d^5)/b/d^5/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^8+2\,abx^6+a^2x^4\right)\sqrt{bx^2+a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^6 + a^2*x^4)*sqrt(b*x^2 + a)/sqrt(d*x
^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

3.961 $\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=436

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{105d^3}-\frac{c^{3/2}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)}{105bd^3\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105bd^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{35d^2}+\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}$$

```
[Out] -((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x
*Sqrt[a + b*x^2])/(105*b*d^3*Sqrt[c + d*x^2]) + ((24*b^2*c^2 - 61
*a*b*c*d + 45*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^
3) - (2*b*(3*b*c - 5*a*d)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3
5*d^2) + (b*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*d) + (Sqrt[
c]*(48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(105*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) - (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(105*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rubi [A] time = 1.27749, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{105d^3}-\frac{c^{3/2}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)}{105bd^3\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105bd^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{35d^2}+\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```


```
[Out] -((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x
*Sqrt[a + b*x^2])/(105*b*d^3*Sqrt[c + d*x^2]) + ((24*b^2*c^2 - 61
*a*b*c*d + 45*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^
3) - (2*b*(3*b*c - 5*a*d)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3
5*d^2) + (b*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*d) + (Sqrt[
c]*(48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(105*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) - (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
```

```
*d)])/(105*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rubi in Sympy [A] time = 149.728, size = 410, normalized size = 0.94

$$\frac{bx^3\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{7d}+\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(5ad-3bc\right)}{35d^2}-\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{105d^{\frac{7}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{105d^3}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^3d^3-103a^2bcd^2+128ab^2c^2d-48b^3c^3\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{105bd^{\frac{7}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\left(15a^3d^3-103a^2bcd^2+128ab^2c^2d-48b^3c^3\right)}{105bd^3\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x**3*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(7*d) + 2*b*x**3*sqrt
(a + b*x**2)*sqrt(c + d*x**2)*(5*a*d - 3*b*c)/(35*d**2) - c**(3/2
)*sqrt(a + b*x**2)*(45*a**2*d**2 - 61*a*b*c*d + 24*b**2*c**2)*ell
iptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(105*d**(7/2)*sqr
t(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)) + x*sqrt(a +
 b*x**2)*sqrt(c + d*x**2)*(45*a**2*d**2 - 61*a*b*c*d + 24*b**2*c*
*2)/(105*d**3) - sqrt(c)*sqrt(a + b*x**2)*(15*a**3*d**3 - 103*a**
2*b*c*d**2 + 128*a*b**2*c**2*d - 48*b**3*c**3)*elliptic_e(atan(sq
rt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(105*b*d**(7/2)*sqrt(c*(a + b*x*
*2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)) + x*sqrt(a + b*x**2)*(15*
a**3*d**3 - 103*a**2*b*c*d**2 + 128*a*b**2*c**2*d - 48*b**3*c**3)
/(105*b*d**3*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.10101, size = 306, normalized size = 0.7

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(45a^2d^2+abd\left(45dx^2-61c\right)+3b^2\left(8c^2-6cdx^2+5d^2x^4\right)\right)+4ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(15a^3d^3-4\right.}{1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(45*a^2*d^2 + a*b*d*(-61*c
 + 45*d*x^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - I*c*(-48*
b^3*c^3 + 128*a*b^2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + (4*I)*c*(-12*b^3*c^3 + 38*a*b^2*c^2*d - 41*a^2*b
*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*Sqrt[b/a]*d^4*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.03, size = 782, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-15*(-b/a)^(1/2)*x^9*b^3*
d^4-60*(-b/a)^(1/2)*x^7*a*b^2*d^4+3*(-b/a)^(1/2)*x^7*b^3*c*d^3-90
*(-b/a)^(1/2)*x^5*a^2*b*d^4+19*(-b/a)^(1/2)*x^5*a*b^2*c*d^3-6*(-b
/a)^(1/2)*x^5*b^3*c^2*d^2-45*(-b/a)^(1/2)*x^3*a^3*d^4-29*(-b/a)^(
1/2)*x^3*a^2*b*c*d^3+55*(-b/a)^(1/2)*x^3*a*b^2*c^2*d^2-24*(-b/a)^
(1/2)*x^3*b^3*c^3*d+60*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3-164*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a^2*b*c^2*d^2+152*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d-48*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*b^3*c^4-15*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+103*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b*c^2*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d+48*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*b^3*c^4-45*(-b/a)^(1/2)*x*a^3*c*d^3+61*(-b/a)^(1/2)*x*a^2*b*c
^2*d^2-24*(-b/a)^(1/2)*x*a*b^2*c^3*d)/d^4/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^4+a^2x^2\right)\sqrt{bx^2+a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^4 + a^2*x^2)*sqrt(b*x^2 + a)/sqrt(d*x
^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}} x^2}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

3.962 $\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt{a+bx^2}\,(-3a^2d^2-7abcd+2b^2c^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{c}+7ab-\frac{2b^2c}{d}\right)}{3\sqrt{c+dx^2}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc-9ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3cd}$$

```
[Out] ((7*a*b - (2*b^2*c)/d + (3*a^2*d)/c)*x*Sqrt[a + b*x^2])/(3*Sqrt[c
 + d*x^2]) + (b*(b*c + 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/
(3*c*d) - (a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x) + ((2*b^2*c
^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(b*c -
9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])
```

Rubi [A] time = 0.781233, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\sqrt{a+bx^2}\,(-3a^2d^2-7abcd+2b^2c^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{c}+7ab-\frac{2b^2c}{d}\right)}{3\sqrt{c+dx^2}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc-9ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3cd}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((7*a*b - (2*b^2*c)/d + (3*a^2*d)/c)*x*Sqrt[a + b*x^2])/(3*Sqrt[c
 + d*x^2]) + (b*(b*c + 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/
(3*c*d) - (a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x) + ((2*b^2*c
^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(b*c -
9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])
```

Rubi in Sympy [A] time = 99.0764, size = 308, normalized size = 0.93

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,(3a^2d^2+7abcd-2b^2c^2)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3cd^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{a\,(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{cx}+\frac{b\sqrt{c}\sqrt{a+bx^2}\,(9ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3d^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3ad+bc)}{3cd}+\frac{bx\sqrt{c+dx^2}\,(3a^2d^2+7abcd-2b^2c^2)}{3cd^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((b*x**2+a)**(5/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*(3*a**2*d**2 + 7*a*b*c*d - 2*b*
*2*c**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*c
*d**2*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)) - a
*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(c*x) + b*sqrt(c)*sqrt(a +
b*x**2)*(9*a*d - b*c)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c
/(a*d))/(3*d**(3/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c
+ d*x**2)) + b*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(3*a*d + b*c)/
(3*c*d) + b*x*sqrt(c + d*x**2)*(3*a**2*d**2 + 7*a*b*c*d - 2*b**2*
c**2)/(3*c*d**2*sqrt(a + b*x**2))
```

Mathematica [C]   time = 0.788659, size = 254, normalized size = 0.77

$$\frac{-2ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-4abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2+7abcd-2b^2\right.}{3cd^2x\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[b/a]*d*(a + b*x^2)*(3*a^2*d - b^2*c*x^2)*(c + d*x^2)) - I
*b*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - (2*I)*b*c*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)])/(3*Sqrt[b/a]*c*d^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]   time = 0.027, size = 568, normalized size = 1.7

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac\right)d^2cx}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^6b^3cd^2-3\sqrt{-\frac{b}{a}}x^4a^2bd^3+\sqrt{-\frac{b}{a}}x^4ab^2cd^2+\sqrt{-\frac{b}{a}}x^4b^3c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^6*b^3*c*d^2-3
*(-b/a)^(1/2)*x^4*a^2*b*d^3+(-b/a)^(1/2)*x^4*a*b^2*c*d^2+(-b/a)^(
1/2)*x^4*b^3*c^2*d+6*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*a^2*b*c*d^2-8*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*x*a*b^2*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*b^3*c^3+3*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*x*a^2*b*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*a*b^2*c^2*d-2*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*x*b^3*c^3-3*(-b/a)^(1/2)*x^2*a^3*d^3-3*(-b/a)^(1/2)*x^2*a^2*b
*c*d^2+(-b/a)^(1/2)*x^2*a*b^2*c^2*d-3*(-b/a)^(1/2)*a^3*c*d^2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)/c/x
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{dx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(sqrt(d*x^2
+ c)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{x^2\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**2*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\sqrt{dx^2 + c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.963 $\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=336

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{3c^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{3c^2x} + \frac{b\sqrt{a+bx^2}(9bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c^2*Sq
rt[c + d*x^2]) - (2*a*(3*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(3*c^2*x) - (a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(3*c*x^3) -
 ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*(9*b*
c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.825867, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{3c^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{3c^2x} + \frac{b\sqrt{a+bx^2}(9bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c^2*Sq
rt[c + d*x^2]) - (2*a*(3*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(3*c^2*x) - (a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(3*c*x^3) -
 ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*(9*b*
c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 99.7162, size = 304, normalized size = 0.9

$$-\frac{a\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{3cx^3} + \frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}\left(ad-3bc\right)}{3c^2x} - \frac{b\sqrt{a+bx^2}\left(ad-9bc\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{x\sqrt{a+bx^2}\left(2a^2d^2-7abcd-3b^2c^2\right)}{3c^2\sqrt{c+dx^2}} + \frac{\sqrt{a+bx^2}\left(2a^2d^2-7abcd-3b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3c^{\frac{3}{2}}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(3*c*x**3) + 2*a*sqrt(a +
 b*x**2)*sqrt(c + d*x**2)*(a*d - 3*b*c)/(3*c**2*x) - b*sqrt(a + b
*x**2)*(a*d - 9*b*c)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/
(a*d))/(3*sqrt(c)*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*s
qrt(c + d*x**2)) - x*sqrt(a + b*x**2)*(2*a**2*d**2 - 7*a*b*c*d -
3*b**2*c**2)/(3*c**2*sqrt(c + d*x**2)) + sqrt(a + b*x**2)*(2*a**2
*d**2 - 7*a*b*c*d - 3*b**2*c**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c
)), 1 - b*c/(a*d))/(3*c**(3/2)*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c
+ d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.814571, size = 261, normalized size = 0.78

$$\frac{-ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+2abcd-3b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-7abcd-3b\right.}{3c^2dx^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 7*b*c*x^2 + 2*a*
d*x^2) + I*b*c*(-3*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*x^3*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - I*b*c*(-3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*x^3*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d*x^3*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])
```

Maple [A] time = 0.026, size = 583, normalized size = 1.7

$$\frac{1}{(3\,bd{x}^{4}+3\,ad{x}^{2}+3\,c{x}^{2}b+3\,ac){c}^{2}{x}^{3}d}\sqrt{b{x}^{2}+a}\sqrt{d{x}^{2}+c}\left(2\,\sqrt{-{\frac{b}{a}}}{x}^{6}{a}^{2}b{d}^{3}-7\,\sqrt{-{\frac{b}{a}}}{x}^{6}a{b}^{2}c{d}^{2}+\sqrt{{\frac{b{x}^{2}+a}{a}}}\sqrt{{\frac{d{x}^{2}+c}{c}}}{\it Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*x^6*a^2*b*d^3
-7*(-b/a)^(1/2)*x^6*a*b^2*c*d^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a^2*b*c*d^2+
2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*x^3*a*b^2*c^2*d-3*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^3*c^3
-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*x^3*a^2*b*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b^2*
c^2*d+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*x^3*b^3*c^3+2*(-b/a)^(1/2)*x^4*a^3*d^3-6
*(-b/a)^(1/2)*x^4*a^2*b*c*d^2-7*(-b/a)^(1/2)*x^4*a*b^2*c^2*d+(-b/
a)^(1/2)*x^2*a^3*c*d^2-8*(-b/a)^(1/2)*x^2*a^2*b*c^2*d-(-b/a)^(1/2
)*a^3*c^2*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/x^3/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(sqrt(d*x^2
+ c)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{x^4\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**4*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\sqrt{dx^2 + c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.964 $\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{7}{135}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{1}{15}\sqrt{2-3x^2}\sqrt{3x^2-1}x^3-\frac{2F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}}-\frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{45\sqrt{3}}$$

```
[Out] (-7*x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/135 - (x^3*Sqrt[2 - 3*x^2
]*Sqrt[-1 + 3*x^2])/15 - (8*EllipticE[ArcCos[Sqrt[3/2]*x], 2])/(4
5*Sqrt[3]) - (2*EllipticF[ArcCos[Sqrt[3/2]*x], 2])/(27*Sqrt[3])
```

Rubi [A] time = 0.298323, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{7}{135}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{1}{15}\sqrt{2-3x^2}\sqrt{3x^2-1}x^3-\frac{2F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}}-\frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{45\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/135 - (x^3*Sqrt[2 - 3*x^2
]*Sqrt[-1 + 3*x^2])/15 - (8*EllipticE[ArcCos[Sqrt[3/2]*x], 2])/(4
5*Sqrt[3]) - (2*EllipticF[ArcCos[Sqrt[3/2]*x], 2])/(27*Sqrt[3])
```

Rubi in Sympy [A] time = 34.8414, size = 92, normalized size = 0.93

$$-\frac{x^3\sqrt{-3x^2+2}\sqrt{3x^2-1}}{15}-\frac{7x\sqrt{-3x^2+2}\sqrt{3x^2-1}}{135}-\frac{8\sqrt{3}E\left(\operatorname{acos}\left(\frac{\sqrt{6}x}{2}\right)\middle|2\right)}{135}-\frac{2\sqrt{3}F\left(\operatorname{acos}\left(\frac{\sqrt{6}x}{2}\right)\middle|2\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] -x**3*sqrt(-3*x**2 + 2)*sqrt(3*x**2 - 1)/15 - 7*x*sqrt(-3*x**2 +
2)*sqrt(3*x**2 - 1)/135 - 8*sqrt(3)*elliptic_e(acos(sqrt(6)*x/2),
 2)/135 - 2*sqrt(3)*elliptic_f(acos(sqrt(6)*x/2), 2)/81
```

Mathematica [A] time = 0.143801, size = 92, normalized size = 0.93

$$\frac{10\sqrt{3-9x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)-24\sqrt{3-9x^2}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)-3x\sqrt{2-3x^2}\left(27x^4+12x^2-7\right)}{405\sqrt{3x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-3*x*Sqrt[2 - 3*x^2]*(-7 + 12*x^2 + 27*x^4) - 24*Sqrt[3 - 9*x^2]
*EllipticE[ArcSin[Sqrt[3/2]*x], 2] + 10*Sqrt[3 - 9*x^2]*EllipticF
[ArcSin[Sqrt[3/2]*x], 2])/(405*Sqrt[-1 + 3*x^2])
```

Maple [A] time = 0.031, size = 135, normalized size = 1.4

$$-\frac{\sqrt{2}}{7290\,x^4-7290\,x^2+1620}\sqrt{3\,x^2-1}\sqrt{-6\,x^2+4}\left(243\,x^7-54\,x^5+5\,\sqrt{3}\sqrt{2}\sqrt{-6\,x^2+4}\sqrt{-3\,x^2+1}EllipticF\left(1/2\,x\sqrt{3}\sqrt{2},\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/810*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(243*x^7-54*x^5+5
*3^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*
x*3^(1/2)*2^(1/2),2^(1/2))-12*3^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(-
3*x^2+1)^(1/2)*EllipticE(1/2*x*3^(1/2)*2^(1/2),2^(1/2))-135*x^3+4
2*x)/(9*x^4-9*x^2+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}x^4}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\,x^2-1}x^4}{\sqrt{-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}x^4}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2),x, algorithm="giac")

[Out] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

3.965 $\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{36}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

```
[Out] (-7*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/72 - (Sqrt[2 - 3*x^2]*(-1 +
 3*x^2)^(3/2))/36 - (7*ArcSin[3 - 6*x^2])/144
```

Rubi [A] time = 0.146526, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{36}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/72 - (Sqrt[2 - 3*x^2]*(-1 +
 3*x^2)^(3/2))/36 - (7*ArcSin[3 - 6*x^2])/144
```

Rubi in Sympy [A] time = 14.324, size = 75, normalized size = 1.15

$$-\frac{\sqrt{-3x^2+2}\left(3x^2-1\right)^{\frac{3}{2}}}{36}-\frac{7\sqrt{-3x^2+2}\sqrt{3x^2-1}}{72}-\frac{7\,\mathrm{atan}\left(\frac{-18x^2+9}{6\sqrt{-9x^4+9x^2-2}}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(-3*x**2 + 2)*(3*x**2 - 1)**(3/2)/36 - 7*sqrt(-3*x**2 + 2)*s
qrt(3*x**2 - 1)/72 - 7*atan((-18*x**2 + 9)/(6*sqrt(-9*x**4 + 9*x*
*2 - 2)))/144
```

Mathematica [A] time = 0.0678985, size = 60, normalized size = 0.92

$$\frac{1}{144}\left(-2\sqrt{-9x^4+9x^2-2}\left(6x^2+5\right)-7\tan^{-1}\left(\frac{3-6x^2}{2\sqrt{-9x^4+9x^2-2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-2*(5 + 6*x^2)*Sqrt[-2 + 9*x^2 - 9*x^4] - 7*ArcTan[(3 - 6*x^2)/(
2*Sqrt[-2 + 9*x^2 - 9*x^4])])/144
```

Maple [A] time = 0.035, size = 81, normalized size = 1.3

$$\frac{1}{144}\sqrt{-3\,x^2+2}\sqrt{3\,x^2-1}\left(-12\,x^2\sqrt{-9\,x^4+9\,x^2-2}+7\,\arcsin\left(6\,x^2-3\right)-10\,\sqrt{-9\,x^4+9\,x^2-2}\right)\frac{1}{\sqrt{-9\,x^4+9\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/144*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(-12*x^2*(-9*x^4+9*x^2-2)^
(1/2)+7*arcsin(6*x^2-3)-10*(-9*x^4+9*x^2-2)^(1/2))/(-9*x^4+9*x^2-
2)^(1/2)
```

Maxima [A] time = 1.51679, size = 62, normalized size = 0.95

$$-\frac{1}{12}\sqrt{-9x^4+9x^2-2}x^2-\frac{5}{72}\sqrt{-9x^4+9x^2-2}+\frac{7}{144}\arcsin\left(6x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(3*x^2 - 1)*x^3/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-9*x^4 + 9*x^2 - 2)*x^2 - 5/72*sqrt(-9*x^4 + 9*x^2 - 2
) + 7/144*arcsin(6*x^2 - 3)
```

Fricas [A] time = 0.240719, size = 78, normalized size = 1.2

$$-\frac{1}{72}\left(6x^2+5\right)\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{7}{144}\arctan\left(\frac{3\left(2x^2-1\right)}{2\sqrt{3x^2-1}\sqrt{-3x^2+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(3*x^2 - 1)*x^3/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 7/144*arctan
(3/2*(2*x^2 - 1)/(sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{3x^2-1}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(3*x**2 - 1)/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [A] time = 0.229491, size = 54, normalized size = 0.83

$$-\frac{1}{72}\left(6x^2+5\right)\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{7}{72}\arcsin\left(\sqrt{3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)*x^3/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 7/72*arcsin(
sqrt(3*x^2 - 1))
```

3.966 $\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{9}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

```
[Out] -(x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/9 - EllipticE[ArcCos[Sqrt[3
/2]*x], 2]/(3*Sqrt[3]) - EllipticF[ArcCos[Sqrt[3/2]*x], 2]/(9*Sqr
t[3])
```

Rubi [A] time = 0.161637, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{1}{9}\sqrt{2-3x^2}\sqrt{3x^2-1}x-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/9 - EllipticE[ArcCos[Sqrt[3
/2]*x], 2]/(3*Sqrt[3]) - EllipticF[ArcCos[Sqrt[3/2]*x], 2]/(9*Sqr
t[3])
```

Rubi in Sympy [A] time = 25.6158, size = 61, normalized size = 0.87

$$-\frac{x\sqrt{-3x^2+2}\sqrt{3x^2-1}}{9}-\frac{\sqrt{3}E\left(\operatorname{acos}\left(\frac{\sqrt{6}x}{2}\right)\middle|2\right)}{9}-\frac{\sqrt{3}F\left(\operatorname{acos}\left(\frac{\sqrt{6}x}{2}\right)\middle|2\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] -x*sqrt(-3*x**2 + 2)*sqrt(3*x**2 - 1)/9 - sqrt(3)*elliptic_e(acos
(sqrt(6)*x/2), 2)/9 - sqrt(3)*elliptic_f(acos(sqrt(6)*x/2), 2)/27
```

Mathematica [A] time = 0.145141, size = 86, normalized size = 1.23

$$\frac{3x\sqrt{2-3x^2}\left(1-3x^2\right)+\sqrt{3-9x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)-3\sqrt{3-9x^2}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (3*x*(1 - 3*x^2)*Sqrt[2 - 3*x^2] - 3*Sqrt[3 - 9*x^2]*EllipticE[Ar
cSin[Sqrt[3/2]*x], 2] + Sqrt[3 - 9*x^2]*EllipticF[ArcSin[Sqrt[3/2
]*x], 2])/(27*Sqrt[-1 + 3*x^2])
```

Maple [A] time = 0.017, size = 129, normalized size = 1.8

$$-\frac{\sqrt{2}}{972\,x^4-972\,x^2+216}\sqrt{3\,x^2-1}\sqrt{-6\,x^2+4}\left(54\,x^5+\sqrt{3}\sqrt{2}\sqrt{-6\,x^2+4}\sqrt{-3\,x^2+1}EllipticF\left(\frac{x\sqrt{3}\sqrt{2}}{2},\sqrt{2}\right)-3\,\sqrt{3}\sqrt{2}\sqrt{-6\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/108*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(54*x^5+3^(1/2)*2
^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*3^(1/2)*
2^(1/2),2^(1/2))-3*3^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1
/2)*EllipticE(1/2*x*3^(1/2)*2^(1/2),2^(1/2))-54*x^3+12*x)/(9*x^4-
9*x^2+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{3x^2-1}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(3*x**2 - 1)/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

3.967 $\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

```
[Out] -(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/6 - ArcSin[3 - 6*x^2]/12
```

Rubi [A] time = 0.0888087, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{1}{6}\sqrt{2-3x^2}\sqrt{3x^2-1}-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/6 - ArcSin[3 - 6*x^2]/12
```

Rubi in Sympy [A] time = 11.111, size = 49, normalized size = 1.26

$$-\frac{\sqrt{-3x^2+2}\sqrt{3x^2-1}}{6}-\frac{\operatorname{atan}\left(\frac{-18x^2+9}{6\sqrt{-9x^4+9x^2-2}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(-3*x**2 + 2)*sqrt(3*x**2 - 1)/6 - atan((-18*x**2 + 9)/(6*sq
rt(-9*x**4 + 9*x**2 - 2)))/12
```

Mathematica [A] time = 0.0302128, size = 37, normalized size = 0.95

$$\frac{1}{6}\left(-\sin^{-1}\left(\sqrt{2-3x^2}\right)-\sqrt{-9x^4+9x^2-2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-Sqrt[-2 + 9*x^2 - 9*x^4] - ArcSin[Sqrt[2 - 3*x^2]])/6
```

Maple [A] time = 0.013, size = 60, normalized size = 1.5

$$\frac{1}{12}\sqrt{-3\,x^2+2}\sqrt{3\,x^2-1}\left(\arcsin\left(6\,x^2-3\right)-2\sqrt{-9\,x^4+9\,x^2-2}\right)\frac{1}{\sqrt{-9\,x^4+9\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/12*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(arcsin(6*x^2-3)-2*(-9*x^4+
9*x^2-2)^(1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

Maxima [A] time = 1.49738, size = 36, normalized size = 0.92

$$-\frac{1}{6}\sqrt{-9x^4+9x^2-2}+\frac{1}{12}\arcsin\left(6x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)*x/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-9*x^4 + 9*x^2 - 2) + 1/12*arcsin(6*x^2 - 3)
```

Fricas [A] time = 0.233216, size = 69, normalized size = 1.77

$$-\frac{1}{6}\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{1}{12}\arctan\left(\frac{3\left(2x^2-1\right)}{2\sqrt{3x^2-1}\sqrt{-3x^2+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)*x/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 1/12*arctan(3/2*(2*x^2 -
1)/(sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)))
```

Sympy [A] time = 10.969, size = 66, normalized size = 1.69

$$\frac{\begin{cases}-\frac{\sqrt{-3x^2+2}\sqrt{3x^2-1}}{2}+\frac{\operatorname{asin}\left(\sqrt{3x^2-1}\right)}{2} & \text{for}\ \left(x\geq\frac{\sqrt{3}}{3}\wedge x<\frac{\sqrt{6}}{3}\right)\vee\left(x\leq-\frac{\sqrt{3}}{3}\wedge x>-\frac{\sqrt{6}}{3}\right)\end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(-3*x**2 + 2)*sqrt(3*x**2 - 1)/2 + asin(sqrt(3*x*
*2 - 1))/2, ((x >= sqrt(3)/3) & (x < sqrt(6)/3)) | ((x <= -sqrt(3
)/3) & (x > -sqrt(6)/3))))/3
```

GIAC/XCAS [A] time = 0.232529, size = 45, normalized size = 1.15

$$-\frac{1}{6}\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{1}{6}\arcsin\left(\sqrt{3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)*x/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 1/6*arcsin(sqrt(3*x^2 - 1
))
```

3.968 $\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{2\sqrt{2}(3b-d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2x(3b-d)\sqrt{bx^2+2}}{3bd\sqrt{dx^2+3}}$$

```
[Out] (-2*(3*b - d)*x*Sqrt[2 + b*x^2])/(3*b*d*Sqrt[3 + d*x^2]) + (x*Sqr
t[2 + b*x^2]*Sqrt[3 + d*x^2])/(3*d) + (2*Sqrt[2]*(3*b - d)*Sqrt[2
 + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)]
)/(3*b*d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) - (
Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1
- (3*b)/(2*d)])/(d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d
*x^2])
```

Rubi [A] time = 0.445376, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{2\sqrt{2}(3b-d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2x(3b-d)\sqrt{bx^2+2}}{3bd\sqrt{dx^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```


```
[Out] (-2*(3*b - d)*x*Sqrt[2 + b*x^2])/(3*b*d*Sqrt[3 + d*x^2]) + (x*Sqr
t[2 + b*x^2]*Sqrt[3 + d*x^2])/(3*d) + (2*Sqrt[2]*(3*b - d)*Sqrt[2
 + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)]
)/(3*b*d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) - (
Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1
- (3*b)/(2*d)])/(d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d
*x^2])
```

Rubi in Sympy [A] time = 51.6652, size = 218, normalized size = 0.9

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{\sqrt{3}\sqrt{bx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{d}x}{3}\right)\middle|-\frac{3b}{2d}+1\right)}{d^{\frac{3}{2}}\sqrt{\frac{3bx^2+6}{2dx^2+6}}\sqrt{dx^2+3}}-\frac{2x\left(3b-d\right)\sqrt{bx^2+2}}{3bd\sqrt{dx^2+3}}+\frac{2\sqrt{3}\left(3b-d\right)\sqrt{bx^2+2}E\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{d}x}{3}\right)\middle|-\frac{3b}{2d}+1\right)}{3bd^{\frac{3}{2}}\sqrt{\frac{3bx^2+6}{2dx^2+6}}\sqrt{dx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)/(3*d) - sqrt(3)*sqrt(b*x**2 +
 2)*elliptic_f(atan(sqrt(3)*sqrt(d)*x/3), -3*b/(2*d) + 1)/(d**(3/
2)*sqrt((3*b*x**2 + 6)/(2*d*x**2 + 6))*sqrt(d*x**2 + 3)) - 2*x*(3
*b - d)*sqrt(b*x**2 + 2)/(3*b*d*sqrt(d*x**2 + 3)) + 2*sqrt(3)*(3*
b - d)*sqrt(b*x**2 + 2)*elliptic_e(atan(sqrt(3)*sqrt(d)*x/3), -3*
b/(2*d) + 1)/(3*b*d**(3/2)*sqrt((3*b*x**2 + 6)/(2*d*x**2 + 6))*sq
```

```
rt(d*x**2 + 3))
```

Mathematica [C] time = 0.190148, size = 127, normalized size = 0.53

$$\frac{\sqrt{b}dx\sqrt{bx^2+2}\sqrt{dx^2+3}-2i\sqrt{3}(3b-2d)F\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)+2i\sqrt{3}(3b-d)E\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{3\sqrt{b}d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] + (2*I)*Sqrt[3]*(3*b
 - d)*EllipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - (2
*I)*Sqrt[3]*(3*b - 2*d)*EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]],
 (2*d)/(3*b)])/(3*Sqrt[b]*d^2)
```

Maple [A] time = 0.025, size = 306, normalized size = 1.3

$$\frac{1}{(3\,bdx^4+9\,bx^2+6\,dx^2+18)\,db}\sqrt{bx^2+2}\sqrt{dx^2+3}\left(x^5b^2d\sqrt{-d}+3\,x^3b^2\sqrt{-d}+2\,x^3bd\sqrt{-d}+3\,EllipticF\left(1/3\,x\sqrt{3}\sqrt{-d},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)*(x^5*b^2*d*(-d)^(1/2)+3*x^3*b
^2*(-d)^(1/2)+2*x^3*b*d*(-d)^(1/2)+3*EllipticF(1/3*x*3^(1/2)*(-d)
^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/2))*2^(1/2)*b*(b*x^2+2)^(1/2)
*(d*x^2+3)^(1/2)-2*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)
*2^(1/2)*(b/d)^(1/2))*2^(1/2)*d*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)-6
*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/
2))*2^(1/2)*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+2*EllipticE(1/3*x*3
^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/2))*2^(1/2)*d*(b*x
^2+2)^(1/2)*(d*x^2+3)^(1/2)+6*x*b*(-d)^(1/2))/(b*d*x^4+3*b*x^2+2*
d*x^2+6)/d/(-d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{(4abcd-3(ad+bc)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8b^2d^2}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}$$

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b^2*d^2) + (x
^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*b*d) - ((4*a*b*c*d - 3*(b*
c + a*d)^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])])/(8*b^(5/2)*d^(5/2))
```

Rubi [A] time = 0.420212, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{(4abcd-3(ad+bc)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8b^2d^2}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b^2*d^2) + (x
^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*b*d) - ((4*a*b*c*d - 3*(b*
c + a*d)^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])])/(8*b^(5/2)*d^(5/2))
```

Rubi in Sympy [A] time = 33.4774, size = 128, normalized size = 0.91

$$\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+bc)}{8b^2d^2}-\frac{\left(abcd-\frac{3(ad+bc)^2}{4}\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{\frac{5}{2}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(4*b*d) - 3*sqrt(a + b*x**
2)*sqrt(c + d*x**2)*(a*d + b*c)/(8*b**2*d**2) - (a*b*c*d - 3*(a*d
 + b*c)**2/4)*atanh(sqrt(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(c + d*
x**2)))/(2*b**(5/2)*d**(5/2))
```

Mathematica [A] time = 0.183064, size = 135, normalized size = 0.96

$$\frac{(3a^2d^2+2abcd+3b^2c^2)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{16b^{5/2}d^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-3ad-3bc+2bdx^2\right)}{8b^2d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c - 3*a*d + 2*b*d*x^2))/(8
*b^2*d^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Log[b*c + a*d +
```

```
2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(
16*b^(5/2)*d^(5/2))
```

Maple [B] time = 0.046, size = 340, normalized size = 2.4

$$\frac{1}{16\,b^2d^2}\left(4\,\sqrt{bdx^4+adx^2+cx^2b+ac}\,x^2db\sqrt{bd}+3\,\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)a^2d^2+2\,\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*d*b*(b*d)^(1/2)+3
*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1
/2)+a*d+b*c)/(b*d)^(1/2))*a^2*d^2+2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c*a*
d*b+3*b^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*a*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*c*b*(b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/d^2
/b^2/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277308, size = 1, normalized size = 0.01

$$\left[\frac{4\left(2\,bdx^2-3\,bc-3\,ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}+\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\log\left(4\left(2\,b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2}\right.}{32\,\sqrt{bd}b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/32*(4*(2*b*d*x^2 - 3*b*c - 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c)*sqrt(b*d) + (3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*log(4*(2*b^2*
d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8
*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d
^2)*x^2)*sqrt(b*d)))/(sqrt(b*d)*b^2*d^2), 1/16*(2*(2*b*d*x^2 - 3*
b*c - 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) + (3*b^2*
c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*s
qrt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/(sqrt(-b*d)*b^2
*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.245114, size = 212, normalized size = 1.5

$$\frac{\sqrt{b^2c + (bx^2 + a)bd - abd}\sqrt{bx^2 + a}\left(\frac{2\,(bx^2+a)}{bd} - \frac{3\,b^2cd+5\,abd^2}{b^2d^3}\right) - \frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\ln\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}d^2}}{8\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*
x^2 + a)/(b*d) - (3*b^2*c*d + 5*a*b*d^2)/(b^2*d^3)) - (3*b^2*c^2
+ 2*a*b*c*d + 3*a^2*d^2)*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2))/(b*abs(b))
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*
d^(3/2))
```

Rubi [A] time = 0.236678, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*
d^(3/2))
```

Rubi in Sympy [A] time = 21.9862, size = 75, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{\frac{3}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(a + b*x**2)*sqrt(c + d*x**2)/(2*b*d) - (a*d + b*c)*atanh(sqr
t(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(c + d*x**2)))/(2*b**(3/2)*d**
(3/2))
```

Mathematica [A] time = 0.0875192, size = 103, normalized size = 1.17

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*Log[b*c
+ a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2]])/(4*b^(3/2)*d^(3/2))
```

Maple [B] time = 0.024, size = 200, normalized size = 2.3

$$-\frac{1}{4bd}\left(a\ln\left(\frac{1}{2}\left(2bdx^2+2\sqrt{bdx^4+adx^2+cx^2b+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)d+b\ln\left(\frac{1}{2}\left(2bdx^2+2\sqrt{bdx^4+adx^2+cx^2b+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(a*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d+b*ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c-2
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/d/(b*d)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263777, size = 1, normalized size = 0.01

$$\left[\frac{(bc+ad)\log\left(-4\left(2b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}+\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2\right)\sqrt{bd}\right.}{8\sqrt{bd}bd}\right.$$
$$\left.-\frac{(bc+ad)\arctan\left(\frac{(2bdx^2+bc+ad)\sqrt{-bd}}{2\sqrt{bx^2+a}\sqrt{dx^2+c}bd}\right)-2\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{4\sqrt{-bd}bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/8*((b*c + a*d)*log(-4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d)) + 4*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c)*sqrt(b*d))/(sqrt(b*d)*b*d), -1/4*((b*c + a
*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a
)*sqrt(d*x^2 + c)*b*d)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
-b*d))/(sqrt(-b*d)*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.241798, size = 140, normalized size = 1.59

$$\frac{\frac{(bc+ad)\ln\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}d}+\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}}{bd}}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*c + a*d)*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c +
 (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d)*sqrt(b*x^2 + a)/(b*d))/abs(b)
```

3.971 $\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqr
t[b]*Sqrt[d])
```

Rubi [A] time = 0.120843, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqr
t[b]*Sqrt[d])
```

Rubi in Sympy [A] time = 14.8411, size = 41, normalized size = 0.91

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] atanh(sqrt(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(sqrt(
b)*sqrt(d))
```

Mathematica [A] time = 0.0361382, size = 63, normalized size = 1.4

$$\frac{\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] Log[b*c + a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2]]/(2*Sqrt[b]*Sqrt[d])
```

Maple [B] time = 0.02, size = 103, normalized size = 2.3

$$\frac{1}{2}\ln\left(\frac{1}{2}\left(2\,bd{x}^{2}+2\,\sqrt{bd{x}^{4}+ad{x}^{2}+c{x}^{2}b+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{b{x}^{2}+a}\sqrt{d{x}^{2}+c}\frac{1}{\sqrt{bd{x}^{4}+ad{x}^{2}+c{x}^{2}b+ac}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2)+a*d+b*c)/(b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.244118, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(4\left(2\,b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}+\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2\right)\sqrt{bd}\right)}{4\sqrt{bd}}, \frac{\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log(4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
8*(b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d))/sqrt(b*d), 1/2*arctan(1/2*(
2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c
)*b*d))/sqrt(-b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.241208, size = 73, normalized size = 1.62

$$-\frac{b\ln\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -b*ln(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b
*d - a*b*d)))/(sqrt(b*d)*abs(b))
```

3.972 $\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(S
qrt[a]*Sqrt[c]))
```

Rubi [A] time = 0.172883, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(S
qrt[a]*Sqrt[c]))
```

Rubi in Sympy [A] time = 16.6223, size = 42, normalized size = 0.91

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt
(a)*sqrt(c))
```

Mathematica [C] time = 0.0834788, size = 153, normalized size = 3.33

$$\frac{2bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*b*d*x^2*AppellF1[1, 1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))])/
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-4*b*d*x^2*AppellF1[1, 1/2, 1/2
, 2, -(a/(b*x^2)), -(c/(d*x^2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -
(a/(b*x^2)), -(c/(d*x^2))] + a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*
x^2)), -(c/(d*x^2))]))
```

Maple [B] time = 0.026, size = 103, normalized size = 2.2

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+2\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2ac\right)\right)\sqrt{dx^2+c}\sqrt{bx^2+a}\frac{1}{\sqrt{ac}}\frac{1}{\sqrt{bdx^4+adx^2+cx^2b+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+2*a*c)/x^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259627, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\left(2a^2c^2+\left(abc^2+a^2cd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}-\left(\left(b^2c^2+6abcd+a^2d^2\right)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2\right)\sqrt{ac}}{x^4}\right)}{4\sqrt{ac}},\ -\frac{\arctan\left(\frac{\left((bc+ad)x^2+2ac\right)\sqrt{-ac}}{2\sqrt{bx^2+a}\sqrt{dx^2+c}ac}\right)}{2\sqrt{-ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(4*(2*a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c) - ((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*
c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(a*c))/x^4)/sqrt(a*c), -1/2*
arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*c)/(sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)*a*c))/sqrt(-a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.229428, size = 120, normalized size = 2.61

$$-\frac{\sqrt{bd}b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b
*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*abs(b))
```

3.973 $\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}$$

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*a*c*x^2) + ((b*c + a*d)*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
3/2)*c^(3/2))
```

Rubi [A] time = 0.268698, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*a*c*x^2) + ((b*c + a*d)*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
3/2)*c^(3/2))
```

Rubi in Sympy [A] time = 22.4453, size = 78, normalized size = 0.86

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}+\frac{(ad+bc)\,\mathrm{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2)*sqrt(c + d*x**2)/(2*a*c*x**2) + (a*d + b*c)*ata
nh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(3/
2)*c**(3/2))
```

Mathematica [C] time = 0.322101, size = 192, normalized size = 2.11

$$\frac{\frac{2bdx^4(ad+bc)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)-bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)-adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}-\left(a+bx^2\right)\left(c+dx^2\right)}{2acx^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a + b*x^2)*(c + d*x^2)) + (2*b*d*(b*c + a*d)*x^4*AppellF1[1,
1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))])/(4*b*d*x^2*AppellF1[1,
1/2, 1/2, 2, -(a/(b*x^2)), -(c/(d*x^2))] - b*c*AppellF1[2, 1/2, 3
/2, 3, -(a/(b*x^2)), -(c/(d*x^2))] - a*d*AppellF1[2, 3/2, 1/2, 3,
 -(a/(b*x^2)), -(c/(d*x^2))]))/(2*a*c*x^2*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

Maple [B] time = 0.03, size = 209, normalized size = 2.3

$$\frac{1}{4\,acx^2}\left(\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+2\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2\,ac\right)\right)x^2ad+\ln\left(\frac{1}{x^2}\left(adx^2+cx^2b+2\sqrt{ac}\sqrt{bdx^4+adx^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a/c*(ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a*d+ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*b*c-2*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))*(d*x^2+c)^(1/2)*(b*x
^2+a)^(1/2)/(a*c)^(1/2)/x^2/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293475, size = 1, normalized size = 0.01

$$\left[\frac{(bc+ad)x^2\log\left(\frac{4\left(2\,a^2c^2+\left(abc^2+a^2cd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}+\left(\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2\right)\sqrt{ac}}{x^4}\right)-4\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{}}{8\sqrt{ac}acx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*((b*c + a*d)*x^2*log((4*(2*a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + ((b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(a*c))/x^4)
- 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/(sqrt(a*c)*a*c*x^2
), 1/4*((b*c + a*d)*x^2*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt
(-a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)) - 2*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c)*sqrt(-a*c))/(sqrt(-a*c)*a*c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.260639, size = 558, normalized size = 6.13

$$\frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd}-\frac{2\left(b^3c^2-2\,ab^2cd+a^2bd^2-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(b\cdots\right.\right.}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\cdots\right.\right.}\right.}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2 - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^2*b*c - (sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*d)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
 (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*a*b^2*c
*d))/abs(b)
```

3.974 $\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=149

$$\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8a^2c^2x^2}-\frac{(3a^2d^2+2abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}$$

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*a*c*x^4) + (3*(b*c + a*d)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a^2*c^2*x^2) - ((3*b^2*c^2 + 2
*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*
Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(5/2))
```

Rubi [A] time = 0.430652, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8a^2c^2x^2}-\frac{(3a^2d^2+2abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*a*c*x^4) + (3*(b*c + a*d)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a^2*c^2*x^2) - ((3*b^2*c^2 + 2
*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*
Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(5/2))
```

Rubi in Sympy [A] time = 46.0844, size = 131, normalized size = 0.88

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+bc)}{8a^2c^2x^2}+\frac{\left(abcd-\frac{3(ad+bc)^2}{4}\right)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2)*sqrt(c + d*x**2)/(4*a*c*x**4) + 3*sqrt(a + b*x*
*2)*sqrt(c + d*x**2)*(a*d + b*c)/(8*a**2*c**2*x**2) + (a*b*c*d -
3*(a*d + b*c)**2/4)*atanh(sqrt(c)*sqrt(a + b*x**2)/(sqrt(a)*sqrt(
c + d*x**2)))/(2*a**(5/2)*c**(5/2))
```

Mathematica [C] time = 0.379321, size = 224, normalized size = 1.5

$$\frac{\frac{2bdx^6(3a^2d^2+2abcd+3b^2c^2)F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{-4bdx^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+bcF_1\left(2;\frac{1}{2},\frac{3}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+adF_1\left(2;\frac{3}{2},\frac{1}{2};3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}+(a+bx^2)(c+dx^2)(-2ac+3adx^2+3bcx^2)}{8a^2c^2x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a + b*x^2)*(c + d*x^2)*(-2*a*c + 3*b*c*x^2 + 3*a*d*x^2) + (2*b*
d*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*x^6*AppellF1[1, 1/2, 1/2, 2
, -(a/(b*x^2)), -(c/(d*x^2))])/(-4*b*d*x^2*AppellF1[1, 1/2, 1/2,
```

```
2, -(a/(b*x^2)), -(c/(d*x^2))] + b*c*AppellF1[2, 1/2, 3/2, 3, -(a
/(b*x^2)), -(c/(d*x^2))] + a*d*AppellF1[2, 3/2, 1/2, 3, -(a/(b*x^
2)), -(c/(d*x^2))]))/(8*a^2*c^2*x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Maple [B] time = 0.034, size = 355, normalized size = 2.4

$$-\frac{1}{16\,a^2c^2x^4}\left(3\,\ln\left(\frac{adx^2+cx^2b+2\,\sqrt{ac}\sqrt{bdx^4+adx^2+cx^2b+ac}+2\,ac}{x^2}\right)x^4a^2d^2+2\,\ln\left(\frac{adx^2+cx^2b+2\,\sqrt{ac}\sqrt{bdx^4+adx}}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16/a^2/c^2*(3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*d^2+2*ln((a*d*x^2+c*x^2
*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*
x^4*a*b*c*d+3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*b^2*c^2-6*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*c*a*(a*c)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/x^
4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.323605, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)x^4\log\left(-\frac{4\left(2\,a^2c^2+\left(abc^2+a^2cd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}-\left(\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2\right)\sqrt{ac}}{x^4}\right)+4\,(}{32\,\sqrt{ac}a^2c^2x^4}\right.$$

$$\left.-\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)x^4\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{-ac}}{2\,\sqrt{bx^2+a}\sqrt{dx^2+c}ac}\right)-2\left(3\,(bc+ad)x^2-2\,ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-ac}}{16\,\sqrt{-ac}a^2c^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^5),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*x^4*log(-(4*(2*a^2*c^2
 + (a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) - ((b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*
c*d)*x^2)*sqrt(a*c))/x^4) + 4*(3*(b*c + a*d)*x^2 - 2*a*c)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/(sqrt(a*c)*a^2*c^2*x^4), -1/1
6*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*x^4*arctan(1/2*((b*c + a*d
)*x^2 + 2*a*c)*sqrt(-a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c))
- 2*(3*(b*c + a*d)*x^2 - 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*s
qrt(-a*c))/(sqrt(-a*c)*a^2*c^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^5),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.975 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=342

$$\frac{x\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)}{15b^3d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4c^{3/2}\sqrt{a+bx^2}(ad+bc)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{15b^2d^2}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}$$

```
[Out] ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^3*d
^2*Sqrt[c + d*x^2]) - (4*(b*c + a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*b^2*d^2) + (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*
d) - (Sqrt[c]*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3
*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
 (4*c^(3/2)*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.854287, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{x\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)}{15b^3d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(8a^2d^2+7abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4c^{3/2}\sqrt{a+bx^2}(ad+bc)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{15b^2d^2}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```


```
[Out] ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^3*d
^2*Sqrt[c + d*x^2]) - (4*(b*c + a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*b^2*d^2) + (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*
d) - (Sqrt[c]*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3
*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
 (4*c^(3/2)*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 97.8215, size = 313, normalized size = 0.92

$$\frac{4a^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad+bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15b^{\frac{5}{2}}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}} - \frac{\sqrt{a}\sqrt{c+dx^2}\,(8a^2d^2+7abcd+8b^2c^2)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15b^{\frac{5}{2}}d^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} - \frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+bc)}{15b^2d^2} + \frac{x\sqrt{c+dx^2}\,(8a^2d^2+7abcd+8b^2c^2)}{15b^2d^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] 4*a**(3/2)*sqrt(c + d*x**2)*(a*d + b*c)*elliptic_f(atan(sqrt(b)*x
/sqrt(a)), -a*d/(b*c) + 1)/(15*b**(5/2)*d**2*sqrt(a*(c + d*x**2)/
(c*(a + b*x**2)))*sqrt(a + b*x**2)) - sqrt(a)*sqrt(c + d*x**2)*(8
*a**2*d**2 + 7*a*b*c*d + 8*b**2*c**2)*elliptic_e(atan(sqrt(b)*x/s
qrt(a)), -a*d/(b*c) + 1)/(15*b**(5/2)*d**3*sqrt(a*(c + d*x**2)/(c
*(a + b*x**2)))*sqrt(a + b*x**2)) + x**3*sqrt(a + b*x**2)*sqrt(c
+ d*x**2)/(5*b*d) - 4*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d +
b*c)/(15*b**2*d**2) + x*sqrt(c + d*x**2)*(8*a**2*d**2 + 7*a*b*c*d
 + 8*b**2*c**2)/(15*b**2*d**3*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.827473, size = 249, normalized size = 0.73

$$\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,(4a^2d^2+3abcd+8b^2c^2)\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right) - ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,(8a^2d^2+7abcd+8b^2c^2)\,E\Big(}{15a^2d^3\left(\frac{b}{a}\right)^{5/2}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*b*c + 4*a*d - 3*b*d*x
^2)) - I*c*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)] + I*c*(8*b^2*c^2 + 3*a*b*c*d + 4*a^2*d^2)*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.036, size = 546, normalized size = 1.6

$$-\frac{1}{15\,b^2d^3\,(bdx^4+adx^2+cx^2b+ac)}\Bigg(-3\sqrt{-\frac{b}{a}}x^7b^2d^3+\sqrt{-\frac{b}{a}}x^5abd^3+\sqrt{-\frac{b}{a}}x^5b^2cd^2+4\sqrt{-\frac{b}{a}}x^3a^2d^3+5\sqrt{-\frac{b}{a}}x^3abcd^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(-3*(-b/a)^(1/2)*x^7*b^2*d^3+(-b/a)^(1/2)*x^5*a*b*d^3+(-b/a
)^(1/2)*x^5*b^2*c*d^2+4*(-b/a)^(1/2)*x^3*a^2*d^3+5*(-b/a)^(1/2)*x
^3*a*b*c*d^2+4*(-b/a)^(1/2)*x^3*b^2*c^2*d+4*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*
c*d^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-8*(
```

```
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^2*c*d^2-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*b^2*c^3+4*(-b/a)^(1/2)*x*a^2*c*d^2+4*(-b/a)^(1/2)*x*a*b*
c^2*d)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3/b^2/(-b/a)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=261

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}(ad+bc)}{3b^2d\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}$$

```
[Out] (-2*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (x
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) + (2*Sqrt[c]*(b*c + a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.502498, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}(ad+bc)}{3b^2d\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```


```
[Out] (-2*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (x
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) + (2*Sqrt[c]*(b*c + a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 63.58, size = 226, normalized size = 0.87

$$-\frac{a^{\frac{3}{2}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3b^{\frac{3}{2}}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{2\sqrt{a}\sqrt{c+dx^2}\,(ad+bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3b^{\frac{3}{2}}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{2x\sqrt{c+dx^2}\,(ad+bc)}{3bd^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**(3/2)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a
*d/(b*c) + 1)/(3*b**(3/2)*d*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))
*sqrt(a + b*x**2)) + 2*sqrt(a)*sqrt(c + d*x**2)*(a*d + b*c)*ellip
tic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*b**(3/2)*d**2*s
qrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)) + x*sqrt(a
```

```
+ b*x**2)*sqrt(c + d*x**2)/(3*b*d) - 2*x*sqrt(c + d*x**2)*(a*d +
b*c)/(3*b*d**2*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.474805, size = 201, normalized size = 0.77

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+2bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+bc)E\left(i\,s\right.}{3bd^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(b*c + a*d)*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - I*c*(2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b
*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 333, normalized size = 1.3

$$\frac{1}{3\,d^2b\left(bdx^4+adx^2+cx^2b+ac\right)}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*((-b/a)^(1/2)*x^5*b*d^2+(-b/a)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x
^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+(-b/a)^(1/2)*x*a*c*d)*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/d^2/b/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.169276, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 22.4463, size = 97, normalized size = 0.84

$$-\frac{\sqrt{a}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x\sqrt{c+dx^2}}{d\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*
d/(b*c) + 1)/(sqrt(b)*d*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqr
t(a + b*x**2)) + x*sqrt(c + d*x**2)/(d*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.112398, size = 122, normalized size = 1.05

$$-\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{d\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)]))/(Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.027, size = 129, normalized size = 1.1

$$\frac{c}{d\left(bd x^{4}+ad x^{2}+c x^{2} b+ac\right)}\left(-EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{\frac{d x^{2}+c}{c}}\sqrt{\frac{b x^{2}+a}{a}}\sqrt{b x^{2}+a}\sqrt{d x^{2}+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2)))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*c*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{b x^{2}+a}\sqrt{d x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{2}}{\sqrt{b x^{2}+a}\sqrt{d x^{2}+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{a+b x^{2}}\sqrt{c+d x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{b x^{2}+a}\sqrt{d x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.978 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=153

$$\frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(a*c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(a*c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.286172, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(a*c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(a*c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 39.3731, size = 126, normalized size = 0.82

$$\frac{bx\sqrt{c+dx^2}}{ac\sqrt{a+bx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{b}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{a}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x*sqrt(c + d*x**2)/(a*c*sqrt(a + b*x**2)) - sqrt(a + b*x**2)*sq
rt(c + d*x**2)/(a*c*x) - sqrt(b)*sqrt(c + d*x**2)*elliptic_e(atan
(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(sqrt(a)*c*sqrt(a*(c + d*x**
2)/(c*(a + b*x**2)))*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.528746, size = 146, normalized size = 0.95

$$\frac{-\frac{(a+bx^2)(c+dx^2)}{cx}-ia\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((a + b*x^2)*(c + d*x^2))/(c*x)) - I*a*Sqrt[b/a]*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
```

```
d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.031, size = 224, normalized size = 1.5

$$\frac{1}{cxa\left(bdx^4+adx^2+cx^2b+ac\right)}\left(-\sqrt{-\frac{b}{a}}x^4bd-bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}xEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-(-b/a)^(1/2)*x^4*b*d-b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))-(-b/a)^(1/2)*x^2*a*d-(-b/a)^(1/2)*x^2*b*c-(-b/a)^(1/2)*a*c)*(
d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/x/c/a/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+cx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

3.979 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=307

$$\frac{2\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3a^2c^2x}-\frac{2dx\sqrt{a+bx^2}(ad+bc)}{3a^2c^2\sqrt{c+dx^2}}-\frac{b\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

```
[Out] (-2*d*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*a^2*c^2*Sqrt[c + d*x^2])
- (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*a*c*x^3) + (2*(b*c + a*d)*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*a^2*c^2*x) + (2*Sqrt[d]*(b*c
+ a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*a^2*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.757315, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3a^2c^2x}-\frac{2dx\sqrt{a+bx^2}(ad+bc)}{3a^2c^2\sqrt{c+dx^2}}-\frac{b\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*d*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*a^2*c^2*Sqrt[c + d*x^2])
- (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*a*c*x^3) + (2*(b*c + a*d)*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*a^2*c^2*x) + (2*Sqrt[d]*(b*c
+ a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*a^2*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 105.076, size = 274, normalized size = 0.89

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}-\frac{2bx\sqrt{c+dx^2}\,(ad+bc)}{3a^2c^2\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+bc)}{3a^2c^2x}-\frac{\sqrt{b}d\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3\sqrt{a}c^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{2\sqrt{b}\sqrt{c+dx^2}\,(ad+bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3a^{\frac{3}{2}}c^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2)*sqrt(c + d*x**2)/(3*a*c*x**3) - 2*b*x*sqrt(c +
d*x**2)*(a*d + b*c)/(3*a**2*c**2*sqrt(a + b*x**2)) + 2*sqrt(a + b
*x**2)*sqrt(c + d*x**2)*(a*d + b*c)/(3*a**2*c**2*x) - sqrt(b)*d*s
```

```
qrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) +
1)/(3*sqrt(a)*c**2*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a +
 b*x**2)) + 2*sqrt(b)*sqrt(c + d*x**2)*(a*d + b*c)*elliptic_e(ata
n(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*a**(3/2)*c**2*sqrt(a*(c
+ d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.595675, size = 229, normalized size = 0.75

$$\frac{\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(-ac+2adx^2+2bcx^2\right)-ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+2bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ibcx^3\sqrt{\frac{bx}{a}}}{3a^2c^2x^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) + 2*b*c*x^2 + 2*a*d*x^
2) + (2*I)*b*c*(b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(2*b
*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*c^2*x^3*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.034, size = 435, normalized size = 1.4

$$\frac{1}{3\,a^2c^2x^3\left(bdx^4+adx^2+cx^2b+ac\right)}\left(2\sqrt{-\frac{b}{a}}x^6abd^2+2\sqrt{-\frac{b}{a}}x^6b^2cd+bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathit{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(2*(-b/a)^(1/2)*x^6*a*b*d^2+2*(-b/a)^(1/2)*x^6*b^2*c*d+b*d*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*x^3*a*c+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2-2*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*x^3*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2+2*(-b/a)^(1/2)
*x^4*a^2*d^2+3*(-b/a)^(1/2)*x^4*a*b*c*d+2*(-b/a)^(1/2)*x^4*b^2*c^
2+(-b/a)^(1/2)*x^2*a^2*c*d+(-b/a)^(1/2)*x^2*a*b*c^2-(-b/a)^(1/2)*
a^2*c^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/x^3/c^2/a^2
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

3.980 $\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}$$

```
[Out] -((a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2])) + (Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(2*b^2*d) - ((b*c + 3*a*d)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(5/2)*
d^(3/2))
```

Rubi [A] time = 0.420238, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2])) + (Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(2*b^2*d) - ((b*c + 3*a*d)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(5/2)*
d^(3/2))
```

Rubi in Sympy [A] time = 35.8505, size = 112, normalized size = 0.87

$$\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}\,(ad-bc)}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d}-\frac{(3ad+bc)\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2b^{\frac{5}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c + d*x**2)/(b**2*sqrt(a + b*x**2)*(a*d - b*c)) + sqrt(
a + b*x**2)*sqrt(c + d*x**2)/(2*b**2*d) - (3*a*d + b*c)*atanh(sqr
t(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(2*b**(5/2)*d**
(3/2))
```

Mathematica [A] time = 0.250161, size = 129, normalized size = 1.

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2a^2}{(a+bx^2)(ad-bc)}+\frac{1}{d}\right)}{2b^2}-\frac{(3ad+bc)\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{4b^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(d^(-1) + (2*a^2)/((-(b*c) + a*d
)*(a + b*x^2))))/(2*b^2) - ((b*c + 3*a*d)*Log[b*c + a*d + 2*b*d*x
^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]])/(4*b^(5/
```

```
2)*d^(3/2))
```

Maple [B] time = 0.054, size = 511, normalized size = 4.

$$-\frac{1}{4\,b^{2}d\,(ad-bc)}\left(3\,\ln\left(1/2\,\frac{2\,bdx^{2}+2\,\sqrt{(bx^{2}+a)\,(dx^{2}+c)}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^{2}a^{2}bd^{2}-2\,\ln\left(1/2\,\frac{2\,bdx^{2}+2\,\sqrt{(bx^{2}+a)\,(dx^{2}}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/
2)+a*d+b*c)/(b*d)^(1/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*b*d*x^2+2*((b*
x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a*b
^2*c*d-ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2
)+a*d+b*c)/(b*d)^(1/2))*x^2*b^3*c^2-2*(b*d)^(1/2)*((b*x^2+a)*(d*x
^2+c))^(1/2)*x^2*a*b*d+2*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*
x^2*b^2*c+3*a^3*d^2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/
2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))-2*ln(1/2*(2*b*d*x^2+2*((b*x^
2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c*d
-ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+
b*c)/(b*d)^(1/2))*a*b^2*c^2-6*a^2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*
d)^(1/2)*d+2*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a*b*c)/b^2*(
d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/d/(b*d)^(1/2)/(a*d-b*c)/((b*x^2+a)
*(d*x^2+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.377386, size = 1, normalized size = 0.01

$$\left[\frac{4\left(abc-3\,a^{2}d+\left(b^{2}c-abd\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{bd}+\left(ab^{2}c^{2}+2\,a^{2}bcd-3\,a^{3}d^{2}+\left(b^{3}c^{2}+2\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{2}\right)\log\left(-\right.}{8\left(ab^{3}cd-a^{2}b^{2}d^{2}+\left(b^{4}cd-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b*c - 3*a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(b*d) + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 +
 (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*log(-4*(2*b^2*d^2*x^2
 + b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*d^
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2
)*sqrt(b*d)))/((a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x
^2)*sqrt(b*d)), 1/4*(2*(a*b*c - 3*a^2*d + (b^2*c - a*b*d)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d) - (a*b^2*c^2 + 2*a^2*b*c
*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*arcta
n(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*b*d)))/((a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)
*x^2)*sqrt(-b*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.559617, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.981 $\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}$$

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(3/2)*Sqrt[d
])
```

Rubi [A] time = 0.231633, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(3/2)*Sqrt[d
])
```

Rubi in Sympy [A] time = 21.1407, size = 71, normalized size = 0.86

$$-\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}\,(ad-bc)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*sqrt(c + d*x**2)/(b*sqrt(a + b*x**2)*(a*d - b*c)) + atanh(sqrt
(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a + b*x**2)))/(b**(3/2)*sqrt(d
))
```

Mathematica [A] time = 0.121371, size = 101, normalized size = 1.22

$$\frac{\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{2b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + Log[b*c + a
*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
]]/(2*b^(3/2)*Sqrt[d])
```

Maple [B] time = 0.031, size = 292, normalized size = 3.5

$$\frac{1}{2\,(ad-bc)\,b}\left(\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{(bx^2+a)\,(dx^2+c)}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)x^2abd-\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{(bx^2+a)\,(dx^2+c)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*x^2*a*b*d-ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d
*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^2*c+ln(1/2
*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b
*d)^(1/2))*a^2*d-ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*
(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b*c-2*a*((b*x^2+a)*(d*x^2+c))
^(1/2)*(b*d)^(1/2))/b*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(b*d)^(1/2)
/(a*d-b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.338405, size = 1, normalized size = 0.01

$$\left[\frac{4\,\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}a+\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\log\left(4\left(2\,b^2d^2x^2+b^2cd+abd^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}+\left(8\,b^2d^2x^4+\right.\right.}{4\,(ab^2c-a^2bd+(b^3c-ab^2d)x^2)\sqrt{bd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)*a + (a*b*c - a^
2*d + (b^2*c - a*b*d)*x^2)*log(4*(2*b^2*d^2*x^2 + b^2*c*d + a*b*d
^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 +
6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d)))/((a*
b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(b*d)), 1/2*(2*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)*a + (a*b*c - a^2*d + (b^2*c
 - a*b*d)*x^2)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/((a*b^2*c - a^2*b*d + (b^3*c
- a*b^2*d)*x^2)*sqrt(-b*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.592749, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.982 $\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(bc-ad)}$$

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Rubi [A] time = 0.0976271, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Rubi in Sympy [A] time = 10.1619, size = 26, normalized size = 0.76

$$\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c + d*x**2)/(sqrt(a + b*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.0421779, size = 33, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/((-(b*c) + a*d)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.007, size = 30, normalized size = 0.9

$$\frac{1}{ad-bc}\sqrt{dx^2+c}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25334, size = 65, normalized size = 1.91

$$-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{abc-a^2d+(b^2c-abd)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)
*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.246796, size = 95, normalized size = 2.79

$$-\frac{2\sqrt{bd}b}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)*abs(b))
```

3.983 $\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^2}(3bc-2ad)}{3b^2\sqrt{a+bx^2}(bc-ad)^2}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2
*a*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)^2*Sqrt[a +
 b*x^2]) + ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*
x^2])]/(b^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.37428, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^2}(3bc-2ad)}{3b^2\sqrt{a+bx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2
*a*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)^2*Sqrt[a +
 b*x^2]) + ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*
x^2])]/(b^(5/2)*Sqrt[d])
```

Rubi in Sympy [A] time = 38.348, size = 124, normalized size = 0.91

$$\frac{a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{\frac{3}{2}}(ad-bc)}-\frac{2a\sqrt{c+dx^2}(2ad-3bc)}{3b^2\sqrt{a+bx^2}(ad-bc)^2}+\frac{\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{\frac{5}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c + d*x**2)/(3*b**2*(a + b*x**2)**(3/2)*(a*d - b*c)) -
2*a*sqrt(c + d*x**2)*(2*a*d - 3*b*c)/(3*b**2*sqrt(a + b*x**2)*(a*
d - b*c)**2) + atanh(sqrt(d)*sqrt(a + b*x**2)/(sqrt(b)*sqrt(c + d
*x**2)))/(b**(5/2)*sqrt(d))
```

Mathematica [A] time = 0.328669, size = 133, normalized size = 0.97

$$\frac{a\sqrt{c+dx^2}\left(-3a^2d+ab\left(5c-4dx^2\right)+6b^2cx^2\right)}{3b^2(a+bx^2)^{3/2}(bc-ad)^2}+\frac{\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\sqrt{c+dx^2}+ad+bc+2bdx^2\right)}{2b^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*(-3*a^2*d + 6*b^2*c*x^2 + a*b*(5*c - 4*d*x^2))
)/(3*b^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + Log[b*c + a*d + 2*b*d
*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]]/(2*b^(5
```

```
/2)*Sqrt[d])
```

Maple [B] time = 0.054, size = 609, normalized size = 4.5

$$\frac{1}{6\,b^{2}\left(ad-bc\right)^{2}\left(bd x^{4}+ad x^{2}+c x^{2} b+ac\right)}\left(-8\,\sqrt{bd}x^{4}a^{2}bd^{2}+12\,x^{4}ab^{2}cd\sqrt{bd}+3\,\ln\left(1/2\,\frac{2\,bd x^{2}+2\,\sqrt{\left(b x^{2}+a\right)\left(d x^{2}+c\right)}\sqrt{b\ldots}}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-8*(b*d)^(1/2)*x^4*a^2*b*d^2+12*x^4*a*b^2*c*d*(b*d)^(1/2)+3*
ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b
*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a^2*b*d^2-6*ln(1
/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/
(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*b^2*c*d+3*ln(1/2*(
2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d
)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*b^3*c^2-6*x^2*a^3*d^2*(b
*d)^(1/2)+2*(b*d)^(1/2)*x^2*a^2*b*c*d+12*x^2*a*b^2*c^2*(b*d)^(1/2
)+3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a
*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*a^3*d^2-6*ln(1/2
*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b
*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*a^2*b*c*d+3*ln(1/2*(2*b*d*
x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2
))*((b*x^2+a)*(d*x^2+c))^(1/2)*a*b^2*c^2-6*a^3*c*d*(b*d)^(1/2)+10
*(b*d)^(1/2)*a^2*b*c^2)*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)/(b*d)
^(1/2)/(a*d-b*c)^2/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.401892, size = 1, normalized size = 0.01

$$\left[\frac{4\left(5\,a^{2}bc-3\,a^{3}d+2\left(3\,ab^{2}c-2\,a^{2}bd\right)x^{2}\right)\sqrt{b x^{2}+a}\sqrt{d x^{2}+c}\sqrt{bd}+3\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)\ldots\right.}{12\left(a^{2}b^{4}c^{2}-2\,a^{3}b^{3}cd+a^{4}b^{2}d^{2}+\left(b\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(5*a^2*b*c - 3*a^3*d + 2*(3*a*b^2*c - 2*a^2*b*d)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d) + 3*(a^2*b^2*c^2 - 2*a^3*
b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(
a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)*log(4*(2*b^2*d^2*x^2
+ b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c) + (8*b^2*d^2
*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2)
*sqrt(b*d)))/((a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c
^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*
d + a^3*b^3*d^2)*x^2)*sqrt(b*d)), 1/6*(2*(5*a^2*b*c - 3*a^3*d + 2
*(3*a*b^2*c - 2*a^2*b*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqr
```

```
t(-b*d) + 3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*
b*d^2)*x^2)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)/(sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*b*d)))/((a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 2*(a*b^
5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2)*sqrt(-b*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.599341, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.984 $\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{a\sqrt{c+dx^2}}{3b\,(a+bx^2)^{3/2}\,(bc-ad)}-\frac{\sqrt{c+dx^2}(3bc-ad)}{3b\sqrt{a+bx^2}(bc-ad)^2}$$

```
[Out] (a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((3*b*c
 - a*d)*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.21821, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{a\sqrt{c+dx^2}}{3b\,(a+bx^2)^{3/2}\,(bc-ad)}-\frac{\sqrt{c+dx^2}(3bc-ad)}{3b\sqrt{a+bx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((3*b*c
 - a*d)*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 21.1663, size = 71, normalized size = 0.8

$$-\frac{a\sqrt{c+dx^2}}{3b\,(a+bx^2)^{\frac{3}{2}}\,(ad-bc)}+\frac{\sqrt{c+dx^2}\,(ad-3bc)}{3b\sqrt{a+bx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*sqrt(c + d*x**2)/(3*b*(a + b*x**2)**(3/2)*(a*d - b*c)) + sqrt(
c + d*x**2)*(a*d - 3*b*c)/(3*b*sqrt(a + b*x**2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.0882337, size = 54, normalized size = 0.61

$$\frac{\sqrt{c+dx^2}\left(-2ac+adx^2-3bcx^2\right)}{3\,(a+bx^2)^{3/2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(3*(b*c - a*d)^2
*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 63, normalized size = 0.7

$$-\frac{-adx^2+3\,cx^2b+2\,ac}{3\,a^2d^2-6\,cabd+3\,b^2c^2}\sqrt{dx^2+c}\,\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(d*x^2+c)^(1/2)*(-a*d*x^2+3*b*c*x^2+2*a*c)/(b*x^2+a)^(3/2)/(
a^2*d^2-2*a*b*c*d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267402, size = 173, normalized size = 1.94

$$-\frac{\left((3\,bc - ad)x^2 + 2\,ac\right)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{3\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^4 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -1/3*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + bx^2)^{\frac{5}{2}}\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.257286, size = 289, normalized size = 3.25

$$-\frac{2\left(3\sqrt{bd}b^5c^2 - 4\sqrt{bd}ab^4cd + \sqrt{bd}a^2b^3d^2 - 6\sqrt{bd}\left(\sqrt{bx^2 + a}\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2 b^3c + 3\sqrt{bd}\left(\sqrt{bx^2 + a}\sqrt{bd}\right.\right.}{3\left(b^2c - abd - \left(\sqrt{bx^2 + a}\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)^3 b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(b*d)*b^5*c^2 - 4*sqrt(b*d)*a*b^4*c*d + sqrt(b*d)*a^2
*b^3*d^2 - 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^2*b^3*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*
```

```
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b)/((b^2*c -
 a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d))^2)^3*b*abs(b))
```

3.985 $\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^2]/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*d*Sqrt[c
+ d*x^2])/(3*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.147639, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*d*Sqrt[c
+ d*x^2])/(3*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 15.3092, size = 61, normalized size = 0.82

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}\,(ad-bc)^2}+\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] 2*d*sqrt(c + d*x**2)/(3*sqrt(a + b*x**2)*(a*d - b*c)**2) + sqrt(c
 + d*x**2)/(3*(a + b*x**2)**(3/2)*(a*d - b*c))
```

Mathematica [A] time = 0.0628408, size = 52, normalized size = 0.7

$$\frac{\sqrt{c+dx^2}\left(3ad-bc+2bdx^2\right)}{3\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(3*(b*c - a*d)^2*(
a + b*x^2)^(3/2))
```

Maple [A] time = 0.007, size = 60, normalized size = 0.8

$$\frac{2\,bdx^2+3\,ad-bc}{3\,a^2d^2-6\,cabd+3\,b^2c^2}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(2*b*d*x^2+3*a*d-b*c)/(b*x^2+a)^(3/2)/(a^2*d^
2-2*a*b*c*d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270516, size = 170, normalized size = 2.3

$$\frac{\left(2\,bdx^2 - bc + 3\,ad\right)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{3\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^4 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^
2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + bx^2)^{\frac{5}{2}}\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.239711, size = 174, normalized size = 2.35

$$\frac{4\left(b^2c - abd - 3\left(\sqrt{bx^2 + a}\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)\sqrt{bd}b^2d}{3\left(b^2c - abd - \left(\sqrt{bx^2 + a}\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 4/3*(b^2*c - a*b*d - 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))
^2)^3*abs(b))
```

3.986 $\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) + (2
*a*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^2*(a + b*
x^2)^(3/2)) - ((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x
^2])/(15*b^2*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.472144, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) + (2
*a*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^2*(a + b*
x^2)^(3/2)) - ((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x
^2])/(15*b^2*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 43.9394, size = 141, normalized size = 0.92

$$\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{\frac{5}{2}}(ad-bc)}-\frac{2a\sqrt{c+dx^2}\,(3ad-5bc)}{15b^2\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)^2}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c + d*x**2)/(5*b**2*(a + b*x**2)**(5/2)*(a*d - b*c)) -
2*a*sqrt(c + d*x**2)*(3*a*d - 5*b*c)/(15*b**2*(a + b*x**2)**(3/2)
*(a*d - b*c)**2) + sqrt(c + d*x**2)*(3*a**2*d**2 - 10*a*b*c*d + 1
5*b**2*c**2)/(15*b**2*sqrt(a + b*x**2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.15295, size = 91, normalized size = 0.59

$$-\frac{\sqrt{c+dx^2}\left(a^2\left(8c^2-4cdx^2+3d^2x^4\right)+10abcx^2\left(2c-dx^2\right)+15b^2c^2x^4\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(2*c - d*x^2) +
a^2*(8*c^2 - 4*c*d*x^2 + 3*d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^
2)^(5/2))
```

Maple [A] time = 0.012, size = 119, normalized size = 0.8

$$\frac{3\,x^4a^2d^2-10\,x^4abcd+15\,x^4b^2c^2-4\,x^2a^2cd+20\,ac^2bx^2+8\,a^2c^2}{15\,a^3d^3-45\,a^2cd^2b+45\,ac^2db^2-15\,c^3b^3}\sqrt{dx^2+c}\,\left(bx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(3*a^2*d^2*x^4-10*a*b*c*d*x^4+15*b^2*c^2*x^4
-4*a^2*c*d*x^2+20*a*b*c^2*x^2+8*a^2*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.326913, size = 354, normalized size = 2.3

$$-\frac{\left(\left(15\,b^2c^2-10\,abcd+3\,a^2d^2\right)x^4+8\,a^2c^2+4\,\left(5\,abc^2-a^2cd\right)x^2\right)\sqrt{bx^2+a}}{15\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^6+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 8*a^2*c^2 + 4*
(5*a*b*c^2 - a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b
^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3
*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3
- 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^4 + 3*(a^2*b
^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.280999, size = 806, normalized size = 5.23

$$-\frac{2\left(15\,\sqrt{bd}b^8c^4-40\,\sqrt{bd}ab^7c^3d+38\,\sqrt{bd}a^2b^6c^2d^2-16\,\sqrt{bd}a^3b^5cd^3+3\,\sqrt{bd}a^4b^4d^4-60\,\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(b*d)*b^8*c^4 - 40*sqrt(b*d)*a*b^7*c^3*d + 38*sqrt(
b*d)*a^2*b^6*c^2*d^2 - 16*sqrt(b*d)*a^3*b^5*c*d^3 + 3*sqrt(b*d)*a
^4*b^4*d^4 - 60*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^2*b^6*c^3 + 80*sqrt(b*d)*(sqrt(b*x^2
 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*
c^2*d - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 90*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c
^2 - 40*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^4*a*b^3*c*d + 30*sqrt(b*d)*(sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^2*d^2
- 60*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^6*b^2*c + 15*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8)/((b^2*c - a*b*d -
(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)^5*b*abs(b))
```

3.987 $\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=138

$$\frac{a\sqrt{c+dx^2}}{5b\,(a+bx^2)^{5/2}\,(bc-ad)}+\frac{2d\sqrt{c+dx^2}(5bc-ad)}{15b\sqrt{a+bx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}(5bc-ad)}{15b\,(a+bx^2)^{3/2}\,(bc-ad)^2}$$

```
[Out] (a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((5*b*c
 - a*d)*Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^2*(a + b*x^2)^(3/2)) +
 (2*d*(5*b*c - a*d)*Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^3*Sqrt[a +
 b*x^2])
```

Rubi [A] time = 0.294932, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{a\sqrt{c+dx^2}}{5b\,(a+bx^2)^{5/2}\,(bc-ad)}+\frac{2d\sqrt{c+dx^2}(5bc-ad)}{15b\sqrt{a+bx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}(5bc-ad)}{15b\,(a+bx^2)^{3/2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((5*b*c
 - a*d)*Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^2*(a + b*x^2)^(3/2)) +
 (2*d*(5*b*c - a*d)*Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^3*Sqrt[a +
 b*x^2])
```

Rubi in Sympy [A] time = 31.0271, size = 116, normalized size = 0.84

$$-\frac{a\sqrt{c+dx^2}}{5b\,(a+bx^2)^{\frac{5}{2}}\,(ad-bc)}+\frac{2d\sqrt{c+dx^2}\,(ad-5bc)}{15b\sqrt{a+bx^2}\,(ad-bc)^3}+\frac{\sqrt{c+dx^2}\,(ad-5bc)}{15b\,(a+bx^2)^{\frac{3}{2}}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a*sqrt(c + d*x**2)/(5*b*(a + b*x**2)**(5/2)*(a*d - b*c)) + 2*d*s
qrt(c + d*x**2)*(a*d - 5*b*c)/(15*b*sqrt(a + b*x**2)*(a*d - b*c)*
*3) + sqrt(c + d*x**2)*(a*d - 5*b*c)/(15*b*(a + b*x**2)**(3/2)*(a
*d - b*c)**2)
```

Mathematica [A] time = 0.119777, size = 91, normalized size = 0.66

$$\frac{\sqrt{c+dx^2}\left(-5a^2d\left(dx^2-2c\right)-2ab\left(c^2-13cdx^2+d^2x^4\right)-5b^2cx^2\left(c-2dx^2\right)\right)}{15\,(a+bx^2)^{5/2}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-5*b^2*c*x^2*(c - 2*d*x^2) - 5*a^2*d*(-2*c + d*
x^2) - 2*a*b*(c^2 - 13*c*d*x^2 + d^2*x^4)))/(15*(b*c - a*d)^3*(a
+ b*x^2)^(5/2))
```

Maple [A] time = 0.013, size = 125, normalized size = 0.9

$$-\frac{-2\,abd^2x^4+10\,b^2cdx^4-5\,a^2d^2x^2+26\,abcdx^2-5\,b^2c^2x^2+10\,a^2cd-2\,abc^2}{15\,a^3d^3-45\,a^2cd^2b+45\,ac^2db^2-15\,c^3b^3}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(-2*a*b*d^2*x^4+10*b^2*c*d*x^4-5*a^2*d^2*x^
2+26*a*b*c*d*x^2-5*b^2*c^2*x^2+10*a^2*c*d-2*a*b*c^2)/(b*x^2+a)^(5
/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.334607, size = 363, normalized size = 2.63

$$\frac{\left(2\left(5\,b^2cd-abd^2\right)x^4-2\,abc^2+10\,a^2cd-\left(5\,b^2c^2-26\,abcd+5\,a^2d^2\right)x^2\right)\sqrt{bx^2}}{15\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^6+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*b^2*c*d - a*b*d^2)*x^4 - 2*a*b*c^2 + 10*a^2*c*d - (5*b
^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (
b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*
(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^4
 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.284648, size = 637, normalized size = 4.62

$$4\left(5\sqrt{bd}b^8c^3d-11\sqrt{bd}ab^7c^2d^2+7\sqrt{bd}a^2b^6cd^3-\sqrt{bd}a^3b^5d^4-25\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^6c^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] 4/15*(5*sqrt(b*d)*b^8*c^3*d - 11*sqrt(b*d)*a*b^7*c^2*d^2 + 7*sqrt
(b*d)*a^2*b^6*c*d^3 - sqrt(b*d)*a^3*b^5*d^4 - 25*sqrt(b*d)*(sqrt(
b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b
^6*c^2*d + 30*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
 (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c*d^2 - 5*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b
^4*d^3 + 35*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^4*b^4*c*d + 5*sqrt(b*d)*(sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^3*d^2 -
 15*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^6*b^2*d)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^5*b*abs(b))
```

3.988 $\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=113

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{5/2}(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^2]/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*d*Sqrt[c
+ d*x^2])/(15*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - (8*d^2*Sqrt[c +
d*x^2])/(15*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.195863, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*d*Sqrt[c
+ d*x^2])/(15*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - (8*d^2*Sqrt[c +
d*x^2])/(15*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 23.1907, size = 97, normalized size = 0.86

$$\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}\,(ad-bc)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)^2}+\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{\frac{5}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] 8*d**2*sqrt(c + d*x**2)/(15*sqrt(a + b*x**2)*(a*d - b*c)**3) + 4*
d*sqrt(c + d*x**2)/(15*(a + b*x**2)**(3/2)*(a*d - b*c)**2) + sqrt
(c + d*x**2)/(5*(a + b*x**2)**(5/2)*(a*d - b*c))
```

Mathematica [A] time = 0.101822, size = 83, normalized size = 0.73

$$-\frac{\sqrt{c+dx^2}\left(15a^2d^2-10abd\left(c-2dx^2\right)+b^2\left(3c^2-4cdx^2+8d^2x^4\right)\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x^2) + b^2*(3*c
^2 - 4*c*d*x^2 + 8*d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2)
)
```

Maple [A] time = 0.01, size = 113, normalized size = 1.

$$\frac{8\,b^2d^2x^4+20\,abd^2x^2-4\,b^2cdx^2+15\,a^2d^2-10\,cabd+3\,b^2c^2}{15\,a^3d^3-45\,a^2cd^2b+45\,ac^2db^2-15\,c^3b^3}\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4+20*a*b*d^2*x^2-4*b^2*c*d*x^2+
15*a^2*d^2-10*a*b*c*d+3*b^2*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.335065, size = 350, normalized size = 3.1

$$-\frac{\left(8\,b^{2}d^{2}x^{4}+3\,b^{2}c^{2}-10\,abcd+15\,a^{2}d^{2}-4\left(b^{2}cd-5\,abd^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{6}+3\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^2*d^2*x^4 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b
^2*c*d - 5*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3
*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a
*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 -
3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^4 + 3*(a^2*b^4
*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^{2}\right)^{\frac{7}{2}}\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.264999, size = 328, normalized size = 2.9

$$-\frac{16\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}-5\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}b^{2}c+5\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bc}\right.\right.}{15\left(b^{2}c-abd-\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-a}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -16/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c + 5*(s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))
^2*a*b*d + 10*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((b^2*c - a*b*d - (sqrt(b*x
^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^5*a
bs(b))
```

3.989 $\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=217

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(7bc-4ad)}{35b^2\left(a+bx^2\right)^{5/2}(bc-ad)^2}$$

```
[Out] -(a^2*Sqrt[c + d*x^2])/(7*b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) + (2
*a*(7*b*c - 4*a*d)*Sqrt[c + d*x^2])/(35*b^2*(b*c - a*d)^2*(a + b*
x^2)^(5/2)) - ((35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x
^2])/(105*b^2*(b*c - a*d)^3*(a + b*x^2)^(3/2)) + (2*d*(35*b^2*c^2
 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(105*b^2*(b*c - a*d)^
4*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.648056, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}(7bc-4ad)}{35b^2\left(a+bx^2\right)^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(7*b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) + (2
*a*(7*b*c - 4*a*d)*Sqrt[c + d*x^2])/(35*b^2*(b*c - a*d)^2*(a + b*
x^2)^(5/2)) - ((35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x
^2])/(105*b^2*(b*c - a*d)^3*(a + b*x^2)^(3/2)) + (2*d*(35*b^2*c^2
 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(105*b^2*(b*c - a*d)^
4*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 62.3711, size = 204, normalized size = 0.94

$$\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{\frac{7}{2}}(ad-bc)}-\frac{2a\sqrt{c+dx^2}\,(4ad-7bc)}{35b^2\left(a+bx^2\right)^{\frac{5}{2}}(ad-bc)^2}+\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}\,(ad-bc)^4}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b*x**2+a)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c + d*x**2)/(7*b**2*(a + b*x**2)**(7/2)*(a*d - b*c)) -
2*a*sqrt(c + d*x**2)*(4*a*d - 7*b*c)/(35*b**2*(a + b*x**2)**(5/2)
*(a*d - b*c)**2) + 2*d*sqrt(c + d*x**2)*(3*a**2*d**2 - 14*a*b*c*d
 + 35*b**2*c**2)/(105*b**2*sqrt(a + b*x**2)*(a*d - b*c)**4) + sqr
t(c + d*x**2)*(3*a**2*d**2 - 14*a*b*c*d + 35*b**2*c**2)/(105*b**2
*(a + b*x**2)**(3/2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.215599, size = 151, normalized size = 0.7

$$\frac{\sqrt{c+dx^2}\left(7a^3d\left(8c^2-4cdx^2+3d^2x^4\right)+a^2b\left(-8c^3+200c^2dx^2-101cd^2x^4+6d^3x^6\right)-7ab^2cx^2\left(4c^2-37cdx^2+4d^2x^4\right)-3\right.}{105\left(a+bx^2\right)^{7/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-35*b^3*c^2*x^4*(c - 2*d*x^2) + 7*a^3*d*(8*c^2
- 4*c*d*x^2 + 3*d^2*x^4) - 7*a*b^2*c*x^2*(4*c^2 - 37*c*d*x^2 + 4*
d^2*x^4) + a^2*b*(-8*c^3 + 200*c^2*d*x^2 - 101*c*d^2*x^4 + 6*d^3*
x^6)))/(105*(b*c - a*d)^4*(a + b*x^2)^(7/2))
```

Maple [A] time = 0.016, size = 213, normalized size = 1.

$$\frac{6\,a^2bd^3x^6-28\,ab^2cd^2x^6+70\,b^3c^2dx^6+21\,a^3d^3x^4-101\,a^2bcd^2x^4+259\,ab^2c^2dx^4-35\,b^3c^3x^4-28\,a^3cd^2x^2+200\,a^2bc^2dx^2}{105\,a^4d^4-420\,a^3bcd^3+630\,a^2c^2d^2b^2-420\,ac^3db^3+105\,c^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(6*a^2*b*d^3*x^6-28*a*b^2*c*d^2*x^6+70*b^3*
c^2*d*x^6+21*a^3*d^3*x^4-101*a^2*b*c*d^2*x^4+259*a*b^2*c^2*d*x^4-
35*b^3*c^3*x^4-28*a^3*c*d^2*x^2+200*a^2*b*c^2*d*x^2-28*a*b^2*c^3*
x^2+56*a^3*c^2*d-8*a^2*b*c^3)/(b*x^2+a)^(7/2)/(a^4*d^4-4*a^3*b*c*
d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(9/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.494261, size = 609, normalized size = 2.81

$$\frac{\left(2\left(35\,b^3c^2d-14\,ab^2cd^2+3\,a^2bd^3\right)x^6-8\,a^2bc^3+56\,a^3c^2d-}{105\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^8+4\left(ab^7c^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((b*x^2 + a)^(9/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*b^3*c^2*d - 14*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^6 - 8*a^
2*b*c^3 + 56*a^3*c^2*d - (35*b^3*c^3 - 259*a*b^2*c^2*d + 101*a^2*
b*c*d^2 - 21*a^3*d^3)*x^4 - 4*(7*a*b^2*c^3 - 50*a^2*b*c^2*d + 7*a
^3*c*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^4*b^4*c^4 - 4*a
^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8
*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*
b^4*d^4)*x^8 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2
 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^6 + 6*(a^2*b^6*c^4 - 4*a^3*b^
5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^4
+ 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^
2*c*d^3 + a^7*b*d^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b*x**2+a)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.31608, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2 + a)^(9/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] Done
```

3.990 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sq
rt[b]*Sqrt[d]))
```

Rubi [A] time = 0.126019, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sq
rt[b]*Sqrt[d]))
```

Rubi in Sympy [A] time = 14.8084, size = 41, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] atan(sqrt(b)*sqrt(c + d*x**2)/(sqrt(d)*sqrt(a - b*x**2)))/(sqrt(b
)*sqrt(d))
```

Mathematica [C] time = 0.126204, size = 72, normalized size = 1.53

$$\frac{i\log\left(2\sqrt{a-bx^2}\sqrt{c+dx^2}-\frac{i(-ad+bc+2bdx^2)}{\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((I/2)*Log[2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2] - (I*(b*c - a*d + 2*
b*d*x^2))/(Sqrt[b]*Sqrt[d])])/(Sqrt[b]*Sqrt[d])
```

Maple [B] time = 0.057, size = 108, normalized size = 2.3

$$\frac{1}{2}\arctan\left(\frac{2\,bdx^2-ad+bc}{2\,bd}\sqrt{bd}\frac{1}{\sqrt{-bdx^4+adx^2-cx^2b+ac}}\right)\sqrt{-bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{bd}}\frac{1}{\sqrt{-bdx^4+adx^2-cx^2b+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(b*d)^(1/2)*(2*b*d*x^2-a*d+b*c)/b/d/(-b*d*x^4+a*d*
x^2-b*c*x^2+a*c)^(1/2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1
/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.241622, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(4\left(2\,b^2d^2x^2+b^2cd-abd^2\right)\sqrt{-bx^2+a}\sqrt{dx^2+c}+\left(8\,b^2d^2x^4+b^2c^2-6\,abcd+a^2d^2+8\left(b^2cd-abd^2\right)x^2\right)\sqrt{-bd}\right)}{4\sqrt{-bd}},\ \text{arct}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log(4*(2*b^2*d^2*x^2 + b^2*c*d - a*b*d^2)*sqrt(-b*x^2 + a)*s
qrt(d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 +
 8*(b^2*c*d - a*b*d^2)*x^2)*sqrt(-b*d))/sqrt(-b*d), 1/2*arctan(1/
2*(2*b*d*x^2 + b*c - a*d)*sqrt(b*d)/(sqrt(-b*x^2 + a)*sqrt(d*x^2
+ c)*b*d))/sqrt(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.239551, size = 77, normalized size = 1.64

$$\frac{b\ln\left(\left|-\sqrt{-bx^2+a}\sqrt{-bd}+\sqrt{b^2c+(bx^2-a)bd+abd}\right|\right)}{\sqrt{-bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] b*ln(abs(-sqrt(-b*x^2 + a)*sqrt(-b*d) + sqrt(b^2*c + (b*x^2 - a)*
b*d + a*b*d)))/(sqrt(-b*d)*abs(b))
```

3.991 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(S
qrt[b]*Sqrt[d]))
```

Rubi [A] time = 0.134121, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(S
qrt[b]*Sqrt[d]))
```

Rubi in Sympy [A] time = 19.1459, size = 42, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c-dx^2}}{\sqrt{d}\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -atanh(sqrt(b)*sqrt(c - d*x**2)/(sqrt(d)*sqrt(a - b*x**2)))/(sqrt
(b)*sqrt(d))
```

Mathematica [A] time = 0.0648391, size = 67, normalized size = 1.4

$$\frac{\log\left(2\sqrt{b}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}-ad-bc+2bdx^2\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] Log[-(b*c) - a*d + 2*b*d*x^2 + 2*Sqrt[b]*Sqrt[d]*Sqrt[a - b*x^2]*
Sqrt[c - d*x^2]]/(2*Sqrt[b]*Sqrt[d])
```

Maple [B] time = 0.053, size = 111, normalized size = 2.3

$$\frac{1}{2}\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4-adx^2-cx^2b+ac}\sqrt{bd}-ad-bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{-bx^2+a}\sqrt{-dx^2+c}\frac{1}{\sqrt{bdx^4-adx^2-cx^2b+ac}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4-a*d*x^2-b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2)-a*d-b*c)/(b*d)^(1/2))*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(b
*d*x^4-a*d*x^2-b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243986, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(4\left(2\,b^2d^2x^2 - b^2cd - abd^2\right)\sqrt{-bx^2+a}\sqrt{-dx^2+c} + \left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 - 8\left(b^2cd + abd^2\right)x^2\right)\sqrt{bd}\right)}{4\sqrt{bd}}, \frac{\text{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log(4*(2*b^2*d^2*x^2 - b^2*c*d - a*b*d^2)*sqrt(-b*x^2 + a)*s
qrt(-d*x^2 + c) + (8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
- 8*(b^2*c*d + a*b*d^2)*x^2)*sqrt(b*d))/sqrt(b*d), 1/2*arctan(1/2
*(2*b*d*x^2 - b*c - a*d)*sqrt(-b*d)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2
 + c)*b*d))/sqrt(-b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a - bx^2}\sqrt{c - dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

GIAC/XCAS [A] time = 0.241354, size = 77, normalized size = 1.6

$$\frac{b\ln\left(\left|-\sqrt{-bx^2+a}\sqrt{bd} + \sqrt{b^2c - (bx^2-a)bd - abd}\right|\right)}{\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] b*ln(abs(-sqrt(-b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c - (b*x^2 - a)*b
*d - a*b*d)))/(sqrt(b*d)*abs(b))
```

3.992 $\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqr
t[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.156639, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqr
t[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi in Sympy [A] time = 19.5007, size = 99, normalized size = 0.9

$$\frac{x\sqrt{dx^2+3}}{d\sqrt{bx^2+2}}-\frac{\sqrt{2}\sqrt{dx^2+3}E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{2}\right)\middle|1-\frac{2d}{3b}\right)}{\sqrt{b}d\sqrt{\frac{2dx^2+6}{3bx^2+6}}\sqrt{bx^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(d*x**2 + 3)/(d*sqrt(b*x**2 + 2)) - sqrt(2)*sqrt(d*x**2 + 3
)*elliptic_e(atan(sqrt(2)*sqrt(b)*x/2), 1 - 2*d/(3*b))/(sqrt(b)*d
*sqrt((2*d*x**2 + 6)/(3*b*x**2 + 6))*sqrt(b*x**2 + 2))
```

Mathematica [C] time = 0.0716663, size = 72, normalized size = 0.65

$$-\frac{i\sqrt{3}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[3]*(EllipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3
*b)] - EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)]))/(
Sqrt[b]*d)
```

Maple [A] time = 0.028, size = 70, normalized size = 0.6

$$\frac{\sqrt{2}}{b}\left(-EllipticF\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{b}{d}}\right)+EllipticE\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{b}{d}}\right)\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] (-EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1
/2))+EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)
^(1/2)))*2^(1/2)/(-d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.993 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d
*x^2)/c]) - (c*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/
c])/(d*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.233913, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d
*x^2)/c]) - (c*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/
c])/(d*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 39.3365, size = 71, normalized size = 0.82

$$-\frac{c\sqrt{1+\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}+\frac{\sqrt{c+dx^2}E\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -c*sqrt(1 + d*x**2/c)*elliptic_f(asin(x/2), -4*d/c)/(d*sqrt(c + d
*x**2)) + sqrt(c + d*x**2)*elliptic_e(asin(x/2), -4*d/c)/(d*sqrt(
1 + d*x**2/c))
```

Mathematica [A] time = 0.082787, size = 59, normalized size = 0.68

$$\frac{c\sqrt{\frac{dx^2}{c}+1}\left(E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)-F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (c*Sqrt[1 + (d*x^2)/c]*(EllipticE[ArcSin[x/2], (-4*d)/c] - Ellipt
icF[ArcSin[x/2], (-4*d)/c]))/(d*Sqrt[c + d*x^2])
```

Maple [A] time = 0.024, size = 59, normalized size = 0.7

$$\frac{c}{d}\sqrt{\frac{dx^2+c}{c}}\left(-EllipticF\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+EllipticE\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(d*x^2+c)^(1/2)*c*((d*x^2+c)/c)^(1/2)*(-EllipticF(1/2*x,2*(-d/c
)^(1/2))+EllipticE(1/2*x,2*(-d/c)^(1/2)))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.994 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*Elliptic
E[ArcTan[x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c
*(4 + x^2))])
```

Rubi [A] time = 0.126099, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*Elliptic
E[ArcTan[x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c
*(4 + x^2))])
```

Rubi in Sympy [A] time = 17.8391, size = 75, normalized size = 0.85

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{2\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|1-\frac{4d}{c}\right)}{d\sqrt{\frac{4c+4dx^2}{c(x^2+4)}}\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x*sqrt(c + d*x**2)/(d*sqrt(x**2 + 4)) - 2*sqrt(c + d*x**2)*ellipt
ic_e(atan(x/2), 1 - 4*d/c)/(d*sqrt((4*c + 4*d*x**2)/(c*(x**2 + 4)
))*sqrt(x**2 + 4))
```

Mathematica [C] time = 0.0722538, size = 70, normalized size = 0.8

$$-\frac{ic\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)-F\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*c*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[x/2], (4*d)/c] -
 EllipticF[I*ArcSinh[x/2], (4*d)/c]))/(d*Sqrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 76, normalized size = 0.9

$$-2\frac{1}{\sqrt{dx^{2}+c}}\sqrt{\frac{dx^{2}+c}{c}}\left(EllipticF\left(x\sqrt{-\frac{d}{c}},1/2\sqrt{\frac{c}{d}}\right)-EllipticE\left(x\sqrt{-\frac{d}{c}},1/2\sqrt{\frac{c}{d}}\right)\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*(EllipticF(x*(-d/c)^(1/2),
1/2*(c/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2)))/(-d/c
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{dx^{2}+c}\sqrt{x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{2}}{\sqrt{dx^{2}+c}\sqrt{x^{2}+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{c+dx^{2}}\sqrt{x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{dx^{2}+c}\sqrt{x^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.995 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 - (Sqrt[2]*EllipticF[ArcSi
n[x], -3/2])/3
```

Rubi [A] time = 0.107223, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 - (Sqrt[2]*EllipticF[ArcSi
n[x], -3/2])/3
```

Rubi in Sympy [A] time = 16.4008, size = 29, normalized size = 0.94

$$\frac{\sqrt{2}E\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{3}-\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_e(asin(x), -3/2)/3 - sqrt(2)*elliptic_f(asin(x),
 -3/2)/3
```

Mathematica [A] time = 0.0441813, size = 24, normalized size = 0.77

$$\frac{1}{3}\sqrt{2}\left(E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)-F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*(EllipticE[ArcSin[x], -3/2] - EllipticF[ArcSin[x], -3/2]
))/3
```

Maple [A] time = 0.022, size = 31, normalized size = 1.

$$-\frac{\left(EllipticF\left(x,\frac{i}{2}\sqrt{3}\sqrt{2}\right)-EllipticE\left(x,\frac{i}{2}\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*(EllipticF(x,1/2*I*3^(1/2)*2^(1/2))-EllipticE(x,1/2*I*3^(1/2
)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.996 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)|\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|\frac{3}{2}\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], 3/2])/3 + (Sqrt[2]*EllipticF[ArcSi
n[x], 3/2])/3
```

Rubi [A] time = 0.109574, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{3}\sqrt{2}F\left(\sin^{-1}(x)|\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], 3/2])/3 + (Sqrt[2]*EllipticF[ArcSi
n[x], 3/2])/3
```

Rubi in Sympy [A] time = 16.7976, size = 26, normalized size = 0.84

$$-\frac{\sqrt{2}E\left(\operatorname{asin}(x)\middle|\frac{3}{2}\right)}{3}+\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|\frac{3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(2)*elliptic_e(asin(x), 3/2)/3 + sqrt(2)*elliptic_f(asin(x),
 3/2)/3
```

Mathematica [A] time = 0.051221, size = 37, normalized size = 1.19

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] (-EllipticE[ArcSin[Sqrt[3/2]*x], 2/3] + EllipticF[ArcSin[Sqrt[3/2
]*x], 2/3])/Sqrt[3]
```

Maple [A] time = 0.018, size = 29, normalized size = 0.9

$$\frac{\sqrt{2}}{3}\left(\mathit{EllipticF}\left(x,\frac{\sqrt{3}\sqrt{2}}{2}\right)-\mathit{EllipticE}\left(x,\frac{\sqrt{3}\sqrt{2}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/3*2^(1/2)*(EllipticF(x,1/2*3^(1/2)*2^(1/2))-EllipticE(x,1/2*3^(
1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(-3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.997 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 - (Sqrt[2]*EllipticF[ArcSi
n[x/2], -6])/3
```

Rubi [A] time = 0.105872, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 - (Sqrt[2]*EllipticF[ArcSi
n[x/2], -6])/3
```

Rubi in Sympy [A] time = 17.268, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}E\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-6\right)}{3}-\frac{\sqrt{2}F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_e(asin(x/2), -6)/3 - sqrt(2)*elliptic_f(asin(x/2
), -6)/3
```

Mathematica [A] time = 0.0427926, size = 28, normalized size = 0.8

$$\frac{1}{3}\sqrt{2}\left(E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*(EllipticE[ArcSin[x/2], -6] - EllipticF[ArcSin[x/2], -6]
))/3
```

Maple [A] time = 0.031, size = 35, normalized size = 1.

$$-\frac{\sqrt{2}}{3}\left(\mathit{EllipticF}\left(\frac{x}{2},i\sqrt{3}\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{x}{2},i\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*(EllipticF(1/2*x,I*3^(1/2)*2^(1/2))-EllipticE(1/2*x,I*3^(1/2
)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

3.998 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], 6])/3 + (Sqrt[2]*EllipticF[ArcSi
n[x/2], 6])/3
```

Rubi [A] time = 0.110379, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], 6])/3 + (Sqrt[2]*EllipticF[ArcSi
n[x/2], 6])/3
```

Rubi in Sympy [A] time = 16.5586, size = 26, normalized size = 0.74

$$-\frac{\sqrt{2}E\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|6\right)}{3}+\frac{\sqrt{2}F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+4)**(1/2),x)
```

```
[Out] -sqrt(2)*elliptic_e(asin(x/2), 6)/3 + sqrt(2)*elliptic_f(asin(x/2
), 6)/3
```

Mathematica [A] time = 0.0465028, size = 38, normalized size = 1.09

$$-\frac{2\left(E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

```
[Out] (-2*(EllipticE[ArcSin[Sqrt[3/2]*x], 1/6] - EllipticF[ArcSin[Sqrt[
3/2]*x], 1/6]))/Sqrt[3]
```

Maple [A] time = 0.031, size = 45, normalized size = 1.3

$$\frac{2\sqrt{3}}{3}\left(\mathit{EllipticF}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{\sqrt{3}\sqrt{2}}{6}\right)-\mathit{EllipticE}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{\sqrt{3}\sqrt{2}}{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x)
```

```
[Out] 2/3*3^(1/2)*(EllipticF(1/2*x*3^(1/2)*2^(1/2),1/6*3^(1/2)*2^(1/2))
-EllipticE(1/2*x*3^(1/2)*2^(1/2),1/6*3^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+4)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(-3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.999 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

```
[Out] EllipticE[ArcSin[2*x], -3/8]/(3*Sqrt[2]) - EllipticF[ArcSin[2*x],
 -3/8]/(3*Sqrt[2])
```

Rubi [A] time = 0.108538, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticE[ArcSin[2*x], -3/8]/(3*Sqrt[2]) - EllipticF[ArcSin[2*x],
 -3/8]/(3*Sqrt[2])
```

Rubi in Sympy [A] time = 18.2143, size = 32, normalized size = 0.91

$$\frac{\sqrt{2}E\left(\operatorname{asin}(2x)\middle|-\frac{3}{8}\right)}{6}-\frac{\sqrt{2}F\left(\operatorname{asin}(2x)\middle|-\frac{3}{8}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_e(asin(2*x), -3/8)/6 - sqrt(2)*elliptic_f(asin(2
*x), -3/8)/6
```

Mathematica [A] time = 0.0455505, size = 28, normalized size = 0.8

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)-F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (EllipticE[ArcSin[2*x], -3/8] - EllipticF[ArcSin[2*x], -3/8])/(3*
Sqrt[2])
```

Maple [A] time = 0.029, size = 35, normalized size = 1.

$$-\frac{\left(\mathit{EllipticF}\left(2\,x,\frac{i}{4}\sqrt{3}\sqrt{2}\right)-\mathit{EllipticE}\left(2\,x,\frac{i}{4}\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/6*(EllipticF(2*x,1/4*I*3^(1/2)*2^(1/2))-EllipticE(2*x,1/4*I*3^
(1/2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.1000 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{F\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}-\frac{E\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}$$

```
[Out] -EllipticE[ArcSin[2*x], 3/8]/(3*Sqrt[2]) + EllipticF[ArcSin[2*x],
 3/8]/(3*Sqrt[2])
```

Rubi [A] time = 0.112258, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{F\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}-\frac{E\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

```
[Out] -EllipticE[ArcSin[2*x], 3/8]/(3*Sqrt[2]) + EllipticF[ArcSin[2*x],
 3/8]/(3*Sqrt[2])
```

Rubi in Sympy [A] time = 16.5826, size = 29, normalized size = 0.83

$$-\frac{\sqrt{2}E\left(\operatorname{asin}(2x)\middle|\frac{3}{8}\right)}{6}+\frac{\sqrt{2}F\left(\operatorname{asin}(2x)\middle|\frac{3}{8}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*elliptic_e(asin(2*x), 3/8)/6 + sqrt(2)*elliptic_f(asin(2
*x), 3/8)/6
```

Mathematica [A] time = 0.0445071, size = 28, normalized size = 0.8

$$\frac{F\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)-E\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

```
[Out] (-EllipticE[ArcSin[2*x], 3/8] + EllipticF[ArcSin[2*x], 3/8])/(3*S
qrt[2])
```

Maple [A] time = 0.033, size = 33, normalized size = 0.9

$$\frac{\sqrt{2}}{6}\left(\mathit{EllipticF}\left(2\,x,\frac{\sqrt{3}\sqrt{2}}{4}\right)-\mathit{EllipticE}\left(2\,x,\frac{\sqrt{3}\sqrt{2}}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(EllipticF(2*x,1/4*3^(1/2)*2^(1/2))-EllipticE(2*x,1/4
*3^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(-3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.1001 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3] - EllipticF[ArcSin[S
qrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.100675, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3] - EllipticF[ArcSin[S
qrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 15.6391, size = 42, normalized size = 1.

$$\frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3}-\frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -2/3)/3 - sqrt(3)*elliptic_
f(asin(sqrt(6)*x/2), -2/3)/3
```

Mathematica [A] time = 0.042918, size = 37, normalized size = 0.88

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] (EllipticE[ArcSin[Sqrt[3/2]*x], -2/3] - EllipticF[ArcSin[Sqrt[3/2
]*x], -2/3])/Sqrt[3]
```

Maple [A] time = 0.02, size = 47, normalized size = 1.1

$$-\frac{\sqrt{3}}{3}\left(\mathit{EllipticF}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{i}{3}\sqrt{3}\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{i}{3}\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/3*3^(1/2)*(EllipticF(1/2*x*3^(1/2)*2^(1/2),1/3*I*3^(1/2)*2^(1/
2))-EllipticE(1/2*x*3^(1/2)*2^(1/2),1/3*I*3^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-3*x**2 + 2)*sqrt(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1002 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3] - (2*EllipticF[A
rcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rubi [A] time = 0.100589, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3] - (2*EllipticF[A
rcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rubi in Sympy [A] time = 15.5012, size = 46, normalized size = 1.07

$$\frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{1}{6}\right)}{3}-\frac{2\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{1}{6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+4)**(1/2),x)
```

```
[Out] 2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -1/6)/3 - 2*sqrt(3)*ellip
tic_f(asin(sqrt(6)*x/2), -1/6)/3
```

Mathematica [A] time = 0.0419968, size = 38, normalized size = 0.88

$$\frac{2\left(E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

```
[Out] (2*(EllipticE[ArcSin[Sqrt[3/2]*x], -1/6] - EllipticF[ArcSin[Sqrt[
3/2]*x], -1/6]))/Sqrt[3]
```

Maple [A] time = 0.027, size = 47, normalized size = 1.1

$$-\frac{2\sqrt{3}}{3}\left(\mathit{EllipticF}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{i}{6}\sqrt{3}\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{i}{6}\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x)
```

```
[Out] -2/3*3^(1/2)*(EllipticF(1/2*x*3^(1/2)*2^(1/2),1/6*I*3^(1/2)*2^(1/
2))-EllipticE(1/2*x*3^(1/2)*2^(1/2),1/6*I*3^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3x^2+2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+4)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-3*x**2 + 2)*sqrt(x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.1003 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}} \, dx$

Optimal. Leaf size=47

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3]) - EllipticF[ArcS
in[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3])
```

Rubi [A] time = 0.110925, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3]) - EllipticF[ArcS
in[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 16.676, size = 42, normalized size = 0.89

$$\frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{8}{3}\right)}{12}-\frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{8}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -8/3)/12 - sqrt(3)*elliptic
_f(asin(sqrt(6)*x/2), -8/3)/12
```

Mathematica [A] time = 0.0464951, size = 40, normalized size = 0.85

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] (EllipticE[ArcSin[Sqrt[3/2]*x], -8/3] - EllipticF[ArcSin[Sqrt[3/2
]*x], -8/3])/(4*Sqrt[3])
```

Maple [A] time = 0.027, size = 47, normalized size = 1.

$$-\frac{\sqrt{3}}{12}\left(\mathit{EllipticF}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{2\,i}{3}\sqrt{3}\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{2\,i}{3}\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x)
```

```
[Out] -1/12*3^(1/2)*(EllipticF(1/2*x*3^(1/2)*2^(1/2),2/3*I*3^(1/2)*2^(1
/2))-EllipticE(1/2*x*3^(1/2)*2^(1/2),2/3*I*3^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{4\,x^2+1}\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-3x^2+2}\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2/(-3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-3*x**2 + 2)*sqrt(4*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1004 $\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1
+ x^2)])
```

Rubi [A] time = 0.0924041, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1
+ x^2)])
```

Rubi in Sympy [A] time = 13.6039, size = 70, normalized size = 0.88

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] x*sqrt(3*x**2 + 2)/(3*sqrt(x**2 + 1)) - sqrt(2)*sqrt(3*x**2 + 2)*
elliptic_e(atan(x), -1/2)/(3*sqrt((3*x**2 + 2)/(x**2 + 1))*sqrt(x
**2 + 1))
```

Mathematica [C] time = 0.0402996, size = 34, normalized size = 0.42

$$-\frac{1}{3}i\sqrt{2}\left(E\left(i\sinh^{-1}(x)|\frac{3}{2}\right)-F\left(i\sinh^{-1}(x)|\frac{3}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (-I/3)*Sqrt[2]*(EllipticE[I*ArcSinh[x], 3/2] - EllipticF[I*ArcSin
h[x], 3/2])
```

Maple [A] time = 0.019, size = 36, normalized size = 0.5

$$\frac{i}{3}\left(EllipticF\left(ix,\frac{\sqrt{3}\sqrt{2}}{2}\right)-EllipticE\left(ix,\frac{\sqrt{3}\sqrt{2}}{2}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*I*(EllipticF(I*x,1/2*3^(1/2)*2^(1/2))-EllipticE(I*x,1/2*3^(1/
2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.1005 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=82

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4
+ x^2)])
```

Rubi [A] time = 0.0952093, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4
+ x^2)])
```

Rubi in Sympy [A] time = 13.8343, size = 68, normalized size = 0.83

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{2\sqrt{3x^2+2}E\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{\frac{12x^2+8}{2x^2+8}}\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] x*sqrt(3*x**2 + 2)/(3*sqrt(x**2 + 4)) - 2*sqrt(3*x**2 + 2)*ellipt
ic_e(atan(x/2), -5)/(3*sqrt((12*x**2 + 8)/(2*x**2 + 8))*sqrt(x**2
 + 4))
```

Mathematica [C] time = 0.039859, size = 38, normalized size = 0.46

$$-\frac{1}{3}i\sqrt{2}\left(E\left(i\sinh^{-1}\left(\frac{x}{2}\right)\middle|6\right)-F\left(i\sinh^{-1}\left(\frac{x}{2}\right)\middle|6\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (-I/3)*Sqrt[2]*(EllipticE[I*ArcSinh[x/2], 6] - EllipticF[I*ArcSin
h[x/2], 6])
```

Maple [A] time = 0.023, size = 34, normalized size = 0.4

$$\frac{i}{3}\left(\mathit{EllipticF}\left(\frac{i}{2}x,\sqrt{3}\sqrt{2}\right)-\mathit{EllipticE}\left(\frac{i}{2}x,\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*I*(EllipticF(1/2*I*x,3^(1/2)*2^(1/2))-EllipticE(1/2*I*x,3^(1/
2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2/(x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x**2 + 4)*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

3.1006 $\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*Ellipt
icE[ArcTan[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*S
qrt[1 + 4*x^2])
```

Rubi [A] time = 0.107021, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*Ellipt
icE[ArcTan[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*S
qrt[1 + 4*x^2])
```

Rubi in Sympy [A] time = 13.7355, size = 70, normalized size = 0.8

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\operatorname{atan}(2x)|\frac{5}{8}\right)}{6\sqrt{\frac{3x^2+2}{8x^2+2}}\sqrt{4x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(3*x**2 + 2)/(3*sqrt(4*x**2 + 1)) - sqrt(3*x**2 + 2)*ellipt
ic_e(atan(2*x), 5/8)/(6*sqrt((3*x**2 + 2)/(8*x**2 + 2))*sqrt(4*x*
*2 + 1))
```

Mathematica [C] time = 0.0430924, size = 50, normalized size = 0.57

$$-\frac{i\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ((-I/4)*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3] - EllipticF[I*Arc
Sinh[Sqrt[3/2]*x], 8/3]))/Sqrt[3]
```

Maple [C] time = 0.025, size = 48, normalized size = 0.6

$$\frac{i}{12}\left(\mathit{EllipticF}\left(\frac{i}{2}\sqrt{3}\,\sqrt{2}\,x, \frac{2\sqrt{3}\,\sqrt{2}}{3}\right)-\mathit{EllipticE}\left(\frac{i}{2}\sqrt{3}\,\sqrt{2}\,x, \frac{2\sqrt{3}\,\sqrt{2}}{3}\right)\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x)
```

```
[Out] 1/12*I*(EllipticF(1/2*I*3^(1/2)*2^(1/2)*x,2/3*3^(1/2)*2^(1/2))-El
lipticE(1/2*I*3^(1/2)*2^(1/2)*x,2/3*3^(1/2)*2^(1/2)))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{4\,x^2+1}\,\sqrt{3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{3x^2+2}\,\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(3*x**2 + 2)*sqrt(4*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{4\,x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

3.1007 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2}F\left(\cos^{-1}(x)\middle|2\right)-\frac{1}{2}E\left(\cos^{-1}(x)\middle|2\right)$$

```
[Out] -EllipticE[ArcCos[x], 2]/2 - EllipticF[ArcCos[x], 2]/2
```

Rubi [A] time = 0.101875, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{1}{2}F\left(\cos^{-1}(x)\middle|2\right)-\frac{1}{2}E\left(\cos^{-1}(x)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]

[Out] -EllipticE[ArcCos[x], 2]/2 - EllipticF[ArcCos[x], 2]/2
```

Rubi in Sympy [A] time = 18.7968, size = 14, normalized size = 0.82

$$-\frac{E\left(\operatorname{acos}(x)|2\right)}{2}-\frac{F\left(\operatorname{acos}(x)|2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)

[Out] -elliptic_e(acos(x), 2)/2 - elliptic_f(acos(x), 2)/2
```

Mathematica [B] time = 0.0518936, size = 37, normalized size = 2.18

$$\frac{\sqrt{1-2x^2}\left(F\left(\sin^{-1}(x)\middle|2\right)-E\left(\sin^{-1}(x)\middle|2\right)\right)}{2\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]

[Out] (Sqrt[1 - 2*x^2]*(-EllipticE[ArcSin[x], 2] + EllipticF[ArcSin[x],
 2]))/(2*Sqrt[-1 + 2*x^2])
```

Maple [A] time = 0.018, size = 34, normalized size = 2.

$$\frac{\mathit{EllipticF}\left(x,\sqrt{2}\right)-\mathit{EllipticE}\left(x,\sqrt{2}\right)}{2}\sqrt{-2\,x^2+1}\frac{1}{\sqrt{2\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x)
```

```
[Out] 1/2*(EllipticF(x,2^(1/2))-EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/
(2*x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.1008 $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=109

$$\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

```
[Out] (3*(1 - x^2)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*Sqrt[3]*ArcTa
n[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - (9*Log[3 + x^2]
)/(4*2^(2/3)) + (27*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.220752, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*Sqrt[3]*ArcTa
n[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - (9*Log[3 + x^2]
)/(4*2^(2/3)) + (27*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi in Sympy [A] time = 13.586, size = 100, normalized size = 0.92

$$\frac{3\left(-x^2+1\right)^{\frac{5}{3}}}{10}+\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{2}-\frac{9\sqrt[3]{2}\log\left(x^2+3\right)}{8}+\frac{27\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{8}+\frac{9\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] 3*(-x**2 + 1)**(5/3)/10 + 3*(-x**2 + 1)**(2/3)/2 - 9*2**(1/3)*log
(x**2 + 3)/8 + 27*2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/8
+ 9*2**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3
+ 1/3))/4
```

Mathematica [C] time = 0.0595504, size = 63, normalized size = 0.58

$$\frac{3\left(-45\sqrt[3]{\frac{x^2-1}{x^2+3}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{4}{x^2+3}\right)+x^4-7x^2+6\right)}{10\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*(6 - 7*x^2 + x^4 - 45*((-1 + x^2)/(3 + x^2))^(1/3)*Hypergeomet
ric2F1[1/3, 1/3, 4/3, 4/(3 + x^2)]))/(10*(1 - x^2)^(1/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{x^5}{x^2+3} \frac{1}{\sqrt[3]{-x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [A] time = 1.49327, size = 146, normalized size = 1.34

$$\frac{9}{8} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3}\left(4^{\frac{1}{3}} + 2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}} - \frac{9}{16} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{9}{8} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{3}{2}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] 9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^
2 + 1)^(1/3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3)
 + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log
(-4^(1/3) + (-x^2 + 1)^(1/3)) + 3/2*(-x^2 + 1)^(2/3)
```

Fricas [A] time = 0.234945, size = 135, normalized size = 1.24

$$-\frac{3}{80} \cdot 4^{\frac{2}{3}}\left(2 \cdot 4^{\frac{1}{3}}\left(x^2-6\right)\left(-x^2+1\right)^{\frac{2}{3}} - 30\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + 2\right)\right) + 15\log\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}} + 4\right) - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3/80*4^(2/3)*(2*4^(1/3)*(x^2 - 6)*(-x^2 + 1)^(2/3) - 30*sqrt(3)*
arctan(1/6*sqrt(3)*(4^(2/3)*(-x^2 + 1)^(1/3) + 2)) + 15*log(4^(2/
3)*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) + 4) - 30*log(4^(2
/3)*(-x^2 + 1)^(1/3) - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1009 $\int \frac{x^3}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=94

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(x^2+3\right)}{4\ 2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3)
)/Sqrt[3]])/(2*2^(2/3)) + (3*Log[3 + x^2])/(4*2^(2/3)) - (9*Log[2
^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.164891, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(x^2+3\right)}{4\ 2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3)
)/Sqrt[3]])/(2*2^(2/3)) + (3*Log[3 + x^2])/(4*2^(2/3)) - (9*Log[2
^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi in Sympy [A] time = 11.5047, size = 88, normalized size = 0.94

$$-\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{4}+\frac{3\sqrt[3]{2}\log\left(x^2+3\right)}{8}-\frac{9\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{8}-\frac{3\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -3*(-x**2 + 1)**(2/3)/4 + 3*2**(1/3)*log(x**2 + 3)/8 - 9*2**(1/3)
*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/8 - 3*2**(1/3)*sqrt(3)*atan(
sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/4
```

Mathematica [C] time = 0.0432703, size = 58, normalized size = 0.62

$$\frac{3\left(6\sqrt[3]{\frac{x^2-1}{x^2+3}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{4}{x^2+3}\right)+x^2-1\right)}{4\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*(-1 + x^2 + 6*((-1 + x^2)/(3 + x^2))^(1/3)*Hypergeometric2F1[1
/3, 1/3, 4/3, 4/(3 + x^2)]))/(4*(1 - x^2)^(1/3))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-x^2+1)^(1/3)/(x^2+3),x)

[Out] int(x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [A] time = 1.50275, size = 131, normalized size = 1.39

$$-\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")

[Out] -3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x
^2 + 1)^(1/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(
1/3) + (-x^2 + 1)^(2/3)) - 3/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^
(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

Fricas [A] time = 0.233519, size = 165, normalized size = 1.76

$$\frac{3}{16}\cdot 4^{\frac{2}{3}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}(-1)^{\frac{1}{3}}\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+2(-1)^{\frac{2}{3}}\right)\right)-(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}}-4(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")

[Out] 3/16*4^(2/3)*(2*sqrt(3)*(-1)^(1/3)*arctan(-1/6*sqrt(3)*(-1)^(1/3)
*(4^(2/3)*(-x^2 + 1)^(1/3) + 2*(-1)^(2/3))) - (-1)^(1/3)*log(4^(2
/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) - 4*(-
1)^(1/3)) + 2*(-1)^(1/3)*log(4^(2/3)*(-x^2 + 1)^(1/3) - 4*(-1)^(2
/3)) - 4^(1/3)*(-x^2 + 1)^(2/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-x**2+1)**(1/3)/(x**2+3),x)

[Out] Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1010 $\int \frac{x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=79

$$-\frac{\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - L
og[3 + x^2]/(4*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2
^(2/3))
```

Rubi [A] time = 0.120327, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - L
og[3 + x^2]/(4*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2
^(2/3))
```

Rubi in Sympy [A] time = 8.78088, size = 73, normalized size = 0.92

$$-\frac{\sqrt[3]{2}\log\left(x^2+3\right)}{8}+\frac{3\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{8}+\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -2**(1/3)*log(x**2 + 3)/8 + 3*2**(1/3)*log(-(-x**2 + 1)**(1/3) +
2**(2/3))/8 + 2**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)
**(1/3)/3 + 1/3))/4
```

Mathematica [A] time = 0.0674892, size = 84, normalized size = 1.06

$$\frac{2\log\left(2-\sqrt[3]{2-2x^2}\right)-\log\left(\left(2-2x^2\right)^{2/3}+2\sqrt[3]{2-2x^2}+4\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 2*Log[2 - (2
 - 2*x^2)^(1/3)] - Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3
)])/(4*2^(2/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x}{x^{2}+3}\frac{1}{\sqrt[3]{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [A] time = 1.50847, size = 116, normalized size = 1.47

$$\frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^
2 + 1)^(1/3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1
/3) + (-x^2 + 1)^(2/3)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(
1/3))
```

Fricas [A] time = 0.236298, size = 109, normalized size = 1.38

$$\frac{1}{16}\cdot 4^{\frac{2}{3}}\left(2\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(4^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+2\right)\right)-\log\left(4^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{2}{3}}+4\right)+2\log\left(4^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/16*4^(2/3)*(2*sqrt(3)*arctan(1/6*sqrt(3)*(4^(2/3)*(-x^2 + 1)^(1
/3) + 2)) - log(4^(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/
3) + 4) + 2*log(4^(2/3)*(-x^2 + 1)^(1/3) - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\left(x^{2}+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1011 $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=136

$$\frac{\log(x^2+3)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}$$

```
[Out] -ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + Ar
cTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/6 + Lo
g[3 + x^2]/(12*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/4 - Log[2^(2/3
) - (1 - x^2)^(1/3)]/(4*2^(2/3))
```

Rubi [A] time = 0.231748, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{\log(x^2+3)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + Ar
cTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/6 + Lo
g[3 + x^2]/(12*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/4 - Log[2^(2/3
) - (1 - x^2)^(1/3)]/(4*2^(2/3))
```

Rubi in Sympy [A] time = 13.6533, size = 121, normalized size = 0.89

$$-\frac{\log(x^2)}{12}+\frac{\sqrt[3]{2}\log(x^2+3)}{24}+\frac{\log\left(-\sqrt[3]{-x^2+1}+1\right)}{4}-\frac{\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{8}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -log(x**2)/12 + 2**(1/3)*log(x**2 + 3)/24 + log(-(-x**2 + 1)**(1/
3) + 1)/4 - 2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/8 - 2**(
1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/
12 + sqrt(3)*atan(sqrt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/6
```

Mathematica [C] time = 0.212917, size = 111, normalized size = 0.82

$$-\frac{21x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}{8\sqrt[3]{1-x^2}\,(x^2+3)\left(7x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)-9F_1\left(\frac{7}{3};\frac{1}{3},2;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)+F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]   Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-21*x^2*AppellF1[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2])/(8*(1 - x^2)
^(1/3)*(3 + x^2)*(7*x^2*AppellF1[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2
] - 9*AppellF1[7/3, 1/3, 2, 10/3, x^(-2), -3/x^2] + AppellF1[7/3,
 4/3, 1, 10/3, x^(-2), -3/x^2]))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^{2}+3\right)}\frac{1}{\sqrt[3]{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^{2}+3)(-x^{2}+1)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x), x)
```

Fricas [A] time = 0.239655, size = 257, normalized size = 1.89

$$-\frac{1}{144}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}\left(-1\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{2}{3}}-4\left(-1\right)^{\frac{1}{3}}\right)-2\sqrt{3}\left(-1\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4\left(-1\right)^{\frac{2}{3}}\right)+4^{\frac{1}{3}}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x),x, algorithm="fricas")
```

```
[Out] -1/144*4^(2/3)*sqrt(3)*(sqrt(3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(2/3)
*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) - 4*(-1)^(1/3)) - 2*
sqrt(3)*(-1)^(1/3)*log(4^(2/3)*(-x^2 + 1)^(1/3) - 4*(-1)^(2/3)) +
 4^(1/3)*sqrt(3)*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 2
*4^(1/3)*sqrt(3)*log((-x^2 + 1)^(1/3) - 1) - 6*(-1)^(1/3)*arctan(
-1/6*(-1)^(1/3)*(4^(2/3)*sqrt(3)*(-x^2 + 1)^(1/3) + 2*sqrt(3)*(-1
)^(2/3))) - 6*4^(1/3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*s
qrt(3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\left(x^{2}+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1012 $\int \frac{1}{x^3\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=97

$$-\frac{(1-x^2)^{2/3}}{6x^2}-\frac{\log(x^2+3)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^2)^(2/3)/(6*x^2) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]
]/(6*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(36*2^(2/3)) + Log[2^(2/3) -
 (1 - x^2)^(1/3)]/(12*2^(2/3))
```

Rubi [A] time = 0.183307, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{(1-x^2)^{2/3}}{6x^2}-\frac{\log(x^2+3)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(1 - x^2)^(2/3)/(6*x^2) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]
]/(6*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(36*2^(2/3)) + Log[2^(2/3) -
 (1 - x^2)^(1/3)]/(12*2^(2/3))
```

Rubi in Sympy [A] time = 13.0103, size = 85, normalized size = 0.88

$$-\frac{\sqrt[3]{2}\log(x^2+3)}{72}+\frac{\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{24}+\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{36}-\frac{(-x^2+1)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -2**(1/3)*log(x**2 + 3)/72 + 2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2
**(2/3))/24 + 2**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)
**(1/3)/3 + 1/3))/36 - (-x**2 + 1)**(2/3)/(6*x**2)
```

Mathematica [C] time = 0.202599, size = 115, normalized size = 1.19

$$\frac{-\frac{2x^4F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(x^2\left(F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)\right)-6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)\right)}+x^2-1}{6x^2\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-1 + x^2 - (2*x^4*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/((3 + x^2
)*(-6*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3] + x^2*(AppellF1[2, 1/3,
 2, 3, x^2, -x^2/3] - AppellF1[2, 4/3, 1, 3, x^2, -x^2/3]))))/(6*
x^2*(1 - x^2)^(1/3))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(x^{2}+3\right)}\frac{1}{\sqrt[3]{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^{2}+3)(-x^{2}+1)^{\frac{1}{3}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3), x)
```

Fricas [A] time = 0.23311, size = 159, normalized size = 1.64

$$-\frac{4^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}x^{2}\log\left(4^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{2}{3}}+4\right)-2\sqrt{3}x^{2}\log\left(4^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4\right)-6\,x^{2}\arctan\left(\frac{1}{6}\cdot4^{\frac{2}{3}}\sqrt{3}(-x^{2}+1)\right.}{432\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3),x, algorithm="fricas")
```

```
[Out] -1/432*4^(2/3)*sqrt(3)*(sqrt(3)*x^2*log(4^(2/3)*(-x^2 + 1)^(1/3)
+ 4^(1/3)*(-x^2 + 1)^(2/3) + 4) - 2*sqrt(3)*x^2*log(4^(2/3)*(-x^2
 + 1)^(1/3) - 4) - 6*x^2*arctan(1/6*4^(2/3)*sqrt(3)*(-x^2 + 1)^(1
/3) + 1/3*sqrt(3)) + 6*4^(1/3)*sqrt(3)*(-x^2 + 1)^(2/3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\sqrt[3]{-(x-1)(x+1)}\,(x^{2}+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1013 $\int \frac{1}{x^5\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=172

$$-\frac{\left(1-x^2\right)^{2/3}}{18x^2}+\frac{\log\left(x^2+3\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\log(x)}{27}$$

```
[Out] -(1 - x^2)^(2/3)/(12*x^4) - (1 - x^2)^(2/3)/(18*x^2) - ArcTan[(1
+ (2 - 2*x^2)^(1/3))/Sqrt[3]]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(1 +
2*(1 - x^2)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[x]/27 + Log[3 + x^2
]/(108*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/18 - Log[2^(2/3) - (1
- x^2)^(1/3)]/(36*2^(2/3))
```

Rubi [A] time = 0.352539, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{\left(1-x^2\right)^{2/3}}{18x^2}+\frac{\log\left(x^2+3\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(1 - x^2)^(2/3)/(12*x^4) - (1 - x^2)^(2/3)/(18*x^2) - ArcTan[(1
+ (2 - 2*x^2)^(1/3))/Sqrt[3]]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(1 +
2*(1 - x^2)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[x]/27 + Log[3 + x^2
]/(108*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/18 - Log[2^(2/3) - (1
- x^2)^(1/3)]/(36*2^(2/3))
```

Rubi in Sympy [A] time = 22.7552, size = 148, normalized size = 0.86

$$-\frac{\log\left(x^2\right)}{54}+\frac{\sqrt[3]{2}\log\left(x^2+3\right)}{216}+\frac{\log\left(-\sqrt[3]{-x^2+1}+1\right)}{18}-\frac{\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{72}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{108}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{27}-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{18x^2}-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -log(x**2)/54 + 2**(1/3)*log(x**2 + 3)/216 + log(-(-x**2 + 1)**(1
/3) + 1)/18 - 2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/72 - 2
**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3
))/108 + sqrt(3)*atan(sqrt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/27
- (-x**2 + 1)**(2/3)/(18*x**2) - (-x**2 + 1)**(2/3)/(12*x**4)
```

Mathematica [C] time = 0.293443, size = 215, normalized size = 1.25

$$-\frac{\frac{4x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(x^2\left(F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)\right)-6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)\right)}-\frac{21x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}{(x^2+3)\left(7x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)-9F_1\left(\frac{7}{3};\frac{1}{3},2;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)+F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};\frac{1}{x^2}\right.\right.}}{36\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2 - 3/x^4 + x^(-2) - (4*x^2*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])
/((3 + x^2)*(-6*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3] + x^2*(Appell
F1[2, 1/3, 2, 3, x^2, -x^2/3] - AppellF1[2, 4/3, 1, 3, x^2, -x^2/
3]))) - (21*x^2*AppellF1[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2])/((3 +
 x^2)*(7*x^2*AppellF1[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2] - 9*Appel
lF1[7/3, 1/3, 2, 10/3, x^(-2), -3/x^2] + AppellF1[7/3, 4/3, 1, 10
/3, x^(-2), -3/x^2])))/(36*(1 - x^2)^(1/3))
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(x^2+3\right)}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5), x)
```

Fricas [A] time = 0.243442, size = 319, normalized size = 1.85

$$-\frac{4^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^4\log\left(4^{\frac{2}{3}}\,(-1)^{\frac{2}{3}}\,\left(-x^2+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}}-4\,(-1)^{\frac{1}{3}}\right)-2\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^4\log\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4\,(-1)^{\frac{2}{3}}\right)+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5),x, algorithm="fricas")
```

```
[Out] -1/1296*4^(2/3)*sqrt(3)*(sqrt(3)*(-1)^(1/3)*x^4*log(4^(2/3)*(-1)^
(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) - 4*(-1)^(1/3))
 - 2*sqrt(3)*(-1)^(1/3)*x^4*log(4^(2/3)*(-x^2 + 1)^(1/3) - 4*(-1)
^(2/3)) + 2*4^(1/3)*sqrt(3)*x^4*log((-x^2 + 1)^(2/3) + (-x^2 + 1)
^(1/3) + 1) - 4*4^(1/3)*sqrt(3)*x^4*log((-x^2 + 1)^(1/3) - 1) - 6
*(-1)^(1/3)*x^4*arctan(-1/6*(-1)^(1/3)*(4^(2/3)*sqrt(3)*(-x^2 + 1
)^(1/3) + 2*sqrt(3)*(-1)^(2/3))) - 12*4^(1/3)*x^4*arctan(2/3*sqrt
```

```
(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 3*4^(1/3)*sqrt(3)*(2*x^2 +
3)*(-x^2 + 1)^(2/3))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1014 $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=536

$$-\frac{3}{7}\left(1-x^2\right)^{2/3}x+\frac{54x}{7\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}}+\frac{9\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}-\frac{18\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}}-\frac{3\tanh^{-1}(x)}{2\,2^{2/3}}$$

```
[Out] (-3*x*(1 - x^2)^(2/3))/7 + (54*x)/(7*(1 - Sqrt[3] - (1 - x^2)^(1/
3))) + (3*Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(2/3)) + (3*Sqrt[3]*Arc
Tan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)) - (3*
ArcTanh[x])/(2*2^(2/3)) + (9*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/
3))])/(2*2^(2/3)) + (27*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^
(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3]
- (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqr
t[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) -
(18*Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/
3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elliptic
F[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.621052, antiderivative size = 536, normalized size of antiderivative = 1., number

of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{3}{7}\left(1-x^2\right)^{2/3}x+\frac{54x}{7\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}}+\frac{9\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}-\frac{18\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}}-\frac{3\tanh^{-1}(x)}{2\,2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*x*(1 - x^2)^(2/3))/7 + (54*x)/(7*(1 - Sqrt[3] - (1 - x^2)^(1/
3))) + (3*Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(2/3)) + (3*Sqrt[3]*Arc
Tan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)) - (3*
ArcTanh[x])/(2*2^(2/3)) + (9*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/
3))])/(2*2^(2/3)) + (27*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^
(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3]
- (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqr
t[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) -
(18*Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/
3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elliptic
F[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 7.5991, size = 19, normalized size = 0.04

$$\frac{x^5\operatorname{appellf}_1\left(\frac{5}{2},\frac{1}{3},1,\frac{7}{2},x^2,-\frac{x^2}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] x**5*appellf1(5/2, 1/3, 1, 7/2, x**2, -x**2/3)/15
```

Mathematica [C] time = 0.334782, size = 236, normalized size = 0.44

$$\frac{3x\left(\frac{30x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)-15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}-\frac{27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2}\right.\right.}\right.}{7\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*x*(-1 + x^2 - (27*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((3
 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(Appe
llF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
x^2, -x^2/3]))) + (30*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3]
)/((3 + x^2)*(-15*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2
*(AppellF1[5/2, 1/3, 2, 7/2, x^2, -x^2/3] - AppellF1[5/2, 4/3, 1,
 7/2, x^2, -x^2/3])))))/(7*(1 - x^2)^(1/3))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{x^4}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1015 $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=515

$$-\frac{3x}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}}-\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}}+\frac{\tanh^{-1}(x)}{2\,2^{2/3}}$$

```
[Out] (-3*x)/(1 - Sqrt[3] - (1 - x^2)^(1/3)) - (Sqrt[3]*ArcTan[Sqrt[3]/
x])/(2*2^(2/3)) - (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)
^(1/3)))/x])/(2*2^(2/3)) + ArcTanh[x]/(2*2^(2/3)) - (3*ArcTanh[x/
(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (3*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 -
x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(
1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(2*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*S
qrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x
^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1
 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(x*Sqrt[-((1 - (
1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.447887, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{3x}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}}-\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}}+\frac{\tanh^{-1}(x)}{2\,2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*x)/(1 - Sqrt[3] - (1 - x^2)^(1/3)) - (Sqrt[3]*ArcTan[Sqrt[3]/
x])/(2*2^(2/3)) - (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)
^(1/3)))/x])/(2*2^(2/3)) + ArcTanh[x]/(2*2^(2/3)) - (3*ArcTanh[x/
(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (3*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 -
x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(
1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(2*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*S
qrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x
^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1
 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(x*Sqrt[-((1 - (
1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 8.7077, size = 19, normalized size = 0.04

$$\frac{x^3 \operatorname{appellf}_1\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] x**3*appellf1(3/2, 1/3, 1, 5/2, x**2, -x**2/3)/9
```

Mathematica [C] time = 0.138656, size = 120, normalized size = 0.23

$$-\frac{5x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)\left(2x^2\left(F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)-15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-5*x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/((1 - x^2)^(1/3)
*(3 + x^2)*(-15*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2*(
AppellF1[5/2, 1/3, 2, 7/2, x^2, -x^2/3] - AppellF1[5/2, 4/3, 1, 7
/2, x^2, -x^2/3])))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^2}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+1)**(1/3)/(x**2+3),x)

[Out] Integral(x**2/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")

[Out] integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1016 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2
/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi [A] time = 0.0606573, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2
/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi in Sympy [A] time = 12.832, size = 144, normalized size = 1.27

$$\frac{\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{-x+1}+(x+1)^{\frac{2}{3}}\right)}{8}-\frac{\sqrt[3]{2}\log\left((-x+1)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{8}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] 2**(1/3)*log(2**(1/3)*(-x + 1)**(1/3) + (x + 1)**(2/3))/8 - 2**(1
/3)*log((-x + 1)**(2/3) + 2**(1/3)*(x + 1)**(1/3))/8 - 2**(1/3)*s
qrt(3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(x + 1)**(2/3)/(3*(-x +
1)**(1/3)))/12 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*(-x + 1)*
*(2/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)/12
```

Mathematica [C] time = 0.0734627, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(
3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(App
ellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
 x^2, -x^2/3])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1017 $\int \frac{1}{x^2\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=538

$$\frac{x}{3\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{6\ 2^{2/3}}-\frac{\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}(x)}{18\ 2^{2/3}}$$

```
[Out] -(1 - x^2)^(2/3)/(3*x) + x/(3*(1 - Sqrt[3] - (1 - x^2)^(1/3))) -
ArcTan[Sqrt[3]/x]/(6*2^(2/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(6*2^(2/3)*Sqrt[3]) + ArcTanh[x]/(18*2^(
2/3)) - ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(6*2^(2/3)) + (S
qrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3)
+ (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[A
rcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (Sqrt[2]*(1 - (1 - x^2)^(
1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1
/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4
)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^
2)])
```

Rubi [A] time = 0.598849, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{x}{3\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{6\ 2^{2/3}}-\frac{\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}(x)}{18\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(1 - x^2)^(2/3)/(3*x) + x/(3*(1 - Sqrt[3] - (1 - x^2)^(1/3))) -
ArcTan[Sqrt[3]/x]/(6*2^(2/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(6*2^(2/3)*Sqrt[3]) + ArcTanh[x]/(18*2^(
2/3)) - ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(6*2^(2/3)) + (S
qrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3)
+ (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[A
rcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (Sqrt[2]*(1 - (1 - x^2)^(
1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1
/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4
)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^
2)])
```

Rubi in Sympy [A] time = 8.22013, size = 20, normalized size = 0.04

$$-\frac{\text{appellf}_1\left(-\frac{1}{2},\frac{1}{3},1,\frac{1}{2},x^2,-\frac{x^2}{3}\right)}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -appellf1(-1/2, 1/3, 1, 1/2, x**2, -x**2/3)/(3*x)
```

Mathematica [C] time = 0.274591, size = 243, normalized size = 0.45

$$\frac{\frac{54x^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}+\frac{5x^4F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)-15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2} \right.\right.}}{9x\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3 + 3*x^2 + (54*x^2*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((
3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(App
ellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
 x^2, -x^2/3]))) + (5*x^4*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3]
)/((3 + x^2)*(-15*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2
*(AppellF1[5/2, 1/3, 2, 7/2, x^2, -x^2/3] - AppellF1[5/2, 4/3, 1,
 7/2, x^2, -x^2/3]))))/(9*x*(1 - x^2)^(1/3))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(x^2+3\right)}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt[3]{-(x-1)(x+1)}(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1018 $\int \frac{1}{x^4\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=556

$$\frac{2x}{27\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{2\left(1-x^2\right)^{2/3}}{27x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{18\ 2^{2/3}}-\frac{2\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\left(1-x^2\right)^{2/3}}{9x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{54\ 2^{2/3}}$$

```
[Out] -(1 - x^2)^(2/3)/(9*x^3) - (2*(1 - x^2)^(2/3))/(27*x) + (2*x)/(27
*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + ArcTan[Sqrt[3]/x]/(18*2^(2/3)
*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(18
*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(54*2^(2/3)) + ArcTanh[x/(1 + 2^(1
/3)*(1 - x^2)^(1/3))]/(18*2^(2/3)) + (Sqrt[2 + Sqrt[3]]*(1 - (1 -
 x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sq
rt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 -
x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9
*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))^2)]) - (2*Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2
)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*x*Sqrt[-((1 - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.810555, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{2x}{27\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{2\left(1-x^2\right)^{2/3}}{27x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{18\ 2^{2/3}}-\frac{2\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\left(1-x^2\right)^{2/3}}{9x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{54\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(1 - x^2)^(2/3)/(9*x^3) - (2*(1 - x^2)^(2/3))/(27*x) + (2*x)/(27
*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + ArcTan[Sqrt[3]/x]/(18*2^(2/3)
*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(18
*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(54*2^(2/3)) + ArcTanh[x/(1 + 2^(1
/3)*(1 - x^2)^(1/3))]/(18*2^(2/3)) + (Sqrt[2 + Sqrt[3]]*(1 - (1 -
 x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sq
rt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 -
x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9
*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))^2)]) - (2*Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2
)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*x*Sqrt[-((1 - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 8.05951, size = 24, normalized size = 0.04

$$-\frac{\text{appellf}_1\left(-\frac{3}{2},\frac{1}{3},1,-\frac{1}{2},x^2,-\frac{x^2}{3}\right)}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -appellf1(-3/2, 1/3, 1, -1/2, x**2, -x**2/3)/(9*x**3)
```

Mathematica [C] time = 0.268365, size = 245, normalized size = 0.44

$$\frac{-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}+\frac{10x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(2x^2\left(F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)-15F_1\left(\frac{3}{2};\frac{1}{3},1\right.\right.}}{81\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9/x^3 + 3/x + 6*x - (27*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/
3])/((3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^
2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1
, 5/2, x^2, -x^2/3]))) + (10*x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2,
-x^2/3])/((3 + x^2)*(-15*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3]
+ 2*x^2*(AppellF1[5/2, 1/3, 2, 7/2, x^2, -x^2/3] - AppellF1[5/2,
4/3, 1, 7/2, x^2, -x^2/3]))))/(81*(1 - x^2)^(1/3))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(x^2+3\right)}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt[3]{-(x-1)(x+1)}(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1019 $\int \frac{x^7}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=133

$$\frac{9\left(1-x^2\right)^{2/3}\left(14x^2+69\right)}{40\left(x^2+3\right)}-\frac{99\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{99\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3\left(1-x^2\right)^{2/3}x^4}{10\left(x^2+3\right)}$$

```
[Out] (-3*x^4*(1 - x^2)^(2/3))/(10*(3 + x^2)) + (9*(1 - x^2)^(2/3)*(69
+ 14*x^2))/(40*(3 + x^2)) + (99*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(
1/3))/Sqrt[3]])/(8*2^(2/3)) - (99*Log[3 + x^2])/(16*2^(2/3)) + (2
97*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rubi [A] time = 0.279958, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{9\left(1-x^2\right)^{2/3}\left(14x^2+69\right)}{40\left(x^2+3\right)}-\frac{99\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{99\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3\left(1-x^2\right)^{2/3}x^4}{10\left(x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*x^4*(1 - x^2)^(2/3))/(10*(3 + x^2)) + (9*(1 - x^2)^(2/3)*(69
+ 14*x^2))/(40*(3 + x^2)) + (99*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(
1/3))/Sqrt[3]])/(8*2^(2/3)) - (99*Log[3 + x^2])/(16*2^(2/3)) + (2
97*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rubi in Sympy [A] time = 15.4049, size = 121, normalized size = 0.91

$$-\frac{3x^4\left(-x^2+1\right)^{\frac{2}{3}}}{10\left(x^2+3\right)}+\frac{9\left(-x^2+1\right)^{\frac{2}{3}}\left(28x^2+138\right)}{80\left(x^2+3\right)}-\frac{99\sqrt[3]{2}\log\left(x^2+3\right)}{32}+\frac{297\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{32}+\frac{99\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**7/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -3*x**4*(-x**2 + 1)**(2/3)/(10*(x**2 + 3)) + 9*(-x**2 + 1)**(2/3)
*(28*x**2 + 138)/(80*(x**2 + 3)) - 99*2**(1/3)*log(x**2 + 3)/32 +
 297*2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/32 + 99*2**(1/3
)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/16
```

Mathematica [C] time = 0.0793001, size = 82, normalized size = 0.62

$$\frac{3\left(-495\sqrt[3]{\frac{x^2-1}{x^2+3}}\left(x^2+3\right)\ {}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{4}{x^2+3}\right)+4x^6-46x^4-165x^2+207\right)}{40\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(207 - 165*x^2 - 46*x^4 + 4*x^6 - 495*((-1 + x^2)/(3 + x^2))^(
1/3)*(3 + x^2)*Hypergeometric2F1[1/3, 1/3, 4/3, 4/(3 + x^2)]))/(4
0*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{x^7}{(x^2+3)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.50764, size = 170, normalized size = 1.28

$$\frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}} - \frac{99}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{99}{32} \cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{15}{4}\left(-x^2+1\right)^{\frac{2}{3}}+\frac{27\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] 99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-
x^2 + 1)^(1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2
/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3
)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8
*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A] time = 0.235657, size = 174, normalized size = 1.31

$$\frac{3\cdot 4^{\frac{2}{3}}\left(330\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\sqrt{3}\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+2\right)\right)-2\cdot 4^{\frac{1}{3}}\left(4x^4-42x^2-207\right)\left(-x^2+1\right)^{\frac{2}{3}}-165\left(x^2+3\right)\log\left(4^{\frac{2}{3}}\left(- \right.\right.\right.}{320\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/320*4^(2/3)*(330*sqrt(3)*(x^2 + 3)*arctan(1/6*sqrt(3)*(4^(2/3)*
(-x^2 + 1)^(1/3) + 2)) - 2*4^(1/3)*(4*x^4 - 42*x^2 - 207)*(-x^2 +
 1)^(2/3) - 165*(x^2 + 3)*log(4^(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*
(-x^2 + 1)^(2/3) + 4) + 330*(x^2 + 3)*log(4^(2/3)*(-x^2 + 1)^(1/3
) - 4))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1020 $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\ 2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (9*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (21*
Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (2
1*Log[3 + x^2])/(16*2^(2/3)) - (63*Log[2^(2/3) - (1 - x^2)^(1/3)]
)/(16*2^(2/3))
```

Rubi [A] time = 0.229092, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\ 2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (9*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (21*
Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (2
1*Log[3 + x^2])/(16*2^(2/3)) - (63*Log[2^(2/3) - (1 - x^2)^(1/3)]
)/(16*2^(2/3))
```

Rubi in Sympy [A] time = 13.9165, size = 105, normalized size = 0.91

$$-\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{4}-\frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}+\frac{21\sqrt[3]{2}\log\left(x^2+3\right)}{32}-\frac{63\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{32}-\frac{21\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -3*(-x**2 + 1)**(2/3)/4 - 9*(-x**2 + 1)**(2/3)/(8*(x**2 + 3)) + 2
1*2**(1/3)*log(x**2 + 3)/32 - 63*2**(1/3)*log(-(-x**2 + 1)**(1/3)
 + 2**(2/3))/32 - 21*2**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x*
*2 + 1)**(1/3)/3 + 1/3))/16
```

Mathematica [C] time = 0.0611024, size = 77, normalized size = 0.66

$$\frac{3\left(21\sqrt[3]{\frac{x^2-1}{x^2+3}}\left(x^2+3\right)\,_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{4}{x^2+3}\right)+2x^4+7x^2-9\right)}{8\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(-9 + 7*x^2 + 2*x^4 + 21*((-1 + x^2)/(3 + x^2))^(1/3)*(3 + x^2
)*Hypergeometric2F1[1/3, 1/3, 4/3, 4/(3 + x^2)]))/(8*(1 - x^2)^(1
/3)*(3 + x^2))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{x^5}{(x^2+3)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.52221, size = 155, normalized size = 1.34

$$-\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}-\frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] -21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(
-x^2 + 1)^(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1
)^(1/3) + (-x^2 + 1)^(2/3)) - 21/32*4^(2/3)*log(-4^(1/3) + (-x^2
+ 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 +
3)
```

Fricas [A] time = 0.238461, size = 204, normalized size = 1.76

$$\frac{3\cdot 4^{\frac{2}{3}}\left(14\sqrt{3}(-1)^{\frac{1}{3}}\left(x^2+3\right)\arctan\left(-\frac{1}{6}\sqrt{3}(-1)^{\frac{1}{3}}\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+2(-1)^{\frac{2}{3}}\right)\right)-7(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}\right.\right.}{64\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/64*4^(2/3)*(14*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(-1/6*sqrt(3)
*(-1)^(1/3)*(4^(2/3)*(-x^2 + 1)^(1/3) + 2*(-1)^(2/3))) - 7*(-1)^(
1/3)*(x^2 + 3)*log(4^(2/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*
(-x^2 + 1)^(2/3) - 4*(-1)^(1/3)) + 14*(-1)^(1/3)*(x^2 + 3)*log(4^
(2/3)*(-x^2 + 1)^(1/3) - 4*(-1)^(2/3)) - 2*4^(1/3)*(2*x^2 + 9)*(-
x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1021 $\int \frac{x^3}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=101

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) - (3*Log[3 + x^2])/(16*2^(2/3)
) + (9*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rubi [A] time = 0.183978, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) - (3*Log[3 + x^2])/(16*2^(2/3)
) + (9*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rubi in Sympy [A] time = 11.417, size = 94, normalized size = 0.93

$$\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}-\frac{3\sqrt[3]{2}\log\left(x^2+3\right)}{32}+\frac{9\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{32}+\frac{3\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] 3*(-x**2 + 1)**(2/3)/(8*(x**2 + 3)) - 3*2**(1/3)*log(x**2 + 3)/32
 + 9*2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/32 + 3*2**(1/3)
*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/16
```

Mathematica [C] time = 0.047249, size = 70, normalized size = 0.69

$$-\frac{3\left(3\sqrt[3]{\frac{x^2-1}{x^2+3}}\left(x^2+3\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{4}{x^2+3}\right)+x^2-1\right)}{8\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(-1 + x^2 + 3*((-1 + x^2)/(3 + x^2))^(1/3)*(3 + x^2)*Hypergeo
metric2F1[1/3, 1/3, 4/3, 4/(3 + x^2)]))/(8*(1 - x^2)^(1/3)*(3 + x
^2))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2+3\right)^2} \frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.49939, size = 140, normalized size = 1.39

$$\frac{3}{32} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3}\left(4^{\frac{1}{3}} + 2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{3}{64} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{3}{32} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x
^2 + 1)^(1/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(
1/3) + (-x^2 + 1)^(2/3)) + 3/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)
^(1/3)) + 3/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A] time = 0.237465, size = 158, normalized size = 1.56

$$\frac{3 \cdot 4^{\frac{2}{3}}\left(2\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\sqrt{3}\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+2\right)\right) - \left(x^2+3\right)\log\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}} + 4\right) + 2\left(x^2+3\right)\log}{64\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/64*4^(2/3)*(2*sqrt(3)*(x^2 + 3)*arctan(1/6*sqrt(3)*(4^(2/3)*(-x
^2 + 1)^(1/3) + 2)) - (x^2 + 3)*log(4^(2/3)*(-x^2 + 1)^(1/3) + 4^
(1/3)*(-x^2 + 1)^(2/3) + 4) + 2*(x^2 + 3)*log(4^(2/3)*(-x^2 + 1)^
(1/3) - 4) + 2*4^(1/3)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.1022 $\int \frac{x}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{(1-x^2)^{2/3}}{8(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^2)^(2/3)/(8*(3 + x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/S
qrt[3]]/(8*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(48*2^(2/3)) + Log[2^(
2/3) - (1 - x^2)^(1/3)]/(16*2^(2/3))
```

Rubi [A] time = 0.158692, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{(1-x^2)^{2/3}}{8(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -(1 - x^2)^(2/3)/(8*(3 + x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/S
qrt[3]]/(8*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(48*2^(2/3)) + Log[2^(
2/3) - (1 - x^2)^(1/3)]/(16*2^(2/3))
```

Rubi in Sympy [A] time = 10.1217, size = 87, normalized size = 0.86

$$-\frac{(-x^2+1)^{\frac{2}{3}}}{8(x^2+3)}-\frac{\sqrt[3]{2}\log(x^2+3)}{96}+\frac{\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{32}+\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -(-x**2 + 1)**(2/3)/(8*(x**2 + 3)) - 2**(1/3)*log(x**2 + 3)/96 +
2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/32 + 2**(1/3)*sqrt(3
)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/48
```

Mathematica [C] time = 0.0433382, size = 70, normalized size = 0.69

$$\frac{-\sqrt[3]{\frac{x^2-1}{x^2+3}}(x^2+3)\ {}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{4}{x^2+3}\right)+x^2-1}{8\sqrt[3]{1-x^2}(x^2+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-1 + x^2 - ((-1 + x^2)/(3 + x^2))^(1/3)*(3 + x^2)*Hypergeometric
2F1[1/3, 1/3, 4/3, 4/(3 + x^2)])/(8*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{x}{(x^2+3)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [A] time = 1.51993, size = 140, normalized size = 1.39

$$\frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x
^2 + 1)^(1/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^
(1/3) + (-x^2 + 1)^(2/3)) + 1/96*4^(2/3)*log(-4^(1/3) + (-x^2 + 1
)^(1/3)) - 1/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A] time = 0.236092, size = 173, normalized size = 1.71

$$-\frac{4^{\frac{2}{3}}\sqrt{3}\left(\sqrt{3}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}}+4\right)-2\sqrt{3}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4\right)-6\left(x^2+3\right)\arctan\left(\frac{1}{6}\right.\right.}{576\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] -1/576*4^(2/3)*sqrt(3)*(sqrt(3)*(x^2 + 3)*log(4^(2/3)*(-x^2 + 1)^
(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) + 4) - 2*sqrt(3)*(x^2 + 3)*log(4
^(2/3)*(-x^2 + 1)^(1/3) - 4) - 6*(x^2 + 3)*arctan(1/6*4^(2/3)*sqr
t(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 6*4^(1/3)*sqrt(3)*(-x^2 +
1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1023 $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=158

$$\frac{(1-x^2)^{2/3}}{24(x^2+3)}+\frac{5\log(x^2+3)}{144\ 2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\log(x)}{18}$$

```
[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) - (5*ArcTan[(1 + (2 - 2*x^2)^(1/3)
)/Sqrt[3]])/(24*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))
/Sqrt[3]]/(6*Sqrt[3]) - Log[x]/18 + (5*Log[3 + x^2])/(144*2^(2/3)
) + Log[1 - (1 - x^2)^(1/3)]/12 - (5*Log[2^(2/3) - (1 - x^2)^(1/3
)])/(48*2^(2/3))
```

Rubi [A] time = 0.315347, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{(1-x^2)^{2/3}}{24(x^2+3)}+\frac{5\log(x^2+3)}{144\ 2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) - (5*ArcTan[(1 + (2 - 2*x^2)^(1/3)
)/Sqrt[3]])/(24*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))
/Sqrt[3]]/(6*Sqrt[3]) - Log[x]/18 + (5*Log[3 + x^2])/(144*2^(2/3)
) + Log[1 - (1 - x^2)^(1/3)]/12 - (5*Log[2^(2/3) - (1 - x^2)^(1/3
)])/(48*2^(2/3))
```

Rubi in Sympy [A] time = 18.459, size = 141, normalized size = 0.89

$$\frac{(-x^2+1)^{\frac{2}{3}}}{24(x^2+3)}-\frac{\log(x^2)}{36}+\frac{5\sqrt[3]{2}\log(x^2+3)}{288}+\frac{\log\left(-\sqrt[3]{-x^2+1}+1\right)}{12}-\frac{5\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{96}-\frac{5\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{144}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] (-x**2 + 1)**(2/3)/(24*(x**2 + 3)) - log(x**2)/36 + 5*2**(1/3)*lo
g(x**2 + 3)/288 + log(-(-x**2 + 1)**(1/3) + 1)/12 - 5*2**(1/3)*lo
g(-(-x**2 + 1)**(1/3) + 2**(2/3))/96 - 5*2**(1/3)*sqrt(3)*atan(sq
rt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/144 + sqrt(3)*atan(s
qrt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/18
```

Mathematica [C] time = 0.152672, size = 205, normalized size = 1.3

$$\frac{\frac{2x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{x^2\left(F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)\right)-6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}-\frac{21x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}{7x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)-9F_1\left(\frac{7}{3};\frac{1}{3},2;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)+F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}-x^2+}{24\sqrt[3]{1-x^2}\,(x^2+3)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2 + (2*x^2*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/(-6*Appell
F1[1, 1/3, 1, 2, x^2, -x^2/3] + x^2*(AppellF1[2, 1/3, 2, 3, x^2,
-x^2/3] - AppellF1[2, 4/3, 1, 3, x^2, -x^2/3])) - (21*x^2*AppellF
1[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2])/(7*x^2*AppellF1[4/3, 1/3, 1,
 7/3, x^(-2), -3/x^2] - 9*AppellF1[7/3, 1/3, 2, 10/3, x^(-2), -3/
x^2] + AppellF1[7/3, 4/3, 1, 10/3, x^(-2), -3/x^2]))/(24*(1 - x^2
)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(x^2+3)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x), x)
```

Fricas [A] time = 0.237826, size = 332, normalized size = 2.1

$$-\frac{4^{\frac{2}{3}}\sqrt{3}\left(5\sqrt{3}(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}}-4(-1)^{\frac{1}{3}}\right)-10\sqrt{3}(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}\left(-x^2+1\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x),x, algorithm="fricas")
```

```
[Out] -1/1728*4^(2/3)*sqrt(3)*(5*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*log(4^(2/
3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) - 4*(-1
)^(1/3)) - 10*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*log(4^(2/3)*(-x^2 + 1)
^(1/3) - 4*(-1)^(2/3)) + 4*4^(1/3)*sqrt(3)*(x^2 + 3)*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 8*4^(1/3)*sqrt(3)*(x^2 + 3)*lo
g((-x^2 + 1)^(1/3) - 1) - 30*(-1)^(1/3)*(x^2 + 3)*arctan(-1/6*(-1
)^(1/3)*(4^(2/3)*sqrt(3)*(-x^2 + 1)^(1/3) + 2*sqrt(3)*(-1)^(2/3))
```

```
) - 24*4^(1/3)*(x^2 + 3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/
3*sqrt(3)) - 6*4^(1/3)*sqrt(3)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1024 $\int \frac{1}{x^3 \sqrt[3]{1-x^2} (3+x^2)^2} \, dx$

Optimal. Leaf size=183

$$-\frac{(1-x^2)^{2/3}}{6x^2(x^2+3)}-\frac{5(1-x^2)^{2/3}}{72(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}-\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\log(x)}{54}$$

```
[Out] (-5*(1 - x^2)^(2/3))/(72*(3 + x^2)) - (1 - x^2)^(2/3)/(6*x^2*(3 +
 x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[
3]) - ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[
x]/54 - Log[3 + x^2]/(48*2^(2/3)) - Log[1 - (1 - x^2)^(1/3)]/36 +
 Log[2^(2/3) - (1 - x^2)^(1/3)]/(16*2^(2/3))
```

Rubi [A] time = 0.387676, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{(1-x^2)^{2/3}}{6x^2(x^2+3)}-\frac{5(1-x^2)^{2/3}}{72(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}-\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\log(x)}{54}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-5*(1 - x^2)^(2/3))/(72*(3 + x^2)) - (1 - x^2)^(2/3)/(6*x^2*(3 +
 x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[
3]) - ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[
x]/54 - Log[3 + x^2]/(48*2^(2/3)) - Log[1 - (1 - x^2)^(1/3)]/36 +
 Log[2^(2/3) - (1 - x^2)^(1/3)]/(16*2^(2/3))
```

Rubi in Sympy [A] time = 22.7324, size = 156, normalized size = 0.85

$$-\frac{5(-x^2+1)^{\frac{2}{3}}}{72(x^2+3)}+\frac{\log(x^2)}{108}-\frac{\sqrt[3]{2}\log(x^2+3)}{96}-\frac{\log\left(-\sqrt[3]{-x^2+1}+1\right)}{36}+\frac{\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{32}+\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{48}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{54}-\frac{(-x^2+1)^{\frac{2}{3}}}{6x^2(x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -5*(-x**2 + 1)**(2/3)/(72*(x**2 + 3)) + log(x**2)/108 - 2**(1/3)*
log(x**2 + 3)/96 - log(-(-x**2 + 1)**(1/3) + 1)/36 + 2**(1/3)*log
(-(-x**2 + 1)**(1/3) + 2**(2/3))/32 + 2**(1/3)*sqrt(3)*atan(sqrt(
3)*(2**(1/3)*(-x**2 + 1)**(1/3)/3 + 1/3))/48 - sqrt(3)*atan(sqrt(
3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/54 - (-x**2 + 1)**(2/3)/(6*x**
2*(x**2 + 3))
```

Mathematica [C] time = 0.280329, size = 213, normalized size = 1.16

$$\frac{\frac{10x^4F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{x^2\left(F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)\right)+6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}+\frac{21x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}{7x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)-9F_1\left(\frac{7}{3};\frac{1}{3},2;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)+F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}+5x^4+}{72x^2\sqrt[3]{1-x^2}\,(x^2+3)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-12 + 7*x^2 + 5*x^4 + (10*x^4*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3
])/(6*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3] + x^2*(-AppellF1[2, 1/3
, 2, 3, x^2, -x^2/3] + AppellF1[2, 4/3, 1, 3, x^2, -x^2/3])) + (2
1*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2])/(7*x^2*AppellF1
[4/3, 1/3, 1, 7/3, x^(-2), -3/x^2] - 9*AppellF1[7/3, 1/3, 2, 10/3
, x^(-2), -3/x^2] + AppellF1[7/3, 4/3, 1, 10/3, x^(-2), -3/x^2]))
/(72*x^2*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3), x)
```

Fricas [A] time = 0.241523, size = 343, normalized size = 1.87

$$4^{\frac{2}{3}}\sqrt{3}\left(4\cdot 4^{\frac{1}{3}}\sqrt{3}\left(x^4+3x^2\right)\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)-8\cdot 4^{\frac{1}{3}}\sqrt{3}\left(x^4+3x^2\right)\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)-6\cdot 4^{\frac{1}{3}}\sqrt{3}\left(5x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3),x, algorithm="fricas")
```

```
[Out] 1/5184*4^(2/3)*sqrt(3)*(4*4^(1/3)*sqrt(3)*(x^4 + 3*x^2)*log((-x^2
 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 8*4^(1/3)*sqrt(3)*(x^4 + 3*
x^2)*log((-x^2 + 1)^(1/3) - 1) - 6*4^(1/3)*sqrt(3)*(5*x^2 + 12)*(
-x^2 + 1)^(2/3) - 9*sqrt(3)*(x^4 + 3*x^2)*log(4^(2/3)*(-x^2 + 1)^
(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) + 4) + 18*sqrt(3)*(x^4 + 3*x^2)*
log(4^(2/3)*(-x^2 + 1)^(1/3) - 4) - 24*4^(1/3)*(x^4 + 3*x^2)*arct
an(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 54*(x^4 + 3*x^2)
```

```
*arctan(1/6*4^(2/3)*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)))/(x^4
 + 3*x^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1025 $\int \frac{1}{x^5 \sqrt[3]{1-x^2}(3+x^2)^2} \, dx$

Optimal. Leaf size=208

$$-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(x^2+3\right)}+\frac{\left(1-x^2\right)^{2/3}}{216\left(x^2+3\right)}+\frac{13\log\left(x^2+3\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}-\frac{13\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(x^2+3\right)}-\frac{\log(x)}{54}$$

```
[Out] (1 - x^2)^(2/3)/(216*(3 + x^2)) - (1 - x^2)^(2/3)/(12*x^4*(3 + x^
2)) - (1 - x^2)^(2/3)/(36*x^2*(3 + x^2)) - (13*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]])/(216*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 -
 x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) - Log[x]/54 + (13*Log[3 + x^2]
)/(1296*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/36 - (13*Log[2^(2/3)
- (1 - x^2)^(1/3)])/(432*2^(2/3))
```

Rubi [A] time = 0.458637, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(x^2+3\right)}+\frac{\left(1-x^2\right)^{2/3}}{216\left(x^2+3\right)}+\frac{13\log\left(x^2+3\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}-\frac{13\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(x^2+3\right)}-\frac{\log(x)}{54}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```


```
[Out] (1 - x^2)^(2/3)/(216*(3 + x^2)) - (1 - x^2)^(2/3)/(12*x^4*(3 + x^
2)) - (1 - x^2)^(2/3)/(36*x^2*(3 + x^2)) - (13*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]])/(216*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 -
 x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) - Log[x]/54 + (13*Log[3 + x^2]
)/(1296*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/36 - (13*Log[2^(2/3)
- (1 - x^2)^(1/3)])/(432*2^(2/3))
```

Rubi in Sympy [A] time = 26.9487, size = 177, normalized size = 0.85

$$-\frac{\log\left(x^2\right)}{108}+\frac{13\sqrt[3]{2}\log\left(x^2+3\right)}{2592}+\frac{\log\left(-\sqrt[3]{-x^2+1}+1\right)}{36}-\frac{13\sqrt[3]{2}\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{864}-\frac{13\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{1296}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{54}+\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{216x^2}-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{24x^2\left(x^2+3\right)}-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{12x^4\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -log(x**2)/108 + 13*2**(1/3)*log(x**2 + 3)/2592 + log(-(-x**2 + 1
)**(1/3) + 1)/36 - 13*2**(1/3)*log(-(-x**2 + 1)**(1/3) + 2**(2/3)
)/864 - 13*2**(1/3)*sqrt(3)*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(
1/3)/3 + 1/3))/1296 + sqrt(3)*atan(sqrt(3)*(2*(-x**2 + 1)**(1/3)/
3 + 1/3))/54 + (-x**2 + 1)**(2/3)/(216*x**2) - (-x**2 + 1)**(2/3)
```

```
/(24*x**2*(x**2 + 3)) - (-x**2 + 1)**(2/3)/(12*x**4*(x**2 + 3))
```

Mathematica [C] time = 0.285954, size = 234, normalized size = 1.12

$$\frac{\frac{2x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{(x^2+3)\left(x^2\left(F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)\right)-6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)\right)}-\frac{63x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)}{(x^2+3)\left(7x^2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)-9F_1\left(\frac{7}{3};\frac{1}{3},2;\frac{10}{3};\frac{1}{x^2},-\frac{3}{x^2}\right)+F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};\frac{1}{x^2},\right.\right.}}{216\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-((18 - 12*x^2 - 7*x^4 + x^6)/(3*x^4 + x^6)) + (2*x^2*AppellF1[1
, 1/3, 1, 2, x^2, -x^2/3])/((3 + x^2)*(-6*AppellF1[1, 1/3, 1, 2,
x^2, -x^2/3] + x^2*(AppellF1[2, 1/3, 2, 3, x^2, -x^2/3] - AppellF
1[2, 4/3, 1, 3, x^2, -x^2/3]))) - (63*x^2*AppellF1[4/3, 1/3, 1, 7
/3, x^(-2), -3/x^2])/((3 + x^2)*(7*x^2*AppellF1[4/3, 1/3, 1, 7/3,
 x^(-2), -3/x^2] - 9*AppellF1[7/3, 1/3, 2, 10/3, x^(-2), -3/x^2]
+ AppellF1[7/3, 4/3, 1, 10/3, x^(-2), -3/x^2])))/(216*(1 - x^2)^(
1/3))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5), x)
```

Fricas [A] time = 0.240921, size = 383, normalized size = 1.84

$$-\frac{4^{\frac{2}{3}}\sqrt{3}\left(13\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(x^6+3x^4\right)\log\left(4^{\frac{2}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{2}{3}}-4\left(-1\right)^{\frac{1}{3}}\right)-26\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(x^6+3x^4\right)\log\left(4^{\frac{2}{3}}\left(-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5),x, algorithm="fricas")
```

```
[Out] -1/15552*4^(2/3)*sqrt(3)*(13*sqrt(3)*(-1)^(1/3)*(x^6 + 3*x^4)*log
(4^(2/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) + 4^(1/3)*(-x^2 + 1)^(2/3) -
```

```
 4*(-1)^(1/3)) - 26*sqrt(3)*(-1)^(1/3)*(x^6 + 3*x^4)*log(4^(2/3)*
(-x^2 + 1)^(1/3) - 4*(-1)^(2/3)) + 12*4^(1/3)*sqrt(3)*(x^6 + 3*x^
4)*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 24*4^(1/3)*sqrt
(3)*(x^6 + 3*x^4)*log((-x^2 + 1)^(1/3) - 1) - 6*4^(1/3)*sqrt(3)*(
x^4 - 6*x^2 - 18)*(-x^2 + 1)^(2/3) - 78*(-1)^(1/3)*(x^6 + 3*x^4)*
arctan(-1/6*(-1)^(1/3)*(4^(2/3)*sqrt(3)*(-x^2 + 1)^(1/3) + 2*sqrt
(3)*(-1)^(2/3))) - 72*4^(1/3)*(x^6 + 3*x^4)*arctan(2/3*sqrt(3)*(-
x^2 + 1)^(1/3) + 1/3*sqrt(3)))/(x^6 + 3*x^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1026 $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=543

$$\frac{3\left(1-x^2\right)^{2/3}x}{8\left(x^2+3\right)}-\frac{27x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}}-\frac{15\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}+\frac{9\,3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}}+\frac{5\tanh^{-1}(x)}{8\,2^{2/3}}$$

```
[Out] (3*x*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (27*x)/(8*(1 - Sqrt[3] - (1
 - x^2)^(1/3))) - (5*Sqrt[3]*ArcTan[Sqrt[3]/x])/(8*2^(2/3)) - (5*
Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(8*2^(
2/3)) + (5*ArcTanh[x])/(8*2^(2/3)) - (15*ArcTanh[x/(1 + 2^(1/3)*(
1 - x^2)^(1/3))])/(8*2^(2/3)) - (27*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(
1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(
1/3))^2)]) + (9*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)
^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*x*Sqrt[-((1 - (1 - x^2)
^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.619787, antiderivative size = 543, normalized size of antiderivative = 1., number

of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{3\left(1-x^2\right)^{2/3}x}{8\left(x^2+3\right)}-\frac{27x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}}-\frac{15\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\ 2^{2/3}}+\frac{9\ 3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}}+\frac{5\tanh^{-1}(x)}{8\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*x*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (27*x)/(8*(1 - Sqrt[3] - (1
 - x^2)^(1/3))) - (5*Sqrt[3]*ArcTan[Sqrt[3]/x])/(8*2^(2/3)) - (5*
Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(8*2^(
2/3)) + (5*ArcTanh[x])/(8*2^(2/3)) - (15*ArcTanh[x/(1 + 2^(1/3)*(
1 - x^2)^(1/3))])/(8*2^(2/3)) - (27*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(
1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(
1/3))^2)]) + (9*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)
^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*x*Sqrt[-((1 - (1 - x^2)
^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 7.28128, size = 19, normalized size = 0.03

$$\frac{x^5\,\mathrm{appellf}_1\left(\frac{5}{2},\frac{1}{3},2,\frac{7}{2},x^2,-\frac{x^2}{3}\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] x**5*appellf1(5/2, 1/3, 2, 7/2, x**2, -x**2/3)/45
```

Mathematica [C] time = 0.223834, size = 231, normalized size = 0.43

$$\frac{3x\left(\frac{15x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)+15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}+\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}\right)}{8\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*x*(1 - x^2 + (9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(-9*A
ppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3
, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])
) + (15*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/(15*AppellF1
[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[5/2, 1/3, 2, 7
/2, x^2, -x^2/3] + AppellF1[5/2, 4/3, 1, 7/2, x^2, -x^2/3]))))/(8
*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1027 $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=543

$$-\frac{(1-x^2)^{2/3}x}{8(x^2+3)}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\,2^{2/3}}$$

```
[Out] -(x*(1 - x^2)^(2/3))/(8*(3 + x^2)) + x/(8*(1 - Sqrt[3] - (1 - x^2
)^(1/3))) + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[
3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTan
h[x]/(24*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2
^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(
1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(
1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sq
rt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1
- x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x
^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt
[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*S
qrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))^2)])
```

Rubi [A] time = 0.630415, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{(1-x^2)^{2/3}x}{8(x^2+3)}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\,2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -(x*(1 - x^2)^(2/3))/(8*(3 + x^2)) + x/(8*(1 - Sqrt[3] - (1 - x^2
)^(1/3))) + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[
3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTan
h[x]/(24*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2
^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(
1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(
1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sq
rt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1
- x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x
^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt
[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*S
qrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 8.43138, size = 19, normalized size = 0.03

$$\frac{x^3 \operatorname{appellf}_1\left(\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, x^2, -\frac{x^2}{3}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] x**3*appellf1(3/2, 1/3, 2, 5/2, x**2, -x**2/3)/27
```

Mathematica [C] time = 0.203204, size = 231, normalized size = 0.43

$$\frac{x\left(\frac{5x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)-15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}+\frac{27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+ \right.}{24\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(-3 + 3*x^2 + (27*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*
AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, x^2, -x^2/3] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3
])) + (5*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/(-15*Appell
F1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2*(AppellF1[5/2, 1/3, 2,
7/2, x^2, -x^2/3] - AppellF1[5/2, 4/3, 1, 7/2, x^2, -x^2/3]))))/(
24*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+3\right)^2} \frac{1}{\sqrt[3]{-x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1028 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=543

$$\frac{(1-x^2)^{2/3}x}{24(x^2+3)}-\frac{x}{24\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}+\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\,3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\,2^{2/3}}$$

```
[Out] (x*(1 - x^2)^(2/3))/(24*(3 + x^2)) - x/(24*(1 - Sqrt[3] - (1 - x^
2)^(1/3))) + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt
[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTa
nh[x]/(24*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*
2^(2/3)) - (Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
- x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*x*Sqrt[-((1 - (1
 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + ((1 - (1 -
x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqr
t[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(12
*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1
 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.594902, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$\frac{(1-x^2)^{2/3}x}{24(x^2+3)}-\frac{x}{24\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\,2^{2/3}}+\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\,3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\,2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(1 - x^2)^(2/3))/(24*(3 + x^2)) - x/(24*(1 - Sqrt[3] - (1 - x^
2)^(1/3))) + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt
[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTa
nh[x]/(24*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*
2^(2/3)) - (Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*x*Sqrt[-((1 - (1
 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + ((1 - (1 -
x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqr
t[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(12
*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1
 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 39.6816, size = 456, normalized size = 0.84

$$\frac{x\left(-x^2+1\right)^{\frac{2}{3}}}{24\left(x^2+3\right)}-\frac{x}{24\left(-\sqrt[3]{-x^2+1}-\sqrt{3}+1\right)}+\frac{\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{-x+1}+(x+1)^{\frac{2}{3}}\right)}{32}-\frac{\sqrt[3]{2}\log\left((-x+1)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{32}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{48}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{48}-\frac{\sqrt[4]{3}\sqrt{\frac{(-x^2+1)^{\frac{2}{3}}+\sqrt[3]{-x^2+1}+1}{\left(-\sqrt[3]{-x^2+1}-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\left(-\sqrt[3]{-x^2+1}+1\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-x^2+1}+1+\sqrt{3}}{-\sqrt[3]{-x^2+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{48x\sqrt{\frac{\sqrt[3]{-x^2+1}-1}{\left(-\sqrt[3]{-x^2+1}-\sqrt{3}+1\right)^2}}}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{(-x^2+1)^{\frac{2}{3}}+\sqrt[3]{-x^2+1}+1}{\left(-\sqrt[3]{-x^2+1}-\sqrt{3}+1\right)^2}}\left(-\sqrt[3]{-x^2+1}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-x^2+1}+1+\sqrt{3}}{-\sqrt[3]{-x^2+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{72x\sqrt{\frac{\sqrt[3]{-x^2+1}-1}{\left(-\sqrt[3]{-x^2+1}-\sqrt{3}+1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] x*(-x**2 + 1)**(2/3)/(24*(x**2 + 3)) - x/(24*(-(-x**2 + 1)**(1/3)
 - sqrt(3) + 1)) + 2**(1/3)*log(2**(1/3)*(-x + 1)**(1/3) + (x + 1
)**(2/3))/32 - 2**(1/3)*log((-x + 1)**(2/3) + 2**(1/3)*(x + 1)**(
1/3))/32 - 2**(1/3)*sqrt(3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(x
+ 1)**(2/3)/(3*(-x + 1)**(1/3)))/48 - 2**(1/3)*sqrt(3)*atan(2**(2
/3)*sqrt(3)*(-x + 1)**(2/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)/48 -
3**(1/4)*sqrt(((-x**2 + 1)**(2/3) + (-x**2 + 1)**(1/3) + 1)/(-(-x
**2 + 1)**(1/3) - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-(-x**2 + 1
)**(1/3) + 1)*elliptic_e(asin((-(-x**2 + 1)**(1/3) + 1 + sqrt(3))
/(-(-x**2 + 1)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(48*x*sqrt
(((-x**2 + 1)**(1/3) - 1)/(-(-x**2 + 1)**(1/3) - sqrt(3) + 1)**2)
) + sqrt(2)*3**(3/4)*sqrt(((-x**2 + 1)**(2/3) + (-x**2 + 1)**(1/3
) + 1)/(-(-x**2 + 1)**(1/3) - sqrt(3) + 1)**2)*(-(-x**2 + 1)**(1/
3) + 1)*elliptic_f(asin((-(-x**2 + 1)**(1/3) + 1 + sqrt(3))/(-(-x
**2 + 1)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(72*x*sqrt(((-x*
*2 + 1)**(1/3) - 1)/(-(-x**2 + 1)**(1/3) - sqrt(3) + 1)**2))
```

Mathematica [C] time = 0.205081, size = 231, normalized size = 0.43

$$\frac{x\left(\frac{5x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)+15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}+\frac{189F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}-\right.}{72\sqrt[3]{1-x^2}\,(x^2+3)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(3 - 3*x^2 + (189*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*
AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, x^2, -x^2/3] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3
])) + (5*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/(15*AppellF
1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[5/2, 1/3, 2,
7/2, x^2, -x^2/3] + AppellF1[5/2, 4/3, 1, 7/2, x^2, -x^2/3]))))/(
72*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1029 $\int \frac{1}{x^2 \sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

Optimal. Leaf size=563

$$\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{8x}+\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)x}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\ 2^{2/3}\sqrt{3}}-\frac{7\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{72\ 2^{2/3}}-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}}+\frac{7\tanh^{-1}(x)}{216\ 2^{2/3}}$$

```
[Out] -(1 - x^2)^(2/3)/(8*x) + (1 - x^2)^(2/3)/(24*x*(3 + x^2)) + x/(8*
(1 - Sqrt[3] - (1 - x^2)^(1/3))) - (7*ArcTan[Sqrt[3]/x])/(72*2^(2
/3)*Sqrt[3]) - (7*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/
x])/(72*2^(2/3)*Sqrt[3]) + (7*ArcTanh[x])/(216*2^(2/3)) - (7*ArcT
anh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(72*2^(2/3)) + (3^(1/4)*Sqr
t[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) +
(1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[Arc
Sin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqr
t[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt[(1 + (
1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3]
 - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*x*Sqrt[
-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.803593, antiderivative size = 563, normalized size of antiderivative = 1., number

of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{8x}+\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)x}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\ 2^{2/3}\sqrt{3}}-\frac{7\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{72\ 2^{2/3}}-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}}+\frac{7\tanh^{-1}(x)}{216\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -(1 - x^2)^(2/3)/(8*x) + (1 - x^2)^(2/3)/(24*x*(3 + x^2)) + x/(8*
(1 - Sqrt[3] - (1 - x^2)^(1/3))) - (7*ArcTan[Sqrt[3]/x])/(72*2^(2
/3)*Sqrt[3]) - (7*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/
x])/(72*2^(2/3)*Sqrt[3]) + (7*ArcTanh[x])/(216*2^(2/3)) - (7*ArcT
anh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(72*2^(2/3)) + (3^(1/4)*Sqr
t[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) +
(1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[Arc
Sin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqr
t[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt[(1 + (
1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3]
 - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*x*Sqrt[
-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 7.82815, size = 20, normalized size = 0.04

$$-\frac{\operatorname{appellf}_1\left(-\frac{1}{2},\frac{1}{3},2,\frac{1}{2},x^2,-\frac{x^2}{3}\right)}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -appellf1(-1/2, 1/3, 2, 1/2, x**2, -x**2/3)/(9*x)
```

Mathematica [C] time = 0.279969, size = 241, normalized size = 0.43

$$\frac{\frac{69x^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+\frac{5x^4F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)-15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}+3x}{24x\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-8 + 5*x^2 + 3*x^4 + (69*x^2*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^
2/3])/(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(Appell
F1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^
2, -x^2/3])) + (5*x^4*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3])/(-
15*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^2*(AppellF1[5/2,
 1/3, 2, 7/2, x^2, -x^2/3] - AppellF1[5/2, 4/3, 1, 7/2, x^2, -x^2
/3])))/(24*x*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(x^2+3\right)^2} \frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)^2 \left(-x^2+1\right)^{\frac{1}{3}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1030 $\int \frac{1}{x^4 \sqrt[3]{1-x^2}(3+x^2)^2} \, dx$

Optimal. Leaf size=581

$$-\frac{11x}{648\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{11\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{216\ 2^{2/3}}+\frac{11\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{11\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{432\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)x^3}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}}-\frac{11\tanh^{-1}(x)}{648\ 2^{2/3}}$$

```
[Out] (-11*(1 - x^2)^(2/3))/(216*x^3) + (11*(1 - x^2)^(2/3))/(648*x) +
(1 - x^2)^(2/3)/(24*x^3*(3 + x^2)) - (11*x)/(648*(1 - Sqrt[3] - (
1 - x^2)^(1/3))) + (11*ArcTan[Sqrt[3]/x])/(216*2^(2/3)*Sqrt[3]) +
 (11*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(216*2^(2
/3)*Sqrt[3]) - (11*ArcTanh[x])/(648*2^(2/3)) + (11*ArcTanh[x/(1 +
 2^(1/3)*(1 - x^2)^(1/3))])/(216*2^(2/3)) - (11*Sqrt[2 + Sqrt[3]]
*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3
))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[
3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(432*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3]
- (1 - x^2)^(1/3))^2)]) + (11*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
- x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*x*Sqrt[
-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi [A] time = 0.974455, antiderivative size = 581, normalized size of antiderivative = 1., number

of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{11x}{648\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{11\left(1-x^2\right)^{2/3}}{648x}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{11\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{216\ 2^{2/3}}+\frac{11\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{11\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{432\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}-\frac{11\left(1-x^2\right)^{2/3}}{216x^3}+\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)x^3}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}}-\frac{11\tanh^{-1}(x)}{648\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-11*(1 - x^2)^(2/3))/(216*x^3) + (11*(1 - x^2)^(2/3))/(648*x) +
(1 - x^2)^(2/3)/(24*x^3*(3 + x^2)) - (11*x)/(648*(1 - Sqrt[3] - (
1 - x^2)^(1/3))) + (11*ArcTan[Sqrt[3]/x])/(216*2^(2/3)*Sqrt[3]) +
 (11*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(216*2^(2
/3)*Sqrt[3]) - (11*ArcTanh[x])/(648*2^(2/3)) + (11*ArcTanh[x/(1 +
 2^(1/3)*(1 - x^2)^(1/3))])/(216*2^(2/3)) - (11*Sqrt[2 + Sqrt[3]]
*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3
))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[
3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(432*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3]
- (1 - x^2)^(1/3))^2)]) + (11*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
- x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*x*Sqrt[
-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 7.60229, size = 24, normalized size = 0.04

$$-\frac{\text{appellf}_1\left(-\frac{3}{2},\frac{1}{3},2,-\frac{1}{2},x^2,-\frac{x^2}{3}\right)}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] -appellf1(-3/2, 1/3, 2, -1/2, x**2, -x**2/3)/(27*x**3)
```

Mathematica [C] time = 0.406027, size = 246, normalized size = 0.42

$$\frac{\frac{55x^6F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};x^2,-\frac{x^2}{3}\right)\right)+15F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)}+\frac{2079x^4F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}-33}{1944x^3\sqrt[3]{1-x^2}\left(x^2+3\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-216 + 216*x^2 + 33*x^4 - 33*x^6 + (2079*x^4*AppellF1[1/2, 1/3,
1, 3/2, x^2, -x^2/3])/(9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3]
+ 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] + AppellF1[3/2,
 4/3, 1, 5/2, x^2, -x^2/3])) + (55*x^6*AppellF1[3/2, 1/3, 1, 5/2,
 x^2, -x^2/3])/(15*AppellF1[3/2, 1/3, 1, 5/2, x^2, -x^2/3] + 2*x^
2*(-AppellF1[5/2, 1/3, 2, 7/2, x^2, -x^2/3] + AppellF1[5/2, 4/3,
1, 7/2, x^2, -x^2/3])))/(1944*x^3*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(x^{2}+3\right)^{2}} \frac{1}{\sqrt[3]{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(x^{2}+3\right)}^{2} {\left(-x^{2}+1\right)}^{\frac{1}{3}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1031 $\int \frac{x^7}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=136

$$\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 -
 3*x^2)^(11/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[
2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/81
```

Rubi [A] time = 0.260037, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 -
 3*x^2)^(11/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[
2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/81
```

Rubi in Sympy [A] time = 35.4572, size = 175, normalized size = 1.29

$$\frac{2\left(-3x^2+2\right)^{\frac{11}{4}}}{891}-\frac{16\left(-3x^2+2\right)^{\frac{7}{4}}}{567}+\frac{56\left(-3x^2+2\right)^{\frac{3}{4}}}{243}-\frac{16\sqrt[4]{2}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{81}+\frac{16\sqrt[4]{2}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{81}-\frac{32\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{81}-\frac{32\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] 2*(-3*x**2 + 2)**(11/4)/891 - 16*(-3*x**2 + 2)**(7/4)/567 + 56*(-
3*x**2 + 2)**(3/4)/243 - 16*2**(1/4)*log(-2**(3/4)*(-3*x**2 + 2)*
*(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/81 + 16*2**(1/4)*log(2**(3/
4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/81 - 32*2*
*(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) - 1)/81 - 32*2**(1/4)*a
tan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1)/81
```

Mathematica [C] time = 0.103542, size = 76, normalized size = 0.56

$$-\frac{2\left(-14784\sqrt[4]{\frac{2-3x^2}{4-3x^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2}{4-3x^2}\right)+567x^6+1242x^4+4056x^2-3424\right)}{18711\sqrt[4]{2-3x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(-3424 + 4056*x^2 + 1242*x^4 + 567*x^6 - 14784*((2 - 3*x^2)/(
4 - 3*x^2))^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, 2/(4 - 3*x^2)]
))/(18711*(2 - 3*x^2)^(1/4))
```

Maple [F]   time = 0.109, size = 0, normalized size = 0.

$$\int \frac{x^7}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A]   time = 1.49637, size = 204, normalized size = 1.5

$$\frac{2}{891}\left(-3x^2+2\right)^{\frac{11}{4}}-\frac{16}{567}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{56}{243}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] 2/891*(-3*x^2 + 2)^(11/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*2^(
1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81
*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) +
16/81*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*
x^2 + 2)) - 16/81*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(
2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2 + 2)^(3/4)
```

Fricas [A]   time = 0.25409, size = 351, normalized size = 2.58

$$\frac{2}{18711}\left(189x^4+540x^2+1712\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{8^{\frac{3}{4}}\sqrt{2}}{8^{\frac{3}{4}}\sqrt{2}+4\sqrt{8^{\frac{3}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}+8\left(-3x^2+2\right)^{\frac{1}{4}}}\right)+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{8^{\frac{3}{4}}\sqrt{2}}{8^{\frac{3}{4}}\sqrt{2}-2\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}}-8\left(-3x^2+2\right)^{\frac{1}{4}}}\right)+\frac{8}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)-\frac{8}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 540*x^2 + 1712)*(-3*x^2 + 2)^(3/4) + 32/81*8^(
1/4)*sqrt(2)*arctan(8^(3/4)*sqrt(2)/(8^(3/4)*sqrt(2) + 4*sqrt(8^(
3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
 + 8*(-3*x^2 + 2)^(1/4))) + 32/81*8^(1/4)*sqrt(2)*arctan(-8^(3/4)
*sqrt(2)/(8^(3/4)*sqrt(2) - 2*sqrt(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2
)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 8*(-3*x^2 + 2)^(1/4
))) + 8/81*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/
4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 8/81*8^(1/4)*sqrt(2)*log
(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*
x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^7}{3x^2\sqrt[4]{-3x^2+2}-4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1
/4)), x)
```

GIAC/XCAS [A] time = 0.247951, size = 216, normalized size = 1.59

$$\frac{2}{891}\left(3x^2-2\right)^2\left(-3x^2+2\right)^{\frac{3}{4}}-\frac{16}{567}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{8}{81}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)$$
$$-\frac{8}{81}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{16}{81}\cdot 2^{\frac{1}{4}}\ln\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$
$$-\frac{16}{81}\cdot 2^{\frac{1}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{56}{243}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 2)^2*(-3*x^2 + 2)^(3/4) - 16/567*(-3*x^2 + 2)^(7/4
) - 8/81*8^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/
4))) - 8/81*8^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)
^(1/4))) + 16/81*2^(1/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2)
+ sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4
) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2 + 2)^(3/4)
```

3.1032 $\int \frac{x^5}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=121

$$-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)
*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])
/27 + (8*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2
- 3*x^2)^(1/4))])/27
```

Rubi [A] time = 0.194378, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)
*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])
/27 + (8*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2
- 3*x^2)^(1/4))])/27
```

Rubi in Sympy [A] time = 32.9676, size = 162, normalized size = 1.34

$$-\frac{2\left(-3x^2+2\right)^{\frac{7}{4}}}{189}+\frac{4\left(-3x^2+2\right)^{\frac{3}{4}}}{27}-\frac{4\sqrt[4]{2}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{27}+\frac{4\sqrt[4]{2}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{27}-\frac{8\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{27}-\frac{8\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2*(-3*x**2 + 2)**(7/4)/189 + 4*(-3*x**2 + 2)**(3/4)/27 - 4*2**(1
/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) + sqrt
(2))/27 + 4*2**(1/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*
x**2 + 2) + sqrt(2))/27 - 8*2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)*
*(1/4) - 1)/27 - 8*2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) +
1)/27
```

Mathematica [C] time = 0.0764158, size = 71, normalized size = 0.59

$$-\frac{2\left(-112\sqrt[4]{\frac{2-3x^2}{4-3x^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2}{4-3x^2}\right)+9x^4+30x^2-24\right)}{189\sqrt[4]{2-3x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(-24 + 30*x^2 + 9*x^4 - 112*((2 - 3*x^2)/(4 - 3*x^2))^(1/4)*H
ypergeometric2F1[1/4, 1/4, 5/4, 2/(4 - 3*x^2)]))/(189*(2 - 3*x^2)
^(1/4))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{x^5}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.50712, size = 189, normalized size = 1.56

$$-\frac{2}{189}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{4}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^5/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) - 8/27*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3
/4) + 2*(-3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*arctan(-1/2*2^(1/4)*(
2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x
^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2
^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 4/27*(-
3*x^2 + 2)^(3/4)
```

Fricas [A] time = 0.25794, size = 342, normalized size = 2.83

$$\frac{2}{63}\left(x^2+4\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{8^{\frac{3}{4}}\sqrt{2}}{8^{\frac{3}{4}}\sqrt{2}+4\sqrt{8^{\frac{3}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}+8\left(-3x^2+2\right)^{\frac{1}{4}}}\right)+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{8^{\frac{3}{4}}\sqrt{2}}{8^{\frac{3}{4}}\sqrt{2}-2\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}}-8\left(-3x^2+2\right)^{\frac{1}{4}}}\right)+\frac{2}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)-\frac{2}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^5/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] 2/63*(x^2 + 4)*(-3*x^2 + 2)^(3/4) + 8/27*8^(1/4)*sqrt(2)*arctan(8
^(3/4)*sqrt(2)/(8^(3/4)*sqrt(2) + 4*sqrt(8^(3/4)*sqrt(2)*(-3*x^2
+ 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 8*(-3*x^2 + 2)^(1/
4))) + 8/27*8^(1/4)*sqrt(2)*arctan(-8^(3/4)*sqrt(2)/(8^(3/4)*sqrt
(2) - 2*sqrt(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) +
 16*sqrt(-3*x^2 + 2)) - 8*(-3*x^2 + 2)^(1/4))) + 2/27*8^(1/4)*sqr
t(2)*log(4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*s
qrt(-3*x^2 + 2)) - 2/27*8^(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*(-
3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5}{3x^2\sqrt[4]{-3x^2+2}-4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1
/4)), x)
```

GIAC/XCAS [A] time = 0.246565, size = 189, normalized size = 1.56

$$-\frac{2}{189}\left(-3\,x^2+2\right)^{\frac{7}{4}}+\frac{1}{27}\cdot 8^{\frac{3}{4}}\ln\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{27}\cdot 8^{\frac{3}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{4}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^5/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) + 1/27*8^(3/4)*ln(2^(3/4)*(-3*x^2 + 2)^
(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/27*8^(3/4)*ln(-2^(3/4)*(-
3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 8/27*2^(1/4)*arc
tan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*
arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 4/27*(-3*
x^2 + 2)^(3/4)
```

3.1033 $\int \frac{x^3}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=106

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqr
t[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```

Rubi [A] time = 0.145825, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqr
t[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```

Rubi in Sympy [A] time = 30.9311, size = 144, normalized size = 1.36

$$\frac{2\left(-3x^2+2\right)^{\frac{3}{4}}}{27}-\frac{\sqrt[4]{2}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{9}+\frac{\sqrt[4]{2}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{9}-\frac{2\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{9}-\frac{2\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] 2*(-3*x**2 + 2)**(3/4)/27 - 2**(1/4)*log(-2**(3/4)*(-3*x**2 + 2)*
*(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/9 + 2**(1/4)*log(2**(3/4)*(
-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/9 - 2*2**(1/4)
*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) - 1)/9 - 2*2**(1/4)*atan(2**(
1/4)*(-3*x**2 + 2)**(1/4) + 1)/9
```

Mathematica [C] time = 0.0465022, size = 66, normalized size = 0.62

$$\frac{24\sqrt[4]{\frac{2-3x^2}{4-3x^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2}{4-3x^2}\right)-6x^2+4}{27\sqrt[4]{2-3x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (4 - 6*x^2 + 24*((2 - 3*x^2)/(4 - 3*x^2))^(1/4)*Hypergeometric2F1
[1/4, 1/4, 5/4, 2/(4 - 3*x^2)])/(27*(2 - 3*x^2)^(1/4))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{x^3}{-3\,x^2+4}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.50078, size = 174, normalized size = 1.64

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4)))
 - 2/9*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4
))) + 1/9*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 1/9*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2 + 2)^(3/4)
```

Fricas [A] time = 0.268293, size = 335, normalized size = 3.16

$$\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{8^{\frac{3}{4}}\sqrt{2}}{8^{\frac{3}{4}}\sqrt{2}+4\sqrt{8^{\frac{3}{4}}\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3\,x^2+2}}+8\left(-3\,x^2+2\right)^{\frac{1}{4}}}\right)+\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{8^{\frac{3}{4}}\sqrt{2}}{8^{\frac{3}{4}}\sqrt{2}-2\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3\,x^2+2}}-8\left(-3\,x^2+2\right)^{\frac{1}{4}}}\right)+\frac{1}{18}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3\,x^2+2}\right)-\frac{1}{18}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] 2/9*8^(1/4)*sqrt(2)*arctan(8^(3/4)*sqrt(2)/(8^(3/4)*sqrt(2) + 4*s
qrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^
2 + 2)) + 8*(-3*x^2 + 2)^(1/4))) + 2/9*8^(1/4)*sqrt(2)*arctan(-8^
(3/4)*sqrt(2)/(8^(3/4)*sqrt(2) - 2*sqrt(-4*8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 8*(-3*x^2 + 2)
^(1/4))) + 1/18*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*x^2 + 2
)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/18*8^(1/4)*sqrt(2
)*log(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqr
```

```
t(-3*x^2 + 2)) + 2/27*(-3*x^2 + 2)^(3/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{3x^2\sqrt[4]{-3x^2+2}-4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1
/4)), x)
```

GIAC/XCAS [A] time = 0.240753, size = 174, normalized size = 1.64

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\ln\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4)))
 - 2/9*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4
))) + 1/9*2^(1/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(
-3*x^2 + 2)) - 1/9*2^(1/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(
2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2 + 2)^(3/4)
```

3.1034 $\int \frac{x}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=91

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

```
[Out] ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(
3*2^(3/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*
x^2)^(1/4))]/(3*2^(3/4))
```

Rubi [A] time = 0.0589236, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(
3*2^(3/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*
x^2)^(1/4))]/(3*2^(3/4))
```

Rubi in Sympy [A] time = 27.6946, size = 128, normalized size = 1.41

$$-\frac{\sqrt[4]{2}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{12}+\frac{\sqrt[4]{2}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{12}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{6}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2**(1/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2)
+ sqrt(2))/12 + 2**(1/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt
(-3*x**2 + 2) + sqrt(2))/12 - 2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2
)**(1/4) - 1)/6 - 2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1
)/6
```

Mathematica [A] time = 0.0718851, size = 100, normalized size = 1.1

$$\frac{-\log\left(\sqrt{4-6x^2}-2\sqrt[4]{4-6x^2}+2\right)+\log\left(\sqrt{4-6x^2}+2\sqrt[4]{4-6x^2}+2\right)+2\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-2\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*ArcTan[1 + (4 - 6*x^2)^(1/4)
] - Log[2 - 2*(4 - 6*x^2)^(1/4) + Sqrt[4 - 6*x^2]] + Log[2 + 2*(4
```

```
 - 6*x^2)^(1/4) + Sqrt[4 - 6*x^2]])/(6*2^(3/4))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x}{-3\,x^{2}+4}\,\frac{1}{\sqrt[4]{-3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.50988, size = 159, normalized size = 1.75

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^{2}+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^{2}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4)))
 - 1/6*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4
))) + 1/12*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) - 1/12*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2))
```

Fricas [A] time = 0.271445, size = 250, normalized size = 2.75

$$\frac{1}{24}\cdot 2^{\frac{3}{4}}\left(4\sqrt{2}\arctan\left(\frac{1}{2^{\frac{1}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+\sqrt{\sqrt{2}\sqrt{-3\,x^{2}+2}+2\cdot 2^{\frac{1}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+2}+1}\right)+4\sqrt{2}\arctan\left(\frac{}{2^{\frac{1}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+\sqrt{\sqrt{2}\sqrt{-3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] 1/24*2^(3/4)*(4*sqrt(2)*arctan(1/(2^(1/4)*(-3*x^2 + 2)^(1/4) + sq
rt(sqrt(2)*sqrt(-3*x^2 + 2) + 2*2^(1/4)*(-3*x^2 + 2)^(1/4) + 2) +
 1)) + 4*sqrt(2)*arctan(1/(2^(1/4)*(-3*x^2 + 2)^(1/4) + sqrt(sqrt
(2)*sqrt(-3*x^2 + 2) - 2*2^(1/4)*(-3*x^2 + 2)^(1/4) + 2) - 1)) +
sqrt(2)*log(4*sqrt(2)*sqrt(-3*x^2 + 2) + 8*2^(1/4)*(-3*x^2 + 2)^(
1/4) + 8) - sqrt(2)*log(4*sqrt(2)*sqrt(-3*x^2 + 2) - 8*2^(1/4)*(-
3*x^2 + 2)^(1/4) + 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{3x^{2}\sqrt[4]{-3x^{2}+2}-4\sqrt[4]{-3x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1/4)
), x)
```

GIAC/XCAS [A] time = 0.237091, size = 159, normalized size = 1.75

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\ln\left(2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4)))
 - 1/6*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4
))) + 1/12*2^(1/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 1/12*2^(1/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2))
```

3.1035 $\int \frac{1}{x\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=145

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] -
 Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcT
anh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + S
qrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4))
```

Rubi [A] time = 0.232747, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] -
 Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcT
anh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + S
qrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4))
```

Rubi in Sympy [A] time = 35.6925, size = 178, normalized size = 1.23

$$-\frac{\sqrt[4]{2}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{16}+\frac{\sqrt[4]{2}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{16}+\frac{2^{\frac{3}{4}}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{8}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{8}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{8}-\frac{2^{\frac{3}{4}}\operatorname{atanh}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2**(1/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2)
+ sqrt(2))/16 + 2**(1/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt
(-3*x**2 + 2) + sqrt(2))/16 + 2**(3/4)*atan(2**(3/4)*(-3*x**2 + 2
)**(1/4)/2)/8 - 2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) - 1)/
8 - 2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1)/8 - 2**(3/4)
*atanh(2**(3/4)*(-3*x**2 + 2)**(1/4)/2)/8
```

Mathematica [C] time = 0.239162, size = 140, normalized size = 0.97

$$\frac{54x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)}{5\sqrt[4]{2-3x^2}\left(3x^2-4\right)\left(27x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+2\left(8F_1\left(\frac{9}{4};\frac{1}{4},2;\frac{13}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+F_1\left(\frac{9}{4};\frac{5}{4},1;\frac{13}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (54*x^2*AppellF1[5/4, 1/4, 1, 9/4, 2/(3*x^2), 4/(3*x^2)])/(5*(2 -
 3*x^2)^(1/4)*(-4 + 3*x^2)*(27*x^2*AppellF1[5/4, 1/4, 1, 9/4, 2/(
3*x^2), 4/(3*x^2)] + 2*(8*AppellF1[9/4, 1/4, 2, 13/4, 2/(3*x^2),
4/(3*x^2)] + AppellF1[9/4, 5/4, 1, 13/4, 2/(3*x^2), 4/(3*x^2)])))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-3\,x^{2}+4\right)}\frac{1}{\sqrt[4]{-3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^{2}-4)(-3\,x^{2}+2)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x), x)
```

Fricas [A] time = 0.261023, size = 344, normalized size = 2.37

$$-\frac{1}{16} \cdot 2^{\frac{1}{4}}\left(4\sqrt{2}\arctan\left(\frac{2}{2^{\frac{3}{4}}(-3\,x^{2}+2)^{\frac{1}{4}}+\sqrt{2\sqrt{2}\sqrt{-3\,x^{2}+2}+4}}\right)+\sqrt{2}\log\left(2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+2\right)-\sqrt{2}\log\left(2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}-2\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x),x, algorithm="fricas")
```

```
[Out] -1/16*2^(1/4)*(4*sqrt(2)*arctan(2/(2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2*sqrt(2)*sqrt(-3*x^2 + 2) + 4))) + sqrt(2)*log(2^(3/4)*(-3*x
^2 + 2)^(1/4) + 2) - sqrt(2)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) - 2)
- 4*arctan(1/(2^(1/4)*(-3*x^2 + 2)^(1/4) + sqrt(sqrt(2)*sqrt(-3*x
^2 + 2) + 2*2^(1/4)*(-3*x^2 + 2)^(1/4) + 2) + 1)) - 4*arctan(1/(2
^(1/4)*(-3*x^2 + 2)^(1/4) + sqrt(sqrt(2)*sqrt(-3*x^2 + 2) - 2*2^(
1/4)*(-3*x^2 + 2)^(1/4) + 2) - 1)) - log(4*sqrt(2)*sqrt(-3*x^2 +
2) + 8*2^(1/4)*(-3*x^2 + 2)^(1/4) + 8) + log(4*sqrt(2)*sqrt(-3*x^
2 + 2) - 8*2^(1/4)*(-3*x^2 + 2)^(1/4) + 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^{3}\sqrt[4]{-3x^{2}+2}-4x\sqrt[4]{-3x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(-3*x**2 + 2)**(1/4) - 4*x*(-3*x**2 + 2)**(1/
4)), x)
```

GIAC/XCAS [A] time = 0.257897, size = 292, normalized size = 2.01

$$-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) + \frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\ln\left(4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right) - \frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\ln\left(-4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right) + \frac{1}{8}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\ln\left(-\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right) - \frac{1}{16}\cdot 4^{\frac{3}{8}}\ln\left(\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x),x, algorithm="giac")
```

```
[Out] -1/16*4^(3/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2)
 + 2*(-3*x^2 + 2)^(1/4))) - 1/16*4^(3/8)*sqrt(2)*arctan(-1/8*4^(7
/8)*sqrt(2)*(4^(1/8)*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) + 1/32*4^(3
/8)*sqrt(2)*ln(4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 +
 2) + 4^(1/4)) - 1/32*4^(3/8)*sqrt(2)*ln(-4^(1/8)*sqrt(2)*(-3*x^2
 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/8*4^(1/8)*sqrt(2)*a
rctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) + 1/16*4^(1/8)*sqrt(2)*ln(-
(-3*x^2 + 2)^(1/4) + 4^(1/8)) - 1/16*4^(3/8)*ln((-3*x^2 + 2)^(1/4
) + 4^(1/8))
```

3.1036 $\int \frac{1}{x^3 \sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=163

$$-\frac{(2-3x^2)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\,2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\,2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(16*x^2) + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)
])/(32*2^(1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*
(2 - 3*x^2)^(1/4))])/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/
2^(1/4)])/(32*2^(1/4)) + (3*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(
2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(3/4))
```

Rubi [A] time = 0.335154, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(2-3x^2)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\,2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(16*x^2) + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)
])/(32*2^(1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*
(2 - 3*x^2)^(1/4))])/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/
2^(1/4)])/(32*2^(1/4)) + (3*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(
2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(3/4))
```

Rubi in Sympy [A] time = 41.6029, size = 204, normalized size = 1.25

$$-\frac{3\sqrt[4]{2}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{64}+\frac{3\sqrt[4]{2}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{64}+\frac{9\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{64}-\frac{3\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{32}-\frac{3\sqrt[4]{2}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{32}-\frac{9\cdot 2^{\frac{3}{4}}\operatorname{atanh}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{64}-\frac{(-3x^2+2)^{\frac{3}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -3*2**(1/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2
) + sqrt(2))/64 + 3*2**(1/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) +
sqrt(-3*x**2 + 2) + sqrt(2))/64 + 9*2**(3/4)*atan(2**(3/4)*(-3*x*
*2 + 2)**(1/4)/2)/64 - 3*2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1
/4) - 1)/32 - 3*2**(1/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1)/
32 - 9*2**(3/4)*atanh(2**(3/4)*(-3*x**2 + 2)**(1/4)/2)/64 - (-3*x
```

```
**2 + 2)**(3/4)/(16*x**2)
```

Mathematica [C] time = 0.440676, size = 263, normalized size = 1.61

$$\frac{\frac{180x^4F_1\left(1;\frac{1}{4},1;2;\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(3x^2\left(2F_1\left(2;\frac{1}{4},2;3;\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(2;\frac{5}{4},1;3;\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+16F_1\left(1;\frac{1}{4},1;2;\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}+\frac{972x^4F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)}{(3x^2-4)\left(27x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+2\left(8F_1\left(\frac{9}{4};\frac{1}{4},2;\frac{13}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+F_1\left(\frac{9}{4};\right)\right)\right)}}{80x^2\sqrt[4]{2-3x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-10 + 15*x^2 + (180*x^4*AppellF1[1, 1/4, 1, 2, (3*x^2)/2, (3*x^2
)/4])/((-4 + 3*x^2)*(16*AppellF1[1, 1/4, 1, 2, (3*x^2)/2, (3*x^2)
/4] + 3*x^2*(2*AppellF1[2, 1/4, 2, 3, (3*x^2)/2, (3*x^2)/4] + App
ellF1[2, 5/4, 1, 3, (3*x^2)/2, (3*x^2)/4]))) + (972*x^4*AppellF1[
5/4, 1/4, 1, 9/4, 2/(3*x^2), 4/(3*x^2)])/((-4 + 3*x^2)*(27*x^2*Ap
pellF1[5/4, 1/4, 1, 9/4, 2/(3*x^2), 4/(3*x^2)] + 2*(8*AppellF1[9/
4, 1/4, 2, 13/4, 2/(3*x^2), 4/(3*x^2)] + AppellF1[9/4, 5/4, 1, 13
/4, 2/(3*x^2), 4/(3*x^2)]))))/(80*x^2*(2 - 3*x^2)^(1/4))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-3\,x^2+4\right)}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3), x)
```

Fricas [A] time = 0.272206, size = 400, normalized size = 2.45

$$-\frac{2^{\frac{1}{4}}\left(36\,\sqrt{2}x^2\arctan\left(\frac{2}{2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2\,\sqrt{2}\sqrt{-3\,x^2+2}+4}}\right)+9\,\sqrt{2}x^2\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+2\right)-9\,\sqrt{2}x^2\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}-2\right)-\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3),x, algorithm="fricas")
```

```
[Out] -1/128*2^(1/4)*(36*sqrt(2)*x^2*arctan(2/(2^(3/4)*(-3*x^2 + 2)^(1/
4) + sqrt(2*sqrt(2)*sqrt(-3*x^2 + 2) + 4))) + 9*sqrt(2)*x^2*log(2
```

```
^(3/4)*(-3*x^2 + 2)^(1/4) + 2) - 9*sqrt(2)*x^2*log(2^(3/4)*(-3*x^
2 + 2)^(1/4) - 2) - 24*x^2*arctan(1/(2^(1/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(sqrt(2)*sqrt(-3*x^2 + 2) + 2*2^(1/4)*(-3*x^2 + 2)^(1/4) + 2
) + 1)) - 24*x^2*arctan(1/(2^(1/4)*(-3*x^2 + 2)^(1/4) + sqrt(sqrt
(2)*sqrt(-3*x^2 + 2) - 2*2^(1/4)*(-3*x^2 + 2)^(1/4) + 2) - 1)) -
6*x^2*log(4*sqrt(2)*sqrt(-3*x^2 + 2) + 8*2^(1/4)*(-3*x^2 + 2)^(1/
4) + 8) + 6*x^2*log(4*sqrt(2)*sqrt(-3*x^2 + 2) - 8*2^(1/4)*(-3*x^
2 + 2)^(1/4) + 8) + 4*2^(3/4)*(-3*x^2 + 2)^(3/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^5\sqrt[4]{-3x^2+2}-4x^3\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(-3*x**2 + 2)**(1/4) - 4*x**3*(-3*x**2 + 2)**
(1/4)), x)
```

GIAC/XCAS [A] time = 0.276308, size = 259, normalized size = 1.59

$$\frac{9}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\frac{9}{128}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+\frac{9}{128}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{1}{4}}-\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{3}{64}\cdot 2^{\frac{1}{4}}\ln\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{3}{64}\cdot 2^{\frac{1}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3),x, algorithm="giac")
```

```
[Out] 9/64*2^(3/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 9/128*2^(3/
4)*ln(2^(1/4) + (-3*x^2 + 2)^(1/4)) + 9/128*2^(3/4)*ln(2^(1/4) -
(-3*x^2 + 2)^(1/4)) - 3/32*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) +
2*(-3*x^2 + 2)^(1/4))) - 3/32*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4
) - 2*(-3*x^2 + 2)^(1/4))) + 3/64*2^(1/4)*ln(2^(3/4)*(-3*x^2 + 2)
^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 3/64*2^(1/4)*ln(-2^(3/4)*(
-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/16*(-3*x^2 +
2)^(3/4)/x^2
```

3.1037 $\int \frac{x^4}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=164

$$\frac{2}{45}\left(2-3x^2\right)^{3/4}x+\frac{4\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{16\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

```
[Out] (2*x*(2 - 3*x^2)^(3/4))/45 + (4*2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)
*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) + (
4*2^(1/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*
(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (16*2^(1/4)*EllipticE[ArcSin[S
qrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rubi [A] time = 0.20539, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2}{45}\left(2-3x^2\right)^{3/4}x+\frac{4\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{16\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2)^(3/4))/45 + (4*2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)
*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) + (
4*2^(1/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*
(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (16*2^(1/4)*EllipticE[ArcSin[S
qrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rubi in Sympy [A] time = 8.29578, size = 27, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}x^5\operatorname{appellf}_1\left(\frac{5}{2},\frac{1}{4},1,\frac{7}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] 2**(3/4)*x**5*appellf1(5/2, 1/4, 1, 7/2, 3*x**2/2, 3*x**2/4)/40
```

Mathematica [C] time = 0.351942, size = 273, normalized size = 1.66

$$\frac{2x\left(-\frac{240x^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+20F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}+\frac{32F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\cdots}}{45\sqrt[4]{2-3x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2 + (32*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^
2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*
```

```
x^2)/4] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4]
 + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4]))) - (240*x^2
*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*
(20*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*x^2*(2*A
ppellF1[5/2, 1/4, 2, 7/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[5/2, 5
/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4])))))/(45*(2 - 3*x^2)^(1/4))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{x^4}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{3x^2\sqrt[4]{-3x^2+2}-4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**4/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1038 $\int \frac{x^2}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

```
[Out] (2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
 - 3*x^2)^(1/4))])/(3*Sqrt[3]) + (2^(1/4)*ArcTanh[(2^(3/4) + 2^(1
/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(3*Sqrt[3])
- (2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(3*Sqrt[3])
```

Rubi [A] time = 0.163983, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
 - 3*x^2)^(1/4))])/(3*Sqrt[3]) + (2^(1/4)*ArcTanh[(2^(3/4) + 2^(1
/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(3*Sqrt[3])
- (2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 9.36029, size = 27, normalized size = 0.18

$$\frac{2^{\frac{3}{4}}x^3\,\mathrm{appellf}_1\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] 2**(3/4)*x**3*appellf1(3/2, 1/4, 1, 5/2, 3*x**2/2, 3*x**2/4)/24
```

Mathematica [C] time = 0.05643, size = 140, normalized size = 0.95

$$-\frac{20x^3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{3\sqrt[4]{2-3x^2}\,(3x^2-4)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+20F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-20*x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(3*(2
- 3*x^2)^(1/4)*(-4 + 3*x^2)*(20*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2
)/2, (3*x^2)/4] + 3*x^2*(2*AppellF1[5/2, 1/4, 2, 7/2, (3*x^2)/2,
(3*x^2)/4] + AppellF1[5/2, 5/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4])))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2+4}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\sqrt[4]{-3x^2+2}-4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")

[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1039 $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^
2)^(1/4))]/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*
x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi [A] time = 0.0607581, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))]/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 75.848, size = 88, normalized size = 0.73

$$-\frac{\sqrt[4]{2}\sqrt{3}i\sqrt{x^2}\left(-i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)\middle|-1\right)}{6x}+\frac{\sqrt[4]{2}\sqrt{3}i\sqrt{x^2}\left(i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)\middle|-1\right)}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2**(1/4)*sqrt(3)*I*sqrt(x**2)*elliptic_pi(-I, asin(2**(3/4)*(-3*
x**2 + 2)**(1/4)/2), -1)/(6*x) + 2**(1/4)*sqrt(3)*I*sqrt(x**2)*el
liptic_pi(I, asin(2**(3/4)*(-3*x**2 + 2)**(1/4)/2), -1)/(6*x)
```

Mathematica [C] time = 0.0596759, size = 135, normalized size = 1.12

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\sqrt[4]{2-3x^2}(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x
^2)^(1/4)*(-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (
3*x^2)/4] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/
4] + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-3 x^2 + 4} \frac{1}{\sqrt[4]{-3 x^2 + 2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3 x^2 - 4)(-3 x^2 + 2)^{\frac{1}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 \sqrt[4]{-3x^2 + 2} - 4 \sqrt[4]{-3x^2 + 2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1/4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3 x^2 - 4)(-3 x^2 + 2)^{\frac{1}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1040 $\int \frac{1}{x^2 \sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=166

$$-\frac{(2-3x^2)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqr
t[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) + (Sqrt
[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3
*x^2)^(1/4))])/(8*2^(3/4)) - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*
x]/2, 2])/(4*2^(3/4))
```

Rubi [A] time = 0.189185, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{(2-3x^2)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqr
t[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) + (Sqrt
[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3
*x^2)^(1/4))])/(8*2^(3/4)) - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*
x]/2, 2])/(4*2^(3/4))
```

Rubi in Sympy [A] time = 8.85089, size = 29, normalized size = 0.17

$$-\frac{2^{\frac{3}{4}}\operatorname{appellf_1}\left(-\frac{1}{2},\frac{1}{4},1,\frac{1}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2**(3/4)*appellf1(-1/2, 1/4, 1, 1/2, 3*x**2/2, 3*x**2/4)/(8*x)
```

Mathematica [C] time = 0.173584, size = 152, normalized size = 0.92

$$\frac{-\frac{30x^4F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+20F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}+3x^2-2}{8x\sqrt[4]{2-3x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2 + 3*x^2 - (30*x^4*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^
2)/4])/((-4 + 3*x^2)*(20*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3
*x^2)/4] + 3*x^2*(2*AppellF1[5/2, 1/4, 2, 7/2, (3*x^2)/2, (3*x^2)
```

```
/4] + AppellF1[5/2, 5/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4]))))/(8*x*(
2 - 3*x^2)^(1/4))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(-3x^2+4)} \frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^4\sqrt[4]{-3x^2+2} - 4x^2\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**4*(-3*x**2 + 2)**(1/4) - 4*x**2*(-3*x**2 + 2)**
(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

3.1041 $\int \frac{1}{x^4 \sqrt[4]{2-3x^2}(4-3x^2)} dx$

Optimal. Leaf size=184

$$-\frac{3\left(2-3x^2\right)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{\left(2-3x^2\right)^{3/4}}{24x^3}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(3/4)/(24*x^3) - (3*(2 - 3*x^2)^(3/4))/(16*x) + (3*S
qrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 -
 3*x^2)^(1/4))])/(32*2^(3/4)) + (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(
1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(32*2^(3/4)
) - (3*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(8*2^(3/4))
```

Rubi [A] time = 0.242954, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{3\left(2-3x^2\right)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{\left(2-3x^2\right)^{3/4}}{24x^3}-\frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(24*x^3) - (3*(2 - 3*x^2)^(3/4))/(16*x) + (3*S
qrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 -
 3*x^2)^(1/4))])/(32*2^(3/4)) + (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(
1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(32*2^(3/4)
) - (3*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(8*2^(3/4))
```

Rubi in Sympy [A] time = 8.98586, size = 32, normalized size = 0.17

$$-\frac{2^{\frac{3}{4}}\,\text{appellf}_1\left(-\frac{3}{2},\frac{1}{4},1,-\frac{1}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2**(3/4)*appellf1(-3/2, 1/4, 1, -1/2, 3*x**2/2, 3*x**2/4)/(24*x*
*3)
```

Mathematica [C] time = 0.247531, size = 156, normalized size = 0.85

$$\frac{1}{8}(2-3x^2)^{3/4}\left(\frac{9xF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)-3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}-\frac{9x^2+2}{6x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ((2 - 3*x^2)^(3/4)*(-(2 + 9*x^2)/(6*x^3) + (9*x*AppellF1[1/2, -3/
4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2,
-3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2, -3/4,
2, 5/2, (3*x^2)/2, (3*x^2)/4] - 3*AppellF1[3/2, 1/4, 1, 5/2, (3*x
^2)/2, (3*x^2)/4])))))/8
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-3x^2+4\right)}\frac{1}{\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^6\sqrt[4]{-3x^2+2}-4x^4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**6*(-3*x**2 + 2)**(1/4) - 4*x**4*(-3*x**2 + 2)**
(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

3.1042 $\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (14*(-1 + 3*x^2)^(3/4))/243 + (8*(-1 + 3*x^2)^(7/4))/567 + (2*(-1
 + 3*x^2)^(11/4))/891 + (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*Ar
cTanh[(-1 + 3*x^2)^(1/4)])/81
```

Rubi [A] time = 0.158577, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(3/4))/243 + (8*(-1 + 3*x^2)^(7/4))/567 + (2*(-1
 + 3*x^2)^(11/4))/891 + (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*Ar
cTanh[(-1 + 3*x^2)^(1/4)])/81
```

Rubi in Sympy [A] time = 17.3132, size = 70, normalized size = 0.9

$$\frac{2\left(3x^2-1\right)^{\frac{11}{4}}}{891}+\frac{8\left(3x^2-1\right)^{\frac{7}{4}}}{567}+\frac{14\left(3x^2-1\right)^{\frac{3}{4}}}{243}+\frac{8\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}-\frac{8\,\mathrm{atanh}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(11/4)/891 + 8*(3*x**2 - 1)**(7/4)/567 + 14*(3*x*
*2 - 1)**(3/4)/243 + 8*atan((3*x**2 - 1)**(1/4))/81 - 8*atanh((3*
x**2 - 1)**(1/4))/81
```

Mathematica [C] time = 0.0821995, size = 74, normalized size = 0.95

$$\frac{2\left(-1848\sqrt[4]{\frac{1-3x^2}{2-3x^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2-3x^2}\right)+567x^6+621x^4+1014x^2-428\right)}{18711\sqrt[4]{3x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-428 + 1014*x^2 + 621*x^4 + 567*x^6 - 1848*((1 - 3*x^2)/(2 -
3*x^2))^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (2 - 3*x^2)^(-1)])
)/(18711*(-1 + 3*x^2)^(1/4))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{x^7}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.49877, size = 100, normalized size = 1.28

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^
2 - 1)^(3/4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 -
 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.234819, size = 86, normalized size = 1.1

$$\frac{2}{18711}\left(189x^4+270x^2+428\right)\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 270*x^2 + 428)*(3*x^2 - 1)^(3/4) + 8/81*arctan
((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log(
(3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(x**7/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [A] time = 0.241554, size = 101, normalized size = 1.29

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^
2 - 1)^(3/4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*ln((3*x^2 -
1)^(1/4) + 1) + 4/81*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1043 $\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*(-1 + 3*x^2)^(7/4))/189 + (4*ArcTa
n[(-1 + 3*x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rubi [A] time = 0.147368, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*(-1 + 3*x^2)^(7/4))/189 + (4*ArcTa
n[(-1 + 3*x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rubi in Sympy [A] time = 16.1955, size = 56, normalized size = 0.89

$$\frac{2\left(3x^2-1\right)^{\frac{7}{4}}}{189}+\frac{2\left(3x^2-1\right)^{\frac{3}{4}}}{27}+\frac{4\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}-\frac{4\,\mathrm{atanh}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(7/4)/189 + 2*(3*x**2 - 1)**(3/4)/27 + 4*atan((3*
x**2 - 1)**(1/4))/27 - 4*atanh((3*x**2 - 1)**(1/4))/27
```

Mathematica [C] time = 0.0645802, size = 69, normalized size = 1.1

$$\frac{2\left(-28\sqrt[4]{\frac{1-3x^2}{2-3x^2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2-3x^2}\right)+9x^4+15x^2-6\right)}{189\sqrt[4]{3x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-6 + 15*x^2 + 9*x^4 - 28*((1 - 3*x^2)/(2 - 3*x^2))^(1/4)*Hype
rgeometric2F1[1/4, 1/4, 5/4, (2 - 3*x^2)^(-1)]))/(189*(-1 + 3*x^2
)^(1/4))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{x^5}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.50731, size = 85, normalized size = 1.35

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3
*x^2 - 1)^(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*
x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.23455, size = 77, normalized size = 1.22

$$\frac{2}{63}\left(3x^2-1\right)^{\frac{3}{4}}\left(x^2+2\right)+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 2/63*(3*x^2 - 1)^(3/4)*(x^2 + 2) + 4/27*arctan((3*x^2 - 1)^(1/4))
 - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) -
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(x**5/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [A] time = 0.239257, size = 86, normalized size = 1.37

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3
*x^2 - 1)^(1/4)) - 2/27*ln((3*x^2 - 1)^(1/4) + 1) + 2/27*ln(abs((
3*x^2 - 1)^(1/4) - 1))
```

3.1044 $\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2
*ArcTanh[(-1 + 3*x^2)^(1/4)])/9
```

Rubi [A] time = 0.118385, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2
*ArcTanh[(-1 + 3*x^2)^(1/4)])/9
```

Rubi in Sympy [A] time = 13.9646, size = 42, normalized size = 0.88

$$\frac{2\left(3x^2-1\right)^{\frac{3}{4}}}{27}+\frac{2\,\text{atan}\left(\sqrt[4]{3x^2-1}\right)}{9}-\frac{2\,\text{atanh}\left(\sqrt[4]{3x^2-1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(3/4)/27 + 2*atan((3*x**2 - 1)**(1/4))/9 - 2*atan
h((3*x**2 - 1)**(1/4))/9
```

Mathematica [C] time = 0.0285521, size = 34, normalized size = 0.71

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}\left(1-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};3x^2-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4)*(1 - 2*Hypergeometric2F1[3/4, 1, 7/4, -1 +
3*x^2]))/27
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{x^3}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)

[Out] int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.49916, size = 70, normalized size = 1.46

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")

[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log(
(3*x^2 - 1)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.235034, size = 70, normalized size = 1.46

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")

[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log(
(3*x^2 - 1)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)

[Out] Integral(x**3/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [A] time = 0.234374, size = 72, normalized size = 1.5

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")

[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*ln((
3*x^2 - 1)^(1/4) + 1) + 1/9*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1045 $\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rubi [A] time = 0.0767655, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rubi in Sympy [A] time = 11.2377, size = 26, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}-\frac{\operatorname{atanh}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] atan((3*x**2 - 1)**(1/4))/3 - atanh((3*x**2 - 1)**(1/4))/3
```

Mathematica [A] time = 0.0151013, size = 55, normalized size = 1.67

$$\frac{1}{6}\log\left(1-\sqrt[4]{3x^2-1}\right)-\frac{1}{6}\log\left(\sqrt[4]{3x^2-1}+1\right)+\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 + Log[1 - (-1 + 3*x^2)^(1/4)]/6 - Lo
g[1 + (-1 + 3*x^2)^(1/4)]/6
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{x}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [A] time = 1.49821, size = 55, normalized size = 1.67

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) +
1/6*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.23772, size = 55, normalized size = 1.67

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) +
1/6*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [A] time = 4.19108, size = 48, normalized size = 1.45

$$\frac{\log\left(-1+\frac{1}{\sqrt[4]{3x^2-1}}\right)}{6}-\frac{\log\left(1+\frac{1}{\sqrt[4]{3x^2-1}}\right)}{6}-\frac{\operatorname{atan}\left(\frac{1}{\sqrt[4]{3x^2-1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] log(-1 + (3*x**2 - 1)**(-1/4))/6 - log(1 + (3*x**2 - 1)**(-1/4))/
6 - atan((3*x**2 - 1)**(-1/4))/3
```

GIAC/XCAS [A] time = 0.238624, size = 57, normalized size = 1.73

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*ln((3*x^2 - 1)^(1/4) + 1) + 1
/6*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1046 $\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=173

$$-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1
/4)]/(2*Sqrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt
[2]) - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 - Log[1 - Sqrt[2]*(-1 + 3*x^
2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 +
3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2])
```

Rubi [A] time = 0.334922, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1
/4)]/(2*Sqrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt
[2]) - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 - Log[1 - Sqrt[2]*(-1 + 3*x^
2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 +
3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 31.7308, size = 148, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{8}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{8}+\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}-1\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4}-\frac{\operatorname{atanh}\left(\sqrt[4]{3x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] -sqrt(2)*log(-sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 - 1) + 1)
/8 + sqrt(2)*log(sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 - 1) +
 1)/8 + atan((3*x**2 - 1)**(1/4))/2 - sqrt(2)*atan(sqrt(2)*(3*x**
2 - 1)**(1/4) - 1)/4 - sqrt(2)*atan(sqrt(2)*(3*x**2 - 1)**(1/4) +
 1)/4 - atanh((3*x**2 - 1)**(1/4))/2
```

Mathematica [C] time = 0.217391, size = 137, normalized size = 0.79

$$-\frac{54x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)}{5\left(3x^2-2\right)\sqrt[4]{3x^2-1}\left(27x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+8F_1\left(\frac{9}{4};\frac{1}{4},2;\frac{13}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+F_1\left(\frac{9}{4};\frac{5}{4},1;\frac{13}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]   Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (-54*x^2*AppellF1[5/4, 1/4, 1, 9/4, 1/(3*x^2), 2/(3*x^2)])/(5*(-2
 + 3*x^2)*(-1 + 3*x^2)^(1/4)*(27*x^2*AppellF1[5/4, 1/4, 1, 9/4, 1
/(3*x^2), 2/(3*x^2)] + 8*AppellF1[9/4, 1/4, 2, 13/4, 1/(3*x^2), 2
/(3*x^2)] + AppellF1[9/4, 5/4, 1, 13/4, 1/(3*x^2), 2/(3*x^2)]))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-2)\,x}\,\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{1}{4}}(3x^2-2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x), x)
```

Fricas [A] time = 0.249561, size = 284, normalized size = 1.64

$$\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\sqrt{2}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+4\arctan\left(\frac{}{\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{2\sqrt{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2*sqrt(2)*arctan((3*x^2 - 1)^(1/4)) - sqrt(2)*log((3
*x^2 - 1)^(1/4) + 1) + sqrt(2)*log((3*x^2 - 1)^(1/4) - 1) + 4*arc
tan(1/(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(2*sqrt(2)*(3*x^2 - 1)^(1/
4) + 2*sqrt(3*x^2 - 1) + 2) + 1)) + 4*arctan(1/(sqrt(2)*(3*x^2 -
1)^(1/4) + sqrt(-2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1)
+ 2) - 1)) + log(2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1)
+ 2) - log(-2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [A] time = 0.240291, size = 209, normalized size = 1.21

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{8}\sqrt{2}\ln\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{1}{8}\sqrt{2}\ln\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)+\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4)))
- 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))
) + 1/8*sqrt(2)*ln(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) +
1) - 1/8*sqrt(2)*ln(-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
+ 1) + 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*ln((3*x^2 - 1)^(1/4) +
 1) + 1/4*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1047 $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{(3x^2-1)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] -(-1 + 3*x^2)^(3/4)/(4*x^2) + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 +
(9*ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (9*ArcTa
n[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 +
 3*x^2)^(1/4)])/4 - (9*Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[
-1 + 3*x^2]])/(16*Sqrt[2]) + (9*Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4
) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rubi [A] time = 0.40319, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542

$$-\frac{(3x^2-1)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(3/4)/(4*x^2) + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 +
(9*ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (9*ArcTa
n[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 +
 3*x^2)^(1/4)])/4 - (9*Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[
-1 + 3*x^2]])/(16*Sqrt[2]) + (9*Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4
) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 37.5625, size = 173, normalized size = 0.91

$$-\frac{9\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{32}+\frac{9\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{32}+\frac{3\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{4}-\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}-1\right)}{16}-\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16}-\frac{3\,\mathrm{atanh}\left(\sqrt[4]{3x^2-1}\right)}{4}-\frac{(3x^2-1)^{\frac{3}{4}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] -9*sqrt(2)*log(-sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 - 1) +
1)/32 + 9*sqrt(2)*log(sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 -
 1) + 1)/32 + 3*atan((3*x**2 - 1)**(1/4))/4 - 9*sqrt(2)*atan(sqrt
(2)*(3*x**2 - 1)**(1/4) - 1)/16 - 9*sqrt(2)*atan(sqrt(2)*(3*x**2
- 1)**(1/4) + 1)/16 - 3*atanh((3*x**2 - 1)**(1/4))/4 - (3*x**2 -
1)**(3/4)/(4*x**2)
```

Mathematica [C] time = 0.370066, size = 252, normalized size = 1.32

$$\frac{-\frac{90x^4F_1\left(1;\frac{1}{4},1;2;3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(3x^2\left(2F_1\left(2;\frac{1}{4},2;3;3x^2,\frac{3x^2}{2}\right)+F_1\left(2;\frac{5}{4},1;3;3x^2,\frac{3x^2}{2}\right)\right)+8F_1\left(1;\frac{1}{4},1;2;3x^2,\frac{3x^2}{2}\right)\right)}-\frac{486x^4F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)}{(3x^2-2)\left(27x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+8F_1\left(\frac{9}{4};\frac{1}{4},2;\frac{13}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+F_1\left(\frac{9}{4};\frac{5}{4}\right.\right.}}{20x^2\sqrt[4]{3x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (5 - 15*x^2 - (90*x^4*AppellF1[1, 1/4, 1, 2, 3*x^2, (3*x^2)/2])/(
(-2 + 3*x^2)*(8*AppellF1[1, 1/4, 1, 2, 3*x^2, (3*x^2)/2] + 3*x^2*
(2*AppellF1[2, 1/4, 2, 3, 3*x^2, (3*x^2)/2] + AppellF1[2, 5/4, 1,
 3, 3*x^2, (3*x^2)/2]))) - (486*x^4*AppellF1[5/4, 1/4, 1, 9/4, 1/
(3*x^2), 2/(3*x^2)])/((-2 + 3*x^2)*(27*x^2*AppellF1[5/4, 1/4, 1,
9/4, 1/(3*x^2), 2/(3*x^2)] + 8*AppellF1[9/4, 1/4, 2, 13/4, 1/(3*x
^2), 2/(3*x^2)] + AppellF1[9/4, 5/4, 1, 13/4, 1/(3*x^2), 2/(3*x^2
)])))/(20*x^2*(-1 + 3*x^2)^(1/4))
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{x^3(3x^2-2)}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{1}{4}}(3x^2-2)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3), x)
```

Fricas [A] time = 0.251907, size = 335, normalized size = 1.75

$$\frac{36\sqrt{2}x^2\arctan\left(\frac{1}{\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{2\sqrt{2}(3x^2-1)^{\frac{1}{4}}+2\sqrt{3x^2-1}+2}+1}\right)+36\sqrt{2}x^2\arctan\left(\frac{1}{\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{-2\sqrt{2}(3x^2-1)^{\frac{1}{4}}+2\sqrt{3x^2-1}+2}-1}\right)+9\sqrt{2}x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/32*(36*sqrt(2)*x^2*arctan(1/(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(2
*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2) + 1)) + 36*sq
rt(2)*x^2*arctan(1/(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(-2*sqrt(2)*(
3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2) - 1)) + 9*sqrt(2)*x^2*l
og(2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2) - 9*sqrt(
2)*x^2*log(-2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2)
+ 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3*x^2 - 1)^(1/4)
 + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(3/4))/
x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (3x^2-2) \sqrt[4]{3x^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [A] time = 0.246114, size = 228, normalized size = 1.19

$$-\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{9}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{9}{32}\sqrt{2}\ln\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{9}{32}\sqrt{2}\ln\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{\left(3x^2-1\right)^{\frac{3}{4}}}{4x^2}+\frac{3}{4}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{3}{8}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3),x, algorithm="giac")
```

```
[Out] -9/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4)))
 - 9/16*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4
))) + 9/32*sqrt(2)*ln(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - 9/32*sqrt(2)*ln(-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 -
 1) + 1) - 1/4*(3*x^2 - 1)^(3/4)/x^2 + 3/4*arctan((3*x^2 - 1)^(1/
4)) - 3/8*ln((3*x^2 - 1)^(1/4) + 1) + 3/8*ln(abs((3*x^2 - 1)^(1/4
) - 1))
```

3.1048 $\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=244

$$\frac{2}{45}\left(3x^2-1\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-1}\,x}{15\left(\sqrt{3x^2-1}+1\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{4\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{8\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}$$

```
[Out] (2*x*(-1 + 3*x^2)^(3/4))/45 + (8*x*(-1 + 3*x^2)^(1/4))/(15*(1 + S
qrt[-1 + 3*x^2])) - (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^
(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])
/9 - (8*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])
*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x) + (
4*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*Ellip
ticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x)
```

Rubi [A] time = 0.43382, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{2}{45}\left(3x^2-1\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-1}\,x}{15\left(\sqrt{3x^2-1}+1\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{4\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{8\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(3/4))/45 + (8*x*(-1 + 3*x^2)^(1/4))/(15*(1 + S
qrt[-1 + 3*x^2])) - (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^
(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])
/9 - (8*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])
*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x) + (
4*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*Ellip
ticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x)
```

Rubi in Sympy [A] time = 21.4589, size = 41, normalized size = 0.17

$$\frac{x^5\left(3x^2-1\right)^{\frac{3}{4}}\operatorname{appellf}_1\left(\frac{5}{2},\frac{1}{4},1,\frac{7}{2},3x^2,\frac{3x^2}{2}\right)}{10\left(-3x^2+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] x**5*(3*x**2 - 1)**(3/4)*appellf1(5/2, 1/4, 1, 7/2, 3*x**2, 3*x**
2/2)/(10*(-3*x**2 + 1)**(3/4))
```

Mathematica [C] time = 0.36198, size = 257, normalized size = 1.05

$$\frac{2x\left(\frac{60x^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)\right)+10F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}-\frac{4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2\right.}\right.}{45\sqrt[4]{3x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2 - (4*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2
])/((-2 + 3*x^2)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2]
+ x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[
3/2, 5/4, 1, 5/2, 3*x^2, (3*x^2)/2]))) + (60*x^2*AppellF1[3/2, 1/
4, 1, 5/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(10*AppellF1[3/2, 1/4
, 1, 5/2, 3*x^2, (3*x^2)/2] + 3*x^2*(2*AppellF1[5/2, 1/4, 2, 7/2,
 3*x^2, (3*x^2)/2] + AppellF1[5/2, 5/4, 1, 7/2, 3*x^2, (3*x^2)/2]
)))))/(45*(-1 + 3*x^2)^(1/4))
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \frac{x^4}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] integral(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1049 $\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=224

$$\frac{2\sqrt[4]{3x^2-1}x}{3\left(\sqrt{3x^2-1}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}+\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}-\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}$$

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/(3*(1 + Sqrt[-1 + 3*x^2])) - ArcTan[(Sqr
t[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x
^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1
/4)], 1/2])/(3*Sqrt[3]*x) + (Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(
1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2
])/(3*Sqrt[3]*x)
```

Rubi [A] time = 0.277336, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\sqrt[4]{3x^2-1}x}{3\left(\sqrt{3x^2-1}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}+\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}-\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```


```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/(3*(1 + Sqrt[-1 + 3*x^2])) - ArcTan[(Sqr
t[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x
^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1
/4)], 1/2])/(3*Sqrt[3]*x) + (Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(
1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2
])/(3*Sqrt[3]*x)
```

Rubi in Sympy [A] time = 24.7453, size = 41, normalized size = 0.18

$$\frac{x^3\left(3x^2-1\right)^{\frac{3}{4}}\operatorname{appellf}_1\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)}{6\left(-3x^2+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] x**3*(3*x**2 - 1)**(3/4)*appellf1(3/2, 1/4, 1, 5/2, 3*x**2, 3*x**
2/2)/(6*(-3*x**2 + 1)**(3/4))
```

Mathematica [C] time = 0.054455, size = 132, normalized size = 0.59

$$\frac{10x^3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{3\left(3x^2-2\right)\sqrt[4]{3x^2-1}\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)\right)+10F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (10*x^3*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(3*(-2 + 3*
x^2)*(-1 + 3*x^2)^(1/4)*(10*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*
x^2)/2] + 3*x^2*(2*AppellF1[5/2, 1/4, 2, 7/2, 3*x^2, (3*x^2)/2] +
 AppellF1[5/2, 5/4, 1, 7/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{x^2}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] integral(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1050 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(
Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi [A] time = 0.0405598, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(
Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi in Sympy [A] time = 51.5471, size = 168, normalized size = 2.75

$$\frac{\sqrt{2}x(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{3x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{3x^2-1}+1}\sqrt{i\sqrt{3x^2-1}+1}}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)\middle|\frac{1}{2}\right)}{12x}-\frac{\sqrt{6}\sqrt{x^2}\operatorname{atanh}\left(\frac{\sqrt{6}\sqrt[4]{3x^2-1}}{3\sqrt{x^2}}\right)}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(3*x**2 - 1
)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(3*x**2 - 1) + 1)*sqrt(I*sqrt(3*x
**2 - 1) + 1)) - sqrt(3)*sqrt(x**2/(sqrt(3*x**2 - 1) + 1)**2)*(sq
rt(3*x**2 - 1) + 1)*elliptic_f(2*atan((3*x**2 - 1)**(1/4)), 1/2)/
(12*x) - sqrt(6)*sqrt(x**2)*atanh(sqrt(6)*(3*x**2 - 1)**(1/4)/(3*
sqrt(x**2)))/(12*x)
```

Mathematica [C] time = 0.0479984, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\sqrt[4]{3x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*
(-1 + 3*x^2)^(1/4)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2
] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF
1[3/2, 5/4, 1, 5/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3x^2-2}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{1}{4}}(3x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [A] time = 2.66693, size = 139, normalized size = 2.28

$$\frac{1}{24}\sqrt{6}\left(2\arctan\left(\frac{\sqrt{6}(3x^2-1)^{\frac{1}{4}}}{3x}\right)+\log\left(\frac{36\left(3x^2-1\right)^{\frac{1}{4}}x^3-12\sqrt{6}\sqrt{3x^2-1}x^2+24\left(3x^2-1\right)^{\frac{3}{4}}x-\sqrt{6}(9x^4+12x^2-4)}{9x^4-12x^2+4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*(2*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + log((36
*(3*x^2 - 1)^(1/4)*x^3 - 12*sqrt(6)*sqrt(3*x^2 - 1)*x^2 + 24*(3*x
^2 - 1)^(3/4)*x - sqrt(6)*(9*x^4 + 12*x^2 - 4))/(9*x^4 - 12*x^2 +
 4)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")

[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1051 $\int \frac{1}{x^2(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=246

$$\frac{3\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{\left(3x^2-1\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{4x}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}$$

```
[Out] -(-1 + 3*x^2)^(3/4)/(2*x) + (3*x*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt
[-1 + 3*x^2])) - (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/
4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4
- (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^
2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x) + (Sqrt[3
]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*Ellip
ticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rubi [A] time = 0.319627, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{3\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{\left(3x^2-1\right)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{4x}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```


```
[Out] -(-1 + 3*x^2)^(3/4)/(2*x) + (3*x*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt
[-1 + 3*x^2])) - (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/
4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4
- (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^
2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x) + (Sqrt[3
]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*Ellip
ticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rubi in Sympy [A] time = 23.7882, size = 42, normalized size = 0.17

$$-\frac{\left(3x^2-1\right)^{\frac{3}{4}}\operatorname{appellf}_1\left(-\frac{1}{2},\frac{1}{4},1,\frac{1}{2},3x^2,\frac{3x^2}{2}\right)}{2x\left(-3x^2+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] -(3*x**2 - 1)**(3/4)*appellf1(-1/2, 1/4, 1, 1/2, 3*x**2, 3*x**2/2
)/(2*x*(-3*x**2 + 1)**(3/4))
```

Mathematica [C] time = 0.179085, size = 144, normalized size = 0.59

$$\frac{\frac{15x^4F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)\right)+10F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}-3x^2+1}{2x\sqrt[4]{3x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (1 - 3*x^2 + (15*x^4*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2]
)/((-2 + 3*x^2)*(10*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2]
+ 3*x^2*(2*AppellF1[5/2, 1/4, 2, 7/2, 3*x^2, (3*x^2)/2] + AppellF
1[5/2, 5/4, 1, 7/2, 3*x^2, (3*x^2)/2]))))/(2*x*(-1 + 3*x^2)^(1/4)
)
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(3\,x^2-2\right)}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3\,x^4-2\,x^2)(3\,x^2-1)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((3*x^4 - 2*x^2)*(3*x^2 - 1)^(1/4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,(3x^2-2)\,\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

3.1052 $\int \frac{1}{x^4(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=264

$$\frac{9\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{3\left(3x^2-1\right)^{3/4}}{2x}-\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{4x}-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}-\frac{\left(3x^2-1\right)^{3/4}}{6x^3}$$

```
[Out] -(-1 + 3*x^2)^(3/4)/(6*x^3) - (3*(-1 + 3*x^2)^(3/4))/(2*x) + (9*x
*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^2])) - (3*Sqrt[3/2]*Ar
cTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[
(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3]*Sqrt[x^2/(1 + S
qrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1
 + 3*x^2)^(1/4)], 1/2])/(2*x) + (3*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1
+ 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^
2)^(1/4)], 1/2])/(4*x)
```

Rubi [A] time = 0.522526, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{9\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{3\left(3x^2-1\right)^{3/4}}{2x}-\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{4x}-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}-\frac{\left(3x^2-1\right)^{3/4}}{6x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```


```
[Out] -(-1 + 3*x^2)^(3/4)/(6*x^3) - (3*(-1 + 3*x^2)^(3/4))/(2*x) + (9*x
*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^2])) - (3*Sqrt[3/2]*Ar
cTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[
(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3]*Sqrt[x^2/(1 + S
qrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1
 + 3*x^2)^(1/4)], 1/2])/(2*x) + (3*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1
+ 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^
2)^(1/4)], 1/2])/(4*x)
```

Rubi in Sympy [A] time = 23.8747, size = 46, normalized size = 0.17

$$-\frac{\left(3x^2-1\right)^{\frac{3}{4}}\operatorname{appellf}_1\left(-\frac{3}{2},\frac{1}{4},1,-\frac{1}{2},3x^2,\frac{3x^2}{2}\right)}{6x^3\left(-3x^2+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] -(3*x**2 - 1)**(3/4)*appellf1(-3/2, 1/4, 1, -1/2, 3*x**2, 3*x**2/
2)/(6*x**3*(-3*x**2 + 1)**(3/4))
```

Mathematica [C] time = 0.228995, size = 148, normalized size = 0.56

$$\frac{1}{2}\left(3x^2-1\right)^{3/4}\left(\frac{9xF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(x^2\left(2F_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}-\frac{9x^2+1}{3x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ((-1 + 3*x^2)^(3/4)*(-(1 + 9*x^2)/(3*x^3) + (9*x*AppellF1[1/2, -3
/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(2*AppellF1[1/2, -3/
4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2,
 3*x^2, (3*x^2)/2] - 3*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/
2])))))/2
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(3x^2-2\right)}\frac{1}{\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(3x^6-2x^4\right)\left(3x^2-1\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/((3*x^6 - 2*x^4)*(3*x^2 - 1)^(1/4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (3x^2 - 2) \sqrt[4]{3x^2 - 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2 - 1)^{\frac{1}{4}} (3\,x^2 - 2)x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

3.1053 $\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\,2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 +
 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4) - 2*2^(1
/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*
Sqrt[3])
```

Rubi [A] time = 0.0965152, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\tanh^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{3x^2+2}}{\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{3x^2+2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] -ArcTan[(2^(3/4) + 2^(1/4)*Sqrt[2 + 3*x^2])/(Sqrt[3]*x*(2 + 3*x^2
)^(1/4))]/(3*2^(1/4)*Sqrt[3]) + ArcTanh[(2^(3/4) - 2^(1/4)*Sqrt[2
 + 3*x^2])/(Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 9.65177, size = 31, normalized size = 0.24

$$\frac{\sqrt[4]{2}x^3\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2+2)**(3/4)/(3*x**2+4),x)
```

```
[Out] 2**(1/4)*x**3*appellf1(3/2, 3/4, 1, 5/2, -3*x**2/2, -3*x**2/4)/24
```

Mathematica [C] time = 0.233651, size = 142, normalized size = 1.1

$$-\frac{20x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{3\left(3x^2+2\right)^{3/4}\left(3x^2+4\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)-20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] (-20*x^3*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/2, (-3*x^2)/4])/(3*(
2 + 3*x^2)^(3/4)*(4 + 3*x^2)*(-20*AppellF1[3/2, 3/4, 1, 5/2, (-3*
x^2)/2, (-3*x^2)/4] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, (-3*x^2
)/2, (-3*x^2)/4] + 3*AppellF1[5/2, 7/4, 1, 7/2, (-3*x^2)/2, (-3*x
^2)/4])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^2}{3\,x^2+4}\left(3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x)
```

```
[Out] int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3\,x^2+4)(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

Fricas [A] time = 0.240918, size = 351, normalized size = 2.72

$$\frac{1}{864} \cdot 72^{\frac{3}{4}}\left(4\,\sqrt{2}\arctan\left(\frac{3\,x}{\sqrt{6}x\sqrt{\frac{72^{\frac{1}{4}}\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}\,x+3\,x^2+2\,\sqrt{2}\sqrt{3\,x^2+2}}{x^2}}+72^{\frac{1}{4}}\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}+3\,x}\right)+4\,\sqrt{2}\arctan\left(\frac{}{\sqrt{6}x\sqrt{-\frac{72^{\frac{1}{4}}\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}}\,x-3}{x^2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 1/864*72^(3/4)*(4*sqrt(2)*arctan(3*x/(sqrt(6)*x*sqrt((72^(1/4)*sq
rt(2)*(3*x^2 + 2)^(1/4)*x + 3*x^2 + 2*sqrt(2)*sqrt(3*x^2 + 2))/x^
2) + 72^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4) + 3*x)) + 4*sqrt(2)*arcta
n(3*x/(sqrt(6)*x*sqrt(-(72^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x - 3*
x^2 - 2*sqrt(2)*sqrt(3*x^2 + 2))/x^2) + 72^(1/4)*sqrt(2)*(3*x^2 +
 2)^(1/4) - 3*x)) - sqrt(2)*log(6*(72^(1/4)*sqrt(2)*(3*x^2 + 2)^(
1/4)*x + 3*x^2 + 2*sqrt(2)*sqrt(3*x^2 + 2))/x^2) + sqrt(2)*log(-6
*(72^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x - 3*x^2 - 2*sqrt(2)*sqrt(3
*x^2 + 2))/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2+2)^{\frac{3}{4}}\,(3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2+2)**(3/4)/(3*x**2+4),x)
```

```
[Out] Integral(x**2/((3*x**2 + 2)**(3/4)*(3*x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3\,x^2+4)(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

3.1054 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^
2)^(1/4))]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*
x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.0953642, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 10.4464, size = 27, normalized size = 0.22

$$\frac{\sqrt[4]{2}x^3\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2**(1/4)*x**3*appellf1(3/2, 3/4, 1, 5/2, 3*x**2/2, 3*x**2/4)/24
```

Mathematica [C] time = 0.222895, size = 142, normalized size = 1.18

$$-\frac{20x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{3\left(2-3x^2\right)^{3/4}\left(3x^2-4\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-20*x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(3*(2
- 3*x^2)^(3/4)*(-4 + 3*x^2)*(20*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2
)/2, (3*x^2)/4] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, (3*x^2)/2,
(3*x^2)/4] + 3*AppellF1[5/2, 7/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4]))
)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [A] time = 0.242134, size = 351, normalized size = 2.92

$$\frac{1}{864} \cdot 72^{\frac{3}{4}}\left(4\sqrt{2}\arctan\left(\frac{3x}{\sqrt{6}x\sqrt{\frac{72^{\frac{1}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}x+3x^2+2\sqrt{2}\sqrt{-3x^2+2}}{x^2}}+72^{\frac{1}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+3x}\right)+4\sqrt{2}\arctan\left(\frac{}{\sqrt{6}x\sqrt{-\frac{72^{\frac{1}{4}}\sqrt{2}(-3x^2+2)}{}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 1/864*72^(3/4)*(4*sqrt(2)*arctan(3*x/(sqrt(6)*x*sqrt((72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4)*x + 3*x^2 + 2*sqrt(2)*sqrt(-3*x^2 + 2))/
x^2) + 72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 3*x)) + 4*sqrt(2)*ar
ctan(3*x/(sqrt(6)*x*sqrt(-(72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x
- 3*x^2 - 2*sqrt(2)*sqrt(-3*x^2 + 2))/x^2) + 72^(1/4)*sqrt(2)*(-3
*x^2 + 2)^(1/4) - 3*x)) - sqrt(2)*log(6*(72^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 3*x^2 + 2*sqrt(2)*sqrt(-3*x^2 + 2))/x^2) + sqrt(2
)*log(-6*(72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x - 3*x^2 - 2*sqrt(
2)*sqrt(-3*x^2 + 2))/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}}-4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3\,x^2 - 4)(-3\,x^2 + 2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1055 $\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\,2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

```
[Out] -(ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2
+ b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2*2^(3/4) - 2*2^(1
/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^
(3/2))
```

Rubi [A] time = 0.118746, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\tanh^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{bx^2+2}}{\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{bx^2+2}+2^{3/4}}{\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] -(ArcTan[(2^(3/4) + 2^(1/4)*Sqrt[2 + b*x^2])/(Sqrt[b]*x*(2 + b*x^
2)^(1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2^(3/4) - 2^(1/4)*Sqrt[2
 + b*x^2])/(Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rubi in Sympy [A] time = 10.3375, size = 31, normalized size = 0.25

$$\frac{\sqrt[4]{2}x^3\operatorname{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+2)**(3/4)/(b*x**2+4),x)
```

```
[Out] 2**(1/4)*x**3*appellf1(3/2, 3/4, 1, 5/2, -b*x**2/2, -b*x**2/4)/24
```

Mathematica [C] time = 0.229846, size = 150, normalized size = 1.21

$$-\frac{20x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{3\left(bx^2+2\right)^{3/4}\left(bx^2+4\right)\left(bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)-20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] (-20*x^3*AppellF1[3/2, 3/4, 1, 5/2, -(b*x^2)/2, -(b*x^2)/4])/(3*(
2 + b*x^2)^(3/4)*(4 + b*x^2)*(-20*AppellF1[3/2, 3/4, 1, 5/2, -(b*
x^2)/2, -(b*x^2)/4] + b*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, -(b*x^2
)/2, -(b*x^2)/4] + 3*AppellF1[5/2, 7/4, 1, 7/2, -(b*x^2)/2, -(b*x
^2)/4])))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x^{2}}{b x^{2}+4} \left(b x^{2}+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)

[Out] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{{\left(b x^{2} + 4\right)} {\left(b x^{2} + 2\right)}^{\frac{3}{4}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

Fricas [A] time = 0.24625, size = 532, normalized size = 4.29

$$\sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \frac{1}{b^{6}}^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x}{\sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x + 2 \sqrt{\frac{1}{2}} x \sqrt{\frac{\sqrt{\frac{1}{2}} b^{4} \sqrt{\frac{1}{b^{6}}} x^{2} + 2 \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} (b x^{2} + 2)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x + 2 \sqrt{b x^{2} + 2}}{x^{2}}} + 2 \, (b x^{2} + 2)^{\frac{1}{4}}}\right) + \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \frac{1}{b^{6}}^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x}{\sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x - 2 \sqrt{\frac{1}{2}} x \sqrt{\frac{\sqrt{\frac{1}{2}} b^{4} \sqrt{\frac{1}{b^{6}}} x^{2} - 2 \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} (b x^{2} + 2)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x + 2 \sqrt{b x^{2} + 2}}{x^{2}}} - 2 \, (b x^{2} + 2)^{\frac{1}{4}}}\right) - \frac{1}{4} \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \frac{1}{b^{6}}^{\frac{1}{4}} \log\left(\frac{\sqrt{\frac{1}{2}} b^{4} \sqrt{\frac{1}{b^{6}}} x^{2} + 2 \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \left(b x^{2} + 2\right)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x + 2 \sqrt{b x^{2} + 2}}{2 \, x^{2}}\right) + \frac{1}{4} \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \frac{1}{b^{6}}^{\frac{1}{4}} \log\left(\frac{\sqrt{\frac{1}{2}} b^{4} \sqrt{\frac{1}{b^{6}}} x^{2} - 2 \sqrt{2} \left(\frac{1}{8}\right)^{\frac{1}{4}} \left(b x^{2} + 2\right)^{\frac{1}{4}} b^{2} \frac{1}{b^{6}}^{\frac{1}{4}} x + 2 \sqrt{b x^{2} + 2}}{2 \, x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)),x, algorithm="fricas")

[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan(sqrt(2)*(1/8)^(1/4)*b^2
*(b^(-6))^(1/4)*x/(sqrt(2)*(1/8)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*s
qrt(1/2)*x*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)
^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2)
)/x^2) + 2*(b*x^2 + 2)^(1/4))) + sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/
4)*arctan(-sqrt(2)*(1/8)^(1/4)*b^2*(b^(-6))^(1/4)*x/(sqrt(2)*(1/8
)^(1/4)*b^2*(b^(-6))^(1/4)*x - 2*sqrt(1/2)*x*sqrt((sqrt(1/2)*b^4*
sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b
^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2) - 2*(b*x^2 + 2)^(1/4)))
- 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*s
qrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^
(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2) + 1/4*sqrt(2)*(1/8)^(1/4)
*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(
```

```
2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*
x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+2)^{\frac{3}{4}}(bx^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+2)**(3/4)/(b*x**2+4),x)
```

```
[Out] Integral(x**2/((b*x**2 + 2)**(3/4)*(b*x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+4)(bx^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

3.1056 $\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^
2)^(1/4))]/(2^(1/4)*b^(3/2)) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^
2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rubi [A] time = 0.128891, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-bx^2}}{\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-bx^2}+2^{3/4}}{\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - b*x^2])/(Sqrt[b]*x*(2 - b*x^2)
^(1/4))]/(2^(1/4)*b^(3/2)) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 -
b*x^2])/(Sqrt[b]*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rubi in Sympy [A] time = 11.6322, size = 27, normalized size = 0.23

$$\frac{\sqrt[4]{2}x^3\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{bx^2}{2},\frac{bx^2}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-b*x**2+2)**(3/4)/(-b*x**2+4),x)
```

```
[Out] 2**(1/4)*x**3*appellf1(3/2, 3/4, 1, 5/2, b*x**2/2, b*x**2/4)/24
```

Mathematica [C] time = 0.222151, size = 151, normalized size = 1.27

$$-\frac{20x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)}{3\left(2-bx^2\right)^{3/4}\left(bx^2-4\right)\left(bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)+20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] (-20*x^3*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/2, (b*x^2)/4])/(3*(2
- b*x^2)^(3/4)*(-4 + b*x^2)*(20*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2
)/2, (b*x^2)/4] + b*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, (b*x^2)/2,
(b*x^2)/4] + 3*AppellF1[5/2, 7/4, 1, 7/2, (b*x^2)/2, (b*x^2)/4]))
)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2+4}\left(-bx^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)

[Out] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(bx^2-4)(-bx^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)),x, algorithm="maxima")

[Out] -integrate(x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

Fricas [A] time = 0.246712, size = 545, normalized size = 4.58

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x}{\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{\frac{1}{2}}x\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(-bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{x^2}}+2\left(-bx^2+2\right)^{\frac{1}{4}}}\right)+\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x}{\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x-2\sqrt{\frac{1}{2}}x\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(-bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{x^2}}-2\left(-bx^2+2\right)^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^2+2\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{2x^2}\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^2+2\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)),x, algorithm="fricas")

[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan(sqrt(2)*(1/8)^(1/4)*b^2
*(b^(-6))^(1/4)*x/(sqrt(2)*(1/8)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*s
qrt(1/2)*x*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)
^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 +
2))/x^2) + 2*(-b*x^2 + 2)^(1/4))) + sqrt(2)*(1/8)^(1/4)*(b^(-6))^
(1/4)*arctan(-sqrt(2)*(1/8)^(1/4)*b^2*(b^(-6))^(1/4)*x/(sqrt(2)*(
1/8)^(1/4)*b^2*(b^(-6))^(1/4)*x - 2*sqrt(1/2)*x*sqrt((sqrt(1/2)*b
^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^
2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2) - 2*(-b*x^2 + 2)^(1
/4))) - 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)
*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*
b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2) + 1/4*sqrt(2)*(1/
8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 -
 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x +
2*sqrt(-b*x^2 + 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{bx^2\left(-bx^2+2\right)^{\frac{3}{4}}-4\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-b*x**2+2)**(3/4)/(-b*x**2+4),x)
```

```
[Out] -Integral(x**2/(b*x**2*(-b*x**2 + 2)**(3/4) - 4*(-b*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(bx^2-4)(-bx^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

3.1057 $\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3
*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1
/4))
```

Rubi [A] time = 0.109356, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```


```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3
*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1
/4))
```

Rubi in Sympy [A] time = 32.7863, size = 53, normalized size = 0.44

$$\frac{x^3\sqrt[4]{a+3x^2}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{6a^2\sqrt[4]{1+\frac{3x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2+a)**(3/4)/(3*x**2+2*a),x)
```

```
[Out] x**3*(a + 3*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, -3*x**2/a, -3
*x**2/(2*a))/(6*a**2*(1 + 3*x**2/a)**(1/4))
```

Mathematica [C] time = 0.272825, size = 162, normalized size = 1.35

$$-\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{3\left(a+3x^2\right)^{3/4}\left(2a+3x^2\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] (-10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)]
)/(3*(a + 3*x^2)^(3/4)*(2*a + 3*x^2)*(-10*a*AppellF1[3/2, 3/4, 1,
 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + 3*x^2*(2*AppellF1[5/2, 3/4, 2
, 7/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1, 7/2,
 (-3*x^2)/a, (-3*x^2)/(2*a)])))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{x^{2}}{3 x^{2}+2 a}\left(3 x^{2}+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

```
[Out] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{(3 x^{2}+2 a)(3 x^{2}+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

Fricas [A] time = 0.241943, size = 212, normalized size = 1.77

$$\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}}x\sqrt{\frac{3 x^{2}\sqrt{-\frac{1}{a}}+2\sqrt{3 x^{2}+a}}{x^{2}}}+\left(3 x^{2}+a\right)^{\frac{1}{4}}}\right) - \frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(3 x^{2}+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(3 x^{2}+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(3*(1/36)^(1/4)*x*(-1/a)^(1/4
)/(sqrt(1/2)*x*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(3*x^2 + a))/x^2) +
 (3*x^2 + a)^(1/4))) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/36
)^(1/4)*x*(-1/a)^(1/4) + (3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)
*(-1/a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (3*x^2 + a)^(
1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(a+3x^{2}\right)^{\frac{3}{4}}\left(2a+3x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2+a)**(3/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(x**2/((a + 3*x**2)**(3/4)*(2*a + 3*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

3.1058 $\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*
x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a -
 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/
4))
```

Rubi [A] time = 0.104978, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*
x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a -
 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/
4))
```

Rubi in Sympy [A] time = 33.6522, size = 49, normalized size = 0.41

$$\frac{x^3\sqrt[4]{a-3x^2}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{6a^2\sqrt[4]{1-\frac{3x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+a)**(3/4)/(-3*x**2+2*a),x)
```

```
[Out] x**3*(a - 3*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, 3*x**2/a, 3*x
**2/(2*a))/(6*a**2*(1 - 3*x**2/a)**(1/4))
```

Mathematica [C] time = 0.263197, size = 162, normalized size = 1.35

$$-\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{3\left(a-3x^2\right)^{3/4}\left(3x^2-2a\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] (-10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2*a)])/
(3*(a - 3*x^2)^(3/4)*(-2*a + 3*x^2)*(10*a*AppellF1[3/2, 3/4, 1, 5
/2, (3*x^2)/a, (3*x^2)/(2*a)] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/
2, (3*x^2)/a, (3*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1, 7/2, (3*x^
2)/a, (3*x^2)/(2*a)])))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3\,x^2+2\,a}\left(-3\,x^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

```
[Out] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

Fricas [A] time = 0.240767, size = 212, normalized size = 1.77

$$\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{3\,\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}}x\sqrt{\frac{3\,x^2\sqrt{-\frac{1}{a}}+2\,\sqrt{-3\,x^2+a}}{x^2}}+\left(-3\,x^2+a\right)^{\frac{1}{4}}}\right)-\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\,\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(-3\,x^2+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\,\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(-3\,x^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(3*(1/36)^(1/4)*x*(-1/a)^(1/4
)/(sqrt(1/2)*x*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(-3*x^2 + a))/x^2)
+ (-3*x^2 + a)^(1/4))) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/
36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1
/4)*(-1/a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (-3*x^2 +
a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{-2a\left(a-3x^2\right)^{\frac{3}{4}}+3x^2\left(a-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+a)**(3/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(x**2/(-2*a*(a - 3*x**2)**(3/4) + 3*x**2*(a - 3*x**2)**(
3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3\,x^2 - 2\,a)(-3\,x^2 + a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

3.1059 $\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

```
[Out] -(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a +
b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2
))
```

Rubi [A] time = 0.132644, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] -(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a +
b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2
))
```

Rubi in Sympy [A] time = 36.2143, size = 53, normalized size = 0.46

$$\frac{x^3\sqrt[4]{a+bx^2}\operatorname{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{6a^2\sqrt[4]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**(3/4)/(b*x**2+2*a),x)
```

```
[Out] x**3*(a + b*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, -b*x**2/a, -b
*x**2/(2*a))/(6*a**2*(1 + b*x**2/a)**(1/4))
```

Mathematica [C] time = 0.287666, size = 171, normalized size = 1.49

$$\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{3\left(a+bx^2\right)^{3/4}\left(2a+bx^2\right)\left(10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] (10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -(b*x^2)/(2*a)
])/(3*(a + b*x^2)^(3/4)*(2*a + b*x^2)*(10*a*AppellF1[3/2, 3/4, 1,
 5/2, -((b*x^2)/a), -(b*x^2)/(2*a)] - b*x^2*(2*AppellF1[5/2, 3/4,
 2, 7/2, -((b*x^2)/a), -(b*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1,
7/2, -((b*x^2)/a), -(b*x^2)/(2*a)])))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^2 + 2a}\left(bx^2 + a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

```
[Out] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + 2a)(bx^2 + a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

Fricas [A] time = 0.240366, size = 251, normalized size = 2.18

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}} b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}}x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{bx^2+a}}{x^2}} + \left(bx^2+a\right)^{\frac{1}{4}}}\right) - \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}} b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}} + \left(bx^2+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}} b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}} - \left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan((1/4)^(1/4)*b^2*x*(-1/(a*
b^6))^(1/4)/(sqrt(1/2)*x*sqrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(
b*x^2 + a))/x^2) + (b*x^2 + a)^(1/4))) - 1/2*(1/4)^(1/4)*(-1/(a*b
^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (b*x^2 + a
)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4
)*b^2*x*(-1/(a*b^6))^(1/4) - (b*x^2 + a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^2)^{\frac{3}{4}}(2a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**(3/4)/(b*x**2+2*a),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**(3/4)*(2*a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2+2\,a)(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

3.1060 $\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*
x^2)^(1/4))]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b
*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.136382, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*
x^2)^(1/4))]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b
*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rubi in Sympy [A] time = 40.3722, size = 49, normalized size = 0.41

$$\frac{x^3\sqrt[4]{a-bx^2}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{6a^2\sqrt[4]{1-\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-b*x**2+a)**(3/4)/(-b*x**2+2*a),x)
```

```
[Out] x**3*(a - b*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, b*x**2/a, b*x
**2/(2*a))/(6*a**2*(1 - b*x**2/a)**(1/4))
```

Mathematica [C] time = 0.290312, size = 168, normalized size = 1.41

$$\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{3\left(a-bx^2\right)^{3/4}\left(2a-bx^2\right)\left(bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] (10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2*a)])/(
3*(a - b*x^2)^(3/4)*(2*a - b*x^2)*(10*a*AppellF1[3/2, 3/4, 1, 5/2
```

```
, (b*x^2)/a, (b*x^2)/(2*a)] + b*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2,
 (b*x^2)/a, (b*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1, 7/2, (b*x^2)
/a, (b*x^2)/(2*a)])))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2+2a}\left(-bx^2+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(bx^2-2a\right)\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

Fricas [A] time = 0.239467, size = 257, normalized size = 2.16

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}}x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{-bx^2+a}}{x^2}}+\left(-bx^2+a\right)^{\frac{1}{4}}}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(-bx^2+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(-bx^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan((1/4)^(1/4)*b^2*x*(-1/(a*
b^6))^(1/4)/(sqrt(1/2)*x*sqrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(
-b*x^2 + a))/x^2) + (-b*x^2 + a)^(1/4))) - 1/2*(1/4)^(1/4)*(-1/(a
*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (-b*x^2
+ a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(
1/4)*b^2*x*(-1/(a*b^6))^(1/4) - (-b*x^2 + a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{-2a\left(a-bx^2\right)^{\frac{3}{4}}+bx^2\left(a-bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-b*x**2+a)**(3/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(x**2/(-2*a*(a - b*x**2)**(3/4) + b*x**2*(a - b*x**2)**(
3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(bx^2 - 2\,a)(-bx^2 + a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

3.1061 $\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=188

$$\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}$$
$$+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{16}{81}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{16}{81}$$

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 -
3*x^2)^(9/4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81
 + (16*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(
3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/
81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
- 3*x^2]])/81
```

Rubi [A] time = 0.495926, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}$$
$$+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{16}{81}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{16}{81}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 -
3*x^2)^(9/4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81
 + (16*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(
3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/
81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
- 3*x^2]])/81
```

Rubi in Sympy [A] time = 34.9502, size = 175, normalized size = 0.93

$$\frac{2\left(-3x^2+2\right)^{\frac{9}{4}}}{729}-\frac{16\left(-3x^2+2\right)^{\frac{5}{4}}}{405}+\frac{56\sqrt[4]{-3x^2+2}}{81}$$
$$+\frac{8\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{81}-\frac{8\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{81}$$
$$-\frac{16\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{81}-\frac{16\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2*(-3*x**2 + 2)**(9/4)/729 - 16*(-3*x**2 + 2)**(5/4)/405 + 56*(-3
*x**2 + 2)**(1/4)/81 + 8*2**(3/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1
/4) + sqrt(-3*x**2 + 2) + sqrt(2))/81 - 8*2**(3/4)*log(2**(3/4)*(
-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/81 - 16*2**(3/
4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) - 1)/81 - 16*2**(3/4)*atan(
2**(1/4)*(-3*x**2 + 2)**(1/4) + 1)/81
```

Mathematica [C] time = 0.096009, size = 76, normalized size = 0.4

$$-\frac{2\left(-960\left(\frac{2-3x^2}{4-3x^2}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2}{4-3x^2}\right)+135x^6+378x^4+3096x^2-2272\right)}{3645\left(2-3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(-2272 + 3096*x^2 + 378*x^4 + 135*x^6 - 960*((2 - 3*x^2)/(4 -
 3*x^2))^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, 2/(4 - 3*x^2)]))/
(3645*(2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{x^7}{-3\,x^2+4}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.49938, size = 204, normalized size = 1.09

$$\frac{2}{729}\left(-3\,x^2+2\right)^{\frac{9}{4}} - \frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{16}{405}\left(-3\,x^2+2\right)^{\frac{5}{4}} + \frac{56}{81}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-x^7/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] 2/729*(-3*x^2 + 2)^(9/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3
/4) + 2*(-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*
(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*
x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-
2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/405
*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2 + 2)^(1/4)
```

Fricas [A] time = 0.243048, size = 277, normalized size = 1.47

$$\frac{32}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}+2\sqrt{2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}+2\,(-3\,x^2+2)^{\frac{1}{4}}}\right) + \frac{32}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}-2\sqrt{-2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2\,(-3\,x^2+2)^{\frac{1}{4}}}\right) - \frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{2}{3645}\left(45\,x^4+156\,x^2+1136\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-x^7/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 32/81*2^(3/4)*arctan(2^(3/4)/(2^(3/4) + 2*sqrt(2^(3/4)*(-3*x^2 +
2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2*(-3*x^2 + 2)^(1/4))) +
 32/81*2^(3/4)*arctan(-2^(3/4)/(2^(3/4) - 2*sqrt(-2^(3/4)*(-3*x^2
 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2*(-3*x^2 + 2)^(1/4))
) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(
-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2)) + 2/3645*(45*x^4 + 156*x^2 + 1136)*(-3*x
^2 + 2)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^7}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}}-4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [A] time = 0.242872, size = 216, normalized size = 1.15

$$\frac{2}{729}\left(3x^2-2\right)^2\left(-3x^2+2\right)^{\frac{1}{4}}-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{81}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{8}{81}\cdot 2^{\frac{3}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{16}{405}\left(-3x^2+2\right)^{\frac{5}{4}}+\frac{56}{81}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^7/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 2)^2*(-3*x^2 + 2)^(1/4) - 16/81*2^(3/4)*arctan(1/2
*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan
(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 8/81*2^(3/4)*ln
(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*
2^(3/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 +
2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2 + 2)^(1/4)
```

3.1062 $\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=173

$$-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{4}{27}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{4}{27}$$

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*
ArcTan[1 + (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)
*(2 - 3*x^2)^(1/4)])/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(
3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/27
```

Rubi [A] time = 0.424921, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{4}{27}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{4}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*
ArcTan[1 + (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)
*(2 - 3*x^2)^(1/4)])/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(
3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/27
```

Rubi in Sympy [A] time = 33.0568, size = 162, normalized size = 0.94

$$-\frac{2\left(-3x^2+2\right)^{\frac{5}{4}}}{135}+\frac{4\sqrt[4]{-3x^2+2}}{9}+\frac{2\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{27}-\frac{2\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{27}-\frac{4\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{27}-\frac{4\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -2*(-3*x**2 + 2)**(5/4)/135 + 4*(-3*x**2 + 2)**(1/4)/9 + 2*2**(3/
4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) + sqrt(
2))/27 - 2*2**(3/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x
**2 + 2) + sqrt(2))/27 - 4*2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**
(1/4) - 1)/27 - 4*2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1
)/27
```

Mathematica [C] time = 0.0795577, size = 74, normalized size = 0.43

$$-\frac{2\left(3\left(9x^4+78x^2-56\right)-80\left(\frac{2-3x^2}{4-3x^2}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2}{4-3x^2}\right)\right)}{405\left(2-3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(3*(-56 + 78*x^2 + 9*x^4) - 80*((2 - 3*x^2)/(4 - 3*x^2))^(3/4
)*Hypergeometric2F1[3/4, 3/4, 7/4, 2/(4 - 3*x^2)]))/(405*(2 - 3*x
^2)^(3/4))
```

Maple [F]   time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^5}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A]   time = 1.51104, size = 189, normalized size = 1.09

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{2}{135}\left(-3x^2+2\right)^{\frac{5}{4}}+\frac{4}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^5/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
) - 4/27*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1
/4))) - 2/27*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) + 2/27*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3*x^2 + 2)^(5/4) + 4/9*(-3
*x^2 + 2)^(1/4)
```

Fricas [A]   time = 0.243554, size = 270, normalized size = 1.56

$$\frac{8}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}+2\sqrt{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}+2(-3x^2+2)^{\frac{1}{4}}}\right)+\frac{8}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}-2\sqrt{-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2(-3x^2+2)^{\frac{1}{4}}}\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{135}\left(3x^2+28\right)\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^5/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 8/27*2^(3/4)*arctan(2^(3/4)/(2^(3/4) + 2*sqrt(2^(3/4)*(-3*x^2 + 2
)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2*(-3*x^2 + 2)^(1/4))) +
```

```
8/27*2^(3/4)*arctan(-2^(3/4)/(2^(3/4) - 2*sqrt(-2^(3/4)*(-3*x^2 +
 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2*(-3*x^2 + 2)^(1/4)))
- 2/27*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3
*x^2 + 2)) + 2/27*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(
2) + sqrt(-3*x^2 + 2)) + 2/135*(3*x^2 + 28)*(-3*x^2 + 2)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}}-4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [A] time = 0.249008, size = 189, normalized size = 1.09

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{2}{135}\left(-3\,x^2+2\right)^{\frac{5}{4}}+\frac{4}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^5/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
) - 4/27*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1
/4))) - 2/27*2^(3/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 2/27*2^(3/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3*x^2 + 2)^(5/4) + 4/9*(-3*x
^2 + 2)^(1/4)
```

3.1063 $\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=158

$$\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)$$

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])
/9 + (2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt
[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) -
Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(
1/4))
```

Rubi [A] time = 0.373465, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])
/9 + (2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt
[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) -
Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(
1/4))
```

Rubi in Sympy [A] time = 31.4117, size = 141, normalized size = 0.89

$$\frac{2\sqrt[4]{-3x^2+2}}{9}+\frac{2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{18}-\frac{2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{18}-\frac{2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{9}-\frac{2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2*(-3*x**2 + 2)**(1/4)/9 + 2**(3/4)*log(-2**(3/4)*(-3*x**2 + 2)**
(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/18 - 2**(3/4)*log(2**(3/4)*(
-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) + sqrt(2))/18 - 2**(3/4)*
atan(2**(1/4)*(-3*x**2 + 2)**(1/4) - 1)/9 - 2**(3/4)*atan(2**(1/4
)*(-3*x**2 + 2)**(1/4) + 1)/9
```

Mathematica [C] time = 0.0528609, size = 66, normalized size = 0.42

$$-\frac{2\left(-4\left(\frac{2-3x^2}{4-3x^2}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2}{4-3x^2}\right)+9x^2-6\right)}{27\left(2-3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(-6 + 9*x^2 - 4*((2 - 3*x^2)/(4 - 3*x^2))^(3/4)*Hypergeometri
c2F1[3/4, 3/4, 7/4, 2/(4 - 3*x^2)]))/(27*(2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x^3}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.53668, size = 174, normalized size = 1.1

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) -\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{18} \cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4)))
 - 1/9*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4
))) - 1/18*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) + 1/18*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2 + 2)^(1/4)
```

Fricas [A] time = 0.241383, size = 261, normalized size = 1.65

$$\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}+2\sqrt{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}+2(-3x^2+2)^{\frac{1}{4}}}\right)+\frac{2}{9} \cdot 2^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}-2\sqrt{-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2(-3x^2+2)^{\frac{1}{4}}}\right) -\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{18} \cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 2/9*2^(3/4)*arctan(2^(3/4)/(2^(3/4) + 2*sqrt(2^(3/4)*(-3*x^2 + 2)
^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2*(-3*x^2 + 2)^(1/4))) + 2
/9*2^(3/4)*arctan(-2^(3/4)/(2^(3/4) - 2*sqrt(-2^(3/4)*(-3*x^2 + 2
```

```
)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2*(-3*x^2 + 2)^(1/4))) -
1/18*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x
^2 + 2)) + 1/18*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2)
 + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2 + 2)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}}-4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [A] time = 0.241099, size = 174, normalized size = 1.1

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)$$
$$-\frac{1}{18}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{1}{18}$$
$$\cdot 2^{\frac{3}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4)))
 - 1/9*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4
))) - 1/18*2^(3/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) + 1/18*2^(3/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2 + 2)^(1/4)
```

3.1064 $\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=143

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}}$$

```
[Out] -ArcTan[1 + (4 - 6*x^2)^(1/4)]/(6*2^(1/4)) + ArcTan[1 - 2^(1/4)*(
2 - 3*x^2)^(1/4)]/(6*2^(1/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)
^(1/4) + Sqrt[2 - 3*x^2]]/(12*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2
 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(12*2^(1/4))
```

Rubi [A] time = 0.265978, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -ArcTan[1 + (4 - 6*x^2)^(1/4)]/(6*2^(1/4)) + ArcTan[1 - 2^(1/4)*(
2 - 3*x^2)^(1/4)]/(6*2^(1/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)
^(1/4) + Sqrt[2 - 3*x^2]]/(12*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2
 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(12*2^(1/4))
```

Rubi in Sympy [A] time = 27.3218, size = 128, normalized size = 0.9

$$\frac{2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{24}-\frac{2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{24}-\frac{2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{12}-\frac{2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2**(3/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) +
 sqrt(2))/24 - 2**(3/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(
-3*x**2 + 2) + sqrt(2))/24 - 2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)
**(1/4) - 1)/12 - 2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1
)/12
```

Mathematica [A] time = 0.0761332, size = 100, normalized size = 0.7

$$\frac{\log\left(\sqrt{4-6x^2}-2\sqrt[4]{4-6x^2}+2\right)-\log\left(\sqrt{4-6x^2}+2\sqrt[4]{4-6x^2}+2\right)+2\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-2\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{12\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*ArcTan[1 + (4 - 6*x^2)^(1/4)
] + Log[2 - 2*(4 - 6*x^2)^(1/4) + Sqrt[4 - 6*x^2]] - Log[2 + 2*(4
 - 6*x^2)^(1/4) + Sqrt[4 - 6*x^2]])/(12*2^(1/4))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{x}{-3x^2+4}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [A] time = 1.50882, size = 159, normalized size = 1.11

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)$$
$$-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
) - 1/12*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1
/4))) - 1/24*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) + 1/24*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2))
```

Fricas [A] time = 0.239979, size = 282, normalized size = 1.97

$$\frac{1}{96}\cdot 8^{\frac{3}{4}}\left(4\sqrt{2}\arctan\left(\frac{2}{8^{\frac{1}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+2\sqrt{8^{\frac{1}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-3x^2+2}+2}+2}\right)+4\sqrt{2}\arctan\left(\frac{}{8^{\frac{1}{4}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 1/96*8^(3/4)*(4*sqrt(2)*arctan(2/(8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1
/4) + 2*sqrt(8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(2)*sqrt(-3
*x^2 + 2) + 2) + 2)) + 4*sqrt(2)*arctan(2/(8^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + sqrt(-4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqr
t(2)*sqrt(-3*x^2 + 2) + 8) - 2)) - sqrt(2)*log(4*8^(1/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 4*sqrt(2)*sqrt(-3*x^2 + 2) + 8) + sqrt(2)*lo
g(-4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2)*sqrt(-3*x^2 +
 2) + 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}} - 4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3/4)
), x)
```

GIAC/XCAS [A] time = 0.237306, size = 159, normalized size = 1.11

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)$$
$$-\frac{1}{24}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{24}\cdot 2^{\frac{3}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
) - 1/12*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1
/4))) - 1/24*2^(3/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 1/24*2^(3/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2))
```

3.1065 $\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=197

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}$$

```
[Out] -ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) - ArcTan[1 + (4 -
6*x^2)^(1/4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]
/(8*2^(1/4)) - ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + L
og[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(
1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]
]/(16*2^(1/4))
```

Rubi [A] time = 0.447456, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) - ArcTan[1 + (4 -
6*x^2)^(1/4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]
/(8*2^(1/4)) - ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + L
og[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(
1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]
]/(16*2^(1/4))
```

Rubi in Sympy [A] time = 34.2836, size = 178, normalized size = 0.9

$$\frac{2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{32}-\frac{2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{32}-\frac{\sqrt[4]{2}\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{8}-\frac{2^{\frac{3}{4}}\,\mathrm{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{16}-\frac{2^{\frac{3}{4}}\,\mathrm{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{16}-\frac{\sqrt[4]{2}\,\mathrm{atanh}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2**(3/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2) +
 sqrt(2))/32 - 2**(3/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(
-3*x**2 + 2) + sqrt(2))/32 - 2**(1/4)*atan(2**(3/4)*(-3*x**2 + 2)
**(1/4)/2)/8 - 2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) - 1)/1
6 - 2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1)/16 - 2**(1/4
```

```
)*atanh(2**(3/4)*(-3*x**2 + 2)**(1/4)/2)/8
```

Mathematica [C] time = 0.281857, size = 139, normalized size = 0.71

$$\frac{66x^2F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)}{7\left(2-3x^2\right)^{3/4}\left(3x^2-4\right)\left(33x^2F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+16F_1\left(\frac{11}{4};\frac{3}{4},2;\frac{15}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+6F_1\left(\frac{11}{4};\frac{7}{4},1;\frac{15}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (66*x^2*AppellF1[7/4, 3/4, 1, 11/4, 2/(3*x^2), 4/(3*x^2)])/(7*(2
- 3*x^2)^(3/4)*(-4 + 3*x^2)*(33*x^2*AppellF1[7/4, 3/4, 1, 11/4, 2
/(3*x^2), 4/(3*x^2)] + 16*AppellF1[11/4, 3/4, 2, 15/4, 2/(3*x^2),
 4/(3*x^2)] + 6*AppellF1[11/4, 7/4, 1, 15/4, 2/(3*x^2), 4/(3*x^2)
]))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-3\,x^2+4\right)}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x), x)
```

Fricas [A] time = 0.247947, size = 381, normalized size = 1.93

$$\frac{1}{128}\cdot 8^{\frac{3}{4}}\sqrt{2}\left(4\sqrt{2}\arctan\left(\frac{2}{8^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2\sqrt{2}\sqrt{-3\,x^2+2}+4}}\right)-\sqrt{2}\log\left(8^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+2\right)+\sqrt{2}\log\left(8^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x),x, algorithm="fricas")
```

```
[Out] 1/128*8^(3/4)*sqrt(2)*(4*sqrt(2)*arctan(2/(8^(1/4)*(-3*x^2 + 2)^(
1/4) + sqrt(2*sqrt(2)*sqrt(-3*x^2 + 2) + 4))) - sqrt(2)*log(8^(1/
```

```
4)*(-3*x^2 + 2)^(1/4) + 2) + sqrt(2)*log(8^(1/4)*(-3*x^2 + 2)^(1/
4) - 2) + 4*arctan(2/(8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 2*sqrt
(8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(2)*sqrt(-3*x^2 + 2) +
2) + 2)) + 4*arctan(2/(8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(
-4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2)*sqrt(-3*x^2 + 2
) + 8) - 2)) - log(4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(
2)*sqrt(-3*x^2 + 2) + 8) + log(-4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1
/4) + 4*sqrt(2)*sqrt(-3*x^2 + 2) + 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^3(-3x^2+2)^{\frac{3}{4}} - 4x(-3x^2+2)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(-3*x**2 + 2)**(3/4) - 4*x*(-3*x**2 + 2)**(3/
4)), x)
```

GIAC/XCAS [A] time = 0.257703, size = 284, normalized size = 1.44

$$\begin{aligned}&-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)\\&-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)\\&-\frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\ln\left(4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right)+\frac{1}{32}\\&\cdot 4^{\frac{1}{8}}\sqrt{2}\ln\left(-4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right)-\frac{1}{8}\cdot 4^{\frac{1}{8}}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)\\&-\frac{1}{16}\cdot 4^{\frac{1}{8}}\ln\left(\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)+\frac{1}{16}\cdot 4^{\frac{1}{8}}\ln\left(-\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x),x, algorithm="giac")
```

```
[Out] -1/16*4^(1/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2)
 + 2*(-3*x^2 + 2)^(1/4))) - 1/16*4^(1/8)*sqrt(2)*arctan(-1/8*4^(7
/8)*sqrt(2)*(4^(1/8)*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) - 1/32*4^(1
/8)*sqrt(2)*ln(4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 +
 2) + 4^(1/4)) + 1/32*4^(1/8)*sqrt(2)*ln(-4^(1/8)*sqrt(2)*(-3*x^2
 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/8*4^(1/8)*arctan(1/
4*4^(7/8)*(-3*x^2 + 2)^(1/4)) - 1/16*4^(1/8)*ln((-3*x^2 + 2)^(1/4
) + 4^(1/8)) + 1/16*4^(1/8)*ln(-(-3*x^2 + 2)^(1/4) + 4^(1/8))
```

3.1066 $\int \frac{1}{x^3(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}-\frac{3\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(16*x^2) - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4
)])/(32*2^(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4))
 + (3*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*A
rcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(3/4)) + (3*Log[Sqrt[2]
- 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(64*2^(1/4)) - (3
*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(64*
2^(1/4))
```

Rubi [A] time = 0.558932, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}-\frac{3\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```


```
[Out] -(2 - 3*x^2)^(1/4)/(16*x^2) - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4
)])/(32*2^(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4))
 + (3*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*A
rcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(3/4)) + (3*Log[Sqrt[2]
- 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(64*2^(1/4)) - (3
*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(64*
2^(1/4))
```

Rubi in Sympy [A] time = 40.1182, size = 204, normalized size = 0.95

$$\frac{3\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{128}-\frac{3\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}+\sqrt{-3x^2+2}+\sqrt{2}\right)}{128}-\frac{15\sqrt[4]{2}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{64}-\frac{3\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}-1\right)}{64}-\frac{3\cdot 2^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{2}\sqrt[4]{-3x^2+2}+1\right)}{64}-\frac{15\sqrt[4]{2}\operatorname{atanh}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)}{64}-\frac{\sqrt[4]{-3x^2+2}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 3*2**(3/4)*log(-2**(3/4)*(-3*x**2 + 2)**(1/4) + sqrt(-3*x**2 + 2)
 + sqrt(2))/128 - 3*2**(3/4)*log(2**(3/4)*(-3*x**2 + 2)**(1/4) +
sqrt(-3*x**2 + 2) + sqrt(2))/128 - 15*2**(1/4)*atan(2**(3/4)*(-3*
x**2 + 2)**(1/4)/2)/64 - 3*2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**
(1/4) - 1)/64 - 3*2**(3/4)*atan(2**(1/4)*(-3*x**2 + 2)**(1/4) + 1
)/64 - 15*2**(1/4)*atanh(2**(3/4)*(-3*x**2 + 2)**(1/4)/2)/64 - (-
3*x**2 + 2)**(1/4)/(16*x**2)
```

Mathematica [C] time = 0.311, size = 136, normalized size = 0.63

$$\frac{90F_1\left(\frac{11}{4};\frac{3}{4},1;\frac{15}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)}{11\left(2-3x^2\right)^{3/4}\left(3x^2-4\right)\left(45x^2F_1\left(\frac{11}{4};\frac{3}{4},1;\frac{15}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+16F_1\left(\frac{15}{4};\frac{3}{4},2;\frac{19}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)+6F_1\left(\frac{15}{4};\frac{7}{4},1;\frac{19}{4};\frac{2}{3x^2},\frac{4}{3x^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (90*AppellF1[11/4, 3/4, 1, 15/4, 2/(3*x^2), 4/(3*x^2)])/(11*(2 -
3*x^2)^(3/4)*(-4 + 3*x^2)*(45*x^2*AppellF1[11/4, 3/4, 1, 15/4, 2/
(3*x^2), 4/(3*x^2)] + 16*AppellF1[15/4, 3/4, 2, 19/4, 2/(3*x^2),
4/(3*x^2)] + 6*AppellF1[15/4, 7/4, 1, 19/4, 2/(3*x^2), 4/(3*x^2)]
))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-3x^2+4\right)}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3x^2-4\right)\left(-3x^2+2\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3), x)
```

Fricas [A] time = 0.248981, size = 440, normalized size = 2.05

$$8^{\frac{3}{4}}\sqrt{2}\left(60\sqrt{2}x^2\arctan\left(\frac{2}{8^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2\sqrt{2}\sqrt{-3x^2+2}+4}}\right)-15\sqrt{2}x^2\log\left(8^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+2\right)+15\sqrt{2}x^2\log\left(8^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3),x, algorithm="fricas")
```

```
[Out] 1/1024*8^(3/4)*sqrt(2)*(60*sqrt(2)*x^2*arctan(2/(8^(1/4)*(-3*x^2
+ 2)^(1/4) + sqrt(2*sqrt(2)*sqrt(-3*x^2 + 2) + 4))) - 15*sqrt(2)*
x^2*log(8^(1/4)*(-3*x^2 + 2)^(1/4) + 2) + 15*sqrt(2)*x^2*log(8^(1
/4)*(-3*x^2 + 2)^(1/4) - 2) + 24*x^2*arctan(2/(8^(1/4)*sqrt(2)*(-
3*x^2 + 2)^(1/4) + 2*sqrt(8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sq
rt(2)*sqrt(-3*x^2 + 2) + 2) + 2)) + 24*x^2*arctan(2/(8^(1/4)*sqrt
(2)*(-3*x^2 + 2)^(1/4) + sqrt(-4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/
4) + 4*sqrt(2)*sqrt(-3*x^2 + 2) + 8) - 2)) - 6*x^2*log(4*8^(1/4)*
sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2)*sqrt(-3*x^2 + 2) + 8) + 6*
x^2*log(-4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2)*sqrt(-3
*x^2 + 2) + 8) - 4*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^5\left(-3x^2+2\right)^{\frac{3}{4}} - 4x^3\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(-3*x**2 + 2)**(3/4) - 4*x**3*(-3*x**2 + 2)**
(3/4)), x)
```

GIAC/XCAS [A] time = 0.26306, size = 259, normalized size = 1.2

$$-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{128}\cdot 2^{\frac{3}{4}}\ln\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{3}{128}\cdot 2^{\frac{3}{4}}\ln\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{15}{64}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\frac{15}{128}\cdot 2^{\frac{1}{4}}\ln\left(2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+\frac{15}{128}\cdot 2^{\frac{1}{4}}\ln\left(2^{\frac{1}{4}}-\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3),x, algorithm="giac")
```

```
[Out] -3/64*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
) - 3/64*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1
/4))) - 3/128*2^(3/4)*ln(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) + 3/128*2^(3/4)*ln(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) - 15/64*2^(1/4)*arctan(1/2*2^(3/4)*(
-3*x^2 + 2)^(1/4)) - 15/128*2^(1/4)*ln(2^(1/4) + (-3*x^2 + 2)^(1/
4)) + 15/128*2^(1/4)*ln(2^(1/4) - (-3*x^2 + 2)^(1/4)) - 1/16*(-3*
x^2 + 2)^(1/4)/x^2
```

3.1067 $\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=182

$$\frac{80}{567}\sqrt[4]{2-3x^2}x+\frac{8\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{8\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}+\frac{2}{63}\sqrt[4]{2-3x^2}x^3-\frac{160\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{567\sqrt{3}}$$

```
[Out] (80*x*(2 - 3*x^2)^(1/4))/567 + (2*x^3*(2 - 3*x^2)^(1/4))/63 + (8*
2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
- 3*x^2)^(1/4))])/(27*Sqrt[3]) - (8*2^(3/4)*ArcTanh[(2^(3/4) + 2^
(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(27*Sqrt[3
]) - (160*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(567*Sqrt[
3])
```

Rubi [A] time = 0.337396, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{80}{567}\sqrt[4]{2-3x^2}x+\frac{8\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{8\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}+\frac{2}{63}\sqrt[4]{2-3x^2}x^3-\frac{160\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{567\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (80*x*(2 - 3*x^2)^(1/4))/567 + (2*x^3*(2 - 3*x^2)^(1/4))/63 + (8*
2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
- 3*x^2)^(1/4))])/(27*Sqrt[3]) - (8*2^(3/4)*ArcTanh[(2^(3/4) + 2^
(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(27*Sqrt[3
]) - (160*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(567*Sqrt[
3])
```

Rubi in Sympy [A] time = 8.66633, size = 27, normalized size = 0.15

$$\frac{\sqrt[4]{2}x^7\operatorname{appellf}_1\left(\frac{7}{2},\frac{3}{4},1,\frac{9}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2**(1/4)*x**7*appellf1(7/2, 3/4, 1, 9/2, 3*x**2/2, 3*x**2/4)/56
```

Mathematica [C] time = 0.531382, size = 282, normalized size = 1.55

$$\frac{2x\left(-\frac{4960x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(6x^2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+9x^2F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}+\frac{1280F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right.\right.}\right.}{567\left(2-3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(80 - 102*x^2 - 27*x^4 + (1280*AppellF1[1/2, 3/4, 1, 3/2, (3
*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, 3/4, 1, 3/2,
(3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2, 3/4, 2, 5/2, (3*x^2)
/2, (3*x^2)/4] + 3*AppellF1[3/2, 7/4, 1, 5/2, (3*x^2)/2, (3*x^2)/
4]))) - (4960*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4
])/((-4 + 3*x^2)*(20*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2
)/4] + 6*x^2*AppellF1[5/2, 3/4, 2, 7/2, (3*x^2)/2, (3*x^2)/4] + 9
*x^2*AppellF1[5/2, 7/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4]))))/(567*(2
 - 3*x^2)^(3/4))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{x^{6}}{-3\,x^{2}+4}\left(-3\,x^{2}+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{6}}{(3\,x^{2}-4)(-3\,x^{2}+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{6}}{3x^{2}\left(-3x^{2}+2\right)^{\frac{3}{4}}-4\left(-3x^{2}+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**6/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^6}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1068 $\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=164

$$\frac{2}{27}\sqrt[4]{2-3x^2}\,x+\frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}}$$

```
[Out] (2*x*(2 - 3*x^2)^(1/4))/27 + (2*2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)
*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (
2*2^(3/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*
(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (4*2^(3/4)*EllipticF[ArcSin[Sq
rt[3/2]*x]/2, 2])/(27*Sqrt[3])
```

Rubi [A] time = 0.28236, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{2}{27}\sqrt[4]{2-3x^2}\,x+\frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2)^(1/4))/27 + (2*2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)
*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (
2*2^(3/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*
(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (4*2^(3/4)*EllipticF[ArcSin[Sq
rt[3/2]*x]/2, 2])/(27*Sqrt[3])
```

Rubi in Sympy [A] time = 8.60811, size = 27, normalized size = 0.16

$$\frac{\sqrt[4]{2}x^5\,\mathrm{appellf}_1\left(\frac{5}{2},\frac{3}{4},1,\frac{7}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2**(1/4)*x**5*appellf1(5/2, 3/4, 1, 7/2, 3*x**2/2, 3*x**2/4)/40
```

Mathematica [C] time = 0.195354, size = 277, normalized size = 1.69

$$\frac{2x\left(-\frac{160x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(6x^2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+9x^2F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}+\frac{32F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right.\right.}\right.}{27\left(2-3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2 + (32*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^
2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*
x^2)/4] + x^2*(2*AppellF1[3/2, 3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4]
```

```
 + 3*AppellF1[3/2, 7/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4]))) - (160*x
^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2
)*(20*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + 6*x^2*Ap
pellF1[5/2, 3/4, 2, 7/2, (3*x^2)/2, (3*x^2)/4] + 9*x^2*AppellF1[5
/2, 7/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4]))))/(27*(2 - 3*x^2)^(3/4))
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{x^4}{-3\,x^2+4}\left(-3\,x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}}-4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**4/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1069 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^
2)^(1/4))]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*
x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.090545, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 9.76285, size = 27, normalized size = 0.22

$$\frac{\sqrt[4]{2}x^3\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] 2**(1/4)*x**3*appellf1(3/2, 3/4, 1, 5/2, 3*x**2/2, 3*x**2/4)/24
```

Mathematica [C] time = 0.0715191, size = 142, normalized size = 1.18

$$-\frac{20x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{3\left(2-3x^2\right)^{3/4}\left(3x^2-4\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+20F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-20*x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(3*(2
- 3*x^2)^(3/4)*(-4 + 3*x^2)*(20*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2
)/2, (3*x^2)/4] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, (3*x^2)/2,
(3*x^2)/4] + 3*AppellF1[5/2, 7/4, 1, 7/2, (3*x^2)/2, (3*x^2)/4]))
)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{-3\,x^2+4}\left(-3\,x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [A] time = 0.237617, size = 351, normalized size = 2.92

$$\frac{1}{864}\cdot 72^{\frac{3}{4}}\left(4\sqrt{2}\arctan\left(\frac{3\,x}{\sqrt{6}x\sqrt{\frac{72^{\frac{1}{4}}\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}x+3\,x^2+2\sqrt{2}\sqrt{-3\,x^2+2}}{x^2}}+72^{\frac{1}{4}}\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+3\,x}\right)+4\sqrt{2}\arctan\left(\frac{}{\sqrt{6}x\sqrt{-\frac{72^{\frac{1}{4}}\sqrt{2}(-3\,x^2+2)}{}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")
```

```
[Out] 1/864*72^(3/4)*(4*sqrt(2)*arctan(3*x/(sqrt(6)*x*sqrt((72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4)*x + 3*x^2 + 2*sqrt(2)*sqrt(-3*x^2 + 2))/
x^2) + 72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 3*x)) + 4*sqrt(2)*ar
ctan(3*x/(sqrt(6)*x*sqrt(-(72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x
- 3*x^2 - 2*sqrt(2)*sqrt(-3*x^2 + 2))/x^2) + 72^(1/4)*sqrt(2)*(-3
*x^2 + 2)^(1/4) - 3*x)) - sqrt(2)*log(6*(72^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 3*x^2 + 2*sqrt(2)*sqrt(-3*x^2 + 2))/x^2) + sqrt(2
)*log(-6*(72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x - 3*x^2 - 2*sqrt(
2)*sqrt(-3*x^2 + 2))/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\left(-3x^2+2\right)^{\frac{3}{4}}-4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1070 $\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}+\frac{F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))]/(4*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(4*2^(1/4)*Sqrt[3]) + El
lipticF[ArcSin[Sqrt[3/2]*x]/2, 2]/(2*2^(1/4)*Sqrt[3])
```

Rubi [A] time = 0.13442, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}+\frac{F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))]/(4*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(4*2^(1/4)*Sqrt[3]) + El
lipticF[ArcSin[Sqrt[3/2]*x]/2, 2]/(2*2^(1/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 81.7798, size = 87, normalized size = 0.59

$$-\frac{2^{\frac{3}{4}}\sqrt{3}\sqrt{x^2}\left(-i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)\middle|-1\right)}{12x}-\frac{2^{\frac{3}{4}}\sqrt{3}\sqrt{x^2}\left(i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)\middle|-1\right)}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -2**(3/4)*sqrt(3)*sqrt(x**2)*elliptic_pi(-I, asin(2**(3/4)*(-3*x*
*2 + 2)**(1/4)/2), -1)/(12*x) - 2**(3/4)*sqrt(3)*sqrt(x**2)*ellip
tic_pi(I, asin(2**(3/4)*(-3*x**2 + 2)**(1/4)/2), -1)/(12*x)
```

Mathematica [C] time = 0.185931, size = 137, normalized size = 0.93

$$-\frac{4xF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(2-3x^2)^{3/4}(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x
^2)^(3/4)*(-4 + 3*x^2)*(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (
3*x^2)/4] + x^2*(2*AppellF1[3/2, 3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/
```

```
4] + 3*AppellF1[3/2, 7/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4]))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{-3 x^2 + 4} \left(-3 x^2 + 2\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)

[Out] int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3 x^2 - 4)(-3 x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="maxima")

[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 (-3x^2 + 2)^{\frac{3}{4}} - 4 (-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)

[Out] -Integral(1/(3*x**2*(-3*x**2 + 2)**(3/4) - 4*(-3*x**2 + 2)**(3/4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3 x^2 - 4)(-3 x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)),x, algorithm="giac")

[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1071 $\int \frac{1}{x^2(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\sqrt[4]{2}}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqr
t[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) - (Sqr
t[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 -
3*x^2)^(1/4))])/(16*2^(1/4)) + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2
]*x]/2, 2])/(4*2^(1/4))
```

Rubi [A] time = 0.266509, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(8*x) + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqr
t[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) - (Sqr
t[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 -
3*x^2)^(1/4))])/(16*2^(1/4)) + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2
]*x]/2, 2])/(4*2^(1/4))
```

Rubi in Sympy [A] time = 9.00535, size = 29, normalized size = 0.17

$$-\frac{\sqrt[4]{2}\,\mathrm{appellf}_1\left(-\frac{1}{2},\frac{3}{4},1,\frac{1}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -2**(1/4)*appellf1(-1/2, 3/4, 1, 1/2, 3*x**2/2, 3*x**2/4)/(8*x)
```

Mathematica [C] time = 0.252745, size = 140, normalized size = 0.84

$$\frac{4F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{x\left(2-3x^2\right)^{3/4}\left(3x^2-4\right)\left(3x^2\left(2F_1\left(\frac{1}{2};\frac{3}{4},2;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{1}{2};\frac{7}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*AppellF1[-1/2, 3/4, 1, 1/2, (3*x^2)/2, (3*x^2)/4])/(x*(2 - 3*x
^2)^(3/4)*(-4 + 3*x^2)*(4*AppellF1[-1/2, 3/4, 1, 1/2, (3*x^2)/2,
(3*x^2)/4] + 3*x^2*(2*AppellF1[1/2, 3/4, 2, 3/2, (3*x^2)/2, (3*x^^
```

```
2)/4] + 3*AppellF1[1/2, 7/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(-3x^2+4)}\left(-3x^2+2\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)

[Out] int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2),x, algorithm="maxima")

[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^4(-3x^2+2)^{\frac{3}{4}}-4x^2(-3x^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)

[Out] -Integral(1/(3*x**4*(-3*x**2 + 2)**(3/4) - 4*x**2*(-3*x**2 + 2)**
(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2),x, algorithm="giac")

[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

3.1072 $\int \frac{1}{x^4(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt[4]{2-3x^2}}{4x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{24x^3}+\frac{11\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

```
[Out] -(2 - 3*x^2)^(1/4)/(24*x^3) - (2 - 3*x^2)^(1/4)/(4*x) + (3*Sqrt[3
]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^
2)^(1/4))])/(64*2^(1/4)) - (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*
Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(64*2^(1/4)) + (
11*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(32*2^(1/4))
```

Rubi [A] time = 0.32583, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{\sqrt[4]{2-3x^2}}{4x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{24x^3}+\frac{11\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(24*x^3) - (2 - 3*x^2)^(1/4)/(4*x) + (3*Sqrt[3
]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^
2)^(1/4))])/(64*2^(1/4)) - (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*
Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(64*2^(1/4)) + (
11*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(32*2^(1/4))
```

Rubi in Sympy [A] time = 9.19939, size = 32, normalized size = 0.17

$$-\frac{\sqrt[4]{2}\,\text{appellf}_1\left(-\frac{3}{2},\frac{3}{4},1,-\frac{1}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -2**(1/4)*appellf1(-3/2, 3/4, 1, -1/2, 3*x**2/2, 3*x**2/4)/(24*x*
*3)
```

Mathematica [C] time = 0.273325, size = 142, normalized size = 0.77

$$-\frac{4F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{3x^3\left(2-3x^2\right)^{3/4}\left(3x^2-4\right)\left(3x^2\left(2F_1\left(-\frac{1}{2};\frac{3}{4},2;\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(-\frac{1}{2};\frac{7}{4},1;\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)-4F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*AppellF1[-3/2, 3/4, 1, -1/2, (3*x^2)/2, (3*x^2)/4])/(3*x^3*(2
 - 3*x^2)^(3/4)*(-4 + 3*x^2)*(-4*AppellF1[-3/2, 3/4, 1, -1/2, (3*
x^2)/2, (3*x^2)/4] + 3*x^2*(2*AppellF1[-1/2, 3/4, 2, 1/2, (3*x^2)
/2, (3*x^2)/4] + 3*AppellF1[-1/2, 7/4, 1, 1/2, (3*x^2)/2, (3*x^2)
/4])))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-3x^2+4\right)}\left(-3x^2+2\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^6\left(-3x^2+2\right)^{\frac{3}{4}}-4x^4\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**6*(-3*x**2 + 2)**(3/4) - 4*x**4*(-3*x**2 + 2)**
(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

3.1073 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi [A] time = 0.0654231, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi in Sympy [A] time = 25.7036, size = 41, normalized size = 0.67

$$\frac{x^3\sqrt[4]{3x^2-1}\,\text{appellf}_1\left(\frac{3}{2}, \frac{3}{4}, 1, \frac{5}{2}, 3x^2, \frac{3x^2}{2}\right)}{6\sqrt[4]{-3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] x**3*(3*x**2 - 1)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, 3*x**2, 3*x**
2/2)/(6*(-3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.243664, size = 134, normalized size = 2.2

$$\frac{10x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{3\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)\right)+10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (10*x^3*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(3*(-2 + 3*
x^2)*(-1 + 3*x^2)^(3/4)*(10*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*
x^2)/2] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, 3*x^2, (3*x^2)/2] +
 3*AppellF1[5/2, 7/4, 1, 7/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{x^2}{3\,x^2-2}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)

[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")

[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [A] time = 0.226986, size = 115, normalized size = 1.89

$$-\frac{1}{36}\,\sqrt{6}\left(2\,\arctan\left(\frac{\sqrt{6}\left(3\,x^2-1\right)^{\frac{1}{4}}}{3\,x}\right)-\log\left(-\frac{3\,\sqrt{6}x^2-12\,\left(3\,x^2-1\right)^{\frac{1}{4}}x+2\,\sqrt{6}\sqrt{3\,x^2-1}}{3\,x^2-2\,\sqrt{3\,x^2-1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")

[Out] -1/36*sqrt(6)*(2*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) - log(-(
3*sqrt(6)*x^2 - 12*(3*x^2 - 1)^(1/4)*x + 2*sqrt(6)*sqrt(3*x^2 - 1
))/(3*x^2 - 2*sqrt(3*x^2 - 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)

[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")

[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1074 $\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi [A] time = 0.0685388, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi in Sympy [A] time = 24.9799, size = 46, normalized size = 0.75

$$\frac{x^3\sqrt[4]{-3x^2-1}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-3x^2,-\frac{3x^2}{2}\right)}{6\sqrt[4]{3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2-2)/(-3*x**2-1)**(3/4),x)
```

```
[Out] x**3*(-3*x**2 - 1)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, -3*x**2, -3*
x**2/2)/(6*(3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.229369, size = 134, normalized size = 2.2

$$\frac{10x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)}{3\left(-3x^2-1\right)^{3/4}\left(3x^2+2\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-3x^2,-\frac{3x^2}{2}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-3x^2,-\frac{3x^2}{2}\right)\right)-10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] (10*x^3*AppellF1[3/2, 3/4, 1, 5/2, -3*x^2, (-3*x^2)/2])/(3*(-1 -
3*x^2)^(3/4)*(2 + 3*x^2)*(-10*AppellF1[3/2, 3/4, 1, 5/2, -3*x^2,
(-3*x^2)/2] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, -3*x^2, (-3*x^2
)/2] + 3*AppellF1[5/2, 7/4, 1, 7/2, -3*x^2, (-3*x^2)/2])))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3\,x^2-2}\left(-3\,x^2-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3\,x^2+2)(-3\,x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

Fricas [A] time = 0.224488, size = 158, normalized size = 2.59

$$-\frac{1}{36}\sqrt{6}\left(\log\left(\frac{\sqrt{6}\left(3\,x+\sqrt{6}\left(-3\,x^2-1\right)^{\frac{1}{4}}\right)}{6\,x}\right)-\log\left(-\frac{\sqrt{6}\left(3\,x-\sqrt{6}\left(-3\,x^2-1\right)^{\frac{1}{4}}\right)}{6\,x}\right)+i\,\log\left(\frac{\sqrt{6}\left(3i\,x+\sqrt{6}\left(-3\,x^2-1\right)^{\frac{1}{4}}\right)}{6\,x}\right)-i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*(log(1/6*sqrt(6)*(3*x + sqrt(6)*(-3*x^2 - 1)^(1/4))
/x) - log(-1/6*sqrt(6)*(3*x - sqrt(6)*(-3*x^2 - 1)^(1/4))/x) + I*
log(1/6*sqrt(6)*(3*I*x + sqrt(6)*(-3*x^2 - 1)^(1/4))/x) - I*log(1
/6*sqrt(6)*(-3*I*x + sqrt(6)*(-3*x^2 - 1)^(1/4))/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{3x^2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2-2)/(-3*x**2-1)**(3/4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(-3*x**2 - 1)**(3/4) + 2*(-3*x**2 - 1)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3\,x^2+2)(-3\,x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

3.1075 $\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}} \, dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)
) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^
(3/2))
```

Rubi [A] time = 0.091415, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)
) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^
(3/2))
```

Rubi in Sympy [A] time = 28.7619, size = 41, normalized size = 0.57

$$\frac{x^3\sqrt[4]{bx^2-1}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},bx^2,\frac{bx^2}{2}\right)}{6\sqrt[4]{-bx^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2-2)/(b*x**2-1)**(3/4),x)
```

```
[Out] x**3*(b*x**2 - 1)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, b*x**2, b*x**
2/2)/(6*(-b*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.297, size = 138, normalized size = 1.92

$$\frac{10x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)}{3\left(bx^2-2\right)\left(bx^2-1\right)^{3/4}\left(bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};bx^2,\frac{bx^2}{2}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};bx^2,\frac{bx^2}{2}\right)\right)+10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] (10*x^3*AppellF1[3/2, 3/4, 1, 5/2, b*x^2, (b*x^2)/2])/(3*(-2 + b*
x^2)*(-1 + b*x^2)^(3/4)*(10*AppellF1[3/2, 3/4, 1, 5/2, b*x^2, (b*
x^2)/2] + b*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, b*x^2, (b*x^2)/2] +
 3*AppellF1[5/2, 7/4, 1, 7/2, b*x^2, (b*x^2)/2])))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x^2}{b x^2-2}\left(b x^2-1\right)^{-\frac{3}{4}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b x^2-1\right)^{\frac{3}{4}}\left(b x^2-2\right)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

Fricas [A] time = 0.234159, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{2}\left(2 \arctan\left(\frac{\sqrt{2}\left(b x^2-1\right)^{\frac{1}{4}}}{\sqrt{b} x}\right)-\log\left(-\frac{\sqrt{2} b^{\frac{3}{2}} x^2-4\left(b x^2-1\right)^{\frac{1}{4}} b x+2 \sqrt{2} \sqrt{b x^2-1} \sqrt{b}}{b x^2-2 \sqrt{b x^2-1}}\right)\right)}{4 b^{\frac{3}{2}}}, -\frac{\sqrt{2}\left(2 \arctan\left(\frac{\sqrt{2}\left(b x^2-1\right)^{\frac{1}{4}} \sqrt{-b}}{b x}\right)-\log\left(-\frac{\sqrt{2} \sqrt{-b} b x^2+4\left(b x^2-1\right)^{\frac{1}{4}} b x-2 \sqrt{2} \sqrt{b x^2-1} \sqrt{-b}}{b x^2+2 \sqrt{b x^2-1}}\right)\right)}{4 \sqrt{-b} b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*(2*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) -
log(-(sqrt(2)*b^(3/2)*x^2 - 4*(b*x^2 - 1)^(1/4)*b*x + 2*sqrt(2)*s
qrt(b*x^2 - 1)*sqrt(b))/(b*x^2 - 2*sqrt(b*x^2 - 1))))/b^(3/2), -1
/4*sqrt(2)*(2*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)*sqrt(-b)/(b*x)) -
log(-(sqrt(2)*sqrt(-b)*b*x^2 + 4*(b*x^2 - 1)^(1/4)*b*x - 2*sqrt(2
)*sqrt(b*x^2 - 1)*sqrt(-b))/(b*x^2 + 2*sqrt(b*x^2 - 1))))/(sqrt(-
b)*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b x^2-2\right)\left(b x^2-1\right)^{\frac{3}{4}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2-2)/(b*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((b*x**2 - 2)*(b*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2-1)^{\frac{3}{4}}(bx^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

3.1076 $\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)
) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^
(3/2))
```

Rubi [A] time = 0.0990811, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)
) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^
(3/2))
```

Rubi in Sympy [A] time = 29.5794, size = 46, normalized size = 0.62

$$\frac{x^3\sqrt[4]{-bx^2-1}\,\mathrm{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-bx^2,-\frac{bx^2}{2}\right)}{6\sqrt[4]{bx^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-b*x**2-2)/(-b*x**2-1)**(3/4),x)
```

```
[Out] x**3*(-b*x**2 - 1)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, -b*x**2, -b*
x**2/2)/(6*(b*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.242502, size = 143, normalized size = 1.93

$$\frac{10x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)}{3\left(-bx^2-1\right)^{3/4}\left(bx^2+2\right)\left(bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-bx^2,-\frac{bx^2}{2}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-bx^2,-\frac{bx^2}{2}\right)\right)-10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] (10*x^3*AppellF1[3/2, 3/4, 1, 5/2, -(b*x^2), -(b*x^2)/2])/(3*(-1
- b*x^2)^(3/4)*(2 + b*x^2)*(-10*AppellF1[3/2, 3/4, 1, 5/2, -(b*x^
2), -(b*x^2)/2] + b*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, -(b*x^2), -
(b*x^2)/2] + 3*AppellF1[5/2, 7/4, 1, 7/2, -(b*x^2), -(b*x^2)/2]))
)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2-2}\left(-bx^2-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(bx^2+2)(-bx^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

Fricas [A] time = 0.233858, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)-\log\left(\frac{4(-bx^2-1)^{\frac{1}{4}}bx-\sqrt{2}\left(bx^2+2\sqrt{-bx^2-1}\right)\sqrt{b}}{bx^2-2\sqrt{-bx^2-1}}\right)\right)}{4b^{\frac{3}{2}}},\ -\frac{\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\log\left(-\frac{4(-bx^2-1)^{\frac{1}{4}}bx+\sqrt{2}\left(bx^2-2\sqrt{-bx^2-1}\right)\sqrt{-b}}{bx^2+2\sqrt{-bx^2-1}}\right)\right)}{4\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*(2*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) -
 log((4*(-b*x^2 - 1)^(1/4)*b*x - sqrt(2)*(b*x^2 + 2*sqrt(-b*x^2 -
 1))*sqrt(b))/(b*x^2 - 2*sqrt(-b*x^2 - 1))))/b^(3/2), -1/4*sqrt(2
)*(2*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - log(-(4*
(-b*x^2 - 1)^(1/4)*b*x + sqrt(2)*(b*x^2 - 2*sqrt(-b*x^2 - 1))*sqr
t(-b))/(b*x^2 + 2*sqrt(-b*x^2 - 1))))/(sqrt(-b)*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{bx^2\left(-bx^2-1\right)^{\frac{3}{4}}+2\left(-bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-b*x**2-2)/(-b*x**2-1)**(3/4),x)
```

```
[Out] -Integral(x**2/(b*x**2*(-b*x**2 - 1)**(3/4) + 2*(-b*x**2 - 1)**(3
/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(bx^2+2)(-bx^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

3.1077 $\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(
1/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sq
rt[6]*a^(1/4))
```

Rubi [A] time = 0.101892, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(
1/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sq
rt[6]*a^(1/4))
```

Rubi in Sympy [A] time = 33.717, size = 49, normalized size = 0.58

$$\frac{x^3\sqrt[4]{-a+3x^2}\operatorname{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{6a^2\sqrt[4]{1-\frac{3x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2-2*a)/(3*x**2-a)**(3/4),x)
```

```
[Out] x**3*(-a + 3*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, 3*x**2/a, 3*
x**2/(2*a))/(6*a**2*(1 - 3*x**2/a)**(1/4))
```

Mathematica [C] time = 0.30744, size = 164, normalized size = 1.93

$$\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{3\left(3x^2-2a\right)\left(3x^2-a\right)^{3/4}\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] (10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2*a)])/(
3*(-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)*(10*a*AppellF1[3/2, 3/4, 1, 5
/2, (3*x^2)/a, (3*x^2)/(2*a)] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/
```

```
2, (3*x^2)/a, (3*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1, 7/2, (3*x^
2)/a, (3*x^2)/(2*a)])))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{x^{2}}{3 x^{2}-2 a} \left(3 x^{2}-a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)

[Out] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{{\left(3 x^{2}-a\right)}^{\frac{3}{4}} {\left(3 x^{2}-2 a\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)),x, algorithm="maxima")

[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

Fricas [A] time = 0.23318, size = 185, normalized size = 2.18

$$\frac{2 \left(\frac{1}{36}\right)^{\frac{1}{4}} \arctan\left(\frac{3 \left(\frac{1}{36}\right)^{\frac{1}{4}} x}{\left(\sqrt{\frac{1}{2}} x \sqrt{\frac{\frac{3 x^{2}}{\sqrt{a}}+2 \sqrt{3 x^{2}-a}}{x^{2}}}+\left(3 x^{2}-a\right)^{\frac{1}{4}}\right) a^{\frac{1}{4}}}\right)}{3 a^{\frac{1}{4}}} - \frac{\left(\frac{1}{36}\right)^{\frac{1}{4}} \log\left(\frac{\frac{3 \left(\frac{1}{36}\right)^{\frac{1}{4}} x}{a^{\frac{1}{4}}}+\left(3 x^{2}-a\right)^{\frac{1}{4}}}{x}\right)}{6 a^{\frac{1}{4}}} + \frac{\left(\frac{1}{36}\right)^{\frac{1}{4}} \log\left(-\frac{\frac{3 \left(\frac{1}{36}\right)^{\frac{1}{4}} x}{a^{\frac{1}{4}}}-\left(3 x^{2}-a\right)^{\frac{1}{4}}}{x}\right)}{6 a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)),x, algorithm="fricas")

[Out] 2/3*(1/36)^(1/4)*arctan(3*(1/36)^(1/4)*x/((sqrt(1/2)*x*sqrt((3*x^
2/sqrt(a) + 2*sqrt(3*x^2 - a))/x^2) + (3*x^2 - a)^(1/4))*a^(1/4))
)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (3*x
^2 - a)^(1/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)
*x/a^(1/4) - (3*x^2 - a)^(1/4))/x)/a^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(-2a + 3x^{2}\right) \left(-a + 3x^{2}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2-2*a)/(3*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + 3*x**2)*(-a + 3*x**2)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3\,x^2-a)^{\frac{3}{4}}(3\,x^2-2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

3.1078 $\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(
1/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sq
rt[6]*a^(1/4))
```

Rubi [A] time = 0.0956848, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(
1/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sq
rt[6]*a^(1/4))
```

Rubi in Sympy [A] time = 34.3596, size = 54, normalized size = 0.64

$$\frac{x^3\sqrt[4]{-a-3x^2}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{6a^2\sqrt[4]{1+\frac{3x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3*x**2-2*a)/(-3*x**2-a)**(3/4),x)
```

```
[Out] x**3*(-a - 3*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, -3*x**2/a, -
3*x**2/(2*a))/(6*a**2*(1 + 3*x**2/a)**(1/4))
```

Mathematica [C] time = 0.276683, size = 164, normalized size = 1.93

$$\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{3\left(-a-3x^2\right)^{3/4}\left(2a+3x^2\right)\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] (10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)])
/(3*(-a - 3*x^2)^(3/4)*(2*a + 3*x^2)*(-10*a*AppellF1[3/2, 3/4, 1,
 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + 3*x^2*(2*AppellF1[5/2, 3/4, 2
```

```
, 7/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1, 7/2,
 (-3*x^2)/a, (-3*x^2)/(2*a)])))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{x^2}{-3x^2-2a}\left(-3x^2-a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)

[Out] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(3x^2+2a)(-3x^2-a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)),x, algorithm="maxima")

[Out] -integrate(x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

Fricas [A] time = 0.235559, size = 185, normalized size = 2.18

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}}\arctan\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{\frac{3x^2}{\sqrt{a}}+2\sqrt{-3x^2-a}}{x^2}}+(-3x^2-a)^{\frac{1}{4}}\right)a^{\frac{1}{4}}}\right)}{3a^{\frac{1}{4}}}-\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)),x, algorithm="fricas")

[Out] 2/3*(1/36)^(1/4)*arctan(3*(1/36)^(1/4)*x/((sqrt(1/2)*x*sqrt((3*x^
2/sqrt(a) + 2*sqrt(-3*x^2 - a))/x^2) + (-3*x^2 - a)^(1/4))*a^(1/4
)))/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (-
3*x^2 - a)^(1/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1
/4)*x/a^(1/4) - (-3*x^2 - a)^(1/4))/x)/a^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{2a\left(-a-3x^2\right)^{\frac{3}{4}}+3x^2\left(-a-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3*x**2-2*a)/(-3*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - 3*x**2)**(3/4) + 3*x**2*(-a - 3*x**2)**
(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

3.1079 $\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]
*a^(1/4)*b^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*
x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.132292, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]
*a^(1/4)*b^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*
x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b^(3/2))
```

Rubi in Sympy [A] time = 38.3381, size = 49, normalized size = 0.51

$$\frac{x^3\sqrt[4]{-a+bx^2}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{6a^2\sqrt[4]{1-\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2-2*a)/(b*x**2-a)**(3/4),x)
```

```
[Out] x**3*(-a + b*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, b*x**2/a, b*
x**2/(2*a))/(6*a**2*(1 - b*x**2/a)**(1/4))
```

Mathematica [C] time = 0.346934, size = 169, normalized size = 1.76

$$-\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{3(2a-bx^2)(bx^2-a)^{3/4}\left(bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] (-10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2*a)])/
(3*(2*a - b*x^2)*(-a + b*x^2)^(3/4)*(10*a*AppellF1[3/2, 3/4, 1, 5
/2, (b*x^2)/a, (b*x^2)/(2*a)] + b*x^2*(2*AppellF1[5/2, 3/4, 2, 7/
2, (b*x^2)/a, (b*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1, 7/2, (b*x^
```

```
2)/a, (b*x^2)/(2*a)])))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^2-2a}\left(bx^2-a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2-a)^{\frac{3}{4}}(bx^2-2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

Fricas [A] time = 0.233096, size = 252, normalized size = 2.62

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}}x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{bx^2-a}}{x^2}}+\left(bx^2-a\right)^{\frac{1}{4}}}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan((1/4)^(1/4)*b^2*x*(1/(a*b^
6))^(1/4)/(sqrt(1/2)*x*sqrt((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(b*x
^2 - a))/x^2) + (b*x^2 - a)^(1/4))) - 1/2*(1/4)^(1/4)*(1/(a*b^6))
^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (b*x^2 - a)^(1/
4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*
x*(1/(a*b^6))^(1/4) - (b*x^2 - a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2-2*a)/(b*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + b*x**2)*(-a + b*x**2)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2-a)^{\frac{3}{4}}(bx^2-2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

3.1080 $\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]
*a^(1/4)*b^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*
x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b^(3/2))
```

Rubi [A] time = 0.133258, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]
*a^(1/4)*b^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*
x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b^(3/2))
```

Rubi in Sympy [A] time = 39.1104, size = 54, normalized size = 0.55

$$\frac{x^3\sqrt[4]{-a-bx^2}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{6a^2\sqrt[4]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-b*x**2-2*a)/(-b*x**2-a)**(3/4),x)
```

```
[Out] x**3*(-a - b*x**2)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, -b*x**2/a, -
b*x**2/(2*a))/(6*a**2*(1 + b*x**2/a)**(1/4))
```

Mathematica [C] time = 0.293479, size = 174, normalized size = 1.78

$$-\frac{10ax^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{3\left(-a-bx^2\right)^{3/4}\left(2a+bx^2\right)\left(10aF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] (-10*a*x^3*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -(b*x^2)/(2*a
)])/(3*(-a - b*x^2)^(3/4)*(2*a + b*x^2)*(10*a*AppellF1[3/2, 3/4,
1, 5/2, -((b*x^2)/a), -(b*x^2)/(2*a)] - b*x^2*(2*AppellF1[5/2, 3/
4, 2, 7/2, -((b*x^2)/a), -(b*x^2)/(2*a)] + 3*AppellF1[5/2, 7/4, 1
```

```
, 7/2, -((b*x^2)/a), -(b*x^2)/(2*a)])))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bx^2-2a}\left(-bx^2-a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)

[Out] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{(bx^2+2a)(-bx^2-a)^{\frac{3}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)),x, algorithm="maxima")

[Out] -integrate(x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

Fricas [A] time = 0.232164, size = 258, normalized size = 2.63

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}}x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{-bx^2-a}}{x^2}}+\left(-bx^2-a\right)^{\frac{1}{4}}}\right) - \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(-bx^2-a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(-bx^2-a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)),x, algorithm="fricas")

[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan((1/4)^(1/4)*b^2*x*(1/(a*b^
6))^(1/4)/(sqrt(1/2)*x*sqrt((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(-b*
x^2 - a))/x^2) + (-b*x^2 - a)^(1/4))) - 1/2*(1/4)^(1/4)*(1/(a*b^6
))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (-b*x^2 - a)^
(1/4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b
^2*x*(1/(a*b^6))^(1/4) - (-b*x^2 - a)^(1/4))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{2a\left(-a-bx^2\right)^{\frac{3}{4}}+bx^2\left(-a-bx^2\right)^{\frac{3}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-b*x**2-2*a)/(-b*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - b*x**2)**(3/4) + b*x**2*(-a - b*x**2)**
(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(bx^2 + 2\,a)(-bx^2 - a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

3.1081 $\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (14*(-1 + 3*x^2)^(1/4))/81 + (8*(-1 + 3*x^2)^(5/4))/405 + (2*(-1
+ 3*x^2)^(9/4))/729 - (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcT
anh[(-1 + 3*x^2)^(1/4)])/81
```

Rubi [A] time = 0.159706, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(1/4))/81 + (8*(-1 + 3*x^2)^(5/4))/405 + (2*(-1
+ 3*x^2)^(9/4))/729 - (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcT
anh[(-1 + 3*x^2)^(1/4)])/81
```

Rubi in Sympy [A] time = 15.8558, size = 70, normalized size = 0.9

$$\frac{2\left(3x^2-1\right)^{\frac{9}{4}}}{729}+\frac{8\left(3x^2-1\right)^{\frac{5}{4}}}{405}+\frac{14\sqrt[4]{3x^2-1}}{81}-\frac{8\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}-\frac{8\,\mathrm{atanh}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(9/4)/729 + 8*(3*x**2 - 1)**(5/4)/405 + 14*(3*x**
2 - 1)**(1/4)/81 - 8*atan((3*x**2 - 1)**(1/4))/81 - 8*atanh((3*x*
*2 - 1)**(1/4))/81
```

Mathematica [C] time = 0.0881131, size = 74, normalized size = 0.95

$$\frac{2\left(-120\left(\frac{1-3x^2}{2-3x^2}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2-3x^2}\right)+135x^6+189x^4+774x^2-284\right)}{3645\left(3x^2-1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-284 + 774*x^2 + 189*x^4 + 135*x^6 - 120*((1 - 3*x^2)/(2 - 3*
x^2))^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2 - 3*x^2)^(-1)]))/
(3645*(-1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{x^7}{3\,x^2-2}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.5056, size = 100, normalized size = 1.28

$$\frac{2}{729}\left(3x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2
- 1)^(1/4) - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1
)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.22634, size = 86, normalized size = 1.1

$$\frac{2}{3645}\left(45x^4+78x^2+284\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 2/3645*(45*x^4 + 78*x^2 + 284)*(3*x^2 - 1)^(1/4) - 8/81*arctan((3
*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*
x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^7}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**7/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [A] time = 0.236653, size = 101, normalized size = 1.29

$$\frac{2}{729}\left(3x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2
- 1)^(1/4) - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*ln((3*x^2 - 1)
^(1/4) + 1) + 4/81*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1082 $\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 + (2*(-1 + 3*x^2)^(5/4))/135 - (4*ArcTan
[(-1 + 3*x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rubi [A] time = 0.146888, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 + (2*(-1 + 3*x^2)^(5/4))/135 - (4*ArcTan
[(-1 + 3*x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rubi in Sympy [A] time = 14.5365, size = 56, normalized size = 0.89

$$\frac{2\left(3x^2-1\right)^{\frac{5}{4}}}{135}+\frac{2\sqrt[4]{3x^2-1}}{9}-\frac{4\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}-\frac{4\operatorname{atanh}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(5/4)/135 + 2*(3*x**2 - 1)**(1/4)/9 - 4*atan((3*x
**2 - 1)**(1/4))/27 - 4*atanh((3*x**2 - 1)**(1/4))/27
```

Mathematica [C] time = 0.0706535, size = 69, normalized size = 1.1

$$\frac{2\left(-20\left(\frac{1-3x^2}{2-3x^2}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2-3x^2}\right)+27x^4+117x^2-42\right)}{405\left(3x^2-1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-42 + 117*x^2 + 27*x^4 - 20*((1 - 3*x^2)/(2 - 3*x^2))^(3/4)*H
ypergeometric2F1[3/4, 3/4, 7/4, (2 - 3*x^2)^(-1)]))/(405*(-1 + 3*
x^2)^(3/4))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{x^5}{3\,x^2-2}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.50096, size = 85, normalized size = 1.35

$$\frac{2}{135}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*
x^2 - 1)^(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x
^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.22489, size = 80, normalized size = 1.27

$$\frac{2}{135}\left(3x^2+14\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 2/135*(3*x^2 + 14)*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(1
/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/
4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**5/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [A] time = 0.249449, size = 86, normalized size = 1.37

$$\frac{2}{135}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*
x^2 - 1)^(1/4)) - 2/27*ln((3*x^2 - 1)^(1/4) + 1) + 2/27*ln(abs((3
*x^2 - 1)^(1/4) - 1))
```

3.1083 $\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*
ArcTanh[(-1 + 3*x^2)^(1/4)])/9
```

Rubi [A] time = 0.115188, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*
ArcTanh[(-1 + 3*x^2)^(1/4)])/9
```

Rubi in Sympy [A] time = 12.2907, size = 42, normalized size = 0.88

$$\frac{2\sqrt[4]{3x^2-1}}{9}-\frac{2\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{9}-\frac{2\,\mathrm{atanh}\left(\sqrt[4]{3x^2-1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(1/4)/9 - 2*atan((3*x**2 - 1)**(1/4))/9 - 2*atanh
((3*x**2 - 1)**(1/4))/9
```

Mathematica [C] time = 0.0280881, size = 34, normalized size = 0.71

$$\frac{2}{9}\sqrt[4]{3x^2-1}\left(1-2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};3x^2-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(1 - 2*Hypergeometric2F1[1/4, 1, 5/4, -1 +
3*x^2]))/9
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{x^3}{3\,x^2-2}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)

[Out] int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.50785, size = 70, normalized size = 1.46

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")

[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((
3*x^2 - 1)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.226596, size = 70, normalized size = 1.46

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")

[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((
3*x^2 - 1)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)

[Out] Integral(x**3/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [A] time = 0.23715, size = 72, normalized size = 1.5

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")

[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*ln((3
*x^2 - 1)^(1/4) + 1) + 1/9*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1084 $\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rubi [A] time = 0.070066, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]

[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rubi in Sympy [A] time = 9.60551, size = 27, normalized size = 0.82

$$-\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}-\frac{\operatorname{atanh}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)

[Out] -atan((3*x**2 - 1)**(1/4))/3 - atanh((3*x**2 - 1)**(1/4))/3
```

Mathematica [A] time = 0.0131593, size = 55, normalized size = 1.67

$$\frac{1}{6}\log\left(1-\sqrt[4]{3x^2-1}\right)-\frac{1}{6}\log\left(\sqrt[4]{3x^2-1}+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]

[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 + Log[1 - (-1 + 3*x^2)^(1/4)]/6 - L
og[1 + (-1 + 3*x^2)^(1/4)]/6
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{x}{3\,x^2-2}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x)

[Out] int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [A] time = 1.50719, size = 55, normalized size = 1.67

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) +
 1/6*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A] time = 0.224549, size = 55, normalized size = 1.67

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) +
 1/6*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [A] time = 0.234157, size = 57, normalized size = 1.73

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*ln((3*x^2 - 1)^(1/4) + 1) +
1/6*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1085 $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(
1/4)]/(2*Sqrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqr
t[2]) - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 + Log[1 - Sqrt[2]*(-1 + 3*x
^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 +
 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2])
```

Rubi [A] time = 0.333392, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(
1/4)]/(2*Sqrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqr
t[2]) - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 + Log[1 - Sqrt[2]*(-1 + 3*x
^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 +
 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 28.4012, size = 148, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{8}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{8}-\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}-1\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4}-\frac{\operatorname{atanh}\left(\sqrt[4]{3x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] sqrt(2)*log(-sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 - 1) + 1)/
8 - sqrt(2)*log(sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 - 1) +
1)/8 - atan((3*x**2 - 1)**(1/4))/2 - sqrt(2)*atan(sqrt(2)*(3*x**2
 - 1)**(1/4) - 1)/4 - sqrt(2)*atan(sqrt(2)*(3*x**2 - 1)**(1/4) +
1)/4 - atanh((3*x**2 - 1)**(1/4))/2
```

Mathematica [C] time = 0.282196, size = 139, normalized size = 0.8

$$-\frac{66x^2F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)}{7\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}\left(33x^2F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+8F_1\left(\frac{11}{4};\frac{3}{4},2;\frac{15}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+3F_1\left(\frac{11}{4};\frac{7}{4},1;\frac{15}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (-66*x^2*AppellF1[7/4, 3/4, 1, 11/4, 1/(3*x^2), 2/(3*x^2)])/(7*(-
2 + 3*x^2)*(-1 + 3*x^2)^(3/4)*(33*x^2*AppellF1[7/4, 3/4, 1, 11/4,
 1/(3*x^2), 2/(3*x^2)] + 8*AppellF1[11/4, 3/4, 2, 15/4, 1/(3*x^2)
, 2/(3*x^2)] + 3*AppellF1[11/4, 7/4, 1, 15/4, 1/(3*x^2), 2/(3*x^2
)]))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{\left(3 x^{2}-2\right) x}\left(3 x^{2}-1\right)^{-\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(3 \, x^{2} - 1\right)}^{\frac{3}{4}} {\left(3 \, x^{2} - 2\right)} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x), x)
```

Fricas [A] time = 0.238999, size = 284, normalized size = 1.64

$$-\frac{1}{8} \, \sqrt{2} {\left(2 \, \sqrt{2} \arctan\left({\left(3 \, x^{2} - 1\right)}^{\frac{1}{4}}\right) + \sqrt{2} \log\left({\left(3 \, x^{2} - 1\right)}^{\frac{1}{4}} + 1\right) - \sqrt{2} \log\left({\left(3 \, x^{2} - 1\right)}^{\frac{1}{4}} - 1\right) - 4 \, \arctan\left(\frac{1}{\sqrt{2} {\left(3 \, x^{2} - 1\right)}^{\frac{1}{4}} + \sqrt{2 \cdots}}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(2*sqrt(2)*arctan((3*x^2 - 1)^(1/4)) + sqrt(2)*log((
3*x^2 - 1)^(1/4) + 1) - sqrt(2)*log((3*x^2 - 1)^(1/4) - 1) - 4*ar
ctan(1/(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(2*sqrt(2)*(3*x^2 - 1)^(1
/4) + 2*sqrt(3*x^2 - 1) + 2) + 1)) - 4*arctan(1/(sqrt(2)*(3*x^2 -
 1)^(1/4) + sqrt(-2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1)
 + 2) - 1)) + log(2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1)
 + 2) - log(-2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(3 x^{2} - 2\right) \left(3 x^{2} - 1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [A] time = 0.241391, size = 209, normalized size = 1.21

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^{2}-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^{2}-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\ln\left(\sqrt{2}\left(3x^{2}-1\right)^{\frac{1}{4}}+\sqrt{3x^{2}-1}+1\right)+\frac{1}{8}\sqrt{2}\ln\left(-\sqrt{2}\left(3x^{2}-1\right)^{\frac{1}{4}}+\sqrt{3x^{2}-1}+1\right)-\frac{1}{2}\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\ln\left(\left(3x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\ln\left(\left|\left(3x^{2}-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4)))
- 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))
) - 1/8*sqrt(2)*ln(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) +
1) + 1/8*sqrt(2)*ln(-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
+ 1) - 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*ln((3*x^2 - 1)^(1/4) +
 1) + 1/4*ln(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1086 $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{15\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{15\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

```
[Out] -(-1 + 3*x^2)^(1/4)/(4*x^2) - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 +
(15*ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (15*Arc
Tan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1
 + 3*x^2)^(1/4)])/4 + (15*Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sq
rt[-1 + 3*x^2]])/(16*Sqrt[2]) - (15*Log[1 + Sqrt[2]*(-1 + 3*x^2)^
(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rubi [A] time = 0.399893, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{15\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{15\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(1/4)/(4*x^2) - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 +
(15*ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (15*Arc
Tan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1
 + 3*x^2)^(1/4)])/4 + (15*Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sq
rt[-1 + 3*x^2]])/(16*Sqrt[2]) - (15*Log[1 + Sqrt[2]*(-1 + 3*x^2)^
(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 35.0296, size = 173, normalized size = 0.91

$$\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{32}-\frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{3x^2-1}+\sqrt{3x^2-1}+1\right)}{32}-\frac{3\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{4}-\frac{15\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}-1\right)}{16}-\frac{15\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16}-\frac{3\operatorname{atanh}\left(\sqrt[4]{3x^2-1}\right)}{4}-\frac{\sqrt[4]{3x^2-1}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] 15*sqrt(2)*log(-sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2 - 1) +
1)/32 - 15*sqrt(2)*log(sqrt(2)*(3*x**2 - 1)**(1/4) + sqrt(3*x**2
```

```
- 1) + 1)/32 - 3*atan((3*x**2 - 1)**(1/4))/4 - 15*sqrt(2)*atan(sq
rt(2)*(3*x**2 - 1)**(1/4) - 1)/16 - 15*sqrt(2)*atan(sqrt(2)*(3*x*
*2 - 1)**(1/4) + 1)/16 - 3*atanh((3*x**2 - 1)**(1/4))/4 - (3*x**2
 - 1)**(1/4)/(4*x**2)
```

Mathematica [C] time = 0.303299, size = 136, normalized size = 0.71

$$-\frac{90F_1\left(\frac{11}{4};\frac{3}{4},1;\frac{15}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)}{11\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}\left(45x^2F_1\left(\frac{11}{4};\frac{3}{4},1;\frac{15}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+8F_1\left(\frac{15}{4};\frac{3}{4},2;\frac{19}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)+3F_1\left(\frac{15}{4};\frac{7}{4},1;\frac{19}{4};\frac{1}{3x^2},\frac{2}{3x^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (-90*AppellF1[11/4, 3/4, 1, 15/4, 1/(3*x^2), 2/(3*x^2)])/(11*(-2
+ 3*x^2)*(-1 + 3*x^2)^(3/4)*(45*x^2*AppellF1[11/4, 3/4, 1, 15/4,
1/(3*x^2), 2/(3*x^2)] + 8*AppellF1[15/4, 3/4, 2, 19/4, 1/(3*x^2),
 2/(3*x^2)] + 3*AppellF1[15/4, 7/4, 1, 19/4, 1/(3*x^2), 2/(3*x^2)
]))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(3\,x^2-2\right)}\left(3\,x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3), x)
```

Fricas [A] time = 0.241737, size = 335, normalized size = 1.75

$$60\sqrt{2}x^2\arctan\left(\frac{1}{\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}+2\sqrt{3\,x^2-1}+2}+1}\right)+60\sqrt{2}x^2\arctan\left(\frac{1}{\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}+\sqrt{-2\sqrt{2}\left(3\,x^2-1\right)^{\frac{1}{4}}+2\sqrt{3\,x^2-1}+2}-1}\right)-15\sqrt{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/32*(60*sqrt(2)*x^2*arctan(1/(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(2
*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2) + 1)) + 60*sq
```

```
rt(2)*x^2*arctan(1/(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(-2*sqrt(2)*(
3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2) - 1)) - 15*sqrt(2)*x^2*
log(2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2) + 15*sqr
t(2)*x^2*log(-2*sqrt(2)*(3*x^2 - 1)^(1/4) + 2*sqrt(3*x^2 - 1) + 2
) - 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3*x^2 - 1)^(1/
4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(1/4)
)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [A] time = 0.241478, size = 228, normalized size = 1.19

$$-\frac{15}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{15}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)$$
$$-\frac{15}{32}\sqrt{2}\ln\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)+\frac{15}{32}\sqrt{2}\ln\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)$$
$$-\frac{\left(3x^2-1\right)^{\frac{1}{4}}}{4x^2}-\frac{3}{4}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\ln\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{3}{8}\ln\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3),x, algorithm="giac")
```

```
[Out] -15/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))
) - 15/16*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1
/4))) - 15/32*sqrt(2)*ln(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 -
 1) + 1) + 15/32*sqrt(2)*ln(-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x
^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(1/4)/x^2 - 3/4*arctan((3*x^2 - 1)
^(1/4)) - 3/8*ln((3*x^2 - 1)^(1/4) + 1) + 3/8*ln(abs((3*x^2 - 1)^
(1/4) - 1))
```

3.1087 $\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=165

$$\frac{40}{567}\sqrt[4]{3x^2-1}x+\frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{40\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)\mid\frac{1}{2}\right)}{567\sqrt{3}x}+\frac{2}{63}\sqrt[4]{3x^2-1}x^3$$

```
[Out] (40*x*(-1 + 3*x^2)^(1/4))/567 + (2*x^3*(-1 + 3*x^2)^(1/4))/63 + (
2*Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 - (2*Sqr
t[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 + (40*Sqrt[x
^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*A
rcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(567*Sqrt[3]*x)
```

Rubi [A] time = 0.437282, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{40}{567}\sqrt[4]{3x^2-1}x+\frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{40\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)\mid\frac{1}{2}\right)}{567\sqrt{3}x}+\frac{2}{63}\sqrt[4]{3x^2-1}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (40*x*(-1 + 3*x^2)^(1/4))/567 + (2*x^3*(-1 + 3*x^2)^(1/4))/63 + (
2*Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 - (2*Sqr
t[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 + (40*Sqrt[x
^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*A
rcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(567*Sqrt[3]*x)
```

Rubi in Sympy [A] time = 21.6824, size = 41, normalized size = 0.25

$$\frac{x^7\sqrt[4]{3x^2-1}\,\text{appellf}_1\left(\frac{7}{2},\frac{3}{4},1,\frac{9}{2},3x^2,\frac{3x^2}{2}\right)}{14\sqrt[4]{-3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] x**7*(3*x**2 - 1)**(1/4)*appellf1(7/2, 3/4, 1, 9/2, 3*x**2, 3*x**
2/2)/(14*(-3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.497821, size = 266, normalized size = 1.61

$$\frac{2x\left(\frac{620x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(6x^2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+9x^2F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}-\frac{80F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+\right.}\right.}{567\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-20 + 51*x^2 + 27*x^4 - (80*AppellF1[1/2, 3/4, 1, 3/2, 3*x^
2, (3*x^2)/2])/((-2 + 3*x^2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2,
 (3*x^2)/2] + x^2*(2*AppellF1[3/2, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2]
 + 3*AppellF1[3/2, 7/4, 1, 5/2, 3*x^2, (3*x^2)/2]))) + (620*x^2*A
ppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(10*Ap
pellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2] + 6*x^2*AppellF1[5/2,
3/4, 2, 7/2, 3*x^2, (3*x^2)/2] + 9*x^2*AppellF1[5/2, 7/4, 1, 7/2,
 3*x^2, (3*x^2)/2]))))/(567*(-1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{x^6}{3x^2-2}\left(3x^2-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] integral(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**6/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1088 $\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=147

$$\frac{2}{27}\sqrt[4]{3x^2-1}x+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{27\sqrt{3}x}$$

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/27 + (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1
 + 3*x^2)^(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^
2)^(1/4)])/9 + (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(27*Sqrt
[3]*x)
```

Rubi [A] time = 0.348402, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{27}\sqrt[4]{3x^2-1}x+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{27\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/27 + (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1
 + 3*x^2)^(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^
2)^(1/4)])/9 + (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(27*Sqrt
[3]*x)
```

Rubi in Sympy [A] time = 21.4953, size = 41, normalized size = 0.28

$$\frac{x^5\sqrt[4]{3x^2-1}\,\text{appellf}_1\left(\frac{5}{2},\frac{3}{4},1,\frac{7}{2},3x^2,\frac{3x^2}{2}\right)}{10\sqrt[4]{-3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] x**5*(3*x**2 - 1)**(1/4)*appellf1(5/2, 3/4, 1, 7/2, 3*x**2, 3*x**
2/2)/(10*(-3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.216773, size = 261, normalized size = 1.78

$$\frac{2x\left(\frac{40x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(6x^2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+9x^2F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}-\frac{4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+ \cdots}\right.}{27\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2 - (4*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2
])/((-2 + 3*x^2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2]
+ x^2*(2*AppellF1[3/2, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF
1[3/2, 7/4, 1, 5/2, 3*x^2, (3*x^2)/2]))) + (40*x^2*AppellF1[3/2,
3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(10*AppellF1[3/2, 3
/4, 1, 5/2, 3*x^2, (3*x^2)/2] + 6*x^2*AppellF1[5/2, 3/4, 2, 7/2,
3*x^2, (3*x^2)/2] + 9*x^2*AppellF1[5/2, 7/4, 1, 7/2, 3*x^2, (3*x^
2)/2]))))/(27*(-1 + 3*x^2)^(3/4))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{x^{4}}{3\,x^{2}-2}\left(3\,x^{2}-1\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(3\,x^{2}-1\right)^{\frac{3}{4}}\left(3\,x^{2}-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{4}}{\left(3\,x^{2}-1\right)^{\frac{3}{4}}\left(3\,x^{2}-2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] integral(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(3x^{2}-2\right)\left(3x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1089 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi [A] time = 0.0695003, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]

[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rubi in Sympy [A] time = 24.9294, size = 41, normalized size = 0.67

$$\frac{x^3\sqrt[4]{3x^2-1}\,\text{appellf}_1\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)}{6\sqrt[4]{-3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)

[Out] x**3*(3*x**2 - 1)**(1/4)*appellf1(3/2, 3/4, 1, 5/2, 3*x**2, 3*x**
2/2)/(6*(-3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.072639, size = 134, normalized size = 2.2

$$\frac{10x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{3\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}\left(3x^2\left(2F_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};3x^2,\frac{3x^2}{2}\right)\right)+10F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]

[Out] (10*x^3*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(3*(-2 + 3*
x^2)*(-1 + 3*x^2)^(3/4)*(10*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*
x^2)/2] + 3*x^2*(2*AppellF1[5/2, 3/4, 2, 7/2, 3*x^2, (3*x^2)/2] +
 3*AppellF1[5/2, 7/4, 1, 7/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{3x^{2}-2}\left(3x^{2}-1\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{(3x^{2}-1)^{\frac{3}{4}}(3x^{2}-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [A] time = 0.219538, size = 115, normalized size = 1.89

$$-\frac{1}{36}\sqrt{6}\left(2\arctan\left(\frac{\sqrt{6}\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x}\right)-\log\left(-\frac{3\sqrt{6}x^{2}-12\left(3x^{2}-1\right)^{\frac{1}{4}}x+2\sqrt{6}\sqrt{3x^{2}-1}}{3x^{2}-2\sqrt{3x^{2}-1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*(2*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) - log(-(
3*sqrt(6)*x^2 - 12*(3*x^2 - 1)^(1/4)*x + 2*sqrt(6)*sqrt(3*x^2 - 1
))/(3*x^2 - 2*sqrt(3*x^2 - 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{(3x^{2}-2)(3x^{2}-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{(3x^{2}-1)^{\frac{3}{4}}(3x^{2}-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")

[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1090 $\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=127

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2\sqrt{3}x}$$

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - (Sqrt[x^2/(1 + Sqrt
[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 +
3*x^2)^(1/4)], 1/2])/(2*Sqrt[3]*x)
```

Rubi [A] time = 0.143443, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(S
qrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - (Sqrt[x^2/(1 + Sqrt
[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 +
3*x^2)^(1/4)], 1/2])/(2*Sqrt[3]*x)
```

Rubi in Sympy [A] time = 49.1533, size = 170, normalized size = 1.34

$$-\frac{\sqrt{2}x(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{3x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{3x^2-1}+1}\sqrt{i\sqrt{3x^2-1}+1}}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)\middle|\frac{1}{2}\right)}{12x}-\frac{\sqrt{6}\sqrt{x^2}\operatorname{atanh}\left(\frac{\sqrt{6}\sqrt[4]{3x^2-1}}{3\sqrt{x^2}}\right)}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] -sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(3*x**2 -
1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(3*x**2 - 1) + 1)*sqrt(I*sqrt(3*
x**2 - 1) + 1)) - sqrt(3)*sqrt(x**2/(sqrt(3*x**2 - 1) + 1)**2)*(s
qrt(3*x**2 - 1) + 1)*elliptic_f(2*atan((3*x**2 - 1)**(1/4)), 1/2)
/(12*x) - sqrt(6)*sqrt(x**2)*atanh(sqrt(6)*(3*x**2 - 1)**(1/4)/(3
*sqrt(x**2)))/(12*x)
```

Mathematica [C] time = 0.159596, size = 129, normalized size = 1.02

$$\frac{2xF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)(3x^2-1)^{3/4}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*
(-1 + 3*x^2)^(3/4)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2
] + x^2*(2*AppellF1[3/2, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*Appel
lF1[3/2, 7/4, 1, 5/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{3 x^{2}-2}\left(3 x^{2}-1\right)^{-\frac{3}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(3 \, x^{2} - 1\right)}^{\frac{3}{4}} {\left(3 \, x^{2} - 2\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(3 \, x^{2} - 1\right)}^{\frac{3}{4}} {\left(3 \, x^{2} - 2\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] integral(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3 x^{2} - 2\right) \left(3 x^{2} - 1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{3}{4}}(3x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x, algorithm="giac")

[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1091 $\int \frac{1}{x^2(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt[4]{3x^2-1}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}$$

```
[Out] -(-1 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 +
 3*x^2)^(1/4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)
^(1/4)])/4 - (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqr
t[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x
)
```

Rubi [A] time = 0.307079, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt[4]{3x^2-1}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 +
 3*x^2)^(1/4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)
^(1/4)])/4 - (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqr
t[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x
)
```

Rubi in Sympy [A] time = 22.1584, size = 42, normalized size = 0.28

$$-\frac{\sqrt[4]{3x^2-1}\,\mathrm{appellf}_1\left(-\frac{1}{2},\frac{3}{4},1,\frac{1}{2},3x^2,\frac{3x^2}{2}\right)}{2x\sqrt[4]{-3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] -(3*x**2 - 1)**(1/4)*appellf1(-1/2, 3/4, 1, 1/2, 3*x**2, 3*x**2/2
)/(2*x*(-3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.278853, size = 132, normalized size = 0.89

$$-\frac{2F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};3x^2,\frac{3x^2}{2}\right)}{x\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}\left(3x^2\left(2F_1\left(\frac{1}{2};\frac{3}{4},2;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{1}{2};\frac{7}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2*AppellF1[-1/2, 3/4, 1, 1/2, 3*x^2, (3*x^2)/2])/(x*(-2 + 3*x^2
)*(-1 + 3*x^2)^(3/4)*(2*AppellF1[-1/2, 3/4, 1, 1/2, 3*x^2, (3*x^2
)/2] + 3*x^2*(2*AppellF1[1/2, 3/4, 2, 3/2, 3*x^2, (3*x^2)/2] + 3*
AppellF1[1/2, 7/4, 1, 3/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(3 x^{2}-2\right)} \left(3 x^{2}-1\right)^{-\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(3 x^{2}-1\right)}^{\frac{3}{4}} {\left(3 x^{2}-2\right)} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{{\left(3 x^{4}-2 x^{2}\right)} {\left(3 x^{2}-1\right)}^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((3*x^4 - 2*x^2)*(3*x^2 - 1)^(3/4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(3x^{2}-2\right) \left(3x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

3.1092 $\int \frac{1}{x^4(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt[4]{3x^2-1}}{x}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)\middle|\frac{1}{2}\right)}{8x}-\frac{\sqrt[4]{3x^2-1}}{6x^3}$$

```
[Out] -(-1 + 3*x^2)^(1/4)/(6*x^3) - (2*(-1 + 3*x^2)^(1/4))/x + (3*Sqrt[
3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*A
rcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (11*Sqrt[3]*Sqrt[x^
2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*Ar
cTan[(-1 + 3*x^2)^(1/4)], 1/2])/(8*x)
```

Rubi [A] time = 0.403795, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{2\sqrt[4]{3x^2-1}}{x}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)\middle|\frac{1}{2}\right)}{8x}-\frac{\sqrt[4]{3x^2-1}}{6x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(1/4)/(6*x^3) - (2*(-1 + 3*x^2)^(1/4))/x + (3*Sqrt[
3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*A
rcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (11*Sqrt[3]*Sqrt[x^
2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*Ar
cTan[(-1 + 3*x^2)^(1/4)], 1/2])/(8*x)
```

Rubi in Sympy [A] time = 22.6108, size = 46, normalized size = 0.28

$$-\frac{\sqrt[4]{3x^2-1}\,\mathrm{appellf}_1\left(-\frac{3}{2},\frac{3}{4},1,-\frac{1}{2},3x^2,\frac{3x^2}{2}\right)}{6x^3\sqrt[4]{-3x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] -(3*x**2 - 1)**(1/4)*appellf1(-3/2, 3/4, 1, -1/2, 3*x**2, 3*x**2/
2)/(6*x**3*(-3*x**2 + 1)**(1/4))
```

Mathematica [C] time = 0.277936, size = 134, normalized size = 0.81

$$\frac{2F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};3x^2,\frac{3x^2}{2}\right)}{3x^3\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}\left(3x^2\left(2F_1\left(-\frac{1}{2};\frac{3}{4},2;\frac{1}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(-\frac{1}{2};\frac{7}{4},1;\frac{1}{2};3x^2,\frac{3x^2}{2}\right)\right)-2F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*AppellF1[-3/2, 3/4, 1, -1/2, 3*x^2, (3*x^2)/2])/(3*x^3*(-2 + 3
*x^2)*(-1 + 3*x^2)^(3/4)*(-2*AppellF1[-3/2, 3/4, 1, -1/2, 3*x^2,
(3*x^2)/2] + 3*x^2*(2*AppellF1[-1/2, 3/4, 2, 1/2, 3*x^2, (3*x^2)/
2] + 3*AppellF1[-1/2, 7/4, 1, 1/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(3 x^{2}-2\right)} \left(3 x^{2}-1\right)^{-\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(3 \, x^{2} - 1\right)}^{\frac{3}{4}} {\left(3 \, x^{2} - 2\right)} x^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(3 \, x^{6} - 2 \, x^{4}\right)} {\left(3 \, x^{2} - 1\right)}^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/((3*x^6 - 2*x^4)*(3*x^2 - 1)^(3/4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(3 x^{2} - 2\right) \left(3 x^{2} - 1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

3.1093 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{3ae^{5/2}(8bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(8bc-7ad)}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

```
[Out] ((8*b*c - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b^2) + (d*(
e*x)^(7/2)*(a + b*x^2)^(1/4))/(4*b*e) + (3*a*(8*b*c - 7*a*d)*e^(5
/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*
b^(11/4)) - (3*a*(8*b*c - 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^(11/4))
```

Rubi [A] time = 0.360219, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{3ae^{5/2}(8bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(8bc-7ad)}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((8*b*c - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b^2) + (d*(
e*x)^(7/2)*(a + b*x^2)^(1/4))/(4*b*e) + (3*a*(8*b*c - 7*a*d)*e^(5
/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*
b^(11/4)) - (3*a*(8*b*c - 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^(11/4))
```

Rubi in Sympy [A] time = 35.4988, size = 165, normalized size = 0.95

$$-\frac{3ae^{\frac{5}{2}}(7ad-8bc)\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{\frac{11}{4}}}+\frac{3ae^{\frac{5}{2}}(7ad-8bc)\operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{\frac{11}{4}}}+\frac{d(ex)^{\frac{7}{2}}\sqrt[4]{a+bx^2}}{4be}-\frac{e(ex)^{\frac{3}{2}}\sqrt[4]{a+bx^2}(7ad-8bc)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] -3*a*e**(5/2)*(7*a*d - 8*b*c)*atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a
 + b*x**2)**(1/4)))/(32*b**(11/4)) + 3*a*e**(5/2)*(7*a*d - 8*b*c)
*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(32*b**(
11/4)) + d*(e*x)**(7/2)*(a + b*x**2)**(1/4)/(4*b*e) - e*(e*x)**(3
/2)*(a + b*x**2)**(1/4)*(7*a*d - 8*b*c)/(16*b**2)
```

Mathematica [C] time = 0.13174, size = 97, normalized size = 0.56

$$\frac{e(ex)^{3/2}\left(a\left(\frac{bx^2}{a}+1\right)^{3/4}(7ad-8bc)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)\left(7ad-4b\left(2c+dx^2\right)\right)\right)}{16b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(-((a + b*x^2)*(7*a*d - 4*b*(2*c + d*x^2))) + a*(-
8*b*c + 7*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/4, 3/4,
7/4, -((b*x^2)/a)]))/(16*b^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int (dx^2 + c) (ex)^{\frac{5}{2}} \left(bx^2 + a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right) (ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

3.1094 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{e}(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt
[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b
^(7/4)) + ((4*b*c - 3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rubi [A] time = 0.270842, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{\sqrt{e}(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```


```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt
[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b
^(7/4)) + ((4*b*c - 3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rubi in Sympy [A] time = 29.1738, size = 124, normalized size = 0.91

$$\frac{d\,(ex)^{\frac{3}{2}}\,\sqrt[4]{a+bx^2}}{2be}+\frac{\sqrt{e}\,(3ad-4bc)\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{7}{4}}}-\frac{\sqrt{e}\,(3ad-4bc)\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] d*(e*x)**(3/2)*(a + b*x**2)**(1/4)/(2*b*e) + sqrt(e)*(3*a*d - 4*b
*c)*atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(4*b**
(7/4)) - sqrt(e)*(3*a*d - 4*b*c)*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e
)*(a + b*x**2)**(1/4)))/(4*b**(7/4))
```

Mathematica [C] time = 0.0974838, size = 80, normalized size = 0.59

$$\frac{x\sqrt{ex}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(4bc-3ad)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+3d\left(a+bx^2\right)\right)}{6b\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (x*Sqrt[e*x]*(3*d*(a + b*x^2) + (4*b*c - 3*a*d)*(1 + (b*x^2)/a)^(
3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -((b*x^2)/a)]))/(6*b*(a + b
```

```
*x^2)^(3/4))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (dx^2+c)\sqrt{ex}\,(bx^2+a)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 5.8518, size = 92, normalized size = 0.68

$$\frac{c\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e\,\Gamma\left(\frac{7}{4}\right)}+\frac{d\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^3\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*(e*x)**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(3/4)*e*gamma(7/4)) + d*(e*x)**(7/2)*gamma(7/4
)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4
)*e**3*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

3.1095 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=113

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*Sqr
t[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2)) + (d*ArcT
anh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^
(3/2))
```

Rubi [A] time = 0.236559, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*Sqr
t[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2)) + (d*ArcT
anh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^
(3/2))
```

Rubi in Sympy [A] time = 28.3293, size = 104, normalized size = 0.92

$$-\frac{d\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{3}{4}}e^{\frac{3}{2}}}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{3}{4}}e^{\frac{3}{2}}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -d*atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(b**(3/
4)*e**(3/2)) + d*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**
(1/4)))/(b**(3/4)*e**(3/2)) - 2*c*(a + b*x**2)**(1/4)/(a*e*sqrt(e
*x))
```

Mathematica [C] time = 0.0712407, size = 77, normalized size = 0.68

$$\frac{x\left(2adx^2\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-6c\left(a+bx^2\right)\right)}{3a(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-6*c*(a + b*x^2) + 2*a*d*x^2*(1 + (b*x^2)/a)^(3/4)*Hypergeome
tric2F1[3/4, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*(a + b*x^
```

```
2)^(3/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{3}{2}} \left(bx^2+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 40.7813, size = 85, normalized size = 0.75

$$\frac{\sqrt[4]{b}c\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(3/2)*gamm
a(3/4)) + d*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*
exp_polar(I*pi)/a)/(2*a**(3/4)*e**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

3.1096 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt[4]{a+bx^2}(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)
*(a + b*x^2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

Rubi [A] time = 0.114783, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2\sqrt[4]{a+bx^2}(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```


```
[Out] (-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)
*(a + b*x^2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

Rubi in Sympy [A] time = 12.1354, size = 63, normalized size = 0.94

$$-\frac{2c\sqrt[4]{a+bx^2}}{5ae\,(ex)^{\frac{5}{2}}}-\frac{2\sqrt[4]{a+bx^2}\,(5ad-4bc)}{5a^2e^3\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -2*c*(a + b*x**2)**(1/4)/(5*a*e*(e*x)**(5/2)) - 2*(a + b*x**2)**(
1/4)*(5*a*d - 4*b*c)/(5*a**2*e**3*sqrt(e*x))
```

Mathematica [A] time = 0.0657754, size = 44, normalized size = 0.66

$$-\frac{2x\sqrt[4]{a+bx^2}\left(a\left(c+5dx^2\right)-4bcx^2\right)}{5a^2(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*(-4*b*c*x^2 + a*(c + 5*d*x^2)))/(5*a^2*(e
*x)^(7/2))
```

Maple [A] time = 0.008, size = 39, normalized size = 0.6

$$-\frac{2\,x\left(5\,adx^2-4\,cx^2b+ac\right)}{5\,a^2}\sqrt[4]{bx^2+a}\,(ex)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*(b*x^2+a)^(1/4)*x*(5*a*d*x^2-4*b*c*x^2+a*c)/a^2/(e*x)^(7/2)
```

Maxima [A] time = 1.43469, size = 82, normalized size = 1.22

$$\frac{2\,c\left(\frac{5\,(bx^2+a)^{\frac{1}{4}}b}{\sqrt{x}} - \frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{5\,a^2e^{\frac{7}{2}}} - \frac{2\,(bx^2+a)^{\frac{1}{4}}d}{ae^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] 2/5*c*(5*(b*x^2 + a)^(1/4)*b/sqrt(x) - (b*x^2 + a)^(5/4)/x^(5/2))
/(a^2*e^(7/2)) - 2*(b*x^2 + a)^(1/4)*d/(a*e^(7/2)*sqrt(x))
```

Fricas [A] time = 0.226485, size = 58, normalized size = 0.87

$$\frac{2\left((4\,bc - 5\,ad)x^2 - ac\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{5\,a^2e^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c - 5*a*d)*x^2 - a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*
e^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2+a)^{\frac{3}{4}}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

3.1097 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{8\left(a+bx^2\right)^{5/4}(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) + (2*(8*b*c - 9*a*d)
*(a + b*x^2)^(1/4))/(9*a^2*e^3*(e*x)^(5/2)) - (8*(8*b*c - 9*a*d)*
(a + b*x^2)^(5/4))/(45*a^3*e^3*(e*x)^(5/2))
```

Rubi [A] time = 0.169665, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{8\left(a+bx^2\right)^{5/4}(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) + (2*(8*b*c - 9*a*d)
*(a + b*x^2)^(1/4))/(9*a^2*e^3*(e*x)^(5/2)) - (8*(8*b*c - 9*a*d)*
(a + b*x^2)^(5/4))/(45*a^3*e^3*(e*x)^(5/2))
```

Rubi in Sympy [A] time = 17.6922, size = 99, normalized size = 0.95

$$-\frac{2c\sqrt[4]{a+bx^2}}{9ae\,(ex)^{\frac{9}{2}}}-\frac{2\sqrt[4]{a+bx^2}\,(9ad-8bc)}{9a^2e^3\,(ex)^{\frac{5}{2}}}+\frac{8\left(a+bx^2\right)^{\frac{5}{4}}(9ad-8bc)}{45a^3e^3\,(ex)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -2*c*(a + b*x**2)**(1/4)/(9*a*e*(e*x)**(9/2)) - 2*(a + b*x**2)**(
1/4)*(9*a*d - 8*b*c)/(9*a**2*e**3*(e*x)**(5/2)) + 8*(a + b*x**2)*
*(5/4)*(9*a*d - 8*b*c)/(45*a**3*e**3*(e*x)**(5/2))
```

Mathematica [A] time = 0.100227, size = 72, normalized size = 0.69

$$-\frac{2\sqrt{ex}\sqrt[4]{a+bx^2}\left(a^2\left(5c+9dx^2\right)-4abx^2\left(2c+9dx^2\right)+32b^2cx^4\right)}{45a^3e^6x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(a + b*x^2)^(1/4)*(32*b^2*c*x^4 - 4*a*b*x^2*(2*c +
9*d*x^2) + a^2*(5*c + 9*d*x^2)))/(45*a^3*e^6*x^5)
```

Maple [A] time = 0.01, size = 62, normalized size = 0.6

$$-\frac{2\,x\left(-36\,x^4abd+32\,b^2cx^4+9\,x^2a^2d-8\,abcx^2+5\,a^2c\right)}{45\,a^3}\sqrt[4]{bx^2+a}\,(ex)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*(b*x^2+a)^(1/4)*x*(-36*a*b*d*x^4+32*b^2*c*x^4+9*a^2*d*x^2-8
*a*b*c*x^2+5*a^2*c)/a^3/(e*x)^(11/2)
```

Maxima [A] time = 1.42257, size = 130, normalized size = 1.25

$$\frac{2\,d\left(\frac{5\,(bx^2+a)^{\frac{1}{4}}\,b}{\sqrt{x}}-\frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{5\,a^2e^{\frac{11}{2}}}-\frac{2\left(\frac{45\,(bx^2+a)^{\frac{1}{4}}\,b^2}{\sqrt{x}}-\frac{18\,(bx^2+a)^{\frac{5}{4}}\,b}{x^{\frac{5}{2}}}+\frac{5\,(bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}}\right)c}{45\,a^3e^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)),x, algorithm="maxima")
```

```
[Out] 2/5*d*(5*(b*x^2 + a)^(1/4)*b/sqrt(x) - (b*x^2 + a)^(5/4)/x^(5/2))
/(a^2*e^(11/2)) - 2/45*(45*(b*x^2 + a)^(1/4)*b^2/sqrt(x) - 18*(b*
x^2 + a)^(5/4)*b/x^(5/2) + 5*(b*x^2 + a)^(9/4)/x^(9/2))*c/(a^3*e^
(11/2))
```

Fricas [A] time = 0.224322, size = 89, normalized size = 0.86

$$-\frac{2\left(4\left(8\,b^2c-9\,abd\right)x^4+5\,a^2c-\left(8\,abc-9\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{45\,a^3e^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)),x, algorithm="fricas")
```

```
[Out] -2/45*(4*(8*b^2*c - 9*a*b*d)*x^4 + 5*a^2*c - (8*a*b*c - 9*a^2*d)*
x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^3*e^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}\,(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

3.1098 $\int \frac{c+dx^2}{(ex)^{15/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=141

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) + (2*(12*b*c - 13*
a*d)*(a + b*x^2)^(1/4))/(13*a^2*e^3*(e*x)^(9/2)) - (16*(12*b*c -
13*a*d)*(a + b*x^2)^(5/4))/(65*a^3*e^3*(e*x)^(9/2)) + (64*(12*b*c
 - 13*a*d)*(a + b*x^2)^(9/4))/(585*a^4*e^3*(e*x)^(9/2))
```

Rubi [A] time = 0.222218, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) + (2*(12*b*c - 13*
a*d)*(a + b*x^2)^(1/4))/(13*a^2*e^3*(e*x)^(9/2)) - (16*(12*b*c -
13*a*d)*(a + b*x^2)^(5/4))/(65*a^3*e^3*(e*x)^(9/2)) + (64*(12*b*c
 - 13*a*d)*(a + b*x^2)^(9/4))/(585*a^4*e^3*(e*x)^(9/2))
```

Rubi in Sympy [A] time = 23.4163, size = 136, normalized size = 0.96

$$-\frac{2c\sqrt[4]{a+bx^2}}{13ae\,(ex)^{\frac{13}{2}}}-\frac{2\sqrt[4]{a+bx^2}\,(13ad-12bc)}{13a^2e^3\,(ex)^{\frac{9}{2}}}+\frac{16\left(a+bx^2\right)^{\frac{5}{4}}(13ad-12bc)}{65a^3e^3\,(ex)^{\frac{9}{2}}}-\frac{64\left(a+bx^2\right)^{\frac{9}{4}}(13ad-12bc)}{585a^4e^3\,(ex)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(15/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -2*c*(a + b*x**2)**(1/4)/(13*a*e*(e*x)**(13/2)) - 2*(a + b*x**2)*
*(1/4)*(13*a*d - 12*b*c)/(13*a**2*e**3*(e*x)**(9/2)) + 16*(a + b*
x**2)**(5/4)*(13*a*d - 12*b*c)/(65*a**3*e**3*(e*x)**(9/2)) - 64*(
a + b*x**2)**(9/4)*(13*a*d - 12*b*c)/(585*a**4*e**3*(e*x)**(9/2))
```

Mathematica [A] time = 0.119092, size = 94, normalized size = 0.67

$$-\frac{2\sqrt{ex}\sqrt[4]{a+bx^2}\left(5a^3\left(9c+13dx^2\right)-4a^2bx^2\left(15c+26dx^2\right)+32ab^2x^4\left(3c+13dx^2\right)-384b^3cx^6\right)}{585a^4e^8x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(a + b*x^2)^(1/4)*(-384*b^3*c*x^6 + 32*a*b^2*x^4*(3
*c + 13*d*x^2) + 5*a^3*(9*c + 13*d*x^2) - 4*a^2*b*x^2*(15*c + 26*
d*x^2)))/(585*a^4*e^8*x^7)
```

Maple [A] time = 0.009, size = 86, normalized size = 0.6

$$-\frac{2x\left(416ab^{2}dx^{6}-384b^{3}cx^{6}-104a^{2}bdx^{4}+96ab^{2}cx^{4}+65a^{3}dx^{2}-60a^{2}bcx^{2}+45ca^{3}\right)}{585a^{4}}\sqrt[4]{bx^{2}+a}\,(ex)^{-\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/585*(b*x^2+a)^(1/4)*x*(416*a*b^2*d*x^6-384*b^3*c*x^6-104*a^2*b
*d*x^4+96*a*b^2*c*x^4+65*a^3*d*x^2-60*a^2*b*c*x^2+45*a^3*c)/a^4/(
e*x)^(15/2)
```

Maxima [A] time = 1.43236, size = 176, normalized size = 1.25

$$-\frac{2\left(\frac{45\left(bx^{2}+a\right)^{\frac{1}{4}}b^{2}}{\sqrt{x}}-\frac{18\left(bx^{2}+a\right)^{\frac{5}{4}}b}{x^{\frac{5}{2}}}+\frac{5\left(bx^{2}+a\right)^{\frac{9}{4}}}{x^{\frac{9}{2}}}\right)d}{45a^{3}e^{\frac{15}{2}}}+\frac{2\left(\frac{195\left(bx^{2}+a\right)^{\frac{1}{4}}b^{3}}{\sqrt{x}}-\frac{117\left(bx^{2}+a\right)^{\frac{5}{4}}b^{2}}{x^{\frac{5}{2}}}+\frac{65\left(bx^{2}+a\right)^{\frac{9}{4}}b}{x^{\frac{9}{2}}}-\frac{15\left(bx^{2}+a\right)^{\frac{13}{4}}}{x^{\frac{13}{2}}}\right)c}{195a^{4}e^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)),x, algorithm="maxima")
```

```
[Out] -2/45*(45*(b*x^2 + a)^(1/4)*b^2/sqrt(x) - 18*(b*x^2 + a)^(5/4)*b/
x^(5/2) + 5*(b*x^2 + a)^(9/4)/x^(9/2))*d/(a^3*e^(15/2)) + 2/195*(
195*(b*x^2 + a)^(1/4)*b^3/sqrt(x) - 117*(b*x^2 + a)^(5/4)*b^2/x^(
5/2) + 65*(b*x^2 + a)^(9/4)*b/x^(9/2) - 15*(b*x^2 + a)^(13/4)/x^(
13/2))*c/(a^4*e^(15/2))
```

Fricas [A] time = 0.222813, size = 122, normalized size = 0.87

$$\frac{2\left(32\left(12b^{3}c-13ab^{2}d\right)x^{6}-8\left(12ab^{2}c-13a^{2}bd\right)x^{4}-45a^{3}c+5\left(12a^{2}bc-13a^{3}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{ex}}{585a^{4}e^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)),x, algorithm="fricas")
```

```
[Out] 2/585*(32*(12*b^3*c - 13*a*b^2*d)*x^6 - 8*(12*a*b^2*c - 13*a^2*b*
d)*x^4 - 45*a^3*c + 5*(12*a^2*b*c - 13*a^3*d)*x^2)*(b*x^2 + a)^(1
/4)*sqrt(e*x)/(a^4*e^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(15/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{3}{4}} (ex)^{\frac{15}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

3.1099 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=180

$$-\frac{a^{3/2}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-9ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{ae^3\sqrt{ex}\sqrt[4]{a+bx^2}(10bc-9ad)}{12b^3}+\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(10bc-9ad)}{30b^2}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

```
[Out] -(a*(10*b*c - 9*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(12*b^3) +
((10*b*c - 9*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(30*b^2) + (d*
(e*x)^(9/2)*(a + b*x^2)^(1/4))/(5*b*e) - (a^(3/2)*(10*b*c - 9*a*d
)*e^2*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(12*b^(5/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.40864, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{a^{3/2}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-9ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{ae^3\sqrt{ex}\sqrt[4]{a+bx^2}(10bc-9ad)}{12b^3}+\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(10bc-9ad)}{30b^2}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] -(a*(10*b*c - 9*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(12*b^3) +
((10*b*c - 9*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(30*b^2) + (d*
(e*x)^(9/2)*(a + b*x^2)^(1/4))/(5*b*e) - (a^(3/2)*(10*b*c - 9*a*d
)*e^2*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(12*b^(5/2)*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 40.8395, size = 165, normalized size = 0.92

$$\frac{a^{\frac{3}{2}}e^2\left(ex\right)^{\frac{3}{2}}\left(9ad-10bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{12b^{\frac{5}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{ae^3\sqrt{ex}\sqrt[4]{a+bx^2}\left(9ad-10bc\right)}{12b^3}+\frac{d\left(ex\right)^{\frac{9}{2}}\sqrt[4]{a+bx^2}}{5be}-\frac{e\left(ex\right)^{\frac{5}{2}}\sqrt[4]{a+bx^2}\left(9ad-10bc\right)}{30b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/2)*e**2*(e*x)**(3/2)*(9*a*d - 10*b*c)*(a/(b*x**2) + 1)**(3/
4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(12*b**(5/2)*(a + b
*x**2)**(3/4)) + a*e**3*sqrt(e*x)*(a + b*x**2)**(1/4)*(9*a*d - 10
*b*c)/(12*b**3) + d*(e*x)**(9/2)*(a + b*x**2)**(1/4)/(5*b*e) - e*
(e*x)**(5/2)*(a + b*x**2)**(1/4)*(9*a*d - 10*b*c)/(30*b**2)
```

Mathematica [C] time = 0.156891, size = 123, normalized size = 0.68

$$\frac{e^3\sqrt{ex}\left(\left(a+bx^2\right)\left(45a^2d-2ab\left(25c+9dx^2\right)+4b^2x^2\left(5c+3dx^2\right)\right)+5a^2\left(\frac{bx^2}{a}+1\right)^{3/4}(10bc-9ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{60b^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*((a + b*x^2)*(45*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2)
 - 2*a*b*(25*c + 9*d*x^2)) + 5*a^2*(10*b*c - 9*a*d)*(1 + (b*x^2)/
a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(60*b^3
*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int (dx^2 + c)(ex)^{\frac{7}{2}} \left(bx^2 + a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(de^3x^5 + ce^3x^3\right)\sqrt{ex}}{(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^3*x^5 + c*e^3*x^3)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4), x)
```

3.1100 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(6bc-5ad)}{6b^2}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

```
[Out] ((6*b*c - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^2) + (d*(e*x
)^(5/2)*(a + b*x^2)^(1/4))/(3*b*e) + (Sqrt[a]*(6*b*c - 5*a*d)*(1
+ a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(6*b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.319791, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(6bc-5ad)}{6b^2}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((6*b*c - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^2) + (d*(e*x
)^(5/2)*(a + b*x^2)^(1/4))/(3*b*e) + (Sqrt[a]*(6*b*c - 5*a*d)*(1
+ a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(6*b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 33.1938, size = 124, normalized size = 0.89

$$-\frac{\sqrt{a}\,(ex)^{\frac{3}{2}}\,(5ad-6bc)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{6b^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{d\,(ex)^{\frac{5}{2}}\sqrt[4]{a+bx^2}}{3be}-\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}\,(5ad-6bc)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] -sqrt(a)*(e*x)**(3/2)*(5*a*d - 6*b*c)*(a/(b*x**2) + 1)**(3/4)*ell
iptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(6*b**(3/2)*(a + b*x**2)*
*(3/4)) + d*(e*x)**(5/2)*(a + b*x**2)**(1/4)/(3*b*e) - e*sqrt(e*x
)*(a + b*x**2)**(1/4)*(5*a*d - 6*b*c)/(6*b**2)
```

Mathematica [C] time = 0.115883, size = 97, normalized size = 0.7

$$\frac{e\sqrt{ex}\left(a\left(\frac{bx^2}{a}+1\right)^{3/4}(5ad-6bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(a+bx^2\right)\left(5ad-2b\left(3c+dx^2\right)\right)\right)}{6b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-((a + b*x^2)*(5*a*d - 2*b*(3*c + d*x^2))) + a*(-6*
b*c + 5*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/
```

```
4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \left(e x\right)^{\frac{3}{2}} \left(b \,x^{2}+a\right)^{-\frac{3}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)

[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{2} + c\right) \left(ex\right)^{\frac{3}{2}}}{\left(bx^{2} + a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dex^{3} + cex\right)\sqrt{ex}}{\left(bx^{2} + a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4),x, algorithm="fricas")

[Out] integral((d*e*x^3 + c*e*x)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

Sympy [A] time = 59.3142, size = 94, normalized size = 0.68

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)

[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*a**(3/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*g
amma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(3/4)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

3.1101 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=102

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] (d*Sqrt[e*x]*(a + b*x^2)^(1/4))/(b*e) - ((2*b*c - a*d)*(1 + a/(b*
x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[a]*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.263758, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (d*Sqrt[e*x]*(a + b*x^2)^(1/4))/(b*e) - ((2*b*c - a*d)*(1 + a/(b*
x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[a]*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 28.3887, size = 90, normalized size = 0.88

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}+\frac{2\left(ex\right)^{\frac{3}{2}}\left(\frac{ad}{2}-bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] d*sqrt(e*x)*(a + b*x**2)**(1/4)/(b*e) + 2*(e*x)**(3/2)*(a*d/2 - b
*c)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/
2, 2)/(sqrt(a)*sqrt(b)*e**2*(a + b*x**2)**(3/4))
```

Mathematica [C] time = 0.0939675, size = 77, normalized size = 0.75

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/4}(2bc-ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+dx\left(a+bx^2\right)}{b\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (d*x*(a + b*x^2) + (2*b*c - a*d)*x*(1 + (b*x^2)/a)^(3/4)*Hypergeo
metric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)])/(b*Sqrt[e*x]*(a + b*x^2)^
(3/4))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \frac{1}{\sqrt{e x}} \left(b x^{2}+a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d x^{2}+c}{\left(b x^{2}+a\right)^{\frac{3}{4}} \sqrt{e x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d x^{2}+c}{\left(b x^{2}+a\right)^{\frac{3}{4}} \sqrt{e x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

Sympy [A] time = 10.5639, size = 78, normalized size = 0.76

$$-\frac{c {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{a e^{i \pi}}{b x^{2}}} \right)}{b^{\frac{3}{4}} \sqrt{e x}} + \frac{d x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 a^{\frac{3}{4}} \sqrt{e} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -c*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4
)*sqrt(e)*x) + d*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*
x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*sqrt(e)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d x^{2}+c}{\left(b x^{2}+a\right)^{\frac{3}{4}} \sqrt{e x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

3.1102 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (2*Sqrt[b]*(2*b*c
- 3*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.271071, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (2*Sqrt[b]*(2*b*c
- 3*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 29.8702, size = 99, normalized size = 0.93

$$-\frac{2c\sqrt[4]{a+bx^2}}{3ae\,(ex)^{\frac{3}{2}}}-\frac{4\sqrt{b}\,(ex)^{\frac{3}{2}}\left(\frac{3ad}{2}-bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{3a^{\frac{3}{2}}e^4\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -2*c*(a + b*x**2)**(1/4)/(3*a*e*(e*x)**(3/2)) - 4*sqrt(b)*(e*x)**
(3/2)*(3*a*d/2 - b*c)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqr
t(a)/(sqrt(b)*x))/2, 2)/(3*a**(3/2)*e**4*(a + b*x**2)**(3/4))
```

Mathematica [C] time = 0.10591, size = 84, normalized size = 0.79

$$\frac{x\left(2x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-2c\left(a+bx^2\right)\right)}{3a(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-2*c*(a + b*x^2) + 2*(-2*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/
4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5
/2)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{-\frac{5}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)

[Out] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}\sqrt{ex}e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)),x, algorithm="fricas")

[Out] integral((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(3/4),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)),x, algorithm="giac")

[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)), x)
```

3.1103 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{a+bx^2}(6bc-7ad)}{21a^2e^3(ex)^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) + (2*(6*b*c - 7*a*d)
*(a + b*x^2)^(1/4))/(21*a^2*e^3*(e*x)^(3/2)) - (4*b^(3/2)*(6*b*c
- 7*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/(21*a^(5/2)*e^6*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.337427, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{a+bx^2}(6bc-7ad)}{21a^2e^3(ex)^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) + (2*(6*b*c - 7*a*d)
*(a + b*x^2)^(1/4))/(21*a^2*e^3*(e*x)^(3/2)) - (4*b^(3/2)*(6*b*c
- 7*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/(21*a^(5/2)*e^6*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 35.9221, size = 134, normalized size = 0.93

$$-\frac{2c\sqrt[4]{a+bx^2}}{7ae\left(ex\right)^{\frac{7}{2}}}-\frac{2\sqrt[4]{a+bx^2}\left(7ad-6bc\right)}{21a^2e^3\left(ex\right)^{\frac{3}{2}}}+\frac{4b^{\frac{3}{2}}\left(ex\right)^{\frac{3}{2}}\left(7ad-6bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{21a^{\frac{5}{2}}e^6\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -2*c*(a + b*x**2)**(1/4)/(7*a*e*(e*x)**(7/2)) - 2*(a + b*x**2)**(
1/4)*(7*a*d - 6*b*c)/(21*a**2*e**3*(e*x)**(3/2)) + 4*b**(3/2)*(e*
x)**(3/2)*(7*a*d - 6*b*c)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan
(sqrt(a)/(sqrt(b)*x))/2, 2)/(21*a**(5/2)*e**6*(a + b*x**2)**(3/4)
)
```

Mathematica [C] time = 0.185749, size = 107, normalized size = 0.74

$$-\frac{2\sqrt{ex}\left(2bx^4\left(\frac{bx^2}{a}+1\right)^{3/4}(7ad-6bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+\left(a+bx^2\right)\left(3ac+7adx^2-6bcx^2\right)\right)}{21a^2e^5x^4\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*((a + b*x^2)*(3*a*c - 6*b*c*x^2 + 7*a*d*x^2) + 2*b*
(-6*b*c + 7*a*d)*x^4*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4,
 3/4, 5/4, -((b*x^2)/a)]))/(21*a^2*e^5*x^4*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int (dx^2 + c)(ex)^{-\frac{9}{2}}\left(bx^2 + a\right)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{3}{4}}(ex)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2 + c}{(bx^2 + a)^{\frac{3}{4}}\sqrt{ex}e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{3}{4}}(ex)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)), x)
```

3.1104 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=182

$$\frac{8b^{5/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}e^8\left(a+bx^2\right)^{3/4}} - \frac{4b\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^3e^5(ex)^{3/2}} + \frac{2\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^2e^3(ex)^{7/2}} - \frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(11*a*e*(e*x)^(11/2)) + (2*(10*b*c - 11*
a*d)*(a + b*x^2)^(1/4))/(77*a^2*e^3*(e*x)^(7/2)) - (4*b*(10*b*c -
 11*a*d)*(a + b*x^2)^(1/4))/(77*a^3*e^5*(e*x)^(3/2)) + (8*b^(5/2)
*(10*b*c - 11*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[Ar
cCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*e^8*(a + b*x^2)^(3/4
))
```

Rubi [A] time = 0.402303, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{8b^{5/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}e^8\left(a+bx^2\right)^{3/4}} - \frac{4b\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^3e^5(ex)^{3/2}} + \frac{2\sqrt[4]{a+bx^2}(10bc-11ad)}{77a^2e^3(ex)^{7/2}} - \frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(11*a*e*(e*x)^(11/2)) + (2*(10*b*c - 11*
a*d)*(a + b*x^2)^(1/4))/(77*a^2*e^3*(e*x)^(7/2)) - (4*b*(10*b*c -
 11*a*d)*(a + b*x^2)^(1/4))/(77*a^3*e^5*(e*x)^(3/2)) + (8*b^(5/2)
*(10*b*c - 11*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[Ar
cCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*e^8*(a + b*x^2)^(3/4
))
```

Rubi in Sympy [A] time = 43.2482, size = 173, normalized size = 0.95

$$-\frac{2c\sqrt[4]{a+bx^2}}{11ae\,(ex)^{\frac{11}{2}}} - \frac{2\sqrt[4]{a+bx^2}\,(11ad-10bc)}{77a^2e^3\,(ex)^{\frac{7}{2}}} + \frac{4b\sqrt[4]{a+bx^2}\,(11ad-10bc)}{77a^3e^5\,(ex)^{\frac{3}{2}}} - \frac{8b^{\frac{5}{2}}\,(ex)^{\frac{3}{2}}\,(11ad-10bc)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{77a^{\frac{7}{2}}e^8\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -2*c*(a + b*x**2)**(1/4)/(11*a*e*(e*x)**(11/2)) - 2*(a + b*x**2)*
*(1/4)*(11*a*d - 10*b*c)/(77*a**2*e**3*(e*x)**(7/2)) + 4*b*(a + b
*x**2)**(1/4)*(11*a*d - 10*b*c)/(77*a**3*e**5*(e*x)**(3/2)) - 8*b
**(5/2)*(e*x)**(3/2)*(11*a*d - 10*b*c)*(a/(b*x**2) + 1)**(3/4)*el
liptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(77*a**(7/2)*e**8*(a + b
*x**2)**(3/4))
```

Mathematica [C] time = 0.218361, size = 132, normalized size = 0.73

$$\frac{\sqrt{ex}\left(8b^2x^6\left(\frac{bx^2}{a}+1\right)^{3/4}(11ad-10bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-2\left(a+bx^2\right)\left(a^2\left(7c+11dx^2\right)-2abx^2\left(5c+11dx^2\right)+20b^2cx^4\right.\right.}{77a^3e^7x^6\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (Sqrt[e*x]*(-2*(a + b*x^2)*(20*b^2*c*x^4 - 2*a*b*x^2*(5*c + 11*d*
x^2) + a^2*(7*c + 11*d*x^2)) + 8*b^2*(-10*b*c + 11*a*d)*x^6*(1 +
(b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))
/(77*a^3*e^7*x^6*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{13}{2}}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}e^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)*e^6*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{3}{4}} (ex)^{\frac{13}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)), x)
```

3.1105 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=171

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

```
[Out] -((4*b*c - 5*a*d)*e*Sqrt[e*x])/(2*b^2*(a + b*x^2)^(1/4)) + (d*(e*
x)^(5/2))/(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*e^(3/2)*Ar
cTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4)
) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]
*(a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rubi [A] time = 0.304934, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] -((4*b*c - 5*a*d)*e*Sqrt[e*x])/(2*b^2*(a + b*x^2)^(1/4)) + (d*(e*
x)^(5/2))/(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*e^(3/2)*Ar
cTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4)
) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]
*(a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rubi in Sympy [A] time = 33.3324, size = 158, normalized size = 0.92

$$\frac{d\,(ex)^{\frac{5}{2}}}{2be\sqrt[4]{a+bx^2}}+\frac{e\sqrt{ex}\,(5ad-4bc)}{2b^2\sqrt[4]{a+bx^2}}-\frac{e^{\frac{3}{2}}\,(5ad-4bc)\,\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{9}{4}}}-\frac{e^{\frac{3}{2}}\,(5ad-4bc)\,\text{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] d*(e*x)**(5/2)/(2*b*e*(a + b*x**2)**(1/4)) + e*sqrt(e*x)*(5*a*d -
 4*b*c)/(2*b**2*(a + b*x**2)**(1/4)) - e**(3/2)*(5*a*d - 4*b*c)*a
tan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(4*b**(9/4)
) - e**(3/2)*(5*a*d - 4*b*c)*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a
 + b*x**2)**(1/4)))/(4*b**(9/4))
```

Mathematica [C] time = 0.121992, size = 84, normalized size = 0.49

$$\frac{e\sqrt{ex}\left(\sqrt[4]{\frac{bx^2}{a}+1}(4bc-5ad)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+5ad-4bc+bdx^2\right)}{2b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-4*b*c + 5*a*d + b*d*x^2 + (4*b*c - 5*a*d)*(1 + (b*
x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -((b*x^2)/a)]))/(2
*b^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256652, size = 999, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*d*e*x^2 - (4*b*c - 5*a*d)*e)*(b*x^2 + a)^(3/4)*sqrt(e*x
) + 4*(b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a
^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*a
rctan(-(b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*
a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)/
((b*x^2 + a)^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e - (b*x^2 + a)*sqrt
(((16*b^2*c^2 - 40*a*b*c*d + 25*a^2*d^2)*sqrt(b*x^2 + a)*e^3*x +
(b^5*x^2 + a*b^4)*sqrt((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2
*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9))/(b*x^2 +
 a)))) + (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 240
0*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4
)*log(-((b*x^2 + a)^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e + (b^3*x^2
+ a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2
- 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4))/(b*x^2 + a)) -
(b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2
*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((
b*x^2 + a)^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e - (b^3*x^2 + a*b^2)*
((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^
3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4))/(b*x^2 + a)))/(b^3*x^2 +
 a*b^2)
```

Sympy [A] time = 162.9, size = 94, normalized size = 0.55

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*a**(5/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*g
amma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(5/4)*gamma(13/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

3.1106 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=122

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (d*ArcTan[(
b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e]
) + (d*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/
(b^(5/4)*Sqrt[e])
```

Rubi [A] time = 0.20268, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (d*ArcTan[(
b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e]
) + (d*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/
(b^(5/4)*Sqrt[e])
```

Rubi in Sympy [A] time = 26.3039, size = 110, normalized size = 0.9

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{5}{4}}\sqrt{e}}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{5}{4}}\sqrt{e}}-\frac{2\sqrt{ex}\,(ad-bc)}{abe\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] d*atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(b**(5/4
)*sqrt(e)) + d*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1
/4)))/(b**(5/4)*sqrt(e)) - 2*sqrt(e*x)*(a*d - b*c)/(a*b*e*(a + b*
x**2)**(1/4))
```

Mathematica [C] time = 0.0665856, size = 71, normalized size = 0.58

$$\frac{2x\left(ad\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-ad+bc\right)}{ab\sqrt{ex}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x*(b*c - a*d + a*d*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4
, 1/4, 5/4, -((b*x^2)/a)]))/(a*b*Sqrt[e*x]*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int (dx^2+c)\frac{1}{\sqrt{ex}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d\int \frac{x^{\frac{3}{2}}}{\left(b\sqrt{e}x^2+a\sqrt{e}\right)\left(bx^2+a\right)^{\frac{1}{4}}}\,dx+\frac{2\,c\sqrt{x}}{\left(bx^2+a\right)^{\frac{1}{4}}a\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] d*integrate(x^(3/2)/((b*sqrt(e)*x^2 + a*sqrt(e))*(b*x^2 + a)^(1/4
)), x) + 2*c*sqrt(x)/((b*x^2 + a)^(1/4)*a*sqrt(e))
```

Fricas [A] time = 0.250231, size = 482, normalized size = 3.95

$$\frac{4\left(bx^2+a\right)^{\frac{3}{4}}(bc-ad)\sqrt{ex}-4\left(ab^2ex^2+a^2be\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(b^2ex^2+abe\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,d+\left(bx^2+a\right)\sqrt{\frac{\sqrt{bx^2+a}\,d^2ex+\left(b^3e^2x^2+ab^2e^2\right)\sqrt{\frac{d^4}{b^5e^2}}}{bx^2+a}}}\right)+\left(ab^2e\right.}{2\left(ab^2ex\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(b*x^2 + a)^(3/4)*(b*c - a*d)*sqrt(e*x) - 4*(a*b^2*e*x^2 +
 a^2*b*e)*(d^4/(b^5*e^2))^(1/4)*arctan((b^2*e*x^2 + a*b*e)*(d^4/(
b^5*e^2))^(1/4)/((b*x^2 + a)^(3/4)*sqrt(e*x)*d + (b*x^2 + a)*sqrt
((sqrt(b*x^2 + a)*d^2*e*x + (b^3*e^2*x^2 + a*b^2*e^2)*sqrt(d^4/(b
^5*e^2)))/(b*x^2 + a)))) + (a*b^2*e*x^2 + a^2*b*e)*(d^4/(b^5*e^2)
)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d + (b^2*e*x^2 + a*b*e)*
(d^4/(b^5*e^2))^(1/4))/(b*x^2 + a)) - (a*b^2*e*x^2 + a^2*b*e)*(d^
4/(b^5*e^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d - (b^2*e*x^
2 + a*b*e)*(d^4/(b^5*e^2))^(1/4))/(b*x^2 + a)))/(a*b^2*e*x^2 + a^
2*b*e)
```

Sympy [A] time = 60.7568, size = 83, normalized size = 0.68

$$\frac{c\,\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dx^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*gamma(1/4)/(2*a*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(
5/4)) + d*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*a**(5/4)*sqrt(e)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

3.1107 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\sqrt{ex}(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)
*Sqrt[e*x])/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.113632, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{2\sqrt{ex}(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)
*Sqrt[e*x])/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

Rubi in Sympy [A] time = 13.0666, size = 61, normalized size = 0.91

$$-\frac{2c}{3ae\,(ex)^{\frac{3}{2}}\,\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}\,(3ad-4bc)}{3a^2e^3\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -2*c/(3*a*e*(e*x)**(3/2)*(a + b*x**2)**(1/4)) + 2*sqrt(e*x)*(3*a*
d - 4*b*c)/(3*a**2*e**3*(a + b*x**2)**(1/4))
```

Mathematica [A] time = 0.0670166, size = 45, normalized size = 0.67

$$\frac{x\left(-2ac+6adx^2-8bcx^2\right)}{3a^2(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-2*a*c - 8*b*c*x^2 + 6*a*d*x^2))/(3*a^2*(e*x)^(5/2)*(a + b*x^
2)^(1/4))
```

Maple [A] time = 0.008, size = 39, normalized size = 0.6

$$-\frac{2\,x\left(-3\,adx^2+4\,cx^2b+ac\right)}{3\,a^2}\frac{1}{\sqrt[4]{bx^2+a}}(ex)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/3*x*(-3*a*d*x^2+4*b*c*x^2+a*c)/(b*x^2+a)^(1/4)/a^2/(e*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

Fricas [A] time = 0.222907, size = 57, normalized size = 0.85

$$-\frac{2\left((4\,bc - 3\,ad)x^2 + ac\right)}{3\,(bx^2 + a)^{\frac{1}{4}}\sqrt{ex}a^2e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - 3*a*d)*x^2 + a*c)/((b*x^2 + a)^(1/4)*sqrt(e*x)*a^2
*e^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

3.1108 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - (2*(8*b*c - 7*a*d)
)/(7*a^2*e^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (8*(8*b*c - 7*a*d)*
(a + b*x^2)^(3/4))/(21*a^3*e^3*(e*x)^(3/2))
```

Rubi [A] time = 0.164755, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - (2*(8*b*c - 7*a*d)
)/(7*a^2*e^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (8*(8*b*c - 7*a*d)*
(a + b*x^2)^(3/4))/(21*a^3*e^3*(e*x)^(3/2))
```

Rubi in Sympy [A] time = 17.5452, size = 97, normalized size = 0.93

$$-\frac{2c}{7ae\,(ex)^{\frac{7}{2}}\sqrt[4]{a+bx^2}}+\frac{2\,(7ad-8bc)}{7a^2e^3\,(ex)^{\frac{3}{2}}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{\frac{3}{4}}(7ad-8bc)}{21a^3e^3\,(ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -2*c/(7*a*e*(e*x)**(7/2)*(a + b*x**2)**(1/4)) + 2*(7*a*d - 8*b*c)
/(7*a**2*e**3*(e*x)**(3/2)*(a + b*x**2)**(1/4)) - 8*(a + b*x**2)*
*(3/4)*(7*a*d - 8*b*c)/(21*a**3*e**3*(e*x)**(3/2))
```

Mathematica [A] time = 0.105062, size = 71, normalized size = 0.68

$$-\frac{2\sqrt{ex}\left(a^2\left(3c+7dx^2\right)+ab\left(28dx^4-8cx^2\right)-32b^2cx^4\right)}{21a^3e^5x^4\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(-32*b^2*c*x^4 + a^2*(3*c + 7*d*x^2) + a*b*(-8*c*x^
2 + 28*d*x^4)))/(21*a^3*e^5*x^4*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.009, size = 62, normalized size = 0.6

$$-\frac{2\,x\left(28\,x^4abd-32\,b^2cx^4+7\,x^2a^2d-8\,abcx^2+3\,a^2c\right)}{21\,a^3}\frac{1}{\sqrt[4]{bx^2+a}}(ex)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/21*x*(28*a*b*d*x^4-32*b^2*c*x^4+7*a^2*d*x^2-8*a*b*c*x^2+3*a^2*
c)/(b*x^2+a)^(1/4)/a^3/(e*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{9}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

Fricas [A] time = 0.217903, size = 88, normalized size = 0.85

$$\frac{2 \left(4 \left(8 b^2 c - 7 abd\right) x^4 - 3 a^2 c + \left(8 abc - 7 a^2 d\right) x^2\right)}{21 (bx^2 + a)^{\frac{1}{4}} \sqrt{ex} a^3 e^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(8*b^2*c - 7*a*b*d)*x^4 - 3*a^2*c + (8*a*b*c - 7*a^2*d)*x
^2)/((b*x^2 + a)^(1/4)*sqrt(e*x)*a^3*e^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{9}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

3.1109 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - (2*(12*b*c - 11*
a*d))/(11*a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (16*(12*b*c -
11*a*d)*(a + b*x^2)^(3/4))/(33*a^3*e^3*(e*x)^(7/2)) - (64*(12*b*c
 - 11*a*d)*(a + b*x^2)^(7/4))/(231*a^4*e^3*(e*x)^(7/2))
```

Rubi [A] time = 0.219422, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - (2*(12*b*c - 11*
a*d))/(11*a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (16*(12*b*c -
11*a*d)*(a + b*x^2)^(3/4))/(33*a^3*e^3*(e*x)^(7/2)) - (64*(12*b*c
 - 11*a*d)*(a + b*x^2)^(7/4))/(231*a^4*e^3*(e*x)^(7/2))
```

Rubi in Sympy [A] time = 23.334, size = 134, normalized size = 0.95

$$-\frac{2c}{11ae\,(ex)^{\frac{11}{2}}\sqrt[4]{a+bx^2}}+\frac{2\,(11ad-12bc)}{11a^2e^3\,(ex)^{\frac{7}{2}}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{\frac{3}{4}}(11ad-12bc)}{33a^3e^3\,(ex)^{\frac{7}{2}}}+\frac{64\left(a+bx^2\right)^{\frac{7}{4}}(11ad-12bc)}{231a^4e^3\,(ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -2*c/(11*a*e*(e*x)**(11/2)*(a + b*x**2)**(1/4)) + 2*(11*a*d - 12*
b*c)/(11*a**2*e**3*(e*x)**(7/2)*(a + b*x**2)**(1/4)) - 16*(a + b*
x**2)**(3/4)*(11*a*d - 12*b*c)/(33*a**3*e**3*(e*x)**(7/2)) + 64*(
a + b*x**2)**(7/4)*(11*a*d - 12*b*c)/(231*a**4*e**3*(e*x)**(7/2))
```

Mathematica [A] time = 0.159672, size = 94, normalized size = 0.67

$$\frac{\sqrt{ex}\left(-6a^3\left(7c+11dx^2\right)+8a^2bx^2\left(9c+22dx^2\right)+64ab^2x^4\left(11dx^2-3c\right)-768b^3cx^6\right)}{231a^4e^7x^6\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (Sqrt[e*x]*(-768*b^3*c*x^6 + 64*a*b^2*x^4*(-3*c + 11*d*x^2) - 6*a
^3*(7*c + 11*d*x^2) + 8*a^2*b*x^2*(9*c + 22*d*x^2)))/(231*a^4*e^7
*x^6*(a + b*x^2)^(1/4))
```

Maple [A] time = 0.009, size = 86, normalized size = 0.6

$$-\frac{2x\left(-352\,ab^2dx^6+384\,b^3cx^6-88\,a^2bdx^4+96\,ab^2cx^4+33\,a^3dx^2-36\,a^2bcx^2+21\,ca^3\right)}{231\,a^4}\frac{1}{\sqrt[4]{bx^2+a}}(ex)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/231*x*(-352*a*b^2*d*x^6+384*b^3*c*x^6-88*a^2*b*d*x^4+96*a*b^2*
c*x^4+33*a^3*d*x^2-36*a^2*b*c*x^2+21*a^3*c)/(b*x^2+a)^(1/4)/a^4/(
e*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

Fricas [A] time = 0.218447, size = 122, normalized size = 0.87

$$-\frac{2\left(32\left(12\,b^3c-11\,ab^2d\right)x^6+8\left(12\,ab^2c-11\,a^2bd\right)x^4+21\,a^3c-3\left(12\,a^2bc-11\,a^3d\right)x^2\right)}{231\,(bx^2+a)^{\frac{1}{4}}\sqrt{ex}a^4e^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)),x, algorithm="fricas")
```

```
[Out] -2/231*(32*(12*b^3*c - 11*a*b^2*d)*x^6 + 8*(12*a*b^2*c - 11*a^2*b
*d)*x^4 + 21*a^3*c - 3*(12*a^2*b*c - 11*a^3*d)*x^2)/((b*x^2 + a)^
(1/4)*sqrt(e*x)*a^4*e^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{13}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

3.1110 $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}} dx$

Optimal. Leaf size=180

$$-\frac{7a^{3/2}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}} - \frac{7ae^3(ex)^{3/2}(10bc-11ad)}{60b^3\sqrt[4]{a+bx^2}} + \frac{e(ex)^{7/2}(10bc-11ad)}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

```
[Out] (-7*a*(10*b*c - 11*a*d)*e^3*(e*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4
)) + ((10*b*c - 11*a*d)*e*(e*x)^(7/2))/(30*b^2*(a + b*x^2)^(1/4))
 + (d*(e*x)^(11/2))/(5*b*e*(a + b*x^2)^(1/4)) - (7*a^(3/2)*(10*b*
c - 11*a*d)*e^4*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.310552, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{7a^{3/2}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}} - \frac{7ae^3(ex)^{3/2}(10bc-11ad)}{60b^3\sqrt[4]{a+bx^2}} + \frac{e(ex)^{7/2}(10bc-11ad)}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (-7*a*(10*b*c - 11*a*d)*e^3*(e*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4
)) + ((10*b*c - 11*a*d)*e*(e*x)^(7/2))/(30*b^2*(a + b*x^2)^(1/4))
 + (d*(e*x)^(11/2))/(5*b*e*(a + b*x^2)^(1/4)) - (7*a^(3/2)*(10*b*
c - 11*a*d)*e^4*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(7/2)*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{7a^2e^4\sqrt{ex}\,(11ad-10bc)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}\,dx}{40b^4\sqrt[4]{a+bx^2}} + \frac{7ae^3\,(ex)^{\frac{3}{2}}\,(11ad-10bc)}{60b^3\sqrt[4]{a+bx^2}} + \frac{d\,(ex)^{\frac{11}{2}}}{5be\sqrt[4]{a+bx^2}} - \frac{e\,(ex)^{\frac{7}{2}}\,(11ad-10bc)}{30b^2\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] 7*a**2*e**4*sqrt(e*x)*(11*a*d - 10*b*c)*(a/(b*x**2) + 1)**(1/4)*I
ntegral((a*x**2/b + 1)**(-5/4), (x, 1/x))/(40*b**4*(a + b*x**2)**
(1/4)) + 7*a*e**3*(e*x)**(3/2)*(11*a*d - 10*b*c)/(60*b**3*(a + b*
x**2)**(1/4)) + d*(e*x)**(11/2)/(5*b*e*(a + b*x**2)**(1/4)) - e*(
e*x)**(7/2)*(11*a*d - 10*b*c)/(30*b**2*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.165026, size = 111, normalized size = 0.62

$$\frac{e^3(ex)^{3/2}\left(-77a^2d+7a\sqrt[4]{\frac{bx^2}{a}+1}(11ad-10bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+ab\left(70c-11dx^2\right)+2b^2x^2\left(5c+3dx^2\right)\right)}{30b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(-77*a^2*d + a*b*(70*c - 11*d*x^2) + 2*b^2*x^2*(
5*c + 3*d*x^2) + 7*a*(-10*b*c + 11*a*d)*(1 + (b*x^2)/a)^(1/4)*Hyp
ergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(30*b^3*(a + b*x^2)
^(1/4))
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{\frac{9}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(de^4x^6+ce^4x^4\right)\sqrt{ex}}{(bx^2+a)^{\frac{5}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^4*x^6 + c*e^4*x^4)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

3.1111 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}(6bc-7ad)}{6b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

```
[Out] ((6*b*c - 7*a*d)*e*(e*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (d*(e
*x)^(7/2))/(3*b*e*(a + b*x^2)^(1/4)) + (Sqrt[a]*(6*b*c - 7*a*d)*e
^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/S
qrt[a]]/2, 2])/(2*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.243145, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}(6bc-7ad)}{6b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] ((6*b*c - 7*a*d)*e*(e*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (d*(e
*x)^(7/2))/(3*b*e*(a + b*x^2)^(1/4)) + (Sqrt[a]*(6*b*c - 7*a*d)*e
^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/S
qrt[a]]/2, 2])/(2*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ae^2\sqrt{ex}\,(7ad-6bc)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{4b^3\sqrt[4]{a+bx^2}}-\frac{ae^2\sqrt{ex}\,(7ad-6bc)}{2b^3x\sqrt[4]{a+bx^2}}+\frac{d\,(ex)^{\frac{7}{2}}}{3be\sqrt[4]{a+bx^2}}-\frac{e\,(ex)^{\frac{3}{2}}\,(7ad-6bc)}{6b^2\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] a*e**2*sqrt(e*x)*(7*a*d - 6*b*c)*(a/(b*x**2) + 1)**(1/4)*Integral
((a*x**2/b + 1)**(-1/4), (x, 1/x))/(4*b**3*(a + b*x**2)**(1/4)) -
 a*e**2*sqrt(e*x)*(7*a*d - 6*b*c)/(2*b**3*x*(a + b*x**2)**(1/4))
+ d*(e*x)**(7/2)/(3*b*e*(a + b*x**2)**(1/4)) - e*(e*x)**(3/2)*(7*
a*d - 6*b*c)/(6*b**2*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.125334, size = 84, normalized size = 0.59

$$\frac{e(ex)^{3/2}\left(\sqrt[4]{\frac{bx^2}{a}+1}(6bc-7ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+7ad-6bc+bdx^2\right)}{3b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(-6*b*c + 7*a*d + b*d*x^2 + (6*b*c - 7*a*d)*(1 + (
b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/
(3*b^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \left(e x\right)^{\frac{5}{2}} \left(b x^{2}+a\right)^{-\frac{5}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right) \left(e x\right)^{\frac{5}{2}}}{\left(b x^{2}+a\right)^{\frac{5}{4}}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\left(d e^{2} x^{4}+c e^{2} x^{2}\right) \sqrt{e x}}{\left(b x^{2}+a\right)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^2*x^4 + c*e^2*x^2)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right) \left(e x\right)^{\frac{5}{2}}}{\left(b x^{2}+a\right)^{\frac{5}{4}}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

3.1112 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=99

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (d*(e*x)^(3/2))/(b*e*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a
/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/(Sqrt[a]*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.166838, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (d*(e*x)^(3/2))/(b*e*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a
/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/(Sqrt[a]*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\,(ex)^{\frac{3}{2}}}{be\sqrt[4]{a+bx^2}}+\frac{\sqrt{ex}\left(\frac{3ad}{2}-bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}\,dx}{b^2\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] d*(e*x)**(3/2)/(b*e*(a + b*x**2)**(1/4)) + sqrt(e*x)*(3*a*d/2 - b
*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-5/4), (x,
1/x))/(b**2*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.0964026, size = 81, normalized size = 0.82

$$\frac{2x\sqrt{ex}\left(\sqrt[4]{\frac{bx^2}{a}+1}(3ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-3ad+3bc\right)}{3ab\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*b*c - 3*a*d + (-2*b*c + 3*a*d)*(1 + (b*x^2)/a)^
(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*b*(a
```

```
+ b*x^2)^(1/4))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (dx^2+c)\sqrt{ex}\,(bx^2+a)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

Sympy [A] time = 32.6515, size = 94, normalized size = 0.95

$$\frac{c\sqrt{e}x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\,\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}x^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*a**(5/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gam
ma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(5/4)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)\sqrt{ex}}{(bx^2 + a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

3.1113 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (2*(2*b*c - a*d)*(1 +
a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.179151, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (2*(2*b*c - a*d)*(1 +
a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}\left(\frac{ad}{2}-bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}\,dx}{abe^2\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -2*c/(a*e*sqrt(e*x)*(a + b*x**2)**(1/4)) - 2*sqrt(e*x)*(a*d/2 - b
*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-5/4), (x,
1/x))/(a*b*e**2*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.120509, size = 93, normalized size = 0.9

$$\frac{x\left(-4x^2\sqrt[4]{\frac{bx^2}{a}+1}(ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-6\left(a\left(c-dx^2\right)+2bcx^2\right)\right)}{3a^2(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-6*(2*b*c*x^2 + a*(c - d*x^2)) - 4*(-2*b*c + a*d)*x^2*(1 + (b
*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(
```

```
3*a^2*(e*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{-\frac{3}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2+c}{(bex^3+aex)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*e*x^3 + a*e*x)*(b*x^2 + a)^(1/4)*sqrt(e*
x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)), x)
```

3.1114 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-5ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}+\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) + (2*(6*b*c - 5*a*d)
)/(5*a^2*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (4*Sqrt[b]*(6*b*c - 5
*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x
)/Sqrt[a]]/2, 2])/(5*a^(5/2)*e^4*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.246005, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-5ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}+\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) + (2*(6*b*c - 5*a*d)
)/(5*a^2*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (4*Sqrt[b]*(6*b*c - 5
*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x
)/Sqrt[a]]/2, 2])/(5*a^(5/2)*e^4*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{5ae\left(ex\right)^{\frac{5}{2}}\sqrt[4]{a+bx^2}}-\frac{2\left(5ad-6bc\right)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}\left(5ad-6bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{5a^2e^4\sqrt[4]{a+bx^2}}+\frac{4\sqrt{ex}\left(5ad-6bc\right)}{5a^2e^4x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -2*c/(5*a*e*(e*x)**(5/2)*(a + b*x**2)**(1/4)) - 2*(5*a*d - 6*b*c)
/(5*a**2*e**3*sqrt(e*x)*(a + b*x**2)**(1/4)) - 2*sqrt(e*x)*(5*a*d
 - 6*b*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-1/4)
, (x, 1/x))/(5*a**2*e**4*(a + b*x**2)**(1/4)) + 4*sqrt(e*x)*(5*a*
d - 6*b*c)/(5*a**2*e**4*x*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.178048, size = 114, normalized size = 0.79

$$\frac{x\left(-6a^2\left(c+5dx^2\right)+8bx^4\sqrt[4]{\frac{bx^2}{a}+1}(5ad-6bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+12ab\left(3cx^2-5dx^4\right)+72b^2cx^4\right)}{15a^3(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(72*b^2*c*x^4 - 6*a^2*(c + 5*d*x^2) + 12*a*b*(3*c*x^2 - 5*d*x^
4) + 8*b*(-6*b*c + 5*a*d)*x^4*(1 + (b*x^2)/a)^(1/4)*Hypergeometri
c2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(15*a^3*(e*x)^(7/2)*(a + b*x^
2)^(1/4))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \left(e x\right)^{-\frac{7}{2}} \left(b \,x^{2}+a\right)^{-\frac{5}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d x^{2} + c}{{\left(b x^{2} + a\right)}^{\frac{5}{4}} \left(e x\right)^{\frac{7}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{d x^{2} + c}{{\left(b e^{3} x^{5} + a e^{3} x^{3}\right)} {\left(b x^{2} + a\right)}^{\frac{1}{4}} \sqrt{e x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*e^3*x^5 + a*e^3*x^3)*(b*x^2 + a)^(1/4)*s
qrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)), x)
```

3.1115 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=182

$$\frac{8b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}e^6\sqrt[4]{a+bx^2}}-\frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(1/4)) + (2*(10*b*c - 9*a*d
))/(45*a^2*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (4*b*(10*b*c - 9*
a*d))/(15*a^3*e^5*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (8*b^(3/2)*(10*b
*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqr
t[b]*x)/Sqrt[a]]/2, 2])/(15*a^(7/2)*e^6*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.310752, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{8b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(10bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}e^6\sqrt[4]{a+bx^2}}-\frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(1/4)) + (2*(10*b*c - 9*a*d
))/(45*a^2*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (4*b*(10*b*c - 9*
a*d))/(15*a^3*e^5*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (8*b^(3/2)*(10*b
*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqr
t[b]*x)/Sqrt[a]]/2, 2])/(15*a^(7/2)*e^6*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{9ae\,(ex)^{\frac{9}{2}}\sqrt[4]{a+bx^2}}-\frac{2\,(9ad-10bc)}{45a^2e^3\,(ex)^{\frac{5}{2}}\sqrt[4]{a+bx^2}}+\frac{4b\,(9ad-10bc)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{4b\sqrt{ex}\,(9ad-10bc)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}\,dx}{15a^3e^6\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -2*c/(9*a*e*(e*x)**(9/2)*(a + b*x**2)**(1/4)) - 2*(9*a*d - 10*b*c
)/(45*a**2*e**3*(e*x)**(5/2)*(a + b*x**2)**(1/4)) + 4*b*(9*a*d -
10*b*c)/(15*a**3*e**5*sqrt(e*x)*(a + b*x**2)**(1/4)) - 4*b*sqrt(e
*x)*(9*a*d - 10*b*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b +
 1)**(-5/4), (x, 1/x))/(15*a**3*e**6*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.235699, size = 143, normalized size = 0.79

$$-\frac{2\sqrt{ex}\left(a^3\left(5c+9dx^2\right)-2a^2bx^2\left(5c+27dx^2\right)+8b^2x^6\sqrt[4]{\frac{bx^2}{a}+1}(9ad-10bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+12ab^2x^4\left(5c-9dx^2\right)+\right.}{45a^4e^6x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(120*b^3*c*x^6 + 12*a*b^2*x^4*(5*c - 9*d*x^2) + a^3
*(5*c + 9*d*x^2) - 2*a^2*b*x^2*(5*c + 27*d*x^2) + 8*b^2*(-10*b*c
+ 9*a*d)*x^6*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/
4, -((b*x^2)/a)]))/(45*a^4*e^6*x^5*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{11}{2}}\left(bx^2+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx^2+c}{(be^5x^7+ae^5x^5)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*e^5*x^7 + a*e^5*x^5)*(b*x^2 + a)^(1/4)*s
qrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{11}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)), x)
```

3.1116 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=184

$$-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c
 - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(6*a*b^2) - ((4*b*c -
7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1
/4))])/(4*b^(11/4)) + ((4*b*c - 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*S
qrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(11/4))
```

Rubi [A] time = 0.372869, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c
 - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(6*a*b^2) - ((4*b*c -
7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1
/4))])/(4*b^(11/4)) + ((4*b*c - 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*S
qrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(11/4))
```

Rubi in Sympy [A] time = 36.135, size = 158, normalized size = 0.86

$$\frac{d\,(ex)^{\frac{7}{2}}}{2be\,(a+bx^2)^{\frac{3}{4}}}+\frac{e\,(ex)^{\frac{3}{2}}\,(7ad-4bc)}{6b^2\,(a+bx^2)^{\frac{3}{4}}}+\frac{e^{\frac{5}{2}}\,(7ad-4bc)\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{11}{4}}}-\frac{e^{\frac{5}{2}}\,(7ad-4bc)\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] d*(e*x)**(7/2)/(2*b*e*(a + b*x**2)**(3/4)) + e*(e*x)**(3/2)*(7*a*
d - 4*b*c)/(6*b**2*(a + b*x**2)**(3/4)) + e**(5/2)*(7*a*d - 4*b*c
)*atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(4*b**(1
1/4)) - e**(5/2)*(7*a*d - 4*b*c)*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e
)*(a + b*x**2)**(1/4)))/(4*b**(11/4))
```

Mathematica [C] time = 0.143481, size = 85, normalized size = 0.46

$$\frac{e(ex)^{3/2}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(4bc-7ad)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+7ad-4bc+3bdx^2\right)}{6b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(-4*b*c + 7*a*d + 3*b*d*x^2 + (4*b*c - 7*a*d)*(1 +
 (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -((b*x^2)/a)])
)/(6*b^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{5}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

3.1117 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=125

$$-\frac{d\sqrt{e}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - (d*Sqrt
[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7
/4) + (d*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)
^(1/4))])/b^(7/4)
```

Rubi [A] time = 0.243671, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{d\sqrt{e}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - (d*Sqrt
[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7
/4) + (d*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)
^(1/4))])/b^(7/4)
```

Rubi in Sympy [A] time = 29.3493, size = 112, normalized size = 0.9

$$-\frac{d\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{7}{4}}}+\frac{d\sqrt{e}\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{7}{4}}}-\frac{2\,(ex)^{\frac{3}{2}}\,(ad-bc)}{3abe\,(a+bx^2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] -d*sqrt(e)*atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))
/b**(7/4) + d*sqrt(e)*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x*
*2)**(1/4)))/b**(7/4) - 2*(e*x)**(3/2)*(a*d - b*c)/(3*a*b*e*(a +
b*x**2)**(3/4))
```

Mathematica [C] time = 0.0663421, size = 73, normalized size = 0.58

$$\frac{2x\sqrt{ex}\left(ad\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-ad+bc\right)}{3ab\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*x*Sqrt[e*x]*(b*c - a*d + a*d*(1 + (b*x^2)/a)^(3/4)*Hypergeomet
ric2F1[3/4, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c\right) \sqrt{e x}\, \left(b x^{2}+a\right)^{-\frac{7}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d\sqrt{e}\int\frac{x^{\frac{5}{2}}}{\left(bx^{2}+a\right)^{\frac{7}{4}}}\,dx+\frac{2\,c\sqrt{e}x^{\frac{3}{2}}}{3\,\left(bx^{2}+a\right)^{\frac{3}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4),x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(x^(5/2)/(b*x^2 + a)^(7/4), x) + 2/3*c*sqrt(e)
*x^(3/2)/((b*x^2 + a)^(3/4)*a)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 173.65, size = 87, normalized size = 0.7

$$\frac{c\sqrt{e}x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)}{2a^{\frac{7}{4}}\left(1+\frac{bx^{2}}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}x^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{7}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)/(2*a**(7/4)*(1 + b*x**2/a)**(3/4)*g
amma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/
4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(11/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)\sqrt{ex}}{(bx^2 + a)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

3.1118 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}} \, dx$

Optimal. Leaf size=65

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3(a+bx^2)^{3/4}}-\frac{2c}{ae\sqrt{ex}(a+bx^2)^{3/4}}$$

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)
^(3/2))/(3*a^2*e^3*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.117032, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3(a+bx^2)^{3/4}}-\frac{2c}{ae\sqrt{ex}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)
^(3/2))/(3*a^2*e^3*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 11.8793, size = 58, normalized size = 0.89

$$-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{\frac{3}{4}}}+\frac{2\,(ex)^{\frac{3}{2}}\,(ad-4bc)}{3a^2e^3\,(a+bx^2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -2*c/(a*e*sqrt(e*x)*(a + b*x**2)**(3/4)) + 2*(e*x)**(3/2)*(a*d -
4*b*c)/(3*a**2*e**3*(a + b*x**2)**(3/4))
```

Mathematica [A] time = 0.0570357, size = 44, normalized size = 0.68

$$\frac{2x\left(-3ac+adx^2-4bcx^2\right)}{3a^2(ex)^{3/2}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*x*(-3*a*c - 4*b*c*x^2 + a*d*x^2))/(3*a^2*(e*x)^(3/2)*(a + b*x^
2)^(3/4))
```

Maple [A] time = 0.009, size = 40, normalized size = 0.6

$$-\frac{2\,x\left(-adx^2+4\,cx^2b+3\,ac\right)}{3\,a^2}\left(bx^2+a\right)^{-\frac{3}{4}}(ex)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/3*x*(-a*d*x^2+4*b*c*x^2+3*a*c)/(b*x^2+a)^(3/4)/a^2/(e*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{7}{4}} (ex)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

Fricas [A] time = 0.242057, size = 76, normalized size = 1.17

$$-\frac{2 \left((4\,bc - ad)x^2 + 3\,ac\right) \left(bx^2 + a\right)^{\frac{1}{4}} \sqrt{ex}}{3\,(a^2be^2x^3 + a^3e^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - a*d)*x^2 + 3*a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2
*b*e^2*x^3 + a^3*e^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{7}{4}} (ex)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

3.1119 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}} \, dx$

Optimal. Leaf size=104

$$\frac{8\sqrt[4]{a+bx^2}(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\,(a+bx^2)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\,(a+bx^2)^{3/4}}$$

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - (2*(8*b*c - 5*a*d)
)/(15*a^2*e^3*Sqrt[e*x]*(a + b*x^2)^(3/4)) + (8*(8*b*c - 5*a*d)*(
a + b*x^2)^(1/4))/(15*a^3*e^3*Sqrt[e*x])
```

Rubi [A] time = 0.170531, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{8\sqrt[4]{a+bx^2}(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\,(a+bx^2)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - (2*(8*b*c - 5*a*d)
)/(15*a^2*e^3*Sqrt[e*x]*(a + b*x^2)^(3/4)) + (8*(8*b*c - 5*a*d)*(
a + b*x^2)^(1/4))/(15*a^3*e^3*Sqrt[e*x])
```

Rubi in Sympy [A] time = 17.1046, size = 97, normalized size = 0.93

$$-\frac{2c}{5ae\,(ex)^{\frac{5}{2}}\,(a+bx^2)^{\frac{3}{4}}}+\frac{2\,(5ad-8bc)}{15a^2e^3\sqrt{ex}\,(a+bx^2)^{\frac{3}{4}}}-\frac{8\sqrt[4]{a+bx^2}\,(5ad-8bc)}{15a^3e^3\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -2*c/(5*a*e*(e*x)**(5/2)*(a + b*x**2)**(3/4)) + 2*(5*a*d - 8*b*c)
/(15*a**2*e**3*sqrt(e*x)*(a + b*x**2)**(3/4)) - 8*(a + b*x**2)**(
1/4)*(5*a*d - 8*b*c)/(15*a**3*e**3*sqrt(e*x))
```

Mathematica [A] time = 0.100183, size = 66, normalized size = 0.63

$$\frac{x\left(-6a^2\left(c+5dx^2\right)+8abx^2\left(6c-5dx^2\right)+64b^2cx^4\right)}{15a^3(ex)^{7/2}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (x*(64*b^2*c*x^4 + 8*a*b*x^2*(6*c - 5*d*x^2) - 6*a^2*(c + 5*d*x^2
)))/(15*a^3*(e*x)^(7/2)*(a + b*x^2)^(3/4))
```

Maple [A] time = 0.009, size = 62, normalized size = 0.6

$$-\frac{2\,x\left(20\,x^4abd-32\,b^2cx^4+15\,x^2a^2d-24\,abcx^2+3\,a^2c\right)}{15\,a^3}\left(bx^2+a\right)^{-\frac{3}{4}}(ex)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/15*x*(20*a*b*d*x^4-32*b^2*c*x^4+15*a^2*d*x^2-24*a*b*c*x^2+3*a^
2*c)/(b*x^2+a)^(3/4)/a^3/(e*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{7}{4}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

Fricas [A] time = 0.237579, size = 109, normalized size = 1.05

$$\frac{2\left(4\left(8\,b^2c - 5\,abd\right)x^4 - 3\,a^2c + 3\left(8\,abc - 5\,a^2d\right)x^2\right)\left(bx^2 + a\right)^{\frac{1}{4}}\sqrt{ex}}{15\left(a^3be^4x^5 + a^4e^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] 2/15*(4*(8*b^2*c - 5*a*b*d)*x^4 - 3*a^2*c + 3*(8*a*b*c - 5*a^2*d)
*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^3*b*e^4*x^5 + a^4*e^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{7}{4}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

3.1120 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{64\left(a+bx^2\right)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - (2*(4*b*c - 3*a*d)
)/(9*a^2*e^3*(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (16*(4*b*c - 3*a*d)
*(a + b*x^2)^(1/4))/(9*a^3*e^3*(e*x)^(5/2)) - (64*(4*b*c - 3*a*d)
*(a + b*x^2)^(5/4))/(45*a^4*e^3*(e*x)^(5/2))
```

Rubi [A] time = 0.220013, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{64\left(a+bx^2\right)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - (2*(4*b*c - 3*a*d)
)/(9*a^2*e^3*(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (16*(4*b*c - 3*a*d)
*(a + b*x^2)^(1/4))/(9*a^3*e^3*(e*x)^(5/2)) - (64*(4*b*c - 3*a*d)
*(a + b*x^2)^(5/4))/(45*a^4*e^3*(e*x)^(5/2))
```

Rubi in Sympy [A] time = 22.7234, size = 134, normalized size = 0.95

$$-\frac{2c}{9ae\,(ex)^{\frac{9}{2}}\,(a+bx^2)^{\frac{3}{4}}}+\frac{2\,(3ad-4bc)}{9a^2e^3\,(ex)^{\frac{5}{2}}\,(a+bx^2)^{\frac{3}{4}}}-\frac{16\sqrt[4]{a+bx^2}\,(3ad-4bc)}{9a^3e^3\,(ex)^{\frac{5}{2}}}+\frac{64\left(a+bx^2\right)^{\frac{5}{4}}(3ad-4bc)}{45a^4e^3\,(ex)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -2*c/(9*a*e*(e*x)**(9/2)*(a + b*x**2)**(3/4)) + 2*(3*a*d - 4*b*c)
/(9*a**2*e**3*(e*x)**(5/2)*(a + b*x**2)**(3/4)) - 16*(a + b*x**2)
**(1/4)*(3*a*d - 4*b*c)/(9*a**3*e**3*(e*x)**(5/2)) + 64*(a + b*x
*2)**(5/4)*(3*a*d - 4*b*c)/(45*a**4*e**3*(e*x)**(5/2))
```

Mathematica [A] time = 0.149288, size = 89, normalized size = 0.63

$$-\frac{2\sqrt{ex}\left(a^3\left(5c+9dx^2\right)-12a^2bx^2\left(c+6dx^2\right)+96ab^2x^4\left(c-dx^2\right)+128b^3cx^6\right)}{45a^4e^6x^5\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(128*b^3*c*x^6 + 96*a*b^2*x^4*(c - d*x^2) - 12*a^2*
b*x^2*(c + 6*d*x^2) + a^3*(5*c + 9*d*x^2)))/(45*a^4*e^6*x^5*(a +
b*x^2)^(3/4))
```

Maple [A] time = 0.01, size = 86, normalized size = 0.6

$$-\frac{2x\left(-96ab^2dx^6+128b^3cx^6-72a^2bdx^4+96ab^2cx^4+9a^3dx^2-12a^2bcx^2+5ca^3\right)}{45a^4}\left(bx^2+a\right)^{-\frac{3}{4}}(ex)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/45*x*(-96*a*b^2*d*x^6+128*b^3*c*x^6-72*a^2*b*d*x^4+96*a*b^2*c*
x^4+9*a^3*d*x^2-12*a^2*b*c*x^2+5*a^3*c)/(b*x^2+a)^(3/4)/a^4/(e*x)
^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

Fricas [A] time = 0.251963, size = 142, normalized size = 1.01

$$-\frac{2\left(32\left(4b^3c-3ab^2d\right)x^6+24\left(4ab^2c-3a^2bd\right)x^4+5a^3c-3\left(4a^2bc-3a^3d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{45\left(a^4be^6x^7+a^5e^6x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)),x, algorithm="fricas")
```

```
[Out] -2/45*(32*(4*b^3*c - 3*a*b^2*d)*x^6 + 24*(4*a*b^2*c - 3*a^2*b*d)*
x^4 + 5*a^3*c - 3*(4*a^2*b*c - 3*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sq
rt(e*x)/(a^4*b*e^6*x^7 + a^5*e^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)),x, algorithm="giac")

[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

3.1121 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=192

$$\frac{5\sqrt{a}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{5e^3\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-3ad)}{6b^3}-\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(2bc-3ad)}{3ab^2}+\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(3*a*b*e*(a + b*x^2)^(3/4)) + (5*(2*b
*c - 3*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^3) - ((2*b*c -
3*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(3*a*b^2) + (5*Sqrt[a]*(2
*b*c - 3*a*d)*e^2*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[Arc
Cot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(5/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.419147, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{5\sqrt{a}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{5e^3\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-3ad)}{6b^3}-\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}(2bc-3ad)}{3ab^2}+\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(3*a*b*e*(a + b*x^2)^(3/4)) + (5*(2*b
*c - 3*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^3) - ((2*b*c -
3*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(3*a*b^2) + (5*Sqrt[a]*(2
*b*c - 3*a*d)*e^2*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[Arc
Cot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(5/2)*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 40.7811, size = 167, normalized size = 0.87

$$-\frac{5\sqrt{a}e^2\left(ex\right)^{\frac{3}{2}}\left(\frac{3ad}{2}-bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{3b^{\frac{5}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{d\left(ex\right)^{\frac{9}{2}}}{3be\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{e\left(ex\right)^{\frac{5}{2}}\left(3ad-2bc\right)}{3b^2\left(a+bx^2\right)^{\frac{3}{4}}}-\frac{5e^3\sqrt{ex}\sqrt[4]{a+bx^2}\left(\frac{3ad}{2}-bc\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] -5*sqrt(a)*e**2*(e*x)**(3/2)*(3*a*d/2 - b*c)*(a/(b*x**2) + 1)**(3
/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(3*b**(5/2)*(a + b
*x**2)**(3/4)) + d*(e*x)**(9/2)/(3*b*e*(a + b*x**2)**(3/4)) + e*(
e*x)**(5/2)*(3*a*d - 2*b*c)/(3*b**2*(a + b*x**2)**(3/4)) - 5*e**3
*sqrt(e*x)*(a + b*x**2)**(1/4)*(3*a*d/2 - b*c)/(3*b**3)
```

Mathematica [C] time = 0.145395, size = 110, normalized size = 0.57

$$\frac{e^3\sqrt{ex}\left(-15a^2d+5a\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+ab\left(10c-9dx^2\right)+2b^2x^2\left(3c+dx^2\right)\right)}{6b^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-15*a^2*d + a*b*(10*c - 9*d*x^2) + 2*b^2*x^2*(3*c
 + d*x^2) + 5*a*(-2*b*c + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeome
tric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{7}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(de^3x^5+ce^3x^3\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{7}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^3*x^5 + c*e^3*x^3)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4), x)
```

3.1122 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=152

$$-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-5ad)}{3ab^2}+\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c
 - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(3*a*b^2) - ((2*b*c - 5*
a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.334732, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}(2bc-5ad)}{3ab^2}+\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c
 - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(3*a*b^2) - ((2*b*c - 5*
a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*b^(3/2)*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 35.0309, size = 124, normalized size = 0.82

$$\frac{d\,(ex)^{\frac{5}{2}}}{be\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{e\sqrt{ex}\,(5ad-2bc)}{3b^2\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{2\,(ex)^{\frac{3}{2}}\left(\frac{5ad}{2}-bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{3\sqrt{a}b^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] d*(e*x)**(5/2)/(b*e*(a + b*x**2)**(3/4)) + e*sqrt(e*x)*(5*a*d - 2
*b*c)/(3*b**2*(a + b*x**2)**(3/4)) + 2*(e*x)**(3/2)*(5*a*d/2 - b*
c)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/2
, 2)/(3*sqrt(a)*b**(3/2)*(a + b*x**2)**(3/4))
```

Mathematica [C] time = 0.112707, size = 85, normalized size = 0.56

$$\frac{e\sqrt{ex}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(2bc-5ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+5ad-2bc+3bdx^2\right)}{3b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-2*b*c + 5*a*d + 3*b*d*x^2 + (2*b*c - 5*a*d)*(1 + (
b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/
```

```
(3*b^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \left(d x^{2}+c \right) \left(e x \right)^{\frac{3}{2}} \left(b \,x^{2}+a \right)^{-\frac{7}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)

[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right) \left(e x\right)^{\frac{3}{2}}}{\left(b x^{2}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d e x^{3}+c e x\right) \sqrt{e x}}{\left(b x^{2}+a\right)^{\frac{7}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4),x, algorithm="fricas")

[Out] integral((d*e*x^3 + c*e*x)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{2}+c\right) \left(e x\right)^{\frac{3}{2}}}{\left(b x^{2}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4), x)
```

3.1123 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=116

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\sqrt{b}e^2\,(a+bx^2)^{3/4}}$$

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/4)) - (2*(2*b*c
 + a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[
b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.275211, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\sqrt{b}e^2\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/4)) - (2*(2*b*c
 + a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[
b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 34.7218, size = 126, normalized size = 1.09

$$-\frac{d\sqrt{ex}}{be\,(a+bx^2)^{\frac{3}{4}}}+\frac{2\sqrt{ex}\left(\frac{ad}{2}+bc\right)}{3abe\,(a+bx^2)^{\frac{3}{4}}}-\frac{4\,(ex)^{\frac{3}{2}}\left(\frac{ad}{2}+bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{3a^{\frac{3}{2}}\sqrt{b}e^2\,(a+bx^2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -d*sqrt(e*x)/(b*e*(a + b*x**2)**(3/4)) + 2*sqrt(e*x)*(a*d/2 + b*c
)/(3*a*b*e*(a + b*x**2)**(3/4)) - 4*(e*x)**(3/2)*(a*d/2 + b*c)*(a
/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/
(3*a**(3/2)*sqrt(b)*e**2*(a + b*x**2)**(3/4))
```

Mathematica [C] time = 0.0991147, size = 79, normalized size = 0.68

$$\frac{2x\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-ad+bc\right)}{3ab\sqrt{ex}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*x*(b*c - a*d + (2*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeomet
ric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[e*x]*(a + b*x^2
)^(3/4))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int (dx^2+c)\frac{1}{\sqrt{ex}}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

3.1124 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=144

$$\frac{4\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{ex}(2bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/4)) - (2*(2*b*c - a*d)*S
qrt[e*x])/(3*a^2*e^3*(a + b*x^2)^(3/4)) + (4*Sqrt[b]*(2*b*c - a*d
)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(3*a^(5/2)*e^4*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.328439, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{4\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{ex}(2bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/4)) - (2*(2*b*c - a*d)*S
qrt[e*x])/(3*a^2*e^3*(a + b*x^2)^(3/4)) + (4*Sqrt[b]*(2*b*c - a*d
)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(3*a^(5/2)*e^4*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 36.2747, size = 131, normalized size = 0.91

$$-\frac{2c}{3ae\,(ex)^{\frac{3}{2}}\,(a+bx^2)^{\frac{3}{4}}}+\frac{4\sqrt{ex}\left(\frac{ad}{2}-bc\right)}{3a^2e^3\,(a+bx^2)^{\frac{3}{4}}}-\frac{8\sqrt{b}\,(ex)^{\frac{3}{2}}\left(\frac{ad}{2}-bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{3a^{\frac{5}{2}}e^4\,(a+bx^2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -2*c/(3*a*e*(e*x)**(3/2)*(a + b*x**2)**(3/4)) + 4*sqrt(e*x)*(a*d/
2 - b*c)/(3*a**2*e**3*(a + b*x**2)**(3/4)) - 8*sqrt(b)*(e*x)**(3/
2)*(a*d/2 - b*c)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/
(sqrt(b)*x))/2, 2)/(3*a**(5/2)*e**4*(a + b*x**2)**(3/4))
```

Mathematica [C] time = 0.138028, size = 91, normalized size = 0.63

$$\frac{x\left(4x^2\left(\frac{bx^2}{a}+1\right)^{3/4}(ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-2ac+2adx^2-4bcx^2\right)}{3a^2(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (x*(-2*a*c - 4*b*c*x^2 + 2*a*d*x^2 + 4*(-2*b*c + a*d)*x^2*(1 + (b
*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(
```

```
3*a^2*(e*x)^(5/2)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{-\frac{5}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2+c}{(be^2x^4+ae^2x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*e^2*x^4 + a*e^2*x^2)*(b*x^2 + a)^(3/4)*s
qrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)), x)
```

3.1125 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{7/4}} dx$

Optimal. Leaf size=181

$$-\frac{8b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{7/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt[4]{a+bx^2}(10bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(3/4)) - (2*(10*b*c - 7*a*d
))/(21*a^2*e^3*(e*x)^(3/2)*(a + b*x^2)^(3/4)) + (4*(10*b*c - 7*a*
d)*(a + b*x^2)^(1/4))/(21*a^3*e^3*(e*x)^(3/2)) - (8*b^(3/2)*(10*b
*c - 7*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(7/2)*e^6*(a + b*x^2)^(3/4))
```

Rubi [A] time = 0.393772, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$-\frac{8b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{7/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt[4]{a+bx^2}(10bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(3/4)) - (2*(10*b*c - 7*a*d
))/(21*a^2*e^3*(e*x)^(3/2)*(a + b*x^2)^(3/4)) + (4*(10*b*c - 7*a*
d)*(a + b*x^2)^(1/4))/(21*a^3*e^3*(e*x)^(3/2)) - (8*b^(3/2)*(10*b
*c - 7*a*d)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(7/2)*e^6*(a + b*x^2)^(3/4))
```

Rubi in Sympy [A] time = 42.5803, size = 170, normalized size = 0.94

$$-\frac{2c}{7ae\left(ex\right)^{\frac{7}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{2\left(7ad-10bc\right)}{21a^2e^3\left(ex\right)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}-\frac{4\sqrt[4]{a+bx^2}\left(7ad-10bc\right)}{21a^3e^3\left(ex\right)^{\frac{3}{2}}}+\frac{8b^{\frac{3}{2}}\left(ex\right)^{\frac{3}{2}}\left(7ad-10bc\right)\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}\middle|2\right)}{21a^{\frac{7}{2}}e^6\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -2*c/(7*a*e*(e*x)**(7/2)*(a + b*x**2)**(3/4)) + 2*(7*a*d - 10*b*c
)/(21*a**2*e**3*(e*x)**(3/2)*(a + b*x**2)**(3/4)) - 4*(a + b*x**2
)**(1/4)*(7*a*d - 10*b*c)/(21*a**3*e**3*(e*x)**(3/2)) + 8*b**(3/2
)*(e*x)**(3/2)*(7*a*d - 10*b*c)*(a/(b*x**2) + 1)**(3/4)*elliptic_
f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(21*a**(7/2)*e**6*(a + b*x**2)*
*(3/4))
```

Mathematica [C] time = 0.241079, size = 121, normalized size = 0.67

$$\frac{\sqrt{ex}\left(-2a^2\left(3c+7dx^2\right)+8bx^4\left(\frac{bx^2}{a}+1\right)^{3/4}(10bc-7ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+4abx^2\left(5c-7dx^2\right)+40b^2cx^4\right)}{21a^3e^5x^4\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (Sqrt[e*x]*(40*b^2*c*x^4 + 4*a*b*x^2*(5*c - 7*d*x^2) - 2*a^2*(3*c
 + 7*d*x^2) + 8*b*(10*b*c - 7*a*d)*x^4*(1 + (b*x^2)/a)^(3/4)*Hype
rgeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(21*a^3*e^5*x^4*(a +
 b*x^2)^(3/4))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{9}{2}}\left(bx^2+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx^2+c}{(be^4x^6+ae^4x^4)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b*e^4*x^6 + a*e^4*x^4)*(b*x^2 + a)^(3/4)*s
qrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{7}{4}} (ex)^{\frac{9}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)), x)
```

3.1126 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=221

$$\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e^3\sqrt{ex}(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c
 - 9*a*d)*e^3*Sqrt[e*x])/(2*b^3*(a + b*x^2)^(1/4)) - ((4*b*c - 9*
a*d)*e*(e*x)^(5/2))/(10*a*b^2*(a + b*x^2)^(1/4)) + ((4*b*c - 9*a*
d)*e^(7/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))
])/(4*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTanh[(b^(1/4)*Sqrt[
e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4))
```

Rubi [A] time = 0.391884, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e^3\sqrt{ex}(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c
 - 9*a*d)*e^3*Sqrt[e*x])/(2*b^3*(a + b*x^2)^(1/4)) - ((4*b*c - 9*
a*d)*e*(e*x)^(5/2))/(10*a*b^2*(a + b*x^2)^(1/4)) + ((4*b*c - 9*a*
d)*e^(7/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))
])/(4*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTanh[(b^(1/4)*Sqrt[
e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4))
```

Rubi in Sympy [A] time = 41.5549, size = 194, normalized size = 0.88

$$\frac{d\,(ex)^{\frac{9}{2}}}{2be\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{e\,(ex)^{\frac{5}{2}}\,(9ad-4bc)}{10b^2\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{e^3\sqrt{ex}\,(9ad-4bc)}{2b^3\sqrt[4]{a+bx^2}}-\frac{e^{\frac{7}{2}}\,(9ad-4bc)\,\mathrm{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{13}{4}}}-\frac{e^{\frac{7}{2}}\,(9ad-4bc)\,\mathrm{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] d*(e*x)**(9/2)/(2*b*e*(a + b*x**2)**(5/4)) + e*(e*x)**(5/2)*(9*a*
d - 4*b*c)/(10*b**2*(a + b*x**2)**(5/4)) + e**3*sqrt(e*x)*(9*a*d
- 4*b*c)/(2*b**3*(a + b*x**2)**(1/4)) - e**(7/2)*(9*a*d - 4*b*c)*
atan(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/(4*b**(13/
4)) - e**(7/2)*(9*a*d - 4*b*c)*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*
(a + b*x**2)**(1/4)))/(4*b**(13/4))
```

Mathematica [C] time = 0.186947, size = 116, normalized size = 0.52

$$\frac{e^3\sqrt{ex}\left(45a^2d+5\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(4bc-9ad)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+ab\left(54dx^2-20c\right)+b^2x^2\left(5dx^2-24c\right)\right)}{10b^3\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*(45*a^2*d + b^2*x^2*(-24*c + 5*d*x^2) + a*b*(-20*c
 + 54*d*x^2) + 5*(4*b*c - 9*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4
)*Hypergeometric2F1[1/4, 1/4, 5/4, -((b*x^2)/a)]))/(10*b^3*(a + b
*x^2)^(5/4))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{7}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [A] time = 0.284408, size = 1108, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/40*(4*(5*b^2*d*e^3*x^4 - 6*(4*b^2*c - 9*a*b*d)*e^3*x^2 - 5*(4*a
*b*c - 9*a^2*d)*e^3)*(b*x^2 + a)^(3/4)*sqrt(e*x) + 20*(b^5*x^4 +
2*a*b^4*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^
2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4
)*arctan(-(b^4*x^2 + a*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 77
76*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)
^(1/4)/((b*x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 - (b*x^2
+ a)*sqrt(((16*b^2*c^2 - 72*a*b*c*d + 81*a^2*d^2)*sqrt(b*x^2 + a)
*e^7*x + (b^7*x^2 + a*b^6)*sqrt((256*b^4*c^4 - 2304*a*b^3*c^3*d +
 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^
13))/(b*x^2 + a)))) + 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*((256*b
```

```
^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*
d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b
*c - 9*a*d)*sqrt(e*x)*e^3 + (b^4*x^2 + a*b^3)*((256*b^4*c^4 - 230
4*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a
^4*d^4)*e^14/b^13)^(1/4))/(b*x^2 + a)) - 5*(b^5*x^4 + 2*a*b^4*x^2
 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d
^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-((b*
x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 - (b^4*x^2 + a*b^3)*
((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a
^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4))/(b*x^2 + a)))/(b^5*x
^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)\left(ex\right)^{\frac{7}{2}}}{\left(bx^2 + a\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

3.1127 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=149

$$\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*d*e*
Sqrt[e*x])/(b^2*(a + b*x^2)^(1/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*S
qrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(9/4) + (d*e^(3/2)*ArcT
anh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(9/4)
```

Rubi [A] time = 0.247068, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*d*e*
Sqrt[e*x])/(b^2*(a + b*x^2)^(1/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*S
qrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(9/4) + (d*e^(3/2)*ArcT
anh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(9/4)
```

Rubi in Sympy [A] time = 33.3222, size = 138, normalized size = 0.93

$$-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{de^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{9}{4}}}+\frac{de^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{\frac{9}{4}}}-\frac{2\,(ex)^{\frac{5}{2}}\,(ad-bc)}{5abe\,(a+bx^2)^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*d*e*sqrt(e*x)/(b**2*(a + b*x**2)**(1/4)) + d*e**(3/2)*atan(b**
(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/b**(9/4) + d*e**(3
/2)*atanh(b**(1/4)*sqrt(e*x)/(sqrt(e)*(a + b*x**2)**(1/4)))/b**(9
/4) - 2*(e*x)**(5/2)*(a*d - b*c)/(5*a*b*e*(a + b*x**2)**(5/4))
```

Mathematica [C] time = 0.127835, size = 96, normalized size = 0.64

$$\frac{2e\sqrt{ex}\left(-5a^2d+5ad\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-6abdx^2+b^2cx^2\right)}{5ab^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*e*Sqrt[e*x]*(-5*a^2*d + b^2*c*x^2 - 6*a*b*d*x^2 + 5*a*d*(a + b
*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -((b
*x^2)/a)]))/(5*a*b^2*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{3}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [A] time = 0.271478, size = 564, normalized size = 3.79

$$-\frac{4\left(5\,a^2de-\left(b^2c-6\,abd\right)ex^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}+20\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(b^3x^2+ab^2\right)\left(\frac{d}{\cdots}\right.}{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}de+\left(bx^2+a\right)\sqrt{\frac{\sqrt{bx^2+\cdots}}{\cdots}}}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/10*(4*(5*a^2*d*e - (b^2*c - 6*a*b*d)*e*x^2)*(b*x^2 + a)^(3/4)*
sqrt(e*x) + 20*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9
)^(1/4)*arctan((b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4)/((b*x^2 + a)
^(3/4)*sqrt(e*x)*d*e + (b*x^2 + a)*sqrt((sqrt(b*x^2 + a)*d^2*e^3*
x + (b^5*x^2 + a*b^4)*sqrt(d^4*e^6/b^9))/(b*x^2 + a)))) - 5*(a*b^
4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((b*x^2
+ a)^(3/4)*sqrt(e*x)*d*e + (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))
/(b*x^2 + a)) + 5*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/
b^9)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d*e - (b^3*x^2 + a*b^
2)*(d^4*e^6/b^9)^(1/4))/(b*x^2 + a)))/(a*b^4*x^4 + 2*a^2*b^3*x^2
+ a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

3.1128 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{ex}(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c
 + a*d)*Sqrt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.128344, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2\sqrt{ex}(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c
 + a*d)*Sqrt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

Rubi in Sympy [A] time = 16.3389, size = 90, normalized size = 1.14

$$-\frac{d\sqrt{ex}}{2be\,(a+bx^2)^{\frac{5}{4}}}+\frac{\sqrt{ex}\,(ad+4bc)}{10abe\,(a+bx^2)^{\frac{5}{4}}}+\frac{2\sqrt{ex}\,(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -d*sqrt(e*x)/(2*b*e*(a + b*x**2)**(5/4)) + sqrt(e*x)*(a*d + 4*b*c
)/(10*a*b*e*(a + b*x**2)**(5/4)) + 2*sqrt(e*x)*(a*d + 4*b*c)/(5*a
**2*b*e*(a + b*x**2)**(1/4))
```

Mathematica [A] time = 0.0539472, size = 44, normalized size = 0.56

$$\frac{2x\left(5ac+adx^2+4bcx^2\right)}{5a^2\sqrt{ex}\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(5*a*c + 4*b*c*x^2 + a*d*x^2))/(5*a^2*Sqrt[e*x]*(a + b*x^2)^
(5/4))
```

Maple [A] time = 0.007, size = 39, normalized size = 0.5

$$\frac{2\,x\left(adx^2+4\,cx^2b+5\,ac\right)}{5\,a^2}\left(bx^2+a\right)^{-\frac{5}{4}}\frac{1}{\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] 2/5*x*(a*d*x^2+4*b*c*x^2+5*a*c)/(b*x^2+a)^(5/4)/a^2/(e*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

Fricas [A] time = 0.238873, size = 84, normalized size = 1.06

$$\frac{2\left((4\,bc + ad)x^2 + 5\,ac\right)\left(bx^2 + a\right)^{\frac{3}{4}}\sqrt{ex}}{5\left(a^2b^2ex^4 + 2\,a^3bex^2 + a^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c + a*d)*x^2 + 5*a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*
b^2*e*x^4 + 2*a^3*b*e*x^2 + a^4*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

3.1129 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{8\sqrt{ex}(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (2*(8*b*c - 3*a*d)
*Sqrt[e*x])/(15*a^2*e^3*(a + b*x^2)^(5/4)) - (8*(8*b*c - 3*a*d)*S
qrt[e*x])/(15*a^3*e^3*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.169736, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{8\sqrt{ex}(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (2*(8*b*c - 3*a*d)
*Sqrt[e*x])/(15*a^2*e^3*(a + b*x^2)^(5/4)) - (8*(8*b*c - 3*a*d)*S
qrt[e*x])/(15*a^3*e^3*(a + b*x^2)^(1/4))
```

Rubi in Sympy [A] time = 17.435, size = 99, normalized size = 0.95

$$-\frac{2c}{3ae\left(ex\right)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{2\sqrt{ex}\left(3ad-8bc\right)}{15a^2e^3\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{8\sqrt{ex}\left(3ad-8bc\right)}{15a^3e^3\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*c/(3*a*e*(e*x)**(3/2)*(a + b*x**2)**(5/4)) + 2*sqrt(e*x)*(3*a*
d - 8*b*c)/(15*a**2*e**3*(a + b*x**2)**(5/4)) + 8*sqrt(e*x)*(3*a*
d - 8*b*c)/(15*a**3*e**3*(a + b*x**2)**(1/4))
```

Mathematica [A] time = 0.0903344, size = 65, normalized size = 0.62

$$\frac{x\left(-10a^2\left(c-3dx^2\right)+ab\left(24dx^4-80cx^2\right)-64b^2cx^4\right)}{15a^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (x*(-64*b^2*c*x^4 - 10*a^2*(c - 3*d*x^2) + a*b*(-80*c*x^2 + 24*d*
x^4)))/(15*a^3*(e*x)^(5/2)*(a + b*x^2)^(5/4))
```

Maple [A] time = 0.01, size = 62, normalized size = 0.6

$$-\frac{2x\left(-12x^4abd+32b^2cx^4-15x^2a^2d+40abcx^2+5a^2c\right)}{15a^3}\left(bx^2+a\right)^{-\frac{5}{4}}(ex)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/15*x*(-12*a*b*d*x^4+32*b^2*c*x^4-15*a^2*d*x^2+40*a*b*c*x^2+5*a
^2*c)/(b*x^2+a)^(5/4)/a^3/(e*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

Fricas [A] time = 0.24599, size = 107, normalized size = 1.03

$$-\frac{2\left(4\left(8\,b^2c-3\,abd\right)x^4+5\,a^2c+5\left(8\,abc-3\,a^2d\right)x^2\right)}{15\left(a^3be^2x^3+a^4e^2x\right)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] -2/15*(4*(8*b^2*c - 3*a*b*d)*x^4 + 5*a^2*c + 5*(8*a*b*c - 3*a^2*d
)*x^2)/((a^3*b*e^2*x^3 + a^4*e^2*x)*(b*x^2 + a)^(1/4)*sqrt(e*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

3.1130 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=141

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (2*(12*b*c - 7*a*d
))/(35*a^2*e^3*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (16*(12*b*c - 7*a
*d))/(35*a^3*e^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (64*(12*b*c - 7
*a*d)*(a + b*x^2)^(3/4))/(105*a^4*e^3*(e*x)^(3/2))
```

Rubi [A] time = 0.21835, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```


```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (2*(12*b*c - 7*a*d
))/(35*a^2*e^3*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (16*(12*b*c - 7*a
*d))/(35*a^3*e^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (64*(12*b*c - 7
*a*d)*(a + b*x^2)^(3/4))/(105*a^4*e^3*(e*x)^(3/2))
```

Rubi in Sympy [A] time = 22.8129, size = 133, normalized size = 0.94

$$-\frac{2c}{7ae\,(ex)^{\frac{7}{2}}\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{2\,(7ad-12bc)}{35a^2e^3\,(ex)^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{16\,(7ad-12bc)}{35a^3e^3\,(ex)^{\frac{3}{2}}\sqrt[4]{a+bx^2}}-\frac{64\left(a+bx^2\right)^{\frac{3}{4}}(7ad-12bc)}{105a^4e^3\,(ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*c/(7*a*e*(e*x)**(7/2)*(a + b*x**2)**(5/4)) + 2*(7*a*d - 12*b*c
)/(35*a**2*e**3*(e*x)**(3/2)*(a + b*x**2)**(5/4)) + 16*(7*a*d - 1
2*b*c)/(35*a**3*e**3*(e*x)**(3/2)*(a + b*x**2)**(1/4)) - 64*(a +
b*x**2)**(3/4)*(7*a*d - 12*b*c)/(105*a**4*e**3*(e*x)**(3/2))
```

Mathematica [A] time = 0.148512, size = 94, normalized size = 0.67

$$\frac{\sqrt{ex}\left(-10a^3\left(3c+7dx^2\right)+40a^2bx^2\left(3c-14dx^2\right)+64ab^2x^4\left(15c-7dx^2\right)+768b^3cx^6\right)}{105a^4e^5x^4\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (Sqrt[e*x]*(768*b^3*c*x^6 + 40*a^2*b*x^2*(3*c - 14*d*x^2) + 64*a*
b^2*x^4*(15*c - 7*d*x^2) - 10*a^3*(3*c + 7*d*x^2)))/(105*a^4*e^5*
x^4*(a + b*x^2)^(5/4))
```

Maple [A] time = 0.009, size = 86, normalized size = 0.6

$$-\frac{2x\left(224ab^2dx^6-384b^3cx^6+280a^2bdx^4-480ab^2cx^4+35a^3dx^2-60a^2bcx^2+15ca^3\right)}{105a^4}\left(bx^2+a\right)^{-\frac{5}{4}}(ex)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/105*x*(224*a*b^2*d*x^6-384*b^3*c*x^6+280*a^2*b*d*x^4-480*a*b^2
*c*x^4+35*a^3*d*x^2-60*a^2*b*c*x^2+15*a^3*c)/(b*x^2+a)^(5/4)/a^4/
(e*x)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

Fricas [A] time = 0.245767, size = 142, normalized size = 1.01

$$\frac{2\left(32\left(12b^3c-7ab^2d\right)x^6+40\left(12ab^2c-7a^2bd\right)x^4-15a^3c+5\left(12a^2bc-7a^3d\right)x^2\right)}{105\,(a^4be^4x^5+a^5e^4x^3)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)),x, algorithm="fricas")
```

```
[Out] 2/105*(32*(12*b^3*c - 7*a*b^2*d)*x^6 + 40*(12*a*b^2*c - 7*a^2*b*d
)*x^4 - 15*a^3*c + 5*(12*a^2*b*c - 7*a^3*d)*x^2)/((a^4*b*e^4*x^5
+ a^5*e^4*x^3)*(b*x^2 + a)^(1/4)*sqrt(e*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)),x, algorithm="giac")

[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

3.1131 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=178

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - (2*(16*b*c - 11*
a*d))/(55*a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (24*(16*b*c -
11*a*d))/(55*a^3*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (64*(16*b*c
 - 11*a*d)*(a + b*x^2)^(3/4))/(55*a^4*e^3*(e*x)^(7/2)) - (256*(16
*b*c - 11*a*d)*(a + b*x^2)^(7/4))/(385*a^5*e^3*(e*x)^(7/2))
```

Rubi [A] time = 0.277489, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - (2*(16*b*c - 11*
a*d))/(55*a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (24*(16*b*c -
11*a*d))/(55*a^3*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (64*(16*b*c
 - 11*a*d)*(a + b*x^2)^(3/4))/(55*a^4*e^3*(e*x)^(7/2)) - (256*(16
*b*c - 11*a*d)*(a + b*x^2)^(7/4))/(385*a^5*e^3*(e*x)^(7/2))
```

Rubi in Sympy [A] time = 29.0576, size = 170, normalized size = 0.96

$$-\frac{2c}{11ae\,(ex)^{\frac{11}{2}}\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{2\,(11ad-16bc)}{55a^2e^3\,(ex)^{\frac{7}{2}}\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{24\,(11ad-16bc)}{55a^3e^3\,(ex)^{\frac{7}{2}}\sqrt[4]{a+bx^2}}-\frac{64\left(a+bx^2\right)^{\frac{3}{4}}(11ad-16bc)}{55a^4e^3\,(ex)^{\frac{7}{2}}}+\frac{256\left(a+bx^2\right)^{\frac{7}{4}}(11ad-16bc)}{385a^5e^3\,(ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*c/(11*a*e*(e*x)**(11/2)*(a + b*x**2)**(5/4)) + 2*(11*a*d - 16*
b*c)/(55*a**2*e**3*(e*x)**(7/2)*(a + b*x**2)**(5/4)) + 24*(11*a*d
 - 16*b*c)/(55*a**3*e**3*(e*x)**(7/2)*(a + b*x**2)**(1/4)) - 64*(
a + b*x**2)**(3/4)*(11*a*d - 16*b*c)/(55*a**4*e**3*(e*x)**(7/2))
+ 256*(a + b*x**2)**(7/4)*(11*a*d - 16*b*c)/(385*a**5*e**3*(e*x)*
*(7/2))
```

Mathematica [A] time = 0.209614, size = 115, normalized size = 0.65

$$-\frac{2\sqrt{ex}\left(5a^4\left(7c+11dx^2\right)-20a^3bx^2\left(4c+11dx^2\right)+160a^2b^2x^4\left(2c-11dx^2\right)+128ab^3x^6\left(20c-11dx^2\right)+2048b^4cx^8\right)}{385a^5e^7x^6\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(2048*b^4*c*x^8 + 160*a^2*b^2*x^4*(2*c - 11*d*x^2)
+ 128*a*b^3*x^6*(20*c - 11*d*x^2) - 20*a^3*b*x^2*(4*c + 11*d*x^2)
 + 5*a^4*(7*c + 11*d*x^2)))/(385*a^5*e^7*x^6*(a + b*x^2)^(5/4))
```

Maple [A] time = 0.01, size = 110, normalized size = 0.6

$$-\frac{2x\left(-1408\,ab^{3}dx^{8}+2048\,b^{4}cx^{8}-1760\,a^{2}b^{2}dx^{6}+2560\,ab^{3}cx^{6}-220\,a^{3}bdx^{4}+320\,a^{2}b^{2}cx^{4}+55\,a^{4}dx^{2}-80\,a^{3}bcx^{2}+35\,c\right.}{385\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/385*x*(-1408*a*b^3*d*x^8+2048*b^4*c*x^8-1760*a^2*b^2*d*x^6+256
0*a*b^3*c*x^6-220*a^3*b*d*x^4+320*a^2*b^2*c*x^4+55*a^4*d*x^2-80*a
^3*b*c*x^2+35*a^4*c)/(b*x^2+a)^(5/4)/a^5/(e*x)^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{9}{4}}\left(ex\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

Fricas [A] time = 0.24504, size = 174, normalized size = 0.98

$$-\frac{2\left(128\left(16\,b^{4}c-11\,ab^{3}d\right)x^{8}+160\left(16\,ab^{3}c-11\,a^{2}b^{2}d\right)x^{6}+35\,a^{4}c+20\left(16\,a^{2}b^{2}c-11\,a^{3}bd\right)x^{4}-5\left(16\,a^{3}bc-11\,a^{4}d\right)x^{2}\right)}{385\,(a^{5}be^{6}x^{7}+a^{6}e^{6}x^{5})(bx^{2}+a)^{\frac{1}{4}}\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)),x, algorithm="fricas")
```

```
[Out] -2/385*(128*(16*b^4*c - 11*a*b^3*d)*x^8 + 160*(16*a*b^3*c - 11*a^
2*b^2*d)*x^6 + 35*a^4*c + 20*(16*a^2*b^2*c - 11*a^3*b*d)*x^4 - 5*
(16*a^3*b*c - 11*a^4*d)*x^2)/((a^5*b*e^6*x^7 + a^6*e^6*x^5)*(b*x^
2 + a)^(1/4)*sqrt(e*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

3.1132 $\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=230

$$-\frac{77a^{3/2}e^6\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{9/2}\sqrt[4]{a+bx^2}}-\frac{77ae^5(ex)^{3/2}(2bc-3ad)}{60b^4\sqrt[4]{a+bx^2}}+\frac{11e^3(ex)^{7/2}(2bc-3ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{11/2}(2bc-3ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{15/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(15/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (77*a*
(2*b*c - 3*a*d)*e^5*(e*x)^(3/2))/(60*b^4*(a + b*x^2)^(1/4)) + (11
*(2*b*c - 3*a*d)*e^3*(e*x)^(7/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((
2*b*c - 3*a*d)*e*(e*x)^(11/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (77*
a^(3/2)*(2*b*c - 3*a*d)*e^6*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*Ellip
ticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(9/2)*(a + b*x^2)^(
1/4))
```

Rubi [A] time = 0.404226, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{77a^{3/2}e^6\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{9/2}\sqrt[4]{a+bx^2}}-\frac{77ae^5(ex)^{3/2}(2bc-3ad)}{60b^4\sqrt[4]{a+bx^2}}+\frac{11e^3(ex)^{7/2}(2bc-3ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{11/2}(2bc-3ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{15/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(15/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (77*a*
(2*b*c - 3*a*d)*e^5*(e*x)^(3/2))/(60*b^4*(a + b*x^2)^(1/4)) + (11
*(2*b*c - 3*a*d)*e^3*(e*x)^(7/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((
2*b*c - 3*a*d)*e*(e*x)^(11/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (77*
a^(3/2)*(2*b*c - 3*a*d)*e^6*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*Ellip
ticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(9/2)*(a + b*x^2)^(
1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{77a^2e^6\sqrt{ex}\left(\frac{3ad}{2}-bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{20b^5\sqrt[4]{a+bx^2}}+\frac{77a^2e^6\sqrt{ex}\left(\frac{3ad}{2}-bc\right)}{10b^5x\sqrt[4]{a+bx^2}}+\frac{77ae^5\,(ex)^{\frac{3}{2}}\,(3ad-2bc)}{60b^4\sqrt[4]{a+bx^2}}+\frac{d\,(ex)^{\frac{15}{2}}}{5be\,(a+bx^2)^{\frac{5}{4}}}+\frac{e\,(ex)^{\frac{11}{2}}\,(3ad-2bc)}{5b^2\,(a+bx^2)^{\frac{5}{4}}}-\frac{11e^3\,(ex)^{\frac{7}{2}}\left(\frac{3ad}{2}-bc\right)}{15b^3\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(13/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] -77*a**2*e**6*sqrt(e*x)*(3*a*d/2 - b*c)*(a/(b*x**2) + 1)**(1/4)*I
ntegral((a*x**2/b + 1)**(-1/4), (x, 1/x))/(20*b**5*(a + b*x**2)**
(1/4)) + 77*a**2*e**6*sqrt(e*x)*(3*a*d/2 - b*c)/(10*b**5*x*(a + b
*x**2)**(1/4)) + 77*a*e**5*(e*x)**(3/2)*(3*a*d - 2*b*c)/(60*b**4*
(a + b*x**2)**(1/4)) + d*(e*x)**(15/2)/(5*b*e*(a + b*x**2)**(5/4)
```

```
) + e*(e*x)**(11/2)*(3*a*d - 2*b*c)/(5*b**2*(a + b*x**2)**(5/4))
- 11*e**3*(e*x)**(7/2)*(3*a*d/2 - b*c)/(15*b**3*(a + b*x**2)**(1/
4))
```

Mathematica [C] time = 0.232949, size = 139, normalized size = 0.6

$$\frac{e^5(ex)^{3/2}\left(-231a^3d+22a^2b\left(7c-12dx^2\right)+ab^2x^2\left(176c-15dx^2\right)+77a\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(3ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right.}{30b^4\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^5*(e*x)^(3/2)*(-231*a^3*d + a*b^2*x^2*(176*c - 15*d*x^2) + 22*
a^2*b*(7*c - 12*d*x^2) + 2*b^3*x^4*(5*c + 3*d*x^2) + 77*a*(-2*b*c
 + 3*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4
, 3/4, 7/4, -((b*x^2)/a)]))/(30*b^4*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{\frac{13}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{13}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(de^6x^8+ce^6x^6\right)\sqrt{ex}}{(b^2x^4+2\,abx^2+a^2)(bx^2+a)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^6*x^8 + c*e^6*x^6)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2
+ a^2)*(b*x^2 + a)^(1/4)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(13/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{13}{2}}}{(bx^2 + a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4), x)
```

3.1133 $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=192

$$\frac{7\sqrt{a}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10b^{7/2}\sqrt[4]{a+bx^2}}+\frac{7e^3(ex)^{3/2}(6bc-11ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{7/2}(6bc-11ad)}{15ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{11/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(11/2))/(5*a*b*e*(a + b*x^2)^(5/4)) + (7*(6*
b*c - 11*a*d)*e^3*(e*x)^(3/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((6*b
*c - 11*a*d)*e*(e*x)^(7/2))/(15*a*b^2*(a + b*x^2)^(1/4)) + (7*Sqr
t[a]*(6*b*c - 11*a*d)*e^4*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*Ellipti
cE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(10*b^(7/2)*(a + b*x^2)^(1/
4))
```

Rubi [A] time = 0.322031, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{7\sqrt{a}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10b^{7/2}\sqrt[4]{a+bx^2}}+\frac{7e^3(ex)^{3/2}(6bc-11ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{7/2}(6bc-11ad)}{15ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{11/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(11/2))/(5*a*b*e*(a + b*x^2)^(5/4)) + (7*(6*
b*c - 11*a*d)*e^3*(e*x)^(3/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((6*b
*c - 11*a*d)*e*(e*x)^(7/2))/(15*a*b^2*(a + b*x^2)^(1/4)) + (7*Sqr
t[a]*(6*b*c - 11*a*d)*e^4*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*Ellipti
cE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(10*b^(7/2)*(a + b*x^2)^(1/
4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7ae^4\sqrt{ex}\,(11ad-6bc)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}\,dx}{20b^4\sqrt[4]{a+bx^2}}+\frac{d\,(ex)^{\frac{11}{2}}}{3be\,(a+bx^2)^{\frac{5}{4}}}+\frac{e\,(ex)^{\frac{7}{2}}\,(11ad-6bc)}{15b^2\,(a+bx^2)^{\frac{5}{4}}}-\frac{7e^3\,(ex)^{\frac{3}{2}}\,(11ad-6bc)}{30b^3\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] -7*a*e**4*sqrt(e*x)*(11*a*d - 6*b*c)*(a/(b*x**2) + 1)**(1/4)*Inte
gral((a*x**2/b + 1)**(-5/4), (x, 1/x))/(20*b**4*(a + b*x**2)**(1/
4)) + d*(e*x)**(11/2)/(3*b*e*(a + b*x**2)**(5/4)) + e*(e*x)**(7/2
)*(11*a*d - 6*b*c)/(15*b**2*(a + b*x**2)**(5/4)) - 7*e**3*(e*x)**
(3/2)*(11*a*d - 6*b*c)/(30*b**3*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.195065, size = 116, normalized size = 0.6

$$\frac{e^3(ex)^{3/2}\left(77a^2d+7\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(6bc-11ad)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+ab\left(88dx^2-42c\right)+b^2x^2\left(5dx^2-48c\right)\right)}{15b^3\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(77*a^2*d + b^2*x^2*(-48*c + 5*d*x^2) + a*b*(-42
*c + 88*d*x^2) + 7*(6*b*c - 11*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(
1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(15*b^3*(a
+ b*x^2)^(5/4))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int (dx^2+c)(ex)^{\frac{9}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(de^4x^6+ce^4x^4\right)\sqrt{ex}}{(b^2x^4+2\,abx^2+a^2)(bx^2+a)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^4*x^6 + c*e^4*x^4)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2
+ a^2)*(b*x^2 + a)^(1/4)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(ex)^{\frac{9}{2}}}{(bx^2 + a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4), x)
```

3.1134 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=155

$$-\frac{3e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{3/2}(2bc-7ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c
 - 7*a*d)*e*(e*x)^(3/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (3*(2*b*c
- 7*a*d)*e^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[a]*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.25518, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{3e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{3/2}(2bc-7ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```


```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c
 - 7*a*d)*e*(e*x)^(3/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (3*(2*b*c
- 7*a*d)*e^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[a]*b^(5/2)*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\,(ex)^{\frac{7}{2}}}{be\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{e\,(ex)^{\frac{3}{2}}\,(7ad-2bc)}{5b^2\left(a+bx^2\right)^{\frac{5}{4}}}-\frac{3e^2\sqrt{ex}\left(\frac{7ad}{2}-bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{5b^3\sqrt[4]{a+bx^2}}+\frac{6e^2\sqrt{ex}\left(\frac{7ad}{2}-bc\right)}{5b^3x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] d*(e*x)**(7/2)/(b*e*(a + b*x**2)**(5/4)) + e*(e*x)**(3/2)*(7*a*d
- 2*b*c)/(5*b**2*(a + b*x**2)**(5/4)) - 3*e**2*sqrt(e*x)*(7*a*d/2
 - b*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-1/4),
(x, 1/x))/(5*b**3*(a + b*x**2)**(1/4)) + 6*e**2*sqrt(e*x)*(7*a*d/
2 - b*c)/(5*b**3*x*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.179201, size = 107, normalized size = 0.69

$$\frac{2e(ex)^{3/2}\left(-7a^2d+\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(7ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+2ab\left(c-4dx^2\right)+3b^2cx^2\right)}{5ab^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*e*(e*x)^(3/2)*(-7*a^2*d + 3*b^2*c*x^2 + 2*a*b*(c - 4*d*x^2) +
(-2*b*c + 7*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(5*a*b^2*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int (dx^2 + c)(ex)^{\frac{5}{2}} \left(bx^2 + a\right)^{-\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(de^2x^4 + ce^2x^2\right)\sqrt{ex}}{(b^2x^4 + 2\,abx^2 + a^2)(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*e^2*x^4 + c*e^2*x^2)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2
+ a^2)*(b*x^2 + a)^(1/4)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)(ex)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4), x)
```

3.1135 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=114

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*(2*b
*c + 3*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqr
t[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.185584, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*(2*b
*c + 3*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqr
t[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*b^(3/2)*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d\,(ex)^{\frac{3}{2}}}{be\,(a+bx^2)^{\frac{5}{4}}}+\frac{2\,(ex)^{\frac{3}{2}}\left(\frac{3ad}{2}+bc\right)}{5abe\,(a+bx^2)^{\frac{5}{4}}}+\frac{2\sqrt{ex}\left(\frac{3ad}{2}+bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{5ab^2\sqrt[4]{a+bx^2}}-\frac{4\sqrt{ex}\left(\frac{3ad}{2}+bc\right)}{5ab^2x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] -d*(e*x)**(3/2)/(b*e*(a + b*x**2)**(5/4)) + 2*(e*x)**(3/2)*(3*a*d
/2 + b*c)/(5*a*b*e*(a + b*x**2)**(5/4)) + 2*sqrt(e*x)*(3*a*d/2 +
b*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-1/4), (x,
 1/x))/(5*a*b**2*(a + b*x**2)**(1/4)) - 4*sqrt(e*x)*(3*a*d/2 + b*
c)/(5*a*b**2*x*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.157829, size = 111, normalized size = 0.97

$$-\frac{2\sqrt{ex}\left(2x\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(3ad+2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-3x\left(2a^2d+3ab\left(c+dx^2\right)+2b^2cx^2\right)\right)}{15a^2b\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (-2*Sqrt[e*x]*(-3*x*(2*a^2*d + 2*b^2*c*x^2 + 3*a*b*(c + d*x^2)) +
 2*(2*b*c + 3*a*d)*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeome
tric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(15*a^2*b*(a + b*x^2)^(5/4
))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (dx^2+c)\sqrt{ex}\left(bx^2+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^2+c\right)\sqrt{ex}}{(b^2x^4+2\,abx^2+a^2)(bx^2+a)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt(e*x)/((b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^
2 + a)^(1/4)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4), x)
```

3.1136 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=142

$$\frac{4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2(ex)^{3/2}(6bc-ad)}{5a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(5/4)) - (2*(6*b*c - a*d)*(e*x)
^(3/2))/(5*a^2*e^3*(a + b*x^2)^(5/4)) + (4*(6*b*c - a*d)*(1 + a/(
b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(5*a^(5/2)*Sqrt[b]*e^2*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.239385, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(6bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2(ex)^{3/2}(6bc-ad)}{5a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(5/4)) - (2*(6*b*c - a*d)*(e*x)
^(3/2))/(5*a^2*e^3*(a + b*x^2)^(5/4)) + (4*(6*b*c - a*d)*(1 + a/(
b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(5*a^(5/2)*Sqrt[b]*e^2*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{2\left(ex\right)^{\frac{3}{2}}\left(ad-6bc\right)}{5a^2e^3\left(a+bx^2\right)^{\frac{5}{4}}}+\frac{2\sqrt{ex}\left(ad-6bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{5a^2be^2\sqrt[4]{a+bx^2}}-\frac{4\sqrt{ex}\left(ad-6bc\right)}{5a^2be^2x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*c/(a*e*sqrt(e*x)*(a + b*x**2)**(5/4)) + 2*(e*x)**(3/2)*(a*d -
6*b*c)/(5*a**2*e**3*(a + b*x**2)**(5/4)) + 2*sqrt(e*x)*(a*d - 6*b
*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-1/4), (x,
1/x))/(5*a**2*b*e**2*(a + b*x**2)**(1/4)) - 4*sqrt(e*x)*(a*d - 6*
b*c)/(5*a**2*b*e**2*x*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.184469, size = 120, normalized size = 0.85

$$\frac{x\left(-6a^2\left(5c-3dx^2\right)-8x^2\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(ad-6bc)\,_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+12abx^2\left(dx^2-9c\right)-72b^2cx^4\right)}{15a^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (x*(-72*b^2*c*x^4 - 6*a^2*(5*c - 3*d*x^2) + 12*a*b*x^2*(-9*c + d*
x^2) - 8*(-6*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyp
ergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(15*a^3*(e*x)^(3/2)
*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int (dx^2 + c)(ex)^{-\frac{3}{2}} (bx^2 + a)^{-\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}} (ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2 + c}{(b^2ex^5 + 2\,abex^3 + a^2ex)(bx^2 + a)^{\frac{1}{4}}\sqrt{ex}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b^2*e*x^5 + 2*a*b*e*x^3 + a^2*e*x)*(b*x^2
+ a)^(1/4)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}} (ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)), x)
```

3.1137 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=181

$$-\frac{24\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}e^4\sqrt[4]{a+bx^2}}+\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\,(a+bx^2)^{5/4}}-\frac{2c}{5ae(ex)^{5/2}\,(a+bx^2)^{5/4}}$$

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(5/4)) - (2*(2*b*c - a*d))/
(5*a^2*e^3*Sqrt[e*x]*(a + b*x^2)^(5/4)) + (12*(2*b*c - a*d))/(5*a
^3*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (24*Sqrt[b]*(2*b*c - a*d)*(
1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(5*a^(7/2)*e^4*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.30389, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{24\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}e^4\sqrt[4]{a+bx^2}}+\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\,(a+bx^2)^{5/4}}-\frac{2c}{5ae(ex)^{5/2}\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(5/4)) - (2*(2*b*c - a*d))/
(5*a^2*e^3*Sqrt[e*x]*(a + b*x^2)^(5/4)) + (12*(2*b*c - a*d))/(5*a
^3*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (24*Sqrt[b]*(2*b*c - a*d)*(
1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(5*a^(7/2)*e^4*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{5ae\,(ex)^{\frac{5}{2}}\,(a+bx^2)^{\frac{5}{4}}}+\frac{4\left(\frac{ad}{2}-bc\right)}{5a^2e^3\sqrt{ex}\,(a+bx^2)^{\frac{5}{4}}}-\frac{24\left(\frac{ad}{2}-bc\right)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{24\sqrt{ex}\left(\frac{ad}{2}-bc\right)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}\,dx}{5a^3e^4\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*c/(5*a*e*(e*x)**(5/2)*(a + b*x**2)**(5/4)) + 4*(a*d/2 - b*c)/(
5*a**2*e**3*sqrt(e*x)*(a + b*x**2)**(5/4)) - 24*(a*d/2 - b*c)/(5*
a**3*e**3*sqrt(e*x)*(a + b*x**2)**(1/4)) + 24*sqrt(e*x)*(a*d/2 -
b*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2/b + 1)**(-5/4), (x,
 1/x))/(5*a**3*e**4*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 0.244187, size = 140, normalized size = 0.77

$$\frac{x\left(-2a^3\left(c+5dx^2\right)-4a^2bx^2\left(9dx^2-5c\right)-24ab^2x^4\left(dx^2-3c\right)+16bx^4\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}(ad-2bc)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{5a^4(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (x*(48*b^3*c*x^6 - 24*a*b^2*x^4*(-3*c + d*x^2) - 2*a^3*(c + 5*d*x
^2) - 4*a^2*b*x^2*(-5*c + 9*d*x^2) + 16*b*(-2*b*c + a*d)*x^4*(a +
 b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -(
(b*x^2)/a)]))/(5*a^4*(e*x)^(7/2)*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{7}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^2+c}{(b^2e^3x^7+2\,abe^3x^5+a^2e^3x^3)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b^2*e^3*x^7 + 2*a*b*e^3*x^5 + a^2*e^3*x^3)
*(b*x^2 + a)^(1/4)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}} (ex)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)), x)
```

3.1138 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=219

$$\frac{16b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(14bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}e^6\sqrt[4]{a+bx^2}}-\frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(5/4)) - (2*(14*b*c - 9*a*d
))/(45*a^2*e^3*(e*x)^(5/2)*(a + b*x^2)^(5/4)) + (4*(14*b*c - 9*a*
d))/(45*a^3*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (8*b*(14*b*c - 9
*a*d))/(15*a^4*e^5*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (16*b^(3/2)*(14
*b*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(9/2)*e^6*(a + b*x^2)^(1/4))
```

Rubi [A] time = 0.385339, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{16b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(14bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}e^6\sqrt[4]{a+bx^2}}-\frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(5/4)) - (2*(14*b*c - 9*a*d
))/(45*a^2*e^3*(e*x)^(5/2)*(a + b*x^2)^(5/4)) + (4*(14*b*c - 9*a*
d))/(45*a^3*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (8*b*(14*b*c - 9
*a*d))/(15*a^4*e^5*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (16*b^(3/2)*(14
*b*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(9/2)*e^6*(a + b*x^2)^(1/4))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{9ae\,(ex)^{\frac{9}{2}}\,(a+bx^2)^{\frac{5}{4}}}+\frac{2\,(9ad-14bc)}{45a^2e^3\,(ex)^{\frac{5}{2}}\,(a+bx^2)^{\frac{5}{4}}}-\frac{4\,(9ad-14bc)}{45a^3e^3\,(ex)^{\frac{5}{2}}\,\sqrt[4]{a+bx^2}}+\frac{8b\,(9ad-14bc)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{8b\sqrt{ex}\,(9ad-14bc)\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}\,dx}{15a^4e^6\sqrt[4]{a+bx^2}}-\frac{16b\sqrt{ex}\,(9ad-14bc)}{15a^4e^6x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] -2*c/(9*a*e*(e*x)**(9/2)*(a + b*x**2)**(5/4)) + 2*(9*a*d - 14*b*c
)/(45*a**2*e**3*(e*x)**(5/2)*(a + b*x**2)**(5/4)) - 4*(9*a*d - 14
*b*c)/(45*a**3*e**3*(e*x)**(5/2)*(a + b*x**2)**(1/4)) + 8*b*(9*a*
d - 14*b*c)/(15*a**4*e**5*sqrt(e*x)*(a + b*x**2)**(1/4)) + 8*b*sq
rt(e*x)*(9*a*d - 14*b*c)*(a/(b*x**2) + 1)**(1/4)*Integral((a*x**2
/b + 1)**(-1/4), (x, 1/x))/(15*a**4*e**6*(a + b*x**2)**(1/4)) - 1
6*b*sqrt(e*x)*(9*a*d - 14*b*c)/(15*a**4*e**6*x*(a + b*x**2)**(1/4
))
```

Mathematica [C] time = 0.462106, size = 171, normalized size = 0.78

$$-\frac{2\sqrt{ex}\left(a^4\left(5c+9dx^2\right)-2a^3bx^2\left(7c+45dx^2\right)+4a^2b^2x^4\left(35c-81dx^2\right)+72ab^3x^6\left(7c-3dx^2\right)+16b^2x^6\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}\cdots}\right.}{45a^5e^6x^5\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(336*b^4*c*x^8 + 4*a^2*b^2*x^4*(35*c - 81*d*x^2) +
72*a*b^3*x^6*(7*c - 3*d*x^2) + a^4*(5*c + 9*d*x^2) - 2*a^3*b*x^2*
(7*c + 45*d*x^2) + 16*b^2*(-14*b*c + 9*a*d)*x^6*(a + b*x^2)*(1 +
(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))
/(45*a^5*e^6*x^5*(a + b*x^2)^(5/4))
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)(ex)^{-\frac{11}{2}}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx^2+c}{(b^2e^5x^9+2\,abe^5x^7+a^2e^5x^5)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)/((b^2*e^5*x^9 + 2*a*b*e^5*x^7 + a^2*e^5*x^5)
*(b*x^2 + a)^(1/4)*sqrt(e*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}} (ex)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)), x)
```

3.1139 $\int (ex)^m \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=101

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{m+1}{2};-p,-q;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p
, -q, (3 + m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + m)*(1 + (b*
x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.234007, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{m+1}{2};-p,-q;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p
, -q, (3 + m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + m)*(1 + (b*
x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi in Sympy [A] time = 32.7982, size = 78, normalized size = 0.77

$$\frac{(ex)^{m+1}\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q \operatorname{appellf}_1\left(\frac{m}{2}+\frac{1}{2},-p,-q,\frac{m}{2}+\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**m*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] (e*x)**(m + 1)*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x
**2)**p*(c + d*x**2)**q*appellf1(m/2 + 1/2, -p, -q, m/2 + 3/2, -b
*x**2/a, -d*x**2/c)/(e*(m + 1))
```

Mathematica [B] time = 0.606023, size = 218, normalized size = 2.16

$$\frac{ac(m+3)x(ex)^m\left(a+bx^2\right)^p\left(c+dx^2\right)^q F_1\left(\frac{m+1}{2};-p,-q;\frac{m+3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(m+1)\left(2x^2\left(bcpF_1\left(\frac{m+3}{2};1-p,-q;\frac{m+5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{m+3}{2};-p,1-q;\frac{m+5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+ac(m+3)F_1\left(\frac{m+1}{2};-p\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (a*c*(3 + m)*x*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 +
m)/2, -p, -q, (3 + m)/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + m)*(a
*c*(3 + m)*AppellF1[(1 + m)/2, -p, -q, (3 + m)/2, -((b*x^2)/a), -
((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[(3 + m)/2, 1 - p, -q, (5 + m
)/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[(3 + m)/2, -p,
1 - q, (5 + m)/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**m*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

3.1140 $\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=84

$$\frac{1}{5}x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*
x^2)/a), -((d*x^2)/c)])/(5*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.221904, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{5}x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*
x^2)/a), -((d*x^2)/c)])/(5*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi in Sympy [A] time = 29.1757, size = 65, normalized size = 0.77

$$\frac{x^5 \left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \operatorname{appellf}_1\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] x**5*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c
 + d*x**2)**q*appellf1(5/2, -p, -q, 7/2, -b*x**2/a, -d*x**2/c)/5
```

Mathematica [B] time = 0.359984, size = 176, normalized size = 2.1

$$\frac{7acx^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5\left(2x^2\left(bcpF_1\left(\frac{7}{2};1-p,-q;\frac{9}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{7}{2};-p,1-q;\frac{9}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+7acF_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (7*a*c*x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2,
 -((b*x^2)/a), -((d*x^2)/c)])/(5*(7*a*c*AppellF1[5/2, -p, -q, 7/2
, -((b*x^2)/a), -((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[7/2, 1 - p,
 -q, 9/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[7/2, -p, 1
 - q, 9/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x^4 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)

[Out] int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

3.1141 $\int x^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*
x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.216247, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*
x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi in Sympy [A] time = 35.4811, size = 65, normalized size = 0.77

$$\frac{x^3 \left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \operatorname{appellf}_1\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] x**3*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c
 + d*x**2)**q*appellf1(3/2, -p, -q, 5/2, -b*x**2/a, -d*x**2/c)/3
```

Mathematica [B] time = 0.374183, size = 174, normalized size = 2.07

$$\frac{5acx^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{6x^2 \left(bcpF_1\left(\frac{5}{2};1-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{5}{2};-p,1-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+15acF_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (5*a*c*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)])/(15*a*c*AppellF1[3/2, -p, -q, 5/2,
-((b*x^2)/a), -((d*x^2)/c)] + 6*x^2*(b*c*p*AppellF1[5/2, 1 - p, -
q, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[5/2, -p, 1 -
 q, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

3.1142 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q\,dx$

Optimal. Leaf size=79

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^
2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.118171, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^
2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi in Sympy [A] time = 29.119, size = 61, normalized size = 0.77

$$x\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] x*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c +
d*x**2)**q*appellf1(1/2, -p, -q, 3/2, -b*x**2/a, -d*x**2/c)
```

Mathematica [B] time = 0.113224, size = 172, normalized size = 2.18

$$\frac{3acx\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{2x^2\left(bcpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3acF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -
((b*x^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((
b*x^2)/a), -((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q,
 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p (dx^2 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p (dx^2 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p (dx^2 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1143 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x
^2)/a), -((d*x^2)/c)])/(x*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))
```

Rubi [A] time = 0.202976, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x
^2)/a), -((d*x^2)/c)])/(x*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))
```

Rubi in Sympy [A] time = 29.3653, size = 65, normalized size = 0.79

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}(a+bx^2)^p(c+dx^2)^q\,\mathrm{appellf}_1\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/x**2,x)
```

```
[Out] -(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c + d
*x**2)**q*appellf1(-1/2, -p, -q, 1/2, -b*x**2/a, -d*x**2/c)/x
```

Mathematica [B] time = 0.355485, size = 171, normalized size = 2.09

$$-\frac{ac(a+bx^2)^p(c+dx^2)^qF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{acxF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^3\left(bcpF_1\left(\frac{1}{2};1-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{1}{2};-p,1-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

```
[Out] -((a*c*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -(
(b*x^2)/a), -((d*x^2)/c)])/(a*c*x*AppellF1[-1/2, -p, -q, 1/2, -((
b*x^2)/a), -((d*x^2)/c)] + 2*x^3*(b*c*p*AppellF1[1/2, 1 - p, -q,
3/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[1/2, -p, 1 - q,
 3/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p (dx^2+c)^q}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

3.1144 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4} \, dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

```
[Out] -((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x
^2)/a), -((d*x^2)/c)])/(3*x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q
)
```

Rubi [A] time = 0.208077, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

```
[Out] -((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x
^2)/a), -((d*x^2)/c)])/(3*x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q
)
```

Rubi in Sympy [A] time = 28.9303, size = 70, normalized size = 0.83

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} (a+bx^2)^p (c+dx^2)^q \operatorname{appellf}_1\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/x**4,x)
```

```
[Out] -(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c + d
*x**2)**q*appellf1(-3/2, -p, -q, -1/2, -b*x**2/a, -d*x**2/c)/(3*x
**3)
```

Mathematica [B] time = 0.457249, size = 173, normalized size = 2.06

$$\frac{ac(a+bx^2)^p (c+dx^2)^q F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{6x^5\left(bcpF_1\left(-\frac{1}{2};1-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(-\frac{1}{2};-p,1-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acx^3F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

```
[Out] (a*c*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((
b*x^2)/a), -((d*x^2)/c)])/(-3*a*c*x^3*AppellF1[-3/2, -p, -q, -1/2
, -((b*x^2)/a), -((d*x^2)/c)] + 6*x^5*(b*c*p*AppellF1[-1/2, 1 - p
, -q, 1/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[-1/2, -p,
 1 - q, 1/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{\left(b x^2+a\right)^p \left(d x^2+c\right)^q}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^2+a\right)^p \left(d x^2+c\right)^q}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^2+a\right)^p \left(d x^2+c\right)^q}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/x**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^2+a\right)^p \left(d x^2+c\right)^q}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

3.1145 $\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=242

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(a^2d^2\left(q^2+3q+2\right)+2abcd(p+1)(q+1)+b^2c^2\left(p^2+3p+2\right)\right)\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\right.}{2b^3d^2(p+1)(p+q+2)(p+q+3)}$$
$$-\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(ad(q+2)+bc(p+2))}{2b^2d^2(p+q+2)(p+q+3)}+\frac{x^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd(p+q+3)}$$

```
[Out] -((b*c*(2 + p) + a*d*(2 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1
+ q))/(2*b^2*d^2*(2 + p + q)*(3 + p + q)) + (x^2*(a + b*x^2)^(1 +
 p)*(c + d*x^2)^(1 + q))/(2*b*d*(3 + p + q)) + ((b^2*c^2*(2 + 3*p
 + p^2) + 2*a*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*(a
 + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 +
p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^3*d^2*(1 + p)*(2 + p + q
)*(3 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.717857, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(a^2d^2\left(q^2+3q+2\right)+2abcd(p+1)(q+1)+b^2c^2\left(p^2+3p+2\right)\right)\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\right.}{2b^3d^2(p+1)(p+q+2)(p+q+3)}$$
$$-\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(ad(q+2)+bc(p+2))}{2b^2d^2(p+q+2)(p+q+3)}+\frac{x^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd(p+q+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] -((b*c*(2 + p) + a*d*(2 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1
+ q))/(2*b^2*d^2*(2 + p + q)*(3 + p + q)) + (x^2*(a + b*x^2)^(1 +
 p)*(c + d*x^2)^(1 + q))/(2*b*d*(3 + p + q)) + ((b^2*c^2*(2 + 3*p
 + p^2) + 2*a*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*(a
 + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 +
p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^3*d^2*(1 + p)*(2 + p + q
)*(3 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi in Sympy [A] time = 101.978, size = 206, normalized size = 0.85

$$\frac{x^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd\left(p+q+3\right)}-\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}\left(ad\left(q+2\right)+bc\left(p+2\right)\right)}{2b^2d^2\left(p+q+2\right)\left(p+q+3\right)}$$
$$+\frac{\left(\frac{b(-c-dx^2)}{ad-bc}\right)^{-q}\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(-abcd\left(p+q+2\right)+\left(ad\left(q+1\right)+bc\left(p+1\right)\right)\left(ad\left(q+2\right)+bc\left(p+2\right)\right)\right){}_2F_1\left(\begin{matrix}-q,p+1\\p+2\end{matrix}\right|}{2b^3d^2\left(p+1\right)\left(p+q+2\right)\left(p+q+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] x**2*(a + b*x**2)**(p + 1)*(c + d*x**2)**(q + 1)/(2*b*d*(p + q +
3)) - (a + b*x**2)**(p + 1)*(c + d*x**2)**(q + 1)*(a*d*(q + 2) +
b*c*(p + 2))/(2*b**2*d**2*(p + q + 2)*(p + q + 3)) + (b*(-c - d*x
**2)/(a*d - b*c))**(-q)*(a + b*x**2)**(p + 1)*(c + d*x**2)**q*(-a
*b*c*d*(p + q + 2) + (a*d*(q + 1) + b*c*(p + 1))*(a*d*(q + 2) + b
*c*(p + 2)))*hyper((-q, p + 1), (p + 2,), d*(a + b*x**2)/(a*d - b
*c))/(2*b**3*d**2*(p + 1)*(p + q + 2)*(p + q + 3))
```

Mathematica [C] time = 0.442012, size = 160, normalized size = 0.66

$$\frac{2acx^6\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(3;-p,-q;4;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3\left(bcpx^2F_1\left(4;1-p,-q;5;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqx^2F_1\left(4;-p,1-q;5;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+4acF_1\left(3;-p,-q;4;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*a*c*x^6*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3, -p, -q, 4, -((
b*x^2)/a), -((d*x^2)/c)])/(3*(4*a*c*AppellF1[3, -p, -q, 4, -((b*x
^2)/a), -((d*x^2)/c)] + b*c*p*x^2*AppellF1[4, 1 - p, -q, 5, -((b*
x^2)/a), -((d*x^2)/c)] + a*d*q*x^2*AppellF1[4, -p, 1 - q, 5, -((b
*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x^5\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)^qx^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p\left(dx^2+c\right)^qx^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p (dx^2 + c)^q x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

3.1146 $\int x^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=146

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd(p+q+2)} - \frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q (ad(q+1)+bc(p+1))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b^2d(p+1)(p+q+2)}$$

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b*d*(2 + p + q)) - (
(b*c*(1 + p) + a*d*(1 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hyp
ergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])
/(2*b^2*d*(1 + p)*(2 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.318021, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd(p+q+2)} - \frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q (ad(q+1)+bc(p+1))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b^2d(p+1)(p+q+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b*d*(2 + p + q)) - (
(b*c*(1 + p) + a*d*(1 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hyp
ergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])
/(2*b^2*d*(1 + p)*(2 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi in Sympy [A] time = 46.5865, size = 117, normalized size = 0.8

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{2bd\left(p+q+2\right)} - \frac{\left(\frac{b(-c-dx^2)}{ad-bc}\right)^{-q}\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(ad\left(q+1\right)+bc\left(p+1\right)\right){}_2F_1\left(\begin{matrix} -q, p+1 \\ p+2 \end{matrix}\middle| \frac{d(a+bx^2)}{ad-bc}\right)}{2b^2d\left(p+1\right)\left(p+q+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] (a + b*x**2)**(p + 1)*(c + d*x**2)**(q + 1)/(2*b*d*(p + q + 2)) -
 (b*(-c - d*x**2)/(a*d - b*c))**(-q)*(a + b*x**2)**(p + 1)*(c + d
*x**2)**q*(a*d*(q + 1) + b*c*(p + 1))*hyper((-q, p + 1), (p + 2,)
, d*(a + b*x**2)/(a*d - b*c))/(2*b**2*d*(p + 1)*(p + q + 2))
```

Mathematica [C] time = 0.410162, size = 159, normalized size = 1.09

$$\frac{3acx^4\left(a+bx^2\right)^p\left(c+dx^2\right)^q F_1\left(2;-p,-q;3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{4\left(x^2\left(bcpF_1\left(3;1-p,-q;4;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(3;-p,1-q;4;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3acF_1\left(2;-p,-q;3;-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x^4*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[2, -p, -q, 3, -((
b*x^2)/a), -((d*x^2)/c)])/(4*(3*a*c*AppellF1[2, -p, -q, 3, -((b*x
^2)/a), -((d*x^2)/c)] + x^2*(b*c*p*AppellF1[3, 1 - p, -q, 4, -((b
*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[3, -p, 1 - q, 4, -((b*x^
2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int x^{3} \left(b x^{2}+a \right)^{p} \left(d x^{2}+c \right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q} x^{3}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q} x^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q} x^{3}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

3.1147 $\int x \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=85

$$\frac{\left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b(p+1)}$$

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2
 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d*x^
2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.160307, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{\left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2
 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d*x^
2))/(b*c - a*d))^q)
```

Rubi in Sympy [A] time = 27.0313, size = 65, normalized size = 0.76

$$\frac{\left(\frac{b(-c-dx^2)}{ad-bc}\right)^{-q} \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^q {}_2F_1\left(\begin{matrix} -q, p+1 \\ p+2 \end{matrix}\middle| \frac{d(a+bx^2)}{ad-bc}\right)}{2b\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] (b*(-c - d*x**2)/(a*d - b*c))**(-q)*(a + b*x**2)**(p + 1)*(c + d*
x**2)**q*hyper((-q, p + 1), (p + 2,), d*(a + b*x**2)/(a*d - b*c))
/(2*b*(p + 1))
```

Mathematica [A] time = 0.115828, size = 84, normalized size = 0.99

$$\frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^{q+1} \left(\frac{d(a+bx^2)}{ad-bc}\right)^{-p} {}_2F_1\left(-p,q+1;q+2;\frac{b(dx^2+c)}{bc-ad}\right)}{2d(q+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^p*(c + d*x^2)^(1 + q)*Hypergeometric2F1[-p, 1 + q, 2
 + q, (b*(c + d*x^2))/(b*c - a*d)])/(2*d*(1 + q)*((d*(a + b*x^2))
/(-(b*c) + a*d))^p)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x \left(b x^{2}+a \right)^{p} \left(d x^{2}+c \right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)

[Out] int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)**p*(d*x**2+c)**q,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

3.1148 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx$

Optimal. Leaf size=97

$$-\frac{(a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(p+1;-q,1;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a(p+1)}$$

```
[Out] -((a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p,
 -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a*(1 + p)*((b
*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.245747, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{(a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(p+1;-q,1;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

```
[Out] -((a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p,
 -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a*(1 + p)*((b
*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi in Sympy [A] time = 29.5256, size = 73, normalized size = 0.75

$$-\frac{\left(\frac{b(-c-dx^2)}{ad-bc}\right)^{-q} (a+bx^2)^{p+1} (c+dx^2)^q \operatorname{appellf}_1\left(p+1,1,-q,p+2,\frac{a+bx^2}{a},\frac{d(a+bx^2)}{ad-bc}\right)}{2a\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/x,x)
```

```
[Out] -(b*(-c - d*x**2)/(a*d - b*c))**(-q)*(a + b*x**2)**(p + 1)*(c + d
*x**2)**q*appellf1(p + 1, 1, -q, p + 2, (a + b*x**2)/a, d*(a + b*
x**2)/(a*d - b*c))/(2*a*(p + 1))
```

Mathematica [B] time = 0.427312, size = 225, normalized size = 2.32

$$\frac{bdx^2(p+q-1)(a+bx^2)^p (c+dx^2)^q F_1\left(-p-q;-p,-q;-p-q+1;-\frac{a}{b\ldots}\right.}{2(p+q)\left(bdx^2(p+q-1)F_1\left(-p-q;-p,-q;-p-q+1;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)-adpF_1\left(-p-q+1;1-p,-q;-p-q+2;-\frac{a}{bx^2},-\frac{c}{dx\ldots}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

```
[Out] (b*d*(-1 + p + q)*x^2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-p - q
, -p, -q, 1 - p - q, -(a/(b*x^2)), -(c/(d*x^2))])/(2*(p + q)*(b*d
*(-1 + p + q)*x^2*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^2)
), -(c/(d*x^2))] - a*d*p*AppellF1[1 - p - q, 1 - p, -q, 2 - p - q
, -(a/(b*x^2)), -(c/(d*x^2))] - b*c*q*AppellF1[1 - p - q, -p, 1 -
 q, 2 - p - q, -(a/(b*x^2)), -(c/(d*x^2))]))
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p (dx^2+c)^q}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

3.1149 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx$

Optimal. Leaf size=98

$$\frac{b\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^2(p+1)}$$

```
[Out] (b*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 2, 2 + p
, -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^2*(1 + p)*
((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.260212, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{b\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 2, 2 + p
, -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^2*(1 + p)*
((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi in Sympy [A] time = 29.0642, size = 75, normalized size = 0.77

$$\frac{b\left(\frac{b(-c-dx^2)}{ad-bc}\right)^{-q}\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\operatorname{appellf}_1\left(p+1,2,-q,p+2,\frac{a+bx^2}{a},\frac{d(a+bx^2)}{ad-bc}\right)}{2a^2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/x**3,x)
```

```
[Out] b*(b*(-c - d*x**2)/(a*d - b*c))**(-q)*(a + b*x**2)**(p + 1)*(c +
d*x**2)**q*appellf1(p + 1, 2, -q, p + 2, (a + b*x**2)/a, d*(a + b
*x**2)/(a*d - b*c))/(2*a**2*(p + 1))
```

Mathematica [B] time = 0.470141, size = 225, normalized size = 2.3

$$\frac{bd(p+q-2)\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(-p-q+1;-p,-q;-p-q+2\right.}{2(p+q-1)\left(bdx^2(p+q-2)F_1\left(-p-q+1;-p,-q;-p-q+2;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)-adpF_1\left(-p-q+2;1-p,-q;-p-q+3;-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

```
[Out] (b*d*(-2 + p + q)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1 - p - q,
 -p, -q, 2 - p - q, -(a/(b*x^2)), -(c/(d*x^2))])/(2*(-1 + p + q)*
(b*d*(-2 + p + q)*x^2*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(a/
(b*x^2)), -(c/(d*x^2))] - a*d*p*AppellF1[2 - p - q, 1 - p, -q, 3
- p - q, -(a/(b*x^2)), -(c/(d*x^2))] - b*c*q*AppellF1[2 - p - q,
-p, 1 - q, 3 - p - q, -(a/(b*x^2)), -(c/(d*x^2))]))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)

[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p (dx^2+c)^q}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/x**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

3.1150 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2 (a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(p+1;-q,3;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^3(p+1)}$$

```
[Out] -(b^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 3, 2
+ p, -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^3*(1 +
p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.260977, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{b^2 (a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(p+1;-q,3;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

```
[Out] -(b^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 3, 2
+ p, -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^3*(1 +
p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rubi in Sympy [A] time = 30.6111, size = 78, normalized size = 0.78

$$-\frac{b^2 \left(\frac{b(-c-dx^2)}{ad-bc}\right)^{-q} (a+bx^2)^{p+1} (c+dx^2)^q \operatorname{appellf}_1\left(p+1,3,-q,p+2,\frac{a+bx^2}{a},\frac{d(a+bx^2)}{ad-bc}\right)}{2a^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/x**5,x)
```

```
[Out] -b**2*(b*(-c - d*x**2)/(a*d - b*c))**(-q)*(a + b*x**2)**(p + 1)*(
c + d*x**2)**q*appellf1(p + 1, 3, -q, p + 2, (a + b*x**2)/a, d*(a
 + b*x**2)/(a*d - b*c))/(2*a**3*(p + 1))
```

Mathematica [B] time = 0.527402, size = 228, normalized size = 2.28

$$\frac{bd(p+q-3)(a+bx^2)^p (c+dx^2)^q F_1\left(-p-q+2;-p,-q;-p-q+\right.}{2x^2(p+q-2)\left(bdx^2(p+q-3)F_1\left(-p-q+2;-p,-q;-p-q+3;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)-adpF_1\left(-p-q+3;1-p,-q;-p-q+4;\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

```
[Out] (b*d*(-3 + p + q)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[2 - p - q,
 -p, -q, 3 - p - q, -(a/(b*x^2)), -(c/(d*x^2))])/(2*(-2 + p + q)*
x^2*(b*d*(-3 + p + q)*x^2*AppellF1[2 - p - q, -p, -q, 3 - p - q,
-(a/(b*x^2)), -(c/(d*x^2))] - a*d*p*AppellF1[3 - p - q, 1 - p, -q
, 4 - p - q, -(a/(b*x^2)), -(c/(d*x^2))] - b*c*q*AppellF1[3 - p -
 q, -p, 1 - q, 4 - p - q, -(a/(b*x^2)), -(c/(d*x^2))]))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)

[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p (dx^2+c)^q}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/x**5,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

3.1151 $\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

```
[Out] (2*(e*x)^(7/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q,
11/4, -((b*x^2)/a), -((d*x^2)/c)])/(7*e*(1 + (b*x^2)/a)^p*(1 + (d
*x^2)/c)^q)
```

Rubi [A] time = 0.219663, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(7/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q,
11/4, -((b*x^2)/a), -((d*x^2)/c)])/(7*e*(1 + (b*x^2)/a)^p*(1 + (d
*x^2)/c)^q)
```

Rubi in Sympy [A] time = 32.6929, size = 71, normalized size = 0.78

$$\frac{2\,(ex)^{\frac{7}{2}} \left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \operatorname{appellf_1}\left(\frac{7}{4},-p,-q,\frac{11}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(5/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] 2*(e*x)**(7/2)*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x
**2)**p*(c + d*x**2)**q*appellf1(7/4, -p, -q, 11/4, -b*x**2/a, -d
*x**2/c)/(7*e)
```

Mathematica [A] time = 0.407122, size = 179, normalized size = 1.97

$$\frac{22acx(ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{28x^2 \left(bcpF_1\left(\frac{11}{4};1-p,-q;\frac{15}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{11}{4};-p,1-q;\frac{15}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+77acF_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (22*a*c*x*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -
p, -q, 11/4, -((b*x^2)/a), -((d*x^2)/c)])/(77*a*c*AppellF1[7/4, -
p, -q, 11/4, -((b*x^2)/a), -((d*x^2)/c)] + 28*x^2*(b*c*p*AppellF1
[11/4, 1 - p, -q, 15/4, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*Appel
lF1[11/4, -p, 1 - q, 15/4, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}} \left(b x^2+a\right)^p \left(d x^2+c\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)

[Out] int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}} \left(b x^2+a\right)^p \left(d x^2+c\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="maxima")

[Out] integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex}\left(b x^2+a\right)^p \left(d x^2+c\right)^q e^2 x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="fricas")

[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q*e^2*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(5/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}} \left(b x^2+a\right)^p \left(d x^2+c\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="giac")

[Out] integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1152 $\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

```
[Out] (2*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q,
9/4, -((b*x^2)/a), -((d*x^2)/c)])/(5*e*(1 + (b*x^2)/a)^p*(1 + (d*
x^2)/c)^q)
```

Rubi [A] time = 0.217305, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2(ex)^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In]  Int[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q,
9/4, -((b*x^2)/a), -((d*x^2)/c)])/(5*e*(1 + (b*x^2)/a)^p*(1 + (d*
x^2)/c)^q)
```

Rubi in Sympy [A] time = 33.0135, size = 71, normalized size = 0.78

$$\frac{2\left(ex\right)^{\frac{5}{2}}\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\operatorname{appellf_1}\left(\frac{5}{4},-p,-q,\frac{9}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(3/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] 2*(e*x)**(5/2)*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x
**2)**p*(c + d*x**2)**q*appellf1(5/4, -p, -q, 9/4, -b*x**2/a, -d*
x**2/c)/(5*e)
```

Mathematica [A] time = 0.365757, size = 181, normalized size = 1.99

$$\frac{18acx(ex)^{3/2}\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5\left(4x^2\left(bcpF_1\left(\frac{9}{4};1-p,-q;\frac{13}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{9}{4};-p,1-q;\frac{13}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+9acF_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (18*a*c*x*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -
p, -q, 9/4, -((b*x^2)/a), -((d*x^2)/c)])/(5*(9*a*c*AppellF1[5/4,
-p, -q, 9/4, -((b*x^2)/a), -((d*x^2)/c)] + 4*x^2*(b*c*p*AppellF1[
9/4, 1 - p, -q, 13/4, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF
1[9/4, -p, 1 - q, 13/4, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)

[Out] int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="maxima")

[Out] integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex}\left(bx^2+a\right)^p \left(dx^2+c\right)^q ex, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="fricas")

[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q*e*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(3/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="giac")

[Out] integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1153 $\int \sqrt{ex}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

```
[Out] (2*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q,
7/4, -((b*x^2)/a), -((d*x^2)/c)])/(3*e*(1 + (b*x^2)/a)^p*(1 + (d*
x^2)/c)^q)
```

Rubi [A] time = 0.215807, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2(ex)^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q,
7/4, -((b*x^2)/a), -((d*x^2)/c)])/(3*e*(1 + (b*x^2)/a)^p*(1 + (d*
x^2)/c)^q)
```

Rubi in Sympy [A] time = 32.4106, size = 71, normalized size = 0.78

$$\frac{2\left(ex\right)^{\frac{3}{2}}\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\operatorname{appellf_1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x)**(1/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] 2*(e*x)**(3/2)*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x
**2)**p*(c + d*x**2)**q*appellf1(3/4, -p, -q, 7/4, -b*x**2/a, -d*
x**2/c)/(3*e)
```

Mathematica [A] time = 0.365913, size = 181, normalized size = 1.99

$$\frac{14acx\sqrt{ex}\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3\left(4x^2\left(bcpF_1\left(\frac{7}{4};1-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{7}{4};-p,1-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+7acF_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (14*a*c*x*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p,
 -q, 7/4, -((b*x^2)/a), -((d*x^2)/c)])/(3*(7*a*c*AppellF1[3/4, -p
, -q, 7/4, -((b*x^2)/a), -((d*x^2)/c)] + 4*x^2*(b*c*p*AppellF1[7/
4, 1 - p, -q, 11/4, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[
7/4, -p, 1 - q, 11/4, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(b x^{2}+a\right)^{p} \left(d x^{2}+c\right)^{q} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)

[Out] int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="maxima")

[Out] integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex}\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="fricas")

[Out] integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x)**(1/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="giac")

[Out] integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1154 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{ex}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

```
[Out] (2*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/
4, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)
/c)^q)
```

Rubi [A] time = 0.216459, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2\sqrt{ex}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/
4, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)
/c)^q)
```

Rubi in Sympy [A] time = 32.1572, size = 70, normalized size = 0.79

$$\frac{2\sqrt{ex}\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\operatorname{appellf}_1\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(1/2),x)
```

```
[Out] 2*sqrt(e*x)*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2
)**p*(c + d*x**2)**q*appellf1(1/4, -p, -q, 5/4, -b*x**2/a, -d*x**
2/c)/e
```

Mathematica [B] time = 0.374481, size = 179, normalized size = 2.01

$$\frac{10acx\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt{ex}\left(4x^2\left(bcpF_1\left(\frac{5}{4};1-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{5}{4};-p,1-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+5acF_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (10*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4,
-((b*x^2)/a), -((d*x^2)/c)])/(Sqrt[e*x]*(5*a*c*AppellF1[1/4, -p,
-q, 5/4, -((b*x^2)/a), -((d*x^2)/c)] + 4*x^2*(b*c*p*AppellF1[5/4,
 1 - p, -q, 9/4, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[5/4
, -p, 1 - q, 9/4, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q} \frac{1}{\sqrt{e x}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q}}{\sqrt{e x}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q}}{\sqrt{e x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q}}{\sqrt{e x}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

3.1155 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}} \, dx$

Optimal. Leaf size=89

$$-\frac{2 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*
x^2)/a), -((d*x^2)/c)])/(e*Sqrt[e*x]*(1 + (b*x^2)/a)^p*(1 + (d*x^
2)/c)^q)
```

Rubi [A] time = 0.216652, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{2 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*
x^2)/a), -((d*x^2)/c)])/(e*Sqrt[e*x]*(1 + (b*x^2)/a)^p*(1 + (d*x^
2)/c)^q)
```

Rubi in Sympy [A] time = 32.863, size = 73, normalized size = 0.82

$$-\frac{2 \left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \operatorname{appellf_1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(3/2),x)
```

```
[Out] -2*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c +
 d*x**2)**q*appellf1(-1/4, -p, -q, 3/4, -b*x**2/a, -d*x**2/c)/(e*
sqrt(e*x))
```

Mathematica [B] time = 0.387302, size = 179, normalized size = 2.01

$$-\frac{6acx\left(a+bx^2\right)^p\left(c+dx^2\right)^q F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(ex)^{3/2}\left(4x^2\left(bcpF_1\left(\frac{3}{4};1-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{4};-p,1-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3acF_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-6*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4,
 -((b*x^2)/a), -((d*x^2)/c)])/((e*x)^(3/2)*(3*a*c*AppellF1[-1/4,
-p, -q, 3/4, -((b*x^2)/a), -((d*x^2)/c)] + 4*x^2*(b*c*p*AppellF1[
3/4, 1 - p, -q, 7/4, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1
[3/4, -p, 1 - q, 7/4, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{p} \left(d x^{2}+c\right)^{q} \left(e x\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}}{\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}}{\sqrt{ex}ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/(sqrt(e*x)*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}}{\left(ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2), x)
```

3.1156 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*e*(e*x)^(3/2)*(1 + (b*x^2)/a)^p*(1 + (
d*x^2)/c)^q)
```

Rubi [A] time = 0.220991, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*e*(e*x)^(3/2)*(1 + (b*x^2)/a)^p*(1 + (
d*x^2)/c)^q)
```

Rubi in Sympy [A] time = 33.0525, size = 75, normalized size = 0.82

$$-\frac{2\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\operatorname{appellf}_1\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e\left(ex\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(5/2),x)
```

```
[Out] -2*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c +
 d*x**2)**q*appellf1(-3/4, -p, -q, 1/4, -b*x**2/a, -d*x**2/c)/(3*
e*(e*x)**(3/2))
```

Mathematica [A] time = 0.383027, size = 180, normalized size = 1.98

$$-\frac{2acx\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3(ex)^{5/2}\left(4x^2\left(bcpF_1\left(\frac{1}{4};1-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{1}{4};-p,1-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+acF_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4,
 -((b*x^2)/a), -((d*x^2)/c)])/(3*(e*x)^(5/2)*(a*c*AppellF1[-3/4,
-p, -q, 1/4, -((b*x^2)/a), -((d*x^2)/c)] + 4*x^2*(b*c*p*AppellF1[
1/4, 1 - p, -q, 5/4, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1
[1/4, -p, 1 - q, 5/4, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q}{\sqrt{ex}e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/(sqrt(e*x)*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2), x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```